Joint Antenna-Array Calibration and Direction of Arrival Estimation for Automotive Radars

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***Abstract*—We present an iterative method for joint antenna-array calibration and direction of arrival estimation using millimeter-wave (mm-Wave) radar operating at 77 GHz. The calibration compensates for antenna-array coupling, and phase and gain errors, and does not require any training data. This method is well suited for applications, such as automotive radars, where multiple antenna elements are packaged on a chip and where offline calibration is either expensive or is not possible. The proposed method is highly effective when the array coupling is a function of direction of arriving waves from the object. The novel optimization method proposed in the paper allows it to be used for a 2D array of any shape. Experiments using real data collected from a four-element array on a single-chip radar demonstrate the viability of the algorithm.**

***Index Terms*—Antenna Array, Coupling, Calibration, Direction of Arrival, mmWave, Radar**

I. Introduction

Recent years have witnessed widespread use of millimeter-wave (mm-Wave) radars for advanced driver assistance system (ADAS) applications [1], [2], [3]. Compared with other sensing modalities such as camera, a radar has the ability to perform equally well during different times of the day and can be deployed out of sight behind the car bumper or the doors. In many ADAS applications such as parking, cruise control, and braking, the radar is primarily used to find the three-dimensional location of objects around the vehicle. This includes range, azimuth angle, and elevation angle. The range is computed from the round trip delay of the transmitted signal and the two-dimensional (2D) angle is estimated by using the data collected by a radar antenna array and employing a beamforming-based or an eigen-decomposition based high-resolution frequency estimation method. It is well known that coupling between antenna elements of an array may adversely affect 2D angle estimation [4]. This effect is compounded if there is phase and gain mismatch between array elements. The impact of these non-idealities is pronounced when the antennas are placed very close to each other, which generally, is the case in automotive radars [1]; in fact, it is known that the antenna coupling is not negligible if the antenna separation is closer than a few wavelengths [5]. Moreover, in automotive applications, because of space limitation on the chip, the antennas may not be arranged in a linear, rectangular, or circular array. This, in turn, does not allow to make any assumption about the structure of coupling matrix. In order to mitigate the impact of antenna coupling and gain and phase mismatch on angle estimation, the signal received by an antenna array is calibrated prior to or during its processing.

Calibration for antenna coupling has been widely studied in the past and many methods have been proposed [6], [7], [8], [9]. Most calibration methods are based on training data collected from objects placed at known directions with respect to the antenna array. For example, $L \times L$ calibration matrix is estimated using training data collected by L-element antenna array; this matrix is then applied to signal received by the array. While this methodology works well for many cases, it is not very well suited for automotive radar applications primarily because of the following reasons: First, the antenna coupling for a 2D array changes with angle of arrival from objects. This, in fact, is true for any application. It is, therefore, not possible to estimate a calibration matrix that would be applicable for all directions of arriving waves in the radar field of view. Secondly, in ADAS applications, where multiple radars are placed around the car and are thus produced in high volume, it is desired that the calibration is done online without any need for training data. This also allows re-calibration if antenna coupling changes over time.

In this paper we will present a joint array calibration and 2D angle estimation method for multiple objects around the vehicle. The method does not require any

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training data and need minimal supervision. Whereas in the past, the joint estimation problem was solved for specific array shapes [6], the problem formulation and optimization proposed in this paper can be applied to any array design and shape. We will present experimental results using data collected from a 77-GHz radar with a four-element antenna array to show efficacy of the proposed method. In Section II, the problem of direction of arrival estimation in the presence of coupling and array gain and phase errors is discussed with reference to automotive applications. Section III presents the proposed method to jointly estimate direction of arrival and calibration matrix. The method also estimates the number of objects around the vehicle. Experiments using the radar with four antennas are presented in Section IV. Finally, some conclusions are drawn in Section V.

II. Problem Formulation

Consider an array of L antennas receiving signals from K objects. These objects are located at azimuth and elevation angles of $\{(\theta_k, \phi_k)\}_{k=1}^{K}$, respectively, with respect to the array. The signal received by L sensors at time n is given by

$$\mathbf{x}(n) = \sum_{k=1}^{K} \mathbf{a}(\theta_k, \phi_k) s_k(n), \tag{1}$$

where $s_k(n)$ is complex signal amplitude from kth source, $\mathbf{a}(\theta_k, \phi_k)$ is array response from direction (θ_k, ϕ_k), and $\mathbf{x}(n) = [x_1(n), x_2(n), \ldots, x_L(n)]^T$, where $(\cdot)^T$ denotes complex transposition. The location information of each source is embedded in its array response, which is commonly referred to as the steering vector. In direction of arrival (DOA) estimation, the objective is to find source locations given multiple measurement snapshots. Techniques ranging from classical methods based on beamforming to high-resolution method such as MUSIC and ESPRIT have been proposed to solve this problem [10].

In antenna arrays, the total energy arriving at an antenna from an object is the vector sum of energy arriving directly at that antenna and energy reflected from other array elements. The amount of energy reflected from an antenna depends on its terminal impedance and location [11]. This dependence of energy received by an antenna on other antenna in the array is referred to as coupling. Mutual coupling between antennas affects the array steering response, which is then denoted by $\tilde{\mathbf{a}}(\theta, \phi)$ and is modeled as

$$\tilde{\mathbf{a}}(\theta_k, \phi_k) = \mathbf{C}\mathbf{a}(\theta_k, \phi_k), \tag{2}$$

where $\mathbf{C}$ is a $L \times L$ coupling matrix. The (i, j)th element of matrix $\mathbf{C}$ denotes the coupling effect of antenna element j on element i. It is known that the structure of coupling matrix $\mathbf{C}$ depends on array shape; e.g., for a uniform linear array, $\mathbf{C}$ is banded Toeplitz, whereas for circular arrays, it consists of three bands [6].

In addition to antenna coupling, each antenna element has its own gain and phase that has to be compensated before the signal received at these antennas are processed further. The impact of antenna gain and phase is accounted for by forming a $L \times L$ diagonal matrix

$$\mathbf{\Gamma} = \mathrm{diag}\left[\alpha_1 e^{-j\omega\psi_1}, \alpha_2 e^{-j\omega\psi_2}, \ldots, \alpha_L e^{-j\omega\psi_L}\right], \tag{3}$$

where $\mathrm{diag}[\cdot]$ is a diagonal matrix formed from the elements of its arguments. In above, the parameters α_l and ψ_l are gain and delay associated with lth sensor.

Incorporating mutual coupling and antenna gain and phase in the array response, the received signal $\mathbf{x}(n)$ in (1) transforms to

$$\mathbf{x}(n) = \sum_{k=1}^{K} \mathbf{C}\mathbf{\Gamma}\mathbf{a}(\theta_k, \phi_k) s_k(n) = \mathbf{C}\mathbf{\Gamma}\mathbf{A}\mathbf{s}(n), \tag{4}$$

where

$$\mathbf{A} = [\mathbf{a}(\theta_1, \phi_1), \mathbf{a}(\theta_2, \phi_2), \ldots, \mathbf{a}(\theta_K, \phi_K)] \tag{5}$$

is a $L \times K$ matrix of steering vectors and $\mathbf{s}(n)$ is a vector of complex signal amplitudes, given by

$$\mathbf{s}(n) = [s_1(n), s_2(n), \ldots, s_K(n)]^T. \tag{6}$$

For a generic 2D array considered in this paper and shown in Fig. 1, the column steering vector $\mathbf{a}(\theta_k, \phi_k)$ is given by

$$\begin{aligned}\mathbf{a}(\theta_k, \phi_k) = \Big[1,\ & e^{-j\frac{2\pi}{\lambda} d_x \sin(\theta_k)\cos(\phi_k)}, \ldots, \\ & e^{-j\frac{2(L-1)\pi}{\lambda} d_x \sin(\theta_k)\cos(\phi_k)}, \\ & e^{-j\left(\frac{2\pi}{\lambda} d_x \sin(\theta_k)\cos(\phi_k) + \frac{2\pi}{\lambda} d_y \sin(\phi_k)\right)}\Big]^T,\end{aligned} \tag{7}$$

where d_x and d_y are antenna spacing in x and y directions of the array, respectively. Note that the spacing in either direction does not necessarily have to be equal or consistent.

It should be noted that even though an L-shaped 2D array is considered in this paper, the analysis can straightforwardly be extended to any array shape.

Methods that model the mutual coupling between antenna array elements assume that the coupling is independent of the direction of arrival [12]. For the

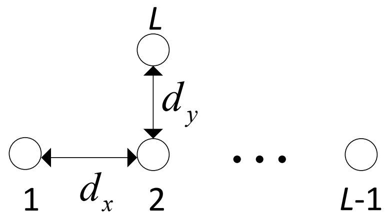

Fig. 1. L-element array.

small on-chip antennas used in automotive applications, the coupling may become direction dependent [13]. As a result, traditional method that make use of training data collected from various directions in the field of view of radar may not necessarily work. Furthermore, it is desired that the calibration is done online since the antenna coupling tends to change with time, temperature, and component life [6].

The method proposed in [13] finds global calibration matrix, which is considered independent of the DOA. The authors also proposed a local calibration method, where a smaller set of probable DOAs are selected and the cost function is evaluated over that set of DOAs. As expected, the local optimization method works better, but is expensive to evaluate every time DOA estimation is desired.

In this paper, we propose an online joint array calibration and DOA estimation method. The method is based on iterative optimization and estimates the calibration matrix, antenna array gain and phase, and angle of arrivals at each iteration. Moreover, in contrast to [6], the method does not assume any array shape and works well for any 2D array shape as long as the array response in (5) is properly constructed. The proposed method uses linear algebraic techniques to transform the coupling matrix estimation to a linear least-squares problem. We will illustrate the performance of the proposed method using a 2D array, with only two antennas capturing elevation angle and four antennas for azimuth angle estimation.

III. Calibration and DOA Estimation

Let λ_i and $\mathbf{u}_i$, $i = 1, 2, \ldots, L$ be the eigenvalues and eigenvectors of the received data sample covariance matrix $\mathbf{R_x} = E\{\mathbf{x}\mathbf{x}^H\}$. Assuming K ($K < L$) sources, collect the set of eigenvectors belonging to noise subspace in $L \times (L-K)$ matrix

$$\mathbf{U} = [\mathbf{u}_{K+1}, \mathbf{u}_{K+2}, \ldots, \mathbf{u}_L]. \tag{8}$$

The unknown DOAs are obtained by minimizing the following cost function over θ and ϕ

$$J = \sum_{k=1}^{K} \|\mathbf{U}^H \mathbf{C} \boldsymbol{\Gamma} \mathbf{a}(\theta_k, \phi_k)\|^2, \tag{9}$$

where $\|\cdot\|$ denotes Frobenius norm.

It is clear that unless the matrices $\mathbf{C}$ and $\boldsymbol{\Gamma}$ are known, accurate DOA estimation is not possible.

This cost function in (9) is similar to what is employed in MUSIC [10], with the addition of matrices $\mathbf{C}$ and $\boldsymbol{\Gamma}$ in the argument. Optimization of J in the presence of unknown $\mathbf{C}$ and $\boldsymbol{\Gamma}$ becomes a challenging problem to solve. Friedlander and Weiss in [6] proposed a joint calibration and angle estimation method using a cost function similar to (9). Their work assumed linear and circular arrays and the optimization of $\mathbf{C}$ was based on these two structures. In automotive applications, because of limited die space, the antennas in an array are not necessarily arranged in a known configuration. It is, therefore, important that no structure should be associated with the calibration matrix. Furthermore, the authors of this paper are not aware of a joint calibration and angle estimation method for an arbitrary shape when the array coupling may change with the angle of arriving waves.

In our proposed method, the matrices $\boldsymbol{\Gamma}$ and $\mathbf{C}$, and the angles (θ, ϕ) are estimated using the following iterative method:

1) Initialization:

$$i = 0; \quad \text{Set } \mathbf{C}^i \text{ and } \boldsymbol{\Gamma}^i \text{ to their initial values.}$$

2) Estimate sample data covariance matrix

$$\widehat{\mathbf{R}}_{\mathbf{x}} = \frac{1}{N} \sum_{n=1}^{N} \mathbf{x}(n)\mathbf{x}(n)^H. \tag{10}$$

3) Eigen-decompose $\widehat{\mathbf{R}}_{\mathbf{x}}$, find $\mathbf{U}$ and search for K peaks in the 2D spectrum defined by

$$P^i(\theta, \phi) = \|\mathbf{U}^H \mathbf{C}^i \boldsymbol{\Gamma}^i \mathbf{a}(\theta, \phi)\|^{-2}. \tag{11}$$

The peaks of (11) correspond to the DOA estimates $\left\{(\theta_k^i, \phi_k^i)\right\}_{k=1}^K$.

4) Use the estimated DOA to form the matrix $\mathbf{A}$ using (5) and (7).

The $\left\{(\theta_k^i, \phi_k^i)\right\}_{k=1}^K$ and $\mathbf{C}^i$ are set as the matrix $\boldsymbol{\Gamma}^{i+1}$ is estimated.

5) Under the constraint $\gamma^H \mathbf{w} = 1$, where

$$\mathbf{w} = [1, 0, 0, \ldots, 0]^T, \tag{12}$$

estimate the $L \times 1$ vector γ [6]:

$$\gamma = \mathbf{Z}^{-1}\mathbf{w} / \left(\mathbf{w}^T\mathbf{Z}^{-1}\mathbf{w}\right), \tag{13}$$

where the matrix $\mathbf{Z}$ is given by

$$\mathbf{Z} = \sum_{k=1}^{K} \mathbf{Q}_k^H \mathbf{C}^{i^H} \mathbf{U}\mathbf{U}^H \mathbf{C}^i \mathbf{Q}_k, \tag{14}$$

and the diagonal matrix $\mathbf{Q}_k$ is formed using

$$\mathbf{Q}_k = \mathrm{diag}\left[\mathbf{a}(\theta_k, \phi_k)\right]. \tag{15}$$

6) Update the estimate of $\mathbf{\Gamma}$ using the diagonal elements of γ as $\mathbf{\Gamma}^{i+1} = \mathrm{diag}[\gamma]$. The estimates $\mathbf{\Gamma}^{i+1}$ and $\left\{(\theta_k^i, \phi_k^i)\right\}_{k=1}^{K}$ are then fixed as coupling matrix $\mathbf{C}^{i+1}$ is estimated.
7) The cost function in (9) is minimized in the least-squares sense to solve for $\mathbf{C}^{i+1}$ under the constraint $C_{11}^{i+1} = 1$. This optimization is carried out as follows. More details are available in Appendix.
   - Compute $K(L-K) \times L^2$ matrix

     $$\mathbf{M} = (\mathbf{A}^T \mathbf{\Gamma}^{i+1^T}) \otimes \mathbf{U}^H, \tag{16}$$

     where $\otimes$ defines the Kronecker product.
   - Extract a $K(L-K) \times 1$ vector

     $$\mathbf{m} = \mathbf{M}(:, 1) \tag{17}$$

     and a $K(L-K) \times (L^2 - 1)$ matrix

     $$\tilde{\mathbf{M}} = \mathbf{M}(:, 2:\mathrm{end}); \tag{18}$$

     i.e., $\mathbf{m}$ contains only the first column of $\mathbf{M}$ and $\tilde{\mathbf{M}}$ is $\mathbf{M}$, except for its first column.
   - Compute $(L^2 - 1) \times 1$ vector

     $$\mathbf{c} = -\tilde{\mathbf{M}}^{\#}\mathbf{m}, \tag{19}$$

     where $(\cdot)^{\#}$ denotes pseudo-inverse.
   - Compute $L^2 \times 1$ vector

     $$\tilde{\mathbf{c}} = [1 \quad \mathbf{c}^T]^T. \tag{20}$$

   - Re-arrange $\tilde{\mathbf{c}}$ in rows of L to form updated $\mathbf{C}^{i+1}$.

The iterative calibration and angle estimation is continued until the cost function (9) at $(i+1)$th iteration is smaller than what it was at the ith iteration by a pre-set threshold.

IV. Experimental Results

We used a Frequency-Modulated Continuous Wave (FMCW) radar with four-element array $(L = 4)$ of shape in Fig. 1; it has three elements in one direction and the fourth element in orthogonal direction. Carrier frequency of 77 GHz was used. The inter-element spacing in either direction was 2 mm. An object was placed at a fixed elevation angle of -20 degrees and moved along azimuth direction from -40 degrees to 40 degrees in increments of 10 degrees. A total of ten data snapshots were collected at each object location to compute the covariance matrix in (10). The time-difference between two snapshots is 125 ms. Fewer snapshots may affect the estimation, so it is desired to include more data in estimating $\widehat{\mathbf{R}}_{\mathbf{x}}$. However, care has to be taken that object movement is not captured during the time that snapshots are collected. The matrices $\mathbf{C}$ and $\mathbf{\Gamma}$ were initialized with identity matrices. At each location of the object, the joint iterative algorithm was used to estimate the coupling matrix. It took a maximum of 5 iterations for the algorithm to converge to an estimate. Fig. 2 shows the azimuth and elevation angles plotted against the azimuth angles. The raw angle estimates show that the effect of coupling is different at different azimuth angles, generally becoming more pronounced at extreme angles, when the estimates deviate more from the true values. Moreover, it is noticed that azimuth angle estimation is, in general, better than the elevation estimation; this is to be expected as there are more antennas available to estimate azimuth angle. The DOA estimation improvement using the proposed method is also shown.

V. Conclusion

A joint calibration and angle estimation algorithm was presented. The method is especially suited for automotive applications where multiple sensors are installed around the vehicle and online calibration and angle estimation is highly desired. The method does not impose any constraint on the array shape and coupling-matrix structure and takes only a few iterations to converge.

Appendix

We can rewrite the product $\mathbf{U}^H\mathbf{C}\mathbf{\Gamma}\mathbf{a}$ as follows

$$\mathbf{U}^H\mathbf{C}\mathbf{\Gamma}\mathbf{a} = \left[(\mathbf{\Gamma}\mathbf{a})^T \otimes \mathbf{U}^H\right]\mathbf{c}, \tag{21}$$

where $\mathbf{c}$ is vectorized form of $\mathbf{C}$; i.e.,

$$\mathbf{c} = \left[C_{11},\ C_{21},\ \ldots,\ C_{LL}\right], \tag{22}$$

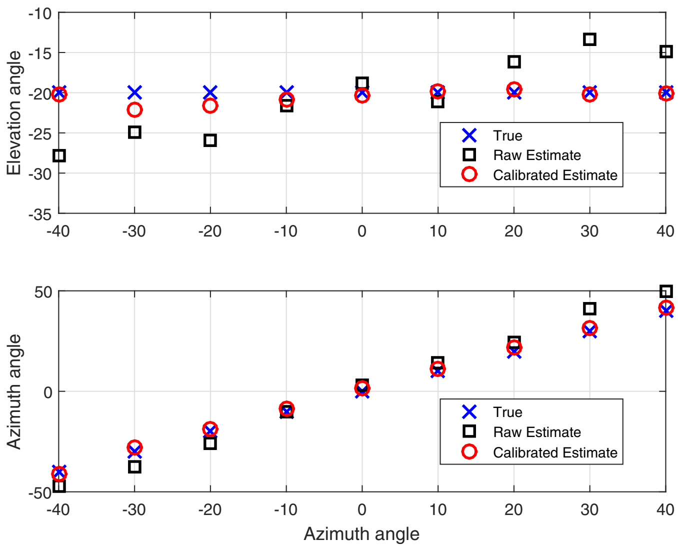

Fig. 2. Estimated and true elevation and azimuth angles plotted against azimuth angle. All angles in degrees.

and $\otimes$ denotes Kronecker product. Using (21), the solution to optimization problem

$$\min_{\mathbf{C}, C_{11}=1} \sum_{k=1}^{K} \|\mathbf{U}^H \mathbf{C} \mathbf{\Gamma} \mathbf{a}(\theta_k, \phi_k)\|^2 \tag{23}$$

is equivalent to

$$\min_{\tilde{\mathbf{c}}} \|\tilde{\mathbf{M}}\tilde{\mathbf{c}} + \mathbf{m}\|^2, \tag{24}$$

where the matrix $\tilde{\mathbf{M}}$ is obtained from $\left[(\mathbf{\Gamma}\mathbf{a})^T \otimes \mathbf{U}^H\right]$ by removing its first column. Similarly, the vector $\mathbf{m}$ is obtained by extracting the first column of $\left[(\mathbf{\Gamma}\mathbf{a})^T \otimes \mathbf{U}^H\right]$. The vector $\tilde{\mathbf{c}}$ is obtained from $\mathbf{c}$ by removing its first element.

The optimization in (24) is solved using $\tilde{\mathbf{c}} = -\tilde{\mathbf{M}}^{\#}\mathbf{m}$.

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