Université de Montréal

Programming tools for intelligent systems

with a case study in autonomous robotics

par

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Summary

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Introduction

Intelligent system: A computer system that uses techniques derived from artificial intelligence, particularly one in which such techniques are central to the operation of the system.

-Wikipedia

In computer science, we are mostly concerned with algorithmic complexity. Computer scientists have developed numerous tools for describing complexity using analysis and information theory, such as Kolmogorov complexity and big-O notation. The field of software engineering is primarily interested in a different kind of complexity - the complexity of building software. One sort of software complexity is the mental effort required to understand a program, and can be approximated by metrics like cyclomatic complexity or Halstead complexity, which try to characterize the mental effort required to work with code.

Our objective is to reduce the mental effort required to build intelligent systems, using developer tools, programming language abstractions, automated testing, and containers.

The goal of this thesis is show it is possible to develop tools that reduce the mental burden of building intelligent software systems. First, we demonstrate an integrated development environment that assists users when writing robotics applications. This project was created to reduce the complexity of working with robotics code as raw text. Next, we demonstrate a type-safe domain specific language for differentiable programming, an emerging paradigm in deep learning. To test this application, we use a set of techniques borrowed from property-based testing [5]. We then use Docker containers [11]to automate the process of building, testing and deploying reproducible robotics applications to heterogeneous hardware platforms. Finally, we demonstrate a functional intelligent system built using the above tools which is comprised of a mobile autonomous vehicle and an Android mobile application.

1.1. Stages in the software development lifecycle

In traditional software engineering, the Waterfall Method is a classic engineering design process model that comprises of five stages. We propose contributions to four: design, implementation, verification and maintenance.

1.1.1. Design

When designing intelligent systems, we need to iterate between requirements and design constraints. It is not sufficient for requirements engineers to hand an objective to the design team, or the design team to hand a design to the implementors. Rather, they must work in concert to produce a system that has the desired properties. Sometimes this means compromising, or redesigning the performance metrics to become more flexible.

1.1.2. Implementation

When implementing intelligent systems, we need to think carefully about languages and abstractions we use. If developers are implementing back-propagation, they will have little time to think about the high-level characteristics of these systems. This is no different from traditional software engineering - we need to use the right abstractions for the job. With machine learning, the necessity of choosing appropriate implementations is even more pressing.

1.1.3. Verification

Because the space of many problems is intractably large, it is difficult to prove the correctness of a given solution. Instead, we need tools to verify the properties of a system under a given budget. In self-driving vehicles, human drivers average one fatality per hundred million miles (cite), this is incredibly difficult to test given our current methodology. Instead, we need better ways to probe the effectiveness of a candidate solution.

1.1.4. Maintenance

In many ways, machine learning shares the same fundamental issues as traditional software maintenance, with dependency management, source code management, documentation and so forth. We can imagine the current process of training a deep learning model as a very long compilation step that requires a great deal of trial-and-error. However, it differs in that the user-facing code for the program is a very high level meta-language that does not directly correspond to the computational performed in a particular instance, but rather configures a directed acyclic graph with randomly initialized edge weights for further optimization. With emerging techniques in meta-learning, even the structure of this computation graph is not directly specified by the user so much as loosely parameterized, and optimized by a search procedure.

While deep learning has many specialty software, unlike general purpose programming, is a fundamentally hardware-agnostic model of computation. As long as a computer can add and multiply, it has all the necessary capabilities to train and run a deep neural network. But many machine learning models can be frustratingly hard to reproduce on different hardware, even when using the same source code and dependencies. While there are some emerging methods for a truly portable model format, if we are not careful about how we design our models, they may simply not run, or produce very different results when run on different hardware. This disconnect is at the heart of machine learning software development, as most open source software libraries are produced by competing industry vendors, with competing standards. These include ONNX, NNEF, OpenVINO, ... Others have tried to leverage existing compiler frameworks such as Haskell and LLVM. While these are typically not optimized for running on GPUs, it allows them to take advantage of existing optimizations without directly.

Chapitre 2

Design: Programming tools for robotics systems

Programming tools are a bicycle for the mind. Complex systems need better tools for developers.

2.1. Software architecture of a robotics application

https://github.com/duckietown/hatchery

2.2. Structure of a ROS application

The Robot Operating System (ROS) is a popular middleware for robotics applications. At its core, ROS provides software infrastructure for distributed messaging, but more broadly encompasses a set of community-developed libraries and graphical plugins for building robotics applications. While ROS is not an operating system in the traditional sense, it extends certain operating systems features like shared memory and inter-process communication for robotics development. Unlike pure message-oriented middleware like DDS and ZMQ, in addition to the message broker, ROS provides specific features for building decentralized robotic systems, particularly those which are capable of mobility.

According to one community census in 2018, 55% of ROS applications on GitHub are written in C/C++, followed by Python at around 25% [9].

2.3. Foundations of a modern IDE

IDEs are more than just a souped-up text editor.

2.3.1. The parser

We can parse URDF, package and launch XML, and srv files.

2.3.2. Refactoring

Refactoring support is implemented.

2.3.3. Running and debugging

Assistance for running ROS applications.

2.4. More ROS Tools

Detecting and managing ROS installations.

Implementation: languages and compilers

"Programs must be written for people to read, and only incidentally for machines to execute."

-Abelson & Sussman, Structure and Interpretation of Computer Programs

In this chapter, we will discuss the theory and implementation of a type safe domain specific language for automatic differentiation (AD). AD is useful for gradient descent and has a variety of applications in numerical optimization and machine learning. The key idea behind AD is fairly simple. A small set of primitive operations form the basis for all modern computers: addition, subtraction, multiplication and division - by composing these operations over the real numbers in an orderly fashion, one can compute any computable function. In machine learning, it is often the case we are given a computable function, i.e. a program¹, that does not work properly. Perhaps the function was implemented poorly. Or perhaps implementation was correct, but the function was not given the right set of inputs - had it been, then the program may have produced a better answer. Regardless of the implementation, we would like to determine how to change the input slightly, so as to produce a more suitable output.

Given some input to a function, AD tells us how to change the input by a minimal amount, in order to maximally change the outputs. Suppose we are handed a function $P: \mathbb{R} \to \mathbb{R}$, composed of a series of nested functions:

 $^{^{1}}$ n.b. Not all programs represent computable functions, but all computable functions are programs.

$$P(p_0) = p_n \circ p_{n-1} \circ p_{n-2} \circ \dots \circ p_1 \circ p_0 \tag{3.0.1}$$

From the chain rule of calculus, we know that:

$$\frac{dP}{dp_0} = \prod_{i=1}^{n} \frac{dp_i}{dp_{i-1}} \tag{3.0.2}$$

Likewise, if we have a function $Q(q_0, q_1, \dots, q_n) : \mathbb{R}^n \to \mathbb{R}$, the gradient ∇Q tells us:

$$\nabla Q = \left(\frac{\partial Q}{\partial q_1}, \dots, \frac{\partial Q}{\partial q_n}\right) \tag{3.0.3}$$

More generally, if we have a function $\mathbf{f}: \mathbb{R}^n \to \mathbb{R}^m$, the Jacobian \mathbf{J} is defined like so:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial \mathbf{f}}{\partial x_1} & \cdots & \frac{\partial \mathbf{f}}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$
(3.0.4)

We can use these tools to compute the direction to adjust the inputs of a computable function, in order to maximally change that function's output, i.e. the direction of steepest descent.

Sometimes a function has the property that given an input x, no matter how we update x, the output stays the same. We say that such functions have zero gradient for that input.

$$(\nabla F)(x) = \mathbf{0} \tag{3.0.5}$$

For some classes of computable functions, small changes to the input can produce a sudden large change in output. We say that such functions are non-differentiable.

$$|\nabla F| = \pm \infty \tag{3.0.6}$$

It is an open question whether non-differentiable functions exist in the real world [3]. At the current physical (10nm) and temporal (10ns) scale of modern computing, there exist no such functions, but modern computers are not equipped with the capability to accurately report the true value of their discrete approximations, so for all intents and purposes, programs implemented by most physical computers are discrete functions. Nevertheless, computers are capable of approximating bounded subsets of \mathbb{R}^n to arbitrary precision given enough time and space. For most ordinary applications, a fixed precision approximation is sufficient.

There exists at the heart of machine learning a theorem which states a simple class of functions, which compute a weighted sum of non-linear functions composed with a linear function, can approximate any bounded function on \mathbb{R}^m to arbitrary precision. More precisely, the universal approximation theorem[10] states that for all continuous functions $f: C(\mathbb{I}_m)$, where $\mathbb{I}_m = [0,1]^m$, there exists a function F, parameterized by constants $n \in \mathbb{N}, \beta, b \in \mathbb{R}^n, \epsilon \in \mathbb{R}^+$ and $W \in \mathbb{R}^{m \times n}$:

$$F(x) = \beta \varphi \left(W^T x + b \right)$$

$$\forall x \in I_m, |F(x) - f(x)| < \varepsilon$$
(3.0.7)

This theorem does not put an upper bound on the constant n, or how to find w, somewhat limiting its practical applicability. But for reasons which are not yet fully understood, empirical results suggest it is possible to obtain reasonably precise approximations to many naturally-arising functions, once thought to be intractable, in a comparatively short time by composing these non-linear and linear functions in an alternating fashion and iteratively updating w in the direction suggested by $\nabla_w F(x)$.

3.1. Static and dynamic languages

Most programs in machine learning and scientific computing are written in dynamic languages, such as Python. In contrast, most of the industry uses statically typed languages [14].

Dynamically typed languages are commonly used for experimentation and prototyping. But are they scalable to production systems?

According to some studies, type errors account for over 15% of bugs [6]. While the causal connections between statically typed languages in general and fewer is not widely established, types are often necessary to build more powerful static analyses and tools for program understanding.

Strong, static types are important for reasoning about the behavior of complex programs. Statically typed languages offer a number of benefits to users, such as eliminating of a broad class of runtime errors by virtue of the language alone. Furthermore, a carefully designed statically typed API can eliminate the potential for incorrect API usages by enforcing certain

usage patterns. Statically typed languages also provide several benefits for static code analysis, as tools can offer more relevant autocompletion suggestions, and provide early warnings for compile and probable runtime errors.

3.2. Imperative and functional languages

Most programs are written in procedural languages, due in part to the prevalence of the Von Neumann architecture. Lambda calculus provides an equivalent formalism for computing, and one which is more amenable to the automatic calculation of derivatives.

3.3. Kotlin

Kotlin is a strong, statically typed language. It is well suited for building cross-platform applications, with implementations in native, JVM, and JavaScript.

3.4. Kotlin ∇

Prior work has shown it is possible to encode a deterministic context-free grammar as a fluent interface [7] in Java. This result was strengthened to prove Java's type system is Turing complete [8]. As a practical consequence, we can use the same technique to perform shape-safe automatic differentiation (AD) in Java, using type-level programming. A similar technique is feasible in any language with generic types. We use Kotlin, whose type system is less expressive, but fully compatible with Java.

Differentiable programming has a rich history among dynamic languages like Python, Lua and JavaScript, with early implementations including projects like Theano, Torch, and TensorFlow. Similar ideas have been implemented in statically typed, functional languages, such as Haskell's Stalin ∇ [13], DiffSharp in F# [1] and recently Swift [16]. However, the majority of existing automatic differentiation (AD) frameworks use a loosely-typed DSL, and few offer shape-safe tensor operations in a widely-used programming language.

Existing AD implementations for the JVM include Lantern [15], Nexus [4] and DeepLearning.scala [2], however these are Scala-based and do not interoperate with other JVM languages. Kotlin ∇ is fully interoperable with vanilla Java, enabling broader adoption in neighboring languages. To our knowledge, Kotlin has no prior AD implementation. However, the language contains a number of desirable features for implementing a native

AD framework. In addition to type-safety and interoperability, Kotlin ∇ primarily relies on the following language features:

Operator overloading and infix functions allow a concise notation for defining arithmetic operations on tensor-algebraic structures, i.e. groups, rings and fields. λ -functions and coroutines support backpropagation with lambdas and shift-reset continuations, following [12] and [15]. Extension functions support extending classes with new fields and methods and can be exposed to external callers without requiring sub-classing or inheritance.

3.5. Usage

Kotlin ∇ allows users to implement differentiable programs by composing operations on fields to form algebraic expressions. Expressions are lazily evaluated inside a numerical context, which may imported on a per-file basis or lexically scoped for finer-grained control over the runtime behavior.

Listing 3.1. Simple code listing.

```
import edu.umontreal.kotlingrad.numerics.DoublePrecision
with(DoublePrecision) { // Use double-precision protocol
  val x = variable("x") // Declare immutable vars (these
  val y = variable("y") // are just symbolic constructs)
  val z = sin(10 * (x * x + pow(y, 2))) / 10 // Lazy exp
  val dz_dx = d(z) / d(x) // Leibniz derivative notation
  val d2z_dxdy = d(dz_dx) / d(y) // Mixed higher partial
  val d3z_d2xdy = grad(d2z_dxdy)[x] // Indexing gradient
  plot3D(d3z_d2xdy, -1.0, 1.0) // Plot in -1 < x,y,z < 1
}</pre>
```

Fig. 3.1. A basic Kotlin ∇ program with two inputs and one output.

Above, we define a function with two variables and take a series of partial derivatives with respect to each variable. The function is numerically evaluated on the interval (-1,1) in each dimension and rendered in 3-space. We can also plot higher dimensional manifolds (e.g. the loss surface of a neural network), projected into four dimensions, and rendered in three, where one axis is represented by time.

$$z = \sin\left(10(x*x+y^2)\right)/10, \, \operatorname{plot}(\tfrac{\partial^3 z}{\partial x^2 \partial y})$$

Fig. 3.2. Output generated by the program shown in Figure 3.1.

Kotlin ∇ treats mathematical functions and programming functions with the same underlying abstraction. Expressions are composed recursively to form a data-flow graph (DFG). An expression is simply a **Function**, which is only evaluated once invoked with numerical values, e.g. z(0, 0).

Listing 3.2. Simple code listing.

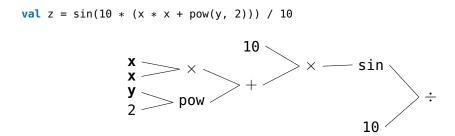


Fig. 3.3. Implicit DFG constructed by the original expression, z.

Kotlin ∇ supports shape-shafe tensor operations by encoding tensor rank as a parameter of the operand's type signature. By enumerating type-level integer literals, we can define tensor operations just once using the highest literal, and rely on Liskov substitution to preserve shape safety for subtypes.

Listing 3.3. Shape safe tensor addition for rank-1 tensors, $\forall L \leq 2$.

```
// Literals have reified values for runtime comparison
open class '0'(override val value: Int = 0): '1'(0)
open class '1'(override val value: Int = 1): '2'(1)
class '2'(open val value: Int = 2) // Greatest literal
// <L: '2'> will accept L <= 2 via Liskov substitution
class Vec<E, L: '2'>(len: L, cts: List<E> = listOf())
// Define addition for two vectors of type Vec<Int, L>
operator fun <L: '2', V: Vec<Int, L>> V.plus(v: V) =
Vec<Int, L>(len, cts.zip(v.cts).map { it.l + it.r })
// Type-checked vector addition with shape inference
val Y = Vec('2', listOf(1, 2)) + Vec('2', listOf(3, 4))
val X = Vec('1', listOf(1, 2)) + Vec('3') // Undefined!
```

It is possible to enforce shape-safe vector construction as well as checked vector arithmetic up to a fixed L, but the full implementation is omitted for brevity. A similar pattern can be applied to matrices and higher rank tensors, where the type signature encodes the shape of the operand at runtime.

With these basic ingredients, we have almost all the features necessary to build an expressive shape-safe AD, but unlike prior implementations using Scala or Haskell, in a language that is fully interoperable with Java, while also capable of compiling to JVM bytecode, JavaScript, and native code.

In future work, we intend to implement a full grammar of differentiable primitives including matrix convolution, control flow and recursion. While $Kotlin\nabla$ currently implements arithmetic manually, we plan to wrap a BLAS such as cuBLAS or native linear algebra library for performance.

3.6. Type system

Describing the Kotlin ∇ type system (formally).

3.7. Operator overloading

Operator overloading enables concise notation for arithmetic on abstract types, where the types encode algebraic structures, e.g. Group, Ring, and Field. These abstractions are extensible to other kinds of mathematical structures, such as complex numbers and quaternions.

For example, suppose we have an interface Group, which overloads the operators + and *, and is defined like so:

Listing 3.4. Simple code listing.

```
interface Group<T: Group<T>> {
    operator fun plus(addend: T): T
    operator fun times(multiplicand: T): T
}
```

Here, we specify a recursive type bound using a method known as F-bounded quantification to ensure that operations return the concrete type variable T, rather than something more generic like **Group**. Imagine a class **Expr** which has implemented **Group**. It can be used as follows:

Listing 3.5. Simple code listing.

```
fun <T: Group<T>> cubed(t: T): T = t * t * t
fun <E: Expr<E>> twiceExprCubed(e: E): E = cubed(e) + cubed(e)
```

Like Python, Kotlin supports overloading a limited set of operators, which are evaluated using a fixed precedence. In the current version of Kotlin, operators do not perform any computation, they simply construct a directed acyclic graph representing the symbolic expression. Expressions are only evaluated when invoked as a function. First-class functions

With higher-order functions and lambdas, Kotlin treats functions as first-class citizens. This allows us to represent mathematical functions and programming functions with the same underlying abstractions (typed FP). A number of recent papers have demonstrated the expressiveness of this paradigm for automatic differentiation.

In Kotlin ∇ , all expressions can be treated as functions. For example:

Listing 3.6. Simple code listing.

```
fun <T: Group<T>> makePoly(x: Var<T>, y: Var<T>) = x * y + y * y + x * x

val x: Var<Double> = Var(1.0)

val f = makePoly(x, y)
```

```
val z = f(1.0, 2.0) // Returns a value
println(z) // Prints: 7
```

Currently, it is only possible to represent functions where all inputs and outputs share a single type. In future iterations, it is possible to extend support for building functions with varying input/output types and enforce constraints on both, using covariant and contravariant type bounds.

3.8. Coroutines

Coroutines are a generalization of subroutines for non-preemptive multitasking, typically implemented using continuations. One form of continuation, known as shift-reset a.k.a. delimited continuations, are sufficient for implementing reverse mode AD with operator overloading alone (without any additional data structures) as described by Wang et al. in Shift/Reset the Penultimate Backpropagator and later in Backpropagation with Continuation Callbacks. Delimited continuations can be implemented using Kotlin coroutines and would be an interesting extension to this work. Please stay tuned!

3.9. Extension Functions

Extension functions augment external classes with new fields and methods. Via context oriented programming, Kotlin can expose its custom extensions (e.g. in DoublePrecision) to consumers without requiring subclasses or inheritance.

Listing 3.7. Simple code listing.

```
data class Const<T: Group<T>>(val number: Double) : Expr()

data class Sum<T: Group<T>>(val e1: Expr, val e2: Expr) : Expr()

data class Prod<T: Group<T>>(val e1: Expr, val e2: Expr) : Expr()

class Expr<T: Group<T>>: Group<Expr<T>> {
    operator fun plus(addend: Expr<T>) = Sum(this, addend)
    operator fun times(multiplicand: Expr<T>) = Prod(this, multiplicand)
}

object DoubleContext {
    operator fun Number.times(expr: Expr<Double>) = Const(toDouble()) * expr
```

}

Now, we can use the context to define another extension, Expr.multiplyByTwo, which computes the product inside a DoubleContext, using the operator overload we defined above:

Listing 3.8. Simple code listing.

```
fun Expr<Double>.multiplyByTwo() = with(DoubleContext) { 2 * this }
```

Extensions can also be defined in another file or context and imported on demand.

3.10. Algebraic data types

Algebraic data types (ADTs) in the form of sealed classes (a.k.a. sum types) allows creating a closed set of internal subclasses to guarantee an exhaustive control flow over the concrete types of an abstract class. At runtime, we can branch on the concrete type of the abstract class. For example, suppose we have the following classes:

Listing 3.9. Simple code listing.

```
sealed class Expr<T: Group<T>>: Group<Expr<T>>> {
   fun diff() = when(expr) {
       is Const -> Zero
       // Smart casting allows us to access members of a checked typed without explicit casting
       is Sum -> e1.diff() + e2.diff()
       // Product rule: d(u*v)/dx = du/dx * v + u * dv/dx
       is Prod -> e1.diff() * e2 + e1 * e2.diff()
       is Var -> One
       // Since the subclasses of Expr are a closed set, no 'else -> ...' is required.
   }
   operator fun plus(addend: Expr<T>) = Sum(this, addend)
   operator fun times(multiplicand: Expr<T>) = Prod(this, multiplicand)
}
data class Const<T: Group<T>>(val number: Double) : Expr()
data class Sum<T: Group<T>>(val e1: Expr, val e2: Expr) : Expr()
data class Prod<T: Group<T>>(val e1: Expr, val e2: Expr) : Expr()
class Var<T: Group<T>>: Expr()
```

```
class Zero<T: Group<T>>: Const<T>
class One<T: Group<T>>: Const<T>
```

Users are forced to handle all subclasses when branching on the type of a sealed class, as incomplete control flow will not compile (instead of say, failing silently at runtime).

Smart-casting allows us to treat the abstract type Expr as a concrete type, e.g. Sum after performing an is Sum check. Otherwise, we would need to write (expr as Sum).e1 in order to access its field, e1. Performing a cast without checking would throw a runtime exception, if the type were incorrect. Using sealed classes helps avoid casting, thus avoiding ClassCastExceptions. Multiple Dispatch

In conjunction with ADTs, Kotlin also uses multiple dispatch to instantiate the most specific result type of applying an operator based on the type of its operands. While multiple dispatch is not an explicit language feature, it can be emulated using inheritance.

Building on the previous example, a common task in AD is to simplify a graph. This is useful in order to minimize the number of calculations required, or to improve numerical stability. We can eagerly simplify expressions based on algebraic rules of replacement. Smart casting allows us to access members of a class after checking its type, without explicitly casting it:

Listing 3.10. Simple code listing.

```
override fun times(multiplicand: Function<X>): Function<X> =
    when {
        this == zero -> this
        this == one -> multiplicand
        multiplicand == one -> this
        multiplicand == zero -> multiplicand
        this == multiplicand -> pow(two)
        this is Const && multiplicand is Const -> const(value * multiplicand.value)
        // Further simplification is possible using rules of replacement
        else -> Prod(this, multiplicand)
    }

val result = Const(2.0) * Sum(Var(2.0), Const(3.0))
// = Sum(Prod(Const(2.0), Var(2.0)), Const(6.0))
```

This allows us to put all related control flow on a single abstract class which is inherited by subclasses, simplifying readability, debugging and refactoring.

3.11. Shape-safe Tensor Operations

While first-class dependent types are useful for ensuring arbitrary shape safety (e.g. when concatenating and reshaping matrices), they are unnecessary for simple equality checking (such as when multiplying two matrices).* When the shape of a tensor is known at compile time, it is possible to encode this information using a less powerful type system, as long as it supports subtyping and parametric polymorphism (a.k.a. generics). In practice, we can implement a shape-checked tensor arithmetic in languages like Java, Kotlin, C++, C or Typescript, which accept generic type parameters. In Kotlin, whose type system is less expressive than Java, we use the following strategy.

First, we enumerate a list of integer type literals as a chain of subtypes, so that $0 <: 1 <: 2 <: 3 <: \dots <: C$, where C is the largest fixed-length dimension we wish to represent. Using this encoding, we are guaranteed linear growth in space and time for subtype checking. C can be specified by the user, but they will need to rebuild this project from scratch.

Listing 3.11. Simple code listing.

```
open class '0'(override val i: Int = 0): '1'(i) { companion object: '0'(), Nat<'0'> }
open class '1'(override val i: Int = 1): '2'(i) { companion object: '1'(), Nat<'1'> }
open class '2'(override val i: Int = 2): '3'(i) { companion object: '2'(), Nat<'2'> }
open class '3'(override val i: Int = 3): '4'(i) { companion object: '3'(), Nat<'3'> }
//...This is generated
sealed class '100'(open val i: Int = 100) { companion object: '100'(), Nat<'100'> }
interface Nat<T: '100'> { val i: Int } // Used for certain type bounds
```

Kotlin ∇ supports shape-safe tensor operations by encoding tensor rank as a parameter of the operand's type signature. Since integer literals are a chain of subtypes, we need only define tensor operations once using the highest literal, and can rely on Liskov substitution to preserve shape safety for all subtypes. For instance, consider the rank-1 tensor (i.e. vector) case:

Listing 3.12. Simple code listing.

```
infix operator fun <C: '100', V: Vec<Float, C>> V.plus(v: V): Vec<Float, C> =
```

```
Vec(length, contents.zip(v.contents).map { it.first + it.second })
```

This technique can be easily extended to additional infix operators. We can also define a shape-safe vector initializer by overloading the invoke operator on a companion object like so:

Listing 3.13. Simple code listing.

```
open class Vec<E, MaxLength: '100'> constructor(
    val length: Nat<MaxLength>,
    val contents: List<E> = listOf()
) {
    operator fun get(i: '100'): E = contents[i.i]
    operator fun get(i: Int): E = contents[i]

    companion object {
        operator fun <T> invoke(t: T): Vec<T, '1'> = Vec('1', arrayListOf(t))
        operator fun <T> invoke(t0: T, t1: T): Vec<T, '2'> = Vec('2', arrayListOf(t0, t1))
        operator fun <T> invoke(t0: T, t1: T, t2: T): Vec<T, '3'> = Vec('3', arrayListOf(t0, t1, t2))
        //...
}
```

The initializer may be omitted in favor of dynamic construction, although this may fail at runtime. For example:

Listing 3.14. Simple code listing.

```
val one = Vec('3', 1, 2, 3) + Vec('3', 1, 2, 3)  // Always runs safely
val add = Vec('3', 1, 2, 3) + Vec('3', listOf(t)) // May fail at runtime
val vec = Vec('2', 1, 2, 3)  // Does not compile
val sum = Vec('2', 1, 2) + add  // Does not compile
```

A similar syntax is possible for matrices and higher-rank tensors. For example, Kotlin can infer the shape of multiplying two matrices, and will not compile if their inner dimensions do not match:

Listing 3.15. Simple code listing.

```
// Inferred type: Mat<Int, '4', '4'>
val l = Mat('4', '4',
    1, 2, 3, 4,
   5, 6, 7, 8,
    9, 0, 0, 0,
    9, 0, 0, 0
)
// Inferred type: Mat<Int, '4', '3'>
val m = Mat('4', '3',
    1, 1, 1,
    2, 2, 2,
    3, 3, 3,
    4, 4, 4
)
// Inferred type: Mat<Int, '4', '3'>
val lm = l * m
// m * m // Does not compile
```

Further examples are provided for shape-safe matrix operations such as addition, subtraction and transposition.

A similar technique is possible in Haskell, which is capable of a more powerful form of type-level computation, type arithmetic. Type arithmetic makes it easy to express convolutional arithmetic and other arithmetic operations on shape variables (say, splitting a vector in half), which is currently not possible, or would require enumerating every possible combination of type literals.

Many less powerful type systems are still capable of performing arbitrary computation in the type checker. As specified, Java's type system is known to be Turing Complete. It may be possible to emulate a limited form of dependent types in Java by exploiting this property, although this may not computationally tractable due to the practical limitations noted by Grigore.

3.12. Differentiable programming

The renaissance of modern deep learning is widely credited to progress in three research areas: algorithms, data and hardware. Among algorithms, most research has focused on deep learning architectures and representation learning. Equally important, arguably, is the role that automatic differentiation (AD) has played in facilitating the implementation of these ideas. Prior to the adoption of general-purpose AD libraries like Theano, PyTorch and TensorFlow, gradients needed to be programmed manually. The emergence of these and other libraries specifically tailored to deep learning simplified and accelerated the pace of machine learning, allowing researchers to focus on gradient-based optimization, network architectures and learning representations. Some of these ideas in turn, formed the basis for new methods in automatic differentiation, which continues to be an active area of research.

In deep learning, research primarily focuses on differentiable representations. In contrast, many problems we would like to solve are non-differentiable in their natural domain. For example, the structure of character-based representations are not easily differentiable as small changes to a word's symbolic representation can cause large differences to its semantic meaning. A key insight from deep learning research is that many problems require posing a good representation. For example, if we can represent words as a vector of real numbers, so that relations between words are spatially preserved by the vector representation, then we can perform gradient descent on such a space. Many classes of discrete problems can be relaxed to continuous surrogates by introducing a good representation.

As more domains found efficient representations, researchers observed neural networks were part of a broader class of differentiable architectures that could be built and interpreted in a manner not unlike computer programming [1], [2]. Hence the term differentiable programming was born.

Math	Infix	Prefix	Postfix	Туре
A + B	a + b a.plus(b)	plus(a, b)		$(a:\mathbb{R}^{\tau}\to\mathbb{R}^{\pi},b:\mathbb{R}^{\lambda}\to\mathbb{R}^{\pi})\to(\mathbb{R}^{?}\to\mathbb{R}^{\pi})$
A - B	a - b a.minus(b)	minus(a, b)		$(a:\mathbb{R}^{ au} o\mathbb{R}^{\pi},b:\mathbb{R}^{\lambda} o\mathbb{R}^{\pi}) o(\mathbb{R}^{?} o\mathbb{R}^{\pi})$
AB	a * b a.times(b)	times(a, b)		$(a:\mathbb{R}^{\tau}\to\mathbb{R}^{m*n},b:\mathbb{R}^{\lambda}\to\mathbb{R}^{n*p})\to(\mathbb{R}^?\to\mathbb{R}^{m*p})$
$\frac{\frac{A}{B}}{AB^{-1}}$	a / b a.div(b)	div(a, b)		$(a:\mathbb{R}^{\tau}\to\mathbb{R}^{m*n},b:\mathbb{R}^{\lambda}\to\mathbb{R}^{p*n})\to(\mathbb{R}^?\to\mathbb{R}^{m*p})$
-A		-a	a.unaryMinus()	$(a:\mathbb{R}^ au o\mathbb{R}^\pi) o(\mathbb{R}^ au o\mathbb{R}^\pi)$
+A		+a	a.unaryPlus()	
A+1	a + 1	++a	a++, a.inc()	$(a:\mathbb{R}^ au o\mathbb{R}^{m*m}) o(\mathbb{R}^ au o\mathbb{R}^{m*m})$
A-1	a - 1	- a	a-, a.dec()	
sin(a)		sin(a)	a.sin()	
cos(a)		cos(a)	a.cos()	$(a:\mathbb{R} o \mathbb{R}) o (\mathbb{R} o \mathbb{R})$
tan(a)		tan(a)	a.tan()	
$\ln(A)$		ln(a)	a.ln()	$(a:\mathbb{R}^ au o\mathbb{R}^{m*m}) o(\mathbb{R}^ au o\mathbb{R}^{m*m})$
		log(a)	a.log()	
$\log_b A$	a.log(b)	log(a, b)		$(a:\mathbb{R}^{\tau}\to\mathbb{R}^{m*m},b:\mathbb{R}^{\lambda}\to\mathbb{R}^{m*m})\to(\mathbb{R}^?\to\mathbb{R})$
A^b	a.pow(b)	pow(a, b)		$(a:\mathbb{R}^{\tau}\to\mathbb{R}^{m*m},b:\mathbb{R}^{\lambda}\to\mathbb{R})\to(\mathbb{R}^?\to\mathbb{R}^{m*m})$
\sqrt{a}	a.pow(1.0/2)	a.pow(1.0/2)	a.sqrt()	$(a:\mathbb{R}^ au o\mathbb{R}^{m*m}) o(\mathbb{R} o\mathbb{R}^{m*m})$
sqrt[3](a)	a.root(3)	a.root(3)	a.cbrt()	
$\frac{\frac{da}{db}}{a'(b)}$	a.diff(b)	grad(a)[b]	d(a) / d(b)	$(\mathbf{a}:C(\mathbb{R}^m)^*,\mathbf{b}:\mathbb{R}\to\mathbb{R})\to(\mathbb{R}^m\to\mathbb{R})$
∇a		grad(a)	a.grad()	$(a:C(\mathbb{R}^m)^*) o (\mathbb{R}^m o \mathbb{R}^m)$

Chapitre 4

Verification and validation

How do we test a machine that learns?

4.1. Adversarial test case generation

Neural networks and differentiable programming has provided a powerful new set of optimization tools for training learning algorithms. However these methods are often brittle to small variations in the input space, and have difficulty with generalization. In contrast, these same techniques used for probing the failure modes of neural networks can be applied to adversarial test case generation for traditional programs.

4.2. Background

Suppose we have a program $P: \mathbb{R} \to \mathbb{R}$ where:

$$P(x) = p_n \circ p_{n-1} \circ p_{n-2} \circ \dots \circ p_1 \circ p_0 \tag{4.2.1}$$

From the chain rule of calculus, we know that:

$$\frac{dP}{dp_0} = \prod_{i=1}^{n} \frac{dp_i}{dp_{i-1}} \tag{4.2.2}$$

Imagine a single test $T: \mathbb{R} \to \mathbb{B}$. Consider the following example::

$$T: \forall x \in (0,1), P(x) < C \tag{4.2.3}$$

How can we find a set of inputs that break the test under a fixed computational budget (i.e. constant number of program evaluations)? In other words:

$$D_T: \{x^i \sim \mathbb{R}(0,1) \mid P(x^i) \implies \neg T\}, maximize|D_T|$$
 (4.2.4)

If we have no information about the program implementation or its input distribution, D_P , we can do no better than random search (cf. no free lunch). However, if we know something about the input distribution, we could re-parameterize the distribution to incorporate our knowledge. Assuming the program has been tested on common inputs, we might consider sampling $x \sim \frac{1}{D_P}$ for inputs that are infrequent. If we knew how P were implemented, we could prioritize our search in input regions leading towards internal discontinuities (e.g. edge cases in software testing). However for functions that are continuous and differentiable, these heuristics are almost certainly insufficient.

Another strategy, independent of how candidate inputs are selected, is to use some form of gradient based optimization in the search procedure. For example in (3) we could have a loss function:

$$\mathcal{L}(P,x) = C - P(x) \tag{4.2.5}$$

The gradient of the loss w.r.t. x (assuming P is fixed¹) is:

$$\nabla \mathcal{L}(P, x) = -\frac{dP}{dx} \tag{4.2.6}$$

Where the vanilla gradient update step is defined as:

$$x_{n+1}^i = x_n^i - \alpha \frac{dP}{dr} \tag{4.2.7}$$

We hypothesize that if the implementation of P were flawed and a counterexample to (3) existed, as sample size increased, a subset of gradient descent trajectories would fail to converge, a portion would converge to local minima, and a subset of trajectories would discover inputs violating the program specification. How would such a search procedure look in practice? Consider the following algorithm:

4.3. Regression testing: a tool to prevent catastrophic forgetting

An endemic problem in modern deep learning is the problem of forgetting. In order to combat this issue, we turn to a classic software testing tool: regression testing.

Regular regression testing gives clear diagnostics about the behavior of intelligent systems.

¹In contrast with backpropagation, where the parameters are updated.

```
Input: Program P, specification T, evaluation budget Budget
Output: D_T, the set of inputs which cause P to fail on T
D_T = [];
evalCount = 0;
while evalCount \leq Budget do
   sample candidate input x^i according to selection strategy S;
   if P(x^i) \implies \neg T then
   | append x^i to D_T
   else
     x_n^i = x^i; while n \leq C \wedge evalCount \leq Budget \wedge \neg converged do
   end
          evalCount++;
    end
    |evalCount++; i++;
   end
```

Algorithm 1: Algorithm for finding test failures. First select a candidate input x^i according to sampling strategy S (e.g. uniform random, or a neural network which takes P and T as input). If $P(x^i)$ violates T, we can append x^i to D_T and repeat. Otherwise, we follow the gradient of $\mathcal{L}(P,x)$ with respect to x and repeat until test failure, gradient descent convergence, or a fixed number of G.D. steps C are reached before resampling x^{i+1} from the initial sampling strategy S to ensure each G.D. trajectory will terminate before exhausting our budget.

Software reproducibility

In this chapter, we will discuss the challenges of software reproducibility and how best practices in software engineering like continuous integration and delivery can help researchers mitigate the variability associated with building and running software. Docker [11] is one technical solution we will discuss. Our work is roughly related to determinism, and does not consider the variability associated with training data and statistical variability.

One of the challenges of building intelligent systems and programming at large, is the problem of reproducibility. Software reproducibility has a number of subtle aspects, including hardware compatibility, operating systems, file systems, build reproducibility, and runtime determinism. While at one point in the history of computing, it may have been sufficient to write down a program and feed it directly into a computer, the source code of a modern program is far too removed from its mechanical implementation to be meaningfully executed in isolation. Today's handwritten programs are like the schematics for a traffic light. Built by compilers, interpreted by virtual machines, communicating with other programs, running inside an operating system, they are essentially meaningless abstractions without a factory to build them, a city-worth of infrastructure, cars, and traffic laws.

As necessary in any good schematic, much of the information required to build a program is divided into layers of nested abstractions. Most of the instructions executed by a computer between when a programmer starts a program and the output becomes visible are not intended to be read by the programmer, and have long since been automated and forgotten. In a modern programming language like Java, C# or Python, the total information required to compile and run a simple program numbers in the trillions of bits. A portion of the data pertains to the software for building and running programs, including the build

system, software dependencies, and development tools. Part of the data pertains to the operating system, firmware, drivers, and embedded software. And for most programs, such as those found in a typical GitHub repository, a vanishingly small fraction corresponds to the handwritten program itself.

Researchers would often like to reproduce the work of other researchers, but the mental effort of re-implementing their abstractions can be tedious and detrimental towards scientific progress. Similarly in programming, it is often necessary to utilize programs written by other developers, and would be convenient if we had tools for reproducibility and incremental development. This is the same problem software developers have been attempting to solve for many years, via the open source community. But source control alone is insufficient, since they are primarily intended to store text. While text-based representations may be stable for a time, as software is updated and rebuilt, important details about the original development environment can be misplaced. To reproduce the program in its original environment, we need a snapshot of all digital information present on the computer at the time of its execution. Short of that, minimum required information to run a program is required.

5.1. Operating systems and virtualization

In 2006, Linux introduced a kernel feature called cgroups. This enabled a kind of light-weight virtualization for containers, where a fully virtualized machine was no longer necessary to get many of the benefits of VMs. This paved the way for the tools that are now known as containers. Unlike VMs, containers do not run their own operating systems, but rather share resources with the host operating system, while remaining isolated from the host and sibling containers. While VMs heavy resource requirements often limits their deployment to server-class hardware, containers' lightweight footprint enabled them to run on a far broader class of mobile and embedded platforms.

5.2. Dependency management

Packages are one source of variability. Dependency hell is NP Complete: https://research.swtch.com/version-sat

5.3. Containerization

One of the challenges of distributed software development across heterogeneous platforms is the problem of variability. With the increasing pace of software development comes the added burden of software maintenance. As hardware and software stacks evolve, so too must source code be updated to build and run correctly. Maintaining a stable and well documented codebase can be a considerable challenge, especially in a robotics setting where contributors are frequently joining and leaving the project. Together, these challenges present significant obstacles to experimental reproducibility and scientific collaboration.

In order to address the issue of software reproducibility, we developed a set of tools and development workflows that draw on best practices in software engineering. These tools are primarily built around containerization, a widely adopted virtualization technology in the software industry. In order to lower the barrier of entry for participants and minimize variability across platforms (e.g. simulators, robotariums, Duckiebots), we provide a state-of-the-art container infrastructure based on Docker, a popular container engine. Docker allows us to construct versioned deployment artifacts that represent the entire filesystem and to manage resource constraints via a sandboxed runtime environment.

The Duckietown platform supports two primary instruction set architectures: x86 and ARM. To ensure the runtime compatibility of Duckietown packages, we cross-build using hardware virtualization to ensure build artifacts can be run on all target architectures. Runtime emulation of foreign artifacts is also possible, using a similar technique. For performance and simplicity, we only build ARM artifacts and use emulation where necessary (e.g., on x86 devices). On ARM-native, the base operating system is HypriotOS, a light-weight Debian distribution with built-in support for Docker. For both x86 and ARM-native, Docker is the underlying container platform upon which all user applications are run, inside a container.

Docker containers are sandboxed runtime environments that are portable, reproducible and version controlled. Each environment contains all the software dependencies necessary to run the packaged application(s), but remains isolated from the host OS and file system. Docker provides a mechanism to control the resources each container is permitted to access,

¹For more information, this technique is described in further depth at the following URL: https://www.balena.io/blog/building-arm-containers-on-any-x86-machine-even-dockerhub/.

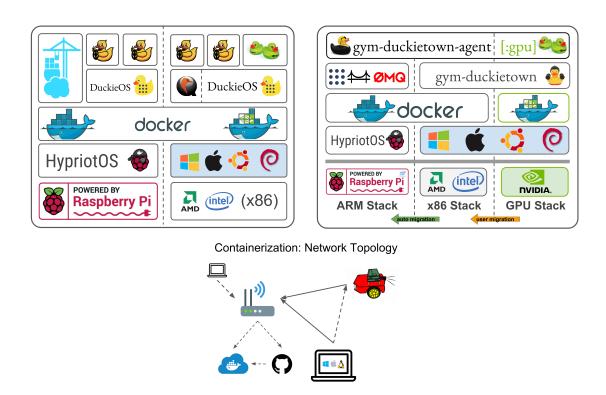


Fig. 5.1. AI-DO container infrastructure. Left: The ROS stack targets two primary architectures, x86 and ARM. To simplify the build process, we only build ARM artifacts, and emulate ARM on x86. Right: Reinforcement learning stack. Build artifacts are typically trained on a GPU, and transferred to CPU for evaluation. Deep learning models, depending on their specific architecture, may be run on an ARM device using an Intel NCS.

and a separate Linux namespace for each container, isolating the network, users, and file system mounts. Unlike virtual machines, container-based virtualization like Docker only requires a lightweight kernel, and can support running many simultaneous containers with close to zero overhead. A single Raspberry Pi is capable of supporting hundreds of running containers.

While containerization considerably simplifies the process of building and deploying applications, it also introduces some additional complexity to the software development lifecycle. Docker, like most container platforms, uses a layered filesystem. This enables Docker to take an existing "image" and change it by installing new dependencies or modifying its functionality. Images may be based on a number of lower layers, which must periodically be updated.

Care must be taken when designing the development pipeline to ensure that such updates do not silently break a subsequent layer as described earlier in Sec. ??.

One issue encountered is the matter of whether to package source code directly inside the container, or to store it separately. If source code is stored separately, a developer can use a shared volume on the host OS for build purposes. In this case, while submissions may be reproducible, they are not easily modified or inspected. The second method is to ship code directly inside the container, where any changes to the source code will trigger a subsequent rebuild, effectively tying the sources and the build artifacts together. Including source code alongside build artifacts also has the benefit of reproducibility and diagnostics. If a competitor requires assistance, troubleshooting becomes much easier when source code is directly accessible. However doing so adds some friction during development, which has caused competitors to struggle with environment setup. One solution is to store all sources on the local development environment and rebuild the Docker environment periodically, copying sources into the image.

5.4. Docker and ROS

Prior work in Dockerization of ROS [17]

Chapitre 6

Case study: application for autonomous robotics

As a case study in the feasibility of these tools, we implement a case study in building a mobile application using ROS, Docker, Android and sHere, we implement a mobile application using the above toolchain.

6.1. Design

Designed with Hatchery.

6.2. Implementation

Implementation includes Kotlin ∇

6.3. Verification

Verified using property-based testing.

6.4. Maintenance

Deployed and CI-tested using Docker.

Chapitre 7

Conclusion

7.1. Future work

7.1.1. Requirements Engineering

Often it is not possible, or desirable to summarize the performance of a complex system using a single variable. In multi-objective optimization, we have the notion of pareto-efficiency...

Traditional software engineering has followed a rigorous process model and testing methodology. This model has guided the development of traditional software engineering, intelligent systems will require a re-imagining of these ideas to build systems that adapts to its environment during operation. Intelligent systems are designed with objective functions, which are typically one- or low-dimensional metrics for evaluating the performance of the system. Most often, these take the form of a single criteria, such as an *error* or *loss* which can represent descriptive phenomena such as latency, safety, energy efficiency or any number of objective measures.

For example, in the design of a web based advertisement recommendation system, we can optimize for various objectives such as click rate, engagement, sales conversion. So long as we can measure these parameters, with today's powerful function approximators, we can optimize for any singly criterion or combination thereof. Much of the work involved in machine learning is to find representations which are amenable to learning, and preventing unintended consequences. For example, by optimizing for click rate, we create an artificial market for click bots. Similarly, in self driving cars, we often want to optimize for passenger

safety. However by doing so naively, we create a vehicle that never moves, or always yields to nearby vehicles.

When building intelligent system developers must first ask, what are the requirements of the system? This process is often the most troublesome part, because the requirements must not be fuzzy specifications like traditional software engineering, but precise, programmable directives. "I would like the system to be fast," is not sufficiently precise. These kinds of requirements must be translated into statistical loss functions, so we must be very precise about how we specify our requirements. If we simply say, "The system must produce a valid response as quickly as possible, in less than 100ms," is better, but leaves open the possibility of returning an empty response.

In traditional software engineering, it is reasonable to assume the people who are implementing the system have some implicit knowledge and are generally well-intentioned human beings working towards the same goal. When building an intelligent system, a more reasonable assumption is that the entity implementing our requirements is a naive but powerful genie, and possibly an adversary. If we are to give it an optimization metric, it will take every conceivable shortcut to achieve that metric. If we are not careful about requirements engineering, this entity can produce a system that does not work, or has unintended consequences.

In the strictest sense, designing a good set of requirements is indistinguishable from implementing the system. With the right language abstractions (e.g. declarative programming), requirements and implementation can be the same thing. These ideas have been explored in recent decades with languages like SQL and Prolog. While these are toy systems, neural networks can express much larger classes of functions than traditional software engineering.

7.1.2. Continuous Delivery and Continual Learning

An overall trend in software and systems engineering is the transition away from long development cycles towards continuous integration and deployment. Development teams in the industry are encouraged to iterate in a series of short sprints between feature development and deployment. In some cases, software is shipped to users on a nightly basis, with automated testing and deployment. Similarly, intelligent systems have a need to continuously adapt to their environment, and will change their code on an even shorter basis.

Incremental updates will grow increasingly smaller, until the program starts to alter itself after every input it processes.

We need tools to more effectively harness the stochasticity of these learning systems.

7.1.3. Developers, Operations, and the DevOps toolchain

Software engineers have begun to realize the value of bespoke tools that facilitate the process of shipping software, in addition to the software itself.

Teams building software are cybernetic systems, and require meta-programs for building code and organizational processes which enable them to ship code more efficiently.

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