Definition: Norm

Let (V, < , >) be an inner product space.

The length, or norm, of a vector v in V is given by

$$\|\underline{v}\| = \sqrt{\langle \underline{v}, \underline{v} \rangle}$$

Ex. $V = P_2([-1, 1])$ $\qquad \langle f, g \rangle = \int_{-1}^{1} f(x) g(x)\, dx$

$\underline{v} = 1 - 2x + 3x^2$

$$\|\underline{v}\| = \left(\frac{184}{15}\right)^{1/2}$$

Lemma: Let (V, < , >) be an inner product space. Then:

- For all vectors $\underline{v}$ in V, $\|\underline{v}\| \geq 0$ and $\|\underline{v}\| = 0$ if and only if $\underline{v} = \underline{0}$
- For all constants c, $\|c\underline{v}\| = |c| \cdot \|\underline{v}\|$

Definition: Distance

Let (V, < , >) be an inner product space, and let $\underline{v}$ and $\underline{w}$ be in V.

Then, the *distance* from $\underline{v}$ to $\underline{w}$ is $\text{dist}(\underline{v}, \underline{w}) = \|\underline{v} - \underline{w}\|$

The distance from v to w is the same as the distance from w to v.