Intermediate Value Theorem

If f is continuous over $[a, b]$, and $f(a) < \alpha < f(b)$ or $f(a) > \alpha > f(b)$ $(\alpha \in \mathbb{R})$, there exists a real number c in $[a, b]$ such that $f(c) = \alpha$.

In English: if f is continuous over an interval $[a, b]$, it intersects every y-coordinate in $[f(a), f(b)]$.

IVT problem guide

1. Check if f is continuous over $[a, b]$
2. Calculate $f(a)$ and $f(b)$
   - Check if problem conditions are satisfied: for example, if looking for root, $f(a) < 0 < f(b)$
   - If conditions are satisfied, choose a point c in $[a, b]$ and repeat step 2 with either $f(a)$ and $f(c)$ or $f(b)$ and $f(c)$.

Extreme Value Theorem

If a function f is continuous over a CLOSED interval $[a, b]$, it has a global min/max on $[a, b]$

If $f(x)$, $g(x)$ are continuous over $[a, b]$, $g(f(x))$ does not necessarily have a global max/min over $[a, b]$

$g = \frac{1}{x-4}$ $\quad f = x^2$

g is continuous over $[1, 3]$
f is continuous over $[1, 3]$

$g(f(x)) = \frac{1}{x^2-4}$, which is not continuous at $x = 2$

Important limits

$\lim_{x\to 0} \frac{\sin x}{x} = 1$ $\lim_{x\to\infty} \frac{\ln x}{x^p} = 0 \quad (p > 0)$ $\lim_{x\to\infty} \frac{x^p}{e^x} = 0$

$\lim_{x\to 0} \frac{\cos x}{x}$ DNE

$(x \to 0^+, \infty;\ x \to 0^-, -\infty)$

$\lim_{x\to\infty} \frac{\sin x}{x} = \lim_{x\to\infty} \frac{\cos x}{x} = 0$ (by squeeze theorem)

Solving ln limits (strategies)

- Divide all terms by x
- Use log rules:

  $\ln(x) - \ln(y) = \ln\left(\frac{x}{y}\right)$

$x^2 + x\cos x = 1$

x^2, x, $\cos x$ all continuous over $\mathbb{R}$

Using the closed interval $[-\frac{\pi}{2}, 0]$:

If $x = -\frac{\pi}{2}$:

$\left(-\frac{\pi}{2}\right)^2 - \frac{\pi}{2}\cos\frac{\pi}{2} = \frac{\pi^2}{4} > 1$

$(0)^2 - 0 = 0 < 1$

So, by IVT, has a root over $[-\frac{\pi}{2}, 0]$

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SSI bal nom

$$\frac{2(n-1)}{1+\sqrt{n}} = \frac{2n-2}{1+\sqrt{n}} = \frac{2-\frac{2}{n}}{\frac{1}{n}+\frac{\sqrt{n}}{n}} = \infty$$

$$\frac{2n-2}{1+\sqrt{n}} \cdot \frac{1-\sqrt{n}}{1-\sqrt{n}} = \frac{2n-2n\sqrt{n}-2+2\sqrt{n}}{1-n}$$

$$=$$