$$G(x) = \sum_{n=0}^{\infty} c_n x_n$$

We do *not* care about convergence — we CANNOT evaluate G(c) for some c

$$G(x) + H(x) = \sum_{n=0}^{\infty} (c_n + d_n) x^n$$

$$G(x)H(x) = \sum_{n=0}^{\infty} \left(\sum_{i=0}^{n} c_i d_{n-i} \right) x^n$$

Notation: $[x^n]\,G(x)$ – coefficient of x^n in $G(x)$

Some power series have *multiplicative inverses*:

$$(1 + x + x^2 + \cdots)(1 - x) = (1 + x + x^2 + \cdots) - (x - x^2 - x^3 \cdots)$$
$$= 1$$

In general:

$$G(x) \cdot H(x) = 1$$

$$G(x) = \frac{1}{H(x)} \stackrel{?}{=} \frac{1}{1 - (1 - H(x))}$$

$$\stackrel{?}{=} \sum_{n=0}^{\infty} (1 - H(x))^n$$

$H(x)$ can be expressed as $\sum_{n=0}^{\infty}$ (something)

So if we evaluate the first term of $\sum_{n=0}^{\infty} (1 - H(x))^n$, unless $H(x)$ has 1 as a constant term, it evaluates to 0, leaving us with an infinite sum

Ex. 2. $G(x) = 1 + 2x + 4x^2 + 8x^3 + \cdots$

$$G(x)^{-1} = \frac{1}{1 + 2x + 4x^2 + 8x^3 + \cdots}$$

$$= \frac{1}{1 - (-2x - 4x^2 - \cdots)} = \sum_{n=0}^{\infty} (-2x - 4x^2 - \cdots)$$

As such, G(x) can only have a well-defined inverse if its constant term is 1

Generating Series

Sum Lemma

$$\Phi_{A \cup B}(x) = \Phi_A(x) + \Phi_B(x)$$

Product Lemma

$$\Phi^{\eta}_{A \times B}(x) = \Phi^{\omega}_{A}(x) + \Phi^{\nu}_{B}(x) \quad \text{where } \eta(\alpha, \beta) = \omega(\alpha) + \nu(\beta)$$

↳ Cartesian product: $\{(a, b) : a \in A, \ b \in b\}$

or every possible pair of an element in A with an element in b