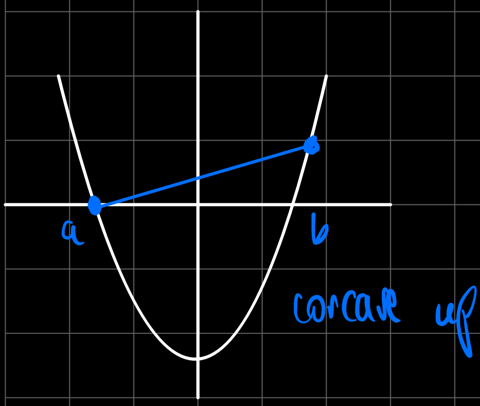
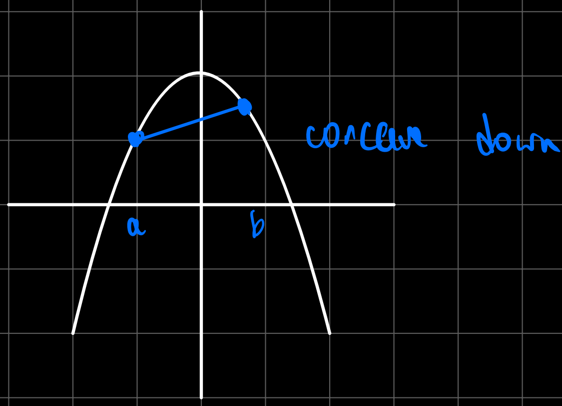
Concavity

- A function f is concave up on an interval I if for all a,b in I, the secant line that joins (a, f(a)) and (b,f(b)) lies *above* the graph.

  no such thing as concavity at a point

- f is concave down if the secant line lies *below* the graph.





Lines, such as y=x or y = |x|, are not concave anything on any interval

If f is concave up, we can see that f’ is increasing, and if f is concave down, f’ is decreasing. This gives:

Theorem

- If f”(x) > 0 for all x in an interval I, then f is concave up on I.
- f”(x) < 0 - concave down

Converse is false because the theorem doesn’t account for when f”(x) = 0: if f” exists and f is concave up, then f”(x) ≥ 0; for example, f(x) = x^4 at x=0 is concave up but f”(0) = 0

Inflection Points

A point x=c is an inflection point if:

- It is continuous at x=c
- Concavity changes

Theorem

If x=c is an inflection point of f and f” is continuous at c, then f”(c) = 0

Ex. Find the intervals of concavity and inflection points of

$$f(x) = x^4 - 6x^2$$
$$f'(x) = 4x^3 - 12x$$
$$f''(x) = 12x^2 - 12$$
$$= 12(x+1)(x-1)$$

$$\Rightarrow f''(x) = 0 \quad @ \quad x = \pm 1$$