Definition of absolute value

$$x = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

x - distance from x to 0
Distance from a to b is given by $|b-a| = |a-b|$

Triangle Inequality

For $x, y \in \mathbb{R}$

$$|x-y| \leq |y-z| + |x-z|$$

proof on OneNote

Triangle Inequality II

Substituting in y = -y and z=0 (for 2 dimensions):

$$|x+y| = |-y+0| + |x-0|$$
$$\therefore |x+y| = |x| + |y|$$

Common absolute value inequalities

$$|x-a| \leq \delta \rightarrow x \in [a-\delta, a+\delta]$$
$$0 < |x-a| \leq \delta \rightarrow x \in [a-\delta, a+\delta], x \neq a$$

$|x-a| \neq 0$
$\therefore x \neq a$

With actual numbers:

$$2 \leq |x-4| < 4$$

Here, the distance between x and 4 must be greater than 2 but less than 4
Therefore the solution set is

$$(0,2] \cup [6,8)$$

(2) $|x-1| + |x+2| \geq 4$

Case 1: $x < -2$

Here, both absolute values are negative

$$-(x-1)-(x+2) \geq 4$$
$$-x+1-x-2 \geq 4$$
$$-2x \geq 5$$
$$x \leq -\frac{5}{2}$$

Case 2: $-2 \leq x \leq 1$

Here, |x+2| is positive while |x-1| is negative

$$-(x-1)+x+2 \geq 4$$
$$-x+1+x+2 \geq 4$$

$\times \rightarrow 3 \geq 4$

Case 3: $x > 1$

Here, both are positive

$$x-1+x+2 \geq 4$$
$$2x \geq 3$$
$$x \geq \frac{3}{2}$$

Final Answer: $(-\infty,-\frac{5}{2}] \cup [\frac{3}{2},\infty)$