

Let X and Y be two binary variables, where the conditional probability of Y given X is expressed as follows:

$$p(Y|X) = p_{y|x}^{I(X=0)I(Y=0)} p_{\bar{y}|x}^{I(X=0)I(Y=1)} p_{y|\bar{x}}^{I(X=1)I(Y=0)} p_{\bar{y}|\bar{x}}^{I(X=1)I(Y=1)}$$

This conditional probability can be expressed in different exponential forms as follows:

$$\begin{aligned} \ln p(Y|X) &= \theta(X)^T s(Y) - A(X) \\ &= \begin{pmatrix} I(X=0) \ln p_{y|x} + I(X=1) \ln p_{\bar{y}|x} \\ I(X=0) \ln p_{y|\bar{x}} + I(X=1) \ln p_{\bar{y}|\bar{x}} \end{pmatrix}^T \begin{pmatrix} I(Y=0) \\ I(Y=1) \end{pmatrix} - 0 \end{aligned}$$

$$\begin{aligned} \ln p(Y|X) &= \theta(Y)^T s(X) - A(Y) \\ &= \begin{pmatrix} I(Y=0) \ln p_{y|x} + I(Y=1) \ln p_{\bar{y}|x} \\ I(Y=0) \ln p_{y|\bar{x}} + I(Y=1) \ln p_{\bar{y}|\bar{x}} \end{pmatrix}^T \begin{pmatrix} I(X=0) \\ I(X=1) \end{pmatrix} - 0 \end{aligned}$$

$$\begin{aligned} \ln p(Y|X) &= \theta^T s(Y, X) - A(\theta) \\ &= \begin{pmatrix} \ln p_{y|x} \\ \ln p_{\bar{y}|x} \\ \ln p_{y|\bar{x}} \\ \ln p_{\bar{y}|\bar{x}} \end{pmatrix}^T \begin{pmatrix} I(Y=0)I(X=0) \\ I(Y=1)I(X=0) \\ I(Y=0)I(X=1) \\ I(Y=1)I(X=1) \end{pmatrix} - 0 \end{aligned}$$