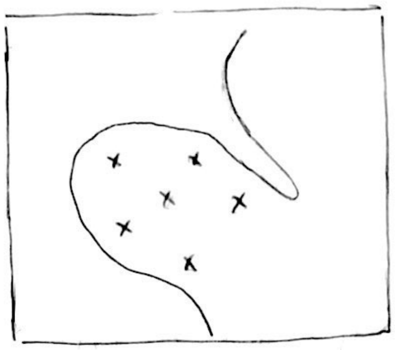
SENSORS

$u \in \mathbb{R}^m$ IS THE STATE SPACE $\quad m >> 1$

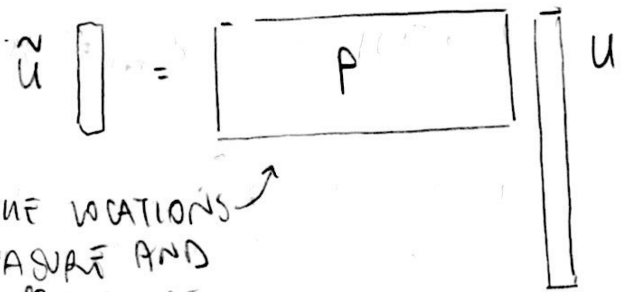
$\tilde{u} \in \mathbb{R}^d$ ARE THE MEASUREMENTS $\quad d << m$

WE WANNA STUDY THE CIRCULATION INSIDE THE GOLFO OF MEXICO



WITH A FINITE SET OF MEASUREMENTS WE WANT RECONSTRUCTING THE STATE SPACE EVERYWHERE.

$$\tilde{u} = P u$$



P HAS 1 IN THE LOCATIONS WHERE WE MEASURE AND 0 IN OTHER POSITIONS

▷ COMPRESSED SENSING

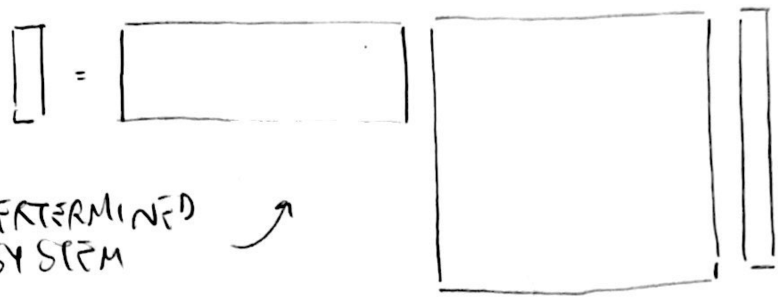
HOW WE MAY SETUP THIS INTERPOLATION PROBLEM?. WE START EXPRESSING THE STATE SPACE AS AN EXPANSION OF MODES.

$$u = \sum_{k=1}^{m} a_k \psi_k = \underline{\underline{\Psi}}\, \underline{a}$$

$$\underline{\underline{\Psi}} = \begin{bmatrix} | & | & & | \\ \psi_1 & \psi_2 & \dots & \psi_m \\ | & | & & | \end{bmatrix}$$

WE CAN EASILY COMPUTE a SINCE THIS IS A LINEAR MODEL, SO, WE CAN USE FOR EXAMPLE BACKSLASH IN MATLAB

$$\tilde{u} = P u = P \Psi a$$



UNDERDETERMINED SYSTEM

COMPRESSED SENSING APPROACH: THE PROBLEM TRYING TO FIND A SPARSE SOLUTION:

$$\min \|a\|_1$$

▷ GAPPY POD

THE PROBLEM HERE IS WHERE MEASURE THE SYSTEM

$$u = \sum_{k=1}^{M} a_k \psi_k = \Psi a \approx U_r a$$

SUBSPACE FOUND WITH SVD

DOT PRODUCT

$$M_{kj} = \langle \psi_k , \psi_j \rangle = \delta_{ij}$$

$$M_{kj} = \langle \psi_k , \psi_j \rangle_{s[\tilde{u}]} \neq 0$$ ORTHOGONALITY NO LONGER HOLDS.

SUPPORT SPACE

LET'S DEFINE:

$$f = \langle u , \psi_k \rangle_{s[\tilde{u}]}$$

$$M \tilde{a} = f$$

INCREASING THE MEASUREMENT SPACE: $\tilde{a} \to a$

WHERE WE HAVE TO PUT SENSORS?

- WILCOX → MINIMIZE THE CONDITION NUMBER
- KARNIADAKIS → MIN / MAX V_r
- EIM / DEIM ← ROM COMMUNITY
- QR / QDEIM

Δ

WE WANT TO BUILD A MAP:

$$\hat{x} = f(s)$$

APPROXIMATION OF THE STATE

MEASUREMENTS

IT IS AN OPTIMIZATION PROBLEM:

SET OF MODEL

$$f \in \underset{\tilde{f} \in \mathcal{F}}{\operatorname{argmin}} \sum^{m} \| \underline{x}_i - \tilde{f}(s_i) \|_2^2$$

WHERE m IS THE SIZE OF THE TRAINING DATA. WE SEARCH FOR THE BEST MODEL.

$$X = [x_1 \dots x_m] \quad \text{DATA}$$

$$X \approx \Phi \Sigma V^* \quad \text{rank-}r$$

$$s = Hx \approx H\Phi \nu$$

LEAST SQUARES SOLUTION: $\nu \in \underset{\tilde{\nu}}{\operatorname{argmin}} \| s - H\Phi\tilde{\nu} \|_2^2 \rightarrow \nu = (H\Phi)^+ s$

WE CAN ALSO SOLVE THE PROBLEM BY WEIGHTING THE POSSIBLE POSITIONS DIFFERENTLY. FOR EXAMPLE A SENSOR IN THE OPEN SEA CAN COST FAR MORE THAN A SENSOR IN THE COASTLINE.

- COPERNICUS → COORDINATE SYSTEM
- KEPLER → MOTION
- NEWTON → MODEL OF MOTION
- EINSTEIN → ALL MODELS ARE WRONG BUT ...

- WILLIAM OF OCCAM → # OF TERMS
- PARETO → # OF DIMENSIONS

KEY CHALLENGES:

1) LIMITED MEASUREMENTS AND DATA
2) COORDINATE SYSTEMS
3) NOISE
4) MULTISCALE - PHYSICS
5) LATENT VARIABLES
6) PARAMETRIC DEPENDENCIES
7) STOCHASTIC SYSTEMS