特征选择：$X=\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$

从N个特征中选M个使识别率最高

选取数：$C_N^M = \frac{N!}{M!(N-M)!}$

自适应提升算法（AdaBoost）

数据集：$dataSet = \{(x_1,y_1)\dots(x_N,y_N)\}$

二分类问题：$y_i = \{-1,+1\}$

算法流程：

输入：dataSet

输出：分类器 $G(x)=\pm1$

① 初始化采样权值

$$D_1=(w_{11}, w_{12}, \dots w_{1N}) \quad w_{1i}=\frac{1}{N} \ (i=1\sim N)$$

② 对 $m=1,2,\dots,M$

（M是弱分类器个数）

用D_m采样N个样本，在训练样本上获得弱分类器 $G_m(x)=\pm1$

③ 计算加权错误率

$$e_m = P(G_m(x_i) \neq y_i) \quad (e_m < \frac{1}{2})$$

$$= \sum_{i=1}^{N} w_{mi} I(G_m(x_i) \neq y_i)$$

$$\alpha_m = \frac{1}{2}\log\frac{1-e_m}{e_m} \quad (\text{识别器} G_m(x_i) \text{的权重}) \quad (\alpha_m > 0)$$

④ 更新权值分布

$$D_{m+1} = (w_{m+1,1}, \dots, w_{m+1,N})$$

$$w_{m+1,i} = \frac{w_{mi}}{Z_m} \exp\{-\alpha_m y_i G_m(x_i)\}$$

$$Z_m = \sum_{i=1}^{N} w_{mi} \exp\{-\alpha_m y_i G_m(x_i)\}$$

⑤ 回到②

⑥ 最终识别器 $G(x)$

$$f(x) = \sum_{m=1}^{M} \alpha_m G_m(x)$$

$$G(x) = \mathrm{sign}(f(x)) = \mathrm{sign}\left(\sum_{m=1}^{M} \alpha_m G_m(x)\right)$$

定理：随着M增加 Adaboost最终分类器 $G(x)$ 在训练集上错误将会越来越小

证明：

错误率：$E = \frac{1}{N}\sum_{i=1}^{N} I(G(x_i) \neq y_i) \leq \frac{1}{N}\sum_{i=1}^{N} \exp\{-y_i f(x_i)\}$

$$\left\{\begin{array}{ll} 若 (G(x_i) = y_i) & \\ 则 I(G(x_i) \neq y_i) = 0 & 0< \quad <1 \\ & \\ 若 (G(x_i) \neq y_i) & \\ 则 I(G(x_i) \neq y_i) = 1 & >1 \end{array}\right\}$$

$$E = \prod_{m=1}^{M} Z_m$$

$$\because E \leq \frac{1}{N}\sum_{i=1}^{N} \exp\{-y_i \sum_{m=1}^{M} \alpha_m G_m(x)\}$$

$$= \sum_{i=1}^{N} w_{1i} \overbrace{\prod_{m=1}^{M} \exp\{-\alpha_m y_i G_m(x_i)\}} \quad (w_{1i} = \tfrac{1}{N})$$

$$= \sum_{i=1}^{M} \left(w_{1i} \exp[-\alpha_1 y_i G_1(x_i)]\right) \left(\prod_{m=2}^{M} \exp\{-\alpha_m y_i G_m(x_i)\}\right)$$

$$= \sum_{i=1}^{M} \left(w_{2i} \cdot Z_1\right)\left(\prod_{m=2}^{M} \exp\{-\alpha_m \cdot y_i G_m(x_i)\}\right)$$

$$= Z_1 \sum_{i=1}^{M} w_{2i} \left(\prod_{m=2}^{M} \exp\{-\alpha_m \cdot y_i G_m(x_i)\}\right)$$

$$= \prod_{m=1}^{M} Z_m$$

证明: $Z_m = 2\sqrt{e_m(1-e_m)}$

$$Z_m = \sum_{i=1}^{N} w_{mi} \exp\{-\alpha_i y_i G_m(x_i)\}$$

$$= \sum_{\substack{i=1 \\ y_i = G_m(x_i)}}^{N} w_{mi} e^{-\alpha_m} + \sum_{\substack{i=1 \\ y_i \neq G_m(x_i)}}^{N} w_{mi} e^{\alpha_m}$$

$$= (1-e_m) e^{-\alpha_m} + e_m e^{\alpha_m}$$

$\alpha_m = \frac{1}{2} \log \frac{1-e_m}{e_m}$ 代入

$Z_m \leq 2\sqrt{e_m(1-e_m)}$

若 $e_m < \frac{1}{2}$ 则: $Z_m < 1$