university of groningen

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# Tutorial 7: Z-transform

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**Signals and Systems**
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# Overview

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# P 7.3

Suppose that an LTI system has a system function

$$H(z) = 1 + 5z^{-1} - 3z^{-2} + 2.5z^{-3} + 4z^{-8}$$

a. Determine the difference equation that relates the output $y[n]$ of the system to the input $x[n]$

b. Determine and plot the output sequence $y[n]$ when the input is $x[n] = \delta[n]$.

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# P 7.4

An LTI system is described by the difference equation

$$y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$$

a. Determine the system function $H(z)$ for this system.

b. Plot the poles and zeroes of $H(z)$ in the $z$-plane.

c. From $H(z)$ obtain an expression for $H(e^{j\hat{\omega}})$, the frequency response of this system.

d. Sketch the frequency response (magnitude and phase) as a function of frequency for $-\pi \leqslant \hat{\omega} \leqslant \pi$.

e. Determine the output if the input is

$$x[n] = 4 + \cos[0.25\pi(n-1)] - 3\cos[(2\pi/3)n]$$

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# P 7.5

Consider an LTI system whose system function is the product of the five terms

$$\begin{aligned} H(z) &= (1 - z^{-1})(1 - e^{j\pi/2}z^{-1})(1 - e^{-j\pi/2}z^{-1}) \\ &\quad (1 - 0.9e^{-j\pi/3}z^{-1})(1 - 0.9e^{j\pi/3}z^{-1}) \end{aligned}$$

a. Write the difference equation that gives the relation between input $x[n]$ and the output $y[n]$.

b. Plot the poles and zeros of $H(z)$ in the complex $z$-plane.

c. If the input is of the form $x[n] = Ae^{j\phi}e^{jn\hat{\omega}}$, for what values of $-\pi < \hat{\omega} \leqslant \hat{\pi}$ is it true that $y[n] = 0$?

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# P 7.14

An LTI system has system function

$$(1 + z^{-2})(1 - 4z^{-2}) = 1 - 3z^{-2} - 4z^{-4}$$

The input to this system is

$$x[n] = 20 - 20\delta[n] + 20\cos(0.5\pi n + \pi/4)$$

for $-\infty < n < \infty$. Determine the output of the system $y[n]$ corresponding to the above input $x[n]$. Give an equation for $y[n]$ that is valid for all $n$.

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# P 7.15

The input to the C-to-D converter below is

$$x(t) = 4 + \cos(250\pi t - \pi/4) - 3\cos((2000\pi/3)t)$$

The system function to the LTI system is

$$H(z) = \frac{1}{3}(1 + z^{-1} + z^{-2})$$

If $f_s = 1000$ samples/s, determine an expression for $y(t)$, the output of the D-to-C converter.

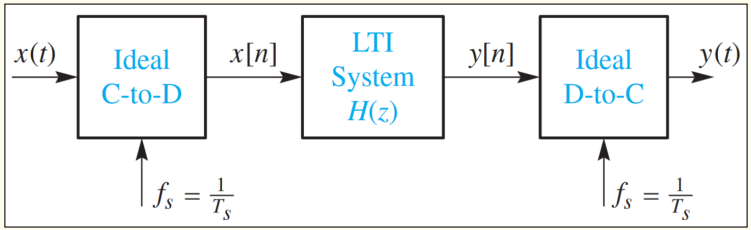
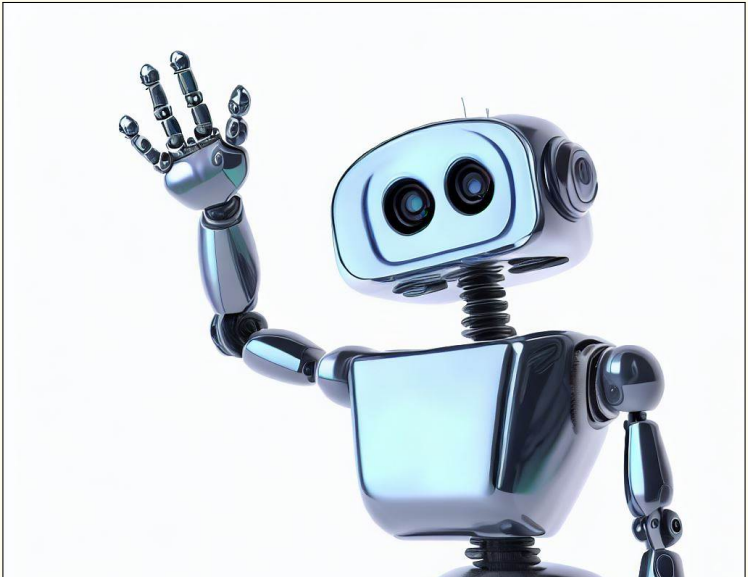



Figure: Proposed system.

# Have a nice day!



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## Disclaimer

- Questions and images are based in Schafer, R. W., Yoder, M. A., & McClellan, J. H. (2003). Signal Processing First. Prentice Hall.
- Grammar was checked with Grammarly and Grammar checker GPT.
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