

# **Architectures of Intelligence**

Lecture 10

**Neural Networks:  
from Spikes to Symbols**

Jelmer Borst



**university of  
groningen**



# Today

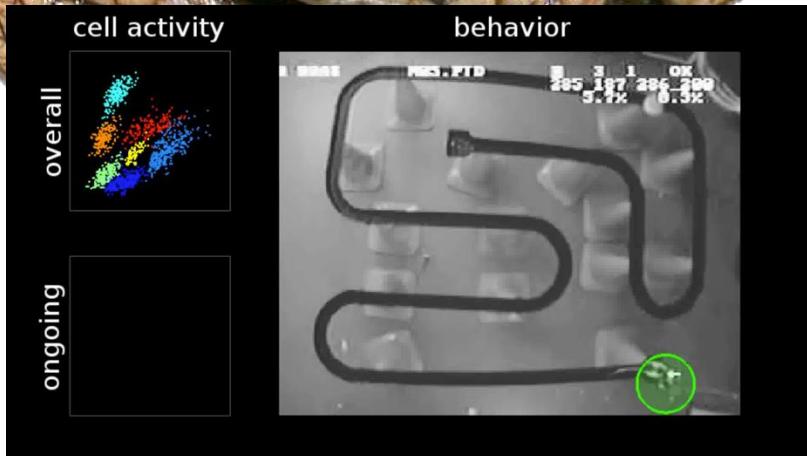
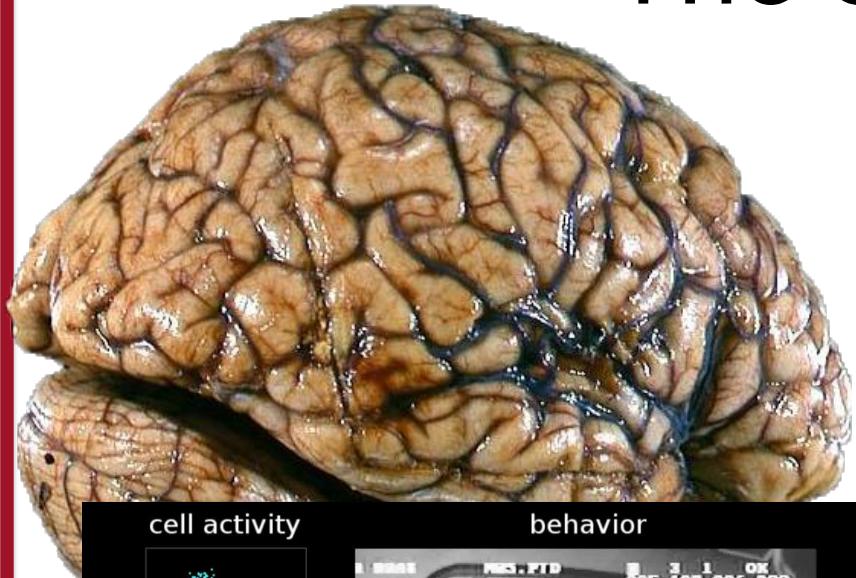
- Neural Networks II: from spikes to symbols
  - Spikes & Symbols
  - Symbol manipulation
  - Cognitive Control
  - Learning in Nengo
- Zbrodoff in Nengo

A photograph of handwritten mathematical work on a grid-lined notebook page. The work shows the steps to solve a system of linear equations. At the top, there is a small diagram with two circles and the number 5 between them. Below this, the equations are written:

$$(4x - (2 - 5y + 2x) + 2y) = 0$$
$$\frac{(x - 1)}{6} = \frac{(x + 5)}{5} = -50$$

The final result is  $x + 3y = 0$ .

# The Challenge



Higher-level  
cognitive functions

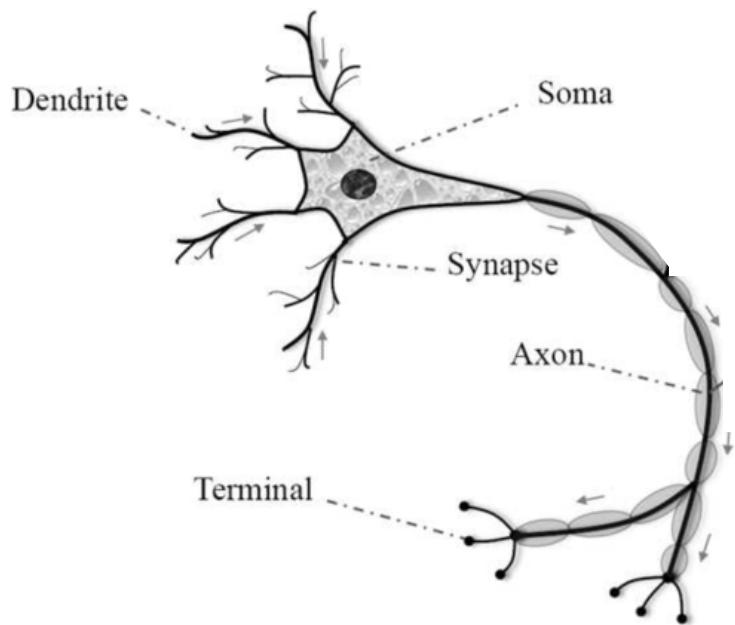
A handwritten mathematical equation on a grid-lined notebook page. The equation is:

$$\frac{(x-1)}{6} = \frac{(x+5)}{5} = -50$$

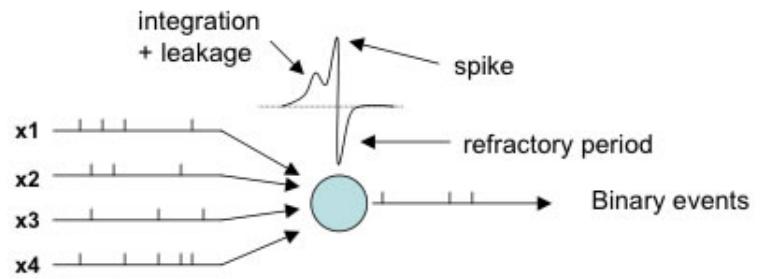
Below the equation, the value  $x = 32$  is written.

86 billion spiking  
neurons

# Recap: Spiking Neurons & Values

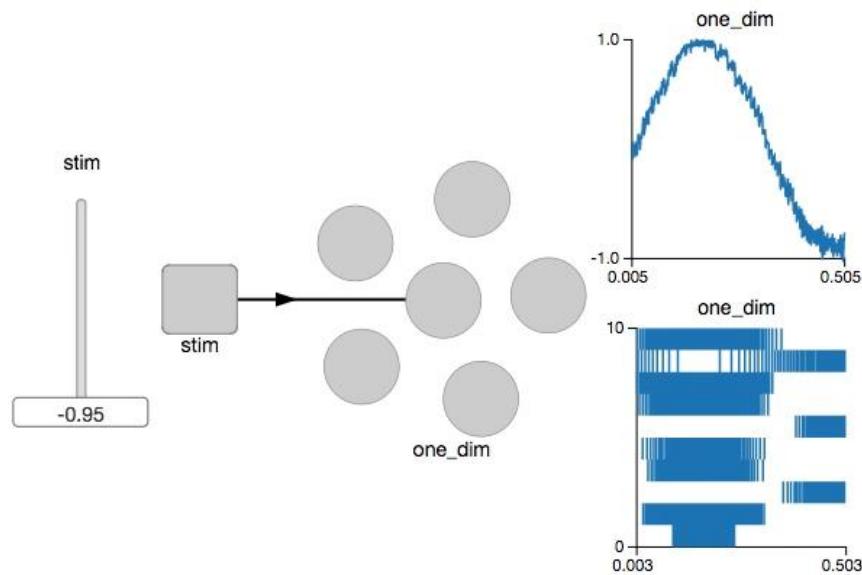


**Artificial spiking neurons**  
(leaky-integrate-and-fire neurons)



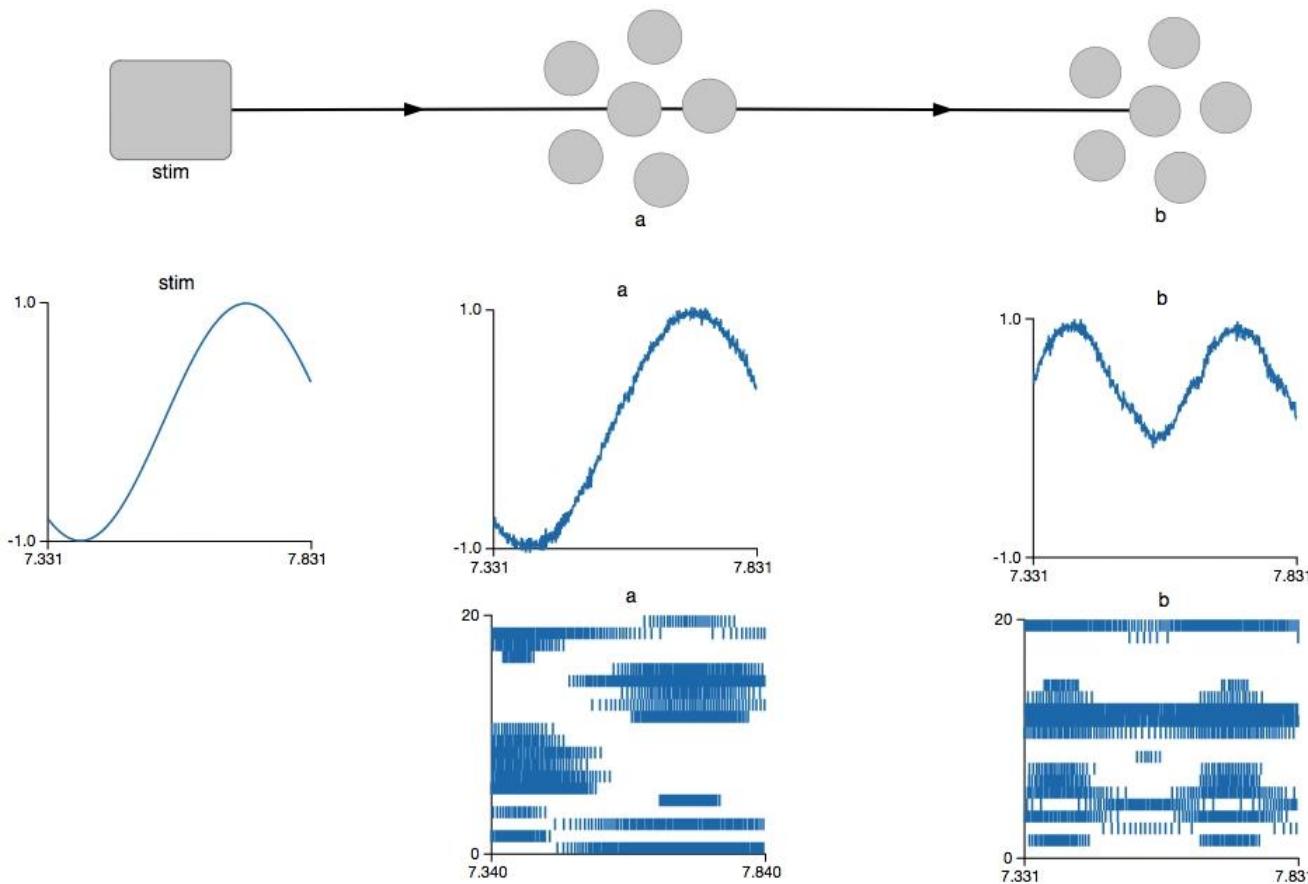
# Representing values

- Ensembles of neurons represent values



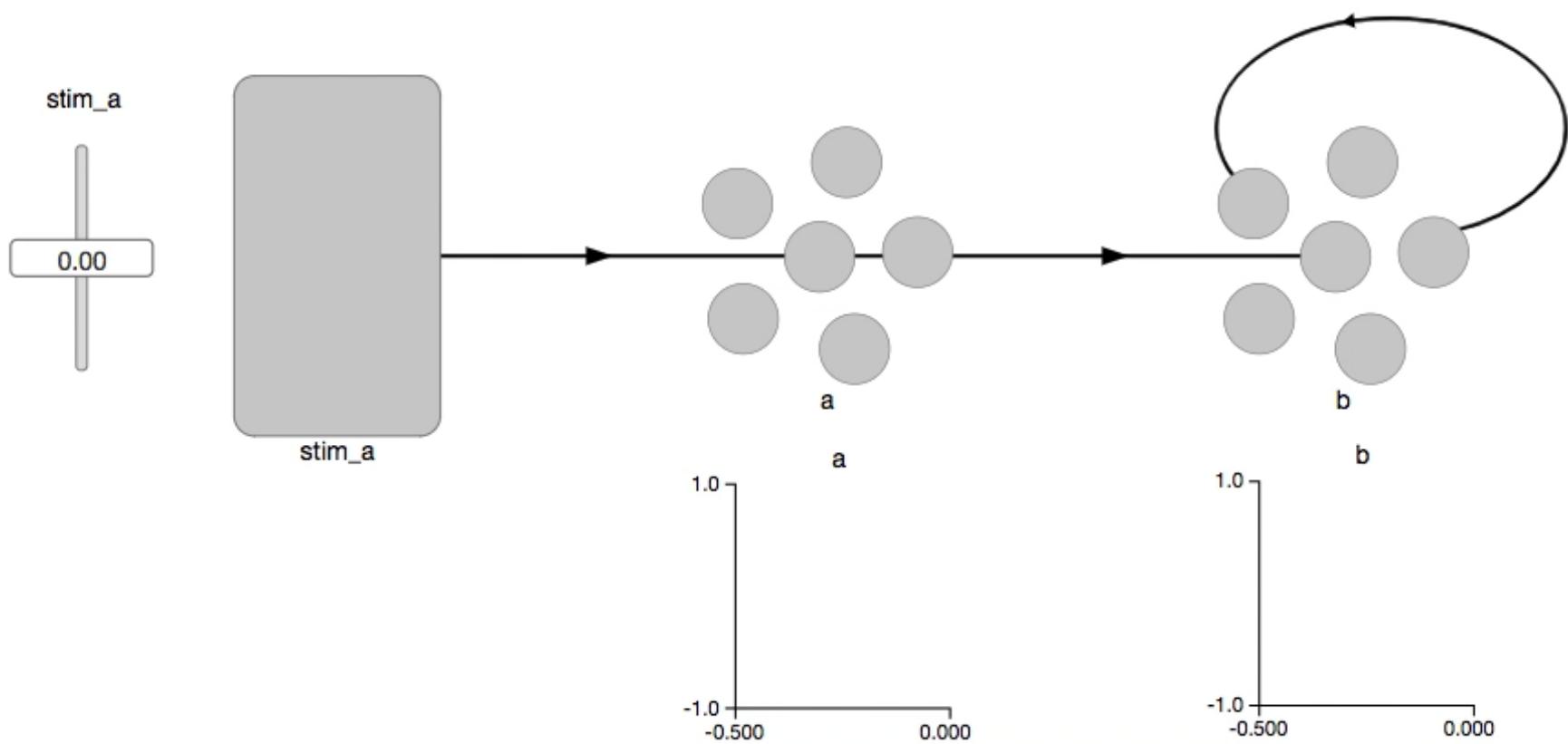
# Representing values

- Values can be communicated and transformed

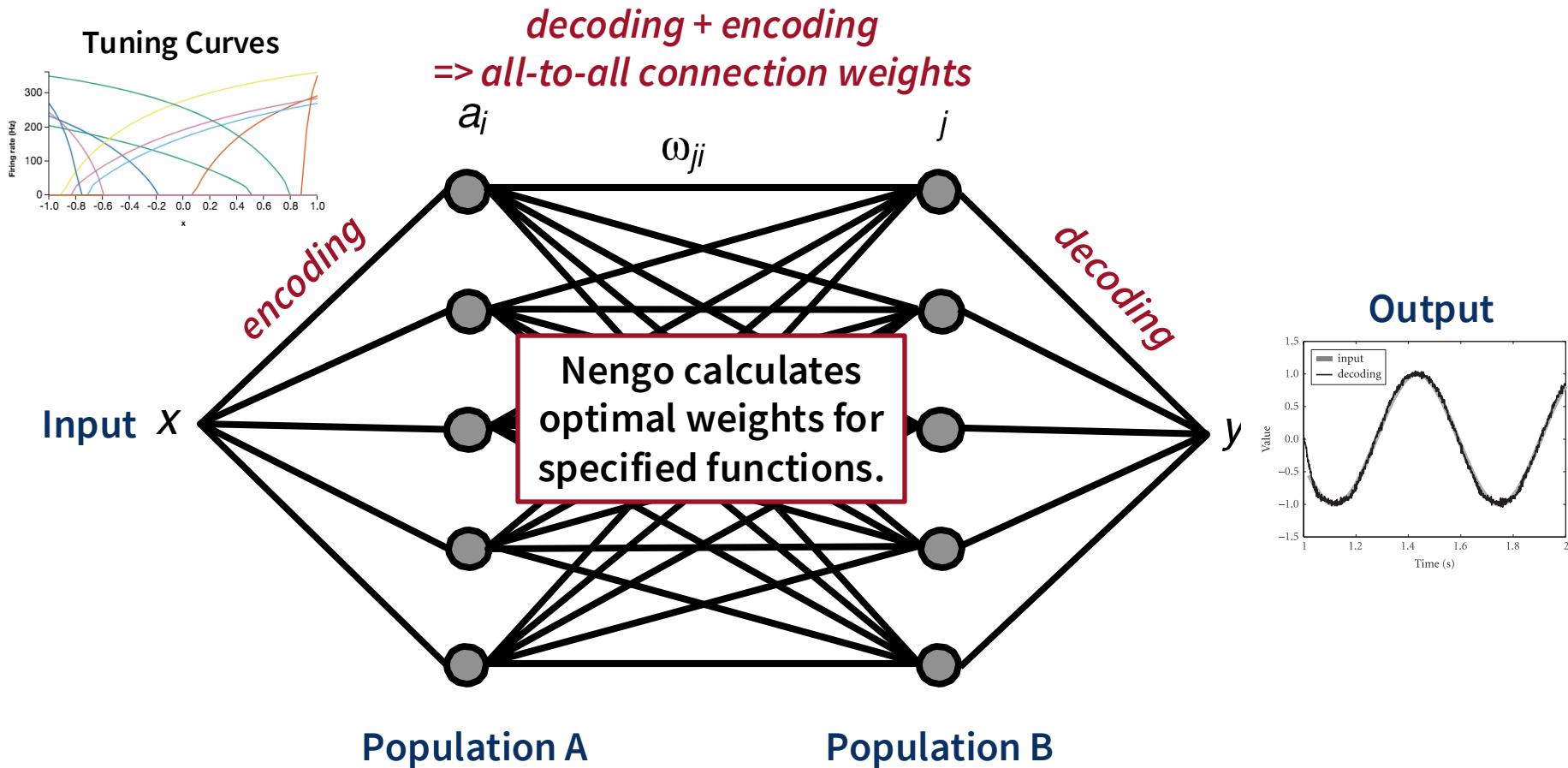


# Representing values

- and stored through recurrent connections



# Encoding and Decoding





# Challenges for (Spiking) Neurons

- How to represent values?
  - How to communicate values?
  - How can we store values?
- 
- How to represent symbols?
  - How to ground symbols in the world?

# Challenges for (Spiking) Neurons

- How to represent values?
- How to communicate values?
- How can we store values?
- How to represent symbols?
- How to ground symbols in the world?

*What are symbols  
and why do we  
need them?*

# Symbols in Spiking Neurons

- Symbols are different from values
- Easy to represent in multiple dimensions – same thing in neurons!

... or are they?

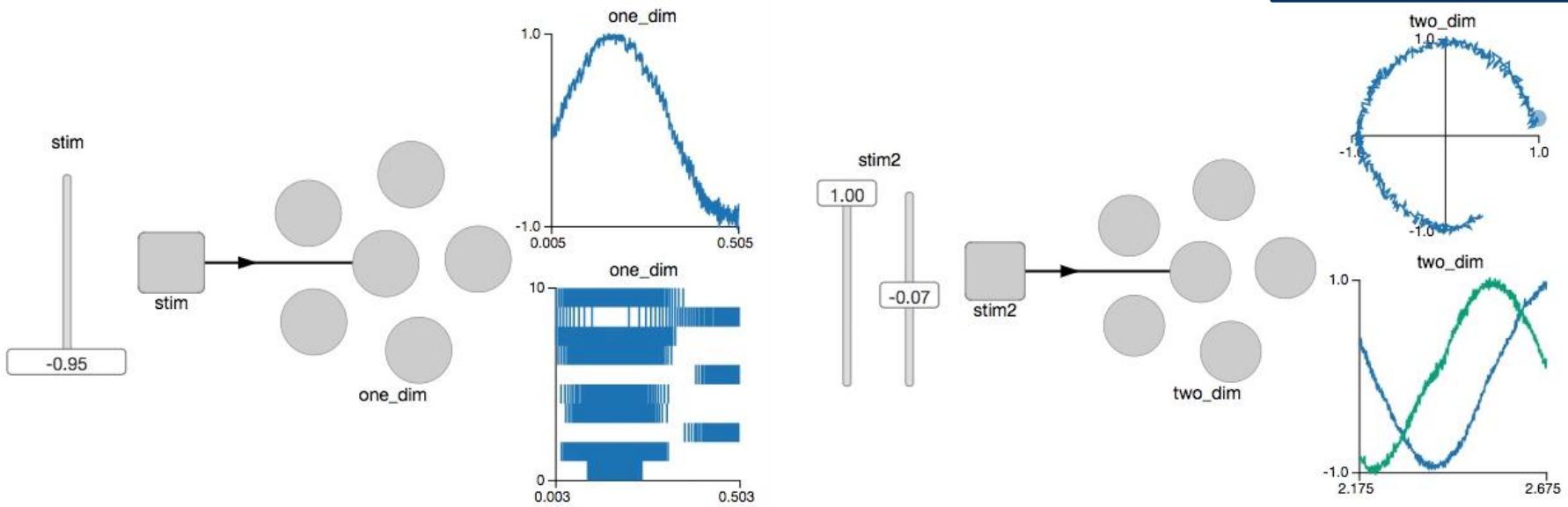
1 = chair

2 = table

[1,0] = chair

[0,-1] = table

[1, 1] = couch

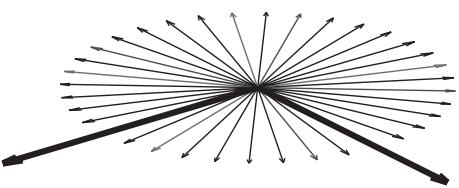


# Symbols in Spiking Neurons

- Symbols are different from values
- Easy to represent in multiple dimensions – same thing in neurons!
- Two dimensions are insufficient

A.

2-D: 36 vectors



# Symbols in Spiking Neurons

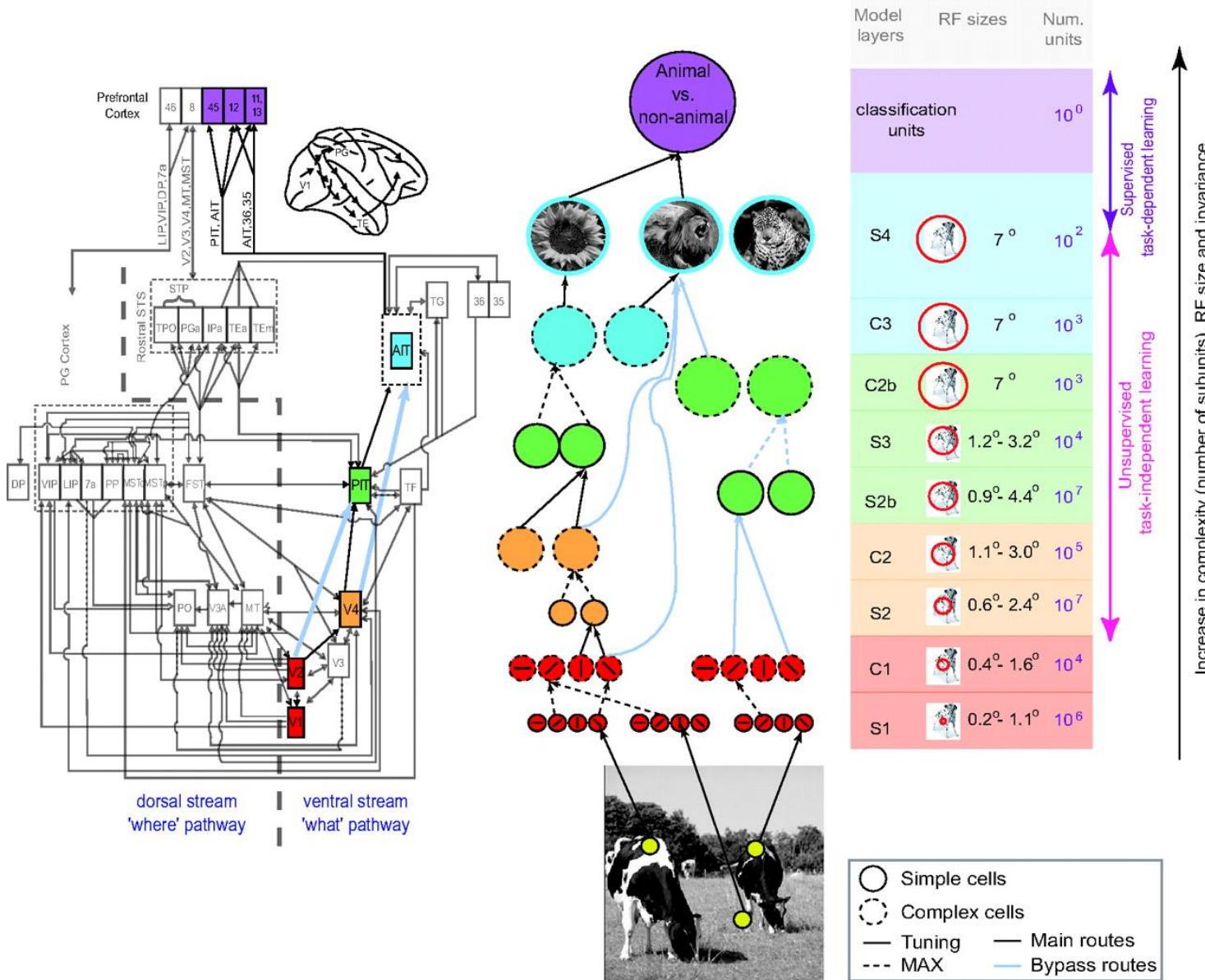
- Symbols are different from values
- Easy to represent in multiple dimensions – same thing in neurons!
- Two dimensions are insufficient
- With ~700 dimensions we can store the whole human vocabulary of ~100,000 concepts in ~40,000 neurons (= .000047% of the brain)
- Symbols are referred to as Semantic Pointers

**Demo!**

# Challenges for (Spiking) Neurons

- How to represent values?
  - How to communicate values?
  - How can we store values?
- 
- How to represent symbols?
  - How to ground symbols in the world?

# SPA: Semantic Pointer Architecture



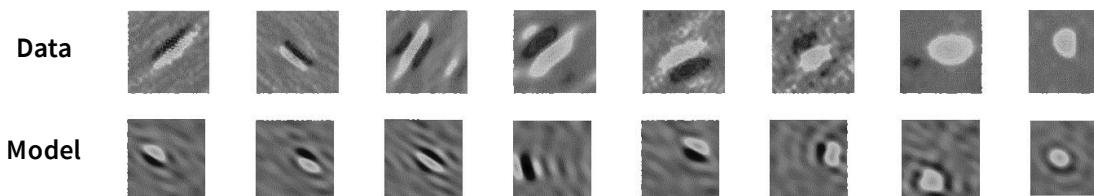
# SPA: Semantic Pointer Architecture

- High-dimensional vectors are referred to as semantic pointers (=symbols)
- The pointer is the activity of the top level of a standard visual processing hierarchy
- The pointer is a compressed representation of the full percept ( $\approx$  jpeg)
- The pointer can be used for ‘symbol manipulation’ ( $\approx$  jpeg file)
- It can also be used to reactivate a full visual representation (lossy ‘decode’)

Visual = Perceptual

# Semantic Pointers applied to Numbers

## Tuning Curves in V1



## Input

**compressed to  
50 dimensional**

## Decompression of 50 dimensions

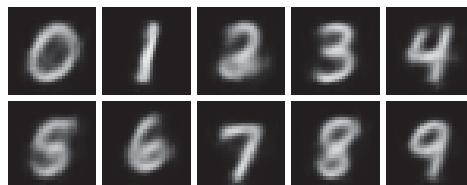
6	7	2	4	5	2	1	8	1	1
8	3	7	0	8	9	3	7	9	0
1	0	1	4	1	9	5	6	2	1

A large red arrow pointing to the right, indicating the direction of the next section.

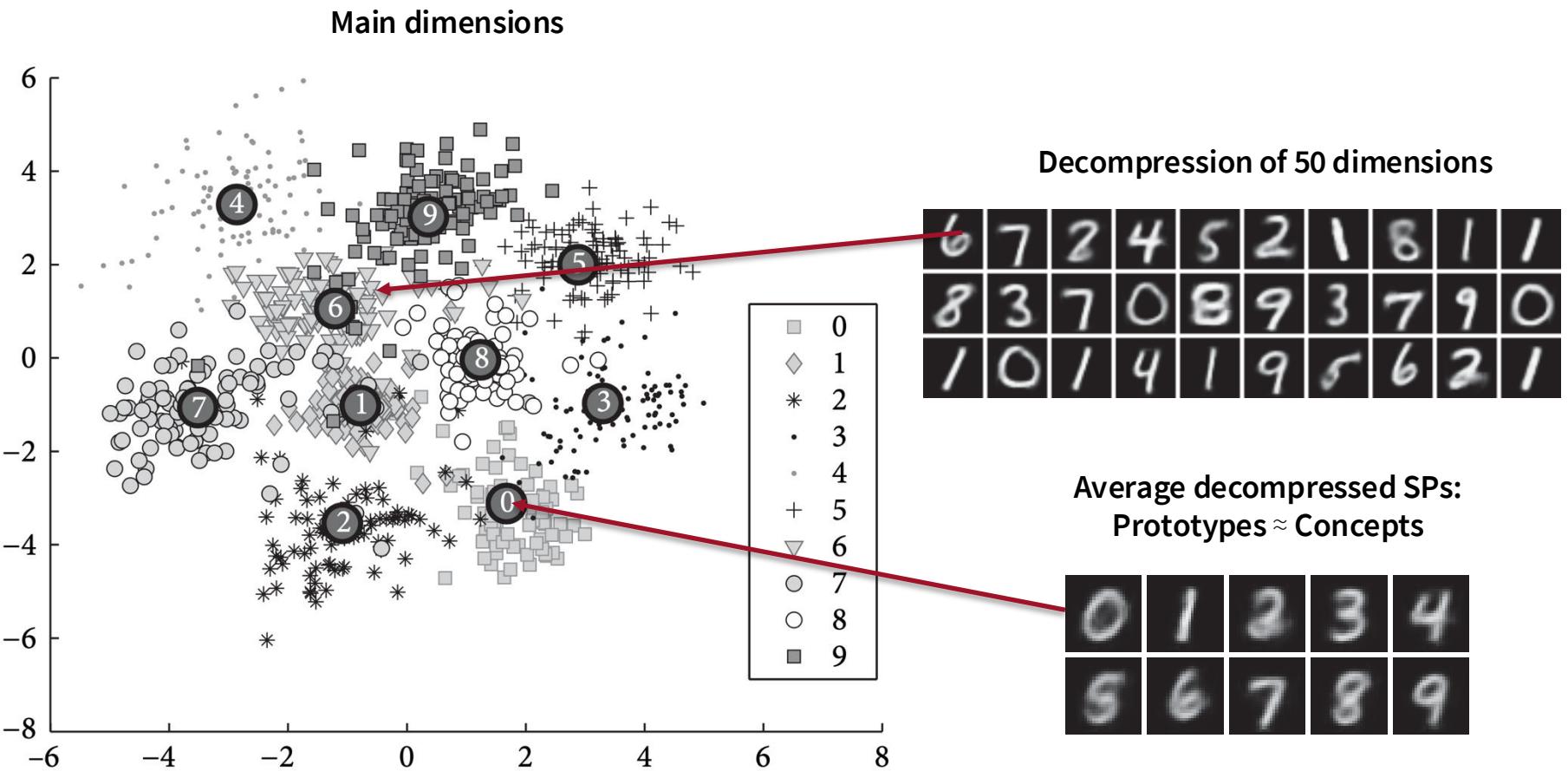
6	7	2	4	5	2	1	8	1	1
8	3	7	0	8	9	3	7	9	0
1	0	1	4	1	9	5	6	2	1

28x28 pixels = 784 dimensions

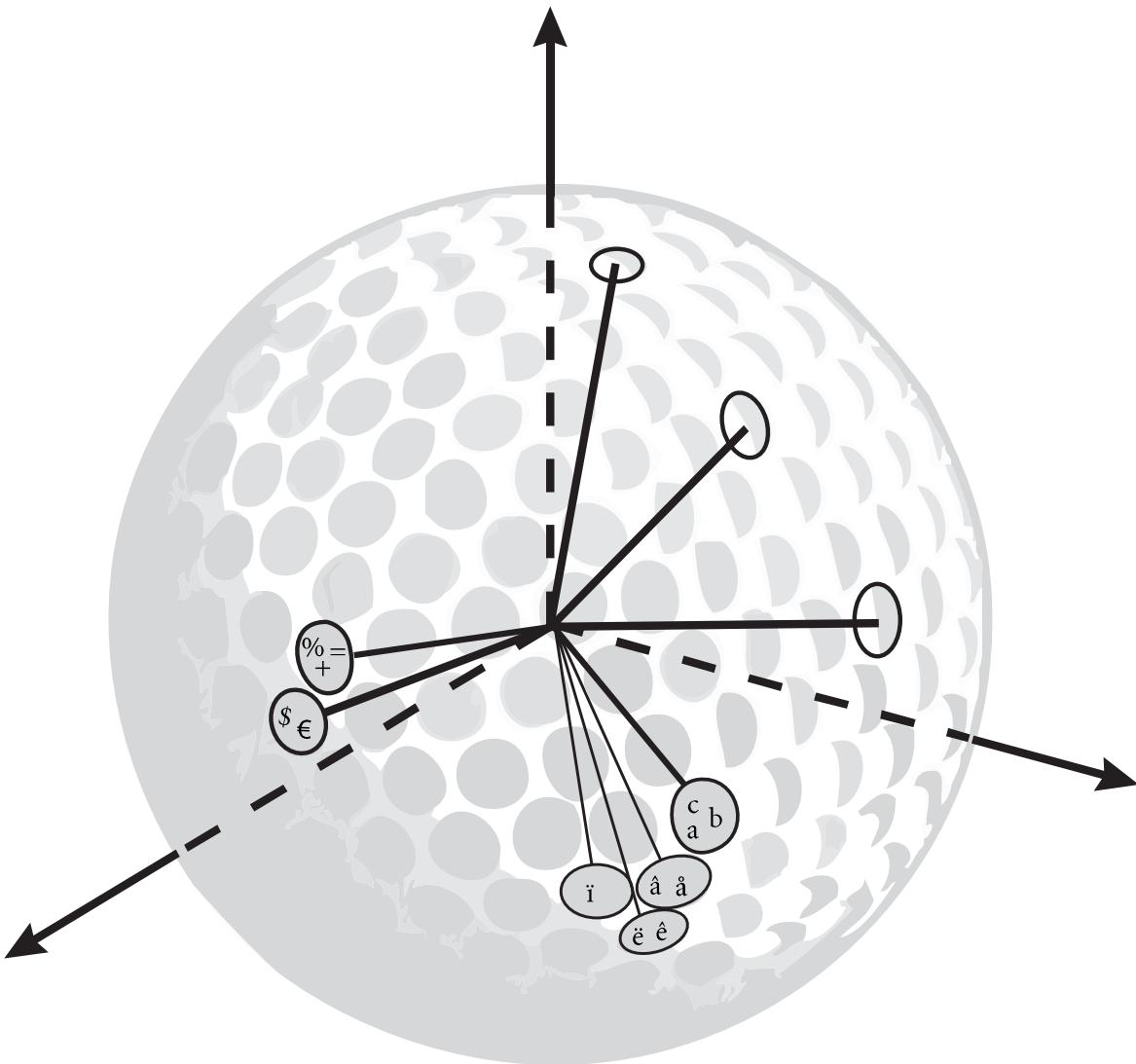
## Average decompressed SPs: Prototypes $\approx$ Concepts



# Semantic Pointers applied to Numbers



# Semantic Pointer Similarity



# Summary

- Symbols are represented by high-dimensional vectors called Semantic Pointers
- Semantic Pointers ultimately point at concepts in the world => they are grounded

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  - Spikes & Symbols
  - Symbol manipulation
  - Cognitive Control
  - Learning in Nengo
- Zbrodoff in Nengo

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The next step shows the equation:

$$\frac{(x-1)}{6} = \frac{(x+5)}{5}$$

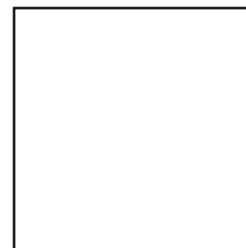
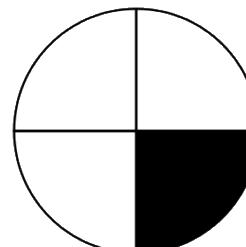
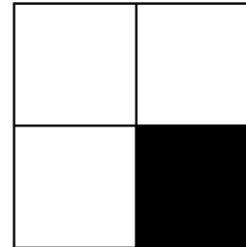
With the final result:

$$3x = 0$$

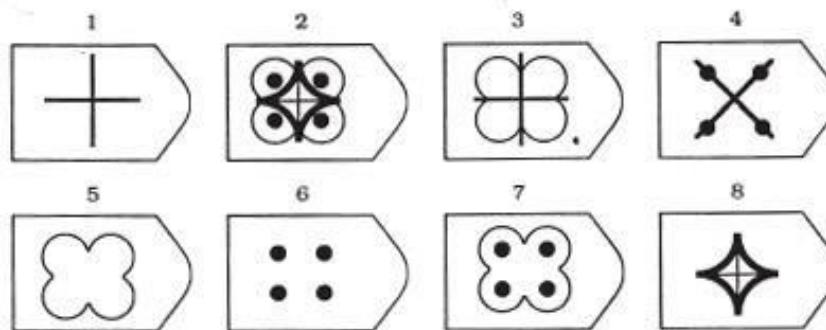
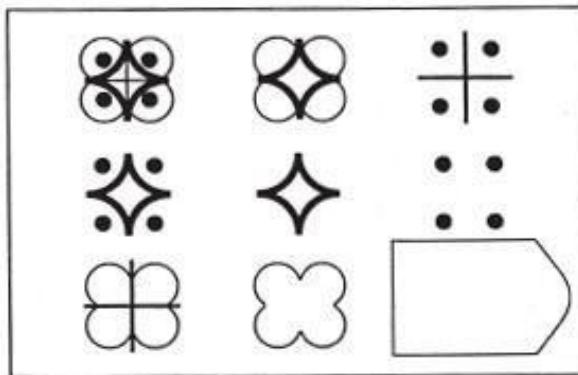
# Raven's Progressive Matrices



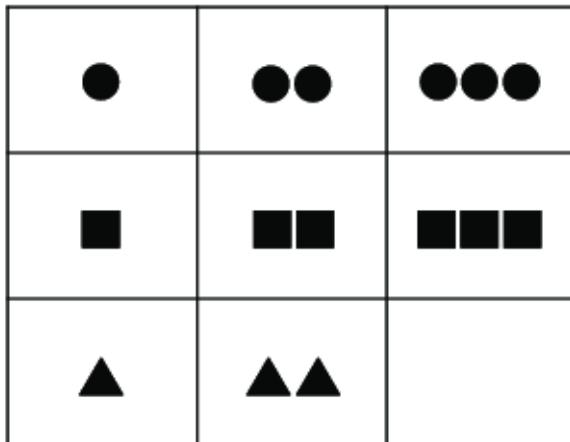
John C. Raven (1902-1970)



# Raven's Progressive Matrices



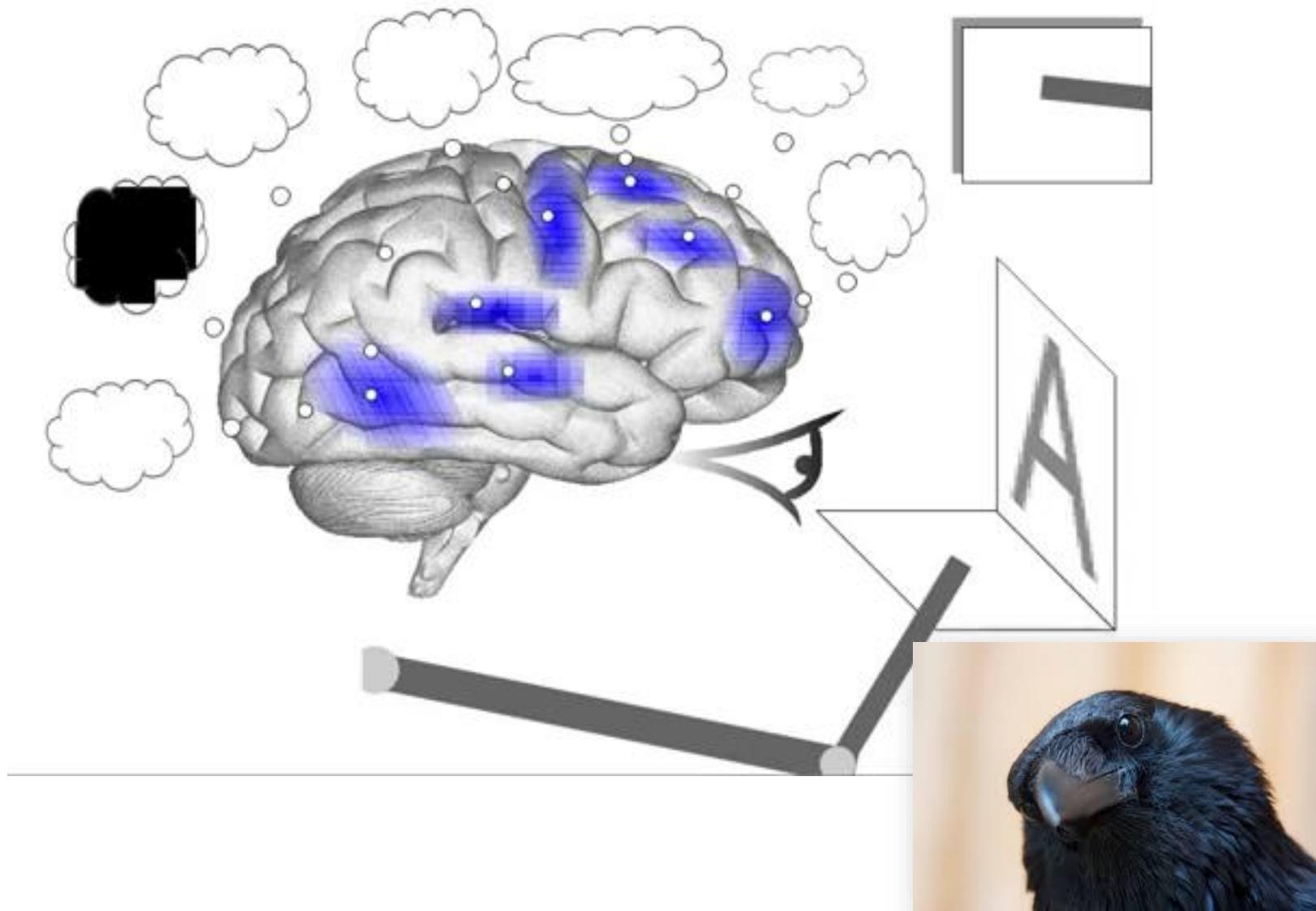
# Raven's Progressive Matrices



1	2	3	4
▲▲▲	▲	■■■	▲▲
▲▲▲▲	■	★★★	■■■■



# Raven's Progressive Matrices

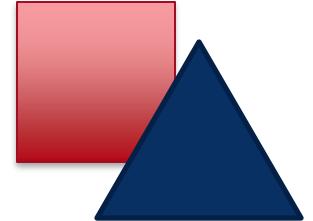


# Symbol manipulation

- Combining symbols:  
RED SQUARE and  
GREEN TRIANGLE
- Role tagging (here:  
DOG CHASES CAT)  
vs  
CAT CHASES DOG



# Binding in Nengo



- Each symbol is represented by a random vector
- Binding through circular convolution:
  - red circle = RED  $\circledast$  CIRCLE = new vector
  - Compare to:  $2 \times 4 = 8$
- Unbinding through inverse:
  - What is red?
    - What was 2 multiplied with?
    - $(2 \times 4) / 2 = (2 \times 4) \times 2^{-1} = 4$
  - $(\text{RED } \circledast \text{ CIRCLE}) \circledast \text{RED}^{-1} = \text{CIRCLE}$ 
    - Deconvolution through convolution with inverse

# Composition in Nengo

- Red circle and blue square =  
RED  $\circledast$  CIRCLE + BLUE  $\circledast$  SQUARE
  - Why not RED + CIRCLE + BLUE + SQUARE?
- Dog chases cat =  
SUBJECT  $\circledast$  DOG + VERB  $\circledast$  CHASES + OBJECT  $\circledast$  CAT



- We can unbind directly from  
  - What is the object?
  - (SUBJECT  $\circledast$  DOG + VERB  $\circledast$  CHASE  
OBJECT<sup>-1</sup> = CAT+ noise



# Pattern Completion

1	11	111
4	44	444
5	55	?

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The next part of the work shows the division of the second equation by 2:

$$\frac{(x-1)}{6} = \frac{(x+5)}{5}$$

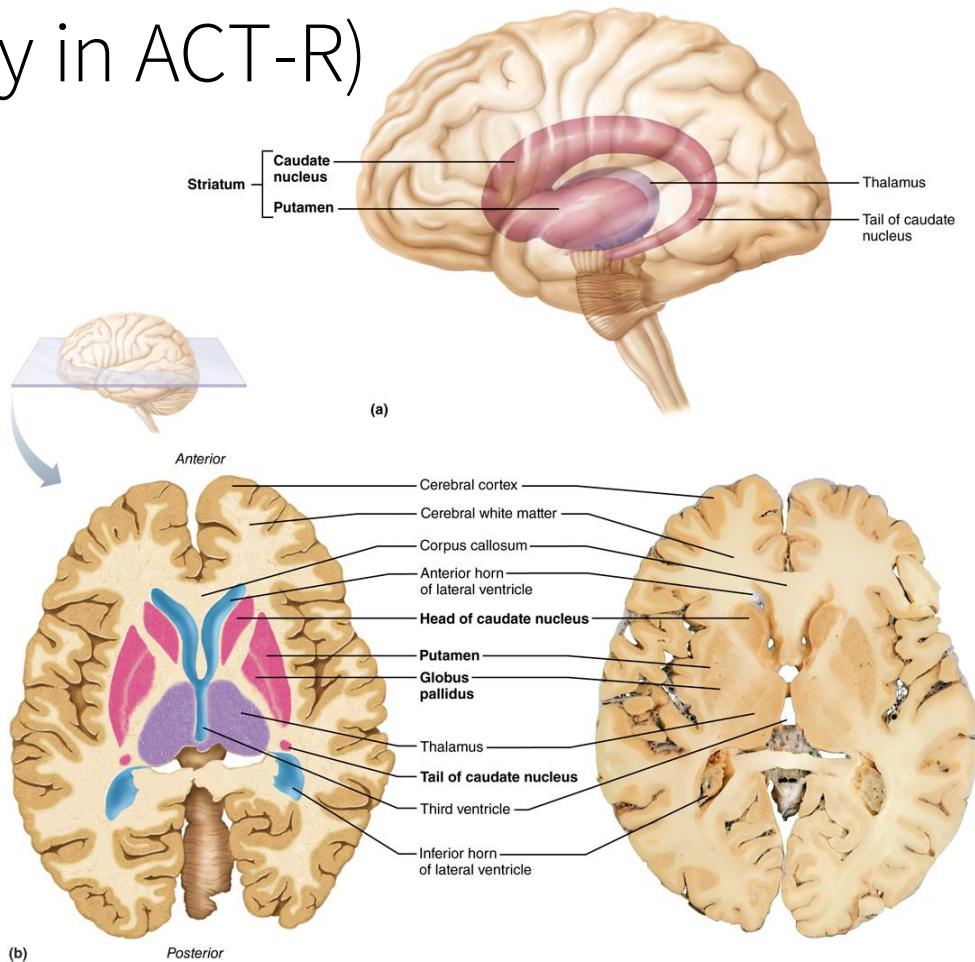
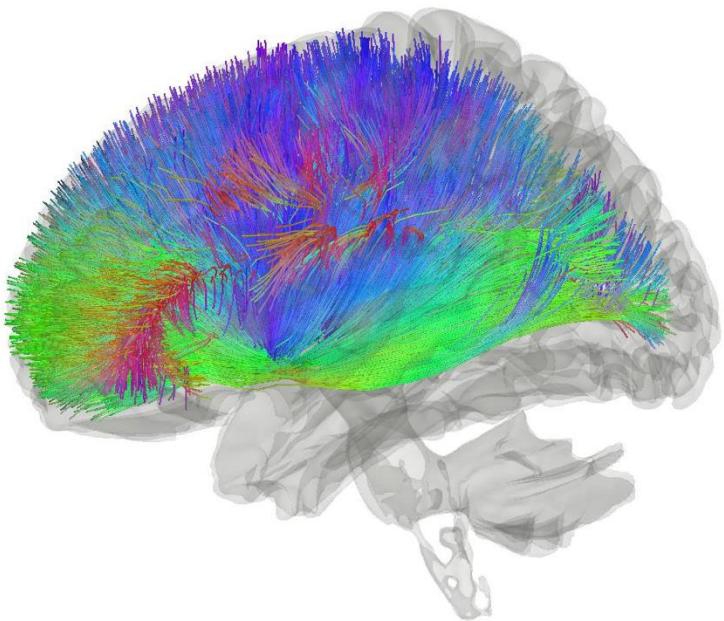
Finally, the work concludes with the equation:

$$3x = 0$$

# Basal Ganglia

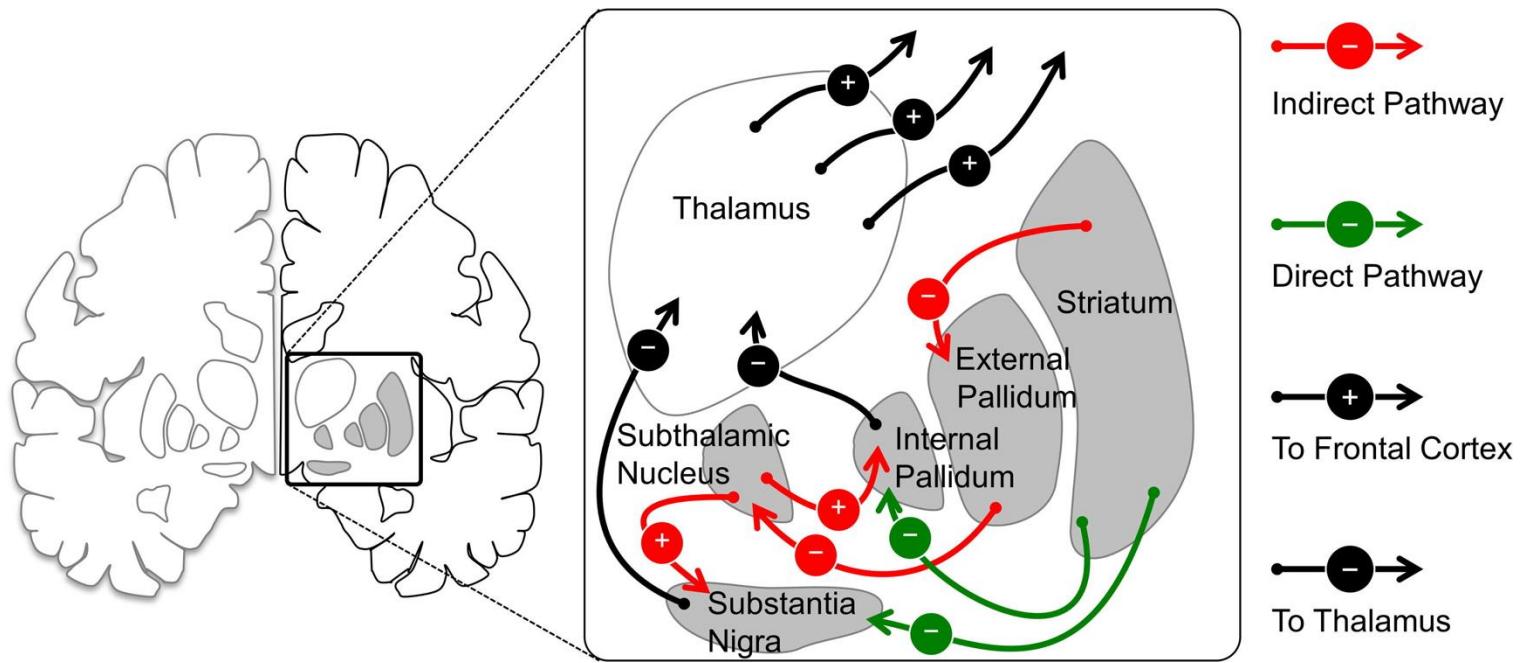
- Action-selection mechanism  
(~ procedural memory in ACT-R)

## Connectivity to Striatum



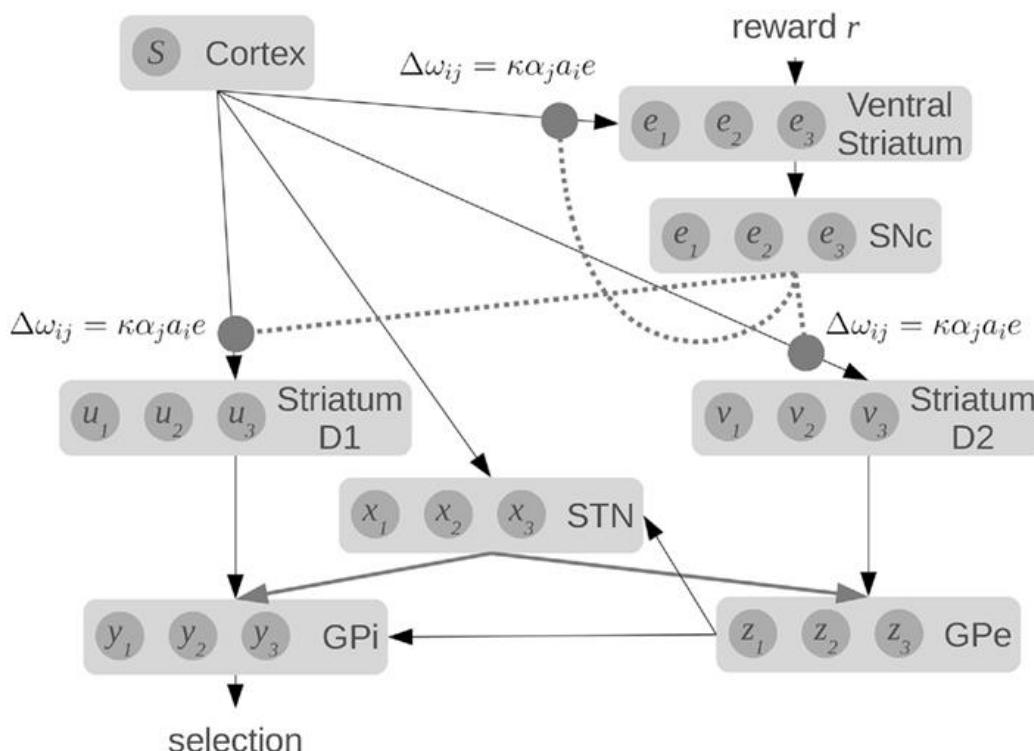
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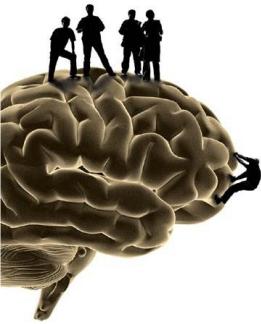
(a) Direct cortico-cortical communication

(b) Basal ganglia

**Not an ACT-R or Nengo assumption**, cornerstone of almost all neural models that do goal-directed processing: e.g., Eliasmith et al., 2012; Hazy et al., 2007; Kriete et al., 2013; O'Reilly & Frank, 2006; Redgrave, Prescott, & Gurney, 1999; Stewart et al., 2012; Stocco, 2017; Stocco et al., 2010, and many others...)

Basal ganglia

Demo!

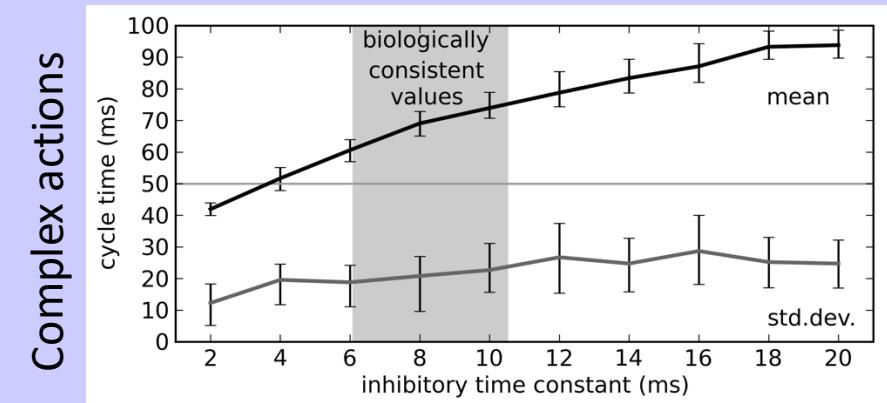
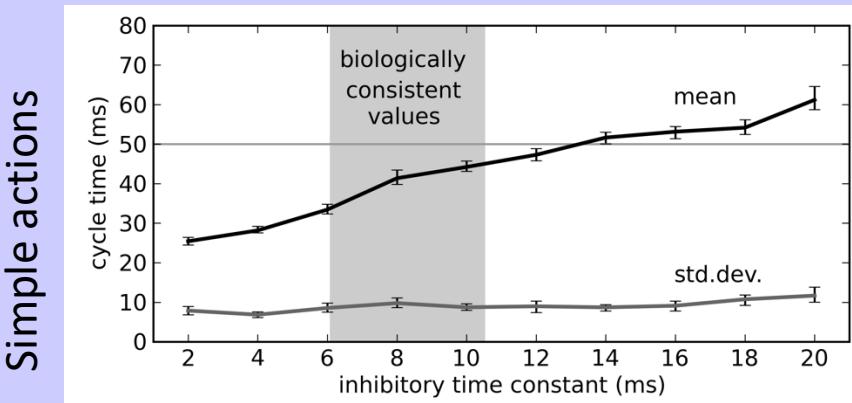


# Connecting Psychology and Neuroscience



- Example

- Standard model-fitting in psychology (e.g. ACT-R) says it takes 50 milliseconds to select and execute an action
- Set neural parameters from neuroscience



- Average: 50 milliseconds (no fitting!)
- 40 for some, 70 for others (new theory!)

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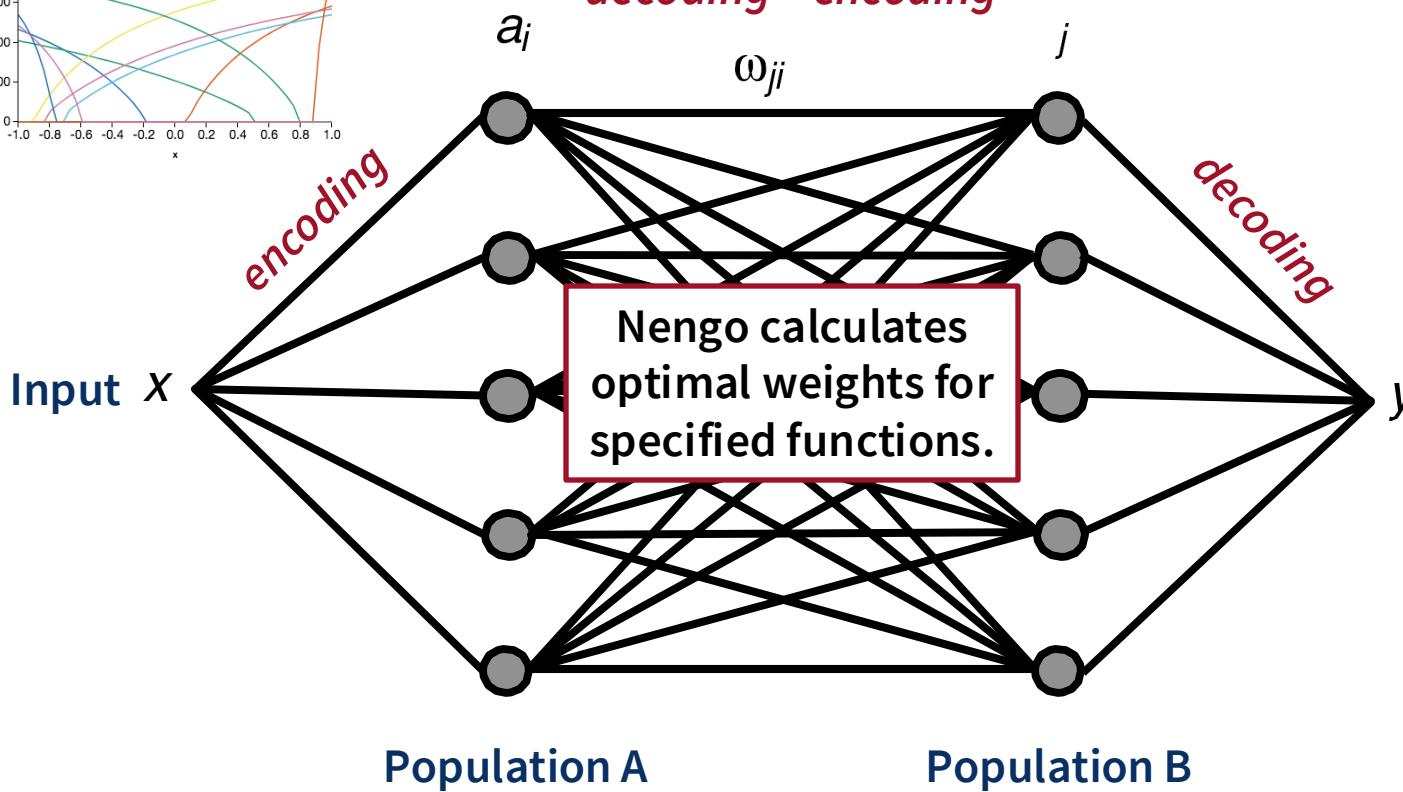
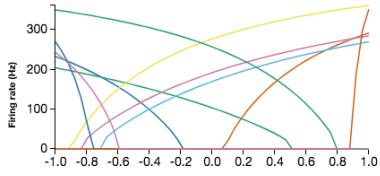
$$\frac{(x-1)}{6} = \frac{(x+5)}{5}$$

And finally, the equation:

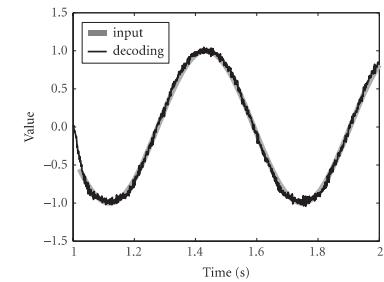
$$x^2 = 0$$

# Learning

Tuning Curves



Output



# Learning

- Learning in neurons: changing connection weights
- Nengo advantage over standard NNs:  
it calculates optimal encoders and decoders  
(= connection weights)
- So why do we need learning?
  - Explain how we got the connection weights in the first place (i.e. no intelligent design)
  - Explain how online learning works
  - Sometimes it is difficult to pre-specify weights (especially in basal ganglia), or they might not be optimal

# Learning in ACT-R

- Instance-based learning
  - Repetition yields stronger memories
  - Nengo:
    - Hebbian learning (unsupervised)
    - PES (supervised)
- Utility learning
  - Reinforcement learning: rewards result in higher utilities
  - Nengo: Q-learning

# Hebbian learning

- Hebbian learning:
  - Neurons that fire together, wire together
  - Unsupervised
- Nengo: BCM rule (Bienenstock, Cooper, & Munro, 1982)

$$\Delta w_{ij} = k a_i a_j (a_j - q)$$

learning rate

activity neurons  $i$  and  $j$

modulation depending on activity  $j$

# PES learning

- ~ Delta rule in classical NNs
    - Supervised learning
    - Requires knowing the error between what you represent and what you want to represent
  - Nengo: PES rule (Prescribed Error Sensitivity)

$$\Delta\omega_{ij} = \kappa a_i (\alpha_j E \cdot e_j)$$

learning rate

activity neuron  $i$

error given properties of neuron  $j$

## Demo!

# Learning in ACT-R

- Instance-based learning
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  - Nengo:
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# Utility learning

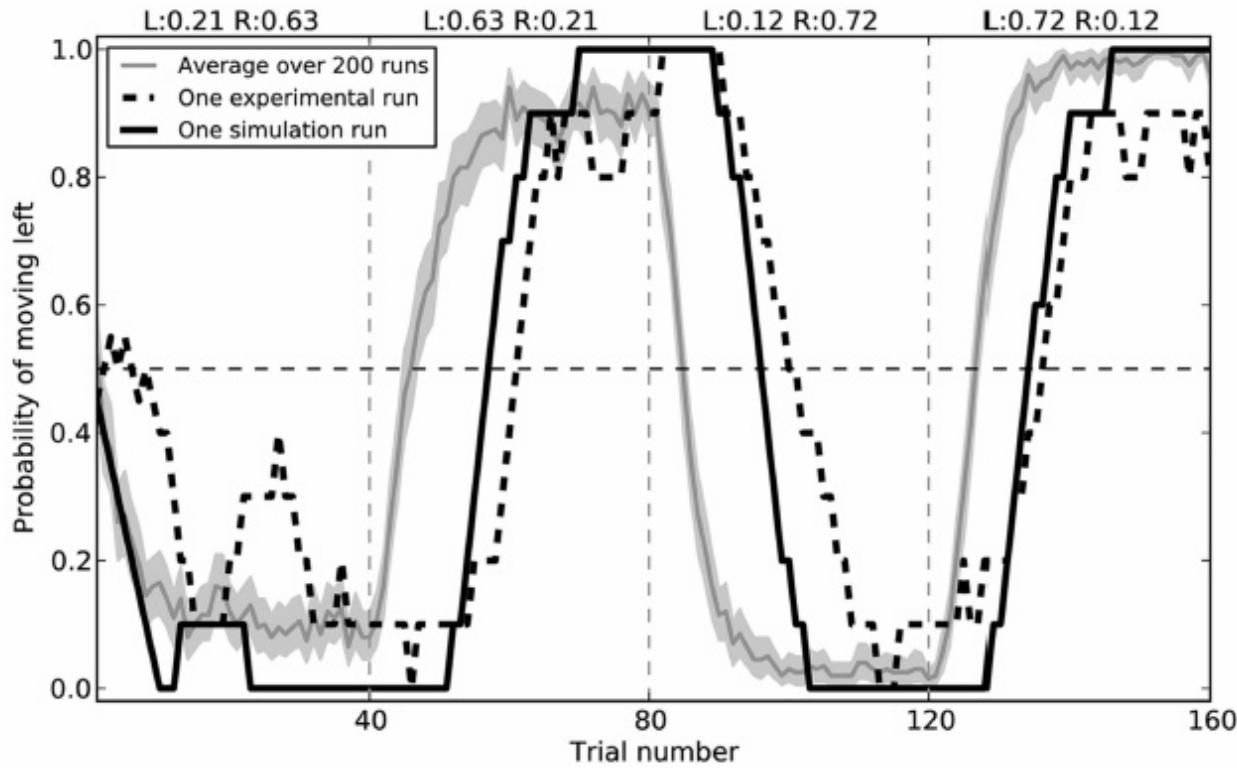
- Learning how to act (vs new knowledge)
- Reinforcement learning (Q-learning)
  - requires a reward signal
- Basal Ganglia
- Dopamine indicates (unexpected) reward
- Example: probability learning in rats

# Choose a button

A

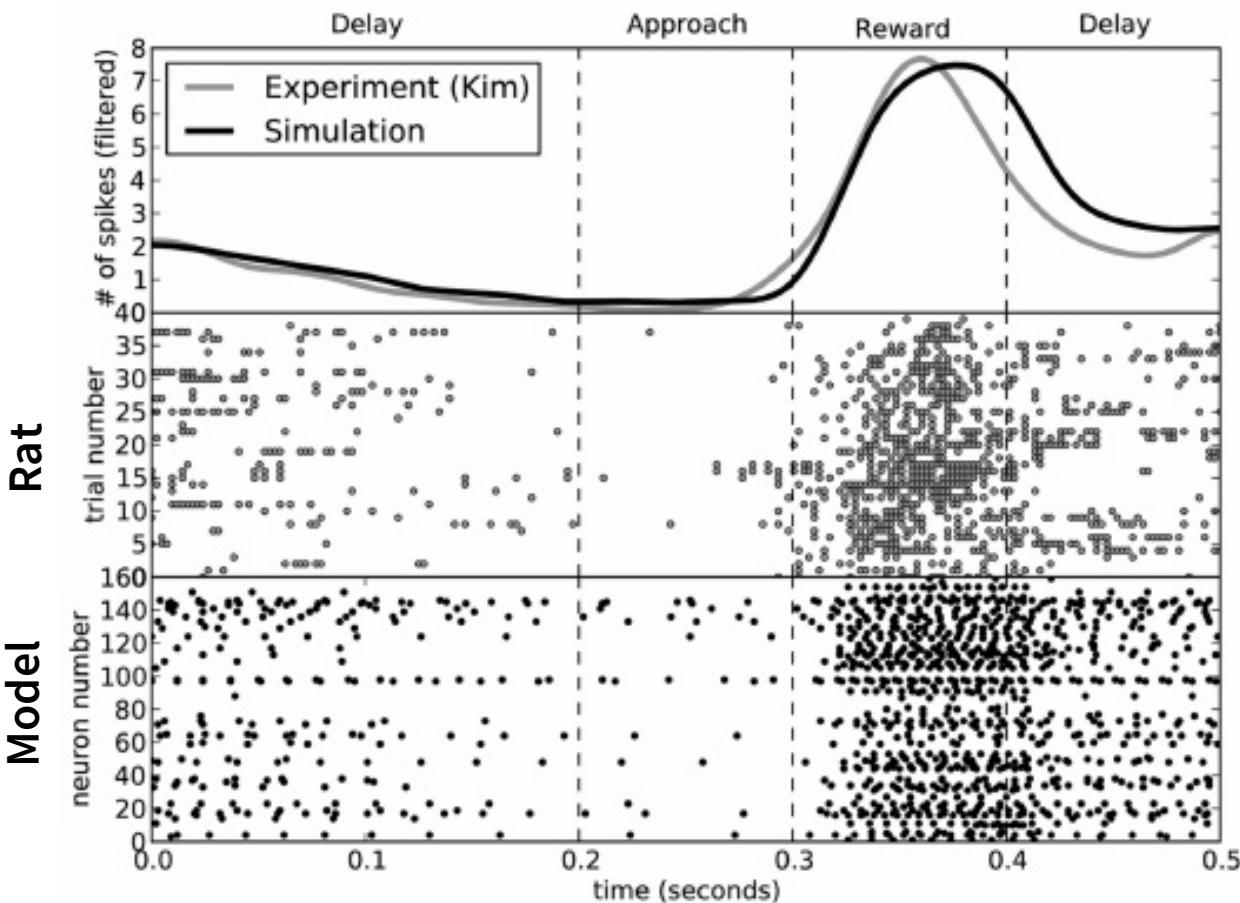
B

# Probability learning



Demo!

# Utility learning: Firing Rates in Striatum



# What about memory?

- Build-in memory system (without learning)
  - Tries to match input
    - cf. +retrieval>

isa	addition-fact
addend1	2
addend2	4
    - Could be learned through PES learning  
=> associative recognition example
- However, some things are missing:
  - No activation levels
  - No different retrieval times
  - No online learning:  
no recency or frequency effects

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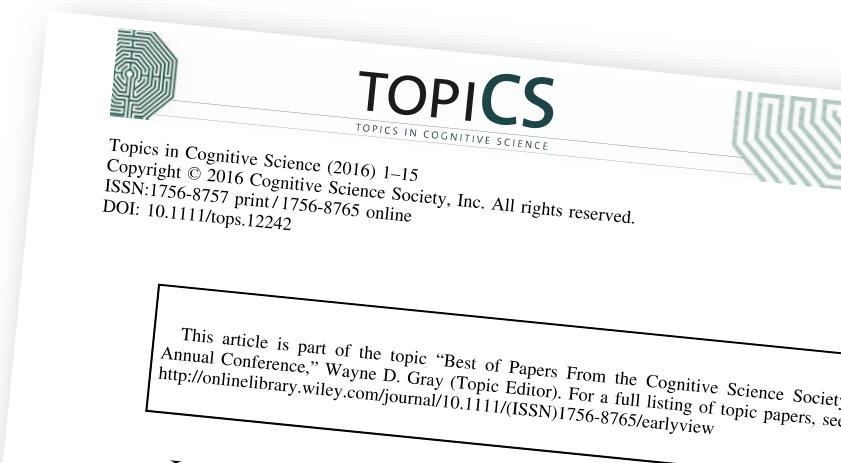
With the final result:

$$3x = 0$$

# Nengo model of mathematical development

- Task: learning addition ( $4 + 5 = 9$ )
  - Two learning phases:
    - Counting
    - Retrieval

$\approx$  Zbrodoff model in ACT-R



# Improving With Practice: A Neural Model of Mathematical Development

Sean Aubin, Aaron R. Voelker, Chris Eliasmith  
*Centre for Theoretical Neuroscience*

Received 7 October 2016; accepted 19 October 2016

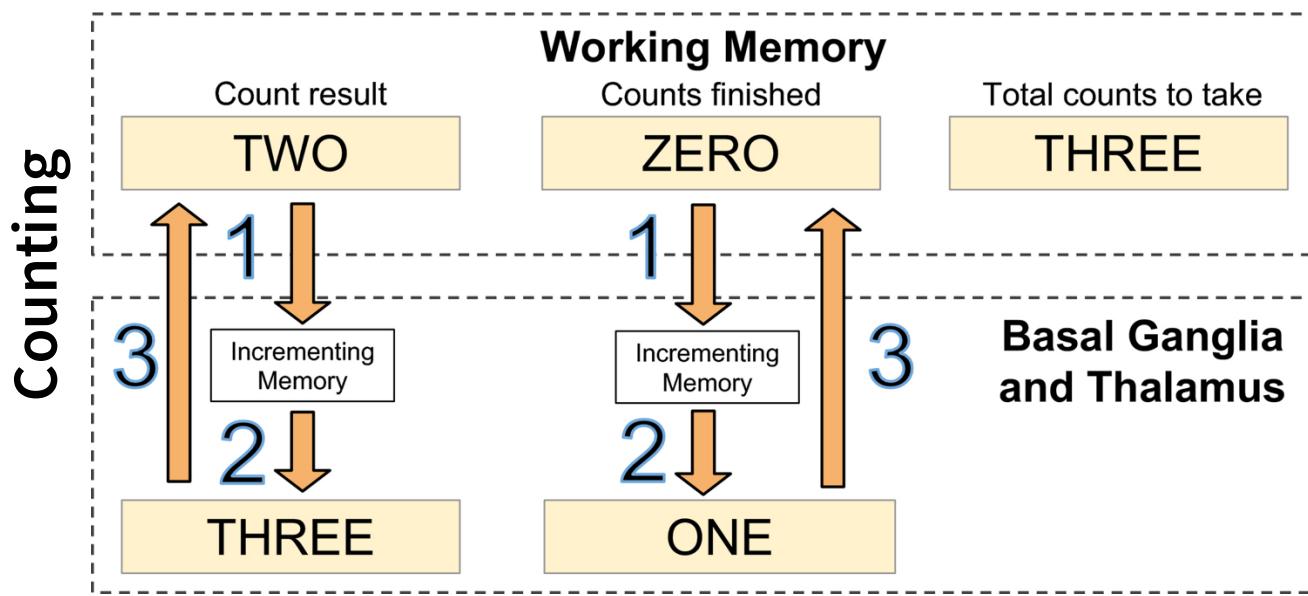
### **Abstract**

The ability to improve in speed and accuracy as a result of repeating some task is an important hallmark of intelligent biological systems. Although gradual behavioral improvements in practice have been modeled in spiking neural networks, few such models have attempted to explain cognitive development of a task as complex as addition. In this work, we model the progression from a counting-based strategy for addition to a recall-based strategy. The model consists of two networks working in parallel: a slower basal ganglia loop and a faster network. The slow network methodically computes the sum of the digits, while the fast network responds to the addition of the digits.

# The model

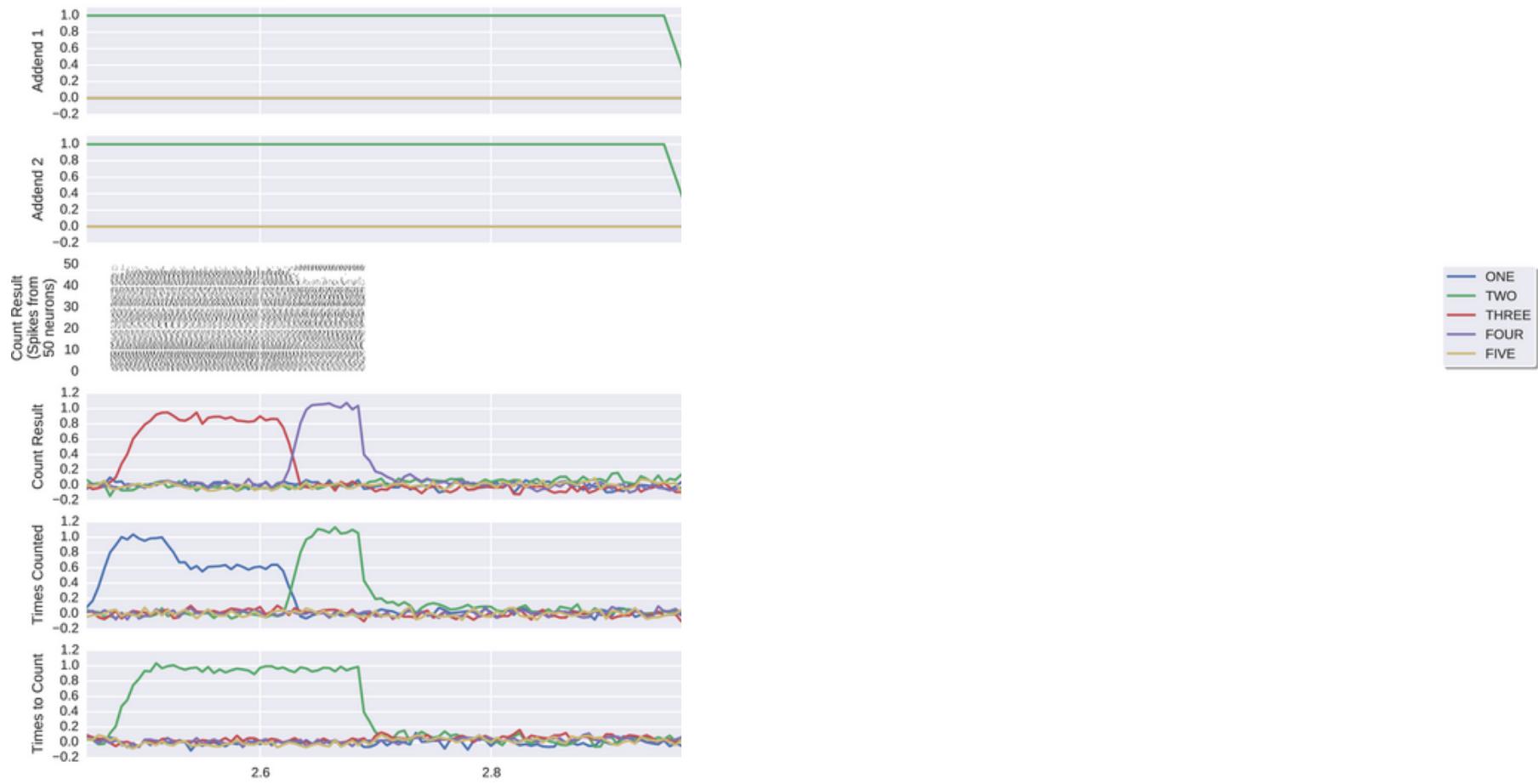
- Two networks:
  - Slow counting
  - Fast retrieval

# The model: Counting

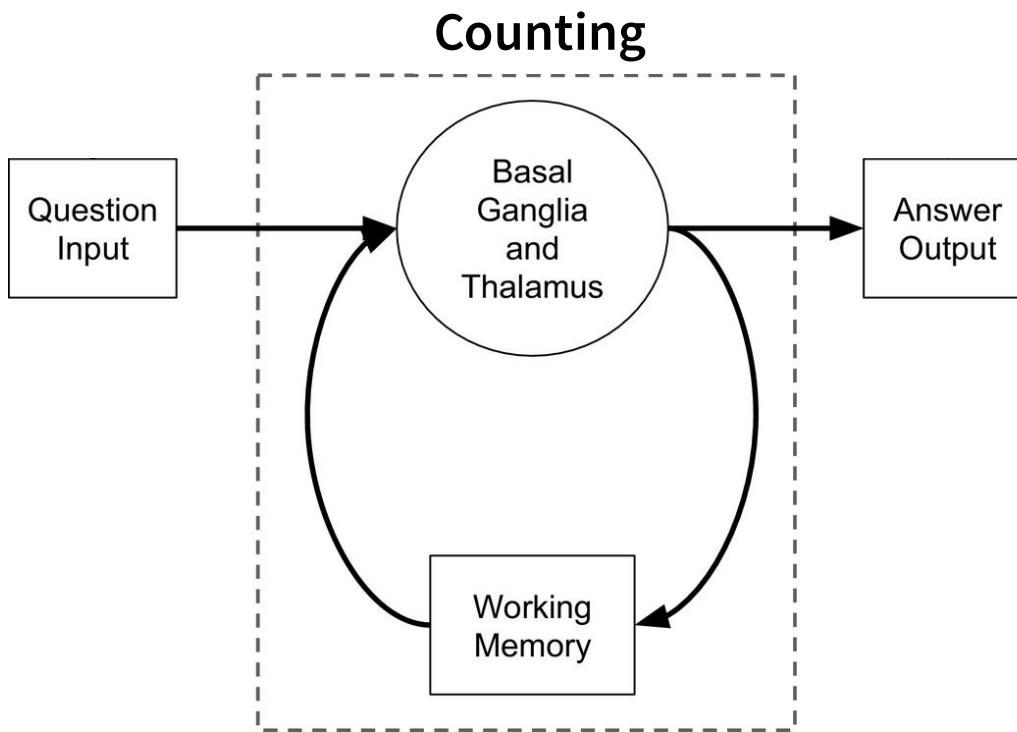


# Results: Counting

2+2

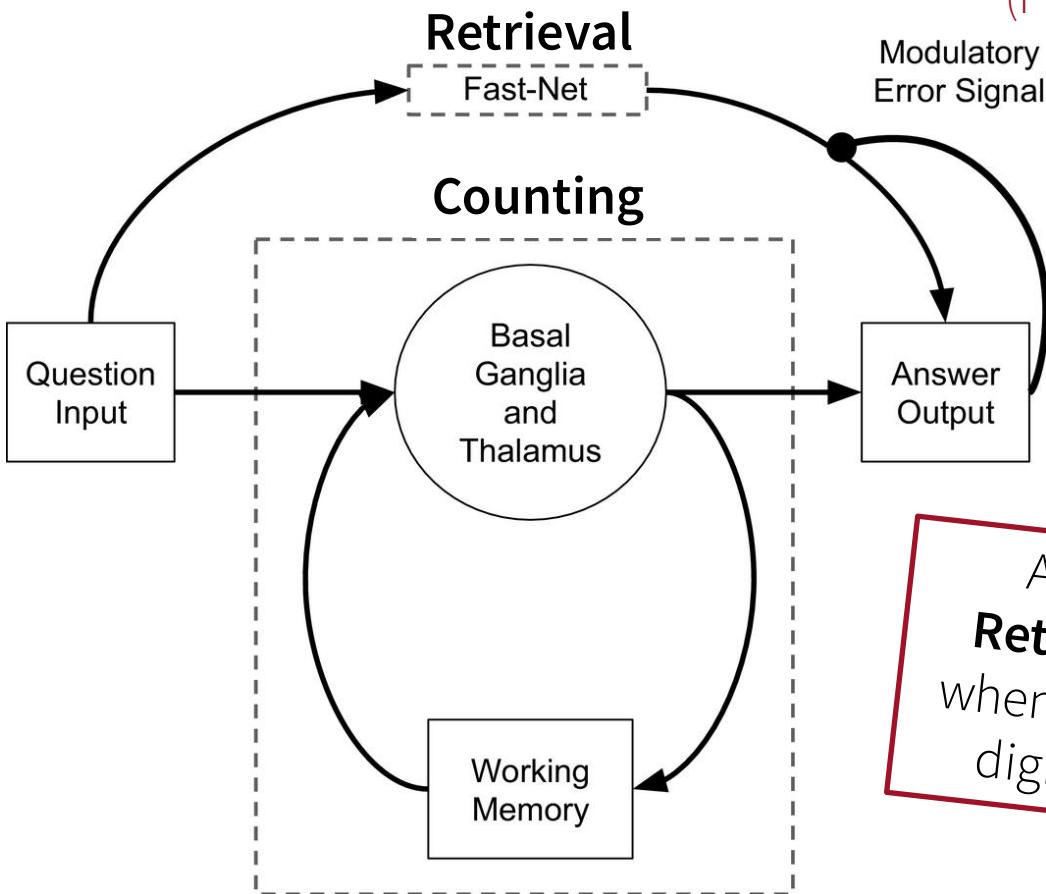


# The model



# The model

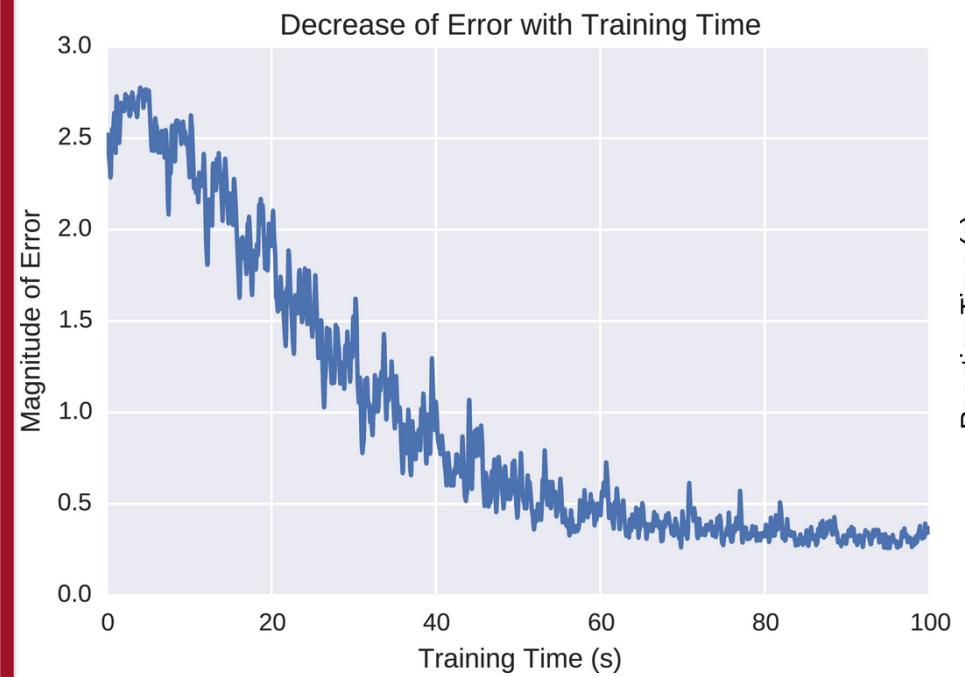
= supervised learning  
(PES rule)



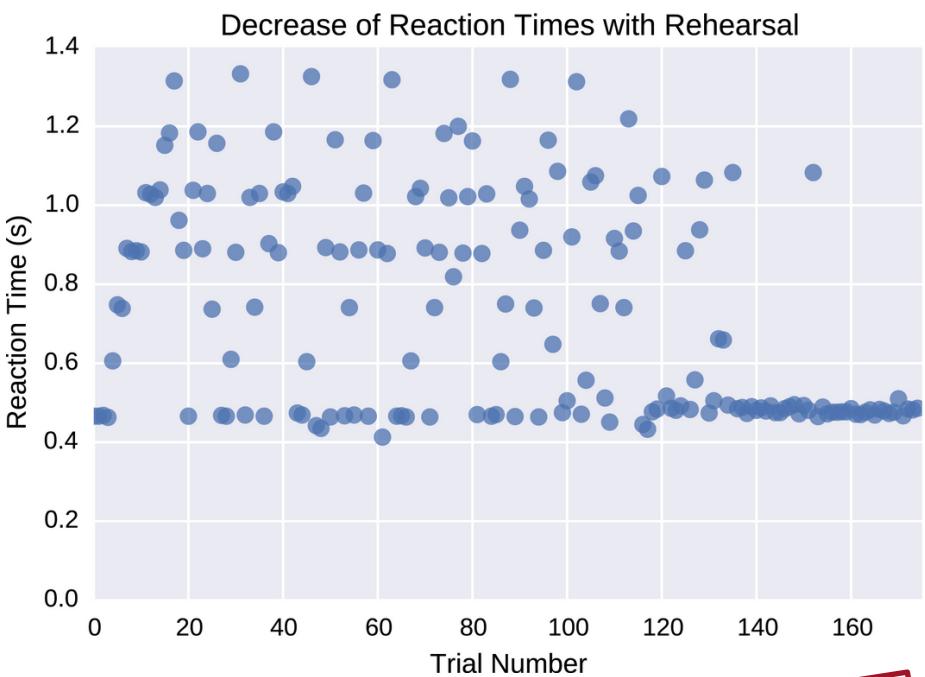
Answer from  
**Retrieval** is used  
when its output is a  
digit-like vector

# Results: Retrieval

Error in memory



Decrease in RT



What is missing here?

# The actual model

Seanny123 / counting\_to\_addition

Code Issues 0 Pull requests 0 Projects 0 Insights

Neural model of progression from counting to memorization addition strategies

nengo neural-networks neuroscience

75 commits 1 branch 1 release 1 contributor

Branch: master New pull request Find file Clone or download

Seanny123 added link to Latex Latest commit 89a7ccb on Mar 19, 2017

This didn't work...  
only counting did... sometimes.

File	Description	Date
tests	beginning cleanup	10 months ago
.gitignore	beginning cleanup	10 months ago
README.md	added link to Latex	2 years ago
adder_env.py	beginning cleanup	a year ago
autoens_learning.py	beginning cleanup	a year ago
better_pred_run.py		a year ago
constants.py		a year ago
cou		a year ago
coul		a year ago
heter		a year ago
hetmem	per submission	a year ago
learn_fa	fixed error plot	a year ago
learn_fig.ipynb	fixed fig	a year ago
learn_fig.m	setup slow run	a year ago
mem_net.py	beginning cleanup	10 months ago
ntmdists.py	got the full network to build	2 years ago
paper count fig.ipynb	clean up repository for paper submission	a year ago
paper err fig.ipynb	add plot notebooks	a year ago
paper pred fig.ipynb	fixed plot	a year ago
paper react fig.ipynb	submitted to CogSci TopICS	a year ago
requirements.txt	clean up repository for paper submission	a year ago
sobol_seq.py	got the full network to build	2 years ago
utils.py	fixed nitpicks	a year ago
with_feedback.py	beginning cleanup	10 months ago
with_feedback.py.cfg	making plots	2 years ago

# Addition in ACT-R

## Declarative Memory

```
(clear-all)
(define-model addition
(sgp :esc t :lf .05)
(chunk-type count-order first second)
(chunk-type add arg1 arg2 sum count)

(add-dm
  (a ISA count-order first 0 second 1)
  (b ISA count-order first 1 second 2)
  (c ISA count-order first 2 second 3)
  (d ISA count-order first 3 second 4)
  (e ISA count-order first 4 second 5)
  (f ISA count-order first 5 second 6)
  (g ISA count-order first 6 second 7)
  (h ISA count-order first 7 second 8)
  (i ISA count-order first 8 second 9)
  (j ISA count-order first 9 second 10)
  (second-goal ISA add arg1 5 arg2 2))
```

## Procedural Memory

```
(P increment-count
=goal>
  ISA      add
  sum     =sum
  count   =count
=retrieval>
  ISA      count-order
  first    =count
  second   =newcount
==>
=goal>
  ISA      add
  count   =sum
  +retrieval>
  ISA      count-order
  first   =newcount
add =newcount

(P initialize-addition
=goal>
  ISA      add
  arg1    =num1
  arg2    =num2
  sum     nil
==>
=goal>
  ISA      add
  sum     =num1
  count   0
  +retrieval>
  ISA      count-order
  first   =num1
)
(P increment-sum
=goal>
  ISA      add
  sum     =sum
  count   =count
  - arg2  =count
=retrieval>
  ISA      count-order
  first   =sum
  second   =count
add =sum

(P terminate-addition
=goal>
  ISA      add
  count   =num
  arg2    =num
  sum     =answer
==>
=goal>
  ISA      add
  sum     =newsum
  +retrieval>
  ISA      count-order
  first   =newsum
add =newsum

)
(P goal-focus second-goal
=goal>
  ISA      add
  count   =count
!output!
  ISA      answer
)
(goal-focus second-goal)
```

# Intermediate Conclusions

- Cognitive models based on spiking neurons:
  - Very complex
  - Hard to control
  - Hard to match behavior (at least it is not shown)
  - Explains how the brain might do cognition
- Cognitive models based on symbolic processing:
  - Less complex
  - Easy to control
  - Good at explaining RTs and accuracy
  - Can predict brain data, but does not explain

# Reminder

- No lecture this Thursday
- No lecture Monday Jan 6
- Final lecture on Thursday Jan 9:
  - Large-scale Nengo model
  - ACT-R vs. Nengo
  - Exam preps
  - Blackjack tournament results

**Happy  
holidays!**



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