data_generation

November 27, 2025

1 Data Generation

Create a script that generates random data from different distributions. Compare: (a) a normal or Gaussian distribution for different values of the variance and mean, (b) a uniformly random distribution, (c) the beta distribution.

[1]:
```
# Generate the data required by problems 1, 2 and 3
# Ouput is a csv file with one column for each combination of distribution and␣
 ↪corresponding variations

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

# General parameters

N = 100000                  # Number of samples
Seed = 100013059          # Seed for reproducibility
output_file_path = f"./data_{N}_{Seed}.csv"
plot = True

rng = np.random.default_rng(seed=Seed)
df = pd.DataFrame()
df.index.name = "sample"

def plot_columns(df, ncols, nrows, title, figsize=(12, 9), max_height=1):
    fig, axes = plt.subplots(nrows, ncols, figsize=figsize)
    fig.suptitle(title)
    for i in range(nrows):
        for j in range(ncols):
            index = i*ncols + j
            data = df[df.columns[index]]
            subtitle = df.columns[index]
            ax = axes[i, j]
            ax.set_title(subtitle)
            ax.set_xlabel("x")
            # split the x range in 50 bins
            counts, bin_edges = np.histogram(data, bins=50, density=True)
```

```
            # use the center of the bins for representation
            x = (bin_edges[:-1] + bin_edges[1:]) / 2
            ax.plot(x,counts,label="PDF")
            ax.set_ylim(0, max_height)
            ax.legend()
    plt.tight_layout()
    plt.show()
```

1.1 Normal/Gaussian distribution

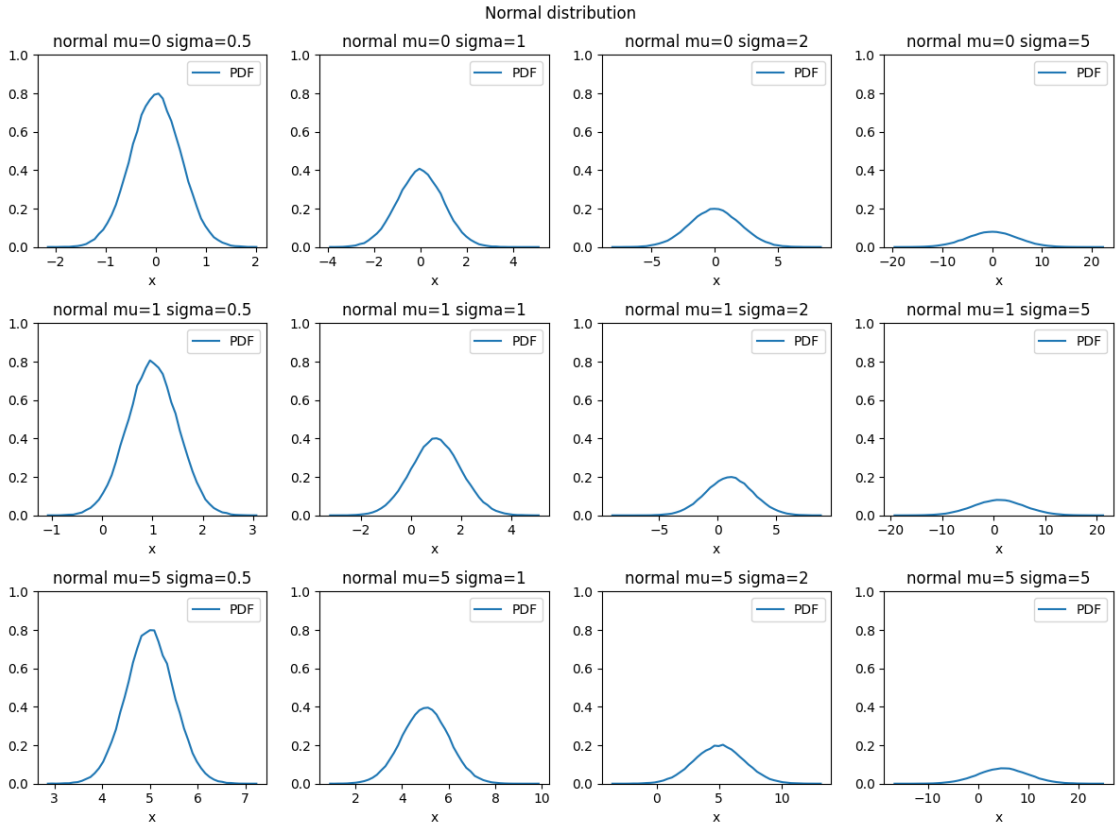
A continuous gaussian distribution with mean μ and variance σ^2 has the following probability density function:

$$f(x|\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp -\frac{(x-\mu)^2}{2\sigma}$$

We can generate a sequence of values following normal distribution with given mean and variance with numpy.random.normal.

[2]:
```
# Normal/Gaussian distribution

normal_mean_values = [0, 1, 5]
normal_sigma_values = [0.5, 1, 2, 5]
for mu in normal_mean_values:
    for sigma in normal_sigma_values:
        df[f"normal mu={mu} sigma={sigma}"] = rng.normal(mu, sigma, size=N)
plot_columns(df.filter(like="normal", axis=1),4,3,"Normal distribution")
```



The probability density function of gaussian distribution is a bell-shaped function centered on the mean value. Its height is inversely proportional to the standard deviation σ. The larger the variance the wider the bell.

1.2 Uniform distribution

A continuous uniform distribution within interval [a,b] has the following probability density function

$$f(x) = \frac{1}{b-a}$$

for $a \leq x < b$ and 0 otherwise

The mean is:

$$\mu = \frac{a+b}{2}$$

And the variance is

$$\sigma^2 = \frac{(b-a)^2}{12}$$

Ref: Continuous uniform distribution

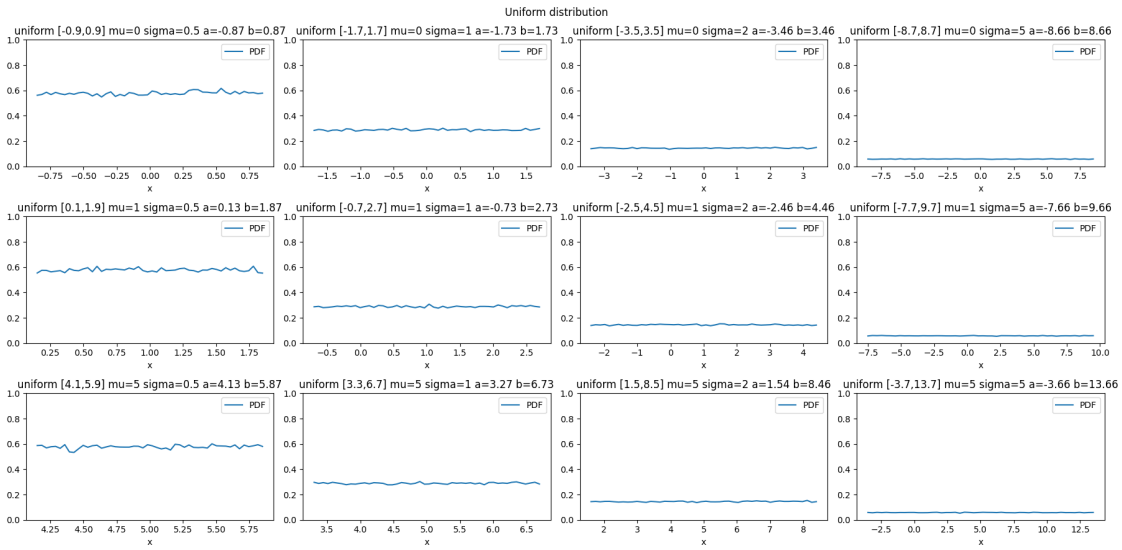
For a given pair of mean and variance values we can choose an interval that satisfies the above relations

$$a = \mu - \sqrt{3}\sigma$$
$$b = \mu + \sqrt{3}\sigma$$

We can generate a sequence of values following uniform distribution within interval [a,b) with numpy.random.uniform.

[6]:
```
# Uniformly random distribution between 0 and 1

uniform_mean_values = [0, 1, 5]
uniform_sigma_values = [0.5, 1, 2, 5]
for mu in uniform_mean_values:
    for sigma in uniform_sigma_values:
        a = mu-np.sqrt(3)*sigma
        b = mu+np.sqrt(3)*sigma
        df[f"uniform [{a:.1f},{b:.1f}] mu={mu} sigma={sigma} a={a:.2f} b={b:.
 ↪2f}"] = rng.uniform(a, b, size=N)
plot_columns(df.filter(like="uniform", axis=1),4,3,"Uniform␣
 ↪distribution",figsize=(18, 9))
```



The probability density function of the uniform dstribution is a horizontal segment between the limits of the interval [a,b], centered horizontally in the middle point of the interval. Its height is inversely proportional to the interval width (b-a). Its standard deviation is directly proportional to the inerval width.

1.3 Beta distribution

A continuous beta distribution has the following probability distribution function:

$$f_X(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$$

for $0 < x < 1$ with beta function defined as:

$$B(\alpha,\beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\,dx$$

Mean:

$$\mu = \frac{\alpha}{\alpha+\beta}$$

Variance:

$$\sigma^2 = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$$

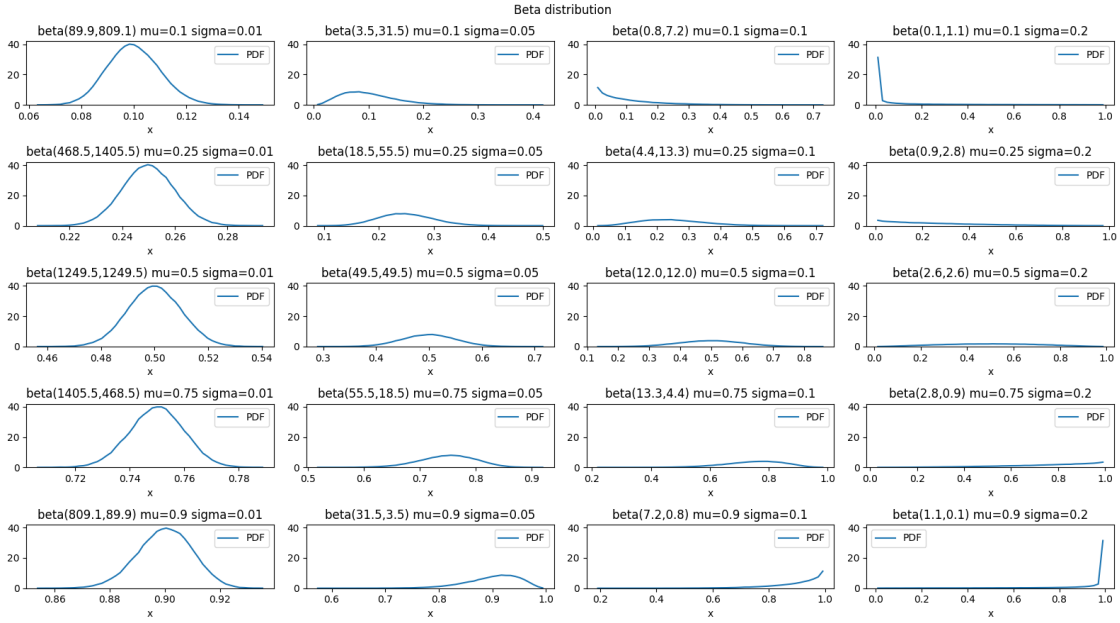
For a given pair of mean and variance values we can choose a pair of α and β that satisfies the above relations

$$\alpha = \mu(\frac{\mu(1-\mu)}{\sigma^2} - 1)$$
$$\beta = (1-\mu)(\frac{\mu(1-\mu)}{\sigma^2} - 1))$$

We can generate a sequence of values following uniform distribution within interval [a,b) with numpy.random.beta.

[4]:
```
# Beta distribution

beta_mean_values = [0.1, 0.25, 0.5, 0.75, 0.9]
beta_sigma_values = [0.01, 0.05, 0.1, 0.2]
for mu in beta_mean_values:
    for sigma in beta_sigma_values:
        alpha = mu*(mu*(1-mu)/(sigma*sigma)-1)
        beta = alpha*(1-mu)/mu
        df[f"beta({alpha:.1f},{beta:.1f}) mu={mu} sigma={sigma}"] = rng.
 ↪beta(alpha, beta, size=N)
plot_columns(df.filter(like="beta", axis=1),4,5,"Beta distribution",␣
 ↪figsize=(16, 9), max_height=42)
```



The probability density function of a beta distribution is defined in the [0, 1] interval. The center of mass is determined by the mean. It is symmetric with respect to the mean when $\alpha = \beta$. The larger the variance the flatter the shape

1.4 Save to file

[5]:
```
df.to_csv(output_file_path)
```