Tutorial 2 – Spectrum Representation, Periodic Signals, and the Fourier Series

1. *(p.111, ex. P-3.2)* A signal composed of sinusoids is given by the equation

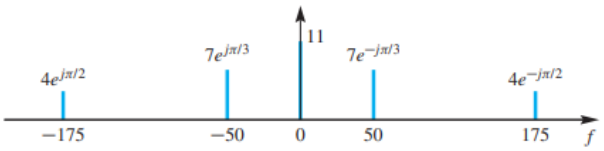
$$x(t) = 16\cos(500\pi t + \pi/4) + 9\cos(1000\pi t - \pi/3) - 5\cos(750\pi t).$$

   (a) Sketch the spectrum of this signal, indicating the complex amplitude of each frequency component
   (b) Is $x(t)$ periodic? If so, what is the period?
   (c) Now consider a new signal defined as

$$y(t) = 12 - 5\sqrt{3}\cos(750\pi t + \pi/2) + 5\cos(1500\pi t + \pi/2).$$

   Draw the spectrum of this new signal. Is y(t) periodic? If so, what is the period?

2. *(p. 110, ex. P-3.1)* A signal x(t) has the two-sided spectrum representation shown in the figure below.



   (a) Write an equation for $x(t)$ as a sum of cosines.
   (b) Is $x(t)$ a periodic signal? If so, determine its fundamental period and its fundamental frequency.
   (c) Explain why negative frequencies are needed in the spectrum.

3. *(p.113, ex. P-3.11)* A periodic signal is given by the equation

$$x(t) = 2 + 4\cos(40\pi t - \pi/5) + 3\sin(60\pi t) + 4\cos(120\pi t - \pi/3).$$

   (a) Determine the fundamental frequency ω_0, the fundamental period T_0, the number of terms N, and the coefficients a_k in the finite Fourier representation for the signal $x(t)$ above. It is possible to do this without evaluating any integrals.
   (b) Sketch the spectrum of this signal indicating the complex amplitude of each frequency component.
   (c) Now, consider the signal

$$y(t) = x(t) + 10\cos(50\pi t - \pi/6).$$

   How is the spectrum changed? Is $y(t)$ still periodic? If so, what is the fundamental period? Plot it.

4. *(p. 88 ex. 3.5)* Show that one possible period of the complex exponential signal

$$v_k(t) = e^{j2\pi kF_0 t}$$

   is $T_0 = \frac{1}{F_0}$ and that the fundamental period is $\frac{1}{kF_0}$.

5. *(p.113, P-3.10)* Consider a signal $x(t)$ such that

$$x(t) = 2\cos(\omega_1 t)cos(\omega_2 t) = cos[(\omega_2 + \omega_1)t] + cos[(\omega_2 - \omega_1)t],$$

   for $0 < \omega_1 < \omega_2$.

(a) What is the general condition that must be satisfied by $\omega_2 - \omega_1$ and $\omega_2 + \omega_1$ so that $x(t)$ is periodic with period T_0?

(b) What does the result of (a) imply about ω_1 and ω_2? For example, must ω_2 be an integer multiple of ω_1?

6. *(p. 111, ex. P-3.4)* Define $x(t)$ as

$$x(t) = \sin^3(54\pi t) + \sin^2(36\pi t)$$

(a) Determine a formula for $x(t)$ as the real part of a sum of complex exponentials.

(b) Determine the fundamental period for $x(t)$.

(c) Plot the spectrum for $x(t)$.