# Massachusetts Institute of Technology

# MIT NULL

Benjamin Qi, Spencer Compton, Zhezheng Luo

adapted from KACTL and MIT NULL

2019-10-17

# Contest (1)

## template.cpp

55 lines

```cpp
#include <bits/stdc++.h>

using namespace std;

typedef long long ll;
typedef long double ld;
typedef double db;
typedef string str;

typedef pair<int, int> pi;
typedef pair<ll,ll> pl;
typedef pair<ld,ld> pd;
typedef complex<ld> cd;

typedef vector<int> vi;
typedef vector<ll> vl;
typedef vector<ld> vd;
typedef vector<str> vs;
typedef vector<pi> vpi;
typedef vector<pl> vpl;
typedef vector<cd> vcd;

#define FOR(i,a,b) for (int i = (a); i < (b); ++i)
#define F0R(i,a) FOR(i,0,a)
#define ROF(i,a,b) for (int i = (b)-1; i >= (a); --i)
#define R0F(i,a) ROF(i,0,a)
#define trav(a,x) for (auto& a : x)

#define mp make_pair
#define pb push_back
#define eb emplace_back
#define f first
#define s second
#define lb lower_bound
#define ub upper_bound

#define sz(x) (int)x.size()
#define all(x) begin(x), end(x)
#define rall(x) rbegin(x), rend(x)
#define rsz resize
#define ins insert

const int MOD = 1e9+7; // 998244353 = (119<<23)+1
const ll INF = 1e18;
const int MX = 2e5+5;
const ld PI = 4*atan((ld)1);

template<class T> bool ckmin(T& a, const T& b) { return a > b ? a = b, 1 : 0; }
template<class T> bool ckmax(T& a, const T& b) { return a < b ? a = b, 1 : 0; }

mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());

int main() {
    cin.sync_with_stdio(0); cin.tie(0);
}
```

## .bashrc

6 lines

```bash
co() {
    g++ -std=c++11 -O2 -Wall -Wl,-stack_size -Wl,0x10000000 -o $1 $1.cc
}
run() {
    co $1 && ./$1
}
```

## .vimrc

4 lines

```
set cin aw ai is ts=4 sw=4 tm=50 nu noeb ru cul
sy on | im jk <esc> | im kj <esc>
set mouse=a
set ww+=<,>,[,]
```

## hash.sh

3 lines

```bash
# Hashes a file, ignoring all whitespace and comments. Use for
# verifying that code was correctly typed.
cpp -dD -P -fpreprocessed | tr -d '[:space:]'| md5sum |cut -c-6
```

## troubleshoot.txt

52 lines

```
Pre-submit:
Write a few simple test cases, if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Make sure to submit the right file.

Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all datastructures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
Confusing N and M, i and j, etc.?
Are you sure your algorithm works?
What special cases have you not thought of?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some testcases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a team mate.
Ask the team mate to look at your code.
Go for a small walk, e.g. to the toilet.
Is your output format correct? (including whitespace)
Rewrite your solution from the start or let a team mate do it.

Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
How big is the input and output? (consider scanf)
Avoid vector, map. (use arrays/unordered_map)
What do your team mates think about your algorithm?

Memory limit exceeded:
What is the max amount of memory your algorithm should need?
Are you clearing all datastructures between test cases?
```

# Data Structures (2)

## 2.1 STL

### MapComparator.h

**Description:** custom comparator for map / set

d0cc31, 8 lines

```cpp
struct cmp {
  bool operator()(const int& l, const int& r) const {
    return l > r;
  }
};

set<int,cmp> s; // FOR(i,10) s.insert(rand()); trav(i,s) ps(i);
map<int,int,cmp> m;
```

### CustomHash.h

**Description:** faster than standard unordered map

e7c12c, 23 lines

```cpp
struct chash {
  static uint64_t splitmix64(uint64_t x) {
    // http://xorshift.di.unimi.it/splitmix64.c
    x += 0x9e3779b97f4a7c15;
    x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
    x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
    return x ^ (x >> 31);
  }

  size_t operator()(uint64_t x) const {
    static const uint64_t FIXED_RANDOM =
      chrono::steady_clock::now()
      .time_since_epoch().count();
    return splitmix64(x + FIXED_RANDOM);
  }
};

template<class K, class V> using um = unordered_map<K, V, chash>;
template<class K, class V> using ht = gp_hash_table<K, V, chash>;

template<class K, class V> V get(ht<K,V>& u, K x) {
  return u.find(x) == end(u) ? 0 : u[x];
}
```

### OrderStatisticTree.h

**Description:** A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change null_type.
**Time:** $\mathcal{O}(\log N)$

<ext/pb_ds/tree_policy.hpp>, <ext/pb_ds/assoc_container.hpp> c5d6f2, 17 lines

```cpp
using namespace __gnu_pbds;

template <class T> using Tree = tree<T, null_type, less<T>,
  rb_tree_tag, tree_order_statistics_node_update>;

#define ook order_of_key
#define fbo find_by_order

void treeExample() {
  Tree<int> t, t2; t.insert(8);
  auto it = t.insert(10).f;
  assert(it == t.lb(9));
  assert(t.ook(10) == 1);
```

```
  assert(t.ook(11) == 2);
  assert(*t.fbo(0) == 8);
  t.join(t2); // assuming T < T2 or T > T2, merge t2 into t
}
```

### Rope.h

**Description:** insert element at n-th position, cut a substring and re-insert somewhere else
**Time:** $\mathcal{O}(\log N)$ per operation?

<ext/rope> a2a5b5, 13 lines

```
using namespace __gnu_cxx;

void ropeExample() {
  rope<int> v(5, 0);
  FOR(i,sz(v)) v.mutable_reference_at(i) = i+1; // or push_back
  rope<int> cur = v.substr(1,2); v.erase(1,2);
  FOR(i,sz(v)) cout << v[i] << " ";  // 1 4 5
  cout << "\n";
  v.insert(v.mutable_begin()+2,cur);
  for (rope<int>::iterator it = v.mutable_begin(); it != v.mutable_end(); ++it)
    cout << *it << " "; // 1 4 2 3 5
  cout << "\n";
}
```

### LineContainer.h

**Description:** Given set of lines, computes greatest y-coordinate for any x
**Time:** $\mathcal{O}(\log N)$

157a83, 31 lines

```
struct Line {
  mutable ll k, m, p; // slope, y-intercept, last optimal x
  ll eval (ll x) { return k*x+m; }
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; }
};

struct LC : multiset<Line,less<>> {
  // for doubles, use inf = 1/.0, div(a,b) = a/b
  const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { return a/b-((a^b) < 0 && a%b); } // floored division
  ll bet(const Line& x, const Line& y) { // last x such that first line is better
    if (x.k == y.k) return x.m >= y.m ? inf : -inf;
    return div(y.m-x.m,x.k-y.k);
  }
  bool isect(iterator x, iterator y) { // updates x->p, determines if y is unneeded
    if (y == end()) { x->p = inf; return 0; }
    x->p = bet(*x,*y); return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k,m,0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p) isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

## 2.2 1D Range Queries

### RMQ.h

**Description:** 1D range minimum query
**Time:** $\mathcal{O}(N \log N)$ build, $\mathcal{O}(1)$ query

0a1f4a, 25 lines

```
template<class T> struct RMQ {
  constexpr static int level(int x) {
    return 31-__builtin_clz(x);
  } // floor(log_2(x))
  vector<vi> jmp;
  vector<T> v;
  int comb(int a, int b) {
    return v[a] == v[b] ? min(a,b) : (v[a] < v[b] ? a : b);
  } // index of minimum

  void init(const vector<T>& _v) {
    v = _v; jmp = {vi(sz(v))}; iota(all(jmp[0]),0);
    for (int j = 1; 1<<j <= sz(v); ++j) {
      jmp.pb(vi(sz(v)-(1<<j)+1));
      FOR(i,sz(jmp[j])) jmp[j][i] = comb(jmp[j-1][i],
                  jmp[j-1][i+(1<<(j-1))]);
    }
  }

  int index(int l, int r) { // get index of min element
    int d = level(r-l+1);
    return comb(jmp[d][l],jmp[d][r-(1<<d)+1]);
  }
  T query(int l, int r) { return v[index(l,r)]; }
};
```

### BIT.h

**Description:** N-D range sum query with point update
**Time:** $\mathcal{O}\left((\log N)^D\right)$

88cda4, 31 lines

```
template <class T, int ...Ns> struct BIT {
  T val = 0;
  void upd(T v) { val += v; }
  T query() { return val; }
};

template <class T, int N, int... Ns> struct BIT<T, N, Ns...> {
  BIT<T,Ns...> bit[N+1];
  template<typename... Args> void upd(int pos, Args... args) {
    for (; pos <= N; pos += (pos&-pos)) bit[pos].upd(args...);
  }
  template<typename... Args> T sum(int r, Args... args) {
    T res = 0; for (; r; r -= (r&-r)) res += bit[r].query(args...);
    return res;
  }
  template<typename... Args> T query(int l, int r, Args... args) {
    return sum(r,args...)-sum(l-1,args...);
  }
}; // BIT<int,10,10> gives a 2D BIT

template<class T, int SZ> struct BITrange {
  BIT<T,SZ> bit[2]; // piecewise linear functions
  // let cum[x] = sum_{i=1}^{x}a[i]
  void upd(int hi, T val) { // add val to a[1..hi]
    bit[1].upd(1,val), bit[1].upd(hi+1,-val); // if x <= hi, cum[x] += val*x
    bit[0].upd(hi+1,hi*val); // if x > hi, cum[x] += val*hi
  }
  void upd(int lo, int hi, T val) { upd(lo-1,-val), upd(hi,val); }
  T sum(int x) { return bit[1].sum(x)*x+bit[0].sum(x); } // get cum[x]
  T query(int x, int y) { return sum(y)-sum(x-1); }
}; // equivalent to 1D lazy segment tree for sum
```

### SegTree.h

**Description:** 1D point update, range query
**Time:** $\mathcal{O}(\log N)$

bf15d6, 21 lines

```
template<class T> struct Seg {
  const T ID = 0; // comb(ID,b) must equal b
  T comb(T a, T b) { return a+b; } // easily change this to min or max
  int n; vector<T> seg;
  void init(int _n) { n = _n; seg.rsz(2*n); }

  void pull(int p) { seg[p] = comb(seg[2*p],seg[2*p+1]); }
  void upd(int p, T value) {  // set value at position p
    seg[p += n] = value;
    for (p /= 2; p; p /= 2) pull(p);
  }

  T query(int l, int r) {  // sum on interval [l, r]
    T ra = ID, rb = ID; // make sure non-commutative operations work
    for (l += n, r += n+1; l < r; l /= 2, r /= 2) {
      if (l&1) ra = comb(ra,seg[l++]);
      if (r&1) rb = comb(seg[--r],rb);
    }
    return comb(ra,rb);
  }
};
```

### PersSegTree.h

**Description:** persistent segtree with lazy updates, assumes that lazy[cur] is included in val[cur] before propagating cur

ed6e9b, 60 lines

```
template<class T, int SZ> struct pseg {
  static const int LIMIT = 10000000; // adjust
  int l[LIMIT], r[LIMIT], nex = 0;
  T val[LIMIT], lazy[LIMIT];

  int copy(int cur) {
    int x = nex++;
    val[x] = val[cur], l[x] = l[cur], r[x] = r[cur], lazy[x] = lazy[cur];
    return x;
  }
  T comb(T a, T b) { return min(a,b); }
  void pull(int x) { val[x] = comb(val[l[x]],val[r[x]]); }
  void push(int cur, int L, int R) {
    if (!lazy[cur]) return;
    if (L != R) {
      l[cur] = copy(l[cur]);
      val[l[cur]] += lazy[cur];
      lazy[l[cur]] += lazy[cur];

      r[cur] = copy(r[cur]);
      val[r[cur]] += lazy[cur];
      lazy[r[cur]] += lazy[cur];
    }
    lazy[cur] = 0;
  }

  T query(int cur, int lo, int hi, int L, int R) {
    if (lo <= L && R <= hi) return val[cur];
    if (R < lo || hi < L) return INF;
    int M = (L+R)/2;
```

```
    return lazy[cur]+comb(query(l[cur],lo,hi,L,M), query(r[cur
        ↪],lo,hi,M+1,R));
  }
  int upd(int cur, int lo, int hi, T v, int L, int R) {
    if (R < lo || hi < L) return cur;

    int x = copy(cur);
    if (lo <= L && R <= hi) { val[x] += v, lazy[x] += v; return
        ↪ x; }
    push(x,L,R);

    int M = (L+R)/2;
    l[x] = upd(l[x],lo,hi,v,L,M), r[x] = upd(r[x],lo,hi,v,M+1,R
        ↪);
    pull(x); return x;
  }
  int build(vector<T>& arr, int L, int R) {
    int cur = nex++;
    if (L == R) {
      if (L < sz(arr)) val[cur] = arr[L];
      return cur;
    }

    int M = (L+R)/2;
    l[cur] = build(arr,L,M), r[cur] = build(arr,M+1,R);
    pull(cur); return cur;
  }

  vi loc;
  void upd(int lo, int hi, T v) { loc.pb(upd(loc.back(),lo,hi,v
      ↪,0,SZ-1)); }
  T query(int ti, int lo, int hi) { return query(loc[ti],lo,hi
      ↪,0,SZ-1); }
  void build(vector<T>& arr) { loc.pb(build(arr,0,SZ-1)); }
};
```

## SegTreeBeats.h

**Description:** Interval min modifications
**Time:** ?

5688c0, 74 lines

```
template<int SZ> struct SegTreeBeats {
  int N;
  ll sum[2*SZ];
  int mx[2][2*SZ], maxCnt[2*SZ];

  void pull(int ind) {
    mx[0][ind] = max(mx[0][2*ind],mx[0][2*ind+1]);
    mx[1][ind] = max(mx[1][2*ind],mx[1][2*ind+1]);
    maxCnt[ind] = 0;

    F0R(i,2) {
      if (mx[0][2*ind^i] == mx[0][ind]) maxCnt[ind] += maxCnt
          ↪[2*ind^i];
      else mx[1][ind] = max(mx[1][ind],mx[0][2*ind^i]);
    }

    sum[ind] = sum[2*ind]+sum[2*ind+1];
  }

  void build(vi& a, int ind = 1, int L = 0, int R = -1) {
    if (R == -1) R += N;
    if (L == R) {
      mx[0][ind] = sum[ind] = a[L];
      maxCnt[ind] = 1; mx[1][ind] = -1;
      return;
    }

    int M = (L+R)/2;
    build(a,2*ind,L,M); build(a,2*ind+1,M+1,R); pull(ind);
  }

  void push(int ind, int L, int R) {
    if (L == R) return;
    F0R(i,2)
      if (mx[0][2*ind^i] > mx[0][ind]) {
        sum[2*ind^i] -= (ll)maxCnt[2*ind^i]*
                (mx[0][2*ind^i]-mx[0][ind]);
        mx[0][2*ind^i] = mx[0][ind];
      }
  }

  void upd(int x, int y, int t, int ind = 1, int L = 0, int R =
      ↪ -1) { // set a_i = min(a_i,t)
    if (R == -1) R += N;
    if (R < x || y < L || mx[0][ind] <= t) return;
    push(ind,L,R);
    if (x <= L && R <= y && mx[1][ind] < t) {
      sum[ind] -= (ll)maxCnt[ind]*(mx[0][ind]-t);
      mx[0][ind] = t;
      return;
    }
    if (L == R) return;
    int M = (L+R)/2;
    upd(x,y,t,2*ind,L,M); upd(x,y,t,2*ind+1,M+1,R); pull(ind);
  }

  ll qsum(int x, int y, int ind = 1, int L = 0, int R = -1) {
    if (R == -1) R += N;
    if (R < x || y < L) return 0;
    push(ind,L,R);
    if (x <= L && R <= y) return sum[ind];

    int M = (L+R)/2;
    return qsum(x,y,2*ind,L,M)+qsum(x,y,2*ind+1,M+1,R);
  }

  int qmax(int x, int y, int ind = 1, int L = 0, int R = -1) {
    if (R == -1) R += N;
    if (R < x || y < L) return -1;
    push(ind,L,R);
    if (x <= L && R <= y) return mx[0][ind];

    int M = (L+R)/2;
    return max(qmax(x,y,2*ind,L,M), qmax(x,y,2*ind+1,M+1,R));
  }
};
```

## Treap.h

**Description:** Easiest BBST
**Time:** $\mathcal{O}(\log N)$

b45b6a, 77 lines

```
typedef struct tnode* pt;

struct tnode {
  int pri, val; pt c[2]; // essential
  int sz; ll sum; // for range queries
  bool flip; // lazy update

  tnode (int _val) {
    pri = rand()+(rand()<<15); val = _val; c[0] = c[1] = NULL;
    sz = 1; sum = val;
    flip = 0;
  }
};

int getsz(pt x) { return x?x->sz:0; }
ll getsum(pt x) { return x?x->sum:0; }

pt prop(pt x) {
  if (!x || !x->flip) return x;
  swap(x->c[0],x->c[1]);
  x->flip = 0;
  F0R(i,2) if (x->c[i]) x->c[i]->flip ^= 1;
  return x;
}

pt calc(pt x) {
  assert(!x->flip);
  prop(x->c[0]), prop(x->c[1]);
  x->sz = 1+getsz(x->c[0])+getsz(x->c[1]);
  x->sum = x->val+getsum(x->c[0])+getsum(x->c[1]);
  return x;
}
void tour(pt x, vi& v) {
  if (!x) return;
  prop(x);
  tour(x->c[0],v); v.pb(x->val); tour(x->c[1],v);
}

pair<pt,pt> split(pt t, int v) { // >= v goes to the right
  if (!t) return {t,t};
  prop(t);
  if (t->val >= v) {
    auto p = split(t->c[0], v); t->c[0] = p.s;
    return {p.f, calc(t)};
  } else {
    auto p = split(t->c[1], v); t->c[1] = p.f;
    return {calc(t), p.s};
  }
}
pair<pt,pt> splitsz(pt t, int sz) { // leftmost sz nodes go to
    ↪left
  if (!t) return {t,t};
  prop(t);
  if (getsz(t->c[0]) >= sz) {
    auto p = splitsz(t->c[0], sz); t->c[0] = p.s;
    return {p.f, calc(t)};
  } else {
    auto p = splitsz(t->c[1], sz-getsz(t->c[0])-1); t->c[1] = p
        ↪.f;
    return {calc(t), p.s};
  }
}

pt merge(pt l, pt r) {
  if (!l || !r) return l ? l : r;
  prop(l), prop(r);
  pt t;
  if (l->pri > r->pri) l->c[1] = merge(l->c[1],r), t = l;
  else r->c[0] = merge(l,r->c[0]), t = r;
  return calc(t);
}
pt ins(pt x, int v) { // insert v
  auto a = split(x,v), b = split(a.s,v+1);
  return merge(a.f,merge(new tnode(v),b.s));
}
pt del(pt x, int v) { // delete v
  auto a = split(x,v), b = split(a.s,v+1);
  return merge(a.f,b.s);
}
```

# Number Theory (3)

## 3.1 Modular Arithmetic

### Modular.h

**Description:** operations with modular arithmetic

20589d, 41 lines

```
template<class T> struct modular {
  T val;
  explicit operator T() const { return val; }
  modular() { val = 0; }
  modular(const ll& v) {
    val = (-MOD <= v && v <= MOD) ? v : v % MOD;
    if (val < 0) val += MOD;
  }

  // friend ostream& operator<<(ostream& os, const modular& a)
    ↪{ return os << a.val; }
  friend void pr(const modular& a) { pr(a.val); }
  friend void re(modular& a) { ll x; re(x); a = modular(x); }

  friend bool operator==(const modular& a, const modular& b) {
    ↪return a.val == b.val; }
  friend bool operator!=(const modular& a, const modular& b) {
    ↪return !(a == b); }
  friend bool operator<(const modular& a, const modular& b) {
    ↪return a.val < b.val; }

  modular operator-() const { return modular(-val); }
  modular& operator+=(const modular& m) { if ((val += m.val) >=
    ↪ MOD) val -= MOD; return *this; }
  modular& operator-=(const modular& m) { if ((val -= m.val) <
    ↪0) val += MOD; return *this; }
  modular& operator*=(const modular& m) { val = (ll)val*m.val%
    ↪MOD; return *this; }
  friend modular pow(modular a, ll p) {
    modular ans = 1; for (; p; p /= 2, a *= a) if (p&1) ans *=
      ↪a;
    return ans;
  }
  friend modular inv(const modular& a) {
    assert(a != 0); return exp(a,MOD-2);
  }
  modular& operator/=(const modular& m) { return (*this) *= inv
    ↪(m); }

  friend modular operator+(modular a, const modular& b) {
    ↪return a += b; }
  friend modular operator-(modular a, const modular& b) {
    ↪return a -= b; }
  friend modular operator*(modular a, const modular& b) {
    ↪return a *= b; }

  friend modular operator/(modular a, const modular& b) {
    ↪return a /= b; }
};

typedef modular<int> mi;
typedef pair<mi,mi> pmi;
typedef vector<mi> vmi;
typedef vector<pmi> vpmi;
```

### ModInv.h

**Description:** pre-compute factorial mod inverses for MOD in linear time assume MOD is prime and SZ < MOD

f88b07, 10 lines

```
vl inv, fac, ifac;
void genInv(int SZ) {
  inv.rsz(SZ), fac.rsz(SZ), ifac.rsz(SZ);
  inv[1] = 1; FOR(i,2,SZ) inv[i] = MOD-MOD/i*inv[MOD%i]%MOD;
  fac[0] = ifac[0] = 1;
  FOR(i,1,SZ) {
    fac[i] = fac[i-1]*i%MOD;
    ifac[i] = ifac[i-1]*inv[i]%MOD;
  }
}
```

### ModMulLL.h

**Description:** multiply two 64-bit integers mod another if 128-bit is not available works for $0 \le a, b < mod < 2^{63}$

cc0f9d, 14 lines

```
typedef unsigned long long ul;

// equivalent to (ul)(__int128(a)*b%mod)
ul modMul(ul a, ul b, const ul mod) {
  ll ret = a*b-mod*(ul)((ld)a*b/mod);
  return ret+((ret<0)-(ret>=(ll)mod))*mod;
}
ul modPow(ul a, ul b, const ul mod) {
  if (b == 0) return 1;
  ul res = modPow(a,b/2,mod);
  res = modMul(res,res,mod);
  if (b&1) return modMul(res,a,mod);
  return res;
}
```

### ModSqrt.h

**Description:** find sqrt of integer mod a prime

**Time:** ?

a9a4c4, 26 lines

```
template<class T> T sqrt(modular<T> a) {
  auto p = pow(a,(MOD-1)/2); if (p != 1) return p == 0 ? 0 :
    ↪-1; // check if zero or does not have sqrt
  T s = MOD-1, e = 0; while (s % 2 == 0) s /= 2, e ++;
  modular<T> n = 1; while (pow(n,(MOD-1)/2) == 1) n = (T)(n)+1;
    ↪ // find non-square residue

  auto x = pow(a,(s+1)/2), b = pow(a,s), g = pow(n,s);
  int r = e;
  while (1) {
    auto B = b; int m = 0; while (B != 1) B *= B, m ++;
    if (m == 0) return min((T)x,MOD-(T)x);
    FOR(i,r-m-1) g *= g;
    x *= g; g *= g; b *= g; r = m;
  }
}

/* Explanation:
 * Initially, x^2=ab, ord(b) = 2^m, ord(g) = 2^r where m<r
 * g = g^{2^{r-m-1}} -> ord(g) = 2^{m+1}
 * if x'=x*g, then b' = b*g^2
     (b')^{2^{m-1}} = (b*g^2)^{2^{m-1}}
                    = b^{2^{m-1}}*g^{2^m}
                    = -1*-1
                    = 1
  -> ord(b')|ord(b)/2
 * m decreases by at least one each iteration
 */
```

### ModSum.h

**Description:** Sums of mod'ed arithmetic progressions

50ee96, 15 lines

```
typedef unsigned long long ul;

ul sumsq(ul to) { return (to-1)*to/2; } // sum of 0..to-1
ul divsum(ul to, ul c, ul k, ul m) { // sum_{i=0}^{to-1}floor((
    ↪ki+c)/m)
  ul res = k/m*sumsq(to)+c/m*to;
  k %= m; c %= m; if (!k) return res;
  ul to2 = (to*k+c)/m;
  return res+(to-1)*to2-divsum(to2,m-1-c,m,k);
}

ll modsum(ul to, ll c, ll k, ll m) {
  c = (c%m+m)%m, k = (k%m+m)%m;
  return to*c+k*sumsq(to)-m*divsum(to,c,k,m);
}
```

## 3.2 Primality

### PrimeSieve.h

**Description:** tests primality up to n

**Time:** $\mathcal{O}(N \log \log N)$

5464fb, 13 lines

```
template<int SZ> struct Sieve {
  bitset<SZ> isprime;
  vi pr;

  Sieve() {
    isprime.set(); isprime[0] = isprime[1] = 0;
    for (int i = 4; i < SZ; i += 2) isprime[i] = 0;
    for (int i = 3; i*i < SZ; i += 2) if (isprime[i])
      for (int j = i*i; j < SZ; j += i*2) isprime[j] = 0;
    vi pr;
    FOR(i,2,SZ) if (isprime[i]) pr.pb(i);
  }
};
```

### FactorFast.h

**Description:** Factors integers up to $2^{60}$

**Time:** ?

"PrimeSieve.h" 936bee, 46 lines

```
Sieve<1<<20> S = Sieve<1<<20>(); // should take care of all
    ↪primes up to n^(1/3)

bool millerRabin(ll p) { // test primality
  if (p == 2) return true;
  if (p == 1 || p % 2 == 0) return false;
  ll s = p - 1; while (s % 2 == 0) s /= 2;
  FOR(i,30) { // strong liar with probability <= 1/4
    ll a = rand() % (p - 1) + 1, tmp = s;
    ll mod = mod_pow(a, tmp, p);
    while (tmp != p - 1 && mod != 1 && mod != p - 1) {
      mod = mod_mul(mod, mod, p);
      tmp *= 2;
    }
    if (mod != p - 1 && tmp % 2 == 0) return false;
  }
  return true;
}

ll f(ll a, ll n, ll &has) { return (mod_mul(a, a, n) + has) % n
    ↪; }

vpl pollardsRho(ll d) {
  vpl res;
  auto& pr = S.pr;
  for (int i = 0; i < sz(pr) && pr[i]*pr[i] <= d; i++) if (d %
      ↪pr[i] == 0) {
    int co = 0; while (d % pr[i] == 0) d /= pr[i], co ++;
    res.pb({pr[i],co});
  }
```

```
  if (d > 1) { // d is now a product of at most 2 primes.
    if (millerRabin(d)) res.pb({d,1});
    else while (1) {
      ll has = rand() % 2321 + 47;
      ll x = 2, y = 2, c = 1;
      for (; c == 1; c = __gcd(abs(x-y), d)) {
        x = f(x, d, has);
        y = f(f(y, d, has), d, has);
      } // should cycle in ~sqrt(smallest nontrivial divisor) turns
      if (c != d) {
        d /= c; if (d > c) swap(d,c);
        if (c == d) res.pb({c,2});
        else res.pb({c,1}), res.pb({d,1});
        break;
      }
    }
  }
  return res;
}
```

## 3.3 Divisibility

### Euclid.h

**Description:** Euclidean Algorithm

338527, 9 lines

```
pl euclid(ll a, ll b) { // returns {x,y} such that a*x+b*y=gcd(a,b)
  if (!b) return {1,0};
  pl p = euclid(b,a%b);
  return {p.s,p.f-a/b*p.s};
}
ll invGeneral(ll a, ll b) {
  pl p = euclid(a,b); assert(p.f*a+p.s*b == 1);
  return p.f+(p.f<0)*b;
}
```

### CRT.h

**Description:** Chinese Remainder Theorem

"Euclid.h" 4bf0b2, 7 lines

```
pl solve(pl a, pl b) {
  auto g = __gcd(a.s,b.s), l = a.s/g*b.s;
  if ((b.f-a.f) % g != 0) return {-1,-1};
  auto A = a.s/g, B = b.s/g;
  auto mul = (b.f-a.f)/g*invGeneral(A,B) % B;
  return {((mul*a.s+a.f)%l+l)%l,l};
}
```

# Combinatorial (4)

### IntPerm.h

**Description:** convert permutation of $\{0,1,\dots,n-1\}$ to integer in $[0,n!)$

**Usage:** `assert(encode(decode(5,37)) == 37);`

**Time:** $\mathcal{O}(n)$

f295dd, 20 lines

```
vi decode(int n, int a) {
  vi el(n), b; iota(all(el),0);
  FOR(i,n) {
    int z = a%sz(el);
    b.pb(el[z]); a /= sz(el);
    swap(el[z],el.back()); el.pop_back();
  }
  return b;
}

int encode(vi b) {
  int n = sz(b), a = 0, mul = 1;
  vi pos(n); iota(all(pos),0); vi el = pos;
  FOR(i,n) {
    int z = pos[b[i]]; a += mul*z; mul *= sz(el);
    swap(pos[el[z]],pos[el.back()]);
    swap(el[z],el.back()); el.pop_back();
  }
  return a;
}
```

### MatroidIntersect.h

**Description:** max size of independent set in both graphic + colorful matroids

**Time:** ?

"DSU.h" 40170e, 108 lines

```
int R;
map<int,int> m;

struct Element {
  pi ed;
  int col;
  bool in_independent_set = 0;
  int independent_set_position;
  Element(int u, int v, int c) { ed = {u,v}; col = c; }
};

vi independent_set;
vector<Element> ground_set;
bool col_used[300];

struct GBasis {
  DSU D;
  void reset() { D.init(sz(m)); }
  void add(pi v) { assert(D.unite(v.f,v.s)); }
  bool independent_with(pi v) { return !D.sameSet(v.f,v.s); }
};

GBasis basis, basis_wo[300];

bool graph_oracle(int inserted) {
  return basis.independent_with(ground_set[inserted].ed);
}
bool graph_oracle(int inserted, int removed) {
  int wi = ground_set[removed].independent_set_position;
  return basis_wo[wi].independent_with(ground_set[inserted].ed);
}
void prepare_graph_oracle() {
  basis.reset();
  FOR(i,sz(independent_set)) basis_wo[i].reset();
  FOR(i,sz(independent_set)) {
    pi v = ground_set[independent_set[i]].ed; basis.add(v);
    FOR(j,sz(independent_set)) if (i != j) basis_wo[j].add(v);
  }
}

bool colorful_oracle(int ins) {
  ins = ground_set[ins].col;
  return !col_used[ins];
}
bool colorful_oracle(int ins, int rem) {
  ins = ground_set[ins].col;
  rem = ground_set[rem].col;
  return !col_used[ins] || ins == rem;
}
void prepare_colorful_oracle() {
  FOR(i,R) col_used[i] = 0;
  trav(t,independent_set) col_used[ground_set[t].col] = 1;
}

bool augment() {
  prepare_graph_oracle();
  prepare_colorful_oracle();

  vi par(sz(ground_set),MOD);
  queue<int> q;
  FOR(i,sz(ground_set)) if (colorful_oracle(i)) {
    assert(!ground_set[i].in_independent_set);
    par[i] = -1; q.push(i);
  }
  int lst = -1;
  while (sz(q)) {
    int cur = q.front(); q.pop();
    if (ground_set[cur].in_independent_set) {
      FOR(to,sz(ground_set)) if (par[to] == MOD) {
        if (!colorful_oracle(to,cur)) continue;
        par[to] = cur; q.push(to);
      }
    } else {
      if (graph_oracle(cur)) { lst = cur; break;  }
      trav(to,independent_set) if (par[to] == MOD) {
        if (!graph_oracle(cur,to)) continue;
        par[to] = cur; q.push(to);
      }
    }
  }
  if (lst == -1) return 0;
  do {
    ground_set[lst].in_independent_set ^= 1;
    lst = par[lst];
  } while (lst != -1);
  independent_set.clear();
  FOR(i,sz(ground_set)) if (ground_set[i].in_independent_set) {
    ground_set[i].independent_set_position = sz(independent_set);
    independent_set.pb(i);
  }
  return 1;
}

void solve() {
  re(R); if (R == 0) exit(0);
  m.clear(); ground_set.clear(); independent_set.clear();
  FOR(i,R) {
    int a,b,c,d; re(a,b,c,d);
    ground_set.pb(Element(a,b,i));
    ground_set.pb(Element(c,d,i));
    m[a] = m[b] = m[c] = m[d] = 0;
  }
  int co = 0;
  trav(t,m) t.s = co++;
  trav(t,ground_set) t.ed.f = m[t.ed.f], t.ed.s = m[t.ed.s];
  while (augment());
  ps(2*sz(independent_set));
}
```

### PermGroup.h

**Description:** Schreier-Sims, count number of permutations in group and test whether permutation is a member of group

**Time:** ?

054283, 51 lines

```
const int N = 15;
int n;

vi inv(vi v) { vi V(sz(v)); FOR(i,sz(v)) V[v[i]] = i; return V; }
vi id() { vi v(n); iota(all(v),0); return v; }
```

```
vi operator*(const vi& a, const vi& b) {
  vi c(sz(a)); FOR(i,sz(a)) c[i] = a[b[i]];
  return c;
}

struct Group {
  bool flag[N];
  vi sigma[N]; // sigma[t][k] = t, sigma[t][x] = x if x > k
  vector<vi> gen;
  void clear(int p) {
    memset(flag,0, sizeof flag);
    flag[p] = 1; sigma[p] = id();
    gen.clear();
  }
} g[N];

bool check(const vi& cur, int k) {
  if (!k) return 1;
  int t = cur[k];
  return g[k].flag[t] ? check(inv(g[k].sigma[t])*cur,k-1) : 0;
}
void updateX(const vi& cur, int k);
void ins(const vi& cur, int k) {
  if (check(cur,k)) return;
  g[k].gen.pb(cur);
  FOR(i,n) if (g[k].flag[i]) updateX(cur*g[k].sigma[i],k);
}
void updateX(const vi& cur, int k) {
  int t = cur[k];
  if (g[k].flag[t]) ins(inv(g[k].sigma[t])*cur,k-1); // fixes k
    ↪ -> k
  else {
    g[k].flag[t] = 1, g[k].sigma[t] = cur;
    trav(x,g[k].gen) updateX(x*cur,k);
  }
}

ll order(vector<vi> gen) {
  assert(sz(gen)); n = sz(gen[0]); FOR(i,n) g[i].clear(i);
  trav(a,gen) ins(a,n-1); // insert perms into group one by one
  ll tot = 1;
  FOR(i,n) {
    int cnt = 0; FOR(j,i+1) cnt += g[i].flag[j];
    tot *= cnt;
  }
  return tot;
}
```

# Numerical (5)

## 5.1 Matrix

### Matrix.h

**Description:** 2D matrix operations

c6abe5, 36 lines

```
template<class T> struct Mat {
  int r,c;
  vector<vector<T>> d;
  Mat(int _r, int _c) : r(_r), c(_c) { d.assign(r,vector<T>(c))
    ↪; }
  Mat() : Mat(0,0) {}
  Mat(const vector<vector<T>>& _d) : r(sz(_d)), c(sz(_d[0])) {
    ↪d = _d; }
  friend void pr(const Mat& m) { pr(m.d); }

  Mat& operator+=(const Mat& m) {
    assert(r == m.r && c == m.c);
    FOR(i,r) FOR(j,c) d[i][j] += m.d[i][j];
    return *this;
  }
  Mat& operator-=(const Mat& m) {
    assert(r == m.r && c == m.c);
    FOR(i,r) FOR(j,c) d[i][j] -= m.d[i][j];
    return *this;
  }
  Mat operator*(const Mat& m) {
    assert(c == m.r); Mat x(r,m.c);
    FOR(i,r) FOR(j,c) FOR(k,m.c) x.d[i][k] += d[i][j]*m.d[j][k
      ↪];
    return x;
  }

  Mat operator+(const Mat& m) { return Mat(*this)+=m; }
  Mat operator-(const Mat& m) { return Mat(*this)-=m; }
  Mat& operator*=(const Mat& m) { return *this = (*this)*m; }

  friend Mat pow(Mat m, ll p) {
    assert(m.r == m.c);
    Mat r(m.r,m.c);
    FOR(i,m.r) r.d[i][i] = 1;
    for (; p; p /= 2, m *= m) if (p&1) r *= m;
    return r;
  }
};
```

### MatrixInv.h

**Description:** calculates determinant via gaussian elimination

**Time:** $\mathcal{O}\left(N^3\right)$

00ad8c, 31 lines

```
template<class T> T gauss(Mat<T>& m) { // determinant of 1000
  ↪x1000 Matrix in ~1s
  int n = m.r;
  T prod = 1; int nex = 0;
  FOR(i,n) {
    int row = -1; // for ld use EPS rather than 0
    FOR(j,nex,n) if (m.d[j][i] != 0) { row = j; break; }
    if (row == -1) { prod = 0; continue; }
    if (row != nex) prod *= -1, swap(m.d[row],m.d[nex]);
    prod *= m.d[nex][i];
    auto x = 1/m.d[nex][i]; FOR(k,i,m.c) m.d[nex][k] *= x;
    FOR(j,n) if (j != nex) {
      auto v = m.d[j][i];
      if (v != 0) FOR(k,i,m.c) m.d[j][k] -= v*m.d[nex][k];
    }
    nex ++;
  }
  return prod;
}

template<class T> Mat<T> inv(Mat<T> m) {
  int n = m.r;
  Mat<T> x(n,2*n);
  FOR(i,n) {
    x.d[i][i+n] = 1;
    FOR(j,n) x.d[i][j] = m.d[i][j];
  }
  if (gauss(x) == 0) return Mat<T>(0,0);
  Mat<T> r(n,n);
  FOR(i,n) FOR(j,n) r.d[i][j] = x.d[i][j+n];
  return r;
}
```

### MatrixTree.h

**Description:** Kirchhoff's Matrix Tree Theorem: given adjacency matrix, calculates # of spanning trees

cdb606, 13 lines

```
mi numSpan(Mat<mi> m) {
  int n = m.r;
  Mat<mi> res(n-1,n-1);
  FOR(i,n) FOR(j,i+1,n) {
    mi ed = m.d[i][j];
    res.d[i][i] += ed;
    if (j != n-1) {
      res.d[j][j] += ed;
      res.d[i][j] -= ed, res.d[j][i] -= ed;
    }
  }
  return gauss(res);
}
```

## 5.2 Polynomials

### Karatsuba.h

**Description:** multiply two polynomials

**Time:** $\mathcal{O}\left(N^{\log_2 3}\right)$

21f372, 26 lines

```
int size(int s) { return s > 1 ? 32-__builtin_clz(s-1) : 0; }

void karatsuba(ll *a, ll *b, ll *c, ll *t, int n) {
  int ca = 0, cb = 0; FOR(i,n) ca += !!a[i], cb += !!b[i];
  if (min(ca, cb) <= 1500/n) { // few numbers to multiply
    if (ca > cb) swap(a, b);
    FOR(i,n) if (a[i]) FOR(j,n) c[i+j] += a[i]*b[j];
  } else {
    int h = n >> 1;
    karatsuba(a, b, c, t, h); // a0*b0
    karatsuba(a+h, b+h, c+n, t, h); // a1*b1
    FOR(i,h) a[i] += a[i+h], b[i] += b[i+h];
    karatsuba(a, b, t, t+n, h); // (a0+a1)*(b0+b1)
    FOR(i,h) a[i] -= a[i+h], b[i] -= b[i+h];
    FOR(i,n) t[i] -= c[i]+c[i+n];
    FOR(i,n) c[i+h] += t[i], t[i] = 0;
  }
}

vl conv(vl a, vl b) {
  int sa = sz(a), sb = sz(b); if (!sa || !sb) return {};
  int n = 1<<size(max(sa,sb)); a.rsz(n), b.rsz(n);
  vl c(2*n), t(2*n); FOR(i,2*n) t[i] = 0;
  karatsuba(&a[0], &b[0], &c[0], &t[0], n);
  c.rsz(sa+sb-1); return c;
}
```

### FFT.h

**Description:** multiply two polynomials

**Time:** $\mathcal{O}(N \log N)$

"Modular.h" 44f949, 40 lines

```
typedef complex<db> cd;
const int MOD = (119 << 23) + 1, root = 3; // = 998244353
// NTT: For p < 2^30 there is also e.g. (5 << 25, 3), (7 << 26,
  ↪ 3),
// (479 << 21, 3) and (483 << 21, 5). The last two are > 10^9.

constexpr int size(int s) { return s > 1 ? 32-__builtin_clz(s
  ↪-1) : 0; }
void genRoots(vmi& roots) { // primitive n-th roots of unity
  int n = sz(roots); mi r = pow(mi(root),(MOD-1)/n);
  roots[0] = 1; FOR(i,1,n) roots[i] = roots[i-1]*r;
}
void genRoots(vcd& roots) { // change cd to complex<double>
  ↪instead?
  int n = sz(roots); double ang = 2*PI/n;
```

```
  FOR(i,n) roots[i] = cd(cos(ang*i),sin(ang*i)); // is there a way to do this more quickly?
}

template<class T> void fft(vector<T>& a, const vector<T>& roots, bool inv = 0) {
  int n = sz(a);
  for (int i = 1, j = 0; i < n; i++) { // sort by reverse bit representation
    int bit = n >> 1;
    for (; j&bit; bit >>= 1) j ^= bit;
    j ^= bit; if (i < j) swap(a[i], a[j]);
  }
  for (int len = 2; len <= n; len <<= 1)
    for (int i = 0; i < n; i += len)
      FOR(j,len/2) {
        int ind = n/len*j; if (inv && ind) ind = n-ind;
        auto u = a[i+j], v = a[i+j+len/2]*roots[ind];
        a[i+j] = u+v, a[i+j+len/2] = u-v;
      }
  if (inv) { T i = T(1)/T(n); trav(x,a) x *= i; }
}

template<class T> vector<T> mult(vector<T> a, vector<T> b) {
  int s = sz(a)+sz(b)-1, n = 1<<size(s);
  vector<T> roots(n); genRoots(roots);
  a.rsz(n), fft(a,roots);
  b.rsz(n), fft(b,roots);
  FOR(i,n) a[i] *= b[i];
  fft(a,roots,1); return a;
}
```

### FFTmod.h

**Description:** multiply two polynomials with arbitrary MOD ensures precision by splitting in half

"FFT.h" a8a6ed, 27 lines

```
vl multMod(const vl& a, const vl& b) {
  if (!min(sz(a),sz(b))) return {};
  int s = sz(a)+sz(b)-1, n = 1<<size(s), cut = sqrt(MOD);
  vcd roots(n); genRoots(roots);

  vcd ax(n), bx(n);
  FOR(i,sz(a)) ax[i] = cd((int)a[i]/cut, (int)a[i]%cut); // ax(x)=a1(x)+i*a0(x)
  FOR(i,sz(b)) bx[i] = cd((int)b[i]/cut, (int)b[i]%cut); // bx(x)=b1(x)+i*b0(x)
  fft(ax,roots), fft(bx,roots);

  vcd v1(n), v0(n);
  FOR(i,n) {
    int j = (i ? (n-i) : i);
    v1[i] = (ax[i]+conj(ax[j]))*cd(0.5,0)*bx[i]; // v1 = a1*(b1+b0*cd(0,1));
    v0[i] = (ax[i]-conj(ax[j]))*cd(0,-0.5)*bx[i]; // v0 = a0*(b1+b0*cd(0,1));
  }
  fft(v1,roots,1), fft(v0,roots,1);

  vl ret(n);
  FOR(i,n) {
    ll V2 = (ll)round(v1[i].real()); // a1*b1
    ll V1 = (ll)round(v1[i].imag())+(ll)round(v0[i].real()); // a0*b1+a1*b0
    ll V0 = (ll)round(v0[i].imag()); // a0*b0
    ret[i] = ((V2%MOD*cut+V1)%MOD*cut+V0)%MOD;
  }
  ret.rsz(s); return ret;
} // ~0.8s when sz(a)=sz(b)=1<<19
```

### PolyInv.h

**Description:** ?

**Time:** ?

"PolyConv.h" a5fd4a, 11 lines

```
template<class T> vector<T> inv(vector<T> v, int p) { // compute inverse of v mod x^p, where v[0] = 1
  v.rsz(p); vector<T> a = {T(1)/v[0]};
  for (int i = 1; i < p; i *= 2) {
    if (2*i > p) v.rsz(2*i);
    auto l = vector<T>(begin(v),begin(v)+i), r = vector<T>(begin(v)+i,begin(v)+2*i);
    auto c = conv(a,l); c = vector<T>(begin(c)+i,end(c));
    auto b = conv(a*T(-1),conv(a,r)+c); b.rsz(i);
    a.insert(end(a),all(b));
  }
  a.rsz(p); return a;
}
```

### PolyDiv.h

**Description:** divide two polynomials

**Time:** $\mathcal{O}(N \log N)$?

"PolyInv.h" 05bb2d, 7 lines

```
template<class T> pair<vector<T>,vector<T>> divi(const vector<T>& f, const vector<T>& g) { // f = q*g+r
  if (sz(f) < sz(g)) return {{},f};
  auto q = conv(inv(rev(g),sz(f)-sz(g)+1),rev(f));
  q.rsz(sz(f)-sz(g)+1); q = rev(q);
  auto r = f-conv(q,g); r.rsz(sz(g)-1);
  return {q,r};
}
```

### PolySqrt.h

**Description:** find sqrt of polynomial

**Time:** $\mathcal{O}(N \log N)$?

"PolyInv.h" 784e58, 8 lines

```
template<class T> vector<T> sqrt(vector<T> v, int p) { // S*S = v mod x^p, p is power of 2
  assert(v[0] == 1); if (p == 1) return {1};
  v.rsz(p);
  auto S = sqrt(v,p/2);
  auto ans = S+conv(v,inv(S,p));
  ans.rsz(p); ans *= T(1)/T(2);
  return ans;
}
```

## 5.3 Misc

### LinRec.h

**Description:** Berlekamp-Massey: computes linear recurrence of order n for sequence of 2n terms

**Time:** ?

49e624, 35 lines

```
using namespace vecOp;

struct LinRec {
  vmi x; // original sequence
  vmi C, rC;
  void init(const vmi& _x) {
    x = _x; int n = sz(x), m = 0;
    vmi B; B = C = {1}; // B is fail vector

    mi b = 1; // B gives 0,0,0,...,b
    FOR(i,n) {
      m ++;
      mi d = x[i]; FOR(j,1,sz(C)) d += C[j]*x[i-j];
      if (d == 0) continue; // recurrence still works
      auto _B = C; C.rsz(max(sz(C),m+sz(B)));
      mi coef = d/b; FOR(j,m,m+sz(B)) C[j] -= coef*B[j-m]; // recurrence that gives 0,0,0,...,d
      if (sz(_B) < m+sz(B)) { B = _B; b = d; m = 0; }
    }

    rC = C; reverse(all(rC)); // polynomial for getPo
    C.erase(begin(C)); trav(t,C) t *= -1; // x[i]=sum_{j=0}^{sz(C)-1}C[j]*x[i-j-1]
  }

  vmi getPo(int n) {
    if (n == 0) return {1};
    vmi x = getPo(n/2); x = rem(x*x,rC);
    if (n&1) { vmi v = {0,1}; x = rem(x*v,rC); }
    return x;
  }
  mi eval(int n) {
    vmi t = getPo(n);
    mi ans = 0; FOR(i,sz(t)) ans += t[i]*x[i];
    return ans;
  }
};
```

### Integrate.h

**Description:** ?

693e87, 8 lines

```
// db f(db x) { return x*x+3*x+1; }

db quad(db (*f)(db), db a, db b) {
  const int n = 1000;
  db dif = (b-a)/2/n, tot = f(a)+f(b);
  FOR(i,1,2*n) tot += f(a+i*dif)*(i&1?4:2);
  return tot*dif/3;
}
```

### IntegrateAdaptive.h

**Description:** ?

b48168, 19 lines

```
// db f(db x) { return x*x+3*x+1; }

db simpson(db (*f)(db), db a, db b) {
  db c = (a+b) / 2;
  return (f(a) + 4*f(c) + f(b)) * (b-a) / 6;
}

db rec(db (*f)(db), db a, db b, db eps, db S) {
  db c = (a+b) / 2;
  db S1 = simpson(f, a, c);
  db S2 = simpson(f, c, b), T = S1 + S2;
  if (abs(T - S) <= 15*eps || b-a < 1e-10)
    return T + (T - S) / 15;
  return rec(f, a, c, eps/2, S1) + rec(f, c, b, eps/2, S2);
}

db quad(db (*f)(db), db a, db b, db eps = 1e-8) {
  return rec(f, a, b, eps, simpson(f, a, b));
}
```

### Simplex.h

**Description:** Simplex algorithm for linear programming, maximize $c^T x$ subject to $Ax \leq b, x \geq 0$

**Time:** ?

3ddcbc, 73 lines

```
typedef double T;
typedef vector<T> vd;
typedef vector<vd> vvd;
const T eps = 1e-8, inf = 1/.0;
```

```
#define ltj(X) if (s == -1 || mp(X[j],N[j]) < mp(X[s],N[s])) s=
    ↪j

struct LPSolver {
  int m, n;
  vi N, B;
  vvd D;

  LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
      FOR(i,m) FOR(j,n) D[i][j] = A[i][j];
      FOR(i,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i]; }
        ↪// B[i] -> basic variables, col n+1 is for constants
        ↪, why D[i][n]=-1?
      FOR(j,n) { N[j] = j; D[m][j] = -c[j]; } // N[j] -> non-
        ↪basic variables, all zero
      N[n] = -1; D[m+1][n] = 1;
    }

  void print() {
    ps("D");
    trav(t,D) ps(t);
    ps();
    ps("B",B);
    ps("N",N);
    ps();
  }

  void pivot(int r, int s) { // row, column
    T *a = D[r].data(), inv = 1/a[s]; // eliminate col s from
      ↪consideration
    FOR(i,m+2) if (i != r && abs(D[i][s]) > eps) {
      T *b = D[i].data(), inv2 = b[s]*inv;
      FOR(j,n+2) b[j] -= a[j]*inv2;
      b[s] = a[s]*inv2;
    }
    FOR(j,n+2) if (j != s) D[r][j] *= inv;
    FOR(i,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv; swap(B[r], N[s]); // swap a basic and non-
      ↪basic variable
  }

  bool simplex(int phase) {
    int x = m+phase-1;
    for (;;) {
      int s = -1; FOR(j,n+1) if (N[j] != -phase) ltj(D[x]); //
        ↪find most negative col
      if (D[x][s] >= -eps) return true; // have best solution
      int r = -1;
      FOR(i,m) {
        if (D[i][s] <= eps) continue;
        if (r == -1 || mp(D[i][n+1] / D[i][s], B[i])
              < mp(D[r][n+1] / D[r][s], B[r])) r = i; // find
                ↪smallest positive ratio
      }
      if (r == -1) return false; // unbounded
      pivot(r, s);
    }
  }

  T solve(vd &x) {
    int r = 0; FOR(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) { // x=0 is not a solution
      pivot(r, n); // -1 is artificial variable, initially set
        ↪to smth large but want to get to 0
      if (!simplex(2) || D[m+1][n+1] < -eps) return -inf; // no
        ↪ solution
      // D[m+1][n+1] is max possible value of the negation of
        ↪artificial variable, starts negative but should get
        ↪to zero
      FOR(i,m) if (B[i] == -1) {
        int s = 0; FOR(j,1,n+1) ltj(D[i]);
        pivot(i,s);
      }
    }
    bool ok = simplex(1); x = vd(n);
    FOR(i,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
  }
};
```

# Graphs (6)

## 6.1 Fundamentals

### DSU.h

**Description:** ?
**Time:** $\mathcal{O}(N\alpha(N))$

cbfb79, 22 lines

```
struct DSU {
  vi e;
  void init(int n) { e = vi(n,-1); }
  int get(int x) { return e[x] < 0 ? x : e[x] = get(e[x]); } //
    ↪ path compression
  bool sameSet(int a, int b) { return get(a) == get(b); }
  int size(int x) { return -e[get(x)]; }
  bool unite(int x, int y) { // union-by-rank
    x = get(x), y = get(y); if (x == y) return 0;
    if (e[x] > e[y]) swap(x,y);
    e[x] += e[y]; e[y] = x;
    return 1;
  }
};

// computes the minimum spanning tree in O(ElogE) time

template<class T> T kruskal(int n, vector<pair<T,pi>> edge) {
  sort(all(edge));
  T ans = 0; DSU D; D.init(n);
  trav(a,edge) if (D.unite(a.s.f,a.s.s)) ans += a.f; // edge is
    ↪ in MST
  return ans;
}
```

### ManhattanMST.h

**Description:** Compute MST of points where edges are manhattan distances
**Time:** $\mathcal{O}(N \log N)$

"DSU.h" 6f801e, 62 lines

```
int N;
vector<array<int,3>> cur;
vector<pair<ll,pi>> ed;
vi ind;

struct {
  map<int,pi> m;
  void upd(int a, pi b) {
    auto it = m.lb(a);
    if (it != m.end() && it->s <= b) return;
    m[a] = b; it = m.find(a);
    while (it != m.begin() && prev(it)->s >= b) m.erase(prev(it
      ↪));
  }
  pi query(int y) { // for all a > y find min possible value of
    ↪ b
    auto it = m.ub(y);
    if (it == m.end()) return {2*MOD,2*MOD};
    return it->s;
  }
} S;

void solve() {
  sort(all(ind),[](int a, int b) { return cur[a][0] > cur[b
    ↪][0]; });
  S.m.clear();
  int nex = 0;
  trav(x,ind) { // cur[x][0] <= ?, cur[x][1] < ?
    while (nex < N && cur[ind[nex]][0] >= cur[x][0]) {
      int b = ind[nex++];
      S.upd(cur[b][1],{cur[b][2],b});
    }
    pi t = S.query(cur[x][1]);
    if (t.s != 2*MOD) ed.pb({(ll)t.f-cur[x][2],{x,t.s}});
  }
}

ll mst(vpi v) {
  N = sz(v); cur.resz(N); ed.clear();
  ind.clear(); FOR(i,N) ind.pb(i);
  sort(all(ind),[&v](int a, int b) { return v[a] < v[b]; });
  FOR(i,N-1) if (v[ind[i]] == v[ind[i+1]]) ed.pb({0,{ind[i],ind
    ↪[i+1]}});

  FOR(i,2) { // it's probably ok to consider just two quadrants
    ↪?
    FOR(i,N) {
      auto a = v[i];
      cur[i][2] = a.f+a.s;
    }
    FOR(i,N) { // first octant
      auto a = v[i];
      cur[i][0] = a.f-a.s;
      cur[i][1] = a.s;
    }
    solve();
    FOR(i,N) { // second octant
      auto a = v[i];
      cur[i][0] = a.f;
      cur[i][1] = a.s-a.f;
    }
    solve();
    trav(a,v) a = {a.s,-a.f}; // rotate 90 degrees, repeat
  }

  return kruskal(ed);
}
```

## 6.2 Trees

### LCAjumps.h

**Description:** calculates least common ancestor in tree with binary jumping
**Time:** $\mathcal{O}(N \log N)$

a5a7dd, 33 lines

```
template<int SZ> struct LCA {
  static const int BITS = 32-__builtin_clz(SZ);
  int N, R = 1; // vertices from 1 to N, R = root
  vi adj[SZ];
  int par[BITS][SZ], depth[SZ];

  // INITIALIZE
  void addEdge(int u, int v) { adj[u].pb(v), adj[v].pb(u); }
```

```
  void dfs(int u, int prev){
    par[0][u] = prev;
    depth[u] = depth[prev]+1;
    trav(v,adj[u]) if (v != prev) dfs(v, u);
  }
  void init(int _N) {
    N = _N; dfs(R, 0);
    FOR(k,1,BITS) FOR(i,1,N+1) par[k][i] = par[k-1][par[k-1][i
      ↪]];
  }

  // QUERY
  int getPar(int a, int b) {
    R0F(k,BITS) if (b&(1<<k)) a = par[k][a];
    return a;
  }
  int lca(int u, int v){
    if (depth[u] < depth[v]) swap(u,v);
    u = getPar(u,depth[u]-depth[v]);
    R0F(k,BITS) if (par[k][u] != par[k][v]) u = par[k][u], v =
      ↪par[k][v];
    return u == v ? u : par[0][u];
  }
  int dist(int u, int v) {
    return depth[u]+depth[v]-2*depth[lca(u,v)];
  }
};
```

## CentroidDecomp.h

**Description:** can support tree path queries and updates
**Time:** $\mathcal{O}(N \log N)$

81e9e4, 45 lines

```
template<int SZ> struct CD {
  vi adj[SZ];
  bool done[SZ];
  int sub[SZ], par[SZ];
  vl dist[SZ];
  pi cen[SZ];
  void addEdge(int a, int b) { adj[a].pb(b), adj[b].pb(a); }

  void dfs (int x) {
    sub[x] = 1;
    trav(y,adj[x]) if (!done[y] && y != par[x]) {
      par[y] = x; dfs(y);
      sub[x] += sub[y];
    }
  }
  int centroid(int x) {
    par[x] = -1; dfs(x);
    for (int sz = sub[x];;) {
      pi mx = {0,0};
      trav(y,adj[x]) if (!done[y] && y != par[x])
        ckmax(mx,{sub[y],y});
      if (mx.f*2 <= sz) return x;
      x = mx.s;
    }
  }

  void genDist(int x, int p) {
    dist[x].pb(dist[p].back()+1);
    trav(y,adj[x]) if (!done[y] && y != p) {
      cen[y] = cen[x];
      genDist(y,x);
    }
  }
  void gen(int x, bool fst = 0) {
    done[x = centroid(x)] = 1; dist[x].pb(0);
    if (fst) cen[x].f = -1;
    int co = 0;
    trav(y,adj[x]) if (!done[y]) {
      cen[y] = {x,co++};
      genDist(y,x);
    }
    trav(y,adj[x]) if (!done[y]) gen(y);
  }
  void init() { gen(1,1); }
};
```

## HLD.h

**Description:** Heavy Light Decomposition
**Time:** $\mathcal{O}(\log^2 N)$ per path operations

69f40a, 50 lines

```
template<int SZ, bool VALUES_IN_EDGES> struct HLD {
  int N; vi adj[SZ];
  int par[SZ], sz[SZ], depth[SZ];
  int root[SZ], pos[SZ];
  LazySegTree<ll,SZ> tree;
  void addEdge(int a, int b) { adj[a].pb(b), adj[b].pb(a); }

  void dfs_sz(int v = 1) {
    if (par[v]) adj[v].erase(find(all(adj[v]),par[v]));
    sz[v] = 1;
    trav(u,adj[v]) {
      par[u] = v; depth[u] = depth[v]+1;
      dfs_sz(u); sz[v] += sz[u];
      if (sz[u] > sz[adj[v][0]]) swap(u, adj[v][0]);
    }
  }
  void dfs_hld(int v = 1) {
    static int t = 0;
    pos[v] = t++;
    trav(u,adj[v]) {
      root[u] = (u == adj[v][0] ? root[v] : u);
      dfs_hld(u);
    }
  }
  void init(int _N) {
    N = _N; par[1] = depth[1] = 0; root[1] = 1;
    dfs_sz(); dfs_hld();
  }

  template <class BinaryOperation>
  void processPath(int u, int v, BinaryOperation op) {
    for (; root[u] != root[v]; v = par[root[v]]) {
      if (depth[root[u]] > depth[root[v]]) swap(u, v);
      op(pos[root[v]], pos[v]);
    }
    if (depth[u] > depth[v]) swap(u, v);
    op(pos[u]+VALUES_IN_EDGES, pos[v]);
  }

  void modifyPath(int u, int v, int val) { // add val to
    ↪vertices/edges along path
    processPath(u, v, [this, &val](int l, int r) { tree.upd(l,
      ↪r, val); });
  }
  void modifySubtree(int v, int val) { // add val to vertices/
    ↪edges in subtree
    tree.upd(pos[v]+VALUES_IN_EDGES,pos[v]+sz[v]-1,val);
  }
  ll queryPath(int u, int v) { // query sum of path
    ll res = 0; processPath(u, v, [this, &res](int l, int r) {
      ↪res += tree.qsum(l, r); });
    return res;
  }
};
```

## 6.3 DFS Algorithms

## SCC.h

**Description:** Kosaraju's Algorithm does DFS two times to generate SCC in topological order
**Time:** $\mathcal{O}(N + M)$

f53f41, 24 lines

```
template<int SZ> struct SCC {
  int N, comp[SZ];
  vi adj[SZ], radj[SZ], todo, allComp;
  bitset<SZ> visit;
  void addEdge(int a, int b) { adj[a].pb(b), radj[b].pb(a); }

  void dfs(int v) {
    visit[v] = 1;
    trav(w,adj[v]) if (!visit[w]) dfs(w);
    todo.pb(v);
  }
  void dfs2(int v, int val) {
    comp[v] = val;
    trav(w,radj[v]) if (comp[w] == -1) dfs2(w,val);
  }

  void init(int _N) { // fills allComp
    N = _N;
    F0R(i,N) comp[i] = -1, visit[i] = 0;
    F0R(i,N) if (!visit[i]) dfs(i);
    reverse(all(todo)); // now todo stores vertices in order of
      ↪ topological sort
    trav(i,todo) if (comp[i] == -1) dfs2(i,i), allComp.pb(i);
  }
};
```

## 2SAT.h

**Description:** ?

"SCC.h" 6c209d, 38 lines

```
template<int SZ> struct TwoSat {
  SCC<2*SZ> S;
  bitset<SZ> ans;
  int N = 0;
  int addVar() { return N++; }

  void either(int x, int y) {
    x = max(2*x,-1-2*x), y = max(2*y,-1-2*y);
    S.addEdge(x^1,y); S.addEdge(y^1,x);
  }
  void implies(int x, int y) { either(~x,y); }
  void setVal(int x) { either(x,x); }
  void atMostOne(const vi& li) {
    if (sz(li) <= 1) return;
    int cur = ~li[0];
    FOR(i,2,sz(li)) {
      int next = addVar();
      either(cur,~li[i]);
      either(cur,next);
      either(~li[i],next);
      cur = ~next;
    }
    either(cur,~li[1]);
  }

  bool solve(int _N) {
    if (_N != -1) N = _N;
    S.init(2*N);
    for (int i = 0; i < 2*N; i += 2)
      if (S.comp[i] == S.comp[i^1]) return 0;
    reverse(all(S.allComp));
    vi tmp(2*N);
```

```
    trav(i,S.allComp) if (tmp[i] == 0)
      tmp[i] = 1, tmp[S.comp[i^1]] = -1;
    FOR(i,N) if (tmp[S.comp[2*i]] == 1) ans[i] = 1;
    return 1;
  }
};
```

## EulerPath.h

**Description:** Eulerian Path for both directed and undirected graphs
**Time:** $\mathcal{O}(N+M)$

fd7ad7, 30 lines

```
template<int SZ, bool directed> struct Euler {
  int N, M = 0;
  vpi adj[SZ];
  vpi::iterator its[SZ];
  vector<bool> used;

  void addEdge(int a, int b) {
    if (directed) adj[a].pb({b,M});
    else adj[a].pb({b,M}), adj[b].pb({a,M});
    used.pb(0); M ++;
  }

  vpi solve(int _N, int src = 1) {
    N = _N;
    FOR(i,1,N+1) its[i] = begin(adj[i]);
    vector<pair<pi,int>> ret, s = {{{src,-1},-1}};
    while (sz(s)) {
      int x = s.back().f.f;
      auto& it = its[x], end = adj[x].end();
      while (it != end && used[it->s]) it ++;
      if (it == end) {
        if (sz(ret) && ret.back().f.s != s.back().f.f) return
          {}; // path isn't valid
        ret.pb(s.back()), s.pop_back();
      } else { s.pb({{it->f,x},it->s}); used[it->s] = 1; }
    }
    if (sz(ret) != M+1) return {};
    vpi ans; trav(t,ret) ans.pb({t.f.f,t.s});
    reverse(all(ans)); return ans;
  }
};
```

## BCC.h

**Description:** computes biconnected components
**Time:** $\mathcal{O}(N+M)$

393aff, 37 lines

```
template<int SZ> struct BCC {
  int N;
  vpi adj[SZ], ed;
  void addEdge(int u, int v) {
    adj[u].pb({v,sz(ed)}), adj[v].pb({u,sz(ed)});
    ed.pb({u,v});
  }

  int disc[SZ];
  vi st; vector<vi> fin;
  int bcc(int u, int p = -1) { // return lowest disc
    static int ti = 0;
    disc[u] = ++ti; int low = disc[u];
    int child = 0;
    trav(i,adj[u]) if (i.s != p)
      if (!disc[i.f]) {
        child ++; st.pb(i.s);
        int LOW = bcc(i.f,i.s); ckmin(low,LOW);
        // disc[u] < LOW -> bridge
        if (disc[u] <= LOW) {
          // if (p != -1 || child > 1) -> u is articulation
            point
          vi tmp; while (st.back() != i.s) tmp.pb(st.back()),
            st.pop_back();
          tmp.pb(st.back()), st.pop_back();
          fin.pb(tmp);
        }
      } else if (disc[i.f] < disc[u]) {
        ckmin(low,disc[i.f]);
        st.pb(i.s);
      }
    return low;
  }

  void init(int _N) {
    N = _N; FOR(i,N) disc[i] = 0;
    FOR(i,N) if (!disc[i]) bcc(i); // st should be empty after
      each iteration
  }
};
```

# 6.4 Flows

## Dinic.h

**Description:** faster flow
**Time:** $\mathcal{O}\left(N^2M\right)$ flow, $\mathcal{O}\left(M\sqrt{N}\right)$ bipartite matching

f1366f, 47 lines

```
template<int SZ> struct Dinic {
  typedef ll F; // flow type
  struct Edge { int to, rev; F f, c; };

  int N,s,t;
  vector<Edge> adj[SZ];
  typename vector<Edge>::iterator cur[SZ];
  void addEdge(int u, int v, F cap) {
    assert(cap >= 0); // don't try smth dumb
    Edge a{v, sz(adj[v]), 0, cap}, b{u, sz(adj[u]), 0, 0};
    adj[u].pb(a), adj[v].pb(b);
  }

  int level[SZ];
  bool bfs() { // level = shortest distance from source
    // after computing flow, edges {u,v} such that level[u] \
      neq -1, level[v] = -1 are part of min cut
    FOR(i,N) level[i] = -1, cur[i] = begin(adj[i]);
    queue<int> q({s}); level[s] = 0;
    while (sz(q)) {
      int u = q.front(); q.pop();
      trav(e,adj[u]) if (level[e.to] < 0 && e.f < e.c) {
        level[e.to] = level[u]+1; q.push(e.to);
      }
    }
    return level[t] >= 0;
  }

  F sendFlow(int v, F flow) {
    if (v == t) return flow;
    for (; cur[v] != end(adj[v]); cur[v]++) {
      Edge& e = *cur[v];
      if (level[e.to] != level[v]+1 || e.f == e.c) continue;
      auto df = sendFlow(e.to,min(flow,e.c-e.f));
      if (df) { // saturated at least one edge
        e.f += df; adj[e.to][e.rev].f -= df;
        return df;
      }
    }
    return 0;
  }
  F maxFlow(int _N, int _s, int _t) {
    N = _N, s = _s, t = _t; if (s == t) return -1;
    F tot = 0;
    while (bfs()) while (auto flow = sendFlow(s,numeric_limits<
      F>::max())) tot += flow;
    return tot;
  }
};
```

## MCMF.h

**Description:** Min-Cost Max Flow, no negative cycles allowed
**Time:** ?

f67674, 56 lines

```
template<class T> using pqg = priority_queue<T,vector<T>,
  greater<T>>;
template<class T> T poll(pqg<T>& x) {
  T y = x.top(); x.pop();
  return y;
}

template<int SZ> struct mcmf {
  struct Edge { int to, rev; ll f, c, cost; };
  vector<Edge> adj[SZ];
  void addEdge(int u, int v, ll cap, ll cost) {
    assert(cap >= 0);
    Edge a{v, sz(adj[v]), 0, cap, cost}, b{u, sz(adj[u]), 0, 0,
       -cost};
    adj[u].pb(a), adj[v].pb(b);
  }

  int N, s, t;
  pi pre[SZ]; // previous vertex, edge label on path
  pl cost[SZ]; // tot cost of path, amount of flow
  ll totFlow, totCost, curCost;
  void reweight() { // ensures all non-negative edge weights
    FOR(i,N) trav(p,adj[i]) p.cost += cost[i].f-cost[p.to].f;
  }
  bool spfa() { // reweighting will ensure that there will be
    negative weights only during the first time you run this
    FOR(i,N) cost[i] = {INF,0};
    cost[s] = {0,INF};
    pqg<pair<ll,int>> todo({{0,s}});

    while (sz(todo)) {
      auto x = poll(todo); if (x.f > cost[x.s].f) continue;
      trav(a,adj[x.s]) if (x.f+a.cost < cost[a.to].f && a.f < a
        .c) {
        pre[a.to] = {x.s,a.rev};
        cost[a.to] = {x.f+a.cost, min(a.c-a.f,cost[x.s].s)};
        todo.push({cost[a.to].f,a.to});
      }
    }

    curCost += cost[t].f; return cost[t].s;
  }
  void backtrack() {
    auto f = cost[t].s; totFlow += f, totCost += curCost*f;
    for (int x = t; x != s; x = pre[x].f) {
      adj[x][pre[x].s].f -= f;
      adj[pre[x].f][adj[x][pre[x].s].rev].f += f;
    }
  }

  pl calc(int _N, int _s, int _t) {
    N = _N; s = _s, t = _t; totFlow = totCost = curCost = 0;
    spfa();
    while (1) {
      reweight();
      if (!spfa()) return {totFlow, totCost};
      backtrack();
    }
```

```
  }
};
```

## GomoryHu.h

**Description:** Compute max flow between every pair of vertices of undirected graph

fe44db, 56 lines

```
template<int SZ> struct GomoryHu {
  int N;
  vector<pair<pi,int>> ed;
  void addEdge(int a, int b, int c) { ed.pb({{a,b},c}); }

  vector<vi> cor = {{}}; // groups of vertices
  map<int,int> adj[2*SZ]; // current edges of tree
  int side[SZ];

  int gen(vector<vi> cc) {
    Dinic<SZ> D = Dinic<SZ>();
    vi comp(N+1); FOR(i,sz(cc)) trav(t,cc[i]) comp[t] = i;
    trav(t,ed) if (comp[t.f.f] != comp[t.f.s]) {
      D.addEdge(comp[t.f.f],comp[t.f.s],t.s);
      D.addEdge(comp[t.f.s],comp[t.f.f],t.s);
    }
    int f = D.maxFlow(0,1);
    FOR(i,sz(cc)) trav(j,cc[i]) side[j] = D.level[i] >= 0; //
      ↪min cut
    return f;
  }

  void fill(vi& v, int a, int b) {
    trav(t,cor[a]) v.pb(t);
    trav(t,adj[a]) if (t.f != b) fill(v,t.f,a);
  }

  void addTree(int a, int b, int c) { adj[a][b] = c, adj[b][a]
    ↪= c; }
  void delTree(int a, int b) { adj[a].erase(b), adj[b].erase(a)
    ↪; }

  vector<pair<pi,int>> init(int _N) { // returns edges of
    ↪Gomory-Hu Tree
    N = _N;
    FOR(i,1,N+1) cor[0].pb(i);
    queue<int> todo; todo.push(0);
    while (sz(todo)) {
      int x = todo.front(); todo.pop();
      vector<vi> cc; trav(t,cor[x]) cc.pb({t});
      trav(t,adj[x]) {
        cc.pb({});
        fill(cc.back(),t.f,x);
      }
      int f = gen(cc); // run max flow
      cor.pb({}), cor.pb({});
      trav(t,cor[x]) cor[sz(cor)-2+side[t]].pb(t);
      FOR(i,2) if (sz(cor[sz(cor)-2+i]) > 1) todo.push(sz(cor)
        ↪-2+i);
      FOR(i,sz(cor)-2) if (i != x && adj[i].count(x)) {
        addTree(i,sz(cor)-2+side[cor[i][0]],adj[i][x]);
        delTree(i,x);
      } // modify tree edges
      addTree(sz(cor)-2,sz(cor)-1,f);
    }
    vector<pair<pi,int>> ans;
    FOR(i,sz(cor)) trav(j,adj[i]) if (i < j.f)
      ans.pb({{cor[i][0],cor[j.f][0]},j.s});
    return ans;
  }
};
```

# 6.5 Matching

## DFSmatch.h

**Description:** naive bipartite matching

**Time:** $\mathcal{O}(NM)$

37ad8b, 26 lines

```
template<int SZ> struct MaxMatch {
  int N, flow = 0, match[SZ], rmatch[SZ];
  bitset<SZ> vis;
  vi adj[SZ];
  MaxMatch() {
    memset(match,0,sizeof match);
    memset(rmatch,0,sizeof rmatch);
  }

  void connect(int a, int b, bool c = 1) {
    if (c) match[a] = b, rmatch[b] = a;
    else match[a] = rmatch[b] = 0;
  }
  bool dfs(int x) {
    if (!x) return 1;
    if (vis[x]) return 0;
    vis[x] = 1;
    trav(t,adj[x]) if (t != match[x] && dfs(rmatch[t]))
      return connect(x,t),1;
    return 0;
  }
  void tri(int x) { vis.reset(); flow += dfs(x); }
  void init(int _N) {
    N = _N; FOR(i,1,N+1) if (!match[i]) tri(i);
  }
};
```

## Hungarian.h

**Description:** finds min cost to complete n jobs w/ m workers each worker is assigned to at most one job (n <= m)

**Time:** ?

12f135, 28 lines

```
int HungarianMatch(const vector<vi>& a) { // cost array,
  ↪negative values are ok
  int n = sz(a)-1, m = sz(a[0])-1; // jobs 1..n, workers 1..m
  vi u(n+1), v(m+1), p(m+1); // p[j] -> job picked by worker j
  FOR(i,1,n+1) { // find alternating path with job i
    p[0] = i; int j0 = 0;
    vi dist(m+1, MOD), pre(m+1,-1); // dist, previous vertex on
      ↪ shortest path
    vector<bool> done(m+1, false);
    do {
      done[j0] = true;
      int i0 = p[j0], j1; int delta = MOD;
      FOR(j,1,m+1) if (!done[j]) {
        auto cur = a[i0][j]-u[i0]-v[j];
        if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
        if (dist[j] < delta) delta = dist[j], j1 = j;
      }
      FOR(j,m+1) // just dijkstra with potentials
        if (done[j]) u[p[j]] += delta, v[j] -= delta;
        else dist[j] -= delta;
      j0 = j1;
    } while (p[j0]);
    do { // update values on alternating path
      int j1 = pre[j0];
      p[j0] = p[j1];
      j0 = j1;
    } while (j0);
  }
  return -v[0]; // min cost
}
```

## UnweightedMatch.h

**Description:** general unweighted matching

**Time:** ?

c24787, 79 lines

```
template<int SZ> struct UnweightedMatch {
  int vis[SZ], par[SZ], orig[SZ], match[SZ], aux[SZ], t, N; //
    ↪1-based index
  vi adj[SZ];
  queue<int> Q;
  void addEdge(int u, int v) {
    adj[u].pb(v); adj[v].pb(u);
  }

  void init(int n) {
    N = n; t = 0;
    FOR(i,N+1) {
      adj[i].clear();
      match[i] = aux[i] = par[i] = 0;
    }
  }

  void augment(int u, int v) {
    int pv = v, nv;
    do {
      pv = par[v]; nv = match[pv];
      match[v] = pv; match[pv] = v;
      v = nv;
    } while(u != pv);
  }

  int lca(int v, int w) {
    ++t;
    while (1) {
      if (v) {
        if (aux[v] == t) return v; aux[v] = t;
        v = orig[par[match[v]]];
      }
      swap(v, w);
    }
  }

  void blossom(int v, int w, int a) {
    while (orig[v] != a) {
      par[v] = w; w = match[v];
      if (vis[w] == 1) Q.push(w), vis[w] = 0;
      orig[v] = orig[w] = a;
      v = par[w];
    }
  }

  bool bfs(int u) {
    fill(vis+1, vis+1+N, -1); iota(orig + 1, orig + N + 1, 1);
    Q = queue<int> (); Q.push(u); vis[u] = 0;
    while (sz(Q)) {
      int v = Q.front(); Q.pop();
      trav(x,adj[v]) {
        if (vis[x] == -1) {
          par[x] = v; vis[x] = 1;
          if (!match[x]) return augment(u, x), true;
          Q.push(match[x]); vis[match[x]] = 0;
        } else if (vis[x] == 0 && orig[v] != orig[x]) {
          int a = lca(orig[v], orig[x]);
          blossom(x, v, a); blossom(v, x, a);
        }
      }
    }
    return false;
  }
```

```
  int match() {
    int ans = 0;
    // find random matching (not necessary, constant
      ↪improvement)
    vi V(N-1); iota(all(V), 1);
    shuffle(all(V), mt19937(0x94949));
    trav(x,V) if(!match[x])
      trav(y,adj[x]) if (!match[y]) {
        match[x] = y, match[y] = x;
        ++ans; break;
      }

    FOR(i,1,N+1) if (!match[i] && bfs(i)) ++ans;
    return ans;
  }
};
```

## 6.6 Misc

### MaximalCliques.h

**Description:** Finds all maximal cliques

**Time:** $\mathcal{O}\left(3^{n/3}\right)$

f70515, 19 lines

```
typedef bitset<128> B;
int N;
B adj[128];

void cliques(B P = ~B(), B X={}, B R={}) { // possibly in
  ↪clique, not in clique, in clique
  if (!P.any()) {
    if (!X.any()) {
      // do smth with maximal clique
    }
    return;
  }
  auto q = (P|X)._Find_first();
  auto cands = P&~eds[q]; // clique must contain q or non-
    ↪neighbor of q
  FOR(i,N) if (cands[i]) {
    R[i] = 1;
    cliques(eds, f, P & eds[i], X & eds[i], R);
    R[i] = P[i] = 0; X[i] = 1;
  }
}
```

### LCT.h

**Description:** Link-Cut Tree, use vir for subtree size queries

**Time:** $\mathcal{O}(\log N)$

06a240, 96 lines

```
typedef struct snode* sn;

struct snode {
  sn p, c[2]; // parent, children
  int val; // value in node
  int sum, mn, mx; // sum of values in subtree, min and max
    ↪prefix sum
  bool flip = 0;
  // int vir = 0; stores sum of virtual children

  snode(int v) {
    p = c[0] = c[1] = NULL;
    val = v; calc();
  }

  friend int getSum(sn x) { return x?x->sum:0; }
  friend int getMn(sn x) { return x?x->mn:0; }
  friend int getMx(sn x) { return x?x->mx:0; }

  void prop() {
    if (!flip) return;
    swap(c[0],c[1]); tie(mn,mx) = mp(sum-mx,sum-mn);
    FOR(i,2) if (c[i]) c[i]->flip ^= 1;
    flip = 0;
  }
  void calc() {
    FOR(i,2) if (c[i]) c[i]->prop();
    int s0 = getSum(c[0]), s1 = getSum(c[1]); sum = s0+val+s1;
      ↪// +vir
    mn = min(getMn(c[0]),s0+val+getMn(c[1]));
    mx = max(getMx(c[0]),s0+val+getMx(c[1]));
  }

  int dir() {
    if (!p) return -2;
    FOR(i,2) if (p->c[i] == this) return i;
    return -1; // p is path-parent pointer, not in current
      ↪splay tree
  }
  bool isRoot() { return dir() < 0; }

  friend void setLink(sn x, sn y, int d) {
    if (y) y->p = x;
    if (d >= 0) x->c[d] = y;
  }
  void rot() { // assume p and p->p propagated
    assert(!isRoot()); int x = dir(); sn pa = p;
    setLink(pa->p, this, pa->dir());
    setLink(pa, c[x^1], x);
    setLink(this, pa, x^1);
    pa->calc(); calc();
  }
  void splay() {
    while (!isRoot() && !p->isRoot()) {
      p->p->prop(), p->prop(), prop();
      dir() == p->dir() ? p->rot() : rot();
      rot();
    }
    if (!isRoot()) p->prop(), prop(), rot();
    prop();
  }

  void access() { // bring this to top of tree
    for (sn v = this, pre = NULL; v; v = v->p) {
      v->splay();
      // if (pre) v->vir -= pre->sz;
      // if (v->c[1]) v->vir += v->c[1]->sz;
      v->c[1] = pre; v->calc();
      pre = v;
      // v->sz should remain the same if using vir
    }
    splay(); assert(!c[1]); // left subtree of this is now path
      ↪ to root, right subtree is empty
  }
  void makeRoot() { access(); flip ^= 1; }
  void set(int v) { splay(); val = v; calc(); } // change value
    ↪ in node, splay suffices instead of access because it
    ↪doesn't affect values in nodes above it

  friend sn lca(sn x, sn y) {
    if (x == y) return x;
    x->access(), y->access(); if (!x->p) return NULL; // access
      ↪ at y did not affect x, so they must not be connected
    x->splay(); return x->p ? x->p : x;
  }
  friend bool connected(sn x, sn y) { return lca(x,y); }
  friend int balanced(sn x, sn y) {
    x->makeRoot(); y->access();
    return y->sum-2*y->mn;
  }

  friend bool link(sn x, sn y) { // make x parent of y
    if (connected(x,y)) return 0; // don't induce cycle
    y->makeRoot(); y->p = x;
    // x->access(); x->sz += y->sz; x->vir += y->sz;
    return 1; // success!
  }
  friend bool cut(sn x, sn y) { // x is originally parent of y
    x->makeRoot(); y->access();
    if (y->c[0] != x || x->c[0] || x->c[1]) return 0; // splay
      ↪tree with y should not contain anything else besides x
    x->p = y->c[0] = NULL; y->calc(); return 1; // calc is
      ↪redundant as it will be called elsewhere anyways?
  }
};
```

### DirectedMST.h

**Description:** computes the minimum directed spanning tree

**Time:** $\mathcal{O}(M \log M)$

8fe6d9, 47 lines

```
struct Edge { int a, b; ll w; };
struct Node {
  Edge key;
  Node *l, *r;
  ll delta;
  void prop() {
    key.w += delta;
    if (l) l->delta += delta;
    if (r) r->delta += delta;
    delta = 0;
  }
  Edge top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
  if (!a || !b) return a ?: b;
  a->prop(), b->prop();
  if (a->key.w > b->key.w) swap(a, b);
  swap(a->l, (a->r = merge(b, a->r)));
  return a;
}
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

ll dmst(int n, int r, vector<Edge>& g) {
  DSU dsu; dsu.init(n);
  vector<Node*> heap(n);
  trav(e, g) heap[e.b] = merge(heap[e.b], new Node{e});
  ll res = 0;
  vi seen(n, -1), path(n); seen[r] = r;
  FOR(s,n) {
    int u = s, qi = 0, w;
    while (seen[u] < 0) {
      path[qi++] = u, seen[u] = s;
      if (!heap[u]) return -1;
      Edge e = heap[u]->top();
      heap[u]->delta -= e.w, pop(heap[u]);
      res += e.w, u = dsu.get(e.a);
      if (seen[u] == s) {
        Node* cyc = 0;
        do cyc = merge(cyc, heap[w = path[--qi]]);
        while (dsu.unite(u, w));
        u = dsu.get(u);
        heap[u] = cyc, seen[u] = -1;
      }
    }
  }
  return res;
```

```
}
```

### DominatorTree.h

**Description:** a dominates b iff every path from 1 to b passes through a
**Time:** ?

8cba41, 47 lines

```
template<int SZ> struct Dominator {
  vi adj[SZ], ans[SZ]; // input edges, edges of dominator tree
  vi radj[SZ], child[SZ], sdomChild[SZ];
  int label[SZ], rlabel[SZ], sdom[SZ], dom[SZ], co;
  int root = 1;

  int par[SZ], bes[SZ];
  int get(int x) {
    // DSU with path compression
    // get vertex with smallest sdom on path to root
    if (par[x] != x) {
      int t = get(par[x]); par[x] = par[par[x]];
      if (sdom[t] < sdom[bes[x]]) bes[x] = t;
    }
    return bes[x];
  }

  void dfs(int x) { // create DFS tree
    label[x] = ++co; rlabel[co] = x;
    sdom[co] = par[co] = bes[co] = co;
    trav(y,adj[x]) {
      if (!label[y]) {
        dfs(y);
        child[label[x]].pb(label[y]);
      }
      radj[label[y]].pb(label[x]);
    }
  }

  void init() {
    dfs(root);
    FORd(i,1,co+1) {
      trav(j,radj[i]) ckmin(sdom[i],sdom[get(j)]);
      if (i > 1) sdomChild[sdom[i]].pb(i);
      trav(j,sdomChild[i]) {
        int k = get(j);
        if (sdom[j] == sdom[k]) dom[j] = sdom[j];
        else dom[j] = k;
      }
      trav(j,child[i]) par[j] = i;
    }
    FOR(i,2,co+1) {
      if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
      ans[rlabel[dom[i]]].pb(rlabel[i]);
    }
  }
};
```

### EdgeColoring.h

**Description:** Vizing's Theorem: If max degree in simple undirected graph is d, then it can be edge colored with d+1 colors
**Time:** ?

bd4b3d, 94 lines

```
template<int SZ> struct EdgeColor {
  int n, adjVert[SZ][SZ], adjCol[SZ][SZ];
  int deg[SZ], maxDeg;

  EdgeColor(int _n) {
    n = _n; maxDeg = 0;
    FOR(i,n) {
      deg[i] = 0;
      FOR(j,n) adjVert[i][j] = adjCol[i][j] = -1;
    }
  }

  void delEdge(int x, int y) {
    if (adjVert[x][y] == -1) return;
    int C = adjVert[x][y];
    adjCol[x][C] = adjCol[y][C] = adjVert[x][y] = adjVert[y][x]
      ↪ = -1;
  }

  void setEdge(int x, int y, int c) { // delete previous value
    ↪if it had one
    delEdge(x,y); assert(adjCol[x][c] == -1 && adjCol[y][c] ==
      ↪-1);
    adjVert[x][y] = adjVert[y][x] = c, adjCol[x][c] = y, adjCol
      ↪[y][c] = x;
  }

  void shiftPath(int x, vi p) {
    FORd(i,sz(p)) setEdge(x,p[i],notAdj[p[i]]);
  }

  vi getPath(int st, int c0, int c1) {
    vi res = {st};
    for (int nex = 0; ; nex ^= 1) {
      int c = (nex == 0 ? c0 : c1);
      if (adjCol[res.back()][c] == -1) return res;
      res.pb(adjCol[res.back()][c]);
    }
  }

  void flipPath(vi p, int c0, int c1) {
    FOR(i,sz(p)-1) delEdge(p[i],p[i+1]);
    FOR(i,sz(p)-1) {
      if (i&1) setEdge(p[i],p[i+1],c0);
      else setEdge(p[i],p[i+1],c1);
    }
  }

  int notAdj[SZ];

  void addEdge(int x, int y) {
    maxDeg = max(maxDeg,max(++deg[x],++deg[y]));

    // generate a color which is not adjacent to each vertex
    FOR(i,n) {
      FOR(j,maxDeg+1) if (adjCol[i][j] == -1) {
        notAdj[i] = j;
        break;
      }
    }

    vi nex(n);
    FOR(i,n) if (adjVert[x][i] != -1) nex[i] = adjCol[x][notAdj
      ↪[i]];
    nex[y] = adjCol[x][notAdj[y]];

    // generate sequence of neighbors
    vi vis(n), seq = {y};
    while (seq.back() != -1 && !vis[seq.back()]) {
      vis[seq.back()] = 1;
      seq.pb(nex[seq.back()]);
    }

    // case 1: easy
    if (seq.back() == -1) {
      seq.pop_back(), shiftPath(x,seq);
      return;
    }

    // separate into path and cycle
    int ind = 0; while (seq[ind] != seq.back()) ind ++;
    seq.pop_back();
    vi path = vi(seq.begin(),seq.begin()+ind);
    vi cyc = vi(seq.begin()+ind,seq.end());
    int c0 = notAdj[x], c1 = notAdj[cyc.back()];

    // case based on a/b path
    vi p = getPath(cyc.back(),c0,c1);
    if (p.back() != path.back()) {
      if (p.back() == x) { p.pop_back(), delEdge(x,p.back()); }
      flipPath(p,c0,c1);
      notAdj[seq.back()] = c0; shiftPath(x,seq);
    } else {
      reverse(all(p));
      flipPath(p,c0,c1);
      notAdj[path.back()] = c0; shiftPath(x,path);
    }
  }
};
```

# Geometry (7)

## 7.1 Primitives

### Point.h

**Description:** Easy Geo

708158, 47 lines

```
typedef ld T;
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }

namespace Point {
  typedef pair<T,T> P;
  typedef vector<P> vP;

  P dir(T ang) {
    auto c = exp(ang*complex<T>(0,1));
    return P(c.real(),c.imag());
  }

  T norm(P x) { return x.f*x.f+x.s*x.s; }
  T abs(P x) { return sqrt(norm(x)); }
  T angle(P x) { return atan2(x.s,x.f); }
  P conj(P x) { return P(x.f,-x.s); }

  P operator+(const P& l, const P& r) { return P(l.f+r.f,l.s+r.
    ↪s); }
  P operator-(const P& l, const P& r) { return P(l.f-r.f,l.s-r.
    ↪s); }
  P operator*(const P& l, const T& r) { return P(l.f*r,l.s*r);
    ↪}
  P operator*(const T& l, const P& r) { return r*l; }
  P operator/(const P& l, const T& r) { return P(l.f/r,l.s/r);
    ↪}
  P operator*(const P& l, const P& r) { return P(l.f*r.f-l.s*r.
    ↪s,l.s*r.f+l.f*r.s); }
  P operator/(const P& l, const P& r) { return l*conj(r)/norm(r
    ↪); }

  P& operator+=(P& l, const P& r) { return l = l+r; }
  P& operator-=(P& l, const P& r) { return l = l-r; }
  P& operator*=(P& l, const T& r) { return l = l*r; }
  P& operator/=(P& l, const T& r) { return l = l/r; }
  P& operator*=(P& l, const P& r) { return l = l*r; }
  P& operator/=(P& l, const P& r) { return l = l/r; }

  P unit(P x) { return x/abs(x); }
```

```
  T dot(P a, P b) { return (conj(a)*b).f; }
  T cross(P a, P b) { return (conj(a)*b).s; }
  T cross(P p, P a, P b) { return cross(a-p,b-p); }
  P rotate(P a, T b) { return a*P(cos(b),sin(b)); }

  T dist(P p, P a, P b) { return std::abs(cross(p,a,b))/abs(a-b); }
  P reflect(P p, P a, P b) { return a+conj((p-a)/(b-a))*(b-a); }
  P foot(P p, P a, P b) { return (p+reflect(p,a,b))/(T)2; }
  bool onSeg(P p, P a, P b) {
    return cross(a,b,p) == 0 && dot(p-a,p-b) <= 0;
  }
};

using namespace Point;
```

### AngleCmp.h

**Description:** sorts points according to atan2

fccaee, 5 lines

```
template<class T> int half(pair<T,T> x) { return mp(x.s,x.f) > mp((T)0,(T)0); }
bool angleCmp(P a, P b) {
  int A = half(a), B = half(b);
  return A == B ? cross(a,b) > 0 : A < B;
}
```

### SegIntersect.h

**Description:** computes the intersection point(s) of line segments $AB$, $CD$

e5e3ff1, 15 lines

```
P extension(P a, P b, P c, P d) { // computes the intersection of lines AB, CD
  T x = cross(a,b,c), y = cross(a,b,d);
  return (d*x-c*y)/(x-y);
}
vP segIntersect(P a, P b, P c, P d) {
  T x = cross(a,b,c), y = cross(a,b,d);
  T X = cross(c,d,a), Y = cross(c,d,b);
  if (sgn(x)*sgn(y) < 0 && sgn(X)*sgn(Y) < 0) return {(d*x-c*y)/(x-y)};
  set<P> s;
  if (onSeg(a,c,d)) s.insert(a);
  if (onSeg(b,c,d)) s.insert(b);
  if (onSeg(c,a,b)) s.insert(c);
  if (onSeg(d,a,b)) s.insert(d);
  return {all(s)};
}
```

## 7.2 Polygons

### Area.h

**Description:** computes area + the center of mass of a polygon with constant mass per unit area

456d03, 16 lines

```
T area(const vP& v) {
  T area = 0;
  F0R(i,sz(v)) {
    int j = (i+1)%sz(v); T a = cross(v[i],v[j]);
    area += a;
  }
  return std::abs(area)/2;
}
P centroid(const vP& v) {
  P cen(0,0); T area = 0; // 2*signed area
  F0R(i,sz(v)) {
    int j = (i+1)%sz(v); T a = cross(v[i],v[j]);
    cen += a*(v[i]+v[j]); area += a;
  }
  return cen/area/(T)3;
}
```

### InPoly.h

**Description:** tests whether a point is inside, on, or outside the perimeter of any polygon

8f2d6a, 10 lines

```
string inPoly(const vP& p, P z) {
  int n = sz(p), ans = 0;
  F0R(i,n) {
    P x = p[i], y = p[(i+1)%n];
    if (onSeg(z,x,y)) return "on";
    if (x.s > y.s) swap(x,y);
    if (x.s <= z.s && y.s > z.s && cross(z,x,y) > 0) ans ^= 1;
  }
  return ans ? "in" : "out";
}
```

### ConvexHull.h

**Description:** Top-bottom convex hull

**Time:** $\mathcal{O}(N \log N)$

9be106, 37 lines

```
// typedef ll T;

using namespace Point;

pair<vi,vi> ulHull(const vP& P) {
  vi p(sz(P)), u, l; iota(all(p), 0);
  sort(all(p), [&P](int a, int b) { return P[a] < P[b]; });
  trav(i,p) {
    #define ADDP(C, cmp) while (sz(C) > 1 && cross(\
      P[C[sz(C)-2]],P[C.back()],P[i]) cmp 0) C.pop_back(); C.pb(i);
    ADDP(u, >=); ADDP(l, <=);
  }
  return {u,l};
}

vi hullInd(const vP& P) {
  vi u,l; tie(u,l) = ulHull(P);
  if (sz(l) <= 1) return l;
  if (P[l[0]] == P[l[1]]) return {0};
  l.insert(end(l),rbegin(u)+1,rend(u)-1); return l;
}
vP hull(const vP& P) {
  vi v = hullInd(P);
  vP res; trav(t,v) res.pb(P[t]);
  return res;
}

ld diameter(vP P) { // rotating calipers
  P = hull(P);
  int n = sz(P), ind = 1; ld ans = 0;
  F0R(i,n)
    for (int j = (i+1)%n;;ind = (ind+1)%n) {
      ckmax(ans,abs(P[i]-P[ind]));
      if (cross(P[j]-P[i],P[(ind+1)%n]-P[ind]) <= 0) break;
    }
  return ans;
}
```

## 7.3 Circles

### Circles.h

**Description:** misc operations with two circles

c4314f, 52 lines

```
using namespace Point;

namespace Circles {
  typedef pair<P,T> circ;
  bool on(circ x, P y) { return abs(y-x.f) == x.s; }
  bool in(circ x, P y) {  return abs(y-x.f) <= x.s; }
  T arcLength(circ x, P a, P b) {
    P d = (a-x.f)/(b-x.f);
    return x.s*acos(d.f);
  }

  P intersectPoint(circ x, circ y, int t = 0) { // assumes intersection points exist
    T d = abs(x.f-y.f); // distance between centers
    T theta = acos((x.s*x.s+d*d-y.s*y.s)/(2*x.s*d)); // law of cosines
    P tmp = (y.f-x.f)/d*x.s;
    return x.f+tmp*dir(t == 0 ? theta : -theta);
  }
  T intersectArea(circ x, circ y) { // not thoroughly tested
    T d = abs(x.f-y.f), a = x.s, b = y.s; if (a < b) swap(a,b);
    if (d >= a+b) return 0;
    if (d <= a-b) return PI*b*b;
    auto ca = (a*a+d*d-b*b)/(2*a*d), cb = (b*b+d*d-a*a)/(2*b*d);
    auto s = (a+b+d)/2, h = 2*sqrt(s*(s-a)*(s-b)*(s-d))/d;
    return a*a*acos(ca)+b*b*acos(cb)-d*h;
  }

  P tangent(P x, circ y, int t = 0) {
    y.s = abs(y.s); // abs needed because internal calls y.s < 0
    if (y.s == 0) return y.f;
    T d = abs(x-y.f);
    P a = pow(y.s/d,2)*(x-y.f)+y.f;
    P b = sqrt(d*d-y.s*y.s)/d*y.s*unit(x-y.f)*dir(PI/2);
    return t == 0 ? a+b : a-b;
  }
  vector<pair<P,P>> external(circ x, circ y) { // external tangents
    vector<pair<P,P>> v;
    if (x.s == y.s) {
      P tmp = unit(x.f-y.f)*x.s*dir(PI/2);
      v.pb(mp(x.f+tmp,y.f+tmp));
      v.pb(mp(x.f-tmp,y.f-tmp));
    } else {
      P p = (y.s*x.f-x.s*y.f)/(y.s-x.s);
      F0R(i,2) v.pb({tangent(p,x,i),tangent(p,y,i)});
    }
    return v;
  }
  vector<pair<P,P>> internal(circ x, circ y) { // internal tangents
    x.s *= -1; return external(x,y);
  }
}

using namespace Circles;
```

### Circumcenter.h

**Description:** circumcenter

0d49ba, 5 lines

```
pair<P,T> ccCenter(P a, P b, P c) { // circumcenter, radius
  b -= a; c -= a;
  P res = b*c*(conj(c)-conj(b))/(b*conj(c)-conj(b)*c);
  return {a+res,abs(res)};
}
```

### MinEnclosingCircle.h

**Description:** computes minimum enclosing circle

"Circumcenter.h" 63f976, 13 lines

```
pair<P, T> mec(vP ps) {
  shuffle(all(ps), mt19937(time(0)));
  P o = ps[0]; T r = 0, EPS = 1 + 1e-8;
  FOR(i,sz(ps)) if (abs(o-ps[i]) > r*EPS) {
    o = ps[i], r = 0;
    FOR(j,i) if (abs(o-ps[j]) > r*EPS) {
      o = (ps[i]+ps[j])/2, r = abs(o-ps[i]);
      FOR(k,j) if (abs(o-ps[k]) > r*EPS)
        tie(o,r) = ccCenter(ps[i],ps[j],ps[k]);
    }
  }
  return {o,r};
}
```

## 7.4 Misc

### ClosestPair.h

**Description:** $O(N \log N)$ line sweep to find two closest points out of N

b5ed46, 21 lines

```
using namespace Point;

pair<P,P> solve(vP v) {
  pair<ld,pair<P,P>> bes; bes.f = INF;
  set<P> S; int ind = 0;

  sort(all(v));
  FOR(i,sz(v)) {
    if (i && v[i] == v[i-1]) return {v[i],v[i]};
    for (; v[i].f-v[ind].f >= bes.f; ++ind)
      S.erase({v[ind].s,v[ind].f});
    for (auto it = S.ub({v[i].s-bes.f,INF});
      it != end(S) && it->f < v[i].s+bes.f; ++it) {
      P t = {it->s,it->f};
      ckmin(bes,{abs(t-v[i]),{t,v[i]}});
    }
    S.insert({v[i].s,v[i].f});
  }

  return bes.s;
}
```

### DelaunayFast.h

**Description:** Delaunay Triangulation, concyclic points are OK (but not all collinear)

"Point.h" 765ba9, 94 lines

```
typedef ll T;

typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point

struct Quad {
  bool mark; Q o, rot; P p;
  P F() { return r()->p; }
  Q r() { return rot->rot; }
  Q prev() { return rot->o->rot; }
  Q next() { return r()->prev(); }
};

// test if p is in the circumcircle
bool circ(P p, P a, P b, P c) {
  ll ar = cross(a,b,c); assert(ar); if (ar < 0) swap(a,b);
  lll p2 = norm(p), A = norm(a)-p2,
    B = norm(b)-p2, C = norm(c)-p2;
  return cross(p,a,b)*C+cross(p,b,c)*A+cross(p,c,a)*B > 0;
}

Q makeEdge(P orig, P dest) {
  Q q[] = {new Quad{0,0,0,orig}, new Quad{0,0,0,arb},
       new Quad{0,0,0,dest}, new Quad{0,0,0,arb}};
  FOR(i,4) q[i]->o = q[-i & 3], q[i]->rot = q[(i+1) & 3];
  return *q;
}
void splice(Q a, Q b) {
  swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next());
  splice(q->r(), b);
  return q;
}

pair<Q,Q> rec(const vector<P>& s) {
  if (sz(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
    if (sz(s) == 2) return { a, a->r() };
    splice(a->r(), b);
    auto side = cross(s[0], s[1], s[2]);
    Q c = side ? connect(b, a) : 0;
    return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
  }

#define H(e) e->F(), e->p
#define valid(e) (cross(e->F(),H(base)) > 0)
  Q A, B, ra, rb;
  int half = sz(s) / 2;
  tie(ra, A) = rec({all(s) - half});
  tie(B, rb) = rec({sz(s) - half + all(s)});
  while ((cross(B->p,H(A)) < 0 && (A = A->next())) ||
       (cross(A->p,H(B)) > 0 && (B = B->r()->o)));
  Q base = connect(B->r(), A);
  if (A->p == ra->p) ra = base->r();
  if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
      Q t = e->dir; \
      splice(e, e->prev()); \
      splice(e->r(), e->r()->prev()); \
      e = t; \
    }
  for (;;) {
    DEL(LC, base->r(), o);  DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
      base = connect(RC, base->r());
    else
      base = connect(base->r(), LC->r());
  }
  return {ra, rb};
}

vector<array<P,3>> triangulate(vector<P> pts) {
  sort(all(pts)); assert(unique(all(pts)) == pts.end());
  if (sz(pts) < 2) return {};

  Q e = rec(pts).f; vector<Q> q = {e};
  int qi = 0;
  while (cross(e->o->F(), e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
  q.push_back(c->r()); c = c->next(); } while (c != e); }
  ADD; pts.clear();
  while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;

  vector<array<P,3>> ret;
  FOR(i,sz(pts)/3) ret.pb({pts[3*i],pts[3*i+1],pts[3*i+2]});
  return ret;
}
```

## 7.5 3D

### Point3D.h

**Description:** Basic 3D Geometry

a4d471, 45 lines

```
typedef ld T;

namespace Point3D {
  typedef array<T,3> P3;
  typedef vector<P3> vP3;

  T norm(const P3& x) {
    T sum = 0; FOR(i,sz(x)) sum += x[i]*x[i];
    return sum;
  }
  T abs(const P3& x) { return sqrt(norm(x)); }

  P3& operator+=(P3& l, const P3& r) { FOR(i,3) l[i] += r[i];
    ↪return l; }
  P3& operator-=(P3& l, const P3& r) { FOR(i,3) l[i] -= r[i];
    ↪return l; }
  P3& operator*=(P3& l, const T& r) { FOR(i,3) l[i] *= r;
    ↪return l; }
  P3& operator/=(P3& l, const T& r) { FOR(i,3) l[i] /= r;
    ↪return l; }

  P3 operator+(P3 l, const P3& r) { return l += r; }
  P3 operator-(P3 l, const P3& r) { return l -= r; }
  P3 operator*(P3 l, const T& r) { return l *= r; }
  P3 operator*(const T& r, const P3& l) { return l*r; }
  P3 operator/(P3 l, const T& r) { return l /= r; }

  T dot(const P3& a, const P3& b) {
    T sum = 0; FOR(i,3) sum += a[i]*b[i];
    return sum;
  }
  P3 cross(const P3& a, const P3& b) {
    return {a[1]*b[2]-a[2]*b[1],
        a[2]*b[0]-a[0]*b[2],
        a[0]*b[1]-a[1]*b[0]};
  }

  bool isMult(const P3& a, const P3& b) {
    auto c = cross(a,b);
    FOR(i,sz(c)) if (c[i] != 0) return 0;
    return 1;
  }
  bool collinear(const P3& a, const P3& b, const P3& c) {
    ↪return isMult(b-a,c-a); }
  bool coplanar(const P3& a, const P3& b, const P3& c, const P3
    ↪& d) {
    return isMult(cross(b-a,c-a),cross(b-a,d-a));
  }
}

using namespace Point3D;
```

### Hull3D.h

**Description:** 3D Convex Hull + Polyedron Volume

**Time:** $\mathcal{O}(N^2)$

d201e5, 50 lines

```
using namespace Point3D;

struct ED {
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }
  int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vP3& A) {
  assert(sz(A) >= 4);
  vector<vector<ED>> E(sz(A), vector<ED>(sz(A), {-1, -1}));
  #define E(x,y) E[f.x][f.y]
  vector<F> FS; // faces
  auto mf = [&](int i, int j, int k, int l) { // make face
    P3 q = cross(A[j]-A[i],A[k]-A[i]);
    if (dot(q,A[l]) > dot(q,A[i])) q *= -1; // make sure q
      ↪points outward
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.pb(f);
  };
  FOR(i,4) FOR(j,i+1,4) FOR(k,j+1,4) mf(i, j, k, 6-i-j-k);

  FOR(i,4,sz(A)) {
    FOR(j,sz(FS)) {
      F f = FS[j];
      if (dot(f.q,A[i]) > dot(f.q,A[f.a])) { // face is visible
        ↪, remove edges
        E(a,b).rem(f.c), E(a,c).rem(f.b), E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();
      }
    }
    FOR(j,sz(FS)) { // add faces with new point
      F f = FS[j];
      #define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i,
        ↪ f.c);
      C(a, b, c); C(a, c, b); C(b, c, a);
    }
  }
  trav(it, FS) if (dot(cross(A[it.b]-A[it.a],A[it.c]-A[it.a]),
    ↪it.q) <= 0)
    swap(it.c, it.b);
  return FS;
} // computes hull where no four are coplanar

T signedPolyVolume(const vP3& p, const vector<F>& trilist) {
  T v = 0;
  trav(i,trilist) v += dot(cross(p[i.a],p[i.b]),p[i.c]);
  return v/6;
}
```

# Strings (8)

## 8.1 Lightweight

### KMP.h

**Description:** f[i] equals the length of the longest proper suffix of the i-th prefix of s that is a prefix of s

**Time:** $\mathcal{O}(N)$

08f252, 15 lines

```
vi kmp(string s) {
  int N = sz(s); vi f(N+1); f[0] = -1;
  FOR(i,1,N+1) {
    f[i] = f[i-1];
    while (f[i] != -1 && s[f[i]] != s[i-1]) f[i] = f[f[i]];
    f[i] ++;
  }
  return f;
}

vi getOc(string a, string b) { // find occurrences of a in b
  vi f = kmp(a+"@"+b), ret;
  FOR(i,sz(a),sz(b)+1) if (f[i+sz(a)+1] == sz(a)) ret.pb(i-sz(a
    ↪));
  return ret;
}
```

### Z.h

**Description:** similar to KMP

**Time:** $\mathcal{O}(N)$

a4e01c, 19 lines

```
vi z(string s) {
  int N = sz(s); s += '#';
  vi ans(N); ans[0] = N;
  int L = 1, R = 0;
  FOR(i,1,N) {
    if (i <= R) ans[i] = min(R-i+1,ans[i-L]);
    while (s[i+ans[i]] == s[ans[i]]) ans[i] ++;
    if (i+ans[i]-1 > R) L = i, R = i+ans[i]-1;
  }
  return ans;
}

vi getPrefix(string a, string b) { // find prefixes of a in b
  vi t = z(a+b), T(sz(b));
  FOR(i,sz(T)) T[i] = min(t[i+sz(a)],sz(a));
  return T;
}

// pr(z("abcababcabcaba"),getPrefix("abcab","uwetrabcerabcab"))
  ↪;
```

### Manacher.h

**Description:** Calculates length of largest palindrome centered at each character of string

**Time:** $\mathcal{O}(N)$

34a78b, 18 lines

```
vi manacher(string s) {
  string s1 = "@";
  trav(c,s) s1 += c, s1 += "#";
  s1[sz(s1)-1] = '&';

  vi ans(sz(s1)-1);
  int lo = 0, hi = 0;
  FOR(i,1,sz(s1)-1) {
    if (i != 1) ans[i] = min(hi-i,ans[hi-i+lo]);
    while (s1[i-ans[i]-1] == s1[i+ans[i]+1]) ans[i] ++;
    if (i+ans[i] > hi) lo = i-ans[i], hi = i+ans[i];
  }

  ans.erase(begin(ans));
  FOR(i,sz(ans)) if ((i&1) == (ans[i]&1)) ans[i] ++; // adjust
    ↪lengths
  return ans;
}
// ps(manacher("abacaba"))
```

### MinRotation.h

**Description:** minimum rotation of string

**Time:** $\mathcal{O}(N)$

483a1a, 8 lines

```
int minRotation(string s) {
  int a = 0, N = sz(s); s += s;
  FOR(b,N) FOR(i,N) { // a is current best rotation found up to
    ↪ b-1
    if (a+i == b || s[a+i] < s[b+i]) { b += max(0, i-1); break;
      ↪ } // b to b+i-1 can't be better than a to a+i-1
    if (s[a+i] > s[b+i]) { a = b; break; } // new best found
  }
  return a;
}
```

### LyndonFactorization.h

**Description:** A string is "simple" if it is strictly smaller than any of its own nontrivial suffixes. The Lyndon factorization of the string $s$ is a factorization $s = w_1 w_2 \dots w_k$ where all strings $w_i$ are simple and $w_1 \geq w_2 \geq \cdots \geq w_k$

**Time:** $\mathcal{O}(N)$

ff5520, 20 lines

```
vector<string> duval(const string& s) {
  int n = sz(s); vector<string> factors;
  for (int i = 0; i < n; ) {
    int j = i + 1, k = i;
    for (; j < n && s[k] <= s[j]; j++) {
      if (s[k] < s[j]) k = i;
      else k ++;
    }
    for (; i <= k; i += j-k) factors.pb(s.substr(i, j-k));
  }
  return factors;
}

int minRotation(string s) { // get min index i such that cyclic
  ↪ shift starting at i is min rotation
  int n = sz(s); s += s;
  auto d = duval(s); int ind = 0, ans = 0;
  while (ans+sz(d[ind]) < n) ans += sz(d[ind++]);
  while (ind && d[ind] == d[ind-1]) ans -= sz(d[ind--]);
  return ans;
}
```

## 8.2 Suffix Structures

### ACfixed.h

**Description:** for each prefix, stores link to max length suffix which is also a prefix

**Time:** $\mathcal{O}(N\sum)$

2337c9, 36 lines

```
struct ACfixed { // fixed alphabet
  struct node {
    array<int,26> to;
    int link;
  };
  vector<node> d;
  ACfixed() { d.emplace_back(); }

  int add(string s) { // add word
    int v = 0;
    trav(C,s) {
      int c = C-'a';
      if (!d[v].to[c]) {
        d[v].to[c] = sz(d);
        d.emplace_back();
      }
      v = d[v].to[c];
    }
    return v;
  }
```

```
  void init() { // generate links
    d[0].link = -1;
    queue<int> q; q.push(0);
    while (sz(q)) {
      int v = q.front(); q.pop();
      FOR(c,26) {
        int u = d[v].to[c]; if (!u) continue;
        d[u].link = d[v].link == -1 ? 0 : d[d[v].link].to[c];
        q.push(u);
      }
      if (v) FOR(c,26) if (!d[v].to[c])
        d[v].to[c] = d[d[v].link].to[c];
    }
  }
};
```

## PalTree.h

**Description:** palindromic tree
**Time:** $\mathcal{O}(N)$

36a5a4, 26 lines

```
template<int SZ> struct PalTree {
  static const int sigma = 26;
  int s[SZ], len[SZ], link[SZ], to[SZ][sigma], oc[SZ];
  int n, last, sz;
  PalTree() { s[n++] = -1; link[0] = 1; len[1] = -1; sz = 2; }

  int getLink(int v) {
    while (s[n-len[v]-2] != s[n-1]) v = link[v];
    return v;
  }
  void addChar(int c) {
    s[n++] = c;
    last = getLink(last);
    if (!to[last][c]) {
      len[sz] = len[last]+2;
      link[sz] = to[getLink(link[last])][c];
      to[last][c] = sz++;
    }
    last = to[last][c]; oc[last] ++;
  }
  void init() { // number of occurrences of each palindrome
    vpi v; FOR(i,2,sz) v.pb({len[i],i});
    sort(all(v)); reverse(all(v));
    trav(a,v) oc[link[a.s]] += oc[a.s];
  }
};
```

## SuffixArray.h

**Description:** ?
**Time:** $\mathcal{O}(N \log N)$

dbc6b9, 51 lines

```
template<int SZ> struct SuffixArray {
  string S; int N;
  void init(const string& _S) {
    S = _S; N = sz(S);
    genSa(); genLcp();
    // R.init(lcp);
  }

  vi sa, isa;
  void genSa() { // http://ekzlib.herokuapp.com
    sa.rsz(N); vi classes(N);
    FOR(i,N) sa[i] = N-1-i, classes[i] = S[i];
    stable_sort(all(sa), [this](int i, int j) { return S[i] < S
      ↪[j]; });
    for (int len = 1; len < N; len *= 2) {
      vi c(classes);
      FOR(i,N) { // compare first len characters of each suffix
        bool same = i && sa[i-1] + len < N
                    && c[sa[i]] == c[sa[i-1]]
                    && c[sa[i]+len/2] == c[sa[i-1]+len/2];
        classes[sa[i]] = same ? classes[sa[i-1]] : i;
      }
      vi nex(N), s(sa); iota(all(nex),0); // suffixes with <=
        ↪len chars will not change pos
      FOR(i,N) {
        int s1 = s[i]-len;
        if (s1 >= 0) sa[nex[classes[s1]]++] = s1; // order
          ↪pairs w/ same first len chars by next len chars
      }
    }
    isa.rsz(N); FOR(i,N) isa[sa[i]] = i;
  }

  vi lcp;
  void genLcp() { // KACTL
    lcp = vi(N-1);
    int h = 0;
    FOR(i,N) if (isa[i]) {
      int pre = sa[isa[i]-1];
      while (max(i,pre)+h < N && S[i+h] == S[pre+h]) h++;
      lcp[isa[i]-1] = h; // lcp of suffixes starting at pre and
        ↪ i
      if (h) h--; // if we cut off first chars of two strings
        ↪with lcp h, then remaining portions still have lcp h
        ↪-1
    }
  }

  /*RMQ<int,SZ> R;
  int getLCP(int a, int b) {
    if (max(a,b) >= N) return 0;
    if (a == b) return N-a;
    int t0 = isa[a], t1 = isa[b];
    if (t0 > t1) swap(t0,t1);
    return R.query(t0,t1-1);
  }*/
};
```

## ReverseBW.h

**Description:** Reverse Burrows-Wheeler

13b6b0, 8 lines

```
string reverseBW(string s) {
  int nex[sz(s)];
  vector<pair<char,int>> v; FOR(i,sz(s)) v.pb({s[i],i});
  sort(all(v)); FOR(i,sz(v)) nex[i] = v[i].s;
  int cur = nex[0]; string ret;
  for (; cur;cur = nex[cur]) ret += v[cur].f;
  return ret;
}
```

## SuffixAutomaton.h

**Description:** Suffix Automaton
**Time:** $\mathcal{O}(N \log \sum)$

1cb9d7, 72 lines

```
struct SuffixAutomaton {
  struct state {
    int len = 0, firstPos = -1, link = -1;
    bool isClone = 0;
    map<char, int> next;
    vi invLink;
  };

  vector<state> st;
  int last = 0;
  void extend(char c) {
    int cur = sz(st); st.eb();
    st[cur].len = st[last].len+1, st[cur].firstPos = st[cur].
      ↪len-1;
    int p = last;
    while (p != -1 && !st[p].next.count(c)) {
      st[p].next[c] = cur;
      p = st[p].link;
    }
    if (p == -1) {
      st[cur].link = 0;
    } else {
      int q = st[p].next[c];
      if (st[p].len+1 == st[q].len) {
        st[cur].link = q;
      } else {
        int clone = sz(st); st.pb(st[q]);
        st[clone].len = st[p].len+1, st[clone].isClone = 1;
        while (p != -1 && st[p].next[c] == q) {
          st[p].next[c] = clone;
          p = st[p].link;
        }
        st[q].link = st[cur].link = clone;
      }
    }
    last = cur;
  }
  void init(string s) {
    st.eb(); trav(x,s) extend(x);
    FOR(v,1,sz(st)) st[st[v].link].invLink.pb(v);
  }

  void getAllOccur(vi& oc, int v) {
    if (!st[v].isClone) oc.pb(st[v].firstPos);
    trav(u,st[v].invLink) getAllOccur(oc,u);
  }
  vi allOccur(string s) {
    int cur = 0;
    trav(x,s) {
      if (!st[cur].next.count(x)) return {};
      cur = st[cur].next[x];
    }
    vi oc; getAllOccur(oc,cur); trav(t,oc) t += 1-sz(s);
    sort(all(oc)); return oc;
  }

  vl distinct;
  ll getDistinct(int x) {
    if (distinct[x]) return distinct[x];
    distinct[x] = 1;
    trav(y,st[x].next) distinct[x] += getDistinct(y.s);
    return distinct[x];
  }
  ll numDistinct() { // # of distinct substrings, including
    ↪empty
    distinct.rsz(sz(st));
    return getDistinct(0);
  }
  ll numDistinct2() { // another way to get # of distinct
    ↪substrings
    ll ans = 1;
    FOR(i,1,sz(st)) ans += st[i].len-st[st[i].link].len;
    return ans;
  }
};
```

## SuffixTree.h

**Description:** ?
**Time:** $\mathcal{O}(N \sum)$

61394a, 50 lines

```
struct SuffixTree {
```

```
  enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
  int toi(char c) { return c - 'a'; }
  string a; // v = cur node, q = cur position
  int t[N][ALPHA],l[N],r[N],p[N],s[N],v=0,q=0,m=2;

  void ukkadd(int i, int c) { suff:
    if (r[v]<=q) {
      if (t[v][c]==-1) { t[v][c]=m;  l[m]=i;
        p[m++]=v; v=s[v]; q=r[v];  goto suff; }
      v=t[v][c]; q=l[v];
    }
    if (q==-1 || c==toi(a[q])) q++; else {
      l[m+1]=i;  p[m+1]=m;  l[m]=l[v];  r[m]=q;
      p[m]=p[v];  t[m][c]=m+1;  t[m][toi(a[q])]=v;
      l[v]=q;  p[v]=m;  t[p[m]][toi(a[l[m]])]=m;
      v=s[p[m]];  q=l[m];
      while (q<r[m]) { v=t[v][toi(a[q])];  q+=r[v]-l[v]; }
      if (q==r[m])  s[m]=v;  else s[m]=m+2;
      q=r[v]-(q-r[m]);  m+=2;  goto suff;
    }
  }

  SuffixTree(string a) : a(a) {
    fill(r,r+N,sz(a));
    memset(s, 0, sizeof s);
    memset(t, -1, sizeof t);
    fill(t[1],t[1]+ALPHA,0);
    s[0] = 1; l[0] = l[1] = -1; r[0] = r[1] = p[0] = p[1] = 0;
    FOR(i,sz(a)) ukkadd(i, toi(a[i]));
  }

  // example: find longest common substring (uses ALPHA = 28)
  pi best;
  int lcs(int node, int i1, int i2, int olen) {
    if (l[node] <= i1 && i1 < r[node]) return 1;
    if (l[node] <= i2 && i2 < r[node]) return 2;
    int mask = 0, len = node ? olen + (r[node] - l[node]) : 0;
    FOR(c,ALPHA) if (t[node][c] != -1)
      mask |= lcs(t[node][c], i1, i2, len);
    if (mask == 3)
      best = max(best, {len, r[node] - len});
    return mask;
  }
  static pi LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2));
    st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
    return st.best;
  }
};
```