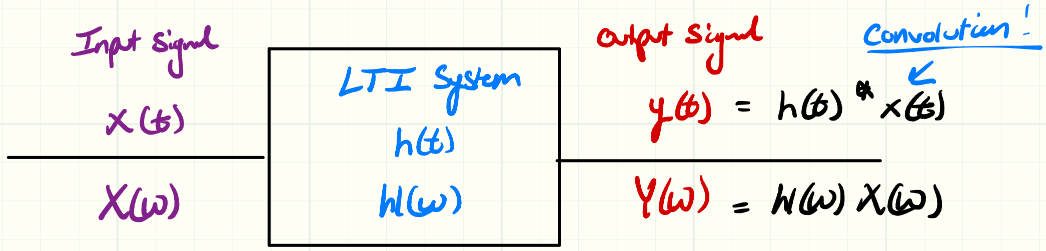
Lecture 3 Learning Outcomes

- Define LTI systems and calculate Unit Impulse Response
- Be familiar with different Filters and calculate their Bandwidths
- Calculate Energy Spectral Density (ESD)
- Calculate Power Spectral Density (PSD)

Linear Time Invariant (LTI) System

- This is a black-box that converts an input signal $x(t)$ into an output signal $y(t)$



- LTI systems have two main properties
  1. The sum of the input signals yields the sum of the outputs
  2. The system does not change with time
     ↳ $x(t-t_0)$ yields $y(t-t_0)$

- In this course, we are only interested in:
  Bounded - Input - Bounded - Output - Stable - LTI systems

- LTI systems provide good/accurate models for a large class of Comm Systems → Tx, Rx, filters, Amps, Equalizers, etc...

- To define your LTI system → you need unit impulse response
  → when $x(t) = \delta(t)$ → $y(t) = \delta(t) * h(t)$
  → $y(t) = h(t)$

* $H(\omega)$ → Transfer function or frequency response

Input signal $x(t)$, $X(\omega)$ → LTI System $h(t)$, $H(\omega)$ → output signal

$y(t) = h(t) * x(t)$ ← Convolution!

$Y(\omega) = H(\omega)X(\omega)$

- Typically, $H(\omega)$ is a complex function:

$$H(\omega) = |H(\omega)|e^{j\theta\omega}$$

Amplitude Response ($|H(\omega)|$); Phase Response ($e^{j\theta\omega}$)

Band Pass Signal

Bandpass Signal: A signal that has a band of frequencies ranging from some non-zero value to another non-zero value

- This means the signal will often hang-around frequencies not equal to zero

$$g_{bp}(t) = g_c(t)\cos(\omega_0 t) + g_s \sin(\omega_0 t)$$

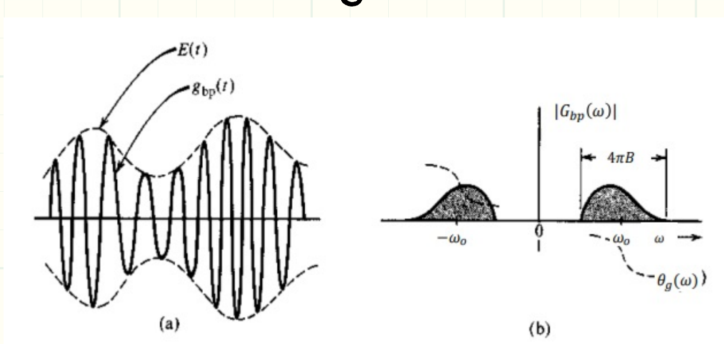
- We say the spectrum of $g_{bp}(t)$ is centered at $\pm\omega_0$ with a bandwidth (B) of $4\pi B$ if $B_1 = B_2$

- With our trig identities, we can simplify $g_{bp}(t)$ as:

$$g_{bp}(t) = E(t)\cos(\omega_0 t + \Psi(t))$$

$$E(t) = \sqrt{g_c^2(t) + g_s^2(t)}$$

$$\Psi(t) = -\tan^{-1}\left(\frac{g_s(t)}{g_c(t)}\right)$$



(a) (b)

Ideal Filters :

- In our class, there are two main filters we examine :
  - Low Pass and Band Pass
- Ideal filters allow distortionless transmission of a certain band of frequencies and suppress all remaining frequencies

Low Pass Filter :

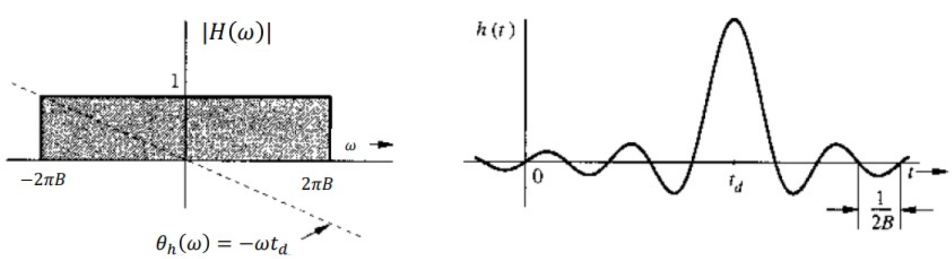
- Low-Pass filters → allow low frequencies to pass through
- The frequency response is given by:

$$H(\omega) = \text{rect}\left(\frac{\omega}{2\pi}\right)e^{-j\omega t_d}$$

- The time domain impulse response is given by:

$$h(t) = 2B\,\text{sinc}\left(2\pi B\left(t - t_d\right)\right)$$

Time-shift property

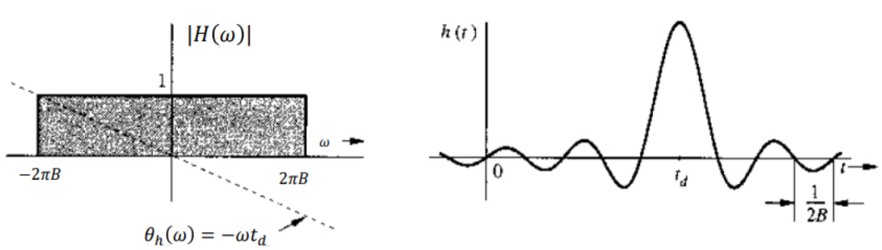


Ideal low-pass filter

Band Pass Filter :

- Bandpass is very useful if you capture a modulated signal and that signal has a frequency centered @ ω_0

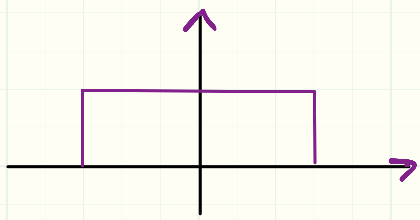
* The most important thing you need to know is the bandwidth because normally the amplitude is just 1 → scalable! *



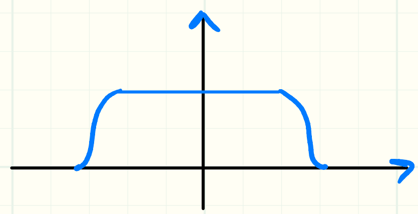
Ideal low-pass filter

Real-World (Practical) Filters:

- In real-life, you don't have a nice rectangular (90° edge) on the frequency response! → Not realizable (sinc is not fully realizable)
- We call real-world filters → Practical Filter → approximated



Ideal

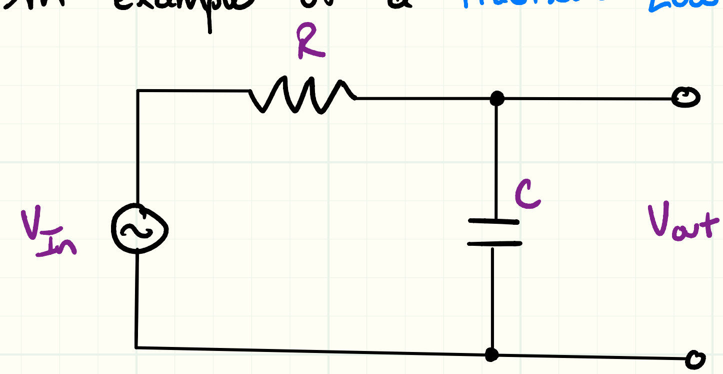


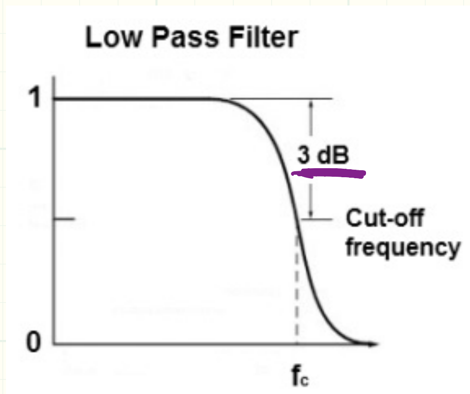
Practical

- The Impulse Response of the Practical Filter is bounded by

$$h(t) = 0 \text{ for } t < 0$$

- An example of a Practical LowPass filter is:





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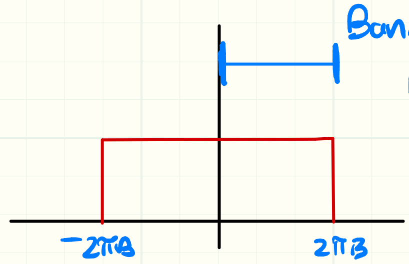


Bandwidth → $2\pi B$

* only positive, not negative *

- In realworld applications, we utilize the $-3dB$ bandwidth

↳ This half-power bandwidth is the band of frequencies at which the magnitude is at least $1/\sqrt{2}$ of max value.

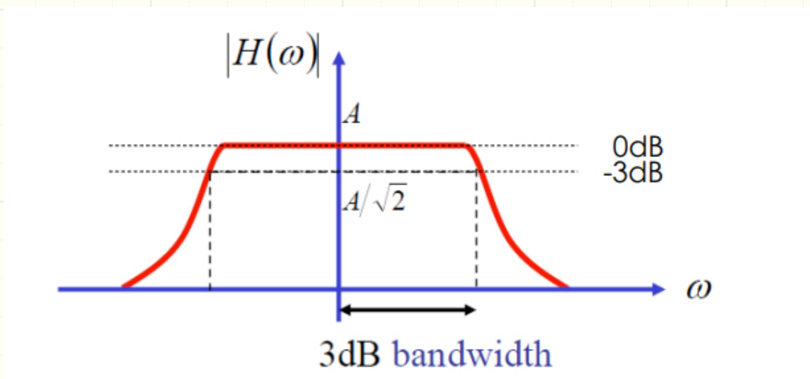
* 0.707 *

* Remember → $P = \frac{V^2}{R}$ → If $V = \frac{1}{\sqrt{2}}$ and $R = 1\Omega$

→ $P = \dfrac{\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)}{1} = \dfrac{\frac{1}{2}}{\frac{1}{2}} = \frac{1}{2}$

* Also → $10\log_{10}\left(\frac{1}{2}\right) = -3dB$ - Power Signal

→ $20\log_{10}\left(\frac{1}{\sqrt{2}}\right) = -3dB$ - Voltage Signal



Signal Distortion

- For an ideal distortionless transmission, your output for LTI system should be:

$$y(t) = kx(t-\tau)$$

* k and τ are just constants *

$$Y(\omega) = kX(\omega)e^{-j\omega\tau}$$

- To make this happen, we need our system transfer function:

$$H(\omega) = ke^{-j\omega\tau}$$

* constant magnitude response

* Phase shift Linear with frequency

How Does Distortion Happen?

1. Linear / Nonlinear Distortion
2. Random Noise
3. Interference from other Transmitters
4. Self Interferences (Reflections / Multi-Path)