NORMAL DISTRIBUTION:

$$\log p(x;\mu,\sigma^2) = \begin{pmatrix} \frac{\mu}{\sigma^2} \\ -\frac{1}{2\sigma^2} \end{pmatrix}^T \begin{pmatrix} x \\ x^2 \end{pmatrix} + \frac{1}{2}\left(-2\log\sigma - \frac{\mu^2}{\sigma^2} - \log 2\pi\right) =$$

$$= \frac{\mu}{\sigma^2}x - \frac{x^2}{2\sigma^2} - \log\sigma - \frac{\mu^2}{2\sigma^2} - 0.5\log 2\pi =$$

$$= \log\left(\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}\right) =$$

$$= -\log(\sigma(2\pi)^{1/2}) + \left(-\frac{(x-\mu)^2}{2\sigma^2}\right) =$$

$$= -\log\sigma - 0.5\log(2\pi) - \frac{x^2 + 2x\mu - \mu^2}{2\sigma^2} =$$

$$= \frac{\mu x}{\sigma^2} - \frac{x^2}{2\sigma^2} - \log\sigma - \frac{\mu^2}{2\sigma^2} - 0.5\log(2\pi)$$

2nd form (messages from μ and σ^2 vars. to x)

$$\log p(x|\mu,\sigma^2) = \theta(\mu,\sigma^2)\, s(x) - A(\theta(\mu,\sigma^2)) + h(x) =$$

1st moment param. (not 2nd)

$$= \begin{pmatrix} \frac{\mu}{\sigma^2} \\ -\frac{1}{2\sigma^2} \end{pmatrix}^T \begin{pmatrix} x \\ x^2 \end{pmatrix} - \left(\log\sigma + \frac{\mu^2}{2\sigma^2}\right) - 0.5\log 2\pi$$

μ and σ^2 are the moment param. of these vars received.

3erd form (messages from x to μ): (like before but changing x for μ)

$$\log p(x|\mu,\sigma^2) = \theta(x,\sigma^2)\, s(\mu) - A(\theta(x,\sigma^2)) + h(\sigma^2,\mu) =$$

$$= \begin{pmatrix} \frac{x}{\sigma^2} \\ -\frac{1}{2\sigma^2} \end{pmatrix} \begin{pmatrix} \mu \\ \mu^2 \end{pmatrix} - \left(\log\sigma + \frac{x^2}{2\sigma^2}\right) - 0.5\log 2\pi$$

3rd form (messages from x to σ^{-2})

$$\log(x \mid \mu, \sigma^{-2}) = \theta(x, \mu)\, s(\sigma^{-2}) - A(\theta(x, \mu)) + h(\mu, \sigma^{-2}) =$$

$$= \begin{pmatrix} -\frac{1}{2} \\ -\frac{(x-\mu)^2}{2} \end{pmatrix} \begin{pmatrix} \log \sigma^{-2} \\ \frac{1}{\sigma^{-2}} \end{pmatrix} - 0.5 \ln(2\pi)$$