

Common Taylor polynomials

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$\text{Ex. } \lim_{x \rightarrow 0} \frac{(e^x - 1)(\cos x - 1)x^3}{(e^{x^2} - 1)(\sin(x^2))(\sin^2 x)}$$

TATI Steps

$$e^x = 1 + x + O(x^2)$$

$$e^{x^2} = 1 + x^2 + O(x^4)$$

$$\cos x = 1 - \frac{x^2}{2} + O(x^4)$$

$$\sin(x^2) = x^2 + O(x^6)$$

$$\sin x = x + O(x^3)$$

So the function becomes

$$\lim_{x \rightarrow 0} \frac{(x + O(x^2))\left(\frac{x^2}{2} + O(x^4)\right)x^3}{(x^2 + O(x^4))(x^2 + O(x^6))(x + O(x^3))^2} \rightarrow \dots -\frac{1}{2}$$