Let $X \sim N(\mu, \sigma^2)$ and $Y = aX + b$, where a and b are constants

Then, $E(Y) = a \cdot E(X) = a\mu$

And $Var(Y) = a^2 \cdot Var(X) = a^2 \cdot \sigma^2$

So $Y \sim N(a\mu, a^2\sigma^2)$

If $X \sim (\mu_1, \sigma_1)$ and $Y \sim (\mu_2, \sigma_2)$, then

$$aX + bY \sim N(a\mu_1 + b\mu_2,\ a^2\sigma_1^2 + b^2\sigma_2^2)$$

If $X1, X2, \dots, X\_n$ are independent normal random variables:

Then:

$$\sum_{i=1}^{n} X_i \sim N(n\mu, n\sigma^2)$$

and

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \sim N(\mu, \sigma^2/n)$$

Ex. Let $X \sim N(3,5)$ and $Y \sim N(6,14)$. Find $P(X > Y)$

This is equal to $P(X - Y > 0)$

Let $W = X - Y$

$E(W) = E(X) - E(Y) = 3 - 6 = -3$

$Var(W) = Var(X) + (-1)^2 \cdot Var(Y) = 5 + 14 = 19$

So $W \sim N(-3, 19)$

Now we want to solve for $P(W > 0)$

$$= P\left(\frac{W - E(W)}{\sigma_W} > \frac{0 - (-3)}{\sqrt{19}}\right)$$

$= P(Z > 0.69)$

$= 1 - P(Z \leq 0.69)$

$= 0.2451$