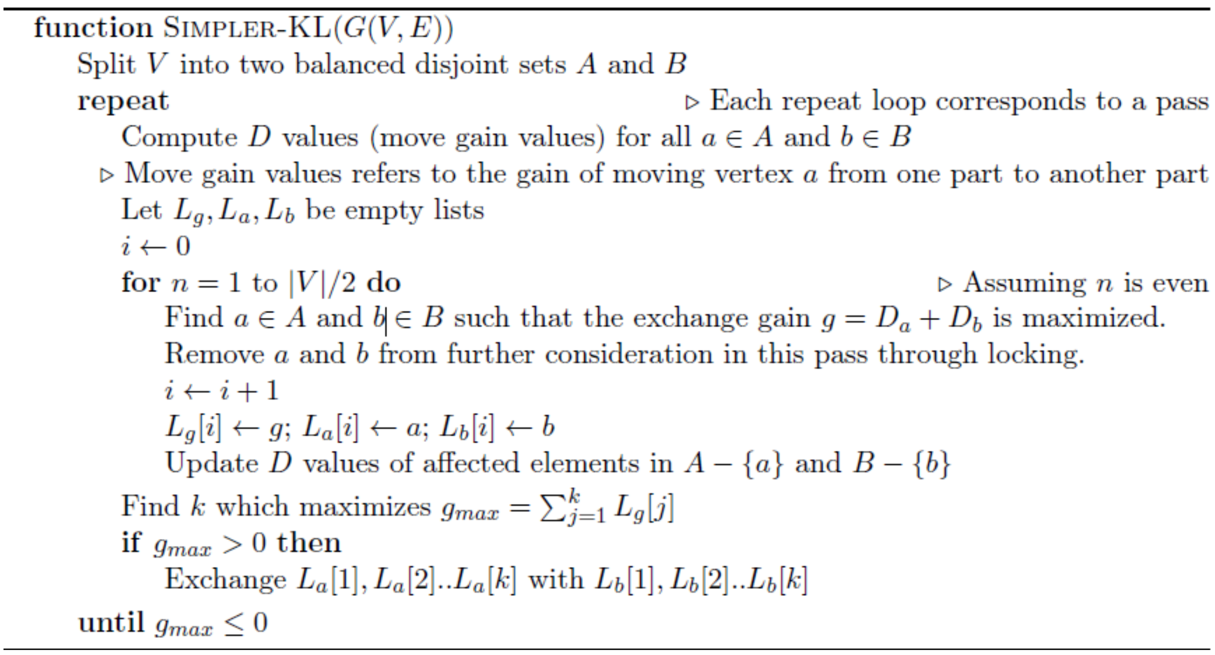
Kernighan-Lin (KL) Algorithm

Kernighan-Lin (KL) algorithm is a famous heuristic algorithm for partitioning (dividing) the vertices of a graph into two balanced parts while minimizing the cutsize (the number of edges crossing between the two parts). The balance criteria is defined such that the number of vertices in each part differ at most by 1 (assuming unit-weight vertices). Read the original paper by Kernighan and Lin in order to understand the algorithm. Consider the following pseudocode of a simplified KL:



```
function SIMPLER-KL($G(V, E)$)
    Split $V$ into two balanced disjoint sets $A$ and $B$
    repeat                                    ▷ Each repeat loop corresponds to a pass
        Compute $D$ values (move gain values) for all $a \in A$ and $b \in B$
       ▷ Move gain values refers to the gain of moving vertex $a$ from one part to another part
        Let $L_g, L_a, L_b$ be empty lists
        $i \leftarrow 0$
        for $n = 1$ to $|V|/2$ do                               ▷ Assuming $n$ is even
            Find $a \in A$ and $b \in B$ such that the exchange gain $g = D_a + D_b$ is maximized.
            Remove $a$ and $b$ from further consideration in this pass through locking.
            $i \leftarrow i + 1$
            $L_g[i] \leftarrow g$; $L_a[i] \leftarrow a$; $L_b[i] \leftarrow b$
            Update $D$ values of affected elements in $A - \{a\}$ and $B - \{b\}$
        Find $k$ which maximizes $g_{max} = \sum_{j=1}^{k} L_g[j]$
        if $g_{max} > 0$ then
            Exchange $L_a[1], L_a[2]..L_a[k]$ with $L_b[1], L_b[2]..L_b[k]$
    until $g_{max} \leq 0$
```

Note that the difference between this simplified version and the original algorithm is the use of a more relaxed definition of exchange gain . That is, in the original KL definition the Exchange gain is defined as g = D_a + D_b – 2 if there is an edge (a,b) ∈ **E,** whereas g = D_a + D_b otherwise.

In the simplified KL, however, we assume that g = D_a + D_b fort he sake of easier implementation.

You are asked to do the following tasks:

1. Implement the above simplified version of the KL algorithm in C considering the following two approaches for finding an exchange with maximum gain:

a) Maintain the vertices in the two parts as binary heaps and perform extract max operation on each heap at each iteration. What is the asymptotic worst-case running time of the algorithm?

b) Go over each part in order to find the maximum D values at each iteration. What is the asymptotic worst-case running time of the algorithm?

Your program should take the name of a graph file as input, and prints three values as output (one line, space or tab separated): initial cutsize, final cutsize, and runtime for both a) and b) on the same line. The input graph file is stored as list of edges, where each line represents an edge as "SRC DST", where SRC and DST are receptively the source and destination vertex ids.

2. Compare your program against Python's NetworkX library implementation of KL in terms of final cutsize and runtime for the following graphs which can be obtained from SuitSparse Matrix Collection [1]:

- Erdos02
- com-DBLP
- rgg_n_2_20_s0

You may use either graphical plots or tables for the comparison. Which runs faster and why?

3. Use GNU profiler [2] (gprof) to analyze the actual running time of your program and to see where most of the time is spent. Compare the asymptotic running times of the steps of your algorithm with the actual runtimes, observe any ineffciencies (for instance, if an O(n)-time operation takes much more time than an O(n^2) operation) and try to improve the overall performance by focusing on improving the most time-consuming operations/steps.

4. Prepare a report of maximum two pages that contains:

_ a brief explanation of your implementation

_ discuss which of 1.a) or 1.b) is more effcient in case the maximum degree (max edges per vertex) of the input graph is limited to a constant C.

_ the comparison requested in task #3

_ your conclusions and improvements done to the code after profiling

_ and any other code-based/asymptotic improvements you have performed and would like to highlight.

- Use GNU Make[3] tool to automate the process (you should be familiar with this from CS-201/202). Your executable is called KL and takes only one argument. Example compile and run:

```
> make                      #This should generate a binary executable file called KL
> ./KL input_graph.mtx      #runs the code with input graph file
> 1530 440 3.6s 1530 440 2.8s        #output (initial_cut final_cut runtime)
```

  Before submission, make sure that your code can be compiled on a Unix-like environment using gcc. If you use Windows OS, you can either use Cygwin or newly added windows subsystem for Linux (WSL) feature of Windows 10.

- The Matrix Market (.mtx) files has the following format for storing an undirected, unweighted graphs:

```
% some comment lines
50 50 230 %#Vertices #Vertices #Edges
1 3 %SRC DST
3 1
...
```

What is needed?

- **Source code files (.c and .h files)**
- **Makefile to be used to compile and build you program**
- **Report.pdf**
- **README.txt file that contains any (potential) technical issues.**
- **Graph input files**

References:

[1] https://sparse.tamu.edu/

[2] https://ftp.gnu.org/old-gnu/Manuals/gprof-2.9.1/html_node/gprof_toc.html

[3] https://www.gnu.org/software/make/

[4] B. W. Kernighan and S. Lin, "An effcient heuristic procedure for partitioning graphs," in The Bell System Technical Journal, vol. 49, no. 2, pp. 291-307, Feb. 1970, doi: 10.1002/j.1538-7305.1970.tb01770.x.