university of groningen

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Tutorial 5: Frequency response of FIR filters

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Signals and Systems
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Overview

1. P 6.1
2. P 6.4
3. P 6.5
4. P 6.7
5. P 6.16 / P 6.17

Table of Contents

P 6.1

Suppose the input signal to an FIR filter is

$$x[n] = e^{j(0.4\pi n - 0.5\pi)}$$

If we define a new signal $y[n]$ to be the first difference $y[n] = x[n] - x[n-1]$, it is possible to express $y[n]$ in the form

$$y[n] = Ae^{j(\hat{\omega}_0 n + \phi)}$$

Determine the numerical values of A, ϕ, and $\hat{\omega}_0$.

Table of Contents

P 6.4

A linear time-invariant system is described by the difference equation

$$y[n] = 2x[n] - 3x[n-1] + 2x[n-2]$$

a Find the frequency response $H(e^{j\hat{\omega}})$; then express it as a mathematical formula, in polar form (magnitude and phase).

b $H(e^{j\hat{\omega}})$ is a periodic function of $\hat{\omega}$; determine the period.

c Plot the magnitude and phase of $H(e^{j\hat{\omega}})$ as a function of $\hat{\omega}$ for $-\pi \leqslant \hat{\omega} \leqslant \pi$.

d Find all frequencies $\hat{\omega}$, for which the output response to the input $e^{j\hat{\omega}n}$ is zero.

e When the input to the system is $x[n] = \sin(\pi n/13)$, determine the output signal and express it in the form $y[n] = A\cos(\hat{\omega}_0 n + \phi)$.

Table of Contents

P 6.5

A linear time-invariant filter is described by the difference equation

$$y[n] = x[n] + 2x[n-1] + x[n-2]$$

- a Obtain an expression for the frequency response of this system.
- b Sketch the frequency response (magnitude and phase) as a function of frequency.
- c Determine the output when the input is $x[n] = 10 + 4\cos(0.5\pi n + \pi/4)$.
- d Determine the output when the input is the unit-impulse sequence, $\delta[n]$.
- e Determine the output when the input is the unit-step sequence, $u[n]$.

Break!

See you at ______________________________

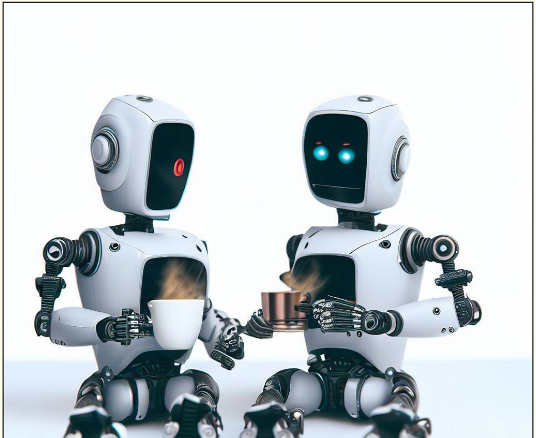


Table of Contents

P 6.7

For each of the following frequency responses determine the corresponding impulse response

a $H(e^{j\hat{\omega}}) = 1 + 2e^{-j3\hat{\omega}}$

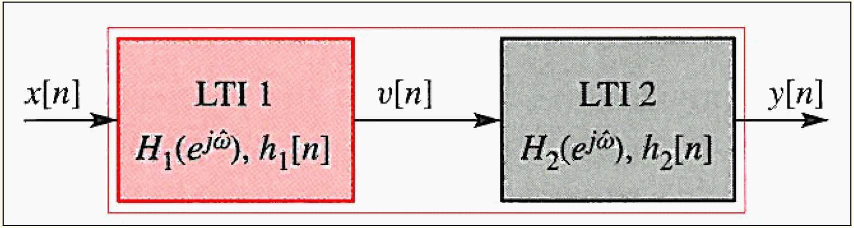
b $H(e^{j\hat{\omega}}) = 2e^{-j3\hat{\omega}}\cos(\hat{\omega})$

c $H(e^{j\hat{\omega}}) = e^{-j4.5\hat{\omega}}\frac{\sin(5\hat{\omega})}{\sin(\hat{\omega}/2)}$

Table of Contents

P 6.16 / P 6.17

The diagram below depicts a cascade connection of two linear time-invariant systems, where the first system is a 3-point moving averager and the second system is a first difference.



a If the input is of the form $x[n] = 10 + x_1[n]$, the output, $y[n]$, of the overall system will be of the form $y[n] = y_1[n]$, where $y_1[n]$ is the output due only to $x_1[n]$. Explain why this is true.

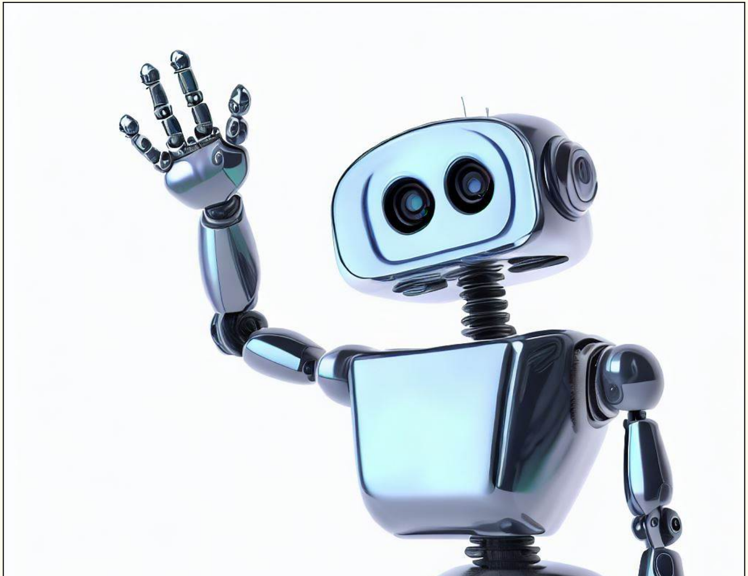
b Determine the frequency-response function of the overall cascade system.

P 6.16 / P 6.17

c Sketch the frequency-response (magnitude and phase) functions of the individual systems and the overall cascade system for $-\pi < \hat{\omega} \leqslant \pi$.

d Obtain a single-difference equation that relates $y[n]$ to $x[n]$ for the overall cascade system.

Have a nice day!



Acknowledgements

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Disclaimer

- Questions and images are based in Schafer, R. W., Yoder, M. A., & McClellan, J. H. (2003). Signal Processing First. Prentice Hall.
- Grammar was checked with Grammarly and Grammar checker GPT.
- Images without source were created with the assistance of DALL·E.