Correction in Winn's thesis

DIRICHLET DISTRIBUTION:

$$\log p(x;\vec{u},\vec{p}) = \begin{bmatrix} u_1-1 \\ \vdots \\ u_K-1 \end{bmatrix}^T \begin{bmatrix} \log p_1 \\ \vdots \\ \log p_K \end{bmatrix} + \log\Gamma(\underset{\sum_{i=1}^{K} u_i}{\underset{\parallel}{u}}) - \sum_{i=1}^{K} \underline{\log}\,\Gamma(u_i) =$$

$$= \vec{u}\log\vec{p} + \log\Gamma\left(\sum_{i=1}^{K} u_i\right) - \sum_{i=1}^{K}\log\Gamma(u_i)$$

(from Wikipedia)

$$= \log\left(\frac{\Gamma\left(\sum_{i=1}^{K} u_i\right)}{\prod_{i=1}^{K}\Gamma(u_i)} \prod_{i=1}^{K} p_i^{u_i-1}\right) =$$

$$= \log\Gamma\left(\sum_{i=1}^{K} u_i\right) - \log\prod_{i=1}^{K}\Gamma(u_i) + \log\prod_{i=1}^{K} p_i^{u_i-1} =$$

$$= \log\Gamma\left(\sum_{i=1}^{K} u_i\right) - \sum_{i=1}^{K}\log\Gamma(u_i) + \sum_{i=1}^{K}(u_i-1)\log p_i =$$

$$= \vec{u}\log\vec{p} + \log\Gamma\left(\sum_{i=1}^{K} u_i\right) - \sum_{i=1}^{K}\log\Gamma(u_i)$$

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* FROM NATURAL TO MOMENT PARAMETERS:

$u_i - 1 = \theta_i \; ; \; u_i = \theta_i + 1$

$$\boxed{\mu_i} = \frac{\partial A(\theta_1, \dots \theta_c)}{\partial \theta_i} = \partial / \partial \theta_i \left(-\log \Gamma\left(\sum_{i=1}^{k} (\theta_i + 1)\right) + \sum_{i=1}^{k} \log \Gamma(\theta_i + 1) \right) =$$

according to Winn's thesis

$$= -\Psi\left(\sum_{i=1}^{k} (\theta_i + 1)\right) + \Psi(\theta_i + 1) =$$

$$= \boxed{\Psi(u_i) - \Psi\left(\sum_{i=1}^{k} u_i\right)}$$

Help: $\frac{\partial \ln \Gamma(g(\theta))}{\partial \theta} = \Psi(g(\theta)) \frac{\partial g(\theta)}{\partial (\theta)}$

* FROM MOMENT TO NATURAL PARAMETERS:

$$\theta^* = \arg\max_{u_i} \begin{pmatrix} u_1 - 1 \\ \vdots \\ u_K - 1 \end{pmatrix} \begin{pmatrix} \log p_1 \\ \vdots \\ \log p_K \end{pmatrix} + \log \Gamma(u) - \sum_{i=1}^{K} \log \Gamma(u_i)$$

$$\frac{\partial \theta^*}{\partial u_i} = \log p_i + \Psi(u_i) - \Psi\left(\sum_{i=1}^{k} u_i\right) = 0$$

assuming $\Psi(u_i) \simeq \ln(u_i)$, $\Psi\left(\sum_{i=1}^{k} u_i\right) = \ln\left(\sum_{i=1}^{k} u_i\right)$

$$\log p_i = -\ln(u_i) + \ln\left(\sum_{i=1}^{k} u_i\right);$$

$$p_i = \frac{\sum_{i=1}^{k} u_i}{u_i}$$