

$$C = a \wedge b$$

a	b	$a \wedge b$	c
0	0	0	0
0	1	0	0
1	0	0	0
1	1	1	1

?  
 $\Leftrightarrow$

"c is true"  
 $C \wedge a \wedge b$   
 OR  
 "c is false"  
 $\neg C \wedge (a \wedge b)$

$$\neg (a \wedge b)$$

$$\Rightarrow \neg a \vee \neg b$$

$$\equiv \neg C \wedge (\neg a \vee \neg b)$$

$$\Rightarrow (\neg C \wedge \neg a) \vee (\neg C \wedge \neg b)$$

$\neg C$	$\neg a$	$\neg b$	$\alpha$ $\neg C \wedge \neg a$	$\beta$ $\neg C \wedge \neg b$	$\alpha \vee \beta$
1	1	1	1	1	1
1	1	0	1	0	1
1	0	1	0	1	1
0	1	1	0	0	0

$s_{True} \text{ or } s_{False}$

$\Downarrow$  CNF

$\neg s_{True} \text{ and } \neg s_{False}$

a b | c s  
1 2 3 4

AND of ORs

$$\begin{aligned} C_{ID} &= 3 \\ C_{VAL} &= a \wedge b \\ C_{TRUE} &= C_{ID} \wedge C_{VAL} \\ C_{FALSE} &= \neg C_{ID} \wedge \neg C_{VAL} \end{aligned}$$

$$\begin{array}{cc} C_{TRUE} & \text{OR} & C_{FALSE} \\ \hline \downarrow \text{CNF} & & \\ \neg C_{TRUE} & \text{AND} & \neg C_{FALSE} \end{array}$$

$$\begin{aligned} \neg C_{TRUE} &= \neg [C_{ID} \wedge C_{VAL}] \\ &= \neg C_{ID} \vee \neg C_{VAL} \\ &\quad \text{canonicalize to AND of ORs} \\ &\quad \downarrow \\ &\text{"AND" of } [( \quad ), ( \quad ), ( \quad ) \dots] \\ &\quad \downarrow \quad \downarrow \quad \downarrow \\ &\quad \text{OR} \quad \text{OR} \quad \text{OR} \\ &\quad \text{then distribute} \\ &\text{map } (1 \text{ ors} \rightarrow \text{ors}; \neg C_{ID}) \neg C_{VAL} \end{aligned}$$

$$\begin{aligned} \Rightarrow A \vee [(B \vee C) \wedge (D \vee E)] \\ (A \vee (B \vee C)) \wedge (A \vee (D \vee E)) \\ \downarrow \\ (A \vee B \vee C) \wedge (A \vee D \vee E) \end{aligned}$$