Derivatives:

- Constant: 0
- Linear equation: If f(x) = ax+b, f'(x) = a
- Quadratics: f'(x) = 2ax + b
- Trigonometric functions:
    - d/dx sin(x) = cos(x)
    - d/dx cos(x) = -sin(x)

Claim: $\lim_{x\to 0} \frac{\cos x - 1}{x} = 0$

Proof: $\lim_{x\to 0} \frac{\cos x - 1}{x} \cdot \frac{\cos x + 1}{\cos x + 1} = \frac{\cos^2 x - 1}{x(\cos x + 1)}$

$$= \frac{-\sin^2 x}{x(\cos x + 1)} \to \lim_{x\to 0} \underbrace{\left(\frac{\sin x}{x}\right)}_{1} \underbrace{\left(\frac{1}{\cos x + 1}\right)}_{0.5} \underbrace{(-\sin x)}_{0} = 0$$

Also: $\frac{d}{dx}\sin x = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$

$$e = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = \sum_{n=0}^{\infty} \frac{1}{n!}$$

Definition: e is the unique number a such that the tangent line to f(x) = a^x has slope 1 at x=0

i.e. $\lim_{h\to 0} \frac{e^h - 1}{h} = 1$

So $(e^x)' = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h\to 0} e^x \cdot \frac{e^h - 1}{h}$ $\left[\lim_{h\to 0}\frac{e^h-1}{h} = 1\right]$

$$= e^x$$