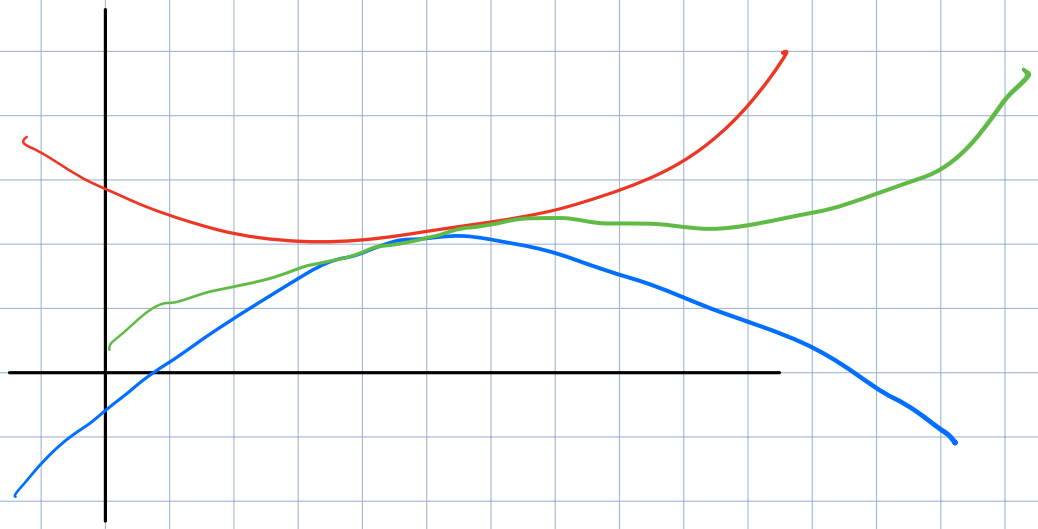
Theorem: Let f, g, h be functions defined on an open interval I containing x=a except possibly at x = a. For x in I, if $f(x) \le g(x) \le h(x)$ and f(x) and h(x) converge to the same limit, then

$$\lim_{x \to a} g(x) = L$$



$0 \le \sin x \le x$ for $x \in (0, \frac{\pi}{2})$

Since $\lim_{x \to 0^+} x = \lim_{x \to 0^+} = 0$, $\lim_{x \to 0^+} \sin x = 0$

Recall $\sin(-x) = -\sin(x)$

$$\lim_{x \to 0^-} \sin x = \lim_{x \to 0^-} -\sin(-x)$$

$$= \lim_{-x \to 0^+} = \sin(-x)$$

$$= -0 \ge 0$$

$\therefore \lim_{x \to 0} \sin(x) = 0$

$$\lim_{x \to 0} \cos x = \lim_{x \to 0} \sqrt{1 - \sin^2 x} = 1$$