04-05-3 非电类期中试卷答案

一、选择题

C　C　B　A

二、填空题

1. $\frac{1}{2} \le x < \frac{3}{2}$　　2. $-2 < x < 4$　　3 $\begin{cases} (1-x-y)^2 = 4(x^2-y) \\ z = 0 \end{cases}$　　4. $dx - \sqrt{2}dy$

5. $8,\quad (2,4,4)$　　6. $\frac{1}{2}$

三、计算题

1. 设平面束方程为 $(2x+3y+9z+5)+\lambda(x+y+z+1)=0$

即 $(2+\lambda)x+(3+\lambda)y+(9+\lambda)z+(5+\lambda)=0$

该平面与直线平行即：$(2+\lambda)2+(3+\lambda)-(9+\lambda)=0$

解得 $\lambda=1$。故所求平面为 $3x+4y+10z+6=0$

2. $$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = 3-\pi$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx=\frac{2}{n^2\pi}[1-(-1)^n]$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx=\frac{1}{n\pi}[1-(-1)^n]$$

$$S(x)=\begin{cases} f(x) & x\in(-\pi,0)\bigcup(0,\pi) \\ \frac{3}{2} & x=0 \\ \frac{3-2\pi}{2} & x=\pm\pi \end{cases}$$

3. $$\frac{\partial z}{\partial x}= f + x(f_1' y + f_2' e^{xy} y) = yf + xy f_1' + xy e^{xy} f_2'$$

$$\frac{\partial^2 z}{\partial y \partial x} = xf_1 + e^{xy} x f_2 + xf_1 + xy^2 f_{11} + x^2 y e^{xy} f_{12} + xe^{xy} f_2 + x^2 y e^{xy} f_2$$

$$+ x^2 y e^{xy} f_{21} + x^2 y e^{2xy} f_{22}$$

$$= 2xf_1 + (2xe^{xy} + x^2 y e^{xy}) f_2 + x^2 y f_{11} + 2x^2 y e^{xy} f_{12} + x^2 y e^{2xy} f_{22}$$

4. $\begin{cases} x' = 4/3 \\ y' = 2t \\ z' = 3t^2 \end{cases}$ 曲线与平面法向垂直即：$\frac{4}{3} + 4t + 3t^2 = 0$ 解得 $t = -\frac{2}{3}$

故所求的点为 $(-\frac{8}{9}, \frac{4}{9}, -\frac{8}{27})$

5. $y = \frac{1}{x^2(4-x^2)} = \frac{1}{4}(\frac{1}{x^2} + \frac{1}{4-x^2}) = \frac{1}{4}\frac{1}{x^2} + \frac{1}{16}\frac{1}{2-x} + \frac{1}{16}\frac{1}{2+x}$

$$= \frac{1}{4}(\frac{-1}{1+(x-1)})' + \frac{1}{16}\frac{1}{1-(x-1)} + \frac{1}{16}\frac{1}{3+(x-1)}$$

$$= -\frac{1}{4}\left(\sum_{n=0}^{\infty}(-1)^n (x-1)^n\right)' + \frac{1}{16}\sum_{n=0}^{\infty}(x-1)^n + \frac{1}{48}\sum_{n=0}^{\infty}(-1)^n \frac{(x-1)^n}{3^n}$$

$$=\frac{1}{4}\sum_{n=1}^{\infty}(-1)^{n-1}n(x-1)^{n-1}+\frac{1}{16}\sum_{n=0}^{\infty}(x-1)^{n}+\frac{1}{48}\sum_{n=0}^{\infty}(-1)^{n}\frac{(x-1)^{n}}{3^{n}}\quad(0<x<2)$$

四． 椭圆上点(x,y,z)到平面的距离为$d=\frac{|2x+2y+z+5|}{3}$

设目标函数为$F(x,y,z,\lambda)=(2x+2y+z+5)^2+\lambda(\frac{x^2}{2}+y^2+\frac{z^2}{4}-1)$

$$\begin{cases}F_x=4(2x+2y+z+5)+\lambda x=0\\F_y=4(2x+2y+z+5)+2\lambda y=0\\F_z=2(2x+2y+z+5)+\frac{1}{2}\lambda y=0\\F_\lambda=\frac{1}{2}x^2+y^2+\frac{1}{4}z^2-1=0\end{cases}$$

解得：$(1,\frac{1}{2},1)$，$(-1,-\frac{1}{2},-1)$，距离为$d_1=3,\ d_2=\frac{1}{3}$

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五． 曲面在点(x_0,y_0,z_0)处的切平面方程为

$$\frac{1}{\sqrt{x_0}}(x-x_0)+\frac{1}{\sqrt{y_0}}(y-y_0)+\frac{1}{\sqrt{z_0}}(z-z_0)=0$$

即 $\frac{1}{\sqrt{x_0}}x+\frac{1}{\sqrt{y_0}}y+\frac{1}{\sqrt{z_0}}z=\sqrt{a}$

则在三坐标轴上的截距为$\sqrt{ax_0},\sqrt{ay_0},\sqrt{az_0}$，和为$\sqrt{a}(\sqrt{x}+\sqrt{y}+\sqrt{z})=a$

六．级数在$[0,1]$上收敛，故在$(0,1)$内可以逐项求导得

$$F(x)=f(x)+f(1-x)+\ln x\ln(1-x)$$

$$F'(x)=\left(\sum_{n=1}^{\infty}\frac{x^n}{n^2}\right)'+\left(\sum_{n=1}^{\infty}\frac{(1-x)^n}{n^2}\right)'+\frac{\ln(1-x)}{x}-\frac{\ln x}{1-x}$$

$$=\sum_{n=1}^{\infty}\frac{x^{n-1}}{n}-\sum_{n=1}^{\infty}\frac{(1-x)^{n-1}}{n}-\frac{1}{x}\sum_{n=1}^{\infty}\frac{x^n}{n}-\frac{1}{1-x}\sum_{n=1}^{\infty}\frac{(-1)^n(x-1)^n}{n}$$

$$=0$$

故$F(x)$为常数C

$$C=\lim_{x\to0^+}F(x)=\lim_{x\to0^+}f(x)+\lim_{x\to0^+}f(1-x)+\lim_{x\to0^+}\ln x\ln(1-x)$$

$$=\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$$