MU calculation / PDD / TMR / TAR

- $$PDD = 100 \times \underbrace{\left(\frac{f+d_m}{f+d}\right)^2}_{\text{IVS}} \times \underbrace{e^{-\mu(d-d_m)}}_{\text{Attenuation (TMR)}} \times \overset{\text{scattering}}{k_s}$$ $\mu = (\frac{\mu}{\rho})\cdot\rho$

$\Rightarrow$ TMR > PDD @ some depth.

EX: Rosemark 2003. Q4.

$\mu/\rho = 0.018\,cm^2/g$, $d = 5cm$, $d_m = 2cm$ $\Rightarrow$ find PDD

$$PDD(d=5, f=100) = 100\times\left(\frac{100+2}{100+5}\right)^2 \cdot e^{-(0.018cm^2/g)\times(1g/cm^3)\times[5-2]cm}$$

$$= 100\times 0.944\times 0.947 = 89.4\%.$$

$\Downarrow$ 10MV attenuation 2%/cm $\Rightarrow$ 3cm $\Rightarrow$ 6%.

$$PDD(f_1, r, d) = 100\left(\frac{f_1+d_m}{f_1+d}\right)^2 e^{-\mu(d-d_m)}\cdot k_{S1}$$
$$PDD(f_2, r, d) = 100\left(\frac{f_2+d_m}{f_2+d}\right)^2 \cdot e^{-\mu(d-d_m)}\cdot k_{S2}$$
$$\Rightarrow \frac{PDD(f_2,r,d)}{PDD(f_1,r,d)} = \underbrace{\left(\frac{f_2+d_m}{f_1+d_m}\right)^2\left(\frac{f_1+d}{f_2+d}\right)^2}_{F}\cdot\frac{k_{S2}}{k_{S1}}$$
$$= \underbrace{\left(\frac{f_2+d_m}{f_1+d_m}\right)^2\left(\frac{f_1+d}{f_2+d}\right)^2}_{F}\cdot\underbrace{\frac{TAR(d, r_{d f_2})}{TAR(d, r_{d f_1})}}_{\text{scattering}}$$

- $$PDD(f,r,d) = TAR(r_d, d)\cdot\frac{1}{BSF(r)}\cdot\left(\frac{f+d_m}{f+d}\right)^2\times 100$$

$$PDD(f,r,d) = TMR(r_d, d)\cdot\frac{S_p(r_d)}{S_p(r_{dm})}\cdot\left(\frac{f+d_m}{f+d}\right)^2\times 100.$$

$$TMR(r_d, d) = \frac{TAR(r_d,d)}{BSF(r_d)} = \frac{TAR(r_d,d)}{BSF(r_{ref})\cdot S_p(r_d)}$$ $$S_p(r_d) = \frac{BSF(r_d)}{BSF(r_{ref})}$$

- MU calculation:

(1) SSD setup.

if output calibration in Air:

$$MU = \frac{D}{Output\cdot S_c(r_c)\, S_p(r)\cdot BSF(r)\cdot \underbrace{PDD(SSD_{tx}, r, d)}_{PDD(SSD_0, r, d)\cdot F}\cdot\left(\frac{SCD}{SSD_{tx}+d_m}\right)^2}$$

if output calibration in water

$$MU = \frac{D}{Output\cdot S_c(r_c)\, S_p(r)\cdot \underbrace{PDD(SSD_{tx}, r, d)}_{PDD(SSD_0, r, d)\cdot F}\cdot\left(\frac{SCD}{SSD_{tx}+d_m}\right)^2}$$

$\begin{cases} r @ \text{surface} \\ r_c @ 100 \end{cases}$

(2) SAD setup

if calibration in Air

$$MU = \frac{D}{Output\; S_c(r)\cdot TAR(r_d, d)\cdot\left(\frac{SCD}{\underset{\hookrightarrow SAD}{f+d}}\right)^2}$$

$$\text{or} = \frac{D}{(Output)\cdot BSF(r_d)\cdot TMR(r_d,d)\left(\frac{SCD}{f+d}\right)^2}$$

if calibration in water:

$$MU = \frac{D}{Output\cdot S_c\cdot S_p(r_d)\cdot TMR(r_d,d)\cdot\left(\frac{SCD}{\underset{\downarrow SAD}{f+d}}\right)^2}$$

- Dose / Dose rate with BSF / PDD

$$D = \underbrace{(R_{fs} \cdot f_{med})} \cdot BSF \cdot PDD$$

- Given two TMR @ d=10 for 10x10, 20x20 fields & measured dose

  find Scp(20)

$\Rightarrow$ $D_1 = (\text{Output} \cdot MU) \cdot Scp(10) \cdot TMR(r=10, d=10) \cdot \left(\frac{100}{100}\right)^2$

$D_2 = (\text{output} \cdot MU) \cdot Scp(20) \cdot TMR(r=20, d=10) \cdot \left(\frac{100}{100}\right)^2$

$$\frac{Scp(20)}{\underset{\downarrow 1}{Scp(10)}} = \frac{D_2 \cdot TMR(10\times10, d=10)}{D_1 \cdot TMR(20\times20, d=10)}$$

- SAD setup

$$D_B = D_A \cdot \frac{TMR(B)}{TMR(A)} \cdot \left(\frac{SAD_A}{SAD_B}\right)^2$$

the FS at each point will be different.

SSD setup

$D_A = D_m \cdot PDD(d_A)$ $\Rightarrow$ $D_B = \frac{D_A}{PDD(d_A)} \cdot PDD(d_B)$

$D_B = D_m \; PDD(d_m)$

the same pdd, same FS @ SSD

- Fm factor (mayneord)

$$\frac{PDD(f_2, r, d)}{PDD(f_1, r, d)} = \underbrace{\frac{TAR(d, r_{df_2})}{TAR(d, r_{df_1})}}_{\text{scattering} < 1} \cdot \underbrace{\left(\frac{f_2+d_m}{f_1+d_m}\right)^2 \cdot \left(\frac{f_1+d}{f_2+d}\right)^2}_{Fm}$$

if $f_2 > f_1$ $\quad r_{df_2} = r \cdot \left(\frac{f_2}{f_1+d}\right)$ $\quad \underline{r_{df_2} < r_{df_1}}$ $\quad TAR(d, r_{df_2}) < TAR(d, r_{df_1})$

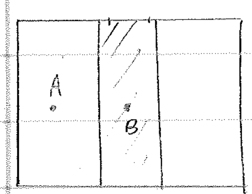
$r_{df_1} = r \cdot \left(\frac{f_1}{f_1+d}\right)$

Fm overestimate PDD changes.

F error ↑ with $\left\{ \begin{array}{l} \text{low E} \\ \text{large FS} \\ \text{large SSD change} \\ \text{large depth.} \end{array} \right.$

6MV 6x6 110SSD
6MV 30x30 150SSD
15MV 6x6 110SSD ✓ F more accurate
15MV 6x6 150SSD

- Dose under block



$D_A = (DR \cdot MU) \cdot TAR(\text{unblock}, d)$

$D_B = (DR \cdot MU)\ TAR(\text{total } A, d) - (1-TF)\ (DR \cdot MU) \cdot TAR(\text{block}, d)$

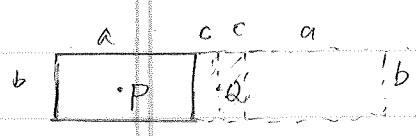
or $D_A = (DR\ MU) \cdot TMR(\text{unblock}, d) \cdot Sp(\text{unblock})$

$D_B = (DR \cdot MU) \cdot TMR(\text{unblock}, d) \cdot Sp(\text{unblock}) - (1-TF)(DR \cdot MU) \cdot TMR(\text{block})\ Sp(\text{block})$

or $D_A = (DR \cdot MU) \cdot BSF(\text{unblock}) \cdot PDD(\text{unblock}, d)$

$D_B = (DR \cdot MU) \cdot BSF(\text{total}) \cdot PDD(\text{total}) - (1-TF) \cdot (DR \cdot MU) \cdot BSF(\text{block}) \cdot PDD(\text{block})$

- Dose outside field:



$\%D_Q = \frac{1}{2} \times \frac{1}{BSF(a \times b)} \cdot [BSF((2a+2c) \times b) \cdot PDD((2a+2c) \times b) - BSF(2c \times b) \cdot PDD(2c \times b, d)]$

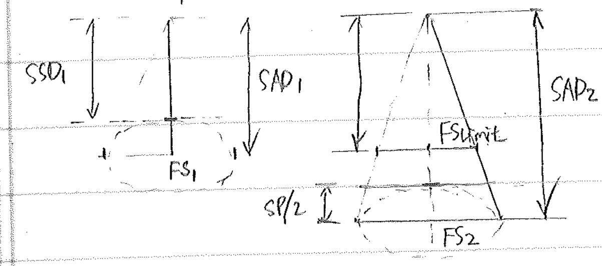
or

$D_Q = (DR \cdot MU) \cdot [TMR((2a+2c) \times b) \cdot Sp((2a+2c) \times b) - TMR(2c \times b) \cdot Sp(2c \times b)] \cdot \frac{1}{2}$

$D_P = (DR \cdot MU) \cdot TMR(a \times b)\ Sp(a \times b)$

Setup (SAD - SSD) issues:

(1) change SAD due to field size limitation (wedge)



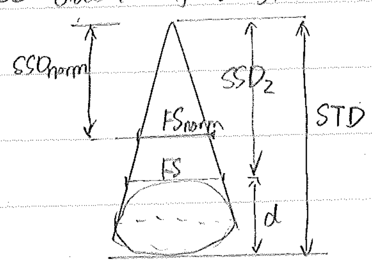
$\boxed{FS_1 = FS_2}$

$$FS_{limits}/SAD_{normal} = FS_2/SAD_2$$

$$SAD_2 = \left(\frac{FS_1}{FS_{limit}} \cdot SAD_1\right)$$

$$SSD_2 = SAD_2 - (SP/2)$$

or change SSD due to field size



$$\frac{FS}{SSD_2} = \frac{FS_{norm}}{SSD_{norm}}$$

$$\Rightarrow SSD_2 = \frac{FS \cdot SSD_{norm}}{FS_{norm}}$$

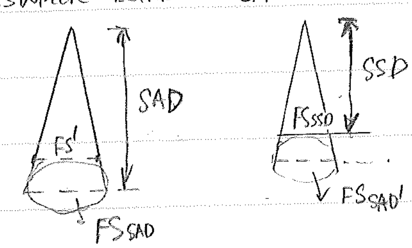
$$d = STD - SSD_2.$$

(2) Simu film / block cut issue

sim film taken @ SSD_1, SFD_1; now try to treat @ SSD_2, what is SFD_2?

$$\Rightarrow mag = \frac{SFD_1}{SSD_1} = \frac{SFD_2}{SSD_2} \Rightarrow SFD_2 = \frac{SFD_1}{SSD_1} \cdot SSD_2$$

(3) Switch between SAD $\Leftrightarrow$ SSD. to make:



$\boxed{FS_{SAD} = FS_{SAD'}}$

$$SAD' = SSD + \frac{SP}{2}$$

$$FS_{SSD}/FS_{SAD'} = \frac{SSD}{SAD'} \Rightarrow FS_{SSD} = FS_{SAD'}\left(\frac{SSD}{SSD + \frac{SP}{2}}\right)$$

$$= FS_{SAD} \cdot \left[\frac{SSD}{SSD + SP/2}\right]$$

Ex: 10x10 @ 80cm SAD and d = 8cm. Now change to SSD = 80cm

$$FS = 10 \times \frac{80}{80+8} = 9\text{cm} \text{ @ } SSD = 80$$

$$MU = \frac{TD}{D_{fs} \cdot S_c \cdot BSF(9)\, PDD(9; d=8) \cdot \left(\frac{f+d_{ref}}{f+d_m}\right)^2}$$

Ex: SAD = 100, Now change to SSD = 80, the FS on skin will be?

$$FS_{skin\text{-}SAD} = FS_{SAD} \cdot \frac{100-d}{100}$$

$$FS_{skin\text{-}SSD} = FS_{SAD'} \cdot \frac{80}{80+d} \Rightarrow \text{depends on } d$$

if it is changed to SAD = 80

$$\left.\begin{aligned} FS_{skin} &= FS_{SAD\,100} \cdot \frac{100-d}{100} \\ FS'_{skin} &= FS_{SAD\,80} \cdot \frac{80-d}{80} \end{aligned}\right\} \Rightarrow FS_{skin} > FS'_{skin}$$

- Ion chamber questions:

(1) $\Rightarrow$ find chamber volume:

$$X = \frac{Q}{m} = \frac{Q}{\rho \cdot V} \qquad \rho = 1.29\,kg/m^3 \text{ for air}$$

Assume $X(R)$ exposure delivered, n (nC) charge measured.

$$X(R) \cdot 2.58\times10^{-4}(C/kg) = \frac{n(C)}{1.29\,kg/m^3 \cdot V(m^3)}$$

$$\Rightarrow V(m^3) = \frac{n(C)}{X(R) \cdot 2.58\times10^{-4} \times 1.29}$$

(2) find current (A) measured:

$$\dot{X}(R/s) = \frac{Q/s}{m} = \frac{Q/s}{\rho \cdot V} \qquad 1\,amp = \frac{1C}{1S}$$

$$\Rightarrow Q/s = X(R/s) \cdot 2.58\times10^{-4}\,C/kg \cdot 1.29\,kg/m^3 \cdot V(m^3)$$

$$A = 3.32\times10^{-4} \cdot X \cdot V\ \underbrace{(C/s)}_{amp}$$

(3). find calibration factor (R/C) or $(\text{Voltage}/R)$

$$X(R) \cdot 2.58\times10^{-4}(C/kg) = \frac{Q(C)}{\rho \cdot V}$$

$$\Rightarrow \frac{X(R)}{Q(C)} = \frac{1}{2.58\times10^{-4}(C/kg) \cdot 1.29\,kg/m^3 \cdot V(m^3)}$$

or $X(R) = \frac{Q}{m} \qquad V = \frac{Q}{C}$

$$\Rightarrow V = \frac{X(R) \cdot m}{C} = \frac{X(R) \cdot 2.58\times10^{-4} \times \rho \times V}{C}$$

$$\Rightarrow \frac{V(vol)}{X(R)} = \frac{2.58\times10^{-4} \times 1.29 \times V}{C}$$

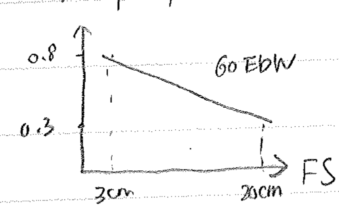
Wedge:

physical wedge:

- ↓ surface dose (7-10% compared with open field) because of filteration ⇒ ↑ beam energy
- ↑ PDD. because photon interaction in wedge ⇒ change spectrum
  - ⇒ 6MV 15x15cm, PDD% ↑ 10% @ d=30cm
  - only 2% difference for EDW.
- wedge factor measured at d=10cm.
- wedge factor ↑ with d ↑. because of ↑ scatter; also with FS ↑. because of scatter
  - ⇓ relative small compared with EDW
- mounted at lease 15cm away from pt skin surface ⇒ skin-sparing

EDW: (Enhanced dynamic wedge)

- skin dose slightly higher (3%) than open field.
- PDD simiar to open field. with 2% from dmax to 30cm.
- reduce dose to contralateral breast by half. (Raphex 2006 Q46)
- lower peripheral dose outside tx field compared to metal wedge ⇒ Varian EDW manual
  - ⇓ reduce peripheral dose by a factor of 2.
  - physical wedge peripheral dose ↑ due to the scatter out of the field generated by inteactions of primary photon beam with physical wedge.
- wedge factor is a smooth & continuous functions of field size!



⇔ the ratio of WF for the largese FS to the smallest fs is 1.012 for 45° physical wedge
1.008 for 60° physical wedge

the smaller variation in wedge factor with FS for physical wedges

Wedge Calculation:

1. Wrong wedge MU

E.X. Treating with parallel opposed wedge field for 60 Gy in 30 fx, the MUs per beam in first 10 fx = 160 MU, then realize WF was not in calc. How many MUs required for remaining 20 fx to get 60 Gy?

Solution:

Dose per fx = 60 Gy / 30 fx = 200 cGy ⇒ 100 cGy per beam

Dose delivered in first 10 fx = 200 × WF × 10 = 2000 · WF (cGy)

$$MU = \frac{Dose}{\text{All-factors} \cdot WF} \Rightarrow 160 = \frac{100}{\text{All-factor}} \Rightarrow \frac{Dose}{MU} = \frac{100}{160} \text{ for no wedge.}$$

Dose error to be compensate: $200 \times 10 \times (1-WF) = 2000\,(1-WF)$.

Dose to be compensate per fx: $\frac{2000\,(1-WF)}{20} = 100\,(1-WF)$

therefore. total dose per fx: $200 + 100\,(1-WF)$

$$MU = \frac{200+100(1-WF)}{(\text{All-factor} \cdot WF) \times 2} = \frac{200+100(1-WF)}{(\frac{100}{160} \times WF) \times \underbrace{2}_{\text{opposed beam}}}$$

if $WF = 0.6 \Rightarrow MU = \frac{200 + 100 \times 0.4}{\frac{100}{160} \times 0.6 \times 2} = 320\,MU$.

to verify:

dose delivered in first 10 fx: $160 \times (\frac{100}{160} \times 0.6) \times 10 \times 2 = 1200$ cGy

dose . . . last 20 fx: $320 \times (\frac{100}{160} \times 0.6) \times 20 \times 2 = 4800$ cGy

6000 cGy.

to generalize:

$$MU_2 = \frac{D_0 + \frac{D_0 \times n_1 \times (1-WF)}{n_2}}{(\frac{(D_0/2)}{MU_1}) \cdot WF \cdot 2}$$

- n_1 – # of fx treated wrong
- n_2 – # of fx to compensate
- D_0 – corrected dose per fx

(2) wedge in wrong field:

two fields weighted W_o (open), W_w (wedge) $W_o + W_w = 1$

if wedge is put in the wrong field, what is real dose.

$$D_w = D \cdot W_w \Rightarrow MU_w = \frac{D \cdot W_w}{WTF \cdot CF}$$

$$D_o = D \cdot W_o \Rightarrow MU_o = \frac{D \cdot W_o}{CF}$$

Now wedge flipped.

$$D_w' = MU_w \cdot CF = \frac{D \cdot W_w}{WTF \cdot \cancel{CF}} \cancel{CF}$$

$$D_o' = MU_o \cdot CF \cdot WTF = \frac{D \cdot W_o}{\cancel{CF}} \cancel{CF} \cdot WTF = D \cdot W_o \cdot WTF$$

$$\text{total dose} = D_w' + D_o' = \left(\frac{D \cdot D_w}{WTF} + D \cdot W_o \cdot WTF \right)$$

(3) wedge in wrong direction:

(4) combination of wedge with open field.

① Dose-weighting for
open field W_o $\qquad W_o + W_W = 1$
wedge field W_W

$$\boxed{W_W = \frac{\tan\theta_{eff}}{\tan\theta_W}}$$

θ_W – angle of wedge. θ_{eff} – angle of synthetic wedge.

⇒ Dose ratio of wedge field to open field:

$$W_W / W_o = \frac{\tan\theta_{eff} / \tan\theta_W}{1 - \tan\theta_{eff}/\tan\theta_W}$$

$$\Rightarrow MU_W = \frac{Dose \cdot W_W}{Output \cdot Sc \cdot Sp\ TMR \cdot WF}$$

$$MU_o = \frac{Dose \cdot W_o}{Output \cdot Sc \cdot Sp \cdot TMR \cdot 1}$$

⇒ MU ratio of wedge field to open field.

$$MU_W / MU_o = \frac{W_W}{WF \cdot W_o}$$

⇒ effective wedge factor:

$$WF_{eff} = \frac{MU_W \cdot WF + MU_o}{MU_W + MU_o} = \frac{\left(\frac{W_W}{WF \cdot W_o} \cdot WF\right) \cdot MU_o + MU_o}{\frac{W_W}{WF \cdot W_o} \cdot MU_o + MU_o} = \frac{\frac{W_W}{W_o} + 1}{\frac{W_W}{WF \cdot W_o} + 1} = \frac{\frac{W_W + W_o}{W_o}}{\frac{W_W + WF\,W_o}{WF \cdot W_o}}$$

$$= \frac{1}{W_o} \cdot \frac{WF\,W_o}{W_W + WF \cdot W_o} = \frac{WF}{W_o \cdot WF + W_W}$$

Ex: $\theta = 60°$. $WF = 0.5$, $\theta_{eff} = 30°$ ⇒ find MU_{ratio}, Dose ratio

$$\frac{W_W}{W_o} = \frac{\tan 30°/\tan 60°}{1 - \tan 30°/\tan 60°} = \frac{0.577/1.732}{1 - 0.333} = \frac{1}{2}.$$

$$\frac{MU_W}{MU_o} = \frac{W_W}{WF \cdot W_o} = \frac{0.333}{0.5 \times 0.666} = \frac{1}{1}$$

② Tatcher's eqution.

$$\boxed{W_W = \frac{\theta_{eff}}{\theta_W}}$$

⇒ Dose rato of wedge to open: $\frac{W_W}{W_o} = \frac{\theta_{eff}/\theta_W}{1 - \frac{\theta_{eff}}{\theta_W}}$

⇒ MU ratio of wedge to open: $\frac{MU_W}{MU_o} = \frac{W_W}{WF \cdot W_o}$

Ex: $\theta = 60°$. $WF = 0.5$ $\theta_{eff} = 30°$

$$\frac{W_W}{W_o} = \frac{30/60}{1 - 30/60} = \frac{0.5}{0.5} = \frac{1}{1}$$

$$\frac{MU_W}{MU_o} = \frac{W_W}{WF \cdot W_o} = \frac{0.5}{0.5 \times 0.5} = \frac{2}{1}$$

# of atoms per mass (# of atoms/g) = $\frac{N_A}{A}$

of e^- per m^3 = $\rho_m \cdot \frac{N_A \cdot Z}{A}$

of e^- per g = $\frac{N_A Z}{A} = N_A \cdot (\frac{Z}{A})$

↳ $\frac{Z}{A} \approx 0.5$ for all materials except H.

(Total energy release per mass)

TERMA : loss of energy from uncharged particles (primaries) by interactions in material.

$$\text{TERMA} = \Psi \cdot \frac{\mu}{\rho} \ [\tfrac{J}{kg}]$$

$$\frac{\mu}{\rho} = \underbrace{\frac{\sigma_R}{\rho}}_{\text{Rayleigh}} + \underbrace{\frac{\sigma}{\rho}}_{\text{Compton}} + \underbrace{\frac{\kappa}{\rho}}_{\text{PE}} + \underbrace{\frac{\tau}{\rho}}_{\text{pair}} + \underbrace{\frac{\eta}{\rho}}_{\text{photonuclear}}$$

(Kinetic energy release per Mass)

KERMA : transfer of energy from uncharged primaries to charged particles

$$\text{KERMA} = \underbrace{\Psi}_{\text{(energy fluence)}} \frac{\mu_{tr}}{\rho} \ [\tfrac{J}{kg}]$$

when CPE.

$K_c \overset{CPE}{=} \text{Dose}$

$$\begin{aligned} K &= K_C + K_R \\ &= K \cdot (1-g) + K \cdot g \\ &= \Psi \frac{\mu_{tr}}{\rho}(1-g) + \Psi \frac{\mu_{tr}}{\rho} \cdot g \\ &= \underbrace{\Psi \frac{\mu_{en}}{\rho}}_{K_C} + \underbrace{\Psi \frac{\mu_{tr}}{\rho} \cdot g}_{K_R} \end{aligned}$$

Exposure : $X = \frac{dQ}{dm} \ [\frac{C}{kg}]$

$$X = (K_c)_{air} / \frac{W}{e}$$

$$(\tfrac{C}{kg}) = (J/kg) / (J/C)$$

or

$$\underbrace{(K_c)_{air}}_{J/kg} = X(\tfrac{C}{kg}) \cdot \frac{W}{e}(\tfrac{J}{C})$$

$$= X(R) \cdot 2.58 \times 10^{-4} (\tfrac{C/kg}{R}) \times 33.97 (\tfrac{J}{C})$$

$$= X(R) \cdot 0.08769 (\tfrac{J}{kg}/R)$$

(Gy/R)

(converted energy per mass)

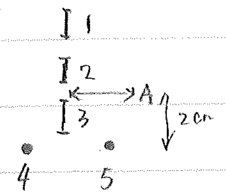
CEMA : energy transfer from primary charged particles to secondary charged particles (δ-ray)

$$C = \int \underbrace{\Phi_E(E)}_{\text{(fluence)}} \cdot \frac{S_{col}(E)}{\rho} dE.$$

when δ-ray equilibrium CEMA $\overset{\delta CPE}{=}$ Dose

restrict CEMA : $C_\Delta = \int \Phi'_E(E) \frac{L_\Delta(E)}{\rho} dE$. $\left(\frac{L_\Delta(E)}{\rho} = \frac{S_{col}}{\rho} - \frac{dE_{ke,\delta}}{\rho} \right)$

- Fletcher tandom and ovoid



(1) source 0.57 cGy / mgRn·hr

(2) source 1.52 cGy / mgRn·hr

(3) source 1.52 cGy / mgRn·hr

(4) source 0.58 cGy / mgRn·hr

(5) source 1.59 cGy / mgRn·hr

point A dose rate = $(0.57 \times A_1) + (1.52 \times A_2) + (152 \times A_3) + (0.58 \times A_4) + (1.59 \times A_5)$