Ex. $\lim_{x\to 0^+} xe^{1/x}$

get derivatives

$$= \lim_{x\to 0^+} \frac{e^{\frac{1}{x}}}{1/x} \to \frac{\left(e^{\frac{1}{x}}\right)\left(\frac{-1}{x^2}\right)}{\frac{-1}{x^2}} = \lim_{x\to 0^+} e^{\frac{1}{x}} = \infty$$

<u>Strategy for indeterminate form ∞-∞</u>

Combine the functions into a single term by simplifying/factoring etc. What remains is likely 0/0, ∞/∞, or 0/∞.

Ex. $\lim_{x\to \frac{\pi}{2}^-} \sec x - \tan x$ (type $\infty - \infty$)

$$= \lim_{x\to\frac{\pi}{2}^-} \frac{1}{\cos x} - \frac{\sin x}{\cos x}$$

$$\Rightarrow \frac{1-\sin x}{\cos x}$$ (type $\frac{0}{0}$)

Getting the derivatives:

$$\Rightarrow \frac{\cos x}{-\sin x} = \frac{0}{1} = 0$$

Ex. 2. $\lim_{x\to\infty} \ln(x) - \ln(3x+2)$

$$= \lim_{x\to\infty} \ln\left(\frac{x}{3x+2}\right)$$

$$= \ln\left(\lim_{x\to\infty}\frac{x}{3x+2}\right) = \ln\left(\frac{1}{3}\right)$$

LHR

Strategy for types 0^0, ∞^0, 1^∞

$f^g = e^{g\ln(f)}$

Ex. $\lim_{x \to 0^+} x^x$ (type 0^0)

$\Rightarrow \lim_{x \to 0^+} e^{x\ln(x)} = e^{\lim_{x \to 0^+} x\ln(x)} = e^0 = 1$

Ex. 2. $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$ (type 1^∞)

$= e^{\lim_{x \to \infty} x\ln\left(1+\frac{1}{x}\right)}$

let $u = \lim_{x \to \infty} x\ln\left(1+\frac{1}{x}\right)$

$= \dfrac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} = \dfrac{\frac{-1}{x^2}\left(1+\frac{1}{x}\right)}{\frac{-1}{x^2}} = 1 + \frac{1}{x} \Rightarrow 1$

$1 \quad 0$

$\Rightarrow e^u = e^1 = e$

(review type ∞^0 before quiz) - $\lim_{x \to \frac{\pi}{2}^-} (\sec(x))^{\cos x}$

LHR can fail - ex. $\lim_{x \to \infty} \dfrac{6^x + 3^x}{5^x + 7^x}$

Getting the derivatives, we have

$\lim_{x \to \infty} \dfrac{6^x \ln(6) + 3^x \ln(3)}{5^x \ln(5) + 7^x \ln(7)}$. worse

$$\lim_{x \to \infty} \left(\frac{x}{x+1} \right) = \frac{1}{e}$$

$$\hookrightarrow \left(1 + \frac{1}{x} \right)^{-1}$$