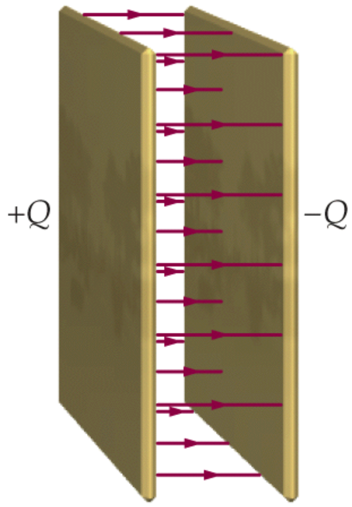
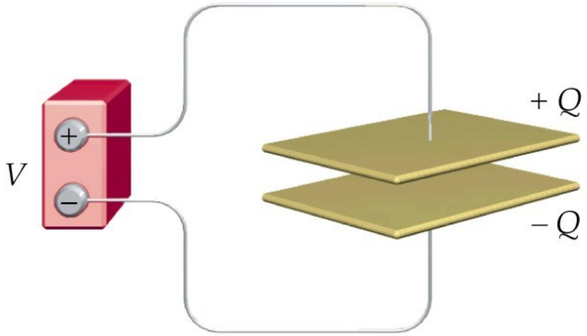
Capacitors and dielectrics

Capacitance

CAPACITOR: System formed by two conductors placed very close to each other.





When a capacitor is connected to a battery, it charges until the ΔV between the plates equals the one of the battery. The plates acquire charges +Q and -Q. There is an E between the plates.

<u>Charge of a capacitor Q</u> : Magnitude of the charge on either conductor.

<u>Potential of a capacitor V</u>: Potential difference between the two plates ($V=V_+-V_-$).

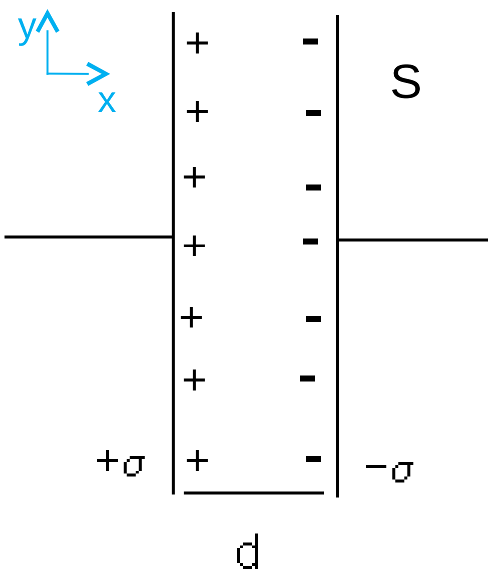
CAPACITANCE (C): The Q stored in a capacitor is proportional to V. The constant of proportionality is C. It measures the capacity to store charge for a given potential difference.

$$C = \frac{Q}{V}$$

SI unit of capacitance: Farad $F= CV^{-1}$

Capacitance of a parallel plate capacitor

The capacitance depends only on the geometry of the conductor and on the properties of the insulating medium.



- Electric field between the plates:

$$\vec{E}_{+} = \frac{\sigma}{2\varepsilon_0}\vec{i} \qquad \vec{E}_{-} = \frac{\sigma}{2\varepsilon_0}\vec{i} \qquad E_{net} = E_{+} + E_{-} = \frac{\sigma}{\varepsilon_0}$$

- Potential <u>difference</u> between the plates:

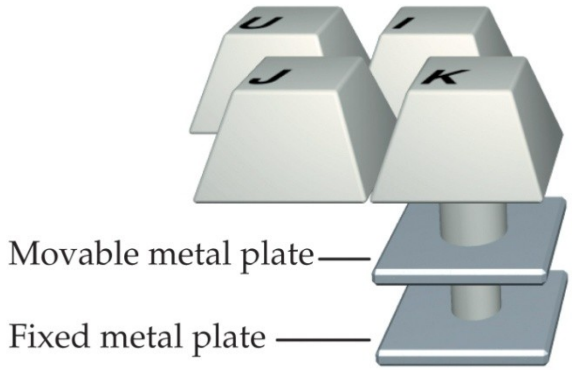
$$V = -\int_{-}^{+} \vec{E}\cdot d\vec{l} = Ed = \frac{Qd}{\varepsilon_0 S}$$

- Capacitance (free space):

$$C = \frac{Q}{V} = \varepsilon_0 \frac{S}{d}$$

C depends on the geometry of the capacitor, and on the insulating medium.

Application:
Keyboard



Capacitance of a parallel plate capacitor

What happens to Q and V when C changes? It depends on the situation:

- If the capacitor is charged, and subsequently disconnected → Q const.
- If the capacitor is still connected → V const.

EXAMPLE:

A parallel-plate capacitor with plates of area A and distance between the plates d is charged by connecting it to a battery which maintains a constant potential difference V_o.

a) Find the capacitance, the charge and the potential of the capacitor supposing that it is disconnected from the battery after being charged and the distance between the plates is decreased in d/2.

b) Find the capacitance, the charge and the potential of the capacitor supposing that the capacitor is still connected to the battery and the distance between the plates is decreased in d/2.

Capacitance of a parallel plate capacitor

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<u>EXAMPLE</u>: Ex 1

A parallel-plate capacitor with plates of area A and distance between the plates d is charged by connecting it to a battery which maintains a constant potential difference V_o.

a) Find the capacitance, the charge and the potential of the capacitor supposing that it is disconnected from the battery after being charged and the distance between the plates is decreased in d/2.

$$C = \frac{2\varepsilon_0 A}{d} \qquad Q = \frac{\varepsilon_0 A V_0}{d} \qquad V = \frac{V_0}{2}$$

b) Find the capacitance, the charge and the potential of the capacitor supposing that the capacitor is still connected to the battery and the distance between the plates is decreased in d/2.

$$C = \frac{2\varepsilon_0 A}{d} \qquad Q = \frac{2\varepsilon_0 A V_0}{d} \qquad V = V_0$$

Energy stored in a capacitor

When we charge a capacitor, we start from a system where V=0 and we increase V, and we also increase U.

A capacitor stores charge <u>and energy</u>.

Energy stored in a capacitor (system formed by two conductors):

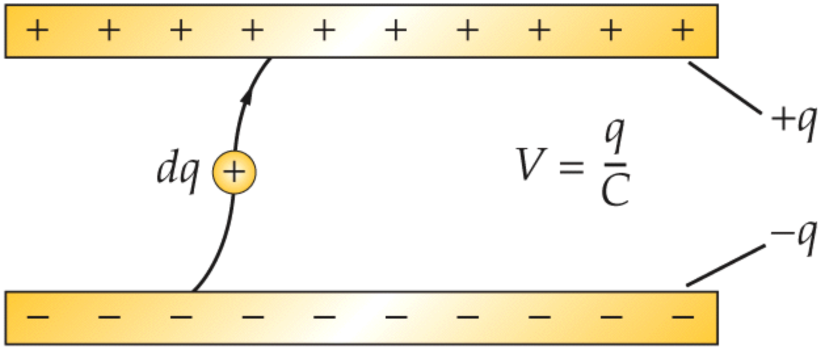
$$U_{capacitor} = \left(\frac{1}{2} Q \cdot V_{+}\right) + \left(\frac{1}{2}(-Q) \cdot V_{-}\right) = \frac{1}{2} QV$$

If the system is formed by several capacitors, the energy stored will depend on how they are connected: in parallel or in series.

Energy stored in a capacitor

When we charge a capacitor, we start from a system where V=0 and we increase V, and we also increase U (dq > 0).

A capacitor stores charge <u>and energy</u>.



$$U = \int_0^Q V dq = \int_0^Q \frac{q}{C} dq = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$$

Energy stored in a capacitor

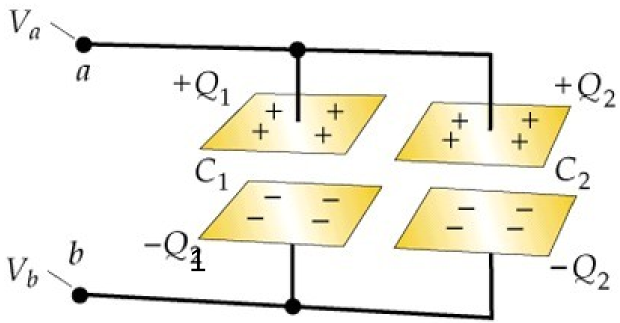
Applications: Capacitor bank used to store energy

The world's largest capacitor bank

(capable of storing 50 MJ)

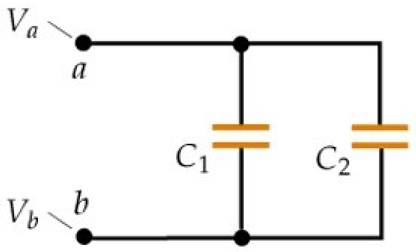
Combination of capacitors

Capacitors connected in parallel:



The upper plates of the capacitors are connected together, so they are at a common potential V_a.

The lower plates of the capacitors are connected together, so they are at a common potential V_b.



The potential difference $V = (V_a - V_b)$ is the same for all capacitors.

The total charge stored is the sum of the ones for each capacitor.

$$Q_{net} = C_{eq}V$$

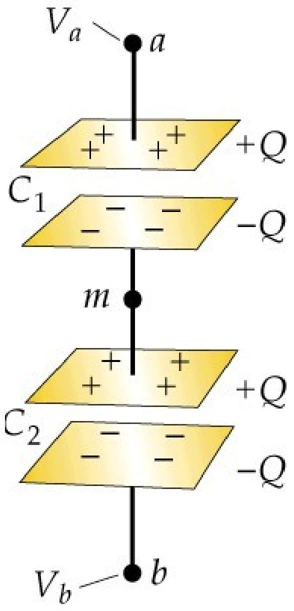
$$Q_{net} = Q_1 + Q_2 = C_1V + C_2V$$

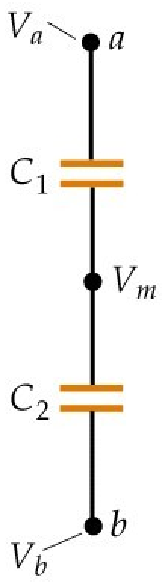
$$\text{So then}: C_{eq} = C_1 + C_2$$

$$C_{eq} = \sum C_i$$

Combination of capacitors

Capacitors connected in series:





A plate of one capacitor is connected to a plate of a second capacitor by a wire containing no junctions.

All capacitors have the same charge.

The net potential difference is the sum of the ones for each capacitor.

$$Q_{net} = C_{eq} V = C_{eq}(V_1 + V_2) = C_{eq}\left(\frac{Q_1}{C_1} + \frac{Q_2}{C_2}\right)$$

$$Q_{net} = Q_1 = Q_2$$

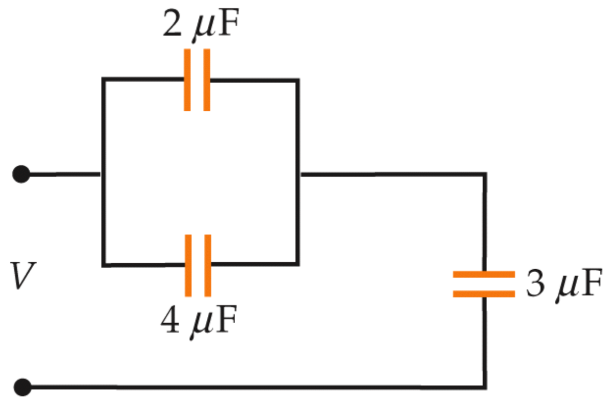
$$\text{So then}: \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_{eq}} = \sum \frac{1}{C_i}$$

Combination of capacitors

EXAMPLE:

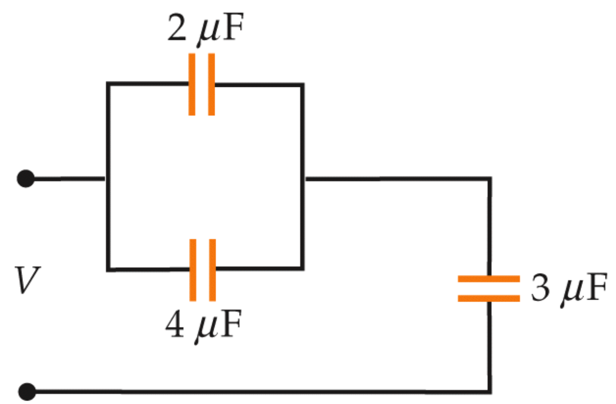
Three capacitors are connected as shown in the figure. (a) Find the equivalent capacitance of the three-capacitor combination. (b) The capacitors are initially uncharged. The combination is then connected to a 6 V battery. Find the potential difference across each capacitor and the charge on each capacitor after the battery is connected and the charges have stopped flowing. (c) Find the net energy stored in the system.



Combination of capacitors

EXAMPLE: Ex 3

Three capacitors are connected as shown in the figure. (a) Find the equivalent capacitance of the three-capacitor combination. (b) The capacitors are initially uncharged. The combination is then connected to a 6 V battery. Find the potential difference across each capacitor and the charge on each capacitor after the battery is connected and the charges have stopped flowing. (c) Find the net energy stored in the system.



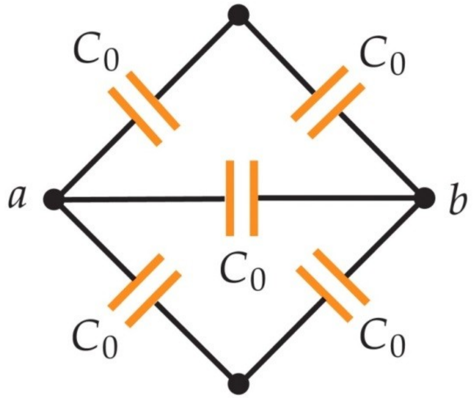
Answer:

(a) C_{eq}=2 μF; (b) V_3=4 V, $V_{2,4}$=2 V, Q_2= 4 μC, Q_3=12 μC, Q_4=8 μC; (c) U=36 μJ

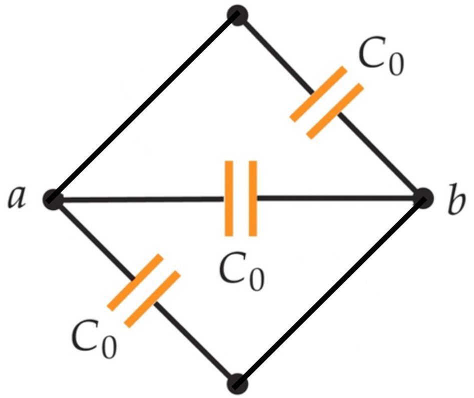
Combination of capacitors

EXAMPLE:

Five identical capacitors of capacitance C_o= 1μF are connected as shown. What is the equivalent capacitance between points a and b?



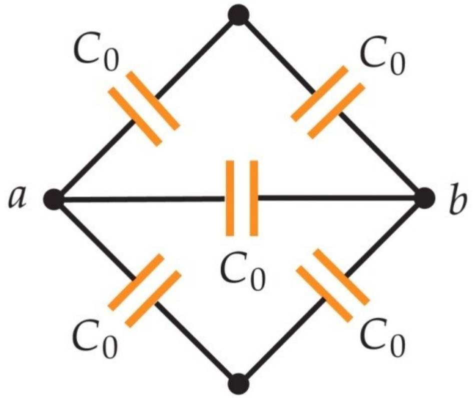
What is the equivalent capacitance between points a and b if we take two capacitors away (see figure)?



Combination of capacitors

EXAMPLE: Ex 5

Five identical capacitors of capacitance C_o= 1μF are connected as shown. What is the equivalent capacitance between points a and b?



ANSWER: C_{eq}= 2 μF

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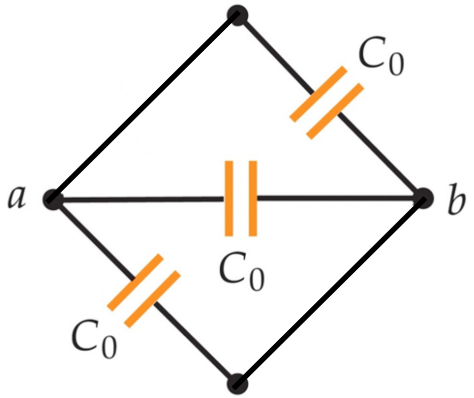


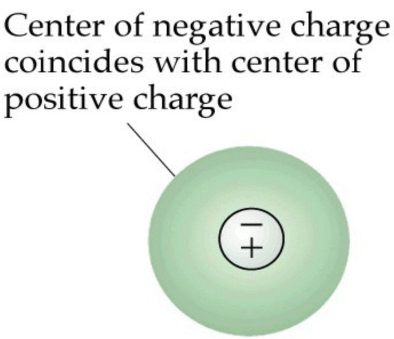
ANSWER: C_{eq}= 3 μF

Capacitors with dielectrics: Polarisation

What happens to a dielectric material in the presence of an electric field?

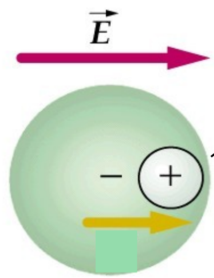
The atoms of the dielectric polarise as a reaction to the external electric field E (forming dipoles).

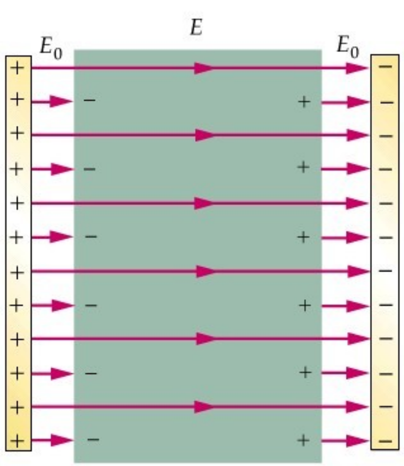
No Electric field:



Electric field applied:

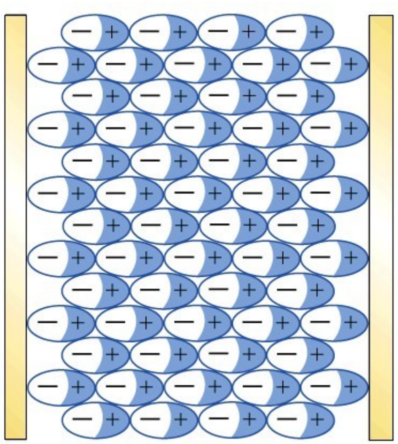
Charges slightly shift from their equilibrium positions (they polarise)





The E due to the dipoles opposes E_{ext}.

As a result, $E_{net} < E_{ext}$



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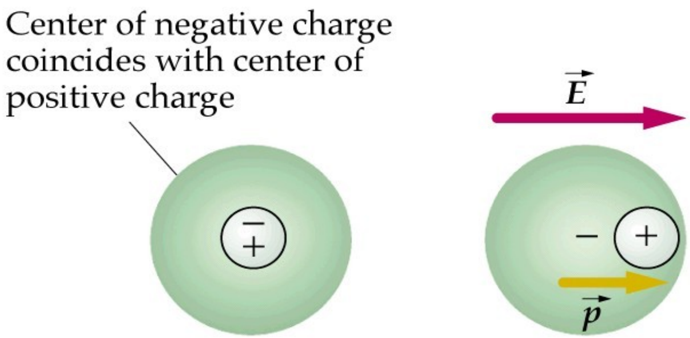

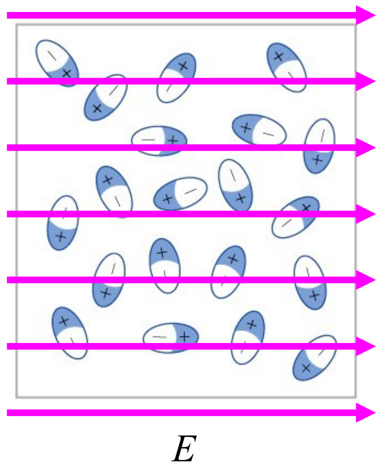

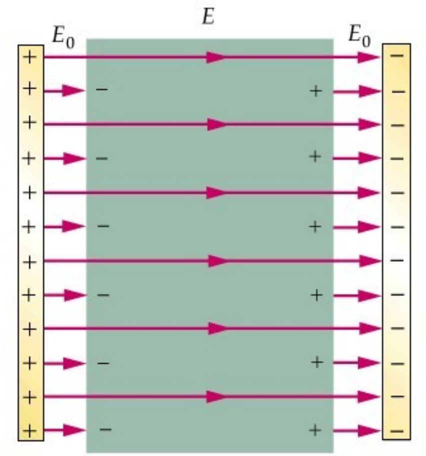

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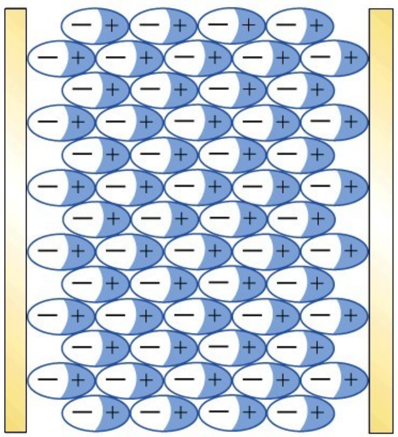
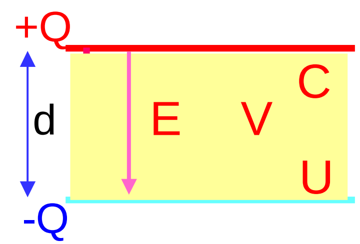


Capacitors with dielectrics

CHARGED CAPACITOR (DISCONNECTED FROM BATTERY):

What happens with Q, E, V, U and C?



1. The charge remains constant (the capacitor is not connected to the battery) $Q = Q_0$

2. The E between the plates becomes weaker $E = \dfrac{E_0}{k}$ k (>1) = dielectric constant

3. The potential difference decreases $V = Ed = \dfrac{E_0}{k} d = \dfrac{V_0}{k}$

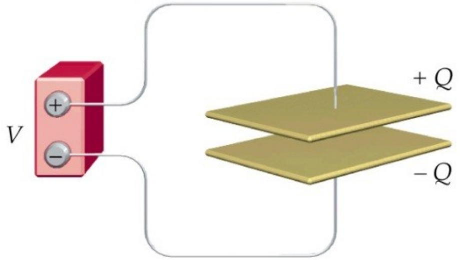
4. The capacitance increases $C = \dfrac{Q}{V} = \dfrac{Q}{V_0 / k} = k \dfrac{Q}{V_0} = kC_0 \longrightarrow C = k\varepsilon_0 \dfrac{S}{d} = \varepsilon \dfrac{S}{d}$

5. The energy decreases $U = \dfrac{1}{2} CV^2 = \dfrac{1}{2} k \dfrac{Q}{V_0} \left(\dfrac{V_0}{k} \right)^2 = \dfrac{1}{2} \dfrac{QV_0}{k} = \dfrac{U_0}{k}$

Capacitors with dielectrics

$\varepsilon = \kappa \varepsilon_0$ ε =dielectric permitivity $\varepsilon_r = \kappa =$ relative permitivity=dielectric const

CHARGED CAPACITOR, CONNECTED TO A BATTERY:



The battery mantains a constant potential difference V, so:

1. The E between the plates remains constant $E = E_0$

2. The charge increases $Q = Q_0 k$

3. The capacitance increases $C = \frac{Q}{V} = \frac{kQ_0}{V} = kC_0$

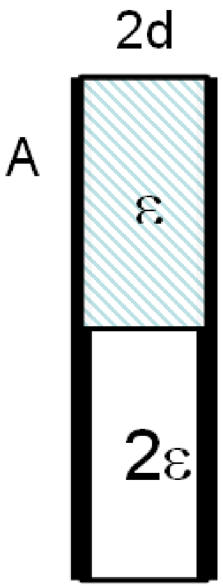
4. The energy increases $U = \frac{1}{2} QV = \frac{1}{2} Q_0 kV = U_0 k$

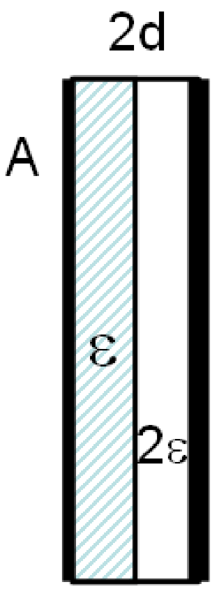
Capacitors with dielectrics

What happens if there are several media between the plates?

Example: (Ex 8) Two parallel-plate capacitors of area A and distance between plates 2d are half-filled with two dielectrics of permitivities ε and 2ε.

Find the equivalent capacitance of each combination.





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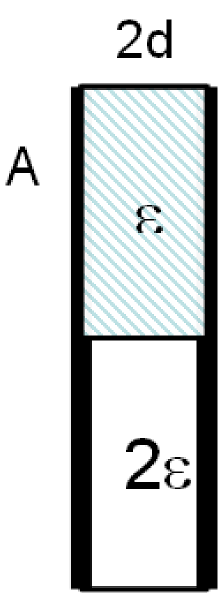
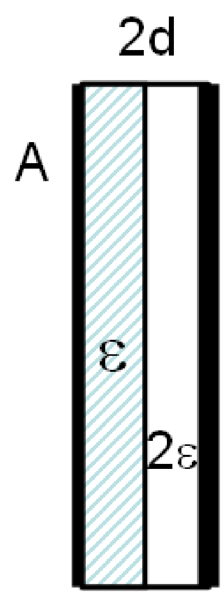


Equivalent to two capacitors connected in parallel

$$C = \frac{3\varepsilon A}{4d}$$

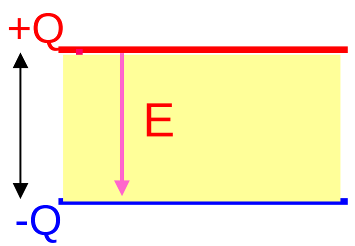


Equivalent to two capacitors connected in series

$$C = \frac{2\varepsilon A}{3d}$$

Capacitors with dielectrics: dielectric breakdown

Effects of very large Electric Fields:



When charging a capacitor, E increases as Q increases.

If there is a dielectric between the plates, E can be so large as to rip the ions inside the dielectric, making it a conductor →DIELECTRIC BREAKDOWN.

For air (dielectric), this phenomena occurs for E > 3 MV/m. This E_{max} ~ 3 MV/m is called <u>dielectric strength</u> of air.

V_{max}=(E_{max}·d) is the <u>breakdown voltage</u>.

Capacitors with dielectrics: dielectric breakdown

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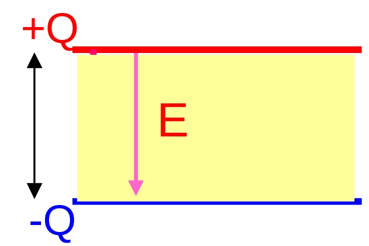

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EXAMPLE: Lightning bolts in a thunderstorm