Can be beneficial to examine a limit on one side instead of both sides

Definition - right-sided limit

$\lim_{x \to a^+} f(x) = L$ if for all $\varepsilon > 0$, there exists a $\delta > 0$ such that if $0 < |x - a| < \delta$ and $x > a$, then $|f(x) - L| < \varepsilon$

$x < a$ for LHS

Theorem: If f is a function defined on an open interval containing x=a except possibly at x=a, then the following are equivalent:

1) $\lim_{x \to a} f(x) = L$

2) $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = L$

Ex. $\lim_{x \to 0} \frac{|x|}{x}$

$\lim_{x \to 0^+} \frac{|x|}{x} = 1$

$\lim_{x \to 0^-} \frac{|x|}{x} = -1$

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Ex.

$$f(x) = \begin{cases} 7 & x \leq -1 \\ x+1 & -1 < x < 2 \\ x^2 & x > 2 \end{cases}$$

$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} x+1 = 0$

$(x+1)$ approaches -1 from right side because its domain is $(-1, 2]$

$\lim_{x \to -1} f(x) = \lim_{x \to -1} 7 = 7$

Since limit LHS and RHS are different, the limit as x approaches -1 does not exist