SASTRA DEEMED UNIVERSITY

(A University under section 3 of the UGC Act, 1956)

End Semester Examinations

February 2023

Course Code: **MAT133**

Course: **DISCRETE MATHEMATICS**

QP No. :**U017R-1**

Duration: **3 hours**

Max. Marks:**100**

PART - A

Answer all the questions **10 x 2 = 20 Marks**

1. State the idempotent and dominance laws of Boolean algebra.

2. Find the complement of the Boolean expression $a(bc + b'c')$

3. Find $\lim_{x\to 0} \frac{\sqrt{x^2+9} - 3}{x^2}$

4. Evaluate $\int_0^{\pi/2} \int_0^{sin\theta} r d\theta dr$

5. If $f: R \rightarrow R$ is given by $f(x) = 3x - 7$, find inverse of f.

6. If every element of a group (G,*) is its own inverse, prove that G is abelian.

7. Prove that the additive inverse of every element of the ring is unique.

8. How many different strings can be made from the letters of the " SUCCESS" using all the letters?

9. Use mathematical induction to show that $n! \geq 2^{n-1}$, for n = 1,2,3,...

10. Among 200 people, how many of them were born on the same month?

PART - B

Answer all the questions **4 x 15 = 60 Marks**

11. (a) In any Boolean algebra, show that
$$ab' + a'b = 0 \text{ if and only if } a = b. \quad (7)$$
(b) Simplify the following Boolean function, by Karnaugh map method.
$$f(a, b, c, d) = \sum(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 11) \quad (8)$$

(OR)

12. (a) Find the conjunctive normal forms of the following Boolean expression f(x, y, z) = (x + z)y using (i) truth table method & (ii) algebraic method. (8)
(b) In any Boolean algebra, show that
$$(xy'z' + xy'z + xyz + xyz')(x + y) = x \quad (7)$$

13. (a) Change the order of integration in $\int_0^1 \int_{x^2}^{2-y} xydydx$ and hence evaluate. (8)
(b) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dzdydx}{\sqrt{a^2-x^2-y^2-z^2}}$ (7)

(OR)

14. (a) Show that the area between the parabolas
$$y^2 = 4ax \text{ and } x^2 = 4ay \text{ is } \frac{16a^2}{3} \quad (8)$$
(b) Find the values of a and b that makes f(x) is continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2}, & x < 2 \\ ax^2 - bx + c, & 2 \le x < 3 \\ 2x - a + b, & x \ge 3 \end{cases}$$

(7)

15. (a) State and prove Lagrange's theorem (8)

(b) If R is the relation on the set of integers such that (a, b) ∈ R, if and only if 3a + 4b = 7n for some integer n, prove that R is an equivalence relation. (7)

(OR)

16. (a) Prove that every group of order 4 is abelian. (7)

(b) Determine whether or not each of the following defines a one-to-one and/or onto function with justification.

(i) $f: N \times N \rightarrow N$, defined by f(m, n) = 2m + 3n,

(ii) $f: Z \times Z \rightarrow Z$, defined by f(m, n) = 2m + 3n (8)

17. (a) Use method of generating function, solve the recurrence relation

$a_n = 4a_{n-1} - 4a_{n-2} + 4^n;\ \ n \ge 2,$ given that $a_0 = 2\ and\ a_1 = 8$

(8)

(b) Use mathematical induction to prove that

$H_{2^n} \ge 1 + \frac{n}{2},\ where\ H_j = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{j}$ (7)

(OR)

18. (a) Solve the recurrence relation $a_{n+1} - a_n = 3n^2 - n;\ \ n \ge 0,\ a_0 = 3$

(7)

(b) (i) Assuming that repetitions are not permitted, how many four-digit numbers can be formed from the six digits 1, 2, 3, 5, 7, 8?

(ii) How many of these numbers are less than 4000?

(iii) How many of the numbers in (i) are even?

(iv) How many of the numbers in (i) are odd?

(v) How many of the numbers in (i) are multiples of 5? (8)

PART – C

Answer the following **1 x 20 = 20 Marks**

19. (a) If * is the binary operation on the set R of real numbers defined by $a*b = a + b + 2ab$.

    (i) Find if { R, * } is a semigroup. Is it commutative?

    (ii) Find the identity element, if exists.

    (iii) Which elements have inverses and what are they? (10)

    (b) Simplify the following Boolean expression using Boolean algebra.

    $x\,[\,y + z\,(\,xy + xz)']$ (10)

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