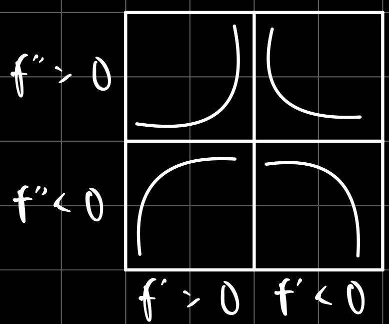
To sketch f(x):

1. Find the domain of f
2. Find x and y intercepts
3. Find vertical and horizontal asymptotes
4. Find f'(x) and all CPs (x and y coordinates)
5. Find f"(x) and any points where f" = 0 or f" does not exist
6. Test points to find intervals of increasing/decreasing concavity, inflection points, and local extrema (make a huge table)
7. Plot interesting points and asymptotes on a graph
8. Connect the dots using the following table:



Ex. $x^4 - 16x^2$

1. Domain: $\mathbb{R}$
2. Intercepts: $0 = x^4 - 16x^2$
   $0 = x^2(x^2 - 16) \rightarrow (0, 0), (4, 0), (-4, 0)$
   $0 = x^2(x+4)(x-4)$

3. No asymptotes
4. $f'(x) = 4x^3 - 32x$
   $\Rightarrow 0 = 4x(x^2 - 8)$
   $\Rightarrow$ CPs: $x = 0, \pm\sqrt{8}$
   $\Rightarrow (0, 0), (\sqrt{8}, -64), (-\sqrt{8}, -64)$

5. $f''(x) = 12x^2 - 32 = 0$

$\Rightarrow \left(\pm\sqrt{\frac{8}{3}}, \ \frac{-320}{9}\right)$

6.

		local min	IP	max	IP	max
		$-\sqrt{8}$	$-\sqrt{\frac{8}{3}}$	0	$\sqrt{\frac{8}{3}}$	$\sqrt{8}$
f''		+		−		+
f'	−	+		−		+
f	↘ C.D.	↗ C.U.	↗ C.D.			
C						

Ex. 2. $x^4 + 6x^3 - 60x^2 + 6x - 60$

$f'(x) = 4x^3 + 18x^2 - 120x + 6$
$f''(x) = 12x^2 + 36x - 120$

Inflection points: $12x^2 + 36x - 120 = 0$
$\Rightarrow 12(x^2 + 3x - 10) = 0$
$12(x+5)(x-2) = 0$
$x = -5, \ x = 2$

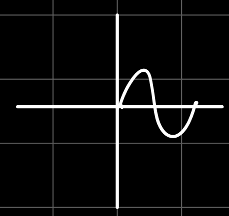
If $x < -5$, $f'' > 0$. So f is concave up.
If $-5 < x < 2$, $f'' < 0$. So f is concave down.
If $x > 2$, $f'' > 0$. So f is concave up.

Ex. 3. $f(x) = \dfrac{\sin x}{1+\cos x}$

$$f'(x) = \frac{(1+\cos x)(\cos x) - (\sin x)(-\sin x)}{(1+\cos x)^2}$$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} = \frac{1+\cos x}{(1+\cos x)^2} = \frac{1}{1+\cos x}$$

$$f''(x) = \frac{-1}{(1+\cos x)^2} \cdot (-\sin x) = \frac{\sin x}{(1+\cos x)^2}$$



Inflection points: $x = 0$, $x = k\pi$ $(\forall x \in \mathbb{Z})$

But the function is not continuous at all $x = k\pi$ where k is odd

$f''(x) < 0$ when $x \in ((2k-1)\pi, 2k\pi]$ for all $k \in \mathbb{Z}$

$f''(x) > 0$ when $x \in [2k\pi, (2k+1)\pi)$ for all $k \in \mathbb{Z}$