university of groningen / faculty of science and engineering

Lecture 3: Sampling and Aliasing

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Signals and Systems
1B - 2024/2025

Preliminaries

- The first lab assignment is available now. And its deadline is on Friday, December 6, at 17:30.
- We will upload a video and slides with the answers to the tutorial problems on Friday. The video for the tutorial about sinusoids is available. However, it is **strongly recommended that you attend the tutorials**, as the TAs will provide close support.
- **Recall:** Lectures on Tuesdays; Tutorials on Mondays and Wednesdays, depending on your group; Office hours on Fridays.
- Lab assignment 2 will open next Monday, December 2, 2024, and the deadline is December 20, 2024 at 17:30.
- A few comments about the first lab assignment: Max. pages, style, and clean code-plagiarism tests - See **sections 1.2 to 1.4** in the assignament.

Overview

1. Recap
2. Amplitude and Frecuency Modulation
3. Sampling
4. Reconstruction
5. Closing Remarks

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Recap

- A spectrum depicts a frequency-domain representation of a signal
  - Obtained using the inverse Euler relations
- If the sum of sinusoids is periodic, their frequencies are harmonically related

Recap

- Any periodic function can be expressed as a (possible infinite) sum of harmonically related sinusoids

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/T_0)kt} \tag{1}$$

- The values of a_k are obtained through

$$a_k = \frac{1}{T_0} \int_0^{T_0} x(t) e^{j(2\pi/T_0)kt} dt, \text{ for all } k \in \mathbb{Z} \tag{2}$$

- An approximation of the signal $x(t)$ can be obtained by taking a finite sum

$$x_N(t) = \sum_{k=-N}^{N} a_k e^{j(2\pi/T_0)kt} \tag{3}$$

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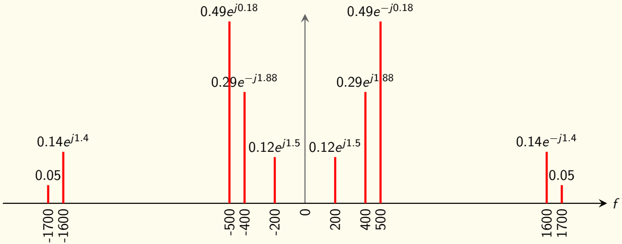
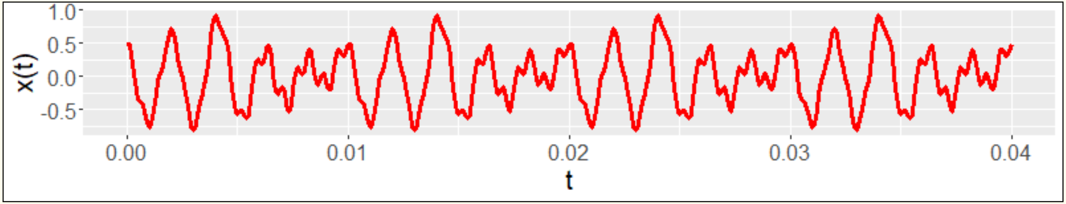
Spectrum

A spectrum depicts a frequency-domain representation of a signal

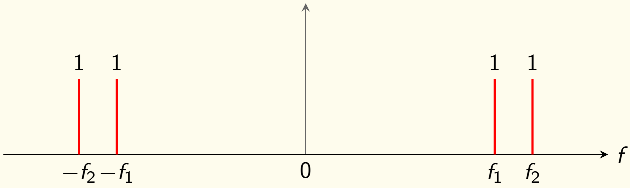




Beat Notes

Beat notes[1] are combinations of two sinusoids $x_1(t)$ and $x_2(t)$

- $x_1(t)$ and $x_2(t)$ have equal amplitude
- $x_1(t)$ and $x_2(t)$ have near identical frecuencies



[1] Taken from http://dspfirst.gatech.edu/chapters/03spect/demos/beatcon/index.html

Beat Notes

Beat notes can also be represented as a product of two sinusoids

$$x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$$

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Combine the real parts:

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Combine the real parts:

$$x(t) = \Re\left\{e^{j2\pi f_1 t} + e^{j2\pi f_2 t}\right\}$$

Express the sum in terms of average and difference frequencies:

$$x(t) = \Re\left\{e^{j2\pi\left(\frac{f_1+f_2}{2}\right)t} e^{j2\pi\left(\frac{f_1-f_2}{2}\right)t} + e^{j2\pi\left(\frac{f_2+f_1}{2}\right)t} e^{j2\pi\left(\frac{f_2-f_1}{2}\right)t}\right\}$$

Beat Notes (Cont.)

$$x(t) = \Re\left\{ e^{j2\pi\left(\frac{f_1+f_2}{2}\right)t} e^{j2\pi\left(\frac{f_1-f_2}{2}\right)t} + e^{j2\pi\left(\frac{f_2+f_1}{2}\right)t} e^{j2\pi\left(\frac{f_2-f_1}{2}\right)t} \right\}$$

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Introduce shorthand terms: $f_c = f_1 + f_2$; $f_\Delta = f_1 - f_2$:

$$x(t) = \Re\left\{ e^{j2\pi f_c t} e^{j2\pi f_\Delta t} + e^{j2\pi f_c t} e^{j2\pi(-f_\Delta)t} \right\}$$

Beat Notes (Cont.)

$$x(t) = \Re\left\{ e^{j2\pi\left(\frac{f_1+f_2}{2}\right)t} e^{j2\pi\left(\frac{f_1-f_2}{2}\right)t} + e^{j2\pi\left(\frac{f_2+f_1}{2}\right)t} e^{j2\pi\left(\frac{f_2-f_1}{2}\right)t} \right\}$$

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Factorize common terms:

$$x(t) = \Re\left\{ e^{j2\pi f_c t} \left(e^{j2\pi f_\Delta t} + e^{-j2\pi f_\Delta t} \right) \right\}$$

Beat Notes (Cont.)

$$x(t) = \Re\left\{ e^{j2\pi\left(\frac{f_1+f_2}{2}\right)t} e^{j2\pi\left(\frac{f_1-f_2}{2}\right)t} + e^{j2\pi\left(\frac{f_2+f_1}{2}\right)t} e^{j2\pi\left(\frac{f_2-f_1}{2}\right)t} \right\}$$

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Factorize common terms:

$$x(t) = \Re\left\{ e^{j2\pi f_c t} \left(e^{j2\pi f_\Delta t} + e^{-j2\pi f_\Delta t} \right) \right\}$$

Simplify the expression inside parentheses:

$$x(t) = \Re\left\{ e^{j2\pi f_c t} \cdot 2\cos(2\pi f_\Delta t) \right\}$$

Beat Notes (Cont.)

Simplify the expression inside parentheses:

$$x(t) = \Re\left\{e^{j2\pi f_c t} \cdot 2\cos(2\pi f_\Delta t)\right\}$$

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Simplify the expression inside parentheses:

$$x(t) = \Re\left\{e^{j2\pi f_c t} \cdot 2\cos(2\pi f_\Delta t)\right\}$$

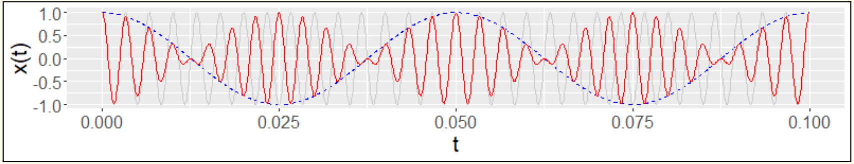
Finally, expand the real part:

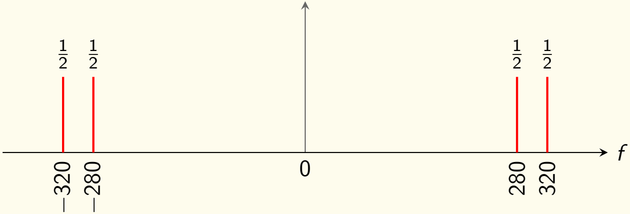
$$x(t) = 2\cos(2\pi f_c t)\cos(2\pi f_\Delta t)$$

Beat Notes

$$
\begin{aligned}
x(t) &= \frac{1}{2}\left(\cos(2\pi(f_c + f_\Delta)t) + \cos(2\pi(f_c - f_\Delta)t)\right) \\
&= 2\cos(j2\pi f_c t)\cos(2\pi f_\Delta t)
\end{aligned} \tag{4}
$$

For $f_c = 300$ and $f_\Delta = 20$, the waveform is





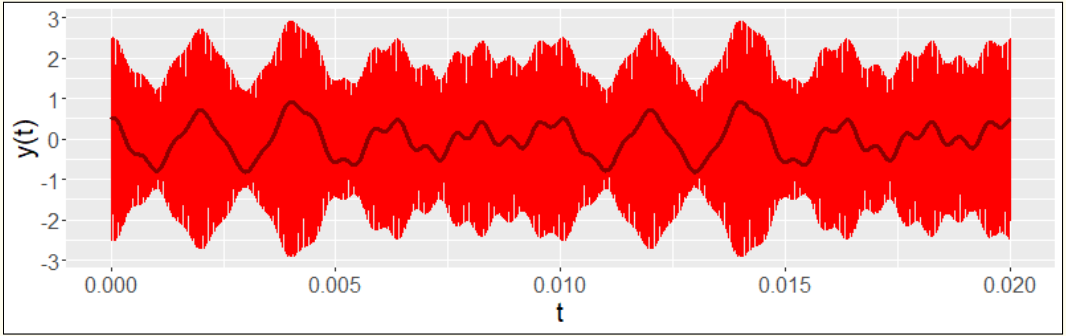
Amplitude modulation

Broadcast AM radio takes the form

$$x(t) = v(t)\cos(2\pi f_c t)$$

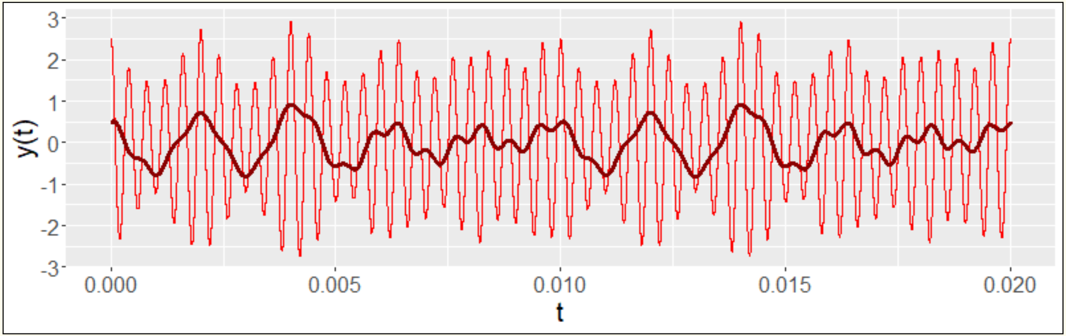
- Information $v(t)$ is encoded in the amplitude of a carrier signal with frecuency f_c.
  - $v(t)$ modulates the amplitude of $cos(2\pi f_c t)$.
- The frecuency f_c should be much higher than any frecuency in the spectrum of $v(t)$.

Amplitude modulation example



- The synthetic vowel (dark red graph) is encoded in the amplitude of a 25 kHz cosine wave
- Since 25 kHz is much higher than the highest frequency in the signal (1700 Hz), the shape of the synthetic vowel is clearly visible in the amplitude

Amplitude modulation example



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- Since 25 kHz is much higher than the highest frequency in the signal (1700 Hz), the shape of the synthetic vowel is clearly visible in the amplitude.

Frequency modulation

Broadcast FM radio takes the form

$$x(t) = \cos(\psi(t))$$

- The information is encoded in changes in the frequency of the carrier signal.
  - The angle function $\psi(t)$ modulates the frequency of the carrier wave.
- The spectrum of the signal changes over time.
- We need something to represent a spectrum that changes over time.

Time-frequency spectrum

A **spectogram** shows variation in the spectrum over time

- Similar to sheet music
  - Horizontal axis represents time
  - Vertical axis represents frequency

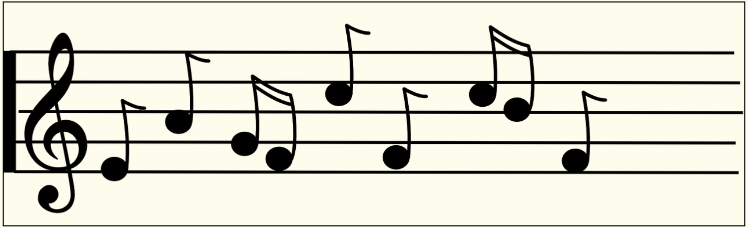
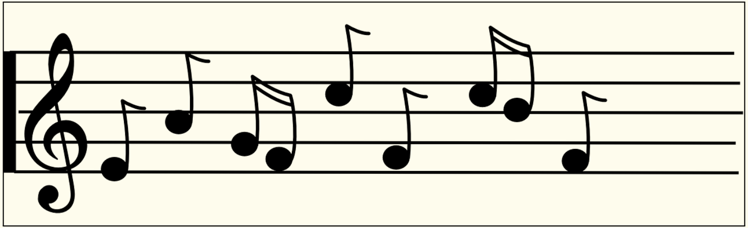


Figure: Clker-Free-Vector-Images, Music, pixabay.com

Time-frequency spectrum



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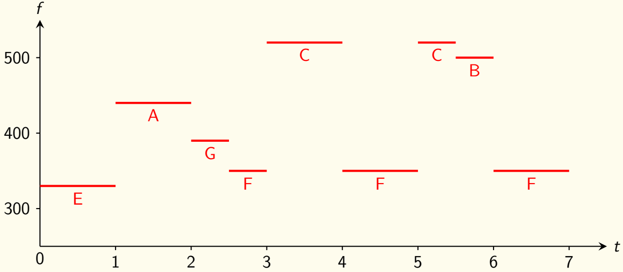
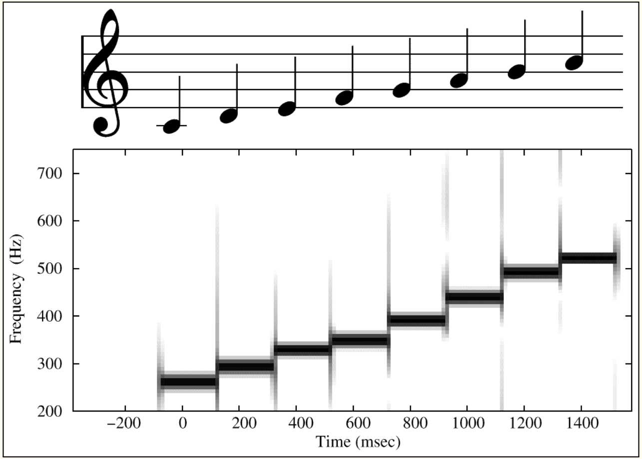


Time-frequency spectrum



A spectrogram identifies frequency content over time

- Darker shades indicate higher amplitudes
- Note the artefacts at note transitions

Chirp signals

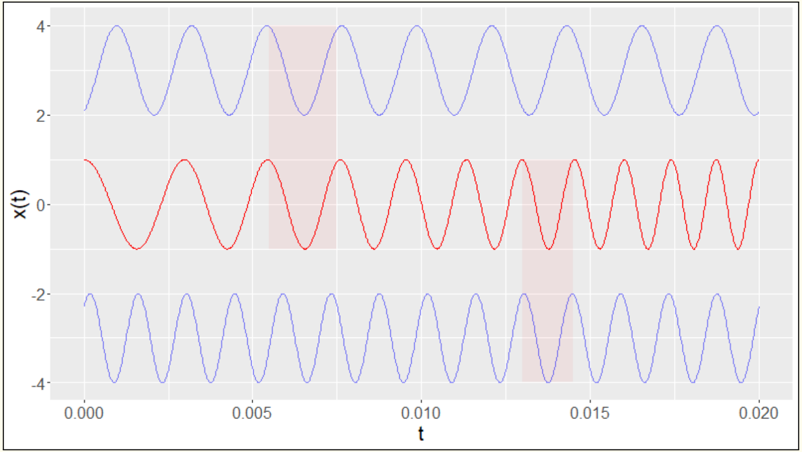
A chirp[2] signal changes frequency over time

- The change in frequency is linear
- Represented as a quadratic angle function $\psi(t)$

$$\begin{aligned} x(t) &= A\cos(\psi(t)) \\ &= A\cos(2\pi\mu t^2 + 2\pi f_0 t + \phi) \end{aligned}$$

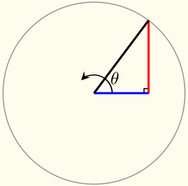
[2] Taken from http://dspfirst.gatech.edu/chapters/03spect/demos/spectrog/chirps/index.html

Example



- Top signal: $\cos(2\pi(450)t + \phi_1)$
- Chirp signal: $\cos(2\pi(300 + 12500t)t)$
- Bottom signal: $\cos(2\pi(700)t + \phi_2)$

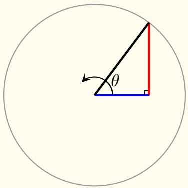
Instantaneous frequency



Frequency measures the rate of change of angle θ

- For constant signal $A\cos(\omega_0 t + \phi)$, the rate of change is ω_0

Instantaneous frequency



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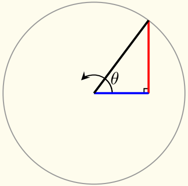
- For FM signal $A\cos(\psi(t))$, the rate is $\omega_i(t) = \frac{d}{dt}\psi(t)$.

Instantaneous frequency



Frequency measures the rate of change of angle θ

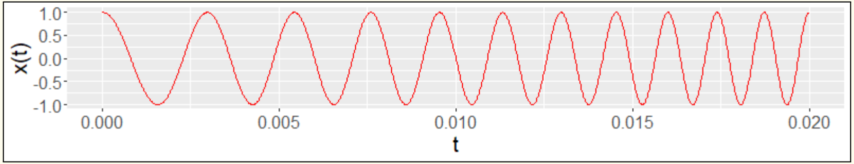
- For constant signal $A\cos(\omega_0 t + \phi)$, the rate of change is ω_0
- For FM signal $A\cos(\psi(t))$, the rate is $\omega_i(t) = \frac{d}{dt}\psi(t)$.
- For chirp signal, this **instantaneous frequency** is

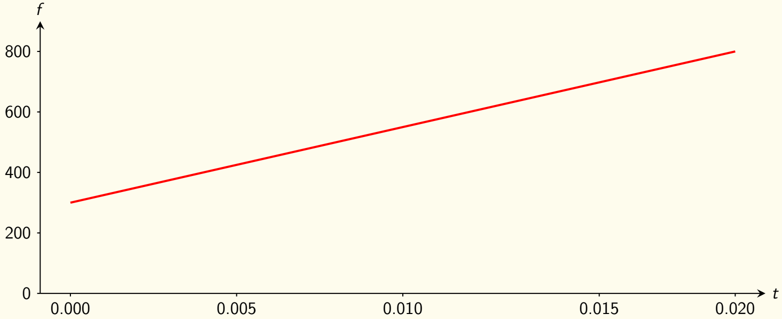
$$\omega_i(t) = \frac{d}{dt}\psi(t) = \frac{d}{dt}(2\pi\mu t^2 + 2\pi f_0 t + \phi) = 4\pi\mu t + 2\pi f_0$$

$$f_i(t) = \frac{1}{2\pi}\omega_i(t) = 2\mu t + f_0$$

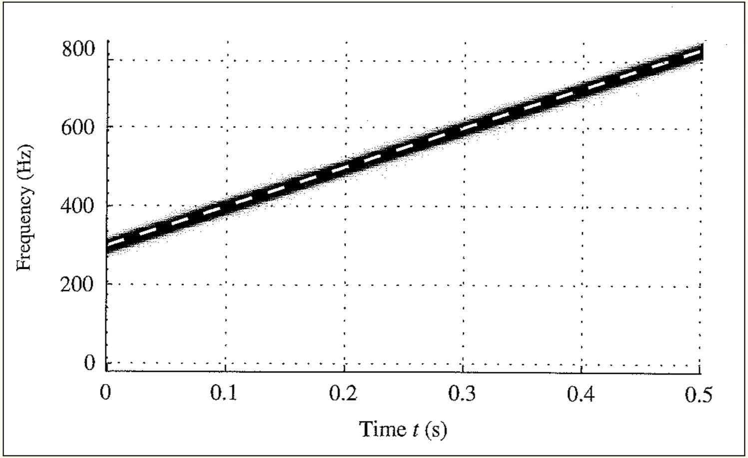
Example

$$x(t) = \cos(2\pi(12500)t^2 + 2\pi(300)t) \to f_i = 25000t + 300$$





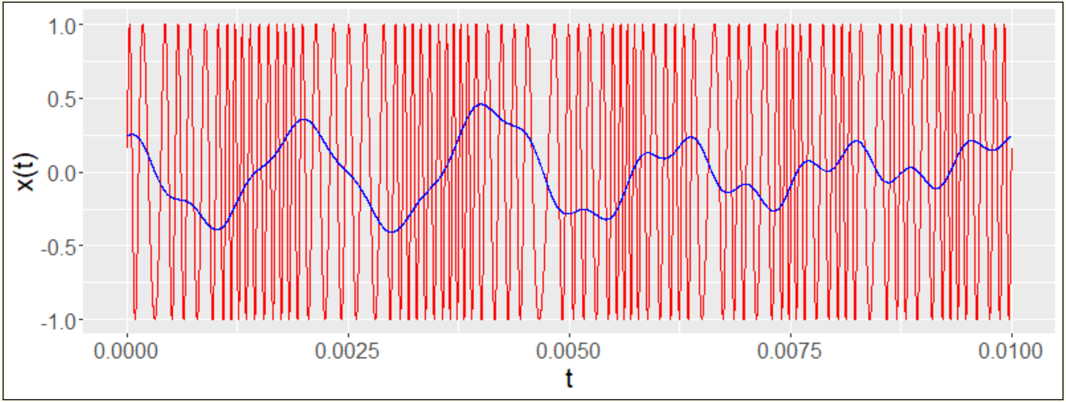
Chirp spectogram



A spectogram using Fourier analysis is performed over short periods of time

- Local frequency rather than instanteous frequency

Frecuency modulation

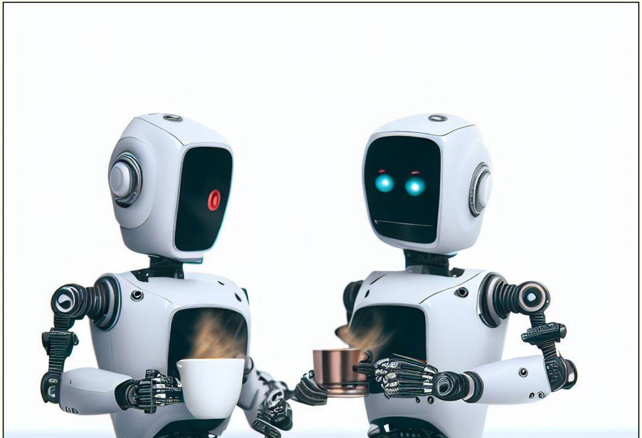


Broadcast FM radio encodes information in the frequency of the carrier signal

- **Synthetic vowel** (in blue) encoded in the frequency of a **carrier wave** (in red)

Break!

See you at ________________________________



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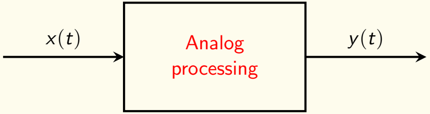
Systems



So far, we have discussed continuous signals

- Manipulations can be performed by analog systems.
- E.g. electronic components, physics.

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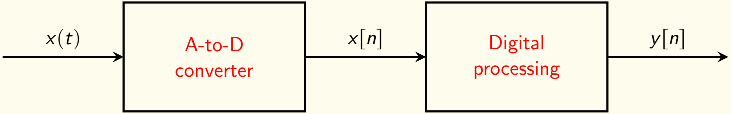
Most of our applications are computer-based

- Manipulations are done on discrete signals.
- Continuous signals have to be discretized.
- Digital signals have to be made continuous.

Systems



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Sampling

Analog-to-Digital converter

- Samples the continuous signal at fixed intervals T_s

$$x[n] = x(nT_s) = x(n/f_s) \tag{5}$$

  - A common sampling rate for audio is $f_s = 44100$ Hz
- We consider theoretical real-valued discrete signals $x[n]$
  - We ignore that samples have finite precision
  - We ignore jitter in sampling frequency

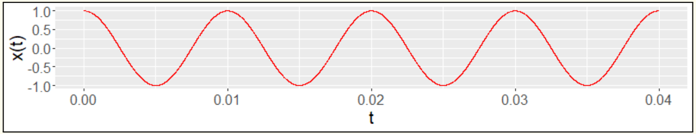
Sampling sinusoidal signals

For a sinusoidal signal $x(t) = A\cos(\omega t + \phi)$,

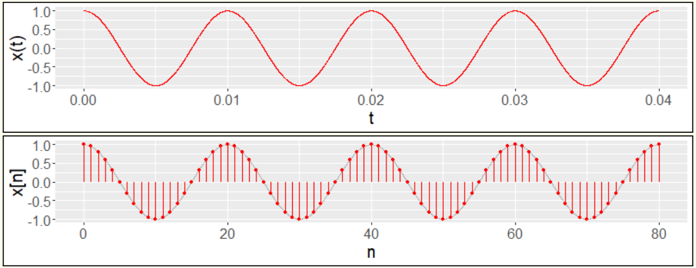
$$x[n] = x(nT_s) = A\cos(\omega n T_s + \phi) =: A\cos(\hat{\omega} n + \phi).$$

- $x[n]$ is a discrete-time sinusoid with discrete-time frecuency $\hat{\omega}$.
  - Note that ω is measured in *rad*/*s* and $\hat{\omega}$ in *rad*.
- $\hat{\omega} := \omega T_s$ is called **normalized radian frequency**

Example

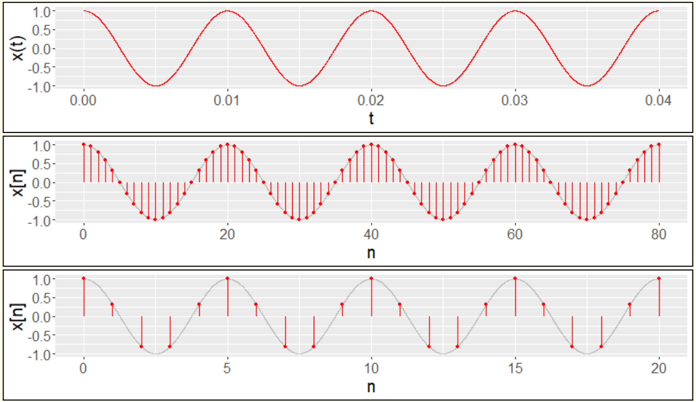


Example



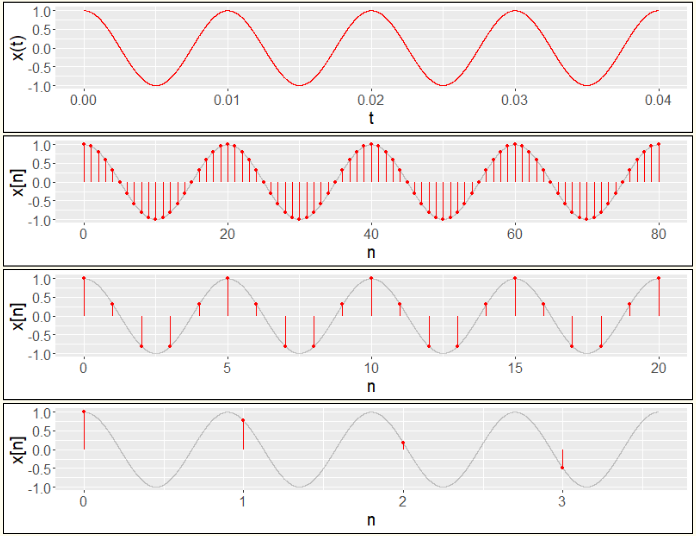
1.0
0.5
0.0
-0.5
-1.0
x(t)
0.00
0.01
0.02
0.03
0.04
t
1.0
0.5
0.0
-0.5
-1.0
x[n]
0
20
40
60
80
n

Example



x(t)
1.0
0.5
0.0
-0.5
-1.0
0.00
0.01
0.02
0.03
0.04
t
x[n]
1.0
0.5
0.0
-0.5
-1.0
0
20
40
60
80
n
x[n]
1.0
0.5
0.0
-0.5
-1.0
0
5
10
15
20
n

Example



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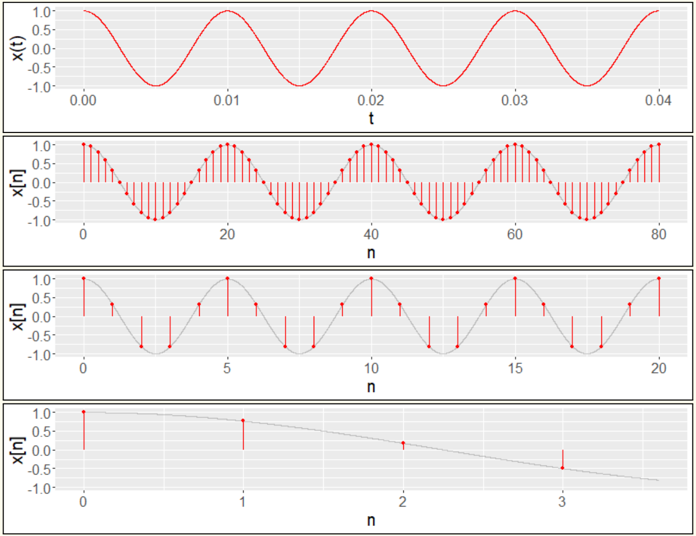
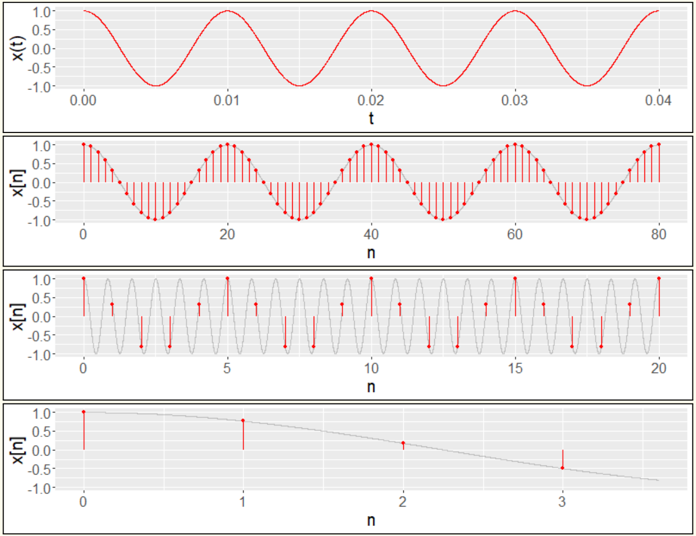


1.0
0.5
0.0
-0.5
-1.0
x(t)
0.00
0.01
0.02
0.03
0.04
t
x[n]
0
20
40
60
80
n
x[n]
0
5
10
15
20
n
x[n]
0
1
2
3
n

Example



x(t)
1.0
0.5
0.0
-0.5
-1.0
0.00
0.01
0.02
0.03
0.04
t
x[n]
0
20
40
60
80
n
x[n]
0
5
10
15
20
n
x[n]
0
1
2
3
n

Aliasing

Different continuous-time frequencies ω may yield the same discrete-time frequency $\hat{\omega}$

- Suppose we sample $\cos(2\pi(100)t)$ at $f_s = 500$

$$x_1[n] = \cos(2\pi(100)n/500) = \cos(0.4\pi n)$$

Aliasing

Different continuous-time frequencies ω may yield the same discrete-time frequency $\hat{\omega}$

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$$x_1[n] = \cos(2\pi(100)n/500) = \cos(0.4\pi n)$$

- Suppose we also sample $\cos(2\pi(600)t)$ at $f_s = 500$

$$x_1[n] = \cos(2\pi(600)n/500) = \cos(2.4\pi n) = \cos(0.4\pi n)$$

Aliasing

Different continuous-time frequencies ω may yield the same discrete-time frequency $\hat{\omega}$

- Suppose we sample $\cos(2\pi(100)t)$ at $f_s = 500$

$$x_1[n] = \cos(2\pi(100)n/500) = \cos(0.4\pi n)$$

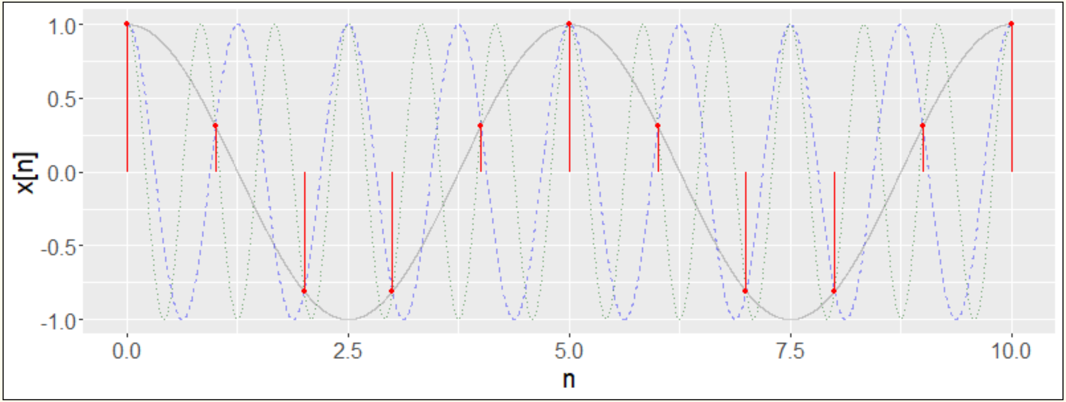
- Suppose we also sample $\cos(2\pi(600)t)$ at $f_s = 500$

$$x_1[n] = \cos(2\pi(600)n/500) = \cos(2.4\pi n) = \cos(0.4\pi n)$$

- Suppose we also sample $\cos(2\pi(400)t)$ at $f_s = 500$

$$x_1[n] = \cos(2\pi(600)n/500) = \cos(1.6\pi n) = \cos(-0.4\pi n) = \cos(0.4\pi n)$$

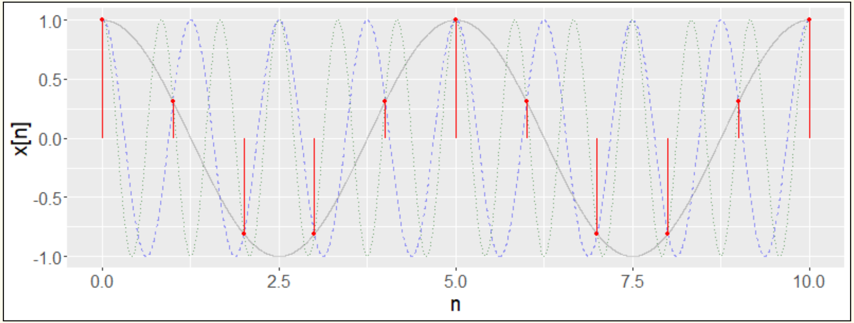
Aliasing

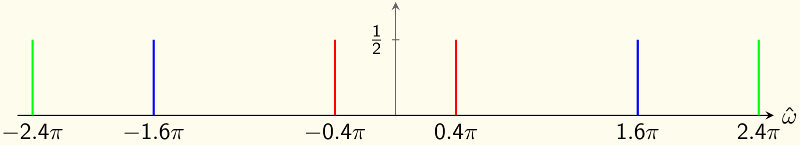


Sampling at $f_s = 500$ Hz

- Gray: $cos(2\pi(100)t)$; blue: $cos(2\pi(400)t)$; green: $cos(2\pi(600)t)$.

Aliasing





The spectrum of a discrete signal contains all its aliases

Aliasing

Aliasing occurs whenever the normalized radian frequency $\hat{\omega} = \omega T_s$ is not equal to the **principal alias** ω_l

- The principal alias ω_l is such that $-\pi < \omega_l \leqslant \pi$
- When aliasing occurs, there is a lower frequency signal that produces the same sampled signal $x[n]$

Aliasing

Shannon sampling theorem

- A continuous-time signal $x(t)$ with frequencies no higher than f_{max} can be reconstructed exactly from its samples $x[n] = x(nT_s)$ if the samples are taken at a rate $f_s > 2f_{max}$
  - I.e. at least twice per oscillation
- Only works for **bandlimited** signal that has a maximum frequency f_{max}
  - The square signal is not bandlimited[3]
- The minimum sampling rate $2f_{max}$ is called the **Nyquist rate**
  - Audio is sampled at 44.1 kHz because it is slightly more than twice the upper limit of human hearing (20 kHz)

---

[3] A bandlimited signal could be obtained by dropping all the very high harmonics, and it might be an almost perfect representation of the FWRS signal.

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Reconstruction

Digital-to-Analog converter

- We assume an ideal D-to-C converter
  - D-to-A converter has no access to the formula $x[n]$

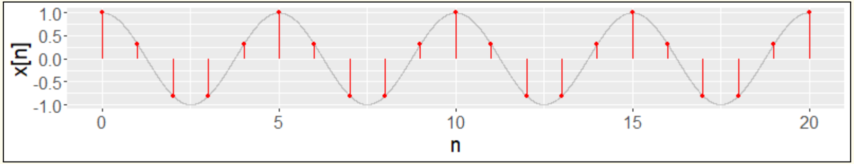
$$y(t) = y[f_s t] = \cos(\hat{\omega} f_s t) = \cos((\omega / f_s) f_s t) = \cos(\omega t)$$

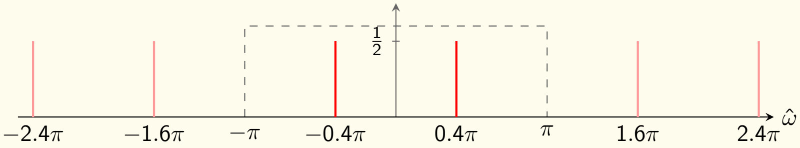
- What about aliases?
  - Select the principal value so that $-\pi < \hat{\omega} \leqslant \pi$

Oversampling $f_s > 2f_{max}$

Suppose we also sample $\cos(2\pi(100)t)$ at $f_s = 500$

$$x[n] = x(n/f_s) = \cos(2\pi(100)n/f_s) = cos(0.4\pi n)$$



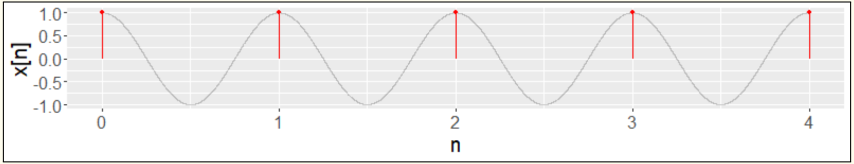


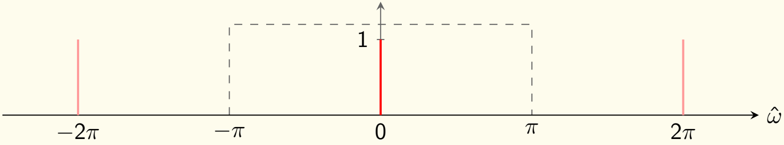
$$x(t) = x[f_s t] = \cos(0.4\pi f_s t) = cos(2\pi(100)t)$$

Extreme aliasing $f_s = f_{max}$

Suppose we also sample $\cos(2\pi(100)t)$ at $f_s = 100$

$$x[n] = x(n/f_s) = \cos(2\pi(100)n/f_s) = cos(2\pi n) = 1$$



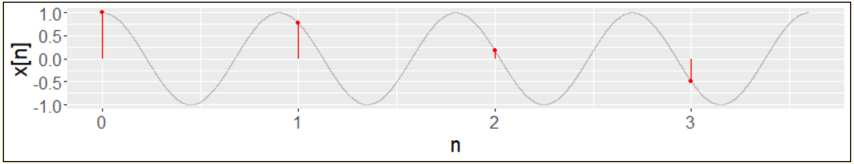


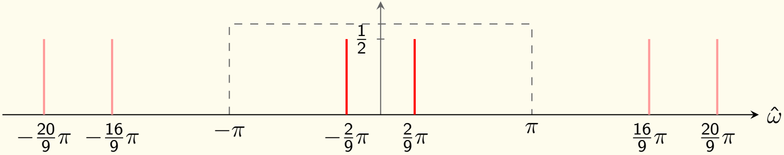
$$x(t) = x[f_s t] = 1$$

Undersampling $f_s < f_{max}$

Suppose we also sample $\cos(2\pi(100)t)$ at $f_s = 90$

$$x[n] = x(n/f_s) = \cos(2\pi(100)n/f_s) = \cos((2/9)\pi n)$$



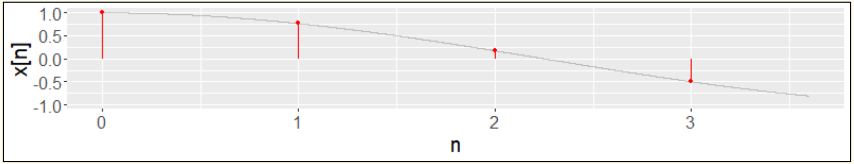


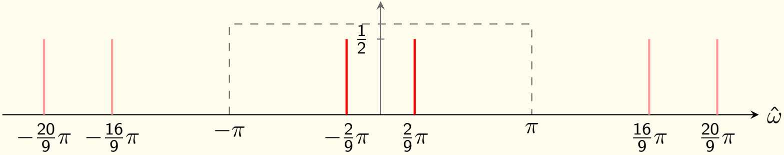
$$x(t) = x[f_s t] = \cos((2/9)\pi f_s t) = \cos(2\pi(10)t)$$

Undersampling $f_s < f_{max}$

Suppose we also sample $\cos(2\pi(100)t)$ at $f_s = 90$

$$x[n] = x(n/f_s) = \cos(2\pi(100)n/f_s) = \cos((2/9)\pi n)$$



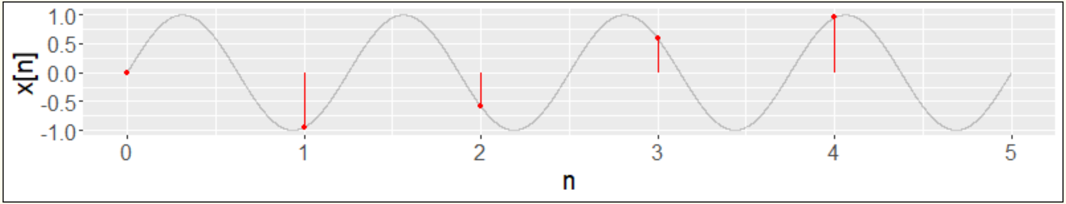


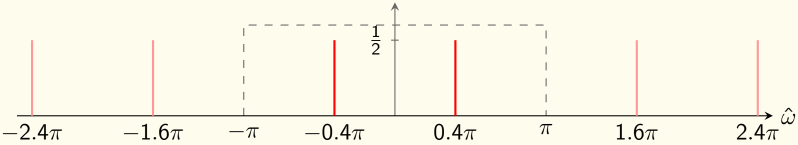
$$x(t) = x[f_s t] = \cos((2/9)\pi f_s t) = \cos(2\pi(10)t)$$

Folding $f_{max} < f_s < 2f_{max}$

Suppose we also sample $\cos(2\pi(100)t)$ at $f_s = 125$

$$x[n] = x(n/f_s) = \cos(2\pi(100)n/f_s) = \cos(1.6\pi n) = \cos(-0.4\pi n)$$



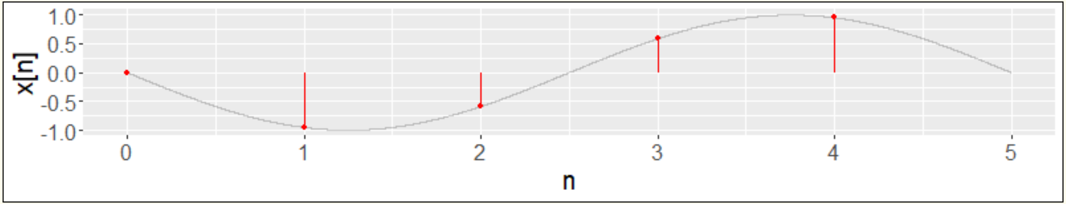


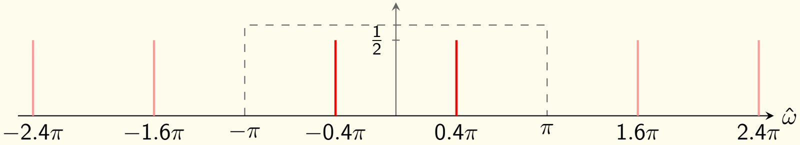
$$x(t) = x[f_s t] = \cos(0.4\pi f_s t) = \cos(2\pi(25)t + \pi)$$

Folding $f_{max} < f_s < 2f_{max}$

Suppose we also sample $\cos(2\pi(100)t)$ at $f_s = 125$

$$x[n] = x(n/f_s) = \cos(2\pi(100)n/f_s) = \cos(1.6\pi n) = \cos(-0.4\pi n)$$





$$x(t) = x[f_s t] = \cos(0.4\pi f_s t) = \cos(2\pi(25)t + \pi)$$

Interpolation pulses

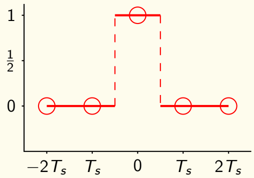
So far, we have considered ideal D-to-C converters that have access to the signal function

- $y[n]$ only contains information for integers n
- How do we interpolate the values between integers?

A broad range of D-to-C converters is described by

$$y(t) = \sum_{n=-\infty}^{\infty} y[n]p(t - nT_s)$$

Zero-order hold interpolation

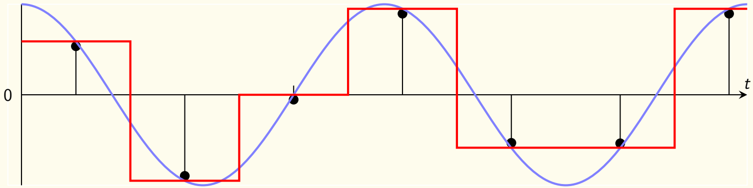


Square pulse

$$p(t) = \begin{cases} 1, & \text{if } -\frac{1}{2}T_s < t \leqslant \frac{1}{2}T_s, \\ 0, & \text{otherwise} \end{cases}$$

- No interpolation
- Simple to implement
- Poor approximation of the original sinusoid
  - Oversampling makes the approximation better

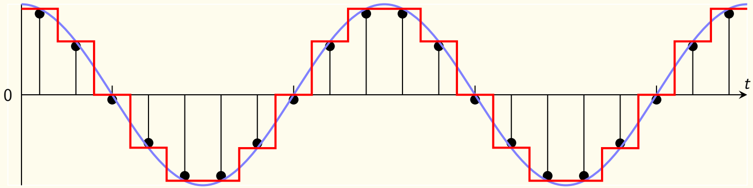
Zero hold interpolation



Original (in blue) and reconstructed (in red) signal using square pulse

- Poor approximation of the original signal
- With oversampling, the approximation becomes better

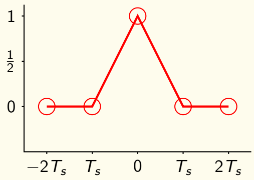
Zero hold interpolation



Original (in blue) and reconstructed (in red) signal using square pulse

- Poor approximation of the original signal
- With oversampling, the approximation becomes better

Linear interpolation

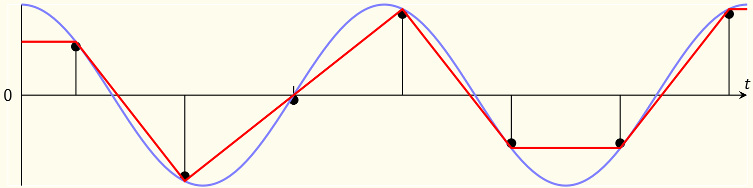


Triangular pulse

$$p(t) = \begin{cases} 1- \mid t \mid / T_s, & \text{if } -\frac{1}{2}T_s < t \leqslant \frac{1}{2}T_s, \\ 0, & \text{otherwise} \end{cases}$$

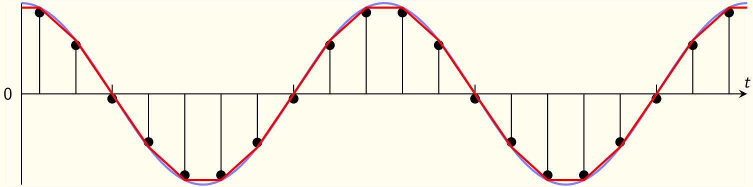
- Linear interpolation
- Note that each point between n and $n+1$ is influenced by both endpoints
- Better approximation than square

Linear interpolation



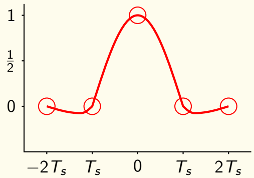
Original (in blue) and reconstructed (in red) signal using triangular pulse

Linear interpolation



Original (in blue) and reconstructed (in red) signal using triangular pulse

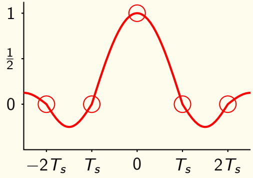
Cubic-spline interpolation



Cube spline

- Smooth interpolation using third-order polynomials
- Individual points have influence over interpolation more than T_s distance

Ideal bandlimited interpolation



Ideal pulse

$$p(t) = \text{sinc}(t/T_s) = \frac{\sin(\pi t/T_s)}{\pi t/T_s}$$

- $p(t)$ has infinity range
  - Interpolation requires processing of the entire sequence
- Reconstructs sinusoids identical to the input if sampling rate f_s is above the Nyquist rate $2f_{max}$

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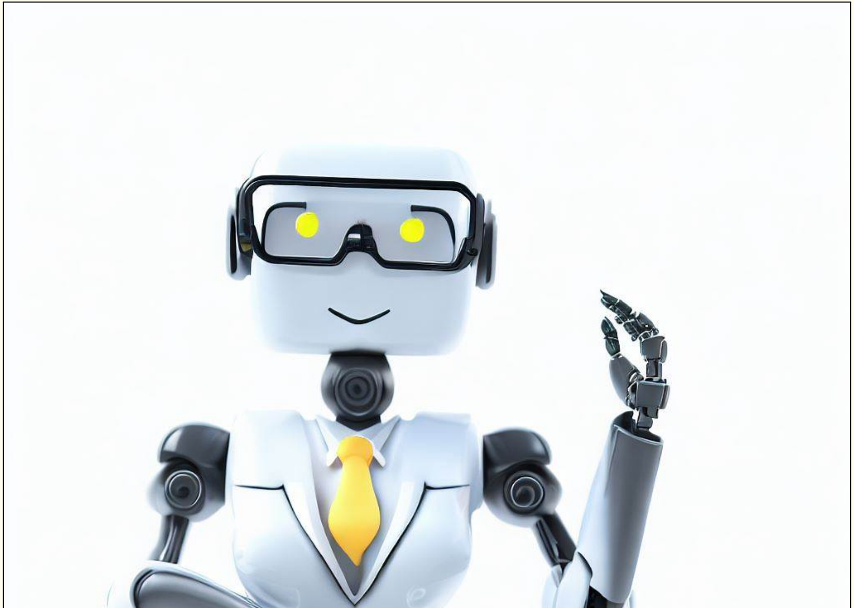
Let us wrap up the lecture!



Take-home Messages

- Sampling is the process of converting a continuous-time signal into a discrete-time signal.
- The Shannon sampling theorem states that a continuous-time signal can be reconstructed from its samples if the sampling rate is greater than twice the maximum frequency of the signal.
- Aliasing occurs when the sampling rate is less than the Nyquist rate.

Practice Questions

The following questions might appear in the final exam:

- What is the different between AM and FM modulation?
- What is the Nyquist rate? Why is it important?
- Why is the square wave signal not bandlimited?
- What is the principal alias?
- What is the Shannon sampling theorem?
- How can we convert a continuous-time signal to a discrete-time signal? And vice-versa?

Tutorial exercises

During the tutorial, the exercises below will be discussed in class

- Attempt to complete the exercises **before** class starts
- As the weeks progress, more time is needed for explanation

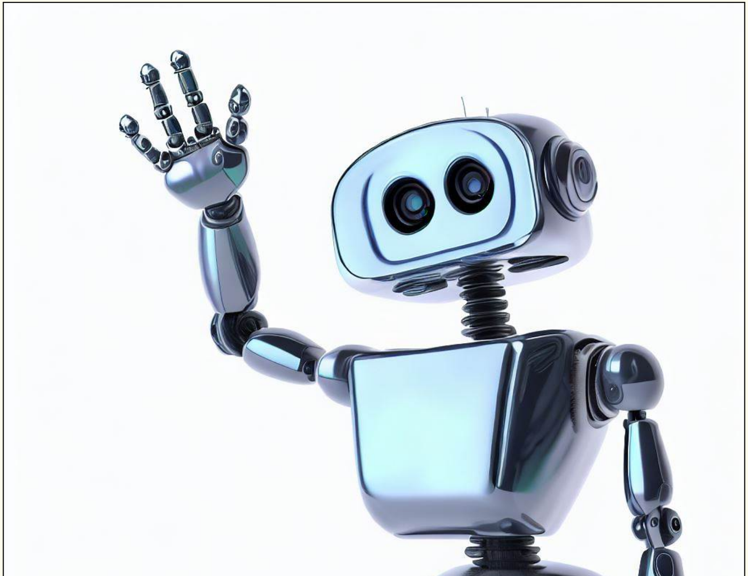
SPF	DSPF
P 4.1 (p. 96)	P 4.11 (p.159)
P 4.2 (p. 96)	P 4.1 (p. 155)
P 4.4 (p. 96)	P 4.14 (p. 160)
P 4.8 (p. 97)	P 4.4 (p. 156)
P 3.17 (p. 69)	P 3.21 (p. 118)

Next Lecture

Let us manipulate digital signals with a special type of filters.

FIR Filters

Have a nice day!



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Disclaimer

- Grammar was checked with Grammarly and Grammar checker GPT.
- Images without source were created with the assistance of DALL·E.