university of groningen / faculty of science and engineering

Lecture 1B: Sinusoids

Juan Diego Cardenas-Cartagena, M.Sc.
(j.d.cardenas.cartagena@rug.nl)

Signals and Systems
1B - 2024/2025

Preliminaries

- You can now enroll with a lab partner in groups for the assignments; and choose your preferred Tutorial schedule in the Tools/Groups tab. The enrollment deadline is Tuesday, November 19th at midnight.
- The first lab assignment is available now. And its deadline is on Friday, December 6, at 17:30.

Preliminaries

AI & CCS/CS Programme Committees event

On **November 20**, the programme committees of CS, AI, and CCS will hold an event where they will introduce themselves and explain their roles. You will have the opportunity to meet your representatives, ask questions, raise any issues you have encountered in your studies, and recommend outstanding lecturers for this year's Teacher of the Year Award.

The event will begin at 12:00 in **BB 5161.0116**, and you are welcome to stop by until 16:00. Free pizza and drinks will be provided.

Overview

Table of Contents

Recap

- A **signal** is a pattern of variation that represents or encodes information.
- A **system** is an operator that transforms signals.

Table of Contents

Sinusoidal signals

A **sinusoidal signal** is a continuous time signal of the form

$$x(t) = A\cos(\omega_0 t + \varphi) \tag{1}$$

Sinusoidal signals

A **sinusoidal signal** is a continuous time signal of the form

$$x(t) = A\cos(\omega_0 t + \varphi) \tag{1}$$

Where,

- A is the **amplitude**

Sinusoidal signals

A **sinusoidal signal** is a continuous time signal of the form

$$x(t) = A\cos(\omega_0 t + \varphi) \tag{1}$$

Where,

- A is the **amplitude**
- ω_0 is the **radian frequency**.

Sinusoidal signals

A **sinusoidal signal** is a continuous time signal of the form

$$x(t) = A\cos(\omega_0 t + \varphi) \tag{1}$$

Where,

- A is the **amplitude**
- ω_0 is the **radian frequency**.
  - A **cyclic frequency** of f Hertz (Hz) corresponds to

$$\omega_0 = 2\pi f \tag{2}$$

  - **Period** in seconds

$$T = \frac{1}{f} = \frac{2\pi}{\omega_0} \tag{3}$$

Sinusoidal signals

A **sinusoidal signal** is a continuous time signal of the form

$$x(t) = A\cos(\omega_0 t + \varphi) \tag{1}$$

Where,

- A is the **amplitude**
- ω_0 is the **radian frequency**.
  - A **cyclic frequency** of f Hertz (Hz) corresponds to

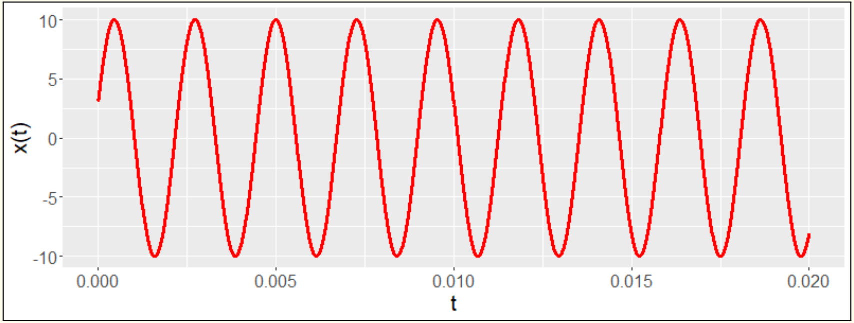
$$\omega_0 = 2\pi f \tag{2}$$

  - **Period** in seconds

$$T = \frac{1}{f} = \frac{2\pi}{\omega_0} \tag{3}$$

- And, φ is the **phase**

Example

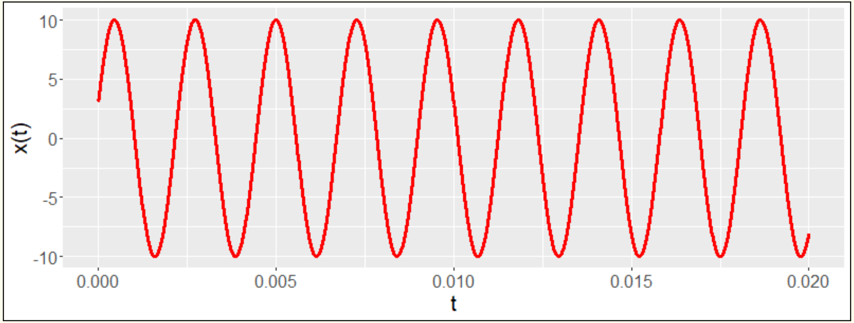


The figure above corresponds to the signal

$$x(t) = 10\cos(2\pi(440)t - 0.4\pi) \tag{4}$$

With oscillation amplitude $A = 10$ and period $T = 1/440$ (~ 0.00227) seconds

Example

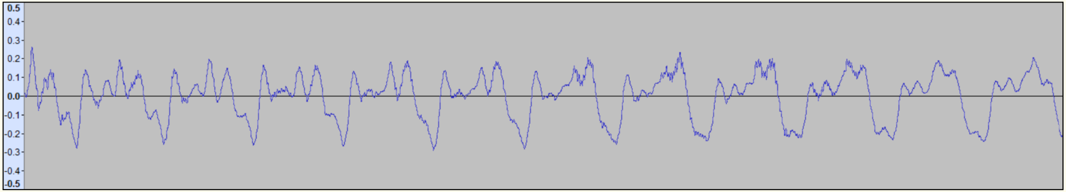


"Pure" tones consist of exactly one frequency

- E.g. produced by a tuning fork
- 440 Hz corresponds to the note A above middle C[1]

[1] http://dspfirst.gatech.edu/chapters/02sines/demos/tuningfo/index.html

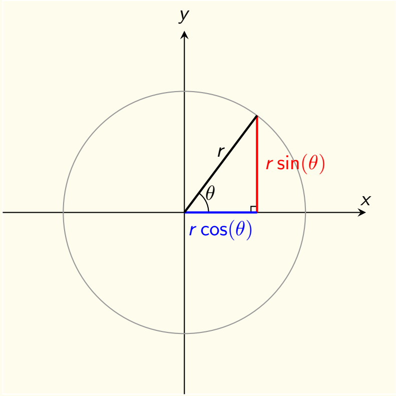
More complex signals



Speech signals tend to be (almost) periodic as well

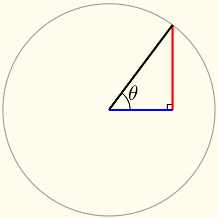
- Definitely not a single sinusoid
- Maybe a combination of a few sinusoids?

Angles

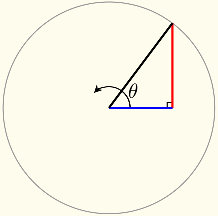


$$\begin{aligned} \cos(\theta) &= \frac{x}{r} && \Rightarrow && x = r\cos(\theta) \\ \sin(\theta) &= \frac{y}{r} && \Rightarrow && y = r\sin(\theta) \end{aligned} \tag{5}$$

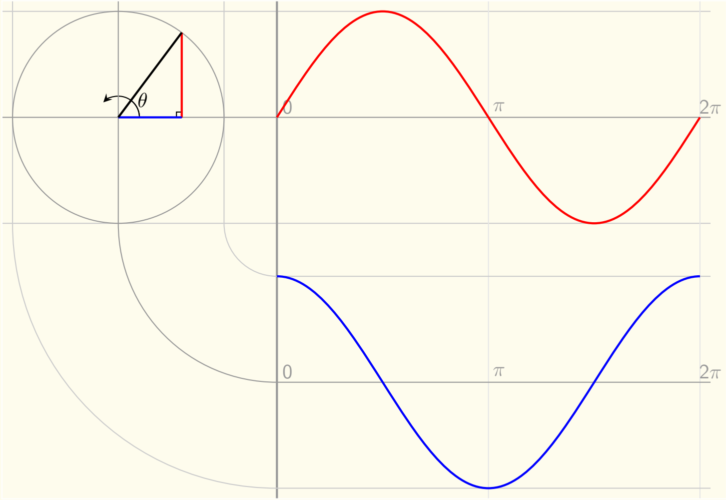
Angles



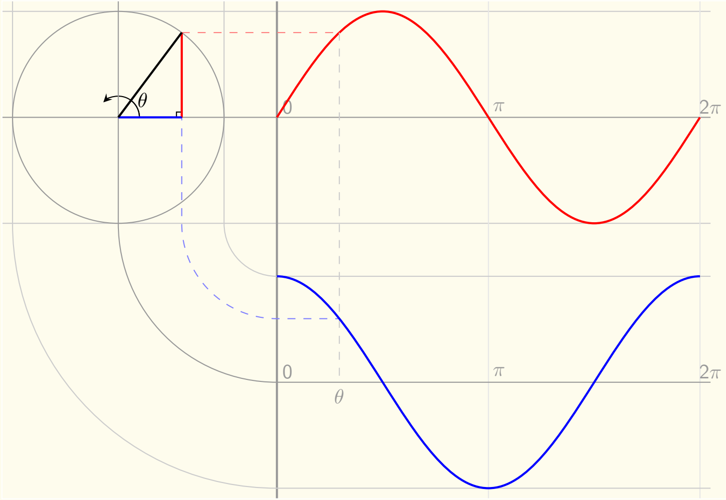
Angles



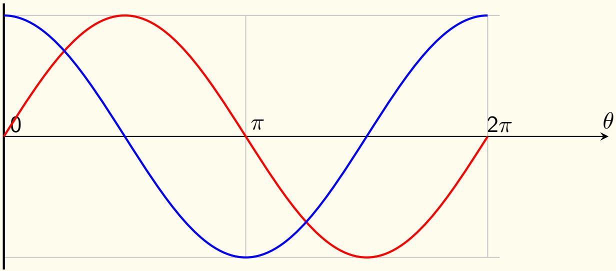
Angles



Angles

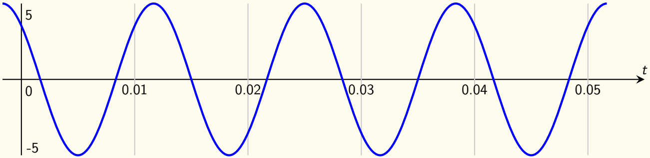


Basic properties of sinusoids



Property	Cosine equation	Sine equation
Equivalence	$\cos(\theta) = \sin(\theta + \pi/2)$	$\sin(\theta) = \cos(\theta - \pi/2)$
Maxima	$\cos(2k\pi) = 1$	$\sin((2k + 1/2)\pi) = 1$
Minima	$\cos((2k + 1)\pi) = -1$	$\sin((2k - 1/2)\pi) = -1$
Zeroes	$\cos((k + 1/2)\pi) = 0$	$\sin(k\pi) = 0$

Reading a sinusoid function from a graph

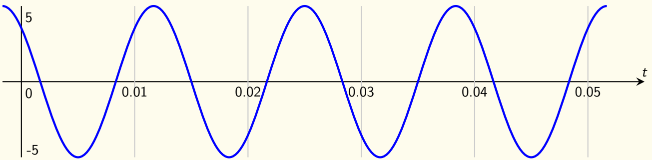


Suppose you are asked to determine the sinusoid from a graph

$$x(t) = A\cos(\omega_0 t + \varphi)$$

- Read A from the vertical axis
- Determine the period T or frequency f_0
- Determine the phase φ

Reading a sinusoid function from a graph

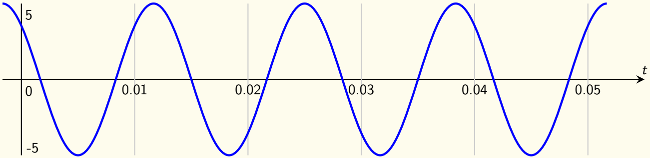


Suppose you are asked to determine the sinusoid from a graph

$$x(t) = A\cos(\omega_0 t + \varphi)$$

- Read A from the vertical axis
- Determine the period T or frequency f_0
- Determine the phase φ

Reading a sinusoid function from a graph

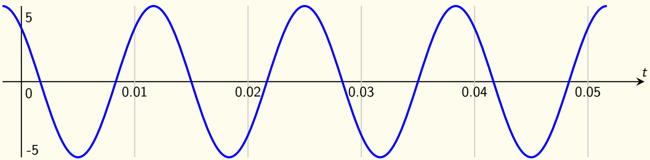


Suppose you are asked to determine the sinusoid from a graph

$$x(t) = 5\cos(\omega_0 t + \varphi)$$

- Read A from the vertical axis
- Determine the period T or frequency f_0
- Determine the phase φ

Reading a sinusoid function from a graph

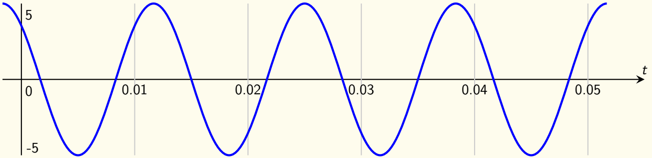


Suppose you are asked to determine the sinusoid from a graph

$$x(t) = 5\cos(\omega_0 t + \varphi)$$

- Read A from the vertical axis
- Determine the period T or frequency f_0
- Determine the phase φ

Reading a sinusoid function from a graph

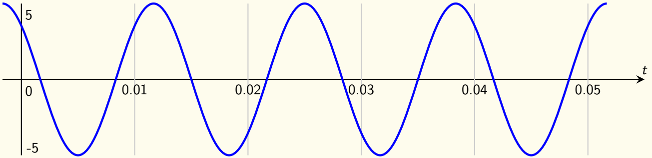


Suppose you are asked to determine the sinusoid from a graph

$$x(t) = 5\cos(\omega_0 t + \varphi)$$

- Read A from the vertical axis
- Determine the period T or frequency f_0
  - From 0.01 to 0.05, the graph goes through 3 iterations
- Determine the phase φ

Reading a sinusoid function from a graph

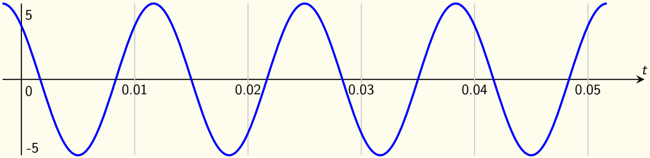


Suppose you are asked to determine the sinusoid from a graph

$$x(t) = 5\cos(\omega_0 t + \varphi)$$

- Read A from the vertical axis
- Determine the period T or frequency f_0
  - $T = (0.05 - 0.01)/3 = 0.04/3 = 4/300$
- Determine the phase φ

Reading a sinusoid function from a graph

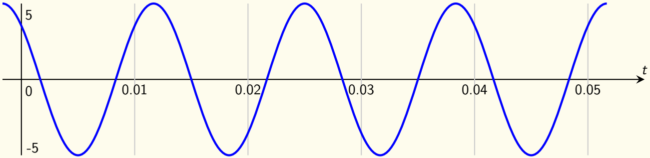


Suppose you are asked to determine the sinusoid from a graph

$$x(t) = 5\cos(\omega_0 t + \varphi)$$

- Read A from the vertical axis
- Determine the period T or frequency f_0
  - $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4/300} = 150\pi$
- Determine the phase φ

Reading a sinusoid function from a graph

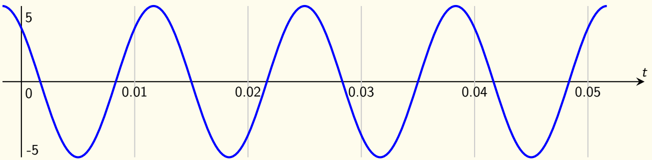


Suppose you are asked to determine the sinusoid from a graph

$$x(t) = 5\cos(150\pi t + \varphi)$$

- Read A from the vertical axis
- Determine the period T or frequency f_0
  - $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4/300} = 150\pi$
- Determine the phase φ

Reading a sinusoid function from a graph

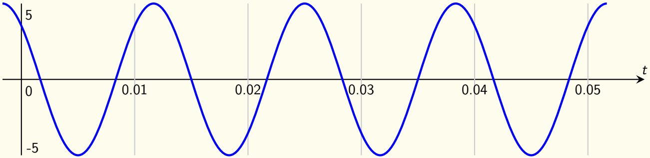


Suppose you are asked to determine the sinusoid from a graph

$$x(t) = 5\cos(150\pi t + \varphi)$$

- Read A from the vertical axis
- Determine the period T or frequency f_0
- Determine the phase φ

Reading a sinusoid function from a graph

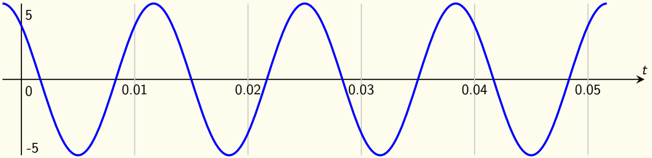


Suppose you are asked to determine the sinusoid from a graph

$$x(t) = 5\cos(150\pi t + \varphi)$$

- Read A from the vertical axis
- Determine the period T or frequency f_0
- Determine the phase φ
  - At $t = 0.045$, 3.5 iterations have passed

Reading a sinusoid function from a graph

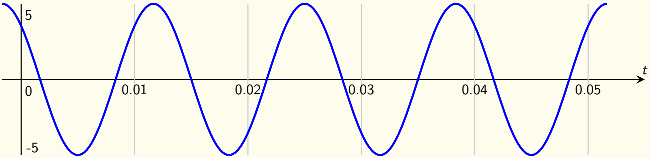


Suppose you are asked to determine the sinusoid from a graph

$$x(t) = 5\cos(150\pi t + \varphi)$$

- Read A from the vertical axis
- Determine the period T or frequency f_0
- Determine the phase φ
  - $150\pi(0.045) + \varphi = 2\pi(3.5) \Rightarrow \varphi = 0.25\pi$

Reading a sinusoid function from a graph

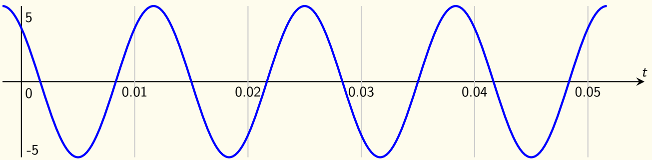


Suppose you are asked to determine the sinusoid from a graph

$$x(t) = 5\cos(150\pi t + 0.25\pi)$$

- Read A from the vertical axis
- Determine the period T or frequency f_0
- Determine the phase φ
  - $150\pi(0.045) + \varphi = 2\pi(3.5) \Rightarrow \varphi = 0.25\pi$

Reading a sinusoid function from a graph



Suppose you are asked to determine the sinusoid from a graph

$$x(t) = 5\cos(150\pi t + 0.25\pi)$$

- Read A from the vertical axis
- Determine the period T or frequency f_0
- Determine the phase φ
  - Read from graph: $\varphi = 0.25\pi$

Sinusoid functions

Question

What would you expect to see if the frequency f_0 becomes zero? Conside the signal

$$x(t) = 5\cos(0.25\pi) \tag{6}$$

Table of Contents

Sinusoids and Complex Exponential Signals

Signals can be represented mathematically as sinusoids. However, the analysis of sinusoid functions is difficult. The analysis is simplified by using complex exponential signals. The following equation relates **complex exponents and sinusoid functions**:

$$Ae^{j(\omega_0 t+\varphi)} = A\cos(\omega_0 t + \varphi) + jA\sin(\omega_0 t + \varphi) \tag{7}$$

where $j^2 = -1$ is the **imaginary unit**. This relation comes from the **Euler's formula**.

Cartesian form complex numbers

In **rectangular form** or **Cartesian form**, complex numbers are represented as points in a two-dimensional plane

$$\begin{aligned} z &= (a, b) \\ &= a + jb \\ &= \Re\{z\} + j(\Im\{z\}) \end{aligned} \tag{8}$$

Where,

- $\Re\{z\}$ is the *real part* of z
- $\Im\{z\}$ is the *imaginary part* of z

Cartesian form complex numbers

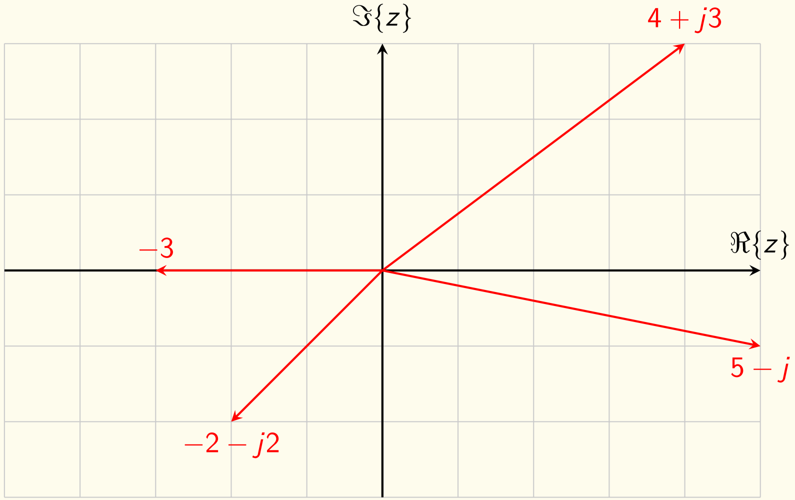


Figure: Representation of different complex numbers in the Cartesian plane

Polar form complex numbers

In **polar form**, complex numbers are represented in lengths and angles

$$\begin{aligned} z &= r\angle\theta \\ &= r\cos(\theta) + jr\sin(\theta) \end{aligned}$$

Where $\theta \in (-\pi, \pi]$ is the angle (in radians) and $r > 0$ is the radius length.

Polar form complex numbers

In **polar form**, complex numbers are represented in lengths and angles

$$
\begin{aligned}
z &= r\angle\theta \\
&= r\cos(\theta) + jr\sin(\theta)
\end{aligned}
$$

Where $\theta \in (-\pi, \pi]$ is the angle (in radians) and $r > 0$ is the radius length.

Relation between Polar and Cartesian coordinates

For $z = (a, b)$, $r = \sqrt{a^2 + b^2}$ and $\theta = \arctan\left(\frac{b}{a}\right)$

Polar form complex numbers

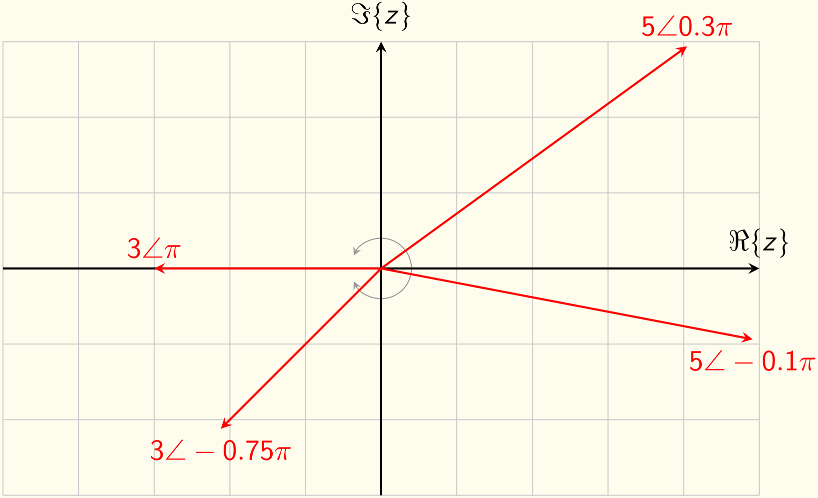


Figure: Representation of different complex numbers in the polar plane

Euler's formula

Euler's formula relates sinusoids to the complex exponential

$$e^{j\theta} = \cos(\theta) + j\sin(\theta) \tag{9}$$

The inverse Euler's formulas switch the relation around

$$\begin{aligned} \cos(\theta) &= \frac{e^{j\theta} + e^{-j\theta}}{2} \\ \sin(\theta) &= \frac{e^{j\theta} - e^{-j\theta}}{2j} \end{aligned}$$

Euler's formula

Proof

- Let us write the exponential function as a power series

$$
\begin{aligned}
e^z &= \sum_{n=0}^{\infty} \frac{z^n}{n!} \\
&= 1 + \frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots
\end{aligned}
\tag{10}
$$

Euler's formula

- Recall the following power series expansions,

$$\cos(z) = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n}}{(2n)!} = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} + \dots, \tag{11}$$

$$\sin(z) = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{(2n+1)!} = z - \frac{z^3}{3!} + \frac{z^5}{5!} + \dots. \tag{12}$$

Euler's formula

- Recall the following power series expansions,

$$\begin{aligned}\cos(z) &= \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n}}{(2n)!} \\ &= 1 - \frac{z^2}{2!} + \frac{z^4}{4!} + \ldots,\end{aligned} \tag{11}$$

$$\begin{aligned}\sin(z) &= \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{(2n+1)!} \\ &= z - \frac{z^3}{3!} + \frac{z^5}{5!} + \ldots.\end{aligned} \tag{12}$$

- And the cycle in the imaginary unit j, i.e.,

$$j = \sqrt{-1}, \qquad j^2 = -1, \qquad j^3 = -j. \qquad j^4 = 1, \qquad j^5 = j, \ldots \tag{13}$$

Euler's formula

- Take $z = jx$ in (10),

$$e^{jx} = 1 + \frac{jx}{1!} + \frac{(jx)^2}{2!} + \frac{(jx)^3}{3!} + \dots \tag{14}$$

Euler's formula

- Take $z = jx$ in (10),

$$e^{jx} = 1 + \frac{jx}{1!} + \frac{(jx)^2}{2!} + \frac{(jx)^3}{3!} + \dots \tag{14}$$

- Use the properties for the imaginary unit, as shown in (13), to simplify (14),

$$\begin{aligned} e^{jx} &= 1 + jx - \frac{x^2}{2!} - \frac{jx^3}{3!} + \frac{x^4}{4!} + \frac{jx^5}{5!} - \frac{x^6}{6!} - \frac{jx^7}{7!} + \dots \\ &= \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots\right) + j\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots\right). \end{aligned} \tag{15}$$

Euler's formula

- Take $z = jx$ in (10),

$$e^{jx} = 1 + \frac{jx}{1!} + \frac{(jx)^2}{2!} + \frac{(jx)^3}{3!} + \dots \tag{14}$$

- Use the properties for the imaginary unit, as shown in (13), to simplify (14),

$$\begin{aligned} e^{jx} &= 1 + jx - \frac{x^2}{2!} - \frac{jx^3}{3!} + \frac{x^4}{4!} + \frac{jx^5}{5!} - \frac{x^6}{6!} - \frac{jx^7}{7!} + \dots \\ &= \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots\right) + j\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots\right). \end{aligned} \tag{15}$$

- Note the grouped terms in (15) have the same structure as in (11) and (12). Hence

$$e^{jx} = \cos(x) + j\sin(x) \quad \square$$

Complex exponential signals

Definition

The complex exponential signal is defined as

$$\begin{aligned} z(t) &= Ae^{j(\omega_0 t+\varphi)} \\ &= A\cos(\omega_0 t + \varphi) + jA\sin(\omega_0 t + \varphi) \end{aligned} \tag{16}$$

Complex exponential signals

Definition

The complex exponential signal is defined as

$$\begin{aligned} z(t) &= Ae^{j(\omega_0 t+\varphi)} \\ &= A\cos(\omega_0 t + \varphi) + jA\sin(\omega_0 t + \varphi) \end{aligned} \tag{16}$$

As a result, a **sinusoid** can be represented by the **real part** of a complex exponential signal

$$\begin{aligned} x(t) &= \Re\{z(t)\} \\ &= \Re\left\{Ae^{j(\omega_0 t+\varphi)}\right\} \\ &= A\cos(\omega_0 t + \varphi) \end{aligned} \tag{17}$$

Complex is easier

Calculations with complex exponentials are easier

$$\cos(\theta_1 + \theta_2)$$

Complex is easier

Calculations with complex exponentials are easier

$$\cos(\theta_1 + \theta_2) \quad = \quad \Re\left\{e^{j(\theta_1+\theta_2)}\right\}$$

Complex is easier

Calculations with complex exponentials are easier

$$\cos(\theta_1 + \theta_2) \quad = \quad \Re\left\{e^{j(\theta_1+\theta_2)}\right\} = \Re\left\{e^{j\theta_1 + j\theta_2}\right\} = \Re\left\{e^{j\theta_1} e^{j\theta_2}\right\}$$

Complex is easier

Calculations with complex exponentials are easier

$$\begin{aligned}\cos(\theta_1+\theta_2) &= \Re\left\{e^{j(\theta_1+\theta_2)}\right\} = \Re\left\{e^{j\theta_1+j\theta_2}\right\} = \Re\left\{e^{j\theta_1}e^{j\theta_2}\right\} \\ &= \Re\left\{\Big(\cos(\theta_1)+j\sin(\theta_1)\Big)\cdot\Big(\cos(\theta_2)+j\sin(\theta_2)\Big)\right\}\end{aligned}$$

Complex is easier

Calculations with complex exponentials are easier

$$
\begin{aligned}
\cos(\theta_1 + \theta_2) &= \Re\left\{e^{j(\theta_1+\theta_2)}\right\} = \Re\left\{e^{j\theta_1+j\theta_2}\right\} = \Re\left\{e^{j\theta_1}e^{j\theta_2}\right\} \\
&= \Re\left\{\Big(\cos(\theta_1) + j\sin(\theta_1)\Big)\cdot\Big(\cos(\theta_2) + j\sin(\theta_2)\Big)\right\} \\
&= \Re\{\cos(\theta_1)\cos(\theta_2) + j\cos(\theta_1)\sin(\theta_2) + \\
&\qquad j\sin(\theta_1)\cos(\theta_2) + j^2\sin(\theta_1)\sin(\theta_2)\}
\end{aligned}
$$

Complex is easier

Calculations with complex exponentials are easier

$$\begin{aligned}
\cos(\theta_1 + \theta_2) &= \Re\left\{e^{j(\theta_1+\theta_2)}\right\} = \Re\left\{e^{j\theta_1+j\theta_2}\right\} = \Re\left\{e^{j\theta_1}e^{j\theta_2}\right\} \\
&= \Re\left\{\Big(\cos(\theta_1) + j\sin(\theta_1)\Big)\cdot\Big(\cos(\theta_2) + j\sin(\theta_2)\Big)\right\} \\
&= \Re\{\cos(\theta_1)\cos(\theta_2) + j\cos(\theta_1)\sin(\theta_2) + \\
&\qquad j\sin(\theta_1)\cos(\theta_2) + j^2\sin(\theta_1)\sin(\theta_2)\} \\
&= \Re\{\cos(\theta_1)\cos(\theta_2) + j\cos(\theta_1)\sin(\theta_2) + \\
&\qquad j\sin(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)\}
\end{aligned}$$

Complex is easier

Calculations with complex exponentials are easier

$$
\begin{aligned}
\cos(\theta_1+\theta_2) &= \Re\left\{e^{j(\theta_1+\theta_2)}\right\} = \Re\left\{e^{j\theta_1+j\theta_2}\right\} = \Re\left\{e^{j\theta_1}e^{j\theta_2}\right\} \\
&= \Re\left\{\Big(\cos(\theta_1)+j\sin(\theta_1)\Big)\cdot\Big(\cos(\theta_2)+j\sin(\theta_2)\Big)\right\} \\
&= \Re\{\cos(\theta_1)\cos(\theta_2)+j\cos(\theta_1)\sin(\theta_2)+ \\
&\qquad j\sin(\theta_1)\cos(\theta_2)+j^2\sin(\theta_1)\sin(\theta_2)\} \\
&= \Re\{\cos(\theta_1)\cos(\theta_2)+j\cos(\theta_1)\sin(\theta_2)+ \\
&\qquad j\sin(\theta_1)\cos(\theta_2)-\sin(\theta_1)\sin(\theta_2)\} \\
&= \cos(\theta_1)\cos(\theta_2)-\sin(\theta_1)\sin(\theta_2)
\end{aligned}
$$

Break!

See you at ____________________

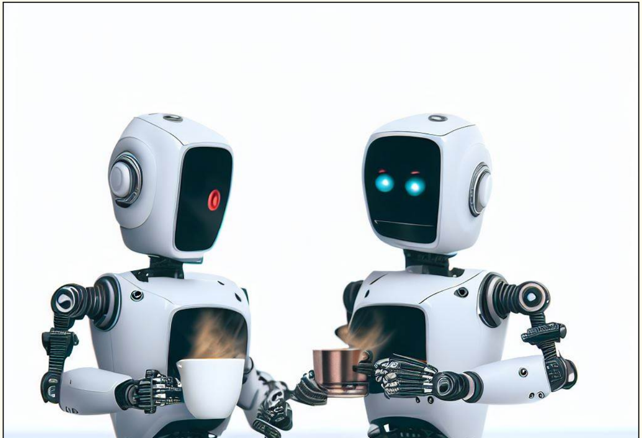
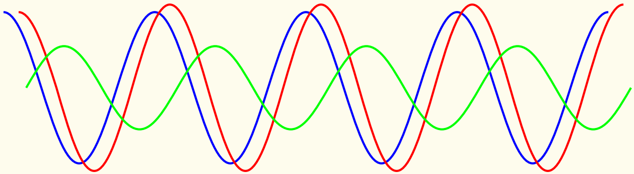


Table of Contents

Phasor addition

Suppose N signals with the **same frequency** ω_0 but different amplitudes A_k and phases φ_k, $k = 1, 2, \ldots, N$ occur at the same time.



What does the resulting signal look like?

Phasor addition

$$\sum_{k=1}^{N} A_k \cos(\omega_0 t + \varphi_k)$$

Phasor addition

$$\sum_{k=1}^{N} A_k \cos(\omega_0 t + \varphi_k) = \sum_{k=1}^{N} \mathfrak{R}\left\{A_k e^{j(\omega_0 t + \varphi_k)}\right\}$$

Phasor addition

$$\begin{aligned}\sum_{k=1}^{N} A_k \cos(\omega_0 t + \varphi_k) &= \sum_{k=1}^{N} \Re\left\{A_k e^{j(\omega_0 t + \varphi_k)}\right\} \\ &= \Re\left\{\sum_{k=1}^{N} A_k e^{j\omega_0 t + j\varphi_k}\right\}\end{aligned}$$

Phasor addition

$$\begin{aligned}
\sum_{k=1}^{N} A_k \cos(\omega_0 t + \varphi_k) &= \sum_{k=1}^{N} \Re\left\{A_k e^{j(\omega_0 t + \varphi_k)}\right\} \\
&= \Re\left\{\sum_{k=1}^{N} A_k e^{j\omega_0 t + j\varphi_k}\right\} \\
&= \Re\left\{\sum_{k=1}^{N} A_k e^{j\omega_0 t} e^{j\varphi_k}\right\}
\end{aligned}$$

Phasor addition

$$
\begin{aligned}
\sum_{k=1}^{N} A_k \cos(\omega_0 t + \varphi_k) &= \sum_{k=1}^{N} \Re\left\{A_k e^{j(\omega_0 t + \varphi_k)}\right\} \\
&= \Re\left\{\sum_{k=1}^{N} A_k e^{j\omega_0 t + j\varphi_k}\right\} \\
&= \Re\left\{\sum_{k=1}^{N} A_k e^{j\omega_0 t} e^{j\varphi_k}\right\} \\
&= \Re\left\{e^{j\omega_0 t}\left(\sum_{k=1}^{N} A_k e^{j\varphi_k}\right)\right\}
\end{aligned}
$$

Adding complex numbers

$$\sum_{k=1}^{N} A_k e^{j\varphi_k} = \sum_{k=1}^{N} A_k \angle \varphi_k \tag{18}$$

Adding complex numbers

$$\sum_{k=1}^{N} A_k e^{j\varphi_k} = \sum_{k=1}^{N} A_k \angle \varphi_k \tag{18}$$

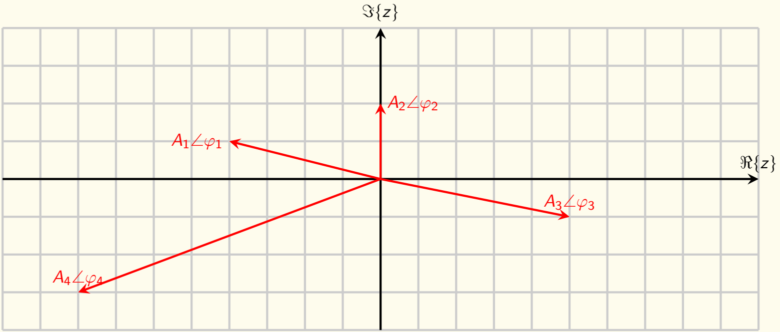


Figure: Representation of summing complex numbers in polar coordinates

Adding complex numbers

$$\sum_{k=1}^{N} A_k e^{j\varphi_k} = \sum_{k=1}^{N} A_k \angle \varphi_k \tag{18}$$

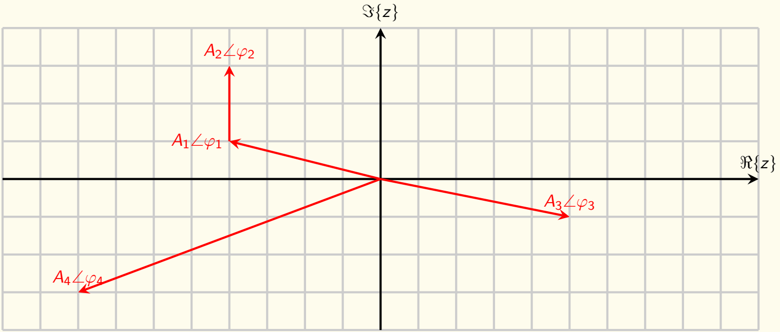


Figure: Representation of summing complex numbers in polar coordinates

Adding complex numbers

$$\sum_{k=1}^{N} A_k e^{j\varphi_k} = \sum_{k=1}^{N} A_k \angle \varphi_k \tag{18}$$

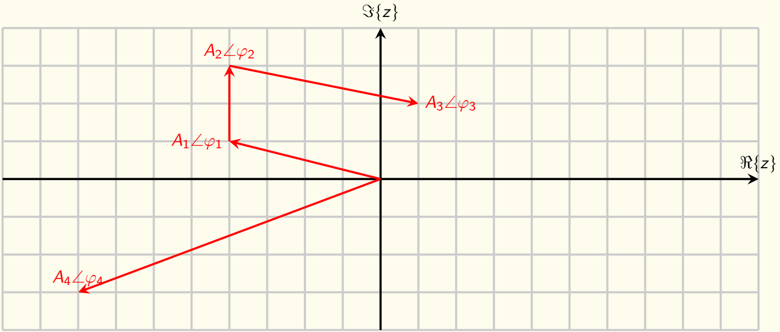


Figure: Representation of summing complex numbers in polar coordinates

Adding complex numbers

$$\sum_{k=1}^{N} A_k e^{j\varphi_k} = \sum_{k=1}^{N} A_k \angle \varphi_k \quad (18)$$

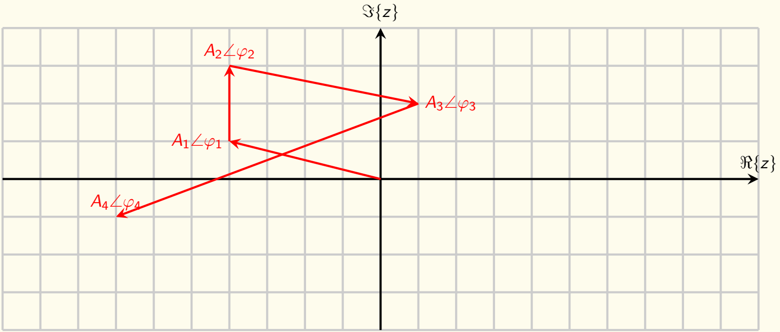


Figure: Representation of summing complex numbers in polar coordinates

Adding complex numbers

$$\sum_{k=1}^{N} A_k e^{j\varphi_k} = \sum_{k=1}^{N} A_k \angle \varphi_k \tag{18}$$

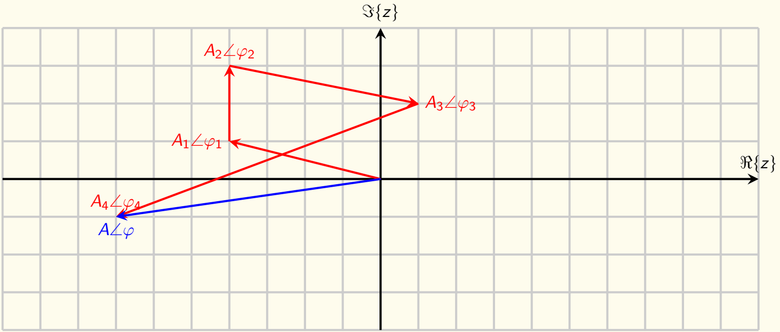


Figure: Representation of summing complex numbers in polar coordinates

Adding complex numbers

$$\sum_{k=1}^{N} A_k e^{j\varphi_k} = \sum_{k=1}^{N} A_k \angle \varphi_k = A \angle \varphi \tag{18}$$

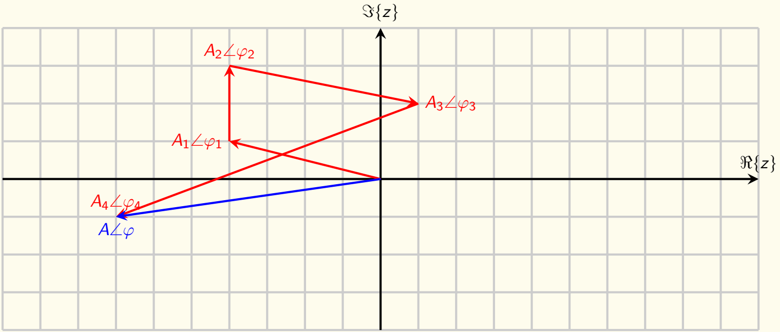


Figure: Representation of summing complex numbers in polar coordinates

Adding complex numbers

$$\sum_{k=1}^{N} A_k e^{j\varphi_k} = \sum_{k=1}^{N} A_k \angle \varphi_k = A\angle\varphi = Ae^{j\varphi} \tag{18}$$

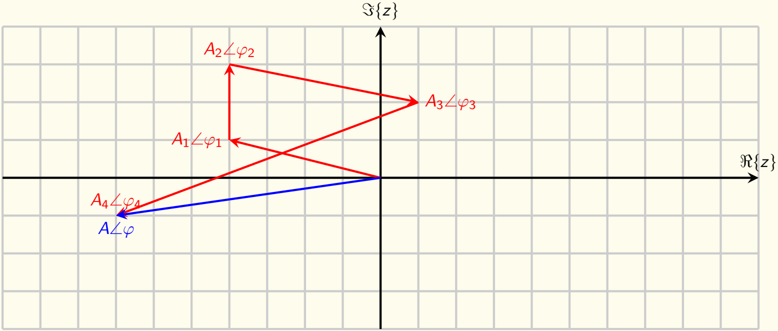


Figure: Representation of summing complex numbers in polar coordinates

Adding same frequency sinusoids

$$
\begin{aligned}
\sum_{k=1}^{N} A_k \cos(\omega_0 t + \varphi_k) &= \Re\left\{ e^{j\omega_0 t} \left(\sum_{k=1}^{N} A_k e^{j\varphi_k} \right) \right\} \\
&= \Re\left\{ e^{j\omega_0 t} A e^{j\varphi} \right\} \\
&= A \cos(\omega_0 t + \varphi)
\end{aligned}
\tag{19}
$$

Example

$$1.7\cos\left(2\pi(10)t + 70\pi/180\right) + 1.9\cos\left(2\pi(10)t + 200\pi/180\right) \tag{20}$$

Example

$$1.7\cos\left(2\pi(10)t + 70\pi/180\right) + 1.9\cos\left(2\pi(10)t + 200\pi/180\right) \tag{20}$$

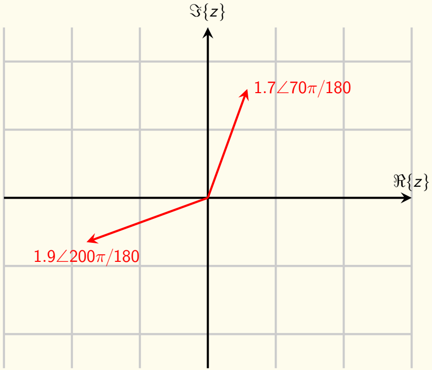


Figure: Sum of two sinusoids with the same frequency in the polar plane

Example

$$1.7\cos\left(2\pi(10)t + 70\pi/180\right) + 1.9\cos\left(2\pi(10)t + 200\pi/180\right) \tag{20}$$

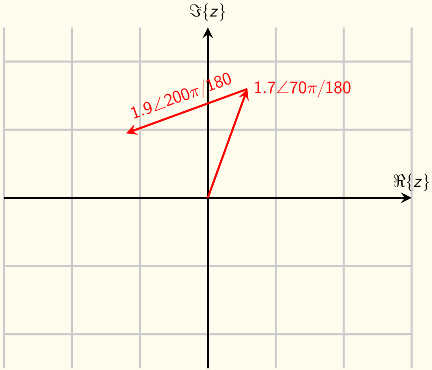


Figure: Sum of two sinusoids with the same frequency in the polar plane

Example

$$\begin{aligned}1.7\cos\left(2\pi(10)t + 70\pi/180\right) + 1.9\cos\left(2\pi(10)t + 200\pi/180\right) \\ = 1.532\cos\left(2\pi(10)t + 141.8\pi/180\right)\end{aligned} \tag{20}$$

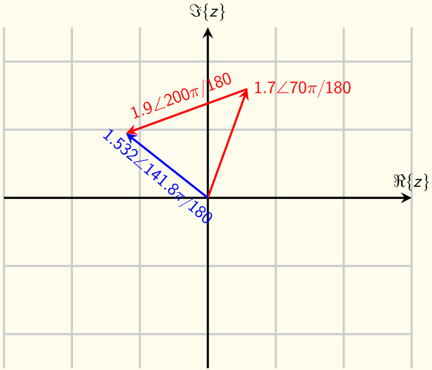


Figure: Sum of two sinusoids with the same frequency in the polar plane

Phasor addition rule

To add signals with the same frequency

$$x(t) = \sum_{k=1}^{N} A_k \cos(\omega_0 t + \varphi_k) = A \cos(\omega_0 t + \varphi) \tag{21}$$

Phasor addition rule

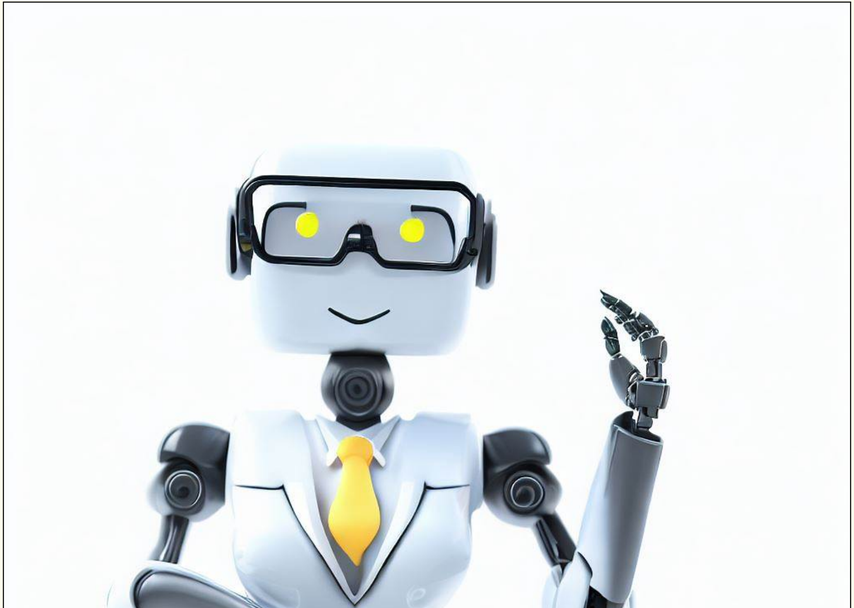
To add signals with the same frequency

$$x(t) = \sum_{k=1}^{N} A_k \cos(\omega_0 t + \varphi_k) = A\cos(\omega_0 t + \varphi) \tag{21}$$

1. Obtain the phasor representations $X_k = A_k e^{j\varphi_k}$ for each term
2. Convert polar to Cartesian coordinates $X_k = a_k + jb_k$
3. Calculate the sum of phasors $X = (\sum a_k) + j(\sum b_k)$
4. Convert Cartesian to polar coordinates $X = Ae^{j\varphi}$
5. Obtain the sinusoid $A\cos(\omega_0 t + \varphi)$

Table of Contents

Let us wrap up the lecture!



Take-home Messages

- A sinusoidal signal is a continuous-time signal represented by sine and cosine functions.
- The Euler's formula relates complex exponents and sinusoid functions. This relation brings us to the complex exponential signal concept.
- A sinusoid can be represented by the real part of a complex exponential signal.
- Summing sinusoid signals with the same frequency follows the phasor addition rule.

Practice Questions

Similar questions might appear in the final exam:

- What are the components of a sinusoidal signal?
- How do you read a sinusoid function?
- How do you sum sinusoid signals with the same frequency?

Tutorial exercises

During the tutorial, the exercises below will be discussed in class. Attempt to complete the exercises **before** class starts.

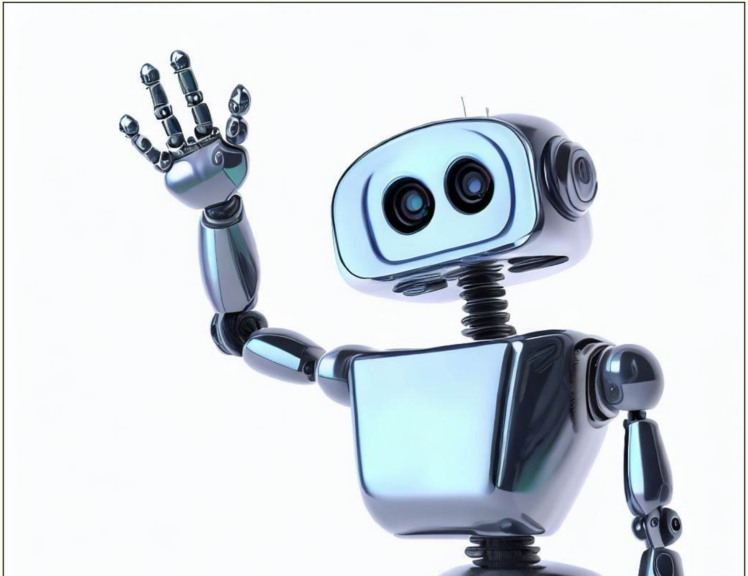
SPF	DSPF
P 2.2 (p. 31)	P 2.2 (p. 60)
Ex 2.1 (p. 11)	Ex 2.1 (p. 34)
Ex 2.6 (p. 19)	Ex 2.6 (p. 44)
Ex 2.9 (p. 26)	Ex 2.8 (p. 54)
P 2.4 (p. 32)	P 2.4 (p. 61)
P 2.6 (p. 32)	P 2.6 (p. 61)
P 2.15 (p. 33)	P 2.15 (p. 63)
P 2.20 (p. 34)	P 2.24 (p. 65)

Next Lecture

Let us analyze signals in the frequency domain.

Spectrum representation

See you on Friday!



Acknowledgements

The material for this lecture series was developed by dr. Arnold Meijster and dr. Harmen de Weerd and modified by Juan Diego Cardenas-Cartagena.

Disclaimer

- Grammar was checked with Grammarly and Grammar checker GPT.
- Images without source were created with the assistance of DALL·E.