LAMMbert: an automated market maker defined by the Lambert W function

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1 Introduction

Two popular automated market makers (AMM) are the constant-sum (CSMM) and constant-product (CPMM) market makers. A simple version of the CSMM is defined by the equation

$$x + y = 2. \tag{1}$$

A simple version of the CPMM is defined by the equation

$$xy = 1. \tag{2}$$

Our goal is to combine these two functions in such a way as to reveal a relationship between AMMs and the Lambert W function. We begin by multiplying through equation (1) by a liquidity concentration parameter $c \geq 0$ to get

$$c(x + y) = 2c. \tag{3}$$

We then take the natural log of both sides of equation (2) to get

$$\ln xy = 0. \tag{4}$$

Then adding equations (3) and (4) we obtain

$$c(x + y) + \ln xy = 2c. \tag{5}$$

This equation combines the CSMM and CPMM equations in an interesting way that is similar to StableSwap [2]. When $c = 0$ we have a pure CPMM equation, when c grows large it tends towards a pure CSMM equation.

We now show that equation (5) can be solved using the Lambert W function. Exponentiating both sides, and rearranging, we have

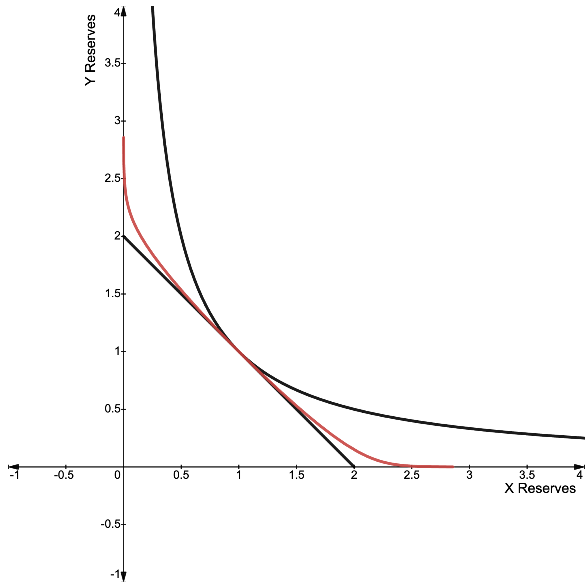

Figure 1: **The LAMMbert automated market maker**. Constant-sum and constant-product market makers can be combined to give an intermediate curve that depends on a concentration parameter. The curve is explicitly defined in terms of the Lambert W function. Desmos link here.

$$xe^{cx} = \frac{e^{2c-y}}{y}. \tag{6}$$

The Lambert W function is defined such that $W(xe^x) = x$. So we can re-write the prior equation by taking W of both sides, to get

$$x = \frac{1}{c} \cdot W\left(\frac{e^{2-y}}{y}\right). \tag{7}$$

Thus equation (6) can be written in explicit form using the Lambert W function. We call this the LAMMbert AMM. A depiction is shown in Fig. 1. Equations of this form appear in the solution of linear constant-coefficient delay equations [1], although it is unclear what, if any, the relationship between those equations and AMMs are.

Note that there are currently no audited methods to use the Lambert W function on chain, so it is unclear whether or not LAMMbert can be brought to life in a live smart contract today. However, a recent proposal to add such a function to the solady repository may change this in the near future.

References

[1] Robert M Corless et al. “On the Lambert W function”. In: *Advances in Computational Mathematics* 5 (1996), pp. 329–359.

[2] Michael Egorov. “Stableswap-efficient mechanism for stablecoin liquidity”. In: (2019).