Bounded Derivative Theorem

If $f(x)$ is continuous and differentiable on $[a,b]$, and $m \le f'(x) \le M$ for all $x \in (a,b)$, then

$$f(a) + m(x-a) \le f(x) \le f(a) + M(x-a)$$

8. Assuming f is differentiable everywhere, which bounds on $f'(x)$ will allow the existence of the provided $f(b)$ for the given $f(a)$:
   (a) if $f(1) = 0$ and $f'(x) \le 2$ then it is possible that $f(2) = 5$
   (b) $f(10) = 10$ and $f'(x) \ge 1$ then it is possible that $f(100) = 99$
   (c) $f(-2) = 2$ and $f'(x) \le -1$ then it is possible that $f(2) = -2$
   (d) $f(-5) = -1$ and $f'(x) \ge -2$ then it is possible that $f(5) = 1$
   (e) $f(6) = 5$ and $f'(x) \le 1$ then it is possible that $f(0) = -5$

$$0 + m(x-1) \le f(x) \le 0 + 2(x-1)$$
$$f(x) \le 2$$

$$10 + (x-10) \le f(x)$$
$$10 + (100-10) \le f(100)$$
$$100 \le f(100) \text{ – FALSE}$$

$a = -2 \quad f(a) = 2 \quad M = -1 \quad f(2) \overset{?}{\le} -2$

$$f(x) \le 2 - 1(x+2)$$
$$f(x) \le -x$$
$$f(2) \le -2 \ \checkmark$$