Differential Equations (DE) Student Coursework Guide

Introduction
The aim of this coursework is to investigate a mathematical model of a real life situation; the model should involve an appropriate differential equation. However, even situations which appear to be simple can produce complicated differential equations. A key part of the process, therefore, is to make (and justify) simplifying assumptions so that the model can be expressed by one, or more, differential equations. This equation is then solved to give a set of outcomes which can be compared to a set of experimental or collected data. It is then expected that the assumptions will be reviewed and adjusted to produce an improved model.

Coursework Task
It is essential that your coursework task follows the modelling cycle and the marking criteria (see the end of this guide).

You are reminded that the differential equation for your model needs to be sufficiently sophisticated so that it reflects the syllabus coverage of this unit. Before embarking upon a particular approach, make sure you discuss it fully with your teacher to ensure that it will be suitable. There is a bank of possible investigations for this coursework and most students do one of these, though it is possible for teachers to develop their own.

The tasks fall into two broad categories:

Scheme A: Modelling: For these tasks the data are provided from an outside source such as the coursework bank, a book, the internet or perhaps a demonstration experiment, or one that has been performed previously. The following are examples of such investigations.

- Aeroplane Landing
- Interacting Species
- Growth Rates

Scheme B: Modelling / Experimental: For these tasks you will actually perform the experiment yourself and use the data that you get from it. The following are examples of such investigations.

- Cascades (Flow of water through vessels)
- Parachutes
- Paper Cups (Investigating the effects of air resistance)

Writing the report
Assume that you are writing the report for a person who has a good understanding of mathematics but no previous knowledge of the task in hand. In other words, explain your work clearly and carefully and don't assume that the reader knows all about the task you have done. Make sure that you have a copy of the marking criteria. These can be found at the end of this guide.

Assessment Scheme A – Work Based on the modelling Cycle

Often for a modelling task it actually improves the flow of the work to introduce the subject and the data at the start, before commencing the modelling. In terms of the marking criteria this is actually the third domain:

Domain 3: The collection of data to verify the model.
You must make clear the source of the data, i.e. where they have come from. It could be the coursework bank, a book, the internet etc. It would also be appropriate, at this stage to present the data, if possible, in both graphical and tabular form. Discuss the accuracy of the data, possible anomalies and missing values, if relevant. If you are doing 'Aeroplane Landing', this would be a good time to establish the time at which the brakes are applied.

Domain 1: Simplifying and setting up of the model.
The assumptions you make in order to set up the initial model are an *essential and fundamental* part of the process of modelling. A common fault in coursework is just to give a bare list of assumptions. As was stated earlier, even situations which appear to be simple are likely to produce very complicated differential equations, so you need to make some assumptions which will allow you to ignore some factors but include others.

For example, an aeroplane is a rigid body, but for simplicity's sake we might decide to consider it as a particle. What implications will this have for our model? How important is this assumption? Each of the assumptions should be analysed and discussed with two criteria in mind: How important is it? How is it relevant to the model?

Finally, in establishing the differential equations, define clearly the variables and parameters you have used.
For example, if you have a simple differential equation such as $m\frac{\mathrm{d}v}{\mathrm{d}t} = mg - kv$, although it may appear obvious, define what the variables *m, v, t* and the parameter *k* represent. A force diagram may help to explain what you mean.

In general, the variables are the measurements you have made or have been given. The parameters tend to be the values that you have calculated, usually from your data. These are necessary to get a specific solution.

If attempting 'Interacting species' the maths bank suggests initial equations:
$\dot{x} = ax + by$
$\dot{y} = cx + dy$
Which of the variables, *x* and *y* will represent the predator population and which the prey? What is the meaning of the parameters *a, b, c* and *d?*

Domain 2: Manipulating the model
Sometimes the initial differential equation is fairly simple. In that case, the revision must provide a differential equation more in line with those presented in the rest of this module

Make it clear how you calculated or chose your parameters. Justify their choice. For example in 'Interacting Species' there is a temptation to choose the parameters by trial and error. Whatever values are chosen, they need to be grounded in assumptions rather than trying a variety of values, in the hope that a reasonable fit may ensue. These need to be defined and chosen carefully. For 'Aeroplane Landing' this may involve sensible choices of time and velocity to produce the relevant parameter.

Having chosen (and justified) your parameters, you should solve the differential equation to produce a set of predicted data that can be used to compare with original data.

Note that a common error in 'Aeroplane Landing' is just to use the initial model to find a solution for the first part of the motion. The model is for the whole motion, before and after braking.

Domain 4: Comparison between the collected data and the predictions of the model.
Compare the data both graphically and in table form if at all possible. Label the graphs so that it is clear which line represents the predictions and which the collected data. Don't forget to title the graph and label the axes.

After comparing the predicted and collected data, the values of the parameters can be varied ,e.g. by using alternative boundary conditions, to see if this is all that is required to make the model fit better. Don't overdo this - a couple of variations are plenty. For the second mark in this domain, either add to the graph or draw a second, with a set of predictions for the varied parameters. Or, it may be appropriate to consider the accuracy of the data and see whether the predicted values fall within these bounds.

Domain 5: Revision of the Model
Having made the comparison, it is likely that the initial model is not accurate enough. You must now return to the assumptions. Adjusting the assumptions will lead to a revised model. This will lead to a new differential equation which will usually be more complicated than the first one.

Justify why you are making the change. Why, for example, might you consider air resistance being proportional to v^2, not some other power of v? Avoid curve fitting. It is essential to *justify* the changes by considering new assumptions rather than attempting to produce a 'look alike' graph.

Domain 6: Assessment of the improvement obtained
Finally, solve the revised model and compare the predictions to the obtained data. Again, use both table and graphical forms for the comparison.

Title the graphs clearly, particularly if all the graphs in the investigation appear together, say in an appendix. At this stage solutions should be analytical if possible (i.e. within the compass of the work covered at this level). If a solution is not possible analytically, as is the case for example in 'Interacting Species', then an appropriate numerical method can be used.

Domain 7: Oral Communication
To conclude, you may be asked to either give a presentation of your work, outlining to the teacher or, perhaps, the other members of the group, what you have investigated and what were the outcomes. Alternatively, your teacher may ask you questions about your work, asking you to explain what you have done and perhaps to clarify some sections of your work.

Assessment Scheme B – Work based on the Experimental / Modelling cycle

Domain 1: Simplifying the situation and setting up the model.
The assumptions you make in order to set up the initial model are an *essential and fundamental* part of the process of modelling. A common fault in coursework is just to give a bare list of assumptions. As was stated earlier, even situations which appear to be simple are likely to produce very complicated differential equations, so you need to make some assumptions which will allow you to ignore some factors but include others.

Each of the assumptions should be analysed and discussed with two criteria in mind: How important is it? How is it relevant to the model?

Finally, in establishing the differential equations define clearly the variables and parameters you have used.

For example, if you have a simple differential equation such as $m\frac{\mathrm{d}v}{\mathrm{d}t} = mg - kv$, although it may appear obvious, define what the variables *m, v, t* and the parameter *k* represent. A force diagram may help to explain what you mean.

In general, the variables are the measurements you have made or have been given. The parameters tend to be the values that you have calculated, usually from your data. These are necessary to get a specific solution.

Domain 2: Manipulating the Model.
Sometimes the initial differential equation is fairly simple. In that case, the revision must provide a differential equation more in line with those presented in the rest of this module

Note, if investigating 'Cascades', a simple experiment and model involving a single container will be useful for establishing parameters but, to meet the criteria, the focus of the work must be the flow through the second container.

Make it clear how you have calculated or chosen your parameters. Justify their choice. Often the parameters will be calculated from the experimental values. If this is the case then use this parameter to predict values which can be compared to experimental results other than those used to calculate the parameter. Otherwise this leads to what is known as a 'circular argument'. That is, using the experimental data to predict the same experimental data. For example, if considering 'Paper Cups', any necessary parameters could be calculated using say two paper cups and then using this to predict the outcomes in the other experiments with other numbers of cups. Again, varying the mass used on a parachute can provide more than one set of data.

A set of numeric predictions must be produced which can be compared with the experimental data. Where possible this should be in table form, although a graph may be produced as well.

Descriptive predictions do not meet this criterion. It is not enough to predict that the greater the mass attached to a parachute the quicker it falls, for example. This does not provide any numeric data for comparison.

Also it is not enough calculate a series of parameters and simply comment upon their consistency or otherwise.

There must be comparison with observed data.

Domain 3: Conducting the experiment
Check through your assumptions and see which ones are relevant to the design of the experiment. For example, if you are assuming that an object is falling vertically, is there any way that you can ensure this?

Describe the conduct of the experiment fully; don't assume that the person reading and marking your script knows anything about it. A diagram is important. Also, what steps have been taken to reduce experimental errors? For example, how can you ensure that a parachute is released from rest? How have the measurements been taken so that errors can be reduced?

Are there sufficient results? As a minimum there should be three repetitions so that an average, a maximum and minimum are possible when comparing. Also, as was mentioned earlier, will there be at least two series of measurements to avoid a circular argument? This will, with a little discussion, also meet the final criterion in this domain.

Domain 4: Comparison between the experimental results and the predictions of the model
Compare the data both graphically and in table form if at all possible. Label the graphs so that it is clear which line represents the predictions and which the collected data. Don't forget to title the graph and label the axes.

For the second mark in this domain, add error bounds to the graph. These are the values; average, maximum and minimum from the repetitions. Then you can see whether the predicted values fall within these bounds.

Domain 5: Revision of the process.
For the revision phase you can either improve the model and/or suggest ways in which the experiment could be improved. Unless the original model was particularly sophisticated and the solution of an improvement outside the confines of this module, it is generally expected that there should be a solution of the revised model.

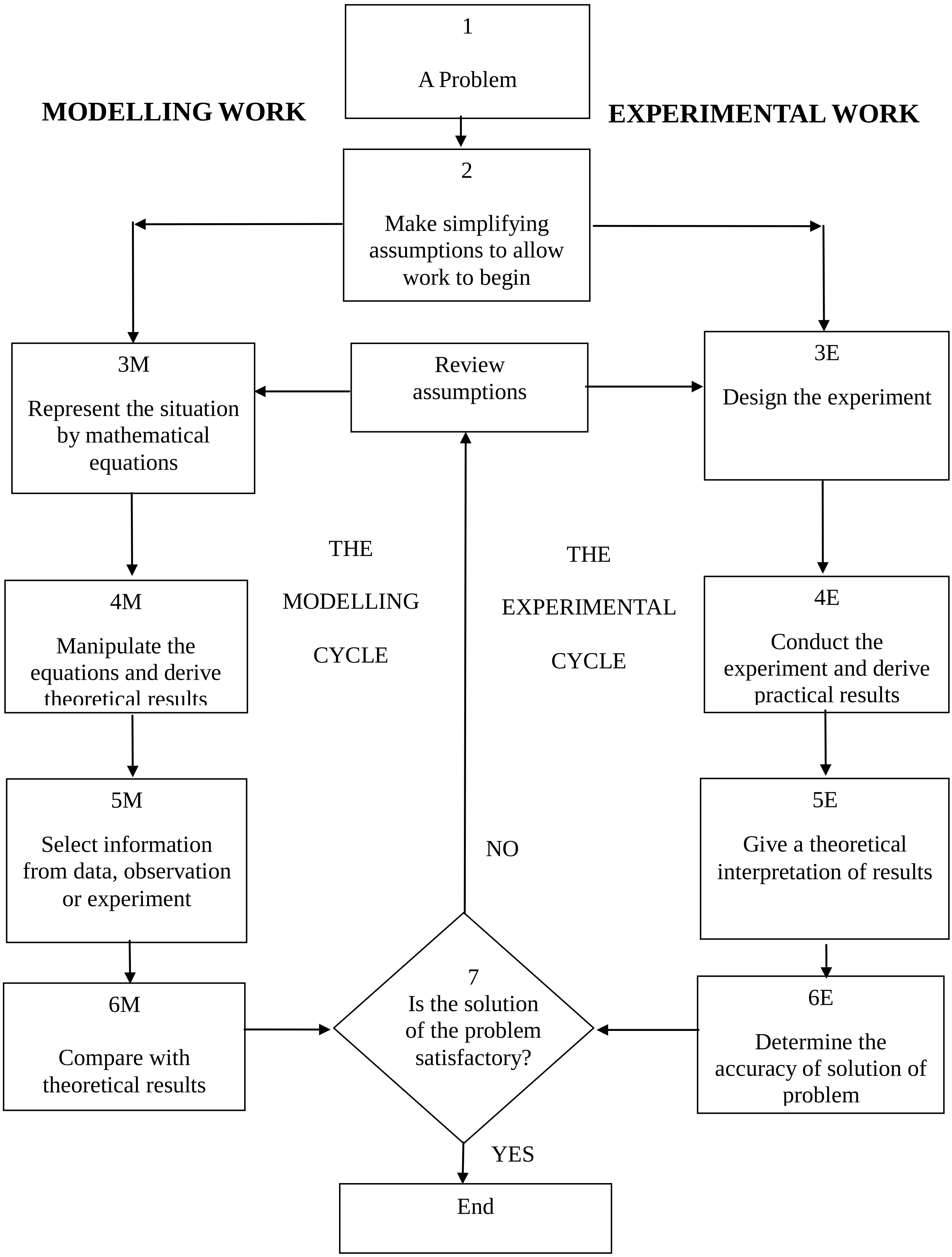
Having made the comparison it is likely that the initial model is not accurate enough. You must now return to the assumptions. What assumptions that have been made need to be changed or, perhaps not ignored? Justify why you are making the change. Why, for example, might you consider air resistance being proportional to v^2, not some other power of v? Avoid curve fitting. It is essential to *justify* the changes by considering new assumptions rather than attempting to produce a 'look alike' graph.

An example of curve fitting in 'Cascades' is to assume that the flow is proportional to some power of h and then finding which power of h fits best.   Referring to laws of fluid dynamics would be more appropriate.

Domain 6: Assessment of the improvement obtained.
Finally solve the improved model. Where possible this will be done analytically but in some cases a numerical method will be employed. If a solution is not possible then try to describe the effect of the proposed changes or your proposed extension.

Domain 7: Oral Communication
To conclude you will be asked to either give a presentation of your work, outlining to the teacher or, perhaps the other members of the group what you have investigated and what were the outcomes. Alternatively, your teacher may ask you questions about your work, asking you to explain what you have done and perhaps to clarify some sections of your work.

Modelling Flowchart



Differential Equations (DE) Coursework: Assessment Sheet (A)
Work based on the Modelling Cycle

Task: Candidates will model a real-life situation of their own choice which requires the use of differential equations.

Coursework Title				
Candidate Name		**Candidate Number**		
Centre Number		**Date**		

Domain	**Mark**	**Description**	**Comment**	**Mark**
Simplifying the situation and setting up the model. (3)	1 1 1	There are clear statements of the assumptions made and an awareness is shown of the relative importance of these assumptions. The relevance of the assumptions to the initial equations of the model is explained clearly. Differential equation(s) to model the situation are established and justified.		
Manipulating the model. (5)	1 1 1 1 1	A correct methods of solution is applied to the differential equation(s). A solution to the differential equation(s) is obtained. Values for the parameters of the equation(s) are chosen and justified. There is a quantitative consideration of the effects of possible variation in the values of the parameters. A set of predictions is produced from the equation(s).		
The collection of data to verify the model. (2)	1 1	The source and means of collecting the data are clearly described and their relevance to the model is demonstrated. The data are presented in a form suitable for comparison with the predictions of the model.		
Comparison between the data collected and the predictions of the model. (2)	1 1	A clear comparison is made, using diagrams and graphs where appropriate. The effects of variation in the parameters are taken into account.		
Revision of the model. (2)	1 1	In the light of the comparison, proposals are made for amending the initial assumptions to improve the quality of the model. New equations are established as a result of the amended assumptions.		
Assessment of the improvement obtained. (2)	1 1	The new equations are manipulated to produce new predictions. A comparison is made between the data and the new predictions, and comments made on whether an improvement has been obtained.		
Oral communication. (2)	2	Presentation ☐ Interview ☐ Discussion ☐	Please tick at least one box and give a brief report.	

Half marks may be awarded but the overall total must be an integer. Please report overleaf on any help that the candidate has received beyond the guidelines.	**Total**	/ 18

Authentication
Teachers should ensure that an OCR Declaration sheet is completed for all candidates and sent with the tasks to the moderator.

Coursework must be available for moderation by OCR.

Differential Equations (DE) Coursework: Assessment Sheet (B)
Work based on a combination of the Modelling and Experimental Cycles

Task: Candidates will model a real-life situation of their own choice which requires the use of differential equations.

Coursework Title			
Candidate Name		**Candidate Number**	
Centre Number		**Date**	

Domain	**Mark**	**Description**	**Comment**	**Mark**
Simplifying the situation and setting up the model. (3)	1 1 1	There are clear statements of the assumptions made and an awareness is shown of the relative importance of these assumptions. The relevance of the assumptions to the initial equations of the model is explained clearly. Differential equation(s) to model the situation are established and justified.		
Manipulating the model. (4)	1 1 1 1	A correct methods of solution is applied to the differential equation(s). A solution to the differential equation(s) is obtained. Values for the parameters of the equation(s) are chosen and justified. A set of predictions is produced from the equation(s).		
Conducting the experiment. (4)	1 1 1 1	The relevance of the assumptions to the design of the experiment is explained clearly. The conduct of the experiment is described clearly, including diagrams of the apparatus used. Any steps taken to reduce experimental error are also described. Sufficient results are obtained, and these are presented clearly and concisely, in a form suitable for comparison with the predictions of the model. There is a discussion of the variability in the measurements taken.		
Comparison between the experimental results and the predictions of the model. (2)	1 1	A clear comparison is made, using diagrams and graphs where appropriate. The effects of variation in the parameters and in the measurements are taken into account.		
Revision of the process. (2)	1 1	In the light of the comparison, a decision is made, and justified, on whether further revision is needed to the modelling process, or to the conduct of the experiment, or both. If revision is needed (or if the original assumptions led to a trivial piece of modelling), detailed proposals are made for amending the assumptions and/or the conduct of the experiment. If not, there is a serious discussion of how the work could be extended to related situations.		
Assessment of the improvement obtained. (1)	1	There is an investigation into applying the effects of the proposed revision/extension. [Note: it is not necessary for the candidate to rework the model completely unless the initial differential equation(s) was (were) trivial.]		
Oral communication. (2)	2	Presentation ☐ Interview ☐ Discussion ☐	Please tick at least one box and give a brief report.	

Half marks may be awarded but the overall total must be an integer. Please report overleaf on any help that the candidate has received beyond the guidelines.	**Total**	/ 18