Chaotic Pendulum

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I. Goals

What we want to achieve

- Understand the movement equations of a double pendulum and compare with experimental results
  a. Construct a compound pendulum
  b. Programme the equations and do some plots
  c. Get experimental data and do some plots
  d. Compare plots
  e. Demonstration

- What is chaos?

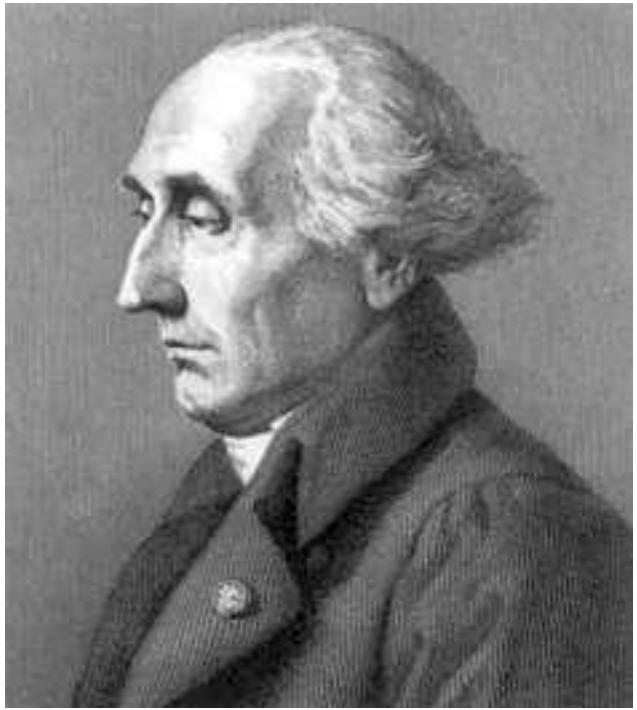
II. Theory

- Chaotic does not mean random

- The path is determined by initial conditions and is very sensitive to them

The Lagrangian

- Lagrangian mechanics is a reformulation of classical mechanics.
- “The Lagrangian”
  - summarizes dynamics of an entire system
  - Differential equation - as function of time
  - Transitioning from coordinates
- Based on minimum action principle



Joseph-Louis Lagrange

1736 - 1813

Formulas to solve

[1]

$$\frac{d^2\theta_1}{dt^2} = \frac{\cos(\theta_1-\theta_2)\left(\frac{g}{l_1}\sin\theta_2-\left(\frac{d\theta_1}{dt}\right)^2\sin(\theta_1-\theta_2)\right)-\frac{l_2}{l_1}\left((m+1)\frac{g}{l_2}\sin\theta_1+\left(\frac{d\theta_2}{dt}\right)^2\sin(\theta_1-\theta_2)\right)}{m+\sin^2(\theta_1-\theta_2)}$$

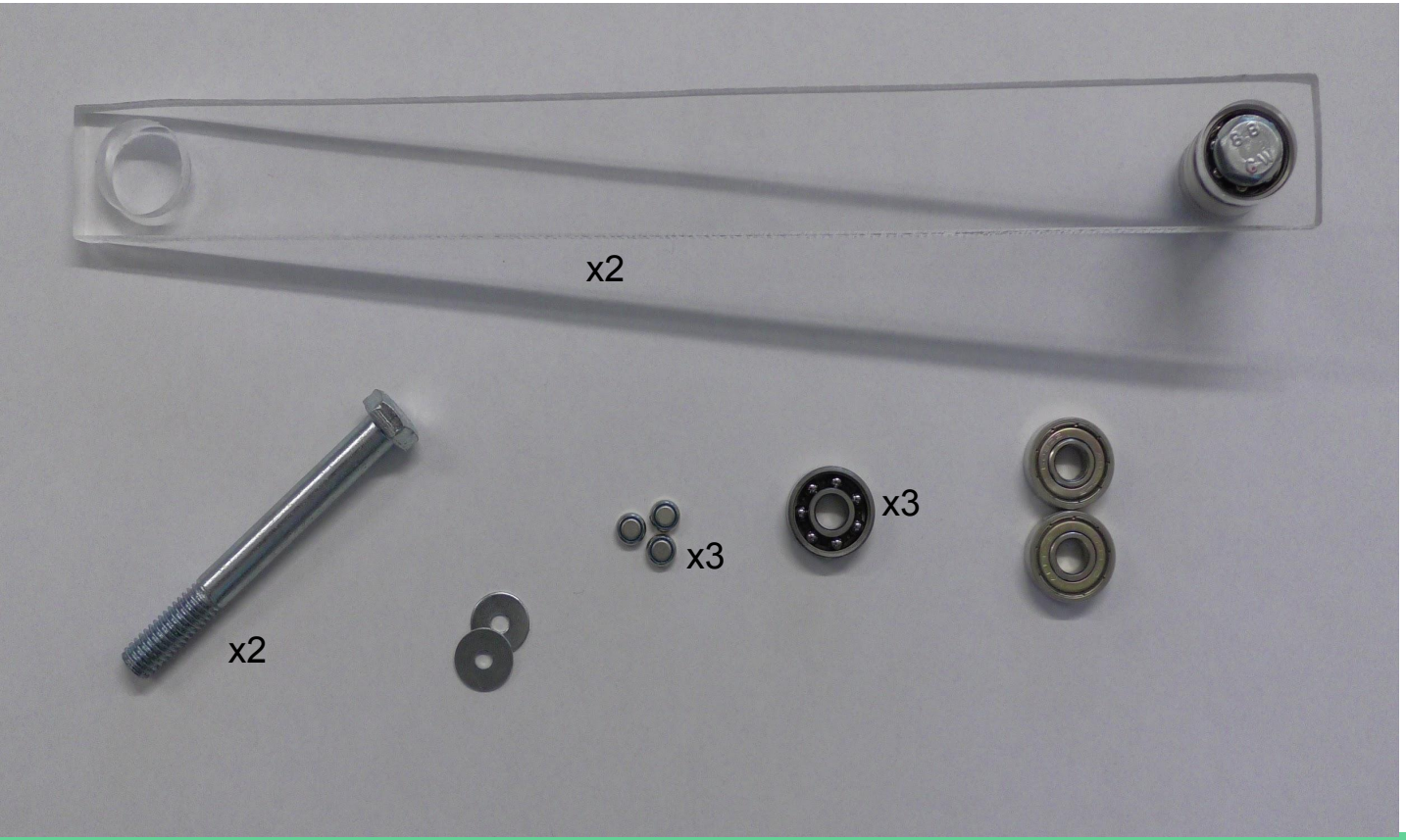
[2]

$$\frac{d^2\theta_2}{dt^2} = \frac{\cos(\theta_1-\theta_2)\left((m+1)\frac{g}{l_2}\sin\theta_1+\left(\frac{d\theta_2}{dt}\right)^2\sin(\theta_1-\theta_2)\right)-(m+1)\frac{l_1}{l_2}\left(\frac{g}{l_1}\sin\theta_2-\left(\frac{d\theta_1}{dt}\right)^2\sin(\theta_1-\theta_2)\right)}{m+\sin^2(\theta_1-\theta_2)}$$

m = m1/m2

Construction

Materials

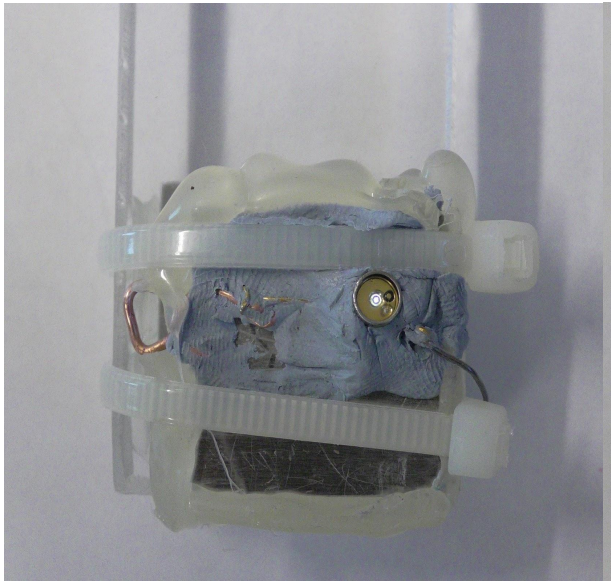


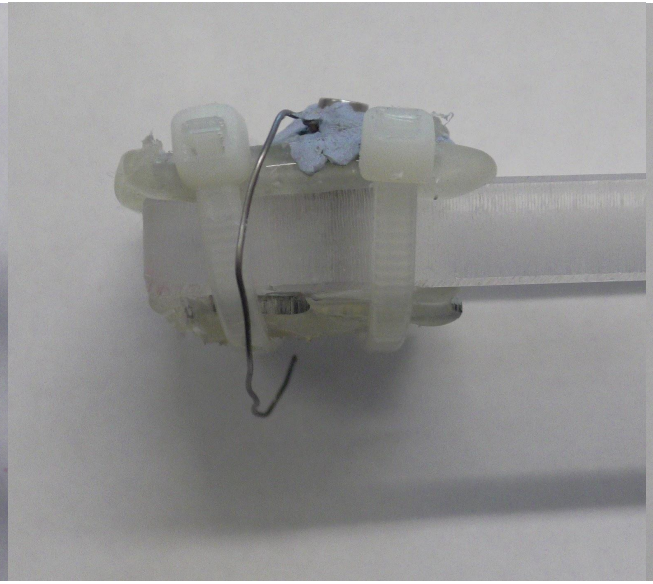
- UV diode
- Clay
- Copper wires
- Conducting plates
- White sticker papers
- Photoluminescent powder - Strontiumaluminate
- Akryl Lak

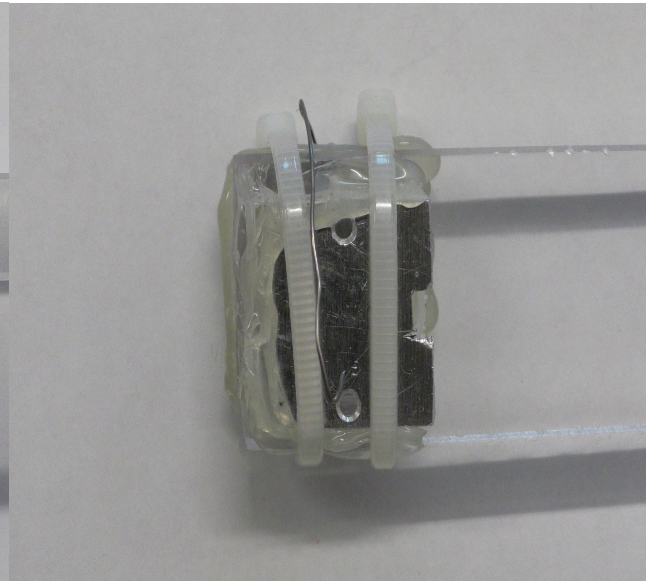
Main Parts

- Assembling the pendulum
  - Insert bearings into openings
  - Adjust rods together
  - Use spacers, extra bearings
- Attaching UV diode onto the pendulum
  - Place batteries in between conducting plates
  - Adjust with glue-gun and tie zips
  - Adjust diode with clay
- Preparing the board
  - Stick papers
  - Paint
  - Fasten on stand

Closer look...







Path tracing\programming

Python

1. Theoretical data
    a. Input: parameters and initial conditions
    b. Applied to equations [1] and [2]
    c. Output: Θ1, Θ2 ⟶ (x1,y1) and (x2,y2)
2. Experimental data
    a. Computer vision ⟶ automatically analyzes pixels
    b. Red spot attached to the end of a rod :
        i. Input: video of an experiment
        ii. Output: position of the red spot per every frame

Python - Theoretical data

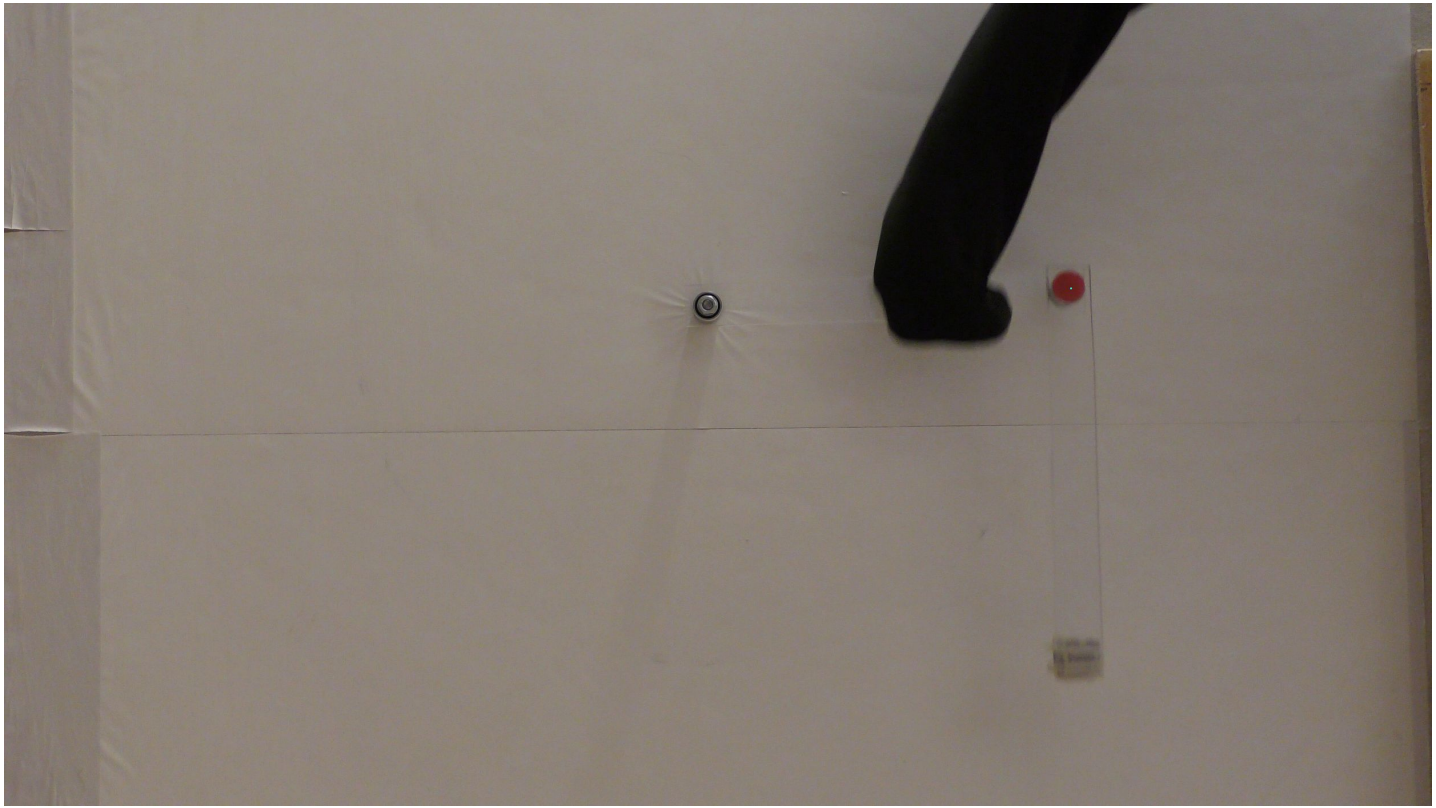
1. Sets g, l1, l2, m1 and m2
2. Sets Θ1, Θ2, ω1 and ω2.
3. Sets the interval time.
4. Numerically solves the differential equations for that time interval (odeint)
5. From Θ1, Θ2 ⟶ (x1,y1) and (x2,y2)
6. Plots:
    a. x vs y
    b. time vx x
    c. time vs y

Python - Experimental data

1. Gets all frames of a video
2. For every frame:
    a. Locates all red pixels
    b. Computes the center of those pixels
    c. (x,y) centre coordinates ⟹ data file
    d. Saves the frame with a turquoise dot in the centre
3. Plots:
    a. x vs y
    b. Frames vs x
    c. Frames vs y

Outcome

Frames



Plots

All based on the following initial conditions:

$\Theta 1 = \pi/2$ rad

$\Theta 2 = 0$ rad

$\omega 1 = 0$ m/s

$\Omega 2 = 0$ m/s

Parameters (theoretical):

$g = 9.81\ m/s^2$

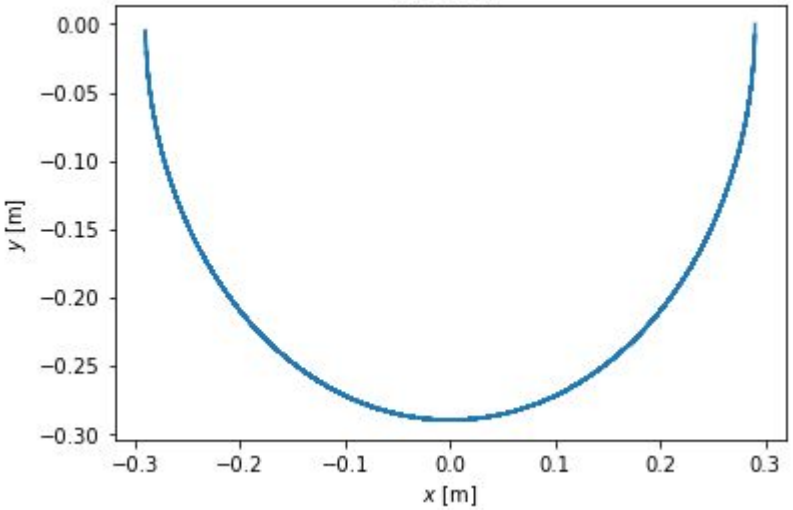
$m1 = 0.190$ kg

$m2 = 0.116$ kg

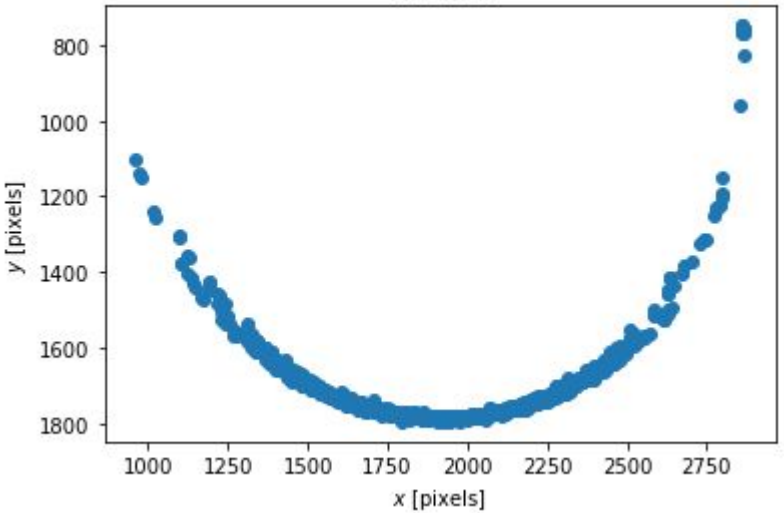
$l1 = l2 = 0.29$ m

First Rod - x vs y

Theoretical

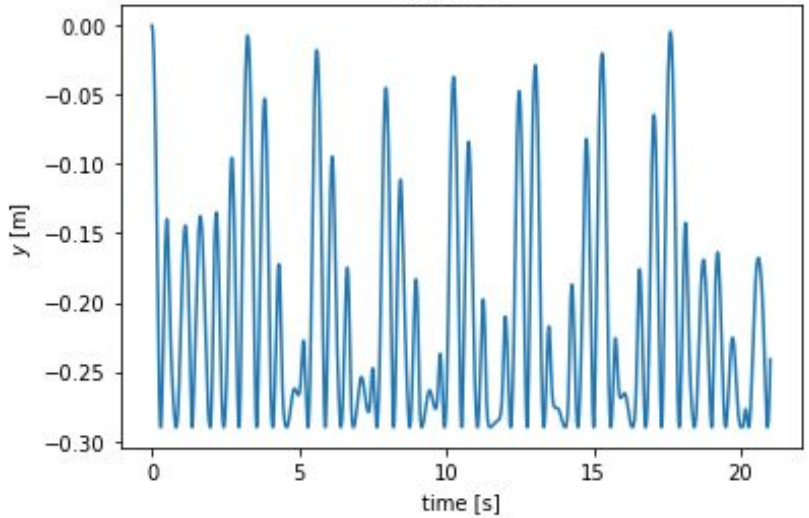


Experimental

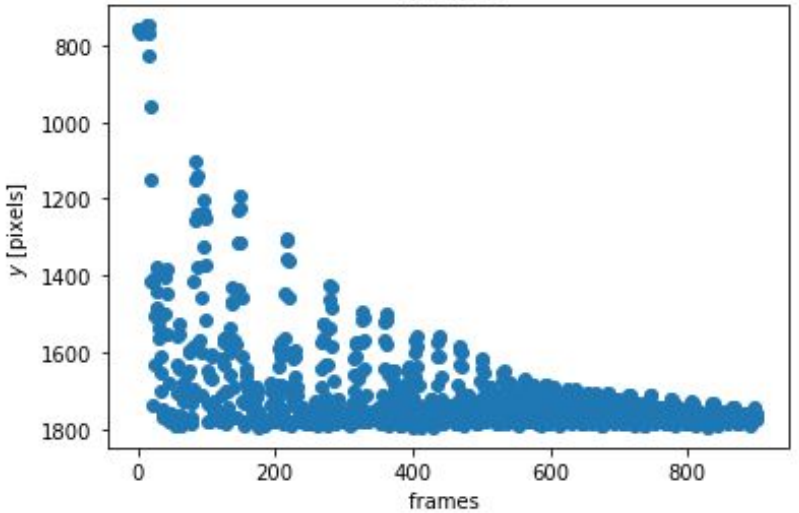


First Rod - time vs y

Theoretical

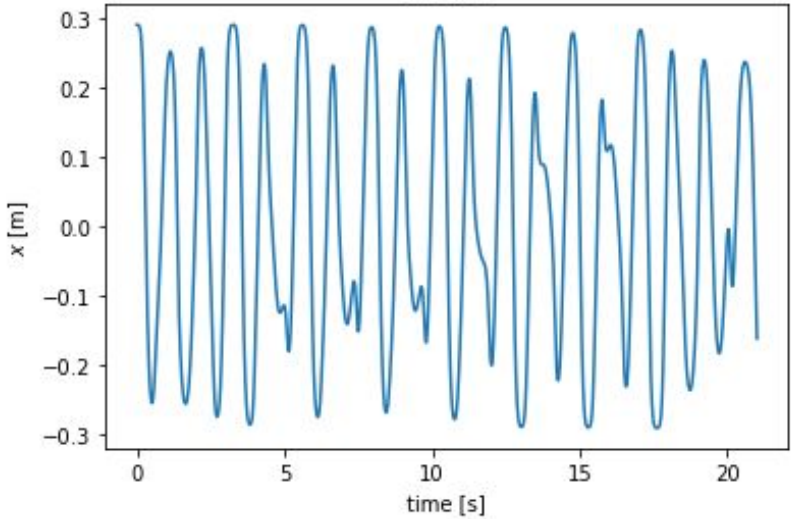


Experimental

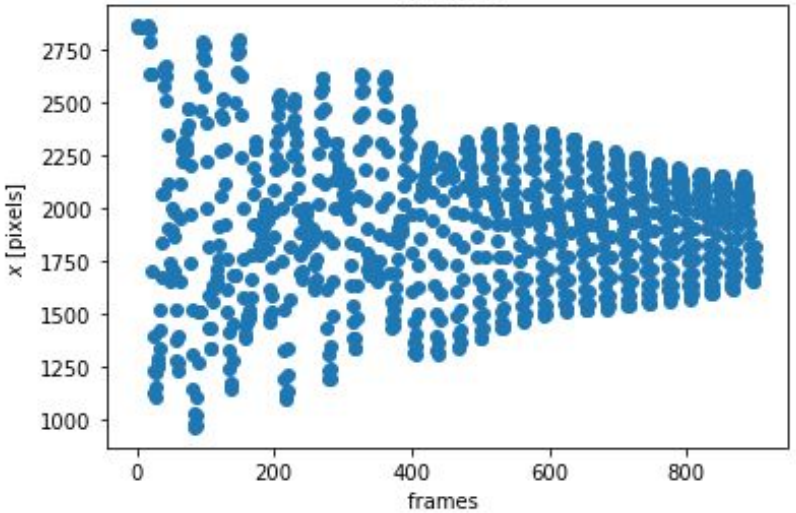


First Rod - time vs x

Theoretical

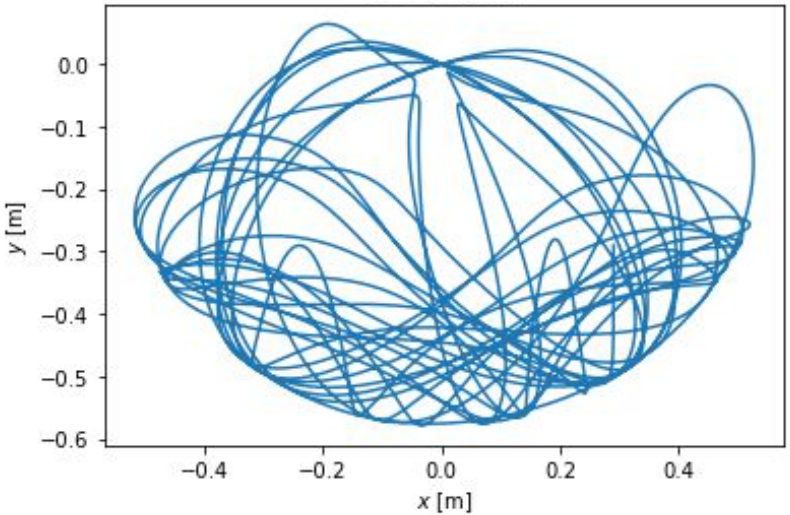


Experimental

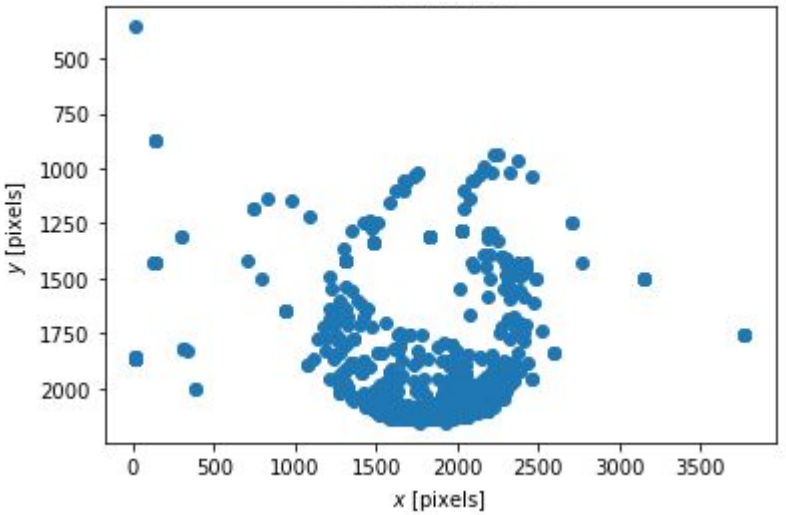


Second Rod - x vs y

Theoretical

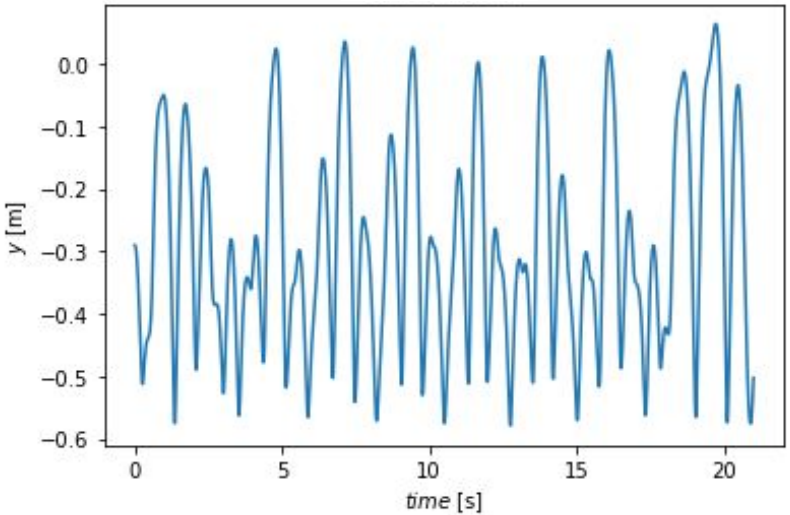


Experimental

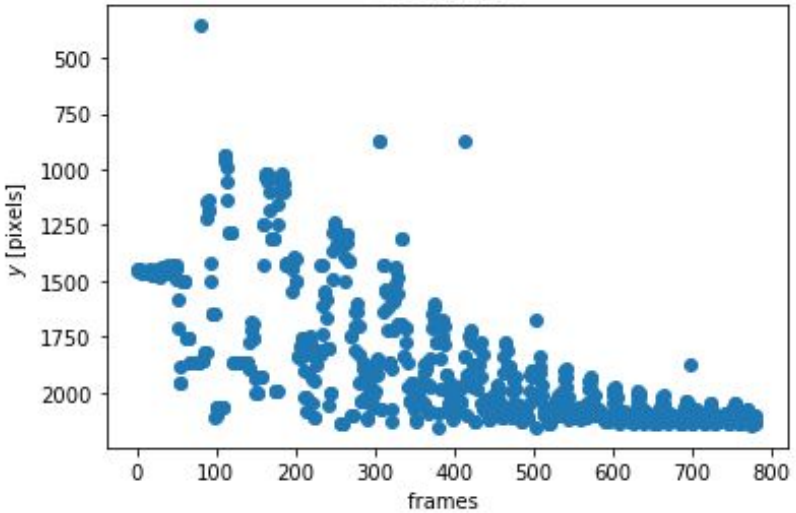


Second Rod - time vs y

Theoretical

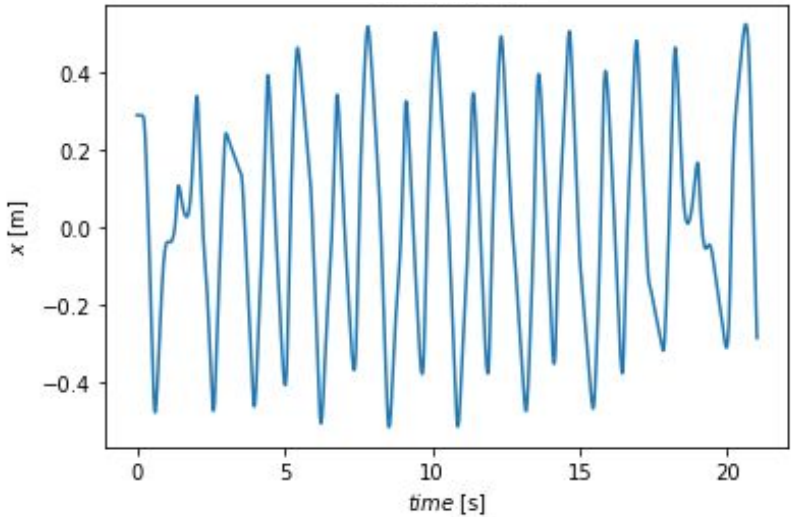


Experimental

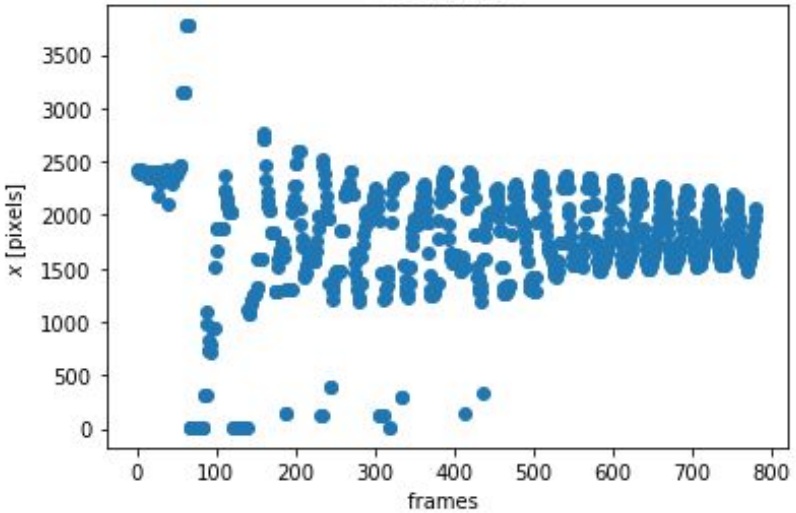


Second Rod - time vs x

Theoretical



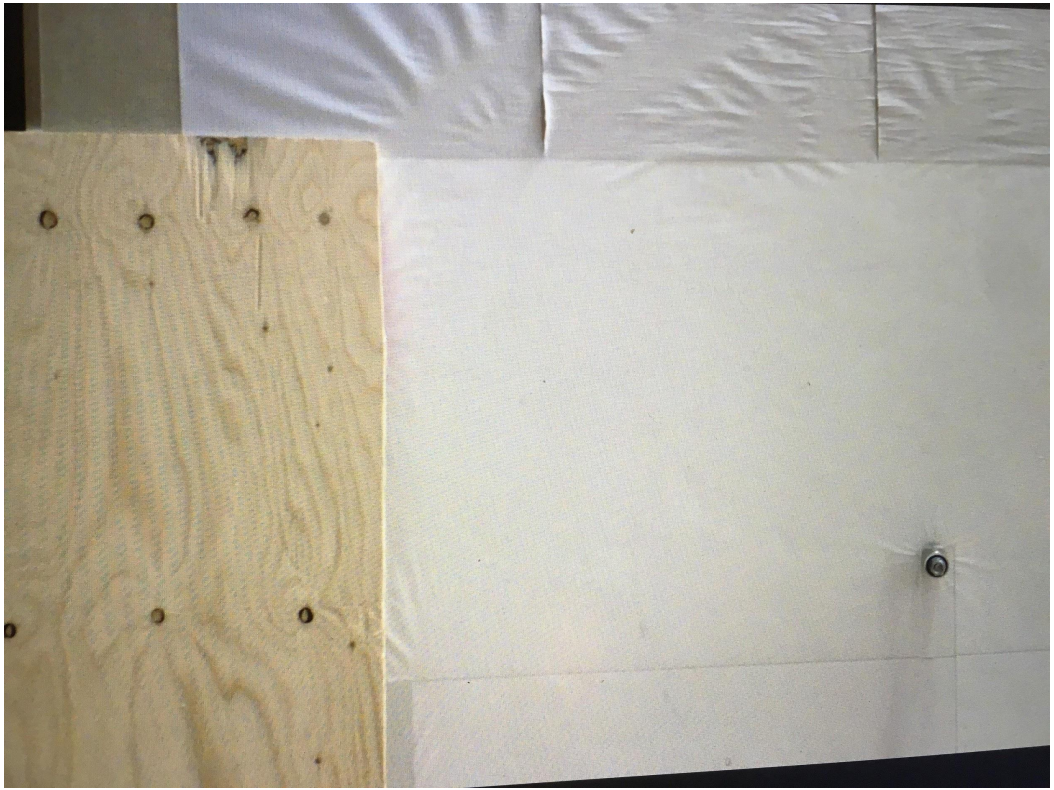
Experimental



Failures\Trial Error

- Failed Arduino idea
  - Too heavy without wi-fi
- Board painted glow in the dark pink - wasted 3-4h
- Complications with adjusting
- Last resort - unit circle coordinate system

The verdict




problem?

Future improvements

- New solvers for differential equations
- Tracing multiple colors instead of one
- Better construction - spacing between rods
- High-definition and speed camera

UV light show

Insert group picture