1 Binary Child given one binary parent

Let X and Y be two binary variables, where the conditional probability of Y given X is expressed as follows:

$$p(Y|X) = p_{y|x}^{I(X=0)I(Y=0)} p_{\bar{y}|x}^{I(X=0)I(Y=1)} p_{y|\bar{x}}^{I(X=1)I(Y=0)} p_{\bar{y}|\bar{x}}^{I(X=1)I(Y=1)}$$

This conditional probability can be expressed in different exponential forms as follows:

$$\begin{aligned}
\ln p(Y|X) &= \theta(X)^T s(Y) - A(X) \\
&= \begin{pmatrix} I(X=0)\ln p_{y|x} + I(X=1)\ln p_{y|\bar{x}} \\ I(X=0)\ln p_{\bar{y}|x} + I(X=1)\ln p_{\bar{y}|\bar{x}} \end{pmatrix}^T \begin{pmatrix} I(Y=0) \\ I(Y=1) \end{pmatrix} - 0
\end{aligned}$$

$$\begin{aligned}
\ln p(Y|X) &= \theta(Y)^T s(X) - A(Y) \\
&= \begin{pmatrix} I(Y=0)\ln p_{y|x} + I(Y=1)\ln p_{\bar{y}|x} \\ I(Y=0)\ln p_{y|\bar{x}} + I(Y=1)\ln p_{\bar{y}|\bar{x}} \end{pmatrix}^T \begin{pmatrix} I(X=0) \\ I(X=1) \end{pmatrix} - 0
\end{aligned}$$

$$\begin{aligned}
\ln p(Y|X) &= \theta^T s(Y,X) - A(\theta) \\
&= \begin{pmatrix} \ln p_{y|x} \\ \ln p_{\bar{y}|x} \\ \ln p_{y|\bar{x}} \\ \ln p_{\bar{y}|\bar{x}} \end{pmatrix}^T \begin{pmatrix} I(Y=0)I(X=0) \\ I(Y=1)I(X=0) \\ I(Y=0)I(X=1) \\ I(Y=1)I(X=1) \end{pmatrix} - 0
\end{aligned}$$

2 Multinomial Child given a set of multinomial parents

Let us Y be a multinomial variable whose state space is $\{y_1, ..., y_k\}$. Let us also be $X_1, \ldots, X_p$ the set of parents of variable Y, all them also multinomial. The log-conditional probability of Y given X can be expressed as follows:

$$\ln p(Y|X_1, \ldots, X_p) =$$

Similarly the above conditional probability can be expressed in the following exponential forms:

$$\ln p(Y|X_1, \ldots, X_p) = \theta(X_1, \ldots, X_p)^T s(Y) - A(X_1, \ldots, X_p)$$

$$\ln p(Y|X_1,\ldots,X_p) \quad = \quad \theta(Y,X_1,\ldots,X_{L-1},X_L,\ldots,X_p)^T s(X_L) - A(Y,X_1,\ldots,X_{L-1},X_L,\ldots,X_p)$$

$$\ln p(Y|X_1,\ldots,X_p) \quad = \quad \theta^T s(Y,X_1,\ldots,X_p) - A(\theta)$$