Some Useful Fourier Transforms

No.	$x(t)$	$X(\omega)$	
1	$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$a>0$
2	$e^{-at}u(-t)$	$\frac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\frac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_o t}$	$2\pi\delta(\omega-\omega_o)$	

Some Useful Fourier Transforms (Cont.)

No.	$x(t)$	$X(\omega)$	
9	$\cos(\omega_o t)$	$\pi[\delta(\omega-\omega_o)+\delta(\omega+\omega_o)]$	
10	$\sin(\omega_o t)$	$j\pi[\delta(\omega+\omega_o)-\delta(\omega-\omega_o)]$	
11	$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	
12	$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$	
13	$cos(\omega_o t)u(t)$	$\frac{\pi}{2}[\delta(\omega-\omega_o)+\delta(\omega+\omega_o)]+\frac{j\omega}{\omega_o^2-\omega^2}$	
14	$sin(\omega_o t)u(t)$	$\frac{\pi}{2j}[\delta(\omega-\omega_o)+\delta(\omega+\omega_o)]+\frac{\omega_o}{\omega_o^2-\omega^2}$	
15	$e^{-at}sin(\omega_o t)u(t)$	$\frac{\omega_o}{(a+j\omega)^2+\omega_o^2}$	$a>0$
16	$e^{-at}cos(\omega_o t)u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_o^2}$	$a>0$

Some Useful Fourier Transforms (Cont.)

No.	$x(t)$	$X(\omega)$	
17	$rect(\frac{t}{\tau})$	$\tau sinc(\frac{\omega\tau}{2})$	
18	$\frac{B}{2\pi} sinc(\frac{Bt}{2})$	$rect(\frac{\omega}{B})$	
19	$\Delta(\frac{t}{\tau})$	$\tau sinc^2(\frac{\omega\tau}{4})$	
20	$\frac{B}{2\pi} sinc^2(\frac{Bt}{2})$	$\Delta(\frac{\omega}{2B})$	
21	$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$\omega_o \sum_{n=-\infty}^{+\infty} \delta(\omega - n\omega_o)$	$\omega_o = \frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi} e^{-\sigma^2\omega^2/2}$	

Summary

▪ Linearity	$af(t) + bg(t) \leftrightarrow aF(\omega) + bG(\omega)$
▪ Duality	$G(t) \leftrightarrow 2\pi g(-\omega)$
▪ Complex conjugate	$G(-\omega) = G^*(\omega)$
▪ Time-scaling	$g(at) \leftrightarrow \frac{1}{\lvert a \rvert} G\left(\frac{\omega}{a}\right), a \neq 0$
▪ Time-shifting	$g(t \pm t_o) \leftrightarrow e^{\pm j\omega t_o} G(\omega)$
▪ Frequency-shifting	$g(t)e^{\pm j\omega_o t} \leftrightarrow G(\omega \mp \omega_o)$
▪ Convolution	$g_1(t) * g_2(t) \leftrightarrow G_1(\omega)G_2(\omega)$ (time)
▪ Time differentiation*	$\frac{dg(t)}{dt} \leftrightarrow j\omega G(\omega)$
▪ Time integration*	$\int_{-\infty}^{t} g(\tau)d\tau \leftrightarrow \frac{1}{j\omega} G(\omega) + \pi G(0)\delta(\omega)$