

CALCULUS

Bachelor in Computer Science and Engineering

Course 2021–2022

Local and global behavior of a function

Problem 8.1. Find and classify the *local* extrema of the following functions.

(a) $f(x) = 2x^3 - 3x^2 - 12x + 5$.

(b) $f(x) = \frac{x+3}{x-2}$.

(c) $f(x) = x^2 e^{-2x}$.

Problem 8.2. Consider the function $f(x) = |x^3(x-4)| - 1$.

- Study where $f(x)$ is (strictly) increasing and where (strictly) decreasing.
- Find and classify *local* maxima and minima of $f(x)$, if any.
- Prove that the equation $f(x) = 0$ has a unique solution for $x \in (0, 1)$.

Problem 8.3. Find the area of the largest rectangle with sides parallel to the x and y axes and inscribed in the ellipse $(x/a)^2 + (y/b)^2 = 1$, where a and b are positive real numbers.

Problem 8.4. What is the *local* behavior of the function $f(x) = x^{101} + x^{51} + x + 1$ close to $x = 0$?

Problem 8.5. Study the concavity of the following functions.

$$f(x) = (x-2)x^{2/3}.$$

$$f(x) = x(x-2)^{3/2}.$$

$$f(x) = |x|e^{|x|}.$$

$$f(x) = \ln(x^2 - 6x + 8).$$

Problem 8.6. Study the *local* behavior of the function

$$f(x) = x^4 \sqrt{1+x^2} (\cos(2x) - 1)^2$$

close to $x = 0$.

Problem 8.7. Let

$$f(x) = \begin{cases} \alpha + x + x^2 & \text{if } x < 0, \\ \beta \sin(x) & \text{if } x \geq 0, \end{cases}$$

where α and β are real parameters.

- (1) Find the intervals where $f(x)$ is decreasing for $x < 0$.
- (2) Find the values of α and β that make $f(x)$ differentiable at $x = 0$.
- (3) Set $\alpha = -1$ and $\beta = 1$. Then, find and classify the *global* extrema of $f(x)$ in \mathbb{R} .

Problem 8.8. Let $f(x) = 3x^4 - 4x^3 + 1$.

- (a) Find and classify the critical points of $f(x)$.
- (b) Determine the intervals where $f(x)$ is increasing or decreasing.
- (c) Find the inflection points of $f(x)$.
- (d) Determine the intervals where $f(x)$ is concave up or down.

Problem 8.9. Find the *global* extrema of the following functions on the indicated intervals.

$$f(x) = \left| \frac{x}{\sqrt{2}} \right| + \cos(x), \quad \text{for } x \in [-\pi, \pi].$$

$$f(x) = 2x^{5/3} + 5x^{2/3}, \quad \text{for } x \in [-2, 1].$$