Definition

$$\frac{\partial f}{\partial x} f(x,y) = \lim_{h \to 0} \frac{f(x+h,\ y) - f(x,y)}{h}$$

$$\frac{\partial f}{\partial y} f(x,y) = \lim_{h \to 0} \frac{f(x,\ y+h) - f(x,y)}{h}$$

Partial derivative exists if this limit exists

Evaluate with respect to one variable and treat the other as a constant

Ex. Determine whether $\frac{\partial f}{\partial x}(0,0)$ exists for $f(x,y) = (x^3+y^3)^{1/3}$.

$$\frac{\partial f}{\partial x} f(x,y) = \frac{(x^3+y^3)^{-2/3}}{3} \cdot 3x^2$$

$y^3 \to 0$

$$= \frac{x^2}{(x^3+y^3)^{2/3}}$$

Undefined at $(0, 0)$

But derivative may still exist

$$\Rightarrow \lim_{h \to 0} \frac{f(x+h,\ y) - f(x,y)}{h}$$

At $(x, y) = (0, 0)$:

$$= \lim_{h \to 0} \frac{f(h, 0) - f(0, 0)}{h}$$

$$= \lim_{h \to 0} \frac{h + 0}{h} = 1$$

Thus, the limit exists

Theorem 1: Clairaut's Theorem

If f_{xy} and f_{yx} are defined in some neighborhood of (a,b) and are both continuous at (a,b), then

$$f_{xy}(a,b) = f_{yx}(a,b)$$

The partial derivative with respect to y of the partial derivative of f with respect to x is equal to the partial derivative with respect to x of the partial derivative of f with respect to y

Essentially, partial derivatives are commutative (kinda?)

Class

$f \in C^k$: 1st, 2nd, ..., k-th partial derivatives are continuous