Definition: Orthogonality

Let (V, < , >) be an inner product space.

We say that two vectors v and w in the IPS are *orthogonal* if <v, w> = 0.

Ex. $p(x) = 1 + 2x + 3x^2$

In $P_2([-1, 1])$, $\langle f, g \rangle = \int_{-1}^{1} f(x)g(x)\,dx$

Find all polynomials q(x) that are orthogonal to p(x).

$[p(x)]_S = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $[q(x)]_S = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

Recall: $\langle \underline{v}, \underline{w} \rangle = \underline{a}^T G_B \underline{b}$

where $\underline{a} = [\underline{v}]_B$, $\underline{b} = [\underline{w}]_B$

$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 & 2/3 \\ 0 & 2/3 & 0 \\ 2/3 & 0 & 2/5 \end{bmatrix} \begin{bmatrix} \bar{a} \\ \bar{b} \\ \bar{c} \end{bmatrix} = 0$; solve

Theorem: Real Pythagorean Theorem

Let (V, < , >) be a real inner product space and let v, w be in V.

If v and w are orthogonal, then ||v + w||^2 = ||v||^2 + ||w||^2

Theorem: Complex Pythagorean Theorem

Let (V, < , >) be a real inner product space and let v, w be in V.

If v and w are orthogonal, then ||v + w||^2 = ||v||^2 + ||w||^2

and Re(<v, w>) = 0.