Formula Sheet

Trigonometric Identities

$$
\begin{aligned}
\cos(\theta) &= \sin(\theta + \pi/2) \\
\cos(\theta) &= \cos(\theta + 2\pi k),\ k \in \mathbb{Z} \\
\cos(\theta) &= \cos(-\theta) \\
\sin(\theta) &= -\sin(\theta) \\
\cos(2\pi k) &= 1,\ k \in \mathbb{Z} \\
\cos(\pi k + \pi/2) &= 0,\ k \in \mathbb{Z} \\
\cos(2\pi k + \pi) &= -1,\ k \in \mathbb{Z} \\
\cos^2(\theta) + \sin^2(\theta) &= 1 \\
\cos(\theta)\cos(\phi) &= \frac{1}{2}\left(\cos(\theta - \phi) + \cos(\theta + \phi)\right) \\
\sin(\theta)\sin(\phi) &= \frac{1}{2}\left(\cos(\theta - \phi) - \cos(\theta + \phi)\right) \\
\sin(\theta)\cos(\phi) &= \frac{1}{2}\left(\sin(\theta - \phi) + \sin(\theta + \phi)\right) \\
\cos(\theta)\sin(\phi) &= \frac{1}{2}\left(\sin(\theta - \phi) - \sin(\theta + \phi)\right)
\end{aligned}
$$

Complex numbers and Euler's formula

$$
\begin{aligned}
j^2 &= -1 \\
\Re(a + jb) &= a \\
\Im(a + jb) &= b \\
(a + jb)^* &= a - jb \\
e^{j\theta} &= \cos(\theta) + j\sin(\theta) \\
\cos(\theta) &= \frac{e^{j\theta} + e^{-j\theta}}{2} \\
\sin(\theta) &= \frac{e^{j\theta} - e^{-j\theta}}{2j}
\end{aligned}
$$

Relevant Integrals and functions

$$
\begin{aligned}
\int e^{\theta t} dt &= \frac{e^{\theta t}}{\theta} + c, \quad \theta \neq 0 \\
\int t e^{\theta t} dt &= \frac{\theta t - 1}{\theta^2} e^{\theta t} + c, \quad \theta \neq 0 \\
\prod_{k=m}^{M} a_k &= a_m a_{m+1} \cdots a_M, \quad m < M
\end{aligned}
$$

Consider the polynomial $ax^2 + bx + c$, where $a, b, c \in \mathbb{R}$. Its roots are given by

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Signal Processing

Let $T_s, f_s > 0$ be the sampling period and frequency, respectively. Let $x[n]$ and $y[n]$ be discrete-time signals, and $h[n]$ be the unit impulse response.

$$
\begin{aligned}
f_s &= \frac{1}{T_s} \\
\hat{\omega} &= \omega T_s \\
x[n] &= x(nT_s) \\
h[n] &= \sum_{k=0}^{M} b_k \delta[n-k] \\
y[n] &= \sum_{k=0}^{M} b_k x[n-k] \\
h[n] * x[n] &= \sum_{k=-\infty}^{\infty} h[k]x[n-k] \\
y[n] &= h[n] * x[n] \\
H(e^{j\hat{\omega}}) &= \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} = \sum_{k=0}^{M} h[k] e^{j\hat{\omega}} \\
h_1[n] * h_2[n] &\longleftrightarrow H_1(e^{j\hat{\omega}}) H_2(e^{j\hat{\omega}})
\end{aligned}
$$

Fourier Analysis

Let $x(t)$ be a continuous-time periodic signal with period T_0 and Fourier series coefficients a_k.

$$
\begin{aligned}
x(t) &= \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/T_0)kt} \\
a_k &= \frac{1}{T_0} \int_0^{T_0} x(t) e^{-j(2\pi/T_0)kt} dt
\end{aligned}
$$

Z-transform

Let $x[n]$ be a discrete-time signal with Z-transform $X(z)$, and $h[n]$ be the unit impulse response.

$$
\begin{aligned}
X(z) &= \sum_{k=0}^{N} x[k] z^{-n} \\
H(z) &= \sum_{k=0}^{N} b_k z^{-k} = \sum_{k=0}^{N} h[k] z^{-k} \\
h[n] * x[n] &\longleftrightarrow H(z)X(z) \\
a x_1[n] + b x_2[n] &\longleftrightarrow a X_1(z) + b X_2(z),\ a, b \in \mathbb{R}
\end{aligned}
$$

Values for trigonometric functions in selected angles

Let $\theta \in [0, \pi]$ be an angle. Relevant values for $\sin(\theta)$, $\cos(\theta)$ and $\tan(\theta)$ are given in Table 1. If the value you look for is not listed in the table, please assign a variable to it, e.g. $\sin(\pi/7) = \alpha$ or $\tan^{-1}(\pi/7) = \beta$. Also, note that relevant trigonometric identities are above to compute values for negative angles.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan(\theta)$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

Table 1: Values for trigonometric functions in selected angles

End of the Exam

Dear student, talking to you about signals and systems and their relation to AI was truly a joy. We, the teaching team, hope that these topics will come in handy for you in your future projects.

If you want to leave any feedback, drawings, comments, or suggestions, please do so in the box below. We appreciate it.

Have a well-deserved weekend!

The teaching team of Signals and Systems for AI