$$S^* = \bigcup_{k=0}^{\infty} S^k$$

Disjoint union of all k-length tuples of a set S

$$E_{x}$$
. If $B = N$, $B^{x} = \{ \text{set of multisets of } k \text{ types} \}$

$$B^{x} = \{ \text{set of multisets of all types} \}$$

Computing the generating series of S*:

• The weight function of S*, w*, is inherited from the weight function w on S (bijection)

So:

$$\mathbf{W}^{*}(\mathbf{a}_{1},\ldots,\mathbf{a}_{k}) = \mathbf{W}(\mathbf{a}_{1}) + \cdots + \mathbf{W}(\mathbf{a}_{k})$$

w* is a valid weight function if and only if there are no elements with weight (w, not w*) 0 in S

$$|\mathbf{v}^{-1}(0) - \mathbf{v}| = |\mathbf{v}|$$

$$|\{c \in S : \mathbf{w}(c) = 0\}| = |\mathbf{v}|$$

$$\{ C \in S^* : \mathbf{U}^*(C) = 0 \}$$

$$= \{ (\alpha_1, \dots, \alpha_k) \in S^* : \mathbf{U}(\alpha_1) + \dots + \mathbf{U}(\alpha_k) = 0 \}$$

$$S^* = \bigcup_{k=0}^{\infty} S^k$$

There exist an infinite amount of tuples whose weight is 0 (For example, {0, 00, 000, 0000 ...}

$$\Rightarrow |\mathbf{v}'(0)| = 0$$

Then, if
$$u^{-1}(n) = \{(a, ..., a_k) : W(a,) + -- + W(a_k) = n \ k \leq n\}$$

$$|\mathbf{u}^{-1}(\mathbf{n})| < \infty$$

(There is a finite number of elements with weight n)

Also, k can be at most n since:

- The multiset has k elements
- Each element is at least 1

Since
$$w^*$$
 counts the size of a multiset, and in u_2 each element is 1, $w_2^*(a) = ik$ length

Lemma 2.14 (The String Lemma.). Let A be a set with a weight function $\omega : A \to \mathbb{N}$ such that there are no elements of A of weight zero. Then

$$\Phi_{\mathcal{A}^*}(x) = \frac{1}{1 - \Phi_{\mathcal{A}}(x)}$$

Proof. By the Infinite Sum and Product Lemmas 2.11 and 2.12,

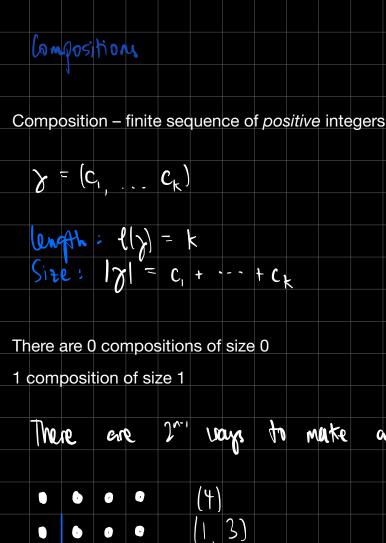
$$\Phi_{A^*}(x) = \sum_{k=0}^{\infty} \Phi_{A^k}(x) = \sum_{k=0}^{\infty} (\Phi_A(x))^k = \frac{1}{1 - \Phi_A(x)}$$

If we apply this weight function to A*, which, in this example, contains multisets of elements that are either 0 or 1, then

$$W_3^*[a_1, ... a_k] = k + \# \text{ of } 1's$$

$$\int_{\Omega} \varphi_{M,r}^{\{0^{r}\}} = X + \chi_{r}^{r}$$

$$\phi_{\{0,1\}*}^{\bullet,*} = \frac{1}{1-x-x^2} \quad (stong lemma)$$



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