

The average velocity, if $S(t)$ denotes the position at time t , from $t=t_0$ to $t=t_1$ is

$$V_{\text{ave}} = \frac{S(t_1) - S(t_0)}{t_1 - t_0}$$

The instantaneous velocity at $t=t_0$ is

$$\begin{aligned} S'(t_0) &= \lim_{t \rightarrow t_0} \frac{S(t) - S(t_0)}{t - t_0} \\ &= \lim_{h \rightarrow 0} \frac{S(t_0 + h) - S(t_0)}{h} \end{aligned}$$

Ex. Find instantaneous velocity at $t=1$ if $S(t) = t^2 + 3t$

$$\begin{aligned} S'(1) &= \lim_{h \rightarrow 0} \frac{S(1+h) - S(1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(1+h)^2 + 3(1+h) - 4}{h} \end{aligned}$$

Ex. 2. Find $S'(t_0)$ for any $t_0 \in \mathbb{R}$

$$\begin{aligned} S'(t_0) &= \lim_{h \rightarrow 0} \frac{S(t_0+h) - S(t_0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(t_0+h)^2 + 3(t_0+h) - t_0^2 - 3t_0}{h} \end{aligned}$$