Let f and g be differentiable at x=a. Then:

1. Constant multiple rule – can factor out a constant c
2. Sum rule – If h(x) = f(x) + g(x), then h’(x) = f’(x) + g’(x)
3. Product Rule – If h(x) = f(x) • g(x), then h is differentiable at x=a and h’(a) = f’(a)g(a) + f(a)g’(a)

Reciprocal Rule

If $g(x) = \frac{1}{f(x)}$, $g(x) = -\frac{f'(x)}{f(x)^2}$

Quotient Rule

If $h(x) = \frac{f(x)}{g(x)}$, then h is differentiable at $x = a$ and

$$h'(a) = \frac{f'(a) \cdot g(a) - f(a) \cdot g'(a)}{g(a)^2}$$

Proof of product rule

$$h(x) = f(x) \cdot g(x)$$

$$h'(a) = \lim_{h \to 0} \frac{f(a+h) \cdot g(a+h) - f(a)g(a)}{h}$$

$$= \lim_{h \to 0} \frac{f(a+h) \cdot g(a+h) \overbrace{- f(a+h)g(a) + f(a+h)g(a)}^{0} - f(a)g(a)}{h}$$

$$= \lim_{h \to 0} f(a+h)\left[\frac{g(a+h) - g(a)}{h}\right] + g(a)\left[\frac{f(a+h) - f(a)}{h}\right]$$

$f(a+0) = f(a)$ $g(a)$ by limit defn of derivatives $g(a)$ $f'(a)$

$$= f'(a)g(a) + g'(a)f(a)$$

Power Rule

Basic

Examples

1. $f(x) = x^2 \sin x$

$f'(x) = 2x\sin x + x^2\cos x$

2. $f(x) = \dfrac{x^4+1}{x-7}$

$$f'(x) = \frac{(x-7)(4x^3) - (x^4+1)(1)}{(x-7)^2} = \frac{4x^4 - 28x^3 - x^4 + 1}{(x-7)^2}$$

$$= \frac{3x^4 - 28x^3}{(x-7)^2}$$

3. $f(x) = e^x \cos x$

$f'(x) = e^x\cos x - e^x \sin x$

product rule: $f'(a)\cdot g(a) - f(a)\cdot g'(a)$

4. $f(x) = \sec x$ quotient rule

$$= \frac{1}{\cos x} = \frac{0 + \sin x}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \tan x \sec x$$