

Bachelor in Computer Science and Engineering

Statistics Problems

IV Probability Models

1. A set of random variables is described below. Specify for each one of them which is their corresponding probability model, binomial, Poisson, exponential or none of those. Indicate the value for the corresponding parameters.
 - a. An average of 120 passengers per hour arrive to the lobby of a train station. Their arrival has a steady average rhythm and they arrive in an independent way. The random variables of our interest are:
 - i. X: Number of passengers arriving in an hour
 - ii. X: Number of passengers arriving in four hours
 - iii. X: Minutes elapsed without passengers arriving
 - b. An electrical component is packed in boxes of 100 elements. If the percentage of defective elements is 0,03%, the random variable of interest is X: number of defective items in a package.
 - c. A building has two access doors, one on the front and another on the side. It is checked that people arrive to both doors with a steady average rhythm. It is assumed that people arrive to the building in an independent way. An average of 3 people per minute arrive to the front door, and an average of 1 person every 5 minutes arrives to the side door. The random variables of interest are:
 - i. X: Number of people that access the building through the front door every hour.
 - ii. X: Number of people that access the building through the side door every hour.
2. A manufacturer of high-end refrigerators accepts only top quality fans in order to minimize acoustic emissions from the appliance. The quality manager is defining the acceptance protocol for the new fan supply and, to facilitate this process, it is intended to reject the lot if a noisy fan is found among the fans contained in a sample selected randomly.
 - a. Which should be the minimum size of the sample so that the probability of accepting a lot with at least 5% of noisy fans be negligible? (less than 1%)
 - b. Inspecting a lot of 100 fans 6% of noisy fans have been found. Which is the probability that a lot contains more than 8 noisy fans?

3. An electrical component is sold in boxes of 4 units. If the manufacturer assures that they work correctly with probability 0,9875:
 - a. Which is the probability that analyzing the components contained in 100 boxes, 2 or more defective components are found?
 - b. The number of components that fail in a month follows a Poisson distribution with an average of 6 failures per month. Calculate the probability that there are no failures in two months.
 - c. After a month without failures, which is the probability that the next failure occurs in the next month?
4. The time (in hours) a machine takes to fail is an exponential random variable, with an average lifetime of 100 hours. In a given workshop there are three machines of this type working independently. It is asked:
 - a. The probability that machine 1 fails in the first 100 hours.
 - b. The probability that at least one of the machines fails in the first 100 hours.
 - c. Calculate the maximum duration that the manufacturer can indicate in the guarantee if he wants that the probability of receiving a claim, for having a failure before that time, be 30%.
5. The number of assistance calls received by a towing service follows a Poisson process, receiving an average of 5 calls per hour.
 - a. Calculate the probability of receiving exactly 10 calls in a period of two hours.
 - b. If the service operators have 30 minutes for lunch, which is the probability that they do not lose any assistance call?
 - c. How many calls are expected during lunch time?
6. An enterprise that produces batteries knows that the probability that none of the batteries produced during an hour be defective is 0,10. It can be considered the number of defective batteries produced per hour as a Poisson random variable with density function given by:

$$P(X = x) = \frac{\lambda^x}{x!} e^{-\lambda}, x = 0, 1, 2 \dots$$

- a. Calculate the value of λ
 - b. Which is the probability that less than three defective batteries are produced in an hour?
 - c. Which is the probability to get the first defective battery in less than an hour?
 - d. After 100 hours, which is the probability that the first defective battery appears in the next hour?
7. The average access time to a given server is 10 seconds during a working hours (30% of the day), and 5 seconds the rest of the day, and it follows an exponential distribution. It is asked:
 - a. The probability that the time to access the server during working hours be greater than 8 seconds.

- b. The probability that the time to access the server at any time of the day be greater than 8 seconds.
 - c. If a user tries to access the server once in a week during working hours, what is the probability that in a year the total access time be greater than 10 minutes? (consider 52 weeks in a year).
8. The life of an electronic component (time elapsed from connection to failure) follows an exponential distribution. The average component life is 200 days.
 - a. Which is the probability that a component lasts more than 100 days?
 - b. If the component has lasted more than 100 days, which is the probability that it lasts more than 200 days?
 - c. An electronic equipment is integrated by 10 of those components working in an independent way, and it is working whenever there are at least four of them working correctly. Which is the probability that such equipment works more than 100 days?
9. A damaged server suffers random disconnections from the network with an average of 3 disconnections per hour. Assuming that the disconnections are independent from each other, it is asked:
 - a. The probability that just one disconnection occurs in 15 minutes
 - b. The probability to have 2 or more disconnections in one hour
 - c. The probability that just one disconnection occurs in 15 minutes if during the previous 15 minutes 2 disconnections had occurred.
 - d. The probability that the time between two consecutive disconnections be greater than 25 minutes, and the mean time between disconnections.
10. During world war two in a German military prison, the French colonel Antonie Poisson, head of the allied escape committee, has to choose what plan to execute from two that have been proposed. He needs to calculate which is more likely to be successful. In order to help making the decision they have been watching the guardians' movements during the last 48 hours. The frequency they pass by the point which is easier to escape is the following:

Number of guardians	Number of hours
0	7
1	9
2	6
3	10
4	8
5	5
6	3

The proposed plans are:

Plan A: To be successful less than three guards during an hour

Plan B: To be successful during a minimum of thirty minutes no guards

- a. Calculate the probability of success for each plan and say which should be chosen assuming that the guards appear in an stable and independent way.
- b. Suspecting there is an escape plan, the german officials decide to increment the number of guards. They act in an independent way from those that were already in the prisión, appearing according to a Poisson process with average 2 guards per hour. Which is the success probability of the chosen plan with this new group of guards?