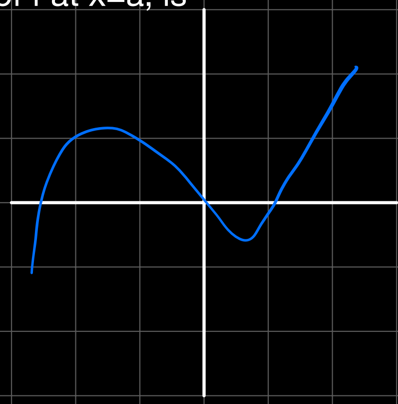
If f(x) is a function, the average rate of change for f(x) from x=a to x=b is

$$f_{ave} = \frac{f(b) - f(a)}{b - a}$$

Meanwhile, the instantaneous rate of change, or derivative of f at x=a, is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$



If the limit exists, then f is differentiable at x=a

Tangent Line

If f is differentiable at x=a, then the tangent line to f at x=a is given by

$$y = f(a) + f'(a)(x - a)$$

Tangent line of $f(x) = x^2 + x + 1$ at $x = 3$:

$$f'(x) = 2x + 1$$
$$f'(3) = 7$$
$$f(3) = 13$$

TL: $13 + 7(x - 3)$

Differentiability vs. Continuity

$f(x) = |x|$

If f is differentiable at x=a, is it continuous at x=a? - *yes.*

Proof: Assume $f'(a) = \lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$ exists

Then $\lim_{x \to a} f(x) = \lim_{x \to a} \left[f(x) - f(a) + f(a) \right]$

$$= \lim_{x \to a} \left[\underbrace{\left(\frac{f(x) - f(a)}{x - a} \right)}_{\text{we assumed this exists}} (x - a) + f(a) \right]$$

$$= \lim_{x \to a} \left[\underbrace{f'(a)}_{\text{exists}} \underbrace{(x - a)}_{0} + \underbrace{f(a)}_{f(a)} \right]$$

$$= f(a) \quad \text{QED}$$

If f is differentiable at x=a, it is also continuous at x=a.

Meanwhile, if it is continuous at x=a, it is not necessarily differentiable at x=a.