Random variables are defined for every outcome of a random experiment

X - random variable

x - actual value of the random variable X (X = x)

For example, if we toss a coin 3 times, and the random variable X is the number of heads obtained, the *range* of X is {0,1,2,3}

- Either 0 heads
- 1 head
- 2 heads
- 3 heads

For *discrete* random variables, the range is either finite or countably infinite

- Countably infinite: when playing cards, there could be an infinite amount of outcomes before finally getting a certain hand

For *continuous* random variables, the range is infinite

Discrete: things we can count

- Number of students present in a class
- Number of times you hit the snooze button

Continuous: things we can measure

- Time taken to get to school on a given day
- Height

A *function* maps each value in a domain into a unique point (one-to-one).

Probability function: since X=x represents some event, its probability is represented by P(X=x)

- The probability that the random variable X takes on the value x

Discrete random variables have probability functions

$f(x) = P(X = x) \ \forall x \in A$

Probability distribution of X:

$\{(x, f(x)) : x \in A\}$

Rules:

- $0 \leq f(x) \leq 1$ for all x
- The sum of all f(x) must be 1

Ex. A fair coin is tossed 3 times. Let X be the number of heads observed.

- f(0) = P(X=0) = P(TTT) = 1/8
- f(1) = P(X=1) = P({HTT, THT, TTH}) = 3/8 (3 choose 1)
- f(2) = P(X=2) = P({HHT, THH, HTH}) = 3/8 (3 choose 2)
- f(3) = P(X=3) = P({HHH}) = 1/8

All probabilities sum to 1, and $0 \leq f(x) \leq 1$ for all x

So this is a valid probability function

Example: The random variable X has p.f. given by

x	0	1	2	3	4
$f(x)$	$0.1c$	$0.2c$	$0.5c$	c	$0.2c$

a) Determine the value of c.

b) Calculate $P(X > 2)$.

(a) All probabilities must sum to 1

So 0.1c+ 0.2c + 0.5c + c + 0.2c = 1 -> c=0.5

(b) P(X>2) = P(X=3) + P(X=4) = 0.5 + (0.2)(0.5) = 0.6

Cumulative Distribution Function

$F(x) = P(X \le x)$ for all real numbers X

- Accumulated probabilities up to and including some real number x

$$\Rightarrow \sum_{u \le x} f(u)$$

where f is a probability function

Properties:

- F(x) is a *non-decreasing function*: $P(X \le 8)$ *cannot be less than* $P(X \le 7)$ because $P(X\le8)$ includes $P(X\le7)$, plus $P(7<X\le8)$
- $0 \le F(x) \le 1$ for all x

$$\lim_{x \to -\infty} F(x) = 0 \qquad \lim_{x \to \infty} F(x) = 1$$

We can also obtain f(x) from F(x), since F(x) is just a summation of f(x)

For example, if X only takes on integer values, then $f(x) = F(x) - F(x-1)$

- $P(X=x) = P(X \le x) - P(X \le x-1)$

Ex. Flipping a coin 3 times:

x	$f(x)$	$F(x)$
0	1/8	1/8
1	3/8	1/2
2	3/8	7/8
3	1/8	1

$f(1) = F(1) - F(0)$

CDF F(x) can be represented using a *step function* (goes up in increments)

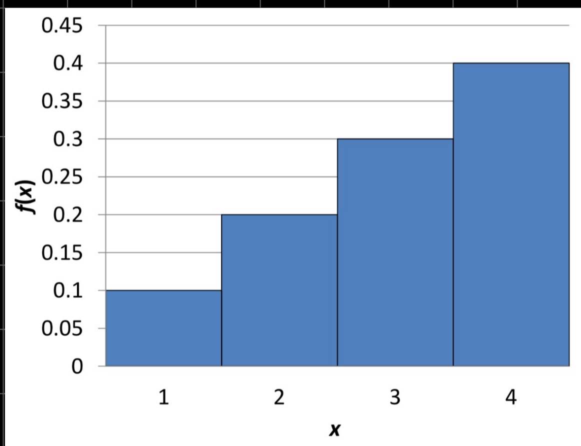
Probability function f(x) can be represented using a *histogram*

Example: Consider the r.v. X such that

x	$f(x)$
1	0.1
2	0.2
3	0.3
4	0.4

Plot $f(x)$ and $F(x)$.

PF



CDF :

$$\left.\begin{aligned} F(1) &= 0.1 \\ F(2) &= 0.3 \\ F(3) &= 0.6 \\ F(4) &= 1 \end{aligned}\right\} \text{strictly non-decreasing}$$