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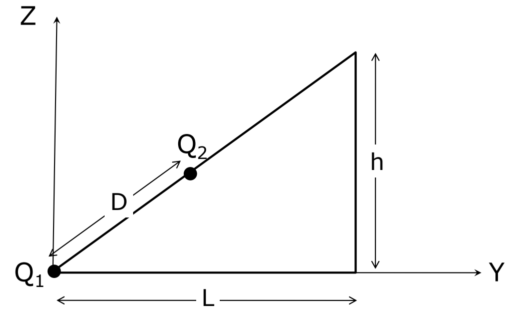
P1. ***(2p)*** A point charge Q_1 is fixed at the base of a ramp with length L and height h (it's position coincides with the origin of the reference frame). A second point charge Q_2 with mass M is placed on the frictionless ramp at a distance D from the first charge.



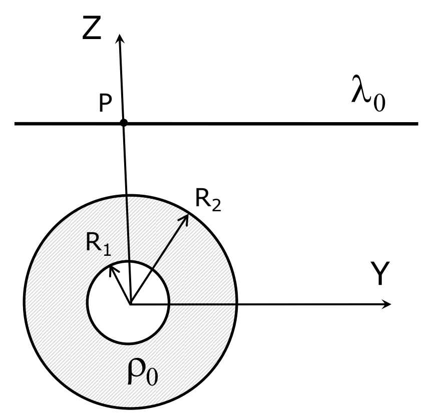
a) Calculate the acceleration vector that the second particle experiences at it's initial position.

b) Calculate the position vector of the second particle when the acceleration it experiences is zero.

c) For case (b) calculate the electrostatic energy stored in the system of charges.

DATA: h= 28 m; L= 40 m; Q_1= -2 μC; Q_2= -4 μC; M= 1.5 g; D= 1.7 m; g=9.8 m/s^2

P2. ***(2p)*** You have the following charge distributions:



- A uniform charge distribution in the volume of a hollow sphere, centered at the coordinates origin, with internal radius R_1= 2 cm and external R_2= 4 cm. The volume charge density is $\rho_0 = 2\times10^{-6}$ C/m^3.

- An infinite straight line with uniform charge density $\lambda_0 = 4\times10^{-6}$ C/m. The line is parallel to the Y axis and passes through the point P (0,0,7).

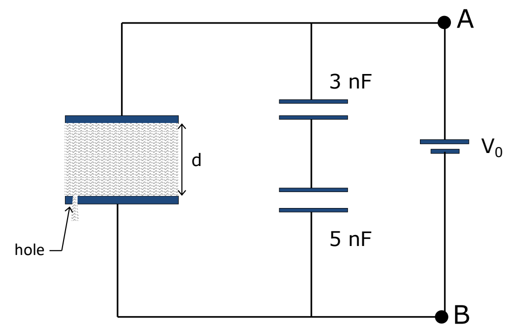
a) Calculate the electric field flux through the closed surface of a cube centered at origin and with edge length 20 cm.

b) Calculate the electric field vector at points A (0,3,0) and B (0, 5,0)

c) Calculate the potential difference ($V_C - V_B$) between B (0,5,0) and C (0,8,0)

NOTE: All coordinates are expressed in cm.

P3. ***(2p)*** A plane capacitor, formed by plates of area A and separated by a distance d, is completely filled with a liquid dielectric of dielectric constant ε_r. This capacitor reservoir is connected to a capacitor circuit, as shown in the figure. Once the circuit is charged, the battery is disconnected. A small hole is then opened in the lower plate of the capacitor, through which liquid starts to leak out.

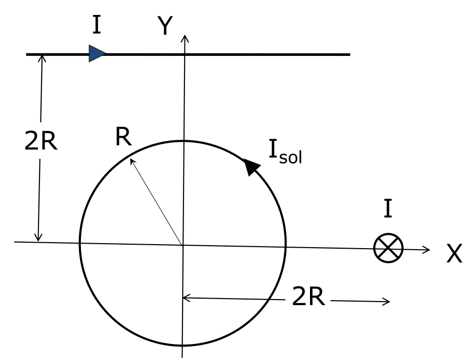


a) Calculate how much volume of dielectric liquid remains in the capacitor reservoir if the potential difference between points A and B is $V_{AB}= 8.25\times10^3$ V

b) Calculate the electrostatic energy stored in the circuit for case (a)

DATA: $A= 0.5$ m^2 ; $d= 5$ cm; $\varepsilon_r= 30$; $V_0= 4\times10^3$ V

P4. ***(2p)*** You have the following currents (the directions of the currents are indicated in the figure):



- An ideal solenoid, coaxial with the Z axis, with a turns density n, and traversed by a current I_{SOL}.
- A straight infinite cable, parallel to the Z axis, passing through the point (2R,0,0) and traversed by a current I.
- A straight infinite cable, parallel to the X axis, passing through the point (0, 2R,0) and traversed by a current I.

a) Calculate the field $\vec{B}$ at point A (-R/2,0,0) and at point C (3R,0,0)

b) Calculate the force vector that would be experienced by an electron placed at point A and with a velocity $\vec{v} = v_0\vec{j}$

c) What should be the value of I so that the electron experiences a zero force when placed at point A with the previous velocity?

DATA: $I_{SOL}= 0.02$ A; $n= 500$ turns/m; $R= 6$ cm; $I= 3$ A, $v_0= 3\times10^6$ m/s

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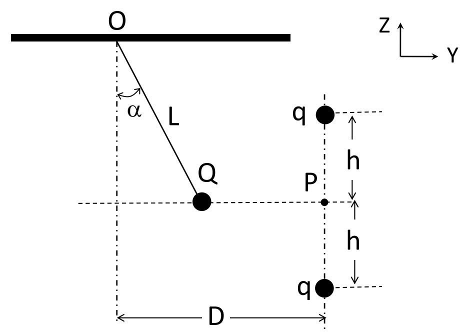
P1. ***(2p)*** A particle of mass M and charge Q is suspended from point O by a thread of length L and negligible mass. In addition, there are two point particles of charge q, which are fixed in the positions indicated in the figure. When the pendulum and the vertical axis form an angle α, the particle of the pendulum is in equilibrium. Knowing that in this configuration the magnitude of the rope's tension is T = 0.179 N

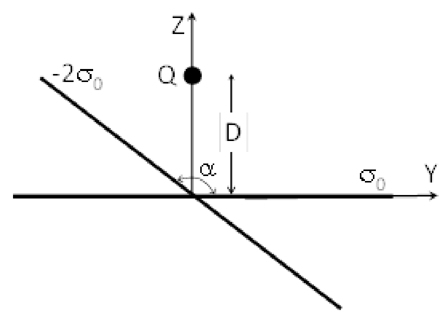


a) Calculate the value of Q

b) Calculate the work needed to bring a particle of charge q' from infinity to the point P indicated in the figure.

DATA: q= -1.25 μC; M= 15 g; L= 2.4 m; α= 35°; D= 4 m; h= 1.2 m; q'= 8 μC; g=9.8 m/s^2

P2. ***(2p)*** The electrostatic system shown in the figure consists of:



- An uniformly charged infinite plane, that coincides with the XY plane and has a charge density σ_0.
- A second, uniformly charged infinite plane with a charge density $-2\sigma_0$, that passes through the origin and forms an angle α with the XY plane (see figure).
- A point charge Q located at (0, 0, D).

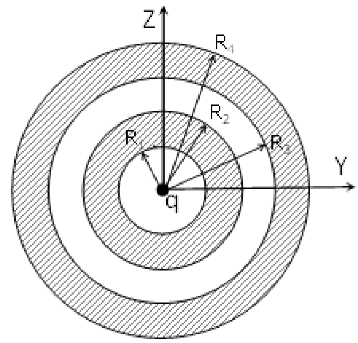
a) Deduce the general expression for the electric field created by an infinite, uniformly charged plane with a surface charge density σ.

b) Calculate the electric field vector $\vec{E}$ at point P (0, D/2, D/2).

c) For which value of Q, the electric field at P is equal to $\vec{E}(P) = E\vec{k}$?

DATA: $\sigma_0 = 1.4\times10^{-6}$ C/m^2; α= 140°; Q= 4.5×10^{-5} C; D= 6 m

P3. ***(2p)*** Consider two hollow conductive spheres placed concentrically, as indicated in the figure, with the center located at the origin of coordinates. The sphere of internal radius R_1 and external radius R_2 has charge Q_1, while the sphere of internal radius R_3 and external radius R_4 has charge Q_2. In addition, a point charge q is placed in the center of the spheres. Given the points A (0, 0, 2) and B(0, 0, 7) and that the ratio of the magnitudes of the electric field at these points is $\frac{E(A)}{E(B)} = 37.69$



a) Calculate the value of Q_1.

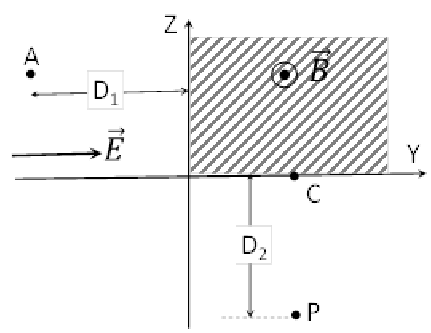
b) Calculate the charge densities on all conductive surfaces.

c) Given the points P1 (0, 0, 15) and P2 (0, 0, 20) calculate the potential difference $\left(V(P2) - V(P1)\right)$

DATA: R_1 = 4 cm; R_2 = 6 cm; R_3 = 8 cm; R_4 = 12 cm; q= 8 μC; Q_2= 3 μC

NOTE: All coordinates are expressed in cm

P4. ***(2p)*** An α particle (a nucleus of He, consisting of 2 protons and 2 neutrons) is placed initially at rest, at point A, see figure. In the region of space defined by y<0, a uniform electric field is applied $\vec{E} = E_0\vec{j}$. In addition, in the region of space defined by y>0 and z>0 (shaded region of the figure) a uniform magnetic field is applied $\vec{B} = B_0\vec{i}$. Knowing that the total time it takes the α particle to go from point A to point P is
t_{total} = 0.145 ms



a) Calculate the value of B_0

b) Calculate the Cartesian coordinates of point C of the figure (point where the α particle trajectory intersects with the Y axis)

DATA: E_0= 2400 N/C; D_1= 450 m; D_2= 350 m

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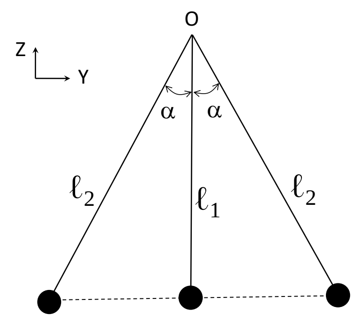


P1. ***(2p)*** Three point charges, each with mass m and charge Q, are suspended from three massless, insulating threads of lengths ℓ_1 and ℓ_2, as indicated in the figure. α is the angle formed between any of the threads of length ℓ_2 and the thread of length ℓ_1.

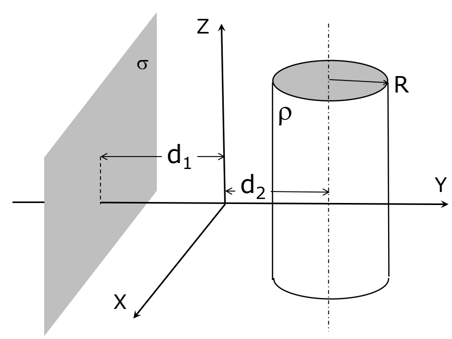
a) Calculate the value of α so that the configuration represented in the figure corresponds to the equilibrium configuration (note that the three charges are aligned along the Y axis).

b) For the configuration (a) described above, calculate the electric field (expressed in Cartesian coordinates) at point O.

c) For configuration (a), calculate the electrostatic energy stored in the system of charges.

DATA: Q= -2.4 μC; m= 15 g; ℓ_1= 3 m; g= 9.8 m/s^2

P2. ***(2p)*** Consider a system with the following charge distributions:



- A uniformly charged infinite plane with surface charge density σ, parallel to the XZ plane and passing through point (0, -d_1,0).
- A uniformly charged cylinder of infinite length, radius R and volume charge density ρ. The cylinder axis is parallel to the Z axis and passes through point (0,d_2,0).

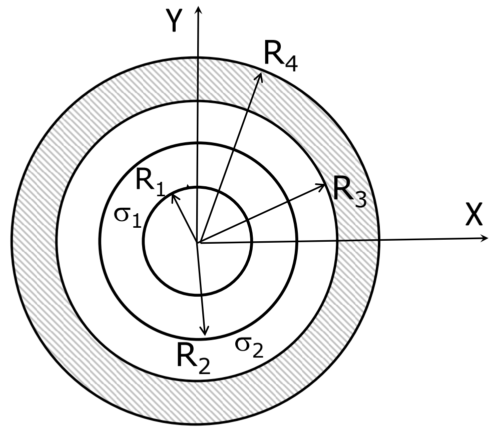
a) Knowing that the force that a proton experiences, if placed at the origin of the coordinate system is zero, calculate the value of σ.

b) Calculate the force that a proton will experience (expressed in Cartesian coordinates) if placed at point (0,-2d_1, -2d_1).

c) Calculate the force that a proton will experience (expressed in Cartesian coordinates) if placed at point (0, 3d_2/2 ,0).

DATA: ρ= 1.7×10^{-8} C/m^3; R= 50 cm; d_1= 30 cm; d_2= 80 cm

P3. ***(2p)*** Consider two concentric spherical shells with radius R_1 and R_2 and corresponding surface charge densities σ_1 and σ_2. They are placed concentrically inside a charged metallic hollow sphere that carries a net charge Q_0 and has inner radius R_3 and outer radius R_4, as depicted in the figure.



a) Given that the electric field value at point (80 cm, 80 cm, 0) is E_0= 3250 N/C, calculate σ_1.

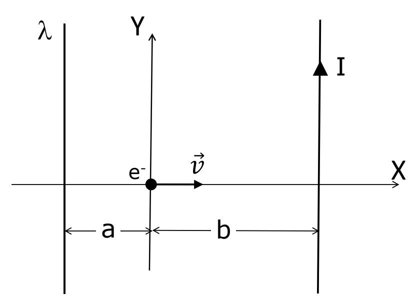
b) Derive the expression for the potential of the metallic hollow sphere and calculate its value.

DATA: σ_2= 4.3×10^{-7} C/m^2; Q_0= 3.2×10^{-7} C; R_1= 15 cm; R_2= 20 cm; R_3= 30 cm; R_4= 40 cm

P4. ***(2p)*** Consider the following system:

- A uniformly charged infinite line, parallel to the Y axis, passing through point (-a,0,0), with a linear charge density λ>0.

- A linear infinite cable, parallel to the Y axis, passing through point (b,0,0), through which current I flows along the direction indicated in the figure.



a) Derive the general expression for the magnetic field $\vec{B}$ created by a linear infinite cable with current I, at all areas of space.

b) If an electron is placed at the coordinates origin point with an initial velocity $\vec{v} = v\,\vec{\imath}$, it experiences a force of magnitude F=2.5×10^{-17} N. Calculate the value of λ.

c) Calculate the force $\vec{F}$ (expressed in Cartesian coordinates) the electron would experience if the current I switches direction.

DATA: a= 20 cm; b= 71 cm; I= 12 A; v= 3×10^7 m/s
NOTE: Neglect gravity

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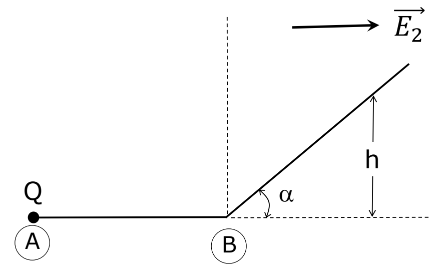
P1. ***(2p)*** A point particle of mass m and charge Q, initially at rest at point A (shown in the figure), accelerates in a homogeneous electric field generated by establishing a potential difference ΔV between points A and B. Upon reaching point B, the particle moves up a ramp inclined at an angle α with respect to the horizontal. During the ascent up the ramp, the particle is subject to the action of a uniform electric field $\overrightarrow{E_2}$, as indicated in the figure.

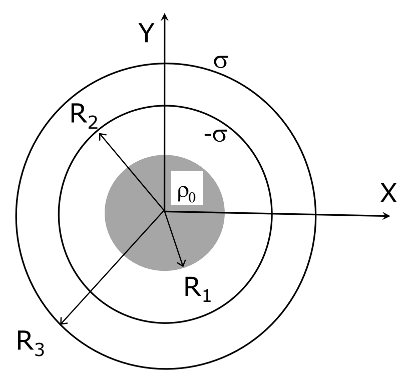


a) Calculate the velocity of the particle at point B.

b) Knowing that the particle ascents to a maximum vertical height h, calculate the magnitude of $\overrightarrow{E_2}$

DATA: Q= -3.4 μC; m= 1.4 g; ΔV= 30 kV; α= 35°; h = 4.18 m; g= 9.8 m/s^2

P2. ***(2p)*** Consider the following system: Charge is distributed uniformly throughout the volume of a sphere with radius R_1, with ρ_0 being the volume charge density. Additionally, charge is distributed uniformly over the surface of spheres with radius R_2 and R_3, with σ_2= -σ and σ_3= σ the corresponding surface charge densities.



a) Calculate the magnitude of the electric field at points A(-5,0,0) and B(10,25,35)

b) Calculate the electric potential difference ($V_C - V_D$) between points C(0,25,0) and D(0,35,0)

DATA: ρ_0= 4.8× 10^{-3} C/m^3; σ= 1.7× 10^{-5} C/m$^{2;}$ R_1 = 10 cm; R_2 = 20 cm; R_3 = 40 cm
NOTE: All coordinates are expressed in cm

P3. ***(2p)*** Consider two metallic spheres. The first one, with radius R_1, has charge $Q_1>0$. The second sphere, with radius R_2, is not charged. They are positioned in such a way that the distance between their centres is much greater than their radii. Then, the two spheres are connected to each other by a very thin conducting wire. Once electrostatic equilibrium is reached, the electrostatic energy stored in the sphere of radius R_2 is U'_2.

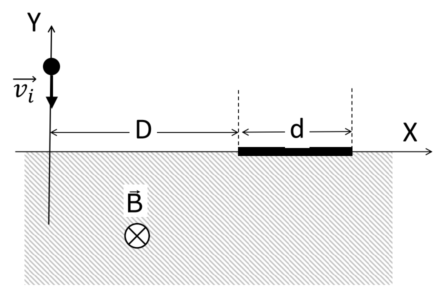
a) Calculate the potential difference (V_1-V_2) between the two spheres before connecting them to each other.

b) If before connecting the two spheres a proton is released from rest at a point on the surface of the sphere with radius R_1 and impacts the sphere of radius R_2 , then calculate the speed with which it will impact the second sphere.

NOTE: The electric potential of an isolated spherical conductor is given by $V = \frac{Q_{sphere}}{4\pi\varepsilon_0 R_{sphere}}$.

DATA: R_1= 45 cm; R_2= 12 cm; U'_2 = 1.7 J

P4. ***(2p)*** A beam of Helium nuclei (He nucleus has two protons and two neutrons) that travel in parallel but with different velocities $\vec{v_i}$, enter through the origin of the coordinate system in an area of space with uniform magnetic field $\vec{B} = -B_0\vec{k}$ (shaded region in the figure). A plate of length d is placed on the X axis, at distance D from the Y axis. In this case, only the nuclei with kinetic energy values between E_{KMIN} and E_{KMAX} impact the plate.



a) If the ratio E_{KMAX}/E_{KMIN} = 1.3, calculate the length of the plate d.

b) For the nuclei that impact the plate, calculate the time spent in the shaded region.

DATA: D= 10 cm; B_0= 0.1 T
NOTE: Neglect gravity

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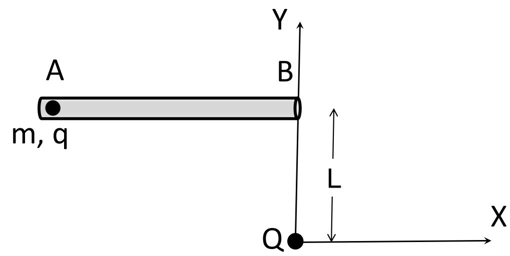
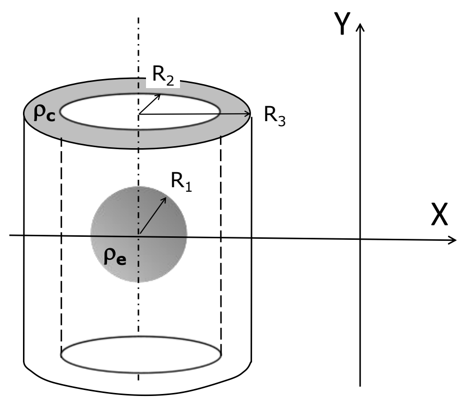


P1. ***(2p)*** A particle of mass m and charge q (q<0) is inside a hollow insulating tube through which it can move without friction. The particle is initially released at point A at the end of the tube with velocity $\vec{v} = \ v_A\,\vec{\imath}$. At the origin of the coordinate system there is a fixed point charge Q (Q>0). Consider that the distance between point A and the coordinate's origin is very large and that the endpoint of the tube B is located at (0,L,0).

a) Calculate the magnitude of the acceleration that the particle experiences as a function of the Cartesian coordinates at an arbitrary point along its trajectory (between A and B).

b) Calculate the speed of the particle as a function of the Cartesian coordinates at an arbitrary point along its trajectory (between A and B).



P2. ***(2p)*** Consider the following charge distributions

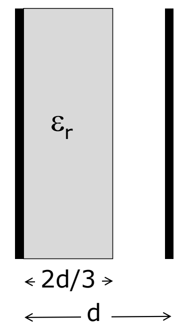
-A uniformly charged sphere of radius R_1, centred at point (-L,0,0) and with volume charge density ρ_e.
-An infinite, uniformly charged hollow cylinder with inner radius R_2 and outer radius R_3, with its axis parallel to the Y axis and passing through x=-L, z=0. The volume charge density of the hollow cylinder is ρ_c.

a) If the electric field at point A (-17,0,0) is $\vec{E}(A) = 2.93 \times 10^3\,\vec{\imath}$ (N/C), calculate the value of ρ_c.

b) Calculate the coordinates of a point P, located on the positive Y axis, for which the electric field is equal to $\vec{E}_P = E\,\vec{\jmath}$

NOTE: all coordinates are expressed in cm
DATA: ρ_e= 5.0×10^{-6} C/m^3; R_1= 10 cm; R_2= 20 cm; R_3= 25 cm; L= 40 cm

P3. ***(2p)*** A parallel-plate capacitor with charge Q has plates with surface area S and separation distance d. Part of the space between the plates contains a dielectric of thickness 2d/3 and unknown dielectric constant ε_r, as indicated in the figure.

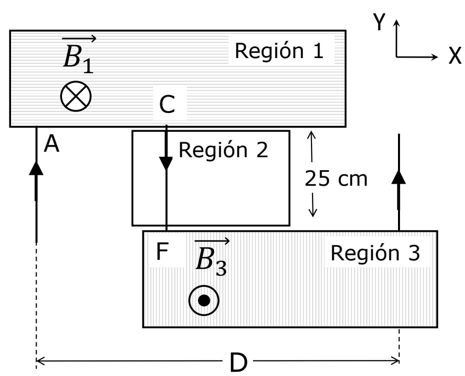


a) Calculate the value of ε_r, knowing that the magnitude of the electric field inside the dielectric is 1/3 times the magnitude of the electric field in the free space area of the capacitor.

b) Calculate the electrostatic energy stored in the capacitor.

c) We observe that if the capacitor is connected to a battery with potential difference V_0, the electrostatic energy of the capacitor does not change. What is the value of V_0?

P4. ***(2p)*** In the Figure below, an electron with an initial kinetic energy of 4.0 keV enters region 1 (point A) at time t=0. That region contains a uniform magnetic field $\overrightarrow{B_1} = -0.01\,\vec{k}$ (T). When exiting region 1, the electron enters region 2 (point C), of length 25.0 cm, where there is an electric potential difference V=2000 V that accelerates the electron. Upon leaving region 2, it enters region 3 (point F) which has a uniform magnetic field $\overrightarrow{B_3} = 0.02\,\vec{k}$ (T).



a) Calculate the distance D along the x-axis between the entry point of the electron in region 1 and the exit point in region 3.

b) Calculate the total time elapsed since the electron enters region 1 until it exits region 3.

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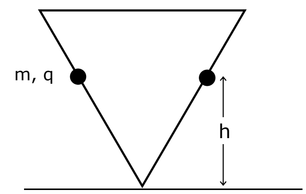
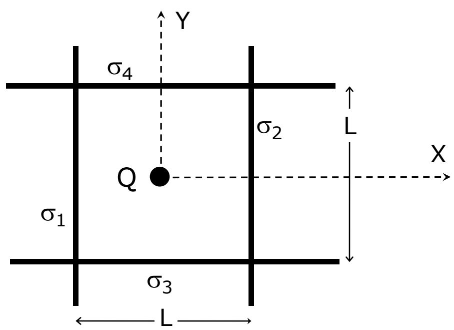
P1 Two identical particles of mass m and charge q are at rest inside an equilateral, triangle-shaped ramp construction. Calculate the height h of each particle with respect to the ground in this equilibrium configuration.



P2 Consider a set of n identical capacitors that are connected in series to a battery with potential a difference V. All the capacitors are filled with a dielectric liquid of dielectric constant ε_r. We disconnect the battery and then we drain completely the liquid from m of these capacitors. Calculate the potential difference across the set of capacitors.

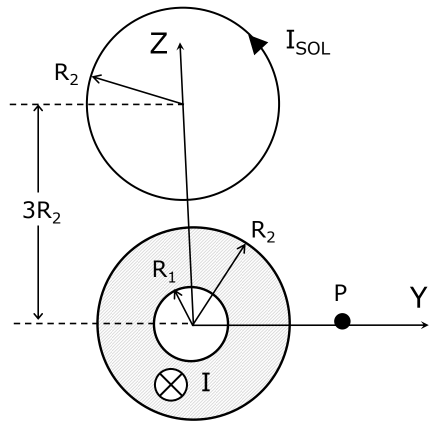
P3 Consider four uniformly charged infinite planes, with charge densities as indicated in the figure. Two planes are parallel to the YZ plane and the other two planes are parallel to the XZ plane. A point charge Q is fixed at the origin of the coordinate system (which coincides with the centre of the square determined by the cross sections of the planes with the XY plane).



a) Calculate the general expression of the electric field vector (in Cartesian coordinates) for a generic point along the Y axis with y>0. Divide space into as many regions as necessary.

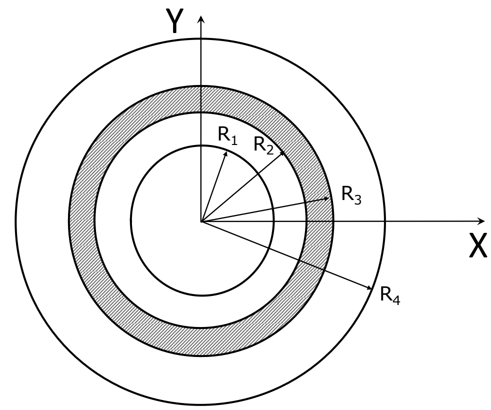
b) Given the points A(L/7,0,0) and B(L/4,0,0) calculate the potential difference (V_A-V_B).

P4 An infinite hollow cylinder, with internal radius R_1 and external radius R_2, has its axis coincident with the X axis and carries a uniformly distributed current I along the direction indicated in the figure. Additionally, an ideal solenoid, with radius R_2 carries current I_{SOL} which circulates in the direction indicated in the figure. The number of wire turns per unit length is n and the axis of the solenoid is parallel to the X axis and passes through the point $(0,0,3R_2)$. Calculate the coordinates of a point P, located on the Y axis ($y>R_2$), at which the magnitude of the magnetic field is one-third the magnitude of the magnetic field at the centre of the solenoid.



JANUARY 2021

P1. ***(2.5 p)*** Consider the following system:



- A uniformly charged spherical surface of radius R_1 and charge density σ_1.
-A hollow conducting sphere with charge Q, inner radius R_2 and outer radius R_3.
-A uniformly charged spherical surface of radius R_4 and charge density σ_4.
The above charge distributions are all concentric and centred at the origin of the Cartesian coordinate system.

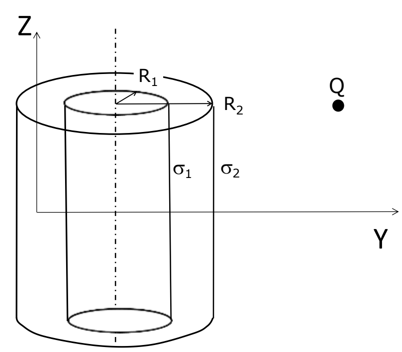
a) If the charge density on the surface of radius R_2 is $\sigma_2 = 6\times10^{-9}$ C/m^2 , calculate the charge on the surface of radius R_1.

b) Calculate the general expression of the electric field vector for any point on the X-axis (with x>0). Use as many regions as necessary.

c) If the electric potential of any point on the spherical surface of radius R_4 is 758.6 V, calculate the electric potential of the conducting sphere.

DATA: R_1= 20 cm; R_2= 40 cm; R_3= 70 cm; R_4= 200 cm; $\sigma_4 = 3\times10^{-9}$ C/m^2 ; Q=3×10^{-8} C

P2. ***(3 p)*** You are given the following charge distributions:



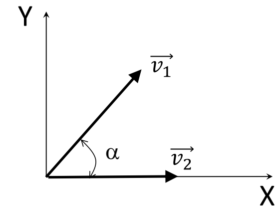
- A uniformly charged cylindrical surface of infinite length and radius R_1, whose axis is parallel to the Z-axis and passes through the point (0,2,0), with surface charge density σ_1.
- A uniformly charged cylindrical surface of infinite length and radius R_2, coaxial to the previous distribution and with surface charge density σ_2.
- A point charge Q >0 located at (0, a, b).

a) If a point charge q is placed at (0,a,0), it experiences an electric force with magnitude F= 4.85×10^{-4} N. Calculate the value of the charge Q.

b) Calculate the electric force (expressed in Cartesian coordinates) that the charge q would experience if it were placed at the origin of coordinates.

DATA: R_1= 0.5 m; R_2= 2.5 m; σ_1= -2.3×10^{-6} C/m^2; σ_2= 5.4×10^{-6} C/m^2; a= 8 m; b=1.5 m; q= 1.3×10^{-9} C

P3. ***(2.5 p)*** A uniform magnetic field $\vec{B}$ is established in a region of space. We know that when a proton is launched from the origin with a velocity $\overrightarrow{v_1}$ that forms an angle α with the X-axis, it does not experience any force that deflects its trajectory. However, if the proton is launched with a velocity $\overrightarrow{v_2}$ of the same magnitude as above but directed in the positive direction of the X-axis it experiences a force $\vec{F} = F\vec{k}$.



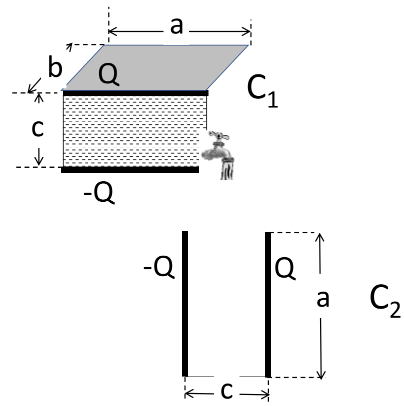
a) Calculate $\vec{B}$.

b) Calculate the radius and pitch of the helix that the proton would describe in the second case.

DATA: $v_1 = v_2 = 2\times10^5$ m/s; $\alpha = 35°$; $F = 5\times10^{-15}$ N

JUNE 2021

P1. ***(2.5 p)*** Consider two identical capacitors of dimensions $a \times b \times c$, which function as small reservoirs of dielectric liquid with dielectric constant ε_r. Both capacitors have charge Q and are isolated. Initially, capacitor C_1 is filled completely with the dielectric liquid and capacitor C_2 is completely empty. At a certain time, liquid starts to flow from one capacitor to the other.



a) Calculate the potential difference between the plates of each of the capacitors when capacitor C_1 contains 25% of its total capacity of dielectric liquid.

b) For configuration a) calculate the electrostatic energy stored in the system.

DATA: a= 40 cm; b= 20 cm; c= 1 cm; ε_r= 80; Q= 3×10^{-7} C

P2. ***(3 p)*** Consider the following charge distributions:

- Charge Q, uniformly distributed over the volume of a sphere centred at the origin and of radius R_1.

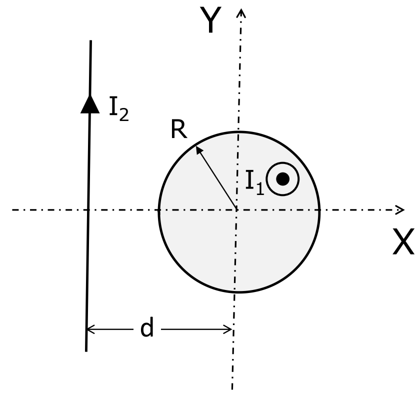
- Charge -Q, uniformly distributed over the volume of a hollow sphere centred at the origin and of inner radius R_1 and outer radius R_2.

a) Calculate the general expression of the electric field vector for any point along the Z axis, with $z \geq 0$. Express the vector in Cartesian components.

b) A proton starts at infinity and travels along the negative Z direction, approaching the two charge distributions. When the proton reaches the point z= R_2 its velocity is $\vec{v} = -v_0 \, \vec{k}$. Calculate the minimum value of v_0 required for the proton to reach the point z= R_1.

DATA: R_1= 15 cm; R_2= 30 cm; Q= 2×10^{-6} C

P3. ***(2.5 p)*** Consider the following distribution of currents:



- Current I_1 uniformly distributed over the cross section of a metallic cylinder of infinite length, radius R, and whose axis coincides with the Z axis of the coordinate system. The direction of the current is indicated in the figure.

- An infinite line, parallel to the Y axis, that carries current I_2 and passes through the point (-d,0,0).

a) Knowing that the magnitude of the magnetic field $\vec{B}$ at point (x_0,0,0) is 4.27 μT, calculate the current density in the cylinder.

b) Calculate the vector $\vec{B}$ at point (0, y_0, 0), expressed in Cartesian components.

c) Calculate the force experienced by a proton located at (-x_0,0,0) with a velocity $\vec{v} = 2 \times 10^5\ (\vec{\imath} + \vec{\jmath})$ m/s .

DATA: I_2= 12 A; R= 20 cm; d= 45 cm; x_0= 40 cm; y_0 = 15 cm