MAST30013 - Techniques in Operations Research: Assignment 1

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Question 1

1(a):

Below is a plot of $f(x) = 8e^{1-x} + 7log(x)$ in the interval $[1, 2]$:

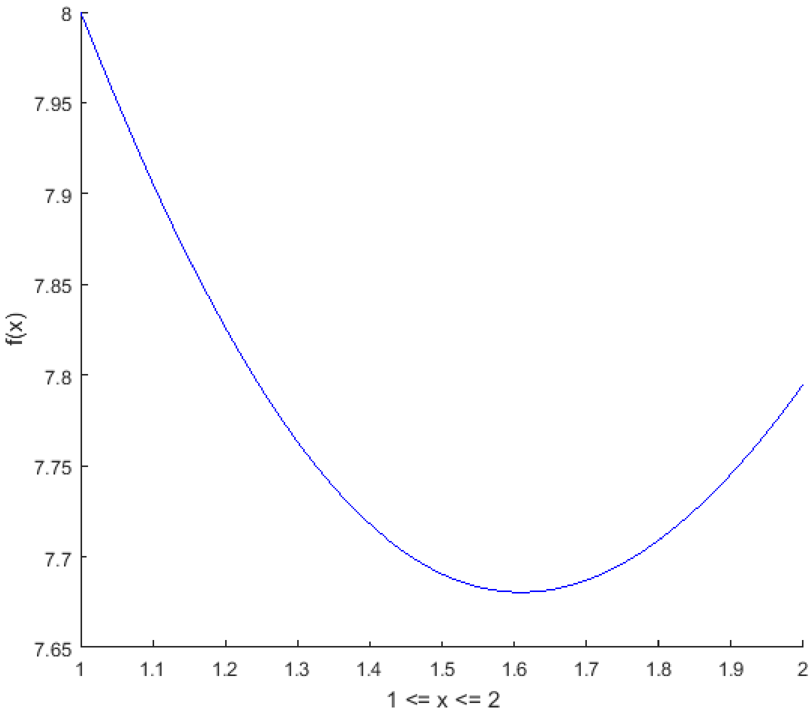

Figure 1: Plot of $f(x) = 8e^{1-x} + 7log(x)$ in the interval $[1, 2]$:

The above plot clearly shows that $f(x)$ is unimodal since it only has one local minima.

1(b):

In order to perform Fibonacci Search we first need to find the smallest n such that $F_n(\alpha) = (b - a)$. Thus, we need to find an n where $\alpha < 2\epsilon$ or we need:

$$(b - a)/F_n < 2\epsilon \tag{1}$$

$$F_n > (b - a)/2\epsilon \tag{2}$$

Setting $b = 2, a = 1$ and $\epsilon = 0.08$ we get:

$$F_n > \frac{2-1}{2 * 0.08} \tag{3}$$

$$F_n > \frac{1}{0.16} \tag{4}$$

$$F_n > 6.25 \tag{5}$$

So, $F_n = 8$ and $n = 5$.

Now, we know $f(x)$, we know the interval is $[1, 2]$ and the initial $k = 5$. We will also use the following formulas for p and q:

$$a = 1 \tag{6}$$

$$b = 2 \tag{7}$$

$$p = b - \frac{F_{k-1}}{Fk}(b - a) \tag{8}$$

$$q = a + \frac{F_{k-1}}{Fk}(b - a) \tag{9}$$

After, that we will use $f(p)$ and $f(q)$ to evaluate how to reduce the intervals.

$$k = n = 5 \tag{10}$$

$$p = 2 - \frac{5}{8} * (1) = 2 - \frac{5}{8} = 1.375 \tag{11}$$

$$q = 1 + \frac{5}{8} * (1) = 1 + \frac{5}{8} = 1.625 \tag{12}$$

$$f(p) = 7.7275 \tag{13}$$

$$f(q) = 7.6806 \tag{14}$$

Now, since $f(p) > f(q)$ the new interval will have:

$$a = p = 1.375 \tag{15}$$

$$b = 2 \tag{16}$$

$$p = q = 1.625 \tag{17}$$

$$k = n = 4 \tag{18}$$

$$q = 1.375 + \frac{3}{5} * (0.625) = 1.75 \tag{19}$$

$$f(p) = 7.6806 \tag{20}$$

$$f(q) = 7.6962 \tag{21}$$

Now, since $f(q) > f(p)$ the new interval will have:

$$a = 1.375 \tag{22}$$
$$b = q = 1.75 \tag{23}$$

$$q = p = 1.625 \tag{24}$$

$$k = n = 3 \tag{25}$$

$$p = 1.75 - \frac{2}{3} * (0.375) = 1.5 \tag{26}$$

$$f(p) = 7.6905 \tag{27}$$

$$f(q) = 7.6806 \tag{28}$$

Now, since $f(p) > f(q)$ the new interval will have:

$$a = p = 1.5 \tag{29}$$
$$b = 1.75 \tag{30}$$

$$p = q = 1.625 \tag{31}$$

$$k = 2 \tag{32}$$

Because $k = 2$ we will have to set q as below:

$$q = a + 2\epsilon = 1.660 \tag{33}$$

$$f(p) = 7.6806 \tag{34}$$

$$f(q) = 7.6825 \tag{35}$$

Now, since $f(p) < f(q)$ the new interval will have:

$$a = 1.5 \tag{36}$$
$$b = q = 1.660 \tag{37}$$

Thus, the final interval found using fibonacci search is $[1.5, 1.66]$ which is withing the tolerance required. Thus $x^* = 1.58 \pm 0.08$.

Question 2

Let us first try to find a b such that $[0, b]$ contains the minimum. But first we need to check that $f(x)$ is unimodal. Given:

$$f(x) = (40x + 1)log(40x + 1)200x \tag{38}$$

Plotting $f(x)$ in the interval $[0, 12]$ we get

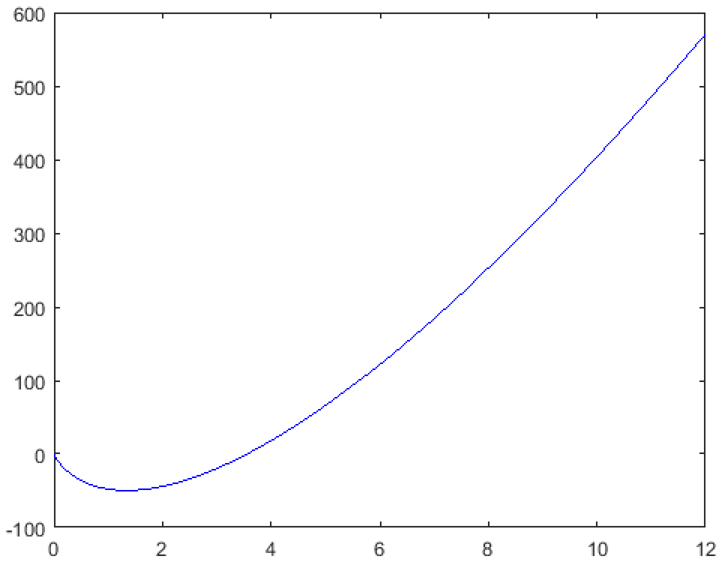

Figure 2: Plot of $f(x) = (40x+1)log(40x+1) - 200x$ in the interval $[0, 12]$:

Also, let f' is:

$$f'(x) = 40log(40x+1) - 160 \tag{39}$$

Setting $f'(x) = 0$:

$$\begin{aligned}
f'(x) &= 0 && (40)\\
40log(40x+1) - 160 &= 0 && (41)\\
log(40x+1) - 4 &= 0 && (42)\\
log(40x+1) &= 4 && (43)\\
x &= \frac{e^4 - 1}{40} && (44)\\
& && (45)
\end{aligned}$$

This shows there is one value for which $f'(x) = 0$ and from the graph we can see this value to be between 0 and 5. Thus, the given function is unimodal and continuous (from the graph and because there are no points where the function wouldn't be defined in $[0, \infty]$ allowing us to use Golden Section Search.

Let us try to find a good interval to do our search in. Assuming T (a small increment): $T = 1$

Set $k = 1$:

$$p = 0 \tag{46}$$

$$q = T \tag{47}$$

$$f(p) = f(0) = 0 \tag{48}$$

$$f(q) = f(1) = -47.7435 \tag{49}$$

So, $f(p) > f(q)$. Set $k = 2$:

$$p = q = 1 \tag{50}$$

$$q = p + 2^{k-1}T = 1 + 2 * 1 = 3 \tag{51}$$

$$f(p) = f(1) = -47.7435 \tag{52}$$

$$f(q) = -19.7093 \tag{53}$$

Thus, $f(p) < f(q)$, meaning the minimum lies in the range: $[0, 3]$. As $f(p) > f(q)$ for $k = 1$ and $f(p) < f(q)$ for $k = 2$.

We can now perform the Golden Section Search in the interval $[0, 3]$. Now, let us determine the number of calculations needed ($a = 0$, $b = 3$, $\epsilon = 0.3$):

$$a = 0 \tag{54}$$

$$b = 3 \tag{55}$$

$$\gamma^n(b-a) < 2\epsilon \tag{56}$$

$$0.6180^n < \frac{0.6}{3} \tag{57}$$

$$0.6180^n < 0.2 \tag{58}$$

$$0.6180^4 = 0.1459 < 0.2 \tag{59}$$

Thus, $n = 4$ which means we need to do 5 f calculations.

$$p = b - \gamma(b-a) = 3 - 0.6180(3) = 1.146 \tag{60}$$

$$q = a + \gamma(b-a) = 0 + 0.6180(3) = 1.854 \tag{61}$$

$$f(p) = -49.0182 \tag{62}$$

$$f(q) = -46.1361 \tag{63}$$

Since, $f(q) > f(p)$, we can set the new $b = q = 1.854$ and $a = 0$ remains the same. We have 3 f calculations left (Also, $q = p = 1.146$):

$$p = 1.854 - 0.6180 * (1.854) = 0.7082 \tag{64}$$

$$f(p) = -42.5542 \tag{65}$$

$$f(q) = -49.0182 \tag{66}$$

Since, $f(p) > f(q)$. We get $a = p = 0.7082$, $p = q = 1.146$ and $b = 1.854$ remains the same. We now have just 2 f calculations left.

Now,

$$q = 0.7082 + 0.6180 * (1.1458) = 1.4163 \tag{67}$$

$$f(p) = -49.0182 \tag{68}$$

$$f(q) = -49.5141 \tag{69}$$

Since, $f(p) > f(q)$, we make $a = p = 1.146$, $p = q = 1.4163$ and $b = 1.854$ remains the same. Doing the last f calculation.

$$q = 1.146 + 0.6180 * (0.708) = 1.5834 \tag{70}$$

$$f(p) = -49.5141 \tag{71}$$

$$f(q) = -48.7760 \tag{72}$$

Finally, since $f(q) > f(p)$ then the final $a = 1.146$ and the final $b = q = 1.5834$. Thus, the interval for minimisation is $[1.146, 1.583]$. Which means $x^* = 1.3645 \pm 0.2185$.

Question 3

Given:
$f(x) = (2x1)^3 + 4(410x)^4$
$f'(x) = 3(2x-1)^2 + 16(4-10x)^3$
Now, let $a = 0$ and $T = 1$ such that,

$$p = a = 0 \tag{73}$$

$$q = a + T = 1 \tag{74}$$

$$f(p) = -1 + 1024 = 1023 \tag{75}$$

$$f(q) = 1 + 5184 = 5185 \tag{76}$$