Formula Sheet

Trigonometric Identities

$$
\begin{aligned}
\cos(\theta) &= \sin(\theta + \pi/2) \\
\cos(\theta) &= \cos(\theta + 2\pi k),\ k \in \mathbb{Z} \\
\cos(\theta) &= \cos(-\theta) \\
\sin(-\theta) &= -\sin(\theta) \\
\cos(2\pi k) &= 1,\ k \in \mathbb{Z} \\
\cos(\pi k + \pi/2) &= 0,\ k \in \mathbb{Z} \\
\cos(2\pi k + \pi) &= -1,\ k \in \mathbb{Z} \\
\cos^2(\theta) + \sin^2(\theta) &= 1 \\
\cos(\theta)\cos(\phi) &= \frac{1}{2}\left(\cos(\theta - \phi) + \cos(\theta + \phi)\right) \\
\sin(\theta)\sin(\phi) &= \frac{1}{2}\left(\cos(\theta - \phi) - \cos(\theta + \phi)\right) \\
\sin(\theta)\cos(\phi) &= \frac{1}{2}\left(\sin(\theta - \phi) + \sin(\theta + \phi)\right) \\
\cos(\theta)\sin(\phi) &= \frac{1}{2}\left(\sin(\theta + \phi) - \sin(\theta - \phi)\right)
\end{aligned}
$$

Complex numbers and Euler's formula

$$
\begin{aligned}
j^2 &= -1 \\
\Re(a + jb) &= a \\
\Im(a + jb) &= b \\
(a + jb)^* &= a - jb \\
e^{j\theta} &= \cos(\theta) + j\sin(\theta) \\
\cos(\theta) &= \frac{e^{j\theta} + e^{-j\theta}}{2} \\
\sin(\theta) &= \frac{e^{j\theta} - e^{-j\theta}}{2j}
\end{aligned}
$$

Relevant Integrals and functions

$$
\begin{aligned}
\int e^{\theta t} dt &= \frac{e^{\theta t}}{\theta} + c, \quad \theta \neq 0 \\
\int t e^{\theta t} dt &= \frac{\theta t - 1}{\theta^2} e^{\theta t} + c, \quad \theta \neq 0 \\
\prod_{k=m}^{M} a_k &= a_m a_{m+1} \cdots a_M, \quad m < M
\end{aligned}
$$

Consider the polynomial $ax^2 + bx + c$, where $a, b, c \in \mathbb{R}$. Its roots are given by

$$
x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

Signal Processing

Let $T_s, f_s > 0$ be the sampling period and frequency, respectively. Let $x[n]$ and $y[n]$ be discrete-time signals, and $h[n]$ be the unit impulse response.

$$
\begin{aligned}
f_s &= \frac{1}{T_s} \\
\hat{\omega} &= \omega T_s \\
x[n] &= x(nT_s) \\
h[n] &= \sum_{k=0}^{M} b_k \delta[n-k] \\
y[n] &= \sum_{k=0}^{M} b_k x[n-k] \\
h[n] * x[n] &= \sum_{k=-\infty}^{\infty} h[k] x[n-k] \\
y[n] &= h[n] * x[n] \\
H(e^{j\hat{\omega}}) &= \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} = \sum_{k=0}^{M} h[k] e^{-j\hat{\omega}k} \\
h_1[n] * h_2[n] &\longleftrightarrow H_1(e^{j\hat{\omega}}) H_2(e^{j\hat{\omega}})
\end{aligned}
$$

Fourier Analysis

Let $x(t)$ be a continuous-time periodic signal with period T_0 and Fourier series coefficients a_k.

$$
\begin{aligned}
x(t) &= \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/T_0)kt} \\
a_k &= \frac{1}{T_0} \int_0^{T_0} x(t) e^{-j(2\pi/T_0)kt} dt
\end{aligned}
$$

Z-transform

Let $x[n]$ be a discrete-time signal with Z-transform $X(z)$, and $h[n]$ be the unit impulse response.

$$
\begin{aligned}
X(z) &= \sum_{k=0}^{N} x[k] z^{-n} \\
H(z) &= \sum_{k=0}^{N} b_k z^{-k} = \sum_{k=0}^{N} h[k] z^{-k} \\
h[n] * x[n] &\longleftrightarrow H(z) X(z) \\
a x_1[n] + b x_2[n] &\longleftrightarrow a X_1(z) + b X_2(z),\ a, b \in \mathbb{R}
\end{aligned}
$$

Values for trigonometric functions in selected angles

Let $\theta \in [0, \pi]$ be an angle. Relevant values for $\sin(\theta)$, $\cos(\theta)$ and $\tan(\theta)$ are given in Table 1. If the value you look for is not listed in the table, please assign a variable to it, e.g. $\sin(\pi/7) = \alpha$ or $\tan^{-1}(\pi/7) = \beta$. Also, note that relevant trigonometric identities are above to compute values for negative angles.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan(\theta)$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

Table 1: Values for trigonometric functions in selected angles