Discrete Math Homework #1

Due Oct, 16, 2018 11:20

Collaboration policy: You can discuss the problem with other students, but you must obtain and write the final solution by yourself. Please specify all of your collaborators (name and student id) for each question. If you solve some problems by yourself, please also specify "no collaborators".

Problem 1 A set S of logical connector is *complete* if any propositional statement has an equivalent statement using only connectors in S. We proved that $\{\vee, \neg\}$ is a complete set.

1. (10 %) Is $\{\rightarrow, \neg\}$ a complete set? Prove your answer.
2. (10 %) Is $\{\wedge, \vee\}$ a complete set? Prove your answer.

Problem 2 Prove the following equivalences. You must follow the exact format used in class. Only the equivalence laws listed on the class website is allowed. Sepcify the law used in every step.

1. (10%) Prove that $[\neg q \wedge (p \rightarrow q)] \rightarrow \neg p$ is a tautology.
2. (10%) Prove that $\big(\neg p \wedge (q \wedge r)\big) \vee \big((\neg p \wedge q) \wedge \neg r\big)$ is equivalent to $q \wedge \neg p$.

Problem 3 (40%) Prove the validity of the following four arguments. You must follow the exact format used in class. Only the rules listed on the class website is allowed. Sepcify the law used in every step. Domain of quantifiers: all students.

1. "If a student does not have a personal computer, then he is not in EE."

   "Every student in CS has a personal computer."

   "Paul is either in EE or in CS."

   Therefore,

   "Paul has a personal computer."

2. "If Superman were able to prevent evil, he would do so."

   "If Superman were unable to prevent evil, he would be impotent."

   "If Superman exists, he is not impotent."

   "If Superman exists, he does not prevent evil."

   Therefore,

   "If Superman exists, it is impossible to learn logic."

3. "Every person who lives in Taipei lives within 100 kilometers to the ocean."

   "Some people who live in Taipei have never eaten seafood."

   Therefore,

   "Some people live within 100 kilometers to the ocean but have never eaten seafood."

4. "Every student in the discrete math class knows calculus."

   "If a student doesnt know C++, then he is not a student in the discrete math class."

   "If Peter knows calculus, then he doesnt know C++."

   Therefore,

   "Peter is not a student in the discrete math class."

Problem 4

1. (10%) Prove that $\neg\big((p \vee q) \wedge (r \vee s)\big) \equiv (\neg s \wedge \neg r) \vee (\neg p \wedge \neg q)$. You must strictly follow the format taught in class.
2. (5%) Prove that $\overline{(A \cup B) \cap (C \cup D)} = (\overline{D} \cap \overline{C}) \cup (\overline{A} \cap \overline{B})$ for any four sets A, B, C, D.

Problem 5 (5%)

Let $S_1 = \{\{1, 2\}\}$ and $S_2 = \phi$. What are the power sets 2^{S_1} and 2^{S_2}? (No explanation needed.)