A function has an inverse if and only if it is both one-to-one and onto.

Proof: (<=)

If T: V -> W is both one-to-one and onto, then there exists two distinct vectors v1 and v2 in V such that T(v1) = w1 and T(v2) = w2, for any two vectors w1 and w2 in W. Then:

$$T^{-1}(C\overrightarrow{u}_1 + C\overrightarrow{u}_2) = T^{-1}(CT(\overrightarrow{v}_1) + T(\overrightarrow{v}_2))$$

$$= T^{-1}(T(C\overrightarrow{v}_1 + \overrightarrow{v}_2))$$

$$= C\overrightarrow{v}_1 + \overrightarrow{v}_2$$

$$= CT^{-1}(\overrightarrow{w}_1) + CT^{-1}(\overrightarrow{w}_2)$$

Lemma 2

Let $T \in \mathcal{L}(V, W)$ be a linear transformation from the finite-dimensional vector space V to the finite dimensional vector space W.

Let B_V and B_W be bases for \tilde{V} and W, respectively. Then

T is invertible **iff** $_{B_W}[T]_{B_1}$ is invertible.

Also, if T is invertible, then $B_V[T^{-1}]B_W = (B_W[T]B_V)^{-1}$.



$$\frac{E_{x}}{E_{x}} \cdot \frac{T(a+bx)}{=} = \frac{a+2b}{3a+4b} \cdot \frac{F_{x}}{=}$$

$$\frac{T(1)}{=} = \frac{1}{3} \cdot \frac{2}{4}$$

Inverse:
$$\frac{-1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} \rightarrow T^{-1}(\vec{x}) = \frac{-1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} \rightarrow P_1(\mathbf{R})$$

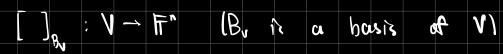
$$=-2a+b+\frac{3}{2}a-\frac{1}{2}b$$

IsomorphismVector spaces V and

Vector spaces V and W are isomorphic if there exists an invertible linear transformation T: V -> W.

Lemma: V and W are isomorphic if and only if dim(V) = dim(W).

As such, if V has n dimensions, then V is isomorphic to F^n since they have the same dimension. This is a linear transformation



(Coordinates of some vector V in B_v)

Definition: Isomorphism

An isomorphism is an invertible linear transformation from V to W.

To prove that T is an isomorphism, we must prove that it is invertible and linear.