Signals and Systems

Let's examine the difference between two other signal types

- Energy Signal
  - Has finite energy → Any signal that is bounded and lasts finite time
  - Capacity to do work / total power used
  - If $x(t)$ is a signal, then Energy is:

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

- For $x(t)$ to be an Energy Signal, $E < \infty$
  - ↳ If $E = \infty$ → might be a Power Signal

Power Signal

- Has finite (and non-zero) power → Any signal that is bounded but does not approach zero

Power, the rate @ which Energy is used

- If $x(t)$ has infinite energy → we check for Power Signal

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

- For a Periodic signal → our period keeps repeating

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt \to \lim_{n \to \infty} \frac{1}{nT_0} \int_{-nT_0/2}^{nT_0/2} |x(t)|^2 dt$$

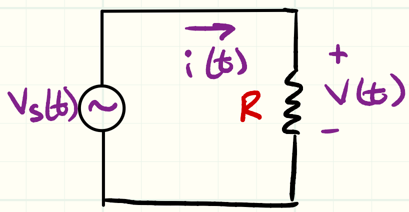
$$= \lim_{n \to \infty} \frac{1}{\cancel{n}T_0} \left[\cancel{n} \int_{T_0} |x(t)|^2 dt \right] = \boxed{\frac{1}{T_0} \int_{T_0} |x(t)|^2 dt}$$

Neither Energy nor Power

- Has infinite power → usually not a "Real System"

Signals and Systems

- Let's examine the following simple electrical circuit:



- Voltage output (across) resistor: $v(t)$
- Current output through resistor: $i(t)$

- What is the Power that is dissipated through R?

$$\text{Power } (P) = \frac{[v(t)]^2}{R} = [i(t)]^2 R \quad (\text{Power} = \text{Watts})$$

- We can see that Power is proportional to $[v(t)]^2$ and $[i(t)]^2$
  ↳ Assuming that $R = 1\,\Omega$
- We can say that for any arbitrary signal $x(t)$
  ↳ The Power (P) is proportional to the magnitude-squared of the signal $x(t) \rightarrow P \propto |x(t)|^2$
  → The absolute value around $x(t)$ is important because it allows us to use the same eq. for complex values
  → The Power we are talking about here is Instantaneous Power

$$\boxed{\text{Instantaneous Power } (P) = |x(t)|^2}$$

What is the Power (Watts) @ time (t)?

- We also know that Power is: Energy per unit time

Note: If Power is Energy per unit time, then:

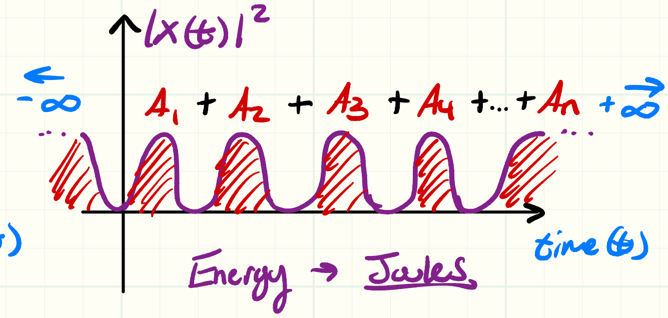
$$\boxed{\text{Total Energy } (E) = \int_{-\infty}^{\infty} |x(t)|^2\, dt}$$

- By integrating the Instantaneous Power over all time, we are essentially summing up the "used" Energy over all time → Energy "used" → Power Dissipation

Signals and Systems

Total Energy $(E) = \int_{-\infty}^{\infty} |x(t)|^2 dt$

- Lets suppose we have a signal and square the Power



- The Energy is equal to the sum of the area under the curve
- We can see that by using $|\square|$, we get positive Energy (J)
- Also, if there are no bounds in time, we can see that there is infinite Energy $\rightarrow E = \infty$ (never stops add..)
- This concept of Energy is only useful when its bounded

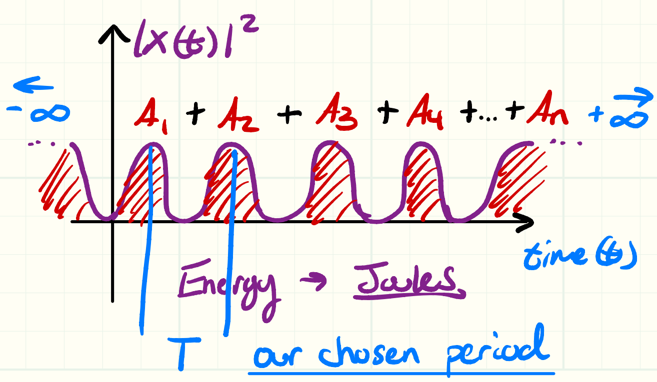
$$0 < E < +\infty$$

- We've examined Instantaneous Power and Energy, let's now look at Average Power (P_{avg}) $\rightarrow$ Power over some period (T)

$$\text{Average Power } (P_{avg}) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

What does this mean?

- Let's choose some period of time (T) ... but why $T/2$?



- We cut our period in half to examine an upper + lower
- By dividing by $\left(\frac{1}{T}\right)$, we get to look at "typical" power used during the time period

Signals and Systems

- We can use the terms Energy and Power to classify different signals.
- If a signal is periodic → repeats for all time (t)
  - ↳ Most Periodic functions would have infinite Energy but finite Average Power (Average Power of $\sin(t) = \frac{A^2}{2}$)
  - ↳ Therefore, they are Power Signals
- If a signal has finite Energy → Energy Signal
- If a signal has infinite Energy and infinite Power, then these signals are neither
- Let's work on example → Next Page

Signals and Systems

Example 2:

The signal $g(t) = A\cos(\omega_0 t + \theta)$, Energy or Power Signal?

Energy $(E) = \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} |A\cos(\omega_0 t + \theta)|^2 dt$

$E = \int_{-\infty}^{\infty} A^2 \cos^2(\omega_0 t + \theta)\, dt$ $\quad * \cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta))$

$E = \int_{-\infty}^{\infty} \frac{A^2}{2}\left[1 + \cos(2\omega_0 t + 2\theta)\right] dt$

$E = \int_{-\infty}^{\infty} \frac{A^2}{2}\, dt + \int_{-\infty}^{\infty} \frac{A^2}{2}\left[\cos(2\omega_0 t + 2\theta)\right] dt$

$E = \frac{A^2}{2} t \Big|_{-\infty}^{\infty} + \ ?$ $\quad * \int \cos(\omega t)\, dt = \frac{1}{\omega}\sin(\omega t)$

$E = \underbrace{\frac{A^2}{2} t \Big|_{-\infty}^{\infty}}_{\text{Goes to } \infty} + \left[\frac{A^2}{4\omega_0} \sin(2\omega_0 t + 2\theta) \Big|_{-\infty}^{\infty}\right]$ Oscillates from -1 to 1

$E = \infty$

We can say that this signal has infinite Energy

↳ Therefore, it is not an Energy signal

Continued on next page →

Signals and Systems

Example ②: continued

The signal $g(t) = A\cos(\omega_0 t + \theta)$, Energy or Power Signal?

Average Power (P_{avg}) $= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |g(t)|^2 dt$

- This signal is periodic → so we can write the eq as:

Periodic → repeats!

Average Power (P_{avg}) $= \frac{1}{T_0}\int_{T_0} |g(t)|^2 dt$

$$P_{avg} = \frac{1}{T}\int_0^T |g(t)|^2 dt = \frac{1}{T}\int_0^T A^2\cos^2(\omega_0 t + \theta)\,dt$$

$$P_{avg} = \frac{A^2}{T}\int_0^T \frac{1}{2}\left[1 + \cos(2\omega_0 t + 2\theta)\right]dt$$

$$P_{avg} = \int_0^T \frac{A^2}{2T}dt + \int_0^T \frac{A^2}{T}\left[\cos(2\omega_0 t + 2\theta)\right]dt$$

$$P_{avg} = \frac{A^2}{2T}\, t\Big|_0^T + \frac{A^2}{2T\,2\omega_0}\left[\sin(2\omega_0 t + 2\theta)\right]\Big|_0^T$$

$$P_{avg} = \frac{A^2}{2\cancel{T}}\left[\cancel{T} - \cancel{0}\right] + \frac{A^2}{2T\,2\omega_0}\left[\sin(2\omega_0 t + 2\theta)\right]\Big|_0^T$$

$\omega_0 = 2\pi f_0$

$\omega_0 = \frac{2\pi}{T}$

$$P_{avg} = \frac{A^2}{2} + \frac{A^2}{2T\,2\omega_0}\left[\sin(2\omega_0 t + 2\theta)\right]\Big|_0^T$$

$T_0 = \frac{2\pi}{\omega_0}$

$\theta = 0°$

$$P_{avg} = \frac{A^2}{2} + \frac{A^2}{2\left(\frac{2\pi}{\omega_0}\right)2\omega_0}\left[\sin\left(2\left(\frac{2\pi}{\cancel{T}}\right)\cancel{T} + \theta\right) - \sin\left(2\cancel{\left(\frac{2\pi}{T}\right)}\overset{0}{\cancel{0}} + \theta\right)\right]$$

$$P_{avg} = \frac{A^2}{2} + \cancel{\frac{A^2}{8\pi}\left(\sin(4\pi) - \sin(0)\right)}^{\,0}$$

$P_{avg} = \frac{A^2}{2}$ → This is a finite quantity!

$$\boxed{\text{Therefore, since } g(t) \text{ has infinite Energy; finite Power; its a Power Signal}}$$

Signals and Systems

- Let's consider two signals : $g(t)$ and $f(t)$
  - What is the total power?
  - If two signals are orthogonal (meaning different freq)
    ↳ Then their total power is :

$$P_{x+y} = P_x + P_y$$

$$x(t) = A_0 \cos(2\pi f_0 t + \theta) + A_1 \cos(2\pi f_1 t + \theta) + \dots + A_n \cos(2\pi f_n t + \theta)$$

$$P_x = \frac{A_0^2}{2} + \frac{A_1^2}{2} + \dots + \frac{A_n^2}{2}$$

- Sometimes we want to represent the ratio of power between two signals → Linear vs. Logarithmic

Linear:

- The Linear ratio is simply the ratio between two Powers

$$\frac{P_2}{P_1}$$

Logarithmic:

- The Logarithmic ratio is measured in decibels (dB)

$$10\log_{10}\left(\frac{P_2}{P_1}\right) = 10\log_{10}(P_2) - 10\log_{10}(P_1)$$

Logarithmic Units of Absolute Power

dBm → dB relative to 1 mW

$$\text{power (dBm)} = 10\log_{10}\left(\frac{P(\text{mW})}{1\ \text{mW}}\right)$$

$$\text{power (dBW)} = 10\log_{10}\left(\frac{P(\text{W})}{1\ \text{W}}\right)$$

What is 6dBm?

3dBm + 3dBm = 6dBm

(2)(2) = 6dBm

4 = 6dBm

29dBm?

10dBm + 10dBm + 3dBm + 3... + 3...

(10) (10) (2) (2) (2) = 800

Quick dB Math

$10\log_{10}\left(\frac{1}{2}\right) = -3\text{dB}$; $10\log_{10}\left(\frac{2}{1}\right) = 3\text{dB}$; $10\log_{10}\left(\frac{1}{10}\right) = -10\text{dB}$

$10\log_{10}(10) = 10\text{dB}$