07-3高A期中试卷答案

一. 填空题(本题共 5 小题，每小题 5 分，满分 2 5 分)

1. $\int_0^2 \mathrm{d}y \int_{-\sqrt{4-y^2}}^{\sqrt{2y-y^2}} f(x,y)\mathrm{d}x$； 2. $\dfrac{xy}{z}$； 3. $\dfrac{\pi}{2}$； 4. $\dfrac{x-1}{0}=\dfrac{y}{1}=\dfrac{z-1}{0}$； 5. $2\sqrt{3}\pi$.

二. 单项选择题(本题共 4 小题，每小题 4 分，满分 16 分)

6. [B] 7. [D] 8. [A] 9. [C]

三. 计算下列各题(本题共 4 小题，每小题 9 分，满分 36 分)

10. 解 $\iint\limits_D \sqrt{x^2+y^2}\mathrm{d}x\mathrm{d}y = \int_0^{\frac{\pi}{2}}\mathrm{d}\varphi\int_0^{2\sin\varphi}\rho^2\mathrm{d}\rho = \dfrac{8}{3}\int_0^{\frac{\pi}{2}}\sin^3\varphi\mathrm{d}\varphi = \dfrac{16}{9}$

11. 解 $\mathrm{grad}u\big|_P = \left\{y\mathrm{e}^{-(xy)^2}, x\mathrm{e}^{-(xy)^2}, -\mathrm{e}^{-z^2}\right\}_P = \left\{\dfrac{1}{\mathrm{e}}, \dfrac{1}{\mathrm{e}}, -\dfrac{1}{\mathrm{e}}\right\}$，单位法向量为

$\mathbf{n} = \pm\left\{\dfrac{3}{\sqrt{14}}, \dfrac{2}{\sqrt{14}}, \dfrac{1}{\sqrt{14}}\right\}$， $\dfrac{\partial u}{\partial n}\Big|_P = \pm\left\{\dfrac{3}{\sqrt{14}}, \dfrac{2}{\sqrt{14}}, \dfrac{1}{\sqrt{14}}\right\}\cdot\left\{\dfrac{1}{\mathrm{e}}, \dfrac{1}{\mathrm{e}}, -\dfrac{1}{\mathrm{e}}\right\} = \pm\dfrac{4}{\sqrt{14}\mathrm{e}}$.

12. 解 $\iiint\limits_\Omega (xy^2+z^2)\mathrm{d}V = \iiint\limits_\Omega z^2\mathrm{d}V = \pi\int_1^4 z^3\mathrm{d}z = \dfrac{255}{4}\pi$.

13. 解 投影区域 $D_{xy} = \left\{(x,y)\,\middle|\, x^2+y^2 \le Ry\right\}$，

$$\iint\limits_\Sigma \frac{x^2+y^2+R^2}{\sqrt{x^2+y^2+z^2}}\mathrm{d}A = 2R\iint\limits_\Sigma \mathrm{d}A - \frac{1}{R}\iint\limits_\Sigma\left(R^2-x^2-y^2\right)\mathrm{d}A$$

$$= 2R^2\iint\limits_{D_{xy}}\frac{1}{\sqrt{R^2-x^2-y^2}}\mathrm{d}x\mathrm{d}y - \iint\limits_{D_{xy}}\sqrt{R^2-x^2-y^2}\mathrm{d}x\mathrm{d}y$$

$$= 2R^2\int_0^\pi \mathrm{d}\varphi\int_0^{R\sin\varphi}\frac{\rho}{\sqrt{R^2-\rho^2}}\mathrm{d}\rho - \int_0^\pi\mathrm{d}\varphi\int_0^{R\sin\varphi}\sqrt{R^2-\rho^2}\,\rho\mathrm{d}\rho$$

$$= \frac{5}{3}\pi R^3 - \frac{32}{9}R^3$$

四（14）解 $m = 12\int_L x\mathrm{d}s = 12\int_0^1 x\sqrt{1+4x^2}\mathrm{d}x = 5\sqrt{5}-1$

五(15)解 $L = x^2 + y^2 + z^2 + \lambda(x^2 + y^2 - z) + \mu(x + y + z - 4)$，$L_x = 2x + 2\lambda x + \mu = 0$，

$L_y = 2y + 2\lambda y + \mu = 0$, $L_z = 2z - \lambda + \mu = 0$, $x^2 + y^2 - z = 0$, $x + y + z - 4 = 0$

解得 $M_1(-2,-2,8), M_2(1,1,2)$，由问题的实际意义知，M_1 为最远点，M_2 为最近点

$d_{\max} = 6\sqrt{2}$, $d_{\min} = \sqrt{6}$

六（16）解 记 $f^2(z) = U(x,y) + \mathrm{i}V(x,y)$, $V(x,y) = 2u(x,y) \cdot v(x,y) = 4xy(x^2 - y^2)$,

$\dfrac{\partial V}{\partial y} = 4x(x^2 - 3y^2) = \dfrac{\partial U}{\partial x}$, $U = x^4 - 6x^2y^2 + \varphi(y)$,

$\dfrac{\partial U}{\partial y} = -12x^2y + \varphi'(y) = -\dfrac{\partial V}{\partial x} = -12x^2y + 4y^3$, 于是 $\varphi(y) = y^4 + C$,

$U = x^4 + y^4 - 6x^2y^2 + C$（$C$ 为常数）,

$f^2(z) = x^4 + y^4 - 6x^2y^2 + C + \mathrm{i}4xy(x^2 - y^2)$，$f^2(x) = x^4 + C$，$f^2(z) = z^4 + C$