Simple Linear Regression

Suppose we have datapoints (x_i, y_i)

Assumption: $Y_i \sim G(\mu(x_i), \sigma^2)$

where $\mu(x_i) = \alpha + \beta x_i$

How can we get estimates for α and β?

Since our distribution is Gaussian:

$$L(\alpha, \beta, \sigma) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-1}{2\sigma^2}(y_i - \alpha - \beta x_i)^2\right]$$

Getting log likelihood and solving

$$\frac{dl}{d\alpha} = 0 \qquad \frac{dl}{d\beta} = 0 \qquad \frac{dl}{d\sigma} = 0$$

we have the MLEs

$$\hat{\beta} = \frac{\sum_{i=1}^{n} x_i (y_i - \bar{y})}{\sum_{i=1}^{n} x_i (x_i - \bar{x})} = \frac{S_{xy}}{S_{xx}}$$

$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$$

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{\alpha} - \hat{\beta}x_i)^2 = \frac{1}{n}\left(S_{yy} - \hat{\beta}S_{xy}\right)$$

s_e^2 is an unbiased estimator for σ^2 since $E(s_e^2) = \sigma^2$