# Application of the Conjugate Lindley's Utility Function to a Three-Parameter Multinomial distribution

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***Abstract*— The problem of this research becomes more complicated when the number of estimated parameters increases, which makes the estimation process numerical because it is difficult to obtain the analytical formulas. the Bayesian method estimates the coefficients of probability distributions with random variables. When estimating by Bayesian method, the estimation is either direct, using loss functions, or using utility functions, unlike other estimation methods. The issue, however gets complicated as the number of estimated parameters increases, which makes the estimation process numerical because it is difficult to obtain analytical formulas. In this paper, the paired Lindley function of three coefficients was used and applied to a multinomial distribution, the parameters were non-numeric, when estimating parameters in this way, the size of the conjugate Lindley utility function increases when the optimal approximate decisions are obtained.**

***Keywords—Conjugate Lindley's Utility function, Utility function, Multinomial distribution.***

## I. Introduction

In this paper, the parameters of the three-parameter multinomial distribution will be estimated similarly to the three-parameter exponential family by maximizing the conjugate Lindley utility function. The results will be for the estimated parameters with close values, which makes the amount of bias increase exponentially, When using the conjugate Lindley utility function, it will reduce the initial values so that it is easier to get the values of the estimated parameters., In 1961 [5],[6] the initial distribution of the conjugate function was proposed, which had the great merit of creating and finding this function, In 1976 [4] the scientist Lindley proposed an important function to be applied to the continuous and discrete probability distributions through which we can find the estimated parameters and it was called the conjugated Lindley utility function, by writing the probability distributions and the possibility function and the primary distribution in the form of the exponential family, In 2011 [2] the researcher worked on extending the Lindley function using it in some continuous and discrete probability distributions using only two parameters, as he developed the steps of the Lindley world using the conjugate Lindley function of the one-variable exponential family and growing it into two variables, The researcher [1] in 1973 used distribution functions such as the causal distribution and the exponential distribution to represent utility functions to get the appropriate decision, The researchers [3] in 2021 estimated the coefficients of the polynomial distribution by suggesting the Bayesian inference method.

## II. Conjugate Lindley's Utility Function:

The conjugated Lindley utility function is one of the important functions developed by the world Lindley in 1976, and the parameters can be estimated using the Bayesian method in the probability distributions and using the conjugated Lindley utility function with the exponential family, we can extend Lindley's idea by using discrete and continuous probability distributions with more than one parameter. And that we are going to extend the function of the conjugated Lindley utility for three parameters in proportion to the exponential family of three parameters. Then we will work to get the approximate optimal resolution of the three parameters which are the parameters of the estimated exponential family.

The density function can be written as $p(x|\underline{\theta})$ in the following form:

$$p(x|\underline{\theta}) = exp\{T_1(x)\omega_1(\underline{\theta}) + T_2(x)\omega_2(\underline{\theta}) + T_3(x)\omega_3(\underline{\theta})\} W(\underline{\theta})H(x) \quad (1)$$

$$W(\underline{\theta})^{-1} = \int exp\{T_1(x)\omega_1(\underline{\theta}) + T_2(x)\omega_2(\underline{\theta}) + T_3(x)\omega_3(\underline{\theta})\} H(x)dx \quad (2)$$

Where: $H(x)$ is nonnegative function.

$\underline{\theta} = \omega_1(\underline{\theta}), \omega_2(\underline{\theta}), \omega_3(\underline{\theta}), \underline{T}(x) = T_1(x), T_2(x), T_3(x)$

are jointly sufficient for ( $\omega_1(\underline{\theta}), \omega_2(\underline{\theta}), \omega_3(\underline{\theta})$ ).

The likelihood function will be written as

$$p(\underline{x}|\underline{\theta}) = exp\{\textstyle\sum_{i=1}^n T_1(x_i)\omega_1(\underline{\theta}) + \sum_{i=1}^n T_2(x_i)\omega_2(\underline{\theta}) + \sum_{i=1}^n T_3(x_i)\omega_3(\underline{\theta})\} \times W(\underline{\theta})^n \prod_{i=1}^n H(x_i) \quad (3)$$

The initial probability distribution function for (θ) can be written in the form of the conjugated exponential family (prior distribution) [5],[6] in the following form: