Theorem:

Say $\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$ $(L, M \in \mathbb{R})$

Then:

1) $\lim_{x \to a} C = C$

2) $\lim_{x \to a} C \cdot f(x) = C \cdot L$

3) $\lim_{x \to a} [f(x) + g(x)] = L + M$

4) $\lim_{x \to a} f(x) g(x) = LM$

5) $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{L}{M}$ if $M \neq 0$

6) $\lim_{x \to a} [f(x)]^{\alpha} = L^{\alpha}$

7) If $M = 0$ and $\lim_{x \to a} \frac{f(x)}{g(x)}$ exists, then $L = 0$

Theorem

If P(x) is a polynomial, then

$$\lim_{x \to a} P(x) = P(a)$$

Limits of rational functions

Consider P(x) / Q(x) where P and Q are polynomials

Case 1: $Q(a) \neq 0$

Just substitute in *a* to get the limit

Case 2: $Q(a) = 0$, $P(a) \neq 0$, then limit doesn't exist (see Rule 7)

Case 3: $P(a) = Q(a) = 0$

If $P(a) = Q(a) = 0$, then $(x - a)$ is a factor of both P and Q - just cancel out $(x - a)$, which is possible since x can never be equal to a (see definition of limits)