f is differentiable on an interval I if f'(a) exists for all a in I and

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

So, we can think of f'(x) as a function on I

Ex. Find $\frac{d}{dx} x^2 + 3x + 2$ and tangents at $x = 3$ and $x = 2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 3(x+h) + 2 - x^2 - 3x - 2}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + 3h}{h}$$

$$= \lim_{h \to 0} 2x + h + 3 = \boxed{2x+3}$$

Ex. 2. Find $f'(x)$ for $x > 0$ if $f(x) = \sqrt{x}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{x+h-x}{h} \cdot \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Higher order derivatives

$$f''(x) \,/\, \frac{d^2 f}{dx^2} \,/\, f^{(2)}(x)$$