USACO Notebook

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Contents

1 Contest

1.1 C++ Template

```
/**
* Sources: various
*/

#include <bits/stdc++.h>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/assoc_container.hpp>

using namespace std;
using namespace __gnu_pbds;

typedef long long ll;
typedef long double ld;
typedef vector<int> vi;
typedef pair<int, int> pi;
typedef pair<ll,ll> pl;
typedef pair<double,double> pd;

template <class T> using Tree = tree<T, null_type,
    less<T>,
    rb_tree_tag,tree_order_statistics_node_update>;

#define FOR(i, a, b) for (int i=a; i<(b); i++)
#define FOR(i, a) for (int i=0; i<(a); i++)
#define FORd(i,a,b) for (int i = (b)-1; i >= a; i--)
#define FORd(i,a) for (int i = (a)-1; i >= 0; i--)

#define sz(x) (int)(x).size()
#define mp make_pair
#define pb push_back
#define f first
#define s second
#define lb lower_bound
#define ub upper_bound
#define all(x) x.begin(), x.end()

const int MOD = 1000000007;
const double PI = 4*atan(1);
const ll INF = 1e18;

int main() {
   ios_base::sync_with_stdio(0); cin.tie(0);

}

// read the question correctly (is y a vowel?)
// look out for special cases (n=1?) and overflow (ll
    vs int?)
```

1.2 FastScanner

```
/**
* Source: Matt Fontaine
*/

class FastScanner {
   private InputStream stream;
   private byte[] buf = new byte[1024];
   private int curChar;
   private int numChars;

   public FastScanner(InputStream stream) {
      this.stream = stream;
   }
   int read() {
      if (numChars == -1)
         throw new InputMismatchException();
      if (curChar >= numChars) {
         curChar = 0;
```

```
            try {
                numChars = stream.read(buf);
            } catch (IOException e) {
                throw new InputMismatchException();
            }
            if (numChars <= 0) return -1;
        }
        return buf[curChar++];
    }

    boolean isSpaceChar(int c) {
        return c == ' ' || c == '\n' || c == '\r' || c
            == '\t' || c == -1;
    }

    boolean isEndline(int c) {
        return c == '\n' || c == '\r' || c == -1;
    }

    public int nextInt() {
        return Integer.parseInt(next());
    }

    public long nextLong() {
        return Long.parseLong(next());
    }

    public double nextDouble() {
        return Double.parseDouble(next());
    }

    public String next() {
        int c = read();
        while (isSpaceChar(c)) c = read();
        StringBuilder res = new StringBuilder();
        do {
            res.appendCodePoint(c);
            c = read();
        } while (!isSpaceChar(c));
        return res.toString();
    }

    public String nextLine() {
        int c = read();
        while (isEndline(c))
            c = read();
        StringBuilder res = new StringBuilder();
        do {
            res.appendCodePoint(c);
            c = read();
        } while (!isEndline(c));
        return res.toString();
    }
}
```

1.3 Troubleshooting

Source: KACTL

Pre-submit:

- Write a few simple test cases, if sample is not enough.
- Are time limits close? If so, generate max cases.
- Is the memory usage fine?
- Could anything overflow?
- Make sure to submit the right file.

Wrong answer:

- Print your solution! Print debug output, as well.
- Are you clearing all datastructures between test cases?
- Can your algorithm handle the whole range of input?
- Read the full problem statement again.
- Do you handle all corner cases correctly?
- Have you understood the problem correctly?
- Any uninitialized variables?
- Any overflows?
- Confusing N and M, i and j, etc.?
- Are you sure your algorithm works?
- What special cases have you not thought of?
- Are you sure the STL functions you use work as you think?
- Add some assertions, maybe resubmit.
- Create some testcases to run your algorithm on.
- Go through the algorithm for a simple case.
- Go through this list again.
- Explain your algorithm to a team mate.
- Ask the team mate to look at your code.
- Go for a small walk, e.g. to the toilet.
- Is your output format correct? (including whitespace)
- Rewrite your solution from the start or let a team mate do it.

Runtime error:

- Have you tested all corner cases locally?
- Any uninitialized variables?
- Are you reading or writing outside the range of any vector?
- Any assertions that might fail?
- Any possible division by 0? (mod 0 for example)

- Any possible infinite recursion?
- Invalidated pointers or iterators?
- Are you using too much memory?
- Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:

- Do you have any possible infinite loops?
- What is the complexity of your algorithm?
- Are you copying a lot of unnecessary data? (References)
- How big is the input and output? (consider scanf)
- Avoid vector, map. (use arrays/unordered map)
- What do your team mates think about your algorithm?

Memory limit exceeded:

- What is the max amount of memory your algorithm should need?
- Are you clearing all data structures between test cases?

2 Sorting And Searching (2)

2.1 Interval Cover

```
/**
* Usage: https://open.kattis.com/problems/intervalcover
* Description: Example of greedy algorithm
*/

double A,B,cur;
vector<pair<pdd,int>> in;
int N,nex;
vi ans;

void solve() {
    nex = 0; ans.clear();
    cin >> N; in.resize(N);
    FOR(i,N) {
        cin >> in[i].f.f >> in[i].f.s;
        in[i].s = i;
    }

    sort(all(in));
    pair<double,int> mx = {-DBL_MAX,-1};

    while (nex < in.size() && in[nex].f.f <= A) {
        mx = max(mx,{in[nex].f.s,in[nex].s});
        nex++;
    }
    if (nex == 0) {
        cout << "impossible\n";
        return;
    }
    ans.pb(mx.s);

    while (mx.f < B) {
        cur = mx.f;
        while (nex < in.size() && in[nex].f.f <= cur) {
            mx = max(mx,{in[nex].f.s,in[nex].s});
            nex++;
        }
        if (mx.f == cur) {
            cout << "impossible\n";
            return;
        }
        ans.pb(mx.s);
    }
    cout << ans.size() << "\n";
    for (int i: ans) cout << i << " ";
    cout << "\n";
}
```

2.2 Binary Search

```
/**
* Description: Basic example of binary search
* Guess the Number
* https://open.kattis.com/problems/guess
*/

int main() {
    int lo = 1, hi = 1000;
    while (1) {
        int mid = (lo+hi)/2;
        cout << mid << endl;
        string res; cin >> res;
        if (res == "correct") return 0;
        else if (res == "lower") hi = mid-1;
        else lo = mid+1;
    }
}
```

3 Data Structures (2)

3.1 Set

3.1.1 Coordinate Compression

```
/**
* Description: Demonstrates use of map
* Verification: POI 12 - The Bus
*/

void compress(vector<array<int,3>>& x, int ind) {
    map<int,int> m;
    for (auto& a: x) m[a[ind]] = 0;
    int co = 0; for (auto& a: m) a.s = co++;
    for (auto& a: x) a[ind] = m[a[ind]];
}
```

3.1.2 Map Customization

```
/**
* Description: Define your own comparator / hash
    function
* Source: StackOverflow
*/

struct cmp {
    bool operator()(const int& l, const int& r) const {
        return l > r;
    }
};

struct hsh {
    size_t operator()(const pii& k) const {
        return k.f^k.s; // bad, but you get the point
    }
};

set<int,cmp> s;
map<int,int,cmp> m;
unordered_map<pii,int,hsh> u;
```

4 DP (3)

4.1 Divide And Conquer (4)

```
/**
* Source: Own
* Usage: CEOI 2004 Two Sawmills
*/

void divi(int lo, int hi, int L, int R) {
    if (lo > hi) return;

    int mid = (lo+hi)/2;
    pair<ll,int> tmp = {1e18,-1};
    FOR(i,max(mid+1,L),R+1)
        tmp = min(tmp,{calc(0,mid)+calc(mid+1,i)
            +calc(i+1,n),i});
    ans = min(ans,tmp.f);

    divi(lo,mid-1,L,tmp.s);
    divi(mid+1,hi,tmp.s,R);
}
```

4.2 Examples

4.2.1 Knapsack

```
// https://open.kattis.com/problems/knapsack

double C;
int n,v[2000],w[2000],dp[2001][2001];

void solve() {
    FOR(i,n) cin >> v[i] >> w[i];
    FOR(i,n) {
        FOR(j,C+1) dp[i+1][j] = dp[i][j];
        FOR(j,C+1) if (w[i]+j <= C) dp[i+1][w[i]+j] =
            max(dp[i+1][w[i]+j],dp[i][j]+v[i]);
    }

    vi ans;
    int x = C;
    FORd(i,n) if (dp[i][x] != dp[i+1][x]) x -= w[i],
        ans.pb(i);

    cout << ans.size() << "\n";
    for (int i: ans) cout << i << " ";
    cout << "\n";
}
```

4.2.2 Longest Common Subsequence

```
/**
* Description: Classic DP example
*/

int dp[1001][1001];
string a,b;

int main() {
        cin >> a >> b;
        FOR(i,sz(a)) FOR(j,b.sz(b)) {
            dp[i+1][j+1] = max(dp[i+1][j],dp[i][j+1]);
            if (a[i] == b[j]) dp[i+1][j+1] =
                max(dp[i+1][j+1],dp[i][j]+1);
        }
        cout << dp[sz(a)][sz(b)];
}
```

4.2.3 Longest Increasing Subsequence

```
/**
* Description: DP with Binary Search
*/

vi bes = {0};
int n;

void ad(int x) {
    int lo = 0, hi = sz(bes)-1;
    while (lo < hi) {
        int mid = (lo+hi+1)/2;
        if (bes[mid] < x) lo = mid;
        else hi = mid-1;
    }
    if (lo == sz(bes)-1) bes.pb(0);
    bes[lo+1] = x;
}

int main() {
    cin >> n;
    FOR(i,n) {
        int x; cin >> x;
        ad(x);
    }
    cout << sz(bes)-1;
}
```

4.2.4 String Removals

```
/**
* Description: DP eliminates overcounting
* Verification: https://cses.fi/problemset/task/1149/
*/

int distinct(string S) {
    vi tot(26);
    int ans = 1;
    for (char c: S) {
        int t = (ans-tot[c-'a']+MOD)%MOD;
        tot[c-'a'] = (tot[c-'a']+t)%MOD;
        ans = (ans+t)%MOD;
    }
    return ans;
}
```

4.2.5 Traveling Salesman (4)

```
/**
* Description: Bitset DP example
* Solves TSP for small N
*/

const int MX = 15;

int N, dp[MX][1<<MX], dist[MX][MX];

int solve() {
    FOR(i,N) FOR(j,1<<N) dp[i][j] = MOD;

    dp[0][1] = 0;
    FOR(j,1<<N) FOR(i,N) if (j&(1<<i))
        FOR(k,N) if (!(j&(1<<k)))
            dp[k][j^(1<<k)] = min(dp[k][j^(1<<k)],
                                  dp[i][j]+dist[i][k]);

    int ans = MOD;
    FOR(j,1,N) ans =
        min(ans,dp[j][(1<<N)-1]+dist[j][0]);
    return ans;
}

int main() {
        int T; cin >> T;
        FOR(i,T) {
            cin >> N; N++;
            FOR(j,N) FOR(k,N) if (j != k) cin >>
                dist[j][k];
            cout << solve() << "\n";
        }
}
```

5 Graphs Easy (2)

5.1 Traversal

5.1.1 BFS on Grid

```
/**
* Note: Use xdir and ydir
*/

int xdir[4] = {0,1,0,-1}, ydir[4] = {1,0,-1,0};
int dist[21][21];
queue<pii> todo;

void process(pii x) {
        FOR(i,4) {
                pii y = {x.f+xdir[i],x.s+ydir[i]};
                if (y.f < 0 || y.f > 20 || y.s < 0 ||
                    y.s > 20) continue; // ignore this
                    point if it's outside of grid
                if (dist[y.f][y.s] == MOD) { // test
                    whether point has been visited or
                    not
                    dist[y.f][y.s] = dist[x.f][x.s]+1;
                    todo.push(y); // push point to queue
                }
        }
}

int main() {
        FOR(i,21) FOR(j,21) dist[i][j] = MOD;
        dist[10][10] = 0; todo.push({10,10}); //
            initialize queue, distances
        while (todo.size()) {
            process(todo.front());
            todo.pop(); // pop point from queue
        }
        cout << dist[4][5]; // 11
}
```

5.1.2 DFS on Graph

```
/**
* Classic
*/

int n, visit[100001];
vi adj[100001];

void dfs(int node) {
    if (visit[node]) return;
    visit[node] = 1;
    for (int i: adj[node]) dfs(i);
    cout << node << "\n";
        // do stuff

}

int main() {
        cin >> n;
        FOR(i,n-1) {
            int a,b; cin >> a >> b;
            adj[a].pb(b);
            adj[b].pb(a);
        }
        dfs(1);
}
```

5.2 Shortest Path (3)

5.2.1 Bellman-Ford

```
/**
* Usage: https://open.kattis.com/problems/shortestpath3
* Description: can be useful with linear programming
* Constraints of the form x_i-x_j<k
*/

const ll INF = 1e18;

int n,m,q,s,bad[1000];
vector<pair<pii,int>> edge;
ll dist[1000];

void solve() {
    edge.clear();
    FOR(i,n) dist[i] = INF, bad[i] = 0;
    dist[s] = 0;
    FOR(i,m) {
        int u,v,w; cin >> u >> v >> w;
        edge.pb({{u,v},w});
    }
    FOR(i,n) for (auto a: edge) if (dist[a.f.f] < INF)
         dist[a.f.s] = min(dist[a.f.s],
         dist[a.f.f]+a.s);
    for (auto a: edge) if (dist[a.f.f] < INF) if
         (dist[a.f.s] > dist[a.f.f]+a.s) bad[a.f.s] = 1;
    FOR(i,n) for (auto a: edge) if (bad[a.f.f])
         bad[a.f.s] = 1;

    FOR(i,q) {
        int x; cin >> x;
        if (bad[x]) cout << "-Infinity\n";
        else if (dist[x] == INF) cout <<
            "Impossible\n";
        else cout << dist[x] << "\n";
    }
    cout << "\n";
}
```

5.2.2 Dijkstra

```
/**
* Description: shortest path!
* Works with negative edge weights (aka SPFA?)
*/

template<int SZ> struct Dijkstra {
    int dist[SZ];
    vector<pii> adj[SZ];
    priority_queue<pii,vector<pii>,greater<pii>> q;

    void gen() {
        fill_n(dist,SZ,MOD); dist[0] = 0;

        q.push({0,0});
        while (q.size()) {
                pii x = q.top(); q.pop();
                if (dist[x.s] < x.f) continue;
                for (pii y: adj[x.s]) if (x.f+y.s <
                    dist[y.f]) {
                       dist[y.f] = x.f+y.s;
                       q.push({dist[y.f],y.f});
                }
        }
    }
};

Dijkstra<100> D;

int main() {
        FOR(i,100) FOR(j,100) if (rand() % 10 == 0)
            D.adj[i].pb({j,rand() % 10+1});
        D.gen();
        FOR(i,100) cout << D.dist[i] << "\n";
}
```

5.2.3 Floyd-Warshall

```
/**
* Usage: https://open.kattis.com/problems/allpairspath
*/

const ll INF = 1e18;

int n,m,q; // vertices, edges, queries
ll dist[150][150], bad[150][150];

void solve() {
    FOR(i,n) FOR(j,n) dist[i][j] = INF, bad[i][j] = 0;
    FOR(i,n) dist[i][i] = 0;
    FOR(i,m) {
        int u,v,w; cin >> u >> v >> w;
        dist[u][v] = min(dist[u][v],(ll)w);
    }
    FOR(k,n) FOR(i,n) FOR(j,n) if (dist[i][k] != INF
         && dist[k][j] != INF)
        dist[i][j] =
            min(dist[i][j],dist[i][k]+dist[k][j]);
    FOR(k,n) FOR(i,n) FOR(j,n) if (dist[i][k] != INF
         && dist[k][j] != INF)
        if (dist[i][j] > dist[i][k]+dist[k][j])
            bad[i][j] = 1;

    FOR(k,n) FOR(i,n) FOR(j,n) {
        if (dist[i][k] < INF && bad[k][j]) bad[i][j] =
             1;
        if (bad[i][k] && dist[k][j] < INF) bad[i][j] =
             1;
    }

    FOR(i,q) {
        int u,v; cin >> u >> v;
        if (bad[u][v]) cout << "-Infinity\n";
        else if (dist[u][v] == INF) cout <<
             "Impossible\n";
        else cout << dist[u][v] << "\n";
    }
    cout << "\n";
}
```

5.3 Topological Sort (3)

```
/**
* Description: sorts vertices such that if there
    exists an edge x->y, then x goes before y
*/

int N,M, in[100001];
vi res, adj[100001];

void topo() {
    queue<int> todo;
    FOR(i,1,N+1) if (in[i] == 0) todo.push(i);
    while (sz(todo)) {
        int x = todo.front(); todo.pop();
        res.pb(x);
        for (int i: adj[x]) {
            in[i] --;
            if (!in[i]) todo.push(i);
        }
    }
}

int main() {
        cin >> N >> M;
        FOR(i,M) {
            int x,y; cin >> x >> y;
            adj[x].pb(y), in[y] ++;
        }
        topo();
        for (int i: res) cout << i << " ";
}
```

5.4 Kruskal (3)

```
/**
* Source: own
* Description: computes the minimum spanning tree in
    O(ElogE) time
* Verification: USACO superbull
*/

template<int SZ> struct DSU {
    int par[SZ], sz[SZ];
    DSU() {
        FOR(i,SZ) par[i] = i, sz[i] = 1;
    }

    int get(int x) { // path compression
        if (par[x] != x) par[x] = get(par[x]);
        return par[x];
    }

    bool unite(int x, int y) { // union-by-rank
        x = get(x), y = get(y);
        if (x == y) return 0;
        if (sz[x] < sz[y]) swap(x,y);
        sz[x] += sz[y], par[y] = x;
        return 1;
    }
};

int ans = 0; // total weight of MST
vector<pair<int,pii>> edge;

DSU<100> D;

void kruskal() {
        sort(all(edge));
        for (auto a: edge) if (D.unite(a.s.f,a.s.s))
            ans += a.f; // edge is in MST
}
```

6 Algorithm Design (2)

6.1 Minimum Deque (3)

```
/**
* Source: own
* Verification: Jan 18 Lifeguards
*/

struct MinDeque {
    int lo = 0, hi = -1;
    deque<pii> d;

    void ins(int x) { // add to back
        while (sz(d) && d.back().f >= x) d.pop_back();
        d.pb({x,++hi});
    }

    void del() { // delete from front
        if (d.front().s == lo++) d.pop_front();
    }

    int get() {
        return sz(d) ? d.front().f : MOD;
    }
};
```

6.2 Ternary Search (4)

```
/**
* Description: use on functions which are strictly
    decreasing then strictly increasing
*/

double eval(double x) {
    return (x-5)*(x-5);
}

double ternary(double l, double r) {
    if (abs(r-l) <= 1e-9) return (l+r)/2;
    double l1 = (2*l+r)/3, r1 = (l+2*r)/3;
    return eval(l1) < eval(r1) ? ternary(l,r1) :
        ternary(l1,r);
}

// ternary(-100,100) = 5
```

7 Range Queries (2)

7.1 Demos (3)

7.1.1

```
/**
* Link: http://www.spoj.com/problems/MATSUM/ (modified)
* Description: Use with 2D BIT, 2D SegBIT, 2D SegTree
*/

int main() {
    BIT2D<int,1024> B = BIT2D<int,1024>();
    Node<int> S = Node<int>();

    FOR(i,100000) {
        int c = rand()&1;
        if (c == 0) {
            int x = rand() % SZ, y = rand() % SZ, num =
                rand() % 100;
            S.upd(x,y,num);
            x++, y++;
            B.upd(x,y,num);
        } else if (c == 1) {
            int x1 = rand() % SZ, y1 = rand() % SZ, x2
                = rand() % SZ, y2 = rand() % SZ;
            if (x1 > x2) swap(x1,x2);
            if (y1 > y2) swap(y1,y2);
            int a = S.query(x1,x2,y1,y2);
            x1 ++, y1 ++, x2 ++, y2++;
            int b = B.query(x1,x2,y1,y2);
            assert(a == b);
        } else break;
    }
}
```

7.1.2 BBST Demo (4)

```
/**
* Link: http://www.spoj.com/problems/ORDERSET/
* Description: Use with treap, splay tree
*/

int main() {
        int Q; cin >> Q;
        FOR(i,Q) {
            char c; int d; cin >> c >> d;
                if (c == 'I') root = ins(root,d);
                else if (c == 'D') root = del(root,d);
                else if (c == 'K') {
                        if (!root || root->sz < d) cout
                            << "invalid\n";
                        else cout << find_by_order(d) <<
                            "\n";
                } else cout << order_of_key(d) << "\n";
        }
}
```

7.1.3 Point Update Demo

```
/*
* Link: http://www.spoj.com/problems/FENTREE/
* Description: Use with SegTree, BIT, Sparse SegTree
*/

Seg<ll,1<<20> B;

int main() {
        int N; cin >> N;
        FOR(i,1,N+1) {
            int x; cin >> x;
            B.upd(i,x);
        }
        int q; cin >> q;
        FOR(i,q) {
            char c; int a, b;
            cin >> c >> a >> b;
            if (c == 'q') cout << B.query(a,b) << "\n";
            else B.upd(a,b);
        }
}
```

7.1.4 Range Update Demo (4)

```
/**
* Link: http://www.spoj.com/problems/HORRIBLE/
* Description: Use with range BIT, lazy segtree
*/

int main() {
    int T; cin >> T;
    FOR(i,T) {
        LazySegTree<ll,1<<17> B =
            LazySegTree<ll,1<<17>();
        int N, C; cin >> N >> C;
        FOR(j,C) {
            int t; cin >> t;
            if (t == 0) {
                int p,q,v; cin >> p >> q >> v;
                B.upd(p,q,v);
            } else {
                int p,q; cin >> p >> q;
                cout << B.qsum(p,q) << "\n";
            }
        }
    }
}
```

7.2 Static Array Queries

7.2.1 Prefix Sums

```
/**
 * Description: Calculates rectangle sums in constant
     time
 * Verification: POI 16 Ticket Inspector
 */

template<class T, int SZ> struct sums {
    T sum[SZ][SZ];
    sums () { memset(sum,0,sizeof sum); }
    void init() {
        FOR(i,1,SZ) FOR(j,1,SZ)
            sum[i][j] += sum[i][j-1]
            +sum[i-1][j]-sum[i-1][j-1];
    }
    T get(int X1, int X2, int Y1, int Y2) {
        return sum[X2][Y2]-sum[X1-1][Y2]
                -sum[X2][Y1-1]+sum[X1-1][Y1-1];
    }
};
```

7.2.2 Range Minimum Query (3)

```
/**
* Description: Supports 1D range minimum query in
    constant time.
* Verification: Problem Tournament from IOI 2012:
    http://wcipeg.com/problem/ioi1223
* Source code: https://pastebin.com/ChpniVZL
*/

template<class T, int SZ> struct RMQ {
    T stor[SZ][32-__builtin_clz(SZ)];

    T comb(T a, T b) {
        return min(a,b);
    }

    void build(vector<T>& x) {
        FOR(i,sz(x)) stor[i][0] = x[i];
        FOR(j,1,32-__builtin_clz(SZ))
            FOR(i,SZ-(1<<(j-1)))
           stor[i][j] = comb(stor[i][j-1],
                     stor[i+(1<<(j-1))][j-1]);
    }

    T query(int l, int r) {
        int x = 31-__builtin_clz(r-l+1);
        return comb(stor[l][x],stor[r-(1<<x)+1][x]);
    }
};
```

7.2.3 Wavelet Tree (6)

```
/**
* Description: Segment tree on values instead of
    indices
* Verification: http://www.spoj.com/problems/MKTHNUM/
*/

int N,Q, A[100000];
map<int,int> m;
vi revm;

void input() {
        cin >> N >> Q;
        FOR(i,N) cin >> A[i];
}

void compress() {
    FOR(i,N) m[A[i]] = 0;
    int nex = 0;
    for (auto& a: m) {
        a.s = nex++;
        revm.pb(a.f);
    }
    FOR(i,N) A[i] = m[A[i]];
}

template<int SZ> struct wavelet {
    vi mapl[2*SZ], mapr[2*SZ], val[2*SZ];

    void build(int ind = 1, int L = 0, int R = SZ-1) {
         // build a wavelet tree
        if (ind == 1) { FOR(i,N) val[ind].pb(i); }

        if (L < R) {
            int M = (L+R)/2;
            for (int i: val[ind]) {
                val[2*ind+(A[i] > M)].pb(i);
                mapl[ind].pb(sz(val[2*ind])-1);
                mapr[ind].pb(sz(val[2*ind+1])-1);
            }
            build(2*ind,L,M);
            build(2*ind+1,M+1,R);
        }
    }

    int getl(int ind, int x) { return x < 0 ? -1 :
        mapl[ind][x]; }

    int getr(int ind, int x) { return x < 0 ? -1 :
        mapr[ind][x]; }

    int query(int lind, int rind, int k, int ind = 1,
        int L = 0, int R = SZ-1) { // how many <= mid
        with index <= r
        if (L == R) return L;

        int M = (L+R)/2;
        int t = getl(ind,rind)-getl(ind,lind-1);
        if (t >= k) return query(getl(ind,lind-1)+1,
                                 getl(ind,rind),k,2*ind,L,M);
        return query(getr(ind,lind-1)+1,
                     getr(ind,rind),k-t,2*ind+1,M+1,R);
    }
};

wavelet<1<<17> w;

int main() {
    input();
    compress();
    w.build();

    FOR(i,Q) {
        int l,r,k; cin >> l >> r >> k;
        cout << revm[w.query(l-1,r-1,k)] << "\n";
    }
}
```

7.3 1D Range Queries (3)

7.3.1 BIT with Range Update (4)

```
/**
* Source: GeeksForGeeks?
* Description: 1D range update, range query
* Alternative to lazy segment tree
*/

// BIT template

template<class T, int SZ> struct BITrange {
    BIT<T,SZ> bit[2]; // sums piecewise linear
        functions

    void upd(int hi, T val) {
        bit[1].upd(1,val), bit[1].upd(hi+1,-val);
        bit[0].upd(hi+1,hi*val);
    }
    void upd(int lo, int hi, T val) { upd(lo-1,-val),
        upd(hi,val); }
    T query(int x) { return
        bit[1].query(x)*x+bit[0].query(x); }
    T query(int x, int y) { return
        query(y)-query(x-1); }
};
```

7.3.2 Binary Indexed Tree

```
/**
* Description: 1D range sum query with point update
* Verification: SPOJ Fenwick
*/

template<class T, int SZ> struct BIT {
    T bit[SZ+1];

    BIT() { memset(bit,0,sizeof bit); }

    void upd(int k, T val) { // add val to index k
        for( ;k <= SZ; k += (k&-k)) bit[k] += val;
    }

    T query(int k) {
        T temp = 0;
        for (;k > 0;k -= (k&-k)) temp += bit[k];
        return temp;
    }
    T query(int l, int r) { return
        query(r)-query(l-1); } // range query [l,r]
};
```

7.3.3 Lazy SegTree (4)

```
/**
* Description: 1D range update, range query
* Verification: SPOJ Horrible
*/

const ll INF = 1e18; // setting this to MOD can be
    disastrous :(

template<class T, int SZ> struct LazySegTree {
    T sum[2*SZ], mn[2*SZ], lazy[2*SZ]; // set SZ to a
        power of 2

    LazySegTree() {
        memset (sum,0,sizeof sum);
        memset (mn,0,sizeof mn);
        memset (lazy,0,sizeof lazy);
    }

    void push(int ind, int L, int R) {
        sum[ind] += (R-L+1)*lazy[ind];
        mn[ind] += lazy[ind];
        if (L != R) lazy[2*ind] += lazy[ind],
            lazy[2*ind+1] += lazy[ind];
        lazy[ind] = 0;
    }

    void pull(int ind) {
        sum[ind] = sum[2*ind]+sum[2*ind+1];
        mn[ind] = min(mn[2*ind],mn[2*ind+1]);
    }

    void build() {
        FORd(i,SZ) pull(i);
    }

    T qsum(int lo, int hi, int ind = 1, int L = 0, int
         R = SZ-1) {
        push(ind,L,R);
        if (lo > R || L > hi) return 0;
        if (lo <= L && R <= hi) return sum[ind];

        int M = (L+R)/2;
        return qsum(lo,hi,2*ind,L,M) +
            qsum(lo,hi,2*ind+1,M+1,R);
    }

    T qmin(int lo, int hi, int ind = 1, int L = 0, int
         R = SZ-1) {
        push(ind,L,R);
        if (lo > R || L > hi) return INF;
        if (lo <= L && R <= hi) return mn[ind];

        int M = (L+R)/2;
        return min(qmin(lo,hi,2*ind,L,M),
            qmin(lo,hi,2*ind+1,M+1,R));
    }

    void upd(int lo, int hi, ll inc, int ind = 1, int
         L = 0, int R = SZ-1) {
        push(ind,L,R);
        if (hi < L || R < lo) return;
        if (lo <= L && R <= hi) {
            lazy[ind] = inc;
            push(ind,L,R);
            return;
        }

        int M = (L+R)/2;
        upd(lo,hi,inc,2*ind,L,M);
            upd(lo,hi,inc,2*ind+1,M+1,R);
        pull(ind);
    }
};
```

7.3.4 SegTree Beats (6)

```
/**
 * Description: Interval min modifications
 * Verification:
     http://acm.hdu.edu.cn/showproblem.php?pid=5306
 */

const int MX = 1<<20;

int N,M, a[MX];

struct Seg {
    ll sum[2*MX];
    int mx1[2*MX], mx2[2*MX], maxCnt[2*MX];

    void pull(int ind) {
        mx1[ind] = max(mx1[2*ind],mx1[2*ind+1]);
        mx2[ind] = max(mx2[2*ind],mx2[2*ind+1]);
        maxCnt[ind] = 0;

        if (mx1[2*ind] == mx1[ind]) maxCnt[ind] +=
             maxCnt[2*ind];
        else mx2[ind] = max(mx2[ind],mx1[2*ind]);

        if (mx1[2*ind+1] == mx1[ind]) maxCnt[ind] +=
             maxCnt[2*ind+1];
        else mx2[ind] = max(mx2[ind],mx1[2*ind+1]);

        sum[ind] = sum[2*ind]+sum[2*ind+1];
    }

    void build(int ind = 1, int L = 0, int R = N-1) {
        if (L == R) {
            mx1[ind] = sum[ind] = a[L];
            maxCnt[ind] = 1;
            mx2[ind] = -1;
            return;
        }

        int M = (L+R)/2;
        build(2*ind,L,M); build(2*ind+1,M+1,R);
        pull(ind);
    }

    void push(int ind, int L, int R) {
        if (L == R) return;
        if (mx1[2*ind] > mx1[ind]) {
            sum[2*ind] -=
                (ll)maxCnt[2*ind]*(mx1[2*ind]-mx1[ind]);
            mx1[2*ind] = mx1[ind];
        }
        if (mx1[2*ind+1] > mx1[ind]) {
            sum[2*ind+1] -=
                (ll)maxCnt[2*ind+1]*(mx1[2*ind+1]-mx1[ind]);
            mx1[2*ind+1] = mx1[ind];
        }
    }

    void modify(int x, int y, int t, int ind = 1, int
         L = 0, int R = N-1) {
        if (R < x || y < L || mx1[ind] <= t) return;
        push(ind,L,R);
        if (x <= L && R <= y && mx2[ind] < t) {
            sum[ind] -= (ll)maxCnt[ind]*(mx1[ind]-t);
            mx1[ind] = t;
            return;
        }
        if (L == R) return;
        int M = (L+R)/2;
        modify(x,y,t,2*ind,L,M);
        modify(x,y,t,2*ind+1,M+1,R);
        pull(ind);
    }

    ll qsum(int x, int y, int ind = 1, int L = 0, int
        R = N-1) {
        if (R < x || y < L) return 0;
        push(ind,L,R);
        if (x <= L && R <= y) return sum[ind];

        int M = (L+R)/2;
        return
            qsum(x,y,2*ind,L,M)+qsum(x,y,2*ind+1,M+1,R);
    }

    int qmax(int x, int y, int ind = 1, int L = 0, int
        R = N-1) {
        if (R < x || y < L) return -1;
        push(ind,L,R);
        if (x <= L && R <= y) return mx1[ind];

        int M = (L+R)/2;
        return
            max(qmax(x,y,2*ind,L,M),qmax(x,y,2*ind+1,M+1,R));
    }
};

Seg S = Seg();

void solve() {
        cin >> N >> M;
        FOR(i,N) cin >> a[i];
        S.build();

        FOR(i,M) {
            int t; cin >> t;
            if (t == 0) {
                int x,y,z; cin >> x >> y >> z;
                S.modify(x-1,y-1,z);
            } else if (t == 1) {
                int x,y; cin >> x >> y;
                cout << S.qmax(x-1,y-1) << "\n";
            } else {
                int x,y; cin >> x >> y;
                cout << S.qsum(x-1,y-1) << "\n";
            }
        }
}
```

7.3.5 SegTree

```
/*
* Source: http://codeforces.com/blog/entry/18051
* Description: 1D point update, range query
* Verification: SPOJ Fenwick
*/

template<class T, int SZ> struct Seg {
    T seg[2*SZ], MN = 0;

    Seg() {
        memset(seg,0,sizeof seg);
    }

    T comb(T a, T b) { return a+b; } // easily change
        this to min or max

    void upd(int p, T value) { // set value at
        position p
        for (seg[p += SZ] = value; p > 1; p >>= 1)
            seg[p>>1] = comb(seg[(p|1)^1],seg[p|1]); //
                non-commutative operations
    }

    void build() {
        FORd(i,SZ) seg[i] = comb(seg[2*i],seg[2*i+1]);
    }

    T query(int l, int r) { // sum on interval [l, r]
        T res1 = MN, res2 = MN; r++;
        for (l += SZ, r += SZ; l < r; l >>= 1, r >>=
            1) {
            if (l&1) res1 = comb(res1,seg[l++]);
            if (r&1) res2 = comb(seg[--r],res2);
        }
        return comb(res1,res2);
    }
};
```

7.3.6 Sparse SegTree (4)

```
/**
* Source: Own
*/

const int SZ = 1<<20;

template<class T> struct node {
    T val;
    node<T>* c[2];

    node() {
        val = 0;
        c[0] = c[1] = NULL;
    }

    void upd(int ind, T v, int L = 0, int R = SZ-1) {
        // add v
        if (L == ind && R == ind) { val += v; return; }

        int M = (L+R)/2;
        if (ind <= M) {
            if (!c[0]) c[0] = new node();
            c[0]->upd(ind,v,L,M);
        } else {
            if (!c[1]) c[1] = new node();
            c[1]->upd(ind,v,M+1,R);
        }

        val = 0;
        if (c[0]) val += c[0]->val;
        if (c[1]) val += c[1]->val;
    }

    T query(int low, int high, int L = 0, int R =
        SZ-1) { // query sum of segment
        if (low <= L && R <= high) return val;
        if (high < L || R < low) return 0;

        int M = (L+R)/2;
        T t = 0;
        if (c[0]) t += c[0]->query(low,high,L,M);
        if (c[1]) t += c[1]->query(low,high,M+1,R);
        return t;
    }

    void UPD(int ind, node* c0, node* c1, int L = 0,
        int R = SZ-1) { // for 2D segtree
        if (L != R) {
            int M = (L+R)/2;
            if (ind <= M) {
                if (!c[0]) c[0] = new node();
                c[0]->UPD(ind,c0 ? c0->c[0] : NULL,c1 ?
                    c1->c[0] : NULL,L,M);
            } else {
                if (!c[1]) c[1] = new node();
                c[1]->UPD(ind,c0 ? c0->c[1] : NULL,c1 ?
                    c1->c[1] : NULL,M+1,R);
            }
        }
        val = 0;
        if (c0) val += c0->val;
        if (c1) val += c1->val;
    }
};
```

7.4 2D Range Queries (4)

7.4.1

```
/**
* Description: Supports point update & range query,
    can be extended to range update
* Verification: SPOJ matsum
* Dependency: Binary indexed tree
*/

template<class T, int SZ> struct BIT2D {
    BIT<T,SZ> bit[SZ+1];
    void upd(int X, int Y, T val) {
        for (; X <= SZ; X += (X&-X)) bit[X].upd(Y,val);
    }
    T query(int X, int Y) {
        T ans = 0;
        for (; X > 0; X -= (X&-X)) ans +=
            bit[X].query(Y);
        return ans;
    }
    T query(int X1, int X2, int Y1, int Y2) {
        return query(X2,Y2)-query(X1-1,Y2)
            -query(X2,Y1-1)+query(X1-1,Y1-1);
    }
};

int main() {
        int T; cin >> T;
        FOR(i,T) {
            int N; cin >> N;
            BIT2D<ll,1024> B = BIT2D<ll,1024>();
            while (1) {
                string c; cin >> c;
                if (c == "SET") {
                    int x, y,num; cin >> x >> y >> num;
                    x++, y++;
                    B.upd(x,y,num-B.query(x,x,y,y));
                } else if (c == "SUM") {
                    int x1, y1, x2, y2; cin >> x1 >> y1
                        >> x2 >> y2;
                    x1 ++, y1 ++, x2 ++, y2++;
                    cout << B.query(x1,x2,y1,y2) << "\n";
                } else break;
            }
        }
}
```

7.4.2

```
/**
* Source: USACO Mowing the Field
* Dependency: Sparse SegTree
*/

const int SZ = 1<<17;

template<class T> struct SegBit {
    node<T> seg[SZ+1];

    SegBit() {
        FOR(i,SZ+1) seg[i] = node<T>();
    }

    void upd(int x, int y, int v) { // add v
        for (x++;x <= SZ; x += (x&-x)) seg[x].upd(y,v);
    }

    T query(int x, int y1, int y2) {
        T ret = 0;
        for (;x > 0; x -= (x&-x)) ret +=
            seg[x].query(y1,y2);
        return ret;
    }

    T query(int x1, int x2, int y1, int y2) { // query
        sum of rectangle
        return query(x2+1,y1,y2)-query(x1,y1,y2);
    }
};
```

7.4.3

```
/**
* Source: USACO Mowing the Field
* Dependency: Sparse SegTree
*/

const int SZ = 1<<17;

template<class T> struct Node {
    node<T> seg;
    Node* c[2];

    void upd(int x, int y, T v, int L = 0, int R =
        SZ-1) { // add v
        if (L == x && R == x) {
            seg.upd(y,v);
            return;
        }

        int M = (L+R)/2;
        if (x <= M) {
            if (!c[0]) c[0] = new Node();
            c[0]->upd(x,y,v,L,M);
        } else {
            if (!c[1]) c[1] = new Node();
            c[1]->upd(x,y,v,M+1,R);
        }

        seg.UPD(y,c[0] ? &c[0]->seg : NULL,c[1] ?
            &c[1]->seg : NULL);
    }

    T query(int x1, int x2, int y1, int y2, int L = 0,
        int R = SZ-1) { // query sum of rectangle
        if (x1 <= L && R <= x2) return
            seg.query(y1,y2);
        if (x2 < L || R < x1) return 0;

        int M = (L+R)/2;
        T t = 0;
        if (c[0]) t += c[0]->query(x1,x2,y1,y2,L,M);
        if (c[1]) t += c[1]->query(x1,x2,y1,y2,M+1,R);
        return t;
    }
};
```

7.4.4 Merge-Sort Tree

```
/**
* Description: Similar to 2D segtree, less memory
* For more complex queries use a customized treap
* Verification:
    http://codeforces.com/contest/785/submission/33953058
*/

template<int SZ> struct mstree {
    Tree<pii> val[SZ+1]; // for offline queries use
        vector with binary search instead

    void upd(int x, int y, int t = 1) { //
        x-coordinate between 1 and SZ inclusive
        for (int X = x; X <= SZ; X += X&-X) {
            if (t == 1) val[X].insert({y,x});
            else val[X].erase({y,x});
        }
    }

    int query(int x, int y) {
        int t = 0;
        for (;x > 0; x -= x&-x) t +=
            val[x].order_of_key({y,MOD});
        return t;
    }

    int query(int lox, int hix, int loy, int hiy) { //
        query number of elements within a rectangle
        return query(hix,hiy)-query(lox-1,hiy)
            -query(hix,loy-1)+query(lox-1,loy-1);
    }
};
```

7.5 BBST (4)

7.5.1 Link-Cut Tree (5)

```
/**
* Source: Dhruv Rohatgi
* Usage: USACO Camp - The Applicant
*/

template<int SZ> struct LCT {
    int p[SZ], pp[SZ], c[SZ][2], sum[SZ];

    LCT () {
        FOR(i,1,SZ) sum[i] = 1;
        memset(p,0,sizeof p);
        memset(pp,0,sizeof pp);
        memset(c,0,sizeof c);
    }

    int getDir(int x, int y) {
        return c[x][0] == y ? 0 : 1;
    }

    void setLink(int x, int y, int d) {
        c[x][d] = y, p[y] = x;
    }

    void rotate(int y, int d) {
        int x = c[y][d], z = p[y];
        setLink(y,c[x][d^1],d);
        setLink(x,y,d^1);
        setLink(z,x,getDir(z,y));

        sum[x] = sum[y];
        sum[y] = sum[c[y][0]]+sum[c[y][1]]+1;
        pp[x] = pp[y]; pp[y] = 0;
    }
void splay(int x) {
    while (p[x]) {
        int y = p[x], z = p[y];
        int dy = getDir(y,x), dz = getDir(z,y);
        if (!z) rotate(y,dy);
        else if (dy == dz) rotate(z,dz),
            rotate(y,dy);
        else rotate(y,dy), rotate(z,dz);
    }
}

void dis(int v, int d) {
    p[c[v][d]] = 0, pp[c[v][d]] = v;
    sum[v] -= sum[c[v][d]];
    c[v][d] = 0;
}

void con(int v, int d) {
    c[pp[v]][d] = v;
    sum[pp[v]] += sum[v];
    p[v] = pp[v], pp[v] = 0;
}

void access(int v) {
    // v is brought to the root of auxiliary tree
    // modify preferred paths

    splay(v);
    dis(v,1);

    while (pp[v]) {
        int w = pp[v]; splay(w);
        dis(w,1), con(v,1);
        splay(v);
    }
}

int find_root(int v) {
    access(v);
    while (c[v][0]) v = c[v][0];
    access(v);
    return v;
}

int find_depth(int v) {
    access(v);
    return sum[c[v][0]];
}

void cut(int v) {
    // cut link between v and par[v]
    access(v);
    pp[c[v][0]] = p[c[v][0]] = 0; // fix
    sum[v] -= sum[c[v][0]];
    c[v][0] = 0;
}

void link(int v, int w) {
    // v, which is root of another tree, is now
        child of w
    access(v), access(w);
        pp[w] = v; con(w,0);
    }

    int anc(int v, int num) {
        if (find_depth(v) < num) return 0;
        access(v);
        v = c[v][0];

        while (1) {
            if (sum[c[v][1]] >= num) v = c[v][1];
            else if (sum[c[v][1]]+1 == num) return v;
            else num -= (sum[c[v][1]]+1), v = c[v][0];
        }
    }

    void print(int x) {
        FOR(i,1,x+1) cout << i << " " << find_root(i)
            << " " << find_depth(i) << " " << anc(i,2)
            << "\n";
        cout << "\n";
    }
};

LCT<100001> L;

int main() {
    L.link(2,1); L.link(3,1); L.link(4,1); L.link(5,4);
    L.link(10,4); L.link(7,6); L.link(8,7);
        L.link(9,8);
    L.print(10);

    L.cut(4); L.link(4,8);
    L.print(10);
}
```

7.5.2 Splay Tree (5)

```
/*
* Description: Based off treap code
* Source:
    https://sites.google.com/site/kc97ble/container/splay-tre
* Verification: http://www.spoj.com/problems/ORDERSET/
*/

struct snode {
    int val, sz;
    snode *p, *c[2];

    snode (int v) {
        val = v, sz = 1;
        c[0] = c[1] = p = NULL;
    }

    void inOrder(bool f = 0) {
        if (c[0]) c[0]->inOrder();
        cout << val << " ";
        if (c[1]) c[1]->inOrder();
        if (f) cout << "\n------------\n";
    }
    void recalc() {
        sz = 1+(c[0]?c[0]->sz:0)+(c[1]?c[1]->sz:0);
    }
};

void setLink(snode* x, snode* y, int d) {
    if (x) x->c[d] = y, x->recalc();
    if (y) y->p = x;
}

snode* unLink(snode* x, int d) {
    snode* y = x->c[d];
    x->c[d] = NULL; x->recalc();
    if (y) y->p = NULL;
    return y;
}

int getDir(snode* x, snode* y) {
    if (!x) return -1;
    return x->c[0] == y ? 0 : 1;
}

void rot(snode* x, int d) {
    snode *y = x->c[d], *z = x->p;
    setLink(x, y->c[d^1], d);
    setLink(y, x, d^1);
    setLink(z, y, getDir(z, x));
}

snode* splay(snode* x) {
    while (x && x->p) {
        snode* y = x->p, *z = y->p;
        int dy = getDir(y, x), dz = getDir(z, y);
        if (!z) rot(y, dy);
        else if (dy == dz) rot(z, dz), rot(y, dy);
        else rot(y, dy), rot(z, dz);
    }
    return x;
}

snode* find(snode *cur, int v) {
    if (!cur) return cur;
    snode* x;
    if (cur->val >= v) x = find(cur->c[0],v);
    else x = find(cur->c[1],v);
    return x?x:cur;
}

snode* getmx(snode* x) {
    return x->c[1]?getmx(x->c[1]):x;
}

pair<snode*,snode*> split(snode* x, int v) {
    if (!x) return {x,x};
    snode* y = find(x,v); y = splay(y);
    if (y->val >= v) return {unLink(y,0),y};
    else return {y,unLink(y,1)};
}

snode* find_by_order(snode* x, int v) {
    int tmp = x->c[0]?x->c[0]->sz:0;
    if (v < tmp) return find_by_order(x->c[0],v);
    else if (v == tmp) return x;
    else return find_by_order(x->c[1],v-tmp-1);
}

pair<snode*,snode*> split_by_order(snode* x, int v) {
     // left subtree has v elements
    if (!x) return {x,x};
    if (v == x->sz) return {x,NULL};
    snode* y = find_by_order(x,v); y = splay(y);
    return {unLink(y,0),y};
}

snode* merge(snode* x, snode* y) {
    if (!x) return y;
    x = splay(getmx(x));
    setLink(x,y,1);
    return x;
}

// same as treap

snode* ins(snode* x, int v) { // insert value v
    auto a = split(x,v);
    auto b = split(a.s,v+1);
    return merge(a.f,merge(new snode(v),b.s));
}

snode* del(snode* x, int v) { // delete all values
     equal to v
    auto a = split(x,v), b = split(a.s,v+1);
    return merge(a.f,b.s);
}

snode* root;

int order_of_key(int x) {
    auto a = split(root,x);
    int t = a.f?a.f->sz:0;
    root = merge(a.f,a.s);
    return t;
}

int find_by_order(int x) {
    auto a = split_by_order(root,x);
    auto b = split_by_order(a.f,x-1);
    int t = b.s->val;
    root = merge(merge(b.f,b.s),a.s);
    return t;
}
```

7.5.3 Treap

```
/*
* Sources: various
* Description: Easiest BBST
* Verification: http://www.spoj.com/problems/ORDERSET/
*/

struct tnode {
    int val, pri, sz;
    tnode *c[2];

    tnode (int v) {
        val = v, sz = 1, pri = rand()+(rand()<<15);
        c[0] = c[1] = NULL;
    }

    void inOrder(bool f = 0) {
        if (c[0]) c[0]->inOrder();
        cout << val << " ";
        if (c[1]) c[1]->inOrder();
        if (f) cout << "\n------------\n";
    }

    void recalc() {
        sz = 1+(c[0]?c[0]->sz:0)+(c[1]?c[1]->sz:0);
    }
};

pair<tnode*,tnode*> split(tnode* t, int v) { // >= v
     goes to the right
    if (!t) return {t,t};

    if (v <= t->val) {
        auto p = split(t->c[0], v);
        t->c[0] = p.s; t->recalc();
        return {p.f, t};
    } else {
        auto p = split(t->c[1], v);
        t->c[1] = p.f; t->recalc();
        return {t, p.s};
    }
}

pair<tnode*,tnode*> split_by_order(tnode* t, int v) {
    if (!t) return {t,t};
    int tmp = t->c[0]?t->c[0]->sz:0;
    if (v <= tmp) {
        auto p = split_by_order(t->c[0], v);
        t->c[0] = p.s; t->recalc();
        return {p.f, t};
    } else {
        auto p = split_by_order(t->c[1], v-tmp-1);
        t->c[1] = p.f; t->recalc();
        return {t, p.s};
    }
}

tnode* merge(tnode* l, tnode* r) {
    if (!l) return r;
    if (!r) return l;

    if (l->pri > r->pri) {
        l->c[1] = merge(l->c[1],r);
        l->recalc();
        return l;
    } else {
        r->c[0] = merge(l,r->c[0]);
        r->recalc();
        return r;
    }
}

tnode* ins(tnode* x, int v) { // insert value v
    auto a = split(x,v);
    auto b = split(a.s,v+1);
    return merge(a.f,merge(new tnode(v),b.s));
}

tnode* del(tnode* x, int v) { // delete all values
     equal to v
    auto a = split(x,v), b = split(a.s,v+1);
    return merge(a.f,b.s);
}

tnode *root;

int order_of_key(int x) {
    auto a = split(root,x);
    int t = a.f?a.f->sz:0;
    root = merge(a.f,a.s);
    return t;
}

int find_by_order(int x) {
    auto a = split_by_order(root,x);
    auto b = split_by_order(a.f,x-1);
    int t = b.s->val;
    root = merge(merge(b.f,b.s),a.s);
    return t;
}
```

7.6 Persistent Queries (5)

7.6.1 Basic Persistent SegTree

```
/**
* Description: persistent segtree node without lazy
    updates
* Verification: Codeforces Problem 893F - Subtree
    Minimum Query
* Implementation:
    http://codeforces.com/contest/893/submission/32652140
*/

struct Node {
    int val = 0;
    Node* c[2];

    Node* copy() {
        Node* x = new Node(); *x = *this;
        return x;
    }

    int query(int low, int high, int L, int R) {
        if (low <= L && R <= high) return val;
        if (R < low || high < L) return MOD;
        int M = (L+R)/2;
        return min(c[0]->query(low,high,L,M),
                   c[1]->query(low,high,M+1,R));
    }
    Node* upd(int ind, int v, int L, int R) {
        if (R < ind || ind < L) return this;
        Node* x = copy();

        if (ind <= L && R <= ind) {
            x->val += v;
            return x;
        }

        int M = (L+R)/2;
        x->c[0] = x->c[0]->upd(ind,v,L,M);
        x->c[1] = x->c[1]->upd(ind,v,M+1,R);
        x->val = min(x->c[0]->val,x->c[1]->val);

        return x;
    }

    void build(vi& arr, int L, int R) {
        if (L == R) {
            if (L < (int)arr.size()) val = arr[L];
            else val = 0;
            return;
        }
        int M = (L+R)/2;
        c[0] = new Node();
        c[0]->build(arr,L,M);
        c[1] = new Node();
        c[1]->build(arr,M+1,R);
        val = min(c[0]->val,c[1]->val);
    }
};

template<int SZ> struct pers {
    Node* loc[SZ+1]; // stores location of root after
        ith update
    int nex = 1;

    pers() { loc[0] = new Node(); }

    void upd(int ind, int val) {
        loc[nex] = loc[nex-1]->upd(ind,val,0,SZ-1);
        nex++;
    }
    void build(vi& arr) {
        loc[0]->build(arr,0,SZ-1);
    }
    int query(int ti, int low, int high) {
        return loc[ti]->query(low,high,0,SZ-1);
    }
};
```

7.6.2 Lazy Persistent SegTree

```
/**
* Source:
    http://codeforces.com/blog/entry/47108?#comment-315047
* Description: Node + lazy updates
*/

struct node {
    int val = 0, lazy = 0;
    node* c[2];

    node* copy() {
        node* x = new node(); *x = *this;
        return x;
    }

    void push() {
        if (!lazy) return;
        FOR(i,2) if (c[i]) {
            c[i] = new node(*c[i]);
            c[i]->lazy += lazy;
        }
        lazy = 0;
    }

    int query(int low, int high, int L, int R) {
        if (low <= L && R <= high) return val;
        if (R < low || high < L) return MOD;
        int M = (L+R)/2;
        return lazy+min(c[0]->query(low,high,L,M),
                        c[1]->query(low,high,M+1,R));
    }

    node* upd(int low, int high, int v, int L, int R) {
        if (R < low || high < L) return this;
        node* x = copy();
        if (low <= L && R <= high) {
            x->lazy += v, x->val += v;
            return x;
        }
        push();

        int M = (L+R)/2;
        x->c[0] = x->c[0]->upd(low,high,v,L,M);
        x->c[1] = x->c[1]->upd(low,high,v,M+1,R);
        x->val = min(x->c[0]->val,x->c[1]->val);

        return x;
    }

    void build(vi& arr, int L, int R) {
        if (L == R) {
            if (L < sz(arr)) val = arr[L];
            else val = 0;
            return;
        }
        int M = (L+R)/2;
        c[0] = new node();
        c[0]->build(arr,L,M);
        c[1] = new node();
        c[1]->build(arr,M+1,R);
        val = min(c[0]->val,c[1]->val);
    }
};

template<int SZ> struct pers {
    node* loc[SZ+1]; // stores location of root after
        ith update
    int nex = 1;
    pers() { loc[0] = new node(); }

    void upd(int low, int high, int val) {
        loc[nex] =
            loc[nex-1]->upd(low,high,val,0,SZ-1);
        nex++;
    }
    void build(vi& arr) {
        loc[0]->build(arr,0,SZ-1);
    }
    int query(int ti, int low, int high) {
        return loc[ti]->query(low,high,0,SZ-1);
    }
};

pers<8> p;

int main() {
    vi arr = {1,7,2,3,5,9,4,6};
    p.build(arr);

    p.upd(1,2,2); // 1 9 4 3 5 9 4 6

    FOR(i,8) {
        FOR(j,i,8) cout << p.query(1,i,j) << " ";
        cout << "\n";
    }
    cout << "\n";

    p.upd(4,7,5); // 1 9 4 3 10 14 9 11
    FOR(i,8) {
        FOR(j,i,8) cout << p.query(2,i,j) << " ";
        cout << "\n";
    }
    cout << "\n";

    FOR(i,8) {
        FOR(j,i,8) cout << p.query(1,i,j) << " ";
        cout << "\n";
    }
    cout << "\n";
}
```

7.6.3 Low-Memory Persistent Segment Tree

```
//uses about 34 MB
const int MAXN = 100100;
int N = 100000;
struct Node {
        ll val;
} SEG[20*MAXN];
int e = 0;
int LFT[20*MAXN], RGT[20*MAXN];

int roots[MAXN];

int build(int l = 0, int r = N - 1) {
        //build from L to R inclusive.
        int x = ++e;
        if (l == r){
                SEG[x].val = 0;
                LFT[x] = -1;
                RGT[x] = -1;
                return x;
        }
        int mid = (l + r)/2;
        LFT[x] = build(l, mid);
        RGT[x] = build(mid + 1, r);
        return x;
}

int upd(int cur, int pos, int set, int l = 0, int r =
    N - 1) {
        //set a[pos] = set in the root cur
        if (r < pos || pos < l) return cur;
        int x = ++e;
        //we're creating a new node
        if (l == r){
                SEG[x].val = set;
                return x;
        }
        int m = (l+r)/2;
        LFT[x] = upd(LFT[cur], pos, set, l, m);
        RGT[x] = upd(RGT[cur], pos, set, m + 1, r);
        SEG[x].val = SEG[LFT[x]].val + SEG[RGT[x]].val;
        return x;
}

ll query(int cur, int L, int R, int l = 0, int r = N -
    1){
        if (r < L || R < l) return 0LL;
        int m = (l + r)/2;
        if (L <= l && r <= R) return SEG[cur].val;
        return query(LFT[cur], L, R, l, m) +
            query(RGT[cur], L, R, m + 1, r);
}
```

8 Trees (4)

8.1 Tree Diameter (5)

```
/**
* Might not be obvious why this works!
* Verification: http://www.spoj.com/problems/PT07Z/
*/

const int MX = 10001;

int n, dist[MX];
vi adj[MX];

void dfs(int cur, int pre) {
    for (int i: adj[cur]) if (i != pre) {
        dist[i] = dist[cur]+1;
        dfs(i,cur);
    }
}

void dfs(int cur) {
    memset(dist,0,sizeof dist);
    dfs(cur,-1);
}

int treeDiameter() {
    dfs(1);
    int bes = 0; FOR(i,1,n+1) if (dist[i] > dist[bes])
        bes = i;
    dfs(bes); FOR(i,1,n+1) if (dist[i] > dist[bes])
        bes = i;
    return dist[bes];
}

int main() {
    cin >> n;
    FOR(i,n-1) {
        int a, b; cin >> a >> b;
        adj[a].pb(b), adj[b].pb(a);
    }
    cout << treeDiameter();
}
```

8.2 Queries (4)

8.2.1 Heavy-Light Set

```
/**
* Description: offline subtree queries in O(Nlog^2N)
* Verification: January Easy 2018 - Shubham & Tree 1
*/

const int MX = 200001;

struct HeavyLightSet {
    int loc[MX], sub[MX], par[MX], val[MX];
    vi child[MX];
    map<int,int> dat[MX];

    void comb(int a, int b) {
        int A = loc[a], B = loc[b];
        if (sz(dat[A]) < sz(dat[B])) swap(a,b),
            swap(A,B);
        for (auto& x: dat[B]) dat[A][x.f] += x.s;
        dat[B].clear(); loc[b] = A;
    }

    void process(int ind) {
        sub[ind] = 1; loc[ind] = ind;
            dat[ind][val[ind]] ++;
        for (int i: child[ind]) {
            process(i);
            comb(i,ind);
            sub[ind] += sub[i];
        }
        // now do stuff with values
    }
};
```

8.2.2 LCA Demo

```
/**
* Debug the Bugs
* Description: Use for both LCA's
*/

LCA L;

int Q;

int main() {
    cin >> L.V >> Q >> L.R;
    FOR(i,L.V-1) {
        int u,v; cin >> u >> v;
        L.addEdge(u,v);
    }
    L.construct();

    FOR(i,Q) {
        int u,v; cin >> u >> v;
        cout << L.lca(u,v) << "\n";
    }
}
```

8.2.3 LCA with Binary Jumps

```
/**
* Source: USACO Camp
* Verification: Debug the Bugs
*/

const int MAXN = 100001, MAXK = 17;

struct LCA {
    int V, R;
    vi edges[MAXN];
    int parK[MAXK][MAXN];
    int depth[MAXN];

    void addEdge(int u, int v) {
        edges[u].pb(v), edges[v].pb(u);
    }

    void dfs(int u, int prev){
        parK[0][u] = prev;
        depth[u] = depth[prev]+1;
        for (int v: edges[u]) if (v != prev) dfs(v, u);
    }

    void construct() {
        dfs(R, 0);
        FOR(k,1,MAXK) FOR(i,1,V+1)
            parK[k][i] = parK[k-1][parK[k-1][i]];
    }

    int lca(int u, int v){
        if (depth[u] < depth[v]) swap(u,v);
        FORd(k,MAXK) if (depth[u] >= depth[v]+(1<<k))
            u = parK[k][u];
        FORd(k,MAXK) if (parK[k][u] != parK[k][v]) u =
            parK[k][u], v = parK[k][v];

        if(u != v) u = parK[0][u], v = parK[0][v];
        return u;
    }

    int dist(int u, int v) {
        return depth[u]+depth[v]-2*depth[lca(u,v)];
    }
};
```

8.3 Advanced (4)

8.3.1 Centroid Decomposition

```
/**
* Source: own
* Verification Problem: Ciel and Commander
    (http://codeforces.com/contest/321/problem/C)
* Code:
    http://codeforces.com/contest/321/submission/33952270
*/

const int MX = 100001;

int N, visit[MX], sub[MX], par[MX];
vi adj[MX];

void dfs (int no) {
    sub[no] = 1;
    for (int i: adj[no]) if (!visit[i] && i !=
        par[no]) {
        par[i] = no;
        dfs(i);
        sub[no] += sub[i];
    }
}

int get_centroid(int x) {
    par[x] = 0;
    dfs(x);
    int sz = sub[x];
    while (1) {
        pii mx = {0,0};
        for (int i: adj[x]) if (!visit[i] && i !=
            par[x]) mx = max(mx,{sub[i],i});
        if (mx.f*2 > sz) x = mx.s;
        else return x;
    }
}

void solve (int x) {
    x = get_centroid(x); visit[x] = 1;
    // do stuff
    cout << x << "\n";
    for (int i: adj[x]) if (!visit[i]) solve(i);
}

int main() {
        cin >> N;
        FOR(i,N-1) {
            int a,b; cin >> a >> b;
            adj[a].pb(b), adj[b].pb(a);
        }
        solve(1);
}
```

8.3.2 Heavy-Light Decomposition

```
/**
* Source: http://codeforces.com/blog/entry/22072
* Dependency: Lazy SegTree
* Verification: USACO Grass Planting
*/

vector<vi> graph;

template <int V> struct HeavyLight { // sum queries,
    sum updates
    int parent[V], heavy[V], depth[V];
    int root[V], treePos[V];
    LazySegTree<V> tree;

    void init() {
        int n = graph.size();
        FOR(i,1,n+1) heavy[i] = -1;
        parent[1] = -1, depth[1] = 0;
        dfs(1);
        for (int i = 1, currentPos = 0; i <= n; ++i)
            if (parent[i] == -1 || heavy[parent[i]]
                != i)
                for (int j = i; j != -1; j =
                    heavy[j]) {
                    root[j] = i;
                    treePos[j] = currentPos++;
                }
    }

    int dfs(int v) {
        int size = 1, maxSubtree = 0;
        for (auto u : graph[v]) if (u != parent[v]) {
            parent[u] = v;
            depth[u] = depth[v] + 1;
            int subtree = dfs(u);
            if (subtree > maxSubtree) heavy[v] = u,
                maxSubtree = subtree;
            size += subtree;
        }
        return size;
    }

    template <class BinaryOperation>
    void processPath(int u, int v, BinaryOperation op)
        {
        for (; root[u] != root[v]; v =
            parent[root[v]]) {
```

```
        if (depth[root[u]] > depth[root[v]])
            swap(u, v);
        op(treePos[root[v]], treePos[v]);
    }
    if (depth[u] > depth[v]) swap(u, v);
    op(treePos[u]+1, treePos[v]); // assumes
        values are stored in edges, not vertices
  }

  void modifyPath(int u, int v, int value) {
    processPath(u, v, [this, &value](int l, int r)
        { tree.upd(l, r, value); });
  }

  ll queryPath(int u, int v) {
    ll res = 0;
    processPath(u, v, [this, &res](int l, int r) {
        res += tree.qsum(l, r); });
    return res;
  }
};

HeavyLight<1<<17> H;
int N,M;

int main() {
    cin >> N >> M;
    graph.resize(N+1);
    FOR(i,N-1) {
        int a,b; cin >> a >> b;
        graph[a].pb(b), graph[b].pb(a);
    }
    H.init();
    FOR(i,M) {
        char c; int A,B;
        cin >> c >> A >> B;
        if (c == 'P') H.modifyPath(A,B,1);
        else cout << H.queryPath(A,B) << "\n";
    }
}
```

9 Math (4)

9.1 Number Theory

9.1.1 CRT (5)

```
/**
* Source: Own
* Verification:
    * Kattis generalchineseremainder
    * POI 9 Rhyme
*/

typedef pair<ll,ll> pll;

struct CRT {
    ll n,m,a,b;
    map<ll,pii> M;
    bool bad;

    ll inv(ll a, ll b) { // 0 < a < b, gcd(a,b) = 1
        a %= b;
        if (a <= 1) return a;
        ll i = inv(b%a,a);
        ll tmp = -((b/a)*i+((b%a)*i)/a) % b;
        while (tmp < 0) tmp += b;
        return tmp;
    }

    ll naive(ll n, ll m, ll a, ll b) {
        ll x = (a-b)*inv(m,n) % n;
        ll ans = (m*x+b) % (m*n);
        while (ans < 0) ans += (m*n);
        return ans;
    }

    void process(ll a, ll n) {
        vector<pii> z;
        for (int i = 2; i*i <= n; ++i) if (n % i == 0)
            {
            int co = 0;
            while (n % i == 0) n /= i, co++;
            z.pb({i,co});
        }
        if (n != 1) z.pb({n,1});
        for (auto A: z) {
            if (M.count(A.f)) {
                pii p1 = M[A.f];
                pii p2 = {A.s,a%(ll)pow(A.f,A.s)};
                if (p1 > p2) swap(p1,p2);
                if (p2.s%(ll)pow(A.f,p1.f) != p1.s) bad
                    = 1;
                M[A.f] = p2;
            } else M[A.f] = {A.s,a%(ll)pow(A.f,A.s)};
        }
    }

    ll po(ll b, ll p) {
        ll z = 1;
        FOR(i,p) z *= b;
        return z;
    }

    pll solve(ll aa, ll nn, ll bb, ll mm) {
        bad = 0, M.clear();
        a = aa, n = nn, b = bb, m = mm;
        process(a,n), process(b,m);
        if (bad) {
            cout << "NIE";
            exit(0);
        }
        ll a1 = 0, a2 = 1;
        for (auto& x: M) {
            a1 = naive(a2,po(x.f,x.s.f),a1,x.s.s);
            a2 *= po(x.f,x.s.f);
        }
        return {a1,a2};
    }
};
```

9.1.2 Eratosthenes' Sieve

```
/**
* Source: KACTL?
* https://open.kattis.com/problems/primesieve
*/

template<int SZ> struct Sieve {
    bitset<SZ+1> comp;
    Sieve() {
        for (int i = 2; i*i <= SZ; ++i) if (!comp[i]) {
            for (int j = i*i; j <= SZ; j += i) comp[j]
                = 1;
        }
    }
    bool isprime(int x) {
        if (x == 1) return 0;
        return !comp[x];
    }
};
```

9.1.3 Phi

```
/**
 * Observation: number of operations needed s.t.
 *              phi(phi(...phi(n)...))=1
 * is O(log n).
 * Euler's theorem: a^{\phi(p)}\equiv 1 (mod p),
    gcd(a,p)=1
 * Verification: CF Power Tower
 */

int phi(int x) {
    if (x == 1) return 1;
    int X = x;

    vi pri;
    for (int i = 2; i*i <= x; ++i) if (x % i == 0) {
        while (x % i == 0) x /= i;
        pri.pb(i);
    }

    if (x > 1) pri.pb(x);
    for (int i: pri) { X /= i; X *= i-1; }
    return X;
}
```

9.2 Combinatorics (5)

9.2.1 Combo Basic

```
/**
* Source: Own
* MOD is a large prime
*/

template<int SZ> struct Combo {
    ll fac[SZ+1], ifac[SZ+1];

    Combo() {
        fac[0] = ifac[0] = 1;
        FOR(i,1,SZ+1) {
            fac[i] = i*fac[i-1] % MOD;
            ifac[i] = inv(fac[i]);
        }
    }

    ll po (ll b, ll p) {
        return !p?1:po(b*b%MOD,p/2)*(p&1?b:1)%MOD;
    }

    ll inv (ll b) { return po(b,MOD-2); }

    ll comb(ll a, ll b) {
        if (a < b) return 0;
        ll tmp = fac[a]*ifac[b] % MOD;
        tmp = tmp*ifac[a-b] % MOD;
        return tmp;
    }
};
```

9.2.2 Combo Plus

```
/**
* Description: Extends combo to a power of a prime
* Verification: https://dmoj.ca/problem/tle17c4p5
*/

typedef pair<ll,ll> pll;

template<int SZ> struct ComboExtended {
    pll fac[SZ+1], ifac[SZ+1], mod;
    ll MOD = 1;

    void init(pll _mod) { // prime, power
        mod = _mod; FOR(i,mod.s) MOD *= mod.f;

        fac[0] = ifac[0] = {1,0};
        FOR(i,1,SZ+1) {
            fac[i] = fac[i-1];
            int I = i, z = 0;
            while (I % mod.f == 0) I /= mod.f, z++;
            fac[i].f = fac[i].f*I%MOD; fac[i].s += z;
            ifac[i] = {inv(fac[i].f,MOD),fac[i].s};
        }
    }

    ll inv(ll a, ll b) { // 0 < a < b, gcd(a,b) = 1
        a %= b;
        if (a <= 1) return a;
        ll i = inv(b%a,a);
        ll tmp = -((b/a)*i+((b%a)*i)/a) % b;
        while (tmp < 0) tmp += b;
        return tmp;
    }

    ll comb(ll a, ll b) {
        if (a < b) return 0;
        ll tmp = (fac[a].f*ifac[b].f%MOD)*ifac[a-b].f
            % MOD;
        ll z = fac[a].s-fac[b].s-fac[a-b].s;
        if (z >= mod.s) return 0;
        FOR(i,z) tmp = tmp*mod.f % MOD;
        return tmp;
    }
};
```

9.3 Matrices

9.3.1 Gaussian Elimination (6)

```
/**
* Description: Gaussian Elimination
* Usage:
    https://open.kattis.com/problems/equationsolverplus
*/

typedef long double ld;
typedef vector<vector<ld>> mat;

ld EPS = 1e-10;
int n;

void elim(mat& a, int i, int j, int k) {
    ld t = a[k][i];
    FOR(ind,n+1) a[k][ind] -= t*a[j][ind];
}

void prin(mat& a) {
    FOR(i,n) {
        FOR(j,n+1) cout << a[i][j] << " ";
        cout << "\n";
    }
    cout << "----\n";
}

void solve() {
    mat a(n); FOR(i,n) a[i].resize(n+1);
    FOR(i,n) FOR(j,n) cin >> a[i][j];
    FOR(i,n) cin >> a[i][n];
    int done[n]; FOR(i,n) done[i] = -1;

    FOR(i,n) {
        FOR(j,n) if (done[j] == -1 && abs(a[j][i]) >
            EPS) {
            ld t = a[j][i];
            FOR(k,n+1) a[j][k] /= t;

            FOR(k,n) if (j != k) elim(a,i,j,k);
            done[j] = i; break;
        }
    }

    int num = 0;
    FOR(i,n) if (done[i] == -1) {
        num ++;
        if (abs(a[i][n]) > EPS) {
            cout << "inconsistent\n";
            return;
        }
    }
    ld ans[n]; FOR(i,n) ans[i] =
        numeric_limits<double>::max();
    FOR(i,n) if (done[i] != -1) {
        bool bad = 0;
        FOR(j,n) if (j != done[i] && abs(a[i][j]) >
            EPS) {
            bad = 1;
            break;
        }
        if (!bad) ans[done[i]] = a[i][n];
    }
    FOR(i,n) {
        if (ans[i] != numeric_limits<double>::max())
            cout << ans[i];
        else cout << "?";
        cout << " ";
    }
    cout << "\n";
}
```

9.3.2 Matrix Exponentiation

```
/**
* Source: KACTL
* Verification: https://dmoj.ca/problem/si17c1p5
*/

template<int SZ> struct mat {
    array<array<ll,SZ>,SZ> d;

    mat() {
        FOR(i,SZ) FOR(j,SZ) d[i][j] = 0;
    }

    mat operator+(const mat& m) {
        mat<SZ> a;
        FOR(i,SZ) FOR(j,SZ) a.d[i][j] =
            (d[i][j]+m.d[i][j]) % MOD;
        return a;
    }

    mat operator*(const mat& m) {
        mat<SZ> a;
        FOR(i,SZ) FOR(j,SZ) FOR(k,SZ)
            a.d[i][k] = (a.d[i][k]+d[i][j]*m.d[j][k]) %
                MOD;
        return a;
    }

    mat operator^(ll p) {
        mat<SZ> a, b(*this);
        FOR(i,SZ) a.d[i][i] = 1;

        while (p) {
            if (p&1) a = a*b;
            b = b*b;
            p /= 2;
        }

        return a;
    }

    void print() {
        FOR(i,SZ) {
            FOR(j,SZ) cout << d[i][j] << " ";
            cout << "\n";
        }
        cout << "------------\n";
    }
};

/*
mat<2> x; x.d[0][0] = 1, x.d[1][0] = 2, x.d[1][1] = 1,
    x.d[0][1] = 3;
mat<2> y = x*x;
mat<2> z = x^5;
x.print(), y.print(), z.print();
*/
```

9.4 FFT

9.4.1 And Convolution

```
/**
* Description: Similar to FWHT
* Source: CSA - FFT And Variations
*/

typedef vector<double> vd;
typedef vector<ll> vl;

int get(int s) {
    return s > 1 ? 32 - __builtin_clz(s - 1) : 0;
}

namespace andConv {
    vd andConv(vd P, bool inv = 0) {
        for (int len = 1; 2 * len <= sz(P); len <<= 1)
            {
            for (int i = 0; i < sz(P); i += 2 * len) {
                for (int j = 0; j < len; j++) {
                    double u = P[i + j];
                    double v = P[i + len + j];

                    if (!inv) {
                        P[i + j] = v;
                        P[i + len + j] = u + v;
                    } else {
                        P[i + j] = -u + v;
                        P[i + len + j] = u;
                    }
                }
            }
        }

        return P;
    }

    vd conv(vd a, vd b) {
        int s = max(sz(a),sz(b)), L = get(s), n = 1<<L;
        if (s <= 0) return {};

        a.resize(n); a = andConv(a);
        b.resize(n); b = andConv(b);

        FOR(i,n) a[i] = a[i]*b[i];
        a = andConv(a,1);
        return a;
    }

    vd orConv(vd a, vd b) {
        int s = max(sz(a),sz(b)), L = get(s), n = 1<<L;
        if (s <= 0) return {};

        a.resize(n); reverse(all(a)); a = andConv(a);
        b.resize(n); reverse(all(b)); b = andConv(b);

        FOR(i,n) a[i] = a[i]*b[i];
        a = andConv(a,1);
        reverse(all(a));

        return a;
    }

    vl orConv(vl a, vl b) {
        vd A; for (ll x: a) A.pb(x);
        vd B; for (ll x: b) B.pb(x);
        vd c = orConv(A,B);
        vl C; for (double x: c) C.pb(round(x));
        return C;
    }

    vl conv(vl a, vl b) {
        vd A; for (ll x: a) A.pb(x);
        vd B; for (ll x: b) B.pb(x);
        vd c = conv(A,B);
        vl C; for (double x: c) C.pb(round(x));
        return C;
    }
}
```

9.4.2 Base Conversion

```
/**
* Description: NTT Application
* Usage: 2017 VT HSPC - Alien Codebreaking
*/

// NTT template

struct Base {
    vl po10[21];
    const int base = 27;

    Base() {
        po10[0] = {10};
        FOR(i,1,21) {
            po10[i] = NTT::conv(po10[i-1],po10[i-1]);
            normalize(po10[i]);
        }
    }

    void normalize(vl& x) {
        FOR(i,sz(x)) if (x[i] >= base) {
            if (i == sz(x)-1) x.pb(0);
            x[i+1] += x[i]/base;
            x[i] %= base;
        }
        while (sz(x) && !x.back()) x.pop_back();
    }

    vl convert(vl in) {
        if (sz(in) == 1) return in;
        vl l =
             convert(vl(in.begin(),in.begin()+sz(in)/2));
        vl r =
             convert(vl(in.begin()+sz(in)/2,in.end()));

        r = NTT::conv(r,po10[get(sz(in))-1]);
        normalize(r);

        int z = max(sz(l),sz(r));
        r.resize(z);
        FOR(i,sz(l)) r[i] += l[i];
        normalize(r);
        return r;
    }
};

Base B;

int main() {
        FOR(i,10) FOR(j,10) FOR(k,10) {
            vl z = {k,j,i};
            vl o = B.transform(z);
            for (ll x: o) cout << x << " ";
            cout << "\n";
        }
}
```

9.4.3 FFT

```
/**
* Sources: KACTL, https://pastebin.com/3Tnj5mRu
* Verification: SPOJ polymul
*/

typedef complex<double> cd;
typedef vector<cd> vcd;
typedef vector<ll> vl;

int get(int s) {
    return s > 1 ? 32 - __builtin_clz(s - 1) : 0;
}

namespace FFT {
    vcd fft(vcd& a) {
        int n = a.size(), x = get(n);
        vcd res, RES(n), roots(n);
        FOR(i,n) roots[i] =
             cd(cos(2*M_PI*i/n),sin(2*M_PI*i/n));

        res = a;
        FOR(i,1,x+1) {
            int inc = n>>i;
            FOR(j,inc) for (int k = 0; k < n; k += inc)
                {
                int t = 2*k%n+j;
                RES[k+j] = res[t]+roots[k]*res[t+inc];
            }
            swap(res,RES);
        }

        return res;
    }

    vcd fft_rev(vcd& a) {
        vcd res = fft(a);
        FOR(i,sz(res)) res[i] /= a.size();
        reverse(res.begin() + 1, res.end());
        return res;
    }

    vcd brute(vcd& a, vcd& b) {
        vcd c(sz(a)+sz(b)-1);
        FOR(i,sz(a)) FOR(j,sz(b)) c[i+j] += a[i]*b[j];
        return c;
    }

    vcd conv(vcd a, vcd b) {
        int s = sz(a)+sz(b)-1, L = get(s), n = 1<<L;
        if (s <= 0) return {};
        if (s <= 200) return brute(a,b);

        a.resize(n); a = fft(a);
        b.resize(n); b = fft(b);

        FOR(i,n) a[i] *= b[i];
        a = fft_rev(a);

        a.resize(s);
        return a;
    }

    vl convll(vl a, vl b) {
        vcd A(sz(a)); FOR(i,sz(a)) A[i] = a[i];
        vcd B(sz(b)); FOR(i,sz(b)) B[i] = b[i];
        vcd X = conv(A,B);
        vl x(sz(X)); FOR(i,sz(X)) x[i] =
             round(X[i].real());
        return x;
    }
}

int main() {
    int T; cin >> T;
    FOR(i,T) {
        int N; cin >> N;
        vl a(N+1), b(N+1);
        FOR(j,N+1) cin >> a[N-j];
        FOR(j,N+1) cin >> b[N-j];
        vl x = FFT::convll(a,b);
        FORd(j,sz(x)) cout << x[j] << " ";
        cout << "\n";
    }
}
```

9.4.4 NTT

```
/**
* Description: Use if you are working with
    non-negative integers
* Verification:
    http://codeforces.com/contest/632/submission/33953285
*/

typedef vector<ll> vl;

int get(int s) {
    return s > 1 ? 32 - __builtin_clz(s - 1) : 0;
}

namespace NTT {
    const ll mod = (119 << 23) + 1, root = 3; // =
        998244353
    // For p < 2^30 there is also e.g. (5 << 25, 3),
        (7 << 26, 3),
    // (479 << 21, 3) and (483 << 21, 5). The last two
        are > 10^9.

    ll modpow(ll b, ll p) { return
        !p?1:modpow(b*b%mod,p/2)*(p&1?b:1)%mod; }

    ll inv (ll b) { return modpow(b,mod-2); }

    vl ntt(vl& a) {
        int n = a.size(), x = get(n);
        vl res, RES(n), roots(n);
        roots[0] = 1, roots[1] =
            modpow(root,(mod-1)/n);
        FOR(i,2,n) roots[i] = roots[i-1]*roots[1] %
            mod;

        res = a;
        FOR(i,1,x+1) {
            int inc = n>>i;
            FOR(j,inc) for (int k = 0; k < n; k += inc)
                {
                int t = 2*k%n+j;
                RES[k+j] = (res[t]+roots[k]*res[t+inc])
                    % mod;
            }
            swap(res,RES);
        }

        return res;
    }

    vl ntt_rev(vl& a) {
        vl res = ntt(a);
        ll in = inv(a.size());
        FOR(i,sz(res)) res[i] = res[i]*in % mod;
        reverse(res.begin() + 1, res.end());
        return res;
    }

    vl brute(vl& a, vl& b) {
        vl c(sz(a)+sz(b)-1);
        FOR(i,sz(a)) FOR(j,sz(b)) c[i+j] =
            (c[i+j]+a[i]*b[j])%mod;
        return c;
    }

    vl conv(vl a, vl b) {
        int s = sz(a)+sz(b)-1, L = get(s), n = 1<<L;
        if (s <= 0) return {};
        if (s <= 200) return brute(a,b);

        a.resize(n); a = ntt(a);
        b.resize(n); b = ntt(b);

        FOR(i,n) a[i] = a[i]*b[i] % mod;
        a = ntt_rev(a);

        a.resize(s);
        return a;
    }
}

int main() {
    vl X = NTT::conv({1,2,3,4,5,6,7,8},
        {1,2,3,4,5,6,7,8});
    for (auto a: X) cout << a << "\n";
}
```

9.4.5 XOR Convolution

```
/**
* Description: FWHT, similar to FFT
* Source: CSA - FFT And Variations
* Verification:
    https://www.hackerrank.com/challenges/xor-subsequence/pro
*/

typedef vector<double> vd;
typedef vector<ll> vl;

int get(int s) {
    return s > 1 ? 32 - __builtin_clz(s - 1) : 0;
}

namespace FWHT {
    vd fwht(vd P) {
        for (int len = 1; 2 * len <= sz(P); len <<= 1)
            {
            for (int i = 0; i < sz(P); i += 2 * len) {
                for (int j = 0; j < len; j++) {
                    double u = P[i + j];
                double v = P[i + len + j];
                P[i + j] = u+v;
                P[i + len + j] = u-v;
            }
        }
    }

    return P;
}


vd fwht_rev(vd& a) {
    vd res = fwht(a);
    FOR(i,sz(res)) res[i] /= a.size();
    return res;
}

vd conv(vd a, vd b) {
    int s = max(sz(a),sz(b)), L = get(s), n = 1<<L;
    if (s <= 0) return {};

    a.resize(n); a = fwht(a);
    b.resize(n); b = fwht(b);

    FOR(i,n) a[i] = a[i]*b[i];
    a = fwht_rev(a);
    return a;
}

vl conv(vl a, vl b) {
    vd A; for (ll x: a) A.pb(x);
    vd B; for (ll x: b) B.pb(x);
    vd c = conv(A,B);
    vl C; for (double x: c) C.pb(round(x));
    return C;
}
}
```

10 Graphs Hard (4)

10.1 Kosaraju

```
/**
* Source: Wikipedia
* Description: generates SCC in topological order,
    support for 2-SAT
* Verification: POI 8 peaceful commission
*/

int rev(int x) {
    return x&1?x+1:x-1;
}

template<int SZ> struct scc {
    vi adj[SZ], radj[SZ], todo, allComp;
    int N, comp[SZ];
    bitset<SZ> visit;

    void dfs(int v) {
        visit[v] = 1;
        for (int w: adj[v]) if (!visit[w]) dfs(w);
        todo.pb(v);
    }

    void dfs2(int v, int val) {
        comp[v] = val;
        for (int w: radj[v]) if (!comp[w]) dfs2(w,val);
    }

    void addEdge(int a, int b) {
            adj[a].pb(b), radj[b].pb(a);
    }

    void genSCC() {
        FOR(i,1,N+1) comp[i] = visit[i] = 0;
        FOR(i,1,N+1) if (!visit[i]) dfs(i);
        reverse(all(todo)); // toposort
        for (int i: todo) if (!comp[i]) {
            dfs2(i,i);
            allComp.pb(i);
        }
    }

    int tmp[SZ];
    bitset<SZ> ans;

    bool twosat() {
        for (int i = 1; i <= N; i += 2) if (comp[i] ==
            comp[rev(i)]) return 0;
        reverse(all(allComp));
        for (int i: allComp) if (tmp[i] == 0) {
            tmp[i] = 1;
            tmp[comp[rev(i)]] = -1;
        }
            FOR(i,1,N+1) if (tmp[comp[i]] == 1) ans[i]
                = 1;
        return 1;
    }
};
```

10.2 Euler Tour (6)

```
/**
* Description: extra log factor
* Usage: https://open.kattis.com/problems/eulerianpath
*/

vi circuit;
multiset<int> adj[10000], adj1[10000];
int N,M, out[10000], in[10000];

void find_circuit(int x) { // directed graph, possible
    that resulting circuit is not valid
    while (adj[x].size()) {
        int j = *adj[x].begin();
            adj[x].erase(adj[x].begin());
        find_circuit(j);
    }
    circuit.pb(x);
}

int a,b,start;

void solve() {
    FOR(i,N) {
        adj[i].clear(), adj1[i].clear();
        out[i] = in[i] = 0;
    }
    circuit.clear();
    FOR(i,M) {
        cin >> a >> b;
        adj[a].insert(b), adj1[a].insert(b);
        out[a] ++, in[b] ++;
    }
    start = a;
    FOR(i,N) if (out[i]-in[i] == 1) start = i;

    find_circuit(start);
    reverse(circuit.begin(),circuit.end());

    if (circuit.size() != M+1) {
        cout << "Impossible\n";
        return;
    }

    FOR(i,M) {
        if (adj1[circuit[i]].find(circuit[i+1]) ==
             adj1[circuit[i]].end()) {
            cout << "Impossible\n";
            return;
        }
        int t = circuit[i];
        adj1[t].erase(adj1[t].find(circuit[i+1]));
    }
    FOR(i,M+1) cout << circuit[i] << " ";
    cout << "\n";
}
```

10.3 Flows

10.3.1 Dinic (5)

```
/**
* Source: GeeksForGeeks
* Verification: Problem Fashion (RMI 2017 Day 1)
* Code: https://pastebin.com/VJxTvEg1
*/

struct Edge {
    int v;
    ll flow, C;
    int rev;
};

template<int SZ> struct Dinic {
    int level[SZ], start[SZ];
    vector<Edge> adj[SZ];

    void addEdge(int u, int v, int C) {
        Edge a{v, 0, C, sz(adj[v])};
        Edge b{u, 0, 0, sz(adj[u])};
        adj[u].pb(a), adj[v].pb(b);
    }

    bool BFS(int s, int t) {
        FOR(i,SZ) level[i] = -1;
        level[s] = 0;

        queue<int> q; q.push(s);
        while (!q.empty()) {
            int u = q.front(); q.pop();
            for (auto e: adj[u])
                if (level[e.v] < 0 && e.flow < e.C) {
                    level[e.v] = level[u] + 1;
                    q.push(e.v);
                }
        }

        return level[t] >= 0;
    }

    ll sendFlow(int u, ll flow, int t) {
        if (u == t) return flow;

        for ( ; start[u] < sz(adj[u]); start[u] ++) {
            Edge &e = adj[u][start[u]];

            if (level[e.v] == level[u]+1 && e.flow <
                 e.C) {
                ll curr_flow = min(flow, e.C - e.flow);
                ll temp_flow = sendFlow(e.v, curr_flow,
                     t);

                if (temp_flow > 0) {
                    e.flow += temp_flow;
                    adj[e.v][e.rev].flow -= temp_flow;
                    return temp_flow;
                }
            }
        }

        return 0;
    }

    ll maxFlow(int s, int t) {
        if (s == t) return -1;
        ll total = 0;

        while (BFS(s, t)) {
            FOR(i,SZ) start[i] = 0;
            while (ll flow = sendFlow(s, INT_MAX, t))
                 total += flow;
        }

        return total;
    }
};
```

10.3.2 Flows Demo

```
/**
* Link: http://www.spoj.com/problems/FASTFLOW/
* Use with Dinic, Push-Relabel
*/

int N,M;
PushRelabel<5001> D;

int main() {
    cin >> N >> M;
    FOR(i,M) {
        int a,b,c; cin >> a >> b >> c;
        D.addEdge(a,b,c);
        D.addEdge(b,a,c);
    }
    cout << D.maxFlow(1,N);
}
```

10.3.3 MinCostFlow (6)

```
/**
* Source: GeeksForGeeks
*/

struct Edge {
    int v, flow, C, rev, cost;
};

template<int SZ> struct mcf {
    pii pre[SZ];
    int cost[SZ], num[SZ], SC, SNC;
    ll flo, ans, ccost;
    vector<Edge> adj[SZ];

    void addEdge(int u, int v, int C, int cost) {
        Edge a{v, 0, C, sz(adj[v]), cost};
        Edge b{u, 0, 0, sz(adj[u]), -cost};
        adj[u].pb(a), adj[v].pb(b);
    }

    void reweight() {
        FOR(i,SZ) {
            for (auto& p: adj[i]) p.cost +=
                cost[i]-cost[p.v];
        }
    }

    bool spfa() {
        FOR(i,SZ) cost[i] = MOD, num[i] = 0;
        cost[SC] = 0, num[SC] = MOD;
        priority_queue<pii,vector<pii>,greater<pii>>
             todo; todo.push({0,SC});

        while (todo.size()) {
            pii x = todo.top(); todo.pop();
            if (x.f > cost[x.s]) continue;
            for (auto a: adj[x.s]) if (x.f+a.cost <
                 cost[a.v] && a.flow < a.C) {
                pre[a.v] = {x.s,a.rev};
                cost[a.v] = x.f+a.cost;
                num[a.v] = min(a.C-a.flow,num[x.s]);
                todo.push({cost[a.v],a.v});
            }
        }

        ccost += cost[SNC];
        return num[SNC] > 0;
    }

    void backtrack() {
        flo += num[SNC], ans += (ll)num[SNC]*ccost;
        for (int x = SNC; x != SC; x = pre[x].f) {
            adj[x][pre[x].s].flow -= num[SNC];
            int t = adj[x][pre[x].s].rev;
            adj[pre[x].f][t].flow += num[SNC];
        }
    }

    pii mincostflow(int sc, int snc) {
        SC = sc, SNC = snc;
        flo = ans = ccost = 0;

        spfa();
        while (1) {
            reweight();
            if (!spfa()) return {flo,ans};
            backtrack();
        }
    }
};

mcf<100> m;

int main() {
    m.addEdge(0, 1, 16, 5);
    m.addEdge(1, 2, 13, 7);
    m.addEdge(1, 2, 13, 8);

    pii x = m.mincostflow(0,2);
    cout << x.f << " " << x.s;
}
```

10.3.4 Push-Relabel (5)

```
/**
 * Source: http://codeforces.com/blog/entry/14378
 * Verification: SPOJ fastflow
 */

struct Edge {
    int v;
    ll flow, C;
    int rev;
};

template <int SZ> struct PushRelabel {
    vector<Edge> adj[SZ];
    ll excess[SZ];
    int dist[SZ], count[SZ+1], b = 0;
    bool active[SZ];
    vi B[SZ];

    void addEdge(int u, int v, int C) {
        Edge a{v, 0, C, sz(adj[v])};
        Edge b{u, 0, 0, sz(adj[u])};
        adj[u].pb(a), adj[v].pb(b);
    }

    void enqueue (int v) {
        if (!active[v] && excess[v] > 0 && dist[v] <
             SZ) {
            active[v] = 1;
            B[dist[v]].pb(v);
            b = max(b, dist[v]);
        }
    }

    void push (int v, Edge &e) {
        ll amt = min(excess[v], e.C-e.flow);
        if (dist[v] == dist[e.v]+1 && amt > 0) {
            e.flow += amt, adj[e.v][e.rev].flow -= amt;
            excess[e.v] += amt, excess[v] -= amt;
            enqueue(e.v);
        }
    }

    void gap (int k) {
        FOR(v,SZ) if (dist[v] >= k) {
            count[dist[v]] --;
            dist[v] = SZ;
            count[dist[v]] ++;
            enqueue(v);
        }
    }

    void relabel (int v) {
        count[dist[v]] --; dist[v] = SZ;
        for (auto e: adj[v]) if (e.C > e.flow) dist[v]
             = min(dist[v], dist[e.v] + 1);
        count[dist[v]] ++;
        enqueue(v);
    }

    void discharge(int v) {
        for (auto &e: adj[v]) {
            if (excess[v] > 0) push(v,e);
            else break;
        }
        if (excess[v] > 0) {
            if (count[dist[v]] == 1) gap(dist[v]);
            else relabel(v);
        }
    }

    ll maxFlow (int s, int t) {
        for (auto &e: adj[s]) excess[s] += e.C;

        count[0] = SZ;
        enqueue(s); active[t] = 1;

        while (b >= 0) {
            if (sz(B[b])) {
                int v = B[b].back(); B[b].pop_back();
                active[v] = 0; discharge(v);
            } else b--;
        }
        return excess[t];
    }
};
```

10.4 Tarjan BCC

```
/**
* Source: GeeksForGeeks (corrected)
* Verification: USACO December 2017, Push a Box
* Code: https://pastebin.com/yUWuzTH8
*/

template<int SZ> struct BCC {
    int N, ti = 0;
    vi adj[SZ];
    int disc[SZ], low[SZ], comp[SZ], par[SZ];
    vector<vector<pii>> fin;
    vector<pii> st;

    void addEdge(int u, int v) {
        adj[u].pb(v), adj[v].pb(u);
    }

    void BCCutil(int u) {
        disc[u] = low[u] = ti++;
        int child = 0;

        for (int i: adj[u]) if (i != par[u]) {
            if (disc[i] == -1) {
                child ++; par[i] = u;
                st.pb({u,i});
                BCCutil(i);
                low[u] = min(low[u],low[i]);

                if ((disc[u] == 0 && child > 1) ||
                    (disc[u] != 0 && disc[u] <=
                    low[i])) { // articulation point!
                   vector<pii> tmp;
                   while (st.back() != mp(u,i))
                       tmp.pb(st.back()),
                       st.pop_back();
                   tmp.pb(st.back()), st.pop_back();
                   fin.pb(tmp);
                }
            } else if (disc[i] < disc[u]) {
                low[u] = min(low[u],disc[i]);
                st.pb({u,i});
            }
        }
    }

    void bcc() {
        FOR(i,1,N+1) par[i] = disc[i] = low[i] = -1;
        FOR(i,1,N+1) if (disc[i] == -1) {
            BCCutil(i);
            if (sz(st)) fin.pb(st);
            st.clear();
        }
    }
};
```

11 Geometry (4)

11.1 Techniques

11.1.1

```
/**
* Description: Basic 3D Geometry
* Usage: AMPPZ 2011 Cross Spider
*/

typedef vector<ll> vl;

typedef long double ld;

int n;
vector<vl> cur;

vl operator-(vl a, vl b) {
    vl c(sz(a)); FOR(i,sz(a)) c[i] = a[i]-b[i];
    return c;
}

bool ismult(vl b, vl c) {
    if ((ld)b[0]*c[1] != (ld)b[1]*c[0]) return 0;
    if ((ld)b[0]*c[2] != (ld)b[2]*c[0]) return 0;
    if ((ld)b[2]*c[1] != (ld)b[1]*c[2]) return 0;
    return 1;
}

bool collinear(vl a, vl b, vl c) {
    b = b-a, c = c-a;
    return ismult(b,c);
}

vl cross(vl a, vl b) {
    return {a[1]*b[2]-a[2]*b[1],
            a[2]*b[0]-a[0]*b[2],
            a[0]*b[1]-a[1]*b[0]};
}

bool coplanar(vl a, vl b, vl c, vl d) {
    b = b-a, c = c-a, d = d-a;
    return ismult(cross(b,c),cross(b,d));
}
```

11.1.2 Circles

```
/**
* Source: Own
* Usage:
    https://codefights.com/tournaments/s8thqrnQL2YPK7XQt/L
*/

typedef complex<double> cd;
typedef pair<cd,double> circle;

cd intersect(circle a, circle b, int x = 0) {
    double d = sqrt(norm(a.f-b.f));
    double co = (a.s*a.s+d*d-b.s*b.s)/(2*a.s*d);
    double theta = acos(co);

    cd tmp = (b.f-a.f)/d;
    if (x == 0) return a.f+tmp*a.s*polar(1.0,theta);
    return a.f+tmp*a.s*polar(1.0,-theta);
}

double arc(circle x, cd a, cd b) {
    cd d = (a-x.f)/(b-x.f);
    return x.s*acos(d.real());
}

bool on (circle x, cd y) {
    return norm(y-x.f) == x.s*x.s;
}

int main() {
    cout << intersect({0,2},{1,1}) << "\n";
    cout << arc({0,1},cd(1,0),cd(0,1)) << "\n";
    cout << on({0,1},1) << "\n";
}
```

11.1.3 Line Segment Intersection (5)

```
/**
* Verification: Kattis segmentintersection
* If numbers are small enough, fractions are
    recommended.
*/

typedef pair<double,double> pdd;

pii A,B,C,D;

pdd operator*(int x, pdd y) {
    return {x*y.f,x*y.s};
}

pdd operator/(pdd y, int x) {
    return {y.f/x,y.s/x};
}

pdd operator+(pdd l, pdd r) {
    return {l.f+r.f,l.s+r.s};
}

int sgn(pii a, pii b, pii c) {
    return (b.s-a.s)*(c.f-a.f)-(b.f-a.f)*(c.s-a.s);
}

pdd get(pii a, pii b, pii c, pii d) {
    return (abs(sgn(a,b,c))*d+abs(sgn(a,b,d))*c)
     /(abs(sgn(a,b,c))+abs(sgn(a,b,d)));
}

void solve() {
    cin >> A.f >> A.s >> B.f >> B.s >> C.f >> C.s >>
        D.f >> D.s;
    if (A > B) swap(A,B);
    if (C > D) swap(C,D);
    int a1 = sgn(A,B,C), a2 = sgn(A,B,D);
    if (a1 > a2) swap(a1,a2);
    if (!(a1 <= 0 && a2 >= 0)) {
        cout << "none\n";
        return;
    }
    if (a1 == 0 && a2 == 0) {
        if (sgn(A,C,D) != 0) {
            cout << "none\n";
            return;
        }
        pii x1 = max(A,C), x2 = min(B,D);
        if (x1 > x2) cout << "none\n";
        else if (x1 == x2) cout << (double)x1.f << " "
            << (double)x1.s << "\n";
        else cout << (double)x1.f << " " <<
            (double)x1.s << " " << (double)x2.f << " "
            << (double)x2.s << "\n";
        return;
    }
    pdd z = get(A,B,C,D);
    if (mp((double)A.f,(double)A.s) <= z && z <=
        mp((double)B.f,(double)B.s)) cout << z.f << "
        " << z.s << "\n";
    else cout << "none\n";
}

int main() {
        int n; cin >> n;
        cout << fixed << setprecision(2);
        FOR(i,n) solve();
}
```

11.1.4 Pair Operators

```
/**
* Source: own
*/

template<class T> pair<T,T> operator+(const pair<T,T>&
    l, const pair<T,T>& r) {
    return {l.f+r.f,l.s+r.s};
}

template<class T> pair<T,T> operator-(const pair<T,T>&
    l, const pair<T,T>& r) {
    return {l.f-r.f,l.s-r.s};
}

template<class T> pair<T,T> operator*(const pair<T,T>&
    l, T r) {
    return {l.f*r,l.s*r};
}

template<class T> pair<T,T> operator/(const pair<T,T>&
    l, T r) {
    return {l.f/r,l.s/r};
}

template<class T> double mag(pair<T,T> p) {
    return sqrt(p.f*p.f+p.s*p.s);
}

template<class T> pair<T,T> operator*(const pair<T,T>&
    l, const pair<T,T>& r) {
    // l.f+l.s*i, r.f+r.s*i
    return {l.f*r.f-l.s*r.s,l.s*r.f+l.f*r.s};
}

template<class T> pair<T,T> operator/(const pair<T,T>&
    l, const pair<T,T>& r) {
    // l.f+l.s*i, r.f+r.s*i
    pair<T,T> z =
        {r.f/(r.f*r.f+r.s*r.s),-r.s/(r.f*r.f+r.s*r.s)};
    return l*z;
}

template<class T> double area(pair<T,T> a, pair<T,T>
    b, pair<T,T> c) {
    b = b-a, c = c-a;
    return (b.f*c.s-b.s*c.f)/2;
}

template<class T> double dist(pair<T,T> l, pair<T,T>
    r) {
    return mag(r-l);
}

template<class T> double dist(pair<T,T> o, pair<T,T>
    x, pair<T,T> d) { // signed distance
    return 2*area(o,x,x+d)/mag(d);
}
```

11.1.5 Point in Polygon (5)

```
/**
* Source: own
* Usage:
    https://open.kattis.com/problems/pointinpolygon
*/

int n,m;
pii p[1000];

int area(pii x, pii y, pii z) {
    return (y.f-x.f)*(z.s-x.s)-(y.s-x.s)*(z.f-x.f);
}
```

```
bool on(pii x, pii y, pii z) {
    if (area(x,y,z) != 0) return 0;
    return min(x,y) <= z && z <= max(x,y);
}

double get(pii x, pii y, int z) {
    return double((z-x.s)*y.f+(y.s-z)*x.f)/(y.s-x.s);
}

void test(pii z) {
    int ans = 0;
    FOR(i,n) {
        pii x = p[i], y = p[(i+1)%n];
        if (on(x,y,z)) {
            cout << "on\n";
            return;
        }
        if (x.s > y.s) swap(x,y);
        if (x.s <= z.s && y.s > z.s) {
            double t = get(x,y,z.s);
            if (t > z.f) ans++;
        }
    }
    if (ans % 2 == 1) cout << "in\n";
    else cout << "out\n";
}

void solve() {
    FOR(i,n) cin >> p[i].f >> p[i].s;
    cin >> m;
    FOR(i,m) {
        pii z; cin >> z.f >> z.s;
        test(z);
    }
}
```

11.1.6 Polygon Area

```
/**
* Description: Shoelace Formula
* Usage: https://open.kattis.com/problems/polygonarea
*/

double area(vector<pii> v) {
    double x = 0;
    FOR(i,sz(v)) {
        int j = (i+1)%sz(v);
        x += (ll)v[i].f*v[j].s;
        x -= (ll)v[j].f*v[i].s;
    }
    return abs(x)/2;
}
```

11.2 Sweep Line

11.2.1 Closest Pair (6)

```
/**
* Source: GeeksForGeeks
* Description: Nlog^2N, can be improved
* Use: https://open.kattis.com/problems/closestpair2
*/

pair<double,pair<pdd,pdd>> MN = {INF,{{0,0},{0,0}}};

int n;

bool cmp(pdd a, pdd b) {
    return a.s < b.s;
}

double dist(pdd a, pdd b) {
    b.f -= a.f, b.s -= a.s;
    return sqrt(b.f*b.f+b.s*b.s);
}

pair<double,pair<pdd,pdd>> strip(vector<pdd> v, double
     di) {
    pair<double,pair<pdd,pdd>> ans = MN;
    FOR(i,v.size()) FOR(j,i+1,v.size()) {
        if (v[i].s+di <= v[j].s) break;
        ans = min(ans,{dist(v[i],v[j]),{v[i],v[j]}});
    }
    return ans;
}

pair<double,pair<pdd,pdd>> bes (vector<pdd> v) {
    if (v.size() == 1) return MN;
    int M = v.size()/2;
    vector<pdd> v1(v.begin(),v.begin()+M),
        v2(v.begin()+M,v.end());
    auto a = bes(v1), b = bes(v2);
    double di = min(a.f,b.f);

    vector<pdd> V;
    FOR(i,v.size()) if (v[i].f > v[M].f-di && v[i].f <
        v[M].f+di) V.pb(v[i]);
    sort(V.begin(),V.end(),cmp);

    auto z = strip(V,di);
    return min(min(a,b),z);
}

int main() {
        cout << fixed << setprecision(2);
        while (cin >> n) {
            if (n == 0) break;
            vector<pdd> v(n);
            FOR(i,n) cin >> v[i].f >> v[i].s;
            sort(v.begin(),v.end());
            auto a = bes(v);
            cout << a.s.f.f << " " << a.s.f.s << " " <<
                a.s.s.f << " " << a.s.s.s << "\n";
        }
}
```

11.2.2 Convex Hull

```
/**
* Source: Wikibooks
* Usage: https://open.kattis.com/problems/convexhull
*/

ll cross(pii O, pii A, pii B) {
    return (ll)(A.f-O.f)*(B.s-O.s)
           -(ll)(A.s-O.s)*(B.f-O.f);
}

vector<pii> convex_hull(vector<pii> P) {
    sort(P.begin(),P.end());
        P.erase(unique(P.begin(),P.end()),P.end());
    if (P.size() == 1) return P;

    int n = P.size();

    vector<pii> bot = {P[0]};
    FOR(i,1,n) {
        while (bot.size() > 1 &&
            cross(bot[bot.size()-2], bot.back(), P[i])
            <= 0) bot.pop_back();
        bot.pb(P[i]);
    }
    bot.pop_back();

    vector<pii> up = {P[n-1]};
    FORd(i,n-1) {
        while (up.size() > 1 && cross(up[up.size()-2],
            up.back(), P[i]) <= 0) up.pop_back();
        up.pb(P[i]);
    }
    up.pop_back();

    bot.insert(bot.end(),all(up));
    return bot;
}

int main() {
    int n;
    while (cin >> n) {
        if (n == 0) break;
        vector<pii> P(n); FOR(i,n) cin >> P[i].f >>
            P[i].s;
        vector<pii> hull = convex_hull(P);

        cout << hull.size() << "\n";
        for (auto a: hull) cout << a.f << " " << a.s
            << "\n";
    }
}
```

11.2.3 LineContainer (6)

```
/**
* Source: KACTL
* Verification: CSA Squared Ends
*/

bool Q;
struct Line {
        mutable ll k, m, p; // slope, y-intercept,
            last optimal x
        bool operator<(const Line& o) const {
                return Q ? p < o.p : k < o.k;
        }
};

struct LineContainer : multiset<Line> {
        const ll inf = LLONG_MAX;
        ll div(ll a, ll b) { // floored division
            if (b < 0) a *= -1, b *= -1;
            if (a >= 0) return a/b;
            return -((-a+b-1)/b);
        }

        // updates x->p, determines if y is unneeded
        bool isect(iterator x, iterator y) {
                if (y == end()) { x->p = inf; return 0;
                    }
                if (x->k == y->k) x->p = x->m > y->m ?
                    inf : -inf;
                else x->p = div(y->m - x->m, x->k -
                    y->k);
                return x->p >= y->p;
        }

        void add(ll k, ll m) {
                auto z = insert({k, m, 0}), y = z++, x
                    = y;
                while (isect(y, z)) z = erase(z);
                if (x != begin() && isect(--x, y))
                    isect(x, y = erase(y));
                while ((y = x) != begin() && (--x)->p
                    >= y->p) isect(x, erase(y));
        }

        ll query(ll x) { // gives max value
                assert(!empty());
                Q = 1; auto l = *lb({0,0,x}); Q = 0;
                return l.k * x + l.m;
        }
};
```

11.3 Max Collinear

```
/**
* Source: own
* Usage: https://open.kattis.com/problems/maxcolinear
*/

int n, mx, ans;
map<pair<pii,int>,int> m;
pii p[1000];

pair<pii,int> getline(pii a, pii b) {
    pii z = {b.f-a.f,b.s-a.s};
    swap(z.f,z.s); z.f *= -1;
    int g = __gcd(z.f,z.s); z.f /= g, z.s /= g;
    if (z.f < 0 || (z.f == 0 && z.s < 0)) z.f *= -1,
         z.s *= -1;
    return {z,z.f*a.f+z.s*a.s};
}

void solve() {
    mx = ans = 0; m.clear();
    FOR(i,n) cin >> p[i].f >> p[i].s;
    FOR(i,n) FOR(j,i+1,n) m[getline(p[i],p[j])] ++;

    for (auto a: m) mx = max(mx,a.s);
    FOR(i,1,n+1) if (i*(i-1)/2 <= mx) ans = i;
    cout << ans << "\n";
}
```

12 Strings (3)

12.1 Bitset Trie (4)

```
/**
* Source: Algorithms Gym
* Verification: January Easy 2018 - Shubham and
    Subarray Xor
*/

template<int MX> struct tri {
    int nex = 0, ans = 0;
    int trie[MX][2]; // easily changed to character

    tri() {
        memset(trie,0,sizeof trie);
    }

    void ins(int x) {
        int cur = 0;
        FORd(i,30) {
            int t = (x&(1<<i))>>i;
            if (!trie[cur][t]) trie[cur][t] = ++nex;
            cur = trie[cur][t];
        }
    }

    void test(int x) {
        int cur = 0;
        FORd(i,30) {
            int t = ((x&(1<<i))>>i) ^ 1;
            if (!trie[cur][t]) t ^= 1;
            cur = trie[cur][t];
            if (t) x ^= (1<<i);
        }
        ans = max(ans,x);
    }
};
```

12.2 Hashing

```
/**
* Source: own
* Description: Pairs reduce frequency of collision
* Verification: Dec 17 Plat 1
*/

typedef pair<ll, ll> pll;

template<class T> pair<T,T> operator+(const pair<T,T>&
     l, const pair<T,T>& r) {
    return {(l.f+r.f)%MOD,(l.s+r.s)%MOD};
}

template<class T> pair<T,T> operator-(const pair<T,T>&
     l, const pair<T,T>& r) {
    return {(l.f-r.f+MOD)%MOD,(l.s-r.s+MOD)%MOD};
}

template<class T> pair<T,T> operator*(const pair<T,T>&
     l, const T& r) {
    return {l.f*r%MOD,l.s*r%MOD};
}

template<class T> pair<T,T> operator*(const pair<T,T>&
     l, const pair<T,T>& r) {
    return {l.f*r.f%MOD,l.s*r.s%MOD};
}

struct hsh {
    string S;
    vector<pll> po, ipo, cum;
    pll base = mp(948392576,573928192);

    ll modpow(ll b, ll p) {
        return !p?1:modpow(b*b%MOD,p/2)*(p&1?b:1)%MOD;
    }

    ll inv(ll x) {
        return modpow(x,MOD-2);
    }

    void gen(string _S) {
        S = _S;
        po.resize(sz(S)), ipo.resize(sz(S)),
             cum.resize(sz(S)+1);
        po[0] = ipo[0] = {1,1};
        FOR(i,1,sz(S)) {
            po[i] = po[i-1]*base;
            ipo[i] = {inv(po[i].f),inv(po[i].s)};
        }
        FOR(i,sz(S)) cum[i+1] =
             cum[i]+po[i]*(ll)(S[i]-'a'+1);
    }

    pll get(int l, int r) {
        return ipo[l]*(cum[r+1]-cum[l]);
    }
};
```

```
int lcp(hsh& a, hsh& b) { // can be used to generate a
     suffix array
    int lo = 0, hi = min(sz(a.S),sz(b.S));
    while (lo < hi) {
        int mid = (lo+hi+1)/2;
        if (a.get(0,mid-1) == b.get(0,mid-1)) lo = mid;
        else hi = mid-1;
    }
    return lo;
}

int main() {
    string _S = "abacaba";
    hsh h; h.gen(_S);
    FOR(i,sz(_S)) FOR(j,i,sz(_S)) cout << i << " " <<
         j << " " << h.get(i,j).f << " " <<
         h.get(i,j).s << "\n";

    hsh H; H.gen("abadaba");
    cout << lcp(h,H);
}
```

12.3 Z (4)

12.3.1 Aho-Corasick

```
/**
 * Source: https://ideone.com/0cMjZJ
 * Usage: Kattis stringmultimatching
*/

template<int SZ> struct Aho {
    int link[SZ], dict[SZ], sz = 1, num = 0;
    vector<pii> ind[SZ];
    map<char,int> to[SZ];
    vi oc[SZ];
    queue<int> q;

    Aho() {
        memset(link,0,sizeof link);
        memset(dict,0,sizeof dict);
    }

    void add(string s) {
        int v = 0;
        for(auto c: s) {
            if (!to[v].count(c)) to[v][c] = sz++;
            v = to[v][c];
        }
        dict[v] = v; ind[v].pb({++num,sz(s)});
    }

    void push_links() {
        link[0] = -1; q.push(0);
        while (sz(q)) {
            int v = q.front(); q.pop();
            for (auto it: to[v]) {
                char c = it.f; int u = it.s, j =
                     link[v];
                while (j != -1 && !to[j].count(c)) j =
                     link[j];
                if (j != -1) {
                    link[u] = to[j][c];
                    if (!dict[u]) dict[u] =
                         dict[link[u]];
                }
                q.push(u);
            }
        }
    }

    void process(int pos, int cur) {
        cur = dict[cur];
        while (cur) {
            for (auto a: ind[cur])
                 oc[a.f].pb(pos-a.s+1);
            cur = dict[link[cur]];
        }
    }

    int nex(int pos, int cur, char c) {
        while (cur != -1 && !to[cur].count(c)) cur =
             link[cur];
        if (cur == -1) cur = 0;
        else cur = to[cur][c];
        process(pos, cur);
        return cur;
    }
};

Aho<100001> A;

int n;

void solve() {
    A = Aho<100001>();
    cin >> n;
    FOR(i,n) {
        string pat; getline(cin,pat); if (!i)
             getline(cin,pat);
        A.add(pat);
    }
    A.push_links();

    string t; getline(cin,t);
    int cur = 0;
    FOR(i,sz(t)) cur = A.nex(i,cur,t[i]);
    FOR(i,1,n+1) {
        for (int j: A.oc[i]) cout << j << " ";
        cout << "\n";
    }
}
```

12.3.2 Manacher (5)

```
/**
* Source: http://codeforces.com/blog/entry/12143
* Description: Calculates length of largest palindrome
    centered at each character of string
```

```
* Verification: http://www.spoj.com/problems/MSUBSTR/
*/

vi manacher(string s) {
    string s1 = "@";
    for (char c: s) s1 += c, s1 += "#";
    s1[s1.length()-1] = '&';

    vi ans(s1.length()-1);
    int lo = 0, hi = 0;
    FOR(i,1,s1.length()-1) {
        ans[i] = min(hi-i,ans[hi-i+lo]);
        while (s1[i-ans[i]-1] == s1[i+ans[i]+1])
            ans[i] ++;
        if (i+ans[i] > hi) lo = i-ans[i], hi =
            i+ans[i];
    }

    ans.erase(ans.begin());
    FOR(i,sz(ans)) if ((i&1) == (ans[i]&1)) ans[i] ++;
        // adjust lengths
    return ans;
}

int main() {
    int T; cin >> T;
    FOR(i,T) {
        pii bes = {0,0};
        string s; cin >> s;
        vi t = manacher(s);
        for (int i: t) {
            if (i > bes.f) bes = {i,1};
            else if (i == bes.f) bes.s++;
        }
        cout << bes.f << " " << bes.s << "\n";
    }
}
```

12.3.3 Minimum Rotation

```
/**
* Source: KACTL
* Unused
*/

int min_rotation(string s) {
        int a=0, N=sz(s); s += s;
        FOR(b,N) FOR(i,N) {
                if (a+i == b || s[a+i] < s[b+i]) {b +=
                    max(0, i-1); break;}
                if (s[a+i] > s[b+i]) { a = b; break; }
        }
        return a;
}
```

12.3.4 Z

```
/**
* Source: http://codeforces.com/blog/entry/3107
* Description: similar to KMP
* Verification: POI 12 Template
*/

vi z(string s) {
    int N = s.length(); s += '#';
    vi ans(N); ans[0] = N;
    while (s[1+ans[1]] == s[ans[1]]) ans[1] ++;

    int L = 1, R = ans[1];
    FOR(i,2,N) {
        if (i <= R) ans[i] = min(R-i+1,ans[i-L]);
        while (s[i+ans[i]] == s[ans[i]]) ans[i] ++;
        if (i+ans[i]-1 > R) L = i, R = i+ans[i]-1;
    }
    return ans;
}

vi get(string a, string b) { // find prefixes of a in b
    string s = a+"@"+b;
    vi t = z(s);
    return vi(t.begin()+a.length()+1,t.end());
}

int main() {
        vi x = z("abcababcabcaba");
        for (int i: x) cout << i << " ";
        cout << "\n";

        x = get("abcab","uwetrabcerabcab");
        for (int i: x) cout << i << " ";
}
```

12.4 Suffix Array (4)

12.4.1 Reverse Burrows-Wheeler (6)

```
/**
* Verification: https://cses.fi/problemset/task/1113/
*/

string transform(string s) {
        vector<pair<char,int>> v;
        int nex[sz(s)];

        FOR(i,sz(s)) v.pb({s[i],i});
        sort(all(v));
        FOR(i,sz(v)) nex[i] = v[i].s;

        int cur = nex[0];
        string ret;
        while (cur != 0) {
                ret += v[cur].f;
                cur = nex[cur];
        }
        return ret;
}
```

12.4.2 Suffix Array

```
/**
* Source: SuprDewd CP Course
* Task: https://open.kattis.com/problems/suffixsorting
* KACTL version is slightly faster
* Verification: USACO December 2017: Standing out from
    the herd:
    http://usaco.org/index.php?page=viewproblem2&cpid=768
* Code to Verify: https://pastebin.com/y2Z9FYr6
*/

struct suffix_array {
    int N;
    vector<vi> P;
    vector<array<int,3>> L;
    vi idx;
    string str;

    /*void bucket(int ind) {
        int mn = MOD, mx = -MOD;

        for (auto a: L) mn = min(mn,a[ind]), mx =
            max(mx,a[ind]);
        vector<array<int,3>> tmp[mx-mn+1];
        FORd(i,sz(L)) tmp[L[i][ind]-mn].pb(L[i]);

        int nex = 0;
        FOR(i,mx-mn+1) for (auto a: tmp[i]) L[nex++] =
            a;
    }

    void bucket_sort() {
        bucket(1), bucket(0);
    }*/

    suffix_array(string _str) {
        str = _str; N = sz(str);
        P.pb(vi(N)); L.resize(N);
        FOR(i,N) P[0][i] = str[i];

        for (int stp = 1, cnt = 1; cnt < N; stp ++,
            cnt *= 2) {
            P.pb(vi(N));
            FOR(i,N) L[i] = {P[stp-1][i],i+cnt < N ?
                P[stp-1][i+cnt] : -1,i};
            sort(all(L));
            // bucket_sort();
            FOR(i,N) {
                if (i && mp(L[i][0],L[i][1]) ==
                    mp(L[i-1][0],L[i-1][1]))
                    P[stp][L[i][2]] = P[stp][L[i-1][2]];
                else P[stp][L[i][2]] = i;
            }
        }

        idx.resize(N);
        FOR(i,sz(P.back())) idx[P.back()[i]] = i;
    }

    int lcp(int x, int y) {
        int res = 0;
        if (x == y) return N-x;
        for (int k = sz(P) - 1; k >= 0 && x < N && y <
            N; k--) {
            if (P[k][x] == P[k][y]) {
                x += 1 << k;
                y += 1 << k;
                res += 1 << k;
            }
        }
        return res;
    }
};
```

13 Additional (4)

13.1 Mo (6)

```
/**
* Source: Codeforces
* Description: Answers queries offline in (N+Q)sqrt(N)
* Also see Mo's on trees
*/

int block = 300; // set ~sqrt(N)

bool cmp(vi a, vi b) {
    if (a[0]/block != b[0]/block) return a[0] < b[0];
    return a[1] < b[1];
}
```

13.2 Misc

13.2.1 Discrete Logarithm

```
/**
 * Description: find k such that primitive^k=x
 * meet in the middle, O(sqrt(MOD))
 * Source: Own
 * Verification: PA 2006 - Professor Laugh's Numbers
 */

const int BLOCK = 32000;

int primitive = 5, invy[BLOCK];
unordered_map<int,int> u;

ll po (ll b, ll p) {
    return !p?1:po(b*b%MOD,p/2)*(p&1?b:1)%MOD;
}

ll inv (ll b) { return po(b,MOD-2); }

ll query(int x) {
        FOR(i,BLOCK) if (u.count(x*invy[i]%MOD))
            return i*BLOCK+u[x*invy[i]%MOD];
        return -1;
}

int main() {
    ll cur = 1;
        FOR(i,BLOCK) {
            u[cur] = i;
            cur = primitive*cur%MOD;
        }
        ll t = 1;
        FOR(i,BLOCK) {
            invy[i] = inv(t);
            t = t*cur%MOD;
        }
        ll x; cin >> x;
        cout << query(x) << "\n";
}
```

13.3 Pragma Optimization (6)

```
/**
 * Source: Misc solutions to CF Nagini
 * Description: 10^{10} operations are ok!
 * Passes the occasional disgusting CF task
 * Also see "Welcome home, Chtholly"
 */

#pragma GCC optimize ("O3")
#pragma GCC target ("sse4")

int q, mx[100001], mn[100001];

int main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);cout.tie(0);
    cin >> q;
    FOR(i,100001) mx[i] = -MOD, mn[i] = MOD;
    FOR(i,q) {
        int t,l,r,k; cin >> t >> l >> r;
        r -= l;

        auto a = mx+l, b = mn+l;
        if (t == 1) {
            cin >> k;
            if (k > 0) FOR(j,r) b[j] = min(b[j],k);
            else FOR(j,r) a[j] = max(a[j],k);
        } else {
            ll ans = 0;
            FOR(j,r) if (a[j] != -MOD && b[j] != MOD)
                ans += b[j]-a[j];
            cout << ans << "\n";
        }
    }
}
```