Breaking the Nonsmooth Barrier: A Scalable Parallel Method for Composite Optimization

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Summary

Optimization methods need to be adapted to the **parallel** setting to leverage modern computer architectures.

Highly efficient variants of stochastic gradient descent have been recently proposed, such as Hogwild [1], Kromagnon [2], ASAGA [3].

They assume that the objective function is smooth, so are inapplicable to problems such as Lasso, optimization with constraints, etc.

Contributions:

1. **Sparse Proximal SAGA**, a sparse variant of the linearly-convergent proximal SAGA algorithm.
2. **ProxASAGA**, the first parallel asynchronous variance-reduced method that supports composite objective functions.

Problem setting

Objective: develop parallel asynchronous method for problems of the form

$$\underset{\boldsymbol{x}\in\mathbb{R}^p}{\text{minimize}} \; \frac{1}{n}\sum_{i=1}^n f_i(\boldsymbol{x}) + h(\boldsymbol{x}) \;,$$

- f_i is differentiable with L-Lipschitz gradient.
- h is block-separable ($h(x) = \sum_B h_B([\boldsymbol{x}]_B)$) and "simple" in the sense that we have access to $\mathbf{prox}_h \stackrel{\text{def}}{=} \mathbf{arg\,min}_{\boldsymbol{x}}\, h(\boldsymbol{x}) + \frac{1}{2}\|\boldsymbol{x} - \boldsymbol{z}\|^2$.

$\Longrightarrow$ includes Lasso, group Lasso or ERM with box constraints.

Variance-reduced stochastic gradient methods are natural candidates: state of the art performance and recent asynchronous variants . The **SAGA** algorithm [4] has an iteration of the form

$$\begin{aligned}&\text{choose random } i \in \{1, \ldots, n\}\\ &\boldsymbol{x}^+ = \mathbf{prox}_{\gamma g}(\boldsymbol{x} - \gamma(\nabla f_i(\boldsymbol{x}) - \boldsymbol{\alpha}_i + \overline{\boldsymbol{\alpha}}))\,,\;\; \boldsymbol{\alpha}_i^+ = \nabla f_i(\boldsymbol{x})\;.\end{aligned} \tag{SAGA}$$

Difficulty of a composite extension

- Existing parallel asynchronous variants of SGD crucially rely on sparse updates.
- Even in the presence of sparse gradients, the (SAGA) update is not sparse due to the presence of $\overline{\boldsymbol{\alpha}}$ and $\mathbf{prox}$.
- Existing convergence proofs crucially rely on the gradient smoothness

A new algorithm: Sparse Proximal SAGA

The algorithm relies on the definitions

- Extended support
- $\boldsymbol{D}_i$
- φ

$$\boldsymbol{v}_i = \nabla f_i(\boldsymbol{x}) - \boldsymbol{\alpha}_i + \boldsymbol{D}_i\overline{\boldsymbol{\alpha}}\,,\;\; \boldsymbol{x}^+ = \mathbf{prox}_{\gamma\varphi_i}(\boldsymbol{x} - \gamma \boldsymbol{v}_i) \tag{SPS}$$

Linear convergence rate

Theorem. Let $\gamma = \frac{a}{5L}$ for any $a \leq 1$ and f be μ-strongly convex ($\mu > 0$). Then Sparse Proximal SAGA converges geometrically in expectation with a rate factor of at least $\rho = \frac{1}{5}\min\{\frac{1}{n}, a\frac{1}{\kappa}\}$. That is, for $\boldsymbol{x}_t$ obtained after t updates, we have the following bound:

$$\mathbb{E}\|\boldsymbol{x}_t - \boldsymbol{x}^*\|^2 \leq (1-\rho)^t C_0\,, \;\; \text{with } C_0 := \|\boldsymbol{x}_0 - \boldsymbol{x}^*\|^2 + \frac{1}{5L^2}\sum_{i=1}^n \|\boldsymbol{\alpha}_i^0 - \nabla f_i(\boldsymbol{x}^*)\|^2 \quad .$$

Proximal Asynchronous SAGA (ProxASAGA)

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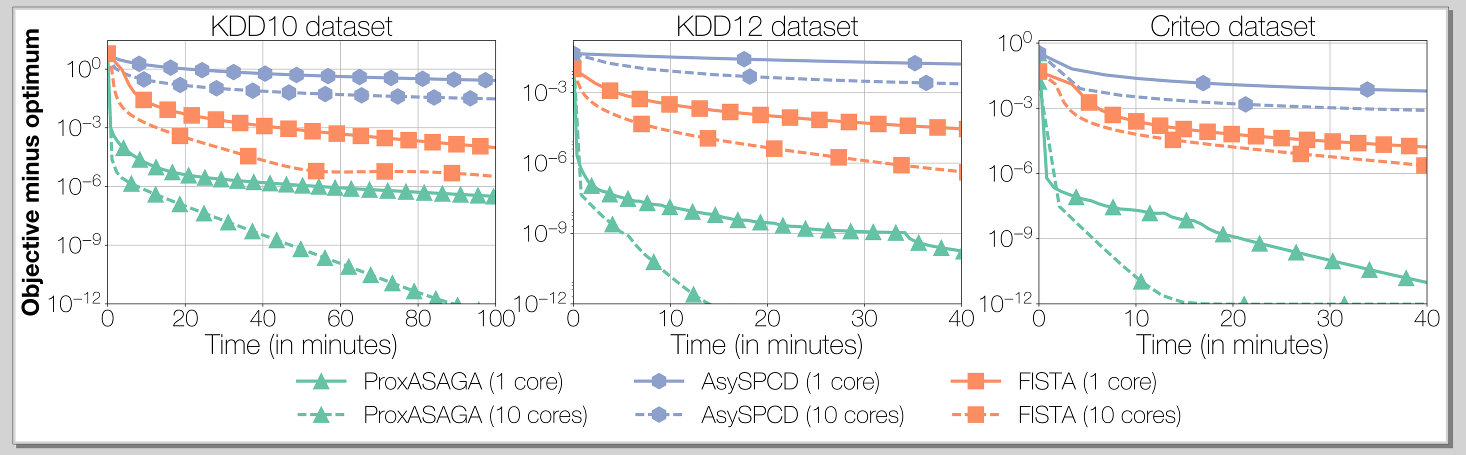
Experimental results

Comparison on 3 large-scale datasets

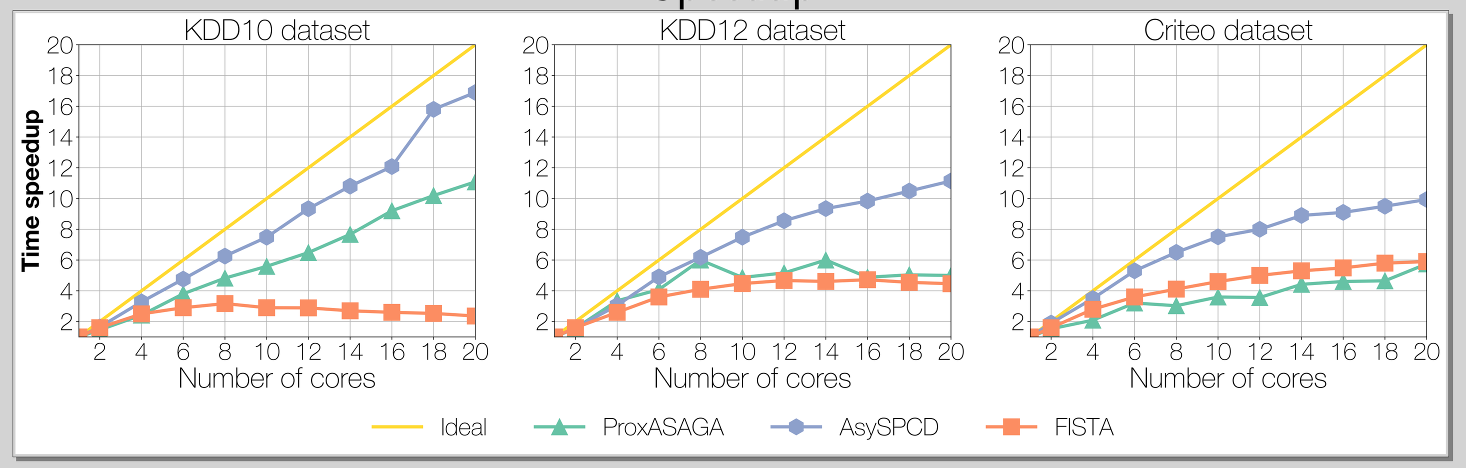
Large-scale logistic regression

Dataset	n	p	density	L	Δ
KDD 2010	19,264,097	1,163,024	10^{-6}	28.12	0.15
KDD 2012	149,639,105	54,686,452	2×10^{-7}	1.25	0.85
Criteo	45,840,617	1,000,000	4×10^{-5}	1.25	0.89

$$\frac{1}{n}\sum_{i=1}^n \log\left(1 + \exp(-b_i \boldsymbol{a}_i^\intercal \boldsymbol{x})\right) + \frac{\lambda_1}{2}\|\boldsymbol{x}\|_2^2 + \lambda_2\|\boldsymbol{x}\|_1 \quad ,$$



Speedup



References

1. Niu, F., Recht, B., Re, C. & Wright, S. *Hogwild: A lock-free approach to parallelizing stochastic gradient descent.* in *NIPS* (2011).
2. Mania, H. *et al.* Perturbed iterate analysis for asynchronous stochastic optimization. *SIAM Journal on Optimization* (2017).
3. Leblond, R., Pedregosa, F. & Lacoste-Julien, S. ASAGA: asynchronous parallel SAGA. *AISTATS* (2017).
4. Defazio, A., Bach, F. & Lacoste-Julien, S. *SAGA: A fast incremental gradient method with support for non-strongly convex composite objectives.* in *NIPS* (2014).