emojicoin dot fun

Sponsored by a grant from the Aptos Foundation

Econia Labs

1 Overview

`emojicoin dot fun` bootstraps liquidity for "fair" (public allocation only, no presale) emojicoin [1] markets using a two-state mechanism popularized by `pump dot fun` [2]. The first state, known as a bonding curve, uses an abridged Concentrated Liquidity Automated Market Maker (CLAMM) with a single price range [3] as described in section 3.

Once the `APT`-denominated [4] market capitalization for an emojicoin reaches a predefined value, a State Transition (ST) occurs whereby `APT` deposits are removed from the bonding curve and locked into a Constant Product Automated Market Maker (CPAMM) [5] together with a remainder of emojicoin supply, as described in section 4.

2 Economic variables

Price p is defined per table 1 and equation 1.

Term	Notation	Asset
Base asset	b	emojicoin
Quote asset	q	`APT`

Table 1: Base and quote asset definitions

$$p = \frac{q}{b} \tag{1}$$

The economic variables in table 2 fully specify the set of numerical values required for the implementation, as derived in sections 3 through 6.

Term	Notation
`APT`-denominated market capitalization	m_a
Circulating emojicoin supply	c_e
`APT`-denominated spot price	p_s

Table 2: Economic variables at time of ST

As derived in section 6.4, the alternative set of economic variables in table 3 can also be used to fully specify numerical values.

Term	Notation
$p_s^{-1} \approx$ emojicoins per 1 `APT` at ST	A
Ratio of p at ST to p at start of bonding curve	R
Total `APT` deposited before ST	T

Table 3: Alternative economic variables

3 Bonding curve state

The bonding curve is represented by a CLAMM, which functions as a CPAMM within the price range $[p_l, p_h]$ as defined by table 4, equation 2, and figure 1.

Term	Notation
Real base reserves	b_r
Real quote reserves	q_r
Virtual base reserves	b_v
Virtual quote reserves	q_v
Liquidity	L
Low price range endpoint	p_l
High price range endpoint	p_h
Real base reserves ceiling	$b_{r,c}$
Real quote reserves ceiling	$q_{r,c}$
Virtual base reserves ceiling	$b_{v,c}$
Virtual quote reserves ceiling	$q_{v,c}$
Virtual base reserves floor	$b_{v,f}$
Virtual quote reserves floor	$q_{v,f}$

Table 4: Terms, CLAMM as a fixed-range CPAMM

$$(b_r + b_{v,f})(q_r + q_{v,f}) = L^2 = b_v q_v \tag{2}$$

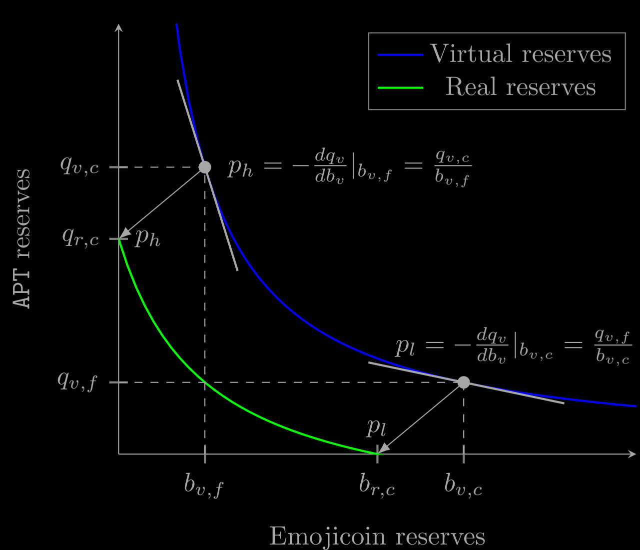

Figure 1: CLAMM as a fixed-range CPAMM

The bonding curve initializes with real emojicoin reserves c_e, which represent only a portion of total supply s_e. The remainder of emojicoin reserves r_e is set aside for the ST which occurs when $q_{r,c}$ of `APT` has been deposited into the bonding curve, per equation 3 as derived in section 6.1.

$$r_e = \frac{m_a - c_e \cdot p_s}{p_s}, s_e = \frac{m_a}{p_s}, q_{r,c} = m_a - c_e \cdot p_s \tag{3}$$

Virtual reserves initialize per equation 4 as derived in section 6.2.

$$b_{v,c} = \frac{c_e^2 \cdot p_s}{2 \cdot c_e \cdot p_s - m_a}, q_{v,f} = \frac{(m_a - c_e \cdot p_s)^2}{2 \cdot c_e \cdot p_s - m_a} \tag{4}$$

The bonding curve price initializes to p_l per equation 5, also derived in section 6.2.

$$p_l = \frac{(m_a - c_e \cdot p_s)^2}{c_e^2 \cdot p_s} \tag{5}$$

During the bonding curve phase, virtual reserve amounts in the CLAMM follow a simple constant product curve invariant for swaps per table 5 and equation 6 as derived in section 6.3.

Term	Swap input	Swap output
Base	b_{in}	b_{out}
Quote	q_{in}	q_{out}

Table 5: Swap input and output definitions

$$b_{out} = \frac{b_0 \cdot q_{in}}{q_0 + q_{in}}, q_{out} = \frac{b_{in} \cdot q_0}{b_0 + b_{in}} \tag{6}$$

The ST occurs when virtual reserves reach the values from equation 7, derived in section 6.2.

$$b_{v,f} = \frac{c_e \cdot (m_a - c_e \cdot p_s)}{2 \cdot c_e \cdot p_s - m_a}, q_{v,c} = \frac{c_e \cdot p_s \cdot (m_a - c_e \cdot p_s)}{2 \cdot c_e \cdot p_s - m_a} \tag{7}$$

Notably, this results in a percent dilution at the time of ST, $d_\%$, defined in equation 8 as derived in section 6.2.

$$d_\% = \frac{c_e \cdot p_s}{m_a} \cdot 100\% \tag{8}$$

4 CPAMM state

At the ST, $q_{r,c}$ is withdrawn from the bonding curve and locked into a CPAMM together with r_e, thus maintaining a constant spot price throughout the ST. This results in the minting of L_i initial Liquidity Provider (LP) tokens, with L_i taken as the geometric mean of the two contributions per equation 9 as derived in section 6.5.

$$L_i = \frac{m_a - c_e \cdot p_s}{\sqrt{p_s}} \tag{9}$$

The initial LP token mint is then held by the protocol, similar in effect to burning LP tokens, thus constituting protocol-owned liquidity.

A pool fee rate f_p denominated in basis points is assessed on the output of a swap to incentivize additional LPs. By assessing the fee on the output amount and reinvesting it in the pool, spot price slippage decreases and liquidity increases for each swap.

For example figure 2 denotes a swap sell, where $f_p \cdot q_{out}$ is deducted from quote proceeds and reinvested in the pool, thus increasing available liquidity.

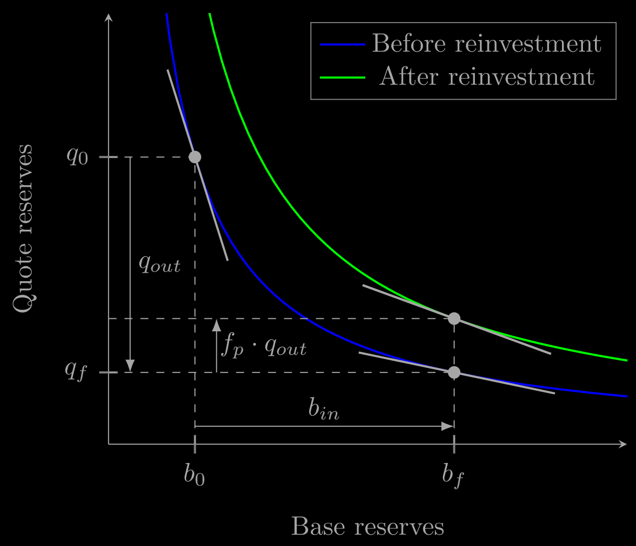

Figure 2: CPAMM swap sell with fee

5 Actual implementation values

Nominal implementation values are given in table 6.

Term	Amount
A	10,000
R	12.25
$T = q_{r,c}$	10,000
m_a	45,000
$c_e = b_{r,c}$	350,000,000
r_e	100,000,000
s_e	450,000,000
$d_\%$	$77.\bar{7}$
$p_s = p_h$	0.0001
p_l	0.000008163 ...
L_i	1,000,000
f_p	25
$b_{v,f}$	140,000,000
$q_{v,f}$	4,000
$b_{v,c}$	490,000,000
$q_{v,c}$	14,000

Table 6: Nominal implementation values

This value set meets the restriction from equation 10, derived in section 6.2.

$$c_e \cdot p_s < m_a < 2 \cdot c_e \cdot p_s \tag{10}$$

Notably, for a given c_e and p_s, larger values of m_a correspond to larger R and larger $d_{\%}$. Hence the alternative economic variable approach described in section 1, which enables direct specification of R.

m_a scales to octa-denominated [4] market capitalization m_o as shown in equation 11.

$$m_o = 10^8 m_a \tag{11}$$

For simplicity emojicoins are taken to have 8 decimals, the same as `APT`. Hence c_e scales to integer emojicoin subunits c_s as shown in equation 12.

$$c_s = 10^8 c_e \tag{12}$$

Applying the same scale factor across variables produces the integer values in table 7.

Constant	Amount
`MARKET_CAP`	4_500_000_000_000
`EMOJICOIN_REMAINDER`	10_000_000_000_000_000
`EMOJICOIN_SUPPLY`	45_000_000_000_000_000
`LP_TOKENS_INITIAL`	100_000_000_000_000
`BASE_REAL_FLOOR`	0
`QUOTE_REAL_FLOOR`	0
`BASE_REAL_CEILING`	35_000_000_000_000_000
`QUOTE_REAL_CEILING`	1_000_000_000_000
`BASE_VIRTUAL_FLOOR`	14_000_000_000_000_000
`QUOTE_VIRTUAL_FLOOR`	400_000_000_000
`BASE_VIRTUAL_CEILING`	49_000_000_000_000_000
`QUOTE_VIRTUAL_CEILING`	1_400_000_000_000
`POOL_FEE_RATE_BPS`	25

Table 7: Integer implementation values

6 Derivations

6.1 Supply amounts

To ensure a constant price during the ST, a portion of emojicoin reserves must be set aside from the initial bonding curve. At the ST this emojicoin remainder r_e is locked into the CPAMM together with all of the `APT` from the bonding curve, yielding:

$$\begin{aligned} p_s &= \frac{q_{r,c}}{r_e} \\ p_s \cdot r_e &= q_{r,c} \\ r_e &= \frac{q_{r,c}}{p_s} \end{aligned} \tag{13}$$

Define $p_s = f(m_a, c_e, r_e)$:

$$p_s = \frac{m_a}{c_e + r_e} \tag{14}$$

Substitute (13) into (14):

$$\begin{aligned} p_s &= \frac{m_a}{c_e + \frac{q_{r,c}}{p_s}} \\ p_s \cdot \left(c_e + \frac{q_{r,c}}{p_s}\right) &= m_a \\ c_e + \frac{q_{r,c}}{p_s} &= \frac{m_a}{p_s} \\ \frac{q_{r,c}}{p_s} &= \frac{m_a}{p_s} - c_e \\ q_{r,c} &= m_a - c_e \cdot p_s \end{aligned} \tag{15}$$

Note $q_{r,c}$ is only positive for:

$$\begin{aligned} m_a - c_e \cdot p_s &> 0 \\ m_a &> c_e \cdot p_s \end{aligned} \tag{16}$$

Substitute (15) into (13):

$$r_e = \frac{m_a - c_e \cdot p_s}{p_s} \tag{17}$$

Hence total supply s_e evaluates to:

$$\begin{aligned} s_e &= c_e + r_e \\ s_e &= c_e + \frac{m_a - c_e \cdot p_s}{p_s} \\ s_e &= \frac{c_e \cdot p_s}{p_s} + \frac{m_a - c_e \cdot p_s}{p_s} \\ s_e &= \frac{c_e \cdot p_s + m_a - c_e \cdot p_s}{p_s} \\ s_e &= \frac{m_a}{p_s} \end{aligned} \tag{18}$$

6.2 Bonding curve amounts

Evaluated at p_l, (2) reduces to:

$$(b_{r,c} + b_{v,f}) \cdot q_{v,f} = L^2 \tag{19}$$

Likewise, (2) evaluated at p_h reduces to:

$$b_{v,f} \cdot (q_{r,c} + q_{v,f}) = L^2 \tag{20}$$

For $b_{r,c} = c_e$, combining (15), (19), and (20) yields:

$$\begin{aligned} (b_{r,c} + b_{v,f}) \cdot q_{v,f} &= b_{v,f} \cdot (q_{r,c} + q_{v,f}) \\ b_{r,c} \cdot q_{v,f} + b_{v,f} \cdot q_{v,f} &= b_{v,f} \cdot q_{r,c} + b_{v,f} \cdot q_{v,f} \\ b_{r,c} \cdot q_{v,f} &= b_{v,f} \cdot q_{r,c} \\ q_{v,f} &= \frac{b_{v,f} \cdot q_{r,c}}{b_{r,c}} \\ q_{v,f} &= \frac{b_{v,f} \cdot (m_a - c_e \cdot p_s)}{c_e} \end{aligned} \tag{21}$$

For $p_h = p_s$, substituting (15) and (21) yields:

$$
\begin{aligned}
p_h &= \frac{q_{v,c}}{b_{v,f}} \\
p_s &= \frac{q_{r,c} + q_{v,f}}{b_{v,f}} \\
b_{v,f} \cdot p_s &= q_{r,c} + q_{v,f} \\
b_{v,f} \cdot p_s - q_{v,f} &= q_{r,c} \\
b_{v,f} \cdot p_s - \frac{b_{v,f} \cdot (m_a - c_e \cdot p_s)}{c_e} &= m_a - c_e \cdot p_s \\
\frac{b_{v,f} \cdot c_e \cdot p_s}{c_e} + \frac{b_{v,f} \cdot (c_e \cdot p_s - m_a)}{c_e} &= m_a - c_e \cdot p_s \\
\frac{b_{v,f} \cdot (2 \cdot c_e \cdot p_s - m_a)}{c_e} &= m_a - c_e \cdot p_s \\
b_{v,f} \cdot (2 \cdot c_e \cdot p_s - m_a) &= c_e \cdot (m_a - c_e \cdot p_s) \\
b_{v,f} &= \frac{c_e \cdot (m_a - c_e \cdot p_s)}{2 \cdot c_e \cdot p_s - m_a} \qquad (22)
\end{aligned}
$$

Since $m_a > c_e \cdot p_s$, the numerator is always positive for positive c_e. However the denominator is only positive if:

$$
\begin{aligned}
2 \cdot c_e \cdot p_s - m_a &> 0 \\
2 \cdot c_e \cdot p_s &> m_a \\
m_a &< 2 \cdot c_e \cdot p_s \qquad (23)
\end{aligned}
$$

Combining (23) and (16) yields:

$$
c_e \cdot p_s < m_a < 2 \cdot c_e \cdot p_s \qquad (24)
$$

Substituting (22) into (21) yields:

$$
\begin{aligned}
q_{v,f} &= \frac{\frac{c_e \cdot (m_a - c_e \cdot p_s)}{2 \cdot c_e \cdot p_s - m_a} \cdot (m_a - c_e \cdot p_s)}{c_e} \\
q_{v,f} &= \frac{(m_a - c_e \cdot p_s)^2}{2 \cdot c_e \cdot p_s - m_a} \qquad (25)
\end{aligned}
$$

For $b_{r,c} = c_e$ and $b_{v,f}$ per (22), $b_{v,c}$ resolves to:

$$
\begin{aligned}
b_{v,c} &= b_{r,c} + b_{v,f} \\
b_{v,c} &= c_e + \frac{c_e \cdot (m_a - c_e \cdot p_s)}{2 \cdot c_e \cdot p_s - m_a} \\
b_{v,c} &= \frac{c_e \cdot (2 \cdot c_e \cdot p_s - m_a)}{2 \cdot c_e \cdot p_s - m_a} + \frac{c_e \cdot (m_a - c_e \cdot p_s)}{2 \cdot c_e \cdot p_s - m_a} \\
b_{v,c} &= \frac{c_e \cdot (c_e \cdot p_s)}{2 \cdot c_e \cdot p_s - m_a} \\
b_{v,c} &= \frac{c_e^2 \cdot p_s}{2 \cdot c_e \cdot p_s - m_a} \qquad (26)
\end{aligned}
$$

For $q_{r,c}$ per (15) and $q_{v,f}$ per (25), $q_{v,c}$ resolves to:

$$
\begin{aligned}
q_{v,c} &= q_{r,c} + q_{v,f} \\
q_{v,c} &= m_a - c_e \cdot p_s + \frac{(m_a - c_e \cdot p_s)^2}{2 \cdot c_e \cdot p_s - m_a} \\
q_{v,c} &= \frac{(2 \cdot c_e \cdot p_s - m_a) \cdot (m_a - c_e \cdot p_s)}{2 \cdot c_e \cdot p_s - m_a} + \frac{(m_a - c_e \cdot p_s)^2}{2 \cdot c_e \cdot p_s - m_a} \\
q_{v,c} &= \frac{(m_a - c_e \cdot p_s) \cdot (2 \cdot c_e \cdot p_s - m_a + m_a - c_e \cdot p_s)}{2 \cdot c_e \cdot p_s - m_a} \\
q_{v,c} &= \frac{(m_a - c_e \cdot p_s) \cdot (c_e \cdot p_s)}{2 \cdot c_e \cdot p_s - m_a} \\
q_{v,c} &= \frac{c_e \cdot p_s \cdot (m_a - c_e \cdot p_s)}{2 \cdot c_e \cdot p_s - m_a} \qquad (27)
\end{aligned}
$$

Hence for $q_{v,f}$ per (25) and $b_{v,c}$ per (26), p_l resolves to:

$$
\begin{aligned}
p_l &= \frac{q_{v,f}}{b_{v,c}} \\
p_l &= \frac{\frac{(m_a - c_e \cdot p_s)^2}{2 \cdot c_e \cdot p_s - m_a}}{\frac{c_e^2 \cdot p_s}{2 \cdot c_e \cdot p_s - m_a}} \\
p_l &= \frac{(m_a - c_e \cdot p_s)^2}{c_e^2 \cdot p_s} \qquad (28)
\end{aligned}
$$

Define dilution percentage at the time of ST, $d_{\%}$ substituting (18):

$$
\begin{aligned}
d_{\%} &= \frac{c_e}{s_e} \cdot 100\% \\
d_{\%} &= \frac{c_e}{\frac{m_a}{p_s}} \cdot 100\% \\
d_{\%} &= \frac{c_e \cdot p_s}{m_a} \cdot 100\%
\end{aligned}
\qquad (29)
$$

6.3 Base in and out for CPAMM swap

Let b_0 and q_0 represent base and quote reserves before a swap, and b_f and q_f represent reserves after a swap. For a feeless swap buy:

$$
\begin{aligned}
b_0 \cdot q_0 &= b_f \cdot q_f \\
b_0 \cdot q_0 &= (b_0 - b_{out}) \cdot (q_0 + q_{in}) \\
b_0 \cdot q_0 &= b_0 \cdot q_0 + b_0 \cdot q_{in} - b_{out} \cdot q_0 - b_{out} \cdot q_{in} \\
b_0 \cdot q_0 &= b_0 \cdot q_0 + b_0 \cdot q_{in} - b_{out} \cdot (q_0 + q_{in}) \\
b_{out} \cdot (q_0 + q_{in}) &= b_0 \cdot q_{in} \\
b_{out} &= \frac{b_0 \cdot q_{in}}{q_0 + q_{in}} \qquad (30)
\end{aligned}
$$

For a feeless swap sell:

$$
\begin{aligned}
b_0 \cdot q_0 &= b_f q_f \\
b_0 \cdot q_0 &= (b_0 + b_{in}) \cdot (q_0 - q_{out}) \\
b_0 \cdot q_0 &= b_0 \cdot q_0 - b_0 \cdot q_{out} + b_{in} \cdot q_0 - b_{in} \cdot q_{out} \\
b_0 \cdot q_0 &= b_0 \cdot q_0 + b_{in} \cdot q_0 - q_{out} \cdot (b_0 + b_{in}) \\
q_{out} \cdot (b_0 + b_{in}) &= b_{in} \cdot q_0 \\
q_{out} &= \frac{b_{in} \cdot q_0}{b_0 + b_{in}} \qquad (31)
\end{aligned}
$$

6.4 Alternative economic variable selection

Define the inverse of spot price, roughly interpreted as the number of emojicoins that 1 `APT` will buy at the ST, A:

$$A = \frac{1}{p_s} \qquad (32)$$

Define the ratio of bonding curve price endpoints, R:

$$R = \frac{p_s}{p_l} \qquad (33)$$

Define the total amount of `APT` required to initiate the ST, T:

$$T = q_{r,c} \qquad (34)$$

Combine (33), (28), and (15):

$$
\begin{aligned}
R &= \frac{p_s}{p_l} \\
R &= \frac{p_s}{\frac{(m_a - c_e \cdot p_s)^2}{c_e^2 \cdot p_s}} \\
R &= \frac{c_e^2 \cdot p_s^2}{(m_a - c_e \cdot p_s)^2} \\
R &= \frac{c_e^2 \cdot (\frac{1}{A})^2}{T^2} \\
R &= \frac{c_e^2}{A^2 \cdot T^2} \\
A^2 \cdot R \cdot T^2 &= c_e^2 \\
c_e^2 &= A^2 \cdot R \cdot T^2 \\
c_e &= A \cdot T \cdot \sqrt{R} \qquad (35)
\end{aligned}
$$

Refactor (32):

$$
\begin{aligned}
A &= \frac{1}{p_s} \\
A \cdot p_s &= \frac{1}{p_s} \\
p_s &= \frac{1}{A} \qquad (36)
\end{aligned}
$$

Combine (34), (15), (35), and (36):

$$
\begin{aligned}
T &= m_a - c_e \cdot p_s \\
T &= m_a - A \cdot T \cdot \sqrt{R} \cdot \frac{1}{A} \\
T &= m_a - T \cdot \sqrt{R} \\
T + T \cdot \sqrt{R} &= m_a \\
m_a &= T + T \cdot \sqrt{R} \\
m_a &= T \cdot (1 + \sqrt{R}) \qquad (37)
\end{aligned}
$$

Combine (16), (35), (36), (37), demonstrating a condition that is always met:

$$
\begin{aligned}
T \cdot (1 + \sqrt{R}) &> A \cdot T \cdot \sqrt{R} \cdot \frac{1}{A} \\
1 + \sqrt{R} &> \sqrt{R} \\
1 &> 0 \qquad (38)
\end{aligned}
$$

Combine (23), (35), (36), (37), demonstrating another condition that is always met:

$$
\begin{aligned}
m_a &< 2 \cdot c_e \cdot p_s \\
T \cdot (1 + \sqrt{R}) &< 2 \cdot A \cdot T \cdot \sqrt{R} \cdot \frac{1}{A} \\
1 + \sqrt{R} &< 2 \cdot \sqrt{R} \\
1 &< \sqrt{R} \\
\sqrt{R} &> 1 \\
R &> 1 \qquad (39)
\end{aligned}
$$

To avoid numerical issues during calculations in a script or similar, substitute (34) and (36) into (13):

$$
\begin{aligned}
r_e &= \frac{q_{r,c}}{p_s} \\
r_e &= \frac{T}{\frac{1}{A}} \\
r_e &= A \cdot T \qquad (40)
\end{aligned}
$$

Similarly, substitute (35), (36), and (37) into (26):

$$
\begin{aligned}
b_{v,c} &= \frac{c_e^2 \cdot p_s}{2 \cdot c_e \cdot p_s - m_a} \\
b_{v,c} &= \frac{(A \cdot T \cdot \sqrt{R})^2 \cdot \frac{1}{A}}{2 \cdot (A \cdot T \cdot \sqrt{R}) \cdot \frac{1}{A} - T \cdot (1 + \sqrt{R})} \\
b_{v,c} &= \frac{A^2 \cdot T^2 \cdot R \cdot \frac{1}{A}}{2 \cdot T \cdot \sqrt{R} - T \cdot (1 + \sqrt{R})} \\
b_{v,c} &= \frac{A \cdot T^2 \cdot R}{T \cdot (2 \cdot \sqrt{R} - (1 + \sqrt{R}))} \\
b_{v,c} &= \frac{A \cdot T \cdot R}{2 \cdot \sqrt{R} - (1 + \sqrt{R})} \\
b_{v,c} &= \frac{A \cdot T \cdot R}{\sqrt{R} - 1} \qquad (41)
\end{aligned}
$$

6.5 Initial LP tokens

Define L_i as the geometric mean of $q_{r,c}$ and r_e, substituting (15) and (17):

$$
\begin{aligned}
L_i &= \sqrt{q_{r,c} \cdot r_e} \\
L_i &= \sqrt{(m_a - c_e \cdot p_s) \cdot \frac{m_a - c_e \cdot p_s}{p_s}} \\
L_i &= \sqrt{\frac{(m_a - c_e \cdot p_s)^2}{p_s}} \\
L_i &= \frac{m_a - c_e \cdot p_s}{\sqrt{p_s}}
\end{aligned} \tag{42}
$$

References

[1] Emojicoins. Retrieved April 11, 2024 from https://x.com/alnoki_/status/1776323147874119915.

[2] Pump. https://www.pump.fun/board, https://x.com/pumpdotfun. Retrieved April 11, 2024.

[3] Hayden Adams, Noah Zinsmeister, Moody Salem, River Keefer, and Dan Robinson. 2021. *Uniswap v3 Core*. Retrieved April 11, 2024 from https://uniswap.org/whitepaper-v3.pdf.

[4] https://aptos.dev/reference/glossary/#gas-unit-price. Retrieved April 11, 2024.

[5] Hayden Adams, Noah Zinsmeister, and Dan Robinson. 2020. *Uniswap v2 Core*. Retrieved April 11, 2024 from https://uniswap.org/whitepaper.pdf.