

CALCULUS

Bachelor in Computer Science and Engineering

Course 2021–2022

Techniques of integration

Problem 10.1. Calculate the following integrals.

$$\int \arctan(3x) \, dx .$$

$$\int e^x \sin(x) \, dx .$$

$$\int \cos(\ln(x)) \, dx .$$

$$\int \cos^2(\ln(x)) \, dx .$$

$$\int \frac{dx}{\sqrt{e^x - 4}} .$$

$$\int \frac{dx}{x \sqrt{x^2 - 1}} .$$

Problem 10.2. Calculate the following integrals by using an appropriate change of variable.

$$\int_1^2 \frac{\sqrt{t^2 - 1}}{t} \, dt .$$

$$\int_0^{\ln(2)} \sqrt{e^t - 1} \, dt .$$

$$\int \frac{dx}{(x + 2)\sqrt{1 + x}} .$$

$$\int \frac{dx}{1 + \sqrt[3]{1 - x}} .$$

Problem 10.3. Calculate the following integrals of *rational* functions.

$$\begin{aligned} & \int \frac{dx}{3x^2 + 4x + 2} \cdot \\ & \int \frac{x^5 - 2x^3}{x^4 - 2x^2 + 1} dx \cdot \\ & \int \frac{x^2 + 1}{x^4 - x^2} dx \cdot \\ & \int \frac{x^3 + 1}{x^2 + 4x + 13} dx \cdot \\ & \int \frac{x^2 + 6x - 1}{x^3 - 7x^2 + 15x - 9} dx \cdot \end{aligned}$$

Problem 10.4. Calculate the following integrals by using the given hints.

1. $\int \cos^3(x) dx$ (change of variable: $u = \sin(x)$)
2. $\int \sin^4(x) dx$ (identities: $\cos(2\alpha) = 1 - 2\sin^2(\alpha) = 2\cos^2(\alpha) - 1$)
3. $\int \frac{e^{4x}}{e^{2x} + 2e^x + 2} dx$ (change of variable: $u = e^x$)
4. $\int \frac{\sin^3(x)}{1 + \cos^2(x)} dx$ (change of variable: $u = \cos(x)$)
5. $\int \frac{dx}{\cos(x)}$ (change of variable: $u = \sin(x)$)
6. $\int \sqrt{a^2 - x^2} dx, a \in \mathbb{R}$ (change of variable: $x = a \sin(u)$)