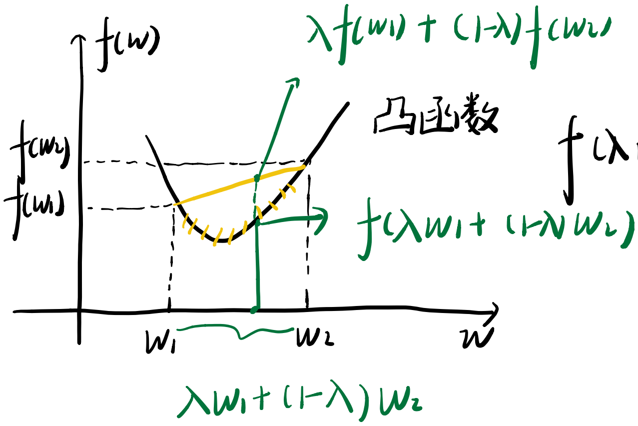
最小化: $\frac{1}{2}\|w\|^2 + C\sum_{i=1}^{N}\xi_i$ → ? (凸函数) $\frac{1}{2}\|w\|^2 - C\sum_{i=1}^{N}\xi_i$

限制条件: ① $y_i[w^T\varphi(x_i)+b] \ge 1-\xi_i$ → $\underbrace{1+\xi_i - y_i w^T\varphi(x_i) - y_i b}_{(w)} \le 0$

② $\xi_i \ge 0$ → $\xi_i \le 0$

? 凸函数



$\forall \underset{\text{向量}}{w_1, w_2}$, $\forall \lambda \in [0,1]$

$$f(\lambda w_1 + (1-\lambda) w_2) \le \lambda f(w_1) + (1-\lambda) f(w_2)$$

$\sum_{i=1}^{K}\alpha_i g_i(w)$

$L(w,\xi,b) =$

对偶问题

最大化 $\theta(\alpha,\beta) = \inf_{\text{所有}(w,\xi,b)} \left\{ \underbrace{\frac{1}{2}\|w\|^2 - C\sum_{i=1}^{N}\xi_i}_{f(w)} + \sum_{i=1}^{N}\beta_i\xi_i + \sum_{i=1}^{N}\alpha_i[1+\xi_i - y_i w^T\varphi(x_i) - y_i b] \right\}$

α

α, k $\sum_{i=1}^{K}\alpha_i g_i(w)$

限制条件: $\alpha_i \ge 0$

$\beta_i \ge 0$ $(i = 1 \sim N)$

$\frac{\partial L}{\partial w} = 0 \Rightarrow w - \sum_{i=1}^{N}\alpha_i y_i \varphi(x_i) = 0$

$w = \sum_{i=1}^{N}\alpha_i y_i \varphi(x_i)$

$\frac{\partial L}{\partial \xi_i} = 0 \Rightarrow -C + \beta_i + \alpha_i = 0$

$\alpha_i + \beta_i = C$

$\frac{\partial L}{\partial b} = 0 \Rightarrow \sum_{i=1}^{N}\alpha_i y_i = 0$

代入得到最小

定义:

$w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_m \end{bmatrix}$ $f(w)$

$\frac{\partial f}{\partial w} = \begin{bmatrix} \frac{\partial f}{\partial w_1} \\ \vdots \\ \frac{\partial f}{\partial w_m} \end{bmatrix}$

若 $f(w) = \frac{1}{2}\|w\|^2$

则: $\frac{\partial f}{\partial w} = w$

若 $f(w) = w^T X$

则: $\frac{\partial L}{\partial w} = X$

$\Downarrow$

$\theta(\alpha, \beta) =$

$$\underbrace{C}_{\alpha_i+\beta_i}$$

$$\Big\{ \frac{1}{2}\|w\|^2 - \cancel{C\sum_{i=1}^{N}\xi_i} + \cancel{\sum_{i=1}^{N}\beta_i\xi_i} + \sum_{i=1}^{N}\alpha_i[1 + \cancel{\xi_i} - y_i w^T\varphi(x_i) - \cancel{y_i b}]$$

$$= \sum_{i=1}^{N}\alpha_i$$

$$\frac{1}{2}\|w\|^2 = \frac{1}{2} w^T w$$

$$= \frac{1}{2}\Big(\sum_{i=1}^{N}\underbrace{\alpha_i y_i}_{\text{常数}}\varphi(x_i)\Big)^T\Big(\sum_{j=1}^{N}\alpha_j y_j\varphi(x_j)\Big)$$

$$= \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j y_i y_j \underbrace{\boxed{\varphi(x_i)^T\varphi(x_j)}}_{=K(x_i, x_j)}$$

$$-\sum_{i=1}^{N}\alpha_i y_i w^T\varphi(x_i)$$

$$= -\sum_{i=1}^{N}\alpha_i y_i\Big(\sum_{j=1}^{N}\alpha_j y_j\varphi(x_j)\Big)^T\varphi(x_i)$$

$$= -\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i y_i\alpha_j y_j \underbrace{\boxed{\varphi(x_j)^T\varphi(x_i)}}_{=K(x_i, x_j)}$$

$\Downarrow$ 整理

$$\theta(\alpha, \beta) = \sum_{i=1}^{N}\alpha_i - \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j y_i y_j K(x_i, x_j)$$

最大化：$\theta(\alpha,\beta)=\sum_{i=1}^{N}\alpha_i-\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j y_i y_j K(X_i,X_j)$

未知　　已知

限制条件：① $0\le\alpha_i\le C$

② $\sum_{i=1}^{N}\alpha_i y_i=0$

(SMO算法)

测试流程

测试样本 X

$$\begin{cases}\text{若 } W^T\varphi(X)+b\ge 0 & \text{则 } y=+1\\ \text{若 } W^T\varphi(X)+b<0 & \text{则 } y=-1\end{cases}$$

$$W^T\varphi(x)=\sum_{i=1}^{N}[\alpha_i y_i\varphi(x_i)]^T\varphi(x) \qquad W=\sum_{i=1}^{N}[\alpha_i y_i\varphi(x_i)]$$

$$=\sum_{i=1}^{N}\alpha_i y_i\varphi(x_i)^T\varphi(x)$$

$$=\sum_{i=1}^{N}\alpha_i y_i K(x_i,x_j)$$

b：KKT条件：$\forall i=1\sim K$　$\alpha_i^*=0$ 或 $g_i^*(w^*)=0$

⇓

$\forall i=1\sim N$

① 要么 $\beta_i=0$ 要么 $\xi_i=0$

② 要么 $\alpha_i=0$ 要么 $1+\xi_i-y_iw^T\varphi(x_i)-y_ib=0$

取一个 $0<\alpha_i<C$ ⇒ $\beta_i=C-\alpha_i>0$

此时 $\beta_i\ne0$ ⇒ $\xi_i=0$

$\alpha_i\ne0$ ⇒ $1+\xi_i-y_iw^T\varphi(x_i)-y_ib=0$

$$\Rightarrow b = \frac{1 - y_i w^T \varphi(x_i)}{y_i}$$

$$= \frac{1 - y_i \sum_{j=1}^{N} \alpha_j y_j k(x_i, x_j)}{y_i}$$