lemma

$${}_{B_W}[T]_{B_V} : \mathcal{L}(V, W) \to M_{m\times n}$$

Getting the change of basis matrix is a linear transformation itself

Lemma 2

Let V, W, X be finite-dimensional vector spaces with bases B_V, B_W, B_X, respectively. Let $T_1 \in \mathcal{L}(V, W)$ and $T_2 \in \mathcal{L}(W, X)$. Then

$${}_{B_X}[T_2 \circ T_1]_{B_V} = {}_{B_X}[T_2]_{B_W}\ {}_{B_W}[T_1]_{B_V}.$$

Lemma 3

Let $T_i : V_i \to V_{i+1}$ be a linear transformation from finite-dimensional vector space V_i to finite-dimensional vector space $V_{i+1},\ i = 1, \dots, n-1$.

Let B_{V_i} be a basis for $V_i,\ i = 1, \dots, n$. Then

$${}_{B_{V_n}}[T_{n-1} \circ T_{n-2} \circ \cdots \circ T_1]_{B_{V_1}} = {}_{B_{V_n}}[T_{n-1}]_{B_{V_{n-1}}}\ {}_{B_{V_{n-1}}}[T_{n-2}]_{B_{V_{n-2}}} \cdots {}_{B_{V_3}}[T_2]_{B_{V_2}}\ {}_{B_{V_2}}[T_1]_{B_{V_1}}.$$