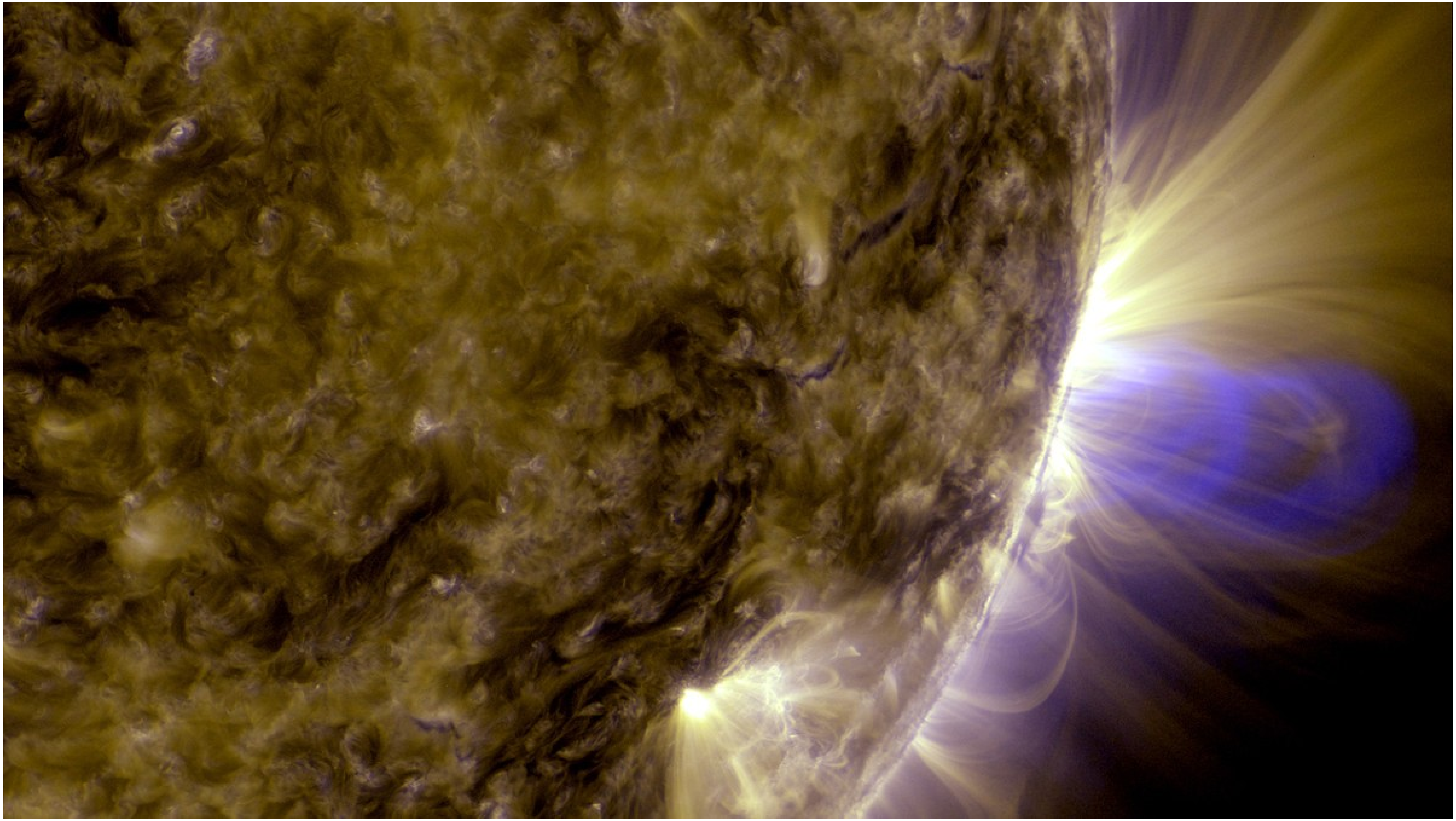
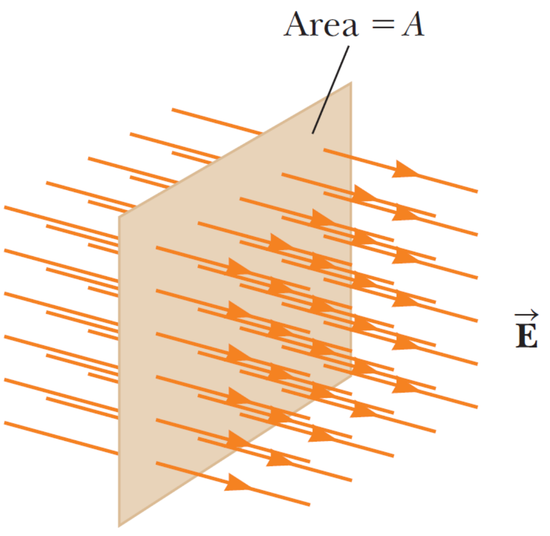
Gauss's law



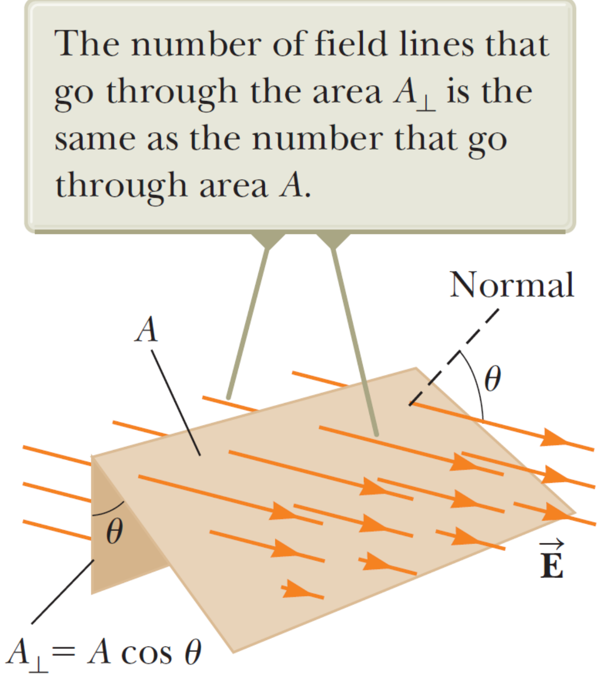
Electric flux



For a plane surface in a uniform electric field

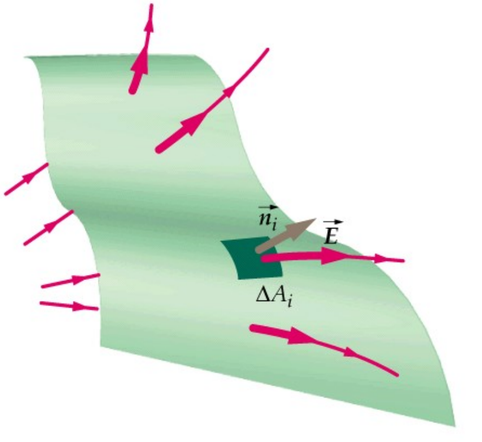
$$\Phi = \vec{E} \cdot \vec{A}$$

UNITS OF ELECTRIC FLUX: $(N.m^2)/C$

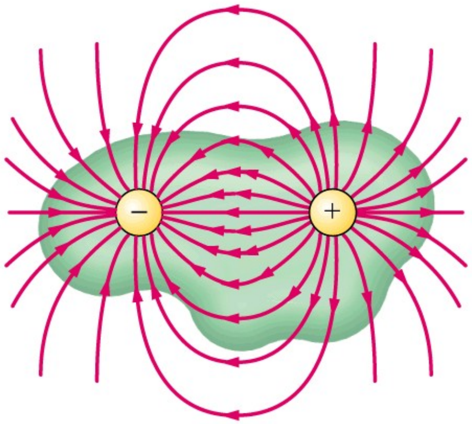


In general, for an arbitrary surface and electric field

$$\Phi = \int_S \vec{E} \cdot d\vec{S}$$



Electric flux and Gauss law

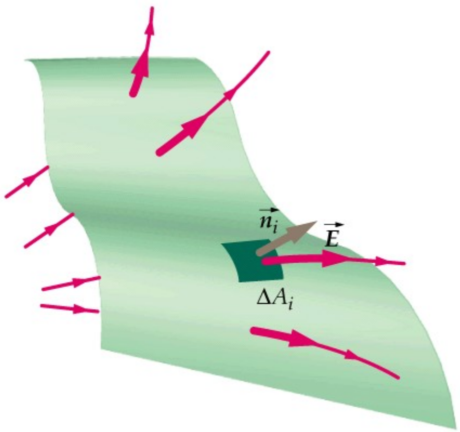


The net number of $\vec{E}$ lines going through a closed surface is proportional to the net charge enclosed by the surface.

The mathematical quantity that corresponds to the net number of field lines passing through a surface is called *electric flux* Φ :

$$\Phi = \int_S \vec{E} \cdot d\vec{S}$$

UNITS OF ELECTRIC FLUX: (N.m^2)/C



Closed surface

GAUSS' LAW

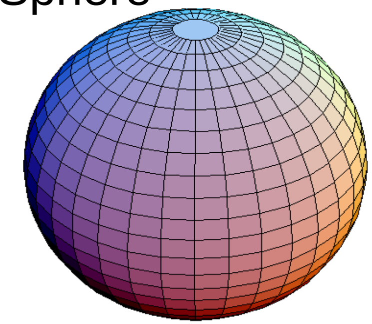
$$\Phi = \oint_S \vec{E} \cdot d\vec{S} = \frac{q_{\text{inside S}}}{\varepsilon_0}$$

Permitivity of free space $\varepsilon_0 = 8.85 \cdot 10^{-12}$ C^2/Nm^2

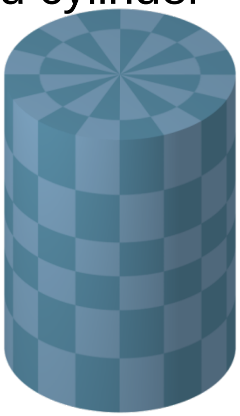
Open and closed surfaces

A closed surface encloses (contains) a volume. EXAMPLES:

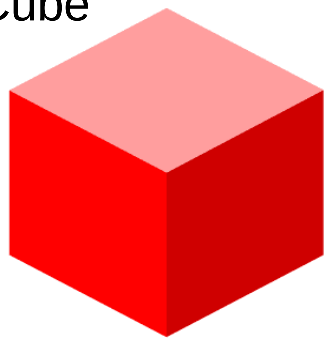
Sphere



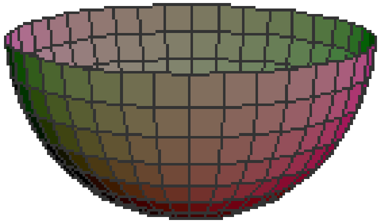
Closed cylinder

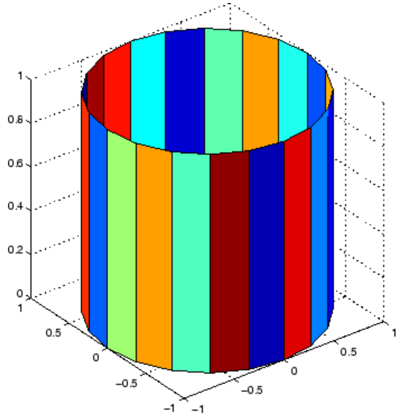


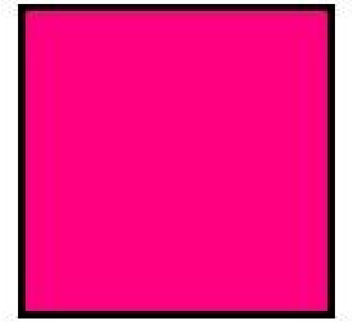
Cube



It is constituted by several open surfaces.







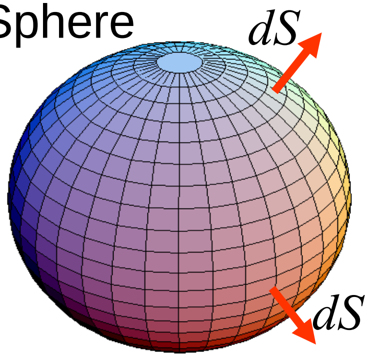
Open and closed surfaces

A closed surface has associated a surface vector $d\vec{S}$ that gives vector character to the surface.

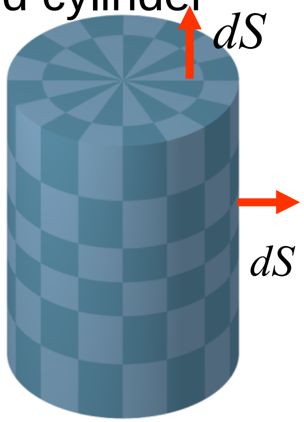
The surface vector is perpendicular to the surface (at each point) and <u>directed towards the outside</u> of the closed surface.

<u>EXAMPLE</u>:

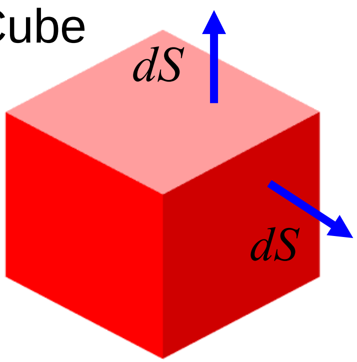
Sphere



Closed cylinder



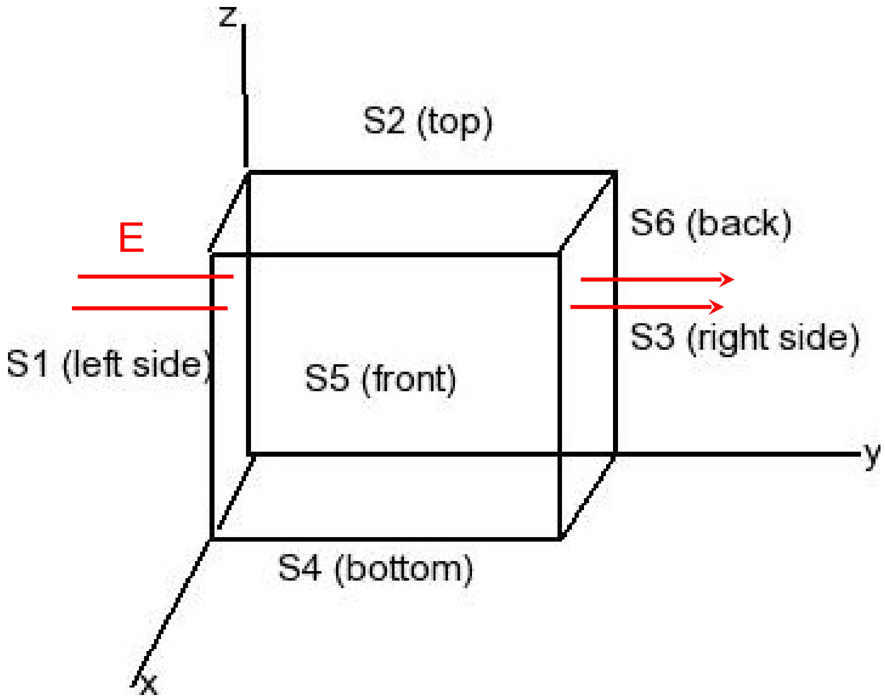
Cube



Electric flux

The flux through a closed surface is calculated by adding the fluxes through each of the open surfaces that constitute the closed surface.

<u>EXAMPLE</u>: Cube

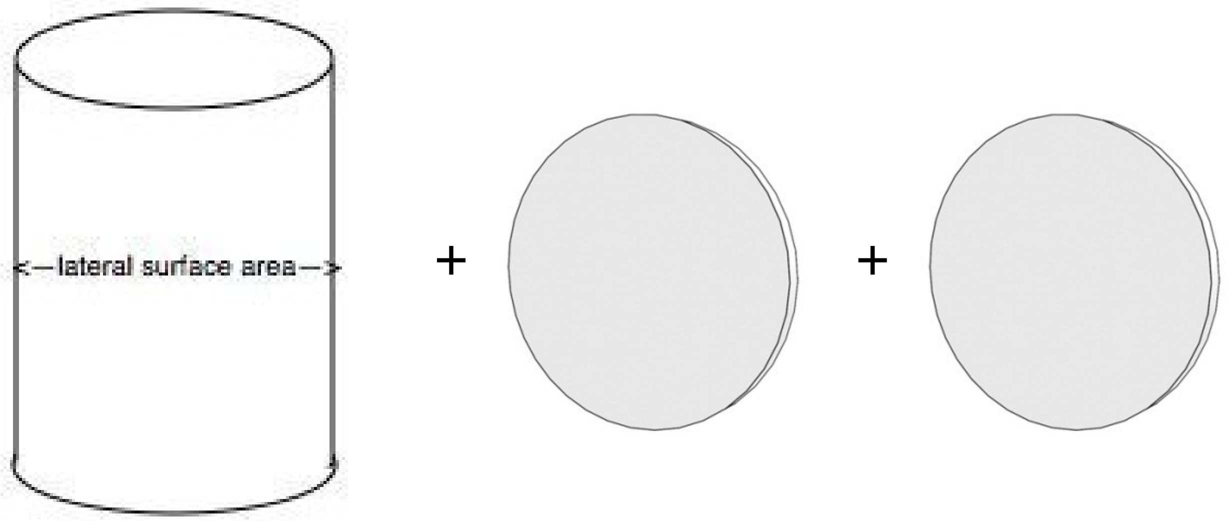


$$\Phi_{total}^{cube} = \Phi_{S1} + \Phi_{S2} + \Phi_{S3} + \Phi_{S4} + \Phi_{S5} + \Phi_{S6}$$

$$\Phi_{total}^{cube} = 0 \text{ if the same number of lines enters and leaves}$$ → General result

Electric flux

EXAMPLE 2: Closed cylinder



$$\Phi_{total}^{closed\ cylinder} = \Phi_{lateral\ area} + \Phi_{base\,1} + \Phi_{base\,2}$$

Electric flux

<u>EXAMPLE</u>:

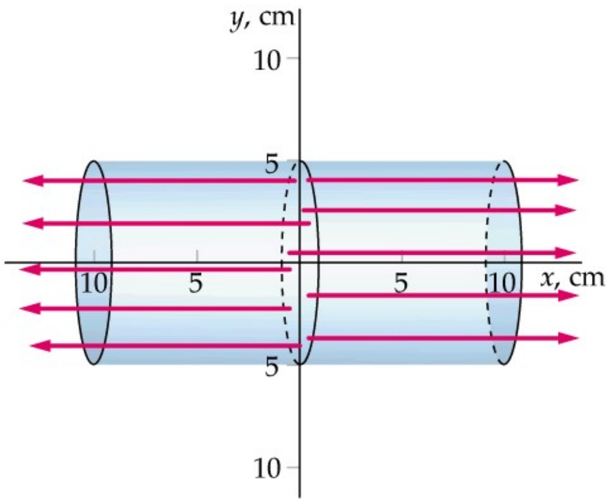
An electric field is E=(200 N/C)i for x>0 and E=(-200 N/C)i for x<0. A closed cylinder of length 20 cm and radius R=5 cm has its centre at the origin and its axis along the x axis, so that one end is at x=10 cm and the other at x=-10 cm.

(a) What is the flux through each end?

(b) What is the flux through the curved surface of the cylinder?

(c) What is the net outward flux through the entire closed surface?

(d) What is the net charge enclosed in the cylinder?



Electric flux

<u>EXAMPLE</u>: **Exercise 3**

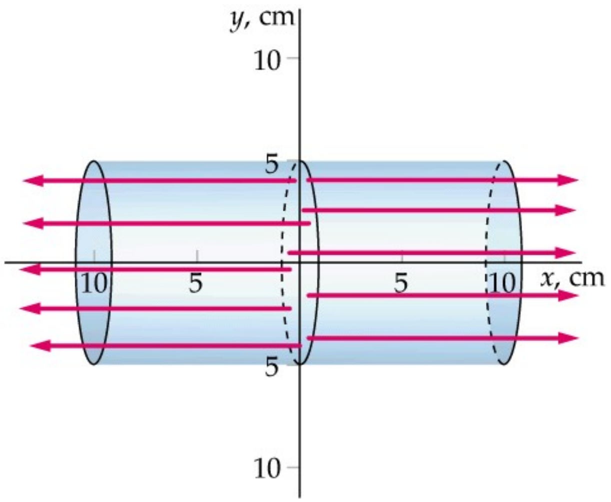
An electric field is E=(200 N/C)i for x>0 and E=(-200 N/C)i for x<0. A closed cylinder of length 20 cm and radius R=5 cm has its centre at the origin and its axis along the x axis, so that one end is at x=10 cm and the other at x=-10 cm.

(a) What is the flux through each end?

(b) What is the flux through the curved surface of the cylinder?

(c) What is the net outward flux through the entire closed surface?

(d) What is the net charge enclosed in the cylinder?



ANSWER: a) 0.5π N m^2/C, b) 0, c) ϕ_{net}= 3.14 N m^2/C d) Q_{inside}=2.78 10^{-11} C

Surface area and volume

THINGS WE NEED TO KNOW:

AREA OF A SPHERE: $4\pi R^2$

VOLUME OF A SPHERE: $(4/3)\pi R^3$

LATERAL AREA OF A CYLINDER: $2\pi RL$

VOLUME OF A CYLINDER: $\pi R^2 L$

AREA OF A DISC (BASE OF THE CYLINDER): πR^2

Using Gauss' law to find E

Gauss' law can be applied to calculate the electric field due to a charge distribution when the distribution has enough symmetry.

HOW TO FIND E AT POINT P:

1. Find the flux through an imaginary closed surface (gaussian surface) passing through P and enclosing the charge distribution.

$$\Phi = \oint_S \vec{E} \cdot d\vec{S}$$

2. Find the total charge enclosed inside the gaussian the surface S.

$$q_{inside\,S}$$

3. Apply Gauss´ Law using the results obtained in 1 and 2.

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{q_{\text{inside S}}}{\varepsilon_0}$$

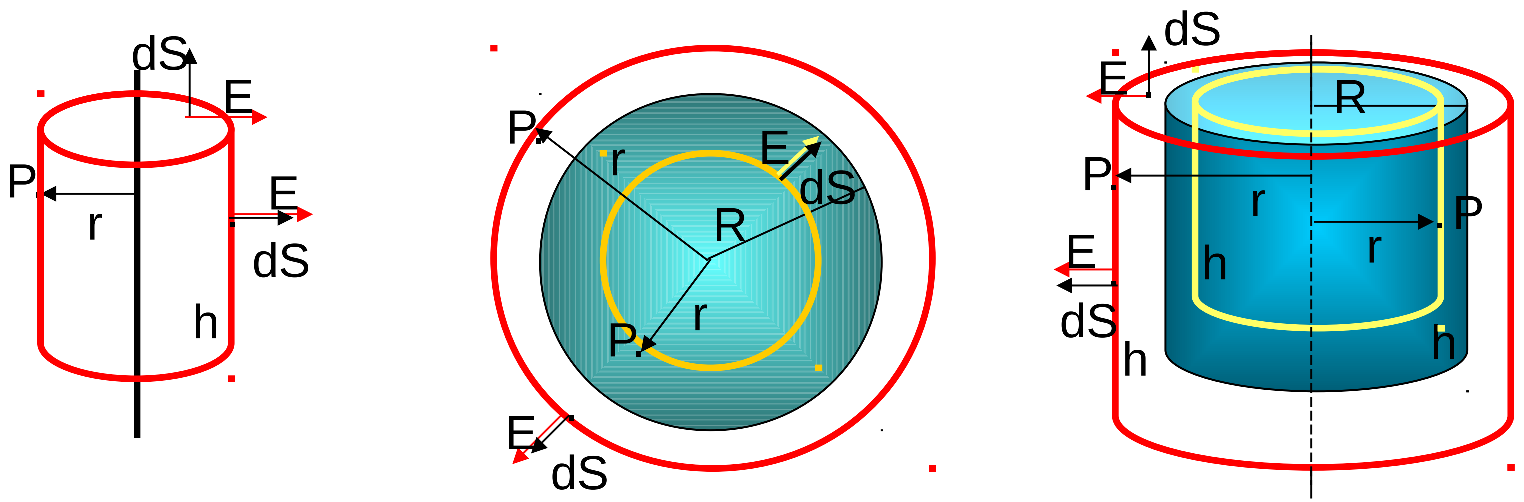
4. Isolate the unknown magnitude (the electric field E). No not forget to add a unit vector indicating the direction of E!

Using Gauss' law to find E

HOW TO FIND E AT POINT P:

1. In order to find the flux: $\Phi = \oint_S \vec{E} \cdot d\vec{S}$

1.1 Choose a closed surface passing through P (Gaussian surface) in which the flux is easy to calculate.



1.2 Determine the direction of E and dS at the surface

1.3 Calculate the scalar product and integrate in S

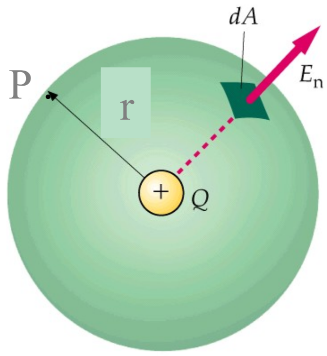
2. In order to find the charge enclosed:

$$q = \int_h \lambda \, dl \qquad q = \int_S \sigma \, dS \qquad q = \int_V \rho \, dV$$

Using Gauss' law to find E

Electric field due to a point charge Q at a point P located at a distance r

1. Find the flux: $\Phi = \oint_S \vec{E} \cdot d\vec{S}$



$$\vec{E} \cdot d\vec{S} = E \cdot dS \;\longrightarrow\; \oint_S |\vec{E}| dS = |\vec{E}| \oint_S dS = |\vec{E}| S = |\vec{E}| 4\pi r^2$$

$\vec{E} || d\vec{s}$ $\quad$ $\vec{E}$ constant along $\vec{S}$

2. Find the charge enclosed: $q_{\text{inside S}} = Q$

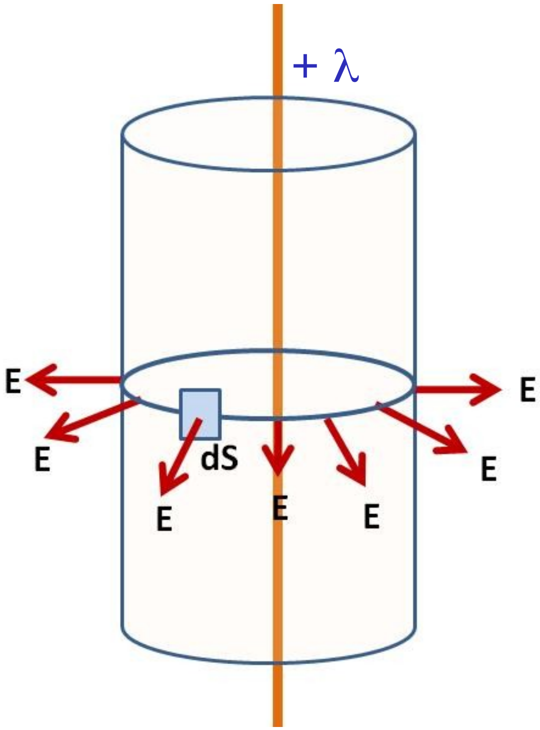
3. Apply Gauss' Law: $\varphi_{net} = |\vec{E}| 4\pi r^2 = \dfrac{Q}{\varepsilon_0}$

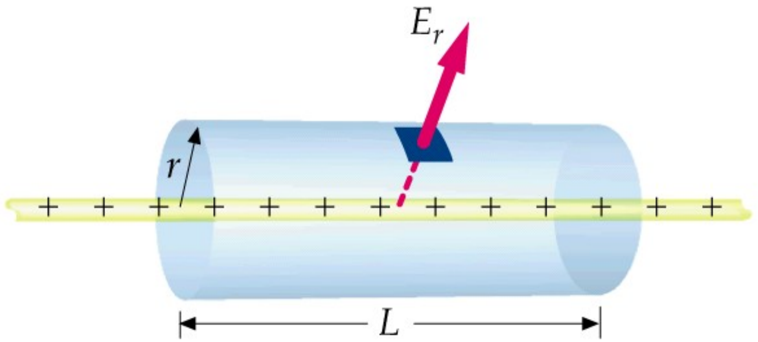
4. Isolate E $\quad |\vec{E}| = \dfrac{Q}{\varepsilon_0 4\pi r^2} = k\dfrac{Q}{r^2}$

$$\vec{E} = k\frac{Q}{r^2}\vec{u}_r$$

Using Gauss' law to find E

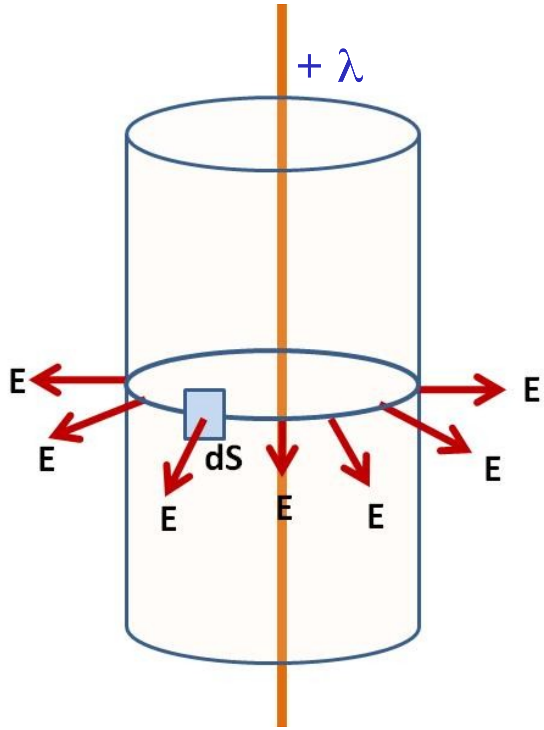
Electric field due to an infinite charged line with constant charge density λ

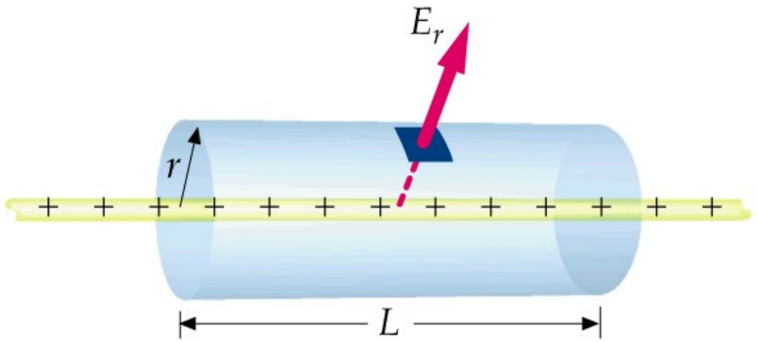




Using Gauss' law to find E

Electric field due to an infinite charged line with constant charge density λ

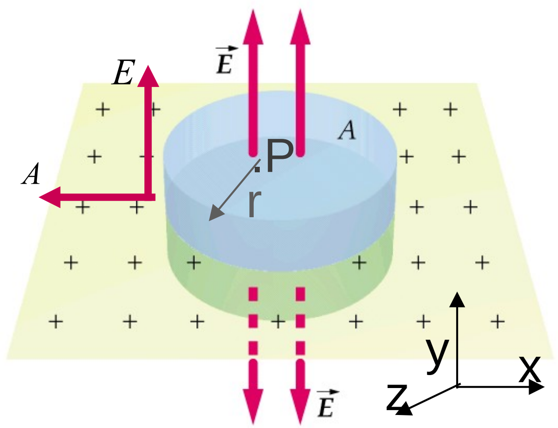




$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\vec{u}_r$$

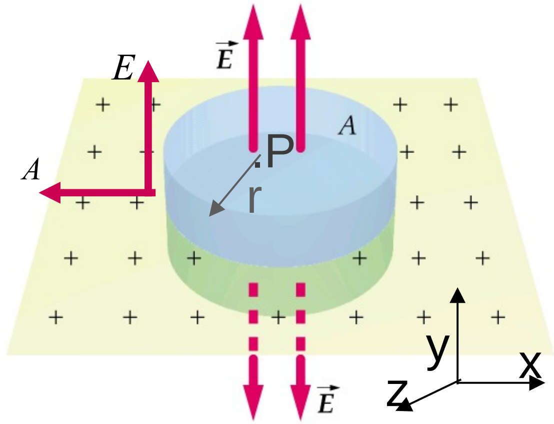
Using Gauss' law to find E

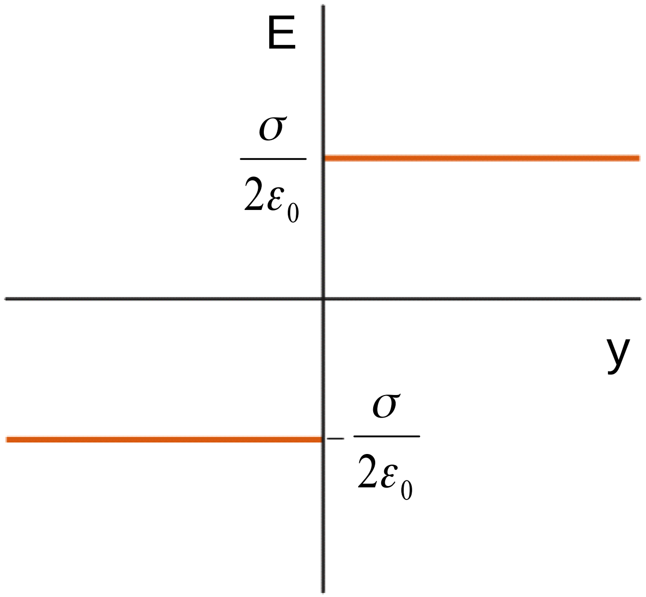
Electric field due to an infinite charged plane with constant charge density σ



Using Gauss' law to find E

Electric field due to an infinite charged plane with constant charge density σ





$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\left(\pm \vec{j}\right)$$

E is constant!

E is discontinuous at the plane.

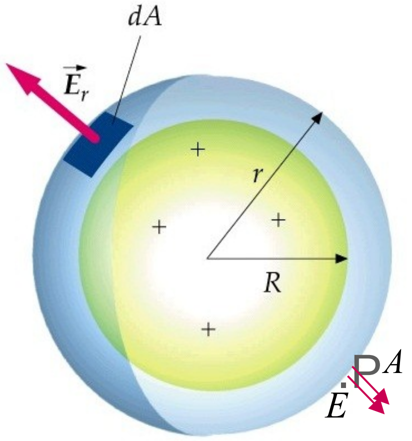
The discontinuity is σ/ε_0.

Using Gauss' law to find E

Electric field due to a uniformly charged solid sphere of radius R and charge density ρ

The whole volume is charged

Outside the sphere:

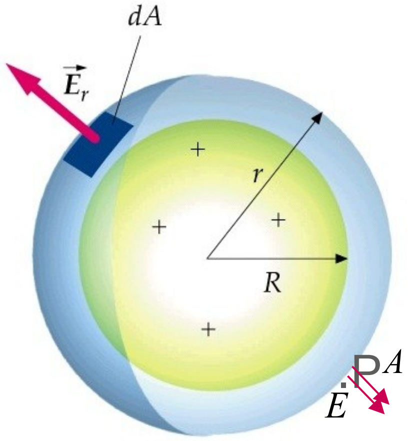


Using Gauss' law to find E

Electric field due to a uniformly charged solid sphere of radius R and charge density ρ

The whole volume is charged

Outside the sphere:



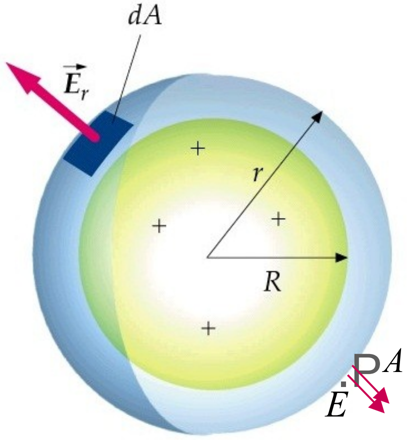
$$\vec{E}=\frac{\rho R^3}{3\varepsilon_0 r^2}\vec{u}_r\,(N/C)$$

Using Gauss' law to find E

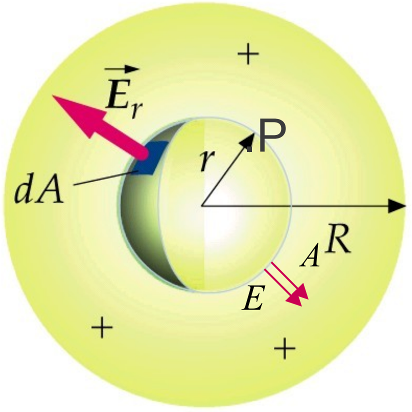
Electric field due to a uniformly charged solid sphere of radius R and charge density ρ

The whole volume is charged

Outside the sphere: $\vec{E} = \dfrac{\rho R^3}{3\varepsilon_0 r^2}\vec{u}_r$



Inside the sphere: $\vec{E} = \dfrac{\rho r}{3\varepsilon_0}\vec{u}_r$

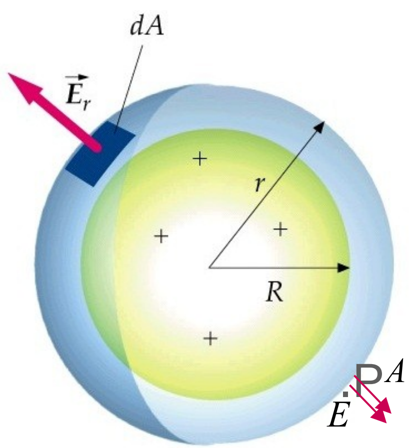


Using Gauss' law to find E

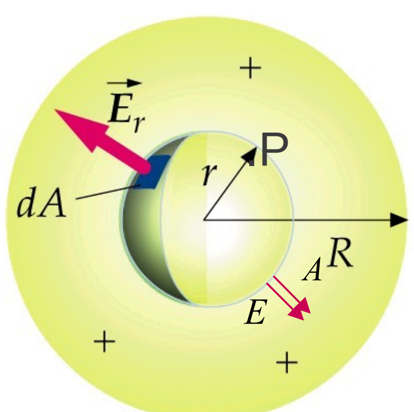
Electric field due to a uniformly charged solid sphere of radius R and charge density ρ

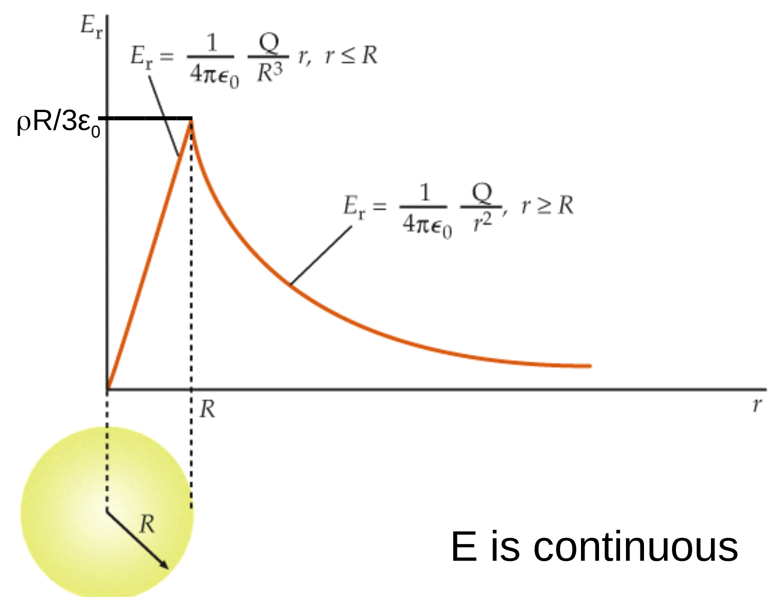
Outside the sphere: $\vec{E} = \dfrac{\rho R^3}{3\varepsilon_0 r^2}\vec{u}_r$

The whole volume is charged



Inside the sphere: $\vec{E} = \dfrac{\rho r}{3\varepsilon_0}\vec{u}_r$



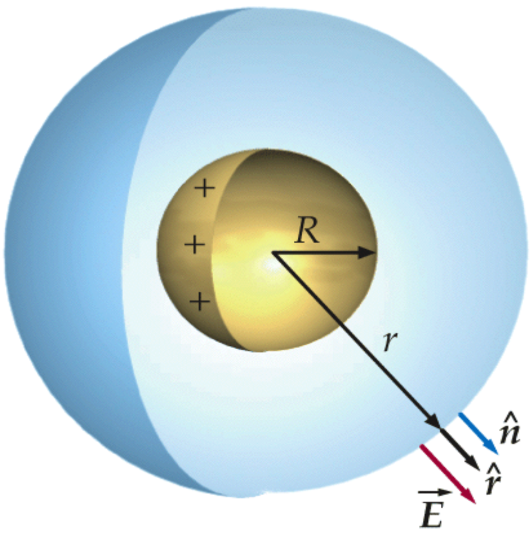


E is continuous

Using Gauss' law to find E

Electric field due to a spherical shell of radius R and constant charge density σ

It is hollow!

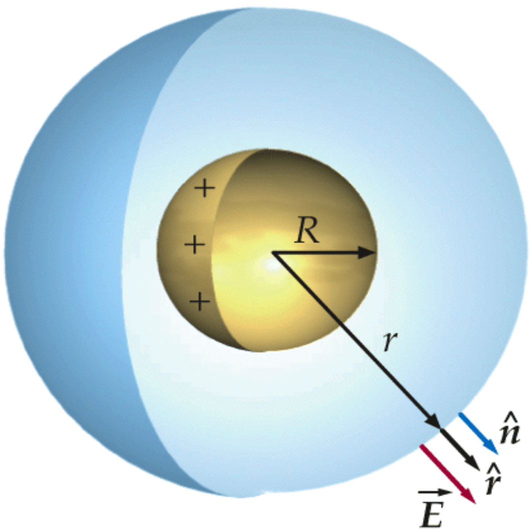


Inside the sphere:

Using Gauss' law to find E

Electric field due to a spherical shell of radius R and constant charge density σ

It is hollow!

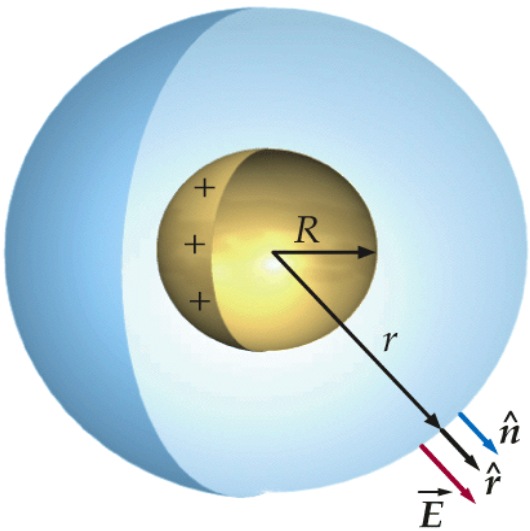


Inside the sphere: $\vec{E} = 0$

Using Gauss' law to find E

Electric field due to a spherical shell of radius R and constant charge density σ

It is hollow!



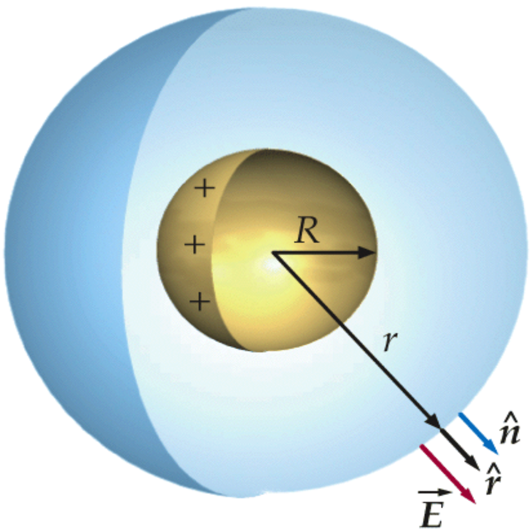
Inside the sphere: $\vec{E} = 0$

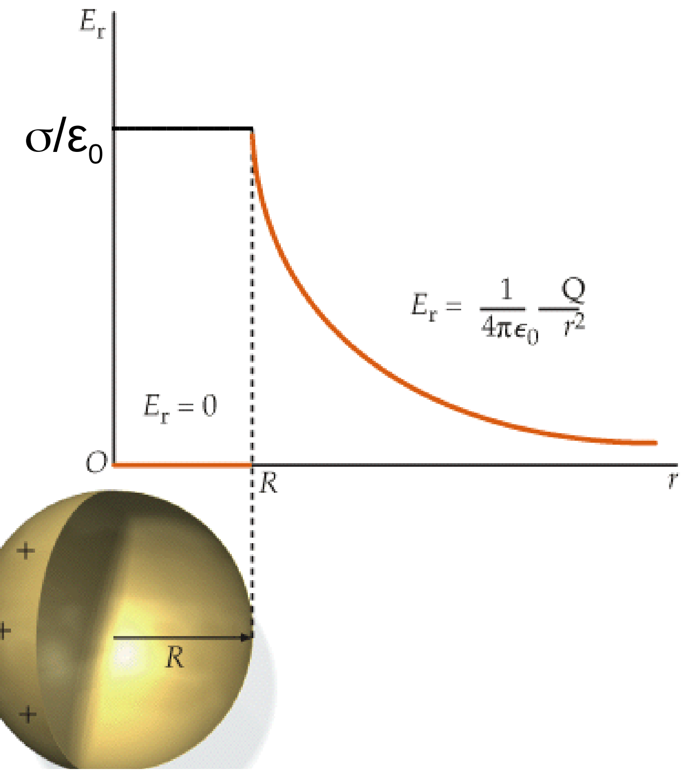
Outside the sphere:

Using Gauss' law to find E

Electric field due to a spherical shell of radius R and constant charge density σ

It is hollow!





Inside the sphere: $\vec{E} = 0$

Outside the sphere: $\vec{E} = \dfrac{\sigma R^2}{\varepsilon_0 r^2}\vec{u}_r$

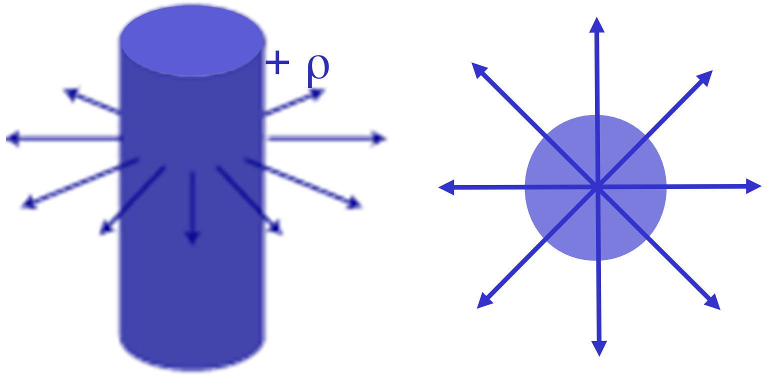
E is discontinuous at the shell.

The discontinuity is σ/ε_0

Using Gauss' law to find E

Electric field due to a uniformly charged infinite solid cylinder of radius R and charge density ρ

The whole volume is charged



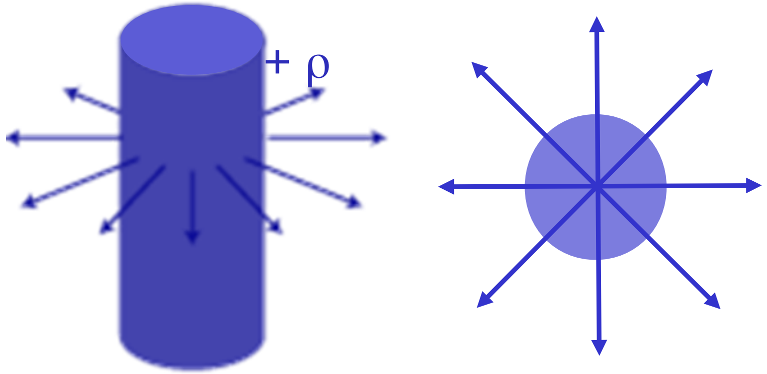
Outside the cylinder:

Inside the cylinder:

Using Gauss' law to find E

Electric field due to a uniformly charged infinite solid cylinder of radius R and charge density ρ

The whole volume is charged



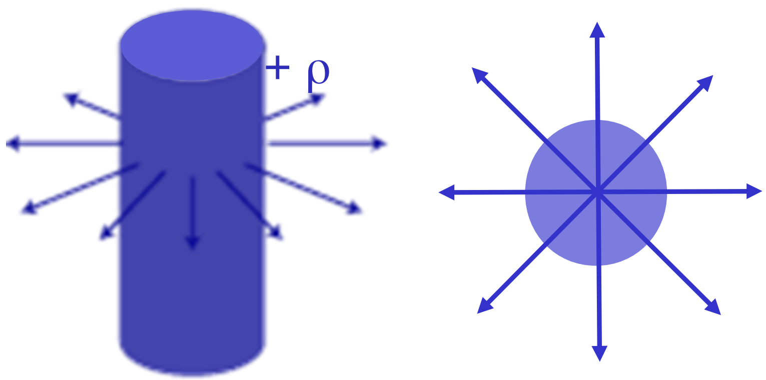
Outside the cylinder: $\vec{E} = \frac{\rho R^2}{2\varepsilon_0 r}\vec{u}_r$

Inside the cylinder:

Using Gauss' law to find E

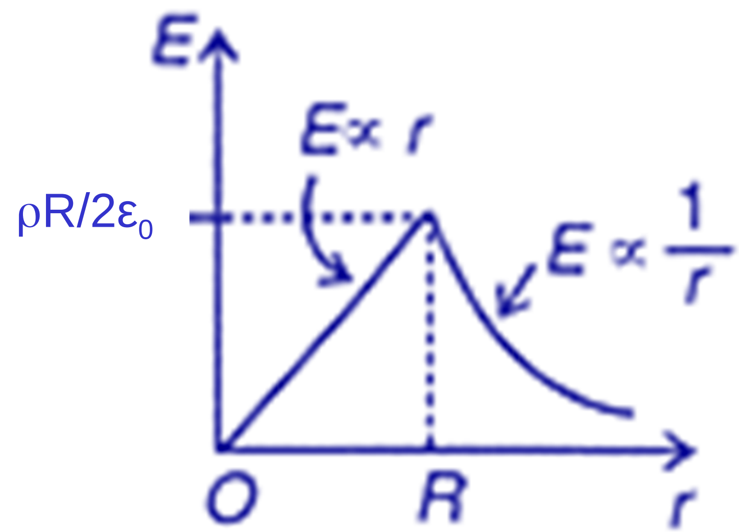
Electric field due to a uniformly charged infinite solid cylinder of radius R and charge density ρ

The whole volume is charged



Outside the cylinder: $$\vec{E} = \frac{\rho R^2}{2\varepsilon_0 r}\vec{u}_r$$

Inside the cylinder: $$\vec{E} = \frac{\rho r}{2\varepsilon_0}\vec{u}_r$$

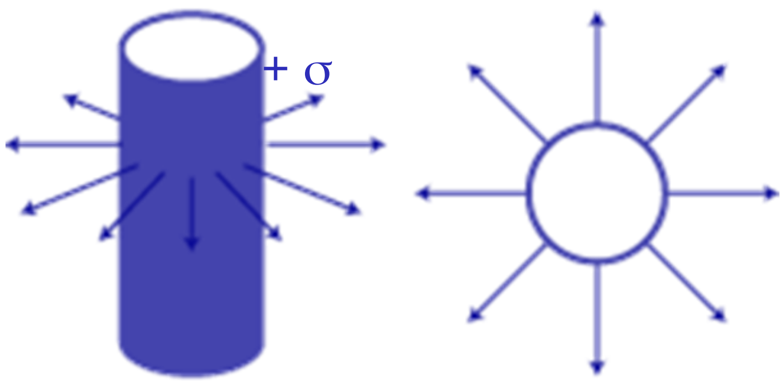


E is continuous

Using Gauss' law to find E

Electric field due to an infinite cylindrical foil of radius R and constant charge density σ

It is hollow!



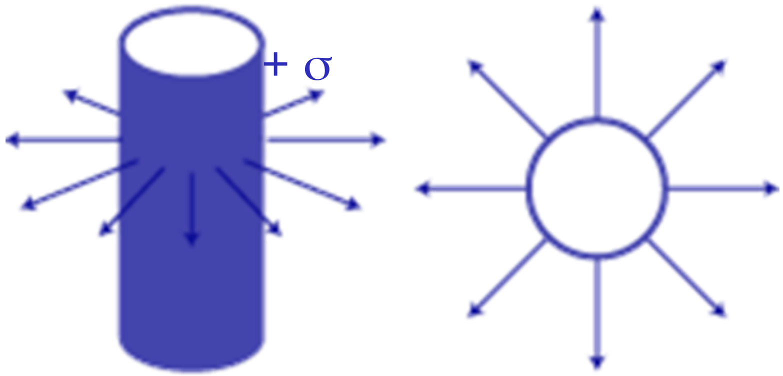
Outside the cylinder:

Inside the cylinder:

Using Gauss' law to find E

Electric field due to an infinite cylindrical foil of radius R and constant charge density σ

It is hollow!



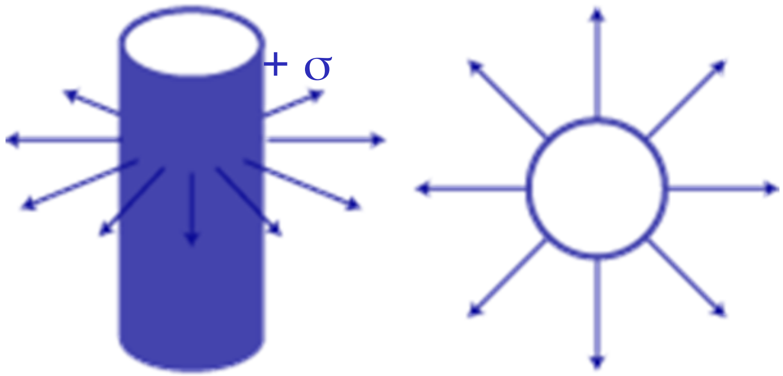
Outside the cylinder: $$\vec{E} = \frac{\sigma R}{\varepsilon_0 r}\vec{u}_r$$

Inside the cylinder:

Using Gauss' law to find E

Electric field due to an infinite cylindrical foil of radius R and constant charge density σ

It is hollow!



Outside the cylinder: $$\vec{E} = \frac{\sigma R}{\varepsilon_0 r} \vec{u}_r$$

Inside the cylinder: $$\vec{E} = 0$$

E is discontinuous at r=R.

The discontinuity is σ/ε_0