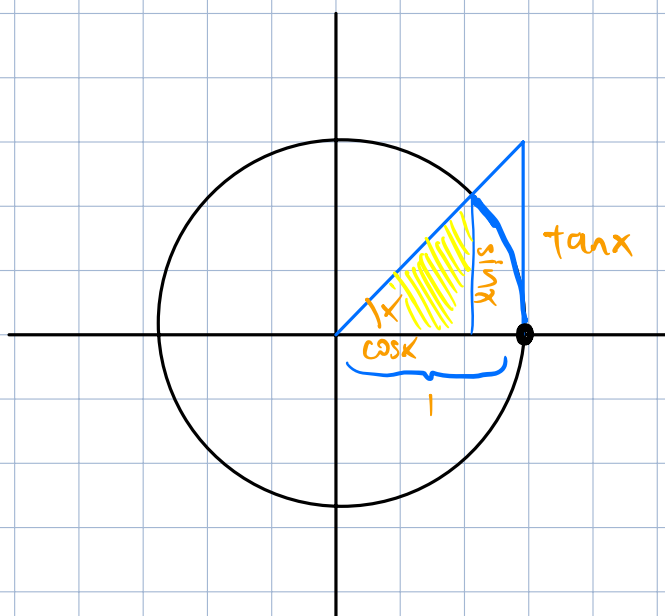
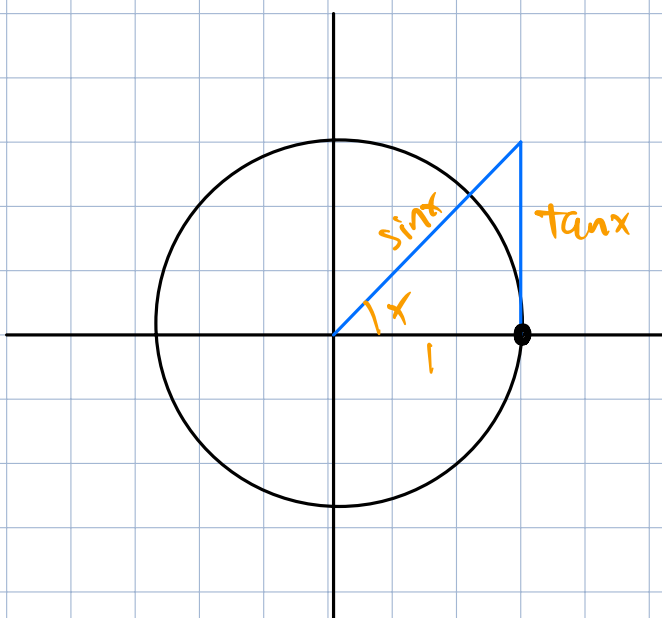


$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$1 \leq \left| \frac{x}{\sin x} \right| \leq |\cos x|$$



Area of smaller Δ : $\frac{\cos x \sin x}{2}$

Area of pizza slice $\Delta = \frac{1}{2} r^2 \theta = \frac{x}{2}$

Area of big Δ : $\frac{\tan x}{2}$

$$\frac{\cos x \sin x}{2} \leq \frac{x}{2} \leq \frac{\tan x}{2}$$

$$\cos x \sin x \leq x \leq \tan x$$

$$\cos x \leq \frac{x}{\sin x} \leq \frac{1}{\cos x}$$

$$\frac{1}{\cos x} \geq \frac{\sin x}{x} \geq \cos x \rightarrow \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

Ex. 1.

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 5x}$$

$$= \frac{\sin 3x}{3x} \cdot \frac{5x}{\sin 5x} \cdot \frac{3x}{5x}$$

$$= 1 \cdot 1 \cdot \frac{3}{5} = \frac{3}{5}$$

Rules:

$$\lim_{x \rightarrow 0} \frac{\sin(ax)}{\sin(bx)} = \frac{a}{b}$$

$$\lim_{x \rightarrow 0} \frac{\sin(ax)}{x} = a$$

$$\lim_{x \rightarrow 0} \frac{\tan(ax)}{\tan(bx)} = \frac{a}{b}$$

$$\lim_{x \rightarrow 0} \frac{\sin(ax)}{\tan(bx)} = \frac{a}{b}$$