Duration: 90 mts   Marks: 50

PART - A

Answer all the questions.   **5x2=10 Marks**

1. There are infinitely many stations on a train route. Suppose that the train stops at the first station and suppose that if the train stops at a station, then it stops at the next station. Does the train stops only few numbers of stations? Justify your answer.

2. How many bit strings of length 10 contain an equal number of 0's and 1's?

3. How many permutations of the letters A B C D E F G contain the strings ABC and CDE.

4. Suppose the number of bacteria in a colony triples every hour. Set up a recurrence relation for the number of bacteria after n hours have elapsed.

5. Among 200 people, how many of them were born on the same month?

PART – B

Answer the following questions.   **4x10=40 Marks**

6. (a) Using mathematical induction to prove that $7^{n+2} + 8^{2n+1}$ is divisible by 57.   (5)

   (b) Find an explicit formula for the Fibonacci number.   (5)

7. Suppose a department consists of eight men and nine women. In how many ways can we select a committee of

   (i)   Three men and four women?   (2)

(ii)	Four persons that has at least one woman?	(2)
(iii)	Four persons that has at most one man?	(2)
(iv)	Four persons that has persons of both sexes?	(2)
(v)	Four persons so that two specific members are not included?	(2)

8. (a) Solve the recurrence relation

$$a_n - 4a_{n-1} + 4a_{n-2} = 3n + 2^n, \quad a_0 = 1, a_1 = 1. \quad (7)$$

(b) A, B, C and D are the subsets of a set U containing 75 elements with the following properties. Each subset contains 28 elements; the intersection of any two of the subsets contains 12 elements; the intersection of any three of the subsets contains 5 elements; the intersection of all four subsets contains 1 element. How many elements belong to none of the four subsets? (3)

9. (a) Use mathematical induction to prove that

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{n}}, \text{ for } n \geq 2. \quad (5)$$

(b) A particle is moving in the horizontal direction. The distance it travels in each second is equal to two times the distance it travelled in the previous second. If a_r denotes the position of the particle in the rth second, determine a_r, given that $a_0 = 3$ and $a_3 = 10$. (5)