05-3高A期中试卷答案

一．填空题（本题共 5 小题，每小题 4 分，满分 20 分）

1. $\dfrac{z\sin x-\cos y}{\cos x-y\sin z}\mathrm{d}x+\dfrac{x\sin y-\cos z}{\cos x-y\sin z}\mathrm{d}y$；**2.** $\mathrm{e}^{\frac{\pi}{2}+2k\pi}$；**3.** $f(2)$；**4.** 0；**5.** -12

二．单项选择题（本题共 4 小题，每小题 4 分，满分 16 分）：6. C；**7.** A；**8.** B；**9.** D

三. 计算下列各题(本题共 5 小题，每小题 7 分，满分 35 分)

10. $\dfrac{\partial z}{\partial x}=f_1\sin y+\dfrac{1}{y}f_2$

$$\frac{\partial^2 z}{\partial x\partial y}=f_1\cos y+\left(f_{11}x\cos y-\frac{x}{y^2}f_{12}\right)\sin y-\frac{1}{y^2}f_2+\frac{1}{y}\left(f_{21}x\cos y-f_{22}\frac{x}{y^2}\right)$$

$$=f_1\cos y-\frac{1}{y^2}f_2+\frac{1}{2}f_{11}x\sin 2y+\frac{x}{y^2}\left(y\cos y-\sin y\right)f_{12}-\frac{x}{y^3}f_{22}$$

11． $\dfrac{\partial v}{\partial y}=\dfrac{\partial u}{\partial x}=\mathrm{e}^x(x\cos y-y\sin y+\cos y)+1$， $v=\mathrm{e}^x(x\sin y+y\cos y)+y+\varphi(x)$，

$\dfrac{\partial u}{\partial y}=-\mathrm{e}^x(x\sin y+\sin y+y\cos y)=-\dfrac{\partial v}{\partial x}$， $\varphi'(x)=0,\varphi(x)=C$，

$v=\mathrm{e}^x(x\sin y+y\cos y)+y+C$， $f(z)=z\mathrm{e}^z+z+C$

12． 由 $\begin{cases}x^2+y^2=1\\x^2+y^2=2x\end{cases}$ 解得 $\varphi=\dfrac{\pi}{3}$，从而得积分区域为：$1\le\rho\le 2\cos\varphi,0\le\varphi\le\dfrac{\pi}{3}$，

$$\iint_D xy\mathrm{d}\sigma=\int_0^{\frac{\pi}{3}}\sin\varphi\cos\varphi\mathrm{d}\varphi\int_1^{2\cos\varphi}\rho^3\mathrm{d}\rho=\frac{9}{16}$$

13. $\displaystyle\iiint_\Omega\left(\sqrt{x^2+y^2+z^2}+x^2y\right)\mathrm{d}V=\iiint_\Omega\sqrt{x^2+y^2+z^2}\,\mathrm{d}V=\int_0^{2\pi}\mathrm{d}\varphi\int_0^{\frac{\pi}{2}}\sin\theta\mathrm{d}\theta\int_0^{2\cos\theta}\rho^3\mathrm{d}\rho$

$=\dfrac{8}{5}\pi$

14. L 的参数方程为 $x=2\cos t,y=2\sin t,z=\sqrt{5}\quad(0\le t\le 2\pi)$，$\mathrm{d}s=2\mathrm{d}t$

$$\int_L x^2\,\mathrm{d}s=8\int_0^{2\pi}\cos^2 t\mathrm{d}t=8\pi$$

四（15）所求曲面在 xOy 平面的投影区域为 $D: x^2+y^2 \le 1$，对曲面 $z = x^2+y^2$ 而言，

$\mathrm{d}S = \sqrt{1+4\left(x^2+y^2\right)}\,\mathrm{d}\,x\mathrm{d}y$，对曲面 $z = 2-\sqrt{x^2+y^2}$ 而言，

$\mathrm{d}S = \sqrt{2}\,\mathrm{d}\,x\mathrm{d}y$，$S = \int_0^{2\pi}\mathrm{d}\varphi\int_0^1\left(\sqrt{1+4\rho^2}+\sqrt{2}\right)\rho\mathrm{d}\rho = \left(\dfrac{5\sqrt{5}-1}{6}+\sqrt{2}\right)\pi$

五（16）设 $P(a,b,c)(a,b,c \ge 0)$ 是椭球面上的一点，切平面方程为

$\dfrac{x}{\frac{4}{a}}+\dfrac{y}{\frac{1}{b}}+\dfrac{z}{\frac{9}{c}}=1$，$V = \dfrac{1}{6}\cdot\dfrac{36}{abc} = \dfrac{6}{abc}$，$F = abc+\lambda\left(\dfrac{a^2}{4}+b^2+\dfrac{c^2}{9}-1\right)$，令

$F_a = bc+\dfrac{a\lambda}{2} = 0, F_b = ac+2b\lambda = 0, F_c = ab+\dfrac{2c\lambda}{9} = 0$，$\dfrac{a^2}{4}+b^2+\dfrac{c^2}{9}=1$

解得唯一驻点：$a = \dfrac{2}{\sqrt{3}}, b = \dfrac{1}{\sqrt{3}}, c = \sqrt{3}$，由于实际问题存在最小值，故点

$P\left(\dfrac{2}{\sqrt{3}},\dfrac{1}{\sqrt{3}},\sqrt{3}\right)$即为所求，$V_{\min} = 3\sqrt{3}$。

六（17）． $\varphi'(x)+\dfrac{\varphi(x)}{x} = \dfrac{\cos x}{x}$，$\varphi(x) = \dfrac{\sin x + C}{x}$，由 $\varphi\left(\dfrac{\pi}{2}\right) = 2$ 得 $C = \pi-1$，

$\varphi(x) = \dfrac{\sin x+\pi-1}{x}$，

$$\int_{(1,0)}^{(\pi,\pi)}\left(\cos x-\frac{\sin x+\pi-1}{x}\right)\frac{y}{x}\mathrm{d}x+\frac{\sin x+\pi-1}{x}\mathrm{d}y = \frac{(\sin x+\pi-1)y}{x}\Big|_{(1,0)}^{(\pi,\pi)} = \pi-1$$

七（18）． Σ 在 yOz 平面的投影区域为 $D_{yz}: y^2+z^2 \le a^2, z \le 0$，

$$I_1 = -2\iint\limits_{D_{yz}}\sqrt{a^2-y^2-z^2}\,\mathrm{d}y\mathrm{d}z = -2\int_{\pi}^{2\pi}\mathrm{d}\,\varphi\int_0^a\sqrt{a^2-\rho^2}\,\rho\mathrm{d}\rho = -\frac{2}{3}\pi a^3,$$

Σ 在 xOy 平面的投影区域为 $D_{xy}: x^2+y^2 \le a^2$，

$$I_2 = \frac{1}{a}\iint\limits_{D_{xy}}\left(a-\sqrt{a^2-x^2-y^2}\right)^2\mathrm{d}x\,\mathrm{d}\,y = \frac{1}{a}\int_0^{2\pi}\mathrm{d}\varphi\int_0^a\left(2a^2-2a\sqrt{a^2-\rho^2}-\rho^2\right)\rho\mathrm{d}\rho = \frac{\pi}{6}a^3$$

$I = I_1+I_2 = -\dfrac{\pi}{2}a^3$.