$P(A|B)$: probability that A occurs if B has already occured

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

"conditional probability" of event A

Product Rules

$$P(AB) = P(A) \cdot P(B|A) = P(B) \cdot P(A|B)$$

$$P(A) \cdot \frac{P(A \cap B)}{P(A)} \qquad P(B) \cdot \frac{P(A \cap B)}{P(B)}$$

$$P(ABC) = P(A) \cdot P(B|A) \cdot P(C|AB)$$

$$P(ABCD) = \underbrace{P(A)}_{\text{prob. of } A} \cdot \underbrace{P(B|A)}_{\text{prob that B occurs, given A}} \cdot \underbrace{P(C|AB)}_{\text{prob that C occurs, given A and B}} \cdot P(D|ABC) \quad \ldots$$

Law of Total Probability

Assume S is partitioned into events $A_1, \ldots, A_k$

Then $P(B) = \sum_{i=1}^{k} P(BA_i)$ probability that A_i occurs, then given A_i, B occurs

$$= \sum_{i=1}^{k} P(A_i)P(B|A_i)$$

The probability of B occurring is equal to the summed probability of (first) each A_i occurring, and then, given each A_i, that B occurs

Bayes' Theorem

$$P(A|B) = \frac{P(A) \cdot P(B|A)}{P(A) \cdot P(B|A) + P(\bar{A}) \cdot P(B|\bar{A})}$$

Proof :

$$P(A|B) = \frac{P(A) \cdot P(B|A)}{P(B)}$$

Independence

If we need to reassess the probability of an event B after an event A has occured, they are dependent

A and B are independent if and only if

$$P(A \cap B) = P(A)P(B)$$

$$\Rightarrow P(A|B)P(B) = P(A)P(B)$$
$$\Rightarrow P(A|B) = P(A)$$

"given B" : doesn't matter