Same-Day Delivery with a Heterogeneous Fleet of Drones and Vehicles

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Abstract

In this paper, we analyze how drones can be combined with regular delivery vehicles to improve same-day delivery performance. To this end, we present a dynamic vehicle routing problem with heterogeneous fleets. Customers order goods over the course of the day. These goods are delivered either by a drone or by a regular transportation vehicle within a delivery deadline. Drones are faster, but have a limited capacity as well as require charging after use. In the same-day context, vehicle capacity is not a constraint, but vehicles are slow due to urban traffic. To decide whether an order is delivered by a drone or by a vehicle, we present a policy function approximation based on geographical districting. Our computational study reveals two major implications. First, geographical districting is highly effective increasing the expected number of same-day deliveries. Second, a combination of drone and vehicle fleets may significantly reduce the required delivery resources.

Keywords: stochastic dynamic vehicle routing, same-day delivery, drone delivery, heterogeneous fleets

1 Introduction

Same-day delivery (SDD) is a powerful tool for online retailers to increase sales (Wahba 2015). In SDD, customers order goods online and receive them the same day. SDD is convenient because customers can order online and do not need to go to the store and wait in lines. Further, customers often receive their purchases within a few hours. Thus, SDD narrows the gap of instant-gratification between online and brick-and-mortar shopping (Anderson 2015). As a result, SDD is experiencing high two-digit growth rates per year (Yahoo! Finance 2016).

While a source of growth, offering SDD leads to significant cost challenges for service providers (Ram 2015). Conventional last-mile delivery already causes the majority of overall delivery costs (Bernau et al. 2016). These costs are increased by the dynamic customer orders and the simultaneous delivery within the deadlines associated with SDD (Ulmer to appear). In many cases, in SDD, dispatchers are unable to consolidate many orders, and delivery vehicles deliver only a few packages per trip to customers often widely spread within the delivery area. In particular, given the slow travel speeds within the city, delivery in areas distant from the depot are costly.

To overcome the cost challenges of SDD, companies have begun to incorporate drones into their SDD operations (Kim 2016). Drones have the advantage that they enable fast and direct delivery from the depot to a customer regardless the traffic conditions. The downside is that drones can transport only one item per trip and require recharging or a battery swap afterward (Conner-Simons 2014). Thus, drones may not be able to entirely replace conventional delivery vehicles, particularly when volumes are high (Wang 2016).

This research analyzes whether and how a combination of road-based delivery vehicles (hereafter referred to as "vehicle") and drones may reduce the required delivery costs and increase the number of customers served in SDD operations. The resulting problem is the *same-day delivery routing problem with heterogeneous fleets of drones and vehicles* (SDDPHF). During a shift, a fleet of vehicles and a fleet of drones deliver goods from a depot to customers. These customers make requests during the shift and are unknown before the time of their order. For each ordering customer, the provider must decide whether or not the order can be served on the same day and whether a vehicle or a drone performs the delivery. If SDD is offered, either a drone is loaded and sent to the customer or a vehicle picks up the order at the depot and delivers it within the delivery deadline. The objective is to maximize the expected number of customers served with SDD.

One main question for the SDDPHF is whether to use a drone or a vehicle for delivery. The question of drone or vehicle needs to account for the impact of the immediate decision on our ability to meet as yet unknown future requests. To do so in problems of this scale where the state and actions spaces of a dynamic programming model grow exponentially, researchers often turn to the techniques of Approximate Dynamic Programming (ADP). ADP seeks to overcome the challenges of exponential state and actions spaces while providing high quality solutions.

In our case, we seek to identify a good heuristic decision making policy. We take advantage of the intuition that, generally, vehicles may be suitable in downtown areas close to the depot and with high customer density while using drones may be beneficial for more distant suburban areas with widely dispersed customers. Based on this idea, to facilitate decision making, we use an ADP approach known as parametric policy function approximation (PFA). In a PFA, one usually seeks to determine the best values for a parameterized policy. In our case, the parameter is a threshold of a vehicle's travel time from the depot. This threshold splits the service area into two zones. Customers in the zone within the threshold are preferably served by vehicles, and customers outside of this threshold are preferably served by drones. Because the threshold for the PFA is determined via offline simulations, it is highly runtime efficient and can offer immediate responses to customers.

This work makes several important contributions to the SDD literature. It is the first to explore the addition of drones to vehicle fleets for the SDD, a business model being considered by companies such as Amazon. Second, this paper is the first to present a delivery scheme for drone delivery operations, answering the call in Otto et al. (to appear) that says that ". . . future research should work out dynamic planning schemes for a range of relevant drone operations." Third, we present a comprehensive Markov decision process model for a complex dynamic vehicle routing problem. Fourth, for a large set of generated and real-world instance settings, our PFA increases the number of same-day services significantly compared to other PFA benchmark policies. We also compare our PFA to a rollout approach and demonstrate that the PFA performs comparably with much less computational effort and much greater transparency. In addition, we use the PFA to evaluate how the combination of vehicles and drones significantly reduce the required delivery resources compared to SDD performed by only drones or by only vehicles. We show that this result holds even when drones are restricted to serving only a subset of the customers orders.

We also derive two main managerial insights:

1. Partitioning the service region in areas preferably served by drones and in areas preferably served by vehicles, which we call geographic districting, improves the overall number of potential services significantly. This is particularly the case if drones are used to serve customers in less populated areas and distant to the depot while vehicles serve populated areas close to the depot.

2. The combination of drones and delivery vehicles has the potential to reduce the required delivery resources. Our results show that, given a fleet solely of vehicles, the addition of only one or two drones to a delivery vehicle fleet can replace the productivity of one vehicle. Further, for a fleet of only drones, adding a single vehicle can replace up to four drones.

The paper is organized as follows. In §2, we present the related literature focusing on same-day delivery routing and routing with heterogeneous fleets. In §3, we present a problem statement and model the SDDPHF as Markov decision process. The PFA and the benchmark policies are defined in §4. We present the test instances and the computational evaluation in §5. The paper concludes with a summary and an outlook.

2 Literature Review

There has been a growing literature exploring how drones can be beneficial in a variety of transportation related contexts. Examples include parking lot utilization (Coifman et al. 2006), traffic management (Huiyuan et al. 2007), traffic monitoring (Chow 2016), and the estimation of origin-destination matrices (Braut et al. 2012). In this literature review, we first discuss the drone literature related to routing. Then, we review the literature related to the problem domain discussed in this paper, same-day delivery, and then present work on dynamic routing applications involving heterogeneous fleets.

2.1 Overview

Table 1 summarizes the related literature. The first part of the table presents literature from drone routing, the second part of the table presents the literature on the SDD, and the third part of the table presents dynamic routing with heterogeneous fleets. The table summarizes each paper with respect to the application, scientific problem model, and solution method. The table uses the following classification:

SDD The paper addresses a same-day delivery problem.

Drones The paper considers drones.

Dynamism The model allows subsequent adaption of decisions.

Table 1: Literature Classification

		Application		Problem Model			Solution Approach	
	Literature	SDD	Drones	Dynamism	Requests	Heterogeneity	Anticipation	Offline
Drones	Murray and Chu (2015)		✓			✓	n/a	n/a
	Ferrandez et al. (2016)		✓			✓	n/a	n/a
	Agatz et al. (2016)		✓			✓	n/a	n/a
	Ha et al. (2016)		✓			✓	n/a	n/a
	Ponza (2016)		✓			✓	n/a	n/a
	Wang et al. (2016)		✓			✓	n/a	n/a
	Dorling et al. (2017)		✓			✓	n/a	n/a
	Tavana et al. (2017)		✓			✓	n/a	n/a
	Poikonen et al. (2017)		✓			✓	n/a	n/a
	Poikonen et al. (to appear)		✓			✓	n/a	n/a
SDD	Azi et al. (2012)	✓		✓	✓		✓	
	Klapp et al. (2018)	✓		✓	✓		✓	
	Klapp et al. (2016)	✓		✓	✓		✓	
	Ulmer et al. (2016)	✓		✓	✓		✓	✓
	Grippa et al. (2017)	✓	✓	✓	✓			
	Ulmer (2017b)	✓		✓	✓		✓	✓
	Voccia et al. (to appear)	✓		✓	✓		✓	
Heterogeneity	Goel and Gruhn (2006)			✓	✓	✓		
	Attanasio et al. (2007)			✓	✓	(✓)	✓	
	Bullo et al. (2011)		(✓)	✓	✓	✓		
	Tirado et al. (2013)			✓	✓	✓	✓	
	Ferrucci and Bock (2014)			✓	✓	✓		
	Agra et al. (2015)			(✓)		✓	✓	n/a
	Schyns (2015)			✓		✓	✓	
	Wang and Kopfer (2015)			✓	✓	✓		
	Basilico et al. (2016)		(✓)	✓	✓	✓		
	Pillac et al. (2018)			✓	✓	✓		
	Zhang et al. (2018)			✓		✓		
	SDDPHF, PFA	✓	✓	✓	✓	✓	✓	✓

Requests The model incorporates stochastic requests.

Heterogeneity The model considers different types of vehicles.

Anticipation The applied method incorporates information about potential future developments. Anticipation is only applicable in dynamic decision problems.

Offline The majority of calculation to achieve anticipation is conducted offline in a learning or training phase. Thus, the required real-time runtime is relatively short.

In some cases such as Attanasio et al. (2007) or Bullo et al. (2011), a binary classification is not possible. We indicate these cases with "(✓)". In the following, we describe the papers in detail.

2.2 Routing with Drones

In this section of the literature review, we focus on work combining conventional vehicles with drones for delivery. For a recent survey on drone routing, the interested reader is referred to Otto et al. (to appear). None of the papers cited in Otto et al. (to appear) or subsequently in this paper include the stochasticity and dynamism of the SDDPHF.

Murray and Chu (2015) introduces the idea combining drones and vehicles. The authors present a (static) traveling salesman problem (TSP) with "sidekick," where the sidekick is a drone carried on a delivery vehicle and the drone is sent to serve customers while the vehicle is on the road. Similar problems are considered in Ferrandez et al. (2016), Agatz et al. (2016), Ha et al. (2016), Ponza (2016), Dorling et al. (2017), Poikonen et al. (2017), and Poikonen et al. (to appear). Murray and Chu (2015) further introduce the parallel drone scheduling TSP (PDSTSP). In the PDSTSP, a subset of customers is served by a vehicle while the remaining customers are served by drones. The PDSTSP can be viewed as static version of the SDDPHF in which all customers are known a priori. Wang et al. (2016) consider an extension of the PDSTSP in which there is a fleet of trucks with potentially multiple sidekicks. The authors present worst-case results for the deterministic problem. Tavana et al. (2017) considers a variant in which drones are used for direct shipping while all other shipments must move through a fleet of over-the-road vehicles and cross docks.

The existing methods for drone routing problems draw on mixed-integer programming and/or metaheuristics to minimize the travel durations. In stochastic dynamic routing problems and the case in this paper, not only do the solution methods necessarily differ, but so do the objectives. In stochastic dynamic problems, time becomes a resource. The objectives seek to maximize some measure of service, which requires effectively using time (Ulmer 2017a, pp.28).

2.3 Same-Day Delivery

In SDD, customer requests are uncertain and occur while the vehicles are already on the road. To serve new customers, the vehicles need to return to the depot to pick up the ordered goods before they can serve the new customer requests. This leads to dynamic vehicle routing problems with depot returns. Work on these problems is relatively scarce, but is increasing with the proliferation of companies offering SDD. Azi et al. (2012) and Voccia et al. (to appear) consider fleets of vehicles and propose solution methods based on the multiple-scenario approach (Bent and Van Hentenryck 2004). Both Klapp et al. (2018) and Klapp et al. (2016) present a dynamic routing problem where a single vehicle delivers good from a depot to requesting customers. They apply a rollout algorithm of an a-priori policy to determine the customers to serve and when to leave the depot. Ulmer et al. (2016) also considers decision making for a single vehicle, but introduces an offline ADP approach based on state-space aggregation.

All of the listed SDD papers consider homogeneous vehicles or fleets. In contrast, this paper is the first to consider heterogeneous fleets in for SDD. Further, this paper is the first to explore the impact of the addition of drones to a fleet for a dynamic vehicle routing problem. In addition, as a result of the combinatorics associated with the heterogeneous fleet, the decision making is more complicated than that found in the literature for SDD. For this reason, we turn to PFA. The PFA allows us to heuristically solve the assignment portion of the problem without computationally expensive online simulations. Most of the approaches for the SDD existing in the literature evaluate the quality of decisions using simulation. Except in the case of Ulmer et al. (2016), these simulations are run at the time of decision making or online. We show that such methods provide only limited improvement in solution quality relative to the PFA while greatly increasing computation. We also show that our PFA can be combined with such an approach.

Grippa et al. (2017) explore same-day delivery exclusively using drones and analyze the system as an $M/G/K$ queue with independent service times. The paper also computationally evaluates the performance of two simple heuristic assignment policies: first-come-first-served and nearest neighbor. These policies do not allow for assignment decisions between vehicles and drones as is necessary in this paper nor do they anticipate future requests.

We note that Ulmer (2017b) explores the pricing of incentives in same-day delivery. As part of the solution method, Ulmer (2017b) introduces a PFA that helps set basis prices for the incentives. The PFA in Ulmer (2017b) and that presented in this paper are similar only in the sense that they are parameterized policies for which the best parameter setting is found by simulation. Further, Ulmer (2017b) models only homogeneous vehicles and does not explore issues associated with assignments in a heterogeneous fleet.

2.4 Dynamic Routing with Heterogeneous Fleets

The presented problem is also related to dynamic vehicle routing with heterogeneous fleets. For heterogeneous fleet problems, vehicles differ in attributes such as travel speed or capacity, and the question arises as to which customers to serve with which type of vehicle. The work on dynamic routing with heterogeneous fleets is limited, and none of the work considers same-day delivery. In addition, unlike the heterogeneous vehicle routing literature, particularly that concerned with maritime and road vehicles, that largely follows traditional routing approaches with additional constraints to account for the fleet heterogeneity, our solution approach is designed specifically to exploit the heterogeneity between drones and vehicles. Notably, drones serve a single customer at a time, but at a higher rate of speed than vehicles. This difference inspires our geographical districting policy that we demonstrate is an effective means of solving the problem.

The vehicle routing literature includes a number of examples of the dynamic routing of heterogeneous fleets of road-based vehicles. Important to the work in this paper, none of the existing work involving heterogeneous fleets of road-based vehicles incorporates information about future requests. The work relies on reoptimization approaches. Goel and Gruhn (2006) present a problem in which trucks serve customers and a percentage of customers are unknown. Trucks differ in their capacity, speed, and travel costs. Goel and Gruhn (2006) present a large neighborhood search to update the routing when new customers request. Ferrucci and Bock (2014) consider a dynamic pickup and delivery problem in which customer requests are dynamically integrated into existing routes. These routes are improved by means of Tabu search algorithms. Ferrucci and Bock (2014) consider different vehicle types varying in speed and capacity. Wang and Kopfer (2015) address a problem in which service providers collaborate to fulfill customer requests in a dynamic pickup and delivery problem. The fleets of the various providers differ in the routing costs. The authors optimize the routing based on current information and analyze the value of receiving order information earlier. Schyns (2015) address the problem of dynamically routing refueling trucks on an airport. Trucks differ in their capacity, speed, and compatibility to different airplanes. Stochasticity can be experienced in the demand and the time windows of the airplanes. The authors apply an ant colony optimization approach based on current information. Chen et al. (2016) and Pillac et al. (2018) consider reoptimization approaches to a technician routing problem in which the heterogeneous fleet result from variation among technicians' skills or technicians' vehicles carrying different sets of spare parts.

We are aware of one paper in the dynamic routing of road vehicle fleets that anticipates future requests. Attanasio et al. (2007) present a dynamic pickup and delivery problem with stochastic requests and homogeneous vehicles, but describe how the methods can be extended to address heterogeneous vehicles. The authors consider information about future requests through repositioning idling vehicles based on real-time forecasts of potential new requests.

Maritime routing is a common application area for dynamic routing of heterogeneous fleets. In contrast to most of the literature on heterogeneous fleets of trucks, the maritime literature does seek to incorporate information about the future. Similar to the problem, Tirado et al. (2013) considers a heterogeneous fleet of ships serving dynamic requests. The authors propose multiple-scenario planning and a branch-and-regret approach. Both of these solution approaches incorporate information about future requests, but do so in an online manner. Such methods are attractive when there is enough time to run the computation before making a decision. In the SDD, such time is not available which is why this paper proposes a PFA. We also note that rollout algorithm extension of the PFA discussed in Section 5.4 can be compared to the methods proposed in Tirado et al. (2013). As we show in Section 5.4, the proposed PFA performs well relative to such methods and can be used in conjunction with such methods.

Other maritime-related work includes Zhang et al. (2018) and Agra et al. (2015). Zhang et al. (2018) implicitly account for travel disruptions by adding constraints to build "flexibility" into the routes of a heterogeneous fleet of ships. Agra et al. (2015) also consider the routing of ships under disruption, but use a stochastic program to solve the problem. Thus, the sequence of visits is not updated as new information is learned. However, the stochastic program accounts for variability in future outcomes.

The application of dynamic routing of heterogeneous fleets also arises in literature related to the routing of robots. An important difference between the problem studied in this paper and those in the area of dynamic routing heterogeneous robot fleets is that we seek to solve a transient problem whereas the robot fleet literature seek to understand steady-state performance. Bullo et al. (2011) consider the problem of routing robots to serve remote locations. In the problems considered, the requests arrive dynamically and are served by heterogeneous robotic drones. Bullo et al. (2011) model the problem as queues and provide performance bounds in light and heavy traffic. Basilico et al. (2016) examines the performance of robotic drones that patrol a given area. The robots are of multiple types. The authors also model the problems as queuing problems and analytically evaluate performance.

3 Problem Description and Model

In this section, we present the mathematical model of the SDDPHF. Because the problem is dynamic and stochastic, we model it as a Markov decision process. Because the MDP for the SDDPHF is complex, we first present a problem narrative and give an illustrative example of a decision state. We then present the MDP-model. Table 3 summarizes the notation associated with both the problem statement and the model.

3.1 Problem Statement

In the SDDPHF, a fleet of m vehicles $\mathcal{V} = \{v_1, \dots, v_m\}$ and a fleet of n drones $\mathcal{D} = \{d_1, \dots, d_n\}$ dynamically deliver goods from the depot N to customers who dynamically request service in the area $\mathfrak{A}$ during a shift $T = [0, t_{\text{max}}]$.

Fleet Characteristics

The vehicles and drones are initially located at the depot. Both drones and vehicles can make multiple return trips to the depot. The two types of fleets differ in several aspects. These aspects are summarized in Table 2. Drones and vehicles differ with respect to their availability, capacity, their requirement for charging, their travel speed, and the network on which they operate.

Due to working hour restrictions, the vehicles need to return to the depot at the end of the shift. Further, the capacity of a drone is one. Thus, drones can serve only a single customer before returning to the depot. Alternatively, given the small size of most of the delivered items (Guglielmo 2013), we model the vehicle as uncapacitated. Thus, vehicles are capable of delivering many customers before returning to the depot.

The time required for a drone to travel between a customer C and the depot D is deterministic and given by the functions $\tau_D(C, D)$ or $\tau_D(D, C)$, respectively. For vehicles, the time required to travel between customers C_1 and C_2 (and/or the depot) is deterministic and given by the function

Table 2: Differences in Fleet Characteristics

Characteristics	Drones	Vehicles
Working Hours	no	yes
Capacity	one	uncapacitated
Charging	yes	no
Speed	fast	slow
Network	direct	road

$\tau_V(C_1, C_2)$. Similarly, the time required for a vehicle to travel between a customer C and the depot D is deterministic and given by the functions $\tau_V(C, D)$ or $\tau_V(D, C)$, respectively. We specify different travel time functions for drones and vehicles to reflect that drones typically travel straight line distances and travel at higher speeds than vehicles. The vehicles on the other hand must travel on road networks and do not travel as fast as drones.

Each package loaded onto a drone requires t_N^D units of time to be loaded. Each drone needs t_C^D time to drop off a package at a customer. The loading time for a vehicle at the depot is t_N^V regardless the number of packages. The service time of a vehicle at a customer is t_C^V. When the drones return to the depot after a delivery, they must charge for t_D units of time. This time may also represent the time required to swap batteries. Vehicles do not have this limitation.

At the same time, it is important that we address flight range limitations. Such limitations have important roles in much of the existing drone routing research. However, even he first drones developed by Amazon and JD.com ahad a flight range of 15 to 20 miles (Lavars 2015, Wingfield and Scott 2016). Such a range is suitable to allow out and back travel in the medium-sized cities in which the authors live, Braunschweig, Germany, and Iowa City, United States. In fact, it is suitable for many larger cities such as Hamburg, Munich, and Paris. Thus, in contrast to other drone applications, in this work, we do not consider flight range as a limiting factor.

Most importantly, the limitation on drone travel distance is not the key question of interest in this paper. In this work, we study a problem inspired by Amazon, undoubtedly the most influential player in retailing in this generation. We make assumptions that fit this interesting new business model, offer a method for solving the problem, and study characteristics of the solution approach.

Customer Orders

During the shift, customers request goods. The requests follow a known spatial-temporal probability distribution, but the customers and their locations are unknown before the request. The time at which a customer C requests is denoted as $t(C)$. Requests can be rejected. Whenever a request is made, the dispatcher needs to respond within seconds as to whether the request can be accepted for service. Accepted orders must be delivered within a hard deadline of $\bar{\delta}$ units of time after $t(C)$. We denote the delivery deadline of customer C as $\delta(C)$. In our same-day delivery problem, we ignore requests once they have been rejected.

Assignments and Routing

For every new request, the dispatcher first determines whether or not to accept the request for same-day delivery. If the request is accepted, the dispatcher must then decide whether to assign the request to a drone or a vehicle. Once made, the assignment of an order to a vehicle or drone is permanent. If the request is assigned to a drone, the dispatcher selects the drone that will provide service. When the drone is available (having possibly served another request and been recharged), it delivers the good to the customer and returns for recharging.

If an order is assigned to a vehicle, the dispatcher decides which vehicle and where in the chosen vehicle's route the order will be delivered. The dispatcher also determines any time that the vehicle will spend waiting at the depot between executing back-to-back tours. For this purpose, the dispatcher maintains and updates a set of planned routes Θ.

A planned route $\theta(v) \in \Theta$ for vehicle v stores information about the time and sequence of customers that are planned to be served by vehicle v. We refer to the routes as "planned" routes as the routes are iteratively updated when new customer requests are accepted for service and assigned to vehicles. Routes are also updated over time to account for the vehicles traveling, serving customers, and returning to the depot. Because we do not allow the vehicle to return to the depot until all loaded packages are delivered, the planned route does not contain stops at loaded customers. It only contains information about depot visits, and the currently assigned but not loaded customers.

For the case in which the vehicle is currently serving customers, the planned route stores the time at which the vehicle will arrive at the depot, the time at which the vehicle is planned to start its next tour from the depot, and the planned sequence of customers the vehicle will visit on its next tour. As a result, each planned route contains at most two depot visits. One visit to load goods for the assigned customers and another visit when the vehicle is planned to return from the delivery tour for these customers. Whenever a vehicle visits the depot and starts a new delivery tour, the loaded customers are removed and the second depot visit becomes the first.

Formally, a planned route for vehicle v is represented by

$$\theta(v) = \left(\left(N_1^\theta, a(N_1^\theta) \to s(N_1^\theta)\right), \left(C_1^\theta, a(C_1^\theta)\right), \ldots, \left(C_h^\theta, a(C_h^\theta)\right), \left(N_2^\theta, a(N_2^\theta) \to t_{\text{max}}\right)\right).$$

The first entry of a route $\theta(v)$ represents the next depot visit N_1^θ, the arrival time of vehicle v at the depot $a(N_1^\theta)$, and the time at which the vehicle starts a new tour $s(N_1^\theta)$. The tour starts with the loading the goods for that tour. Because we do not allow the vehicles to pre-empt tours once a vehicle has left the depot, the return time $a(N_1^\theta)$ is known based on the time that the vehicle left for its current tour and the customers that were loaded onto the vehicle. Further, because we do not pre-empt the tours and thus once the a vehicle leaves the depot it will serve all loaded customers, we do not need to store any of the customers currently being served by a vehicle in the planned route. These customers do not impact any future decisions beyond the time at which the vehicle returns to the depot after serving them. A planned route is feasible if all arrival times at customers are not later than their deadlines and the difference between arrival times reflect loading, service, and travel times.

The first entry additionally contains information about the time at which the vehicle is planned to start loading new goods before departing on the tour, $s(N_1^\theta)$. The difference between $a(N_1^\theta)$ and $s(N_1^\theta)$ is the time the vehicle is currently planned to wait at the depot. The other entries represent the customers not yet loaded but assigned to the vehicle C_i^θ, $i = 1, \ldots, h$, and the according planned arrival times $a(C_i^\theta)$ to those customers. Waiting at customers is not permitted. The last entry represents the return to the depot at time $a(N_2^\theta)$ where the vehicle idles until a new tour is scheduled or the day ends, $s(N_2^\theta) = t_{\text{max}}$. If the dispatcher decides to serve the request by a vehicle, the dispatcher updates the route plans to integrate the new customer.

For the SDDPHF, the objective is to maximize the expected number of customers served the same day. Because the amount of time available for work is fixed, we assume that the driver costs are fixed and omit them from our objective.

Table 3: Problem Notation

Description	Notation
Service area	$\mathfrak{A}$
Depot	$N \in \mathfrak{A}$
Shift	$T = [0, t_{\text{max}}]$
Vehicles	$\mathcal{V} = \{v_1, \ldots, v_m\}$
Drones	$\mathcal{D} = \{d_1, \ldots, d_n\}$
Vehicle travel time function	$\tau_V(\cdot, \cdot)$
Drone travel time function	$\tau_D(\cdot, \cdot)$
Service time at customer for a vehicle	t_C^V
Service time at customer for a drone	t_C^D
Charging time at depot	t_D
Loading time at depot for a vehicle	t_N^V
Loading time at depot for a drone	t_N^D
Customers	$\mathcal{C}$
Request time of C	$t(C) \in T$
Allowed time between order and delivery	$\bar{\delta}$
Delivery deadline of a customer	$\delta(C) = t(C) + \bar{\delta}$
Current planned route of vehicle v	$\theta(v)$
Current availability time for drone d	$\mathcal{A}(d)$
Current arrival time at customer C (or depot)	$a(C)$
Current start of loading at depot N	$s(N)$
Current set of planned routes	$\Theta = \{\theta(v_1), \ldots, \theta(v_m)\}$
Current availability times	$\mathcal{A}(\mathcal{D}) = (\mathcal{A}(d_1), \ldots, \mathcal{A}(d_n))$

3.2 Illustrative Example

To illustrate the SDDPHF, Figure 1 presents the situation 2 hours (120 minutes) into an example shift. For the purpose of example, we assume a Manhattan-style grid with vehicle travel times of 20 minutes and drone travel times of 10 minutes per segment. To simplify this example, we further assume that drones and vehicles operate on the same grid. Though, in our computational study in §5, we assume that the drones fly on Euclidean paths while vehicles travel on an approximated road network.

For the purposes of the example, the depot is located in the center of the service area. Customers are represented by circles. We use dark circles to represent customers whose orders are already on a vehicle which is en route for delivery. We call such customers "loaded customers." Light circles represent customers who have placed an order, had the order accepted for delivery, but which

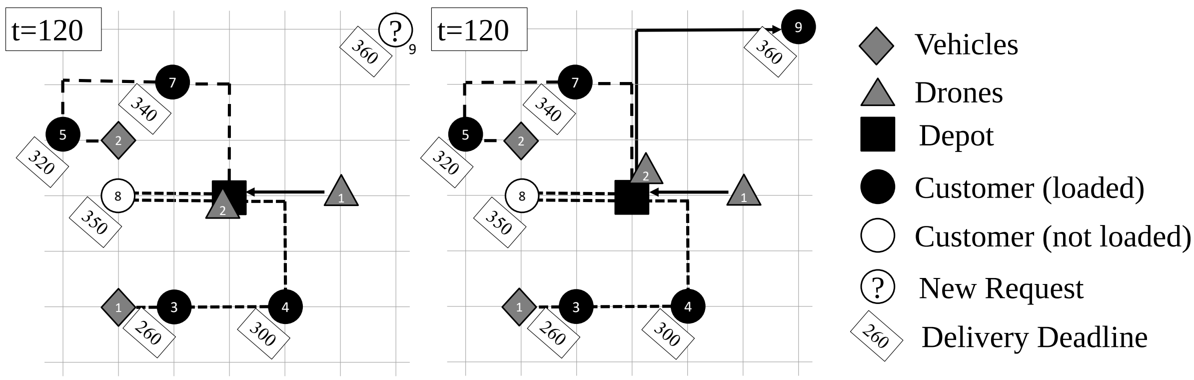

Figure 1: Decision State and Decision

are not yet loaded onto a vehicle. We call such customers "known, but not yet loaded." A new customer request in the upper right corner of the service area is represented by the question mark. The delivery deadline of each customer is represented by the adjacent box. We assume accepted orders must be delivered within 240 minutes of when the order is placed. For example, Customer 4 has a deadline of 300. Thus, the customer requested at time 60.

The left-hand panel of Figure 1, represents the situation just before the decision of whether to accept or reject the new customer request is made. The diamonds indicate vehicles and the triangles drones. In the example, two vehicles and two drones are given. Both vehicles are on the road. The current planned routes are indicated by the dashed lines. The small dashes represent the route for Vehicle 1. The large dashes show the route for Vehicle 2. We assume a delivery and a loading time of 10 minutes each for both drones and vehicles. We assume a charging time of 20 minutes. The first vehicle will serve Customer 3, then Customer 4, and then return to the depot at time $120 + 20 + 10 + 40 + 10 + 60 = 260$. The vehicle is then planned to directly load the goods for Customer 8 and start the next tour to serve Customer 8 at time $260 + 10 + 40 = 310$ before it returns to the depot at time $310 + 10 + 40 = 360$ and waits. The route plan for Vehicle 1 is defined as $\theta(v_1) = ((N, 260 \to 260), (8, 310), (N, 360 \to t_{\max}))$. Note that the plan does not contain the loaded customers, but only the next arrival at the depot. Vehicle 2 will serve Customer 5 at time 140 and Customer 7 at time 210 before returning to the depot at time 280 and waiting. The planned route for Vehicle 2 is therefore $\theta(v_2) = ((N, 280 \to t_{\max}))$. Both planned routes are feasible because all arrival times are lower than the according deadline and the vehicles arrive before the deadline assuming a shift of 12 hours.

Drone 2 is currently idling at the depot. Drone 1 served a customer and is currently on its way to the depot. Hence, Drone 1 will be available in time $120 + 10 + 10 + 20 = 140$.

The center panel of Figure 1 represents the situation just after the decision has been made to accept the new request. We assume that Customer 9 has been assigned to Drone 2. With this decision, the planned routes of the vehicles are unchanged. With this assignment, Drone 2 is next available at $120 + 10 + 120 + 10 + 20 = 280$. The vehicles and drones now follow their plans until the next customer requests.

3.3 Markov Decision Process Model

In this section, we present our Markov decision process model for the SDDPHF. An MDP captures the stochasticity and dynamism of a problem by modeling it as a sequence of states connected by decisions and transitions induced by an exogenous stochastic process.

Decision Point and State

In the SDDPHF, decisions are made when new customers request service. We call this time a decision point. We denote the time of the k^{th} decision point as t_k, noting $t_k = t(C_k)$ where customer C_k is the customer requesting service. The state S_k at the k^{th} decision point summarizes all of the information that is necessary to make feasible decisions, compute rewards, and determine transitions. For this problem, the state must summarize the information about the known, but not yet loaded, customer requests, the vehicles, and the drones. Because the route through all loaded customers is fixed for every vehicle, we can summarize the status of a vehicle out for delivery using only the next arrival of the vehicle to the depot. As such, we also do not need to store loaded customers in the state. Similarly, we can summarize all of the necessary information about customers assigned to drones when we compute each drone's next available time. Thus, we also do not store information about customers assigned to drones.

The state includes the time t_k of the current decision point and the new customer C_k that induced the decision point. The new request is represented by the location of the request. The state also includes the set $\mathcal{C}_k$ that stores the known, but not yet loaded, customers. Each element of the set includes the location of a customer request and its deadline. State S_k contains a set $\mathcal{A}_k = (\mathcal{A}_k(d_1), \ldots, \mathcal{A}_k(d_n))$ that indicates the time at which each drone, $d_1, \ldots, d_n$, will be ready for its next departure. Finally, the state includes the set of previously described planned routes Θ_k. The state at decision point k is then the tuple $S_k = (t_k, C_k, \mathcal{C}_k, \mathcal{A}_k, \Theta_k)$. As an example, the left-hand panel of Figure 1 is represented by the state in which $t_k = 120$, C_k is the location of Customer 9, $\mathcal{C}_k = \{\text{Customer } 8\}$ is the known, but not loaded customer. The state also contains the set $\mathcal{A}_k = \{160, 120\}$, and as stated in the example, the set of planned routes $\Theta_k = \{\theta(v_1) = ((N, 260 \to 260), (8, 310), (N, 360 \to t_{\text{max}})), \theta(v_2) = ((N, 280 \to t_{\text{max}}))\}$.

For the initial state S_0, we assume $t_0 = 0$, $\mathcal{C}_0$ is empty, $\mathcal{A}_0(d) = 0$ for every drone $d_1, \dots, d_n$, and $\theta_0(v) = ((N, 0 \to t_{\text{max}}))$ for every vehicle $v_1, \dots, v_m$.

Decisions and Reward

The decision at decision point k is to determine whether or not to accept the new request for service, and if accepting the request, to assign it to a vehicle or drone, and to determine the resulting routing update or planned time of arrival.

We can represent a decision at decision point k as the tuple $x_k = (\alpha_k, \Theta_k^x, \mathcal{A}_k^x)$. The decision α_k represents whether or not a request is accepted, and if so, whether it is assigned to a vehicle or drone. The acceptance and assignment decision α_k takes the following values:

$$\alpha_k = \begin{cases} 0 & \text{if order is rejected,} \\ 1 & \text{if order is assigned to vehicle,} \\ 2 & \text{if order is assigned to drone.} \end{cases} \tag{1}$$

Based on the acceptance decision α_k, either the arrival time vector or the routing (and set $\mathcal{C}_k$) is updated. We use $\mathcal{A}_k^x$ to represent $\mathcal{A}_k$ after the update has been made, Θ_k^x to represent the updated set of planned routes Θ_k, and $\mathcal{C}_k^x$ to represent $\mathcal{C}_k$ following the update. We call these updated values post-decision values. The updates are:

$\boldsymbol{\alpha_k = 0}$**:** If the order is rejected, $\mathcal{A}_k^x = \mathcal{A}_k$, $\Theta_k^x = \Theta_k$, and $\mathcal{C}_k = \mathcal{C}_k^x$.

$\boldsymbol{\alpha_k = 1}$**:** If the customer C_k is assigned to a vehicle, the set of known, but not yet loaded, customers is updated to $\mathcal{C}_k^x = \mathcal{C}_k \cup \{C_k\}$. Further, the set of planned routes Θ_k is updated to Θ_k^x incorporating C_k. An update of the planned routes is feasible if the following six conditions hold:

1. The set of planned routes in Θ_k^x contains all customers in $\mathcal{C}_k^x$.

2. The planned arrival times for each customer $C \in \mathcal{C}_k^x$ is not later than the deadline $a(C) \leq \delta(C)$.

3. The difference between the arrival times of two consecutive customers in each route θ is the sum of travel time and service time.

4. The start of loading at the depot $s(N_1^\theta)$ is not earlier than the arrival time $a(N_1^\theta)$: $a(N_1^\theta) \leq s(N_1^\theta)$.

5. The difference between the beginning of loading for the next tour $s(N_1^\theta)$ at the depot and the arrival time at the next customer is the sum of travel time and loading time.

6. The vehicles must arrive at the depot before the end of the shift, $a(N_2^\theta) \leq t_{\text{max}}$.

$\boldsymbol{\alpha_k = 2}$**:** If the customer C_k is assigned to a drone, $\mathcal{A}_k$ is updated to $\mathcal{A}_k^x$. This update reflects the assignment of an order to a specific drone and extends the availability time of this drone. Mathematically, the update of $\mathcal{A}_k^x$ is feasible if the following three conditions hold:

1. If the customer is assigned to drone d^*, the customer's deadline is met: $\mathcal{A}_k(d^*) + t_N^D + \tau_D(N, C) \leq t_k + \bar{\delta}$.

2. The availability time $\mathcal{A}_k(d^*)$ for the respective drone is increased to $\mathcal{A}_k^x(d^*) = \mathcal{A}_k(d^*) + t_N^D + \tau_D(N, C) + \tau_D(C, N) + t_C^D + t_D$.

3. All the other availability times remain constant: $\mathcal{A}_k^x(d) = \mathcal{A}_k(d) \, \forall \, d \neq d^*$.

We note that the conditions for feasibility when $\alpha_k = 1$ allow for the vehicle to wait at the depot.

The combination of a state S_k and a decision leads to a known post-decision state S_k^x containing the point of time t_k, the set of customers $\mathcal{C}_k^x$, the set of planned routes of the vehicles Θ_k^x, and the availability vector of the drones $\mathcal{A}_k^x$. The reward of a decision is

$$R(S_k, x_k) = \begin{cases} 0 & \text{if } \alpha_k = 0 \text{ and} \\ 1 & \text{otherwise.} \end{cases} \tag{2}$$

In the example described in the center panel of Figure 1, a decision is being made at the 9^{th} decision point, and the decision is to assign customer C_9 to Drone 2. Thus, $\alpha_9 = 2$, and

$\mathcal{A}_9^x = \{280, 120\}$. The post-decision planned routes are unchanged, and $\Theta_9^x = \Theta_9$. The reward of the decision is $R(S_9, x_9) = 1$.

Exogenous Information and Transition

Once the decision is taken, the vehicles and drones proceed with their planned routes until a new customer request is revealed by an exogenous stochastic process. This process provides the $(k+1)^{st}$ customer request C_{k+1} at time t_{k+1}.

The transition from the post-decision state S_k^x to the next pre-decision state S_{k+1} results from the new requests C_{k+1}. The time of the $(k+1)^{st}$ decision is $t_{k+1} = t(C_{k+1})$, the time at which the new request C_{k+1} arrived. The transition updates the routing plans and the set of customers as follows:

1. In the case that no vehicle has returned to the depot between t_k and t_{k+1}, the planned routes are the same as in the previous post-decision state and thus $\Theta_{k+1} = \Theta_k^x$.

2. If a vehicle v returned to the depot between t_k and t_{k+1} and already started the next trip ($s(N_1^\theta) \leq t_{k+1}$), the customers in the corresponding route $\theta_k(v) \in \Theta_k^x$ are removed from $\mathcal{C}_k^x$ in transition to $\mathcal{C}_{k+1}$ and $\theta_{k+1}(v)$ only contains the next depot return and the respective arrival time.

3. If a vehicle returned to the depot and is currently waiting at the depot ($a(N_1^\theta) \leq t_{k+1} \wedge s(N_1^\theta) > t_{k+1}$), $a(N_1^\theta)$ is set equal to t_{k+1}.

4. In the case that a vehicle finished its route of loaded customers as well as any remaining visits in the planned route between t_k and t_{k+1}, the vehicle idles at the depot, and the route is set $\theta_k(v)$ to $(N_1^\theta, t_{k+1} \rightarrow t_{\text{max}})$.

The arrival time for each drone is set to $\mathcal{A}_{k+1}(d) = \max\{\mathcal{A}_k(d), t_{k+1}\}$. The MDP terminates in state S_K: $t_K = t_{\text{max}}$ with $\theta_K(v) = (N_1^\theta, t_{\text{max}} \rightarrow t_{\text{max}})$ for all vehicles.

Objective Function

A solution for the SDDPHF is a policy $\pi \in \Pi$ assigning a decision to each state. The optimal policy for the SDDPHF maximizes the total expected reward, and the objective can be expressed as

$$\pi^* = \underset{\pi \in \Pi}{\arg\max}\, \mathbb{E}\left[\sum_{k=0}^{K} R(S_k, X_k^{\pi}(S_k))|S_0\right], \tag{3}$$

where $X_k^{\pi}(S_k))$ is the decision taken at decision point k when in state S_k and using policy π.

4 Solution Method

Solving Equation (3) for the SDDPHF is challenging due to the "Curses of Dimensionality" (Powell 2011). The state space is vast because states differ in time, customer locations, vehicle routes, and drone availability times. The decision space is large because each decision consists of assignment and routing. Finally, the information space is vast because customers can request at every point of time at every location in the service area.

A common way to overcome these challenges is through the methods of ADP. A well known example of such a method is value-function approximation (VFA). Such methods are able to generate information about the states and the random transitions by simulation. They are particularly good for overcoming large state spaces and even information spaces. However, these methods do not address the issue of dimensionality in the decision space.

As a result, we turn to an alternative to VFA, and instead focus on PFA. PFA solves the MDP by focusing on a restricted policy class for the problem. As Powell (2011, pp. 232) states, PFAs are suitable if the problem is complex, but "we have a very good idea of how to make a decision, and we can design a function (i.e., a policy) that returns a decision which captures the structure of the problem." In our case, we assume that drones may be suited to deliver orders to dispersed customers in suburban areas. These customers may be isolated, and there may not be the customer density to make delivery by vehicles efficient. We further assume that vehicles may be more effective if they deliver goods in regions with greater customer density, usually in the downtown area of the city. In this section, we present the details of our solution approach derived from this intuition. We also present the details of the assignment and routing heuristic that we use in conjunction with the PFA.

4.1 Policy Function Approximation

To implement that intuition described above, we propose a PFA π^{PFA} parameterized by a threshold τ. A customer C with a vehicle's travel time longer than this threshold ($\tau_D(N, C) > \tau$) is preferably

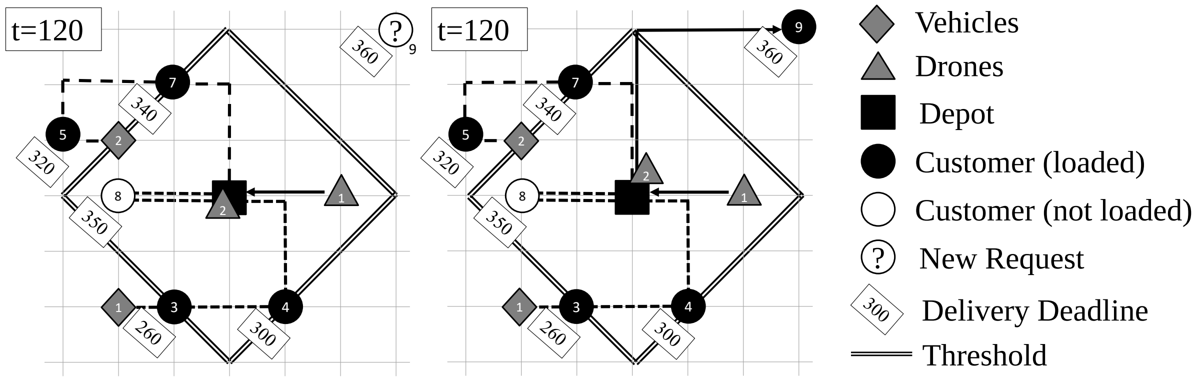

Figure 2: Example for a Threshold Policy

served by drones while a customer with a lower travel time ($\tau_D(N, C) \leq \tau$) is preferably served by a vehicle. We write "preferably" because it is not always feasible to serve a customer with the preferred mode. In the case that a customer can be feasibly served by only a truck or only a drone, the customer is assigned to the feasible option regardless the threshold.

Figure 2 shows the example state presented in Figure 1 and a travel time threshold of $\tau = 60$ minutes. The threshold is indicated by the double line. Requests within this threshold are preferably served by vehicles and customers with longer travel duration by drones. Customer C_5 is such an example. The customer lies outside the threshold, but is assigned to a vehicle. This occurs because no drone is available to serve the customer, but the customer can be feasibly served by a vehicle. In the example, the new request lies outside this threshold and can be feasibly served by a drone. Therefore, the new customer is assigned to a drone.

We define the set of threshold-dependent PFA policies as

$$\widehat{\Pi} = \{\pi_\tau^{\text{PFA}} : 0 \leq \tau\}. \tag{4}$$

To determine the best threshold parameter for a specific instance setting, we use sample average approximation using common random numbers and enumeration (Fu 2015). Specifically, we define a set of potential threshold values. Each potential threshold value defines a policy π, and the set of all candidate policies is $\widehat{\Pi}$. For each instance setting, we then estimate the expected value in Equation (3) for each policy π using H sample path realizations $\omega^1, \dots, \omega^H$. For each instance, we select the PFA leading to the highest average rewards. We denote this policy as π^{PFA}. Mathematically, the optimization for a given instance setting is

$$\pi^{\text{PFA}} = \arg\max_{\pi \in \widehat{\Pi}} \frac{\sum_{i=1}^{H} \left[\sum_{k=0}^{K} R(S_k, X_k^{\pi}(S_k)) | S_0, \omega^i \right]}{H}. \tag{5}$$

For our computational study, we define the potential threshold values minute-by-minute ($\tau = 0, 1, \ldots, 50$) and evaluate each policy using $H = 1000$ trials.

4.2 Routing and Assignment Procedure

As described in §3.3, a decision contains two components, the mode of transportation and the resulting routing updates. While the threshold generally determines the mode for each customer, we simultaneously need to determine a potential assignment and routing update for each mode to check feasibility. Further, the mode determined by the threshold is sometimes not feasible. In that event, we check feasibility with regard to the alternate mode, and as long as there is one feasible option for serving a customer, we accept the customer. These assignment and feasibility decisions can be time consuming, and we employ heuristics to facilitate the real-time decision making necessary in same-day delivery.

We serve customers assigned to drones in a first-in-first-out (FIFO) manner. Thus, we assign a customer to the first drone that becomes available at the depot. Specifically, we assume that the new request is assigned to the drone with the earliest availability. Let that drone be d. Then, we update the availability of d as $\mathcal{A}_k^x(d) = \mathcal{A}_k(d) + t_N^D + \tau_D(N, C) + \tau_D(C, N) + t_C^D + t_D$. If two or more drones are available, we arbitrarily choose one. In the event that the availability of the drones precludes serving C by $\delta(C)$, then C cannot be served by a drone.

For vehicles, we require a fast assignment and routing heuristic because decisions need to be made in real-time. For this purpose, we extend the efficient insertion heuristic presented in Azi et al. (2012). Recall that a route θ in a decision state contains the next depot visit, the set of known, but not yet loaded customers to serve, and a final depot visit. The planned route is expressed as:

$$\theta = ((N_1^\theta, a(N_1^\theta) \rightarrow s(N_1^\theta)), (C_1^\theta, a(C_1^\theta)), \ldots, (C_h^\theta, a(C_h^\theta)), (N_2^\theta, a(N_2^\theta) \rightarrow t_{\text{max}})).$$

For a new customer C, the insertion heuristic applies the following assignment and routing procedure. In the case that a vehicle is currently free and idles at the depot, the routing heuristic selects this vehicle for delivery. If no vehicle is idling, the routing heuristic iterates through all planned routes and determines the feasible route allowing for the "cheapest" insertion. In that context, the procedure may add a subsequent depot visit to a vehicle's routes for any vehicle without customers in θ but still on the road. To this end, we evaluate the cost of inserting the new customer in each position in each route at the position and choose the point of insertion that leads to the smallest increase in travel time. After this insertion, the heuristic updates the arrival times and checks whether all customers of this route are served before their deadline. The feasible route with the smallest extension is selected, and the customer inserted. All other routes remain unaltered. In the case that no feasible route can be found, the customer cannot be served by a delivery vehicle. We omit waiting in our heuristic because previous work indicates that the benefit of waiting is small, particularly when the rate of requests is relatively high (Ulmer et al. 2016, Voccia et al. to appear). Thus, we set $s(N_1^\theta) = a(N_1^\theta)$.

In the case that the mode determined by the threshold is not feasible, we check the alternate mode for a newly requesting customer. In the case of a customer who cannot be served by a drone, we run the just described vehicle routing heuristic to determine whether or not the customer can be inserted in one of the vehicle routes. If not, we reject the customer request. Similarly, in the case of a customer who cannot be feasibly served by a vehicle, we evaluate whether or not there exists a feasible assignment to a drone, again maintaining the FIFO ordering of the drone assignments. If not, we reject the customer.

5 Computational Evaluation

In this section, we present our computational study. First, we describe the instance settings and introduce a set of benchmark PFA policies. We also discuss the benefits and challenges of improving the proposed threshold-based PFA by rollout algorithms. We then compare the solution quality of the policies. Further, we analyze the impact of combining the two delivery fleets highlighting how a combination may result in more efficient deliveries. Finally, we show how the results change in the case that not all customers can be served by drones.

5.1 Instance Settings

In the following, we describe the instance settings. We assume that deliveries occur from 8 am to 8 pm. The time limit is therefore $t_{\text{max}} = 12$ hours. Requests are accepted only until 4 pm, and all orders must be delivered within 4 hours.

The vehicles travel at a speed of 30 km per hour. We assume that vehicles travel on a road network. For all of the geographies, we approximate the distances of a road network using the method described in Boscoe et al. (2012). Using the method, we first determine the point-to-point Euclidean distances for each pair of customers and customers and the depot. We then multiply the resulting Euclidean distances by 1.5 to approximate the effect of a street network. Boscoe et al. (2012) determine the approximation factor using a sample of over 66,000 locations in cities around the United States. We calculate the travel time based on this distance and the vehicle speed.

As in Murray and Chu (2015), we assume drones travel “as the crow flies” or rather that drones travel the Euclidean distance between points. We assume that drones travel at a speed of 40 km per hour. We note that this speed is conservative compared to the 80 km per hour that Amazon expects from its drone fleet (Lavars 2015). Thus, as assumed in Agatz et al. (2016), the net speed of drones is twice as high as for vehicles. For both vehicles and drones, the service and loading times are each 3 minutes. The setup and charging time for the drone is 20 minutes. This assumption follows Grippa et al. (2017), but we use a slightly more conservative estimation to account for the arrival, landing, and dispatching process.

We test the policies for 300, 400, 500, and 800 expected orders. We assume that requests follow a homogeneous Poisson process. We set the load limit of drones to 5 pounds, the weight under which Amazon is believes a package can be served by drone (Guglielmo 2013). Because only a small percentage of e-commerce parcels weights more than 5 pounds, unless otherwise noted, we assume that drones are able deliver all parcels (Guglielmo 2013).

We test two different customer geographies. The first geography is generated to reflect the structure of a medium- to large-sized city with a dense city center and sparse locations in more suburban areas outside of the city center. The x and y coordinates of the orders are independent and identically Normally distributed with the depot at the center center. The standard deviation of each coordinate is 3.0 km. Thus, about 50% of the orders lay within 10 minutes vehicle travel time from the depot, about 95% of the orders lie within 20 minutes, and about 99.9% of the orders lie within

30 minutes or 10 km Euclidean distance. Further, given these values, all customers are within the flight radius of already existing drones (Lavars 2015, Wingfield and Scott 2016).

The second customer geography is generated from real-world data for Iowa City, Iowa, in the United States. Iowa City is a medium-sized city with with nearly 170,000 residents in its metro area (U.S. Census Bureau 2017). The data includes about 32,000 potential locations. The distribution of the locations is depicted in Figure 3. For the purpose of presentation, we depict only one out of every hundred customers with each black circle representing a location. We observe a highly heterogeneous customer distribution with three distinct regions: the suburb of Coralville in the northwest, the suburb University Heights in the southwest, and Iowa City in the east. The depot is not central, but located in the north central part of the map, next to Interstate 80. It is indicated by the red square. The maximum Euclidean distance between the depot and every potential customer is below 10 km. As noted in Section 5, this distance is far below the flight range of commercial delivery drones.

For each customer distribution and number of expected orders, we vary the fleet size for drones between 1 and 20 and for vehicles between 1 and 5. This leads to a set of 800 different instance settings. For each instance setting, after selecting the threshold setting for each instance, we run 1,000 evaluation (or out-of-sample) runs.

5.2 Threshold Benchmark Policies

To evaluate and understand the performance of the proposed PFA, we compare it to three related threshold-type PFAs as benchmark policies. The first policy π^{Vehicles} resembles the idea to preferably serve customers by vehicles and only to use drones in cases a customer cannot be served by a vehicle. It can therefore be associated with π^{PFA} and $\tau = M$ with M a number larger than all potential travel durations. In contrast to π^{Vehicles}, the benchmark policy π^{Drones} preferably serves customers by drones and only uses vehicles when a drone is not available. This policy can therefore be associated with π^{PFA} and $\tau = 0$.

Finally, we define a policy to verify the assumption that far out customers should generally be served by drones and inner city customers generally by vehicles. To this end, we invert the threshold meaning that within the threshold customers are preferably served by drones and customers with longer travel times from the depot should be preferably served by vehicles. We denote this policy

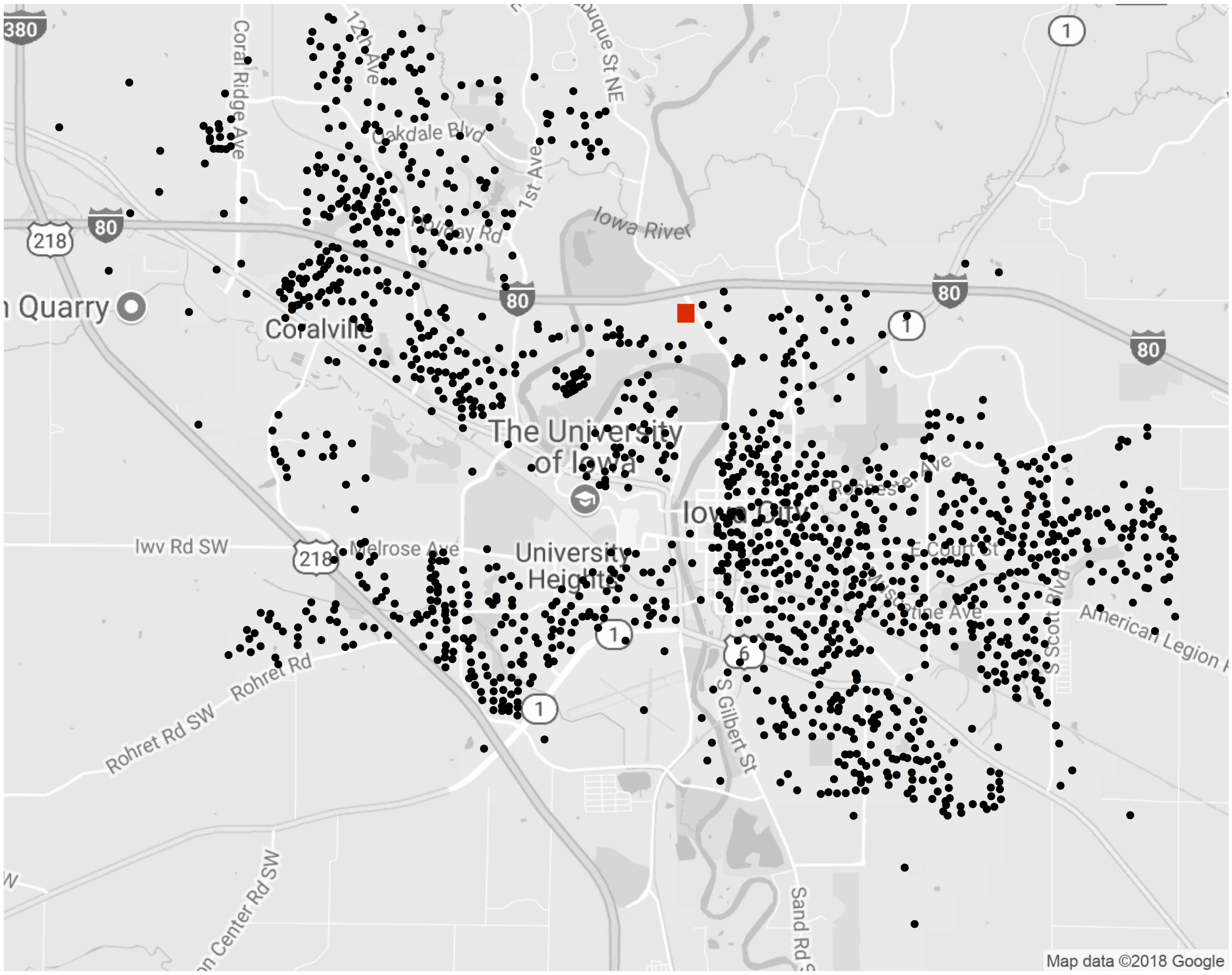

Figure 3: Customer Distribution in Iowa City Area

by π^{Minus}. We tune this policy similar to the tuning of π^{PFA}.

5.3 Solution Quality

In the following, we analyze the solution quality $\mathcal{Q}$ of the different policies. For each PFA policy π that we test, the solution quality is defined as the average percentage of served orders:

$$\mathcal{Q}(\pi) = \frac{\text{Served}}{\text{Requests}}.$$

To compare the different policies, we also define the improvement $\mathcal{I}(\pi^{\text{PFA}}, \pi)$ of π^{PFA} compared to a policy π as

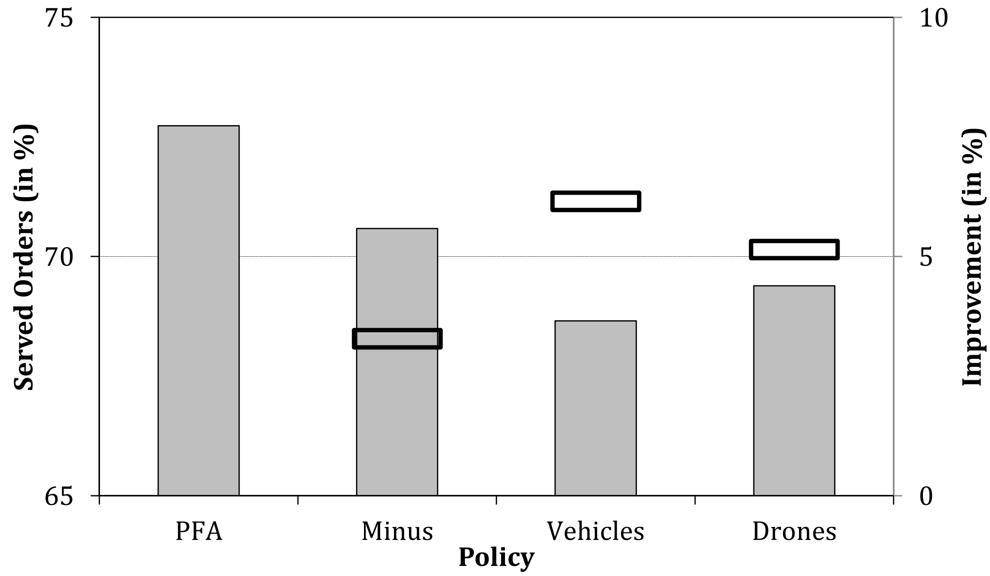

Figure 4: Average Percentage of Served Orders

$$\mathcal{I}(\pi^{\text{PFA}}, \pi) = \frac{\mathcal{Q}(\pi^{\text{PFA}}) - \mathcal{Q}(\pi)}{\mathcal{Q}(\pi)}.$$

Figure 4 summarizes the results over all of the instances for both the generated and Iowa City geographies, 800 instance settings in total. The x-axis shows the policy, the gray bars and the left y-axis the average solution quality, and the black boxes and right y-axis the improvement of π^{PFA} compared to the corresponding policy. The individual results can be found in the Appendix.

The figure shows that policy π^{PFA} achieves the best results, and on average, the improvement of π^{PFA} reaches between 3.3% compared to π^{Minus} and 6.2% compared to π^{Vehicles}. As shown in the Appendix, π^{PFA} also achieves the best performance for almost all of the individual instance settings. In 662 of the 800 instance settings, π^{PFA} outperforms all benchmarks. The improvement is particularly high when resources are scarce, reaching up to 18.5% on some individual instance settings. In 24 cases, π^{Minus} achieves better results than π^{PFA}. In the other 114 cases, all policies serve all orders, and solution quality is 100%. In summary, the districting based on the threshold of the π^{PFA} is highly beneficial.

The policy π^{Minus} performs better than π^{Drones} and π^{Vehicles}. This means that districting is beneficial even when drones serve customers in the inner city while the vehicles serve rural customers. Policy π^{Drones} provides better results than π^{Vehicles}. So, for our instance settings, preferring drones to vehicles is more effective.

Generally, the characterization of the results is similar between the two customer distributions. Policy π^{PFA} achieves the highest solution quality, followed by π^{Minus}, π^{Drones}, and π^{Vehicles}. The results are particularly good for the generated geography. For these instances, policy π^{PFA} achieves 5.5% improvement compared to π^{Minus}, 6.6% compared to π^{Drones}, and 7.9% compared to π^{Vehicles}. The improvement of π^{PFA} over the benchmarks for the Iowa City instances is slightly lower with a 1.0% improvement compared to π^{Minus}, 3.7% compared to π^{Drones}, and 4.4% compared to π^{Vehicles}. This decrease in performance is likely due to the highly heterogeneous distribution of the customers.

5.4 Enhancing PFA via Rollout

In the previous section, we compare the proposed PFA to related PFAs. In this section, we seek to improve the performance of the PFA by combining it with an ADP method known as a rollout algorithm (RA). RAs use heuristic policies to approximate the cost-to-go in a Bellman Equation. For an overview of RAs, the interested reader is referred to Goodson et al. (2017). Here and in the remainder of the section, we focus our analysis only on results from instances using the generated geography.

We compare the policy π^{PFA} with a post-decision state RA. The RA operates on the approximate Bellman Equation, and in this case, a post-decision state approximate Bellman Equation given by:

$$\widehat{V}(S_k) = \max_{x \in X(S_k)} \left\{ R(S_k, x) + \widehat{V}(S_k^x) \right\}. \tag{6}$$

The value $\widehat{V}(S_k^x)$ is the approximate value of the post-decision state. For the SDDPHF, the value represents an estimate of the expected number of future customers that can be served given a post-decision state. Then, the approximate Bellman Equation describes the value of a state as the combination of immediate reward and approximated future rewards and selects the decision x leading to the highest value. The RA selects the decision maximizing the overall value in Equation (6).

To generate estimates of the value of the post-decision state, the RA "rolls out" a base policy over a series of sample paths of the future states. That is, for a given pre-decision state and for each potential decision, RA simulates the future using the base policy to make decisions at each simulated decision point. We use π^{PFA} as base-policy. As shown in Ulmer et al. (to appear), given a

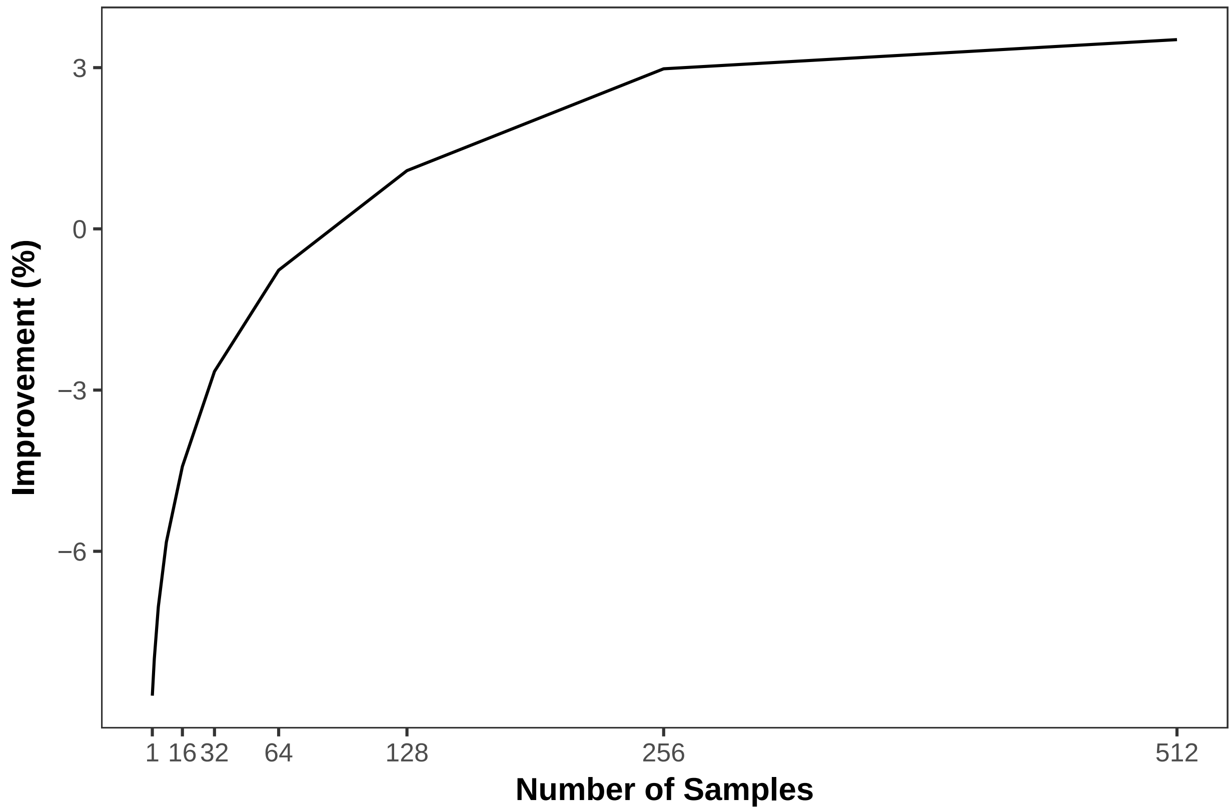

Figure 5: Comparison of RA and π^{PFA} with Varying Number of Simulations

sufficient number of samples, the RA performs at least as well as the base policy. The necessary number of samples though depends on the complexity of the problem. For simple problems with low dimensional MDPs, a few simulations may be sufficient. Rollout algorithms and related sampling methods in dynamic vehicle routing usually draw on a number of samples between 10 and 30 to achieve significant improvements (Bent and Van Hentenryck 2004, Hvattum et al. 2007, Ghiani et al. 2009, Voccia et al. to appear, Ulmer et al. to appear).

In the following, for the SDDPHF, we compare the solution quality for different number of samples for the instance setting with 3 vehicles, 10 drones, and 500 expected orders. Thus, at each decision point, we rollout the base policy with 1, 2, 4, 8, 16, 32, 64, 128, 256, and 512 samples. Figure 5 presents the results averaged over 1,000 evaluation runs for each number of samples. On the x-axis, the number of samples is shown. The y-axis depicts the resulting improvement of the RA compared to π^{PFA}. As expected, we observe an increase in solution quality with an increase in number of samples. Yet, even with 64 sampled simulations, the RA is inferior to π^{PFA}. Only with 128 or more sampled simulations does the solution quality of RA become higher, and only slightly so, than π^{PFA}. The improvement continues with 256 and 512 samples, but does not exceed 3.5% and is leveling out. Thus, a more sophisticated approach than π^{PFA} is capable providing improved performance, but at the loss of managerial simplicity associated with π^{PFA} and requiring additional computational resources.

We note that the proposed RA is capable of making decisions in real-time. That is, we can make a decision at each decision point in only a few seconds. However, while in our example RA demonstrates some improvement over π^{PFA} for a relatively large number of samples, we are challenged to show that the result holds more broadly. Each simulation run of an SDDPHF instance has hundreds of decision points. While each decision in the simulation run takes only seconds, the total runtime required increases linearly with the number of sampled simulations. Eventually, applying the RA with 512 sampled simulations to a single evaluation run takes nearly two hours. Thus, with the authors given resources, testing larger sample sizes over the 1,000 evaluation runs for the RA becomes prohibitive.

5.5 The Impact of Instance Parameters on the Threshold

In this section, we analyze the impact and behavior of the threshold values. First, we analyze the impact of the threshold on solution quality. We then examine the impact of the number of vehicles, the number of drones, and the number of orders on the value of the best threshold. For a detailed analysis of the solutions returned by the policies, we refer to Appendix A.1.

For the instance setting with 500 orders, 3 vehicles, and 10 drones, Figure 6 depicts the solution quality for a varying threshold values. The x-axis shows the threshold in minutes of travel time for a vehicle. The y-axis depicts the according solution quality. A threshold of $\tau = 0$ represents policy π^{Drones}. With an increasing threshold, the policy converges to π^{Vehicles}. We observe a distinct peak at $\tau = 13$ minutes. This suggests that a policy that uses distinct delivery districts is favored over one that favors either drones or vehicles.

We next analyze how the threshold varies with respect to the number of vehicles, number of drones, and number of orders. Figure 7 shows the best found threshold for different instance settings. For the purpose of presentation, we analyze only the instance settings with 2, 3, 4, 5 vehicles and 2, 6, 10, 14, 18 drones. The number of vehicles is depicted on the global x-axis, the number of drones on the global y-axis. Each subfigure shows the best found threshold for the associated expected

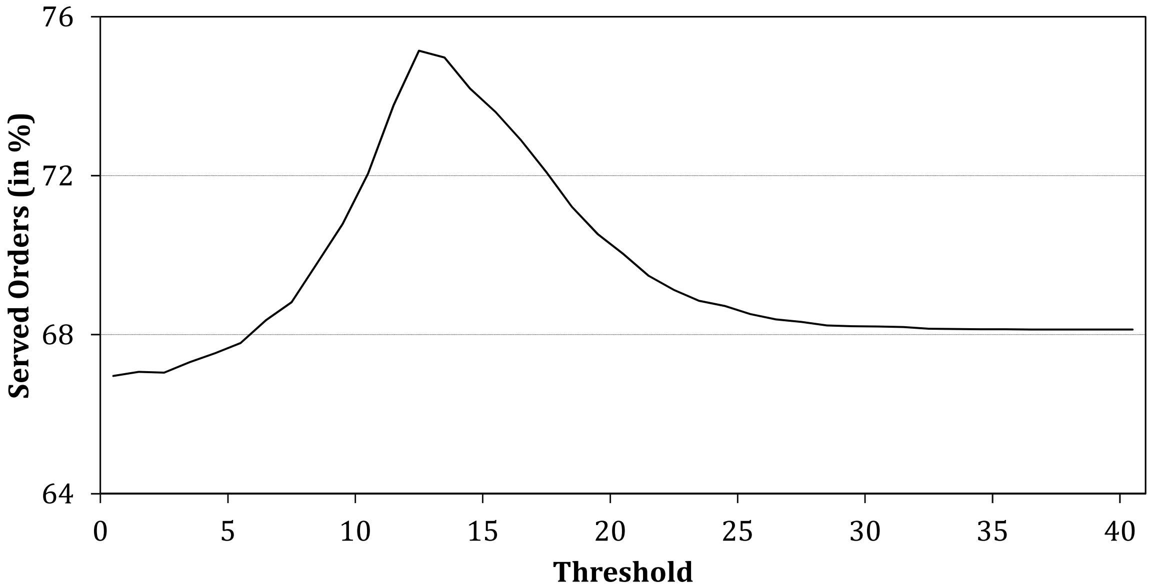

Figure 6: Average Percentage of Served Orders with Varying Threshold

number of orders.

We observe a correlation between the threshold and the ratio of vehicles and drones. Specifically, the threshold increases if we add vehicles and decreases if we increase the number of drones. This behavior is intuitive because, if we have relatively more vehicles than drones, the area preferably served by the vehicles should be larger. There is more vehicle capacity.

Notable is the behavior of the threshold with respect to the number of orders. With a low number of vehicles and/or drones, the thresholds remain nearly constant regardless of the number of expected orders. This observation indicates that the threshold policy is relatively independent of the expected number of orders when the number of vehicles and/or drones is low. The threshold only varies when the number of vehicles and/or drones is high. In this case, we observe an increase in the threshold with an increase in the number of orders. This behavior can be explained by looking at the individual results in the Appendix. For large fleets and only a few orders, all policies serve all orders, and the threshold becomes increasingly irrelevant resulting in a low threshold and eventually thresholds of zero. Thus, when there is a large number of vehicles and/or drones, the threshold increases as the number of orders increases.

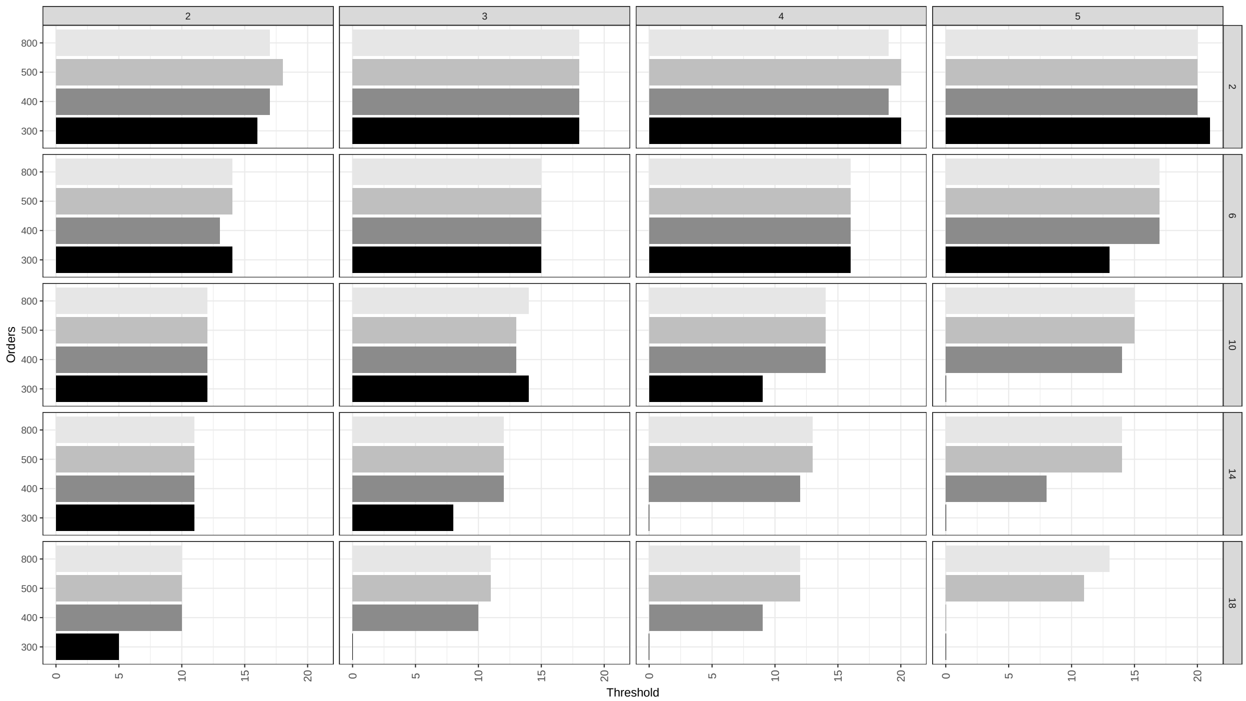

Figure 7: Thresholds with Respect to Number of Orders and Fleet Sizes

5.6 The Value of a Heterogeneous Fleet

In this section, we analyze how the combination of drones and vehicles may reduce delivery resources required to achieve a particular solution quality relative to a homogeneous fleet. To this end, we determine the fleet compositions needed to serve 95% of the customers' orders. We seek to serve only 95% because serving all customers substantially increases the required fleet size due to the stochasticity of the SDDPHF. Specifically, we fix the number of vehicles m and determine the minimum number of drones n_m^* to serve 95% of orders on average over the 1,000 realizations. We represent the optimization as:

$$n_m^* = \arg\min \left\{ n \in \mathbb{N} : Q(\pi^{\text{PFA}}) \geq 95\% |\ m \text{ vehicles}, n \text{ drones} \right\}.$$

Figure 8 shows the required number of drones with respect to different number of vehicles. The x-axis depicts the number of vehicles m and the y-axis the respective number of drones n_m^* for varying expected number of orders. The individual values are depicted in Table A9 in the Appendix. As an example, for 800 orders, we a need a fleet composed solely of 47 drones or solely of 12 vehicles to serve 95% of the orders. Equivalently a combination of 6 vehicles and 17 drones serves

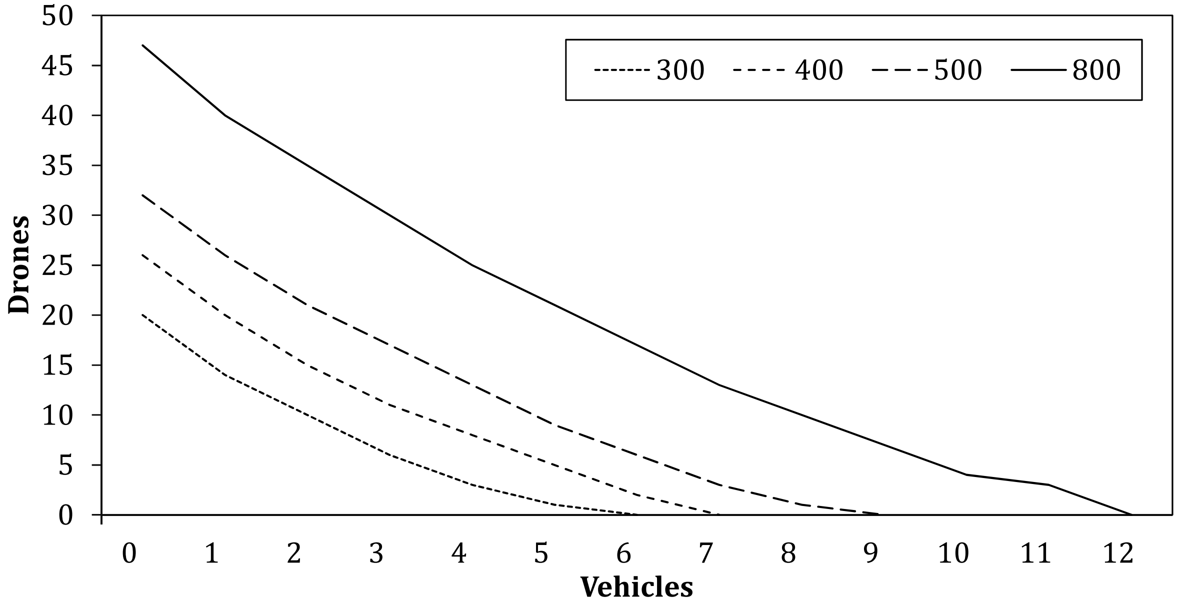

Figure 8: Required Number of Drones and Vehicles to Serve 95% of Customer Orders

95% of the orders. We generally observe a convexity in the functions indicating that a combination may be beneficial. The best composition for an individual company will depend on the combination of fixed and variable costs associated with the drones and vehicles.

It is also useful to note that, as demand increases, the vehicles become more valuable. That is, it takes more drones to replace the productivity of a vehicle as the number of expected orders increases. This increase in value is due to the increasing density of the customer orders as the number of expected orders increases. Not surprisingly, vehicles are becoming more valuable as density increases.

5.7 The Impact of Customer Heterogeneity

In our computational study, we assume that all customers can be served by a drone. This may be the case when the customer owns a dedicated space for drones, notably a garden or backyard (Hern 2016). Such deliveries are also be possible when the customer has access to a nearby drop-off point which can be served by a drone (Chong 2017). Yet, in practice, there may be a variety of reasons why a delivery is not possible. For instance, in addition to a lack of delivery space, some parcels are

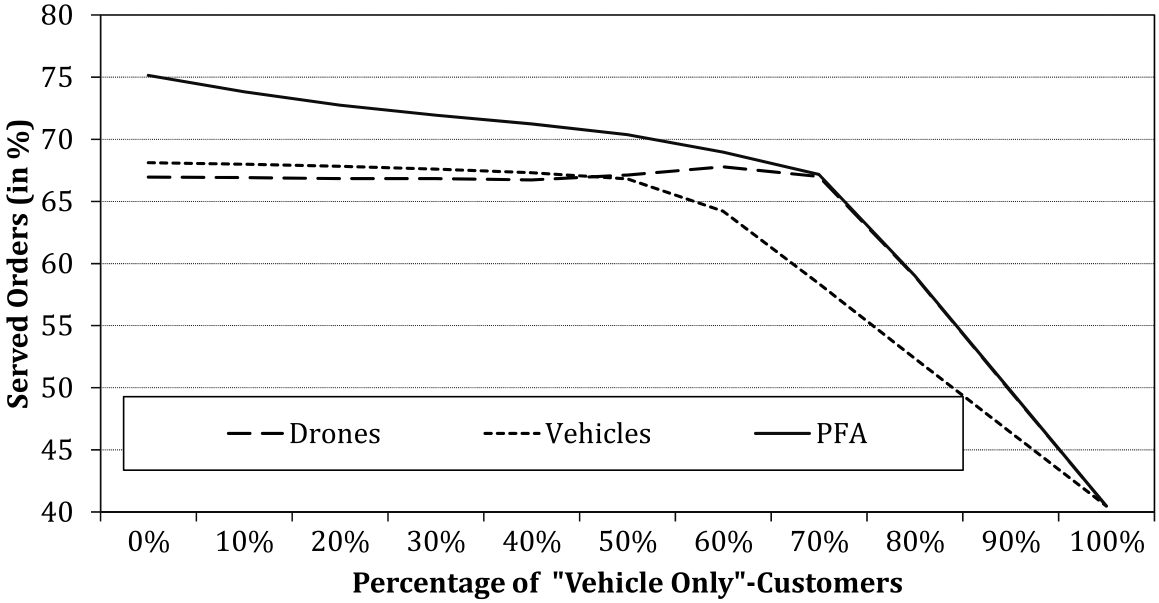

Figure 9: Impact of Heterogeneous Customers

too heavy to be delivered by drones. Such situation requires us to classify customers leaving us with what is known as heterogeneous customers.

In this section, we analyze the impact of customer heterogeneity on the proposed model and methods. We consider the same instance setting as in the previous section with 3 vehicles, 10 drones, and 500 expected orders. In the generation of customer requests, we introduce an expected percentage ρ of "Vehicle Only"-customers that be served by only vehicles and not drones. A percentage of $\rho = 0\%$ leads to the original case where all customers can be served by either vehicles or drones. A percentage of $\rho = 100\%$ leads to instances where none of the customers can be served by drones. When a customer is revealed in our simulations, whether or not the customer is a vehicles-only customer is also revealed. In our experiments, we vary the percentage ρ in steps of 10% between 0% and 100%.

Figure 9 displays the results of the experiments with heterogeneous customers. The x-axis depicts the percentage ρ. The y-axis depicts the solution quality in percent. As expected, we observe a constant decrease for policies π^{PFA} and π^{Vehicles} with increasing ρ. Yet, even for percentages up to 50%, policy π^{PFA} is able to serve more than 70% of customers, a solution quality neither π^{Vehicles}

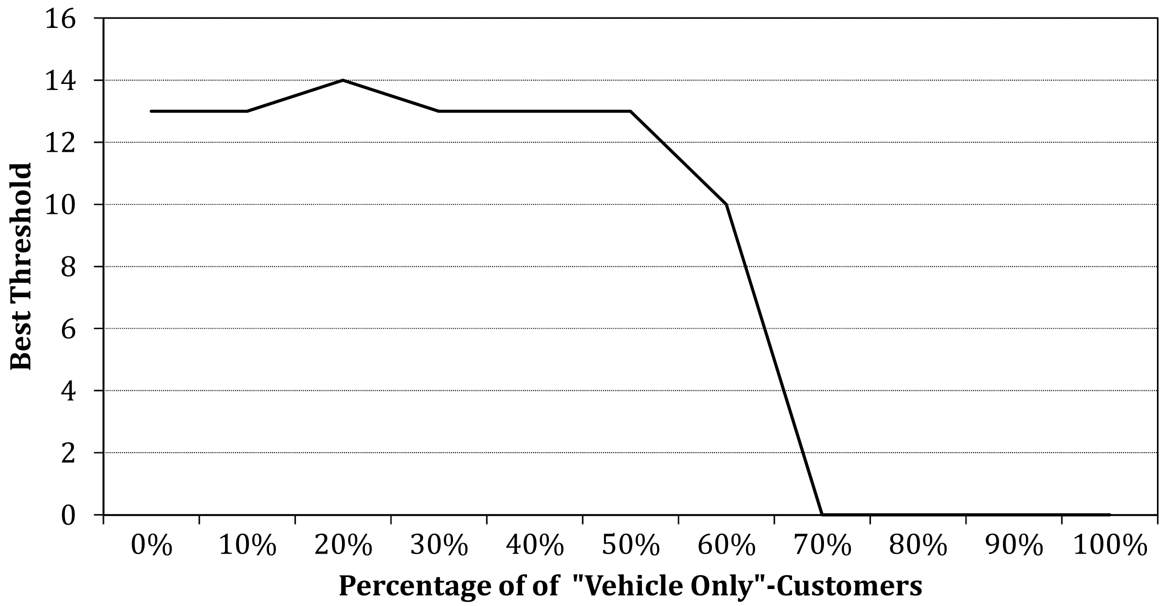

Figure 10: Best Threshold for Instances with Heterogeneous Customers

nor π^{Drones} achieves even for the instances with homogeneous customers, $\rho = 0\%$.

Another interesting observation in Figure 9 is the behavior of Policy π^{Drones}. The performance of the policy is relatively constant up to $\rho = 70\%$ and then decreases. Essentially, at a certain point, so many customers become vehicle-only customers that drones should be used for any customer that is not to reduce the load on the vehicles. Thus, prioritizing drones becomes valuable when the overall number of opportunities to send a drone decreases.

This phenomenon is also reflected in the best threshold of policy π^{PFA} for the respective values of ρ. Figure 10 shows the threshold for each ρ. The x-axis represents ρ, and the y-axis shows the best threshold. We observe that for $\rho \leq 50\%$, the threshold remains relatively constant at 13 minutes and then dramatically drops to zero leading to policy π^{Drones}. Yet, the fact that this drop does not occur until $\rho > 50\%$ indicates that our policy is not only relatively invariant with respect to the number of orders, but that, as long as the majority of customers can be served by both drones and vehicles, it is also relatively invariant to the percentage of customers that can be served by only vehicles.

6 Conclusion and Future Work

In this paper, we analyze and quantify the value of combining drones with regular delivery vehicles for same-day delivery. For this stochastic and dynamic routing problem, we develop a tunable policy function approximation to decide whether a requesting customer is served by a drone or a vehicle. The PFA draws on districting of the service area. The results demonstrate that the method is effective both in a geography in which customers are more tightly clustered around the depot as well as one in which the customer distribution is more heterogeneous. Comparisons to other threshold-type benchmark policies reveal that the districting by the proposed PFA is highly beneficial in significantly increasing the expected number of services by the fleets. Using the the PFA, we show that a combination of drones and vehicles may reduce operational resources required to serve the majority of customers. We also analyze when a combination of drones and delivery can be highly beneficial.

There are a variety of avenues for future research. As shown in the analysis of the Iowa City instances, a global threshold parameter may not be able to capture all detail of highly heterogeneous customer distributions. A reasonable step would be to replace the global threshold parameter by a state-dependent threshold based on a set of state parameters like customer location and the point of time. The SDDPHF may be further extended in several dimensions. Extensions may reflect individual charging times for drones based on parameters like travel distance, load, or weather conditions. Finally, our instances currently do not account for heterogeneous travel times and request rates. As with heterogeneous customer distributions, such cases are also likely to benefit from state-dependent thresholds and would be worthy of future work.

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Appendix

In the Appendix, we show how districting impacts decision making and present the individual results of the instance settings.

A.1 The Impact of Geographic Districting

In this section, we analyze the value of a threshold policy that creates distinct geographic districts for drones and vehicles. To this end, we analyze how the customers are served with respect to the customers' travel durations from the depot and their geographical location. We draw on the aforementioned instance setting with 500 orders, 3 vehicles, and 10 drones.

Figure A1 displays the percentages of customers served by a drone with respect to the customers' travel durations travel duration from the depot. On the x-axis shows the vehicle travel time from depot to the customer. The y-axis represents the percentage of drone services of the served customers. We compare π^{PFA} with π^{Vehicles} as a benchmark. The π^{Vehicles} benchmark represents the case of no geographic districts in which drones serve only customers that cannot be feasibly served by the vehicles.

For π^{Vehicles}, we observe a slight increase in drone services with increasing travel duration. This reflects the lower probability of feasibly inserting a more distant customer into a vehicle's tour. For π^{PFA}, we observe a distinct leap at threshold $\tau = 13$. At $\tau = 13$, customers closer to the depot are served by drones only occasionally while more than 80% of the more distant customers are served by drones. These results demonstrate the value of actively assigning more distant customers to drone rather than using the drones to serve only what the vehicles cannot feasibly serve.

Finally, we analyze the geographic distribution of drone and vehicle services picking an arbitrary sample realization from the 1000 evaluation runs from the instance setting with generated customer distribution, 500 orders, 3 vehicles, and 10 drones. Figure A2 shows the locations of the served customers as well as whether or not they are served by a drone or a vehicle. On the left side, the services by policy π^{Vehicle} are shown. The right side of Figure A2 shows the services of policy π^{PFA}. The square represents the depot. Triangles indicate services by drones and diamonds services by vehicles. The dashed line indicates the threshold of 13 minutes. In this realization, policy π^{PFA} serves 392 orders with 250 by vehicles and 142 by drones. Policy π^{Vehicles} serves only 365 services

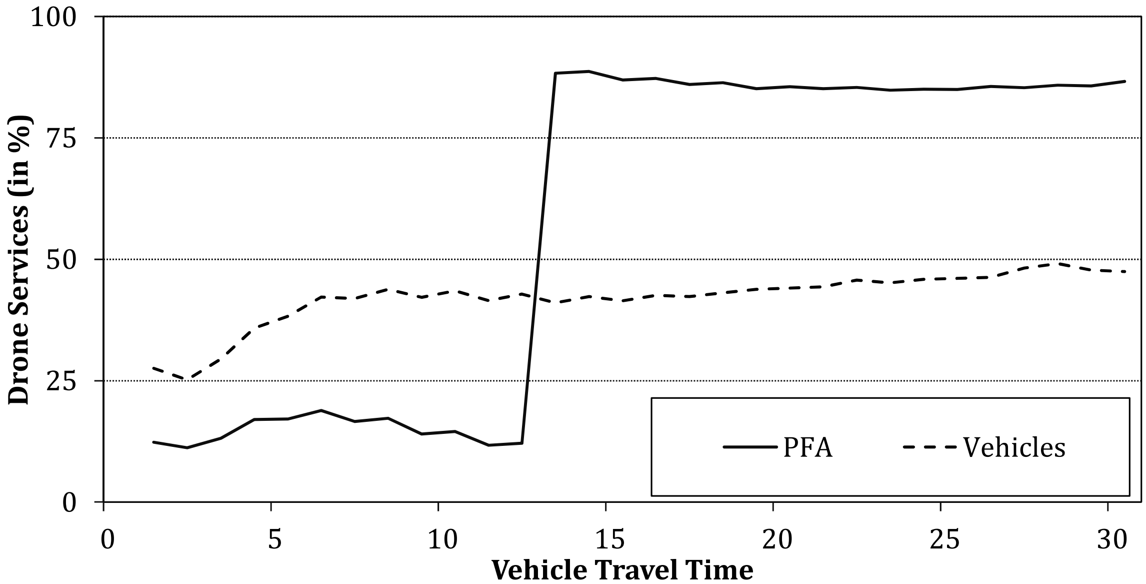
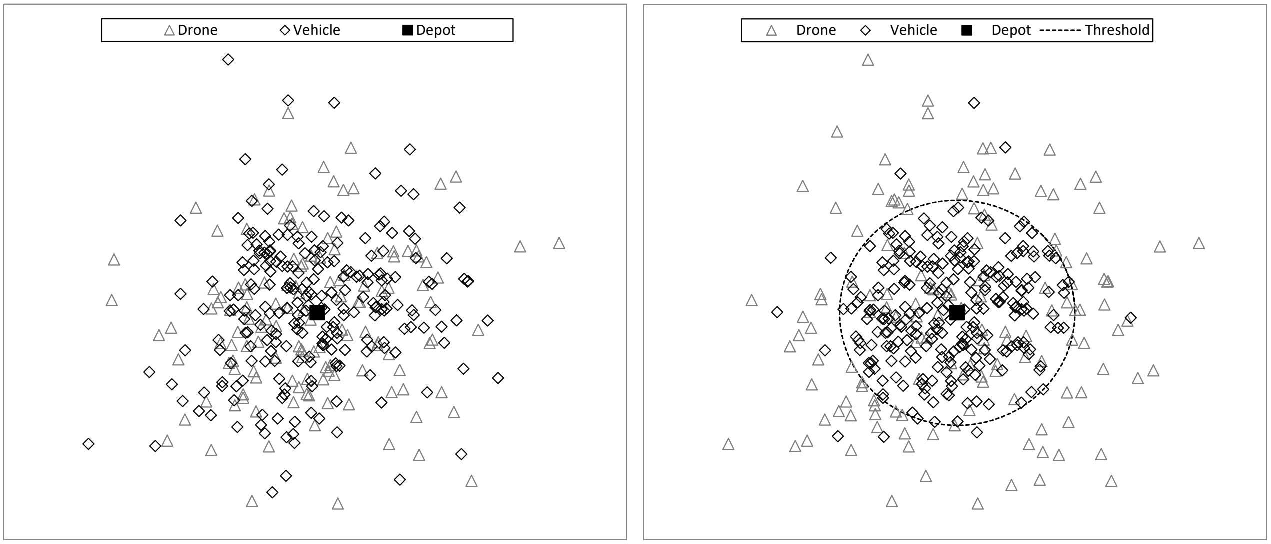


Figure A1: Customers Served by Drones with Respect to Travel Duration

with 231 by vehicles and 134 by drones.

For policy π^{Vehicles}, we observe random distributions of drone and vehicle services in the center. Only the rural customers are served by mainly drones. For policy π^{PFA}, the threshold results in a relatively distinct split of the services with only a few exceptions. These results suggest that even though we occasionally need to serve customers from the "wrong" district, policy π^{PFA} generally enables strict districting and allows for a greater density of vehicle customers close to the depot.



(a) Served Customers by π^{Vehicles} (b) Served Customers by π^{PFA}

Figure A2: Geographical Locations of Served Customers

Table A1: Results: Solution Quality $\mathcal{Q}$ (in %) for 300 Orders and Generated Customer Distribution

Vehicles	1				2				3				4				5			
Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones
1	28.2	27	26.9	26.9	49	47.3	47.2	47.1	68.1	66	65.4	65.8	84.9	82.4	82	82.1	98	96.1	95.7	95.6
2	34.4	32.2	31.8	32	55.1	52.2	51.5	51.7	73.9	70.3	69.3	69.9	91.1	86.6	85.2	85.6	99.7	98.3	97.4	97.4
3	40.2	37.3	36.7	37	61.3	56.9	55.9	56.3	79.9	74.5	73	74	96.2	90.5	88.2	89	100	99.5	98.6	98.7
4	46	42.5	41.7	41.9	66.9	61.6	60.3	60.8	86	78.9	76.6	78	99	94.2	91.3	92.1	100	99.9	99.4	99.5
5	51.7	47.4	46.5	46.8	72.5	66	64.5	65.3	91.5	83.2	80.1	81.9	99.9	97	94	95	100	100	99.9	99.9
6	57.2	52.2	51.3	51.7	77.9	70.5	68.4	69.7	96.4	87.6	83.6	85.8	100	98.9	96.3	97.3	100	100	100	100
7	62.6	56.9	55.8	56.5	83.7	75.1	72.3	74.1	99	91.6	87	89.5	100	99.7	98	98.8	100	100	100	100
8	67.8	61.7	60.5	61.2	89.1	79.5	76.2	78.4	99.9	95.2	90.2	93	100	100	99.1	99.6	100	100	100	100
9	73.2	66.4	64.8	65.9	94.7	84	79.9	82.6	100	97.9	93.1	96.1	100	100	99.7	100	100	100	100	100
10	78.4	71.1	69	70.5	98.1	88.4	83.6	86.8	100	99.3	95.9	98.2	100	100	100	100	100	100	100	100
11	83.6	75.7	73.3	75.1	99.5	92.6	87.4	90.8	100	99.9	98	99.4	100	100	100	100	100	100	100	100
12	88.6	80.3	77.5	79.7	100	96.1	90.9	94.5	100	100	99.2	99.9	100	100	100	100	100	100	100	100
13	93.8	84.8	81.8	84.1	100	98.5	94.4	97.4	100	100	99.9	100	100	100	100	100	100	100	100	100
14	97.4	89.3	86.1	88.5	100	99.6	97.3	99.1	100	100	100	100	100	100	100	100	100	100	100	100
15	99.3	93.5	90.2	92.7	100	100	99	99.8	100	100	100	100	100	100	100	100	100	100	100	100
16	99.9	96.9	94.2	96.2	100	100	99.8	100	100	100	100	100	100	100	100	100	100	100	100	100
17	100	98.9	97.3	98.6	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
18	100	99.8	99.1	99.7	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
19	100	100	99.9	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
20	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100

Table A2: Results: Solution Quality $\mathcal{Q}$ (in %) for 400 Orders and Generated Customer Distribution

Vehicles	1				2				3				4				5			
Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones
1	21.8	21.1	20.9	21	38.1	37.1	36.7	37	53.6	52.2	51.8	51.9	68	66.1	65.8	66.1	80.9	78.9	78.5	78.6
2	26.1	25	24.7	24.9	42.7	40.9	40.3	40.6	58.2	55.7	55.1	55.3	72.5	69.4	68.8	69.2	85.5	82.1	81.2	81.4
3	30.6	29	28.4	28.7	47.1	44.7	43.8	44.2	62.7	59.3	58.3	58.6	76.9	72.6	71.6	72.2	90.3	85.3	83.7	84
4	35	32.8	32.2	32.5	51.5	48.4	47.3	47.8	67	62.7	61.6	61.9	81.2	75.9	74.4	75.2	94.6	88.4	86.1	86.7
5	39.3	36.6	36	36.3	55.9	52	50.8	51.4	71.2	66.2	64.6	65.2	85.4	79.1	76.9	78.2	97.7	91.4	88.5	89.3
6	43.4	40.4	39.7	40.1	60.1	55.6	54.3	54.9	75.4	69.4	67.6	68.5	89.9	82.5	79.5	81.1	99.3	94.3	90.8	91.8
7	47.7	44.2	43.4	43.8	64.4	59.1	57.7	58.4	79.4	72.8	70.4	71.7	94.3	85.8	82.3	84	100	96.9	93	94.2
8	51.7	47.9	47.2	47.6	68.5	62.5	61	61.8	83.9	76.2	73.4	74.9	97.5	89.2	84.8	86.8	100	98.6	95.1	96.3
9	55.8	51.7	50.9	51.2	72.5	66	64.3	65.3	88	79.6	76.2	78.1	99.2	92.3	87.5	89.6	100	99.5	96.8	98
10	59.8	55.3	54.4	54.9	76.7	69.4	67.3	68.7	92.6	82.9	79.1	81.2	99.9	95.1	89.8	92.3	100	99.9	98.3	99.1
11	63.7	58.9	58	58.5	80.5	72.9	70.3	72.1	96.3	86.3	81.7	84.3	100	97.4	92.3	94.9	100	100	99.2	99.7
12	67.5	62.4	61.4	62.1	84.4	76.4	73.3	75.4	98.6	89.6	84.4	87.3	100	98.8	94.5	97	100	100	99.7	100
13	71.5	66	64.6	65.7	88.7	79.8	76.1	78.7	99.7	92.8	86.9	90.3	100	99.6	96.4	98.6	100	100	100	100
14	75.4	69.5	67.8	69.2	92.9	83.2	79.1	82	100	95.7	89.8	93.2	100	100	98	99.5	100	100	100	100
15	79.3	73	71	72.7	96.3	86.6	82.2	85.3	100	97.8	92.4	95.8	100	100	99.1	99.9	100	100	100	100
16	83.1	76.5	74.3	76.1	98.5	89.9	85.2	88.4	100	99.1	94.9	97.9	100	100	99.7	100	100	100	100	100
17	86.8	80	77.7	79.6	99.7	93.2	88.2	91.6	100	99.8	97.1	99.2	100	100	100	100	100	100	100	100
18	90.8	83.5	81.1	83	100	96	91.3	94.5	100	100	98.7	99.8	100	100	100	100	100	100	100	100
19	94.6	86.9	84.3	86.3	100	98.2	94.1	97	100	100	99.6	100	100	100	100	100	100	100	100	100
20	97.5	90.3	87.6	89.6	100	99.4	96.6	98.8	100	100	100	100	100	100	100	100	100	100	100	100

Table A3: Results: Solution Quality $\mathcal{Q}$ (in %) for 500 Orders and Generated Customer Distribution

Vehicles	1				2				3				4				5			
Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones
1	18	17.3	17.2	17.3	31.7	30.6	30.3	30.5	44.5	43.3	43	43.3	56.8	55.5	55.2	55.3	68.3	66.7	66.4	66.5
2	21.4	20.5	20.2	20.5	35.2	33.7	33.2	33.5	48.3	46.2	45.6	46.1	60.7	58.2	57.7	57.9	72.2	69.3	68.7	69
3	24.8	23.6	23.4	23.6	38.6	36.8	36.2	36.5	51.9	49.3	48.6	49	64.3	61	60.2	60.5	75.6	72	71.1	71.4
4	28.4	26.8	26.4	26.7	42.1	39.8	39.1	39.4	55.4	52.2	51.3	51.7	67.8	63.7	62.7	63.2	79.1	74.5	73.1	73.8
5	31.8	29.9	29.5	29.8	45.5	42.9	41.9	42.4	58.7	55	54	54.5	71	66.4	65.3	65.7	82.6	77.3	75.4	76.2
6	35.3	33	32.5	32.9	48.8	45.9	44.9	45.3	62	57.8	56.7	57.3	74.5	69.1	67.5	68.3	86.1	80	77.6	78.6
7	38.7	36.2	35.6	36	52.3	48.9	47.7	48.2	65.4	60.7	59.4	60	77.5	71.7	69.9	70.9	90	82.4	79.7	80.9
8	41.9	39.2	38.6	39	55.7	51.9	50.6	51.1	68.8	63.5	61.9	62.8	80.9	74.4	72.2	73.4	93.2	85	81.9	83.2
9	45.2	42.3	41.6	42	59.2	54.8	53.5	54	71.9	66.2	64.5	65.5	84.2	77.2	74.5	76	96.2	87.8	84	85.5
10	48.5	45.3	44.6	45	62.6	57.7	56.2	56.8	75.1	68.9	67	68.1	88	80	76.7	78.4	98.4	90.4	86.1	87.8
11	51.6	48.3	47.6	48	65.7	60.4	59	59.6	78.5	71.7	69.3	70.8	91.4	82.7	78.9	80.9	99.5	93	88	90
12	54.7	51.3	50.6	51	68.8	63.1	61.7	62.5	81.6	74.5	71.7	73.4	94.4	85.4	81.1	83.4	100	95.2	90.2	92.2
13	58	54.2	53.6	53.9	72.1	65.9	64.4	65.2	85.2	77.2	74	76.1	97.3	88.1	83.4	85.8	100	97.2	92.2	94.3
14	61.1	57.1	56.4	56.9	75.2	68.7	66.8	68	88.8	80.1	76.3	78.7	99.1	90.7	85.5	88.2	100	98.6	94	96.2
15	64.2	59.9	59.2	59.8	78.2	71.5	69.3	70.8	92.2	82.8	78.6	81.2	99.9	93.3	87.6	90.6	100	99.5	95.5	97.8
16	67.2	62.8	61.9	62.7	81.3	74.4	71.6	73.5	95.2	85.5	80.8	83.8	100	95.6	89.7	92.9	100	99.9	97	98.9
17	70.3	65.7	64.5	65.5	84.7	77.3	74	76.2	97.5	88.2	83.2	86.3	100	97.4	91.7	95	100	100	98.3	99.7
18	73.5	68.6	67.1	68.4	88.1	80	76.4	78.9	99.1	90.9	85.4	88.8	100	98.7	93.7	96.9	100	100	99.2	100
19	76.6	71.4	69.7	71.2	91.5	82.7	78.9	81.6	99.9	93.5	87.6	91.3	100	99.5	95.7	98.4	100	100	99.7	100
20	79.7	74.3	72.4	74	94.5	85.5	81.5	84.2	100	95.8	90.1	93.6	100	100	97.3	99.4	100	100	100	100

Table A4: Results: Solution Quality $\mathcal{Q}$ (in %) for 800 Orders and Generated Customer Distribution

Vehicles	1				2				3				4				5			
Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones
1	11.7	11.1	11.1	11.1	20.7	20	19.9	19.8	29.3	28.6	28.4	28.5	37.6	36.9	36.7	36.6	45.6	44.8	44.5	44.7
2	14	13.2	13.1	13.1	23	22	21.8	21.8	31.8	30.6	30.2	30.4	40.2	38.8	38.4	38.5	48.2	46.7	46.3	46.4
3	16.2	15.2	15.1	15.1	25.3	24	23.7	23.7	34.1	32.5	32.1	32.3	42.5	40.6	40.2	40.3	50.6	48.5	48	48.1
4	18.3	17.2	17.1	17.2	27.5	26	25.6	25.7	36.4	34.4	33.8	34.1	44.8	42.5	41.8	42.1	53	50.3	49.6	49.8
5	20.4	19.2	19.1	19.2	29.7	27.9	27.5	27.6	38.5	36.3	35.7	36	47	44.4	43.6	43.8	55.2	52.1	51.2	51.6
6	22.5	21.2	21	21.2	31.7	29.8	29.5	29.5	40.7	38.2	37.5	37.9	49.3	46.3	45.4	45.6	57.4	53.9	52.9	53.3
7	24.6	23.2	23	23.1	33.8	31.8	31.3	31.5	42.8	40.1	39.3	39.7	51.4	48.1	47	47.4	59.6	55.6	54.6	54.9
8	26.7	25.2	24.9	25.1	35.8	33.7	33.1	33.4	44.9	42	41.1	41.5	53.5	49.9	48.8	49.2	61.8	57.3	56.3	56.6
9	28.9	27.1	26.9	27.1	37.9	35.6	35	35.3	47	43.9	42.9	43.4	55.6	51.7	50.5	50.9	63.7	59	57.9	58.3
10	30.9	29.1	28.8	29.1	40	37.5	36.9	37.1	49	45.7	44.7	45.2	57.7	53.5	52.3	52.7	65.8	60.8	59.6	60
11	33	31	30.7	31	42	39.4	38.8	39	51.1	47.5	46.4	47	59.7	55.2	54	54.4	67.8	62.4	61	61.6
12	35.1	33	32.7	32.9	44.1	41.3	40.6	40.9	53.1	49.3	48.3	48.8	61.7	56.9	55.7	56.1	69.7	64.1	62.6	63.3
13	37.1	34.9	34.6	34.9	46.2	43.1	42.5	42.7	55.1	51.2	50.1	50.6	63.7	58.6	57.3	57.8	71.9	65.9	64.1	64.9
14	39.2	36.8	36.5	36.8	48.2	45	44.3	44.6	57.2	53	51.9	52.4	65.7	60.4	59	59.5	74	67.7	65.6	66.5
15	41.1	38.7	38.5	38.7	50.2	46.9	46.2	46.4	59.3	54.8	53.6	54.1	67.7	62.1	60.6	61.2	76	69.6	67	68.2
16	43.1	40.6	40.4	40.6	52.3	48.7	48	48.3	61.2	56.5	55.4	55.9	69.6	63.9	62.2	62.9	77.9	71.3	68.5	69.8
17	45	42.5	42.3	42.5	54.4	50.5	49.8	50.1	63.1	58.2	57.1	57.7	71.5	65.6	63.6	64.6	80.1	73	69.9	71.4
18	47	44.4	44.2	44.4	56.5	52.4	51.7	51.9	65.1	60	58.7	59.4	73.6	67.4	65.1	66.3	82.4	74.8	71.4	73
19	49	46.3	46.1	46.2	58.5	54.2	53.4	53.7	67.2	61.7	60.3	61.1	75.7	69.1	66.5	68	84.5	76.5	72.8	74.6
20	51	48.2	47.9	48.1	60.5	56	55.1	55.5	69.2	63.5	61.9	62.9	77.7	70.9	68	69.6	86.6	78.2	74.2	76.2

Table A5: Results: Solution Quality $\mathcal{Q}$ (in %) for 300 Orders and Iowa City Customer Distribution

Vehicles	1				2				3				4				5			
Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones
1	27.1	27.3	26.5	26.8	48.1	48.3	47.3	47.5	67.9	67.5	66.7	67	85	84.7	83.3	83.8	97.8	97.8	96.9	96.9
2	31.9	32	30.9	31.4	53	53.1	51.5	51.7	72.4	71.8	70	70.7	89.2	88.9	86.2	86.8	99.3	99.1	98.1	98.1
3	37	36.7	35.5	36	57.7	57.6	55.7	55.9	76.6	76	73.4	74.4	93.7	92.8	89.1	89.8	99.8	99.7	99.1	99
4	42.2	41.5	40.3	40.5	62.4	62	60	60	81.2	80.3	76.8	78	97	95.9	91.8	92.6	100	99.9	99.6	99.5
5	47.3	46.1	45	45	67	66.3	64	64.1	85.8	84.4	80	81.6	98.8	98	94.4	95	100	100	99.9	99.8
6	52.2	50.7	49.5	49.4	71.9	70.5	67.4	68.1	90.4	88.3	83	85.1	99.7	99.2	96.4	97.1	100	100	99.9	99.9
7	57	55.1	53.8	53.8	76.6	74.7	70.9	72.1	94.4	92.1	86.1	88.4	99.9	99.7	97.9	98.5	100	100	100	100
8	61.7	59.5	57.9	58.1	81.2	78.9	74.2	76.1	97.6	95.5	88.9	91.7	100	99.9	98.9	99.4	100	100	100	100
9	66.4	63.8	61.9	62.4	86	83.1	77.4	80	99.3	97.7	91.8	94.7	100	100	99.5	99.8	100	100	100	100
10	70.9	68.1	65.7	66.7	90.5	87.2	80.8	83.8	99.8	99.2	94.3	97	100	100	99.8	99.9	100	100	100	100
11	75.5	72.4	69.4	70.9	94.9	91.1	84.1	87.6	100	99.8	96.6	98.7	100	100	99.9	100	100	100	100	100
12	80.2	76.7	73.1	75.1	98	94.5	87.4	91.3	100	99.9	98.3	99.5	100	100	100	100	100	100	100	100
13	84.8	80.9	77.1	79.3	99.4	97.2	90.8	94.6	100	100	99.3	99.9	100	100	100	100	100	100	100	100
14	89.4	85	81.1	83.3	99.9	98.9	94.1	97.2	100	100	99.8	100	100	100	100	100	100	100	100	100
15	93.5	89.1	84.9	87.4	100	99.7	96.9	98.8	100	100	100	100	100	100	100	100	100	100	100	100
16	96.9	93	88.8	91.3	100	99.9	98.6	99.6	100	100	100	100	100	100	100	100	100	100	100	100
17	98.9	96.2	92.5	94.7	100	100	99.6	99.9	100	100	100	100	100	100	100	100	100	100	100	100
18	99.7	98.5	95.8	97.4	100	100	99.9	100	100	100	100	100	100	100	100	100	100	100	100	100
19	99.9	99.5	98.1	99	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
20	100	99.9	99.4	99.7	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100

Table A6: Results: Solution Quality $\mathcal{Q}$ (in %) for 400 Orders and Iowa City Customer Distribution

Vehicles	1				2				3				4				5			
Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones
1	21.3	21.4	20.9	21	37.8	37.8	37.3	37.1	53.8	53.8	53.2	53	68.6	68.4	67.9	67.8	81.8	81.6	80.7	81.1
2	25	25	24.1	24.5	41.5	41.6	40.3	40.4	57.4	57.3	56.2	56.1	72	71.5	70.3	70.6	85.2	84.8	83.1	83.5
3	28.6	28.7	27.5	28.1	45.2	45.2	43.6	43.7	61	60.7	58.9	59.1	75.5	74.8	72.9	73.3	88.7	87.8	85.1	85.8
4	32.2	32.3	31.2	31.5	48.6	48.7	46.9	47	64.5	64	62	62.1	78.8	77.9	75.3	76.1	92.1	91	87.5	88.2
5	35.8	35.9	34.6	35	52.2	52.1	50.3	50.2	68	67.2	64.9	65.1	82.2	81.1	77.6	78.7	95.2	93.9	89.7	90.4
6	39.5	39.4	38.2	38.4	55.6	55.4	53.6	53.5	71.3	70.6	67.6	68.1	85.7	84.4	80.1	81.4	97.6	96.3	91.8	92.6
7	43.3	42.9	41.6	41.9	59.1	58.8	56.9	56.6	74.7	73.8	70.2	71	89	87.3	82.6	84	99	97.9	93.8	94.6
8	47	46.4	45.2	45.2	62.7	62.1	59.8	59.8	78	77	72.9	73.9	92.2	90.3	85.1	86.6	99.6	99	95.5	96.4
9	50.7	49.8	48.6	48.6	66.3	65.4	62.9	63	81.3	80.1	75.4	76.8	95.3	93.2	87.1	89.1	99.9	99.6	97.1	97.8
10	54.3	53.2	52	51.9	69.8	68.6	65.6	66.1	84.6	83.2	77.8	79.7	97.5	95.8	89.3	91.6	99.9	99.8	98.2	98.8
11	58	56.6	55.2	55.2	73.3	71.9	68.3	69.2	88.1	86.3	80.2	82.5	99	97.7	91.4	93.9	99.9	99.9	99	99.4
12	61.5	59.9	58.3	58.5	76.9	75.1	70.9	72.2	91.8	89.4	82.7	85.3	99.7	98.9	93.5	96	99.9	99.9	99.4	99.8
13	65	63.1	61.5	61.8	80.4	78.4	73.5	75.3	95.1	92.5	85	88.1	99.9	99.6	95.4	97.6	99.9	99.9	99.8	99.9
14	68.4	66.5	64.3	65	83.8	81.5	76.2	78.3	97.6	95.1	87.4	90.8	99.9	99.8	97	98.8	99.9	99.9	99.9	99.9
15	71.8	69.7	67.1	68.2	87.2	84.7	78.8	81.3	99	97.2	89.7	93.3	99.9	99.9	98.3	99.5	99.9	99.9	99.9	99.9
16	75.3	72.9	70	71.4	90.7	87.9	81.5	84.3	99.6	98.6	92.3	95.6	99.9	99.9	99.1	99.8	99.9	99.9	99.9	99.9
17	78.7	76.1	73	74.6	94.1	90.9	84.2	87.2	99.9	99.5	94.6	97.5	99.9	99.9	99.7	99.9	99.9	99.9	99.9	99.9
18	82.1	79.4	76.1	77.8	96.8	93.7	87	90.1	99.9	99.8	96.6	98.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9
19	85.5	82.5	79.2	80.9	98.6	96.2	89.9	92.8	99.9	99.9	98.3	99.6	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9
20	89	85.7	82.2	84	99.5	98.1	92.5	95.4	99.9	99.9	99.3	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9

Table A7: Results: Solution Quality $\mathcal{Q}$ (in %) for 500 Orders and Iowa City Customer Distribution

Vehicles	1				2				3				4				5			
Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones
1	17.5	17.6	17.2	17.2	31	31.1	30.7	30.6	44.3	44.5	43.9	43.8	57.4	57.4	56.8	56.7	69.4	69.2	68.8	68.8
2	20.6	20.6	20	20.1	34.3	34.2	33.2	33.3	47.5	47.5	46.6	46.4	60.4	60.2	59.2	59.1	72.3	71.6	70.8	71
3	23.6	23.6	22.8	22.9	37.4	37.1	36	36	50.6	50.5	49	48.9	63.4	62.8	61.6	61.5	74.8	74.3	72.8	73.2
4	26.5	26.4	25.5	25.8	40.2	40	38.8	38.7	53.6	53.2	51.6	51.5	66.1	65.5	63.8	63.9	77.7	76.9	74.8	75.3
5	29.4	29.4	28.2	28.6	43.1	42.9	41.4	41.4	56.4	55.9	54.1	54	69	68.1	66	66.2	80.4	79.4	76.7	77.5
6	32.2	32.4	31.1	31.4	46	45.7	44.2	44.1	59.3	58.7	56.7	56.5	71.7	70.8	68.2	68.6	83.2	82	78.5	79.6
7	35	35.1	34	34.2	48.7	48.5	46.9	46.7	62.1	61.3	59.3	59	74.4	73.4	70.4	70.9	86.1	84.5	80.8	81.7
8	38	37.9	36.9	37	51.5	51.4	49.6	49.4	64.7	63.9	61.9	61.5	77.1	76	72.5	73.2	88.9	87.2	82.6	83.8
9	40.9	40.8	39.7	39.8	54.2	54.1	52.4	52	67.4	66.6	64.2	63.9	79.7	78.7	74.5	75.5	91.4	89.6	84.6	85.8
10	44	43.6	42.6	42.5	56.9	56.8	55.1	54.6	70	69.1	66.3	66.4	82.3	81.1	76.5	77.7	94.1	92.1	86.7	87.8
11	47	46.4	45.3	45.2	59.9	59.4	57.5	57.2	72.8	71.8	68.5	68.8	85	83.6	78.7	80	96.3	94.4	88.2	89.9
12	49.9	49.1	48.1	48	62.7	62.1	60.1	59.7	75.5	74.5	70.6	71.2	87.7	86.1	80.5	82.2	98	96.3	90	91.8
13	52.7	51.9	50.7	50.7	65.6	64.7	62.4	62.3	78.2	77.2	72.7	73.6	90.6	88.7	82.5	84.4	99.1	97.8	91.9	93.7
14	55.6	54.6	53.4	53.3	68.4	67.4	64.6	64.8	80.9	79.8	74.8	76	93.3	91.2	84.2	86.6	99.7	98.8	93.5	95.5
15	58.5	57.2	56.1	56	71.1	70	66.9	67.4	83.7	82.3	76.8	78.4	95.7	93.6	86.1	88.8	99.9	99.5	94.8	97
16	61.4	60	58.6	58.6	73.8	72.7	68.9	69.9	86.4	85	78.6	80.7	97.6	95.7	88.1	90.9	100	99.8	96.4	98.2
17	64.2	62.6	61	61.3	76.7	75.3	71.2	72.4	89.3	87.6	80.7	83.1	99	97.4	89.8	93	100	99.9	97.7	99
18	66.9	65.3	63.4	63.9	79.4	77.9	73.3	74.9	92.2	90	82.7	85.4	99.6	98.6	91.8	94.9	100	100	98.5	99.5
19	69.6	67.9	65.7	66.5	82.2	80.6	75.6	77.3	94.9	92.5	84.8	87.7	99.9	99.3	93.6	96.6	100	100	99.2	99.8
20	72.2	70.5	68.1	69.1	84.9	83.1	77.7	79.8	97.1	94.7	86.9	90	100	99.7	95.4	98	100	100	99.6	99.9

Table A8: Results: Solution Quality $\mathcal{Q}$ (in %) for 800 Orders and Iowa City Customer Distribution

Vehicles	1				2				3				4				5			
Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones	PFA	Minus	Vehicles	Drones
1	11.3	11.3	11	11	20.3	20.4	20.1	20	29.1	29.2	28.9	28.8	37.8	37.8	37.6	37.4	46.4	46.2	46	45.9
2	13.4	13.3	12.9	12.9	22.4	22.3	21.9	21.8	31.2	31.1	30.6	30.5	39.8	39.8	39.1	39	48.3	48.3	47.7	47.5
3	15.3	15.3	14.8	14.7	24.3	24.2	23.7	23.6	33.1	33.1	32.4	32.2	41.8	41.8	40.9	40.7	50.2	50.1	49.3	49
4	17.2	17.2	16.7	16.6	26.2	26.1	25.5	25.3	35.1	34.9	34.2	33.9	43.8	43.6	42.6	42.3	52.1	52.1	51	50.6
5	19.1	19.1	18.5	18.4	28.1	27.9	27.2	27.1	37	36.8	35.8	35.6	45.6	45.4	44.2	43.9	54.1	53.8	52.4	52.2
6	21	21	20.4	20.2	30	29.8	29	28.9	38.8	38.5	37.5	37.3	47.6	47.1	45.9	45.6	56	55.5	54	53.7
7	22.8	22.8	22.1	22	31.8	31.5	30.6	30.6	40.7	40.4	39.2	39	49.4	49	47.5	47.2	57.8	57.2	55.6	55.2
8	24.7	24.6	23.9	23.9	33.6	33.4	32.4	32.4	42.5	42.1	40.9	40.7	51.2	50.7	49.2	48.8	59.6	58.9	57.2	56.8
9	26.5	26.4	25.7	25.6	35.4	35.2	34.1	34.1	44.3	43.9	42.5	42.3	52.9	52.4	50.7	50.4	61.3	60.6	58.6	58.3
10	28.3	28.2	27.4	27.4	37.3	36.9	35.8	35.8	46.2	45.7	44.3	44	54.6	54.1	52.4	51.9	63	62.2	60.1	59.8
11	30	30	29.2	29.2	39	38.7	37.6	37.5	48	47.4	46	45.6	56.3	55.8	54.1	53.5	64.8	63.8	61.5	61.3
12	31.8	31.8	31	30.9	40.8	40.5	39.4	39.2	49.8	49.1	47.7	47.3	58.2	57.4	55.6	55.1	66.5	65.5	62.8	62.8
13	33.6	33.6	32.8	32.7	42.6	42.3	41.2	40.9	51.5	50.9	49.5	48.9	60	59.2	57.2	56.7	68.2	67.3	64.3	64.3
14	35.4	35.3	34.5	34.5	44.4	44	42.9	42.6	53.2	52.6	51	50.5	61.8	60.8	58.7	58.2	69.9	68.9	65.6	65.8
15	37.3	37.1	36.3	36.2	46.1	45.7	44.6	44.3	55	54.2	52.7	52.1	63.4	62.5	60.1	59.8	71.6	70.6	66.8	67.2
16	39.1	38.9	38.1	37.9	47.9	47.5	46.3	46	56.6	55.9	54.3	53.8	65.1	64.1	61.5	61.3	73.3	72.2	68.3	68.7
17	41	40.6	39.9	39.7	49.6	49.2	48.2	47.6	58.3	57.6	55.9	55.4	66.8	65.8	62.9	62.8	74.9	73.8	69.6	70.2
18	42.7	42.4	41.7	41.4	51.3	50.9	49.8	49.3	60	59.2	57.4	57	68.5	67.5	64.1	64.4	76.6	75.5	70.8	71.6
19	44.5	44.1	43.4	43.1	53	52.6	51.4	51	61.7	60.9	58.8	58.6	70.2	69.2	65.6	65.9	78.3	77	72.1	73.1
20	46.4	45.8	45.1	44.9	54.8	54.3	53.1	52.6	63.4	62.7	60.3	60.1	71.9	70.9	66.8	67.4	80	78.6	73.4	74.6

Table A9: Results: Required Drones n_m^* to Serve 95% of the Orders Given m Vehicles

Vehicles	300	400	500	800
0	20	26	32	47
1	14	20	26	40
2	10	15	21	35
3	6	11	17	30
4	3	8	13	25
5	1	5	9	21
6	0	2	6	17
7		0	3	13
8			1	10
9			0	7
10				4
11				3
12				0