If x=c is a local max/min of f, then x=c is a critical point of f.

Local max/mins occur if f(c) exists, and f'(c) = 0 or f'(c) DNE

But not every critical point is a local max/min. As such, to classify critical points, we can use the following methods:

First Derivative Test

If:

- c is a critical point of f
- f is continuous at c
- There exists an interval (a,b) containing c such that
    - f'(x) > 0 for all x in (a,c) and f'(x) < 0 for x in (a,b), then x=c is a local max

f'	+	−
f	inc.	dec.

local max

f'	−	+
f	dec.	inc.

local min

Exercise: Find all local extrema for $f(x) = \frac{x^3}{3} - \frac{3x^2}{2} + 2x + 1$

$f'(x) = x^2 - 3x + 2$

$\Rightarrow (x-2)(x-1) = 0$

@ $x = 1, 2$

		1		2	
f'	+		−		+
f	inc.		dec.		inc.

$x=1$ is a local max

$x=2$ is a local min

x is increasing on $(-\infty, 1]$ and $[2, +\infty)$

decreasing on $[1, 2]$

Both on $x=1$ and $x=2$

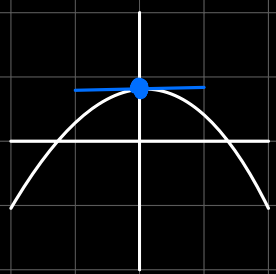
Second Derivative Test (Theorem)

If f'(c) = 0 and f" is continuous at x=c, then:

1. If f"(c) < 0 then x=c is a local max because concave down
2. If f"(c) > 0 then x=c is a local min because concave up
3. If f"(c) = 0, we get no info and you should've used the first derivative test instead you stupid fuck

Ex. Assume f'(c) = 0 (i.e. x=c is a critical point) and f" is continuous at x=c.

Now, f"(c) < 0. This means that the tangent line is horizontal and lies above the curve, meaning that it is a local maximum



Ex. $f(x) = \frac{x^3}{3} + 3x^2 - 7x + 1$

$\Rightarrow f'(x) = x^2 + 6x - 7$

$(x+7)(x-1) = 0$

$\Rightarrow f'(x) = 0$ at $x = -7$, $x = -1$

$f''(x) = 2x + 6$

$f''(-7) = -8 < 0$ - local max

$f''(1) = 8 > 0$ - local min