Theorem: If f is defined on an open interval containing x=a except maybe x=a, then

(1) $\lim_{x \to a} f(x) = L$

(2) If $\{x_n\}$ is a sequence where $\lim_{n \to \infty} x_n = a$ and $x_n = a$ for all n, then

$$\lim_{n \to \infty} f(x_n) = L$$

Theorem: Function limits are unique

(≥ 1 limit DNE)

We also get nice ways to prove that the limit does not exist:

1) Find a sequence $\{x_n\}$ where $x_n \to a$, $x_n \neq a$ and $\lim_{n \to \infty} f(x_n)$ DNE

2) Find 2 sequences

$$\lim_{x \to 0} \frac{|x|}{x} \qquad x_n = \frac{1}{n} \qquad y_n = \frac{-1}{n}$$

Clearly $x_n \to 0$, $y_n \to 0$, $x_n \neq 0$ and $y_n = 0$ for all n

$$\therefore \lim_{n \to \infty} \frac{|x_n|}{x_n} = 1$$

$$\lim_{n \to \infty} \frac{|y_n|}{y_n} = -1$$

$$1 \neq -1$$