

Team notebook

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Contents

1 Contest	1
1.1 C++ Template	1
1.2 FastScanner	2
1.3 troubleshoot	2
2 Data Structures	3
2.1 (2) Coordinate Compression	3
2.2 (2) STL Demo	3
2.3 (4) Matrix	4
2.4 1D Range Queries	4
2.4.1 (3) BIT	4
2.4.2 (3) RMQ	4
2.4.3 (3) SegTree	5
2.4.4 (4) BIT with Range Update	5
2.4.5 (4) Lazy SegTree	5
2.4.6 (5) Lazy Persistent SegTree	6
2.4.7 (6) Wavelet Tree	7
2.5 2D Range Queries	8
2.5.1 (4) 2D BIT	8
2.5.2 (4) 2D Sparse SegTree	8
2.5.3 (4) Merge-Sort Tree	9
2.6 BBST	10
2.6.1 (5) Link-Cut Tree	10
2.6.2 (5) Splay Tree	11
2.6.3 (5) Treap	12
3 Flows	13
3.1 (5) Dinic	13
3.2 (5) MinCostFlow	13
4 Geometry	14
4.1 (3) Pair Operators	14
4.2 (4) Closest Pair	15
4.3 (4) Convex Hull	15
4.4 (4) LineContainer	15
4.5 (4) MaxCollinear	16
4.6 (4) Point in Polygon	16
4.7 (4) Polygon Area	17
4.8 (5) LiChao Segment Tree	17
4.9 (5) Line Segment Intersection	17
4.10 (6) KD Tree	18

5 Graphs	19
5.1 (3) Topological Sort	19
5.2 (5) Biconnected Components	19
5.3 (5) Kosaraju	20
5.4 (6) Euler Tour	20
5.5 Shortest Path	21
5.5.1 (3) Bellman-Ford	21
5.5.2 (3) Dijkstra	21
5.5.3 (3) Floyd-Warshall	21
6 Math	22
6.1 (5) Chinese Remainder Theorem	22
6.2 (5) Combinations	22
6.3 (5) Eratosthenes' Sieve	23
6.4 (5) General Modular Inverse	23
6.5 (6) FFT, NTT	23
7 Strings	24
7.1 (3) Hashing	24
7.2 (4) Bitset Trie	25
7.3 (5) Aho-Corasick	25
7.4 (5) Manacher	26
7.5 (5) Palindromic Tree	26
7.6 (5) Z	26
7.7 (6) Booth	27
7.8 (6) Suffix Array	27
8 Trees	28
8.1 (3) DSU, Kruskal	28
8.2 (4) Centroid Decomposition	28
8.3 (4) HLD	28
8.4 (4) LCA with Binary Jumps	29
8.5 (4) LCA with RMQ	30

1 Contest

1.1 C++ Template

```
#include <bits/stdc++.h>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/assoc_container.hpp>

using namespace std;
using namespace __gnu_pbds;

typedef long long ll;
typedef vector<int> vi;
```

```

typedef pair<int, int> pii;
template <class T> using Tree = tree<T, null_type,
    less<T>,
    rb_tree_tag,tree_order_statistics_node_update>;

#define FOR(i, a, b) for (int i=a; i<(b); i++)
#define FOR(i, a) for (int i=0; i<(a); i++)
#define FORd(i,a,b) for (int i = (b)-1; i >= a; i--)
#define FORd(i,a) for (int i = (a)-1; i >= 0; i--)

#define sz(x) (int)(x).size()
#define mp make_pair
#define pb push_back
#define f first
#define s second
#define lb lower_bound
#define ub upper_bound
#define all(x) x.begin(), x.end()

const int MOD = 1000000007;

int main() {
    ios_base::sync_with_stdio(0);cin.tie(0);

}

// read!read!read!read!read!read!read!
// ll vs. int!

```

1.2 FastScanner

```

/**
 * Source: Matt Fontaine?
 */

class FastScanner {
    private InputStream stream;
    private byte[] buf = new byte[1024];
    private int curChar;
    private int numChars;

    public FastScanner(InputStream stream) {
        this.stream = stream;
    }

    int read() {
        if (numChars == -1)
            throw new InputMismatchException();
        if (curChar >= numChars) {
            curChar = 0;
            try {
                numChars = stream.read(buf);
            } catch (IOException e) {
                throw new InputMismatchException();
            }
            if (numChars <= 0) return -1;
        }
        return buf[curChar++];
    }

    boolean isSpaceChar(int c) {

```

```

        return c == ' ' || c == '\n' || c == '\r' || c
            == '\t' || c == -1;
    }

    boolean isEndline(int c) {
        return c == '\n' || c == '\r' || c == -1;
    }

    public int nextInt() {
        return Integer.parseInt(next());
    }

    public long nextLong() {
        return Long.parseLong(next());
    }

    public double nextDouble() {
        return Double.parseDouble(next());
    }

    public String next() {
        int c = read();
        while (isSpaceChar(c)) c = read();
        StringBuilder res = new StringBuilder();
        do {
            res.appendCodePoint(c);
            c = read();
        } while (!isSpaceChar(c));
        return res.toString();
    }

    public String nextLine() {
        int c = read();
        while (isEndline(c))
            c = read();
        StringBuilder res = new StringBuilder();
        do {
            res.appendCodePoint(c);
            c = read();
        } while (!isEndline(c));
        return res.toString();
    }
}

```

1.3 troubleshoot

Source: KACTL

Pre-submit:

- Write a few simple test cases, if sample is not enough.
- Are time limits close? If so, generate max cases.
- Is the memory usage fine?
- Could anything overflow?
- Make sure to submit the right file.

Wrong answer:

- Print your solution! Print debug output, as well.
- Are you clearing all datastructures between test cases?
- Can your algorithm handle the whole range of input?
- Read the full problem statement again.
- Do you handle all corner cases correctly?
- Have you understood the problem correctly?
- Any uninitialized variables?
- Any overflows?
- Confusing N and M, i and j, etc.?
- Are you sure your algorithm works?
- What special cases have you not thought of?
- Are you sure the STL functions you use work as you think?
- Add some assertions, maybe resubmit.
- Create some testcases to run your algorithm on.
- Go through the algorithm for a simple case.
- Go through this list again.
- Explain your algorithm to a team mate.
- Ask the team mate to look at your code.
- Go for a small walk, e.g. to the toilet.
- Is your output format correct? (including whitespace)
- Rewrite your solution from the start or let a team mate do it.

Runtime error:

- Have you tested all corner cases locally?
- Any uninitialized variables?
- Are you reading or writing outside the range of any vector?
- Any assertions that might fail?
- Any possible division by 0? (mod 0 for example)
- Any possible infinite recursion?
- Invalidated pointers or iterators?
- Are you using too much memory?
- Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:

- Do you have any possible infinite loops?
- What is the complexity of your algorithm?
- Are you copying a lot of unnecessary data? (References)
- How big is the input and output? (consider scanf)
- Avoid vector, map. (use arrays/unordered map)
- What do your team mates think about your algorithm?

Memory limit exceeded:

- What is the max amount of memory your algorithm should need?
- Are you clearing all data structures between test cases?

2 Data Structures

2.1 (2) Coordinate Compression

```

void compress(vi& x) {
    map<int,int> m;
    for (int i: x) m[i] = 0;

    int co = 0;
    for (auto& a: m) a.s = co++;

    for (int& i: x) i = m[i];
}

int main() {
    ios_base::sync_with_stdio(0);cin.tie(0);
    vi z = {2,4,3,6};
    compress(z);
    for (int i: z) cout << i << " ";
}

```

2.2 (2) STL Demo

```

struct cmp {
    bool operator()(const int& l, const int& r) const {
        return l > r;
    }
};

struct hsh {
    size_t operator()(const pii& k) const {
        return
            hash<int>()(k.f)^(hash<int>()(k.s^293849182));
    }
};

set<int,cmp> s;
map<int,int,cmp> m;
unordered_map<pii,int,hsh> u;

```

```

int main() {
    cout << "---\nSET\n---\n";
    s.insert(1), s.insert(2);
    for (int i: s) cout << i << "\n";

    cout << "---\nMAP\n---\n";
    m[1] = 5, m[2] = 10;
    for (pii i: m) cout << i.f << " " << i.s <<
        "\n";
}

```

2.3 (4) Matrix

```

/**
 * Source: KACTL
 */

template<int SZ> struct mat {
    array<array<ll,SZ>,SZ> d;

    mat() {
        FOR(i,SZ) FOR(j,SZ) d[i][j] = 0;
    }

    mat operator+(const mat& m) {
        mat<SZ> a;
        FOR(i,SZ) FOR(j,SZ) a.d[i][j] =
            (d[i][j]+m.d[i][j]) % MOD;
        return a;
    }

    mat operator*(const mat& m) {
        mat<SZ> a;
        FOR(i,SZ) FOR(j,SZ) FOR(k,SZ)
            a.d[i][j] = (a.d[i][k]+d[i][j]*m.d[j][k]) %
                MOD;
        return a;
    }

    mat operator^(ll p) {
        mat<SZ> a, b(*this);
        FOR(i,SZ) a.d[i][i] = 1;

        while (p) {
            if (p&1) a = a*b;
            b = b*b;
            p /= 2;
        }

        return a;
    }

    void print() {
        FOR(i,SZ) {
            FOR(j,SZ) cout << d[i][j] << " ";
            cout << "\n";
        }
        cout << "-----\n";
    }
};

```

```

int main() {
    ios_base::sync_with_stdio(0);cin.tie(0);
    mat<2> x;
    x.d[0][0] = 1, x.d[1][0] = 2, x.d[1][1] = 1,
        x.d[0][1] = 3;
    x.print();

    mat<2> y = x*x;
    y.print();

    mat<2> z = x^5;
    z.print();
}

```

2.4 1D Range Queries

2.4.1 (3) BIT

// 1D point update, range query

```

template<int SZ> struct BIT {
    int bit[SZ+1];

    BIT() {
        memset(bit,0,sizeof bit);
    }

    void upd(int k, int val) {
        for( ;k <= SZ; k += (k&-k)) bit[k] += val;
    }

    int query(int k) {
        int temp = 0;
        for (;k > 0;k -= (k&-k)) temp += bit[k];
        return temp;
    }

    int query(int l, int r) { return
        query(r)-query(l-1); }
};

int main() {
    BIT<1<<17> b;
    b.upd(5,2);
    b.upd(4,1);
    cout << b.query(3,5) << "\n";
}

```

2.4.2 (3) RMQ

```

template<class T, int SZ> struct RMQ {
    T stor[SZ][31-__builtin_clz(SZ)];

    T comb(T a, T b) {
        return min(a,b);
    }

    void build(vector<T>& x) {

```

```

    FOR(i,x.size()) stor[i][0] = x[i];
    FOR(j,1,31-__builtin_clz(SZ))
        FOR(i,SZ-(1<<(j-1)))
            stor[i][j] =
                comb(stor[i][j-1],stor[i+(1<<(j-1))][j-1]);
}

T query(int l, int r) {
    int x = 31-__builtin_clz(r-l+1);
    return comb(stor[l][x],stor[r-(1<<x)+1][x]);
}

};

RMQ<int,100000> R;

int main() {
    ios_base::sync_with_stdio(0);cin.tie(0);
    vi x; FOR(i,10) x.pb(rand()%100);
    FOR(i,10) cout << x[i] << " ";
    cout << "\n";
    R.build(x);
    FOR(i,10) {
        int a = rand() % 10, b = rand() % 10; if (a
            > b) swap(a,b);
        cout << a << " " << b << " " <<
            R.query(a,b) << "\n";
    }
}

```

2.4.3 (3) SegTree

// 1D point update, range query

```

template<class T, int SZ> struct Seg {
    T seg[2*SZ], MN = 0;

    Seg() {
        memset(seg,0,sizeof seg);
    }

    T comb(T a, T b) { return a+b; } // easily change
        this to min or max

    void upd(int p, T value) { // set value at
        position p
        for (seg[p += SZ] = value; p > 1; p >>= 1)
            seg[p>>1] = comb(seg[p],seg[p^1]);
    }

    void build() {
        FORd(i,SZ) seg[i] = comb(seg[2*i],seg[2*i+1]);
    }

    T query(int l, int r) { // sum on interval [l, r]
        T res = MN; r++;
        for (l += SZ, r += SZ; l < r; l >>= 1, r >>=
            1) {
            if (l&1) res = comb(res,seg[l++]);
            if (r&1) res = comb(res,seg[--r]);
        }
    }
}

```

```

        return res;
    }
};

int main() {
    Seg<int,1<<17> s;
    s.upd(5,2);
    s.upd(4,1);
    cout << s.query(3,5) << "\n";
}

```

2.4.4 (4) BIT with Range Update

// 1D range update, range query

```

template<int SZ> struct BIT {
    ll bit[2][SZ+1]; // sums piecewise linear functions

    BIT() { memset(bit,0,sizeof bit); }

    void u(int ind, int hi, int val) {
        for(;hi <= SZ; hi += (hi&-hi)) bit[ind][hi] +=
            val;
    }

    void upd(int hi, int val) {
        u(1,1,val), u(1,hi+1,-val);
        u(0,hi+1,hi*val);
    }

    void upd(int lo, int hi, int val) {
        upd(lo-1,-val), upd(hi,val); }

    ll qsum(int x) {
        ll c1 = 0, c0 = 0;
        for (int x1 = x; x1 > 0; x1 -= (x1&-x1))
            c1 += bit[1][x1], c0 += bit[0][x1];
        return c1*x+c0;
    }

    ll qsum(int x, int y) { return qsum(y)-qsum(x-1); }
};

int main() {
    BIT<1<<17> bit;
    bit.upd(2,5,7);
    cout << bit.qsum(1,2) << " " << bit.qsum(4,6) <<
        "\n"; // 7 14
}

```

2.4.5 (4) Lazy SegTree

// 1D range update, range query
 // Inspiration: USACO Counting Haybales

```

const ll INF = MOD; // change if ll ...

template<class T, int SZ> struct LazySegTree {
    T sum[2*SZ], mn[2*SZ], lazy[2*SZ]; // set SZ to a
        power of 2
}

```

```

LazySegTree() {
    memset (sum,0,sizeof sum);
    memset (mn,0,sizeof mn);
    memset (lazy,0,sizeof lazy);
}

void push(int ind, int L, int R) {
    sum[ind] += (R-L+1)*lazy[ind];
    mn[ind] += lazy[ind];
    if (L != R) lazy[2*ind] += lazy[ind],
        lazy[2*ind+1] += lazy[ind];
    lazy[ind] = 0;
}

void pull(int ind) {
    sum[ind] = sum[2*ind]+sum[2*ind+1];
    mn[ind] = min(mn[2*ind],mn[2*ind+1]);
}

void build() {
    FORd(i,SZ) pull(i);
}

T qsum(int lo, int hi, int ind = 1, int L = 0, int
    R = SZ-1) {
    push(ind,L,R);
    if (lo > R || L > hi) return 0;
    if (lo <= L && R <= hi) return sum[ind];

    int M = (L+R)/2;
    return qsum(lo,hi,2*ind,L,M) +
        qsum(lo,hi,2*ind+1,M+1,R);
}

T qmin(int lo, int hi, int ind = 1, int L = 0, int
    R = SZ-1) {
    push(ind,L,R);
    if (lo > R || L > hi) return INF;
    if (lo <= L && R <= hi) return mn[ind];

    int M = (L+R)/2;
    return min(qmin(lo,hi,2*ind,L,M),
        qmin(lo,hi,2*ind+1,M+1,R));
}

void upd(int lo, int hi, ll inc, int ind = 1, int
    L = 0, int R = SZ-1) {
    push(ind,L,R);
    if (hi < L || R < lo) return;
    if (lo <= L && R <= hi) {
        lazy[ind] = inc;
        push(ind,L,R);
        return;
    }

    int M = (L+R)/2;
    upd(lo,hi,inc,2*ind,L,M);
    upd(lo,hi,inc,2*ind+1,M+1,R);
    pull(ind);
}
};

```

```

int main() {
    LazySegTree<int,1<<17> seg;
    seg.upd(2,5,7);
    cout << seg.qsum(1,2) << " " << seg.qsum(4,6) << "
        " << seg.qmin(3,4) << "\n"; // 7 14 7
}

```

2.4.6 (5) Lazy Persistent SegTree

```

struct Node { // without lazy updates
    int val = 0;
    Node* c[2];

    Node* copy() {
        Node* x = new Node(); *x = *this;
        return x;
    }

    int query(int low, int high, int L, int R) {
        if (low <= L && R <= high) return val;
        if (R < low || high < L) return MOD;
        int M = (L+R)/2;
        return
            min(c[0]->query(low,high,L,M),c[1]->query(low,high,
    }

    Node* upd(int ind, int v, int L, int R) {
        if (R < ind || ind < L) return this;
        Node* x = copy();

        if (ind <= L && R <= ind) {
            x->val += v;
            return x;
        }

        int M = (L+R)/2;
        x->c[0] = x->c[0]->upd(ind,v,L,M);
        x->c[1] = x->c[1]->upd(ind,v,M+1,R);
        x->val = min(x->c[0]->val,x->c[1]->val);

        return x;
    }

    void build(vi& arr, int L, int R) {
        if (L == R) {
            if (L < (int)arr.size()) val = arr[L];
            else val = 0;
            return;
        }
        int M = (L+R)/2;
        c[0] = new Node();
        c[0]->build(arr,L,M);
        c[1] = new Node();
        c[1]->build(arr,M+1,R);
        val = min(c[0]->val,c[1]->val);
    }
};

struct node { // with lazy updates

```

```

int val = 0, lazy = 0;
node* c[2];

node* copy() {
    node* x = new node(); *x = *this;
    return x;
}

void push() {
    if (!lazy) return;
    FOR(i,2) if (c[i]) {
        c[i] = new node(*c[i]);
        c[i]->lazy += lazy;
    }
    lazy = 0;
}

int query(int low, int high, int L, int R) {
    if (low <= L && R <= high) return val+lazy;
    if (R < low || high < L) return MOD;
    int M = (L+R)/2;
    int t =
        min(c[0]->query(low,high,L,M), c[1]->query(low,high,M+1,R));
    return lazy+t;
}

node* upd(int low, int high, int v, int L, int R) {
    if (R < low || high < L) return this;
    if (low <= L && R <= high) {
        x->lazy += v, x->val += v;
        return x;
    }
    push();
    node* x = copy();

    int M = (L+R)/2;
    x->c[0] = x->c[0]->upd(low,high,v,L,M);
    x->c[1] = x->c[1]->upd(low,high,v,M+1,R);
    x->val = min(x->c[0]->val, x->c[1]->val);

    return x;
}

void build(vi& arr, int L, int R) {
    if (L == R) {
        if (L < (int)arr.size()) val = arr[L];
        else val = 0;
        return;
    }
    int M = (L+R)/2;
    c[0] = new node();
    c[0]->build(arr,L,M);
    c[1] = new node();
    c[1]->build(arr,M+1,R);
    val = min(c[0]->val, c[1]->val);
}

};

template<int SZ> struct pers {
    node* loc[SZ+1]; // stores location of root after
                    ith update
    int nex = 1;

```

```

pers() { loc[0] = new node(); }

void upd(int low, int high, int val) {
    loc[nex] =
        loc[nex-1]->upd(low,high,val,0,SZ-1);
    nex++;
}

void build(vi& arr) {
    loc[0]->build(arr,0,SZ-1);
}

int query(int ti, int low, int high) {
    return loc[ti]->query(low,high,0,SZ-1);
}

};

pers<8> p;

int main() {
    vi arr = {1,7,2,3,5,9,4,6};
    p.build(arr);

    FOR(i,8) {
        FOR(j,i,8) cout << p.query(1,i,j) << " ";
        cout << "\n";
    }
    cout << "\n";

    p.upd(4,7,5); // 1 9 4 3 10 14 9 11
    FOR(i,8) {
        FOR(j,i,8) cout << p.query(2,i,j) << " ";
        cout << "\n";
    }
    cout << "\n";

    FOR(i,8) {
        FOR(j,i,8) cout << p.query(1,i,j) << " ";
        cout << "\n";
    }
    cout << "\n";
}

```

2.4.7 (6) Wavelet Tree

```

/**
 * Source: https://ideone.com/Tkters
 * Unused
 */

int MAX = 1000000;
int a[300000];

struct wavelet {
    int lo, hi;
    wavelet *c[2];
    vi b;

    wavelet(int *from, int *to, int x, int y) {

```

```

    lo = x, hi = y;
    if (lo == hi || from >= to) return;
    int mid = (lo+hi)/2;
    auto f = [mid](int x) { return x <= mid; };
    b.pb(0); for (auto it = from; it != to; it++)
        b.pb(b.back()+f(*it));
    auto pivot = stable_partition(from,to,f);
    c[0] = new wavelet(from,pivot,lo,mid);
    c[1] = new wavelet(pivot,to,mid+1,hi);
}

int kth(int l, int r, int k) { // kth number in
    [l,r]
    if (l > r) return 0;
    if (lo == hi) return lo;
    int inLeft = b[r]-b[l-1];
    int lb = b[l-1], rb = b[r];
    if (k <= inLeft) return c[0]->kth(lb+1,rb,k);
    return c[1]->kth(l-rb,r-rb,k-inLeft);
}

int LTE(int l, int r, int k) { // less than or
    equal to k
    if(l > r || k < lo) return 0;
    if(hi <= k) return r-l+1;
    int lb = b[l-1], rb = b[r];
    return c[0]->LTE(lb+1, rb,
        k)+c[1]->LTE(l-rb, r-rb, k);
}

int count(int l, int r, int k) { // equal to k
    if(l > r || k < lo || k > hi) return 0;
    if(lo == hi) return r - l + 1;
    int lb = b[l-1], rb = b[r], mid =
        (lo+hi)/2;
    if(k <= mid) return c[0]->count(lb+1,
        rb, k);
    return c[1]->count(l-lb, r-rb, k);
}

};

int main() {
    ios_base::sync_with_stdio(0);cin.tie(0);
    int i,n,k,j,q,l,r;
    cin >> n;
    FOR(i,n) cin >> a[i+1];
    wavelet T(a+1, a+n+1, 1, MAX);
    cin >> q;
    while (q--){
        int x;
        cin >> x >> l >> r >> k;
        if(x == 0){
            //kth smallest
            cout << "Kth smallest: ";
            cout << T.kth(l, r, k) << endl;
        }
        if(x == 1){
            //less than or equal to K
            cout << "LTE: ";
            cout << T.LTE(l, r, k) << endl;
        }
        if(x == 2){

```

```

            //count occurrence of K in [l, r]
            cout << "Occurrence of K: ";
            cout << T.count(l, r, k) << endl;
        }
    }
}

```

2.5 2D Range Queries

2.5.1 (4) 2D BIT

```

template<int SZ> struct BIT2D {
    int bit[SZ+1][SZ+1];
    void upd(int X, int Y, int val) {
        for (; X <= SZ; X += (X&-X))
            for (int Y1 = Y; Y1 <= SZ; Y1 += (Y1&-Y1))
                bit[X][Y1] += val;
    }
    int query(int X, int Y) {
        int ans = 0;
        for (; X > 0; X -= (X&-X))
            for (int Y1 = Y; Y1 > 0; Y1 -= (Y1&-Y1))
                ans += bit[X][Y1];
        return ans;
    }
    int query(int X1, int X2, int Y1, int Y2) {
        return query(X2,Y2)-query(X1-1,Y2)
            -query(X2,Y1-1)+query(X1-1,Y1-1);
    }
};

int main() {
    BIT2D<1000> x;
    x.upd(2,5,7);
    x.upd(3,6,8);
    x.upd(4,6,9);
    cout << x.query(2,3,5,6);
}

```

2.5.2 (4) 2D Sparse SegTree

```

// 2D Segment Tree and SegBIT
// 2D Point Update, Range Query

const int SZ = 1<<17;

// Sparse 1D SegTree
struct node {
    int val = 0;
    node* c[2];

    void upd(int ind, int v, int L = 0, int R = SZ-1)
    { // set an element equal to v
        if (L == ind && R == ind) { val = v; return; }

        int M = (L+R)/2;
        if (ind <= M) {
            if (!c[0]) c[0] = new node();

```



```

        c[0]->upd(ind,v,L,M);
    } else {
        if (!c[1]) c[1] = new node();
        c[1]->upd(ind,v,M+1,R);
    }

    val = 0;
    if (c[0]) val += c[0]->val;
    if (c[1]) val += c[1]->val;
}

int query(int low, int high, int L = 0, int R =
    SZ-1) { // query sum of segment
    if (low <= L && R <= high) return val;
    if (high < L || R < low) return 0;

    int M = (L+R)/2, t = 0;
    if (c[0]) t += c[0]->query(low,high,L,M);
    if (c[1]) t += c[1]->query(low,high,M+1,R);
    return t;
}
};

// 2D SegTree, sparse segtree of sparse 1D segtrees
struct Node {
    node seg;
    Node* c[2];

    void upd(int x, int y, int v, int L = 0, int R =
        SZ-1) { // set an element equal to v
        seg.upd(y,v);
        if (L == x && R == x) return;

        int M = (L+R)/2;
        if (x <= M) {
            if (!c[0]) c[0] = new Node();
            c[0]->upd(x,y,v,L,M);
        } else {
            if (!c[1]) c[1] = new Node();
            c[1]->upd(x,y,v,M+1,R);
        }
    }

    int query(int x1, int x2, int y1, int y2, int L =
        0, int R = SZ-1) { // query sum of rectangle
        if (x1 <= L && R <= x2) return
            seg.query(y1,y2);
        if (x2 < L || R < x1) return 0;

        int M = (L+R)/2, t = 0;
        if (c[0]) t += c[0]->query(x1,x2,y1,y2,L,M);
        if (c[1]) t += c[1]->query(x1,x2,y1,y2,M+1,R);
        return t;
    }
};

// SegTree + BIT
// Array of Sparse Segtrees
struct SegBit {
    node seg[SZ+1];

```

```

    void upd(int x, int y, int v) { // set an element
        equal to v
        for (x++; x <= SZ; x += (x&-x)) seg[x].upd(y,v);
    }

    int query(int x, int y1, int y2) {
        int ret = 0;
        for (; x > 0; x -= (x&-x)) ret +=
            seg[x].query(y1,y2);
        return ret;
    }

    int query(int x1, int x2, int y1, int y2) { //
        query sum of rectangle
        return query(x2+1,y1,y2)-query(x1,y1,y2);
    }
};

Node n;
SegBit s;

int main() {
    ios_base::sync_with_stdio(0);cin.tie(0);

    n.upd(5,7,2);
    n.upd(3,2,20);
    n.upd(5,8,200);
    cout << n.query(3,5,2,7) << "\n"; // 22

    s.upd(5,7,2);
    s.upd(3,2,20);
    s.upd(5,8,200);
    cout << s.query(3,5,2,7) << "\n"; // 22
}

```

2.5.3 (4) Merge-Sort Tree

```

template<int SZ> struct mstree {
    Tree<pii> val[SZ+1]; // for offline queries use
        vector with binary search instead

    void upd(int x, int y, int t = 1) { //
        x-coordinate between 1 and SZ inclusive
        for (int X = x; X <= SZ; X += X&-X) {
            if (t) val[X].insert({y,x});
            else val[X].erase({y,x});
        }
    }

    int query(int x, int y) {
        int t = 0;
        for (; x > 0; x -= x&-x) t +=
            val[x].order_of_key({y,MOD});
        return t;
    }

    int query(int lox, int hix, int loy, int hiy) { //
        query number of elements within a rectangle
        return query(hix,hiy)-query(lox-1,hiy)
            -query(hix,loy-1)+query(lox-1,loy-1);
    }
}

```

```

    }
};

int main() {
    mstree<100000> m;
    m.upd(3,6); m.upd(4,5);
    cout << m.query(3,5,4,6) << " " <<
        m.query(3,5,4,5); // 2, 1
}

```

2.6 BBST

2.6.1 (5) Link-Cut Tree

```

int p[100001], pp[100001], c[100001][2], sum[100001];

int getDir(int x, int y) {
    return c[x][0] == y ? 0 : 1;
}

void setLink(int x, int y, int d) {
    c[x][d] = y, p[y] = x;
}

void rotate(int y, int d) {
    int x = c[y][d], z = p[y];
    setLink(y, c[x][d^1], d);
    setLink(x, y, d^1);
    setLink(z, x, getDir(z, y));

    sum[x] = sum[y];
    sum[y] = sum[c[y][0]] + sum[c[y][1]] + 1;
    pp[x] = pp[y]; pp[y] = 0;
}

void splay(int x) {
    while (p[x]) {
        int y = p[x], z = p[y];
        int dy = getDir(y, x), dz = getDir(z, y);
        if (!z) rotate(y, dy);
        else if (dy == dz) rotate(z, dz), rotate(y, dy);
        else rotate(y, dy), rotate(z, dz);
    }
}

void dis(int v, int d) { // fix
    p[c[v][d]] = 0, pp[c[v][d]] = v;
    sum[v] -= sum[c[v][d]];
    c[v][d] = 0;
}

void con(int v, int d) { // fix
    c[pp[v]][d] = v;
    sum[pp[v]] += sum[v];
    p[v] = pp[v], pp[v] = 0;
}

void access(int v) {
    // v is brought to the root of auxiliary tree
    // modify preferred paths
}

```

```

    splay(v);
    dis(v, 1);

    while (pp[v]) {
        int w = pp[v]; splay(w);
        dis(w, 1), con(v, 1);
        splay(v);
    }
}

int find_root(int v) {
    access(v);
    while (c[v][0]) v = c[v][0];
    access(v);
    return v;
}

int find_depth(int v) {
    access(v);
    return sum[c[v][0]];
}

void cut(int v) {
    // cut link between v and par[v]
    access(v);
    pp[c[v][0]] = p[c[v][0]] = 0; // fix
    sum[v] -= sum[c[v][0]];
    c[v][0] = 0;
}

void link(int v, int w) {
    // v, which is root of another tree, is now child
    // of w
    access(v), access(w);
    pp[w] = v; con(w, 0);
}

int anc(int v, int num) {
    if (find_depth(v) < num) return 0;
    access(v);
    v = c[v][0];

    while (1) {
        if (sum[c[v][1]] >= num) v = c[v][1];
        else if (sum[c[v][1]] + 1 == num) return v;
        else num -= (sum[c[v][1]] + 1), v = c[v][0];
    }
}

int main() {
    FOR(i, 1, 100001) sum[i] = 1;

    link(2, 1);
    link(3, 1);
    link(4, 1);
    link(5, 4);
    link(10, 4);
    link(7, 6);
    link(8, 7);
    link(9, 8);
}

```

```

FOR(i,1,11) cout << i << " " << find_root(i) << "
    " << find_depth(i) << " " << anc(i,2) << "\n";
cout << "\n";

cut(4);
link(4,8);

FOR(i,1,11) cout << i << " " << find_root(i) << "
    " << find_depth(i) << " " << anc(i,2) << "\n";
}

```

2.6.2 (5) Splay Tree

```

/*
 * Source:
 * http://codeforces.com/blog/entry/18462
 */

struct node{
    int val, sz;
    node *p, *c[2];
    node (int v) {
        val = v, sz = 1;
    }
    void recalc() {
        sz = 1;
        if (c[0]) sz += c[0]->sz;
        if (c[1]) sz += c[1]->sz;
    }
};

node *root;

void setLink(node *x, node *y, int d) {
    if (x) {
        x->c[d] = y;
        x->recalc();
    }
    if (y) y->p = x;
}

int getDir(node *x, node *y) {
    if (!x) return -1;
    return x->c[0] == y ? 0 : 1;
}

void rot(node *x, int d) {
    node *y = x->c[d], *z = x->p;
    setLink(x, y->c[d^1], d);
    setLink(y, x, d^1);
    setLink(z, y, getDir(z, x));
    x->recalc(), y->recalc();
}

void splay(node *x) {
    while (x->p) {
        node *y = x->p, *z = y->p;
        int dy = getDir(y, x), dz = getDir(z, y);
        if (!z) rot(y, dy);
        else if (dy == dz) rot(z, dz), rot(y, dy);
    }
}

```

```

        else rot(y, dy), rot(z, dz);
    }
    root = x;
}

node* ins(node* cur, int v) {
    if (!cur) return cur = new node(v);
    if (cur->val == v) return cur;
    int t = v < cur->val ? 0 : 1;

    if (!cur->c[t]) {
        setLink(cur, new node(v), t);
        return cur->c[t];
    } else {
        node* x = ins(cur->c[t], v);
        cur->recalc();
        return x;
    }
}

void ins(int val) {
    splay(ins(root, val));
}

bool find(node *cur, int v) {
    if (!cur) return 0;
    if (cur->val == v) {
        splay(cur);
        return 1;
    }
    return find(cur->c[v < cur->val ? 0 : 1], v);
}

bool del(int v) {
    if (!find(root, v)) return 0;
    node *N = root, *P = N->c[0];
    if (!P) {
        root = N->c[1]; root->p = NULL, delete N;
        return 1;
    }

    while(P->c[1]) P = P->c[1];
    setLink(P, N->c[1], 1);
    root = N->c[0], root->p = NULL, delete N;

    while (P) {
        P->recalc();
        P = P->p;
    }
    return 1;
}

void inOrder(node* cur) {
    if (!cur) return;
    cout << "NODE " << cur->val << ": SIZE " <<
        cur->sz << "\n";
    if (cur->c[0]) cout << "LEFT: " << cur->c[0]->val
        << "\n";
    if (cur->c[1]) cout << "RIGHT: " << cur->c[1]->val
        << "\n";
    cout << "\n";
    inOrder(cur->c[0]);
}

```

```

    inOrder(cur->c[1]);
}

int main() {
    for (int i = 0; i < 10; ++i) ins(rand() % 50);
    inOrder(root);
    cout << "-----\n\n";
    del(21);
    inOrder(root);
}

```

2.6.3 (5) Treap

```

/**
 * Source: KACTL
 * Description: easiest BBST
 */

struct tnode {
    ll val, lazy;
    int pri, sz;
    tnode *c[2];

    tnode* copy() {
        tnode* x = new tnode(0); *x = *this;
        return x;
    }

    tnode (ll v) {
        val = v, lazy = 0;
        pri = rand()+(rand()<<15), sz = 1;
        c[0] = c[1] = NULL;
    }

    void propogate() {
        if (!lazy) return;
        val += lazy;
        FOR(i,2) if (c[i]) {
            c[i] = c[i]->copy();
            c[i]->lazy += lazy;
        }
        lazy = 0;
    }

    void recalc() {
        sz = 1;
        FOR(i,2) if (c[i]) sz += c[i]->sz;
    }

    void inOrder(bool f = 0) {
        propogate();
        if (c[0]) c[0]->inOrder();
        cout << val << " ";
        if (c[1]) c[1]->inOrder();
        if (f) cout << "\n-----\n";
    }
};

pair<tnode*,tnode*> split(tnode* t, int v) { // >= v
    goes to the right

```

```

    if (!t) return {t,t};

    t->propogate();
    tnode* T = t->copy();

    if (v <= T->val) {
        auto p = split(T->c[0], v);
        T->c[0] = p.s; T->recalc();
        return {p.f, T};
    } else {
        auto p = split(T->c[1], v);
        T->c[1] = p.f; T->recalc();
        return {T, p.s};
    }
}

tnode* merge(tnode* l, tnode* r) {
    if (!l) return r;
    if (!r) return l;

    l->propogate(), r->propogate();
    if (l->pri > r->pri) {
        tnode* L = l->copy();
        L->c[1] = merge(L->c[1],r);
        L->recalc();
        return L;
    } else {
        tnode* R = r->copy();
        R->c[0] = merge(l,R->c[0]);
        R->recalc();
        return R;
    }
}

tnode* ins(tnode* x, int v) { // insert value v
    auto a = split(x,v);
    return merge(merge(a.f, new tnode(v)),a.s);
}

tnode* del(tnode* x, int v) { // delete all values
    equal to v
    auto a = split(x,v);
    auto b = split(a.s,v+1);
    return merge(a.f,b.s);
}

tnode *root, *root1;

int main() {
    ios_base::sync_with_stdio(0);cin.tie(0);

    root = ins(root,1);
    root = ins(root,9);
    root = ins(root,3);

    root1 = root;
    root1->inOrder(1);

    root = ins(root,7);
    root = ins(root,4);
    root = del(root,9);

```

```

root->inOrder(1);
root1->inOrder(1);
cout << root->sz << " " << root1->sz << "\n";
}

```

3 Flows

3.1 (5) Dinic

// source: GeeksForGeeks

```

const int SZ = 1000;

struct Edge {
    int v, flow, C, rev;
};

struct Dinic {
    int level[SZ], start[SZ];
    vector<Edge> adj[SZ];

    void addEdge(int u, int v, int C) {
        Edge a{v, 0, C, (int)adj[v].size()};
        Edge b{u, 0, 0, (int)adj[u].size()};
        adj[u].pb(a);
        adj[v].pb(b);
    }

    bool BFS(int s, int t) {
        FOR(i, SZ) level[i] = -1;
        level[s] = 0;

        queue<int> q; q.push(s);
        while (!q.empty()) {
            int u = q.front(); q.pop();
            for (auto e: adj[u])
                if (level[e.v] < 0 && e.flow < e.C) {
                    level[e.v] = level[u] + 1;
                    q.push(e.v);
                }
        }

        return level[t] >= 0;
    }

    int sendFlow(int u, int flow, int t) {
        if (u == t) return flow;

        for ( ; start[u] < adj[u].size(); start[u]++)
            Edge &e = adj[u][start[u]];

            if (level[e.v] == level[u]+1 && e.flow < e.C) {
                int curr_flow = min(flow, e.C - e.flow);
                int temp_flow = sendFlow(e.v, curr_flow, t);

                if (temp_flow > 0) {

```

```

                    e.flow += temp_flow;
                    adj[e.v][e.rev].flow -= temp_flow;
                    return temp_flow;
                }
            }
        }

        return 0;
    }

    int DinicMaxflow(int s, int t) {
        if (s == t) return -1;
        int total = 0;

        while (BFS(s, t)) {
            FOR(i, SZ) start[i] = 0;
            while (int flow = sendFlow(s, INT_MAX, t))
                total += flow;
        }

        return total;
    }
};

Dinic D;

int main() {
    D.addEdge(0, 1, 16 );
    D.addEdge(0, 2, 13 );
    D.addEdge(1, 2, 10 );
    D.addEdge(1, 3, 12 );
    D.addEdge(2, 1, 4 );
    D.addEdge(2, 4, 14);
    D.addEdge(3, 2, 9 );
    D.addEdge(3, 5, 20 );
    D.addEdge(4, 3, 7 );
    D.addEdge(4, 5, 4);

    cout << "Maximum flow " << D.DinicMaxflow(0, 5);
}

```

3.2 (5) MinCostFlow

// Source: GeeksForGeeks

```

struct Edge {
    int v, flow, C, rev, cost;
};

template<int SZ> struct mcf {
    pii pre[SZ];
    int cost[SZ], num[SZ], SC, SNC;
    ll flo, ans, ccost;
    vector<Edge> adj[SZ];

    void addEdge(int u, int v, int C, int cost) {
        Edge a{v, 0, C, (int)adj[v].size(), cost};
        Edge b{u, 0, 0, (int)adj[u].size(), -cost};
        adj[u].pb(a), adj[v].pb(b);
    }
}

```

```

void reweight() {
    FOR(i,SZ) {
        for (auto& p: adj[i]) p.cost +=
            cost[i]-cost[p.v];
    }
}

bool spfa() {
    FOR(i,SZ) cost[i] = MOD, num[i] = 0;
    cost[SC] = 0, num[SC] = MOD;
    priority_queue<pii,vector<pii>,greater<pii>>
        todo; todo.push({0,SC});

    while (todo.size()) {
        pii x = todo.top(); todo.pop();
        if (x.f > cost[x.s]) continue;
        for (auto a: adj[x.s]) if (x.f+a.cost <
            cost[a.v] && a.flow < a.C) {
            pre[a.v] = {x.s,a.rev};
            cost[a.v] = x.f+a.cost;
            num[a.v] = min(a.C-a.flow,num[x.s]);
            todo.push({cost[a.v],a.v});
        }
    }

    ccost += cost[SNC];
    return num[SNC] > 0;
}

void backtrack() {
    flo += num[SNC], ans += (ll)num[SNC]*ccost;
    for (int x = SNC; x != SC; x = pre[x].f) {
        adj[x][pre[x].s].flow -= num[SNC];
        int t = adj[x][pre[x].s].rev;
        adj[pre[x].f][t].flow += num[SNC];
    }
}

pii mincostflow(int sc, int snc) {
    SC = sc, SNC = snc;
    flo = ans = ccost = 0;

    spfa();
    while (1) {
        reweight();
        if (!spfa()) return {flo,ans};
        backtrack();
    }
}

};

mcf<100> m;

int main() {
    m.addEdge(0, 1, 16, 5);
    m.addEdge(1, 2, 13, 7);
    m.addEdge(1, 2, 13, 8);

    pii x = m.mincostflow(0,2);
    cout << x.f << " " << x.s;
}

```

4 Geometry

4.1 (3) Pair Operators

```

template<class T> pair<T,T> operator+(const pair<T,T>&
    l, const pair<T,T>& r) {
    return {l.f+r.f,l.s+r.s};
}

template<class T> pair<T,T> operator-(const pair<T,T>&
    l, const pair<T,T>& r) {
    return {l.f-r.f,l.s-r.s};
}

template<class T> pair<T,T> operator*(const pair<T,T>&
    l, T r) {
    return {l.f*r,l.s*r};
}

template<class T> pair<T,T> operator/(const pair<T,T>&
    l, T r) {
    return {l.f/r,l.s/r};
}

template<class T> double mag(pair<T,T> p) {
    return sqrt(p.f*p.f+p.s*p.s);
}

template<class T> pair<T,T> operator*(const pair<T,T>&
    l, const pair<T,T>& r) {
    // l.f+l.s*i, r.f+r.s*i
    return {l.f*r.f-l.s*r.s,l.s*r.f+l.f*r.s};
}

template<class T> pair<T,T> operator/(const pair<T,T>&
    l, const pair<T,T>& r) {
    // l.f+l.s*i, r.f+r.s*i
    pair<T,T> z =
        {r.f/(r.f*r.f+r.s*r.s),-r.s/(r.f*r.f+r.s*r.s)};
    return l*z;
}

template<class T> double area(pair<T,T> a, pair<T,T>
    b, pair<T,T> c) {
    b = b-a, c = c-a;
    return (b.f*c.s-b.s*c.f)/2;
}

template<class T> double dist(pair<T,T> l, pair<T,T>
    r) {
    return mag(r-l);
}

template<class T> double dist(pair<T,T> o, pair<T,T>
    x, pair<T,T> d) { // signed distance
    return 2*area(o,x,x+d)/mag(d);
}

```

4.2 (4) Closest Pair

```
// https://open.kattis.com/problems/closestpair2
// Nlog^2N

pair<double, pair<pdd, pdd>> MN = {INF, {{0,0},{0,0}}};

int n;

bool cmp(pdd a, pdd b) {
    return a.s < b.s;
}

double dist(pdd a, pdd b) {
    b.f -= a.f, b.s -= a.s;
    return sqrt(b.f*b.f+b.s*b.s);
}

pair<double, pair<pdd, pdd>> strip(vector<pdd> v, double
di) {
    pair<double, pair<pdd, pdd>> ans = MN;
    FOR(i, v.size()) FOR(j, i+1, v.size()) {
        if (v[i].s+di <= v[j].s) break;
        ans = min(ans, {dist(v[i], v[j]), {v[i], v[j]}});
    }
    return ans;
}

pair<double, pair<pdd, pdd>> bes (vector<pdd> v) {
    if (v.size() == 1) return MN;
    int M = v.size()/2;
    vector<pdd> v1(v.begin(), v.begin()+M),
    v2(v.begin()+M, v.end());
    auto a = bes(v1), b = bes(v2);
    double di = min(a.f, b.f);

    vector<pdd> V;
    FOR(i, v.size()) if (v[i].f > v[M].f-di && v[i].f <
        v[M].f+di) V.pb(v[i]);
    sort(V.begin(), V.end(), cmp);

    auto z = strip(V, di);
    return min(min(a, b), z);
}

int main() {
    ios_base::sync_with_stdio(0); cin.tie(0);
    cout << fixed << setprecision(2);
    while (cin >> n) {
        if (n == 0) break;
        vector<pdd> v(n);
        FOR(i, n) cin >> v[i].f >> v[i].s;
        sort(v.begin(), v.end());
        auto a = bes(v);
        cout << a.s.f << " " << a.s.f.s << " " <<
            a.s.s.f << " " << a.s.s.s << "\n";
    }
}
```

4.3 (4) Convex Hull

```
/**
 * Source: Wikibooks
 * Task: https://open.kattis.com/problems/convexhull
 */

ll cross(pii O, pii A, pii B) {
    return
        (ll)(A.f-O.f)*(B.s-O.s)-(ll)(A.s-O.s)*(B.f-O.f);
}

vector<pii> convex_hull(vector<pii> P) {
    sort(P.begin(), P.end());
    P.erase(unique(P.begin(), P.end()), P.end());
    if (P.size() == 1) return P;

    int n = P.size();

    vector<pii> bot = {P[0]};
    FOR(i, 1, n) {
        while (bot.size() > 1 &&
            cross(bot[bot.size()-2], bot.back(), P[i])
            <= 0) bot.pop_back();
        bot.pb(P[i]);
    }
    bot.pop_back();

    vector<pii> up = {P[n-1]};
    FORd(i, n-1) {
        while (up.size() > 1 && cross(up[up.size()-2],
            up.back(), P[i]) <= 0) up.pop_back();
        up.pb(P[i]);
    }
    up.pop_back();

    bot.insert(bot.end(), up.begin(), up.end());
    return bot;
}

int main() {
    int n;
    while (cin >> n) {
        if (n == 0) break;
        vector<pii> P(n); FOR(i, n) cin >> P[i].f >>
            P[i].s;
        vector<pii> hull = convex_hull(P);

        cout << hull.size() << "\n";
        for (auto a: hull) cout << a.f << " " << a.s
            << "\n";
    }
}
```

4.4 (4) LineContainer

```
/**
 * Source: KACTL
 * Unused
```

```

*/

bool Q;
struct Line {
    mutable ll k, m, p; // slope, y-intercept,
        last optimal x
    bool operator<(const Line& o) const {
        return Q ? p < o.p : k < o.k;
    }
};

struct LineContainer : multiset<Line> {
    const ll inf = LLONG_MAX;
    ll div(ll a, ll b) { // floored division
        if (b < 0) a *= -1, b *= -1;
        if (a >= 0) return a/b;
        return -((-a+b-1)/b);
    }

    // updates x->p, determines if y is unneeded
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = inf; return 0; }
        if (x->k == y->k) x->p = x->m > y->m ?
            inf : -inf;
        else x->p = div(y->m - x->m, x->k -
            y->k);
        return x->p >= y->p;
    }

    void add(ll k, ll m) {
        auto z = insert({k, m, 0}), y = z++, x
            = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y))
            isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p
            >= y->p) isect(x, erase(y));
    }

    ll query(ll x) {
        assert(!empty());
        Q = 1; auto l = *lb({0,0,x}); Q = 0;
        return l.k * x + l.m;
    }
};

int main() {

```

4.5 (4) MaxCollinear

// <https://open.kattis.com/problems/maxcollinear>

```

int n, mx, ans;
map<pair<pii,int>,int> m;
pii p[1000];

pair<pii,int> getline(pii a, pii b) {

```

```

    pii z = {b.f-a.f,b.s-a.s};
    swap(z.f,z.s); z.f *= -1;
    int g = __gcd(z.f,z.s); z.f /= g, z.s /= g;
    if (z.f < 0 || (z.f == 0 && z.s < 0)) z.f *= -1,
        z.s *= -1;
    return {z,z.f*a.f+z.s*a.s};
}

void solve() {
    mx = ans = 0; m.clear();
    FOR(i,n) cin >> p[i].f >> p[i].s;
    FOR(i,n) FOR(j,i+1,n) m[getline(p[i],p[j])] ++;

    for (auto a: m) mx = max(mx,a.s);
    FOR(i,1,n+1) if (i*(i-1)/2 <= mx) ans = i;
    cout << ans << "\n";
}

int main() {
    ios_base::sync_with_stdio(0);cin.tie(0);
    while (cin >> n) {
        if (n == 0) break;
        solve();
    }
}

```

4.6 (4) Point in Polygon

// <https://open.kattis.com/problems/pointinpolygon>

```

int n,m;
pii p[1000];

int area(pii x, pii y, pii z) {
    return (y.f-x.f)*(z.s-x.s)-(y.s-x.s)*(z.f-x.f);
}

bool on(pii x, pii y, pii z) {
    if (area(x,y,z) != 0) return 0;
    return min(x,y) <= z && z <= max(x,y);
}

double get(pii x, pii y, int z) {
    return double((z-x.s)*y.f+(y.s-z)*x.f)/(y.s-x.s);
}

void test(pii z) {
    int ans = 0;
    FOR(i,n) {
        pii x = p[i], y = p[(i+1)%n];
        if (on(x,y,z)) {
            cout << "on\n";
            return;
        }
        if (x.s > y.s) swap(x,y);
        if (x.s <= z.s && y.s > z.s) {
            double t = get(x,y,z.s);
            if (t > z.f) ans++;
        }
    }
}

```



```

    if (ans % 2 == 1) cout << "in\n";
    else cout << "out\n";
}

void solve() {
    FOR(i,n) cin >> p[i].f >> p[i].s;
    cin >> m;
    FOR(i,m) {
        pii z; cin >> z.f >> z.s;
        test(z);
    }
}

```

4.7 (4) Polygon Area

```

/**
 * Description: Shoelace Formula
 * Task: https://open.kattis.com/problems/polygonarea
 */

double area(vector<pii> v) {
    double x = 0;
    FOR(i,sz(v)) {
        int j = (i+1)%sz(v);
        x += (1l)v[i].f*v[j].s;
        x -= (1l)v[j].f*v[i].s;
    }
    return x/2;
}

```

4.8 (5) LiChao Segment Tree

```

/**
 * Source:
 * http://codeforces.com/blog/entry/51275?#comment-351413
 * Unused
 */

const int N = 100000 + 5;

int n, m;
int vis[N << 1];
char op[100];

struct line {
    double k, b;
    line(double _k = 0, double _b = 0) { k = _k; b = _b; }
    double get(double x) { return k * x + b; }
} c[N << 1];

void modify(int x, int l, int r, line v) {
    if (!vis[x]) { vis[x] = 1, c[x] = v; return; }
    if (c[x].get(l) > v.get(l) && c[x].get(r) > v.get(r)) return;
    if (c[x].get(l) < v.get(l) && c[x].get(r) < v.get(r)) { c[x] = v; return; }
}

```

```

int m = (l + r) >> 1;
if (c[x].get(l) < v.get(l)) swap(c[x], v);
if (c[x].get(m) > v.get(m)) modify(x<<1|1, m + 1, r, v);
else {swap(c[x], v); modify(x<<1, l, m, v);}
}

double get(int x, int l, int r, int pos) {
    if (l == r) return c[x].get(l);
    int m = (l + r) >> 1; double ans = c[x].get(pos);
    if (pos <= m) ans = max(ans, get(x<<1, l, m, pos));
    else ans = max(ans, get(x<<1|1, m + 1, r, pos));
    return ans;
}

int main() {
    cin >> n;
    FOR(i,n) {
        cin >> op;
        if (op[0] == 'Q') {
            int x; cin >> x;
            cout << get(1, 1, n, x) << "\n";
        } else {
            double k, b; cin >> b >> k;
            line l = line(k, b);
            modify(1, 1, n, l);
        }
    }
}

```

4.9 (5) Line Segment Intersection

```

/**
 * Source:
 * https://open.kattis.com/problems/segmentintersection
 * If numbers are small enough, fractions are recommended.
 */

typedef pair<double,double> pdd;

pii A,B,C,D;

pdd operator*(int x, pdd y) {
    return {x*y.f,x*y.s};
}

pdd operator/(pdd y, int x) {
    return {y.f/x,y.s/x};
}

pdd operator+(pdd l, pdd r) {
    return {l.f+r.f,l.s+r.s};
}

int sgn(pii a, pii b, pii c) {
    return (b.s-a.s)*(c.f-a.f)-(b.f-a.f)*(c.s-a.s);
}

pdd get(pii a, pii b, pii c, pii d) {

```

```

    return (abs(sgn(a,b,c))*d+abs(sgn(a,b,d))*c)
           /(abs(sgn(a,b,c))+abs(sgn(a,b,d)));
}

void solve() {
    cin >> A.f >> A.s >> B.f >> B.s >> C.f >> C.s >>
        D.f >> D.s;
    if (A > B) swap(A,B);
    if (C > D) swap(C,D);
    int a1 = sgn(A,B,C), a2 = sgn(A,B,D);
    if (a1 > a2) swap(a1,a2);
    if (!(a1 <= 0 && a2 >= 0)) {
        cout << "none\n";
        return;
    }
    if (a1 == 0 && a2 == 0) {
        if (sgn(A,C,D) != 0) {
            cout << "none\n";
            return;
        }
        pii x1 = max(A,C), x2 = min(B,D);
        if (x1 > x2) cout << "none\n";
        else if (x1 == x2) cout << (double)x1.f << " " <<
            << (double)x1.s << "\n";
        else cout << (double)x1.f << " " <<
            << (double)x1.s << " " << (double)x2.f << " " <<
            << (double)x2.s << "\n";
        return;
    }
    pdd z = get(A,B,C,D);
    if (mp((double)A.f,(double)A.s) <= z && z <=
        mp((double)B.f,(double)B.s)) cout << z.f << "
        " << z.s << "\n";
    else cout << "none\n";
}

int main() {
    ios_base::sync_with_stdio(0);cin.tie(0);
    int n; cin >> n;
    cout << fixed << setprecision(2);
    FOR(i,n) solve();
}

```

4.10 (6) KD Tree

```

/**
 * Source: KACTL
 * Description: Supports nearest neighbor query in
 *              O(log n) assuming random distribution
 * unused
 */

int t = 0, cur = 0;

struct point {
    ll d[2];
    point(ll x, ll y) {
        d[0] = x, d[1] = y;
    }
    point() {

```

```

        d[0] = 0, d[1] = 0;
    }
};

ll distance(point a, point b) {
    ll d = 0;
    FOR(i,2) d += (a.d[i]-b.d[i])*(a.d[i]-b.d[i]);
    return d;
}

bool comp(point a, point b) {
    return a.d[cur] < b.d[cur];
}

struct node {
    point* pt = NULL;
    point lo, hi;
    node* c[2];
    int ax = 0;

    ll dist(point p) {
        ll d = 0;
        FOR(i,2) {
            if (p.d[i] < lo.d[i]) d +=
                (p.d[i]-lo.d[i])*(p.d[i]-lo.d[i]);
            else if (p.d[i] > hi.d[i]) d +=
                (p.d[i]-hi.d[i])*(p.d[i]-hi.d[i]);
        }
        return d;
    }
}

node(int axis, point low, point high,
    vector<point> p) {
    lo = low, hi = high, ax = axis;
    if (p.size() > 1) {
        cur = ax;
        sort(p.begin(),p.end(),comp);

        int M = p.size()/2;
        while(M > 0 && p[M].d[ax] == p[M-1].d[ax])
            M--;

        point lo1 = lo; lo1.d[ax] = p[M].d[ax];
        point hi1 = hi; hi1.d[ax] = p[M].d[ax]-1;

        if (M) c[0] = new node((ax+1)%2,lo,hi1,
            {p.begin(),p.begin()+M});
        c[1] = new node((ax+1)%2,lo1,hi,
            {p.begin()+M,p.end()});
    } else if (p.size() == 1) {
        pt = new point(p[0]);
    }
}

point get(point p) {
    if (pt) return *pt;
    if (!c[0]) return c[1]->get(p);

    int t = c[0]->dist(p) < c[1]->dist(p) ? 0 : 1;
    point z = c[t]->get(p);
    if (distance(p,z) <= c[t^1]->dist(p)) return z;
}

```

```

    point z1 = c[t^1]->get(p);
    if (distance(p,z) < distance(p,z1)) return z;
    return z1;
}

};

node* root;

int main() {
    ios_base::sync_with_stdio(0);cin.tie(0);
    vector<point> x;
    FOR(i,100000) x.pb(point(rand() % 1000000000,
        rand() % 1000000000));

    root = new
        node(0,point(-MOD,-MOD),point(MOD,MOD),x);
    FOR(i,100000) {
        point y(rand() % 1000000000, rand() %
            1000000000);
        cout << y.d[0] << " " << y.d[1] << " " <<
            root->get(y).d[0] << " " <<
            root->get(y).d[1] << "\n";
    }
}

```

5 Graphs

5.1 (3) Topological Sort

```

int N,M, in[100001];
vi res, adj[100001];

void topo() {
    queue<int> todo;
    FOR(i,1,N+1) if (in[i] == 0) todo.push(i);
    while (todo.size()) {
        int x = todo.front(); todo.pop();
        res.pb(x);
        for (int i: adj[x]) {
            in[i]--;
            if (in[i] == 0) todo.push(i);
        }
    }
}

int main() {
    ios_base::sync_with_stdio(0);cin.tie(0);
    cin >> N >> M;
    FOR(i,M) {
        int x,y; cin >> x >> y;
        adj[x].pb(y), in[y]++;
    }
    topo();
    for (int i: res) cout << i << " ";
}

```

5.2 (5) Biconnected Components

// <http://www.geeksforgeeks.org/biconnected-components/>

```

struct BCC {
    int V, ti = 0;
    vector<vi> adj;
    vi par, disc, low;
    vector<vector<pii>> fin;
    vector<pii> st;

    void init(int _V) {
        V = _V;
        par.resize(V), disc.resize(V), low.resize(V),
            adj.resize(V);
        FOR(i,V) par[i] = disc[i] = low[i] = -1;
    }

    void addEdge(int u, int v) {
        adj[u].pb(v), adj[v].pb(u);
    }

    void BCCutil(int u) {
        disc[u] = low[u] = ti++;
        int child = 0;

        for (int i: adj[u]) if (i != par[u]) {
            if (disc[i] == -1) {
                child++; par[i] = u;
                st.pb({u,i});
                BCCutil(i);
                low[u] = min(low[u],low[i]);

                if ((disc[u] == 0 && child > 1) ||
                    (disc[u] != 0 && disc[u] <=
                     low[i])) { // checks for
                    articulation point
                    vector<pii> tmp;
                    while (st.back() != mp(u,i))
                        tmp.pb(st.back()),
                            st.pop_back();
                    tmp.pb(st.back()), st.pop_back();
                    fin.pb(tmp);
                }
            } else if (disc[i] < low[u]) {
                low[u] = disc[i];
                st.pb({u,i});
            }
        }
    }

    void bcc() {
        FOR(i,V) if (disc[i] == -1) {
            BCCutil(i);
            if (st.size()) fin.pb(st);
            st.clear();
        }
    }
};

int main() {
    ios_base::sync_with_stdio(0);cin.tie(0);

```

```

    BCC g; g.init(12);
    g.addEdge(0,1);
    g.addEdge(1,2);
    g.addEdge(1,3);
    g.addEdge(2,3);
    g.addEdge(2,4);
    g.addEdge(3,4);
    g.addEdge(1,5);
    g.addEdge(0,6);
    g.addEdge(5,6);
    g.addEdge(5,7);
    g.addEdge(5,8);
    g.addEdge(7,8);
    g.addEdge(8,9);
    g.addEdge(10,11);
    g.bcc();

    for (auto a: g.fin) {
        for (pii b: a) cout << b.f << " " << b.s << "
            | ";
        cout << "\n";
    }
}

```

5.3 (5) Kosaraju

```

const int MX = 100001;

struct scc {
    vi adj[MX], radj[MX], todo;
    int comp[MX], N, M;
    bool visit[MX];

    scc() {
        memset(comp,0,sizeof comp);
        memset(visit,0,sizeof visit);
    }

    void dfs(int v) {
        visit[v] = 1;
        for (int w: adj[v]) if (!visit[w]) dfs(w);
        todo.pb(v);
    }

    void dfs2(int v, int val) {
        comp[v] = val;
        for (int w: radj[v]) if (!comp[w]) dfs2(w,val);
    }

    void addEdge(int a, int b) {
        adj[a].pb(b);
        radj[b].pb(a);
    }

    void genSCC() {
        FOR(i,1,N+1) if (!visit[i]) dfs(i);
        reverse(todo.begin(),todo.end());
        for (int i: todo) if (!comp[i]) dfs2(i,i);
    }
};

```

```

scc S;

int main() {
    cin >> S.N >> S.M;
    FOR(i,S.M) {
        int a,b; cin >> a >> b;
        S.addEdge(a,b);
    }
    S.genSCC();
}

```

5.4 (6) Euler Tour

```

// https://open.kattis.com/problems/eulerianpath
// extra log factor

vi circuit;
multiset<int> adj[10000], adj1[10000];
int N,M, out[10000], in[10000];

void find_circuit(int x) { // directed graph, possible
    that resulting circuit is not valid
    while (adj[x].size()) {
        int j = *adj[x].begin();
        adj[x].erase(adj[x].begin());
        find_circuit(j);
    }
    circuit.pb(x);
}

int a,b,start;

void solve() {
    FOR(i,N) {
        adj[i].clear(), adj1[i].clear();
        out[i] = in[i] = 0;
    }
    circuit.clear();
    FOR(i,M) {
        cin >> a >> b;
        adj[a].insert(b), adj1[a].insert(b);
        out[a] ++, in[b] ++;
    }
    start = a;
    FOR(i,N) if (out[i]-in[i] == 1) start = i;

    find_circuit(start);
    reverse(circuit.begin(),circuit.end());

    if (circuit.size() != M+1) {
        cout << "Impossible\n";
        return;
    }

    FOR(i,M) {
        if (adj1[circuit[i]].find(circuit[i+1]) ==
            adj1[circuit[i]].end()) {
            cout << "Impossible\n";
            return;
        }
    }
}

```

```

    }
    int t = circuit[i];
    adj1[t].erase(adj1[t].find(circuit[i+1]));
}
FOR(i,M+1) cout << circuit[i] << " ";
cout << "\n";
}

int main() {
    ios_base::sync_with_stdio(0);cin.tie(0);
    while (cin >> N >> M) {
        if (N+M == 0) break;
        solve();
    }
}

```

5.5 Shortest Path

5.5.1 (3) Bellman-Ford

```

/**
 * Source:
 *   https://open.kattis.com/problems/shortestpath3
 * Useful with linear programming
 */

const ll INF = 1e18;

int n,m,q,s,bad[1000];
vector<pair<pii,int>> edge;
ll dist[1000];

void solve() {
    edge.clear();
    FOR(i,n) dist[i] = INF, bad[i] = 0;
    dist[s] = 0;
    FOR(i,m) {
        int u,v,w; cin >> u >> v >> w;
        edge.pb({{u,v},w});
    }
    FOR(i,n) for (auto a: edge) if (dist[a.f.f] < INF)
        dist[a.f.s] = min(dist[a.f.s],
            dist[a.f.f]+a.s);
    for (auto a: edge) if (dist[a.f.f] < INF) if
        (dist[a.f.s] > dist[a.f.f]+a.s) bad[a.f.s] = 1;
    FOR(i,n) for (auto a: edge) if (bad[a.f.f])
        bad[a.f.s] = 1;

    FOR(i,q) {
        int x; cin >> x;
        if (bad[x]) cout << "-Infinity\n";
        else if (dist[x] == INF) cout <<
            "Impossible\n";
        else cout << dist[x] << "\n";
    }
    cout << "\n";
}

```

5.5.2 (3) Dijkstra

```

template<int SZ> struct Dijkstra {
    int dist[SZ];
    vector<pii> adj[SZ];
    priority_queue<pii,vector<pii>,greater<pii>> q;

    void gen() {
        fill_n(dist,SZ,MOD); dist[0] = 0;

        q.push({0,0});
        while (q.size()) {
            pii x = q.top(); q.pop();
            if (dist[x.s] < x.f) continue;
            for (pii y: adj[x.s]) if (x.f+y.s <
                dist[y.f]) {
                dist[y.f] = x.f+y.s;
                q.push({dist[y.f],y.f});
            }
        }
    }
};

Dijkstra<100> D;

int main() {
    FOR(i,100) FOR(j,100) if (rand() % 10 == 0)
        D.adj[i].pb({j,rand() % 10+1});
    D.gen();
    FOR(i,100) cout << D.dist[i] << "\n";
}

```

5.5.3 (3) Floyd-Warshall

```

/**
 * Source: https://open.kattis.com/problems/allpairspath
 */

const ll INF = 1e18;

int n,m,q; // vertices, edges, queries
ll dist[150][150], bad[150][150];

void solve() {
    FOR(i,n) FOR(j,n) dist[i][j] = INF, bad[i][j] = 0;
    FOR(i,n) dist[i][i] = 0;
    FOR(i,m) {
        int u,v,w; cin >> u >> v >> w;
        dist[u][v] = min(dist[u][v],(ll)w);
    }
    FOR(k,n) FOR(i,n) FOR(j,n) if (dist[i][k] != INF
        && dist[k][j] != INF)
        dist[i][j] =
            min(dist[i][j],dist[i][k]+dist[k][j]);

    FOR(k,n) FOR(i,n) FOR(j,n) if (dist[i][k] != INF
        && dist[k][j] != INF)
        if (dist[i][j] > dist[i][k]+dist[k][j])
            bad[i][j] = 1;
}

```

```

FOR(k,n) FOR(i,n) FOR(j,n) {
    if (dist[i][k] < INF && bad[k][j]) bad[i][j] = 1;
    if (bad[i][k] && dist[k][j] < INF) bad[i][j] = 1;
}

FOR(i,q) {
    int u,v; cin >> u >> v;
    if (bad[u][v]) cout << "-Infinity\n";
    else if (dist[u][v] == INF) cout << "Impossible\n";
    else cout << dist[u][v] << "\n";
}
cout << "\n";
}

```

6 Math

6.1 (5) Chinese Remainder Theorem

```

/**
 * Task: Kattis generalchineseremainder
 */

ll n,m,a,b;
map<ll,pii> M;
bool bad;

ll inv(ll a, ll b) { // 0 < a < b, gcd(a,b) = 1
    a %= b;
    if (a <= 1) return a;
    ll i = inv(b%a,a);
    ll tmp = -((b/a)*i+((b%a)*i)/a) % b;
    while (tmp < 0) tmp += b;
    return tmp;
}

ll naive(ll n, ll m, ll a, ll b) {
    ll x = (a-b)*inv(m,n) % n;
    ll ans = (m*x+b) % (m*n);
    while (ans < 0) ans += (m*n);
    return ans;
}

void process(ll a, ll n) {
    vector<pii> z;
    for (int i = 2; i*i <= n; ++i) if (n % i == 0) {
        int co = 0;
        while (n % i == 0) n /= i, co++;
        z.pb({i,co});
    }
    if (n != 1) z.pb({n,1});
    for (auto A: z) {
        if (M.count(A.f)) {
            pii p1 = M[A.f];
            pii p2 = {A.s,a%(ll)pow(A.f,A.s)};
            if (p1 > p2) swap(p1,p2);

```

```

                if (p2.s%(ll)pow(A.f,p1.f) != p1.s) bad = 1;
                M[A.f] = p2;
            } else M[A.f] = {A.s,a%(ll)pow(A.f,A.s)};
        }
    }

    ll po(ll b, ll p) {
        ll z = 1;
        FOR(i,p) z *= b;
        return z;
    }

    void solve() {
        bad = 0, M.clear();
        long long aa,nn,bb,mm; cin >> aa >> nn >> bb >> mm;
        a = aa, n = nn, b = bb, m = mm;
        process(a,n), process(b,m);
        if (bad) {
            cout << "no solution\n";
            return;
        }
        ll a1 = 0, a2 = 1;
        for (auto& x: M) {
            a1 = naive(a2,po(x.f,x.s.f),a1,x.s.s);
            a2 *= po(x.f,x.s.f);
        }
        cout << (long long)a1 << " " << (long long)a2 << "\n";
    }

    int main() {
        ios_base::sync_with_stdio(0);cin.tie(0);
        int T; cin >> T;
        FOR(i,T) solve();
    }

```

6.2 (5) Combinations

```

template<int SZ> struct Combo {
    ll fac[SZ+1], ifac[SZ+1];

    Combo() {
        fac[0] = ifac[0] = 1;
        FOR(i,1,SZ+1) {
            fac[i] = i*fac[i-1] % MOD;
            ifac[i] = inv(fac[i]);
        }
    }

    ll po (ll b, ll p) {
        return !p?1:po(b*b%MOD,p/2)*(p&1?b:1)%MOD;
    }

    ll inv (ll b) { return po(b,MOD-2); }

    ll comb(ll a, ll b) {
        if (a < b) return 0;
        ll tmp = fac[a]*ifac[b] % MOD;
        tmp = tmp*ifac[a-b] % MOD;
        return tmp;
    }

```

```

    }
};

```

6.3 (5) Eratosthenes' Sieve

// <https://open.kattis.com/problems/primessieve>

```

template<int SZ> struct Sieve {
    bitset<SZ+1> comp;
    Sieve() {
        for (int i = 2; i*i <= SZ; ++i) if (!comp[i]) {
            for (int j = i*i; j <= SZ; j += i) comp[j]
                = 1;
        }
    }
    bool isprime(int x) {
        if (x == 1) return 0;
        return !comp[x];
    }
};

int n,q,ans=0;

int main() {
    ios_base::sync_with_stdio(0);cin.tie(0);
    Sieve<100000000> s; cin >> n >> q;
    FOR(i,1,n+1) if (s.isprime(i)) ans ++;
    cout << ans << "\n";
    FOR(i,q) {
        int x; cin >> x;
        cout << s.isprime(x) << "\n";
    }
}

```

6.4 (5) General Modular Inverse

```

int inv(int a, int b) { // 0 < a < b, gcd(a,b) = 1
    if (a == 1) return 1;
    ll i = inv(b%a,a);
    ll tmp = -((b/a)*i+((b%a)*i)/a) % b;
    while (tmp < 0) tmp += b;
    return tmp;
}

int main() {
    ios_base::sync_with_stdio(0);cin.tie(0);
    cout << inv(25,43); // 31
}

```

6.5 (6) FFT, NTT

```

/**
 * inspired by KACTL
 * https://pastebin.com/3Tnj5mRu
 */

```

```

typedef complex<double> cd;
typedef vector<cd> vcd;
typedef vector<ll> vl;

namespace Poly {
    int get(int s) {
        return s > 1 ? 32 - __builtin_clz(s - 1) : 0;
    }
}

namespace FFT {
    vcd fft(vcd& a) {
        int n = a.size(), x = get(n);
        vcd res, RES(n), roots(n);
        FOR(i,n) roots[i] =
            cd(cos(2*M_PI*i/n),sin(2*M_PI*i/n));

        res = a;
        FOR(i,1,x+1) {
            int inc = n>>i;
            FOR(j,inc) for (int k = 0; k < n; k +=
                inc) {
                int t = 2*k%n+j;
                RES[k+j] =
                    res[t]+roots[k]*res[t+inc];
            }
            swap(res,RES);
        }

        return res;
    }

    vcd fft_rev(vcd& a) {
        vcd res = fft(a);
        FOR(i,sz(res)) res[i] /= a.size();
        reverse(res.begin() + 1, res.end());
        return res;
    }

    vcd brute(vcd& a, vcd& b) {
        vcd c(sz(a)+sz(b)-1);
        FOR(i,sz(a)) FOR(j,sz(b)) c[i+j] +=
            a[i]*b[j];
        return c;
    }

    vcd conv(vcd a, vcd b) {
        int s = sz(a)+sz(b)-1, L = get(s), n = 1<<L;
        if (s <= 0) return {};
        if (s <= 200) return brute(a,b);

        a.resize(n); a = fft(a);
        b.resize(n); b = fft(b);

        FOR(i,n) a[i] *= b[i];
        a = fft_rev(a);

        a.resize(s);
        return a;
    }
}

```

```

namespace NTT {
    const ll mod = (119 << 23) + 1, root = 3; // =
        998244353
    // For p < 2^30 there is also e.g. (5 << 25,
        3), (7 << 26, 3),
    // (479 << 21, 3) and (483 << 21, 5). The last
        two are > 10^9.

    ll modpow(ll b, ll p) { return
        !p?1:modpow(b*b%mod,p/2)*(p&1?b:1)%mod; }

    ll inv (ll b) { return modpow(b,mod-2); }

    vl ntt(vl& a) {
        int n = a.size(), x = get(n);
        vl res, RES(n), roots(n);
        roots[0] = 1, roots[1] =
            modpow(root,(mod-1)/n);
        FOR(i,2,n) roots[i] = roots[i-1]*roots[1] %
            mod;

        res = a;
        FOR(i,1,x+1) {
            int inc = n>>i;
            FOR(j,inc) for (int k = 0; k < n; k +=
                inc) {
                int t = 2*k%n+j;
                RES[k+j] =
                    (res[t]+roots[k]*res[t+inc]) %
                    mod;
            }
            swap(res,RES);
        }

        return res;
    }

    vl ntt_rev(vl& a) {
        vl res = ntt(a);
        ll in = inv(a.size());
        FOR(i,sz(res)) res[i] = res[i]*in % mod;
        reverse(res.begin() + 1, res.end());
        return res;
    }

    vl brute(vl& a, vl& b) {
        vl c(sz(a)+sz(b)-1);
        FOR(i,sz(a)) FOR(j,sz(b)) c[i+j] =
            (c[i+j]+a[i]*b[j])%mod;
        return c;
    }

    vl conv(vl a, vl b) {
        int s = sz(a)+sz(b)-1, L = get(s), n = 1<<L;
        if (s <= 0) return {};
        if (s <= 200) return brute(a,b);

        a.resize(n); a = ntt(a);
        b.resize(n); b = ntt(b);

        FOR(i,n) a[i] = a[i]*b[i] % mod;
        a = ntt_rev(a);

```

```

        a.resize(s);
        return a;
    }
}

int main() {
    ios_base::sync_with_stdio(0);cin.tie(0);

    vcd x =
        Poly::FFT::conv({1,2,3,4,5,6,7,8},{1,2,3,4,5,6,7,8});
    for (auto a: x) cout << a << "\n";
    cout << "\n";

    vl X =
        Poly::NTT::conv({1,2,3,4,5,6,7,8},{1,2,3,4,5,6,7,8});
    for (auto a: X) cout << a << "\n";
}

```

7 Strings

7.1 (3) Hashing

```

typedef pair<ll, ll> pll;

template<class T> pair<T,T> operator+(const pair<T,T>&
    l, const pair<T,T>& r) {
    return {(l.f+r.f)%MOD,(l.s+r.s)%MOD};
}

template<class T> pair<T,T> operator-(const pair<T,T>&
    l, const pair<T,T>& r) {
    return {(l.f-r.f+MOD)%MOD,(l.s-r.s+MOD)%MOD};
}

template<class T> pair<T,T> operator*(const pair<T,T>&
    l, const T& r) {
    return {l.f*r%MOD,l.s*r%MOD};
}

template<class T> pair<T,T> operator*(const pair<T,T>&
    l, const pair<T,T>& r) {
    return {l.f*r.f%MOD,l.s*r.s%MOD};
}

struct hsh {
    string S;
    vector<pll> po, ipo, cum;
    pll base = mp(948392576,573928192);

    ll modpow(ll b, ll p) {
        return !p?1:modpow(b*b%MOD,p/2)*(p&1?b:1)%MOD;
    }

    ll inv(ll x) {
        return modpow(x,MOD-2);
    }
}

```



```

void gen(string _S) {
    S = _S;
    po.resize(sz(S)), ipo.resize(sz(S)),
    cum.resize(sz(S)+1);
    po[0] = ipo[0] = {1,1};
    FOR(i,1,sz(S)) {
        po[i] = po[i-1]*base;
        ipo[i] = {inv(po[i].f),inv(po[i].s)};
    }
    FOR(i,sz(S)) cum[i+1] =
        cum[i]+po[i]*(11)(S[i]-'a'+1);
}

pll get(int l, int r) {
    return ipo[l]*(cum[r+1]-cum[l]);
}

};

int lcp(hsh& a, hsh& b) {
    int lo = 0, hi = min(sz(a.S),sz(b.S));
    while (lo < hi) {
        int mid = (lo+hi+1)/2;
        if (a.get(0,mid-1) == b.get(0,mid-1)) lo = mid;
        else hi = mid-1;
    }
    return lo;
}

int main() {
    string _S = "abacaba";
    hsh h; h.gen(_S);
    FOR(i,sz(_S)) FOR(j,i,sz(_S)) cout << i << " " <<
        j << " " << h.get(i,j).f << " " <<
        h.get(i,j).s << "\n";

    hsh H; H.gen("abadaba");
    cout << lcp(h,H);
}

```

7.2 (4) Bitset Trie

```

template<int MX> struct tri {
    int nex = 0, ans = 0;
    int trie[MX][2]; // easily changed to character

    tri() {
        memset(trie,0,sizeof trie);
    }

    void ins(int x) {
        int cur = 0;
        FORd(i,30) {
            int t = (x&(1<<i))>>i;
            if (!trie[cur][t]) trie[cur][t] = ++nex;
            cur = trie[cur][t];
        }
    }

    void test(int x) {
        int cur = 0;

```

```

        FORd(i,30) {
            int t = ((x&(1<<i))>>i) ^ 1;
            if (!trie[cur][t]) t ^= 1;
            cur = trie[cur][t];
            if (t) x ^= (1<<i);
        }
        ans = max(ans,x);
    }
};

int main() {
}

```

7.3 (5) Aho-Corasick

```

// Source: GeeksForGeeks
// also see https://ideone.com/0cMjZJ

string arr[200];
int val[200], states = 1;
queue<int> update;
const int MAXS = 201;
const int MAXC = 26;

int n, out[MAXS], f[MAXS], g[MAXS][MAXC];

int buildMatchingMachine() {
    memset(out, 0, sizeof out);
    memset(g, -1, sizeof g);

    FOR(i,n) {
        string word = arr[i];
        int currentState = 0;

        FOR(j,word.size()) {
            int ch = word[j] - 'a';
            if (g[currentState][ch] == -1)
                g[currentState][ch] = states++;
            currentState = g[currentState][ch];
        }

        out[currentState] += val[i];
    }

    FOR(ch,MAXC) if (g[0][ch] == -1) g[0][ch] = 0;
    memset(f, -1, sizeof f);
    queue<int> q;

    FOR(ch,MAXC)
        if (g[0][ch] != 0) {
            f[g[0][ch]] = 0;
            q.push(g[0][ch]);
        }

    while (q.size()) {
        int state = q.front();
        q.pop();

        FOR(ch,MAXC) {

```

```

        if (g[state][ch] != -1) {
            int failure = f[state];
            while (g[failure][ch] == -1) failure =
                f[failure];

            failure = g[failure][ch];
            f[g[state][ch]] = failure;

            out[g[state][ch]] += out[failure];

            q.push(g[state][ch]);
        }
    }

    return states;
}

int findNextState(int currentState, char nextInput) {
    int answer = currentState;
    int ch = nextInput - 'a';

    while (g[answer][ch] == -1){
        update.push(answer);
        answer = f[answer];
    }
    if (update.size()){
        while (update.size()){
            int k = update.front(); update.pop();
            g[k][nextInput-'a'] = g[answer][ch];
            //cache state transitions: often
            //necessary if we don't want to
            //explicitly compute all of them
        }
    }
    return g[answer][ch];
}

int main() {

```

7.4 (5) Manacher

// Source: <http://codeforces.com/blog/entry/12143>
 // Calculates length of largest palindrome centered at
 // each character of string

```

vi manacher(string s) {
    string s1 = "@";
    for (char c: s) s1 += c, s1 += "#";
    s1[s1.length()-1] = '&';

    vi ans(s1.length()-1);
    int lo = 0, hi = 0;
    FOR(i,1,s1.length()-1) {
        ans[i] = min(hi-i,ans[hi-i+lo]);
        while (s1[i-ans[i]-1] == s1[i+ans[i]+1])
            ans[i] ++;
    }
}

```

```

        if (i+ans[i] > hi) lo = i-ans[i], hi =
            i+ans[i];
    }

    ans.erase(ans.begin());
    FOR(i,ans.size()) if ((i&1) == (ans[i]&1)) ans[i]
        ++; // adjust lengths
    return ans;
}

int main() {
    ios_base::sync_with_stdio(0);cin.tie(0);
    vi a1 = manacher("abacaba");
    for (int i: a1) cout << i << " ";
    cout << "\n";

    vi a2 = manacher("aabbaaccaabbbaa");
    for (int i: a2) cout << i << " ";
}

```

7.5 (5) Palindromic Tree

```

/**
 * Source: http://codeforces.com/blog/entry/13959
 * Unused
 */

const int maxn = 1e5, sigma = 26;

int s[maxn], len[maxn], link[maxn], to[maxn][sigma];

int n, last, sz;

void init() {
    s[n++] = -1;
    link[0] = 1;
    len[1] = -1;
    sz = 2;
}

int get_link(int v) {
    while(s[n-len[v]-2] != s[n-1]) v = link[v];
    return v;
}

void add_letter(int c) {
    s[n++] = c;
    last = get_link(last);
    if (!to[last][c]) {
        len[sz] = len[last]+2;
        link[sz] = to[get_link(link[last])][c];
        to[last][c] = sz++;
    }
    last = to[last][c];
}

```

7.6 (5) Z

```
// Source: http://codeforces.com/blog/entry/3107
// better than KMP?
```

```
vi z(string s) {
    int N = s.length(); s += '#';
    vi ans(N); ans[0] = N;
    while (s[1+ans[1]] == s[ans[1]]) ans[1] ++;

    int L = 1, R = ans[1];
    FOR(i,2,N) {
        if (i <= R) ans[i] = min(R-i+1,ans[i-L]);
        while (s[i+ans[i]] == s[ans[i]]) ans[i] ++;
        if (i+ans[i]-1 > R) L = i, R = i+ans[i]-1;
    }
    return ans;
}

vi get(string a, string b) { // find prefixes of a in b
    string s = a+"@"+b;
    vi t = z(s);
    return vi(t.begin()+a.length()+1,t.end());
}

int main() {
    ios_base::sync_with_stdio(0);cin.tie(0);
    vi x = z("abcababcabcaba");
    for (int i: x) cout << i << " ";
    cout << "\n";

    x = get("abcab","uwetrabcerabcab");
    for (int i: x) cout << i << " ";
}
```

7.7 (6) Booth

```
/**
 * Source: Wikipeddia
 * unused
 */

int least_rotation(string S) {
    S += S;
    vi f = vi(S.length(),-1);
    int k = 0;
    FOR(j,1,S.length()) {
        char sj = S[j];
        int i = f[j-k-1];
        while (i != -1 and sj != S[k+i+1]) {
            if (sj < S[k+i+1]) k = j-i-1;
            i = f[i];
        }
        if (sj != S[k+i+1]) {
            if (sj < S[k]) k = j;
            f[j-k] = -1;
        } else f[j-k] = i+1;
    }
    return k;
}
```

```
int main() {
    ios_base::sync_with_stdio(0);cin.tie(0);
    cout << least_rotation("cccaabb");
}
```

7.8 (6) Suffix Array

```
/**
 * Source: SuprDewd CP Course
 * Task: https://open.kattis.com/problems/suffixsorting
 */

struct suffix_array {
    int N;
    vector<vi> P;
    vector<pair<pii,int>> L;
    vi idx;
    string str;

    suffix_array(string _str) {
        str = _str; N = str.length();
        P.pb(vi(N)); L.resize(N);
        FOR(i,N) P[0][i] = str[i];

        for (int stp = 1, cnt = 1; cnt < N; stp ++,
            cnt *= 2) {
            P.pb(vi(N));
            FOR(i,N) L[i] = {{P[stp-1][i],i+cnt < N ?
                P[stp-1][i+cnt] : -1},i};
            sort(L.begin(),L.end());
            FOR(i,N) {
                if (i && L[i].f == L[i-1].f)
                    P[stp][L[i].s] = P[stp][L[i-1].s];
                else P[stp][L[i].s] = i;
            }

            idx.resize(N);
            FOR(i,P[P.size()-1].size())
                idx[P[P.size()-1][i]] = i;
        }

        int lcp(int x, int y) {
            int res = 0;
            if (x == y) return N-x;
            for (int k = P.size() - 1; k >= 0 && x < N &&
                y < N; k--) {
                if (P[k][x] == P[k][y]) {
                    x += 1 << k;
                    y += 1 << k;
                    res += 1 << k;
                }
            }
            return res;
        }
    };

    int main() {
        ios_base::sync_with_stdio(0);cin.tie(0);
        string s;
```

```

while (getline(cin,s)) {
    if (s[s.length()-1] == '\r')
        s.erase(s.end()-1);
    suffix_array sa = suffix_array(s);
    int z; cin >> z;
    FOR(i,z) {
        int x; cin >> x;
        cout << sa.idx[x] << " ";
    }
    cout << "\n";
    getline(cin,s);
}
}

```

8 Trees

8.1 (3) DSU, Kruskal

// compute the minimum spanning tree in $O(E \log E)$ time
// https://en.wikipedia.org/wiki/Kruskal%27s_algorithm

```

template<int SZ> struct DSU {
    int par[SZ], sz[SZ];
    DSU() {
        FOR(i,SZ) par[i] = i, sz[i] = 1;
    }

    int get(int x) { // path compression
        if (par[x] != x) par[x] = get(par[x]);
        return par[x];
    }

    bool unite(int x, int y) { // union-by-rank
        x = get(x), y = get(y);
        if (x == y) return 0;
        if (sz[x] < sz[y]) swap(x,y);
        sz[x] += sz[y], par[y] = x;
        return 1;
    }
};

int ans = 0;
vector<pair<int,pii>> edge;

DSU<100> D;

void kruskal() {
    sort(edge.begin(),edge.end());
    for (auto a: edge) if (D.unite(a.s.f,a.s.s))
        ans += a.f;
}

int main() {
    FOR(i,100) FOR(j,i+1,100) if (rand() % 5 == 0)
        edge.pb({rand() % 100+1,{i,j}});
    kruskal();
    cout << D.sz[D.get(5)] << " " << ans;
}

```

8.2 (4) Centroid Decomposition

```

const int MX = 100001;

int N, visit[MX], sub[MX], par[MX];
vi adj[MX];

void dfs (int no) {
    sub[no] = 1;
    for (int i: adj[no]) if (!visit[i] && i !=
        par[no]) {
        par[i] = no;
        dfs(i);
        sub[no] += sub[i];
    }
}

int get_centroid(int x) {
    par[x] = 0;
    dfs(x);
    int sz = sub[x];
    while (1) {
        pii mx = {0,0};
        for (int i: adj[x]) if (!visit[i] && i !=
            par[x]) mx = max(mx,{sub[i],i});
        if (mx.f*2 > sz) x = mx.s;
        else return x;
    }
}

void solve (int x) {
    x = get_centroid(x); visit[x] = 1;
    // do stuff
    cout << x << "\n";
    for (int i: adj[x]) if (!visit[i]) solve(i);
}

int main() {
    ios_base::sync_with_stdio(0);cin.tie(0);
    cin >> N;
    FOR(i,N-1) {
        int a,b; cin >> a >> b;
        adj[a].pb(b), adj[b].pb(a);
    }
    solve(1);
}

```

8.3 (4) HLD

```

/**
 * Source: http://codeforces.com/blog/entry/22072
 * Task: USACO Grass Planting
 */

// insert LazySegTree Template

vector<vi> graph;

```

```

template <int V> struct HeavyLight { // sum queries,
    sum updates
    int parent[V], heavy[V], depth[V];
    int root[V], treePos[V];
    LazySegTree<V> tree;

    void init() {
        int n = graph.size();
        FOR(i,1,n+1) heavy[i] = -1;
        parent[1] = -1, depth[1] = 0;
        dfs(1);
        for (int i = 1, currentPos = 0; i <= n; ++i)
            if (parent[i] == -1 ||
                heavy[parent[i]] != i)
                for (int j = i; j != -1;
                    j = heavy[j]) {
                    root[j] = i;
                    treePos[j] =
                        currentPos++;
                }
    }

    int dfs(int v) {
        int size = 1, maxSubtree = 0;
        for (auto u : graph[v]) if (u != parent[v]) {
            parent[u] = v;
            depth[u] = depth[v] + 1;
            int subtree = dfs(u);
            if (subtree > maxSubtree) heavy[v] = u,
                maxSubtree = subtree;
            size += subtree;
        }
        return size;
    }

    template <class BinaryOperation>
    void processPath(int u, int v, BinaryOperation op)
    {
        for (; root[u] != root[v]; v =
            parent[root[v]]) {
            if (depth[root[u]] > depth[root[v]])
                swap(u, v);
            op(treePos[root[v]], treePos[v]);
        }
        if (depth[u] > depth[v]) swap(u, v);
        op(treePos[u]+1, treePos[v]); // assumes
            values are stored in edges, not vertices
    }

    void modifyPath(int u, int v, int value) {
        processPath(u, v, [this, &value](int l, int r) {
            tree.upd(l, r, value); });
    }

    ll queryPath(int u, int v) {
        ll res = 0;
        processPath(u, v, [this, &res](int l, int r) {
            res += tree.qsum(l, r); });
        return res;
    }
};

```

```

HeavyLight<1<<17> H;
int N,M;

int main() {
    cin >> N >> M;
    graph.resize(N+1);
    FOR(i,N-1) {
        int a,b; cin >> a >> b;
        graph[a].pb(b), graph[b].pb(a);
    }
    H.init();
    FOR(i,M) {
        char c; int A,B;
        cin >> c >> A >> B;
        if (c == 'P') H.modifyPath(A,B,1);
        else cout << H.queryPath(A,B) << "\n";
    }
}

```

8.4 (4) LCA with Binary Jumps

```

const int MAXN = 100001, MAXK = 17;

int Q;

struct LCA {
    int V;
    vi edges[MAXN];
    int parK[MAXN][MAXN];
    int depth[MAXN];

    void addEdge(int u, int v) {
        edges[u].pb(v), edges[v].pb(u);
    }

    void dfs(int u, int prev){
        parK[0][u] = prev;
        depth[u] = depth[prev]+1;
        for (int v: edges[u]) if (v != prev) dfs(v, u);
    }

    void construct() {
        dfs(1, 0);
        FOR(k,1,MAXK) FOR(i,1,V+1)
            parK[k][i] = parK[k-1][parK[k-1][i]];
    }

    int lca(int u, int v){
        if (depth[u] < depth[v]) swap(u,v);

        FORd(k,MAXK) if (depth[u] >= depth[v]+(1<<k))
            u = parK[k][u];
        FORd(k,MAXK) if (parK[k][u] != parK[k][v]) u =
            parK[k][u], v = parK[k][v];

        if (u != v) u = parK[0][u], v = parK[0][v];
        return u;
    }

    int dist(int u, int v) {

```

```

        return depth[u]+depth[v]-2*depth[lca(u,v)];
    }
};

LCA L;

int main(){
    cin >> L.V >> Q;
    FOR(i,L.V-1) {
        int u,v; cin >> u >> v;
        L.addEdge(u,v);
    }
    L.construct();

    FOR(i,Q) {
        int u,v; cin >> u >> v;
        cout << L.dist(u,v) << "\n";
    }
}

```

8.5 (4) LCA with RMQ

```

// Euler Tour LCA w/ O(1) query

const int MAXN = 100001, MAXK = 17;

int Q;

struct RMQ2 {
    vi edges[MAXN];
    pii rmq[MAXK][2*MAXN];
    int depth[MAXN], pos[MAXN];

    int N, nex=0;

    void addEdge(int u, int v) {
        edges[u].pb(v), edges[v].pb(u);
    }

    void dfs(int u, int prev){
        pos[u] = nex; depth[u] = depth[prev]+1;
        rmq[0][nex++] = {depth[u],u};
        for (int v: edges[u]) if (v != prev) {
            dfs(v, u);
            rmq[0][nex++] = {depth[u],u};
        }
    }

    void construct() {
        dfs(1, 0);
        FOR(k,1,MAXK) FOR(i,nex) if (i+(1<<(k-1)) <
            nex) rmq[k][i] =
            min(rmq[k-1][i],rmq[k-1][i+(1<<(k-1))]);
    }

    int lca(int u, int v){
        u = pos[u], v = pos[v];
        if (u > v) swap(u,v);
        int x = 31-__builtin_clz(v-u+1);
        return min(rmq[x][u],rmq[x][v-(1<<x)+1]).s;
    }
}

```

```

    }

    int dist(int u, int v) {
        return depth[u]+depth[v]-2*depth[lca(u,v)];
    }
};

RMQ2 R;

int main(){
    cin >> R.N >> Q;
    FOR(i,R.N-1) {
        int u,v; cin >> u >> v;
        R.addEdge(u,v);
    }
    R.construct();

    FOR(i,Q) {
        int u,v; cin >> u >> v;
        cout << R.dist(u,v) << "\n";
    }
}

```
