university of groningen

faculty of science and engineering

Tutorial 7: Z-transform

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Signals and Systems
1B - 2024/2025

Overview

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P 7.3

Suppose that an LTI system has a system function

$$H(z) = 1 + 5z^{-1} - 3z^{-2} + 2.5z^{-3} + 4z^{-8}$$

a Determine the difference equation that relates the output $y[n]$ of the system to the input $x[n]$

b Determine and plot the output sequence $y[n]$ when the input is $x[n] = \delta[n]$.

Answer P 7.3

(a) The difference equation is

$$y[n] = x[n] + 5x[n-1] - 3x[n-2] + 2.5x[n-3] + 4x[n-8]$$

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(b) When the input $x[n] = \delta[n]$, the output $y[n] = h[n]$ is the impulse response, which is

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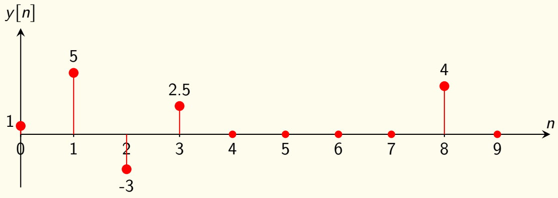

P 9.3

Suppose that an LTI system has a system function

$$H(z) = 1 - 3z^{-2} + 3z^{-4} + 4z^{-6} + 7z^{-7}$$

a Determine the difference equation that relates the output $y[n]$ of the system to the input $x[n]$

b Determine and plot the output sequence $y[n]$ when the input is $x[n] = \delta[n]$.

Answer P 9.3

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Answer P 9.3

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$$y[n] = x[n] - 3x[n-2] + 3x[n-4] + 4x[n-6] + 7x[n-7]$$

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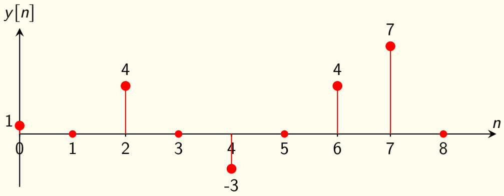

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P 7.4

An LTI system is described by the difference equation

$$y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$$

a. Determine the system function $H(z)$ for this system.
b. Plot the poles and zeroes of $H(z)$ in the z-plane.
c. From $H(z)$ obtain an expression for $H(e^{j\hat{\omega}})$, the frequency response of this system.
d. Sketch the frequency response (magnitude and phase) as a function of frequency for $-\pi \leqslant \hat{\omega} \leqslant \pi$.
e. Determine the output if the input is

$$x[n] = 4 + \cos[0.25\pi(n-1)] - 3\cos[(2\pi/3)n]$$

Answer P 7.4

(a) The system function is

$$H(z) = \frac{1}{3}(1 + z^{-1} + z^{-2})$$

Answer P 7.4

(a) The system function is

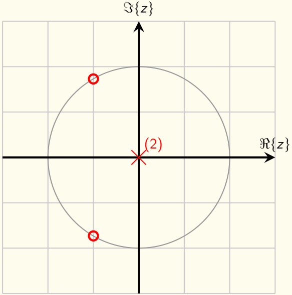
$$\begin{aligned} H(z) &= \frac{1}{3}(1 + z^{-1} + z^{-2}) \\ &= \frac{1}{3}(1 - e^{j2\pi/3}z^{-1})(1 - e^{-j2\pi/3}z^{-1}) \end{aligned}$$

Answer P 7.4

(a) The system function is

$$
\begin{aligned}
H(z) &= \frac{1}{3}(1 + z^{-1} + z^{-2}) \\
&= \frac{1}{3}(1 - e^{j2\pi/3}z^{-1})(1 - e^{-j2\pi/3}z^{-1})
\end{aligned}
$$

(b)



Answer P 7.4

(c) The frequency response is

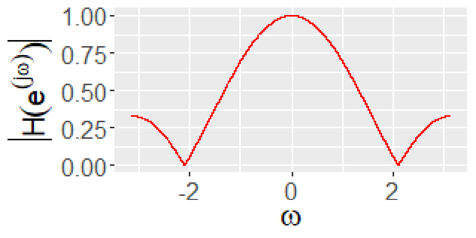
$$H(e^{j\hat{\omega}}) \quad = \quad H(z)\big|_{z=e^{j\hat{\omega}}} = \frac{1}{3}(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}})$$

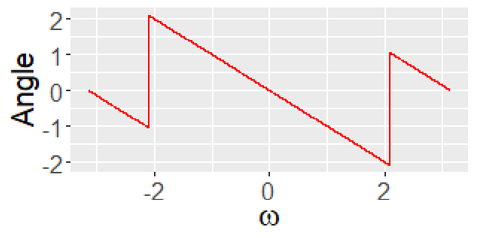
Answer P 7.4

(c) The frequency response is

$$
\begin{aligned}
H(e^{j\hat{\omega}}) &= H(z)\big|_{z=e^{j\hat{\omega}}} = \frac{1}{3}(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}) \\
&= \frac{1}{3}e^{-j\hat{\omega}}(e^{j\hat{\omega}} + 1 + e^{-j\hat{\omega}}) \\
&= \frac{1}{3}e^{-j\hat{\omega}}(1 + 2\cos(\hat{\omega}))
\end{aligned}
$$

(d)





Answer P 7.4

(e) The input

$$x[n] = 4 + \cos[0.25\pi(n-1)] - 3\cos[(2\pi/3)n]$$

has frequency components 0, 0.25π, and $2\pi/3$.

$$\begin{aligned} H(e^{j0}) &= \frac{1}{3}(1 + 2\cos(0)) = 1 \\ H(e^{j0.25\pi}) &= \frac{1}{3}e^{-j0.25\pi}(1 + 2\cos(0.25\pi)) = \frac{1}{3}e^{-j0.25\pi}(1+\sqrt{2}) \\ H(e^{j2\pi/3}) &= \frac{1}{3}e^{-j2\pi/3}(1 + 2\cos(2\pi/3)) = 0 \end{aligned}$$

Combining each component with its frequency response yields

$$y[n] = 4 + \frac{1}{3}(1+\sqrt{2})\cos[0.25\pi(n-2)]$$

P 9.4

An LTI system is described by the difference equation

$$y[n] = 0.2(x[n] - 1.5x[n-1] + 0.5x[n-2])$$

- a Determine the system function $H(z)$ for this system.
- b Plot the poles and zeroes of $H(z)$ in the z-plane.
- c From $H(z)$ obtain an expression for $H(e^{j\hat{\omega}})$, the frequency response of this system.
- d Sketch the frequency response (magnitude and phase) as a function of frequency for $-\pi \leqslant \hat{\omega} \leqslant \pi$.
- e Determine the output if the input is

$$x[n] = 7 - 6\cos[(\pi/3)(n-1)] + 5\cos[0.75\pi n]$$

Answer P 9.4

(a) The system function is

$$H(z) = 0.2(1 - 1.5z^{-1} + 0.5z^{-2}) = 0.2 - 0.3z^{-1} + 0.1z^{-2}$$

Answer P 9.4

(a) The system function is

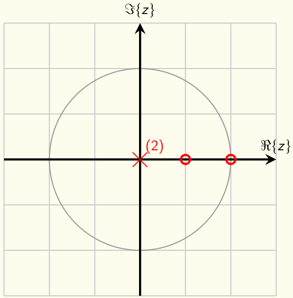
$$\begin{aligned} H(z) &= 0.2(1-1.5z^{-1}+0.5z^{-2}) = 0.2-0.3z^{-1}+0.1z^{-2} \\ &= \frac{1}{5}(1-z^{-1})(1-0.5z^{-1}) \end{aligned}$$

Answer P 9.4

(a) The system function is

$$\begin{aligned} H(z) &= 0.2(1-1.5z^{-1}+0.5z^{-2}) = 0.2-0.3z^{-1}+0.1z^{-2} \\ &= \frac{1}{5}(1-z^{-1})(1-0.5z^{-1}) \end{aligned}$$

(b)



Answer P 9.4

(c) The frequency response is

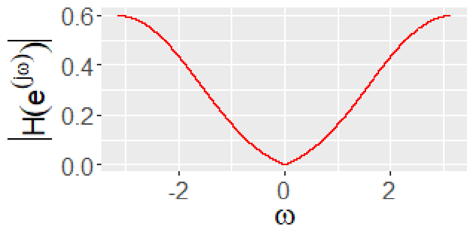
$$H(e^{j\hat{\omega}}) \quad = \quad H(z)\big|_{z=e^{j\hat{\omega}}} = 0.2 - 0.3e^{-j\hat{\omega}} + 0.1e^{-j2\hat{\omega}}$$

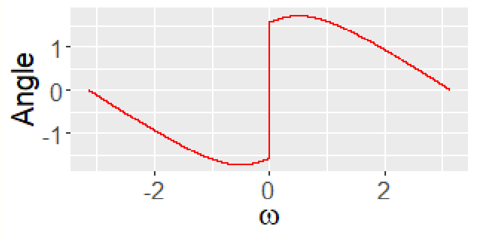
Answer P 9.4

(c) The frequency response is

$$\begin{aligned} H(e^{j\hat{\omega}}) &= H(z)|_{z=e^{j\hat{\omega}}} = 0.2 - 0.3e^{-j\hat{\omega}} + 0.1e^{-j2\hat{\omega}} \\ &= (0.2 - 0.3\cos(-\hat{\omega}) + 0.1\cos(-2\hat{\omega})) + \\ &\quad j(-0.3\sin(-\hat{\omega}) + 0.1\sin(-2\hat{\omega})) \end{aligned}$$

(d)





Answer P 9.4

(e) The input

$$x[n] = 7 - 6\cos[(\pi/3)(n-1)] + 5\cos[0.75\pi n]$$

has frequency components 0, $\pi/3$, and 0.75π.

$$\begin{aligned} H(e^{j0}) &= 0 \\ H(e^{j\pi/3}) &= 0 - j0.1\sqrt{3} = 0.1\sqrt{3}e^{-j\pi/2} \\ H(e^{j0.75\pi}) &= 0.4121 + j0.3121 = 0.5170e^{j0.6482} \end{aligned}$$

Combining each component with its frequency response yields

$$y[n] = -0.6\sqrt{3}\cos[(\pi/3)n - 5\pi/6] + 2.5850\cos[0.75\pi n + 0.6482]$$

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P 7.5

Consider an LTI system whose system function is the product of the five terms

$$\begin{aligned} H(z) &= (1 - z^{-1})(1 - e^{j\pi/2}z^{-1})(1 - e^{-j\pi/2}z^{-1}) \\ &\quad (1 - 0.9e^{-j\pi/3}z^{-1})(1 - 0.9e^{j\pi/3}z^{-1}) \end{aligned}$$

a Write the difference equation that gives the relation between input $x[n]$ and the output $y[n]$.

b Plot the poles and zeros of $H(z)$ in the complex z-plane.

c If the input is of the form $x[n] = Ae^{j\phi}e^{jn\hat{\omega}}$, for what values of $-\pi < \hat{\omega} \leqslant \hat{\pi}$ is it true that $y[n] = 0$?

Answer P 7.5

(a) Writing out the equation yields

$$\begin{aligned} H(z) &= (1 - z^{-1})(1 - e^{-j\pi/2} z^{-1} - e^{j\pi/2} z^{-1} + z^{-2}) \\ &\quad (1 - 0.9 e^{-j\pi/3} z^{-1} - 0.9 e^{j\pi/3} z^{-1} + 0.81 z^{-2}) \end{aligned}$$

Answer P 7.5

(a) Writing out the equation yields

$$\begin{aligned} H(z) &= (1-z^{-1})(1-e^{-j\pi/2}z^{-1}-e^{j\pi/2}z^{-1}+z^{-2}) \\ &\qquad (1-0.9e^{-j\pi/3}z^{-1}-0.9e^{j\pi/3}z^{-1}+0.81z^{-2}) \\ &= (1-z^{-1})(1-2\cos(\pi/2)z^{-1}+z^{-2}) \\ &\qquad (1-1.8\cos(\pi/3)z^{-1}+0.81z^{-2}) \end{aligned}$$

Answer P 7.5

(a) Writing out the equation yields

$$
\begin{aligned}
H(z) &= (1 - z^{-1})(1 - e^{-j\pi/2}z^{-1} - e^{j\pi/2}z^{-1} + z^{-2}) \\
&\qquad (1 - 0.9e^{-j\pi/3}z^{-1} - 0.9e^{j\pi/3}z^{-1} + 0.81z^{-2}) \\
&= (1 - z^{-1})(1 - 2\cos(\pi/2)z^{-1} + z^{-2}) \\
&\qquad (1 - 1.8\cos(\pi/3)z^{-1} + 0.81z^{-2}) \\
&= (1 - z^{-1})(1 + z^{-2})(1 - 0.9z^{-1} + 0.81z^{-2})
\end{aligned}
$$

Answer P 7.5

(a) Writing out the equation yields

$$
\begin{aligned}
H(z) &= (1-z^{-1})(1-e^{-j\pi/2}z^{-1}-e^{j\pi/2}z^{-1}+z^{-2}) \\
&\qquad (1-0.9e^{-j\pi/3}z^{-1}-0.9e^{j\pi/3}z^{-1}+0.81z^{-2}) \\
&= (1-z^{-1})(1-2\cos(\pi/2)z^{-1}+z^{-2}) \\
&\qquad (1-1.8\cos(\pi/3)z^{-1}+0.81z^{-2}) \\
&= (1-z^{-1})(1+z^{-2})(1-0.9z^{-1}+0.81z^{-2}) \\
&= (1-z^{-1}+z^{-2}-z^{-3})(1-0.9z^{-1}+0.81z^{-2})
\end{aligned}
$$

Answer P 7.5

(a) Writing out the equation yields

$$
\begin{aligned}
H(z) &= (1 - z^{-1})(1 - e^{-j\pi/2} z^{-1} - e^{j\pi/2} z^{-1} + z^{-2}) \\
&\qquad (1 - 0.9e^{-j\pi/3} z^{-1} - 0.9e^{j\pi/3} z^{-1} + 0.81 z^{-2}) \\
&= (1 - z^{-1})(1 - 2\cos(\pi/2) z^{-1} + z^{-2}) \\
&\qquad (1 - 1.8\cos(\pi/3) z^{-1} + 0.81 z^{-2}) \\
&= (1 - z^{-1})(1 + z^{-2})(1 - 0.9 z^{-1} + 0.81 z^{-2}) \\
&= (1 - z^{-1} + z^{-2} - z^{-3})(1 - 0.9 z^{-1} + 0.81 z^{-2}) \\
&= 1 - z^{-1} + z^{-2} - z^{-3} - \\
&\qquad 0.9 z^{-1} + 0.9 z^{-2} - 0.9 z^{-3} + 0.9 z^{-4} + \\
&\qquad\quad 0.81 z^{-2} - 0.81 z^{-3} + 0.81 z^{-4} - 0.81 z^{-5}
\end{aligned}
$$

Answer P 7.5

(a) Writing out the equation yields

$$
\begin{aligned}
H(z) &= (1 - z^{-1})(1 - e^{-j\pi/2}z^{-1} - e^{j\pi/2}z^{-1} + z^{-2}) \\
&\qquad (1 - 0.9e^{-j\pi/3}z^{-1} - 0.9e^{j\pi/3}z^{-1} + 0.81z^{-2}) \\
&= (1 - z^{-1})(1 - 2\cos(\pi/2)z^{-1} + z^{-2}) \\
&\qquad (1 - 1.8\cos(\pi/3)z^{-1} + 0.81z^{-2}) \\
&= (1 - z^{-1})(1 + z^{-2})(1 - 0.9z^{-1} + 0.81z^{-2}) \\
&= (1 - z^{-1} + z^{-2} - z^{-3})(1 - 0.9z^{-1} + 0.81z^{-2}) \\
&= 1 - z^{-1} + z^{-2} - z^{-3} - \\
&\qquad 0.9z^{-1} + 0.9z^{-2} - 0.9z^{-3} + 0.9z^{-4} + \\
&\qquad\quad 0.81z^{-2} - 0.81z^{-3} + 0.81z^{-4} - 0.81z^{-5} \\
&= 1 - 1.9z^{-1} + 2.71z^{-2} - 2.71z^{-3} + 1.71z^{-4} - 0.81z^{-5}
\end{aligned}
$$

Answer P 7.5

(a) Writing out the equation yields

$$
\begin{aligned}
H(z) &= (1 - z^{-1})(1 - e^{-j\pi/2}z^{-1} - e^{j\pi/2}z^{-1} + z^{-2}) \\
&\qquad (1 - 0.9e^{-j\pi/3}z^{-1} - 0.9e^{j\pi/3}z^{-1} + 0.81z^{-2}) \\
&= (1 - z^{-1})(1 - 2\cos(\pi/2)z^{-1} + z^{-2}) \\
&\qquad (1 - 1.8\cos(\pi/3)z^{-1} + 0.81z^{-2}) \\
&= (1 - z^{-1})(1 + z^{-2})(1 - 0.9z^{-1} + 0.81z^{-2}) \\
&= (1 - z^{-1} + z^{-2} - z^{-3})(1 - 0.9z^{-1} + 0.81z^{-2}) \\
&= 1 - z^{-1} + z^{-2} - z^{-3} - \\
&\qquad 0.9z^{-1} + 0.9z^{-2} - 0.9z^{-3} + 0.9z^{-4} + \\
&\qquad\quad 0.81z^{-2} - 0.81z^{-3} + 0.81z^{-4} - 0.81z^{-5} \\
&= 1 - 1.9z^{-1} + 2.71z^{-2} - 2.71z^{-3} + 1.71z^{-4} - 0.81z^{-5}
\end{aligned}
$$

So

$y[n] = x[n] - 1.9x[n-1] + 2.71x[n-2] - 2.71x[n-3] + 1.71x[n-4] - 0.81x[n-5]$

Answer P 7.5

(b) The zeros are $1, e^{-j\pi/2}, e^{j\pi/2}, 0.9e^{-j\pi/3}$, and $0.9e^{j\pi/3}$.

Answer P 7.5

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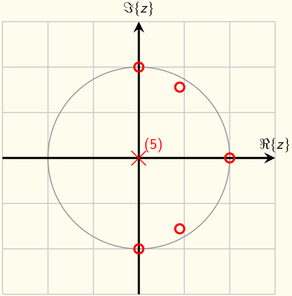
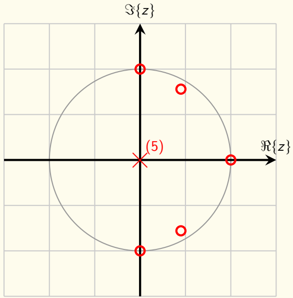


Answer P 7.5

(b) The zeros are $1, e^{-j\pi/2}, e^{j\pi/2}, 0.9e^{-j\pi/3}$, and $0.9e^{j\pi/3}$.



(c) The output $y[n] = 0$ for $\hat{\omega} = 0$, $\hat{\omega} = \pi/2$, and $\hat{\omega} = -\pi/2$

P 9.5

Consider an LTI system whose system function is the product of the five terms

$$\begin{aligned} H(z) \quad = \quad & (1 - z^{-1})(1 - 0.7e^{-j\pi/2}z^{-1})(1 - 0.7e^{j\pi/2}z^{-1}) \\ & (1 + e^{-j2\pi/3}z^{-1})(1 + e^{j2\pi/3}z^{-1}) \end{aligned}$$

a Write the difference equation that gives the relation between input $x[n]$ and the output $y[n]$.

b Plot the poles and zeros of $H(z)$ in the complex z-plane.

c If the input is of the form $x[n] = Ae^{j\phi}e^{jn\hat{\omega}}$, for what values of $-\pi < \hat{\omega} \leqslant \hat{\omega}$ is it true that $y[n] = 0$?

Answer P 9.5

(a) Writing out the equation yields

$$\begin{aligned} H(z) &= (1 - z^{-1})(1 - 0.7e^{-j\pi/2}z^{-1} - 0.7e^{j\pi/2}z^{-1} + 0.49z^{-2}) \\ &\quad (1 + e^{-j2\pi/3}z^{-1} + e^{j2\pi/3}z^{-1} + z^{-2}) \end{aligned}$$

Answer P 9.5

(a) Writing out the equation yields

$$
\begin{aligned}
H(z) &= (1 - z^{-1})(1 - 0.7e^{-j\pi/2}z^{-1} - 0.7e^{j\pi/2}z^{-1} + 0.49z^{-2}) \\
&\qquad (1 + e^{-j2\pi/3}z^{-1} + e^{j2\pi/3}z^{-1} + z^{-2}) \\
&= (1 - z^{-1})(1 - 1.4\cos(\pi/2)z^{-1} + 0.49z^{-2}) \\
&\qquad (1 + 2\cos(2\pi/3)z^{-1} + z^{-2})
\end{aligned}
$$

Answer P 9.5

(a) Writing out the equation yields

$$\begin{aligned} H(z) &= (1-z^{-1})(1-0.7e^{-j\pi/2}z^{-1}-0.7e^{j\pi/2}z^{-1}+0.49z^{-2}) \\ &\qquad (1+e^{-j2\pi/3}z^{-1}+e^{j2\pi/3}z^{-1}+z^{-2}) \\ &= (1-z^{-1})(1-1.4\cos(\pi/2)z^{-1}+0.49z^{-2}) \\ &\qquad (1+2\cos(2\pi/3)z^{-1}+z^{-2}) \\ &= (1-z^{-1})(1+0.49z^{-2})(1-z^{-1}+z^{-2}) \end{aligned}$$

Answer P 9.5

(a) Writing out the equation yields

$$\begin{aligned}
H(z) &= (1 - z^{-1})(1 - 0.7e^{-j\pi/2}z^{-1} - 0.7e^{j\pi/2}z^{-1} + 0.49z^{-2}) \\
&\qquad (1 + e^{-j2\pi/3}z^{-1} + e^{j2\pi/3}z^{-1} + z^{-2}) \\
&= (1 - z^{-1})(1 - 1.4\cos(\pi/2)z^{-1} + 0.49z^{-2}) \\
&\qquad (1 + 2\cos(2\pi/3)z^{-1} + z^{-2}) \\
&= (1 - z^{-1})(1 + 0.49z^{-2})(1 - z^{-1} + z^{-2}) \\
&= (1 - z^{-1} + 0.49z^{-2} - 0.49z^{-3})(1 - z^{-1} + z^{-2})
\end{aligned}$$

Answer P 9.5

(a) Writing out the equation yields

$$
\begin{aligned}
H(z) &= (1-z^{-1})(1-0.7e^{-j\pi/2}z^{-1}-0.7e^{j\pi/2}z^{-1}+0.49z^{-2}) \\
&\qquad (1+e^{-j2\pi/3}z^{-1}+e^{j2\pi/3}z^{-1}+z^{-2}) \\
&= (1-z^{-1})(1-1.4\cos(\pi/2)z^{-1}+0.49z^{-2}) \\
&\qquad (1+2\cos(2\pi/3)z^{-1}+z^{-2}) \\
&= (1-z^{-1})(1+0.49z^{-2})(1-z^{-1}+z^{-2}) \\
&= (1-z^{-1}+0.49z^{-2}-0.49z^{-3})(1-z^{-1}+z^{-2}) \\
&= 1-z^{-1}+0.49z^{-2}-0.49z^{-3}- \\
&\qquad z^{-1}+z^{-2}-0.49z^{-3}+0.49z^{-4})+ \\
&\qquad\quad z^{-2}-z^{-3}+0.49z^{-4}-0.49z^{-5}
\end{aligned}
$$

Answer P 9.5

(a) Writing out the equation yields

$$
\begin{aligned}
H(z) &= (1-z^{-1})(1-0.7e^{-j\pi/2}z^{-1}-0.7e^{j\pi/2}z^{-1}+0.49z^{-2}) \\
&\qquad (1+e^{-j2\pi/3}z^{-1}+e^{j2\pi/3}z^{-1}+z^{-2}) \\
&= (1-z^{-1})(1-1.4\cos(\pi/2)z^{-1}+0.49z^{-2}) \\
&\qquad (1+2\cos(2\pi/3)z^{-1}+z^{-2}) \\
&= (1-z^{-1})(1+0.49z^{-2})(1-z^{-1}+z^{-2}) \\
&= (1-z^{-1}+0.49z^{-2}-0.49z^{-3})(1-z^{-1}+z^{-2}) \\
&= 1-z^{-1}+0.49z^{-2}-0.49z^{-3}- \\
&\qquad z^{-1}+z^{-2}-0.49z^{-3}+0.49z^{-4})+ \\
&\qquad\quad z^{-2}-z^{-3}+0.49z^{-4}-0.49z^{-5} \\
&= 1-2z^{-1}+2.49z^{-2}-1.98z^{-3}+0.98z^{-4}-0.49z^{-5}
\end{aligned}
$$

Answer P 9.5

(a) Writing out the equation yields

$$\begin{aligned}
H(z) &= (1 - z^{-1})(1 - 0.7e^{-j\pi/2}z^{-1} - 0.7e^{j\pi/2}z^{-1} + 0.49z^{-2}) \\
&\qquad (1 + e^{-j2\pi/3}z^{-1} + e^{j2\pi/3}z^{-1} + z^{-2}) \\
&= (1 - z^{-1})(1 - 1.4\cos(\pi/2)z^{-1} + 0.49z^{-2}) \\
&\qquad (1 + 2\cos(2\pi/3)z^{-1} + z^{-2}) \\
&= (1 - z^{-1})(1 + 0.49z^{-2})(1 - z^{-1} + z^{-2}) \\
&= (1 - z^{-1} + 0.49z^{-2} - 0.49z^{-3})(1 - z^{-1} + z^{-2}) \\
&= 1 - z^{-1} + 0.49z^{-2} - 0.49z^{-3} - \\
&\qquad z^{-1} + z^{-2} - 0.49z^{-3} + 0.49z^{-4}) + \\
&\qquad\quad z^{-2} - z^{-3} + 0.49z^{-4} - 0.49z^{-5} \\
&= 1 - 2z^{-1} + 2.49z^{-2} - 1.98z^{-3} + 0.98z^{-4} - 0.49z^{-5}
\end{aligned}$$

So $y[n] = x[n] - 2x[n-1] + 2.49x[n-2] - 1.98x[n-3] + 0.98x[n-4] - 0.49x[n-5]$

Answer P 9.5

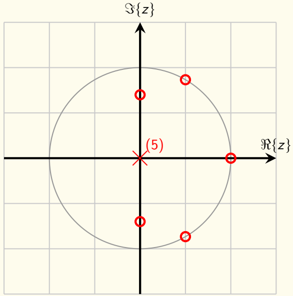
(b) The zeros are $1, 0.7e^{-j\pi/2}, 0.7e^{j\pi/2}, -e^{-j2\pi/3}$, and $-e^{j2\pi/3}$.

Answer P 9.5

(b) The zeros are $1, 0.7e^{-j\pi/2}, 0.7e^{j\pi/2}, -e^{-j2\pi/3}$, and $-e^{j2\pi/3}$. Note that $-e^{-j2\pi/3} = e^{j\pi}e^{-j2\pi/3} = e^{j\pi/3}$

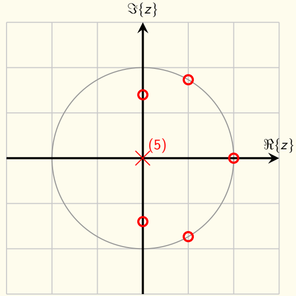
Answer P 9.5

(b) The zeros are $1, 0.7e^{-j\pi/2}, 0.7e^{j\pi/2}, -e^{-j2\pi/3}$, and $-e^{j2\pi/3}$. Note that $-e^{-j2\pi/3} = e^{j\pi}e^{-j2\pi/3} = e^{j\pi/3}$



Answer P 9.5

(b) The zeros are $1, 0.7e^{-j\pi/2}, 0.7e^{j\pi/2}, -e^{-j2\pi/3}$, and $-e^{j2\pi/3}$. Note that $-e^{-j2\pi/3} = e^{j\pi}e^{-j2\pi/3} = e^{j\pi/3}$



(c) The output $y[n] = 0$ for $\hat{\omega} = 0$, $\hat{\omega} = \pi/3$, and $\hat{\omega} = -\pi/3$

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P 7.14

An LTI system has system function

$$(1 + z^{-2})(1 - 4z^{-2}) = 1 - 3z^{-2} - 4z^{-4}$$

The input to this system is

$$x[n] = 20 - 20\delta[n] + 20\cos(0.5\pi n + \pi/4)$$

for $-\infty < n < \infty$. Determine the output of the system $y[n]$ corresponding to the above input $x[n]$. Give an equation for $y[n]$ that is valid for all n.

Answer P 7.14

Since the system is an LTI system, we can separate the input into three parts, calculate the result of applying the system, and combine those results:

Answer P 7.14

Since the system is an LTI system, we can separate the input into three parts, calculate the result of applying the system, and combine those results:

$$\begin{aligned} x_1[n] &= 20 \\ x_2[n] &= -20\delta[n] \\ x_3[n] &= 20\cos(0.5\pi n + \pi/4) \end{aligned}$$

Answer P 7.14

Since the system is an LTI system, we can separate the input into three parts, calculate the result of applying the system, and combine those results:

$$
\begin{aligned}
x_1[n] &= 20 \\
x_2[n] &= -20\delta[n] \\
x_3[n] &= 20\cos(0.5\pi n + \pi/4)
\end{aligned}
$$

For component $x_1[n]$, the frequency response is $1 - 3 - 4 = -6$.

Answer P 7.14

Since the system is an LTI system, we can separate the input into three parts, calculate the result of applying the system, and combine those results:

$$
\begin{aligned}
x_1[n] &= 20 \\
x_2[n] &= -20\delta[n] \\
x_3[n] &= 20\cos(0.5\pi n + \pi/4)
\end{aligned}
$$

For component $x_1[n]$, the frequency response is $1 - 3 - 4 = -6$. That is, $y_1[n] = -120$.

Answer P 9.13

For $x_2[n] = -20\delta[n]$, $y_2[n] = -20h[n]$

Answer P 9.13

For $x_2[n] = -20\delta[n]$, $y_2[n] = -20h[n] = \{-20, 0, 60, 0, 80\}$.

Answer P 9.13

For $x_2[n] = -20\delta[n]$, $y_2[n] = -20h[n] = \{-20, 0, 60, 0, 80\}$.

The component $x_3[n] = 20\cos(0.5\pi n + \pi/4)$ has frequency $\hat{\omega} = 0.5\pi$.

Answer P 9.13

For $x_2[n] = -20\delta[n]$, $y_2[n] = -20h[n] = \{-20, 0, 60, 0, 80\}$.

The component $x_3[n] = 20\cos(0.5\pi n + \pi/4)$ has frequency $\hat{\omega} = 0.5\pi$.
The frequency response is

$$H(e^{j\pi/2}) \;=\; 1 - 3e^{-j\pi} - 4e^{-j2\pi} = 1 - 3(-1) + 4(1) = 0$$

Answer P 9.13

For $x_2[n] = -20\delta[n]$, $y_2[n] = -20h[n] = \{-20, 0, 60, 0, 80\}$.

The component $x_3[n] = 20\cos(0.5\pi n + \pi/4)$ has frequency $\hat{\omega} = 0.5\pi$.
The frequency response is

$$H(e^{j\pi/2}) \quad = \quad 1 - 3e^{-j\pi} - 4e^{-j2\pi} = 1 - 3(-1) + 4(1) = 0$$

$$y[n] = \begin{cases} -140 & n = 0 \\ -60 & n = 2 \\ -40 & n = 4 \\ -120 & \text{otherwise} \end{cases}$$

P 9.13

An LTI system has zeros ± 1 and ± 3, and four poles at $z = 0$. The input to this system is

$$x[n] = 50 - 60\delta[n] + 20\cos(0.5\pi n + \pi/3)$$

for $-\infty < n < \infty$. If $h[0] = 1$, determine the output of the system $y[n]$ corresponding to the above input $x[n]$. Give an equation for $y[n]$ that is valid for all n.

Answer P 9.13

The system function is

$$H(z) = (1 - z^{-1})(1 + z^{-1})(1 - 3z^{-1})(1 + 3z^{-1})$$

Answer P 9.13

The system function is

$$
\begin{aligned}
H(z) &= (1 - z^{-1})(1 + z^{-1})(1 - 3z^{-1})(1 + 3z^{-1}) \\
&= (1 - z^{-2})(1 - 9z^{-2})
\end{aligned}
$$

Answer P 9.13

The system function is

$$\begin{aligned} H(z) &= (1 - z^{-1})(1 + z^{-1})(1 - 3z^{-1})(1 + 3z^{-1}) \\ &= (1 - z^{-2})(1 - 9z^{-2}) = 1 - 10z^{-2} + 9z^{-4} \end{aligned}$$

Answer P 9.13

The system function is

$$
\begin{aligned}
H(z) &= (1 - z^{-1})(1 + z^{-1})(1 - 3z^{-1})(1 + 3z^{-1}) \\
&= (1 - z^{-2})(1 - 9z^{-2}) = 1 - 10z^{-2} + 9z^{-4}
\end{aligned}
$$

Since the system is an LTI system, we can separate the input into three parts, calculate the result of applying the system, and combine those results:

Answer P 9.13

The system function is

$$
\begin{aligned}
H(z) &= (1 - z^{-1})(1 + z^{-1})(1 - 3z^{-1})(1 + 3z^{-1}) \\
&= (1 - z^{-2})(1 - 9z^{-2}) = 1 - 10z^{-2} + 9z^{-4}
\end{aligned}
$$

Since the system is an LTI system, we can separate the input into three parts, calculate the result of applying the system, and combine those results:

$$
\begin{aligned}
x_1[n] &= 50 \\
x_2[n] &= 60\delta[n] \\
x_3[n] &= 20\cos(0.5\pi n + \pi/3)
\end{aligned}
$$

Answer P 9.13

The system function is

$$
\begin{aligned}
H(z) &= (1 - z^{-1})(1 + z^{-1})(1 - 3z^{-1})(1 + 3z^{-1}) \\
&= (1 - z^{-2})(1 - 9z^{-2}) = 1 - 10z^{-2} + 9z^{-4}
\end{aligned}
$$

Since the system is an LTI system, we can separate the input into three parts, calculate the result of applying the system, and combine those results:

$$
\begin{aligned}
x_1[n] &= 50 \\
x_2[n] &= 60\delta[n] \\
x_3[n] &= 20\cos(0.5\pi n + \pi/3)
\end{aligned}
$$

Since 1 is a zero of the system function, constant terms like $x_1[n]$ are eliminated.

Answer P 9.13

The system function is

$$
\begin{aligned}
H(z) &= (1 - z^{-1})(1 + z^{-1})(1 - 3z^{-1})(1 + 3z^{-1}) \\
&= (1 - z^{-2})(1 - 9z^{-2}) = 1 - 10z^{-2} + 9z^{-4}
\end{aligned}
$$

Since the system is an LTI system, we can separate the input into three parts, calculate the result of applying the system, and combine those results:

$$
\begin{aligned}
x_1[n] &= 50 \\
x_2[n] &= 60\delta[n] \\
x_3[n] &= 20\cos(0.5\pi n + \pi/3)
\end{aligned}
$$

Since 1 is a zero of the system function, constant terms like $x_1[n]$ are eliminated.That is, $y_1[n] = 0$.

Answer P 9.13

For $x_2[n] = 60\delta[n]$, $y_2[n] = 60h[n]$

Answer P 9.13

For $x_2[n] = 60\delta[n]$, $y_2[n] = 60h[n] = \{60, 0, -600, 0, 540\}$.

Answer P 9.13

For $x_2[n] = 60\delta[n]$, $y_2[n] = 60h[n] = \{60, 0, -600, 0, 540\}$.

The component $x_3[n] = 20\cos(0.5\pi n + \pi/3)$ has frequency $\hat{\omega} = 0.5\pi$.

Answer P 9.13

For $x_2[n] = 60\delta[n]$, $y_2[n] = 60h[n] = \{60, 0, -600, 0, 540\}$.

The component $x_3[n] = 20\cos(0.5\pi n + \pi/3)$ has frequency $\hat{\omega} = 0.5\pi$.
The frequency response is

$$H(e^{j\pi/2}) = 1 - 10e^{-j\pi} + 9e^{-j2\pi} = 1 - 10(-1) + 9(1) = 20$$

Answer P 9.13

For $x_2[n] = 60\delta[n]$, $y_2[n] = 60h[n] = \{60, 0, -600, 0, 540\}$.

The component $x_3[n] = 20\cos(0.5\pi n + \pi/3)$ has frequency $\hat{\omega} = 0.5\pi$.
The frequency response is

$$H(e^{j\pi/2}) \;=\; 1 - 10e^{-j\pi} + 9e^{-j2\pi} = 1 - 10(-1) + 9(1) = 20$$

$$y[n] = \begin{cases} 60 + 400\cos(0.5\pi n + \pi/3) & n = 0 \\ -600 + 400\cos(0.5\pi n + \pi/3) & n = 2 \\ 540 + 400\cos(0.5\pi n + \pi/3) & n = 4 \\ 400\cos(0.5\pi n + \pi/3) & \text{otherwise} \end{cases}$$

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P 7.15

The input to the C-to-D converter below is

$$x(t) = 4 + \cos(250\pi t - \pi/4) - 3\cos((2000\pi/3)t)$$

The system function to the LTI system is

$$H(z) = \frac{1}{3}(1 + z^{-1} + z^{-2})$$

If $f_s = 1000$ samples/s, determine an expression for $y(t)$, the output of the D-to-C converter.

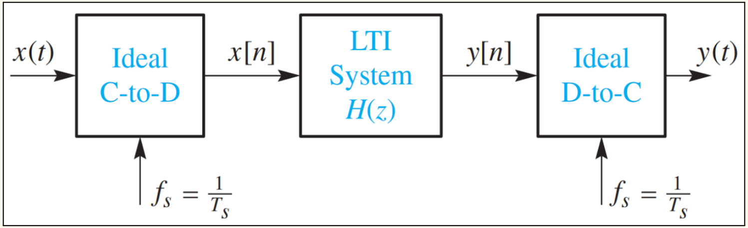


Figure: Proposed system.

P 7.15

The signal $x[n] = x(n/f_s)$ contains frequencies $0, \pi/4$, and $2\pi/3$.

P 7.15

The signal $x[n] = x(n/f_s)$ contains frequencies $0, \pi/4$, and $2\pi/3$.

The frequency response is

$$H(e^{j\hat{\omega}}) = \frac{1}{3}(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}})$$

P 7.15

The signal $x[n] = x(n/f_s)$ contains frequencies $0, \pi/4$, and $2\pi/3$.

The frequency response is

$$
\begin{aligned}
H(e^{j\hat{\omega}}) &= \frac{1}{3}(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}) \\
H(e^{j0}) &= \frac{1}{3}(1 + e^{-j0} + e^{-j2\cdot 0})
\end{aligned}
$$

P 7.15

The signal $x[n] = x(n/f_s)$ contains frequencies $0, \pi/4$, and $2\pi/3$.

The frequency response is

$$
\begin{aligned}
H(e^{j\hat{\omega}}) &= \frac{1}{3}(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}) \\
H(e^{j0}) &= \frac{1}{3}(1 + e^{-j0} + e^{-j2\cdot 0}) = 1
\end{aligned}
$$

P 7.15

The signal $x[n] = x(n/f_s)$ contains frequencies $0, \pi/4$, and $2\pi/3$.

The frequency response is

$$\begin{aligned} H(e^{j\hat{\omega}}) &= \frac{1}{3}(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}) \\ H(e^{j0}) &= \frac{1}{3}(1 + e^{-j0} + e^{-j2\cdot 0}) = 1 \\ H(e^{j\pi/4}) &= \frac{1}{3}(1 + e^{-j\pi/4} + e^{-j2\pi/4}) \end{aligned}$$

P 7.15

The signal $x[n] = x(n/f_s)$ contains frequencies $0, \pi/4$, and $2\pi/3$.

The frequency response is

$$\begin{aligned} H(e^{j\hat{\omega}}) &= \frac{1}{3}(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}) \\ H(e^{j0}) &= \frac{1}{3}(1 + e^{-j0} + e^{-j2\cdot 0}) = 1 \\ H(e^{j\pi/4}) &= \frac{1}{3}(1 + e^{-j\pi/4} + e^{-j2\pi/4}) = \frac{1}{3}(1 + \sqrt{2})e^{-j\pi/4} \end{aligned}$$

P 7.15

The signal $x[n] = x(n/f_s)$ contains frequencies $0, \pi/4$, and $2\pi/3$.

The frequency response is

$$
\begin{aligned}
H(e^{j\hat{\omega}}) &= \frac{1}{3}(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}) \\
H(e^{j0}) &= \frac{1}{3}(1 + e^{-j0} + e^{-j2\cdot 0}) = 1 \\
H(e^{j\pi/4}) &= \frac{1}{3}(1 + e^{-j\pi/4} + e^{-j2\pi/4}) = \frac{1}{3}(1 + \sqrt{2})e^{-j\pi/4} \\
H(e^{j2\pi/3}) &= \frac{1}{3}(1 + e^{-j2\pi/3} + e^{-j4\pi/3})
\end{aligned}
$$

P 7.15

The signal $x[n] = x(n/f_s)$ contains frequencies $0, \pi/4$, and $2\pi/3$.

The frequency response is

$$\begin{aligned}
H(e^{j\hat{\omega}}) &= \frac{1}{3}(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}) \\
H(e^{j0}) &= \frac{1}{3}(1 + e^{-j0} + e^{-j2\cdot 0}) = 1 \\
H(e^{j\pi/4}) &= \frac{1}{3}(1 + e^{-j\pi/4} + e^{-j2\pi/4}) = \frac{1}{3}(1 + \sqrt{2})e^{-j\pi/4} \\
H(e^{j2\pi/3}) &= \frac{1}{3}(1 + e^{-j2\pi/3} + e^{-j4\pi/3}) = 0
\end{aligned}$$

As a result, the output is

$$y[n] =$$

P 7.15

The signal $x[n] = x(n/f_s)$ contains frequencies $0, \pi/4$, and $2\pi/3$.

The frequency response is

$$
\begin{aligned}
H(e^{j\hat{\omega}}) &= \frac{1}{3}(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}) \\
H(e^{j0}) &= \frac{1}{3}(1 + e^{-j0} + e^{-j2\cdot 0}) = 1 \\
H(e^{j\pi/4}) &= \frac{1}{3}(1 + e^{-j\pi/4} + e^{-j2\pi/4}) = \frac{1}{3}(1 + \sqrt{2})e^{-j\pi/4} \\
H(e^{j2\pi/3}) &= \frac{1}{3}(1 + e^{-j2\pi/3} + e^{-j4\pi/3}) = 0
\end{aligned}
$$

As a result, the output is

$$
\begin{aligned}
y[n] &= 4 + \frac{1}{3}(1 + \sqrt{2})\cos(0.25\pi n - \pi/2) \\
y(t) &=
\end{aligned}
$$

P 7.15

The signal $x[n] = x(n/f_s)$ contains frequencies $0, \pi/4$, and $2\pi/3$.

The frequency response is

$$\begin{aligned}
H(e^{j\hat{\omega}}) &= \frac{1}{3}(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}) \\
H(e^{j0}) &= \frac{1}{3}(1 + e^{-j0} + e^{-j2\cdot 0}) = 1 \\
H(e^{j\pi/4}) &= \frac{1}{3}(1 + e^{-j\pi/4} + e^{-j2\pi/4}) = \frac{1}{3}(1 + \sqrt{2})e^{-j\pi/4} \\
H(e^{j2\pi/3}) &= \frac{1}{3}(1 + e^{-j2\pi/3} + e^{-j4\pi/3}) = 0
\end{aligned}$$

As a result, the output is

$$\begin{aligned}
y[n] &= 4 + \frac{1}{3}(1 + \sqrt{2})\cos(0.25\pi n - \pi/2) \\
y(t) &= 4 + \frac{1}{3}(1 + \sqrt{2})\cos(250\pi n - \pi/2)
\end{aligned}$$

P 9.14

The input to the C-to-D converter below is

$$x(t) = 3 + \cos(600\pi t - \pi/6) - 2\cos(1200\pi t)$$

The system function to the LTI system is

$$H(z) = 1 - z^{-1} + z^{-4} - z^{-5}$$

If $f_s = 2400$ samples/s, determine an expression for $y(t)$, the output of the D-to-C converter.

P 9.14

The signal $x[n] = x(n/f_s)$ contains three frequencies: $0, \pi/4$, and $\pi/2$.

P 9.14

The signal $x[n] = x(n/f_s)$ contains three frequencies: $0, \pi/4$, and $\pi/2$.

The frequency response is

$$H(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}} + e^{-j4\hat{\omega}} - e^{-j5\hat{\omega}}$$

P 9.14

The signal $x[n] = x(n/f_s)$ contains three frequencies: $0, \pi/4$, and $\pi/2$.

The frequency response is

$$\begin{aligned} H(e^{j\hat{\omega}}) &= 1 - e^{-j\hat{\omega}} + e^{-j4\hat{\omega}} - e^{-j5\hat{\omega}} \\ H(e^{j0}) &= 1 - e^{-j0} + e^{-j4\cdot 0} - e^{-j5\cdot 0} \end{aligned}$$

P 9.14

The signal $x[n] = x(n/f_s)$ contains three frequencies: $0, \pi/4$, and $\pi/2$.

The frequency response is

$$\begin{aligned} H(e^{j\hat{\omega}}) &= 1 - e^{-j\hat{\omega}} + e^{-j4\hat{\omega}} - e^{-j5\hat{\omega}} \\ H(e^{j0}) &= 1 - e^{-j0} + e^{-j4\cdot 0} - e^{-j5\cdot 0} = 0 \end{aligned}$$

P 9.14

The signal $x[n] = x(n/f_s)$ contains three frequencies: $0, \pi/4$, and $\pi/2$.

The frequency response is

$$
\begin{aligned}
H(e^{j\hat{\omega}}) &= 1 - e^{-j\hat{\omega}} + e^{-j4\hat{\omega}} - e^{-j5\hat{\omega}} \\
H(e^{j0}) &= 1 - e^{-j0} + e^{-j4\cdot 0} - e^{-j5\cdot 0} = 0 \\
H(e^{j\pi/4}) &= 1 - e^{-j\pi/4} + e^{-j4\pi/4} - e^{-j5\pi/4}
\end{aligned}
$$

P 9.14

The signal $x[n] = x(n/f_s)$ contains three frequencies: $0, \pi/4$, and $\pi/2$.

The frequency response is

$$\begin{aligned} H(e^{j\hat{\omega}}) &= 1 - e^{-j\hat{\omega}} + e^{-j4\hat{\omega}} - e^{-j5\hat{\omega}} \\ H(e^{j0}) &= 1 - e^{-j0} + e^{-j4\cdot 0} - e^{-j5\cdot 0} = 0 \\ H(e^{j\pi/4}) &= 1 - e^{-j\pi/4} + e^{-j4\pi/4} - e^{-j5\pi/4} = 0 \end{aligned}$$

P 9.14

The signal $x[n] = x(n/f_s)$ contains three frequencies: $0, \pi/4$, and $\pi/2$.

The frequency response is

$$
\begin{aligned}
H(e^{j\hat{\omega}}) &= 1 - e^{-j\hat{\omega}} + e^{-j4\hat{\omega}} - e^{-j5\hat{\omega}} \\
H(e^{j0}) &= 1 - e^{-j0} + e^{-j4\cdot 0} - e^{-j5\cdot 0} = 0 \\
H(e^{j\pi/4}) &= 1 - e^{-j\pi/4} + e^{-j4\pi/4} - e^{-j5\pi/4} = 0 \\
H(e^{j\pi/2}) &= 1 - e^{-j\pi/2} + e^{-j4\pi/2} - e^{-j5\pi/2}
\end{aligned}
$$

P 9.14

The signal $x[n] = x(n/f_s)$ contains three frequencies: $0, \pi/4$, and $\pi/2$.

The frequency response is

$$\begin{aligned}
H(e^{j\hat{\omega}}) &= 1 - e^{-j\hat{\omega}} + e^{-j4\hat{\omega}} - e^{-j5\hat{\omega}} \\
H(e^{j0}) &= 1 - e^{-j0} + e^{-j4\cdot 0} - e^{-j5\cdot 0} = 0 \\
H(e^{j\pi/4}) &= 1 - e^{-j\pi/4} + e^{-j4\pi/4} - e^{-j5\pi/4} = 0 \\
H(e^{j\pi/2}) &= 1 - e^{-j\pi/2} + e^{-j4\pi/2} - e^{-j5\pi/2} \\
&= 2 - 2e^{-j\pi/2} = 2 + 2j = 2e^{j\pi/4}
\end{aligned}$$

As a result, the output is

$$y[n] =$$

P 9.14

The signal $x[n] = x(n/f_s)$ contains three frequencies: $0, \pi/4$, and $\pi/2$.

The frequency response is

$$\begin{aligned}
H(e^{j\hat{\omega}}) &= 1 - e^{-j\hat{\omega}} + e^{-j4\hat{\omega}} - e^{-j5\hat{\omega}} \\
H(e^{j0}) &= 1 - e^{-j0} + e^{-j4\cdot 0} - e^{-j5\cdot 0} = 0 \\
H(e^{j\pi/4}) &= 1 - e^{-j\pi/4} + e^{-j4\pi/4} - e^{-j5\pi/4} = 0 \\
H(e^{j\pi/2}) &= 1 - e^{-j\pi/2} + e^{-j4\pi/2} - e^{-j5\pi/2} \\
&= 2 - 2e^{-j\pi/2} = 2 + 2j = 2e^{j\pi/4}
\end{aligned}$$

As a result, the output is

$$\begin{aligned}
y[n] &= -4\cos(0.5\pi n + \pi/4) \\
y(t) &=
\end{aligned}$$

P 9.14

The signal $x[n] = x(n/f_s)$ contains three frequencies: $0, \pi/4$, and $\pi/2$.

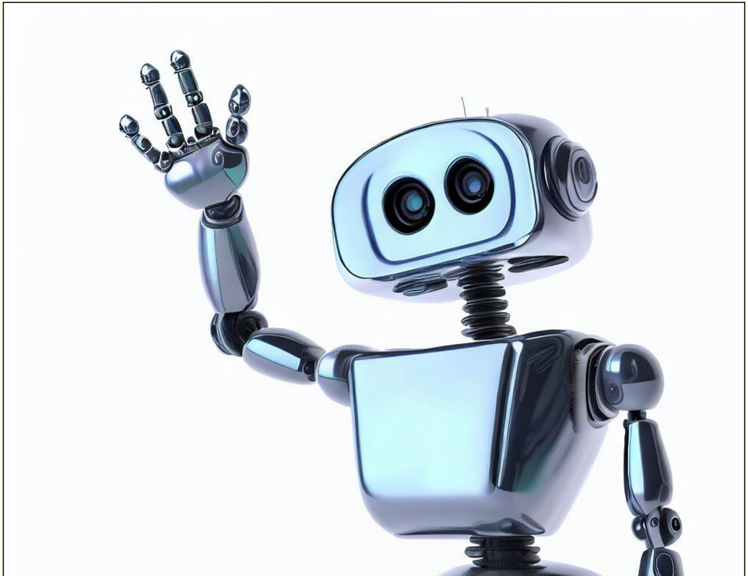
The frequency response is

$$
\begin{aligned}
H(e^{j\hat{\omega}}) &= 1 - e^{-j\hat{\omega}} + e^{-j4\hat{\omega}} - e^{-j5\hat{\omega}} \\
H(e^{j0}) &= 1 - e^{-j0} + e^{-j4\cdot 0} - e^{-j5\cdot 0} = 0 \\
H(e^{j\pi/4}) &= 1 - e^{-j\pi/4} + e^{-j4\pi/4} - e^{-j5\pi/4} = 0 \\
H(e^{j\pi/2}) &= 1 - e^{-j\pi/2} + e^{-j4\pi/2} - e^{-j5\pi/2} \\
&= 2 - 2e^{-j\pi/2} = 2 + 2j = 2e^{j\pi/4}
\end{aligned}
$$

As a result, the output is

$$
\begin{aligned}
y[n] &= -4\cos(0.5\pi n + \pi/4) \\
y(t) &= -4\cos(1200\pi n + 600\pi)
\end{aligned}
$$

Have a nice day!



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Disclaimer

- Questions and images are based in Schafer, R. W., Yoder, M. A., & McClellan, J. H. (2003). Signal Processing First. Prentice Hall.
- Grammar was checked with Grammarly and Grammar checker GPT.
- Images without source were created with the assistance of DALL·E.