A graph is connected if there is a path between any two vertices x and y

```
for x in G:
    for y in G:
        path_exists(x,y)
```

Theorem

Let v be a vertex in G. If there is a path from v to any vertex w in G, then G is connected.

```
v = some vertex in G
for w in G:
    path_exists(v,w)
```

Proof: Assume that there is a path from some vertex v to any vertex x and any vertex y.

Then there is a path from x->v and a path from v->y, meaning that there is a path from x->y for any two vertices x and y.

This satisfies the definition.

Ex. Prove that the n-cube is connected for each n≥0.

let $x \in V(G)$

x has k 1's, in positions $i_1, \dots i_k$ (not necessarily consecutive)

can be $i_1, i_3, i_7, i_{12}, \dots i_k$

Let v_j be a binary string with 1's in positions $i_1, \dots, i_j$

⇒ same positions as in $i_1, \dots i_k$, but cut $(j \leq k)$

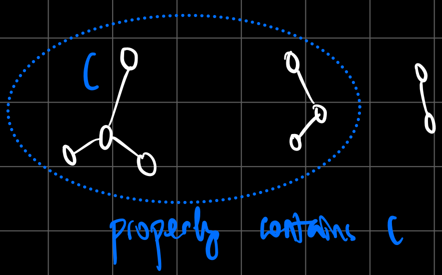
A path from $0^n \to x$ is $v_0 v_1 \dots v_k$

if $x : i_1 i_2 i_7 i_{10}$, $v_7 = i_1 i_2 i_7$

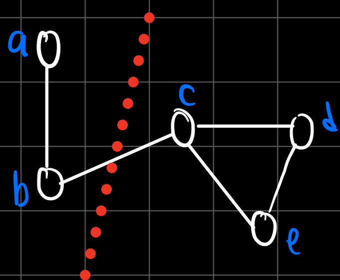
This satisfies the theorem for connectedness

A *component* of G is a subgraph C of G such that

- C is connected
- No subgraph of G that properly contains C is connected ☆
    - A subset of G that contains C must have more than one component



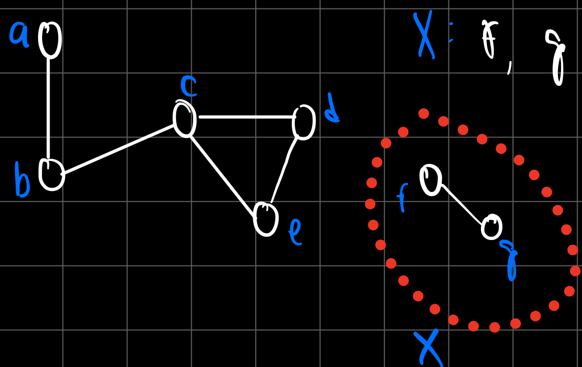
<u>Cut:</u> partition into two disjoint subsets



X : a, b

The cut "induced" by X is the set of edges that have exactly one end in X

A graph is not connected if there exists a proper nonempty subset X of V(G) such that the cut induced by X is empty



X : f, g : no edges with exactly one end in X

X corresponds to a component of a disconnected graph

Summary:

- To show G is connected, find a path between some vertex v and any vertex w
- To show G is not connected, find a cut such that $\mathrm{cut}(X)$ is empty