Tutorial 3 – Sampling and Aliasing

1. *Warm-up* Prove the identity

$$\cos(\theta)\cos(\phi) = \frac{1}{2}(\cos(\theta+\phi) + \cos(\theta-\phi)).$$

2. *p. 159, ex. P-4.11* Consider the wave

$$x(t) = 3\cos(800\pi t),$$

sampled at a rate f_s, after which we obtain the discrete-time signal

$$x[n] = x(n/f_s) = 3\cos(800\pi n/f_s)$$

for $-\infty < n < \infty$. In the following parts, assume that $f_s = 3600$ Hz.

   (a) Determine how many samples are taken in one period of the cosine wave $x(t)$. This answer is the average number of samples per period, which is an integer in this case.

   (b) Now consider another cosine waveform $y(t)$ with a different frequency ω_0:

$$y(t) = 3\cos(\omega_0 t).$$

   Find a value for ω_0, between 7000π and 9999π rad/s, such that the signal samples are identical to $x[n]$ above, that is, $y[n] = y(n/f_s) = x(n/f_s)$ for all n.

   (c) For the frequency found in b), determine the average number of samples taken in one period of $y(t)$.

3. *p. 155, ex. P-4.1* Let

$$x(t) = 10\cos(9\pi t - \pi/5).$$

A discrete time signal $x[n]$ is obtained by sampling $x(t)$ at a rate f_s samples/s. For each part, write the general form of $x[n]$

$$x[n] = A\cos(\omega_0 n + \phi).$$

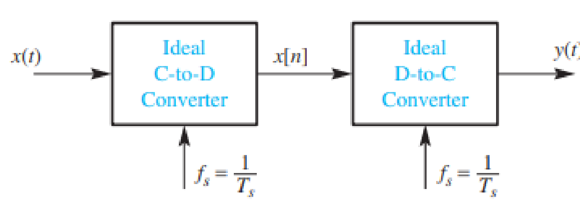
and state whether the signal has been undersampled or oversampled. Where does folding occur?

   (a) $f_s = 11$ samples/s
   (b) $f_s = 7$ samples/s
   (c) $f_s = 4$ samples/s

4. *p. 160, ex. P-4.14* An amplitude-modulated (AM) cosine wave is represented by the formula

$$x(t) = [4 + \sin(6600t)]\cos(2000\pi t).$$

Sketch the two-sided spectrum of this signal. Is the signal periodic? What relation must the sampling rate f_s satisfy so that it is an ideal C-to-D-to-C converter in the figure below?



5. *P.48* Draw and sketch the spectrum of the signal

$$x(t) = \cos(50\pi t)\sin(700\pi t).$$

   Is the signal periodic? Determine the minimum sampling rate to sample $x(t)$ without aliasing any of the components.

6. A chirp signal is one that sweeps in frequency from $\omega_1 = 2\pi_1 f$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$. The general formula of a chirp signal is

$$x(t) = A\cos(\alpha t^2 + \beta t + \phi) = A\cos(\psi(t)).$$

   The instantaneous frequency $w_i(t)$ is the derivative of $\psi(t)$, which is also heard if the frequencies are in the audible range.

   (a) Given an chirp arbitrary signal,

$$x(t) = \Re\left\{e^{j(\alpha t^2+\beta t+\phi)}\right\},$$

   determine its instantaneous frequency in terms of time.

   (b) Determine formulas for ω_1 and ω_2 in terms of α, β, and T_2.

   (c) For the signal

$$x(t) = \Re\left\{e^{j(40t^2+27t+14)}\right\},$$

   plot the instantaneous frequency in Hz versus time over the range $0 \leq t \leq 1$.