Modelling Linear Programming Tasks

Heuristics & Optimization 2017-2018 (Group 89)

Álvaro Cáceres Muñoz (100303602@alumnos.uc3m.es)
Guillermo Escobero Hernández (100346060@alumnos.uc3m.es)

November 1, 2017

Contents

Chapter 1

Introduction

1.1 About this lab assignment

This document contains the report of the work carried out in this lab assignment. It is related with Linear Programming and Dynamic Programming. There are three main parts for this lab assignment, that are organized as follows:

- Part 1 deals with the modelling of a luggage storing problem, by using LP techniques. This part has been implemented using a LP solver bundled with LibreOffice Calc, a spreadsheet software.
- Part 2 models both the previous luggage storing problem and a crew and flight assignment problem, again by using LP techniques. In this case, GLPK has been used instead of LibreOffice for the implementation of the model, as it allows for large-scale LP prolem solving.

1.2 Document contents

This document has been divided in five chapters, following the format requested by our professor. The following list explains what does each chapter show:

- Chapter 1 (this one) explains the contents of the document. It serves as a reference for rapid lookup across this report.
- In chapter 2, the models of the two proposed problems are discussed. This chapter also includes the definition of the objective functions and constraints involved in each of the problems. After that, the results obtained through those models are analyzed and checked.
- Chapter 3 explains the solutions yielded both for Part 1 and Part 2 problems.
- Chapter 4 explains the difficulties we have faced while completing this lab assignment, both from a technical and a conceptual standpoint. It also describes what have we learned (or remembered) thanks to this lab assignment.

Chapter 2

Problem Modelling

2.1 Part 1: Luggage storing

This problem requires a solution for luggage storing in a plane of a certain flight. The company wants to minimize the costs of moving the excess of luggage in the cabin compartments to the plane's hold.

- The luggage is classified by their volume, weight and costs in case of moving to the hold. According to those variables, three classes are created: M_1, M_2 and M_3.
- The plane has six cabin compartments for luggage storing. They are classified by their volume and the maximum weight supported. Compartments are named C_1 to C_6.

2.1.1 Decision variables definition

This model has 18 decision variables. These represent the number of suitcase and bags of each type (M) on each cabin compartment (C). Summary of decision variables is shown on equation 2.1.

$$X_{i,j}^{1)} \rightleftharpoons \#\text{ suitcases and bags of category } j \text{ on compartment } i; \quad i \in C, j \in M; \vec{X^{1)}} \in \mathbb{Z} \tag{2.1}$$

where

M = set of luggage categories

C = set of compartments in the plane

2.1.2 Objective function definition

The objective of this statement is to minimize the costs of moving the excess of luggage.

For this, we sum the number of bags of each class that have been stored in the compartments and then we calculate the number of them that have to be moved to the hold (*total suitcases of class j - suitcases of class j stored in cabin compartments*). After that, we calculate the amount of money needed in that operation (*cost of moving one suitcase of class j times the number of suitcases of class j to be moved to the hold*).

The objective function proposed is:

$$min\ z = \sum_{j \in M} \left(S_j \left(N_j - \sum_{i \in C} X_{i,j}^{1)} \right) \right) \tag{2.2}$$

where

S = set of costs per bag in case it is moved to the hold

N = set of number of bags of each category that have to be loaded into the plane

2.1.3 Constraints

Volume

The sum of the volumes of the different suitcases has to be less or equal than the volume of the compartment where they are stored.

$$\sum_{j \in M} V_j X^{1)}_{i,j} \leq V_{\max_i} \quad \forall i \in C \tag{2.3}$$

Weight

The sum of the weights of the different suitcases stored in each compartment has to be less or equal than the maximum weight supported by the compartment.

$$\sum_{j \in M} W_j X^{1)}_{i,j} \leq W_{\max_i} \quad \forall i \in C \tag{2.4}$$

where

W = set of weights for luggage in the plane

$W_{\max}$ = set of weights compartments can hold

Gravity center of the plane

The sum of the total weight of compartments C_1 and C_4 (nose compartments) has to be at least 10% larger than the sum of the total weight of compartments C_3 and C_6 (tail compartments).

$$\sum_{i \in C^n} \left(\sum_{j \in M} W_j X^{1)}_{i,j} \right) \geq 1.1 \times \sum_{k \in C^t} \left(\sum_{l \in M} W_l X^{1)}_{k,l} \right) \tag{2.5}$$

where

C^n = set of nose compartments in the plane; $C^n \subseteq C$

C^t = set of tail compartments in the plane; $C^t \subseteq C$

Luggage quantity

The sum of the suitcases of each class stored in the compartments, has to be less or equal than the total number of suitcases of each class we are considering.

$$\sum_{i \in C} X^{1)}_{i,j} \leq N_j \quad \forall j \in M \tag{2.6}$$

Decision variables must be greater or equal to 0

$$X^{1)}_{i,j} \geq 0 \quad \forall i \in C \quad \forall j \in M \tag{2.7}$$

2.1.4 Complete LP task

$$\min z = \sum_{j \in M} \left(S_j \left(N_j - \sum_{i \in C} X_{i,j}^{1)} \right) \right) \tag{2.8}$$

$$s.t.$$

$$\sum_{j \in M} V_j X_{i,j}^{1)} \leq V_{\max_i} \quad \forall i \in C$$

$$\sum_{j \in M} W_j X_{i,j}^{1)} \leq W_{\max_i} \quad \forall i \in C$$

$$\sum_{i \in C^n} \left(\sum_{j \in M} W_j X_{i,j}^{1)} \right) \geq 1.1 \times \sum_{k \in C^t} \left(\sum_{l \in M} W_l X_{k,l}^{1)} \right)$$

$$\sum_{i \in C} X_{i,j}^{1)} \leq N_j \quad \forall j \in M$$

$$\sum_{i \in C} X_{i,j}^{1)} \leq N_j \quad \forall j \in M$$

$$X_{i,j}^{1)} \geq 0 \quad \forall i \in C \quad \forall j \in M$$

2.2 Part 2: Crew members assignment

This problem requires a solution for assigning a set of crew members of an airline to a set of scheduled flights. The company wants to minimize the costs, taking into account the amount of money that each crew member earns per flight hour. Also, the model has to fulfill the problem of luggage storing explained in the first problem.

- Crew members are classified in pilots and attendants.
- Earnings per flight hour are different for each crew member and flight.
- Flights do not overlap.
- There are only two cities (Madrid and Valencia).

2.2.1 Decision variables definition

This model has 54 decision variables. 18 of them have been defined in part 2, as that problem is contained in this one.

For the crew assignment problem, we have defined 36 binary variables. These represent if a crew member is assigned to a flight:

$$X_{i,j}^{2)} = \begin{cases} 1 & \text{if crew member } i \text{ is assigned to flight } j \\ 0 & \text{otherwise} \end{cases}, \quad i \in G, \quad j \in F \tag{2.9}$$

where

G: set of crew members

F: set of flights

2.2.2 Objective function definition

The objective of this statement is to minimize the costs of crew assignment based on their earnings per flight hour, as well as the cost of luggage storing.

The first addend is the objective function of part 1, related with luggage distribution already explained. The rest of the function is the one related with crew assignment. We add the earnings per crew member based on the flight duration of the flights assigned. We divided it by 60 as the model computes time in minutes, and data of earnings are based in flight hours.

The objective function proposed is:

$$\min z = \sum_{j \in M} \left(S_j \left(N_j - \sum_{i \in C} X_{i,j}^{1)} \right) \right) + \sum_{i \in G} \left(\sum_{j \in F} X_{i,j}^{2)} \frac{E_{i,j}}{60} D_j \right) \tag{2.10}$$

where

$E_{i,j}$: earnings per flight hour of crew member i and flight j.

D: flight duration of each flight in minutes.

2.2.3 Constraints

Minimum crew members

Each flight shall have assigned at least one pilot, and at least one flight attendant. Pilots and attendants have to be specified in different constraints as long as in each flight both are required.

$$\sum_{i \in P} X_{i,j}^{2)} \geq 1 \quad \forall j \in F \tag{2.11}$$

$$\sum_{i \in A} X_{i,j}^{2)} \geq 1 \quad \forall j \in F \tag{2.12}$$

where

P: set of pilots; $P \subseteq G$.

A: set of flight attendants; $A \subseteq G$.

More flight hours for flight attendants

The number of flight hours of flight attendants will be larger than the number of flight hours of pilots.

$$\sum_{j \in F} \left(D_j \sum_{i \in P} X_{i,j}^{2)} \right) \leq \sum_{j \in F} \left(D_j \sum_{i \in A} X_{i,j}^{2)} \right) \tag{2.13}$$

Breaks for pilots between flights

Each pilot will have a minimum accorded break between flights. For checking this constraint, we have to make sure that the pilot has been assigned to both flights. If the pilot is only assigned to one or none flight, the coefficient will be 0, so the constraint will be true, as we do not need to check that break.

$$B_k(X_{k,i}^{2)} + X_{k,j}^{2)} - 1) \leq T_{i,j} \quad \forall i,j \in F \quad \forall k \in P : i \neq j, i < j \tag{2.14}$$

where

B_k = minimum break between flights accorded for pilot k (in minutes)

$T_{i,j}$ = delta times between two flights i and j (in minutes)

Crew location

No crew member can be assigned to a flight if he/she is not available in the same airport from which the flight departs. Also, all crew members are initially located in Madrid.

The previous statement implies that for each crew member, the flights assigned with origin Madrid, have to be at least 1 more than the flights assigned with origin Valencia. This fulfills the constraint of the initial location.

We also need a second constraint that completes this requirement. As crew members that are not in that airport cannot be assigned to a flight with departure from that airport, the flights assigned for each member with origin Madrid have to be more or equal than the flights origin Valencia.

$$\sum_{k \in F^m : k < j} X^{2)}_{i,k} - \sum_{l \in F^v : l < j} X^{2)}_{i,l} \leq 1 \quad \forall i \in G \quad \forall j \in F \tag{2.15}$$

$$\sum_{k \in F^v : k < j} X^{2)}_{i,k} - \sum_{l \in F^m : l < j} X^{2)}_{i,l} \leq 0 \quad \forall i \in G \quad \forall j \in F \tag{2.16}$$

where

F^m = set of flights with origin Madrid and destination Valencia; $F^m \subseteq F$

F^v = set of flights with origin Valencia and destination Madrid; $F^v \subseteq F$

Decision variables must be 0 or 1 (binary variables)

$$X^{2)}_{i,j} \in \{0,1\} \quad \forall i \in G \quad \forall j \in F \tag{2.17}$$

2.2.4 Complete LP task

$$\min z = \sum_{j \in M} \left(S_j \left(N_j - \sum_{i \in C} X^{1)}_{i,j} \right) \right) + \sum_{i \in G} \left(\sum_{j \in F} X^{2)}_{i,j} \frac{E_{i,j}}{60} D_j \right)$$

$$s.t.$$

$$\sum_{i \in P} X^{2)}_{i,j} \geq 1 \quad \forall j \in F$$

$$\sum_{i \in A} X^{2)}_{i,j} \geq 1 \quad \forall j \in F$$

$$\sum_{j \in F} \left(D_j \sum_{i \in P} X^{2)}_{i,j} \right) \leq \sum_{j \in F} \left(D_j \sum_{i \in A} X^{2)}_{i,j} \right)$$

$$B_k(X^{2)}_{k,i} + X^{2)}_{k,j} - 1) \leq T_{i,j} \quad \forall i,j \in F \quad \forall k \in P : i \neq j, i < j$$

$$\sum_{k \in F^m : k < j} X^{2)}_{i,k} - \sum_{l \in F^v : l < j} X^{2)}_{i,l} \leq 1 \quad \forall i \in G \quad \forall j \in F$$

$$\sum_{k \in F^v : k < j} X^{2)}_{i,k} - \sum_{l \in F^m : l < j} X^{2)}_{i,l} \leq 0 \quad \forall i \in G \quad \forall j \in F$$

$$X^{2)}_{i,j} \in \{0,1\} \quad \forall i \in G \quad \forall j \in F$$

Chapter 3

Analysis of results

3.1 Interpretation of obtained values for decision variables

3.1.1 Part 1

The exact values determined for the decision variables are shown in equation 3.1, which is included as a reference. This section tries to offer an explanation for these values, so that useful insights can be drawn from the output of the Libre Office spreadsheet.

$$\begin{aligned} \vec{X^{1)}} &= \langle 2,2,2,0,2,4,2,2,1,0,5,1,0,5,2,2,2,1\rangle \\ z(\vec{X^{1)}}) &= 160 \end{aligned} \tag{3.1}$$

Regarding the value of the objective function, it means that the airline has to spend 160€ to move luggage that did not fit in the compartments to the hold. The airline has to move 16 suitcase of class M_1 to the hold (the cheapest luggage class to move). Regarding decision variables, they show two patterns worth mentioning. Variables defining the number of bags in compartments 1, 3 and 6 have similar characteristics; they have a relatively equal number of bags from each category (around 2). Compartments 2, 4 and 5 are also similar between one another, as they lack M1 and they have a higher number of M2 or M3 bags (4 M3 bags in the case of compartment 2, and 5 M2 bags in the case of compartments 4 and 5).

3.1.2 Part 2

The exact values determined for the decision variables are shown in table 3.1, which is included as a reference[1]. This section tries to offer an explanation for these values, so that useful insights can be drawn from the output of GLPK.

Table 3.1: Decision variables values

	V_1	V_2	V_3	V_4	V_5	V_6	C_1	C_2	C_3	C_4	C_5	C_6
P_1	1	0	0	1	1	0						
P_2	1	1	1	0	0	1						
P_3	0	0	0	0	0	0						
A_1	0	0	0	0	1	0						
A_2	1	1	1	0	0	1						
A_3	1	0	0	1	0	0						
M_1							2	2	0	2	0	0
M_2							2	2	5	2	2	5
M_3							2	3	0	2	4	0

$$z(\vec{X}) = 502.0833 \tag{3.2}$$

[1]We could have used a vector with the assignment values for the decision variables, but we saw this as cumbersome from a visual point of view, and saw the table representation more clear

The value of the objective function obtained means that the airline has to spend 502.0833€ to move luggage that did not fit in the compartments to the hold in the first flight (160€), and to assign the crew members to the six flights (342.0833€). Regarding decision variables related with luggage distribution, we have obtained different values from the obtained from LibreOffice Calc Solver. The values are different because of the implementation of the solving algorithm. As these variables have been discussed in the previous part, we will start analyzing the binary variables related with crew assignment. In this case, the pilot P3 will not be assigned to any flight.

3.2 Check for correctness

3.2.1 Part 1

To check that the solution is correct, the values obtained for the decision variables are used as an input for the constraints of the LP problem as follows:

Volume This restriction involves constraints 1 to 6:

$$\begin{pmatrix} X^{1)}_{C1,M1} & X^{1)}_{C1,M2} & X^{1)}_{C1,M3} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ X^{1)}_{C6,M1} & X^{1)}_{C6,M2} & X^{1)}_{C6,M3} \end{pmatrix} \begin{pmatrix} 0.006 \\ 0.008 \\ 0.003 \end{pmatrix} = \begin{pmatrix} 0.034 \\ 0.028 \\ 0.031 \\ 0.043 \\ 0.046 \\ 0.031 \end{pmatrix} \leq \begin{pmatrix} 0.1 \\ 0.15 \\ 0.07 \\ 0.1 \\ 0.15 \\ 0.07 \end{pmatrix} \tag{3.3}$$

Weight This restriction involves constraints 7 to 12:

$$\begin{pmatrix} X^{1)}_{C1,M1} & X^{1)}_{C1,M2} & X^{1)}_{C1,M3} \\ \vdots & \vdots & \vdots \\ \\ X^{1)}_{C6,M1} & X^{1)}_{C6,M2} & X^{1)}_{C6,M3} \end{pmatrix} \begin{pmatrix} 7 \\ 8 \\ 10 \end{pmatrix} = \begin{pmatrix} 50 \\ 56 \\ 40 \\ 50 \\ 60 \\ 40 \end{pmatrix} \leq \begin{pmatrix} 50 \\ 60 \\ 40 \\ 50 \\ 60 \\ 40 \end{pmatrix} \tag{3.4}$$

Gravity center This restriction involves constraint 13:

$$\begin{pmatrix} 7 & 8 & 10 \end{pmatrix} \left(\begin{pmatrix} X^{1)}_{C1,M1} \\ \vdots \\ X^{1)}_{C1,M3} \end{pmatrix} + \begin{pmatrix} X^{1)}_{C4,M1} \\ \vdots \\ X^{1)}_{C4,M3} \end{pmatrix} \right) \geq 1.1 \begin{pmatrix} 7 & 8 & 10 \end{pmatrix} \left(\begin{pmatrix} X^{1)}_{C3,M1} \\ \vdots \\ X^{1)}_{C3,M3} \end{pmatrix} + \begin{pmatrix} X^{1)}_{C6,M1} \\ \vdots \\ X^{1)}_{C6,M3} \end{pmatrix} \right) \Rightarrow \\ \Rightarrow 100 \geq 88 \tag{3.5}$$

Number of suitcases and bags This restriction involves constraints 14, 15 and 16:

$$\begin{pmatrix} X^{1)}_{C1,M1} & \cdots & X^{1)}_{C6,M1} \\ \vdots & & \\ X^{1)}_{C1,M3} & \cdots & X^{1)}_{C6,M3} \end{pmatrix} \vec{1}_{6\times 1} = \begin{pmatrix} 6 \\ 18 \\ 11 \end{pmatrix} \leq \begin{pmatrix} 22 \\ 18 \\ 11 \end{pmatrix} \tag{3.6}$$

where $1_{m\times n}$ is a matrix with dimension $m \times n$ filled with ones.

As shown in the previous demonstrations, all constraints are satisfied so that the solution is correct.

3.2.2 Part 2

This problem takes into account the luggage storing problem discussed in Part 1. As long as the Libre Office solver and GLPK solutions are not the same due to the implementation of the algorithm used, we check the GLPK values again.

To check that the solution is correct, the values obtained for the decision variables are used as an input for the constraints of the LP problem as follows:

Volume This restriction involves constraints 1 to 6:

$$\begin{pmatrix} X^{1)}_{C1,M1} & X^{1)}_{C1,M2} & X^{1)}_{C1,M3} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ X^{1)}_{C6,M1} & X^{1)}_{C6,M2} & X^{1)}_{C6,M3} \end{pmatrix} \begin{pmatrix} 0.006 \\ 0.008 \\ 0.003 \end{pmatrix} = \begin{pmatrix} 0.088 \\ 0.118 \\ 0.04 \\ 0.088 \\ 0.136 \\ 0.04 \end{pmatrix} \leq \begin{pmatrix} 0.1 \\ 0.15 \\ 0.07 \\ 0.1 \\ 0.15 \\ 0.07 \end{pmatrix} \tag{3.7}$$

Weight This restriction involves constraints 7 to 12:

$$\begin{pmatrix} X^{1)}_{C1,M1} & X^{1)}_{C1,M2} & X^{1)}_{C1,M3} \\ \vdots & \vdots & \vdots \\ \\ X^{1)}_{C6,M1} & X^{1)}_{C6,M2} & X^{1)}_{C6,M3} \end{pmatrix} \begin{pmatrix} 7 \\ 8 \\ 10 \end{pmatrix} = \begin{pmatrix} 50 \\ 60 \\ 40 \\ 50 \\ 56 \\ 40 \end{pmatrix} \leq \begin{pmatrix} 50 \\ 60 \\ 40 \\ 50 \\ 60 \\ 40 \end{pmatrix} \tag{3.8}$$

Gravity center This restriction involves constraint 13:

$$\begin{pmatrix} 7 & 8 & 10 \end{pmatrix} \left(\begin{pmatrix} X^{1)}_{C1,M1} \\ \vdots \\ X^{1)}_{C1,M3} \end{pmatrix} + \begin{pmatrix} X^{1)}_{C4,M1} \\ \vdots \\ X^{1)}_{C4,M3} \end{pmatrix} \right) \geq 1.1 \begin{pmatrix} 7 & 8 & 10 \end{pmatrix} \left(\begin{pmatrix} X^{1)}_{C3,M1} \\ \vdots \\ X^{1)}_{C3,M3} \end{pmatrix} + \begin{pmatrix} X^{1)}_{C6,M1} \\ \vdots \\ X^{1)}_{C6,M3} \end{pmatrix} \right) \Rightarrow \\ \Rightarrow 100 \geq 88 \tag{3.9}$$

Number of suitcases and bags This restriction involves constraints 14, 15 and 16:

$$\begin{pmatrix} X^{1)}_{C1,M1} & \cdots & X^{1)}_{C6,M1} \\ \vdots & & \\ X^{1)}_{C1,M3} & \cdots & X^{1)}_{C6,M3} \end{pmatrix} \vec{1}_{6\times 1} = \begin{pmatrix} 6 \\ 18 \\ 11 \end{pmatrix} \leq \begin{pmatrix} 22 \\ 18 \\ 11 \end{pmatrix} \tag{3.10}$$

where $1_{m\times n}$ is a matrix with dimension $m \times n$ filled with ones.

Minimum crew members This restriction involves constraints 17 to 28:

$$\begin{pmatrix} X^{2)}_{P1,V1} & X^{2)}_{P2,V1} & X^{2)}_{P3,V1} \\ X^{2)}_{P1,V2} & X^{2)}_{P2,V2} & X^{2)}_{P3,V2} \\ \vdots & \vdots & \vdots \\ X^{2)}_{P1,V6} & X^{2)}_{P2,V6} & X^{2)}_{P3,V6} \end{pmatrix} \vec{1}_{3\times 1} = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \geq \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \tag{3.11}$$

$$\begin{pmatrix} X^{2)}_{A1,V1} & X^{2)}_{A2,V1} & X^{2)}_{A3,V1} \\ X^{2)}_{A1,V2} & X^{2)}_{A2,V2} & X^{2)}_{A3,V2} \\ \vdots & \vdots & \vdots \\ X^{2)}_{A1,V6} & X^{2)}_{A2,V6} & X^{2)}_{A3,V6} \end{pmatrix} \vec{1}_{3\times 1} = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \geq \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \tag{3.12}$$

More flight hours for flight attendants This restriction involves constraint 29:

$$\begin{pmatrix} X^{2)}_{P1,V1} & X^{2)}_{P2,V1} & X^{2)}_{P3,V1} \\ X^{2)}_{P1,V2} & X^{2)}_{P2,V2} & X^{2)}_{P3,V2} \\ \vdots & \vdots & \vdots \\ X^{2)}_{P1,V6} & X^{2)}_{P2,V6} & X^{2)}_{P3,V6} \end{pmatrix} \vec{1}_{3\times1}\vec{D} \le \begin{pmatrix} X^{2)}_{A1,V1} & X^{2)}_{A2,V1} & X^{2)}_{A3,V1} \\ X^{2)}_{A1,V2} & X^{2)}_{A2,V2} & X^{2)}_{A3,V2} \\ \vdots & \vdots & \vdots \\ X^{2)}_{A1,V6} & X^{2)}_{A2,V6} & X^{2)}_{A3,V6} \end{pmatrix} \vec{1}_{3\times1} \Rightarrow \tag{3.13}$$

$$\Rightarrow 0 \le 0$$

where

$$\vec{D} = \langle 75, 90, 105, 65, 75, 70 \rangle$$

Breaks for pilots between flights This restriction involves constraints 30 to 74. As the number of constraints of this requirement is high, we have checked the correctness by seeing if the pilots take non-legal flight combinations. All pilots satisfy the minimum breaks accorded.

Crew location This restriction involves constraints 75 to 146. As the number of constraints of this requirement is high, we have checked the correctness by seeing if all pilots first flight is from Madrid, and that they do not have been assigned to one flight without being in the origin airport. All the first flights of all crew members is from Madrid, and all routes are correct.

As shown in the previous demonstrations, all constraints are satisfied so that the solution is correct.

3.3 Most relevant constraints

In Part 1, weight and volume groups of constraints are more relevant, as they yield a bigger number of constraints in the end, thus restricting more the feasible region. The analogue in Part 2 is the group of crew location constraints, wich number of constraints is notoriously bigger than the rest.

3.4 Problem complexity

We know that the complexity of a Linear Programming task is the number of constraints, not the number of decision variables. Based on that, we have analyzed the complexity of the two problems proposed.

3.4.1 Part 1

For this problem we have defined 18 decision variables and 16 constraints. In this problem, the complexity of the problem will increase if we add more compartments and luggage classes. This is caused by the additional constraints that would be added to the Linear Programming task. If we only add more number of suitcases to allocate, the complexity will remain the same.

In this way, the number of decision variables for m luggage classes and n compartments will be mn. The number of constraints for m luggage classes and n compartments will be $2n + m + 1$, and thus, the number of compartments is the variable that adds more complexity ($2n$).

3.4.2 Part 2

For this problem we have defined decision variables and 176 constraints. In this problem, the complexity of the problem will increase if we add more compartments and luggage classes, as explained in the previous part. This problem have also requirements related with the crew assignment. If more flight and crew members are added, the complexity will increase.This is caused by the additional constraints that would be added to the Linear Programming task.

The number of decision variables for m luggage classes, n compartments, x pilots, y attendants and z flights will be $mn + z(x + y)$. The number of constraints for m luggage classes, n compartments, x pilots, y attendants and z flights will be: $m + 2n + 2z(x + y) + xz^2 + 2z + 2$

The number of flights is the variable that adds more complexity (z^2).

3.5 On strange crew assignments

Flight V1 takes 2 pilots and 2 flight attendants; all remaining flights take 1 pilot and 1 flight attendant. We suspect that it has something to do with hourly salaries, combined with the agreements signed between the airline company and the crew members. If you look at pilot P3, you can see s/he doesn't take any flight, probably because it was expensive to count on him/her taking into account break time preferences. In this case, it makes sense that other pilots may have more responsibility in terms of flights to take, like pilot P2. The analogue can be seen with flight attendants, although in this case, it is not breaks but having to fly more hours what makes them go in the same plane, to our mind.

3.6 Pros and cons of LibreOffice and GLPK

Each software has different advantages and drawbacks. LibreOffice is a graphical tool that gives the user the possibility of using different colors and fonts; visually arranging the problem is great for understanding it. However, LibreOffice becomes cumbersome as the number of constraints and variables increase since, even when using the `sumproduct` function, most problem information has to be written by hand.

MathProg, on the other hand, handles problem scalability better, since the model is specified in a generic and compact way. The downside of this is that it is more difficult to imagine how the problem will look like by just looking at plain text. Also, MathProg is not quite popular among software developers right now, and so most text editors don't have a syntax highlighting option for `.mod` and `.dat` files (with the notable exception of GNU Emacs with its `gmpl-mode`). It must be said that we saved a considerable amount of time when converting our written LP model to Mathprog, compared to doing so in LibreOffice; the reason is that MathProg and the mathematical formulation of the LP task are almost identical.

Chapter 4

Conclusions

4.1 On the difficulty of this lab assignment

It makes sense to divide the difficulties of this problem between two categories: technical difficulties and conceptual difficulties.

Technical difficulties have to do with us struggling with a particular technology or implementation, irrespective of whether we managed to correctly model the proposed problems or not. We did have some bad times with MathProg. Nonetheless, and aside from some syntax errors, we think it was easy to get the programs up and running. It definitely helped us to model the LP tasks in a compact way, using summations and sets; one we had done this, converting the LP tasks to MathProg syntax was fairly simple.

Arguably harder than technical difficulties, we also had conceptual difficulties. These are related to modelling and making sure that we are providing the correct and optimal solution for the proposed problems. Knowing that handling the lab assignment with a wrong model is almost equivalent to failing it felt to us like a sword of Damocles over our heads during this last month. We struggled the most with the infamous three last constraints from Part 2 (pilot break times, crew location, and initial departure from Madrid). We have felt these last three constraints were testing us to think outside the box, while being decisive for the grades of the lab assignment. It is true that our educational system has long been criticized for making us learn by repetition; however, if we only face these sort of problem solving in graded labs and exams, *the cure may be worse than the disease.* Perhaps we could be exposed to these problems during problem sessions, where we can take the risk of being wrong for the first time. This is a suggestion for the rest of this course, and for incoming years.

4.2 On the knowledge we have acquired and/or reinforced

Each part of the lab assignment has helped us in different ways. Part 1 has been great to review basic LP concepts; the problem was simple, which let us better understand what were we doing. To our mind, part 2 stresses the importance of LP solvers, which can handle big amounts of constraints and decision variables by writing just a few, generic constraints that encompass all of them. Also, it is undoubtedly the part that has tested our modelling skills to the greatest extent.