CALCULUS

Bachelor in Computer Science and Engineering

Course 2022–2023

Series of real numbers

Problem 3.1. Analyze the convergence of the following series with positive terms.

a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2+k} = 1$ (prove this).

b) $\displaystyle\sum_{k=1}^{\infty} \frac{3+2\cos(k)}{k^2+k}$.

c) $\displaystyle\sum_{k=1}^{\infty} \frac{k+1}{k^2}$.

d) $\displaystyle\sum_{k=1}^{\infty} \frac{7\sqrt{k}+323}{k^2+\cos(k)}$.

e) $\displaystyle\sum_{k=1}^{\infty} \frac{\arctan(k)}{k^2+7}$.

f) $\displaystyle\sum_{k=1}^{\infty} \frac{2^k}{3^k+(-1)^k}$.

g) $\displaystyle\sum_{k=1}^{\infty} \frac{\ln(k)}{k^4}$.

h) $\displaystyle\sum_{k=1}^{\infty} \frac{\ln(k)}{k}$.

i) $\displaystyle\sum_{k=1}^{\infty} \frac{\ln(k)}{k^2}$.

j) $\displaystyle\sum_{k=1}^{\infty} \frac{(k+1)^k}{k^{k+1}}$.

Problem 3.2. Study whether the following series are convergent or not.

a) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2+k}$.

b) $\displaystyle\sum_{k=1}^{\infty} \frac{\cos(k)}{5^k}$.

c) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{k}$.

d) $\displaystyle\sum_{k=1}^{\infty} \frac{(-4)^k}{4+k!}$.

e) $\displaystyle\sum_{k=1}^{\infty} (-1)^k 3^k 5^{-\sqrt{k}}$.

f) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(\ln(k))^k}$.

g) $\displaystyle\sum_{k=1}^{\infty} \frac{k!}{k^k}$.

h) $\displaystyle\sum_{k=1}^{\infty} \ln\left(\frac{k}{k+1}\right)$.

Problem 3.3. Find *all* values of the parameters $a, b, \alpha \in \mathbb{R}$ for which the following series converge.

1) $\displaystyle\sum_{k=1}^{\infty} \frac{k^a}{b^k}$, with $a > 0$, $b \neq 0$.

2) $\displaystyle\sum_{k=1}^{\infty} \frac{b^k}{k!}$.

3) $\displaystyle\sum_{k=1}^{\infty} (-1)^k \frac{(2\alpha)^{3k}}{7^k \sqrt[3]{k^2+k}}$.