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1 Preamble

```

1 \documentclass[11pt, dvipsnames]{article}
2
3 \usepackage[a4paper, bindingoffset=0 in, left =0.95 in, right=0.95 in, top = 1 in, bottom=1 in, footskip=.25
   in]{geometry} % Paper size
4 %\usepackage[utf8]{inputenc} % Required for inputting international characters
5 %\usepackage[T1]{fontenc} % Output font encoding for international characters
6 \usepackage{amsmath} % Math
7 \usepackage{amssymb} % Math
8 %\usepackage{amsthm} % Math – theorems
9 \usepackage{esint} % Math
10 %\usepackage{physics} % Math
11 \usepackage{mathtools}
12 \usepackage[nointegrals]{wasysym} % Astronomical symbols
13 \usepackage{siunitx} % Units – Can also align tables by decimal point
14 \usepackage{float} % Force table to be placed HERE (\begin{figure}[H] ...)
15 %\usepackage{mathrsfs} % Use \mathscr{} for fancy capital letters
16 \usepackage{caption, subcaption} % Caption for tables, figures, etc. (UB20.04 ??? doesn't work without it
   but complains if it's there.)
17 %\usepackage{dcolumn} % Align decimals in column
18 %\usepackage{parskip} % Skip lines with vertical whitespace (as anyone would expect)
19 \usepackage{indentfirst} % Automatically indent every paragraph
20 %\usepackage{fancyhdr} % Control headers and foots
21 \usepackage{chemformula} % H2O
22 %\usepackage{chngcntr} % Control depth of counters (not toc)
23 \usepackage{tikz}
24 \usepackage{pgfplots,tikz-3dplot}
25 %\usepackage{multirow,tabularx} % Make a multi-columned (or row) entry in tabular
26 %\usepackage{longtable}
27 %\usepackage{listings} % Produce code formatting and font
28 %\usepackage{color} % Color code according to a language (see settings below)
29 \usepackage{tabu}
30 \usepackage{booktabs}
31 \usepackage{xifthen} % Conditionals in tikz
32 \usepackage{algorithm}
33 \usepackage{algpseudocode}
34 \usepackage{pdfscape}
35 \usepackage{graphicx}
36 \usepackage[percent]{overpic}
37 \usepackage{moresize}
38 %\usepackage[toc,page]{appendix} % Can use with \begin{appendices} env but \appendix also works without
   this package
39 \usepackage{scalerel}
40 \PassOptionsToPackage{usenames,dvipsnames}{xcolor} % Give more colors (xcolor, hyp
41 \usepackage{hyperref} % Manage links between section/equation numbers/urls
42 \usepackage{cleveref} % Not sure but linked with hyperref in SE answer (https://tex.
   stackexchange.com/questions/100905/best-practice-for-hyperref-link-colours?rq=1)
43 \usepackage{caption}
44 \usepackage{listings} % Produce code formatting and font
45 \usepackage{color} % Color code according to a language (see settings below)
46
47 \newcommand\myshade{85}
48 \colorlet{mylinkcolor}{violet}
49 \colorlet{mycitecolor}{YellowOrange}
50 \colorlet{myurlcolor}{Aquamarine}
51
52 \hypersetup{
53   colorlinks = true,
54   linkcolor = mylinkcolor!\myshade!black,
55   citecolor = mycitecolor!\myshade!black,
56   urlcolor = myurlcolor!\myshade!black,

```

```

57 linkbordercolor={0 0 1}
58 }
59
60 \bibliographystyle{alpha}
61
62
63
64 % General modifiers
65 % Change default multiplier from {\times (x)} to {\cdot (.)}
66 \sisetup{output-product = \cdot} % Explicitly state 'x' in \SI
67 \sisetup{exponent-product = \cdot} % Implied multiplier with \SI{1e2}{...}
68
69 \protected\def\cosphantom{\qopname\relax o{\vphantom{i}cos}}
70 %\protected\def\arccos{\qopname\relax o{\vphantom{i}arccos}}
71
72 %% Set hbox fuzz (New problem with Ubuntu 20.04 in landscape...)
73 %% (SEE NOTE AT END THE OF \usepackage{caption, subcaption})
74 %\hfuzz=12.002pt
75
76 \pgfplotsset{compat=newest}
77 \usetikzlibrary{arrows.meta, bending, calc, fadings, backgrounds, decorations.pathreplacing, decorations.
    pathmorphing, decorations.shapes, decorations.markings, shapes.geometric, shapes.misc, patterns}
78 %
79 \newcommand{\AxisRotator}[1][rotate=0]{%
80 \tikz [x=0.25cm,y=0.60cm,line width=0.2ex,--stealth,#1] \draw(0,0) arc (-150:150:1 and 1);%
81 }
82 \newcommand\pgfmathsinandcos[3]{%
83 \pgfmathsetmacro#1{sin(#3)}%
84 \pgfmathsetmacro#2{cos(#3)}%
85 }
86 %
87 \newcommand\LongitudePlane[3][current plane]{%
88 \pgfmathsinandcos\sinEl\cosEl{#2} % elevation
89 \pgfmathsinandcos\sint\cost{#3} % azimuth
90 \tikzset{#1/.style={cm={\cost,\sint*\sinEl,0,\cosEl,(0,0)}}}
91 }
92 \newcommand\LatitudePlane[3][current plane]{%
93 \pgfmathsinandcos\sinEl\cosEl{#2} % elevation
94 \pgfmathsinandcos\sint\cost{#3} % latitude
95 \pgfmathsetmacro\yshift{\cosEl*\sint}
96 \tikzset{#1/.style={cm={\cost,0,0,\cost*\sinEl,(0,\yshift)}}} %
97 }
98 \newcommand\DrawLongitudeCircle[2][1]{
99 \LongitudePlane{\angEl}{#2}
100 \tikzset{current plane/.prefix style={scale=#1}}
101 % angle of " visibility "
102 \pgfmathsetmacro\angVis{atan(sin(#2)*cos(\angEl)/sin(\angEl))} %
103 \draw[current plane] (\angVis:1) arc (\angVis:\angVis+180:1);
104 %\draw[current plane,dashed] (\angVis-180:1) arc (\angVis-180:\angVis:1);
105 }
106 \newcommand\DrawLatitudeCircle[2][2]{
107 \LatitudePlane{\angEl}{#2}
108 \tikzset{current plane/.prefix style={scale=#1}}
109 \pgfmathsetmacro\sinVis{sin(#2)/cos(#2)*sin(\angEl)/cos(\angEl)}
110 % angle of " visibility "
111 \pgfmathsetmacro\angVis{asin(min(1,max(\sinVis,-1)))}
112 \draw[current plane] (\angVis:1) arc (\angVis:-\angVis-180:1);
113 % TESTING BELOW
114 % \node[] at (2,2,2) {\angVis};
115 % \draw[current plane] (\angVis:1) arc (\angVis:-\angVis-20:1);
116 % END TESTING
117 %\draw[current plane,dashed] (180-\angVis:1) arc (180-\angVis:\angVis:1);
118 }

```

```

119 \newcommand\DrawLongitudeCircleWithoutSectorHARDCODED[2][1]{
120   \LongitudePlane{\angEl}{#2}
121   \tikzset{current plane/.prefix style={scale=#1}}
122   % angle of " visibility "
123   \pgfmathsetmacro\angVis{atan(sin(#2)*cos(\angEl)/sin(\angEl))} %
124
125   % Begin list of if's checking SPECIFIC (PRE-DETERMINED) INPUT VALUES #2
126   % \foreach \t (#2) in {-5,-20,...,-175} |||| % -5,-20,...,-175
127   \ifthenelse{#2 = -35}% SECOND ELSE / THIRD IF
128   { \draw[current plane] (\angVis:1) arc (\angVis:\angVis+44.8:1);
129     \pgfmathsetmacro\angDis{135.3} % for -35
130     \draw[current plane] (\angVis+\angDis:1) arc (\angVis+\angDis:\angVis+180:1); }% THIRD THEN
131   { \ifthenelse{#2 = -50} % THIRD ELSE / FOURTH IF
132     { \draw[current plane] (\angVis:1) arc (\angVis:\angVis+53:1);
133       \pgfmathsetmacro\angDis{143} % for -50
134       \draw[current plane] (\angVis+\angDis:1) arc (\angVis+\angDis:\angVis+180:1);
135       % Label theta
136       \pgfmathsetmacro\thetaBreak{113}
137       \draw[current plane,color=black] (\angVis+53:1) arc (\angVis+53:\angVis+\thetaBreak:1);
138       \draw[current plane,color=yellow!55!black!90,->,>=stealth,thick] (\angVis+\angDis:1) arc (\angVis+\angDis:\angVis+\thetaBreak:1) node[pos=0.6,anchor=south west,color=black,yshift=-2.35mm]{$\theta$}; } % FOURTH THEN
139   { \ifthenelse{#2 = -65} % FOURTH ELSE / FIFTH IF
140     { \draw[current plane] (\angVis:1) arc (\angVis:\angVis+57.5:1);
141       \pgfmathsetmacro\angDis{148} % for -65
142       \draw[current plane] (\angVis+\angDis:1) arc (\angVis+\angDis:\angVis+180:1); } % FIFTH THEN
143   { \ifthenelse{#2 = -80} % FIFTH ELSE / SIXTH IF
144     { \draw[current plane] (\angVis:1) arc (\angVis:\angVis+59.7:1);
145       \pgfmathsetmacro\angDis{150} % for -80
146       \draw[current plane] (\angVis+\angDis:1) arc (\angVis+\angDis:\angVis+180:1); } % SIXTH THEN
147   { \ifthenelse{#2 = -95} % SIXTH ELSE / SEVENTH IF
148     { \draw[current plane] (\angVis:1) arc (\angVis:\angVis+60:1);
149       \pgfmathsetmacro\angDis{150} % for -95
150       \draw[current plane] (\angVis+\angDis:1) arc (\angVis+\angDis:\angVis+180:1); } % SEVENTH THEN
151   % FINAL ELSE: Normal for -5, -20, -110, -135, etc.
152   { \draw[current plane] (\angVis:1) arc (\angVis:\angVis+180:1); } % ELSE: PLOT NORMALLY
153   }
154   }
155   }
156   }
157   % This one goes in the LAST then
158   %\draw[current plane] (\angVis:1) arc (\angVis:\angVis+180:1);
159 }
160 \newcommand\DrawLatitudeCircleWithoutSectorHARDCODED[2][2]{
161   \LatitudePlane{\angEl}{#2}
162   \tikzset{current plane/.prefix style={scale=#1}}
163   \pgfmathsetmacro\sinVis{sin(#2)/cos(#2)*sin(\angEl)/cos(\angEl)}
164   % angle of " visibility "
165   \pgfmathsetmacro\angVis{asin(min(1,max(\sinVis,-1)))}
166
167   % % \foreach \t (#2) in {-80,-70,...,80} |||| % -80,-70,...,80
168   % % All negative latitudes are fine
169   \ifthenelse{#2 = 0}
170   { \draw[current plane] (\angVis:1) arc (\angVis:-\angVis-50:1);
171     \draw[current plane] (-\angVis-180:1) arc (-\angVis-180:-\angVis-110:1);
172     \draw[current plane,color=yellow!55!black!90,->,>=stealth,thick] (-\angVis-110:1) arc (-\angVis-110:-\angVis-50:1) node[pos=0.5,anchor=south,color=black,yshift=-0.5mm]{$\lambda$}; }
173   { \ifthenelse{#2 = 10}% FIRST IF
174     { \draw[current plane] (\angVis:1) arc (\angVis:-\angVis-14:1);
175       \pgfmathsetmacro\angDis{115.8}
176       \draw[current plane] (\angVis-\angDis:1) arc (\angVis-\angDis:-\angVis-180:1); }% FIRST THEN
177   { \ifthenelse{#2 = 20} % FIRST ELSE / SECOND IF
178     { \draw[current plane] (\angVis:1) arc (\angVis:-\angVis-8:1);

```

```

179 \pgfmathsetmacro{\angDis}{122}
180 \draw[current plane] (\angVis-\angDis:1) arc (\angVis-\angDis:-\angVis-180:1); } % SECOND THEN
181 { \ifthenelse{#2 = 30} % SECOND ELSE / THIRD IF
182 { \draw[current plane] (\angVis:1) arc (\angVis:-\angVis-.6:1);
183 \pgfmathsetmacro{\angDis}{129.5}
184 \draw[current plane] (\angVis-\angDis:1) arc (\angVis-\angDis:-\angVis-180:1); } % THIRD THEN
185 { \ifthenelse{#2 = 40} % THIRD ELSE / FOURTH IF
186 { \draw[current plane] (\angVis:1) arc (\angVis:-\angVis+9:1);
187 \pgfmathsetmacro{\angDis}{139}
188 \draw[current plane] (\angVis-\angDis:1) arc (\angVis-\angDis:-\angVis-180:1); } % FOURTH THEN
189 { \ifthenelse{#2 = 50} % FOURTH ELSE / FIFTH IF
190 { \draw[current plane] (\angVis:1) arc (\angVis:-\angVis+23.5:1);
191 \pgfmathsetmacro{\angDis}{153.7}
192 \draw[current plane] (\angVis-\angDis:1) arc (\angVis-\angDis:-\angVis-180:1); } % FIFTH THEN
193 { \ifthenelse{#2 = 60} % FIFTH ELSE / SIXTH IF
194 { \draw[current plane] (\angVis:1) arc (\angVis:-\angVis+70:1);
195 \draw[current plane,color=black] (-\angVis+70:1) arc (-\angVis+70:-\angVis-20:1);
196 \draw[current plane] (-\angVis-20:1) arc (-\angVis-20:-\angVis-180:1); } % SIXTH THEN
197 { \ifthenelse{#2 = 70} % SIXTH ELSE / SEVENTH IF
198 { \draw[current plane] (250:1) arc (250:-\angVis+70:1); } % SEVENTH THEN
199 { \ifthenelse{#2 = 80}
200 { \draw[current plane] (250:1) arc (250:-\angVis+70:1); } % EIGHTH THEN
201 { \draw[current plane] (\angVis:1) arc (\angVis:-\angVis-180:1); } % EIGHTH ELSE / FINAL ELSE
202 }
203 }
204 }
205 }
206 }
207 }
208 }
209 }
210 % This one goes in the LAST then
211 %\draw[current plane] (\angVis:1) arc (\angVis:-\angVis-180:1);
212 }
213 \newcommand\DrawLongitudeArc[3][1]{
214 \LongitudePlane{\angEl}{#2}
215 \tikzset{current plane/.prefix style={scale=#1}}
216 % angle of " visibility "
217 \pgfmathsetmacro\angVis{atan(sin(#2)*cos(\angEl)/sin(\angEl))} %
218 \draw[current plane,color=#3] (\angVis:1) arc (\angVis:\angVis+180:1);
219 %\draw[current plane,dashed] (\angVis-180:1) arc (\angVis-180:\angVis:1);
220 }
221 \newcommand\DrawLatitudeArc[4][2]{
222 \LatitudePlane{\angEl}{#2}
223 \tikzset{current plane/.prefix style={scale=#1}}
224 \pgfmathsetmacro\sinVis{sin(#2)/cos(#2)*sin(\angEl)/cos(\angEl)}
225 % angle of " visibility "
226 \pgfmathsetmacro\angVis{asin(min(1,max(\sinVis,-1)))}
227 \draw[current plane,color=#3] (\angVis:1) arc (\angVis:-\angVis-#4:1);
228 %\draw[current plane,dashed] (180-\angVis:1) arc (180-\angVis:\angVis:1);
229 }
230 \tikzset{cross/.style={cross out, draw=black, minimum size=2*(#1-\pgflinewidth), inner sep=0pt, outer sep=0pt},cross/.default={1pt}}
231
232 % Define scaling square root
233 \def\depthgrowth{0pt}
234 \def\heightgrowth{2pt}
235 \newsavebox\zbox
236 \newcommand\zsqrt[1]{%
237 \ignoremathstyle
238 \savebox\zbox{$#1\rule{0pt}{.7\baselineskip}$}%
239 \stretchrel*{\sqrt{\phantom{#1}}\kern0.5pt}{%
240 {\rule[-\dimexpr\dp\zbox+\depthgrowth]{0pt}{%

```

```

241 \dimexpr\ht\zbox+\dp\zbox+\depthgrowth+\heightgrowth}}%
242 \kern-\wd\zbox\textstyle#1%
243 }
244
245
246 \DeclarePairedDelimiter{\ceil}{\lceil}{\rceil}
247 \DeclarePairedDelimiter{\floor}{\lfloor}{\rfloor}
248
249 %% Custom commands
250 \newcommand{\raisesym}[2]{\raisebox{#2\depth}{\$#1\$}}
251
252 % Overbar 1.5mu on each side
253 \newcommand{\overbar}[1]{\mkern 1.5mu\overline{\mkern-1.5mu#1\mkern-1.5mu}\mkern 1.5mu}
254
255 % Cell broken into more than 1 line
256 \newcommand{\specialcell}[2][c]{%
257 \begin{tabular}[#1]{@{ }c@{ }}#2\end{tabular}}
258
259
260 \makeatletter
261 \tikzoption{canvas is plane}[]{\@setOxy#1}
262 \def\@setOxy O(#1,#2,#3)x(#4,#5,#6)y(#7,#8,#9)%
263 {\def\tikz@plane@origin{\pgfpointxyz{#1}{#2}{#3}}%
264 \def\tikz@plane@x{\pgfpointxyz{#4}{#5}{#6}}%
265 \def\tikz@plane@y{\pgfpointxyz{#7}{#8}{#9}}%
266 \tikz@canvas@is@plane
267 }
268 \makeatother
269
270
271 % Adjust coding language settings from listings package here.
272 \captionsetup[ lstlisting ]{margin=0cm,format=hang,font=small,format=plain,labelfont={bf,up},textfont={it}}
273 \renewcommand*{\lstlistingname}{Code} %\textcolor{violet}{\textsl{Mathematica}}}
274 % rgb varies 0 to 1 (3 floats), RGB varies 0 to 255 (3 integers)
275 \definecolor{mygreen}{RGB}{28,172,0} % color values Red, Green, Blue
276 \definecolor{mylilas}{RGB}{170,55,241}
277 \definecolor{forblue}{RGB}{0,191,191}
278 \definecolor{gris245}{RGB}{245,245,245}
279 \definecolor{olive}{RGB}{50,140,50}
280 \definecolor{brun}{RGB}{175,100,80}
281 \definecolor{deepred}{rgb}{0.6,0,0}
282 \definecolor{deepgreen}{rgb}{0,0.5,0}
283 \definecolor{deepblue}{rgb}{0,0,0.5}
284 \definecolor{gray}{rgb}{0.2,0.2,0.2}
285 \definecolor{green}{rgb}{0,0.5,0.5}
286 %\definecolor{yellow}{cmyk}{0.5,0.1,0.7,0.1}
287 \lstset {frame=tb,
288 language=Python, % Specify language
289 aboveskip=3 mm,
290 belowskip=3 mm,
291 showstringspaces=false,
292 columns=flexible,
293 basicstyle={\small\ttfamily},
294 numbers=none,
295 numberstyle=\tiny\color{deepgreen},
296 keywordstyle=\color{deepred},
297 commentstyle=\color{gray},
298 stringstyle=\color{yellow},
299 breaklines=true,
300 breakatwhitespace=true,
301 tabsize=3
302 }
303

```

```
304 |
305 | \lstset {
306 | language=TeX,
307 | defaultdialect =empty,
308 | basicstyle=\footnotesize,
309 | numbers=left,
310 | numberstyle=\footnotesize,
311 | stepnumber=1,
312 | numbersep=5pt,
313 | backgroundcolor=\color{white},
314 | showspaces=false,
315 | showstringspaces=false,
316 | showtabs=false,
317 | frame=single,
318 | tabsize=2,
319 | captionpos=b,
320 | breaklines=true,
321 | breakatwhitespace=false
322 | }
```

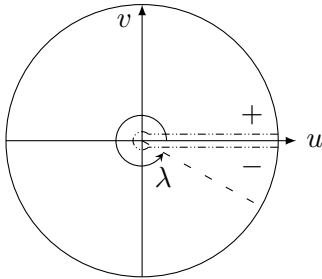

2 Good

2.1 Diagram of Unit Circle with Branch Cut on Positive Real Axis

```

1 \begin{tikzpicture}[scale=1.2,baseline={([yshift=-.5ex]current bounding box.center)}]
2 \pgfmathsetmacro{\R}{1.5}
3 \draw[->,>=latex] (-\R,0) -- (\R+0.2,0) node[right]{$u$};
4 \draw[->,>=latex] (0,-\R) -- (0,\R) node[anchor=north east]{$v$};
5 \draw (0,0) circle [radius=\R cm];
6
7 % Draw keyhole contour for branch cut
8 \pgfmathsetmacro{\r}{\R/15} % \R/35
9 \pgfmathsetmacro{\initAngle}{2.75} % 0.75
10 \pgfmathsetmacro{\initPosX}{\R*cos(\initAngle)}
11 \pgfmathsetmacro{\initPosY}{\R*sin(\initAngle)}
12 \pgfmathsetmacro{\stopAng}{asin((\R/\r) * sin(\initAngle))}
13 \pgfmathsetmacro{\stopPosX}{\r*cos(\stopAng)}
14 \draw[densely dash dot dot] (\initPosX,\initPosY) -- (\stopPosX,\initPosY) node[pos=0.2,above]{$+$};
15 \draw[densely dash dot dot] (\stopPosX,\initPosY) arc [start angle = \stopAng, end angle=360-\stopAng,
    radius=\r];
16 \draw[densely dash dot dot] (\initPosX,-\initPosY) -- (\stopPosX,-\initPosY) node[pos=0.2,below]{$-$};
17
18 \pgfmathsetmacro{\lambdaAng}{331}%244
19 \pgfmathsetmacro{\InitlambdaRx}{\stopPosX+0.2}
20 \pgfmathsetmacro{\InitlambdaRy}{\initPosY}
21 \pgfmathsetmacro{\lambdaR}{sqrt(\InitlambdaRx*\InitlambdaRx + \InitlambdaRy*\InitlambdaRy)}
22 \draw[->,>=stealth] (\InitlambdaRx,0) arc [start angle = 0, end angle=\lambdaAng, radius=\lambdaR cm]
    node[anchor=north,yshift=-0.5mm]{$\lambda$}; %start angle = \stopAng
23 \draw[loosely dashed] (0,0) -- ({\R*cos(\lambdaAng)}, {\R*sin(\lambdaAng)});
24 \end{tikzpicture}

```



2.2 Diagram of Spherical Coordinates

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}[scale=3.809]%3.22 | 3.85
4 \pgfmathsetmacro{\R}{0.9} % sphere radius
5 \pgfmathsetmacro{\angEl}{30} % elevation angle
6 \draw (0,0,0) circle (\R);
7 % VV MOVED TO END VV
8 %\foreach \t in {-5,-20,...,-175} { \DrawLongitudeCircleWithoutSectorHARDCODED[\R]{\t} }
9 %\foreach \t in {-80,-70,...,80} { \DrawLatitudeCircleWithoutSectorHARDCODED[\R]{\t} } %
10 % -80,-70,...,80
11 \tdplotsetmaincoords{70}{100}
12 \tdplotsetrotatedcoords{10}{10}{-20}
13 % Draw axes
14 \draw[densely dashed,thick,tdplot_rotated_coords] (0,0,0) -- (\R,0,0);
15 \draw[densely dashed,thick,tdplot_rotated_coords] (0,0,0) -- (0,\R,0);
16 \draw[densely dashed,thick,tdplot_rotated_coords] (0,0,0) -- (0,0,\R);
17 \pgfmathsetmacro{\BasisScaleR}{1.5}
18 \draw[->, >=latex, thick,tdplot_rotated_coords] (\R,0,0) -- (1.4*\R,0,0) node[right, xshift=0.15mm,yshift
19 =-0.28mm]{$\hat{e}_u$}; %xshift=0.35mm,yshift=-0.2mm
20 \draw[->, >=latex, thick,tdplot_rotated_coords] (0,\R,0) -- (0,\BasisScaleR*\R,0) node[right,xshift=-0.5mm
21 ]{$\hat{e}_v$};
22 \draw[->, >=latex, thick,tdplot_rotated_coords] (0,0,\R) -- (0,0,\BasisScaleR*\R) node[right]{$\hat{e}_w$};
23 %% Draw points of intersection of axes with sphere
24 \draw[tdplot_rotated_coords] (\R,0,0) node[circle, fill, inner sep=1pt]; % MOVED TO END
25 \draw[tdplot_rotated_coords] (0,\R,0) node[circle, fill, inner sep=1pt];
26 % Draw location of spherical coordinate system (r, theta, phi)
27 \pgfmathsetmacro{\r}{\R}
28 \pgfmathsetmacro{\SPHRdelta}{60} % 60,60 | 60,75 | 55,58
29 \pgfmathsetmacro{\SPHRlambda}{60}
30 \pgfmathsetmacro{\SPHRtheta}{90-\SPHRdelta} % For use later
31 \pgfmathsetmacro{\CoordinateX}{\r*cos(\SPHRdelta)*cos(\SPHRlambda)}
32 \pgfmathsetmacro{\CoordinateY}{\r*cos(\SPHRdelta)*sin(\SPHRlambda)}
33 \pgfmathsetmacro{\CoordinateZ}{\r*sin(\SPHRdelta)}
34 % MOVED TO BOTTOM
35 \draw[dashed, thick,tdplot_rotated_coords] (0,0,0) -- (\CoordinateX, \CoordinateY, \CoordinateZ) node[pos
36 =0.6,left]{$r$};
37 \draw[loosely dashed,tdplot_rotated_coords] (0,0,0) -- (\CoordinateX*2, 2*\CoordinateY, 0);
38 \draw[dashed,tdplot_rotated_coords] (\CoordinateX,\CoordinateY,0) -- (\CoordinateX, \CoordinateY, \
39 CoordinateZ) node[pos=0.6,right]{$z$};
40 \draw[dashed,tdplot_rotated_coords] (\CoordinateX,\CoordinateY,0) -- (\CoordinateX, 0, 0) node[pos=0.5,
41 anchor=north,xshift=-1mm,yshift=0.5mm]{$y$};%node[pos=1,anchor=south east,xshift=0.5mm,yshift
42 =-1.75mm]{$x$};
43 \draw[dashed,tdplot_rotated_coords] (\CoordinateX,\CoordinateY,0) -- (0, \CoordinateY, 0) node[pos=0.5,
44 anchor=west,xshift=-0.1mm,yshift=-0.5mm]{$x$};%node[pos=1, anchor=south east,xshift=3mm,yshift
45 =-0.25mm]{$y$};
46 \draw[tdplot_rotated_coords] (\CoordinateX,\CoordinateY,\CoordinateZ) node[circle, fill, inner sep=1pt] node
47 [anchor=south east,xshift=-0.5mm,yshift=-1.7mm]{$\mathfrak{X}$};
48 %% Outline lune
49 \tdplotdrawarc[tdplot_rotated_coords,color=blue]{(0,0,0)}{\R}{0}{90}{}{}
50 \begin{scope}[rotate=90]
51 \tdplotdrawarc[tdplot_rotated_coords,color=blue]{(0,0,0)}{\R}{0}{90}{}{}
52 \end{scope}
53 %
54 %\DrawLatitudeArc[\R]{40}{green}{10}
55 \pgfmathsetmacro{\erX}{sin(\SPHRtheta)*cos(\SPHRlambda)}
56 \pgfmathsetmacro{\erY}{sin(\SPHRtheta)*sin(\SPHRlambda)}

```

```

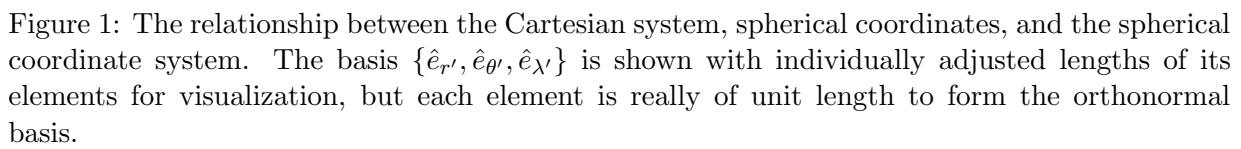
51 \pgfmathsetmacro{\erZ}{cos(\SPHRtheta)}
52 %
53 \pgfmathsetmacro{\ethetaX}{cos(\SPHRtheta)*cos(\SPHRLambda)}
54 \pgfmathsetmacro{\ethetaY}{cos(\SPHRtheta)*sin(\SPHRLambda)}
55 \pgfmathsetmacro{\ethetaZ}{-sin(\SPHRtheta)}
56 %
57 \pgfmathsetmacro{\elambdaX}{-sin(\SPHRLambda)}
58 \pgfmathsetmacro{\elambdaY}{cos(\SPHRLambda)}
59 \pgfmathsetmacro{\elambdaZ}{0}
60 \pgfmathsetmacro{\scaleR}{1} % scale factor for e_r | 0.8
61 \pgfmathsetmacro{\scaleTheta}{0.6} % scale factor for e_theta | 0.6
62 \pgfmathsetmacro{\scaleLambda}{0.6} % scale factor for e_lambda | 0.4
63 % Draw spherical basis
64 \draw[>,>=latex,thick,tdplot_rotated_coords] (\CoordinateX,\CoordinateY,\CoordinateZ) -- (\CoordinateX+\erX*\scaleR,\CoordinateY+\erY*\scaleR,\CoordinateZ+\erZ*\scaleR) node[anchor=south west,xshift=-1.5mm,yshift=-1.5mm]{$\hat{e}_{\mathbf{r}}$};
65 \draw[>,>=latex,thick,tdplot_rotated_coords] (\CoordinateX,\CoordinateY,\CoordinateZ) -- (\CoordinateX+\ethetaX*\scaleTheta,\CoordinateY+\ethetaY*\scaleTheta,\CoordinateZ+\ethetaZ*\scaleTheta) node[pos=1,anchor=north west,xshift=-1.5mm,yshift=0.5mm]{$\hat{e}_{\mathbf{\theta}}$};
66 \draw[>,>=latex,thick,tdplot_rotated_coords] (\CoordinateX,\CoordinateY,\CoordinateZ) -- (\CoordinateX+\elambdaX*\scaleLambda,\CoordinateY+\elambdaY*\scaleLambda,\CoordinateZ+\elambdaZ*\scaleLambda) node[anchor=south west,xshift=-1mm,yshift=-2mm]{$\hat{e}_{\mathbf{\lambda}}$};
67 % Draw new object X' that is referenced in spherical coordinates
68 \pgfmathsetmacro{\XprimeX}{\CoordinateX+0.33} %0.25 | 0.31 | 0.33
69 \pgfmathsetmacro{\XprimeY}{\CoordinateY+0.84} %0.9 | 0.84 | 0.84
70 \pgfmathsetmacro{\XprimeZ}{\CoordinateZ+0.6} %0.1 | 0.5 | 0.6
71 \draw[tdplot_rotated_coords] (\XprimeX,\XprimeY,\XprimeZ) node[circle,fill,inner sep=1pt]{$\mathbf{X}'$};
72 %\draw[tdplot_rotated_coords] (\CoordinateX,\CoordinateY,\CoordinateZ) -- (\XprimeX,\XprimeY,\XprimeZ);
73 \pgfmathsetmacro{\Der}{\erX*(\XprimeX-\CoordinateX) + \erY*(\XprimeY-\CoordinateY) + \erZ*(\XprimeZ-\CoordinateZ)}
74 \pgfmathsetmacro{\XprimeSPHRprojX}{\XprimeX-\Der*\erX}
75 \pgfmathsetmacro{\XprimeSPHRprojY}{\XprimeY-\Der*\erY}
76 \pgfmathsetmacro{\XprimeSPHRprojZ}{\XprimeZ-\Der*\erZ}
77 \draw[dashed,tdplot_rotated_coords] (\XprimeSPHRprojX,\XprimeSPHRprojY,\XprimeSPHRprojZ) -- (\XprimeX,\XprimeY,\XprimeZ) node[pos=0.6,right]{$\mathbf{r}'$};
78 \pgfmathsetmacro{\normSPHRPlanar}{sqrt((\XprimeSPHRprojX-\CoordinateX)*(\XprimeSPHRprojX-\CoordinateX) + (\XprimeSPHRprojY-\CoordinateY)*(\XprimeSPHRprojY-\CoordinateY) + (\XprimeSPHRprojZ-\CoordinateZ)*(\XprimeSPHRprojZ-\CoordinateZ))}
79 \pgfmathsetmacro{\cosTempThetaAngle}{((\XprimeSPHRprojX-\CoordinateX)*\ethetaX + (\XprimeSPHRprojY-\CoordinateY)*\ethetaY + (\XprimeSPHRprojZ-\CoordinateZ)*\ethetaZ) / \normSPHRPlanar}
80 \pgfmathsetmacro{\TempThetaAngle}{acos(\cosTempThetaAngle)}
81 \pgfmathsetmacro{\Detheta}{\normSPHRPlanar*sin(\TempThetaAngle)}
82 \draw[dashed,tdplot_rotated_coords] (\XprimeSPHRprojX,\XprimeSPHRprojY,\XprimeSPHRprojZ) -- (\XprimeSPHRprojX-\Detheta*\elambdaX,\XprimeSPHRprojY-\Detheta*\elambdaY,\XprimeSPHRprojZ-\Detheta*\elambdaZ) node[pos=0.5,anchor=north west,xshift=-0.51mm,yshift=1.5mm]{$\mathbf{\lambda}$};
83 % Repeat but now for lambda axis
84 \pgfmathsetmacro{\cosTempLambdaAngle}{((\XprimeSPHRprojX-\CoordinateX)*\elambdaX + (\XprimeSPHRprojY-\CoordinateY)*\elambdaY + (\XprimeSPHRprojZ-\CoordinateZ)*\elambdaZ) / \normSPHRPlanar}
85 \pgfmathsetmacro{\TempLambdaAngle}{acos(\cosTempLambdaAngle)}
86 \pgfmathsetmacro{\Delambda}{\normSPHRPlanar*sin(\TempLambdaAngle)}
87 \draw[dashed,tdplot_rotated_coords] (\XprimeSPHRprojX,\XprimeSPHRprojY,\XprimeSPHRprojZ) -- (\XprimeSPHRprojX-\Delambda*\ethetaX,\XprimeSPHRprojY-\Delambda*\ethetaY,\XprimeSPHRprojZ-\Delambda*\ethetaZ) node[pos=0.35,right]{$\mathbf{\theta}$};
88 % Draw dashed lines of X' onto xyz axes finally
89 % v This caused some confusion with appearing behind meridian in sphr coords but in front of it in cartesian
90 %\draw[dashed,tdplot_rotated_coords] (\XprimeX,\XprimeY,0) -- (\XprimeX,\XprimeY,\XprimeZ);
91 %\draw[dashed,tdplot_rotated_coords] (\XprimeX,\XprimeY,0) -- (\XprimeX,0,0);
92 %\draw[dashed,tdplot_rotated_coords] (\XprimeX,\XprimeY,0) -- (0,\XprimeY,0);
93

```

```

94
95 % Try clipping on xy plane
96 \begin{scope}[tdplot_rotated_coords]
97 \clip (0,0,0) -- (\R,0,0) arc (0:\SPHRlambda:\R) -- (0,\R,0) -- cycle;
98 % Draw coordinate grid in xy plane
99 \pgfmathsetmacro{\Pd}{1.5}
100 \pgfmathsetmacro{\LowerLim}{0}
101 \pgfmathsetmacro{\stepSize}{0.08}
102 \pgfmathsetmacro{\UpperLim}{\Pd-\stepSize}
103 \foreach \s in {\LowerLim,\stepSize,...,\UpperLim} {
104   \ifthenelse{\NOT \equal{\s}{0}}{%
105     {\draw[black!25,thin] (\s,0,0) -- (\s,\Pd,0);
106       \draw[black!25,thin] (0,\s,0) -- (\Pd,\s,0);}
107   } % No else
108 }
109 \end{scope}
110 \draw[dashed,tdplot_rotated_coords] (0,0,0) -- (\CoordinateX, \CoordinateY, \CoordinateZ) node[pos=0.6,
  left]{\$r\$};
111 \draw[loosely dashed,tdplot_rotated_coords] (0,0,0) -- (\CoordinateX, \CoordinateY, 0);
112 \draw[loosely dashed,tdplot_rotated_coords] (\CoordinateX, \CoordinateY, 0) -- ({\r*cos(\SPHRlambda)},{\r*
  sin(\SPHRlambda)},0);
113 \draw[dashed,tdplot_rotated_coords] (\CoordinateX,\CoordinateY,0) -- (\CoordinateX, \CoordinateY, \
  CoordinateZ) node[pos=0.5,right]{\$w\$};
114 \draw[dashed,tdplot_rotated_coords] (\CoordinateX,\CoordinateY,0) -- (\CoordinateX, 0, 0) node[pos=0.5,
  anchor=north,xshift=-1mm,yshift=0.15mm]{\$v\$};%node[pos=1,anchor=south east,xshift=0.5mm,yshift
  =-1.75mm]{\$x\$};
115 \draw[dashed,tdplot_rotated_coords] (\CoordinateX,\CoordinateY,0) -- (0, \CoordinateY, 0) node[pos=0.5,
  anchor=west,xshift=-0.1mm,yshift=-0.5mm]{\$u\$};%node[pos=1, anchor=south east,xshift=3mm,yshift
  =-0.25mm]{\$y\$};
116 \draw[tdplot_rotated_coords] (\CoordinateX,\CoordinateY,\CoordinateZ) node[circle, fill, inner sep=1]{ } node[
  anchor=south east,xshift=-0.5mm,yshift=-1.7mm]{\$ \mathfrak{X} \$};
117
118 % Moved from beginning to end after image was completed to eliminate overlap
119 \foreach \t in {-5,-20,...,-175} { \DrawLongitudeCircleWithoutSectorHARDCODED[\R]{\t} } %
  -5,-20,...,-175
120 \foreach \t in {-80,-70,...,80} { \DrawLatitudeCircleWithoutSectorHARDCODED[\R]{\t} } %
  -80,-70,...,80
121 %~
122 \draw[tdplot_rotated_coords] (\R,0,0) node[circle, fill, inner sep=1]{ };
123
124 %% Just to check
125 %\node[tdplot_rotated_coords] at (2.5,2.5,2) {\XprimeX, \XprimeY, \XprimeZ};
126 %\pgfmathsetmacro{\coslambda}{cos(\SPHRlambda)}
127 %\pgfmathsetmacro{\sinlambda}{sin(\SPHRlambda)}
128 %\node[tdplot_rotated_coords] at (2.5,2.5,1.8) {\coslambda, \sinlambda, 0};
129 %% This probably has to go last
130 %\tdplotdrawarc[->,>=stealth,tdplot_rotated_coords,thick]{(0,0,0)}{1}{0}{\SPHRlambda}{anchor=south,
  yshift=0.3mm}{\$ \lambda \$}
131 %\tdplotsetthetaplanecoords{\SPHRlambda}
132 %\tdplotdrawarc[->,>=stealth,tdplot_rotated_coords]{(0,0,0)}{0.4}{0}{\SPHRtheta}{above,yshift=-0.3mm
  }{ \$\,,\,,\theta \$}
133 %%\tdplotdrawarc[->,>=stealth,tdplot_rotated_coords]{(0,0,0)}{0.6}{90}{\SPHRdelta}{right}{\$ \delta \$}
134 \end{tikzpicture}
135 \caption{The relationship between the Cartesian system, spherical coordinates, and the spherical coordinate
  system. The basis  $\{\hat{e}_r, \hat{e}_\theta, \hat{e}_\lambda\}$  is shown with individually
  adjusted lengths of its elements for visualization, but each element is really of unit length to form the
  orthonormal basis.}
136 \label{fig:SPHRcoords}
137 \end{figure}

```



2.3 Comparing Perfect Circle with Rugged Ellipse

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}[scale=0.82]
4 \pgfmathsetmacro{\ellipseX}{85}
5 \pgfmathsetmacro{\ellipseY}{75}
6 \draw[decorate, decoration={random steps,segment length=3pt,amplitude=1pt,aspect=0}] (0,0) ellipse[x radius
   =\ellipseX pt, y radius=\ellipseY pt];
7 \draw (0,0) circle [radius=\ellipseX pt];
8
9 % .... Scale = 4:
10 %\draw[decorate, decoration={random steps,segment length=0.25pt,amplitude=0.15pt,aspect=0}] (0,0) ellipse[x
   radius=20pt, y radius=15pt];
11 %\draw (0,0) circle[radius=20pt];
12 \end{tikzpicture}
13 \caption{An exaggerated two-dimensional visualization of the containment of Earth in a sphere of average
   equatorial radius in the cross section along a meridian/anti-meridian great circle.}
14 \end{figure}

```

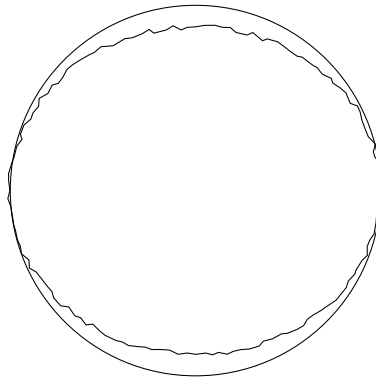


Figure 2: An exaggerated two-dimensional visualization of the containment of Earth in a sphere of average equatorial radius in the cross section along a meridian/anti-meridian great circle.

2.4 Flattening a Circle into an Ellipse and Defining Its Parameters

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}[scale=0.83]
4 \pgfmathsetmacro{\ellipseX}{85}
5 \pgfmathsetmacro{\ellipseY}{60}
6 \draw (0,0) ellipse [x radius=\ellipseX pt, y radius=\ellipseY pt];
7 \draw (0,0) circle [radius=\ellipseX pt];
8 \draw[->,>=stealth] (0 pt,\ellipseX pt) -- (0 pt,\ellipseY pt) node[midway,right]{$f$};
9 \draw[->,>=stealth] (0 pt,-\ellipseX pt) -- (0 pt,-\ellipseY pt) node[midway,right]{$f$};
10
11 % Set focus distance
12 \pgfmathsetmacro{\focusX}{sqrt(\ellipseX^2 - \ellipseY^2)}
13
14 % Draw left-focus to covertex
15 \draw[dashed,color=black!40] (-\focusX pt, 0pt) -- (0 pt, \ellipseY pt) node[pos=0.535,left,xshift=-0.5mm,
16 yshift=0.5mm,black]{$a$};
17 % Draw closed triangle
18 \draw[dashed] (0,0) -- (\focusX pt,0 pt) node[pos=0.45, below,yshift=0.55mm]{$a - x$} -- (0 pt, \ellipseY
19 pt) node[pos=0.535,right,xshift=0.5mm,yshift=0.5mm]{$a$} -- cycle node[midway, left]{$b$};
20 \draw[dashed] (0,0) -- (\focusX pt,0 pt) node[pos=0.425, above,yshift=-0.55mm]{$a - x$};%pos=0.45, below
21 ,yshift=0.55mm
22 \draw[dashed] (\focusX pt,0 pt) -- (0 pt, \ellipseY pt) node[pos=0.535,right,xshift=0.5mm,yshift=0.5mm]{$a
23 $};
24 \draw[dashed] (0 pt, \ellipseY pt) -- (0,0) node[midway, left]{$b$};
25 \draw (\focusX pt,0 pt) -- (\ellipseX pt, 0) node[midway, above]{$x$};
26
27
28
29 \pgfmathsetmacro{\rt}{180}
30 \pgfmathsetmacro{\rx}{\ellipseX*cos(\rt)}
31 \pgfmathsetmacro{\ry}{\ellipseX*sin(\rt)}
32 %\draw[black!40] (0,0) -- (\rx pt, \ry pt) node[pos=0.85, above, black]{$a$}; % pos=0.88, right (270)
33
34
35
36 \pgfmathsetmacro{\OnEllipseAtx}{\ellipseY*sqrt(1-(\focusX/\ellipseX)^2)}
37 \pgfmathsetmacro{\ellipticity}{sqrt(1 - (\ellipseY/\ellipseX)^2)}
38 \pgfmathsetmacro{\semilatusrec}{\ellipseX*(1-\ellipticity^2)}
39 \pgfmathsetmacro{\textAng}{atan(\semilatusrec/(2*\ellipseX*\ellipticity))}
40 \draw[dashed,color=black!40] (\focusX pt, 0) -- (-\focusX pt, -\OnEllipseAtx pt) node[midway,below,black,
41 rotate=\textAng]{$2a - p$};
42 \draw (-\focusX pt, -\OnEllipseAtx pt) -- (-\focusX pt, 0) node[pos=0.55,left]{$p$};
43
44 % Draw circle focus
45 \draw (0,0) node[circle, fill, inner sep=1]{};
46 % Draw ellipse foci
47 \draw (\focusX pt,0 pt) node[circle, fill, inner sep=1]{};
48 \draw (-\focusX pt,0 pt) node[circle, fill, inner sep=1]{};
49 \end{tikzpicture}
50 \caption{A two-dimensional visualization of the containment of the oblate spheroid inside of a sphere showing
51 the definitions of the flattening (above/below ellipse), eccentricity (top-half of ellipse), and semi-
52 latus rectum (bottom-half of ellipse). Here, $x = a(1 - e)$}
53 \label{fig:SPHRELLP}
54 \end{figure}

```

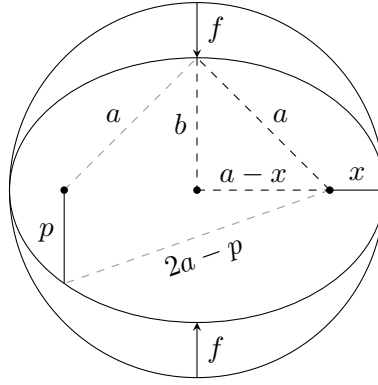


Figure 3: A two-dimensional visualization of the containment of the oblate spheroid inside of a sphere showing the definitions of the flattening (above/below ellipse), eccentricity (top-half of ellipse), and semi-latus rectum (bottom-half of ellipse). Here, $x = a(1 - e)$.

2.5 Circumscribing an Ellipse with a Circle (Reduced Latitude)

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}[scale=0.808]
4 \pgfmathsetmacro{\ellipseMaj}{85}
5 \pgfmathsetmacro{\ellipseMin}{60}
6
7 % Draw axes
8 \draw[>,>=latex,thick] (0,0) -- (\ellipseMaj pt, 0) node[pos=0.85,below]{$a$} node[right]{$\hat{e}\backslash\chi$};
9 \draw[>,>=latex,thick] (0,0) -- (0, \ellipseMin pt) node[pos=0.8,left]{$b$} node[pos=1, above]{$\hat{k}$};
10
11 % Draw circle and ellipse
12 \draw[dashed] (0,0) circle [radius=\ellipseMaj pt];
13 \draw[dashed] (0,0) circle [radius=\ellipseMin pt];
14 \draw (0,0) ellipse [x radius=\ellipseMaj pt, y radius=\ellipseMin pt];
15 %\begin{scope}
16 % \clip (-1,0) rectangle (1,1);
17 % \draw[dashed] (0,0) circle [radius=\ellipseMaj];
18 % \draw (0,0) ellipse [x radius=\ellipseMaj, y radius=\ellipseMin];
19 %\end{scope}
20
21 % Draw lines
22 \pgfmathsetmacro{\circumB}{50}
23 \pgfmathsetmacro{\circY}{\ellipseMaj*sin(\circumB)}
24 \pgfmathsetmacro{\ellipX}{\ellipseMaj*cos(\circumB)}
25 \pgfmathsetmacro{\ellipY}{\ellipseMin*sqrt(1 - (\ellipX/\ellipseMaj)^2)}
26 \draw (0,0) -- (\ellipX pt, \circY pt);
27 \draw[densely dashed] (\ellipX pt, \circY pt) -- (\ellipX pt, \ellipY pt);
28 \pgfmathsetmacro{\SmallerCircX}{\ellipseMin*cos(\circumB)}
29 \pgfmathsetmacro{\SmallerCircY}{\ellipseMin*sin(\circumB)}
30 \draw[densely dashed] (\ellipX pt, \ellipY pt) -- (\SmallerCircX pt, \SmallerCircY pt);
31 %\draw (0,0) -- (\ellipX, \ellipY);
32
33 % Draw angle
34 \pgfmathsetmacro{\rpos}{0.3*\ellipseMaj}
35 \draw[>,>=stealth] (\rpos pt, 0) arc [start angle=0, end angle=\circumB, radius=\rpos pt] node[midway,
    right]{$\beta$};
36
37 %\draw[dashed] (0,0) -- (0,1) node[pos=0.7,solid]{$\backslash$AxisRotator[x=0.15cm,y=0.55cm,>,>=latex,rotate
    =-90]};
38
39 \draw (\ellipX pt,\ellipY pt) node[circle, fill, inner sep=1,]{}; % node[below,xshift=0.8mm,yshift=-0.25mm
    ]{$\mathfrak{X}$};
40
41
42 \draw (\ellipX+25 pt,\ellipY+15 pt) .. controls (\ellipX+15 pt,\ellipY+15 pt) and (\ellipX+10 pt,\ellipY+10
    pt) .. (\ellipX pt,\ellipY pt) node[pos=0, right]{$\mathfrak{X}_s$};
43 \end{tikzpicture}
44 \caption{A two-dimensional visualization of the reduced latitude with inscribing circles of each axis. This
    cross-section is cut along a $\lambda$ meridian of the three-dimensional spheroid}% at the longitude $\lambda$.}
45 \end{figure}

```

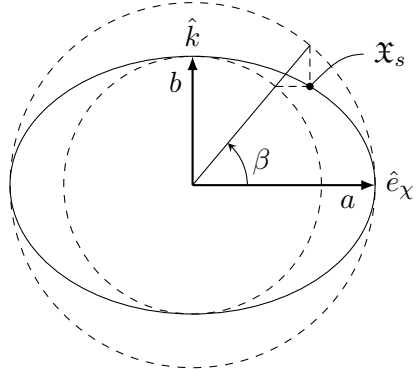


Figure 4: A two-dimensional visualization of the reduced latitude with inscribing circles of each axis. This cross-section is cut along a λ meridian of the three-dimensional spheroid

2.6 Position Measured on an Ellipse (Geocentric Latitude)

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}[scale=0.82]
4 \pgfmathsetmacro{\ellipseMaj}{85}
5 \pgfmathsetmacro{\ellipseMin}{60}
6
7 % Draw axes
8 \draw[>, >=stealth, thick] (0,0) -- (\ellipseMaj pt, 0) node[pos=0.85,below]{$a$} node[right]{\hat{e}-\chi
9 $};
10 \draw[>, >=stealth, thick] (0,0) -- (0, \ellipseMin pt) node[pos=0.8,left]{$b$} node[pos=1, above]{\hat{k}
11 $};
12 % Draw ellipse
13 \draw (0,0) ellipse [x radius=\ellipseMaj pt, y radius=\ellipseMin pt];
14 %\begin{scope}
15 % \clip (-1,0) rectangle (1,1);
16 % \draw[dashed] (0,0) circle [radius=\ellipseMaj];
17 % \draw (0,0) ellipse [x radius=\ellipseMaj, y radius=\ellipseMin];
18 %\end{scope}
19 % Draw lines
20 \pgfmathsetmacro{\circumB}{50}
21 \pgfmathsetmacro{\circY}{\ellipseMaj*sin(\circumB)}
22 \pgfmathsetmacro{\ellipX}{\ellipseMaj*cos(\circumB)}
23 \pgfmathsetmacro{\ellipY}{\ellipseMin*sqrt(1 - (\ellipX/\ellipseMaj)^2)}
24 \draw (0,0) -- (\ellipX pt, \ellipY pt) node[pos=0.7,above,xshift=-0.5mm]{$r_s$};
25
26 % Draw angle
27 \pgfmathsetmacro{\GeocentricAng}{atan(\ellipY/\ellipX)}
28 \pgfmathsetmacro{\rpos}{0.3*\ellipseMaj}
29 \draw[>, >=latex] (\rpos pt, 0) arc [start angle=0, end angle=\GeocentricAng, radius=\rpos pt] node[
30 midway,right]{$\varphi_s$};
31 %\draw[dashed] (0,0) -- (0,1) node[pos=0.7,solid]{\AxisRotator[x=0.15cm,y=0.55cm,>, >=latex,rotate
32 =-90]};
33 \draw (\ellipX pt,\ellipY pt) node[circle, fill, inner sep=1,]{} node[anchor=south west,xshift=-0.5mm,
34 yshift=-0.5mm]{$\mathfrak{X}_s$};
35 \end{tikzpicture}
36 \caption{The geocentric latitude and radius constrained to the spheroid's surface at a longitude $\lambda$.}
37 \end{figure}

```

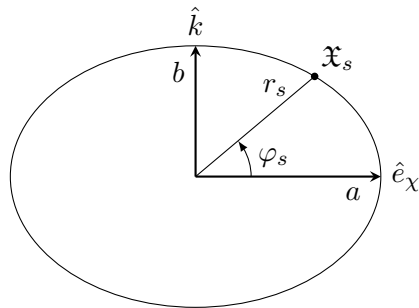


Figure 5: The geocentric latitude and radius constrained to the spheroid's surface at a longitude λ .

2.7 Comparing Geocentric Latitude to Reduced Latitude

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}[scale=0.83]
4 \pgfmathsetmacro{\ellipseMaj}{85}
5 \pgfmathsetmacro{\ellipseMin}{60}
6
7 % Draw axes
8 \draw[>, >=latex, thick] (0,0) -- (\ellipseMaj pt, 0) node[pos=0.85,below]{$a$} node[right]{$\hat{e}\text{-}\chi$};
9 \draw[>, >=latex, thick] (0,0) -- (0, \ellipseMin pt) node[pos=0.8,left]{$b$} node[pos=1, above]{$\hat{k}$};
10
11 % Draw circle and ellipse
12 \draw[dashed] (0,0) circle [radius=\ellipseMaj pt];
13 \draw (0,0) ellipse [x radius=\ellipseMaj pt, y radius=\ellipseMin pt];
14 %\begin{scope}
15 % \clip (-1,0) rectangle (1,1);
16 % \draw[dashed] (0,0) circle [radius=\ellipseMaj];
17 % \draw (0,0) ellipse [x radius=\ellipseMaj, y radius=\ellipseMin];
18 %\end{scope}
19
20 % Draw lines
21 \pgfmathsetmacro{\circumB}{50}
22 \pgfmathsetmacro{\circY}{\ellipseMaj*sin(\circumB)}
23 \pgfmathsetmacro{\ellipX}{\ellipseMaj*cos(\circumB)}
24 \pgfmathsetmacro{\ellipY}{\ellipseMin*sqrt(1 - (\ellipX/\ellipseMaj)^2)}
25 \draw (0,0) -- (\ellipX pt, \circY pt);
26 \draw[densely dashed] (\ellipX pt, \circY pt) -- (\ellipX pt, \ellipY pt);
27 %\draw (0,0) -- (\ellipX, \ellipY);
28 %~
29 \draw (0,0) -- (\ellipX pt,\ellipY pt) node[pos=0.9,below,xshift=1mm]{$r_s$};
30
31 % Draw angles
32 \pgfmathsetmacro{\rpos}{0.35*\ellipseMaj}
33 \draw[>, >=stealth] (\rpos pt, 0) arc [start angle=0, end angle=\circumB, radius=\rpos pt] node[pos=0.35, right,xshift=-0.1mm]{$\beta$};
34 \pgfmathsetmacro{\GeocentricAng}{atan(\ellipY/\ellipX)}
35 \pgfmathsetmacro{\rpos}{0.6*\ellipseMaj}
36 \draw[>, >=stealth] (\rpos pt, 0) arc [start angle=0, end angle=\GeocentricAng, radius=\rpos pt] node[pos=0.45,right,xshift=-0.4mm]{$\varphi_s$};
37
38 %\draw[dashed] (0,0) -- (0,1) node[pos=0.7,solid]{$\text{\AxisRotator}[x=0.15cm,y=0.55cm,>, >=latex,rotate=-90]$};
39
40 \draw (\ellipX pt,\ellipY pt) node[circle, fill, inner sep=1,]{}; % node[below]{$\mathfrak{X}$};
41 \draw (\ellipX+25 pt,\ellipY+15 pt) .. controls (\ellipX+15 pt,\ellipY+15 pt) and (\ellipX+10 pt,\ellipY+10 pt) .. (\ellipX pt,\ellipY pt) node[pos=0, right]{$\mathfrak{X}_s$};
42 \end{tikzpicture}
43 \caption{The reduced latitude and geocentric latitude methods compared together at a longitude $\lambda$ in the spheroid. The effect of the flattening in the relationship between the two angles is seen by the vertical squashing of the circle (sphere) into the ellipse (spheroid).}
44 \label{fig:reducedLatgeocentricLat}
45 \end{figure}

```

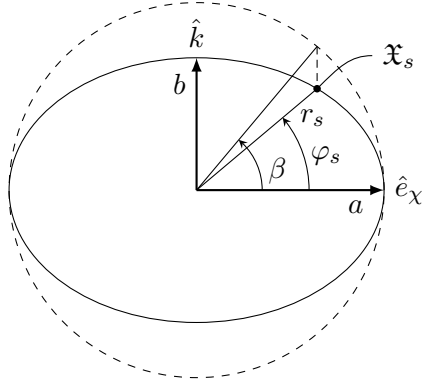


Figure 6: The reduced latitude and geocentric latitude methods compared together at a longitude λ in the spheroid. The effect of the flattening in the relationship between the two angles is seen by the vertical squashing of the circle (sphere) into the ellipse (spheroid).

2.8 Plot of Geocentric Latitudes Varying with the Flattening

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}
4 \begin{axis}[%
5 width=3.229in,
6 height=2.461in,
7 at={(0.542in,0.43in)},
8 scale only axis,
9 unbounded coords=jump,
10 xmin=-90,
11 xmax=90,
12 xtick={-90,-75,-60,-45,-30,-15,0,15,30,45,60,75,90},
13 xticklabels={{-90},{},{-60},{},{-30},{},{0},{},{30},{},{60},{},{90}},
14 xlabel style={font=\color{white!15!black}},
15 xlabel={Reduced Latitude $\beta$ [deg]},
16 ymin=-90,
17 ymax=90,
18 ytick={-90,-75,-60,-45,-30,-15,0,15,30,45,60,75,90},
19 yticklabels={{-90},{},{-60},{},{-30},{},{0},{},{30},{},{60},{},{90}},
20 ylabel style={font=\color{white!15!black}},
21 ylabel={Geocentric Latitude $\varphi_s$ [deg]},
22 axis background/.style={fill=white},
23 title style={font=\bfseries},
24 title={Latitudes of Points on the Spheroid},
25 xmajorgrids,
26 ymajorgrids,
27 legend style={at={(0.03,0.97)}, anchor=north west, legend cell align=left, align=left, draw=white!15!black}
28 ]
29 \addlegendimage{empty legend}
30 \addlegendentry{$f$}
31 %\addplot[domain=-90:90, samples=101, unbounded coords=jump]{atan((1/1)*tan(x))};
32 %\addplot[domain=-90:90, samples=101, unbounded coords=jump, color=green!70!black!100, densely dashed]{
33 %atan((0.75/1)*tan(x))};
34 %\addplot[domain=-90:90, samples=101, unbounded coords=jump, color=blue!70!black!120, dashed]{atan
35 %((0.5/1)*tan(x))};
36 %\addplot[domain=-90:90, samples=101, unbounded coords=jump, red!40!orange!100!black!100, loosely dashed
37 %]{atan((0.25/1)*tan(x))};
38 \addplot[domain=-90:90, samples=101, unbounded coords=jump]{atan((1/1)*tan(x))};
39 \addplot[domain=-90:90, samples=101, unbounded coords=jump, densely dashed]{atan((0.75/1)*tan(x))};
40 \addplot[domain=-90:90, samples=101, unbounded coords=jump, dashed]{atan((0.5/1)*tan(x))};
41 \addplot[domain=-90:90, samples=101, unbounded coords=jump, loosely dashed]{atan((0.25/1)*tan(x))};
42 \addlegendentry{0.00}
43 \addlegendentry{0.25}
44 \addlegendentry{0.50}
45 \addlegendentry{0.75}
46 \end{axis}
47
48 \begin{axis}[%
49 width=4.167in,
50 height=3.125in,
51 at={(0in,0in)},
52 scale only axis,
53 xmin=0,
54 xmax=1,
55 ymin=0,
56 ymax=1,
57 axis line style={draw=none},
58 ticks=none,
59 axis x line*=bottom,
60 axis y line*=left,
61 legend style={legend cell align=left, align=left, draw=white!15!black}

```

```

59| ]
60| \end{axis}
61| \end{tikzpicture}%
62| \caption{Reduced and geocentric latitude relationship as parameterized by the flattening  $f$ .}
63| \label{fig:reducedGeocentricLats}
64| \end{figure}

```

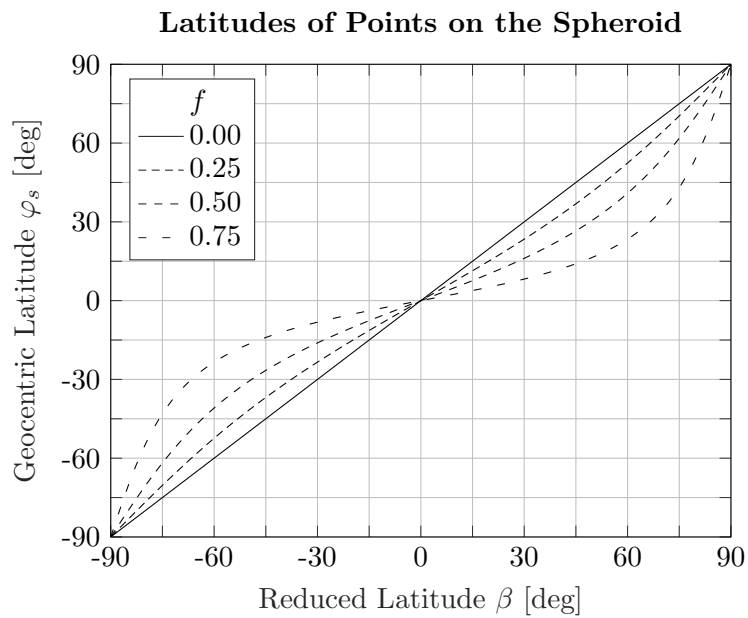


Figure 7: Reduced and geocentric latitude relationship as parameterized by the flattening f .

2.9 Ellipsoidal Coordinates

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}[scale=0.83]
4 \pgfmathsetmacro{\ellipseMaj}{85}
5 \pgfmathsetmacro{\ellipseMin}{60}
6
7 % Draw axes
8 \draw[>, >=latex, thick] (0,0) -- (\ellipseMaj pt, 0) node[pos=0.85,below]{$a$} node[pos=1, right]{$\hat{e}$
9   }-\chi$};
10 \draw[>, >=latex, thick] (0,0) -- (0, \ellipseMin pt) node[pos=0.8,left]{$b$} node[pos=1, above]{$\hat{k}$
11   }$};
12
13 % Draw ellipse
14 \draw (0,0) ellipse [x radius=\ellipseMaj pt, y radius=\ellipseMin pt];
15 %\begin{scope}
16 % \clip (-1,0) rectangle (1,1);
17 % \draw[dashed] (0,0) circle [radius=\ellipseMaj];
18 % \draw (0,0) ellipse [x radius=\ellipseMaj, y radius=\ellipseMin];
19 %\end{scope}
20
21 % Draw lines
22 \pgfmathsetmacro{\circumB}{25}
23 \pgfmathsetmacro{\circY}{\ellipseMaj*sin(\circumB)}
24 \pgfmathsetmacro{\ellipX}{\ellipseMaj*cos(\circumB)}
25 \pgfmathsetmacro{\ellipY}{\ellipseMin*sqrt(1 - (\ellipX/\ellipseMaj)^2)}
26 %\draw (0,0) -- (\ellipX pt, \ellipY pt) node[pos=0.75,anchor=south east,xshift=1mm]{$r$};
27
28 % Draw angle
29 \pgfmathsetmacro{\GeocentricAng}{atan(\ellipY/\ellipX)}
30 \pgfmathsetmacro{\rpos}{0.3*\ellipseMaj}
31 %\draw[>, >=latex] (\rpos pt, 0) arc [start angle=0, end angle=\GeocentricAng, radius=\rpos pt] node[
32   midway,right]{$\varphi$};
33
34 % Draw normal line
35 \pgfmathsetmacro{\normalM}{((\ellipseMaj^2 / \ellipseMin^2) * \ellipY / \ellipX}
36 \pgfmathsetmacro{\geodeticLat}{atan(\normalM)}
37 \pgfmathsetmacro{\yzero}{(1 - \ellipseMaj^2 / \ellipseMin^2) * \ellipY}
38 \draw[dashed] (0, 0) -- (0, \yzero pt);
39 \draw (0, \yzero pt) -- (\ellipX pt, \ellipY pt) node[pos=0.8,above,xshift=-1mm]{$R_n$};
40 \pgfmathsetmacro{\rpos}{0.3*\ellipseMaj}
41 %\draw[>, >=latex] (\rpos*2.06 pt, 0 pt) arc [start angle=0, end angle=\geodeticLat, radius=\rpos pt] node[
42   pos=0.45,right,xshift=-0.4mm]{$\phi$};
43
44 %\draw[dashed] (0, \yzero pt) -- (\ellipX pt, \yzero pt) node[midway, below]{$\chi_s$};
45 \draw[>, >=stealth] (\rpos pt, \yzero pt) arc [start angle=0, end angle=\geodeticLat, radius=\rpos pt] node[
46   pos=0.6,right]{$\phi$};
47
48 % Draw object X
49 %\draw (\ellipX pt,\ellipY pt) node[circle, fill, inner sep=1,]{} node[anchor=east,xshift=-0.35mm,yshift
50   =-0.7mm]{$\mathfrak{X}$};
51 %\draw (\ellipX pt,\ellipY pt) node[circle, fill, inner sep=1,]{} node[below,xshift=0.55mm,yshift=-0.7mm
52   ]{$\mathfrak{X}$};
53 \draw (\ellipX pt,\ellipY pt) node[circle, fill, inner sep=1]{};
54 %node[above]{$\mathfrak{X}_s$}; %node[anchor=south west,xshift=0.5mm,yshift=-3.5mm]{$\mathfrak{X}_s$};
55
56 \draw (\ellipX-10 pt,\ellipY+25 pt) .. controls (\ellipX-10 pt,\ellipY+15 pt) and (\ellipX pt,\ellipY+10 pt)
57   .. (\ellipX pt,\ellipY pt) node[pos=0, above, yshift=-1mm]{$\mathfrak{X}_s$};
58
59 % Draw height line
60 \pgfmathsetmacro{\heightX}{\ellipX + 40}
61 \pgfmathsetmacro{\heightY}{\ellipY + \normalM*(\heightX - \ellipX)}

```



```

53 \draw[dashed] (\ellipX pt,\ellipY pt) -- (\heightX pt,\heightY pt) node[pos=0.6,anchor=south east,yshift
    =-0.7mm]{\h$};
54
55 % Draw object X'
56 \draw (\heightX pt,\heightY pt) node[circle, fill , inner sep=1,]{\} node[anchor=south west, yshift=-1mm]{\mathfrak{X}};
57
58 % Draw chi - chi_s
59 %\draw[dashed] (\ellipX pt, \yzero pt) -- (\heightX pt, \yzero pt) node[midway,below]{\chi - \chi_s};
60 \draw[dashed] (0, \yzero pt) -- (\ellipX pt, \yzero pt) node[midway, below]{\chi - \chi_s} -- (\heightX pt, \yzero pt) node[midway,below,yshift=0.6mm]{\chi - \chi_s} -- (\heightX pt, \heightY pt);
61 \draw (\ellipX pt,-\ellipY pt) node[circle, fill , inner sep=1,]{\};
62 %\pgfmathsetmacro{\tmp}{\yzero-4.25}
63 %\draw[red] (0, \tmp pt) -- (\heightX pt, \tmp pt);
64 \end{tikzpicture}
65 \caption{The geodetic coordinates of an object  $\mathfrak{X}_s$  on the surface of the spheroid at a longitude  $\lambda$  and latitude  $\phi$  and another object  $\mathfrak{X}$  at the same longitude  $\lambda$  and latitude  $\phi$ , but at a nonzero height  $h$  normal to the spheroid's surface.}
66 \label{fig:ellipsoidalCoords}
67 \end{figure}

```

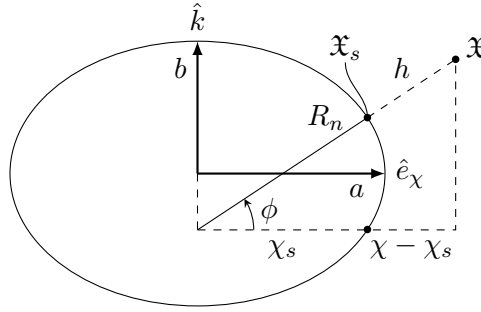


Figure 8: The geodetic coordinates of an object \mathfrak{X}_s on the surface of the spheroid at a longitude λ and latitude ϕ and another object \mathfrak{X} at the same longitude λ and latitude ϕ , but at a nonzero height h normal to the spheroid's surface.

2.10 East-North-Vertical Coordinates

```

1 \begin{figure}[H]
2 \centering
3 \tdplotsetmaincoords{70}{107} %70, 120 | 60,110 | 70,107
4 \begin{tikzpicture}[scale=3.1, tdplot_main_coords, join=bevel] % scale=3.2648 --> same height as sphere
   with a = 1.3, b=1.05
5 % Define spheroid parameters
6 \pgfmathsetmacro{\a}{1.3}
7 \pgfmathsetmacro{\b}{\a}
8 \pgfmathsetmacro{\c}{1.05} %0.9
9
10 % Draw the spheroid
11 \tdplotsetpolarplotrange{0}{180}{0}{360}
12 \tdplotsphericalsurfaceplot {72}{36}{1/sqrt((sin(\tdplottheta)*cos(\tdplotphi) / \a)^2 + (sin(\tdplottheta)*sin
   (\tdplotphi) / \b)^2 + (cos(\tdplottheta) / \c)^2)}{black}{white}{}{}
13 \draw[>, >=latex, thick, color=black] (\a,0,0) -- (\a+0.3, 0, 0) node[left, xshift=1mm,yshift=1.925035mm
   ]{\scalebox{0.94}{\hat{\imath}}}; %node[anchor=north east, xshift=1.7mm, yshift=2.3mm]{\hat{e}-\
   xi$};
14 \draw[>, >=latex, thick, color=black] (0,\b,0) -- (0, \b+0.4, 0) node[right]{\hat{\jmath}};
15 \draw[>, >=latex, thick, color=black] (0,0,\c) -- (0, 0, \c+0.3) node[above,yshift=-0.6mm]{\hat{k}};
16 % ~~~~~~ Draw the wireframe ~~~~~~
17 % Fill in 'lune' (\tdplotsetpolarplotrange{lowertheta}{uppertheta}{lowerphi}{upperphi})
18 \tdplotsetpolarplotrange{0}{90}{0}{90}
19 \tdplotsphericalsurfaceplot {72}{36}{1/sqrt((sin(\tdplottheta)*cos(\tdplotphi) / \a)^2 + (sin(\tdplottheta)*sin
   (\tdplotphi) / \b)^2 + (cos(\tdplottheta) / \c)^2)}{white}{white}{}{}%{black}{red!40!yellow!20!green
   !50!blue!70!white!70}{}{} % Original: red!40!yellow!20!green!50!blue!70
20
21
22 %%%%%%%%%%% Temporary ellipse to help compile time %%%%%%%%%%%
23 %%%%%%%%%%% \draw[red] (0,0) ellipse[x radius=28.5*\a pt, y radius=29.5*\c pt];
24
25 % Draw coordinate system in exposed x-y plane
26 % Try clipping on xy plane
27 \begin{scope}[canvas is xy plane at z=0]
28 \clip (0,0,0) -- (\a,0,0) arc (0:90:\a) -- (0,\b,0) -- cycle;
29 % Draw coordinate grid in xy plane
30 \pgfmathsetmacro{\Pd}{1.5}
31 \pgfmathsetmacro{\LowerLim}{0}
32 \pgfmathsetmacro{\stepSize}{0.08}
33 \pgfmathsetmacro{\UpperLim}{\Pd-\stepSize}
34 \foreach \s in {\LowerLim,\stepSize,...,\UpperLim} {
35 \ifthenelse{\NOT \equal{\s}{0}}%
36 {\draw[black!25,thin] (\s,0,0) -- (\s,\Pd,0);
37 \draw[black!25,thin] (0,\s,0) -- (\Pd,\s,0);}%
38 {} % No else
39 }
40 \end{scope}
41
42 % Draw lat and longs covered in white
43 \tdplotdefinepoints (0,0,0) (\a,0,0) (0,\a,0)
44 \tdplotdrawpolytopearc[black]{\a}{}{}
45 \begin{scope}[canvas is xz plane at y=0]
46 \draw[black] (0:1.3 and \c) arc (0:90:1.3 and \c);
47 \end{scope}
48 \begin{scope}[canvas is yz plane at x=0]
49 \draw[black] (0:1.3 and \c) arc (0:90:1.3 and \c);
50 \end{scope}
51
52
53 % Draw axes inside
54 \draw[densely dashed, thick, color=black] (0,0,0) -- (\a,0,0);
55 \draw[densely dashed, thick, color=black] (0,0,0) -- (0,\b,0);

```

```

56 \draw[densely dashed, thick, color=black] (0,0,0) -- (0,0,\c);
57
58 % Determine where Xs will go
59 \pgfmathsetmacro{\geocentricLatitude}{46.1} % 35.5 | 45.5 | 45.5
60 \pgfmathsetmacro{\geocentricLongitude}{60} % 50 | 50 | 60
61 % Determine geodetic coordinates
62 \pgfmathsetmacro{\h}{0}
63 \pgfmathsetmacro{\geodeticLatitude}{atan((\a / \c) * tan(\geocentricLatitude))}
64 \pgfmathsetmacro{\geodeticLongitude}{\geocentricLongitude}
65 \pgfmathsetmacro{\ellSpheroid}{\a^2 / sqrt((\a * cos(\geodeticLatitude))^2 + (\c * sin(\geodeticLatitude))^2)
66 }
67 \pgfmathsetmacro{\e}{sqrt(1 - (\c / \a)^2)}
68 % Calculate position on equator from longitude
69 \pgfmathsetmacro{\xX}{(\ellSpheroid + \h)*cos(\geodeticLatitude)*cos(\geocentricLongitude)}
70 \pgfmathsetmacro{\yX}{(\ellSpheroid + \h)*cos(\geodeticLatitude)*sin(\geocentricLongitude)}
71 \pgfmathsetmacro{\zX}{((1 - \e^2)*\ellSpheroid + \h)*sin(\geodeticLatitude)}
72
73 %%%
74 % Draw specified latitude and longitude lines
75 \tdplotsetrotatedcoords{0}{0}{\geocentricLongitude}
76 \begin{scope}[tdplot_rotated_coords, canvas is xz plane at y=0]
77 \draw[black] (0:1.3 and \c) arc (0:90:1.3 and \c);
78 \end{scope}
79 \tdplotsetrotatedcoords{0}{0}{0}
80 \begin{scope}[canvas is xy plane at z=\zX]
81 \pgfmathsetmacro{\rtemp}{\a * sqrt(1 - (\zX / \c)^2)}
82 \draw[black] (\rtemp,0) arc (0:90:\rtemp);
83 \end{scope}
84 \tdplotdefinepoints (0,0,0) (0,0,0) (0,0,0)
85 %%%
86
87 % Draw a dot at Xs and label Xs
88 \draw (\xX,\yX,\zX) node[circle, fill, inner sep=1, black]{} node[left, xshift=-0.6mm, yshift=0.91mm]{$\mathfrak{X}_s$, $\mathfrak{X}$};
89
90 %% (MOVED TO BOTTOM) Draw the longitude axis on the equator
91 %\tdplotdefinepoints(0,0,0)(\a,0,0)(\xXs,\yXs,0)
92 %\tdplotdrawpolytopearc[->, >=latex, very thick, yellow!55!black!90]{\a}{anchor=north, yshift=0.6mm}{$\lambda$}
93
94 % Draw geodetic latitude angle
95 \tdplotsetrotatedcoords{0}{0}{\geocentricLongitude}
96 \begin{scope}[tdplot_rotated_coords, canvas is xz plane at y=0]
97 \draw[->, >=stealth, very thick, yellow!55!black!90] (0:1.3 and \c) arc (0:\geocentricLatitude:1.3 and \c)
98 node[pos=0.58, right, black]{$\phi$};
99
100 % Calculate normal line
101 \pgfmathsetmacro{\xXs}{(\ellSpheroid)*cos(\geodeticLatitude)*cos(\geocentricLongitude)}
102 \pgfmathsetmacro{\yXs}{(\ellSpheroid)*cos(\geodeticLatitude)*sin(\geocentricLongitude)}
103 \pgfmathsetmacro{\zXs}{((1 - \e^2)*\ellSpheroid)*sin(\geodeticLatitude)}
104 \pgfmathsetmacro{\chix}{sqrt(\xXs^2 + \yXs^2)}
105 \pgfmathsetmacro{\chiy}{\c * sqrt(1 - (\chix / \a)^2)}
106 \pgfmathsetmacro{\xIntercept}{\e^2 * \chix}
107 \pgfmathsetmacro{\yIntercept}{(1 - (\a / \c)^2) * \chiy}
108
109 % Draw normal line
110 \draw[dashed, black!50] (0,\yIntercept) -- (\xIntercept,0);
111 \draw[dashed, black] (\xIntercept,0) -- (\chix,\chiy) node[pos=0.7, left, black]{$R_n$}; %Original color: red
112 !100!black!50 | red!85!yellow!50
113
114 % Draw planar axis e_chi
115 \draw[thick, dashed, black] (0,0) -- (\a,0);

```

```

114 \draw (\a,0) node[circle, fill, inner sep=1, black]{};
115 \draw[>, >=latex, thick, black] (\a,0) -- (\a+0.4,0) node[right,xshift=-0.25mm, yshift=-1.6mm]{$\hat{e}$
    }-\chi$};
116
117 % Draw e^2*chi component on z axis
118 \draw[dashed, black!50] (0,-0.01) -- (0, \yIntercept);
119
120 % Draw component parallel to e_chi
121 \pgfmithsetmacro{\r}{\a * sqrt(1 - (\yIntercept / \c)^2)}
122 %\draw[dashed, black!25] (0,\yIntercept) -- (\r,\yIntercept);
123
124 %\node at (3,2.5){\xIntercept};
125 %\node at (3,3){\chiy};
126 %\node at (3,3.5){\e};
127 \end{scope}
128
129 % !!!!!!!!
130 % New Basis
131 % !!!!!!!!
132 % Shift origin to Xs
133 \coordinate (Shift) at (\xX,\yX,\zX);
134 \tdplotsetrotatedcoords{0}{0}{0}
135 \tdplotsetrotatedcoordsorigin{(Shift)}
136 % Set scaling factors for each element
137 \pgfmithsetmacro{\scaleh}{1}
138 \pgfmithsetmacro{\scalephi}{0.9}
139 \pgfmithsetmacro{\scalelambda}{1.3}
140 % Rotate coordinates
141 \tdplotsetrotatedcoords{\geodeticLongitude}{90-\geodeticLatitude}{90}
142 % Draw new basis
143 %\draw[tdplot_rotated_coords, >, >=latex, thick, black] (0,0,0) -- (0.8,0,0) node[right, xshift=-1.1mm,
    yshift=1mm]{$\hat{e}$-\lambda$};%node[above]{$\hat{1}$};
144 %\draw[tdplot_rotated_coords, >, >=latex, thick, black] (0,0,0) -- (0,\scalephi,0) node[right, xshift=-0.15
    mm,yshift=1.35mm]{$\hat{e}$-\phi$};%node[above]{$\hat{2}$};
145 %\draw[tdplot_rotated_coords, >, >=latex, thick, black] (0,0,0) -- (0,0,\scaleh) node[above,xshift=1.75mm,
    yshift=-1mm]{$\hat{e}$-h$};%node[above]{$\hat{3}$};
146 \draw[tdplot_rotated_coords, >, >=latex, thick, black] (0,0,0) -- (0.8,0,0) node[right, xshift=-1.1mm,yshift
    =1mm]{$\widehat{E}$};%node[above]{$\hat{1}$};
147 \draw[tdplot_rotated_coords, >, >=latex, thick, black] (0,0,0) -- (0,\scalephi,0) node[right, xshift=-1.65
    mm,yshift=2.5mm]{$\widehat{N}$};%node[above]{$\hat{2}$};
148 \draw[tdplot_rotated_coords, >, >=latex, thick, black] (0,0,0) -- (0,0,\scaleh) node[above,xshift=1.75mm,
    yshift=-1mm]{$\widehat{V}$};%node[above]{$\hat{3}$};
149 % Draw a new object X'
150 \pgfmithsetmacro{\xXp}{0.7} % -0.2
151 \pgfmithsetmacro{\yXp}{0.8} % -0.1 | +0.2
152 \pgfmithsetmacro{\zXp}{0.2} % +0.6
153 \draw[tdplot_rotated_coords] (\xXp,\yXp,\zXp) node[circle, fill, inner sep=1, black]{} node[right]{$\mathfrak{
    X}$};
154 % Draw components of X' in ENV coordinate system
155 \draw[tdplot_rotated_coords, densely dashed, black] (\xXp,\yXp,0) -- (\xXp,\yXp,\zXp) node[pos=0.7,left]{$
    h$};
156 \draw[tdplot_rotated_coords, densely dashed, black] (\xXp,\yXp,0) -- (\xXp,0,0) node[pos=0.2,right]{$\phi
    $};
157 \draw[tdplot_rotated_coords, densely dashed, black] (\xXp,\yXp,0) -- (0,\yXp,0) node[pos=0.5,above]{$\
    lambda$};
158 \tdplotsetrotatedcoords{0}{0}{0}
159 \tdplotresetrotatedcoordsorigin
160
161 % Draw (x,y,z) components of Xs
162 \draw[densely dashed, black] (\xX,\yX,\zX) -- (\xX,\yX,0) node[midway,right]{$z$};
163 \draw[densely dashed, black] (\xX,\yX,0) -- (0,\yX,0) node[midway,right,yshift=-0.5mm]{$x$};
164 \draw[densely dashed, black] (\xX,\yX,0) -- (\xX,0,0) node[pos=0.4,below]{$y$};
165

```

```

166 % Must go last
167 % Draw the longitude axis on the equator
168 \tdplotdefinepoints (0,0,0) (\a,0,0) (\xX,\yX,0)
169 %\tdplotdrawpolytopearc[>, >=stealth, very thick, yellow!55!black!90]{\a}{anchor=north,xshift=-4.025mm,
    yshift=0.751mm}{\lambda$}%{anchor=south,xshift=-3mm,yshift=-1.5mm}{\lambda$}
170 \tdplotdrawpolytopearc[>, >=stealth, very thick, yellow!55!black!90]{\a}{anchor=south,rotate=3,xshift=-4
    mm, yshift=-0.5mm}{\lambda$}%{anchor=south,xshift=-3mm,yshift=-1.5mm}{\lambda$}
171 \draw (\a,0,0) node[circle, fill, inner sep=1, black]{};
172 \draw (0,\b,0) node[circle, fill, inner sep=1, black]{};
173 \end{tikzpicture}
174 \caption{The ENV coordinate system at a position on the surface of the spheroid ($h=0$).}
175 \label{fig:ENVCoords}
176 \end{figure}

```

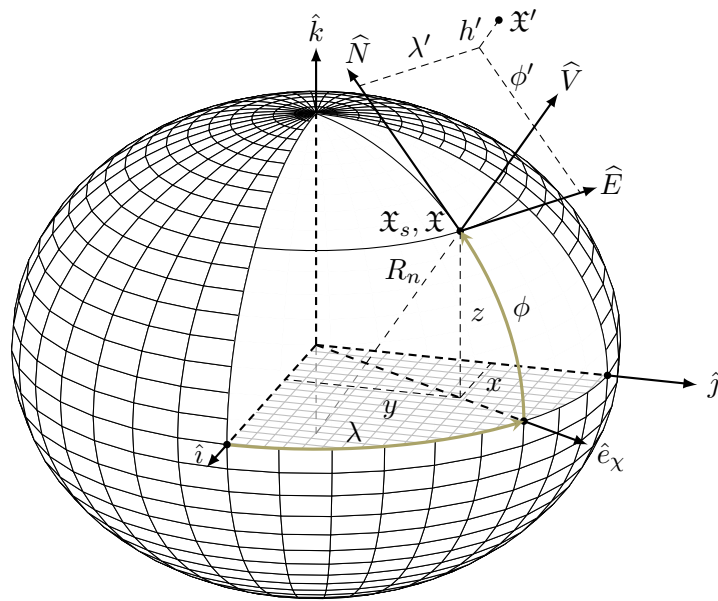


Figure 9: The ENV coordinate system at a position on the surface of the spheroid ($h = 0$).

2.11 Comparing Geocentric and Geodetic Latitudes Varying with the Flattening

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}
4 \definecolor{mycolor1}{rgb}{0.10000,0.80000,0.70000}%
5 \definecolor{mycolor2}{rgb}{0.10000,0.00000,0.80000}%
6 \begin{axis}[%
7 width=3.229in,
8 height=2.461in,
9 at={(0.542in,0.43in)},
10 scale only axis,
11 unbounded coords=jump,
12 xmin=-90,
13 xmax=90,
14 xtick={-90,-75,-60,-45,-30,-15,0,15,30,45,60,75,90},
15 xticklabels={{-90},{},{-60},{},{-30},{},{0},{},{30},{},{60},{},{90}},
16 xlabel style={font=\color{white!15!black}},
17 xlabel={Geodetic Latitude $\phi$ [deg]},
18 ymin=-90,
19 ymax=90,
20 ytick={-90,-75,-60,-45,-30,-15,0,15,30,45,60,75,90},
21 yticklabels={{-90},{},{-60},{},{-30},{},{0},{},{30},{},{60},{},{90}},
22 ylabel style={font=\color{white!15!black}},
23 ylabel={Geocentric Latitude $\varphi_s$ [deg]},
24 axis background/.style={fill=white},
25 title style={font=\bfseries},
26 title={Geocentric and Geodetic Latitude},
27 xmajorgrids,
28 ymajorgrids,
29 legend style={at={(0.03,0.97)}, anchor=north west, legend cell align=left, align=left, draw=white!15!black}
30 ]
31 \addlegendimage{empty legend}
32 \addlegendentry{$f$}
33 %\addplot[domain=-90:90, samples=101, unbounded coords=jump]{atan(tan(x)/((1-0)^2)};
34 %\addplot[domain=-90:90, samples=101, unbounded coords=jump, color=green!70!black!100, densely dashed]{
35 %atan(tan(x)/((1-0.25)^2)};
36 %\addplot[domain=-90:90, samples=101, unbounded coords=jump, color=blue!70!black!120, dashed]{atan(tan(
37 %x)/((1-0.5)^2)};
38 %\addplot[domain=-90:90, samples=101, unbounded coords=jump, red!40!orange!100!black!100, loosely dashed
39 %]{atan(tan(x)/((1-0.75)^2)};
40 \addplot[domain=-90:90, samples=101, unbounded coords=jump]{atan(tan(x)*((1-0)^2)};
41 \addplot[domain=-90:90, samples=101, unbounded coords=jump, densely dashed]{atan(tan(x)*((1-0.25)^2)};
42 \addplot[domain=-90:90, samples=101, unbounded coords=jump, dashed]{atan(tan(x)*((1-0.5)^2)};
43 \addplot[domain=-90:90, samples=101, unbounded coords=jump, loosely dashed]{atan(tan(x)*((1-0.75)^2)};
44 \addlegendentry{0.00}
45 \addlegendentry{0.25}
46 \addlegendentry{0.50}
47 \addlegendentry{0.75}
48 \end{axis}
49
50 \begin{axis}[%
51 width=4.167in,
52 height=3.125in,
53 at={(0in,0in)},
54 scale only axis,
55 xmin=0,
56 xmax=1,
57 ymin=0,
58 ymax=1,
59 axis line style={draw=none},

```

```

57 ticks=none,
58 axis x line*=bottom,
59 axis y line*=left,
60 legend style={legend cell align=left, align=left, draw=white!15!black}
61 ]
62 \end{axis}
63 \end{tikzpicture}%
64 \caption{Geocentric and geodetic latitude relationship as parameterized by the flattening  $f$ .}
65 \label{fig:GeocentricGeodeticLats}
66 \end{figure}

```

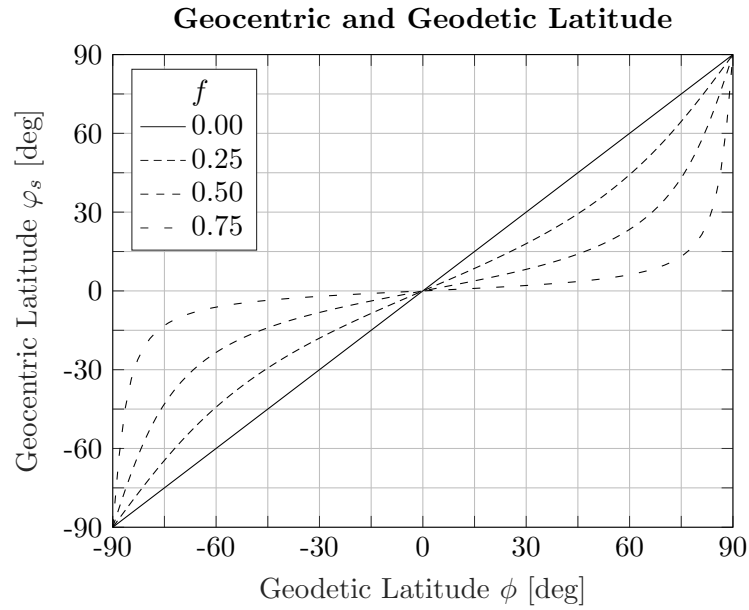


Figure 10: Geocentric and geodetic latitude relationship as parameterized by the flattening f .

2.12 Comparing Perfect Ellipse with Rugged Ellipse

```
1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}[scale=0.82]
4 \pgfmathsetmacro{\ellipseX}{85}
5 \pgfmathsetmacro{\ellipseY}{75}
6
7 \draw (0, 0) ellipse [x radius=\ellipseX pt, y radius=\ellipseY pt]; % (-0.1255 pt, 0), ... x radius=\ellipseX
   -1.05 pt
8 \draw[blue!80!green!80!white!70, decorate, decoration={random steps,segment length=3pt,amplitude=1pt,
   aspect=0}] (0,0) ellipse[x radius=\ellipseX pt, y radius=\ellipseY pt];
9 \end{tikzpicture}
10 \caption{An exaggerated two-dimensional visualization of an ellipsoid (black) fitted to the earth (blue).}
11 \end{figure}
```

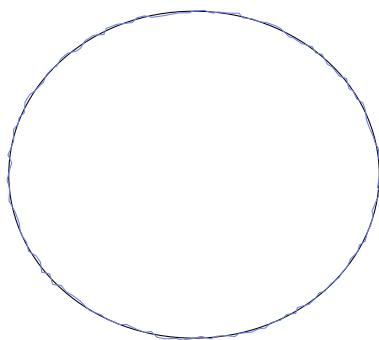


Figure 11: An exaggerated two-dimensional visualization of an ellipsoid (black) fitted to the earth (blue).

2.13 Local Gravity Depiction – Height at Latitude is Not Exactly Aligned with Radius to the Earth’s Center

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}[scale=4]
4 \pgfmathsetmacro{\xlim}{40}
5 \draw[decorate, decoration={random steps,segment length=3pt,amplitude=1pt,aspect=0}] (-\xlim pt,0) -- (\xlim pt,0) node[pos=0.22,above, yshift=-1mm]{Local sea level ( $g_0 = \mathrm{const.}$ )} node[pos=0.9,above,yshift=-1mm]{(Locally flat)}; % Flat ground
6 \pgfmathsetmacro{\xCenterOfEarth}{1.3}
7 \pgfmathsetmacro{\yCenterOfEarth}{-12.5}
8 \pgfmathsetmacro{\xsquig}{0.4*\xCenterOfEarth}
9 \pgfmathsetmacro{\ysquig}{\yCenterOfEarth / \xCenterOfEarth * \xsquig}
10 \draw[decorate, decoration={zigzag,segment length=3pt,amplitude=1pt,aspect=0, pre length = 0.25cm, post length = 1cm}] (\xCenterOfEarth pt, \yCenterOfEarth pt) -- (\xsquig pt, \ysquig pt) -- (0,0) node[pos = 0.2, left]{ $R$ }; % (\xCenterOfEarth pt, \yCenterOfEarth pt) -- (10*10/12.5 pt, \yCenterOfEarth+2.5 pt) -- (0,0) node[pos=0.6,right]{ $R$ };
11 \pgfmathsetmacro{\xX}{12.5}
12 \pgfmathsetmacro{\yX}{8.25}%{8.5}
13 \draw (0,0pt) -- (\xX pt, \yX pt);
14 \draw[dashed] (\xX pt, 0) -- (\xX pt, \yX pt) node[midway, right]{ $H$ };
15
16 % Draw objects X and X'
17 \draw (0, 0) node[circle, fill, inner sep=1,]{} node[anchor=south east]{ $\mathfrak{X}$ };
18 \draw (\xX pt,\yX pt) node[circle, fill, inner sep=1,]{} node[right]{ $\mathfrak{X}'$ };
19 \draw (\xCenterOfEarth pt,\yCenterOfEarth pt) node[circle, fill, inner sep=1,]{} node[below]{Earth center};
20 \end{tikzpicture}
21 \caption{Graphical representation of Newtonian gravity on the surface of Earth. The local elevation above sea level  $H$ , in general, is not in the same direction as the local direction to the earth’s center.}
22 \label{fig:LocalGravity}
23 \end{figure}

```

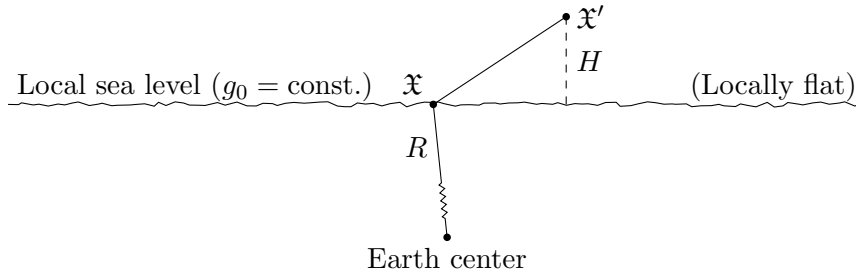


Figure 12: Graphical representation of Newtonian gravity on the surface of Earth. The local elevation above sea level H , in general, is not in the same direction as the local direction to the earth’s center.

2.14 Comparing Ellipsoid to the Geoid to the Actual Topography of Earth's Surface

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}
4 %% Temporary axis (comment out this bit when done)
5 %\draw[red] (0,0) -- (1,0);
6 %\draw[red] (0,0) -- (0,1);
7
8 % Draw the topographic surface, geoid, and ellipsoid
9 \begin{axis}[width=15cm,
10     height=207pt,
11     at={(-6.71cm,0pt)},
12     hide axis,
13 ]
14 % Plot the topographic surface
15 \addplot[domain=-1:1, samples=100, smooth, solid]{cos(deg(pi*x/2)) + (-4*cos(deg(3*pi*x/2)) + 6*sin(deg(pi*
16     *x)) - 3*sin(deg(2*pi*x)))/12};
17 % Plot the geoid surface
18 \addplot[domain=-1:1, samples=100, smooth, densely dashed]{(-0.1 + 0.4*cos(deg(pi*x/2)) - 0.2*cos(deg(pi*
19     x)) + 0.1333333*cos(deg(3*pi*x/2)) - 0.1*cos(deg(2*pi*x)))/1};
20 % Plot the ellipsoid
21 \addplot[domain=-1:1, samples=100, smooth, dashed]{1*sqrt(1 - (x/2)^2) - 1};
22 \end{axis}
23
24 % Add labels as nodes
25 \draw (-2.7, 0.74) -- (-1.7, 0.26) node[right] {Ellipsoid};
26 \draw (-2.7, 1.43) -- (-1.7, 2) node[right] {Geoid};
27 \draw (-2.7, 2.26) -- (-3.7, 3) node[above] {Topography};
28
29 % Draw a plumb line to an object
30 %\draw (\ellipX+25 pt,\ellipY+15 pt) .. controls (\ellipX+15 pt,\ellipY+15 pt) and (\ellipX+10 pt,\ellipY+10
31     pt) .. (\ellipX pt,\ellipY pt) node[pos=0, right]{ $\frac{X}{s}$ };
32 %\draw (2.9,1.4) -- (3,2.4) -- (2.95, 3.5) -- (2.925, 6);
33 %\draw (3,1.34) .. controls (3.3,2.4) and (2.95, 3.5) .. (2.925, 6) node[pos=0.6, right, align=left]{Plumb \
34     Line};
35 %\draw (3,1.34) .. controls (3.3,2.4) and (2.65, 3.5) .. (2.3, 6) node[pos=0.6, right, align=left]{Plumb \
36     Line} node[pos=0.9,left]{ $H$ } node[pos=1, circle, fill, inner sep=1, black]{} node[pos=1, right]{ $\frac{X}{s}$ };
37 \draw (3,1.34) .. controls (3.3,2.4) and (2.65, 3.5) .. (2.3, 4.475) node[pos=0.4, right, align=left, xshift
38     =1mm, yshift=-2.5mm]{Plumb \ Line} node[pos=0.7,right]{ $H$ } node[pos=1, circle, fill, inner sep=1,
39     black]{} node[pos=1, left]{ $\frac{X}{s}$ };
40
41 % Draw geoid undulation
42 \draw (3, 1.34) -- (2.85, 0.75) node[pos=0, circle, fill, inner sep=1, black]{} node[pos=1, circle, fill,
43     inner sep=1, black]{} node[pos=0.55, right]{ $N$ };
44
45 % Draw object height above ellipsoid
46 \draw (2.3, 4.475) -- (2.25, 0.8) node[pos=0.5, left]{ $h$ };
47 \end{tikzpicture}
48 \caption{The exaggerated difference between the ellipsoid and the geoid relative to Earth's surface. The
49     gravitational plumb line, with greatly exaggerated curvature, emanates from the geoid's surface normally.}
50 \end{figure}

```

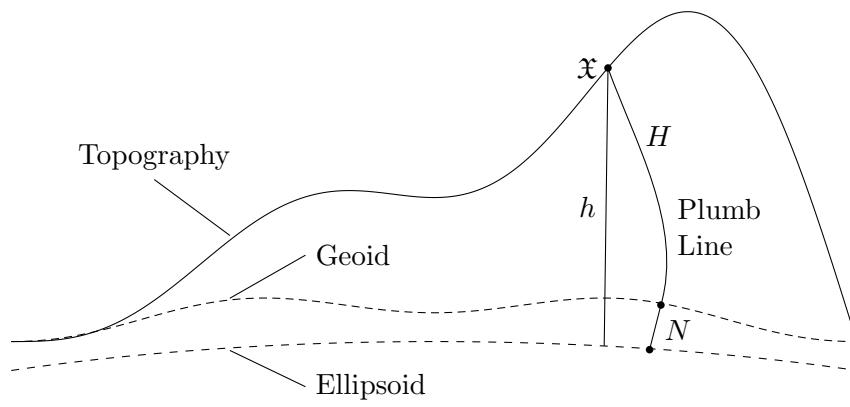


Figure 13: The exaggerated difference between the ellipsoid and the geoid relative to Earth's surface. The gravitational plumb line, with greatly exaggerated curvature, emanates from the geoid's surface normally.

2.15 Variation of Air's Ratio of Heat Capacity (γ) as a Function of Temperature

```

1 \begin{figure}[H]%b!
2 \centering
3 \begin{tikzpicture}
4   \begin{axis}[
5     width=3.229in,
6     height=2.461in,
7     at={(0.542in,0.43in)},
8     scale only axis,
9     %unbounded coords=jump,
10    xmin=200,
11    xmax=1300,
12    xtick={200,350,...,1250},
13    %xticklabels={{-90},{},{-60},{},{-30},{},{0},{},{30},{},{60},{},{90}},
14    xlabel style={font=\color{white!15!black}},
15    xlabel={Temperature $T$ [\si{K}]},
16    %ymin=1.3125,
17    %ymax=1.4075,
18    ytick={1.32,1.33,...,1.41},
19    %yticklabels={{-90},{},{-60},{},{-30},{},{0},{},{30},{},{60},{},{90}},
20    ylabel style={font=\color{white!15!black}},
21    ylabel={Heat Capacity Ratio $\gamma$},
22    axis background/.style={fill=white},
23    title style={font=\bfseries},
24    title={Heat Capacity Ratio of Dry Air},
25    xmajorgrids,
26    ymajorgrids,
27  ]
28  \addplot[mark=x,smooth] coordinates {
29    (233.15, 1.401)
30    (253.15, 1.401)
31    (273.15, 1.401)
32    (278.15, 1.401)
33    (283.15, 1.401)
34    (288.15, 1.401)
35    (293.15, 1.401)
36    (298.15, 1.401)
37    (303.15, 1.400)
38    (313.15, 1.400)
39    (323.15, 1.400)
40    (333.15, 1.399)
41    (343.15, 1.399)
42    (353.15, 1.399)
43    (363.15, 1.398)
44    (373.15, 1.397)
45    (473.15, 1.390)
46    (573.15, 1.379)
47    (673.15, 1.368)
48    (773.15, 1.357)
49    (1273.15, 1.321)
50  };
51 \end{axis}
52 \end{tikzpicture}
53 \caption{The variation of air's heat capacity ratio on temperature (neglecting variations with pressure) [
54   engineering toolbox.]}
54 \label{fig:AirHeatCapacityRatioGraph}
55 \end{figure}

```

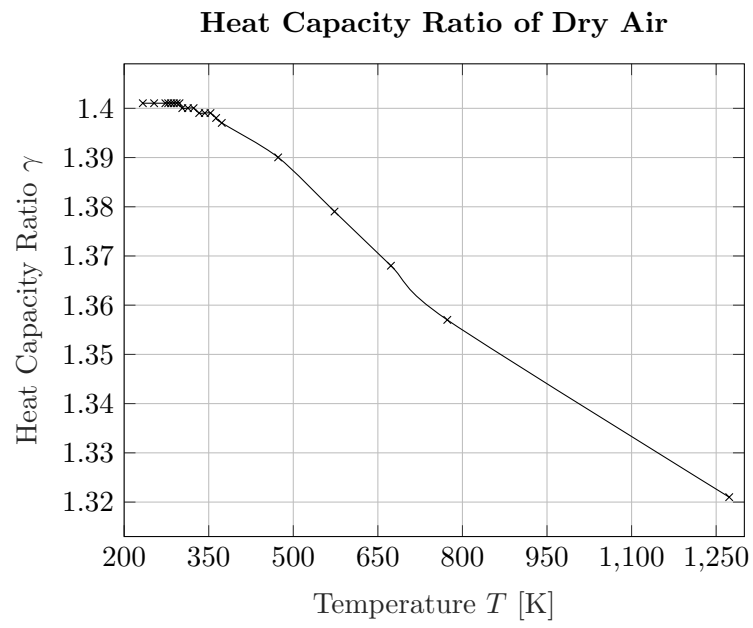


Figure 14: The variation of air's heat capacity ratio on temperature (neglecting variations with pressure) [engineering toolbox].

2.16 Structural Overview of Rocket Body

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}
4 % Originally from propulsion section, modified to fit structures discussion
5 %% Temporary axis (comment out this bit when done)
6 %\draw[red] (0,0) -- (1,0);
7 %\draw[red] (0,0) -- (0,1);
8
9 % Draw the nozzle
10 \begin{axis}[width=15cm,
11     height=207pt,
12     at={(-6.71cm,0pt)},
13     xmin=-8.08, xmax=1.25,
14     ymin=-15, ymax=10,
15     hide axis,
16 ]
17 \tikzset{hatch distance/.store in=\hatchdistance, hatch distance=10pt, hatch thickness/.store in=\hatchthickness, hatch thickness=2pt}
18 \makeatletter
19 \pgfdeclarepatternformonly[\hatchdistance,\hatchthickness]{flexible hatch}
20 {\pgfqpoint{0pt}{0pt}}
21 {\pgfqpoint{\hatchdistance}{\hatchdistance}}
22 {\pgfqpoint{\hatchdistance-1pt}{\hatchdistance-1pt}}{%
23 {
24     \pgfsetcolor{\tikz@pattern@color}
25     \pgfsetlinewidth{\hatchthickness}
26     \pgfpathmoveto{\pgfqpoint{0pt}{0pt}}
27     \pgfpathlineto{\pgfqpoint{\hatchdistance}{\hatchdistance}}
28     \pgfusepath{stroke}
29 }
30 %% Temporary axis (comment out this bit when done)
31 %\draw[blue] (0,0) -- (1,0);
32 %\draw[blue] (0,0) -- (0,1);
33 % Plot the diverging section – must keep curves as trig forms for constants to work (1 is hardcoded)
34 \pgfmathsetmacro{\YOfNozzleThroat}{1} % 1
35 \pgfmathsetmacro{\YOfNozzleExit}{2} % 3
36 \pgfmathsetmacro{\A}{(\YOfNozzleThroat + \YOfNozzleExit) / 2}
37 \pgfmathsetmacro{\B}{(\YOfNozzleThroat - \YOfNozzleExit) / 2}
38 %\addplot[domain=0:1, samples=100, smooth, solid]{\A + \B * cos(deg(pi*x))};
39 %\addplot[domain=0:1, samples=100, smooth, solid]{-\A - \B * cos(deg(pi*x))};
40 % Plot the converging part
41 \pgfmathsetmacro{\XOfBodyNozzleIntersection}{-1.25} % -0.25
42 \pgfmathsetmacro{\YOfBody}{1.5} % 1.25
43 \pgfmathsetmacro{\C}{(\YOfNozzleThroat + \YOfBody) / 2}
44 \pgfmathsetmacro{\D}{(\YOfNozzleThroat - \YOfBody) / 2}
45 \pgfmathsetmacro{\E}{1/\XOfBodyNozzleIntersection}
46 %\addplot[domain=\XOfBodyNozzleIntersection:0, samples=100, smooth, solid]{\C + \D * cos(deg(\E*pi*x))};
47 %\addplot[domain=\XOfBodyNozzleIntersection:0, samples=100, smooth, solid]{-\C - \D * cos(deg(\E*pi*x))};
48 % Plot the body
49 \pgfmathsetmacro{\XofConeBodyIntersection}{-6}
50 %\addplot[domain=\XofConeBodyIntersection:\XOfBodyNozzleIntersection, samples=100, smooth, solid]{\YOfBody};
51 %\addplot[domain=\XofConeBodyIntersection:\XOfBodyNozzleIntersection, samples=100, smooth, solid]{-\YOfBody};
52 % Fill the body
53 % Plot the nose as half of an ellipse
54 \pgfmathsetmacro{\a}{2}
55 \pgfmathsetmacro{\b}{\YOfBody}
56 \pgfmathsetmacro{\aInner}{\a-0.125}
57 \pgfmathsetmacro{\bInner}{\b-0.4}

```

```

58 \pgfmathsetmacro{\XofNoseTip}{\XofConeBodyIntersection-\a}
59 \pgfmathsetmacro{\XofNoseTipInner}{\XofConeBodyIntersection-\aInner}
60
61
62 % Plot the ( elliptical ) nose cone with thickness
63 \addplot[domain=-\YOfBody:\YOfBody, samples=100, smooth, solid, draw=black, postaction={pattern=
    north east lines, pattern color=black}, variable=\y] ({\XofConeBodyIntersection - \a * sqrt(1 - (\y / \b)
    ^2)}, \y);
64 % Corrective white layer to remove lines coming out of the back for some reason
65 \addplot[domain=-\bInner:\bInner, samples=100, smooth, solid, white, fill=white, variable=\y] ({\
    XofConeBodyIntersection+0.05 - \aInner * sqrt(1 - (\y / \bInner)^2)}, \y);
66 \addplot[domain=-\bInner:\bInner, samples=100, smooth, solid, fill=white, variable=\y] ({\
    XofConeBodyIntersection - \aInner * sqrt(1 - (\y / \bInner)^2)}, \y);
67 % Lead into the body
68 \pgfmathsetmacro{\plusSpacing}{0.03}
69 \pgfmathsetmacro{\XofPlusConeBody}{\XofConeBodyIntersection+\plusSpacing}
70 %\node at (\XofPlusConeBody,0){$+$};
71 \pgfmathsetmacro{\XofBody}{\XofPlusConeBody+\plusSpacing}
72 %% Plot the body with thickness
73 \draw[pattern=north east lines, pattern color=black] (\XofBody,-\YOfBody) rectangle (\
    XofBodyNozzleIntersection,-\bInner);
74 \draw[pattern=north east lines, pattern color=black] (\XofBody,\YOfBody) rectangle (\
    XofBodyNozzleIntersection,\bInner);
75 % Lead into the frustum
76 \pgfmathsetmacro{\XofPlusBodyFrustum}{\XofBodyNozzleIntersection+2*\plusSpacing}
77 %\node at (\XofPlusBodyFrustum+0.05,0){$+$};
78 \pgfmathsetmacro{\XofFrustumB}{\XofPlusBodyFrustum+\plusSpacing}
79 % Draw frustum
80 \pgfmathsetmacro{\XOfFrustumT}{0} % 0.5
81 \pgfmathsetmacro{\YOfFrustumT}{1.3*\YOfBody}
82 \pgfmathsetmacro{\YOfFrustumTB}{\YOfFrustumT-\YOfBody+\bInner}
83 \draw[pattern=north east lines, pattern color=black] (\XofFrustumB, \bInner) -- (\XofFrustumB, \YOfBody)
    -- (\XOfFrustumT, \YOfFrustumT) -- (\XOfFrustumT, \YOfFrustumTB) -- cycle;
84 \draw[pattern=north east lines, pattern color=black] (\XofFrustumB, -\bInner) -- (\XofFrustumB, -\
    YOfBody) -- (\XOfFrustumT, -\YOfFrustumT) -- (\XOfFrustumT, -\YOfFrustumTB) -- cycle;
85 % Lead into recursion by drawing another body
86 \pgfmathsetmacro{\XOfNextBody}{\XOfFrustumT + 2.5*\plusSpacing}
87 \pgfmathsetmacro{\XOfFinal}{\XOfNextBody+1.1}
88 \draw[pattern=north east lines, pattern color=black] (\XOfNextBody, \YOfFrustumTB) rectangle (\XOfFinal,
    \YOfFrustumT);
89 \draw[pattern=north east lines, pattern color=black] (\XOfNextBody, -\YOfFrustumTB) rectangle (\
    XOfFinal, -\YOfFrustumT);
90
91 \pgfmathsetmacro{\YOfXLine}{-\YOfBody-2}
92 \pgfmathsetmacro{\tickHeight}{1}
93 \draw (\XofNoseTip, \YOfXLine) -- (\XOfFinal, \YOfXLine);
94 \draw[thick] (\XofNoseTip, \YOfXLine-\tickHeight/2) -- (\XofConeBodyIntersection-\a, \YOfXLine+\
    tickHeight/2); %\node[below, yshift=-2.65mm]{\rho_1}; % yshift=-2.6mm
95 \draw[->, thick, >=latex] (\XofNoseTip, \YOfXLine) -- (\XofNoseTip+\a/2.5, \YOfXLine) node[pos=0.3,
    below]{\rho_1} node[pos=1.1, below]{x_1};
96 %
97 \draw[thick] (\XofBody, \YOfXLine-\tickHeight/2) -- (\XofBody, \YOfXLine+\tickHeight/2);
98 \draw[->, thick, >=latex] (\XofBody, \YOfXLine) -- (\XofBody+\a/2.5, \YOfXLine) node[pos=0.3, below
    ]{\rho_2} node[pos=1.1, below]{x_2};
99 %
100 \draw[thick] (\XofFrustumB, \YOfXLine-\tickHeight/2) -- (\XofFrustumB, \YOfXLine+\tickHeight/2);
101 \draw[->, thick, >=latex] (\XofFrustumB, \YOfXLine) -- (\XofFrustumB+\a/2.5, \YOfXLine) node[pos
    =0.3, below]{\rho_3} node[pos=1.1, below]{x_3};
102 %
103 \draw[thick] (\XOfNextBody, \YOfXLine-\tickHeight/2) -- (\XOfNextBody, \YOfXLine+\tickHeight/2);
104 \draw[->, thick, >=latex] (\XOfNextBody, \YOfXLine) -- (\XOfNextBody+\a/2.5, \YOfXLine) node[pos
    =0.3, below]{\rho_4} node[pos=1.1, below]{x_4};
105 % Center of Mass marker

```

```

106 \pgfmathsetmacro{\XOfCOM}{-2.15}
107 \draw (\XOfCOM, \YOfXLine-\tickHeight/2) -- (\XOfCOM, \YOfXLine+\tickHeight/2) node[below, xshift
    =1mm, yshift=-1mm]{mass center};
108
109 % Draw longitudinal axis from nose
110 \draw[->, >=latex, thick] (\XofNoseTip, 0) -- (\XofNoseTip+4.8, 0) node[right, xshift=-0.5mm]{$\underline{x}$};
111 \draw[->, >=latex, thick] (\XofNoseTip, 0) -- (\XofNoseTip,3); % node[left]{$\underline{y}$}; % Out of
    frame
112 % Draw longitudinal axis from COM
113 \draw[->, >=latex, thick, yellow!55!black!90] (\XOfCOM, 0) -- (\XOfFinal-0.5, 0) node[black, right, xshift
    =-0.5mm]{$x$};
114 \draw[->, >=latex, thick, yellow!55!black!90] (\XOfCOM, 0) -- (\XOfCOM,3) node[black, left]{$y$};
115 \end{axis}
116
117
118 %
119 % Try to draw center of mass
120 % Nose cone
121 \begin{scope}[shift={(-4.75, 2.415)}] % (1.2, 2.415)
122 \pgfmathsetmacro{\Bx}{0}
123 \pgfmathsetmacro{\By}{1}
124 \pgfmathsetmacro{\Br}{0.11}
125 \draw[fill=black] (\Bx,\By) ++(0:\Br) arc (0:90:\Br) -- (\Bx,\By) -- cycle;
126 \draw[fill=white] (\Bx,\By) ++(90:\Br) arc (90:180:\Br) -- (\Bx,\By) -- cycle;
127 \draw[fill=black] (\Bx,\By) ++(180:\Br) arc (180:270:\Br) -- (\Bx,\By) -- cycle;
128 \draw[fill=white] (\Bx,\By) ++(270:\Br) arc (270:360:\Br) -- (\Bx,\By) -- cycle;
129 \end{scope}
130 % Body
131 \begin{scope}[shift={(-0.2, 2.415)}]
132 \pgfmathsetmacro{\Bx}{0}
133 \pgfmathsetmacro{\By}{1}
134 \pgfmathsetmacro{\Br}{0.11}
135 \draw[fill=black] (\Bx,\By) ++(0:\Br) arc (0:90:\Br) -- (\Bx,\By) -- cycle;
136 \draw[fill=white] (\Bx,\By) ++(90:\Br) arc (90:180:\Br) -- (\Bx,\By) -- cycle;
137 \draw[fill=black] (\Bx,\By) ++(180:\Br) arc (180:270:\Br) -- (\Bx,\By) -- cycle;
138 \draw[fill=white] (\Bx,\By) ++(270:\Br) arc (270:360:\Br) -- (\Bx,\By) -- cycle;
139 \end{scope}
140 % Frustum
141 \begin{scope}[shift={(4.3, 2.415)}]
142 \pgfmathsetmacro{\Bx}{0}
143 \pgfmathsetmacro{\By}{1}
144 \pgfmathsetmacro{\Br}{0.11}
145 \draw[fill=black] (\Bx,\By) ++(0:\Br) arc (0:90:\Br) -- (\Bx,\By) -- cycle;
146 \draw[fill=white] (\Bx,\By) ++(90:\Br) arc (90:180:\Br) -- (\Bx,\By) -- cycle;
147 \draw[fill=black] (\Bx,\By) ++(180:\Br) arc (180:270:\Br) -- (\Bx,\By) -- cycle;
148 \draw[fill=white] (\Bx,\By) ++(270:\Br) arc (270:360:\Br) -- (\Bx,\By) -- cycle;
149 \end{scope}
150 % COM
151 \begin{scope}[shift={(1.83, 2.415)}]
152 \pgfmathsetmacro{\Bx}{0}
153 \pgfmathsetmacro{\By}{1}
154 \pgfmathsetmacro{\Br}{0.11}
155 \draw[fill=black] (\Bx,\By) ++(0:\Br) arc (0:90:\Br) -- (\Bx,\By) -- cycle;
156 \draw[fill=yellow!55!black!90] (\Bx,\By) ++(90:\Br) arc (90:180:\Br) -- (\Bx,\By) -- cycle;
157 \draw[fill=black] (\Bx,\By) ++(180:\Br) arc (180:270:\Br) -- (\Bx,\By) -- cycle;
158 \draw[fill=yellow!55!black!90] (\Bx,\By) ++(270:\Br) arc (270:360:\Br) -- (\Bx,\By) -- cycle;
159 \end{scope}
160
161 % Label y_n here to maximize rocket in plot
162 \node at (-6.85, 4.05) {$\underline{y}$}; % y_n -> -6.9 | \underbrace{y} -> 4.05 instead of 4.1
163
164 \end{tikzpicture}

```


165 \caption{Overviewing diagram of the cross-section of an empty, multistaged rocket body with its basic
 components separated from one another for individual analysis. The structure is radially symmetric from
 the longitudinal axis and hollow, containing material only around the outer shell. Measurements are made
 from the nose cone's (outer) tip. Comparitively small components like fins and nozzles are removed from
 consideration.}

166 \label{fig:StructuresRocketNStage}

167 \end{figure}

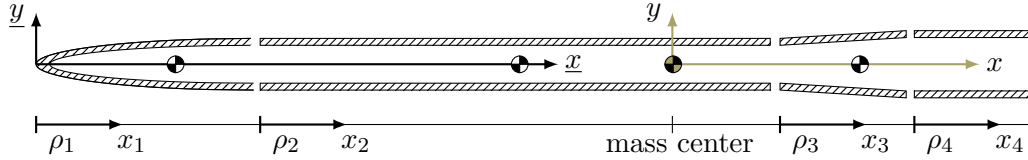


Figure 15: Overviewing diagram of the cross-section of an empty, multistaged rocket body with its basic components separated from one another for individual analysis. The structure is radially symmetric from the longitudinal axis and hollow, containing material only around the outer shell. Measurements are made from the nose cone's (outer) tip. Comparitively small components like fins and nozzles are removed from consideration.

2.17 Solid Cone

```

1 \begin{figure}[H]
2 \centering
3 \tdplotsetmaincoords{-20}{0} % 70 110
4 \tdplotsetrotatedcoords{0}{-20}{0}
5 \begin{tikzpicture}[scale=2, tdplot_main_coords]
6 \pgfmathsetmacro{\m}{0.25}
7 \pgfmathsetmacro{\L}{3}
8 \pgfmathsetmacro{\R}{\m*\L}
9
10 % Draw sloped sides
11 \pgfmathsetmacro{\t}{-20}
12 \pgfmathsetmacro{\Rcost}{\R*cos(\t)}
13 \pgfmathsetmacro{\Rsint}{\R*sin(\t)}
14 \draw[tdplot_rotated_coords] (0, 0, 0) -- (\L, \Rcost, \Rsint);
15 \draw[tdplot_rotated_coords] (0, 0, 0) -- (\L, -\Rcost, -\Rsint);
16 % Draw face
17 \begin{scope}[tdplot_rotated_coords, canvas is yz plane at x=\L]
18 \draw (0, 0) circle [radius=\R];
19 \end{scope}
20 % Draw parameters
21 \draw[tdplot_rotated_coords, dashed] (0, 0, 0) -- (\L, 0, 0) node[pos=0.6, below]{$L$};
22 \draw[tdplot_rotated_coords, dashed] (\L, 0, 0) -- (\L, \R, 0) node[above, xshift=-0.5mm, yshift=0.5mm]{$R$};
23 % Draw half angle
24 \begin{scope}[tdplot_rotated_coords, canvas is xy plane at z=0]
25 \pgfmathsetmacro{\angle}{atan(\R/\L)}
26 \draw[->, >=stealth] (1.2,0) arc (0:\angle:1.225cm) node[midway, right]{$\theta_c$};
27 \end{scope}
28
29 % Draw axes
30 \draw[->, >=latex, thick, tdplot_rotated_coords] (0, 0, 0) -- (1, 0, 0) node[anchor=north west, xshift=-2mm]{$x_1$};
31 \draw[->, >=latex, thick, tdplot_rotated_coords] (0, 0, 0) -- (0, 1, 0) node[pos=1, right]{$y_1$};
32 \draw[->, >=latex, thick, tdplot_rotated_coords] (0, 0, 0) -- (0, 0, 1) node[anchor=north east, xshift=0.75mm, yshift=0.75mm]{$z_1$};
33 \end{tikzpicture}
34 \caption{Solid cone of constant density $\rho$ and half (cone) angle $\theta_c$ satisfying $\tan\theta_c = R/L$. The coordinate frame is labelled appropriately with the index $s = 1$ to reinforce that the section is in agreement with Fig. \ref{fig:StructuresRocketNStage} and will remain so in subsequent figures.}
35 \end{figure}

```

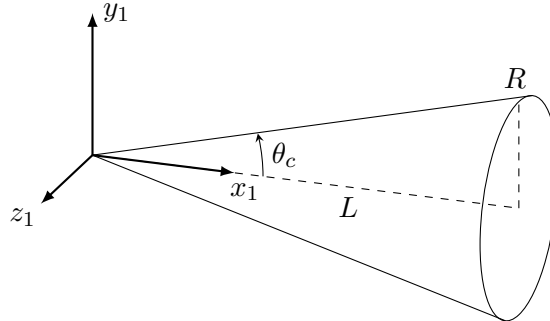


Figure 16: Solid cone of constant density ρ and half (cone) angle θ_c satisfying $\tan\theta_c = R/L$. The coordinate frame is labelled appropriately with the index $s = 1$ to reinforce that the section is in agreement with Fig. 15 and will remain so in subsequent figures.

2.18 Conical Shell

```

1 \begin{figure}[H]
2 \begin{subfigure}[t]{0.5\textwidth}
3 \centering
4 \tdplotsetmaincoords{-20}{0} % 70 110
5 \tdplotsetrotatedcoords{0}{-20}{0}
6 \begin{tikzpicture}[scale=2, tdplot_main_coords]
7 \pgfmathsetmacro{\m}{0.25}
8 \pgfmathsetmacro{\L}{3}
9 \pgfmathsetmacro{\R}{\m*\L}
10
11 % Draw sloped sides
12 \pgfmathsetmacro{\t}{-20}
13 \pgfmathsetmacro{\Rcost}{\R*cos(\t)}
14 \pgfmathsetmacro{\Rsint}{\R*sin(\t)}
15 \draw[tdplot_rotated_coords] (0, 0, 0) -- (\L, \Rcost, \Rsint);
16 \draw[tdplot_rotated_coords] (0, 0, 0) -- (\L, -\Rcost, -\Rsint);
17 % Draw face
18 \begin{scope}[tdplot_rotated_coords, canvas is yz plane at x=\L]
19 \pgfmathsetmacro{\t}{\R/6}
20 \draw (0, 0) circle[radius=\R];
21 \draw (0, 0) circle[radius=\R-\t];
22 \draw[->, >=stealth, thick] (-\R-0.2,0) -- (-\R,0);
23 \draw[->, >=stealth, thick] (-\R+\t+0.4,0) -- (-\R+\t,0) node[pos=1.1, right, xshift=1mm]{$R - r$};
24 \end{scope}
25 % Draw parameters
26 \draw[tdplot_rotated_coords, dashed] (0, 0, 0) -- (\L, 0, 0) node[pos=0.6, below]{$L$};
27 \draw[tdplot_rotated_coords, dashed] (\L, 0, 0) -- (\L, \R, 0) node[above, xshift=-0.5mm, yshift=0.5mm]{$R$};
28 % Draw half angle
29 \begin{scope}[tdplot_rotated_coords, canvas is xy plane at z=0]
30 \pgfmathsetmacro{\angle}{atan(\R/\L)}
31 \draw[->, >=stealth] (1.2,0) arc (0:\angle:1.225cm) node[midway, right]{$\theta_c$};
32 \end{scope}
33
34 % Draw axes
35 \draw[->, >=latex, thick, tdplot_rotated_coords] (0, 0, 0) -- (1, 0, 0) node[anchor=north west, xshift=-2mm]{$x_1$};
36 \draw[->, >=latex, thick, tdplot_rotated_coords] (0, 0, 0) -- (0, 1, 0) node[pos=1, right]{$y_1$};
37 \draw[->, >=latex, thick, tdplot_rotated_coords] (0, 0, 0) -- (0, 0, 1) node[anchor=north east, xshift=0.75mm, yshift=0.75mm]{$z_1$};
38 \end{tikzpicture}% NO SPACE!
39 \end{subfigure}
40 \hspace{1cm}% NO SPACE!
41 \begin{subfigure}[t]{0.5\textwidth}
42 \begin{tikzpicture}[scale=2]
43 % Define the same parameters as above
44 \pgfmathsetmacro{\m}{0.25}
45 \pgfmathsetmacro{\L}{3}
46 \pgfmathsetmacro{\R}{\m*\L}
47 \pgfmathsetmacro{\t}{0.6} %\R/3
48
49 % Draw axes
50 \draw[->, >=latex, thick] (0,0) -- (2.1,0) node[right]{$x_1$}; %1.5
51 \draw[->, >=latex, thick] (0,0) -- (0,1.3) node[left]{$y_1$};
52
53 % Draw angle
54 \pgfmathsetmacro{\structAngle}{atan(\m)}
55 \begin{scope}[shift={(\t,0)}]
56 \draw[->, >=stealth, thick] (1, 0) arc(0:\structAngle:1) node[midway, right]{$\theta_c$};
57 \end{scope}
58

```

```

59 % Outside cone
60 \draw (0,0) -- (\L, \R);
61 \pgfmathsetmacro{\xFort}{\t*0}%0.83
62 \pgfmathsetmacro{\yFort}{\m*\xFort}
63 \draw (\xFort,-\yFort) -- (\L, -\R);
64 % Inside cone
65 \pgfmathsetmacro{\gatL}{\m*(\L-\t)}
66 \draw[dashed] (\t,0) -- (\L, \gatL);
67 \draw[dashed] (\t,0) -- (\L, -\gatL);
68 % Draw t
69 \pgfmathsetmacro{\YofLine}{-0.33} % -0.05
70 \draw[dashed] (0, 0) -- (0, \YofLine);
71 \draw[dashed] (\t, 0) -- (\t, \YofLine);
72 \draw[|-,|] (0,\YofLine) -- (\t,\YofLine) node[pos=0.5, below]{$l$}; % pos=0.6, below, yshift=-2.3mm
73
74 %% Draw vertical thickness
75 \pgfmathsetmacro{\XofVerticalMarker}{0.8*\L}
76 \pgfmathsetmacro{\YofVerticalMarkerOnInside}{\m*(\XofVerticalMarker-\t)}
77 \pgfmathsetmacro{\YofVerticalMarkerOnOutside}{\m*\XofVerticalMarker}
78 \pgfmathsetmacro{\tailLength}{0.2}
79 \draw[->, >=stealth] (\XofVerticalMarker, -\YofVerticalMarkerOnInside+\tailLength) -- (\XofVerticalMarker, -\YofVerticalMarkerOnInside);
80 \draw[->, >=stealth] (\XofVerticalMarker, -\YofVerticalMarkerOnOutside-\tailLength) -- (\XofVerticalMarker, -\YofVerticalMarkerOnOutside) node[left, rotate=-atan(\m), xshift=-1.5mm, yshift=-2.5mm]{$\tan\theta_c$, $l$};
81 % Draw normal thickness
82 \pgfmathsetmacro{\setXOutsideOn}{0.8*\L}
83 \pgfmathsetmacro{\setYOutsideOn}{\m*\setXOutsideOn}
84 \pgfmathsetmacro{\setXOutsideOff}{\setXOutsideOn-0.05}
85 \pgfmathsetmacro{\setYOutsideOff}{\setYOutsideOn-1/\m*(\setXOutsideOff-\setXOutsideOn)}
86 %
87 \pgfmathsetmacro{\setXInsideOn}{\setXOutsideOn+\m^2*\t/(1+\m^2)}
88 \pgfmathsetmacro{\setYInsideOn}{\m*(\setXInsideOn-\t)}
89 \pgfmathsetmacro{\setXInsideOff}{\setXInsideOn+0.05}
90 \pgfmathsetmacro{\setYInsideOff}{\setYInsideOn-1/\m*(\setXInsideOff-\setXInsideOn)}
91 %
92 \draw[->, >=stealth] (\setXOutsideOff, \setYOutsideOff) -- (\setXOutsideOn, \setYOutsideOn) node[left, xshift=-2mm, yshift=2mm, rotate=atan(\m)]{$\sin\theta_c$, $\tan\theta_c$, $l$};
93 \draw[->, >=stealth] (\setXInsideOff, \setYInsideOff) -- (\setXInsideOn, \setYInsideOn);
94
95 \end{tikzpicture}
96 \end{subfigure}
97 \caption{Conical shell of constant density $\rho$, frontal thickness $t$, and half (cone) angle satisfying $\tan\theta_c = R / L$}
98 \label{fig:StructuresConicalShell}
99 \end{figure}

```

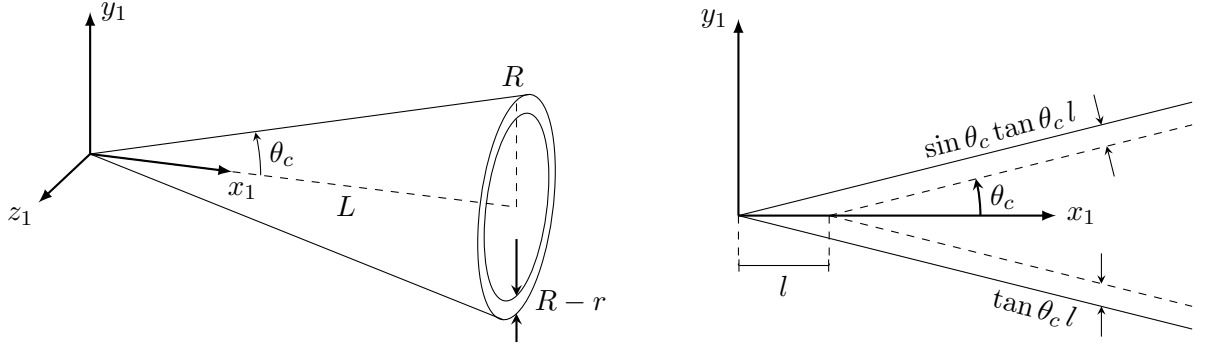


Figure 17: Conical shell of constant density ρ , frontal thickness t , and half (cone) angle satisfying $\tan \theta_c = R/L$.

2.19 Parabolic Nose Cone

```

1 \begin{figure}[H]
2 \begin{subfigure}[t]{0.5\textwidth}
3 \centering
4 \tdplotsetmaincoords{-20}{0} % 70 110
5 \tdplotsetrotatedcoords{0}{-20}{0}
6 \begin{tikzpicture}[scale=2, tdplot_main_coords]
7 \pgfmathsetmacro{\m}{0.25}
8 \pgfmathsetmacro{\L}{3}
9 \pgfmathsetmacro{\R}{\m*\L}
10 \pgfmathsetmacro{\K}{0.75}
11
12 % New approach is to draw curve (parabola) on xy plane and rotate plane until it matches well with base
13 \pgfmathsetmacro{\th}{-20}
14 \pgfmathsetmacro{\cost}{\cos(\th)}
15 \pgfmathsetmacro{\sint}{\sin(\th)}
16 \begin{scope}[tdplot_rotated_coords, canvas is plane={O(0,0,0)x(1,0,0)y(0,\cost,\sint)}]
17 \draw [domain=0:\L, samples=40] plot ({\x}, {((\R / (2 - \K)) * (2*\x/\L - \K * (\x/\L)^2))});
18 \draw [domain=0:\L, samples=40] plot ({\x}, {((-\R / (2 - \K)) * (2*\x/\L - \K * (\x/\L)^2))});
19 \end{scope}
20 % Draw face
21 \begin{scope}[tdplot_rotated_coords, canvas is yz plane at x=\L]
22 \pgfmathsetmacro{\t}{\R/6}
23 \draw (0, 0) circle [radius=\R];
24 \draw (0, 0) circle [radius=\R-\t];
25 \draw[->, >=stealth, thick] (-\R-0.2,0) -- (-\R,0);
26 \draw[->, >=stealth, thick] (-\R+\t+0.4,0) -- (-\R+\t,0) node[pos=1.1, right, xshift=1mm]{$R - r$};
27 \end{scope}
28 % Draw parameters
29 \draw[tdplot_rotated_coords, dashed] (0, 0, 0) -- (\L, 0, 0) node[pos=0.6, below]{$L$};
30 \draw[tdplot_rotated_coords, dashed] (\L, 0, 0) -- (\L, \R, 0) node[above, xshift=-0.5mm, yshift=0.5mm]{$R$};
31 %% Draw half angle
32 %\begin{scope}[tdplot_rotated_coords, canvas is xy plane at z=0]
33 %\pgfmathsetmacro{\angle}{atan(\R/\L)}
34 %\draw[->, >=stealth] (1.2,0) arc (0:\angle:1.225cm) node[midway, right]{$\theta_c$};
35 %\end{scope}
36
37 % Draw axes
38 \draw[->, >=latex, thick, tdplot_rotated_coords] (0, 0, 0) -- (1, 0, 0) node[anchor=north west, xshift=-2mm]{$x_1$};
39 \draw[->, >=latex, thick, tdplot_rotated_coords] (0, 0, 0) -- (0, 1, 0) node[pos=1, right]{$y_1$};
40 \draw[->, >=latex, thick, tdplot_rotated_coords] (0, 0, 0) -- (0, 0, 1) node[anchor=north east, xshift=0.75mm, yshift=0.75mm]{$z_1$};
41 \end{tikzpicture}% NO SPACE!
42 \end{subfigure}
43 \hspace{1cm}% NO SPACE!
44 \begin{subfigure}[t]{0.5\textwidth}
45 \begin{tikzpicture}[scale=2]
46 % Define the same parameters as above
47 \pgfmathsetmacro{\m}{0.25}
48 \pgfmathsetmacro{\L}{3}
49 \pgfmathsetmacro{\R}{\m*\L}
50 \pgfmathsetmacro{\K}{0.75}
51 \pgfmathsetmacro{\t}{0.6} %\R/3
52
53 % Draw axes
54 \draw[->, >=latex, thick] (0,0) -- (2.1,0) node[right]{$x_1$}; %1.5
55 \draw[->, >=latex, thick] (0,0) -- (0,1.3) node[left]{$y_1$};
56
57 %% Draw angle
58 \pgfmathsetmacro{\structAngle}{atan(\m)}

```

```

59 %\begin{scope}[shift={(\t,0)}]
60 %\draw[>-, >=stealth, thick] (1, 0) arc(0:\structAngle:1) node[midway, right]{\theta_c};
61 %\end{scope}
62
63 % Outside cone
64 \draw [domain=0:\L, samples=40] plot ({\x}, {((\R / (2 - \K)) * (2*\x/\L - \K * (\x/\L)^2)});
65 \draw [domain=0:\L, samples=40] plot ({\x}, {((- \R / (2 - \K)) * (2*\x/\L - \K * (\x/\L)^2)});
66 % Inside cone
67 \draw[dashed, domain=\t:\L, samples=40] plot ({\x}, {((\R / (2 - \K)) * (2*(\x-\t)/\L - \K * ((\x-\t)/\L
68 )^2)});
68 \draw[dashed, domain=\t:\L, samples=40] plot ({\x}, {((- \R / (2 - \K)) * (2*(\x-\t)/\L - \K * ((\x-\t)/\L
69 )^2)});
69 % Draw t
70 \pgfmathsetmacro{\YofLine}{-0.33} % -0.05
71 \draw[dashed] (0, 0) -- (0, \YofLine);
72 \draw[dashed] (\t, 0) -- (\t, \YofLine);
73 \draw[| - |] (0, \YofLine) -- (\t, \YofLine) node[pos=0.5, below]{t}; % pos=0.6, below, yshift=-2.3mm
74
75 %%% Draw vertical thickness
76 %\pgfmathsetmacro{\XofVerticalMarker}{0.8*\L}
77 %\pgfmathsetmacro{\YofVerticalMarkerOnInside}{\m*(\XofVerticalMarker-\t)}
78 %\pgfmathsetmacro{\YofVerticalMarkerOnOutside}{\m*\XofVerticalMarker}
79 %\pgfmathsetmacro{\tailLength}{0.2}
80 %\draw[>-, >=stealth] (\XofVerticalMarker, -\YofVerticalMarkerOnInside+\tailLength) -- (\XofVerticalMarker, -\YofVerticalMarkerOnInside);
81 %\draw[>-, >=stealth] (\XofVerticalMarker, -\YofVerticalMarkerOnOutside-\tailLength) -- (\XofVerticalMarker, -\YofVerticalMarkerOnOutside) node[left, rotate=-atan(\m), xshift=-1.5mm, yshift=-2.5mm]{\tan\theta_c \, t};
82 %%% Draw normal thickness
83 %\pgfmathsetmacro{\setXOutsideOn}{0.8*\L}
84 %\pgfmathsetmacro{\setYOutsideOn}{\m*\setXOutsideOn}
85 %\pgfmathsetmacro{\setXOutsideOff}{\setXOutsideOn-0.05}
86 %\pgfmathsetmacro{\setYOutsideOff}{\setYOutsideOn-1/\m*(\setXOutsideOff-\setXOutsideOn)}
87 %%%
88 %\pgfmathsetmacro{\setXInsideOn}{\setXOutsideOn + \m^2*\t/(1+\m^2)}
89 %\pgfmathsetmacro{\setYInsideOn}{\m*(\setXInsideOn-\t)}
90 %\pgfmathsetmacro{\setXInsideOff}{\setXInsideOn+0.05}
91 %\pgfmathsetmacro{\setYInsideOff}{\setYInsideOn-1/\m*(\setXInsideOff-\setXInsideOn)}
92 %%%
93 %\draw[>-, >=stealth] (\setXOutsideOff, \setYOutsideOff) -- (\setXOutsideOn, \setYOutsideOn) node[left, xshift=-2mm, yshift=2mm, rotate=atan(\m)]{\sin\theta_c|\tan\theta_c \, t};
94 %\draw[>-, >=stealth] (\setXInsideOff, \setYInsideOff) -- (\setXInsideOn, \setYInsideOn);
95
96 \end{tikzpicture}
97 \end{subfigure}
98
99 %\begin{subfigure}[t]{0.5\textwidth}
100 %\includegraphics[width=0.9\linewidth]{ParabolicNoseConeAttempt.png}
101 %\end{subfigure}
102 \caption{Parabolic cone of constant density  $\rho$ , frontal thickness  $l$ , and cone constant  $k = 3/4$ , corresponding to a three-quarter parabolic nose cone.}
103 \end{figure}

```

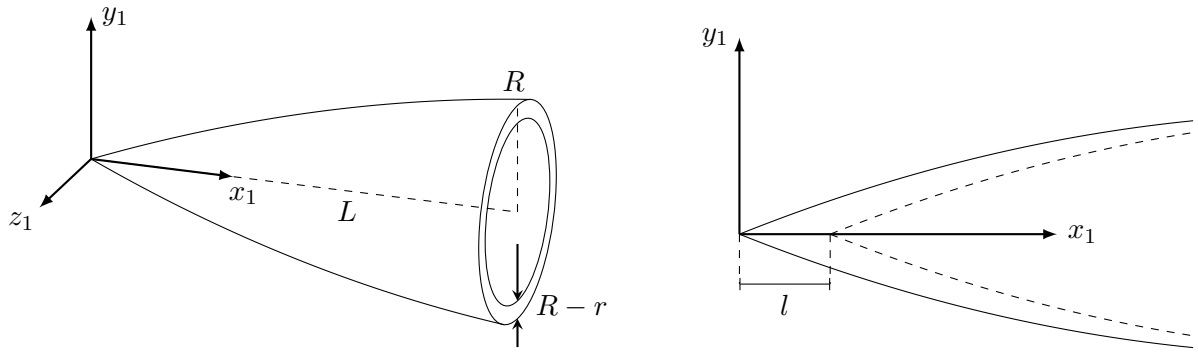


Figure 18: Parabolic cone of constant density ρ , frontal thickness l , and cone constant $k = 3/4$, corresponding to a three-quarter parabolic nose cone.

2.20 Comparing Parabolic Cone to vK Ogive

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}[scale=2]
4 % Define the same parameters as above
5 \pgfmathsetmacro{\m}{0.25}
6 \pgfmathsetmacro{\L}{3}
7 \pgfmathsetmacro{\R}{\m*\L}
8 \pgfmathsetmacro{\K}{0.9}
9 \pgfmathsetmacro{\t}{0.6} %R/3
10
11 % Draw axes
12 \draw[>,>=latex,thick] (0,0) -- (2.1,0) node[right]{$x_1$}; %1.5
13 \draw[>,>=latex,thick] (0,0) -- (0,1.3) node[left]{$y_1$};
14
15 %% Draw angle
16 %\pgfmathsetmacro{\structAngle}{atan(\m)}
17 %\begin{scope}[shift={(\t,0)}]
18 %\draw[>,>=stealth,thick] (1,0) arc(0:\structAngle:1) node[midway,right]{$\theta_c$};
19 %\end{scope}
20
21 % Outside cone
22 \draw[red,domain=0:\L,samples=120] plot ({\x},{((\R/(2-\K))* (2*\x/\L - \K * (\x/\L)^2)});
23 \draw[red,domain=0:\L,samples=120] plot ({\x},{((-\R/(2-\K))* (2*\x/\L - \K * (\x/\L)^2)});
24 % Inside cone
25 \draw[red,dashed,domain=\t:\L,samples=40] plot ({\x},{((\R/(2-\K))* (2*(\x-\t)/\L - \K * ((\x-\t)/\L)^2)});
26 \draw[red,dashed,domain=\t:\L,samples=40] plot ({\x},{((-\R/(2-\K))* (2*(\x-\t)/\L - \K * ((\x-\t)/\L)^2)});
27 % Draw t
28 \pgfmathsetmacro{\YofLine}{-0.33} %-0.05
29 \draw[dashed] (0,0) -- (0,\YofLine);
30 \draw[dashed] (\t,0) -- (\t,\YofLine);
31 \draw[|-,|] (0,\YofLine) -- (\t,\YofLine) node[pos=0.5,below]{$t$}; %pos=0.6,below,yshift=-2.3mm
32
33 % Draw Haack cone
34 \pgfmathsetmacro{\c}{0} % von Karman cone
35 % Outside cone
36 \draw [domain=0:\L,samples=120] plot ({\x},{(\R/sqrt(pi))*sqrt(acos(1-2*\x/\L)*pi/180 - sin(2*acos(1-2*\x/\L))/2 + \c * sin(acos(1-2*\x/\L)^3)});
37 \draw [domain=0:\L,samples=120] plot ({\x},{(-\R/sqrt(pi))*sqrt(acos(1-2*\x/\L)*pi/180 - sin(2*acos(1-2*\x/\L))/2 + \c * sin(acos(1-2*\x/\L)^3)});
38 % Inside cone
39 \draw [dashed,domain=\t:\L,samples=40] plot ({\x},{(\R/sqrt(pi))*sqrt(acos(1-2*(\x-\t)/\L)*pi/180 - sin(2*acos(1-2*(\x-\t)/\L))/2 + \c * sin(acos(1-2*(\x-\t)/\L)^3)});
40 \draw [dashed,domain=\t:\L,samples=40] plot ({\x},{(-\R/sqrt(pi))*sqrt(acos(1-2*(\x-\t)/\L)*pi/180 - sin(2*acos(1-2*(\x-\t)/\L))/2 + \c * sin(acos(1-2*(\x-\t)/\L)^3)});
41
42
43 %%% Draw vertical thickness
44 %\pgfmathsetmacro{\XofVerticalMarker}{0.8*\L}
45 %\pgfmathsetmacro{\YofVerticalMarkerOnInside}{\m*\XofVerticalMarker-\t}
46 %\pgfmathsetmacro{\YofVerticalMarkerOnOutside}{\m*\XofVerticalMarker}
47 %\pgfmathsetmacro{\tailLength}{0.2}
48 %\draw[>,>=stealth] (\XofVerticalMarker,-\YofVerticalMarkerOnInside+\tailLength) -- (\XofVerticalMarker,-\YofVerticalMarkerOnInside);
49 %\draw[>,>=stealth] (\XofVerticalMarker,-\YofVerticalMarkerOnOutside-\tailLength) -- (\XofVerticalMarker,-\YofVerticalMarkerOnOutside) node[left,rotate=-atan(\m),xshift=-1.5mm,yshift=-2.5mm]{$\tan\theta_c$};
50 %%% Draw normal thickness
51 %\pgfmathsetmacro{\setXOutsideOn}{0.8*\L}
52 %\pgfmathsetmacro{\setYOutsideOn}{\m*\setXOutsideOn}

```

```

53 \pgfmathsetmacro{\setXOutsideOff}{\setXOutsideOn-0.05}
54 \pgfmathsetmacro{\setYOutsideOff}{\setYOutsideOn -1/\m * (\setXOutsideOff - \setXOutsideOn)}
55 %%
56 \pgfmathsetmacro{\setXInsideOn}{\setXOutsideOn + \m^2*t/(1+\m^2)}
57 \pgfmathsetmacro{\setYInsideOn}{\m*(\setXInsideOn-\t)}
58 \pgfmathsetmacro{\setXInsideOff}{\setXInsideOn+0.05}
59 \pgfmathsetmacro{\setYInsideOff}{\setYInsideOn -1/\m * (\setXInsideOff - \setXInsideOn)}
60 %%
61 \draw[>,>=stealth] (\setXOutsideOff, \setYOutsideOff) -- (\setXOutsideOn, \setYOutsideOn) node[left,
    xshift=-2mm, yshift=2mm, rotate=atan(\m)]{$|\sin\theta_c|/\tan\theta_c$,t$};
62 \draw[>,>=stealth] (\setXInsideOff, \setYInsideOff) -- (\setXInsideOn, \setYInsideOn);
63
64 \end{tikzpicture}
65
66 \begin{subfigure}[t]{0.5\textwidth}
67 \includegraphics[width=0.9\linewidth]{ParabolicNoseConeAttempt.png}
68 \end{subfigure}
69 \caption{Comparison of the parabolic cone of cone constant  $k = 9/10$  (red, slightly inside) and the von Kármán ogive of the Haack series characterized by the cone constant  $C = 0$  (black, slightly outside).}
70 \end{figure}

```

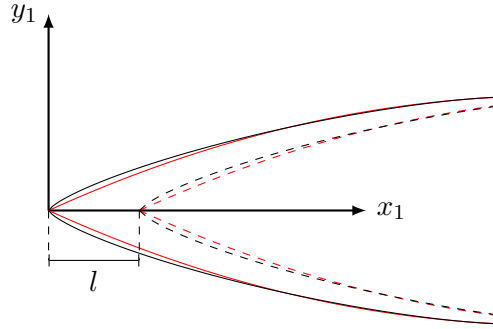


Figure 19: Comparison of the parabolic cone of cone constant $k = 9/10$ (red, slightly inside) and the von Kármán ogive of the Haack series characterized by the cone constant $C = 0$ (black, slightly outside).

2.21 Elliptical Nose Cone

```

1 \begin{figure}[H]
2 \begin{subfigure}[t]{0.5\textwidth}
3 \centering
4 \tdplotsetmaincoords{-20}{0} % 70 110
5 \tdplotsetrotatedcoords{0}{-20}{0}
6 \begin{tikzpicture}[scale=2, tdplot_main_coords]
7 \pgfmathsetmacro{\m}{0.25}
8 \pgfmathsetmacro{\L}{3}
9 \pgfmathsetmacro{\R}{\m*\L}
10 \pgfmathsetmacro{\K}{0.75}
11
12 % New approach is to draw curve (parabola) on xy plane and rotate plane until it matches well with base
13 \pgfmathsetmacro{\th}{-20}
14 \pgfmathsetmacro{\cost}{\cos(\th)}
15 \pgfmathsetmacro{\sint}{\sin(\th)}
16 \begin{scope}[tdplot_rotated_coords, canvas is plane={O(0,0,0)x(1,0,0)y(0,\cost,\sint)}]
17 \draw [domain=0:\L, samples=40] plot ({\x}, {\R * sqrt(1 - ((\x-\L) / \L)^2)});
18 \draw [domain=0:\L, samples=40] plot ({\x}, {-\R * sqrt(1 - ((\x-\L) / \L)^2)});
19 \end{scope}
20 % Draw face
21 \begin{scope}[tdplot_rotated_coords, canvas is yz plane at x=\L]
22 \pgfmathsetmacro{\t}{\R/6}
23 \draw (0, 0) circle [radius=\R];
24 \draw (0, 0) circle [radius=\R-\t];
25 \draw[->, >=stealth, thick] (-\R-0.2,0) -- (-\R,0);
26 \draw[->, >=stealth, thick] (-\R+\t+0.4,0) -- (-\R+\t,0) node[pos=1.1, right, xshift=1mm]{$R - r$};
27 \end{scope}
28 % Draw parameters
29 \draw[tdplot_rotated_coords, dashed] (0, 0, 0) -- (\L, 0, 0) node[pos=0.6, below]{$L$};
30 \draw[tdplot_rotated_coords, dashed] (\L, 0, 0) -- (\L, \R, 0) node[above, xshift=-0.5mm, yshift=0.5mm]{$R$};
31 %% Draw half angle
32 %\begin{scope}[tdplot_rotated_coords, canvas is xy plane at z=0]
33 %\pgfmathsetmacro{\angle}{atan(\R/\L)}
34 %\draw[->, >=stealth] (1.2,0) arc (0:\angle:1.225cm) node[midway, right]{$\theta_c$};
35 %\end{scope}
36
37 % Draw axes
38 \draw[->, >=latex, thick, tdplot_rotated_coords] (0, 0, 0) -- (1, 0, 0) node[anchor=north west, xshift=-2mm]{$x_1$};
39 \draw[->, >=latex, thick, tdplot_rotated_coords] (0, 0, 0) -- (0, 1, 0) node[pos=1, right]{$y_1$};
40 \draw[->, >=latex, thick, tdplot_rotated_coords] (0, 0, 0) -- (0, 0, 1) node[anchor=north east, xshift=0.75mm, yshift=0.75mm]{$z_1$};
41 \end{tikzpicture}% NO SPACE!
42 \end{subfigure}
43 \hspace{1cm}% NO SPACE!
44 \begin{subfigure}[t]{0.5\textwidth}
45 \begin{tikzpicture}[scale=2]
46 % Define the same parameters as above
47 \pgfmathsetmacro{\m}{0.25}
48 \pgfmathsetmacro{\L}{3}
49 \pgfmathsetmacro{\R}{\m*\L}
50 \pgfmathsetmacro{\K}{0.75}
51 \pgfmathsetmacro{\t}{0.6} %\R/3
52
53 % Draw axes
54 \draw[->, >=latex, thick] (0,0) -- (2.1,0) node[right]{$x_1$}; %1.5
55 \draw[->, >=latex, thick] (0,0) -- (0,1.3) node[left]{$y_1$};
56
57 %% Draw angle
58 \pgfmathsetmacro{\structAngle}{atan(\m)}

```

```

59 %\begin{scope}[shift={(\t,0)}]
60 %\draw[>-, >=stealth, thick] (1, 0) arc(0:\structAngle:1) node[midway, right]{\theta_c};
61 %\end{scope}
62
63 % Outside cone
64 \draw [domain=0:\L, samples=40] plot ({\x}, {\R * sqrt(1 - ((\x-\L) / \L)^2)});
65 \draw [domain=0:\L, samples=40] plot ({\x}, {-\R * sqrt(1 - ((\x-\L) / \L)^2)});
66 % Inside cone
67 \draw[dashed, domain=\t:\L, samples=40] plot ({\x}, {\R * sqrt(1 - ((\x-\t-\L) / \L)^2) * (1 - sin(180*\t
/\L)^2)});
68 \draw[dashed, domain=\t:\L, samples=40] plot ({\x}, {-\R * sqrt(1 - ((\x-\t-\L) / \L)^2) * (1 - sin(180*\t
/\L)^2)});
69 % Draw t
70 \pgfmathsetmacro{\YofLine}{-0.65} % -0.05
71 \draw[dashed] (0, 0) -- (0, \YofLine);
72 \draw[dashed] (\t, 0) -- (\t, \YofLine);
73 \draw[| - |] (0, \YofLine) -- (\t, \YofLine) node[pos=0.5, below]{t}; % pos=0.6, below, yshift=-2.3mm
74
75 %%% Draw vertical thickness
76 %\pgfmathsetmacro{\XofVerticalMarker}{0.8*\L}
77 %\pgfmathsetmacro{\YofVerticalMarkerOnInside}{\m*(\XofVerticalMarker-\t)}
78 %\pgfmathsetmacro{\YofVerticalMarkerOnOutside}{\m*\XofVerticalMarker}
79 %\pgfmathsetmacro{\tailLength}{0.2}
80 %\draw[>-, >=stealth] (\XofVerticalMarker, -\YofVerticalMarkerOnInside+\tailLength) -- (\XofVerticalMarker, -\YofVerticalMarkerOnInside);
81 %\draw[>-, >=stealth] (\XofVerticalMarker, -\YofVerticalMarkerOnOutside-\tailLength) -- (\XofVerticalMarker, -\YofVerticalMarkerOnOutside) node[left, rotate=-atan(\m), xshift=-1.5mm, yshift=-2.5mm]{\tan\theta_c \t};
82 %%% Draw normal thickness
83 %\pgfmathsetmacro{\setXOutsideOn}{0.8*\L}
84 %\pgfmathsetmacro{\setYOutsideOn}{\m*\setXOutsideOn}
85 %\pgfmathsetmacro{\setXOutsideOff}{\setXOutsideOn-0.05}
86 %\pgfmathsetmacro{\setYOutsideOff}{\setYOutsideOn - 1/\m * (\setXOutsideOff - \setXOutsideOn)}
87 %%%
88 %\pgfmathsetmacro{\setXInsideOn}{\setXOutsideOn + \m^2*\t/(1+\m^2)}
89 %\pgfmathsetmacro{\setYInsideOn}{\m*(\setXInsideOn-\t)}
90 %\pgfmathsetmacro{\setXInsideOff}{\setXInsideOn+0.05}
91 %\pgfmathsetmacro{\setYInsideOff}{\setYInsideOn - 1/\m * (\setXInsideOff - \setXInsideOn)}
92 %%%
93 %\draw[>-, >=stealth] (\setXOutsideOff, \setYOutsideOff) -- (\setXOutsideOn, \setYOutsideOn) node[left, xshift=-2mm, yshift=2mm, rotate=atan(\m)]{\sin\theta_c \tan\theta_c \t};
94 %\draw[>-, >=stealth] (\setXInsideOff, \setYInsideOff) -- (\setXInsideOn, \setYInsideOn);
95
96 \end{tikzpicture}
97 \end{subfigure}
98
99 %\begin{subfigure}[t]{0.5\textwidth}
100 %\includegraphics[width=0.9\linewidth]{ParabolicNoseConeAttempt.png}
101 %\end{subfigure}
102 \caption{Elliptical cone of constant density  $\rho$  and frontal thickness  $l$ .}
103 \label{fig:StructuresEllipticalCone}
104 \end{figure}

```

2.22 Solid Cylinder

```

1 \begin{figure}[H]
2 \centering
3 \tdplotsetmaincoords{-20}{0} % 70 110
4 \tdplotsetrotatedcoords{0}{-20}{0}
5 \begin{tikzpicture}[scale=2, tdplot_main_coords]
6 \pgfmathsetmacro{\m}{0.25}
7 \pgfmathsetmacro{\L}{3}
8 \pgfmathsetmacro{\R}{\m*\L}
9
10 % Draw sides
11 \pgfmathsetmacro{\t}{-20}
12 \pgfmathsetmacro{\Rcost}{\R*cos(\t)}
13 \pgfmathsetmacro{\Rsint}{\R*sin(\t)}
14 \draw[tdplot_rotated_coords] (0, \Rcost, \Rsint) -- (\L, \Rcost, \Rsint);
15 \draw[tdplot_rotated_coords] (0, -\Rcost, -\Rsint) -- (\L, -\Rcost, -\Rsint);
16 % Draw faces
17 \begin{scope}[tdplot_rotated_coords, canvas is yz plane at x=0]
18 \pgfmathsetmacro{\ttmp}{145}
19 \pgfmathsetmacro{\Rsinttmp}{\R*sin(\ttmp)}
20 \pgfmathsetmacro{\Rcosttmp}{\R*cos(\ttmp)}
21 \draw (\Rcost, \Rsint) arc(\t:\ttmp:\R);
22 \draw[dashed] (\Rcosttmp, \Rsinttmp) arc(\ttmp:360:\R);
23 \end{scope}
24 \begin{scope}[tdplot_rotated_coords, canvas is yz plane at x=\L]
25 \draw (0, 0) circle [radius=\R];
26 \end{scope}
27 % Draw parameters
28 \draw[tdplot_rotated_coords, dashed] (0, 0, 0) -- (\L, 0, 0) node[pos=0.6, below]{$L$};
29 \draw[tdplot_rotated_coords, dashed] (\L, 0, 0) -- (\L, \R, 0) node[above, xshift=-0.5mm, yshift=0.5mm]{$R$};
30
31 % Draw axes
32 \draw[->, >=latex, thick, tdplot_rotated_coords] (0, 0, 0) -- (1, 0, 0) node[anchor=north west, xshift=-2mm]{$x_m$};
33 \draw[->, >=latex, thick, tdplot_rotated_coords] (0, 0, 0) -- (0, 1, 0) node[pos=1, right]{$y_m$};
34 \draw[->, >=latex, thick, tdplot_rotated_coords] (0, 0, 0) -- (0, 0, 1) node[anchor=north east, xshift=0.75mm, yshift=0.75mm]{$z_m$};
35 \end{tikzpicture}
36 \caption{Solid cylinder of constant density $\rho$.}
37 \end{figure}

```

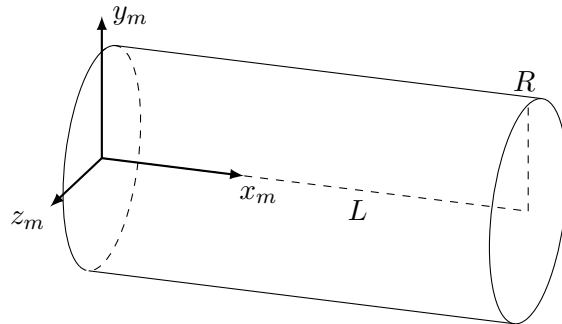


Figure 20: Solid cylinder of constant density ρ .

2.23 Hollow Cylinder

```

1 \begin{figure}[H]
2 \begin{subfigure}[t]{0.5\textwidth}
3 \centering
4 \tdplotsetmaincoords{-20}{0} % 70 110
5 \tdplotsetrotatedcoords{0}{-20}{0}
6 \begin{tikzpicture}[scale=2, tdplot_main_coords]
7 \pgfmathsetmacro{\m}{0.25}
8 \pgfmathsetmacro{\L}{3}
9 \pgfmathsetmacro{\R}{\m*\L}
10
11 % New approach is to draw curve (parabola) on xy plane and rotate plane until it matches well with base
12 \pgfmathsetmacro{\th}{-20}
13 \pgfmathsetmacro{\cost}{\cos(\th)}
14 \pgfmathsetmacro{\sint}{\sin(\th)}
15 \begin{scope}[tdplot_rotated_coords, canvas is plane={O(0,0,0)x(1,0,0)y(0,\cost,\sint)}]
16 \draw [domain=0:\L, samples=3] plot ({\x}, {\R});
17 \draw [domain=0:\L, samples=3] plot ({\x}, {-\R});
18 \end{scope}
19 % Draw face
20 \begin{scope}[tdplot_rotated_coords, canvas is yz plane at x=\L]
21 \pgfmathsetmacro{\t}{\R/6}
22 \draw (0, 0) circle [radius=\R];
23 \draw (0, 0) circle [radius=\R-\t];
24 \draw[->, >=stealth, thick] (-\R-0.2,0) -- (-\R,0);
25 \draw[->, >=stealth, thick] (-\R+\t+0.4,0) -- (-\R+\t,0) node[pos=1.1, right, xshift=2mm]{$\R - r$};
26 \end{scope}
27 % Draw faces
28 %\pgfmathsetmacro{\t}{-20}
29 \pgfmathsetmacro{\Rcost}{\R*\cost}
30 \pgfmathsetmacro{\Rsint}{\R*\sint}
31 \begin{scope}[tdplot_rotated_coords, canvas is yz plane at x=0]
32 \pgfmathsetmacro{\thtmp}{145}
33 \pgfmathsetmacro{\Rsinttmp}{\R*\sin(\thtmp)}
34 \pgfmathsetmacro{\Rcosttmp}{\R*\cos(\thtmp)}
35 \draw (\Rcost, \Rsint) arc(\th:\thtmp:\R);
36 \draw[dashed] (\Rcosttmp, \Rsinttmp) arc(\thtmp:360:\R);
37 \pgfmathsetmacro{\Rminust}{\R*5/6}
38 \draw[dashed] (\Rminust, 0) arc(0:360:\Rminust); % Inside face
39 \end{scope}
40 % Draw parameters
41 \draw[tdplot_rotated_coords, dashed] (0, 0, 0) -- (\L, 0, 0) node[pos=0.6, below]{$\L$};
42 \draw[tdplot_rotated_coords, dashed] (\L, 0, 0) -- (\L, \R, 0) node[above, xshift=-0.5mm, yshift=0.5mm]{$\R$}; % node[midway, right, xshift=-1mm]{$\R$};
43 %% Draw half angle
44 %\begin{scope}[tdplot_rotated_coords, canvas is xy plane at z=0]
45 %\pgfmathsetmacro{\angle}{atan(\R/\L)}
46 %\draw[->, >=stealth] (1.2,0) arc (0:\angle:1.225cm) node[midway, right]{$\theta_c$};
47 %\end{scope}
48
49 % Draw axes
50 \draw[->, >=latex, thick, tdplot_rotated_coords] (0, 0, 0) -- (1, 0, 0) node[anchor=north west, xshift=-2mm]{$x_m$};
51 \draw[->, >=latex, thick, tdplot_rotated_coords] (0, 0, 0) -- (0, 1, 0) node[pos=1, right]{$y_m$};
52 \draw[->, >=latex, thick, tdplot_rotated_coords] (0, 0, 0) -- (0, 0, 1) node[anchor=north east, xshift=0.75mm, yshift=0.75mm]{$z_m$};
53 \end{tikzpicture}% NO SPACE!
54 \end{subfigure}
55 \hspace{1cm}% NO SPACE!
56 \begin{subfigure}[t]{0.5\textwidth}
57 \begin{tikzpicture}[scale=2]
58 % Define the same parameters as above

```

```

59 \pgfmathsetmacro{\m}{0.25}
60 \pgfmathsetmacro{\L}{3}
61 \pgfmathsetmacro{\R}{\m*\L}
62 \pgfmathsetmacro{\t}{\R/6} %\R/3
63
64 % Draw axes
65 \draw[>=>latex, thick] (0,0) -- (2.1,0) node[right]{ $x_m$ }; %1.5
66 \draw[>=>latex, thick] (0,0) -- (0,1.3) node[left]{ $y_m$ };
67
68 %% Draw angle
69 %\pgfmathsetmacro{\structAngle}{atan(\m)}
70 %\begin{scope}[shift={(\t,0)}]
71 %\draw[>=>stealth, thick] (1, 0) arc(0:\structAngle:1) node[midway, right]{ $\theta_c$ };
72 %\end{scope}
73
74 % Outside cylinder
75 \draw [domain=0:\L, samples=3] plot ({\x}, {\R});
76 \draw [domain=0:\L, samples=3] plot ({\x}, {-\R});
77 % Inside cylinder
78 \draw[dashed, domain=0:\L, samples=40] plot ({\x}, {\R-\t});
79 \draw[dashed, domain=0:\L, samples=40] plot ({\x}, {-(\R-\t)});
80 %% Draw t
81 %\pgfmathsetmacro{\YofLine}{-0.33} %-0.05
82 %\draw[dashed] (0, 0) -- (0, \YofLine);
83 %\draw[dashed] (\t, 0) -- (\t, \YofLine);
84 %\draw[| - |] (0,\YofLine) -- (\t,\YofLine) node[pos=0.5, below]{ $t$ }; %pos=0.6, below, yshift=-2.3mm
85
86 %% Draw vertical thickness
87 \pgfmathsetmacro{\XofVerticalMarker}{0.8*\L}
88 \pgfmathsetmacro{\YofVerticalMarkerOnInside}{\R-\t}
89 \pgfmathsetmacro{\YofVerticalMarkerOnOutside}{\R}
90 \pgfmathsetmacro{\tailLength}{0.2}
91 \draw[>=>stealth] (\XofVerticalMarker, \YofVerticalMarkerOnInside-\tailLength) -- (\XofVerticalMarker
, \YofVerticalMarkerOnInside);
92 \draw[>=>stealth] (\XofVerticalMarker, \YofVerticalMarkerOnOutside+\tailLength) -- (\
XofVerticalMarker, \YofVerticalMarkerOnOutside) node[right, xshift=0.5mm, yshift=2.25mm]{ $R - r$ };
93 %% Draw normal thickness
94 %\pgfmathsetmacro{\setXOutsideOn}{0.8*\L}
95 %\pgfmathsetmacro{\setYOutsideOn}{\m*\setXOutsideOn}
96 %\pgfmathsetmacro{\setXOutsideOff}{\setXOutsideOn-0.05}
97 %\pgfmathsetmacro{\setYOutsideOff}{\setYOutsideOn - 1/\m * (\setXOutsideOff - \setXOutsideOn)}
98 %%
99 %\pgfmathsetmacro{\setXInsideOn}{\setXOutsideOn + \m^2*\t/(1+\m^2)}
100 %\pgfmathsetmacro{\setYInsideOn}{\m*(\setXInsideOn-\t)}
101 %\pgfmathsetmacro{\setXInsideOff}{\setXInsideOn+0.05}
102 %\pgfmathsetmacro{\setYInsideOff}{\setYInsideOn - 1/\m * (\setXInsideOff - \setXInsideOn)}
103 %%
104 %\draw[>=>stealth] (\setXOutsideOff, \setYOutsideOff) -- (\setXOutsideOn, \setYOutsideOn) node[left,
xshift=-2mm, yshift=2mm, rotate=atan(\m)]{ $|\sin\theta_c| \tan\theta_c, t$ };
105 %\draw[>=>stealth] (\setXInsideOff, \setYInsideOff) -- (\setXInsideOn, \setYInsideOn);
106
107 \end{tikzpicture}
108 \end{subfigure}
109 \caption{Hollow cylinder of constant density  $\rho$  and wall thickness  $R - r$ .}
110 \end{figure}

```

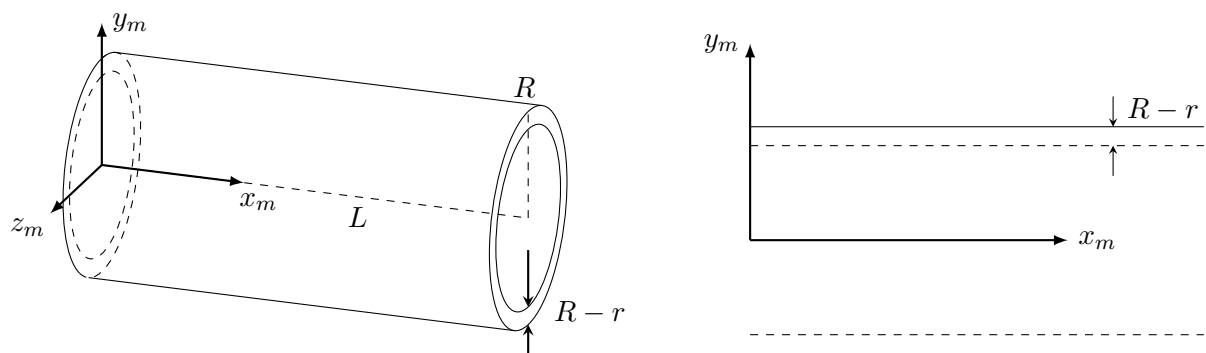


Figure 21: Hollow cylinder of constant density ρ and wall thickness $R - r$.

2.24 Hollow Frustum (Angled Hollow Cylinder)

```

1 \begin{figure}[H]
2 \begin{subfigure}[t]{0.5\textwidth}
3 \centering
4 \tdplotsetmaincoords{-20}{0} % 70 110
5 \tdplotsetrotatedcoords{0}{-20}{0}
6 \begin{tikzpicture}[scale=2, tdplot_main_coords]
7 \pgfmathsetmacro{\m}{0.25}
8 \pgfmathsetmacro{\L}{3}
9 \pgfmathsetmacro{\r}{\m*\L*0.8}
10 \pgfmathsetmacro{\R}{1.6*\r}
11
12 % New approach is to draw curve (parabola) on xy plane and rotate plane until it matches well with base
13 \pgfmathsetmacro{\th}{-20}
14 \pgfmathsetmacro{\cost}{\cos(\th)}
15 \pgfmathsetmacro{\sint}{\sin(\th)}
16 \begin{scope}[tdplot_rotated_coords, canvas is plane={O(0,0,0)x(1,0,0)y(0,\cost,\sint)}]
17 \draw [domain=0:\L, samples=3] plot ({\x}, {\r + ((\R - \r)/\L)*\x});
18 \draw [domain=0:\L, samples=3] plot ({\x}, {-\r + ((\R - \r)/\L)*\x});
19 \end{scope}
20 % Draw face
21 \pgfmathsetmacro{\t}{\R/6} %R/6
22 \begin{scope}[tdplot_rotated_coords, canvas is yz plane at x=\L]
23 \draw (0, 0) circle [radius=\R];
24 \draw (0, 0) circle [radius=\R-\t];
25 \draw[->, >=stealth, thick] (-\R-0.2,0) -- (-\R,0);
26 \draw[->, >=stealth, thick] (-\R+\t+0.4,0) -- (-\R+\t,0) node[pos=1.1, right, xshift=2mm]{$\R - r$};
27 \end{scope}
28 % Draw faces
29 %\pgfmathsetmacro{\t}{-20}
30 \pgfmathsetmacro{\Rcost}{\R*\cost}
31 \pgfmathsetmacro{\Rsint}{\R*\sint}
32 \begin{scope}[tdplot_rotated_coords, canvas is yz plane at x=0]
33 \pgfmathsetmacro{\thtmp}{145}
34 \pgfmathsetmacro{\Rsinttmp}{\r*\sin(\thtmp)}
35 \pgfmathsetmacro{\Rcosttmp}{\r*\cos(\thtmp)}
36 \pgfmathsetmacro{\Rsinttmpforr}{\r*\sin(-180)}
37 \pgfmathsetmacro{\Rcosttmpforr}{\r*\cos(-180)}
38 \draw[dashed] (0,0) -- (\Rcosttmpforr, \Rsinttmpforr) node[below, yshift=-1mm]{$R_0$};
39 \draw (\r*\cost, \r*\sint) arc(\th:\thtmp:\r);
40 \draw[dashed] (\Rcosttmp, \Rsinttmp) arc(\thtmp:360:\r);
41 \pgfmathsetmacro{\Rminust}{\r-\t}
42 \draw[dashed] (\Rminust, 0) arc(0:360:\Rminust); % Inside face
43 \end{scope}
44 %\begin{scope}[tdplot_rotated_coords, canvas is yz plane at x=\L]
45 %\draw (0, 0) circle [radius=\R];
46 %\end{scope}
47 % Draw parameters
48 \draw[tdplot_rotated_coords, dashed] (0, 0, 0) -- (\L, 0, 0) node[pos=0.6, below]{$L$};
49 \draw[tdplot_rotated_coords, dashed] (\L, 0, 0) -- (\L, \R, 0) node[above, xshift=-0.5mm, yshift=0.5mm]{$R$};
50 %% Draw half angle
51 %\begin{scope}[tdplot_rotated_coords, canvas is xy plane at z=0]
52 %\pgfmathsetmacro{\angle}{\atan(\R/\L)}
53 %\draw[->, >=stealth] (1.2,0) arc (0:\angle:1.225cm) node[midway, right]{$\theta_c$};
54 %\end{scope}
55
56 % Draw axes
57 \draw[->, >=latex, thick, tdplot_rotated_coords] (0, 0, 0) -- (1, 0, 0) node[anchor=north west, xshift=-2mm]{$x_m$};
58 \draw[->, >=latex, thick, tdplot_rotated_coords] (0, 0, 0) -- (0, 1, 0) node[pos=1, right]{$y_m$};

```

```

59 \draw[>,>=latex,thick,tdplot_rotated_coords] (0, 0, 0) -- (0, 0, 1) node[anchor=north east, xshift=0.75
mm, yshift=0.75mm]{$z_m$};
60 \end{tikzpicture}% NO SPACE!
61 \end{subfigure}
62 \hspace{1cm}% NO SPACE!
63 \begin{subfigure}[t]{0.5\textwidth}
64 \begin{tikzpicture}[scale=2]
65 % Define the same parameters as above
66 \pgfmathsetmacro{\m}{0.25}
67 \pgfmathsetmacro{\L}{3}
68 \pgfmathsetmacro{\r}{\m*\L*0.8}
69 \pgfmathsetmacro{\R}{1.6*\r}
70 \pgfmathsetmacro{\t}{\R/6} %\R/3
71
72 % Draw axes
73 \draw[>,>=latex,thick] (0,0) -- (2.1,0) node[right]{$x_m$}; %1.5
74 \draw[>,>=latex,thick] (0,0) -- (0,1.3) node[left]{$y_m$};
75
76 %% Draw angle
77 \pgfmathsetmacro{\structAngle}{atan(\m)}
78 %\begin{scope}[shift={(\t,0)}]
79 %\draw[>,>=stealth,thick] (1, 0) arc(0:\structAngle:1) node[midway, right]{$\theta_c$};
80 %\end{scope}
81
82 % Outside cylinder
83 \draw [domain=0:\L, samples=3] plot ({\x}, {\r + ((\R - \r)/\L)*\x});
84 \draw [domain=0:\L, samples=3] plot ({\x}, {-\r + ((\R - \r)/\L)*\x});
85 % Inside cylinder
86 \draw[dashed, domain=0:\L, samples=40] plot ({\x}, {\r-\t + ((\R - \r)/\L)*\x});
87 \draw[dashed, domain=0:\L, samples=40] plot ({\x}, {-\r-\t + ((\R - \r)/\L)*\x});
88 %% Draw t
89 %\pgfmathsetmacro{\YofLine}{-0.33} %-0.05
90 %\draw[dashed] (0, 0) -- (0, \YofLine);
91 %\draw[dashed] (\t, 0) -- (\t, \YofLine);
92 %\draw[| - |] (0, \YofLine) -- (\t, \YofLine) node[pos=0.5, below]{$t$}; %pos=0.6, below, yshift=-2.3mm
93
94 % Draw vertical thickness
95 \pgfmathsetmacro{\XofVerticalMarker}{0.8*\L}
96 \pgfmathsetmacro{\YofVerticalMarkerOnInside}{-(\r+(\R-\r)/\L*\XofVerticalMarker)}
97 \pgfmathsetmacro{\YofVerticalMarkerOnOutside}{-(\r-\t+(\R-\r)/\L*\XofVerticalMarker)}
98 \pgfmathsetmacro{\tailLength}{0.2}
99 \pgfmathsetmacro{\slope}{(\R - \r)/\L}
100 \draw[>,>=stealth] (\XofVerticalMarker, \YofVerticalMarkerOnInside-\tailLength) -- (\XofVerticalMarker
, \YofVerticalMarkerOnInside) node[pos=0,left, rotate=-atan(\slope), xshift=-1.5mm, yshift=1mm]{$\tan
\theta_c$, $t$};
101 \draw[>,>=stealth] (\XofVerticalMarker, \YofVerticalMarkerOnOutside+\tailLength) -- (\
XofVerticalMarker, \YofVerticalMarkerOnOutside);%node[right, xshift=1.5mm, yshift=2.5mm]{$t$};
102 % Draw normal thickness
103 \pgfmathsetmacro{\setXOutsideOn}{0.8*\L}
104 \pgfmathsetmacro{\setYOutsideOn}{\r + (\R - \r)/\L*\setXOutsideOn}
105 \pgfmathsetmacro{\setXOutsideOff}{\setXOutsideOn-0.05}
106 \pgfmathsetmacro{\setYOutsideOff}{\setYOutsideOn - 1/\slope * (\setXOutsideOff - \setXOutsideOn)}
107 %
108 \pgfmathsetmacro{\anglef}{atan(\slope)}
109 \pgfmathsetmacro{\setXInsideOn}{\setXOutsideOn + \t*sin(\anglef)*cos(\anglef)}%\slope^2*t/(1+\slope^2)
}
110 \pgfmathsetmacro{\setYInsideOn}{-3.9*\t+(\r-\t + (\R - \r)/\L)*(\setXInsideOn)}
111 \pgfmathsetmacro{\setXInsideOff}{\setXInsideOn+0.05}
112 \pgfmathsetmacro{\setYInsideOff}{\setYInsideOn - 1/\slope * (\setXInsideOff - \setXInsideOn)}
113 %
114 \draw[>,>=stealth] (\setXOutsideOff, \setYOutsideOff) -- (\setXOutsideOn, \setYOutsideOn) node[left,
xshift=-2mm, yshift=2mm, rotate=atan(\slope)]{$\sin\theta_c$, $\tan\theta_c$, $t$};
115 \draw[>,>=stealth] (\setXInsideOff, \setYInsideOff) -- (\setXInsideOn, \setYInsideOn);

```

```

116 |
117 | % Draw cone angle on x axis
118 | %\pgfmathsetmacro{\xOffset}{\L * \r / (\r + \R)}
119 | \draw[>, >=stealth, thick] (1, 0) arc(0:atan(\slope):4.6) node[midway,right]{\theta_c};
120 |
121 | \end{tikzpicture}
122 | \end{subfigure}
123 | \caption{Hollow conical frustum of constant density  $\rho$  and wall thickness  $l$ .}
124 | \label{fig:StructuresHollowFrustum}
125 | \end{figure}

```

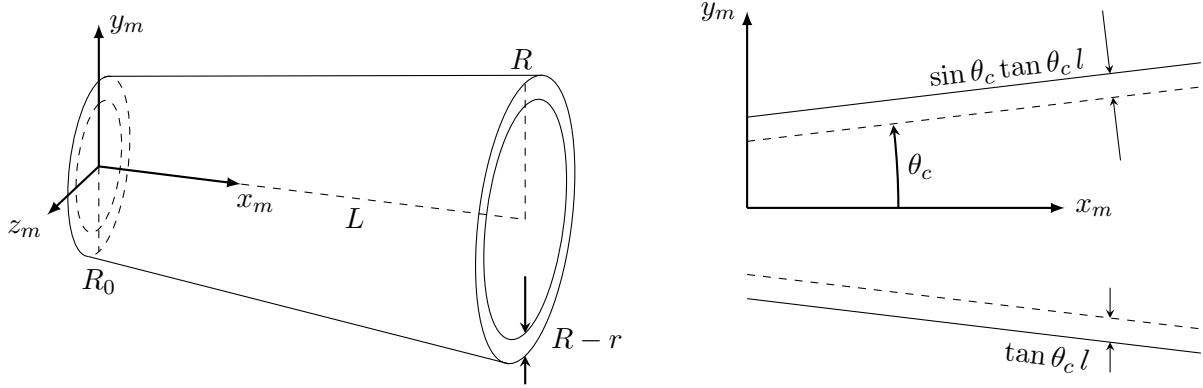


Figure 22: Hollow conical frustum of constant density ρ and wall thickness l .

2.25 Propellant Within the Rocket Body

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}
4 %% Temporary axis (comment out this bit when done)
5 %\draw[red] (0,0) -- (1,0);
6 %\draw[red] (0,0) -- (0,1);
7
8 % Draw the nozzle
9 \begin{axis}[width=15cm,
10     height=207pt,
11     at={(-6.71cm,0pt)},
12     xmin=-8.5, xmax=1.5,
13     ymin=-15, ymax=10,
14     hide axis,
15 ]
16 %% Temporary axis (comment out this bit when done)
17 %\draw[blue] (0,0) -- (1,0);
18 %\draw[blue] (0,0) -- (0,1);
19 % Plot the diverging section - must keep curves as trig forms for constants to work (1 is hardcoded)
20 \pgfmathsetmacro{\YOfNozzleThroat}{1} % 1
21 \pgfmathsetmacro{\YOfNozzleExit}{2} % 3
22 \pgfmathsetmacro{\A}{(\YOfNozzleThroat + \YOfNozzleExit) / 2}
23 \pgfmathsetmacro{\B}{(\YOfNozzleThroat - \YOfNozzleExit) / 2}
24 \addplot[domain=0:1, samples=100, smooth, solid]{\A + \B * cos(deg(pi*x))};
25 \addplot[domain=0:1, samples=100, smooth, solid]{-\A - \B * cos(deg(pi*x))};
26 % Plot the converging part
27 \pgfmathsetmacro{\XOfBodyNozzleIntersection}{-0.25} % -0.25
28 \pgfmathsetmacro{\YOfBody}{1.5} % 1.25
29 \pgfmathsetmacro{\C}{(\YOfNozzleThroat + \YOfBody) / 2}
30 \pgfmathsetmacro{\D}{(\YOfNozzleThroat - \YOfBody) / 2}
31 \pgfmathsetmacro{\E}{1/\XOfBodyNozzleIntersection}
32 \addplot[domain=\XOfBodyNozzleIntersection:0, samples=100, smooth, solid]{\C + \D * cos(deg(\E*pi*x))};
33 \addplot[domain=\XOfBodyNozzleIntersection:0, samples=100, smooth, solid]{-\C - \D * cos(deg(\E*pi*x))};
34 % Plot the body
35 \pgfmathsetmacro{\XofConeBodyIntersection}{-6}
36 \addplot[domain=\XofConeBodyIntersection:\XOfBodyNozzleIntersection, samples=100, smooth, solid]{\YOfBody};
37 \addplot[domain=\XofConeBodyIntersection:\XOfBodyNozzleIntersection, samples=100, smooth, solid]{-\YOfBody};
38 % Plot the nose as half of an ellipse
39 \pgfmathsetmacro{\a}{2}
40 \pgfmathsetmacro{\b}{\YOfBody}
41 \pgfmathsetmacro{\XofNoseTip}{\XofConeBodyIntersection-\a}
42 \addplot[domain=\XofNoseTip:\XofConeBodyIntersection, samples=100, smooth, solid]{\b * sqrt(1 - (x - \XofConeBodyIntersection)^2 / \a^2)};
43 \addplot[domain=\XofNoseTip:\XofConeBodyIntersection, samples=100, smooth, solid]{-\b * sqrt(1 - (x - \XofConeBodyIntersection)^2 / \a^2)};
44
45 % Place an ellipse on the nozzle to show that it's open *** (only shows if axis is hidden) ***
46 \draw (1,-\YOfNozzleExit) arc (-90:90:1.5pt and 13pt);
47 \draw[dashed] (1, \YOfNozzleExit) arc (90:270:1.5pt and 13pt);
48
49 %% Draw a "cutaway" to show the fuel and chamber inside
50 \pgfmathsetmacro{\XOffFuel}{-5}
51 \pgfmathsetmacro{\XOfChamberOffsetFromNozzleTubeIntersection}{0.3}
52 \pgfmathsetmacro{\XOfChamber}{\XOfBodyNozzleIntersection-\XOfChamberOffsetFromNozzleTubeIntersection}
53 \pgfmathsetmacro{\YOffsetFromWall}{0.3}
54 \draw[fill=red!80!black!30] (\XOffFuel,\YOfBody) arc (90:-90:0.6pt and 8.7pt) -- (\XOfChamber, -\YOfBody+\YOffsetFromWall) arc (-90:90:1.2pt and 8.7pt) -- (\XOffFuel,\YOfBody) node[midway, above]{fuel};

```

```

55 \draw[fill=red!80!black!60] (\XOfChamber, \YOfBody) arc (90:-90:1.2pt and 8.7pt) -- (\
    XOfBodyNozzleIntersection, -\YOfBody+\YOffsetFromWall) arc (-90:90:1.4pt and 8.7pt) -- (\
    XOfChamber, \YOfBody) node[midway, above]{chamber};
56
57 % Draw an axis at the bottom to display various points along the rocket (n, t, e)
58 \pgfmathsetmacro{\YOfXLine}{-\YOfBody-1.6}
59 \pgfmathsetmacro{\tickHeight}{1}
60 \pgfmathsetmacro{\XOfChamberTick}{\XOfChamber + 0.16}
61 \draw (\XofNoseTip, \YOfXLine) -- (1, \YOfXLine);
62 \draw (\XofNoseTip, \YOfXLine-\tickHeight/2) -- (\XofConeBodyIntersection-\a, \YOfXLine+\tickHeight
    /2) node[below, yshift=-2.65mm]{n$}; % yshift=-2.6mm
63 \draw (0, \YOfXLine-\tickHeight/2) -- (0, \YOfXLine+\tickHeight/2) node[below, yshift=-2mm]{t$}; %
    yshift=-2mm
64 \draw (\XOfChamberTick, \YOfXLine-\tickHeight/2) -- (\XOfChamberTick, \YOfXLine+\tickHeight/2)
    node[below, yshift=-2.65mm]{c$}; % yshift=-2.75mm
65 \draw (1, \YOfXLine-\tickHeight/2) -- (1, \YOfXLine+\tickHeight/2) node[below, yshift=-2.65mm]{e$};
    % yshift=-2.75mm
66
67 % Draw pressures
68 \draw[->, >=latex] (\XofNoseTip-1.5, 0) -- (\XofNoseTip, 0); % node[above, xshift=-4mm]{p_n$};
69 \draw[->, >=latex] (2.5, \YOfNozzleExit) -- (1, \YOfNozzleExit);
70 \draw[->, >=latex] (2.5, \YOfNozzleExit/2) -- (1, \YOfNozzleExit/2);
71 \draw[->, >=latex] (2.5, 0) -- (1, 0); % node[above, xshift=4mm]{p_e$};
72 \draw[->, >=latex] (2.5, -\YOfNozzleExit) -- (1, -\YOfNozzleExit);
73 \draw[->, >=latex] (2.5, -\YOfNozzleExit/2) -- (1, -\YOfNozzleExit/2);
74
75 % Draw mass flow rate
76 \draw[->, >=latex, dashed] (0,0) -- (0.4, 0) node[right]{\dot{m}};
77
78 %% Draw shock
79 %% Commented this out since it really concerns aerodynamics
80 % \addplot[domain=\XofNoseTip-0.05:\XofNoseTip+1, samples=250, smooth, solid]{sqrt(1 - (x - (\
    XofConeBodyIntersection-0.05))^2 / \a^2) * (\b+1.6)};
81
82 % Draw center of mass for v and F
83 \pgfmathsetmacro{\xcom}{-2.6}
84 \pgfmathsetmacro{\ycom}{0}
85 \pgfmathsetmacro{\rcom}{\YOfBody/4}
86 \draw (\xcom, \ycom) node[circle, fill, inner sep=1pt];
87 \draw[->, >=latex] (\xcom, \ycom) -- (\xcom-0.4, \ycom) node[left]{\hat{v}};
88 % \draw[->, >=latex] (\xcom+0.6, \ycom) -- (\xcom, \ycom) node[pos=0, right]{\vec{F}_T};
89 \end{axis}
90
91 % Try to draw center of mass
92 \begin{scope}[shift={(1.2, 2.4225)}] % (1.2, 2.4225)
93 \pgfmathsetmacro{\Bx}{0}
94 \pgfmathsetmacro{\By}{1}
95 \pgfmathsetmacro{\Br}{0.11}
96 \draw[fill=black] (\Bx, \By) ++(0:\Br) arc (0:90:\Br) -- (\Bx, \By) -- cycle;
97 \draw[fill=white] (\Bx, \By) ++(90:\Br) arc (90:180:\Br) -- (\Bx, \By) -- cycle;
98 \draw[fill=black] (\Bx, \By) ++(180:\Br) arc (180:270:\Br) -- (\Bx, \By) -- cycle;
99 \draw[fill=white] (\Bx, \By) ++(270:\Br) arc (270:360:\Br) -- (\Bx, \By) -- cycle;
100 \end{scope}
101
102 % Draw pressure on nose and exit
103 \node at (7, 3.4){p_e$};
104 \node at (-7, 3.4){p_n$};
105 % more arrows here
106 \end{tikzpicture}
107 \caption{Overviewing diagram of the systems and quantities relevant to the basics of rocket propulsion.
    Particularly, this diagram contains no aerodynamic considerations (shockwaves) in the visualization of p_n$ nor in the exhaust field. The fuel is represented by a block-cutaway to reserve space for either solid propellant or liquid fuel and oxidizer. The chamber, leading into the nozzle, is characterized by a

```

constant total pressure, total temperature, and total density. The exhaust velocity is not shown since its reference frame (the rocket) is different from the reference frame monitoring the velocity and thrust force. The exit area A_e is simply the cross-sectional area of the nozzle at the exit.

108 \label{fig:PropFuelRocket}

109 \end{figure}

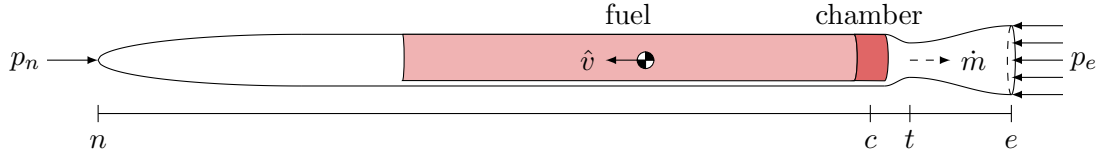


Figure 23: Overviewing diagram of the systems and quantities relevant to the basics of rocket propulsion. Particularly, this diagram contains no aerodynamic considerations (shockwaves) in the visualization of p_n nor in the exhaust field. The fuel is represented by a block-cutaway to reserve space for either solid propellant or liquid fuel and oxidizer. The chamber, leading into the nozzle, is characterized by a constant total pressure, total temperature, and total density. The exhaust velocity is not shown since its reference frame (the rocket) is different from the reference frame monitoring the velocity and thrust force. The exit area A_e is simply the cross-sectional area of the nozzle at the exit.

2.26 Nozzle Flow Coordinates and Symbol Definitions

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}
4 %% Temporary axis (comment out this bit when done)
5 %\draw[red] (0,0) -- (1,0);
6 %\draw[red] (0,0) -- (0,1);
7
8 % Draw the nozzle
9 \begin{axis}[width=15cm,
10     height=207pt,
11     at={(-6.71cm,0pt)},
12     xmin=-1.05, xmax=1.5,
13     ymin=-5, ymax=5,
14     hide axis,
15 ]
16 %% Temporary axis (comment out this bit when done)
17 %\draw[blue] (0,0) -- (1,0);
18 %\draw[blue] (0,0) -- (0,1);
19 % Plot the diverging section - must keep curves as trig forms for constants to work (1 is hardcoded)
20 \pgfmathsetmacro{\YOfNozzleThroat}{1} % 1
21 \pgfmathsetmacro{\YOfNozzleExit}{3} % 3 | 2 | 2.5
22 \pgfmathsetmacro{\A}{(\YOfNozzleThroat + \YOfNozzleExit) / 2}
23 \pgfmathsetmacro{\B}{(\YOfNozzleThroat - \YOfNozzleExit) / 2}
24 \addplot[domain=0:1, samples=100, smooth, solid]{\A + \B * cos(deg(pi*x))};
25 \addplot[domain=0:1, samples=100, smooth, solid]{-\A - \B * cos(deg(pi*x))};
26 % Plot the converging part
27 \pgfmathsetmacro{\XOfBodyNozzleIntersection}{-0.25} % -0.25
28 \pgfmathsetmacro{\YOfBody}{1.5} % 1.25
29 \pgfmathsetmacro{\C}{(\YOfNozzleThroat + \YOfBody) / 2}
30 \pgfmathsetmacro{\D}{(\YOfNozzleThroat - \YOfBody) / 2}
31 \pgfmathsetmacro{\E}{1/\XOfBodyNozzleIntersection}
32 \addplot[domain=\XOfBodyNozzleIntersection:0, samples=100, smooth, solid]{\C + \D * cos(deg(\E*pi*x))};
33 \addplot[domain=\XOfBodyNozzleIntersection:0, samples=100, smooth, solid]{-\C - \D * cos(deg(\E*pi*x))};
34 % Plot the chamber
35 \pgfmathsetmacro{\XOfFuel}{-5}
36 \pgfmathsetmacro{\XOfChamberOffsetFromNozzleTubeIntersection}{0}
37 \pgfmathsetmacro{\XOfChamber}{\XOfBodyNozzleIntersection-\XOfChamberOffsetFromNozzleTubeIntersection}
38 \pgfmathsetmacro{\YOffsetFromWall}{0.3}
39 \pgfmathsetmacro{\XofConeBodyIntersection}{-0.4}
40 \addplot[domain=\XofConeBodyIntersection:\XOfBodyNozzleIntersection, samples=100, smooth, solid]{\YOfBody};
41 \addplot[domain=\XofConeBodyIntersection:\XOfBodyNozzleIntersection, samples=100, smooth, solid]{-\YOfBody};
42 % Plot the nose as half of an ellipse
43 \pgfmathsetmacro{\a}{2}
44 \pgfmathsetmacro{\b}{\YOfBody}
45 %\pgfmathsetmacro{\XofNoseTip}{\XofConeBodyIntersection-\a}
46 %\addplot[domain=\XofNoseTip:\XofConeBodyIntersection, samples=100, smooth, solid]{\b * sqrt(1 - (x - \XofConeBodyIntersection)^2 / \a^2)};
47 %\addplot[domain=\XofNoseTip:\XofConeBodyIntersection, samples=100, smooth, solid]{-\b * sqrt(1 - (x - \XofConeBodyIntersection)^2 / \a^2)};
48
49 % Place an ellipse on the nozzle to show that it's open *** (only shows if axis is hidden)***
50 \draw (1,-\YOfNozzleExit) arc (-90:90:1.5pt and 48.5pt);
51 \draw[dashed] (1, \YOfNozzleExit) arc (90:270:1.5pt and 48.5pt);
52
53 % Place an ellipse on the throat to show that it's open
54 \draw[dashed] (0,-\YOfNozzleThroat) arc (-90:90:1.5pt and 16pt);
55 \draw (0, \YOfNozzleThroat) arc (90:270:1.5pt and 16pt);
56

```

```

57 |
58 | %% Draw a "cutaway" to show the fuel and chamber inside
59 | %\draw[fill=red!80!black!30] (\XOfFuel,\YOfBody) arc (90:-90:0.6pt and 8.7pt) -- (\XOfChamber, -\
    | YOfBody+\YOffsetFromWall) arc (-90:90:1.2pt and 8.7pt) -- (\XOfFuel,\YOfBody) node[midway, above
    | ]{fuel};
60 | %\draw[fill=red!80!black!60] (\XOfChamber, \YOfBody) arc (90:-90:1.2pt and 8.7pt) -- (\
    | XOfBodyNozzleIntersection, -\YOfBody+\YOffsetFromWall) arc (-90:90:1.4pt and 8.7pt) -- (\
    | XOfChamber, \YOfBody) node[midway, above]{chamber};
61 |
62 | % Draw an axis at the bottom to display various points along the rocket (n, t, e)
63 | \pgfmathsetmacro{\YOfXLine}{-\YOfBody-1.6-1}
64 | \pgfmathsetmacro{\tickHeight}{0.6}
65 | \draw (\XOfChamber, \YOfXLine) -- (1, \YOfXLine);
66 | \draw (\XOfChamber, \YOfXLine-\tickHeight/2) -- (\XOfChamber, \YOfXLine+\tickHeight/2) node[below,
    | yshift=-3.7mm]{\$c\$};
67 | \draw (0, \YOfXLine-\tickHeight/2) -- (0, \YOfXLine+\tickHeight/2) node[below, yshift=-3.3mm]{\$t\$}
    | node[above]{\$x = 0\$};
68 | \draw (1, \YOfXLine-\tickHeight/2) -- (1, \YOfXLine+\tickHeight/2) node[below, yshift=-3.7mm]{\$e\$}
    | node[above]{\$x = 1\$};
69 |
70 | % Draw pressure p_e
71 | %\draw[->, >=latex] (2.5, 0) -- (1, 0) node[above, xshift=4mm]{\$p_e\$};
72 |
73 | % Draw mass flow rate
74 | %\draw[->, >=latex, dashed] (0,0) -- (0.4, 0) node[right]{\$ \dot{m} \$};
75 |
76 | %% Draw shock
77 | %% Commented this out since it really concerns aerodynamics
78 | %\addplot[domain=\XofNoseTip-0.05:\XofNoseTip+1, samples=250, smooth, solid]{sqrt(1 - (x - (\
    | XofConeBodyIntersection-0.05))^2 / \a^2) * (\b+1.6)};
79 | \end{axis}
80 | \end{tikzpicture}
81 | \caption{Nondimensionalization of the quasi-unidimensional flow coordinate frame, where the throat is at $x =
    | 0$ and the exit is at $x = 1$. The chamber is not designated a coordinate in terms of $x$, but the
    | subscript $c$ is important nonetheless in the determination of the flow.}
82 | \end{figure}

```

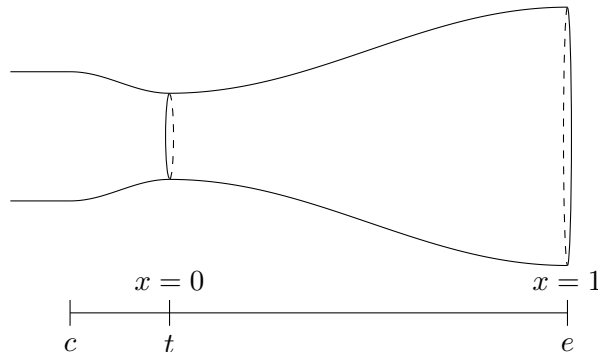


Figure 24: Nondimensionalization of the quasi-unidimensional flow coordinate frame, where the throat is at $x = 0$ and the exit is at $x = 1$. The chamber is not designated a coordinate in terms of x , but the subscript c is important nonetheless in the determination of the flow.

2.27 Normal Shock in Nozzle

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}
4 %% Temporary axis (comment out this bit when done)
5 %\draw[red] (0,0) -- (1,0);
6 %\draw[red] (0,0) -- (0,1);
7
8 % Draw the nozzle
9 \begin{axis}[width=15cm,
10     height=207pt,
11     at={(-6.71cm,0pt)},
12     xmin=-1.05, xmax=1.5,
13     ymin=-5, ymax=5,
14     hide axis,
15 ]
16 %% Temporary axis (comment out this bit when done)
17 %\draw[blue] (0,0) -- (1,0);
18 %\draw[blue] (0,0) -- (0,1);
19 % Plot the diverging section - must keep curves as trig forms for constants to work (1 is hardcoded)
20 \pgfmathsetmacro{\YOfNozzleThroat}{1} % 1
21 \pgfmathsetmacro{\YOfNozzleExit}{3} % 3 | 2 | 2.5
22 \pgfmathsetmacro{\A}{(\YOfNozzleThroat + \YOfNozzleExit) / 2}
23 \pgfmathsetmacro{\B}{(\YOfNozzleThroat - \YOfNozzleExit) / 2}
24 \addplot[domain=0:1, samples=100, smooth, solid]{\A + \B * cos(deg(pi*x))};
25 \addplot[domain=0:1, samples=100, smooth, solid]{-\A - \B * cos(deg(pi*x))};
26 % Plot the converging part
27 \pgfmathsetmacro{\XOfBodyNozzleIntersection}{-0.25} % -0.25
28 \pgfmathsetmacro{\YOfBody}{1.5} % 1.25
29 \pgfmathsetmacro{\C}{(\YOfNozzleThroat + \YOfBody) / 2}
30 \pgfmathsetmacro{\D}{(\YOfNozzleThroat - \YOfBody) / 2}
31 \pgfmathsetmacro{\E}{1/\XOfBodyNozzleIntersection}
32 \addplot[domain=\XOfBodyNozzleIntersection:0, samples=100, smooth, solid]{\C + \D * cos(deg(\E*pi*x))};
33 \addplot[domain=\XOfBodyNozzleIntersection:0, samples=100, smooth, solid]{-\C - \D * cos(deg(\E*pi*x))};
34 % Plot the chamber
35 \pgfmathsetmacro{\XOfFuel}{-5}
36 \pgfmathsetmacro{\XOfChamberOffsetFromNozzleTubeIntersection}{0}
37 \pgfmathsetmacro{\XOfChamber}{\XOfBodyNozzleIntersection-\XOfChamberOffsetFromNozzleTubeIntersection}
38 \pgfmathsetmacro{\YOffsetFromWall}{0.3}
39 \pgfmathsetmacro{\XofConeBodyIntersection}{-0.4}
40 \addplot[domain=\XofConeBodyIntersection:\XOfBodyNozzleIntersection, samples=100, smooth, solid]{\YOfBody};
41 \addplot[domain=\XofConeBodyIntersection:\XOfBodyNozzleIntersection, samples=100, smooth, solid]{-\YOfBody};
42 % Plot the nose as half of an ellipse
43 \pgfmathsetmacro{\a}{2}
44 \pgfmathsetmacro{\b}{\YOfBody}
45 %\pgfmathsetmacro{\XofNoseTip}{\XofConeBodyIntersection-\a}
46 %\addplot[domain=\XofNoseTip:\XofConeBodyIntersection, samples=100, smooth, solid]{\b * sqrt(1 - (x - \XofConeBodyIntersection)^2 / \a^2)};
47 %\addplot[domain=\XofNoseTip:\XofConeBodyIntersection, samples=100, smooth, solid]{-\b * sqrt(1 - (x - \XofConeBodyIntersection)^2 / \a^2)};
48
49 % Place an ellipse on the nozzle to show that it's open *** (only shows if axis is hidden)***
50 \draw (1,-\YOfNozzleExit) arc (-90:90:1.5pt and 48.5pt);
51 \draw[dashed] (1, \YOfNozzleExit) arc (90:270:1.5pt and 48.5pt);
52
53 % Place an ellipse on the throat to show that it's open
54 \draw[dashed] (0,-\YOfNozzleThroat) arc (-90:90:1.5pt and 16pt);
55 \draw (0, \YOfNozzleThroat) arc (90:270:1.5pt and 16pt);
56

```

```

57 |
58 | %% Draw a "cutaway" to show the fuel and chamber inside
59 | %\draw[fill=red!80!black!30] (\XOfFuel,\YOfBody) arc (90:-90:0.6pt and 8.7pt) -- (\XOfChamber, -\
    | YOfBody+\YOffsetFromWall) arc (-90:90:1.2pt and 8.7pt) -- (\XOfFuel,\YOfBody) node[midway, above
    | ]{fuel};
60 | %\draw[fill=red!80!black!60] (\XOfChamber, \YOfBody) arc (90:-90:1.2pt and 8.7pt) -- (\
    | XOfBodyNozzleIntersection, -\YOfBody+\YOffsetFromWall) arc (-90:90:1.4pt and 8.7pt) -- (\
    | XOfChamber, \YOfBody) node[midway, above]{chamber};
61 |
62 | % Draw an axis at the bottom to display various points along the rocket (n, t, e)
63 | \pgfmathsetmacro{\YOfXLine}{-\YOfBody-1.6-1}
64 | \pgfmathsetmacro{\tickHeight}{0.6}
65 | \draw (\XOfChamber, \YOfXLine) -- (1, \YOfXLine);
66 | \draw (\XOfChamber, \YOfXLine-\tickHeight/2) -- (\XOfChamber, \YOfXLine+\tickHeight/2) node[below,
    | yshift=-3.7mm]{c$};
67 | \draw (0, \YOfXLine-\tickHeight/2) -- (0, \YOfXLine+\tickHeight/2) node[below, yshift=-3.3mm]{t$};%
    | node[above]{x = 0$};
68 | \draw (1, \YOfXLine-\tickHeight/2) -- (1, \YOfXLine+\tickHeight/2) node[below, yshift=-3.7mm]{e$};%
    | node[above]{x = 1$};
69 |
70 | % Draw pressure p_e
71 | %\draw[->, >=latex] (2.5, 0) -- (1, 0) node[above, xshift=4mm]{p_e$};
72 |
73 | % Draw mass flow rate
74 | %\draw[->, >=latex, dashed] (0,0) -- (0.4, 0) node[right]{\dot{m}$};
75 |
76 | %% Draw shock in the nozzle
77 | \pgfmathsetmacro{\xOfShock}{0.5}
78 | \pgfmathsetmacro{\YOfShock}{\A + \B * cos(deg(pi*\xOfShock))}
79 | \draw[decorate, decoration={random steps,segment length=3pt,amplitude=1pt,aspect=0}] (\xOfShock, -\
    | YOfShock) -- (\xOfShock, \YOfShock);
80 | \pgfmathsetmacro{\xOfShockSU}{\xOfShock-0.015}
81 | \pgfmathsetmacro{\YOfShockSU}{\A + \B * cos(deg(pi*\xOfShockSU))}
82 | \pgfmathsetmacro{\xOfShockDU}{\xOfShock+0.015}
83 | \pgfmathsetmacro{\YOfShockDU}{\A + \B * cos(deg(pi*\xOfShockDU))}
84 | \draw[red!50, dashed] (\xOfShockSU, -\YOfShockSU) -- (\xOfShockSU, \YOfShockSU) node[pos=0.5, left,
    | black]{M_{s,u}$};
85 | \draw[red!50, dashed] (\xOfShockDU, -\YOfShockDU) -- (\xOfShockDU, \YOfShockDU) node[pos=0.5,
    | right, black]{M_{s,d}$};
86 | % Draw tick
87 | \draw (\xOfShock, \YOfXLine-\tickHeight/2) -- (\xOfShock, \YOfXLine+\tickHeight/2) node[below, yshift
    | =-3.7mm]{s$};
88 | % Draw u and d
89 | \draw[<->, >=stealth, thick] (0, \YOfXLine) -- (\xOfShock, \YOfXLine) node[midway, below, yshift=-1.9
    | mm]{u$} node[midway, above]{M_u\} > 1$};
90 | \draw[<->, >=stealth, thick] (\xOfShock, \YOfXLine) -- (1, \YOfXLine) node[midway, below, yshift=-1mm
    | ]{d$} node[midway, above]{M_d\} < 1$};
91 |
92 | % Draw chamber pressure
93 | \node at (\XOfChamber, 0.8){p_c$};
94 | \node at (\XOfChamber, 0){T_c$};
95 | \node at (\XOfChamber, -0.8){rho_c$};
96 |
97 | % Draw exit flow
98 | \pgfmathsetmacro{\YOfExitTopTip}{\A-\B}
99 | \draw[decorate, decoration={random steps,segment length=3pt,amplitude=1pt,aspect=0}] (1, \YOfExitTopTip)
    | -- (2, \YOfExitTopTip);
100 | \draw[decorate, decoration={random steps,segment length=3pt,amplitude=1pt,aspect=0}] (1, -\
    | YOfExitTopTip) -- (2, -\YOfExitTopTip);
101 |
102 | % Draw back pressure
103 | % (1, \YOfXLine-\tickHeight/2) -- (1, \YOfXLine+\tickHeight/2) node[below, yshift=-3.7mm]
104 | \node[above, yshift=-1.25mm] at (1.25, \YOfXLine){p_b$};

```

```

105 | % Draw exit pressure
106 | \node[right] at (1, 0){$p_e = p_b$};
107 | \end{axis}
108 | \end{tikzpicture}
109 | \caption{Flow resulting from normal shock in the nozzle. The Mach numbers  $M_{s,u}$  and  $M_{s,d}$  are the
      values immediately upstream and immediately downstream of the shock, respectively.}
110 | \end{figure}

```

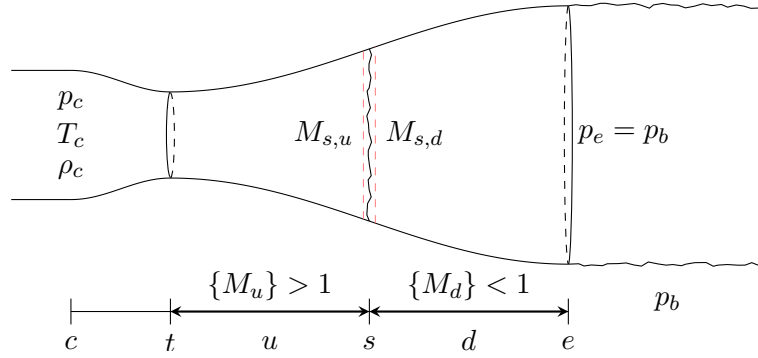


Figure 25: Flow resulting from normal shock in the nozzle. The Mach numbers $M_{s,u}$ and $M_{s,d}$ are the values immediately upstream and immediately downstream of the shock, respectively.

2.28 Oblique Shock from Nozzle Lip

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}
4 %% Temporary axis (comment out this bit when done)
5 %\draw[red] (0,0) -- (1,0);
6 %\draw[red] (0,0) -- (0,1);
7
8 % Draw the nozzle
9 \begin{axis}[width=15cm,
10     height=207pt,
11     at={(-6.71cm,0pt)},
12     xmin=-1.05, xmax=1.5,
13     ymin=-5, ymax=5,
14     hide axis,
15 ]
16 %% Temporary axis (comment out this bit when done)
17 %\draw[blue] (0,0) -- (1,0);
18 %\draw[blue] (0,0) -- (0,1);
19 % Plot the diverging section - must keep curves as trig forms for constants to work (1 is hardcoded)
20 \pgfmathsetmacro{\YOfNozzleThroat}{1} % 1
21 \pgfmathsetmacro{\YOfNozzleExit}{3} % 3 | 2 | 2.5
22 \pgfmathsetmacro{\A}{(\YOfNozzleThroat + \YOfNozzleExit) / 2}
23 \pgfmathsetmacro{\B}{(\YOfNozzleThroat - \YOfNozzleExit) / 2}
24 \addplot[domain=0:1, samples=100, smooth, solid]{\A + \B * cos(deg(pi*x))};
25 \addplot[domain=0:1, samples=100, smooth, solid]{-\A - \B * cos(deg(pi*x))};
26 % Plot the converging part
27 \pgfmathsetmacro{\XOfBodyNozzleIntersection}{-0.25} % -0.25
28 \pgfmathsetmacro{\YOfBody}{1.5} % 1.25
29 \pgfmathsetmacro{\C}{(\YOfNozzleThroat + \YOfBody) / 2}
30 \pgfmathsetmacro{\D}{(\YOfNozzleThroat - \YOfBody) / 2}
31 \pgfmathsetmacro{\E}{1/\XOfBodyNozzleIntersection}
32 \addplot[domain=\XOfBodyNozzleIntersection:0, samples=100, smooth, solid]{\C + \D * cos(deg(\E*pi*x))};
33 \addplot[domain=\XOfBodyNozzleIntersection:0, samples=100, smooth, solid]{-\C - \D * cos(deg(\E*pi*x))};
34 % Plot the chamber
35 \pgfmathsetmacro{\XOfFuel}{-5}
36 \pgfmathsetmacro{\XOfChamberOffsetFromNozzleTubeIntersection}{0}
37 \pgfmathsetmacro{\XOfChamber}{\XOfBodyNozzleIntersection-\XOfChamberOffsetFromNozzleTubeIntersection}
38 \pgfmathsetmacro{\YOffsetFromWall}{0.3}
39 \pgfmathsetmacro{\XofConeBodyIntersection}{-0.4}
40 \addplot[domain=\XofConeBodyIntersection:\XOfBodyNozzleIntersection, samples=100, smooth, solid]{\YOfBody};
41 \addplot[domain=\XofConeBodyIntersection:\XOfBodyNozzleIntersection, samples=100, smooth, solid]{-\YOfBody};
42 % Plot the nose as half of an ellipse
43 \pgfmathsetmacro{\a}{2}
44 \pgfmathsetmacro{\b}{\YOfBody}
45 %\pgfmathsetmacro{\XofNoseTip}{\XofConeBodyIntersection-\a}
46 %\addplot[domain=\XofNoseTip:\XofConeBodyIntersection, samples=100, smooth, solid]{\b * sqrt(1 - (x - \XofConeBodyIntersection)^2 / \a^2)};
47 %\addplot[domain=\XofNoseTip:\XofConeBodyIntersection, samples=100, smooth, solid]{-\b * sqrt(1 - (x - \XofConeBodyIntersection)^2 / \a^2)};
48
49 % Place an ellipse on the nozzle to show that it's open *** (only shows if axis is hidden)***
50 \draw (1,-\YOfNozzleExit) arc (-90:90:1.5pt and 48.5pt);
51 \draw[dashed] (1, \YOfNozzleExit) arc (90:270:1.5pt and 48.5pt);
52
53 % Place an ellipse on the throat to show that it's open
54 \draw[dashed] (0,-\YOfNozzleThroat) arc (-90:90:1.5pt and 16pt);
55 \draw (0, \YOfNozzleThroat) arc (90:270:1.5pt and 16pt);
56

```

```

57 |
58 | %% Draw a "cutaway" to show the fuel and chamber inside
59 | %\draw[fill=red!80!black!30] (\XOfFuel,\YOfBody) arc (90:-90:0.6pt and 8.7pt) -- (\XOfChamber, -\
    | YOfBody+\YOffsetFromWall) arc (-90:90:1.2pt and 8.7pt) -- (\XOfFuel,\YOfBody) node[midway, above
    | ]{fuel};
60 | %\draw[fill=red!80!black!60] (\XOfChamber, \YOfBody) arc (90:-90:1.2pt and 8.7pt) -- (\
    | XOfBodyNozzleIntersection, -\YOfBody+\YOffsetFromWall) arc (-90:90:1.4pt and 8.7pt) -- (\
    | XOfChamber, \YOfBody) node[midway, above]{chamber};
61 |
62 | % Draw an axis at the bottom to display various points along the rocket (n, t, e)
63 | \pgfmathsetmacro{\YOfXLine}{-\YOfBody-1.6-1}
64 | \pgfmathsetmacro{\tickHeight}{0.6}
65 | \draw (\XOfChamber, \YOfXLine) -- (1, \YOfXLine);
66 | \draw (\XOfChamber, \YOfXLine-\tickHeight/2) -- (\XOfChamber, \YOfXLine+\tickHeight/2) node[below,
    | yshift=-3.5mm]{c$};
67 | \draw (0, \YOfXLine-\tickHeight/2) -- (0, \YOfXLine+\tickHeight/2) node[below, yshift=-3.1mm]{t$};%
    | node[above]{x = 0$};
68 | \draw (1, \YOfXLine-\tickHeight/2) -- (1, \YOfXLine+\tickHeight/2) node[below, yshift=-3.7mm]{e$};%
    | node[above]{x = 1$};
69 |
70 | % Draw pressure p_e
71 | %\draw[->, >=latex] (2.5, 0) -- (1, 0) node[above, xshift=4mm]{p_e$};
72 |
73 | % Draw mass flow rate
74 | %\draw[->, >=latex, dashed] (0,0) -- (0.4, 0) node[right]{\dot{m}$};
75 |
76 | % Draw Mach number > 1 in nozzle
77 | \node at (0.55, 0){M > 1$};
78 |
79 | %% Draw shock at the nozzle's exit
80 | \pgfmathsetmacro{\xOfShock}{1}
81 | \pgfmathsetmacro{\YOfShock}{\A + \B * cos(deg(pi*\xOfShock))}
82 | \pgfmathsetmacro{\xOfDisk}{1.15}
83 | \pgfmathsetmacro{\yOfDisk}{\YOfShock/2.5}
84 | \draw[decorate, decoration={random steps,segment length=3pt,amplitude=1pt,aspect=0}] (\xOfShock, \
    | YOfShock) -- (\xOfDisk, \yOfDisk) -- (\xOfDisk, -\yOfDisk) node[midway,right]{Mach disk} -- (\
    | xOfShock, -\YOfShock);
85 | %\pgfmathsetmacro{\xOfShockSU}{\xOfShock-0.015}
86 | %\pgfmathsetmacro{\YOfShockSU}{\A + \B * cos(deg(pi*\xOfShockSU))}
87 | %\pgfmathsetmacro{\xOfShockDU}{\xOfShock+0.015}
88 | %\pgfmathsetmacro{\YOfShockDU}{\A + \B * cos(deg(pi*\xOfShockDU))}
89 | %\draw[red, dashed] (\xOfShockSU, -\YOfShockSU) -- (\xOfShockSU, \YOfShockSU) node[pos=0.5, left,
    | black]{M_{s,u}$};
90 | %\draw[red, dashed] (\xOfShockDU, -\YOfShockDU) -- (\xOfShockDU, \YOfShockDU) node[pos=0.5, right,
    | black]{M_{s,d}$};
91 | % Draw tick
92 | %\draw (\xOfShock, \YOfXLine-\tickHeight/2) -- (\xOfShock, \YOfXLine+\tickHeight/2) node[below,
    | yshift=-3.7mm]{s$};
93 |
94 |
95 | % Draw chamber pressure
96 | \node at (\XOfChamber, 0.8){p_c$};
97 | \node at (\XOfChamber, 0){T_c$};
98 | \node at (\XOfChamber, -0.8){\rho_c$};
99 |
100 | % Draw exit flow
101 | \pgfmathsetmacro{\yOfExitTopTip}{\A-\B}
102 | \draw[decorate, decoration={random steps,segment length=3pt,amplitude=1pt,aspect=0}] (1, \yOfExitTopTip)
    | -- (2, \yOfExitTopTip-1.6);
103 | \draw[decorate, decoration={random steps,segment length=3pt,amplitude=1pt,aspect=0}] (1, -\
    | yOfExitTopTip) -- (2, -\yOfExitTopTip+1.6);
104 |
105 | % Draw back pressure

```

```

106 | %(1, \YOfXLine-\tickHeight/2) -- (1, \YOfXLine+\tickHeight/2) node[below, yshift=-3.7mm]
107 | \node[above, yshift=-2.25mm] at (1.425, \YOfXLine){$p_b$};
108 | % Draw exit pressure
109 | \node[right] at (1, 0){$p_e$};
110 |
111 | % Draw dashed line for theta
112 | \draw[dashed] (\xOfShock, \YOfShock) -- (\xOfShock+1, \YOfShock);
113 | \end{axis}
114 |
115 | % Draw angle beta
116 | %\draw[>, >=stealth, thick] (4.09, 3.7) arc(270:360-45:0.7);% node[midway,below,xshift=1mm]{$\beta$};
117 | %\draw (4.4, 3.7) .. controls (4.7, 3.3) and (5, 3.7) .. (5.2, 3.7) node[pos=1, right]{$\pi/2 - \beta$};
118 | \draw[>, >=stealth, thick] (5, 4.55) arc(0:-45:1.1) node[pos=0.7, right]{$\beta$};
119 | % Draw angle theta
120 | \draw[>, >=stealth, thick] (6.4, 4.55) arc(0:-30:0.7) node[midway, right]{$\theta$};
121 |
122 | % Label dashed line as the nominal flow direction (not a horizontal)
123 | \node at (5.5, 4.75) {sub. flow dir.};
124 | % Label shock wave
125 | \pgfmathsetmacro{\dx}{0.4}
126 | \pgfmathsetmacro{\dy}{-1.75}
127 | \draw[<-, >=stealth] (4.4+\dx, 3.7+\dy) .. controls (4.7+\dx, 3.3+\dy) and (5+\dx, 3.7+\dy) .. (5.2+\dx,
128 | 3.7+\dy) node[pos=1, right]{shock};
129 | % Label supersonic flow
130 | \node[rotate=10] at (5.5, 1.1){flow dir.};
131 | \end{tikzpicture}
132 | \caption{Flow resulting from oblique shocks emanating from the nozzle's edges (resulting from pushing the
normal shock out of the nozzle). Shock reflections from the Mach disk and slip line are not shown. The
angles $\beta$ and $\theta$ are measured from the nominal (subsonic) flow direction.}
\end{figure}

```

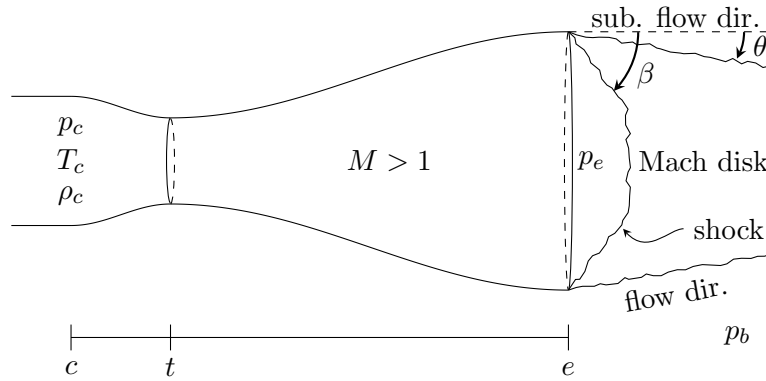


Figure 26: Flow resulting from oblique shocks emanating from the nozzle's edges (resulting from pushing the normal shock out of the nozzle). Shock reflections from the Mach disk and slip line are not shown. The angles β and θ are measured from the nominal (subsonic) flow direction.

2.29 Expansion Fan from Nozzle Lip

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}
4 %% Temporary axis (comment out this bit when done)
5 %\draw[red] (0,0) -- (1,0);
6 %\draw[red] (0,0) -- (0,1);
7
8 % Draw the nozzle
9 \begin{axis}[width=15cm,
10     height=207pt,
11     at={(-6.71cm,0pt)},
12     xmin=-1.05, xmax=1.5,
13     ymin=-5, ymax=5,
14     hide axis,
15 ]
16 %% Temporary axis (comment out this bit when done)
17 %\draw[blue] (0,0) -- (1,0);
18 %\draw[blue] (0,0) -- (0,1);
19 % Plot the diverging section - must keep curves as trig forms for constants to work (1 is hardcoded)
20 \pgfmathsetmacro{\YOfNozzleThroat}{1} % 1
21 \pgfmathsetmacro{\YOfNozzleExit}{3} % 3 | 2 | 2.5
22 \pgfmathsetmacro{\A}{(\YOfNozzleThroat + \YOfNozzleExit) / 2}
23 \pgfmathsetmacro{\B}{(\YOfNozzleThroat - \YOfNozzleExit) / 2}
24 \addplot[domain=0:1, samples=100, smooth, solid]{\A + \B * cos(deg(pi*x))};
25 \addplot[domain=0:1, samples=100, smooth, solid]{-\A - \B * cos(deg(pi*x))};
26 % Plot the converging part
27 \pgfmathsetmacro{\XOfBodyNozzleIntersection}{-0.25} % -0.25
28 \pgfmathsetmacro{\YOfBody}{1.5} % 1.25
29 \pgfmathsetmacro{\C}{(\YOfNozzleThroat + \YOfBody) / 2}
30 \pgfmathsetmacro{\D}{(\YOfNozzleThroat - \YOfBody) / 2}
31 \pgfmathsetmacro{\E}{1/\XOfBodyNozzleIntersection}
32 \addplot[domain=\XOfBodyNozzleIntersection:0, samples=100, smooth, solid]{\C + \D * cos(deg(\E*pi*x))};
33 \addplot[domain=\XOfBodyNozzleIntersection:0, samples=100, smooth, solid]{-\C - \D * cos(deg(\E*pi*x))};
34 % Plot the chamber
35 \pgfmathsetmacro{\XOfFuel}{-5}
36 \pgfmathsetmacro{\XOfChamberOffsetFromNozzleTubeIntersection}{0}
37 \pgfmathsetmacro{\XOfChamber}{\XOfBodyNozzleIntersection-\XOfChamberOffsetFromNozzleTubeIntersection}
38 \pgfmathsetmacro{\YOffsetFromWall}{0.3}
39 \pgfmathsetmacro{\XofConeBodyIntersection}{-0.4}
40 \addplot[domain=\XofConeBodyIntersection:\XOfBodyNozzleIntersection, samples=100, smooth, solid]{\YOfBody};
41 \addplot[domain=\XofConeBodyIntersection:\XOfBodyNozzleIntersection, samples=100, smooth, solid]{-\YOfBody};
42 % Plot the nose as half of an ellipse
43 \pgfmathsetmacro{\a}{2}
44 \pgfmathsetmacro{\b}{\YOfBody}
45 %\pgfmathsetmacro{\XofNoseTip}{\XofConeBodyIntersection-\a}
46 %\addplot[domain=\XofNoseTip:\XofConeBodyIntersection, samples=100, smooth, solid]{\b * sqrt(1 - (x - \XofConeBodyIntersection)^2 / \a^2)};
47 %\addplot[domain=\XofNoseTip:\XofConeBodyIntersection, samples=100, smooth, solid]{-\b * sqrt(1 - (x - \XofConeBodyIntersection)^2 / \a^2)};
48
49 % Place an ellipse on the nozzle to show that it's open *** (only shows if axis is hidden)***
50 \draw (1,-\YOfNozzleExit) arc (-90:90:1.5pt and 48.5pt);
51 \draw[dashed] (1, \YOfNozzleExit) arc (90:270:1.5pt and 48.5pt);
52
53 % Place an ellipse on the throat to show that it's open
54 \draw[dashed] (0,-\YOfNozzleThroat) arc (-90:90:1.5pt and 16pt);
55 \draw (0, \YOfNozzleThroat) arc (90:270:1.5pt and 16pt);
56

```



```

57
58 %% Draw a "cutaway" to show the fuel and chamber inside
59 %\draw[fill=red!80!black!30] (\XOfFuel,\YOfBody) arc (90:-90:0.6pt and 8.7pt) -- (\XOfChamber, -\
    YOfBody+\YOffsetFromWall) arc (-90:90:1.2pt and 8.7pt) -- (\XOfFuel,\YOfBody) node[midway, above
    ]{fuel};
60 %\draw[fill=red!80!black!60] (\XOfChamber, \YOfBody) arc (90:-90:1.2pt and 8.7pt) -- (\
    XOfBodyNozzleIntersection, -\YOfBody+\YOffsetFromWall) arc (-90:90:1.4pt and 8.7pt) -- (\
    XOfChamber, \YOfBody) node[midway, above]{chamber};
61
62 % Draw an axis at the bottom to display various points along the rocket (n, t, e)
63 \pgfmathsetmacro{\YOfXLine}{-\YOfBody-1.6-1}
64 \pgfmathsetmacro{\tickHeight}{0.6}
65 \draw (\XOfChamber, \YOfXLine) -- (1, \YOfXLine);
66 \draw (\XOfChamber, \YOfXLine-\tickHeight/2) -- (\XOfChamber, \YOfXLine+\tickHeight/2) node[below,
    yshift=-3.5mm]{c$};
67 \draw (0, \YOfXLine-\tickHeight/2) -- (0, \YOfXLine+\tickHeight/2) node[below, yshift=-3.1mm]{t$};%
    node[above]{x = 0$};
68 \draw (1, \YOfXLine-\tickHeight/2) -- (1, \YOfXLine+\tickHeight/2) node[below, yshift=-3.7mm]{e$};%
    node[above]{x = 1$};
69
70 % Draw pressure p_e
71 %\draw[->, >=latex] (2.5, 0) -- (1, 0) node[above, xshift=4mm]{p_e$};
72
73 % Draw mass flow rate
74 %\draw[->, >=latex, dashed] (0,0) -- (0.4, 0) node[right]{\dot{m}$};
75
76 % Draw Mach number > 1 in nozzle
77 \node at (0.55, 0){M > 1$};
78
79 %% Draw shock at the nozzle's exit
80 \pgfmathsetmacro{\xOfShock}{1}
81 \pgfmathsetmacro{\YOfShock}{\A + \B * cos(deg(pi*\xOfShock))}
82 \pgfmathsetmacro{\xOfDisk}{1.25}
83 \pgfmathsetmacro{\yOfDisk}{0}
84 \draw[dashed, decorate, decoration={random steps,segment length=3pt,amplitude=1pt,aspect=0}] (\xOfShock,
    \YOfShock) -- (\xOfDisk+0.1, \yOfDisk) -- (\xOfShock, -\YOfShock);
85 \draw[dashed, decorate, decoration={random steps,segment length=3pt,amplitude=1pt,aspect=0}] (\xOfShock,
    \YOfShock) -- (\xOfDisk+0.13, \yOfDisk) -- (\xOfShock, -\YOfShock);
86 \draw[dashed, decorate, decoration={random steps,segment length=3pt,amplitude=1pt,aspect=0}] (\xOfShock,
    \YOfShock) -- (\xOfDisk+0.16, \yOfDisk) -- (\xOfShock, -\YOfShock);
87 \draw[dashed, decorate, decoration={random steps,segment length=3pt,amplitude=1pt,aspect=0}] (\xOfShock,
    \YOfShock) -- (\xOfDisk+0.19, \yOfDisk) -- (\xOfShock, -\YOfShock);
88
89 %\pgfmathsetmacro{\xOfShockSU}{\xOfShock-0.015}
90 %\pgfmathsetmacro{\YOfShockSU}{\A + \B * cos(deg(pi*\xOfShockSU))}
91 %\pgfmathsetmacro{\xOfShockDU}{\xOfShock+0.015}
92 %\pgfmathsetmacro{\YOfShockDU}{\A + \B * cos(deg(pi*\xOfShockDU))}
93 %\draw[red, dashed] (\xOfShockSU, -\YOfShockSU) -- (\xOfShockSU, \YOfShockSU) node[pos=0.5, left,
    black]{M_{s,u}$};
94 %\draw[red, dashed] (\xOfShockDU, -\YOfShockDU) -- (\xOfShockDU, \YOfShockDU) node[pos=0.5, right,
    black]{M_{s,d}$};
95 % Draw tick
96 %\draw (\xOfShock, \YOfXLine-\tickHeight/2) -- (\xOfShock, \YOfXLine+\tickHeight/2) node[below,
    yshift=-3.7mm]{s$};
97
98
99 % Draw chamber pressure
100 \node at (\XOfChamber, 0.8){p_c$};
101 \node at (\XOfChamber, 0){T_c$};
102 \node at (\XOfChamber, -0.8){\rho_c$};
103
104 % Draw exit flow
105 \pgfmathsetmacro{\yOfExitTopTip}{\A-\B}

```



```

106 %%\draw[decorate, decoration={random steps,segment length=3pt,amplitude=1pt,aspect=0}] (1, \
    yOfExitTopTip) -- (2, \yOfExitTopTip+1.4);
107 %%\draw[decorate, decoration={random steps,segment length=3pt,amplitude=1pt,aspect=0}] (1, -\
    yOfExitTopTip) -- (2, -\yOfExitTopTip-1.4);
108 %%\addplot[domain=1:2, samples=10, smooth, solid]{-(x-1)^2 + \yOfExitTopTip};
109 %\pgfmathsetmacro{\XShift}{0.99}
110 %\addplot[domain=1:2, samples=100, red] {3*(sqrt(x-\XShift) - sqrt(1-\XShift)) + \yOfExitTopTip};
111 \draw[decorate, decoration={random steps,segment length=3pt,amplitude=0.51pt,aspect=0}] (1, \
    yOfExitTopTip) .. controls (1.15, \yOfExitTopTip+1.8) and (1.5, \yOfExitTopTip+2) .. (2, \
    yOfExitTopTip+2.5);
112 \draw[decorate, decoration={random steps,segment length=3pt,amplitude=0.51pt,aspect=0}] (1, -\
    yOfExitTopTip) .. controls (1.15, -\yOfExitTopTip-1.8) and (1.5, -\yOfExitTopTip-2) .. (2, -\
    yOfExitTopTip-2.5);
113
114 % Draw back pressure
115 %(1, \YOfXLine-\tickHeight/2) -- (1, \YOfXLine+\tickHeight/2) node[below, yshift=-3.7mm]
116 %\node[above, yshift=-1.25mm] at (1.25, \YOfXLine){$p_b$};
117 \node[below, yshift=-1mm] at (1.1, \YOfXLine){$p_b$};
118 % Draw exit pressure
119 \node[right] at (1, 0){$p_e$};
120
121 % Draw dashed line for theta
122 \draw[dashed] (\xOfShock-0.25, \YOfShock) -- (\xOfShock+1, \YOfShock);
123 \draw[dashed] (\xOfShock, \YOfShock) -- (\xOfShock+0.18, \YOfShock+2);
124
125
126 % Add Mrd and Mru
127 \draw[<-,>=stealth, red] (\xOfShock-0.01, \YOfShock-0.13) .. controls (\xOfShock-0.09, \YOfShock-0.13)
    and (\xOfShock-0.125, \YOfShock-1) .. (\xOfShock-0.15, \YOfShock-1) node[left, xshift=4mm, yshift
    =-1.25mm, black]{$M_{r,u}$};
128 \draw[<-,>=stealth, red] (\xOfShock+0.02, \YOfShock+0.13) .. controls (\xOfShock+0.09, \YOfShock) and (\
    xOfShock+0.125, \YOfShock-1) .. (\xOfShock+0.25, \YOfShock-1) node[right, black]{$M_{r,d}$};
129 \end{axis}
130
131 %% Draw angle beta
132 %%\draw[->, >=stealth, thick] (4.09, 3.7) arc(270:360-45:0.7);% node[midway,below,xshift=1mm]{$\beta$};
133 %%\draw (4.4, 3.7) .. controls (4.7, 3.3) and (5, 3.7) .. (5.2, 3.7) node[pos=1, right]{$\pi/2 - \beta$};
134 %\draw[->, >=stealth, thick] (5, 4.55) arc(0:-45:1.1) node[pos=0.7,right]{$\beta$};
135 % Draw angle theta
136 \draw[->, >=stealth, thick] (3.6, 4.55) arc(180:55:0.5) node[midway,left]{$\dfrac{\pi}{2} - \theta$};
137
138 % Label dashed line as the nominal flow direction (not a horizontal)
139 \node at (5.5, 4.75) {sub. flow dir.};
140 % Label shock wave
141 \pgfmathsetmacro{\dx}{0.4}
142 \pgfmathsetmacro{\dy}{-2}
143 %\draw[<-,>=stealth] (4.4+\dx, 3.7+\dy) .. controls (4.7+\dx, 3.3+\dy) and (5+\dx, 3.7+\dy) .. (5.2+\dx,
    3.7+\dy) node[pos=1, right, align=left]{Rarefaction \ wave};
144 \node[align=left, rotate=38] at (5.2+\dx-0.09, 3.7+\dy+0.2) {Rarefaction wave};
145 % Label supersonic flow
146 \node[rotate=-11] at (6.15, 0.38){flow dir.};
147 \end{tikzpicture}
148 \caption{Flow resulting from expansion fans (rarefaction waves) emanating from the nozzle's edges (resulting
    from pushing the oblique shock past the design condition). Shock reflections from the centerline and slip
    line are not shown.}
149 \end{figure}

```

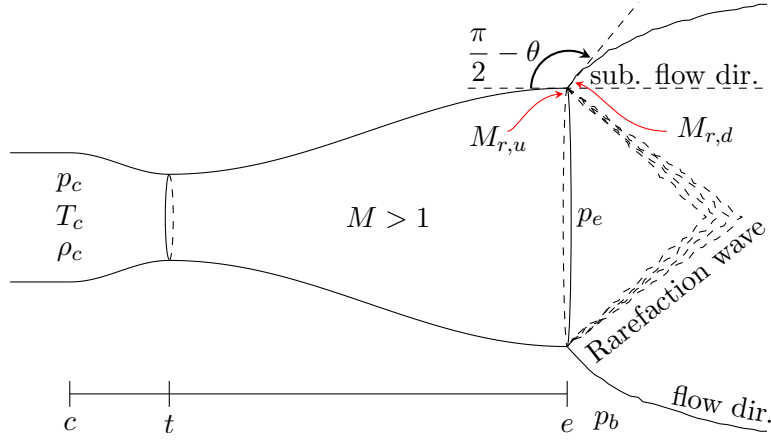


Figure 27: Flow resulting from expansion fans (rarefaction waves) emanating from the nozzle's edges (resulting from pushing the oblique shock past the design condition). Shock reflections from the centerline and slip line are not shown.

2.30 Solid Cylinder of Propellant Grain

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}
4 %\def\blob#1#2{\draw[fill=white,dashed,rounded corners=#1*3mm] (#2) +($ (0:#1*2+#1*rnd)$)
5 %\foreach \a in {20,40,...,350} { -- +($(\a: #1*2+#1*rnd*0.5)$) } -- cycle;}
6 %\blob{0.4}{0,0}
7
8 \pgfmathsetmacro{\R}{1.9}
9 \pgfmathsetmacro{\r}{1.3}
10 \pgfmathsetmacro{\rO}{0.8}
11 \draw[pattern=north east lines, pattern color=black!30] (0,0) circle [radius=\R+0.2];
12 \draw[pattern=north east lines, pattern color=white] (0,0) circle [radius=\R];
13 \draw[pattern=north east lines, pattern color=red!80!black!30] (0,0) circle [radius=\R];
14 \draw[fill=white] (0,0) circle [radius=\r];
15 \draw[dashed] (0,0) circle [radius=\rO];
16
17 % Dimension the cylinder and grain
18 \pgfmathsetmacro{DX}{\R+0.3}
19 \draw[| - |] (-\DX-0.8, -\R-0.2) -- (-\DX-0.8, \R+0.2) node[midway, left]{$2R$};
20 \draw[| - |] (-\DX, -\R) -- (-\DX, \R) node[midway, left]{$2r_1$};
21 \draw[| - |] (\R+0.2, \rO) -- (\R+0.2, \r) node[midway, right]{$W$};
22 \pgfmathsetmacro{\thrO}{150}
23 \pgfmathsetmacro{\thr}{30}
24 \pgfmathsetmacro{\thR}{-120}
25 %
26 \pgfmathsetmacro{\rOx}{\rO*cos(\thrO)}
27 \pgfmathsetmacro{\rOy}{\rO*sin(\thrO)}
28 \pgfmathsetmacro{\rx}{\r*cos(\thr)}
29 \pgfmathsetmacro{\ry}{\r*sin(\thr)}
30 \pgfmathsetmacro{\Rx}{\R*cos(\thR)}
31 \pgfmathsetmacro{\Ry}{\R*sin(\thR)}
32 \draw[->, >=latex, thick] (0,0) -- (\rOx, \rOy) node[pos=0.60, below]{$r_0$}; %{$r_0$}
33 \draw[->, >=latex, thick] (0,0) -- (\rx, \ry) node[pos=0.9, below, yshift=-0.5mm]{$r$};
34 %\draw (0,0) -- (\Rx, \Ry) node[pos=1, anchor=north east]{$R$};
35
36 % Try to draw center of mass
37 \pgfmathsetmacro{Bx}{0}
38 \pgfmathsetmacro{By}{0}
39 \pgfmathsetmacro{Br}{0.11}
40 \draw[fill=black] (\Bx, \By) ++(0:\Br) arc (0:90:\Br) -- (\Bx, \By) -- cycle;
41 \draw[fill=white] (\Bx, \By) ++(90:\Br) arc (90:180:\Br) -- (\Bx, \By) -- cycle;
42 \draw[fill=black] (\Bx, \By) ++(180:\Br) arc (180:270:\Br) -- (\Bx, \By) -- cycle;
43 \draw[fill=white] (\Bx, \By) ++(270:\Br) arc (270:360:\Br) -- (\Bx, \By) -- cycle;
44
45 % Draw A_p
46 \draw[<-, >=stealth] (\R*0.5, -\R*0.75) .. controls (\R*0.6, -\R*0.6) and (\R*0.8, -\R*0.75) .. (\R*1.1, -\R*0.5) node[pos=1, above, xshift=2mm, yshift=-1.25mm]{$A_p$};
47 % Draw casing
48 \draw[<-, >=stealth] (\R*0.17, -\R-0.1) .. controls (\R*0.45, -\R) and (\R*0.55, -\R) .. (\R*0.7, -\R) node[right]{casing};
49
50 % Draw radial arrows point from rp0 to rp at various angles
51 \foreach \t in {75,90,...,330}
52 \pgfmathsetmacro{\xtail}{\rO * cos(\t)}
53 \pgfmathsetmacro{\ytail}{\rO * sin(\t)}
54 \pgfmathsetmacro{\xhead}{\r * cos(\t)}
55 \pgfmathsetmacro{\yhead}{\r * sin(\t)}
56 \draw[->, >=stealth] (\xtail, \ytail) -- (\xhead, \yhead);
57 \end{tikzpicture}

```

58 \caption{Radially symmetric burn rate of BATES grain leaves the center of mass along the longitudinal axis.
The grain borders along, or at least very near, the structural casing for which the inner diameter of the casing is $\sim r$ and the outer diameter is R . The Web thickness here at this instant in time takes on the form $W = r_p - r_0$.}

59 \end{figure}

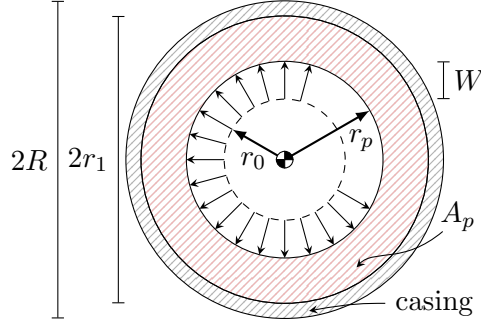


Figure 28: Radially symmetric burn rate of BATES grain leaves the center of mass along the longitudinal axis. The grain borders along, or at least very near, the structural casing for which the inner diameter of the casing is $\sim r$ and the outer diameter is R . The Web thickness here at this instant in time takes on the form $W = r_p - r_0$.

2.31 Supersonic Conical Flow

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}[scale=2]
4 % Define the same parameters as above
5 \pgfmathsetmacro{\m}{0.25}
6 \pgfmathsetmacro{\L}{3}
7 \pgfmathsetmacro{\R}{\m*\L}
8 \pgfmathsetmacro{\t}{0.6} %\R/3
9
10 % Indicate semi-infiniteness
11 %\draw[>,>=latex] (\L, 0) -- (\L+0.5, 0) node[right]{\scriptstyle\infty};
12
13 % Shade inside cone first
14 \pgfmathsetmacro{\ellipser}{21.25}
15 \draw[fill, black!20] (0, 0) -- (\L, \R) arc (90:-90:1.5pt and \ellipser pt) -- cycle;
16 % Outside cone
17 \draw[thick] (\L, \R) -- (0, 0) -- (\L, -\R);
18 % Place an ellipse on the cone face to show it's 3D and circular
19 \draw[thick] (\L, -\R) arc (-90:90:1.5pt and \ellipser pt);
20 \draw[dashed, thick] (\L, \R) arc (90:270:1.5pt and \ellipser pt);
21
22 % Draw axes
23 \draw[>,>=latex, dashed] (0,0) -- (\L-0.2,0) node[pos=1,below]{\scriptstyle\infty};
24
25 % Draw shock
26 \pgfmathsetmacro{\fullShockX}{\L*0.7}
27 \pgfmathsetmacro{\fullShockY}{2.5*\R}
28 \pgfmathsetmacro{\halfShockX}{\L*0.7} %\L*0.4
29 \pgfmathsetmacro{\halfShockY}{-(\halfShockX/\fullShockX)*\fullShockY}
30 \pgfmathsetmacro{\shockAngle}{atan(\fullShockY/\fullShockX)}
31 \draw[decorate, decoration={random steps,segment length=3pt,amplitude=0.5pt,aspect=0}] (0, 0) -- (\fullShockX, \fullShockY);
32 \draw[decorate, decoration={random steps,segment length=3pt,amplitude=0.5pt,aspect=0}] (0, 0) -- (\halfShockX, \halfShockY);
33
34 % Draw coords
35 \pgfmathsetmacro{\px}{2.3}
36 \pgfmathsetmacro{\py}{1}
37 \pgfmathsetmacro{\t}{atan(\py/\px)} % OVERWRITES PREVIOUS t
38 % Draw polar basis
39 \begin{scope}[shift={(\px,\py)}, rotate=\t]
40 \draw[>,>=latex, thick] (0, 0) -- (0.4, 0) node[right]{\mathit{V}_r};
41 \draw[>,>=latex, thick] (0, 0) -- (0, 0.4) node[left]{\mathit{V}_\t};
42 \end{scope}
43 \draw[dashed] (0, 0) -- (\px, \py);
44 \draw (\px,\py) node[circle, fill, inner sep=1,]{};
45
46 % Draw cone, shock, and polar angles
47 \pgfmathsetmacro{\shockAngle}{\shockAngle}
48 \pgfmathsetmacro{\polarAngle}{\t}
49 \pgfmathsetmacro{\structAngle}{atan(\m)}
50 %
51 \pgfmathsetmacro{\sx}{1}
52 \pgfmathsetmacro{\tx}{1.75}
53 \pgfmathsetmacro{\cx}{2.5}
54 %
55 \pgfmathsetmacro{\sr}{(\sx*tan(\shockAngle))}
56 \pgfmathsetmacro{\tr}{(\tx*tan(\polarAngle))}
57 \pgfmathsetmacro{\cr}{(\cx*tan(\structAngle))}
58 \draw[>,>=stealth, thick] (\sx, 0) arc(0:\shockAngle:\sr+0.1) node[pos=0.9, right]{\mathit{V}_\t};
59 \draw[>,>=stealth, thick] (1.75, 0) arc(0:\polarAngle:\tr+0.97) node[pos=0.85, right]{\mathit{V}_\t};

```

```

60 \draw[>-, >=stealth, thick] (2.5, 0) arc(0:\structAngle:\cx) node[pos=0.5, right]{$\theta_c$};
61
62 % Draw second theta ray along the bottom
63 %\draw[dashed] (0, 0) -- (\px, -\py) node[rotate=-\polarAngle;
64 \node at (\px-0.1, -\py){$T,p,\rho,V = \mathrm{const.}$};
65 \node at (\px-0.1, -\py-0.2){along each ray};
66
67
68 % Draw flow
69 \tikzset{set arrow inside/.code={\pgfqkeys{/tikz/arrow inside}{#1}}, set arrow inside={end/.initial=>, opt/.
    initial=}, /pgf/decoration/Mark/.style={mark/.expanded=at position #1 with {\noexpand\arrow[\pgfkeysvalueof{/tikz/arrow inside/opt}]{\pgfkeysvalueof{/tikz/arrow inside/end}}}}, arrow inside/.style 2
    args={set arrow inside={#1}, postaction={decorate,decoration={markings,Mark/.list={#2}}}},}
70 \pgfmathsetmacro{\flowx}{0.4}
71 \pgfmathsetmacro{\flowy}{\flowx*tan(\shockAngle)}
72 \draw[>=latex,domain=-0.4:\flowx,samples=100] plot (\x, -\flowy) [arrow inside={}{0.25,0.5,0.75,1}];
73 \node at (-0.25, -0.6*\flowy){$M_\infty$};
74 \pgfmathsetmacro{\fterminalx}{1}
75 \begin{scope}[shift={(\flowx,-\flowy)}]
76 \draw[>=latex,domain=0:\fterminalx,samples=100] plot (\x, -\m/2*\x^2) [arrow inside={}{0.25,0.5,0.75,1}];
77 \end{scope}
78 \pgfmathsetmacro{\fterminaly}{-\m/2*(\fterminalx)^2} % Ensure f is used as above function
79 \begin{scope}[shift={(\flowx+\fterminalx, -\flowy+\fterminaly)}]
80 \draw[>=latex,domain=0:\fterminalx,samples=100] plot (\x, -\m*\x) [arrow inside={}{0.25,0.5,0.75,1}];
81 \end{scope}
82
83
84
85
86 \end{tikzpicture}
87 \caption{Semi-infinite circular cone with an oblique shock attached at the nose. The incoming flow is
    supersonic ($M_\infty > 1$) and cone is of a constant half-angle $\theta_c$. The flow field behind the
    shock is constant along the ray at an angle $\theta$ from the tip due to the axisymmetric flow field (
    circular cone and $\alpha = 0$).}
88 \label{fig:AerodynamicsConicalFlow}
89 \end{figure}

```

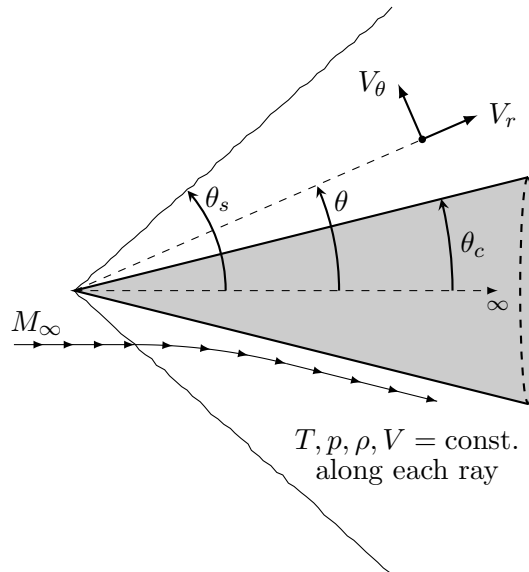


Figure 29: Semi-infinite circular cone with an oblique shock attached at the nose. The incoming flow is supersonic ($M_\infty > 1$) and cone is of a constant half-angle θ_c . The flow field behind the shock is constant along the ray at an angle θ from the tip due to the axisymmetric flow field (circular cone and $\alpha = 0$).

2.32 Rotated Coordinate Frames

```

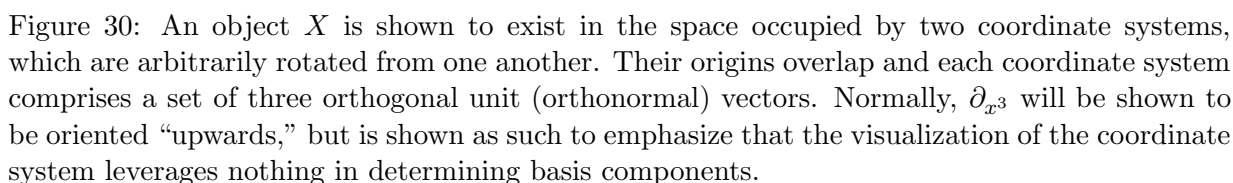
1 \begin{figure}
2 \centering
3 \tdplotsetmaincoords{70}{100}
4 \begin{tikzpicture}[scale=4]
5 % Set viewing angle (?) Not sure if this does anything
6 \tdplotsetrotatedcoords{-90}{-90}{0}
7
8 % Set rotation angles for 213 sequence
9 \pgfmathsetmacro{\phiOne}{-45} % | -45
10 \pgfmathsetmacro{\phiTwo}{-227} % | -227
11 \pgfmathsetmacro{\phiThree}{40} % | 40
12
13 % Set resulting basis components
14 \pgfmathsetmacro{\eOneOne}{cos(\phiTwo)*cos(\phiThree)}
15 \pgfmathsetmacro{\eOneTwo}{sin(\phiThree)}
16 \pgfmathsetmacro{\eOneThree}{-cos(\phiThree)*sin(\phiTwo)}
17 %
18 \pgfmathsetmacro{\eTwoOne}{sin(\phiOne)*sin(\phiTwo) - cos(\phiOne)*cos(\phiTwo)*sin(\phiThree)}
19 \pgfmathsetmacro{\eTwoTwo}{cos(\phiOne)*cos(\phiThree)}
20 \pgfmathsetmacro{\eTwoThree}{cos(\phiTwo)*sin(\phiOne) + cos(\phiOne)*sin(\phiTwo)*sin(\phiThree)}
21 %
22 \pgfmathsetmacro{\eThreeOne}{cos(\phiOne)*sin(\phiTwo) + cos(\phiTwo)*sin(\phiOne)*sin(\phiThree)}
23 \pgfmathsetmacro{\eThreeTwo}{-cos(\phiThree)*sin(\phiOne)}
24 \pgfmathsetmacro{\eThreeThree}{cos(\phiOne)*cos(\phiTwo) - sin(\phiOne)*sin(\phiTwo)*sin(\phiThree)}
25
26 % Set position for object X
27 \pgfmathsetmacro{\XOne}{0.89} % | 0.76
28 \pgfmathsetmacro{\XTwo}{0.2} % | 0.2
29 \pgfmathsetmacro{\XThree}{0.86} % | 0.72
30
31 % Draw grid in the XY plane
32 \pgfmathsetmacro{\Pd}{1.2}
33 \pgfmathsetmacro{\Ld}{1*\Pd}
34 \fill [black!15,opacity=0.25] (\Pd,0,\Pd) -- (\Pd,0,-\Pd) -- (-\Pd+0.2,0,-\Pd) -- (-\Pd+0.2,0,\Pd)
35 -- cycle;
36 \foreach \x in {-1,-0.8,...,1} {
37   \ifthenelse{\NOT -1 = \x}{\draw[black!25,thin] (\x,0,-\Ld) -- (\x,0,\Ld);}{}
38   \draw[black!25,thin] (-\Ld+0.2,0,\x) -- (\Ld,0,\x);
39 }
40
41 % Create coordinates for object X
42 \coordinate (X) at (\XOne,\XTwo,\XThree);
43
44 % Draw axes
45 \draw[->, >=latex, thick] (0,0,0) -- (1,0,0) node[right]{\frac{\partial}{\partial x^1}}; % y
46 \draw[->, >=latex, thick] (0,0,0) -- (0,1,0) node[below right]{\frac{\partial}{\partial x^2}}; % z
47 \draw[->, >=latex, thick] (0,0,0) -- (0,0,1) node[left]{\frac{\partial}{\partial x^3}}; % x
48
49 \draw[->, >=latex, thick] (0,0,0) -- (\eOneOne,\eOneTwo,\eOneThree) node[right]{\hat{e}_1};
50 \draw[->, >=latex, thick] (0,0,0) -- (\eTwoOne,\eTwoTwo,\eTwoThree) node[left]{\hat{e}_2};
51 \draw[->, >=latex, thick] (0,0,0) -- (\eThreeOne,\eThreeTwo,\eThreeThree) node[right]{\hat{e}_3};
52
53 % Draw object X
54 \draw (X) node[circle, fill, inner sep=1pt] node[above, right]{X};
55
56 % Add dashed vertical lines of rotated axes to the XY plane
57 \draw[dashed] (\eOneOne,0,\eOneThree) -- (\eOneOne,\eOneTwo,\eOneThree);
58 \draw[dashed] (\eTwoOne,0,\eTwoThree) -- (\eTwoOne,\eTwoTwo,\eTwoThree);
59 \draw[dashed] (\eThreeOne,0,\eThreeThree) -- (\eThreeOne,\eThreeTwo,\eThreeThree);
60
61 % Add dashed lines from X to the XY plane

```

```

61 \draw[dashed] (\XOne,0,\XThree) -- (\XOne,\XTwo,\XThree) node[midway, left]{$x^2$};
62 %\draw[dashed] (0,0,0) -- (\XOne, 0, \XThree);
63 \draw[dashed] (\XOne, 0, \XThree) -- (0, 0, \XThree) node[midway, below]{$x^1$};
64 \draw[dashed] (\XOne, 0, \XThree) -- (\XOne, 0, 0) node[midway, right]{$x^3$};
65
66 % Add dashed lines from X to the q2q3 plane
67 % ~
68 % Find distance to plane
69 \pgfmathsetmacro{\DtoPlane}{abs(\XOne*\eOneOne + \XTwo*\eOneTwo + \XThree*\eOneThree)};
70 % Label ijk components of this point in the plane
71 \pgfmathsetmacro{\XProjOne}{\XOne+\eOneOne*\DtoPlane};
72 \pgfmathsetmacro{\XProjTwo}{\XTwo+\eOneTwo*\DtoPlane};
73 \pgfmathsetmacro{\XProjThree}{\XThree+\eOneThree*\DtoPlane};
74 % Define its coordinate
75 \coordinate (XProj) at (\XProjOne, \XProjTwo, \XProjThree);
76 % Draw it
77 \draw[dashed] (X) -- (XProj) node[pos=0.4, right]{$q^1$};
78 % Now project XProj onto each of the axes in this plane
79 \pgfmathsetmacro{\DXProjToeTwo}{abs(\XProjOne*\eTwoOne + \XProjTwo*\eTwoTwo + \XProjThree*\eTwoThree)};
80 \pgfmathsetmacro{\DXProjToeThree}{abs(\XProjOne*\eThreeOne + \XProjTwo*\eThreeTwo + \XProjThree*\eThreeThree)};
81 % Set components for projections
82 \pgfmathsetmacro{\XProjToeTwoOne}{\XProjOne-\eTwoOne*\DXProjToeTwo};
83 \pgfmathsetmacro{\XProjToeTwoTwo}{\XProjTwo-\eTwoTwo*\DXProjToeTwo};
84 \pgfmathsetmacro{\XProjToeTwoThree}{\XProjThree-\eTwoThree*\DXProjToeTwo};
85 % ~
86 \pgfmathsetmacro{\XProjToeThreeOne}{\XProjOne-\eThreeOne*\DXProjToeThree};
87 \pgfmathsetmacro{\XProjToeThreeTwo}{\XProjTwo-\eThreeTwo*\DXProjToeThree};
88 \pgfmathsetmacro{\XProjToeThreeThree}{\XProjThree-\eThreeThree*\DXProjToeThree};
89 % Draw
90 \draw[dashed] (XProj) -- (\XProjToeTwoOne, \XProjToeTwoTwo, \XProjToeTwoThree) node[pos=0.6, above]{$q^2$};
91 \draw[dashed] (XProj) -- (\XProjToeThreeOne, \XProjToeThreeTwo, \XProjToeThreeThree) node[pos=0.3, right, above]{$q^3$};
92
93 % Add labels to XY plane and R3
94 \node[cm={1,0,cos(35),sin(55),(0,0)}] at (-0.58*\Pd,0,0.75*\Pd){$\mathbb{R}^2$}; % | -0.55 and 1.1
95 \node at (1.2,0.8,0){$\mathbb{R}^3$};
96 %
97 \end{tikzpicture}
98 \caption{An object  $\mathbb{X}$  is shown to exist in the space occupied by two coordinate systems, which are arbitrarily rotated from one another. Their origins overlap and each coordinate system comprises a set of three orthogonal unit (orthonormal) vectors. Normally,  $\mathbf{\partial_{x^3}}$  will be shown to be oriented “upwards,” but is shown as such to emphasize that the visualization of the coordinate system leverages nothing in determining basis components.}
99 \end{figure}

```

```

1 \begin{equation}
2 \begin{pmatrix}\hat{\textcolor{red}{e}}_1 \\ \hat{\textcolor{red}{e}}_2 \\ \hat{\textcolor{red}{e}}_3\end{pmatrix} = \underbrace{\begin{pmatrix}1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta\end{pmatrix}}_{\textbf{R}_1} \begin{pmatrix}x^1 \\ x^2 \\ x^3\end{pmatrix}
3 \quad\quad\quad\textrm{tdplotsetmaincoords}{70}{200}
4 \begin{tikzpicture}[scale=2.75,baseline={{[yshift=-.5ex]current bounding box.center}},tdplot_main_coords]
5 
6 % Draw grid on plane
7 %\begin{scope}[canvas is xz plane at y=0]
8 %% Draw grid
9 %\draw[black!25,thin,step=2mm] (-1.2,0) grid (0.6,1.2);
10 %\end{scope}
11 
12 \draw[->,>=latex] (0,0,0) -- (-1,0,0) node[right]{$\partial_{x^2}$};
13 \draw[->,>=latex] (0,0,0) -- (0,1,0) node[left]{$\partial_{x^1}$} node[pos=0.9]{AxisRotator[scale=0.65,x
    =0.4cm,y=0.3cm,->,rotate=100]}];
14 \draw[->,>=latex] (0,0,0) -- (0,0,1) node[right]{$\partial_{x^3}$};
15 
16 \pgfmathsetmacro{\t}{30}
17 \tdplotsetrotatedcoords{0}{\t}{0}
18 \draw[->,>=latex,tdplot_rotated_coords] (0,0,0) -- (-1,0,0) node[right]{$\hat{\textcolor{red}{e}}_2$};
19 \draw[->,>=latex,tdplot_rotated_coords] (0,0,0) -- (0,1,0) node[right,xshift=2.5mm]{$\hat{\textcolor{red}{e}}_1$};
20 \draw[->,>=latex,tdplot_rotated_coords] (0,0,0) -- (0,0,1) node[anchor=north east]{$\hat{\textcolor{red}{e}}_3$};
21 
22 % Draw angle
23 \begin{scope}[canvas is xz plane at y=0]
24 \pgfmathsetmacro{\r}{0.5}
25 \draw[->,>=stealth] (-\r, 0) arc [start angle=0, end angle=-\t, radius=-\r] node[midway,right]{$\theta$};
26 \end{scope}
27 \end{tikzpicture}
28 \label{eq:ElementalRotx}
29 \end{equation}
30 ~
31 \begin{equation}
32 \begin{pmatrix}\hat{\textcolor{red}{e}}_1 \\ \hat{\textcolor{red}{e}}_2 \\ \hat{\textcolor{red}{e}}_3\end{pmatrix} = \underbrace{\begin{pmatrix}\cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1\end{pmatrix}}_{\textbf{R}_2} \begin{pmatrix}x^1 \\ x^2 \\ x^3\end{pmatrix}

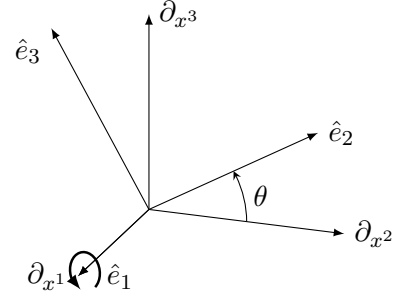
```

```

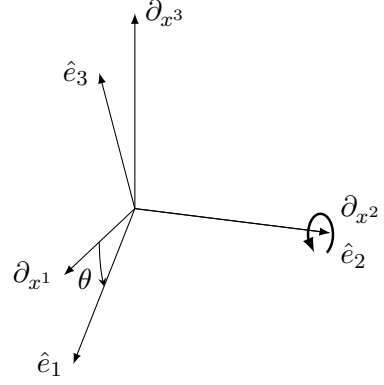
33 \pmatrix{\partial_{x^1} \ \ \ \partial_{x^2} \ \ \ \partial_{x^3}}\end{pmatrix}
34 \qqquad\qqquad\qqquad\tdplotsetmaincoords{70}{110}
35 \begin{tikzpicture}[scale=2.75,baseline=({[yshift=-.5ex]current bounding box.center}),tdplot_main_coords]
36 % Draw grid on plane
37 %\begin{scope}[canvas is xz plane at y=0]
38 %% Draw grid
39 %\draw[black!25,thin,step=2mm] (-1.2,0) grid (0.6,1.2);
40 %\end{scope}
41
42 \draw[->,>=latex] (0,0,0) -- (1,0,0) node[left]{\partial_{x^1}};
43 \draw[->,>=latex] (0,0,0) -- (0,1,0) node[anchor=south west]{\partial_{x^2}} node[pos=0.95]{\
AxisRotator[scale=0.55,x=0.5cm,y=0.3cm,->,rotate=90]};
44 \draw[->,>=latex] (0,0,0) -- (0,0,1) node[right]{\partial_{x^3}};
45
46 \pgfmathsetmacro{\t}{30}
47 \tdplotsetrotatedcoords{0}{\t}{0}
48 \draw[->,>=latex,tdplot_rotated_coords] (0,0,0) -- (1,0,0) node[left]{\hat{e}_1};
49 \draw[->,>=latex,tdplot_rotated_coords] (0,0,0) -- (0,1,0) node[anchor=north west]{\hat{e}_2};
50 \draw[->,>=latex,tdplot_rotated_coords] (0,0,0) -- (0,0,1) node[left]{\hat{e}_3};
51
52 % Draw angle
53 \begin{scope}[canvas is xz plane at y=0]
54 \pgfmathsetmacro{\r}{0.5}
55 \draw[->,>=stealth] (\r, 0) arc [start angle=0, end angle=-\t, radius=\r] node[pos=0.8,left]{\theta};
56 \end{scope}
57 \end{tikzpicture}
58 \label{eq:ElementalRoty}
59 \end{equation}
60 ~
61 \begin{equation}
62 \begin{pmatrix} \hat{e}_1 \ \ \ \hat{e}_2 \ \ \ \hat{e}_3 \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta \\ \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \end{pmatrix}}_{\begin{pmatrix} \partial_{x^1} \ \ \ \partial_{x^2} \ \ \ \partial_{x^3} \end{pmatrix}} \begin{pmatrix} \mathbf{R}_3 \end{pmatrix} \begin{pmatrix} \partial_{x^1} \ \ \ \partial_{x^2} \ \ \ \partial_{x^3} \end{pmatrix}
63 \qqquad\qqquad\qqquad\tdplotsetmaincoords{70}{110}
64 \begin{tikzpicture}[scale=2.75,baseline=({[yshift=-.5ex]current bounding box.center}),tdplot_main_coords]
65
66 % Draw grid on plane
67 %\begin{scope}[canvas is xz plane at y=0]
68 %% Draw grid
69 %\draw[black!25,thin,step=2mm] (-1.2,0) grid (0.6,1.2);
70 %\end{scope}
71
72 \draw[->,>=latex] (0,0,0) -- (1,0,0) node[left]{\partial_{x^1}};
73 \draw[->,>=latex] (0,0,0) -- (0,1,0) node[right]{\partial_{x^2}};
74 \draw[->,>=latex] (0,0,0) -- (0,0,1) node[left]{\partial_{x^3}} node[pos=0.75]{\AxisRotator[scale=0.56,x
=0.3cm,y=0.5cm,->,rotate=-90]};
75
76 \pgfmathsetmacro{\t}{30}
77 \tdplotsetrotatedcoords{0}{0}{\t}
78 \draw[->,>=latex,tdplot_rotated_coords] (0,0,0) -- (1,0,0) node[right]{\hat{e}_1};
79 \draw[->,>=latex,tdplot_rotated_coords] (0,0,0) -- (0,1,0) node[right]{\hat{e}_2};
80 \draw[->,>=latex,tdplot_rotated_coords] (0,0,0) -- (0,0,1) node[right]{\hat{e}_3};
81
82 % Draw angle
83 \begin{scope}[canvas is xy plane at z=0]
84 \pgfmathsetmacro{\r}{0.5}
85 \draw[->,>=stealth] (\r, 0) arc [start angle=0, end angle=\t, radius=\r] node[midway,below]{\theta};
86 \end{scope}
87 \end{tikzpicture}
88 \label{eq:ElementalRotz}
89 \end{equation}

```

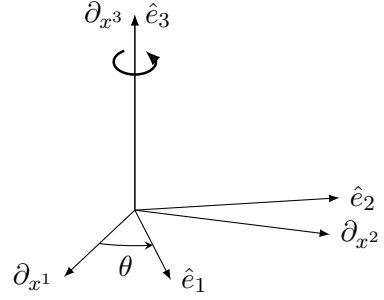
$$\begin{pmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{pmatrix}}_{\mathbf{R}_1} \begin{pmatrix} \partial_{x^1} \\ \partial_{x^2} \\ \partial_{x^3} \end{pmatrix} \quad (1)$$



$$\begin{pmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{pmatrix} = \underbrace{\begin{pmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{pmatrix}}_{\mathbf{R}_2} \begin{pmatrix} \partial_{x^1} \\ \partial_{x^2} \\ \partial_{x^3} \end{pmatrix} \quad (2)$$



$$\begin{pmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{pmatrix} = \underbrace{\begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\mathbf{R}_3} \begin{pmatrix} \partial_{x^1} \\ \partial_{x^2} \\ \partial_{x^3} \end{pmatrix} \quad (3)$$



3 Bad

3.1 Diagram of J2000 Frame

```

1 \begin{figure}[H]
2 \centering
3 \tdplotsetmaincoords{56}{110} % Use this default: 60, 120 | 70, 100 | 56, 140 | 56, 110
4 \begin{tikzpicture}[tdplot_main_coords,scale=4]
5 \pgfmathsetmacro{\thetaz}{35} % Rotation about z axis [deg]
6 \tdplotsetrotatedcoords{0}{0}{\thetaz} % Rotated coordinates (tdplot_rotated_coords)
7
8 %\begin{scope}[tdplot_main_coords, canvas is xy plane at z=0]
9 %\clip[draw] circle [radius=1.2cm];
10 %\fill[black!15,opacity=0.25] circle [radius=2cm];
11 %\draw[black!25,thin,step=0.05cm] (-2cm,-2cm) grid (2cm,2cm);
12 %\end{scope}
13 \pgfmathsetmacro{\Pd}{1.2}
14 \pgfmathsetmacro{\Ld}{1*\Pd}
15 \foreach \x in {-1,-0.8,...,1} {
16 \ifthenelse{\NOT -1 = \x}{\draw[black!25,thin] (\x,-\Ld,0) -- (\x,\Ld,0);}{}
17 \draw[black!25,thin] (-\Ld+0.2,\x,0) -- (\Ld,\x,0);
18 }
19
20 % Draw J2000 ECI axis
21 \draw[->, >=latex, thick] (0,0,0) -- (1,0,0) node[left]{\hat{I}} node[anchor=north west]{\vernal$};
22 \draw[->, >=latex, thick] (0,0,0) -- (0,1,0) node[anchor=bottom, right]{\hat{J}};
23 \draw[->, >=latex, thick] (0,0,0) -- (0,0,1) node[anchor=north east]{\hat{K}};
24 % Draw J2000 ECEF axis
25 \draw[->, >=latex, thick, tdplot_rotated_coords] (0,0,0) -- (1,0,0) node[anchor=west]{\hat{\imath}};
26 \node[tdplot_rotated_coords] at (1.2,0,0) {Greenwich, England};
27 \draw[->, >=latex, thick, tdplot_rotated_coords] (0,0,0) -- (0,1,0) node[anchor=bottom, right]{\hat{\jmath}};
28 \node[tdplot_rotated_coords] at (0.2,1,0) {Bay of Bengal};
29 \draw[->, >=latex, thick, tdplot_rotated_coords] (0,0,0) -- (0,0,1) node[anchor=north west]{\hat{k}} node
[pos=0.7]{\AxisRotator[x=0.18cm,y=0.4cm,->,rotate=-90]} node[pos=0.6, right]{\omega$};
30 % Draw angle of rotation
31 \tdplotdrawarc[->, >=stealth]{(0,0,0)}{0.6}{0}{\thetaz}{anchor=north}{\lambda_G$}
32
33 % Create wire-frame Earth in long and lat
34 %%\shade[shading=ball, ball color=black!0, opacity=0.95] (0,0,0) circle (0.18cm);
35 \pgfmathsetmacro{\sphereStep}{180/8} % Step in theta for wire frame
36 \pgfmathsetmacro{\R}{0.18} % Shere radius
37 \pgfmathsetmacro{\maxLong}{180-\sphereStep}
38 \foreach \sphereTheta in {0,\sphereStep,...,\maxLong} {
39 % Draw equal-latitude lines
40 \tdplotsetrotatedcoords{\sphereTheta}{90}{90};
41 \draw[solid,tdplot_rotated_coords,very thin] (\R,0,0) arc (0:360:\R);
42 }
43 % Reset rotated coordinates
44 \tdplotsetrotatedcoords{0}{0}{\thetaz} % Rotated coordinates (tdplot_rotated_coords)
45 % Do equal latitude lines
46 \pgfmathsetmacro{\negR}{-1*\R}
47 \pgfmathsetmacro{\sphereStep}{\negR+\R/4}
48 \foreach \h in {\negR,\sphereStep,...,\R} {
49 \pgfmathsetmacro{\r}{sqrt(\R*\R - \h*\h)}
50 \draw[solid,tdplot_rotated_coords,very thin] (\r, 0, \h) arc (0:360:\r);
51 }
52
53 % Set position for object X
54 \pgfmathsetmacro{\XOne}{0.36}
55 \pgfmathsetmacro{\XTwo}{0.86}
56 \pgfmathsetmacro{\XThree}{0.93}

```

```

57 \coordinate(X) at (\XOne, \XTwo, \XThree);
58 % Draw object X
59 \draw (X) node[circle, fill, inner sep=1pt] node[above, left]{ $\mathbf{\hat{X}}$ };
60 % Draw X projections onto main coordinate frame
61 \draw[dashed] (\XOne, \XTwo, 0) -- (\XOne, 0, 0) node[pos=0.5, above]{ $\mathbf{\hat{Y}}$ };
62 \draw[dashed] (\XOne, \XTwo, 0) -- (0, \XTwo, 0) node[pos=0.5, right]{ $\mathbf{\hat{X}}$ };
63 \draw[dashed] (\XOne, \XTwo, 0) -- (X) node[pos=0.8, left]{ $\mathbf{\hat{Z}}$ };
64 \draw[dashed] (\XOne, \XTwo, 0) -- (X) node[pos=0.8, right]{ $\mathbf{\hat{z}}$ };
65 % Draw X projections onto rotated coordinate frame
66 \tdplottransformmainrot{\XOne}{\XTwo}{\XThree}
67 \draw[dashed, tdplot_rotated_coords] (\tdplotresx, \tdplotresy, 0) -- (\tdplotresx, 0, 0) node[pos=0.45,
    below]{ $\mathbf{\hat{y}}$ };
68 \draw[dashed, tdplot_rotated_coords] (\tdplotresx, \tdplotresy, 0) -- (0, \tdplotresy, 0) node[pos=0.38, left
    ]{ $\mathbf{\hat{x}}$ };
69
70 % Add label to grid
71 \node[cm={1,0,\cos(55),\sin(75),(0,0)},rotate=-10.5] at ({(0.58-0.16)*\Pd},{(-0.71+0.15)*\Pd},0){J2000};
72 \node[cm={1,0,\cos(55),\sin(75),(0,0)},rotate=-10.5] at ({(0.76-0.16)*\Pd},{(-0.51+0.05)*\Pd},0){Equatorial
    Plane}; % | -0.51 and 0.76
73 \end{tikzpicture}
74 \caption{A graphical representation of the relation between J2000 ECI and J2000 ECEF coordinate systems.
    The earth is depicted as a spherical wire frame of rings corresponding to constant longitudes and latitudes
    --- the 6 unit vectors are exaggerated in length for the sake of illustration .}
75 \end{figure}

```

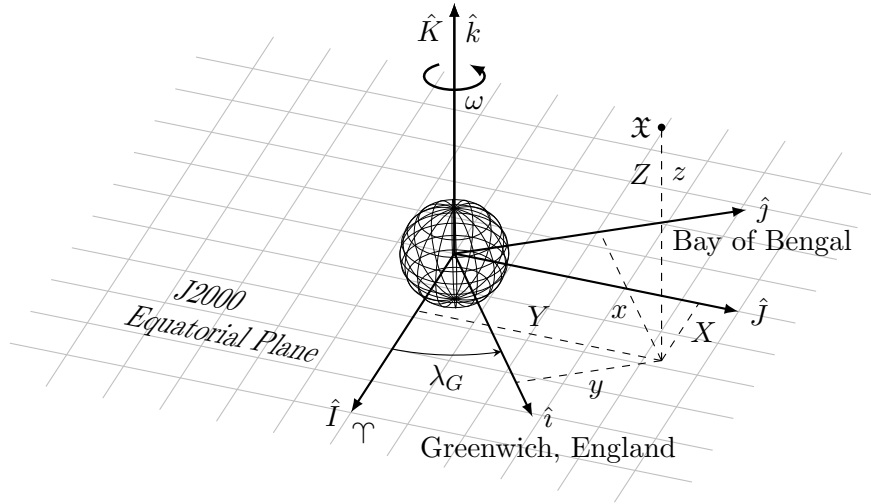


Figure 31: A graphical representation of the relation between J2000 ECI and J2000 ECEF coordinate systems. The earth is depicted as a spherical wire frame of rings corresponding to constant longitudes and latitudes — the 6 unit vectors are exaggerated in length for the sake of illustration.

3.2 Comparing Geocentric and Geodetic Latitude Varying with Height from Surface of Ellipsoid

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}
4 \definecolor{mycolor1}{rgb}{0.10000,0.80000,0.70000}%
5 \definecolor{mycolor2}{rgb}{0.10000,0.00000,0.80000}%
6 \begin{axis}[%
7 width=3.229in,
8 height=2.461in,
9 at={(0.542in,0.43in)},
10 scale only axis,
11 unbounded coords=jump,
12 xmin=-90,
13 xmax=90,
14 xtick={-90,-75,-60,-45,-30,-15,0,15,30,45,60,75,90},
15 xticklabels={{-90},{},{-60},{},{-30},{},{0},{},{30},{},{60},{},{90}},
16 xlabel style={font=\color{white!15!black}},
17 xlabel={Geodetic Latitude $\varphi$ [deg]},
18 ymin=-90,
19 ymax=90,
20 ytick={-90,-75,-60,-45,-30,-15,0,15,30,45,60,75,90},
21 yticklabels={{-90},{},{-60},{},{-30},{},{0},{},{30},{},{60},{},{90}},
22 ylabel style={font=\color{white!15!black}},
23 ylabel={Geocentric Latitude $\varphi_s$ [deg]},
24 axis background/.style={fill=white},
25 title style={font=\bfseries},
26 title={Geocentric and Geodetic Latitude},
27 xmajorgrids,
28 ymajorgrids,
29 legend style={at={(0.03,0.97)}, anchor=north west, legend cell align=left, align=left, draw=white!15!black}
30 ]
31 \addlegendimage{empty legend}
32 \addlegendentry{$h$}
33 \pgfmathsetmacro{\a}{1}
34 \pgfmathsetmacro{\h}{0}
35 \pgfmathsetmacro{\f}{0.5}
36 \addplot[domain=-90:90, samples=101, unbounded coords=jump, solid, color=red!40!orange]{atan(tan(x)
37 *((1-\f)^2 * \a/sqrt(1 - (2*\f - \f^2) * sin(x)^2) + \h)/(\a/sqrt(1 - (2*\f - \f^2) * sin(x)^2) + \h));
38 \pgfmathsetmacro{\h}{0.1}
39 \addplot[domain=-90:90, samples=101, unbounded coords=jump, densely dashed]{atan(tan(x)*((1-\f)^2 * \a/
40 sqrt(1 - (2*\f - \f^2) * sin(x)^2) + \h)/(\a/sqrt(1 - (2*\f - \f^2) * sin(x)^2) + \h));
41 \pgfmathsetmacro{\h}{0.5}
42 \addplot[domain=-90:90, samples=101, unbounded coords=jump, dashed]{atan(tan(x)*((1-\f)^2 * \a/sqrt(1 -
43 (2*\f - \f^2) * sin(x)^2) + \h)/(\a/sqrt(1 - (2*\f - \f^2) * sin(x)^2) + \h));
44 \pgfmathsetmacro{\h}{1}
45 \addplot[domain=-90:90, samples=101, unbounded coords=jump, loosely dashed]{atan(tan(x)*((1-\f)^2 * \a/
46 sqrt(1 - (2*\f - \f^2) * sin(x)^2) + \h)/(\a/sqrt(1 - (2*\f - \f^2) * sin(x)^2) + \h));
47 \end{axis}
48
49 \begin{axis}[%
50 width=4.167in,
51 height=3.125in,
52 at={(0in,0in)},
53 scale only axis,
54 xmin=0,
55 xmax=1,

```

```

56 ymin=0,
57 ymax=1,
58 axis line style={draw=none},
59 ticks=none,
60 axis x line*=bottom,
61 axis y line*=left,
62 legend style={legend cell align=left, align=left, draw=white!15!black}
63 ]
64 \end{axis}
65 \end{tikzpicture}%
66 \caption{test caption}
67 \label{fig:GeocentricGeodeticLats}
68 \end{figure}
69
70 %\begin{figure}[H]
71 \centering
72 \begin{tikzpicture}[scale=0.82]
73 \pgfmathsetmacro{\ellipseMaj}{85}
74 \pgfmathsetmacro{\ellipseMin}{60}
75
76 % Draw axes
77 \draw[>, >=stealth, thick] (0,0) -- (\ellipseMaj pt, 0) node[pos=0.85,below]{$a$} node[right]{$\hat{e}$-$\chi$};
78 \draw[>, >=stealth, thick] (0,0) -- (0, \ellipseMin pt) node[pos=0.8,left]{$b$} node[pos=1, above]{$\hat{k}$};
79
80 % Draw ellipse
81 \draw (0,0) ellipse [x radius=\ellipseMaj pt, y radius=\ellipseMin pt];
82 %\begin{scope}
83 % \clip (-1,0) rectangle (1,1);
84 % \draw[dashed] (0,0) circle [radius=\ellipseMaj];
85 % \draw (0,0) ellipse [x radius=\ellipseMaj, y radius=\ellipseMin];
86 %\end{scope}
87
88 % Draw lines
89 \pgfmathsetmacro{\circumB}{35}
90 \pgfmathsetmacro{\circY}{\ellipseMaj*sin(\circumB)}
91 \pgfmathsetmacro{\ellipX}{\ellipseMaj*cos(\circumB)}
92 \pgfmathsetmacro{\ellipY}{\ellipseMin*sqrt(1 - (\ellipX/\ellipseMaj)^2)}
93 \draw (0,0) -- (\ellipX pt, \ellipY pt) node[pos=0.7,above,xshift=-0.5mm]{$\rho_s$};
94
95 % Draw angle
96 \pgfmathsetmacro{\GeocentricAng}{atan(\ellipY/\ellipX)}
97 \pgfmathsetmacro{\rpos}{0.3*\ellipseMaj}
98 \draw[>, >=latex] (\rpos pt, 0) arc [start angle=0, end angle=\GeocentricAng, radius=\rpos pt] node[
midway,right]{$\varphi_s$};
99
100 %\draw[dashed] (0,0) -- (0,1) node[pos=0.7,solid]{$\backslash$AxisRotator[x=0.15cm,y=0.55cm,>, >=latex,rotate
=-90]};
101
102 \draw (\ellipX pt,\ellipY pt) node[circle, fill, inner sep=1,]{} node[anchor=south west,xshift=-0.5mm,
yshift=-0.5mm]{$\mathfrak{X}_s$};
103 \end{tikzpicture}
104 \caption{Geocentric and geodetic coordinates at a fixed longitude $\lambda$ both on and off of the spheroid's
surface.}
105 \end{figure}

```

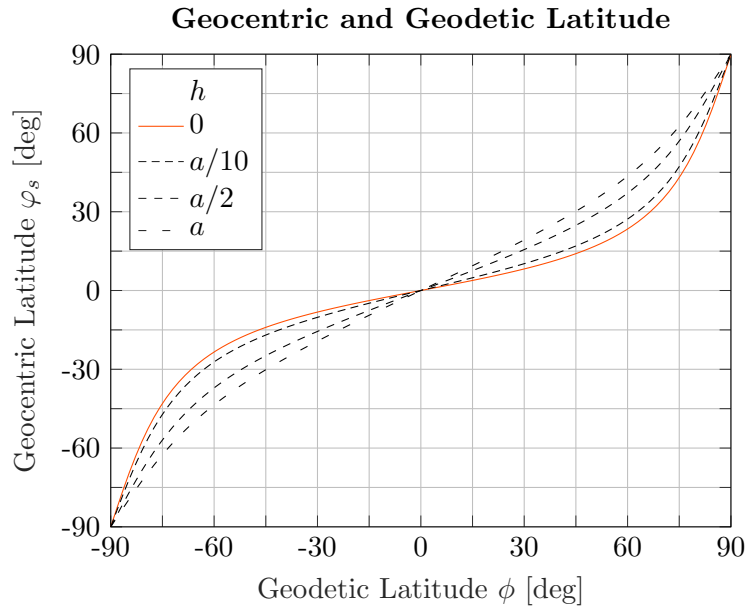


Figure 32: test caption

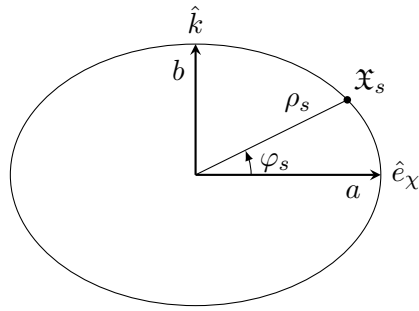


Figure 33: Geocentric and geodetic coordinates at a fixed longitude λ both on and off of the spheroid's surface.

3.3 Infinitesimal Element of Air to Derive Euler's Equation, Probably

```

1 \begin{figure}[H]
2 \centering
3 \tdplotsetmaincoords{70}{110}
4 \begin{tikzpicture}[tdplot_main_coords, scale = 4]
5 % Define pressure box side lengths
6 \pgfmathsetmacro{\s}{0.25}
7 % Draw axis
8 %\draw[dashed] (0,0,0) -- (\s,0,0);
9 %\draw[dashed] (0,0,0) -- (0,\s,0);
10 %\draw[dashed] (0,0,0) -- (0,0,\s);
11 %% Continue axes
12 %\draw[->,>=stealth,black] (\s,0,0) -- (2.5*\s,0,0) node[right]{$x$};
13 %\draw[->,>=stealth,black] (0,\s,0) -- (0,2.5*\s,0) node[right]{$y$};
14 %\draw[->,>=stealth,black] (0,0,\s) -- (0,0,2.5*\s) node[left]{$z$};
15
16 \draw[dashed] (-\s,-\s,-\s) -- (\s,-\s,-\s);
17 \draw[dashed] (-\s,-\s,-\s) -- (-\s,\s,-\s);
18 \draw[dashed] (-\s,-\s,-\s) -- (-\s,-\s,\s);
19 %
20 \draw[->,>=latex,black] (\s,-\s,-\s) -- (1.5*\s,-\s,-\s) node[anchor=north east]{$x$};
21 \draw[->,>=latex,black] (-\s,\s,-\s) -- (-\s,1.5*\s,-\s) node[right]{$y$};
22 \draw[->,>=latex,black] (-\s,-\s,\s) -- (-\s,-\s,1.5*\s) node[left]{$h$};
23
24 % Draw pressure box
25 % Front face
26 \draw (\s,-\s,-\s) -- (\s,\s,-\s) node[pos=0.4, below]{$dy$} -- (\s,\s,\s) -- (\s,-\s,\s) -- (\s,-\s,-\s)
    node[pos=0.5,left]{$dh$};
27 % Top face
28 \draw (\s,-\s,\s) -- (-\s,-\s,\s) -- (-\s,\s,\s) -- (\s,\s,\s);
29 % Right face
30 \draw (\s,\s,-\s) -- (-\s,\s,-\s) node[pos=0.4,right]{$dx$} -- (-\s,\s,\s);
31 %% Bottom face
32 %\draw[dashed] (\s,-\s,-\s) -- (-\s,-\s,-\s) -- (-\s,\s,-\s) -- (\s,\s,-\s);
33 %% Back edge
34 %\draw[dashed] (-\s,-\s,-\s) -- (-\s,-\s,\s);
35
36 % Draw pressure
37 \pgfmathsetmacro{\tmpx}{-1.5*\s}
38 \draw (0,0,{-(1+0.95)*\s}) -- (0,0,\tmpx) node[pos=0.05,right] {$p$};
39 \draw[->,>=stealth,densely dashed] (0,0,\tmpx) -- (0,0,-\s);
40 \draw[->,>=stealth] (0,0,{(1+0.75)*\s}) -- (0,0,\s) node[pos=0.015,right] {$p + dp$};
41 % Draw gravity
42 \draw[->,>=stealth,densely dashed] (0,0,0) -- (0,0,-\s/2.2) node[pos=0.5,right]{$g$};
43
44 %\draw (0,0) .. controls (-2*\s,1*\s) and (2*\s,2*\s) .. (0,3*\s) node[right]{$\rho$};
45 %\draw[dashed] (0,0) .. controls (-1*\s,0.5*\s) and (1*\s,1*\s) .. (0,1.5*\s) node[right]{$\rho$};
46 \node[left] at (0,0,0) {$\rho$};
47
48 \end{tikzpicture}
49 \caption{An infinitesimal element of air with density $\rho$ in which pressure has a differential change on
    the altitude—faces and gravity acts on the center of mass.}
50 \end{figure}

```

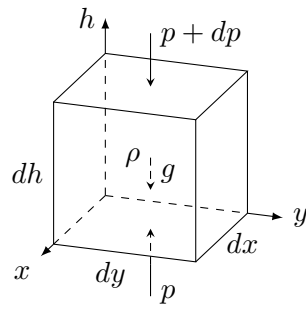


Figure 34: An infinitesimal element of air with density ρ in which pressure has a differential change on the altitude-faces and gravity acts on the center of mass.

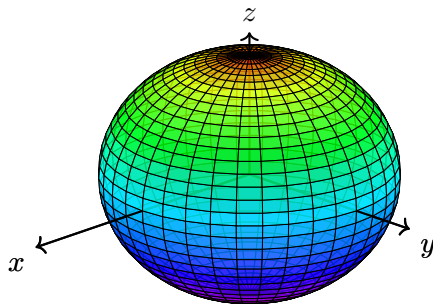
3.4 Colored Ellipsoid in r

4 Terrible (Graveyard)

```

1 \begin{figure}[H]
2 \newcommand{\asa}{1}
3 \newcommand{\bsa}{1}
4 \newcommand{\csa}{0.7}
5 % view angle
6 \tdplotsetmaincoords{70}{135}
7 %
8 \begin{tikzpicture}[scale=2,tdplot_main_coords,line join=bevel,fill opacity=.8]
9   \pgfsetlinewidth{.1pt}
10  \tdplotsphericalsurfaceplot [ parametricfill ] {72}{36}%
11    {1/sqrt((sin(\tdplottheta))^2*(cos(\tdplotphi))^2/\asa+
12      (sin(\tdplottheta))^2*(sin(\tdplotphi))^2/\bsa + (cos(\tdplottheta))^2/\csa)} % function defining
      radius
13    {black} % line color
14    {2*\tdplottheta} % fill
15    {\draw[color=black,thick,->] (0,0,0) -- (2,0,0) node[anchor=north east]{$x$};}% x-axis
16    {\draw[color=black,thick,->] (0,0,0) -- (0,1.5,0) node[anchor=north west]{$y$};}% y-axis
17    {\draw[color=black,thick,->] (0,0,0) -- (0,0,1) node[anchor=south]{$z$};}% z-axis
18 \end{tikzpicture}
19 \end{figure}

```



4.1 J2000 ECI and J2000 ECEF bases - the view is slightly scuffed just enough to matter and have to start over

```

1 \begin{figure}[H]
2 \centering
3 \tdplotsetmaincoords{70}{110} %Use this in the future with [tdplot_main_coords]
4 \begin{tikzpicture}[scale=4]
5 % Set viewing angle (?) Not sure if this does anything
6 \tdplotsetrotatedcoords{0}{0}{-9}
7
8 % Set rotation angles for 213 sequence
9 \pgfmathsetmacro{\phiOne}{0} % | 0
10 \pgfmathsetmacro{\phiTwo}{30} % | 45
11 \pgfmathsetmacro{\phiThree}{0} % | 0
12
13 % Set resulting basis components
14 \pgfmathsetmacro{\eOneOne}{cos(\phiTwo)*cos(\phiThree)}
15 \pgfmathsetmacro{\eOneTwo}{sin(\phiThree)}
16 \pgfmathsetmacro{\eOneThree}{-cos(\phiThree)*sin(\phiTwo)}
17 %
18 \pgfmathsetmacro{\eTwoOne}{sin(\phiOne)*sin(\phiTwo) - cos(\phiOne)*cos(\phiTwo)*sin(\phiThree)}
19 \pgfmathsetmacro{\eTwoTwo}{cos(\phiOne)*cos(\phiThree)}
20 \pgfmathsetmacro{\eTwoThree}{cos(\phiTwo)*sin(\phiOne) + cos(\phiOne)*sin(\phiTwo)*sin(\phiThree)}
21 %
22 \pgfmathsetmacro{\eThreeOne}{cos(\phiOne)*sin(\phiTwo) + cos(\phiTwo)*sin(\phiOne)*sin(\phiThree)}
23 \pgfmathsetmacro{\eThreeTwo}{-cos(\phiThree)*sin(\phiOne)}
24 \pgfmathsetmacro{\eThreeThree}{cos(\phiOne)*cos(\phiTwo) - sin(\phiOne)*sin(\phiTwo)*sin(\phiThree)}
25
26 % Set position for object X
27 \pgfmathsetmacro{\XOne}{0.9} % | 0.34
28 \pgfmathsetmacro{\XTwo}{0.93} % | 0.6
29 \pgfmathsetmacro{\XThree}{0.46} % | 0.9
30
31 % Draw grid in the XY plane
32 \begin{scope}[canvas is xz plane at y=0]
33 \fill [black!15,opacity=0.25] circle [radius=1.2];
34 \clip [draw] circle [radius=1.2];
35 \draw[black!25,thin] [step=0.2] (-10,-10) grid (10,10);
36 \end{scope}
37 \pgfmathsetmacro{\Pd}{1.2}
38 \pgfmathsetmacro{\Ld}{1*\Pd}
39 %\fill [black!15,opacity=0.25] (\Pd,0,\Pd) -- (\Pd,0,-\Pd) -- (-\Pd+0.2,0,-\Pd) -- (-\Pd+0.2,0,\Pd)
40 %\foreach \x in {-1,-0.8,...,1} {
41 % \ifthenelse {\NOT -1 = \x}{\draw[black!25,thin] (\x,0,-\Ld) -- (\x,0,\Ld);}{ }
42 % \draw[black!25,thin] (-\Ld+0.2,0,\x) -- (\Ld,0,\x);
43 %}
44
45 % Create coordinates for object X
46 \coordinate (X) at (\XOne,\XTwo,\XThree);
47
48 % Draw axes
49 \draw[->, >=latex, thick] (0,0,0) -- (1,0,0) node[right]{\hat{jmath}}; % y
50 \draw[->, >=latex, thick] (0,0,0) -- (0,1,0) node[below left]{\hat{k}}; % z
51 \draw[->, >=latex, thick] (0,0,0) -- (0,0,1) node[left]{\hat{i}} node[right]{\ \vernal}; % x
52 %
53 \draw[->, >=latex, thick] (0,0,0) -- (\eOneOne,\eOneTwo,\eOneThree) node[right]{\hat{J}};
54 \draw[->, >=latex, thick] (0,0,0) -- (\eTwoOne,\eTwoTwo,\eTwoThree) node[below right]{\hat{K}};
55 \draw[->, >=latex, thick] (0,0,0) -- (\eThreeOne,\eThreeTwo,\eThreeThree) node[right]{\hat{I}};
56
57 % Draw object X
58 \draw (X) node[circle, fill, inner sep=1pt] node[above, left]{\mathfrak{X}};

```

```

59 |
60 | %% Add dashed vertical lines of rotated axes to the XY plane
61 | %\draw[dashed] (\eOneOne,0,\eOneThree) -- (\eOneOne,\eOneTwo,\eOneThree);
62 | %\draw[dashed] (\eTwoOne,0,\eTwoThree) -- (\eTwoOne,\eTwoTwo,\eTwoThree);
63 | %\draw[dashed] (\eThreeOne,0,\eThreeThree) -- (\eThreeOne,\eThreeTwo,\eThreeThree);
64 |
65 | % Add dashed lines from X to the XY plane
66 | \draw[dashed] (\XOne,0,\XThree) -- (\XOne,\XTwo,\XThree) node[pos=0.8, left]{$z$};
67 | %\draw[dashed] (0,0,0) -- (\XOne, 0, \XThree);
68 | \draw[dashed] (\XOne, 0, \XThree) -- (0, 0, \XThree) node[midway, above]{$y$};
69 | \draw[dashed] (\XOne, 0, \XThree) -- (\XOne, 0, 0) node[midway, right]{$x$};
70 |
71 | % Add dashed lines from X to the q2q3 plane
72 | % ~
73 | % Define its coordinate
74 | \coordinate (XProj) at (\XOne, 0, \XThree);
75 |
76 | \pgfmathsetmacro{\ECEFOne}{\XThree*cos(\phiTwo)+\XOne*sin(\phiTwo)};
77 | \pgfmathsetmacro{\ECEFThree}{-\XThree*sin(\phiTwo)+\XOne*cos(\phiTwo)};
78 | % Draw
79 | \draw[dashed] (XProj) -- (\XOne, \XTwo, \XThree) node[pos=0.8, right]{$Z$};
80 | \draw[dashed] (XProj) -- (\ECEFOne*\eThreeOne, 0, \ECEFOne*\eThreeThree) node[pos=0.5, below]{$Y$};
81 | \draw[dashed] (XProj) -- (\ECEFThree*\eOneOne, 0, \ECEFThree*\eOneThree) node[pos=0.3, left]{$X$};
82 |
83 |
84 | % Add labels to XY plane and R3
85 | \node[cm={1,0,cos(35),sin(55),(0,0)}] at ({(-0.71+0.15)*\Pd},0,{(0.58-0.16)*\Pd}){J2000};
86 | \node[cm={1,0,cos(35),sin(55),(0,0)}] at ({(-0.51+0.05)*\Pd},0,{(0.76-0.16)*\Pd}){Equatorial Plane}; % |
87 | -0.51 and 0.76
88 |
89 | % Draw angle arc between i and I axis (3-axis)
90 | \tdplotdrawarc[->,tdplot_rotated_coords]{{(0,0,0)}{0.72}{0}{\phiTwo-1.6}{anchor=north}}{$\lambda_G$}
91 |
92 | % Try to draw a (spherical) Earth at origin
93 | \shade[ball color=blue!40, opacity=0.4](0,0) circle (0.15);
94 | \end{tikzpicture}
95 | \caption{An object $W$ is shown to exist and be expressed in the J2000 (stationary basis $\{\hat{\imath}, \hat{\jmath}, \hat{k}\}$) and ECEF (rotating basis $\{\hat{I}, \hat{J}, \hat{K}\}$) coordinate systems}
96 | \end{figure}

```

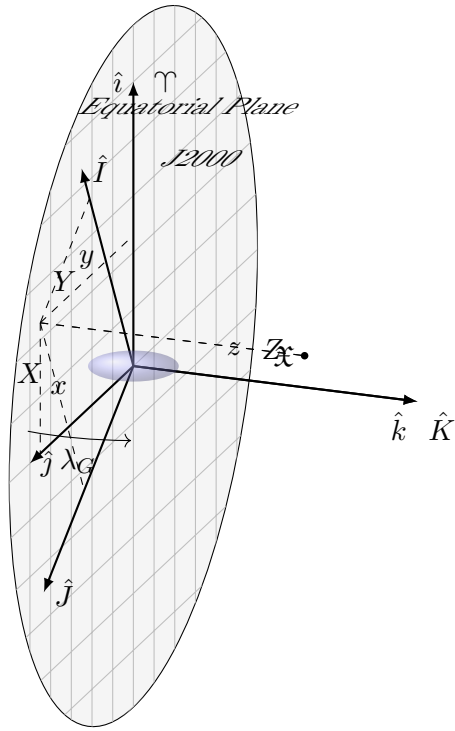


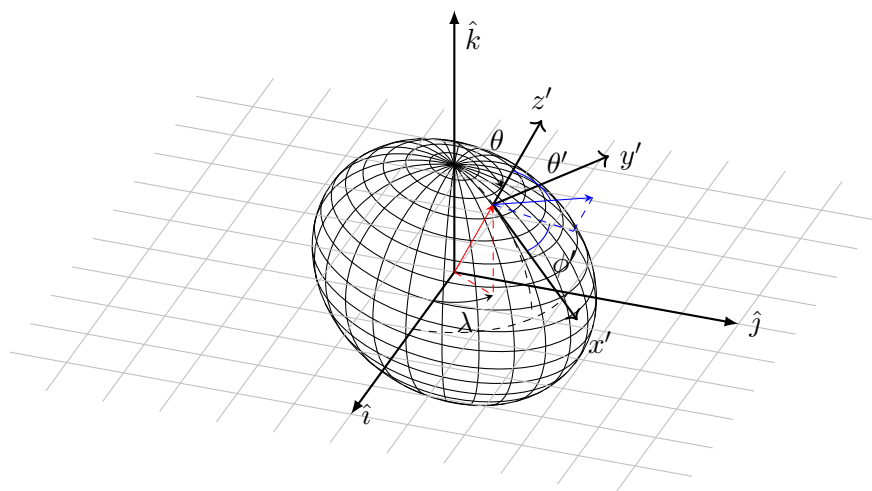
Figure 35: An object W is shown to exist and be expressed in the J2000 (stationary basis $\{\hat{i}, \hat{j}, \hat{k}\}$) and ECEF (rotating basis $\{\hat{I}, \hat{J}, \hat{K}\}$) coordinate systems.

4.2 (failure) J2000 ECEF and SPHR bases - Couldn't figure out how to get 3d projections of both sphere and axes

```

1 \begin{figure}[H]
2 \centering
3 \pgfmathsetmacro{\viewO}{60}
4 \pgfmathsetmacro{\viewT}{110}
5 \tdplotsetmaincoords{\viewO}{\viewT} % 50, 135 | 60, 75 gives the same view of ECEF wrt ECI
6 %
7 \pgfmathsetmacro{\rvec}{.4}
8 \pgfmathsetmacro{\thetavec}{30}
9 \pgfmathsetmacro{\phivec}{60}
10 %
11 \begin{tikzpicture}[tdplot_main_coords, scale=4]
12 \begin{scope}[canvas is yz plane at x=0]
13 \pgfmathsetmacro{\R}{0.5} % sphere radius
14 \pgfmathsetmacro{\angEl}{35} % elevation angle
15 \draw (0,0) circle (\R);
16 \foreach \t in {-80,-70,...,80} { \DrawLatitudeCircle[\R]{\t} }
17 \foreach \t in {-5,-20,...,-180} { \DrawLongitudeCircle[\R]{\t} }
18 \end{scope}
19
20 % Draw coordinate grid in xy plane
21 \pgfmathsetmacro{\Pd}{1.2}
22 \pgfmathsetmacro{\Ld}{1*\Pd}
23 \foreach \x in {-1,-0.8,...,1} {
24 \ifthenelse{\NOT -1 = \x}{\draw[black!25,thin] (\x,-\Ld,0) -- (\x,\Ld,0);}{}
25 \draw[black!25,thin] (-\Ld+0.2,\x,0) -- (\Ld,\x,0);
26 }
27
28 \draw[->, >=latex, thick] (0,0,0) -- (1,0,0) node[right]{$\hat{\imath}$};
29 \draw[->, >=latex, thick] (0,0,0) -- (0,1,0) node[anchor=bottom, right]{$\hat{\jmath}$};
30 \draw[->, >=latex, thick] (0,0,0) -- (0,0,1) node[anchor=north west]{$\hat{k}$};
31 \tdplotsetcoord{P}{\rvec}{\thetavec}{\phivec}
32 \draw[-stealth,color=red] (0,0,0) -- (P);
33 \draw[dashed, color=red] (0,0,0) -- (Pxy);
34 \draw[dashed, color=red] (P) -- (Pxy);
35 \tdplotdrawarc[->, >=stealth]{(0,0,0)}{0.2}{0}{\phivec}{anchor=north}{$\lambda$}
36 \tdplotsetthetaplanecoords{\phivec}
37 \tdplotdrawarc[->, >=stealth, tdplot_rotated_coords]{(0,0,0)}{0.5}{0}{\thetavec}{anchor=south west}{$\theta$}
38 \draw[dashed,tdplot_rotated_coords] (\rvec,0,0) arc (0:90:\rvec);
39 \draw[dashed] (\rvec,0,0) arc (0:90:\rvec);
40 \tdplotsetrotatedcoords{\phivec}{\thetavec}{0}
41 \tdplotsetrotatedcoordsorigin{(P)}
42 \draw[thick,tdplot_rotated_coords,->] (0,0,0) -- (.5,0,0) node[anchor=north west]{$x'$};
43 \draw[thick,tdplot_rotated_coords,->] (0,0,0) -- (0,.5,0) node[anchor=west]{$y'$};
44 \draw[thick,tdplot_rotated_coords,->] (0,0,0) -- (0,0,.5) node[anchor=south]{$z'$};
45 \draw[-stealth,color=blue,tdplot_rotated_coords] (0,0,0) -- (.2,.2,.2);
46 \draw[dashed,color=blue,tdplot_rotated_coords] (0,0,0) -- (.2,.2,0);
47 \draw[dashed,color=blue,tdplot_rotated_coords] (.2,.2,0) -- (.2,.2,.2);
48 \tdplotdrawarc[tdplot_rotated_coords,color=blue]{(0,0,0)}{0.2}{0}{45}{anchor=north west,color=black}{$\phi$}
49 \tdplotsetrotatedthetaplanecoords{45}
50 \tdplotdrawarc[tdplot_rotated_coords,color=blue]{(0,0,0)}{0.2}{0}{55}{anchor=south west,color=black}{$\theta$}
51 \end{tikzpicture}
52 \end{figure}

```

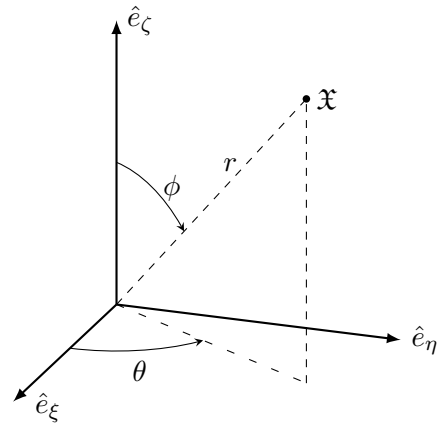
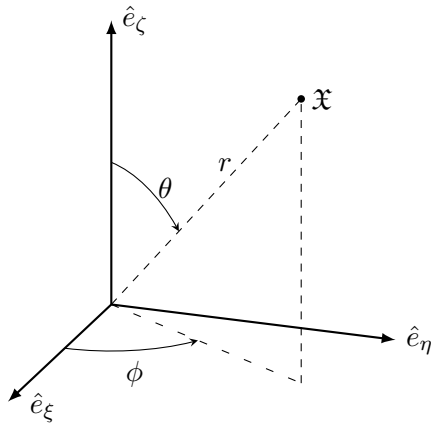


4.3 Physics vs Math SPHR coords. - Didn't like - settled on words

```

1 \begin{center}
2 \tdplotsetmaincoords{70}{110}
3 \pgfmithsetmacro{\ax}{0.5}
4 \pgfmithsetmacro{\ay}{0.85}
5 \pgfmithsetmacro{\az}{1}
6 \pgfmithsetmacro{r}{sqrt(\ax*\ax + \ay*\ay + \az*\az)}
7 \pgfmithsetmacro{\SPHRtheta}{acos(\az/r)}
8 \pgfmithsetmacro{\SPHRLambda}{atan(\ay/\ax)}
9 \begin{tikzpicture}[scale=4,tdplot_main_coords]
10 \draw[->, >=latex, thick] (0,0,0) -- (1,0,0) node[anchor=west, xshift=1.5mm, yshift=-0.75mm]{$\hat{e}_x$};
11 \draw[->, >=latex, thick] (0,0,0) -- (0,1,0) node[anchor=west]{$\hat{e}_y$};
12 \draw[->, >=latex, thick] (0,0,0) -- (0,0,1) node[anchor=west]{$\hat{e}_z$};
13 \draw[dashed] (0,0,0) -- (\ax,\ay,\az) node[pos=0.6,above]{$r$} node[right]{$\mathbf{X}$};
14 \draw[] (\ax,\ay,\az) node[circle, fill, inner sep=1pt]{};
15 \draw[loosely dashed] (0,0,0) -- (\ax,\ay,0);
16 \draw[dashed] (\ax,\ay,0) -- (\ax,\ay,\az);
17 \tdplotdrawarc[->, >=stealth]{(0,0,0)}{0.45}{0}{\SPHRLambda}{below}{$\phi$}
18 \tdplotgetpolarcoords{\ax}{\ay}{\az}
19 \tdplotsetthetaplanecoords{\tdplotresphi}
20 \tdplotdrawarc[->, >=stealth,tdplot_rotated_coords]{(0,0,0)}{0.5}{0}{\tdplotrestheta}{anchor=west}{$\theta$}
21 \end{tikzpicture}
22 \hspace{20ex}
23 \begin{tikzpicture}[scale=4,tdplot_main_coords]
24 \draw[->, >=latex, thick] (0,0,0) -- (1,0,0) node[anchor=west, xshift=1.5mm, yshift=-0.75mm]{$\hat{e}_x$};
25 \draw[->, >=latex, thick] (0,0,0) -- (0,1,0) node[anchor=west]{$\hat{e}_y$};
26 \draw[->, >=latex, thick] (0,0,0) -- (0,0,1) node[anchor=west]{$\hat{e}_z$};
27 \draw[dashed] (0,0,0) -- (\ax,\ay,\az) node[pos=0.6,above]{$r$} node[right]{$\mathbf{X}$};
28 \draw[] (\ax,\ay,\az) node[circle, fill, inner sep=1pt]{};
29 \draw[loosely dashed] (0,0,0) -- (\ax,\ay,0);
30 \draw[dashed] (\ax,\ay,0) -- (\ax,\ay,\az);
31 \tdplotdrawarc[->, >=stealth]{(0,0,0)}{0.45}{0}{\SPHRLambda}{below}{$\phi$}
32 \tdplotgetpolarcoords{\ax}{\ay}{\az}
33 \tdplotsetthetaplanecoords{\tdplotresphi}
34 \tdplotdrawarc[->, >=stealth,tdplot_rotated_coords]{(0,0,0)}{0.5}{0}{\tdplotrestheta}{anchor=west}{$\theta$}
35 \end{tikzpicture}
36 \end{center}

```

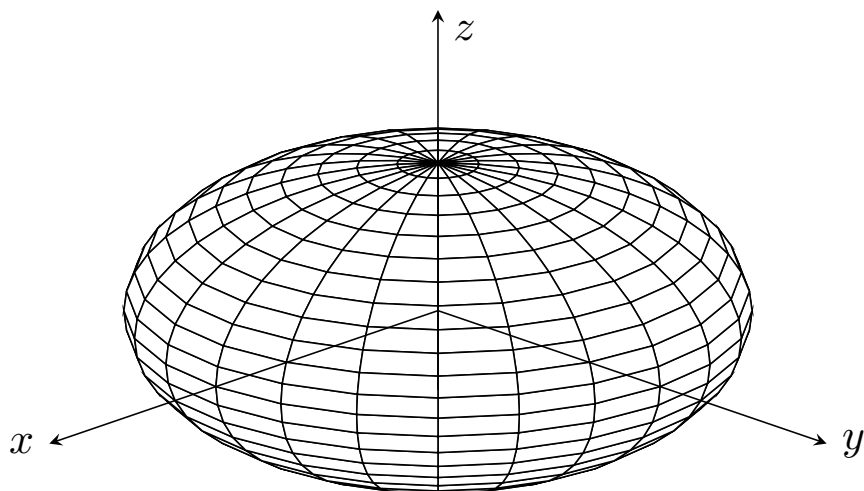


4.4 Kind of failed attempt at 3D ellipse using pgfplots

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}[scale=1.5]
4   \begin{axis}[%
5     width=0.8\textwidth,
6     axis equal,
7     axis lines = center,
8     x label style={at={(axis cs:1.75,0,0)},anchor=east},
9     y label style={at={(axis cs:0,1.75,0)},anchor=west},
10    z label style={at={(axis cs:0,0,0.95)},anchor=west},
11    xlabel = {$x$},
12    ylabel = {$y$},
13    zlabel = {$z$},
14    xmin=0,
15    ymin=0,
16    zmin=0,
17    xmax=1.75,
18    ymax=1.75,
19    zmax=0.75,
20    ticks=none,
21    colormap={}{ gray(0cm)=(1); gray(1cm)=(1);},
22    view/h=135,
23    view/v=20,
24    axis on top
25  ]
26  %% \begin{axis}[hide axis,colormap={}{ gray(0cm)=(1); gray(1cm)=(1);}]
27  % \addplot3[ fill opacity=0.7,surf, domain=0:2*pi, y domain=0:pi,z buffer=sort,faceted color=black] ({1*
28    cos(deg(x))*sin(deg(y))}, {1*sin(deg(x))*sin(deg(y))}, {0.5*cos(deg(y))});
29  % \end{axis}
30  %\begin{axis}[hide axis,colormap={}{ gray(0cm)=(1); gray(1cm)=(1);}]
31  %\addplot3[ fill opacity=1,surf, domain=0:2*pi, y domain=0:pi,z buffer=sort,faceted color=black] ({1*cos
32    (deg(x))*sin(deg(y))}, {1*sin(deg(x))*sin(deg(y))}, {0.5*cos(deg(y))});
33  \end{axis}
34 \end{tikzpicture}
35 \end{figure}

```



4.5 2D ellipse - didn't like it

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}[scale=3]
4 \pgfmathsetmacro{\a}{65} %65
5 \pgfmathsetmacro{\b}{50} % 10
6 \pgfmathsetmacro{\e}{sqrt(1 - (\b / \a)^2)}
7 \pgfmathsetmacro{\DrawAngStart}{20} %20
8 \pgfmathsetmacro{\DrawAngEnd}{180-\DrawAngStart}
9 \pgfmathsetmacro{\DrawAngStart}{0}
10 \pgfmathsetmacro{\DrawAngEnd}{360}
11
12 % Calculate initial starting position with reference to (0,0)
13 \pgfmathsetmacro{\DrawStartX}{\a * cos(\DrawAngStart)}
14 \pgfmathsetmacro{\DrawStartY}{\b * sin(\DrawAngStart)}
15
16 % Draw ellipse and Earth surface
17 \draw (\DrawStartX pt, \DrawStartY pt) arc (\DrawAngStart:\DrawAngEnd:\a pt and \b pt);
18 %\draw[decorate, decoration={coil, segment length=35pt, amplitude=10pt, raise=0pt, aspect=0.8, pre=curveto,
19 %    post=curveto, post length = 0cm, pre length = 1cm}, color=green!40!blue!40!black!60] (\DrawStartX pt, \
20 %    DrawStartY pt) arc (\DrawAngStart:\DrawAngStart+60:\a pt and \b pt); % random steps, 25, 15
21 %\pgfmathsetmacro{\newStartAng}{180-\DrawAngStart-60}
22 %\pgfmathsetmacro{\newStartx}{\a * cos(\newStartAng)}
23 %\pgfmathsetmacro{\newStarty}{\b * sin(\newStartAng)}
24 %\draw[decorate, decoration={random steps, segment length=30pt, amplitude=15pt, raise=-0.5pt, aspect=0,
25 %    pre=curveto, post=curveto, post length = 0.8cm, pre length = 0cm}, color=green!40!blue!40!black!60] (\
26 %    newStartx pt, \newStarty pt) arc (\newStartAng:\DrawAngEnd:\a pt and \b pt); % random steps, 25, 15
27 \draw[decorate, decoration={random steps, segment length=25pt, amplitude=10pt, raise=0pt, aspect=0.8, pre=
28 %    curveto, post=curveto, post length = 0cm, pre length = 0cm}, color=green!40!blue!40!black!60] (\
29 %    DrawStartX pt, \DrawStartY pt) arc (\DrawAngStart:\DrawAngEnd:\a pt and \b pt); % random steps,
30 %    25, 15
31 % Draw north pole line
32 \draw[dashed] (0,\b pt) -- (0, \b+10 pt) node[above]{N};
33
34 % Calculate position of Xs
35 \pgfmathsetmacro{\geocentricLat}{49.8}%50.75
36 \pgfmathsetmacro{\Xsx}{\a * cos(\geocentricLat)}
37 \pgfmathsetmacro{\Xsy}{\b * sin(\geocentricLat)}
38
39 % Calculate position of X
40 \pgfmathsetmacro{\h}{0} % Set height
41 \pgfmathsetmacro{\geodeticLat}{atan((\a / \b) * tan(\geocentricLat))}
42 \pgfmathsetmacro{\ell}{\a^2 / sqrt(\a^2 * cos(\geodeticLat)^2 + \b^2 * sin(\geodeticLat)^2)}
43 \pgfmathsetmacro{\Xx}{(\ell + \h) * cos(\geodeticLat)}
44 \pgfmathsetmacro{\Xy}{((1 - \e^2) * \ell + \h) * sin(\geodeticLat)}
45
46 % Set position of X'
47 \pgfmathsetmacro{\Xpx}{\Xx + 0.3}
48 \pgfmathsetmacro{\Xpy}{\Xy + 0.2}
49
50 % Draw object Xs
51 \draw (\Xsx pt, \Xsy pt) node[circle, fill, inner sep=1,]{} node[below]{$\mathfrak{X}_{.s}$};
52 % Draw object X
53 \draw (\Xx pt, \Xy pt) node[circle, fill, inner sep=1,]{} node[anchor = south west]{$\mathfrak{X}$};
54 \draw[dashed] (\Xsx pt, \Xsy pt) -- (\Xx pt, \Xy pt) node[pos=0.5, left]{$h$};
55
56 % Draw axes in cross section
57 \draw[>, >=stealth] (\Xx pt, \Xy pt) -- (\Xx+1.1 pt, \Xy+10.2 pt) node[pos=1, above]{$\hat{e}_h$};
58 \draw[>, >=stealth] (\Xx pt, \Xy pt) -- (\Xx-10.2 pt, \Xy+1.1 pt) node[pos=1, left]{$\hat{e}_{-\phi}$};
59 \draw (\Xx pt, \Xy pt) node[cross, scale=5, rotate=-10]{} node[right, xshift=0.25mm, yshift=-1.5mm]{$\hat{e}_{-\lambda da}$};
60
61 % Check coordinates - all temp below
62 \draw[dashed] (-\a pt, 0) -- (\a pt, 0);

```

```

54 \draw (\Xx pt, \Xy-10 pt) node[circle, fill, inner sep=1,]{} node[below]{$\phi = \text{\geodeticLat}$};
55 \pgfmathsetmacro{\testX}{\Xsx}
56 \pgfmathsetmacro{\testY}{\b * sqrt(1 - (\testX / \a)^2)}
57 \pgfmathsetmacro{\lineX}{\Xsx}
58 \pgfmathsetmacro{\lineY}{\testY - (\a / \b)^2 * \testY * (\lineX - \testX) / \testX}
59 \pgfmathsetmacro{\lineYzero}{\testY - (\a / \b)^2 * \testY}
60 \draw[dashed] (0, \lineYzero pt) -- (\lineX pt, \lineY pt);
61 \draw[dashed] (0, \lineYzero pt) -- (\testX pt, \testY pt);
62 \draw (0, \lineYzero pt) node[yshift=-5mm]{\lineYzero};
63 %%%tmp
64 \pgfmathsetmacro{\tangeodeticLat}{tan(\geodeticLat)}
65 \pgfmathsetmacro{\dxdy}{(\a / \b)^2 * \Xsy / \Xsx}
66 \draw (0, \lineYzero pt) node[yshift=-10mm]{\tangeodeticLat};
67 \draw (0, \lineYzero pt) node[yshift=-15mm]{\dxdy};
68 \pgfmathsetmacro{\tangeocentricLatabsquared}{(\a / \b)^1 * tan(\geocentricLat)}
69 \draw (0, \lineYzero pt) node[yshift=-20mm]{\tangeocentricLatabsquared};
70 \end{tikzpicture}
71 \caption{Example words example words example words example words example words example words example
       words example words example words example words example words example words.}
72 \end{figure}

```

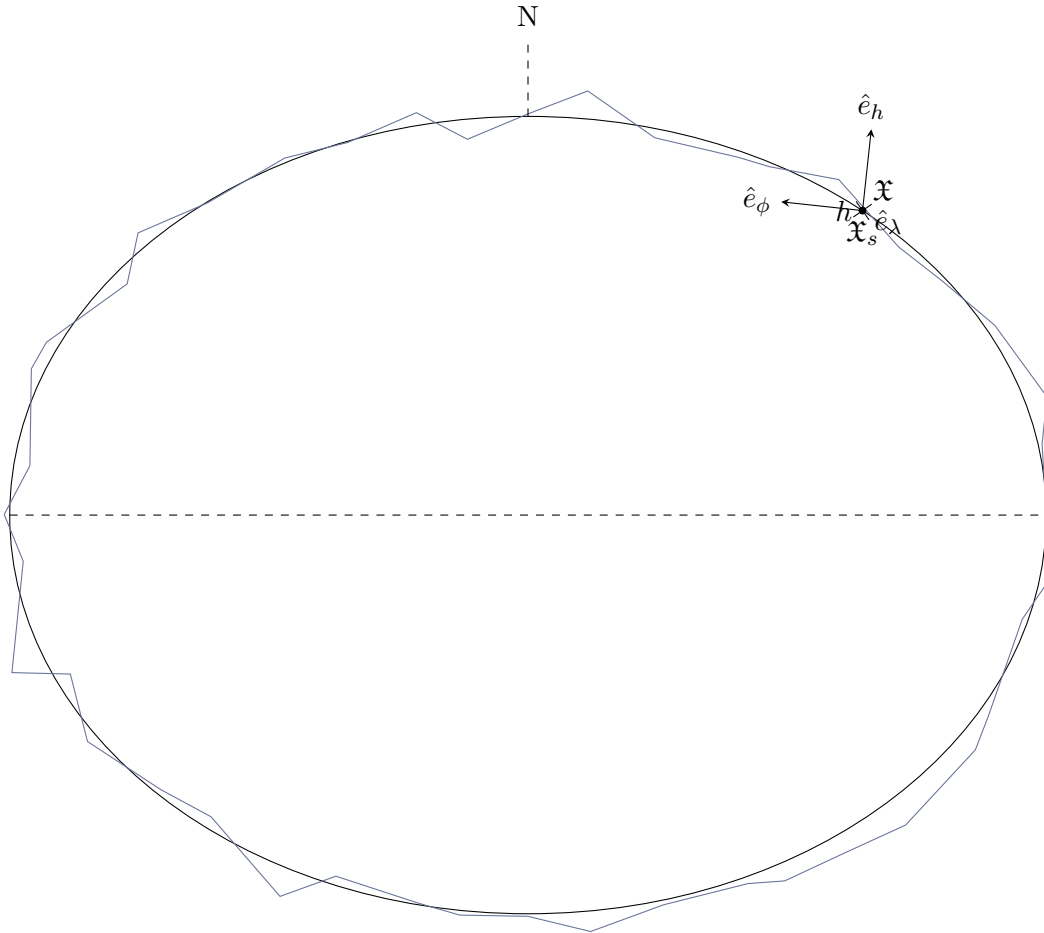


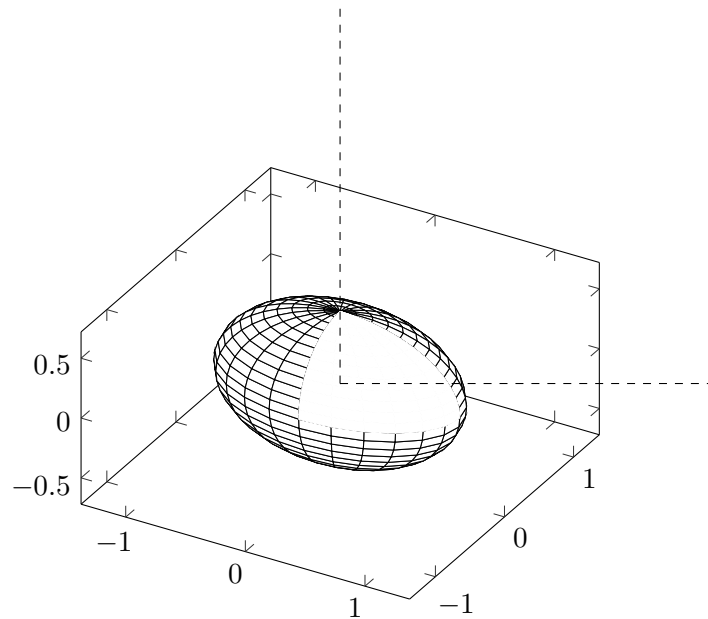
Figure 36: Example words example words example words example words example words example words example words example words example words example words example words example words.

4.6 Playing around with pgfplots for 3D ellipse - too slow and not general enough

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}[scale=1]
4 \tdplotsetmaincoords{0}{0} % 60, 110 --> shift = (1, 4, 4.4)
5 \begin{axis}[view={30}{30}, axis equal]
6 \pgfmathsetmacro{\a}{1}
7 \pgfmathsetmacro{\b}{\a}
8 \pgfmathsetmacro{\b}{0.6}
9 \pgfmathsetmacro{\minLongitude}{0}
10 \pgfmathsetmacro{\maxLongitude}{270}
11 \pgfmathsetmacro{\minGeodeticLat}{0}
12 \pgfmathsetmacro{\maxGeodeticLat}{180}
13 \addplot3[surf, color=black!0, opacity=1, faceted color=black, samples=25, domain=0:180, y domain=\minLongitude:360, z buffer = sort]({\a*sin(x)*cos(y)}, {\b*sin(x)*sin(y)}, {\b*cos(x)});
14 \addplot3[surf, color=black!0, opacity=1, faceted color=black!0, samples=25, domain=0:90, y domain=270:360, z buffer = sort]({\a*sin(x)*cos(y)}, {\b*sin(x)*sin(y)}, {\b*cos(x)});
15 \end{axis}
16 \tdplotsetrotatedcoords{0}{0}{0}
17 \begin{scope}[tdplot_rotated_coords, shift={(3.425,2.85,0)}]
18 \draw[dashed] (0,0,0) -- (5,0,0);
19 \draw[dashed] (0,0,0) -- (0,5,0);
20 \draw[dashed] (0,0,0) -- (0,0,5);
21 \end{scope}
22 \end{tikzpicture}
23 \end{figure}

```



4.7 Playing around with matlab2tikz

```

1 \begin{tikzpicture}
2 \definecolor{mycolor1}{rgb}{0.00000,0.44700,0.74100}%
3 \definecolor{mycolor2}{rgb}{0.85000,0.32500,0.09800}%
4 \definecolor{mycolor3}{rgb}{0.92900,0.69400,0.12500}%
5 \definecolor{mycolor4}{rgb}{0.49400,0.18400,0.55600}%
6 \begin{axis}[%
7 width=3.31in,
8 height=2.886in,
9 at={(0.555in,0.428in)},
10 scale only axis,
11 xmin=-90,
12 xmax=90,
13 xtick={-90, -60, -30, 0, 30, 60, 90},
14 xlabel style={font=\color{white!15!black}},
15 xlabel={Reduced Latitude $\beta$ [deg]},
16 ymin=-90,
17 ymax=90,
18 ytick={-90, -60, -30, 0, 30, 60, 90},
19 ylabel style={font=\color{white!15!black}},
20 ylabel={Geocentric Latitude $\varphi_s$ [deg]},
21 axis background/.style={fill=white},
22 title style={font=\bfseries},
23 title={Latitudes of Points on the Spheroid},
24 xmajorgrids,
25 ymajorgrids,
26 legend style={at={(0.16,0.661)}, anchor=south west, legend cell align=left, align=left, draw=white!15!black}
27 ]
28 \addplot [color=mycolor1]
29 table[row sep=crcr]{%
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31 -88.1818181818182 -88.1818181818182\\
32 -86.3636363636364 -86.3636363636364\\
33 -84.5454545454545 -84.5454545454545\\
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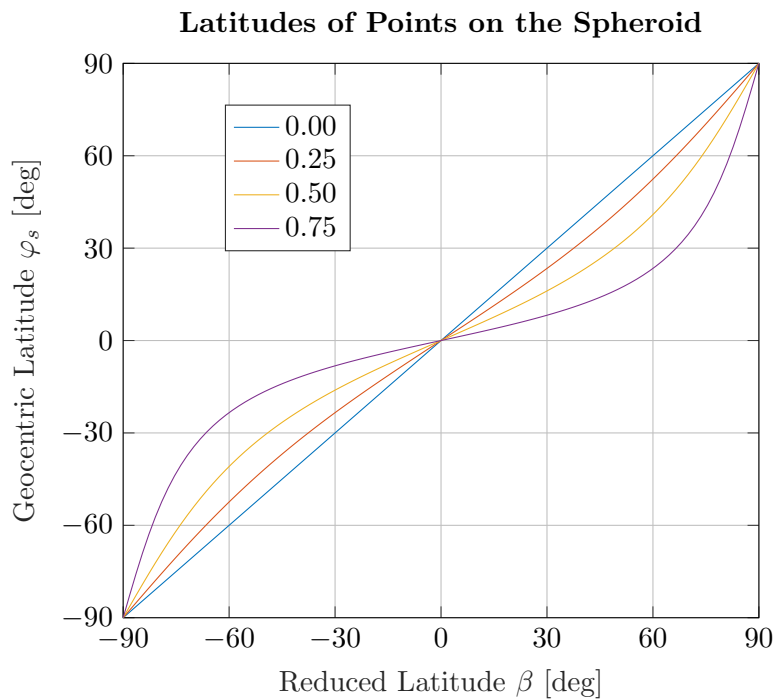
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380 -26.3636363636364 -7.06314271747983\\
381 -24.5454545454545 -6.51332363137864\\
382 -22.7272727272727 -5.97806115307392\\
383 -20.9090909090909 -5.45583704278187\\
384 -19.0909090909091 -4.94524883244532\\
385 -17.2727272727273 -4.44499440381682\\
386 -15.4545454545455 -3.95385839181483\\
387 -13.6363636363636 -3.47070011878603\\
388 -11.8181818181818 -2.99444281010783\\
389 -10 -2.5240638780266\\
390 -8.18181818181819 -2.05858609023242\\
391 -6.36363636363636 -1.59706946359634\\
392 -4.54545454545455 -1.13860374267482\\
393 -2.72727272727273 -0.682301337747689\\
394 -0.909090909090907 -0.227290608871468\\
395 0.909090909090907 0.227290608871468\\
396 2.72727272727273 0.682301337747689\\
397 4.54545454545455 1.13860374267482\\
398 6.36363636363636 1.59706946359634\\
399 8.18181818181819 2.05858609023242\\
400 10 2.5240638780266\\
401 11.8181818181818 2.99444281010783\\
402 13.6363636363636 3.47070011878603\\
403 15.4545454545455 3.95385839181483\\
404 17.2727272727273 4.44499440381682\\
405 19.0909090909091 4.94524883244532\\
406 20.9090909090909 5.45583704278187\\
407 22.7272727272727 5.97806115307392\\
408 24.5454545454545 6.51332363137864\\
409 26.3636363636364 7.06314271747983\\
410 28.1818181818182 7.62917001944639\\
411 30 8.21321070173819\\
412 31.8181818181818 8.81724676476177\\
413 33.6363636363636 9.4434640178922\\
414 35.4545454545455 10.0942834737724\\
415 37.2727272727273 10.7723980468535\\
416 39.0909090909091 11.48081563067\\
417 40.9090909090909 12.2229098648573\\
418 42.7272727272727 13.002480194835\\
419 44.5454545454545 13.8238231867252\\
420 46.3636363636364 14.6918175015794\\
421 48.1818181818182 15.6120254715742\\
422 50 16.5908148710897\\
423 51.8181818181818 17.6355052476766\\
424 53.6363636363636 18.7545440700683\\
425 55.4545454545455 19.9577189345131\\
426 57.2727272727273 21.2564130658582\\
427 59.0909090909091 22.6639121740725\\
428 60.9090909090909 24.195771011661\\
429 62.7272727272727 25.8702470176554\\
430 64.5454545454545 27.7088049319964\\
431 66.3636363636364 29.7366879059489\\
432 68.1818181818182 31.9835334212028\\
433 70 34.4839796200599\\
434 71.8181818181818 37.2781490247475\\
435 73.6363636363636 40.4117972764444\\
436 75.4545454545455 43.935757313024\\
437 77.2727272727273 47.9040849723235\\
438 79.0909090909091 52.3700475464057\\
439 80.9090909090909 57.3789171403246\\

```

440 82.7272727272727 62.9567476419973\\
441 84.5454545454545 69.0954586718637\\
442 86.3636363636364 75.7370918758521\\
443 88.1818181818182 82.7635553502856\\
444 90 90\\
445 };
446 \addlegendentry{$0.75$}
447
448 \end{axis}
449
450 \begin{axis}[%
451 width=4.271in,
452 height=3.583in,
453 at={(0in,0in)},
454 scale only axis,
455 xmin=0,
456 xmax=1,
457 ymin=0,
458 ymax=1,
459 axis line style={draw=none},
460 ticks=none,
461 axis x line*=bottom,
462 axis y line*=left,
463 legend style={legend cell align=left, align=left, draw=white!15!black}
464 ]
465 \end{axis}
466 \end{tikzpicture}%

```

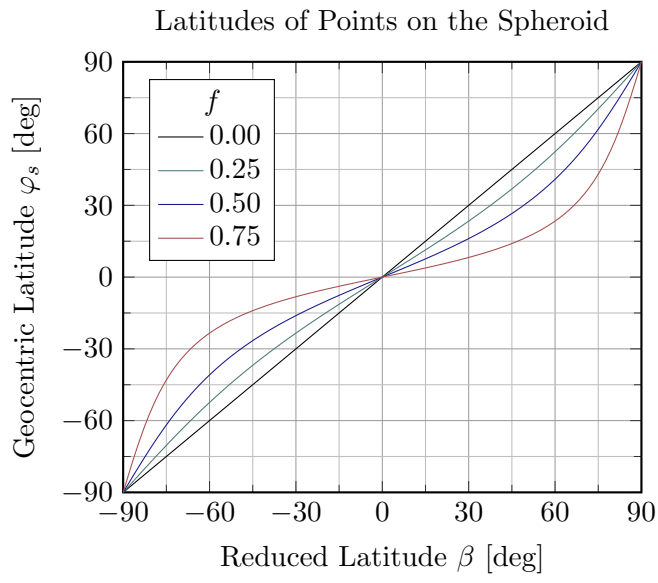


4.8 Replaced this custom tikz with modified output from matlab2tikz

```

1 \begin{figure}[H]
2 \begin{tikzpicture}
3 \begin{axis}[
4     title=Latitudes of Points on the Spheroid,
5     xlabel={Reduced Latitude $\beta$ [deg]},
6     ylabel={Geocentric Latitude $\varphi_s$ [deg]},
7     xmin=-90, xmax=90,
8     ymin=-90, ymax=90,
9     xtick={-90,-60,...,90},
10    ytick={-90,-60,...,90},
11    grid=both,
12    major grid style={line width=0.2pt,draw=gray!50},
13    minor tick num=1,
14    axis line style={latex-latex, thick},
15    legend pos=north west,
16    % legend style={draw=none},
17    legend style={at={(0.05, 0.96)}}
18 ]
19 \addlegendimage{empty legend}
20
21 % Add axes at x=0, y=0
22 %\draw[thin] (axis cs:\pgfkeysvalueof{/pgfplots/xmin},0) -- (axis cs:\pgfkeysvalueof{/pgfplots/xmax},0);
23 %\draw[thin] (axis cs:0,\pgfkeysvalueof{/pgfplots/ymin}) -- (axis cs:0,\pgfkeysvalueof{/pgfplots/ymax});
24
25 % Draw the 4 curves --- x = reduced latitude
26 \addplot[domain=-90:90, samples=101, unbounded coords=jump]{atan((1/1)*tan(x))};
27 \addplot[domain=-90:90, samples=101, unbounded coords=jump, green!50!black!70]{atan((0.75/1)*tan(x))};
28 \addplot[domain=-90:90, samples=101, unbounded coords=jump, blue!50!black!90]{atan((0.5/1)*tan(x))};
29 \addplot[domain=-90:90, samples=101, unbounded coords=jump, red!50!black!70]{atan((0.25/1)*tan(x))};
30
31 % Add legend
32 %\legend{0, 0.25, 0.5, 0.75} % Flattening values (not customizable)
33 \addlegendentry{\hspace{-0.6cm}$f$}
34 \addlegendentry{$0.00$}
35 \addlegendentry{$0.25$}
36 \addlegendentry{$0.50$}
37 \addlegendentry{$0.75$}
38 \end{axis}
39 \end{tikzpicture}
40 \end{figure}

```



4.9 Decided to put relation of geocentric/geodetic angle in instead - this plot doesn't tell really much

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}
4 \begin{axis}[
5     title=Geocentric and Geodetic Radius,
6     xlabel={Latitude ( $\varphi_s$  and  $\phi$ ) [deg]},
7     ylabel={ $\rho_s/a$   $q$   $q$   $\ell/a$ },
8     xmin=-90, xmax=90,
9     ymin=0, ymax=2.1,
10    xtick={-90,-60,...,90},
11    ytick={0,0.2,...,2.1},
12    % xticklabel={
13    %     \ifdim \tick pt < 0 pt
14    %         % If yes
15    %         \pgfmathparse{abs(\tick)}%
16    %         % and print
17    %         \llap{$-{\}}\pgfmathprintnumber{\pgfmathresult}
18    %     \else
19    %         % if no, print as usual
20    %         \pgfmathprintnumber{\tick}
21    %     \fi
22    % },
23    grid=both,
24    major grid style={line width=0.2pt,draw=gray!50},
25    minor tick num=1,
26    axis line style={latex-latex, thick},
27    legend pos=north west,
28    % legend style={draw=none},
29    legend style={at={(0.225, 0.375)}}
30 ]
31 \addlegendimage{empty legend}
32 \addlegendimage{empty legend}
33
34 % Add axes at x=0, y=0
35 %\draw[thin] (axis cs:\pgfkeysvalueof{/pgfplots/xmin},0) -- (axis cs:\pgfkeysvalueof{/pgfplots/xmax},0);
36 %\draw[thin] (axis cs:0,\pgfkeysvalueof{/pgfplots/ymin}) -- (axis cs:0,\pgfkeysvalueof{/pgfplots/ymax});
37
38 % Draw the 4 curves --- x = reduced latitude
39 % \pgfmathsetmacro{\f}{0}
40 % \addplot[domain=-90:90, samples=101, unbounded coords=jump]{sqrt((cos(x)^2 + (1 - \f)^4 * sin(x)^2) /
41 %     (cos(x)^2 + (1 - \f)^2 * sin(x)^2));
42 % \pgfmathsetmacro{\f}{0.25}
43 % \addplot[domain=-90:90, samples=101, unbounded coords=jump, dashed, green!50!black!70]{sqrt((cos(x)^2 +
44 %     (1 - \f)^4 * sin(x)^2) / (cos(x)^2 + (1 - \f)^2 * sin(x)^2));
45 % \pgfmathsetmacro{\f}{0.5}
46 % \addplot[domain=-90:90, samples=101, unbounded coords=jump, dashed, blue!50!black!90]{sqrt((cos(x)^2 +
47 %     (1 - \f)^4 * sin(x)^2) / (cos(x)^2 + (1 - \f)^2 * sin(x)^2));
48 % \pgfmathsetmacro{\f}{0.75}
49 % \addplot[domain=-90:90, samples=101, unbounded coords=jump]{sqrt((cos(x)^2 + (1 - \f)^4 * sin(x)^2) /
50 %     (cos(x)^2 + (1 - \f)^2 * sin(x)^2));
51 % \addplot[domain=-90:90, samples=2, unbounded coords=jump]{1};
52
53 % % Add legend
54 % \legend{0, 0.25, 0.5, 0.75} % Flattening values (not customizable)
55 \addlegendentry{\hspace{-0.6cm}Geodetic Latitude  $\phi$ }

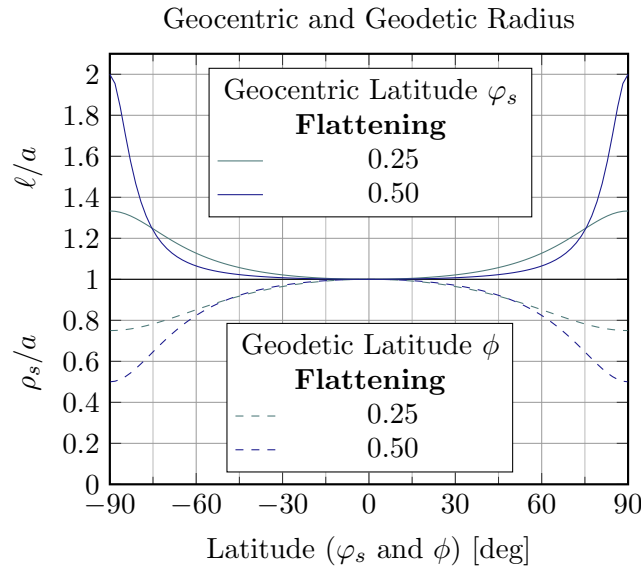
```



```

54 \addlegendentry{\hspace{-0.6cm}\textbf{Flattening}}
55 % \addlegendentry{\$0.00\$}
56 \addlegendentry{\$0.25\$}
57 \addlegendentry{\$0.50\$}
58 % \addlegendentry{\$0.75\$}
59 \end{axis}
60 \begin{axis}[
61     xmin=-90, xmax=90,
62     ymin=0, ymax=2.1,
63     legend pos=south west,
64     ticks=none,
65 %     legend style={draw=none},
66     legend style={at={(0.1905, 0.62)}}
67 ]
68 \addlegendimage{empty legend}
69 \addlegendimage{empty legend}
70 \pgfmathsetmacro{\f}{0.25}
71 \addplot[domain=-90:90, samples=101, unbounded coords=jump, solid, green!50!black!70]{sqrt((cos(x)^2 +
72     (1 - \f)^4 * sin(x)^2) / (cos(x)^2 + (1 - \f)^6 * sin(x)^2))};
73 \pgfmathsetmacro{\f}{0.5}
74 \addplot[domain=-90:90, samples=101, unbounded coords=jump, solid, blue!50!black!90]{sqrt((cos(x)^2 + (1
75     - \f)^4 * sin(x)^2) / (cos(x)^2 + (1 - \f)^6 * sin(x)^2))};
76 \addlegendentry{\hspace{-0.6cm}Geocentric Latitude $\varphi_s$}
77 \addlegendentry{\hspace{-0.6cm}\textbf{Flattening}}
78 % \addlegendentry{\$0.00\$}
79 \addlegendentry{\$0.25\$}
80 \addlegendentry{\$0.50\$}
81 % \addlegendentry{\$0.75\$}
82 \end{axis}
83 \end{tikzpicture}
84 \end{figure}

```



4.10 This is what replaced the last one but this was taken out and its details were filled into a custom matlab2tikz axis settings made from before

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}
4 \begin{axis}[
5     title=Geocentric and Geodetic Latitude,
6     xlabel={Geocentric Latitude $\varphi_s$ [deg]},
7     ylabel={Geodetic Latitude $\phi$ [deg]},
8     xmin=-90, xmax=90,
9     ymin=-90, ymax=90,
10    xtick={-90,-60,...,90},
11    ytick={-90,-60,...,90},
12    % xticklabel={
13    %     \ifdim \tick pt < 0 pt
14    %         % If yes
15    %         \pgfmathparse{abs(\tick)}%
16    %         % and print
17    %         \llap{$-{\}}\pgfmathprintnumber{\pgfmathresult}
18    %     \else
19    %         % if no, print as usual
20    %         \pgfmathprintnumber{\tick}
21    %     \fi
22    % },
23    grid=both,
24    major grid style={line width=0.2pt,draw=gray!50},
25    minor tick num=1,
26    axis line style={latex-latex, thick},
27    legend pos=north west,
28    % legend style={draw=none},
29    legend style={at={(0.225, 0.375)}}
30 ]
31
32 % Add axes at x=0, y=0
33 %\draw[thin] (axis cs:\pgfkeysvalueof{/pgfplots/xmin},0) -- (axis cs:\pgfkeysvalueof{/pgfplots/xmax},0);
34 %\draw[thin] (axis cs:0,\pgfkeysvalueof{/pgfplots/ymin}) -- (axis cs:0,\pgfkeysvalueof{/pgfplots/ymax});
35
36 % Draw the curve
37 \addplot[domain=-90:90, samples=101, unbounded coords=jump]{atan(tan(x)/((1-0)^2));
38 \addplot[domain=-90:90, samples=101, unbounded coords=jump, green!50!black!70]{atan(tan(x)/((1-0.25)
39 ^2));
40 \addplot[domain=-90:90, samples=101, unbounded coords=jump, blue!50!black!90]{atan(tan(x)/((1-0.5)^2)
41 };
42 \addplot[domain=-90:90, samples=101, unbounded coords=jump, red!50!black!70]{atan(tan(x)/((1-0.75)^2)
43 };
44
45 %% % Add legend
46 %% %\legend{0, 0.25, 0.5, 0.75} % Flattening values (not customizable)
47 % \addlegendentry{\hspace{-0.6cm}Geodetic Latitude $\phi$}
48 % \addlegendentry{\hspace{-0.6cm}\textbf{Flattening}}
49 % \addlegendentry{$0.00$}
50 % \addlegendentry{$0.25$}
51 % \addlegendentry{$0.50$}
52 % \addlegendentry{$0.75$}
53 \end{axis}
54 \end{tikzpicture}
55 \caption{Relation between the reduced and geocentric latitude over spheroids of flattening $f$}
56 \label{fig:geocentricdeteticLats}
57 \end{figure}

```

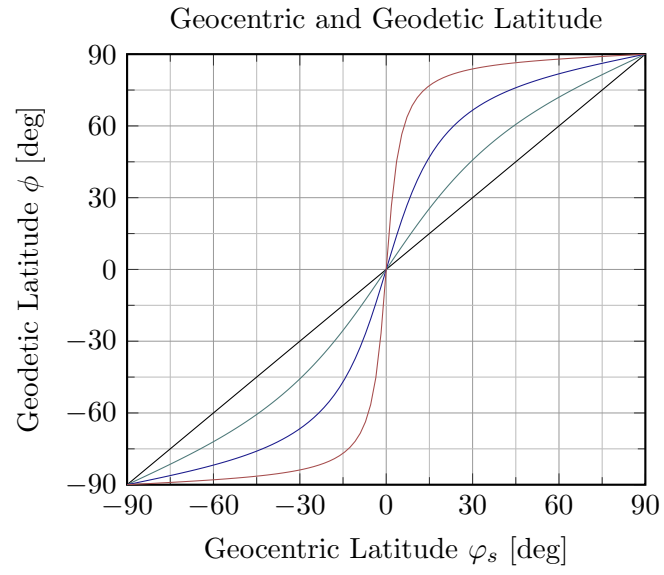


Figure 37: Relation between the reduced and geocentric latitude over spheroids of flattening f .

4.11 Subplots (2,1)

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}
4 \definecolor{mycolor1}{rgb}{0.00000,0.44700,0.74100}%
5 \begin{axis}[%
6 width=3.31in,
7 height=0.956in,
8 at={(0.555in,1.894in)},
9 scale only axis,
10 xmin=-1,
11 xmax=1,
12 xtick={ -1, -0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8, 1},
13 xlabel style={font=\color{white!15!black}},
14 xlabel={Sample data $x = \textcolor{red}{\phi}$ [deg]},
15 ymin=-0.5,
16 ymax=1,
17 ylabel style={font=\color{white!15!black}},
18 ylabel={Sample data $y = \textcolor{red}{\ell}$ [L]},
19 axis background/.style={fill=white},
20 title style={font=\bfseries},
21 title={Test Title},
22 xmajorgrids,
23 xminorgrids,
24 ymajorgrids,
25 yminorgrids,
26 legend style={legend cell align=left, align=left, draw=white!15!black}
27 ]
28 \addplot [color=mycolor1]
29 table[row sep=crcr]{%
30 -1 -0.126275409952829\\
31 -0.97979797979798 -0.16961634666975\\
32 -0.95959595959596 -0.206776323739601\\
33 -0.939393939393939 -0.237990089607263\\
34 -0.919191919191919 -0.263536121297353\\
35 -0.898989898989899 -0.283729740720497\\
36 -0.878787878787879 -0.298916209886257\\
37 -0.858585858585859 -0.309463923551148\\
38 -0.838383838383838 -0.3157578071431\\
39 -0.818181818181818 -0.318193015554922\\
40 -0.797979797979798 -0.317169015005677\\
41 -0.777777777777778 -0.31308411604653\\
42 -0.757575757575758 -0.306330511339174\\
43 -0.737373737373737 -0.297289857438873\\
44 -0.717171717171717 -0.286329425815665\\
45 -0.696969696969697 -0.273798835050538\\
46 -0.676767676767677 -0.260027363806808\\
47 -0.656565656565657 -0.24532183300912\\
48 -0.636363636363636 -0.229965035821282\\
49 -0.616161616161616 -0.214214685606553\\
50 -0.595959595959596 -0.198302845138425\\
51 -0.575757575757576 -0.18243579491897\\
52 -0.555555555555556 -0.166794294527354\\
53 -0.535353535353535 -0.151534188398503\\
54 -0.515151515151515 -0.136787306227036\\
55 -0.494949494949495 -0.122662608185573\\
56 -0.474747474747475 -0.109247526203197\\
57 -0.454545454545455 -0.096609454520618\\
58 -0.434343434343434 -0.0847973454688326\\
59 -0.414141414141414 -0.0738433697519661\\
60 -0.393939393939394 -0.0637646043000602\\
61 -0.373737373737374 -0.0545647148486293\\

```

62 -0.353535353535353 -0.0462356046640295\\
63 -0.333333333333333 -0.0387590051449078\\
64 -0.313131313131313 -0.0321079882824689\\
65 -0.292929292929293 -0.0262483850634747\\
66 -0.272727272727273 -0.0211400977728038\\
67 -0.252525252525252 -0.016738297735237\\
68 -0.232323232323232 -0.0129945032813788\\
69 -0.212121212121212 -0.00985753559556462\\
70 -0.191919191919192 -0.00727435258037154\\
71 -0.171717171717172 -0.00519076293716438\\
72 -0.151515151515151 -0.00355202430366575\\
73 -0.131313131313131 -0.00230333049561469\\
74 -0.111111111111111 -0.00139019364853383\\
75 -0.0909090909090909 -0.0007587273010494\\
76 -0.0707070707070707 -0.000355836100792252\\
77 -0.0505050505050505 -0.000129316616819239\\
78 -0.0303030303030303 -2.78711495070658e-05\\
79 -0.0101010101010101 -1.03085176178584e-06\\
80 0.0101010101010102 1.03085177502495e-06\\
81 0.0303030303030303 2.78711784967036e-05\\
82 0.0505050505050506 0.000129317654399262\\
83 0.0707070707070707 0.000355847064574649\\
84 0.0909090909090908 0.000758791150467142\\
85 0.111111111111111 0.00139045455763947\\
86 0.131313131313131 0.00230417321456293\\
87 0.151515151515152 0.00355432618231649\\
88 0.171717171717172 0.00519630852364105\\
89 0.191919191919192 0.00728647008744538\\
90 0.212121212121212 0.00988202366239757\\
91 0.232323232323232 0.0130409251228202\\
92 0.252525252525253 0.0168217277047785\\
93 0.272727272727273 0.0212834122747217\\
94 0.292929292929293 0.0264851953121158\\
95 0.313131313131313 0.0324863162762148\\
96 0.333333333333333 0.0393458060368238\\
97 0.353535353535354 0.0471222381003296\\
98 0.373737373737374 0.0558734644417698\\
99 0.393939393939394 0.0656563378511116\\
100 0.414141414141414 0.0765264228093408\\
101 0.434343434343434 0.0885376970210822\\
102 0.454545454545455 0.101742245839968\\
103 0.474747474747475 0.116189951926382\\
104 0.494949494949495 0.131928182570626\\
105 0.515151515151515 0.149001477194551\\
106 0.535353535353535 0.167451237608438\\
107 0.555555555555556 0.187315423644657\\
108 0.575757575757576 0.208628256813433\\
109 0.595959595959596 0.231419934626913\\
110 0.616161616161616 0.255716358214439\\
111 0.636363636363636 0.281538875803314\\
112 0.656565656565657 0.30890404456492\\
113 0.676767676767677 0.337823413225497\\
114 0.696969696969697 0.368303327714464\\
115 0.717171717171717 0.400344761971377\\
116 0.737373737373737 0.433943175856551\\
117 0.757575757575758 0.469088401911235\\
118 0.777777777777778 0.505764562492914\\
119 0.797979797979798 0.543950018571706\\
120 0.818181818181818 0.583617351217461\\
121 0.838383838383838 0.624733376536576\\
122 0.858585858585859 0.6672591945357\\
123 0.878787878787879 0.711150272099464\\
124 0.898989898989899 0.756356559974398\\

```

125 0.919191919191919 0.802822643354659\\
126 0.939393939393939 0.850487925370447\\
127 0.959595959595959 0.899286842490513\\
128 0.979797979797979 0.949149110569272\\
129 1 1\\
130 };
131 \addlegendentry{data1}
132
133 \end{axis}
134
135 \begin{axis}[%
136 width=3.31in,
137 height=0.956in,
138 at={(0.555in,0.413in)},
139 scale only axis,
140 xmin=-1,
141 xmax=1,
142 xlabel style={font=\color{white!15!black}},
143 xlabel={Second label},
144 ymin=0,
145 ymax=1,
146 ylabel style={font=\color{white!15!black}},
147 ylabel={Second y label},
148 axis background/.style={fill=white},
149 %title style={font=\bfseries},
150 %title={Test title 2},
151 legend style={legend cell align=left, align=left, draw=white!15!black}
152 ]
153 \addplot [color=mycolor1]
154 table[row sep=crcr]{%
155 -1 0.991995222186702\\
156 -0.979797979797979 0.933870616861269\\
157 -0.959595959595959 0.875191623027376\\
158 -0.939393939393939 0.816511131858994\\
159 -0.919191919191919 0.758335617951913\\
160 -0.898989898989899 0.701122442977675\\
161 -0.878787878787879 0.645278086544379\\
162 -0.858585858585859 0.591157265634755\\
163 -0.838383838383838 0.539062889639949\\
164 -0.818181818181818 0.489246785880245\\
165 -0.797979797979798 0.441911120729597\\
166 -0.777777777777778 0.397210434093834\\
167 -0.757575757575758 0.355254200017841\\
168 -0.737373737373737 0.316109823536763\\
169 -0.717171717171717 0.279805983406341\\
170 -0.696969696969697 0.246336231867169\\
171 -0.676767676767677 0.215662765898537\\
172 -0.656565656565657 0.187720289254016\\
173 -0.636363636363636 0.162419890679407\\
174 -0.616161616161616 0.13965287082241\\
175 -0.595959595959596 0.119294458181135\\
176 -0.575757575757576 0.101207362742515\\
177 -0.555555555555556 0.0852451244840417\\
178 -0.535353535353535 0.07125522242618\\
179 -0.515151515151515 0.0590819182263851\\
180 -0.494949494949495 0.0485688162250324\\
181 -0.474747474747475 0.0395611292449653\\
182 -0.454545454545455 0.0319076461963125\\
183 -0.434343434343434 0.0254624035636156\\
184 -0.414141414141414 0.0200860680987697\\
185 -0.393939393939394 0.0156470424833591\\
186 -0.373737373737374 0.0120223093544248\\
187 -0.353535353535353 0.00909803192650618\\

```

```

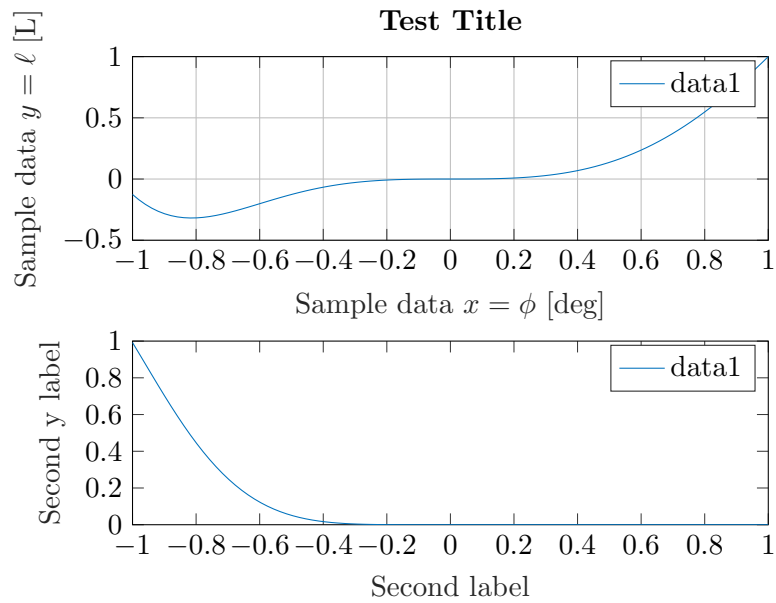
188 -0.3333333333333333 0.00676993152582624\\
189 -0.3131313131313131 0.00494346373012878\\
190 -0.292929292929293 0.00353381554137632\\
191 -0.272727272727273 0.00246574617754412\\
192 -0.252525252525252 0.00167329372803275\\
193 -0.232323232323232 0.00109936915057625\\
194 -0.212121212121212 0.000695257971088315\\
195 -0.191919191919192 0.000420048653942053\\
196 -0.171717171717172 0.000240005006250302\\
197 -0.151515151515151 0.000127898226994373\\
198 -0.131313131313131 6.23123637863848e-05\\
199 -0.111111111111111 2.69350410535447e-05\\
200 -0.0909090909090909 9.84340741546244e-06\\
201 -0.0707070707070707 2.7933383899727e-06\\
202 -0.0505050505050505 5.18028718187105e-07\\
203 -0.0303030303030303 4.0198879308291e-08\\
204 -0.0101010101010101 1.65212162738095e-10\\
205 0.0101010101010102 0\\
206 0.0303030303030303 0\\
207 0.0505050505050506 0\\
208 0.0707070707070707 0\\
209 0.0909090909090908 0\\
210 0.111111111111111 0\\
211 0.131313131313131 0\\
212 0.151515151515152 0\\
213 0.171717171717172 0\\
214 0.191919191919192 0\\
215 0.212121212121212 0\\
216 0.232323232323232 0\\
217 0.252525252525253 0\\
218 0.272727272727273 0\\
219 0.292929292929293 0\\
220 0.313131313131313 0\\
221 0.333333333333333 0\\
222 0.353535353535354 0\\
223 0.373737373737374 0\\
224 0.393939393939394 0\\
225 0.414141414141414 0\\
226 0.434343434343434 0\\
227 0.454545454545455 0\\
228 0.474747474747475 0\\
229 0.494949494949495 0\\
230 0.515151515151515 0\\
231 0.535353535353535 0\\
232 0.555555555555556 0\\
233 0.575757575757576 0\\
234 0.595959595959596 0\\
235 0.616161616161616 0\\
236 0.636363636363636 0\\
237 0.656565656565657 0\\
238 0.676767676767677 0\\
239 0.696969696969697 0\\
240 0.717171717171717 0\\
241 0.737373737373737 0\\
242 0.757575757575758 0\\
243 0.777777777777778 0\\
244 0.797979797979798 0\\
245 0.818181818181818 0\\
246 0.838383838383838 0\\
247 0.858585858585859 0\\
248 0.878787878787879 0\\
249 0.898989898989899 0\\
250 0.919191919191919 0\\

```

```

251 0.939393939393939 0\\
252 0.9595959595959596 0\\
253 0.9797979797979798 0\\
254 1 0\\
255 };
256 \addlegendentry{data1}
257
258 \end{axis}
259
260 \begin{axis}[%
261 width=4.271in,
262 height=3.125in,
263 at={(0in,0in)},
264 scale only axis,
265 xmin=0,
266 xmax=1,
267 ymin=0,
268 ymax=1,
269 axis line style={draw=none},
270 ticks=none,
271 axis x line*=bottom,
272 axis y line*=left,
273 legend style={legend cell align=left, align=left, draw=white!15!black}
274 ]
275 \end{axis}
276 \end{tikzpicture}%
277 \end{figure}

```

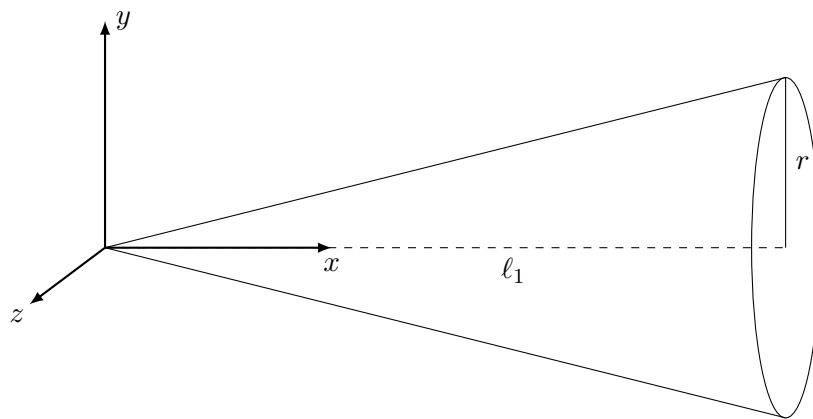


4.12 Solid Cone

```

1 \begin{figure}[H]
2 \centering
3 \begin{tikzpicture}[scale=3]
4 % \draw[dashed] (0,0) arc (170:10:2cm and 0.4cm)coordinate[pos=0] (a);
5 % \draw (0,0) arc (-170:-10:2cm and 0.4cm)coordinate (b);
6 % \draw[densely dashed] ([yshift=4cm]$(a)!0.5!(b)$) -- node[right,font=\footnotesize] {$h$}coordinate[pos=0.95] (aa)($ (a)!0.5!(b)$)
7 % -- node[above,font=\footnotesize] {$r$}coordinate[pos=0.1] (bb) (b);
8 % \draw (aa) -- (bb);
9 % \draw (a) -- ([yshift=4cm]$(a)!0.5!(b)$) -- (b);
10
11 % Draw sloped sides
12 \pgfmathsetmacro{\m}{0.25}
13 \pgfmathsetmacro{\L}{3}
14 \pgfmathsetmacro{\yAtL}{\m*\L}
15 \draw (0, 0) -- (\L, \yAtL);
16 \draw (0, 0) -- (\L, -\yAtL);
17
18 % Draw circle on end for perspective
19 \pgfmathsetmacro{\R}{\yAtL}
20 \draw (\L,\R) arc (90:270:0.15cm and 0.75cm);
21 \draw (\L,\R) arc (90:-90:0.15cm and 0.75cm);
22
23 % Place parameters
24 \draw[dashed] (0,0) -- (\L, 0) node[pos=0.6, below]{$\ell_1$};
25 \draw (\L, 0) -- (\L, \R) node[midway, right]{$r$};
26
27
28 % Draw axes
29 \draw[->, >=latex, thick] (0, 0) -- (1, 0) node[below]{$x$};
30 \draw[->, >=latex, thick] (0, 0) -- (0, 1) node[right]{$y$};
31 \draw[->, >=latex, thick] (0, 0) -- (-1/3, -1/4) node[anchor=north east, xshift=0.75mm, yshift=0.75mm]{$z$};
32 \end{tikzpicture}
33 \end{figure}

```



4.13 Attempt at Trying to Plot Parabolic Nose Cone

```

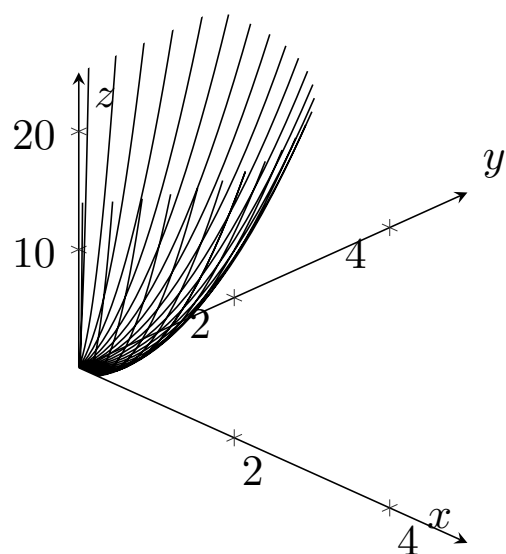
1 \begin{figure}[H]
2 \begin{tikzpicture}[scale=1.5]
3 \pgfmathsetmacro{\R}{0.1}
4 \pgfmathsetmacro{\L}{5*\R}
5 \pgfmathsetmacro{\K}{0.5}
6 %\begin{axis}[
7 %grid=major,
8 %3d box=complete,
9 %enlargeticks=false,
10 %colormap/cool,
11 %xlabel=$x$,
12 %ylabel=$y$,
13 %zlabel=$z$,
14 %zlabel style = {sloped like x axis}
15 %]
16 %\addplot3 [
17 %surf,
18 %shader=faceted,
19 %samples=20,
20 %] ({\R / (2 - \K) * (2 * (sqrt(x^2 + y^2) / \L) - \K * (sqrt(x^2 + y^2) / \L)^2}, {x}, {y});
21 %\end{axis}
22 \begin{axis}[
23 %x post scale=2,
24 % y post scale=2,
25 % z post scale=2,
26 scale=1,
27 axis x line=middle,
28 axis y line=middle,
29 axis z line=middle,
30 colormap/jet,
31 samples=100,
32 view={45}{40},
33 domain=0:5,
34 y domain=0:5,
35 restrict z to domain=0:25,
36 grid=both,
37 xlabel={$x$},
38 ylabel={$y$},
39 zlabel={$z$},
40 xmax=5,
41 ymax=5,
42 zmax=25,
43 xmin=0,
44 ymin=0,
45 zmin=0,
46 % xtick={-10,...,10},
47 % ytick={-10,...,10},
48 % ztick={-10,...,10},
49 % every axis x label/.style={
50 % at={(ticklabel* cs:1)}},
51 % anchor=west,},
52 % every axis y label/.style={
53 % at={(ticklabel* cs:1)}},
54 % anchor=south,},
55 % every axis z label/.style={
56 % at={(ticklabel* cs:1)}},
57 % anchor=west,}
58 ]
59 \addplot3 [samples=50, domain=0:5, y domain=0:360]
60 ({x * cos(y)}, {x * sin(y)}, {5*x^2});
61 \end{axis}

```

```

62 \end{tikzpicture}
63 \end{figure}

```



4.14 Skip here to bottom