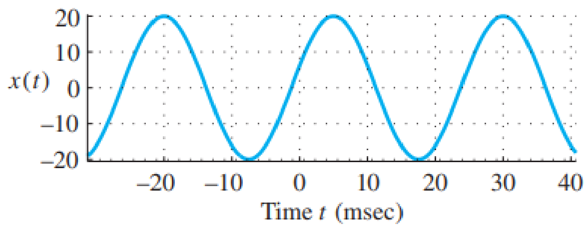
Tutorial 1 – Signals, Sinusoids, and Euler's formula

1. *(p. 60, ex. P-2.2)* Determine values for the amplitude (A), phase (ϕ), and frequency (ω_0) needed in the representation:

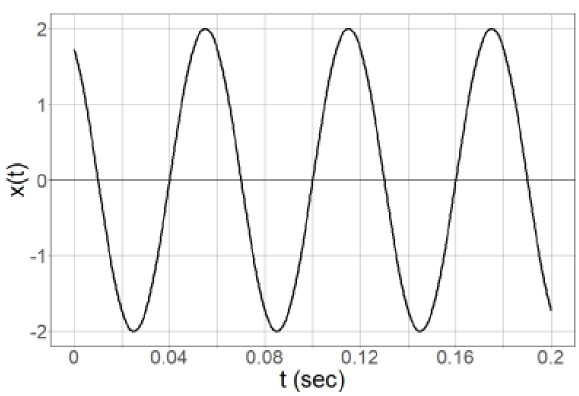
$$x(t) = Acos(\omega_0 t + \phi).$$

Give the answer as numerical values, including the units where applicable.



2. *2023 Exam q. 1* Which of the following is correct?

   (a) $x(t) = 2\cos(2\pi(100/6)t + \pi/6)$
   (b) $x(t) = 2\cos(2\pi(100/3)t + \pi/6)$
   (c) $x(t) = 4\cos(2\pi(100/6)t - \pi/3)$
   (d) $x(t) = 4cos(2\pi(100/3)t - \pi/3)$



3. *(p. 54, ex. 2.8)* Consider the two sinusoids

$$x_1(t) = 5\cos(2\pi(100)t + \pi/3)$$
$$x_2(t) = 4\cos(2\pi(100)t - \pi/4)$$

Prove that

$$x_1(t) + x_2(t) = 5.536\cos(2\pi(100)t + 0.2747).$$

4. *(p. 61, ex. P-2.6)* Use Euler's formula for the complex exponential to prove DeMoivre's formula:

$$(\cos\theta + j\sin\theta)^n = \cos(n\theta) + j\sin(n\theta),$$

and use it to evaluate

$$(\frac{3}{5} + j\frac{4}{5})^{100}.$$

Use that $\tan(0.92723) = \frac{4}{3}$ (or a calculator).

5. *(p. 64, ex. P-2.15)* Given

$$x(t) = 5\cos(\omega t + \frac{1}{3}\pi) + 7\cos(\omega t - \frac{5}{4}\pi) + 3\cos(\omega t),$$

express $x(t)$ in the form $x(t) = A\cos(\omega t + \phi)$

6. *(p.44, ex. 2.6)* Using Euler's formula and the properties of the exponential,

$$e^{j(\alpha+\beta)} = e^{j\alpha}e^{j\beta}$$

show that

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta),$$

and similarly,

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta).$$

7. *(p. 34 ex. 2.1)* Use the previous exercise to derive a formula for $\cos 8\theta$ in terms of $\cos\theta$, $\cos 9\theta$, and $\cos 7\theta$.

8. *(p. 61 ex. P-2.4)* Using Taylor series, prove Euler's formula, namely

$$e^{j\theta} = \cos\theta + j\sin(\theta).$$

Hint: expand e^x for $x = j\theta$.

The Taylor series for e^x, $\sin x$, and $\cos x$ are given below.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

Might be useful for your cheatsheet:

- A – amplitude
- ω_0 – radian frequency
- f – cyclic frequency
- T – period
- ϕ– phase
- $T = (t_1 - t_0)/n$, n– number of iterations
- $\omega_0 = 2\pi f = \frac{2\pi}{T}$
- $T = \frac{1}{f} = \frac{2\pi}{\omega_0}$
- $\phi = -2\pi \frac{t_1}{T} = -\omega_0 t_1$, adjust in $(-\pi, \pi]$, t_1 is the first positive peak after 0.