(a) (l_1 and l_2-norm SVM Classification):

Minimize: $\frac{1}{2}\|\omega\|^2 + C_1\sum_{i=1}^{N}\delta_i + C_2\sum_{i=1}^{N}\delta_i^2$

Subject to: $\delta_i \geq 0, i = 1, 2, \ldots, N$

$y_i\left[\omega^T\varphi(x_i)+b\right] \geq 1-\delta_i$

(Prime Problem): $f(w, \delta) = \frac{1}{2}\|w\|^2 + C_1\sum_{i=1}^{N}\delta_i + C_2\sum_{i=1}^{N}\delta_i^2$

S.t.: $\delta_i \geq 0$

$y_i[w^T\varphi(x_i)+b] \geq 1-\delta_i$

$f(w)$

S.t. $g_i(w) \leq 0$

$h_i(w) = 0$

$f(w,\delta,b) = \frac{1}{2}\|w\|^2 - C_1\sum_{i=1}^{N}\delta_i + C_2\sum_{i=1}^{N}\delta_i^2$

S.t.: $\delta_i \leq 0 \quad i = 1 \sim N$ $\quad g_i(w) \leq 0$

$1+\delta_i - y_i w^T\varphi(x_i) - y_i b \leq 0$

$L(w, a, \beta) = f(w) + \sum_{i=1}^{N} a_i g_i(w) + \sum_{i=1}^{N}\beta_i h_i(w)$

$\frac{1}{2}\|w\|^2 - C_1\sum_{i=1}^{N}\delta_i + C_2\sum_{i=1}^{N}\delta_i^2 + \sum_{i=1}^{N}\beta_i\delta_i$ $\quad w\begin{cases} w \\ \delta \\ b \end{cases}$

$+\sum_{i=1}^{N} a_i[1+\delta_i - y_i w^T\varphi(x_i) - y_i b]$ $\quad a\begin{cases} \beta_i \\ a_i \end{cases}$

Maximize: $\theta(a, \beta) = \inf_{w}\{L(w, a, \beta)\}$ $\quad \beta$: 无

$$\theta(\alpha,\beta) = \inf_{(w,\delta,b)} \left\{ \frac{1}{2}\|w\|^2 - C_1 \sum_{i=1}^{N} \delta_i + C_2 \sum_{i=1}^{N} \delta_i^2 + \sum_{i=1}^{N} \beta_i \delta_i + \sum_{i=1}^{N} \alpha_i \left[1 + \delta_i - y_i w^T \varphi(x_i) - y_i b \right] \right\}$$

$$\frac{\partial L}{\partial w}: \quad w - \sum_{i=1}^{N} \alpha_i y_i \varphi(x_i) = 0$$

$$w = \sum_{i=1}^{N} \alpha_i y_i \varphi(x_i) \quad ①$$

$$\frac{\partial L}{\partial \delta_i}: \quad -C_1 + 2C_2 \delta_i + \beta_i + \alpha_i = 0$$

$$\delta_i = \frac{C_1 - \alpha_i - \beta_i}{2C_2} \quad ②$$

$$\frac{\partial L}{\partial b}: \quad -\sum_{i=1}^{N} \alpha_i y_i = 0 \qquad \sum_{i=1}^{N} \alpha_i y_i = 0 \quad ③$$

①②③代入原式：

$$\theta(\alpha,\beta) = \inf_{(w,\delta,b)} \left\{ \frac{1}{2}\|w\|^2 - C_1 \sum_{i=1}^{N} \delta_i + C_2 \sum_{i=1}^{N} \delta_i^2 + \sum_{i=1}^{N} \beta_i \delta_i + \sum_{i=1}^{N} \alpha_i \left[1 + \delta_i - y_i w^T \varphi(x_i) - y_i b \right] \right\}$$

得（我自己真算不出来）

$$= \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j y_i y_j \varphi(x_i)^T \varphi(x_j) - \frac{(C_1 - \alpha_i - \beta_i)^2}{4C_2}$$

故 Dual Problem：

maximize：

$$\theta(\alpha,\beta) = \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j y_i y_j \varphi(x_i)^T \varphi(x_j) - \frac{(C_1 - \alpha_i - \beta_i)}{4C_2}$$

核函数： $K(x_1, x_2) = \varphi(x_1)^T \varphi(x_2)$

s.t.: ① $a_i \geq 0$ $\beta_i \geq 0$

② $\sum_{i=1}^{N} a_i y_i = 0$ $i = 1 \sim N$

求出 (a, β) 后 寻找 $a_i \neq 0$ 且 $\beta_i \neq 0$ 的 i 值 根据 KKT 条件，一定有 $\delta_i = 0$ 且 $1 + \delta_i - y_i w^T \varphi(x_i) - y_i b = 0$

KKT: $\forall i = 1 \sim k$

$a_i^* = 0$ 或 $g_i(w^*) = 0$

即: $b = \frac{1 - y_i w^T \varphi(x_i)}{y_i}$ 将 $w = \sum_{j=1}^{N} a_j y_j \varphi(x_j)$

$= 1 - \sum_{j=1}^{N} a_j y_i y_j K(x_i, x_j)$

因此: $w^T \varphi(x) + b = \sum_{i=1}^{N} a_i y_i K(x_i, x) + b$

判决标准:

$x \in C_1$ 若 $\sum_{i=1}^{N} a_i y_i K(x_i, x) + b \geq 0$

$x \in C_2$ 若 $\sum_{i=1}^{N} a_i y_i K(x_i, x) + b < 0$