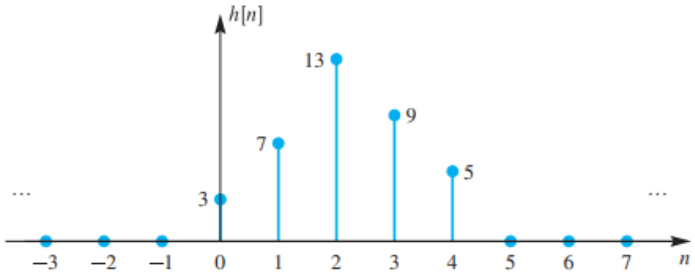
Tutorial 5 – FIR Filters

1. *P 5.1 (p. 126)* The impulse response $h[n]$ of an FIR filter is
$$h[n] = \delta[n-1] - 2\delta[n-4].$$
What is its difference equation?

2. *P 5.2 (p.126)* The running average is given by the formula
$$y[n] = \frac{1}{L}\sum_{k=0}^{L-1} x[n-k].$$
The running average finds lots of applications when dealing with jagged graphs (e.g. in Reinforcement Learning, rewards from an optimal policy might have some variance). For a better comparison, we use the running average to smooth out the graph.
Evaluate the running average of the unit step function, namely,
$$x[n] = u[n] = \begin{cases} 0 & \text{for } n < 0 \\ 1 & \text{for } n \geq 0 \end{cases}$$
To do so, follow this order:
   (a) Make a plot of $u[n]$.
   (b) For $L = 5$, compute the values of $y[n]$ for $-5 \leq n \leq 10$.
   (c) Can you derive a general formula for $y[n]$ for any $n \geq 0$ and $L \in \mathbb{N} \setminus \{0\}$? *Hint:* Yes, you can.

3. *P 5.3 (p.126)* An LTI system is described by the difference equation
$$y[n] = 2x[n] - 3x[n-1] + 2x[n-2].$$
The input to the system is
$$x[n] = \begin{cases} 0 & n < 0 \\ n+1 & n = 0, 1, 2 \\ 5-n & n = 3, 4 \\ 1 & n \geq 5 \end{cases}$$
Compute the values of $y[n]$ for $0 \leq n \leq 10$. What is the difference equation of $y[n]$ to the unit impulse?

4. *P 5.8 (p. 209)* The impulse response $h[n]$ is shown in the figure below. Determine the filter coefficients $\{b_k\}$ of the difference equation for the FIR filter. What is the output of the system when $x[n] = [2, 1, -1]$ is applied?



5. *P 5.9 (p.128)* For each of the systems, determine if they are linear, time-invariant, or causal.
   (a) $y[n] = x[n]\cos(0.2\pi n)$
   (b) $y[n] = x[n] - x[n-1]$
   (c) $y[n] = |x[n]|$
   (d) $y[n] = Ax[n] + B$, where $A, B \in \mathbb{R} \setminus \{0\}$.

6. *P 5.12 (p.128)* For an LTI system, when the input x_1 is the unit step, the output is $y_1[n] = \delta[n] + 2\delta[n-1] - \delta[n-2]$. What is the output when $x_2[n] = 3u[n] - 2u[n-4]$?

7. *P 5.14 (p.128)*
   (a) The output of an FIR filter with $h[n] = \delta[n-2]$ is $y[n] = u[n-3] - u[n-6]$. What was the input signal $x[n]$?
   (b) The output of a first-difference FIR filter is $y[n] = \delta[n] - \delta[n-4]$. Determine the input signal.
   (c) The output of a 4-point averager is $y[n] = -5\delta[n] - 5\delta[n-2]$. What is its input signal $x[n]$?
   Hint: You can assume that $|x|$ is countably infinite.

8. *P 5.17 (p.128)* Three systems are connected consecutively in a cascade. The three systems are as follows:

$$\begin{aligned} \mathcal{S}_1: &\quad y_1[n] = x_1[n] - x_1[n-1] \\ \mathcal{S}_2: &\quad y_2[n] = x_2[n] + x_2[n-2] \\ \mathcal{S}_3: &\quad y_3[n] = x_3[n-1] + x_3[n-2] \end{aligned}$$

   What is the impulse response $h_i[n]$ for each $\mathcal{S}_i, \imath \in \{1, 2, 3\}$? What is the overall system impulse response? What is its overall difference equation?