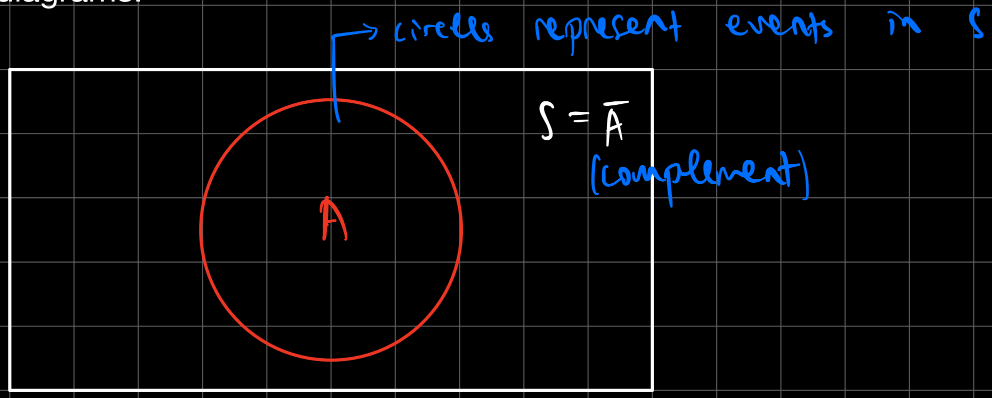
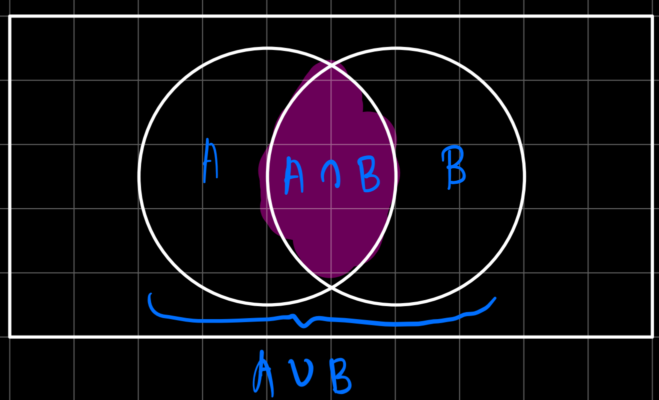
Recall: for some sample space S and event A:

- P(S) = 1, since it consists of *all possible outcomes*
- $0 \le P(A) \le 1$
- If B is another event where A is a subset of B, $P(A) \le P(B)$

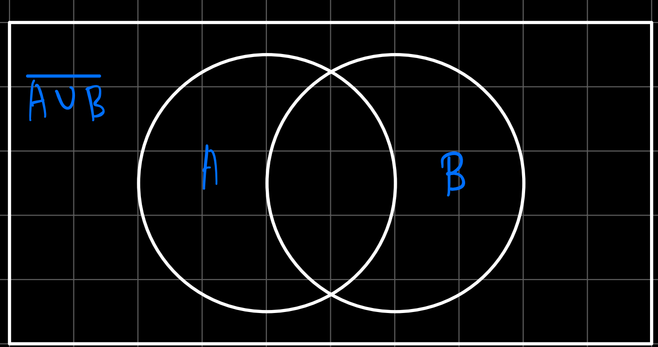
Venn diagrams:





<u>De Morgan's Laws</u>

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$



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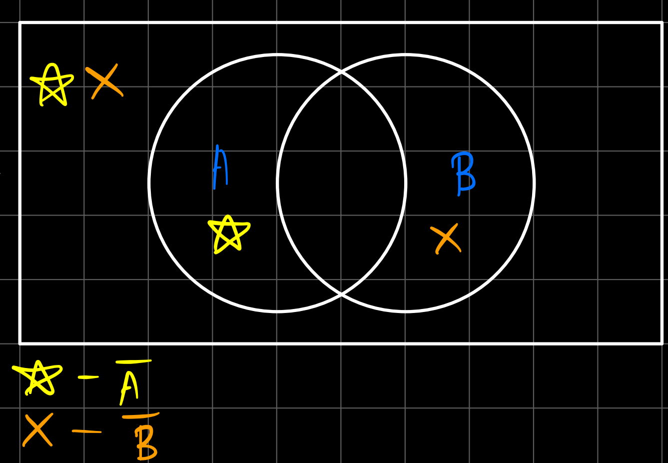
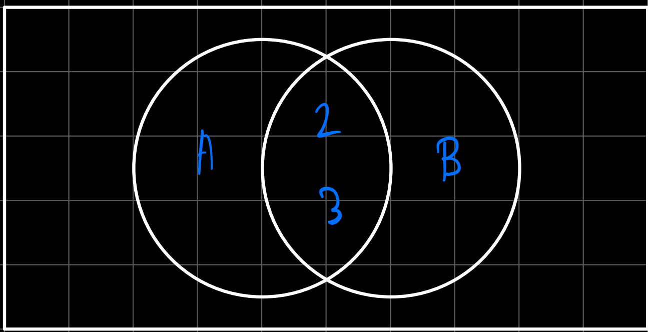


$$\overline{A \cap B} = \bar{A} \cup \bar{B}$$

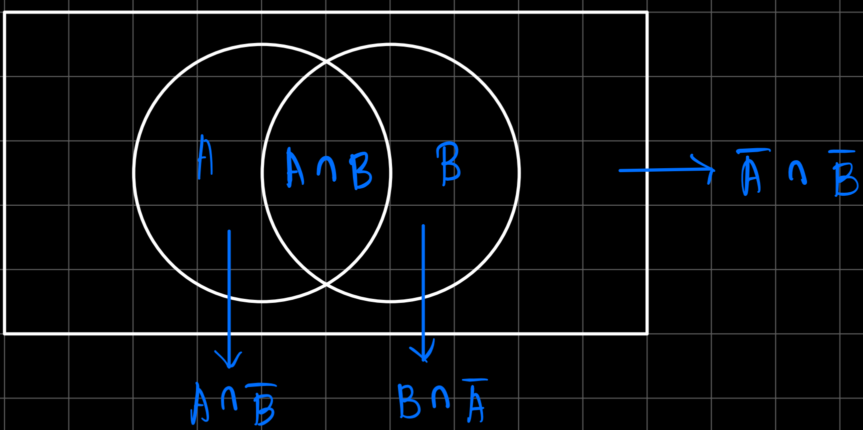
Ex. Let S = {1,2,3,4,5}. Let A = {1,2,3}, B = {2,3,4,5}.



Rules for Unions of Events

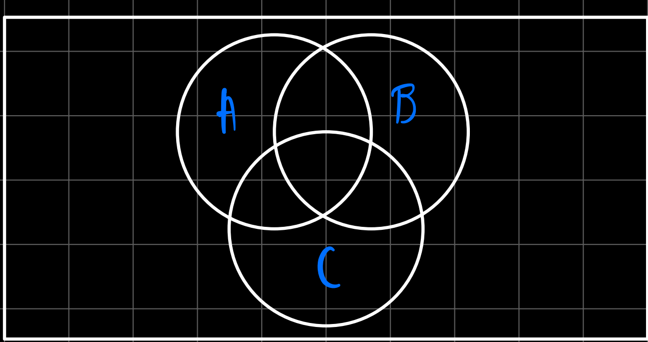
$$P(A \cup B) = \underbrace{P(A) + P(B)}_{\text{these both count } A \cap B} - \underbrace{P(A \cap B)}_{\text{now } A \cap B \text{ is only counted once}}$$

Alternate formula, only in terms of intersections:



$$\Rightarrow P(A \cup B) = P(A \cap \bar{B}) + P(A \cap B) + P(B \cap \bar{A})$$

Union of three events:



$$\begin{aligned} P(A \cup B \cup C) = {} & P(A) + P(B) + P(C) \\ & - P(A \cap B) - P(B \cap C) \\ & - P(A \cap C) \\ & + P(A \cap B \cap C) \end{aligned}$$

Ex. The probability that a randomly selected male is colorblind is 0.05, whereas the probability that a randomly selected female is colorblind is 0.0025.

Let M be the event that a male is chosen

Let C be the event that a colorblind person is chosen

Let F be the event that a female is chosen

Given:

- P(M) = 0.05
- P(C | M) = 0.05 "C given M"
- P(C | $\overline{\text{M}}$) = 0.025
- P($\overline{\text{C}}$ | M) = 0.95

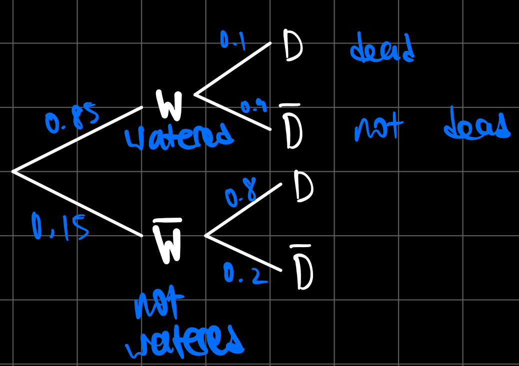
We want P(C)

$$
\begin{aligned}
P(C) &= P(C \cap \overline{M}) + P(M \cap C) \\
&= P(C \mid M) \cdot P(M) + P(C \mid \overline{M}) \cdot P(\overline{M}) \\
&= (0.05)(0.5) + (0.0025)(0.5)
\end{aligned}
$$

Ex.2. Suppose you ask your roommate to water a sickly plant. Without water, the plant will die with probability 0.8 and with water, it will die with probability 0.1.

Roommate will remember to water the plant with probability 0.85.

$$P(D) = \underbrace{(0.85)(0.1) + (0.15)(0.8)}_{\text{multiplication rule}}$$



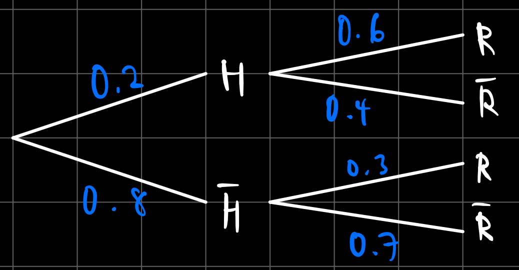
Ex. In a typical year, 20% of the days have a high temperature >22°C. On 40% of these days, there is no rain. During the rest of the year, when the temperature is ≤22°C, 70% of the days have no rain.

Let H = days with a high temperature (>22°C)

Let R = days having rain

Given:

- $P(H) = 0.2$; $P(\overline{H}) = 0.8$
- $P(\overline{R} \mid H) = 0.4 \rightarrow P(R \mid H) = 0.6$
- $P(\overline{R} \mid \overline{H}) = 0.7 \rightarrow P(R \mid \overline{H}) = 0.3$



<u>Bayes' Theorem</u>

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B \mid A)P(\overline{A}) + P(B \mid A)P(A)}$$

Proof: $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B \mid A)\,P(A)}{P(A \cap B) + P(\overline{A} \cap B)} \longrightarrow P(B \mid A) = \frac{P(A \cap B)}{P(A)}$

$\Rightarrow P(A \cap B) = P(A)P(B \mid A)$

$P(\overline{A} \cap B) = P(\overline{A})P(B \mid A)$

<u>Ex.</u> Three methods, A, B, and C, are available for teaching a certain industrial skill. The failure rates are 20%, 10%, and 5% for each of methods A, B, and C, respectively. C is a more expensive method and is only used 20% of the time. The other two methods are used equally often.

Suppose a worker is taught the skill by one of the methods but fails to learn it correctly. What is the probability that he/she was taught by method A? 0.13

F - Failed

$$P(A|F) = \frac{P(A \cap F)}{P(F \cap A) + P(F \cap B) + P(F \cap C)}$$