Grado en Informática

Formal Languages and Automata Theory

DEPARTAMENTO DE INFORMÁTICA
UNIVERSIDAD CARLOS III DE MADRID

Formal Languages and Automata Theory
Exercises Push-Down Automata
Unit 6

1. Design a Push-Down Automaton for each one of the following languages:
   a. $L = \{ a^n \bullet b^n \mid n \geq 0\}$
   b. $L = \{ a^n \bullet b^{2n} \mid n \geq 0\}$
   c. $L = \{ a^{2n} \bullet b^n \mid n \geq 0\}$

2. Design a Push-Down Automaton for each one of the following languages:
   a. $L = \{ a^{n+1} \bullet b^n \mid n > 0\}$
   b. $L = \{ a^{2n+1} \bullet b^n \mid n > 0\}$

3. Design a Push-Down Automaton for the language $L = \{a^{n+m} \cdot b^{m+t} \cdot a^t \cdot b^n \mid n, t > 0\ m \geq 0\}$

4. Design a Push-Down Automaton for the language $L = \{ a^{n+m} \cdot b^{m+t} \cdot a^t \cdot b^n \mid n, t, m > 0\}$

5. Design, directly and without calculating the PDA, a grammar to generate each one of the following languages.
   a. $L = \{ a^n \bullet b^n \mid n \geq 0\}$
   b. $L = \{ a^n \bullet b^{2n} \mid n \geq 0\}$
   c. $L = \{ a^{2n} \bullet b^n \mid n \geq 0\}$
   d. $L = \{ a^{2n} \bullet b^n \mid n \geq 0\}$
   e. $L = \{ a^{n+1} \bullet b^n \mid n > 0\}$
   f. $L = \{ a^{2n+1} \bullet b^n \mid n > 0\}$
   g. $L = \{a^{n+m} \cdot b^{m+t} \cdot a^t \cdot b^n \mid n, t > 0\ m \geq 0\}$
   h. $L = \{ a^{n+m} \cdot b^{m+t} \cdot a^t \cdot b^n \mid n, t, m > 0\}$

- Compare these grammars with the ones obtained for the previous exercise.
- Transform each grammar into a PDA, and compare them with the ones obtained ones for the previous exercise.
- Select a word included in the language $L(G_i)$ and verify this by showing the complete derivation. Verify that this word is also recognized by the equivalent automaton by showing the movements included in the instantaneous descriptions.

6. Design a Push-Down Automaton which recognizes the language of arithmetical expressions with the following alphabet Σ={ 0, 1 , + , * , (,) }

7. Obtain the PDA_E corresponding to the grammar

   G_{FNG}=({a,b,c,d}, {S,A,B}, S, P), with the following production rules:

   S ::= a S B | b A | b | d
   A ::= b A | b
   B ::= c

8. Obtain formally the PDA_F equivalent to the following PDA_E. *NOTE: the PDA_E given in this exercise are the solution of the sections a and b of Exercise 8.*

   a. [Isasi, Martínez, Borrajo; pp. 258] PDA_{Ea}=({1,2}, {A,B,B',C}, {q}, A, q, f, {Φ}), where f is given by:

      f(q,2,A) = (q, BC)
      f(q,1,A) = (q,B)
      f(q,λ,A) = (q, λ)
      f(q,1,B) = {(q,B'), (q,C), (q, λ)}
      f(q,2,B') = {(q,B'), (q,C)}
      f(q,2,C) = (q, λ)

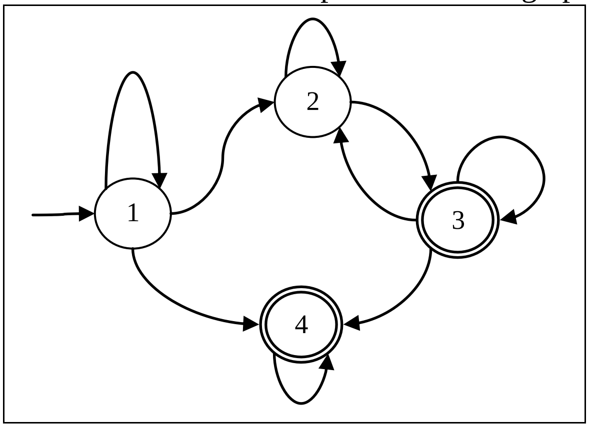
   b. [Isasi, Martínez, Borrajo; pp. 272-73] PDA_{Eb}=({x,y}, {A,B,C,S}, {q}, S, q, f, {Φ}), where f is given by:

      f(q,x,S) = {(q,AC), (q,BCC), (q,C), (q,CC)}
      f(q,λ,S) = (q, λ)
      f(q,x,A) = {(q,AA), (q,C)}
      f(q,x,B) = {(q,BCC), (q,CC)}
      f(q,x,C) = (q, λ)

9. Obtain formally the PDAE equivalent to the following PDA_F:

   a. $PDAF_a$=(Σ, {0,1,A0}, {1,2,3,4}, A0, 1, f, {3,4}), where f is given by:

      Note: Additional information not provided in the graph is not required.



   b. [Exam Problem Feb 1999] PDA_{Fb}=({a,b}, {A,B}, {q1,q2,q3,q4}, A, q1, f, {q4}), where f is given by:

      f(q1,a,A) = {(q2,BA), (q4,A)}
      f(q1,λ,A) = {(q4, λ)}
      f(q2,a,B) = {(q2,BB)}
      f(q2,b,B) = {(q3, λ)}
      f(q3,λ,A) = {(q4,A)}
      f(q3,b,B) = {(q3, λ)}

10. Obtain formally the G2 which generates the same language recognized by the following PDA_E:

   a. [Alfonseca – pp. 230-231]] $PDAE_a$=({a,b}, {A,B}, {p,q}, A, p, f, {Φ}), where f is given by:

      f(p,a,A) = (p,BA)
      f(p,a,B) = (p,BB)
      f(p,b,B) = (q, λ)
      f(q,b,B) = (q, λ)
      f(q,λ,B) = (q, λ)
      f(q,λ,A) = (q, λ)

   b. [Isasi, Martínez, Borrajo; AP_1, pp. 250 y 261] $PDAE_b$=({0,1}, {A,1,0}, {q0,q1}, A, q0, f, {Φ}), where f is given by:

      f(q0,1,A) = (q0,1A)
      f(q0,1,1) = (q0,11)
      f(q0,0,1) = (q1, λ)
      f(q1,0,1) = (q1, λ)
      f(q1,λ,A) = (q1, λ)

Select a word included in the language $L(G_i)$ and verify this by showing the complete derivation. Verify that this word is also recognized by the equivalent automaton by showing the movements included in the instantaneous descriptions.

11. Obtain formally the PDA_E which recognizes the same language generated by each one of the following G2:

    a. [Isasi, Martínez, Borrajo; G_{13}, pp. 258] G_a=({1,2}, {A,B,B',C}, A, p), where p is given by:

        p={ A::=2BC | 1B | λ
            B::=1B' | 1C | 1
            B'::=2B' | 2C
            C::=2
            }

    b. [Isasi, Martínez, Borrajo; Exercise 4.9, pp. 272-274] G_b=({x,y}, {A,B,C,S}, S, p), where p is given by:

        p={ S::=xAC | xBCC | xC | xCC | λ
            A::=xAA | xC
            B::=xBCC | xCC
            C::=y
            }

    Select a word included in the language $L(G_i)$ and verify this by showing the complete derivation. Verify that this word is also recognized by the equivalent automaton by showing the movements included in the instantaneous descriptions.

12. Indicate which of the following statements are correct:

    a. When obtaining a G2 from a PDAF, this grammar will be in GNF.
    b. It is possible to transform a G3 in a PDAE.
    c. Given a non-deterministic PDA, there are algorithms to transform it into a deterministic PDA.
    d. In a deterministic PDA, given a state, a read symbol and a symbol on the top of the stack, it transits to the same state, with the possibility of introducing in the stack two different sets of symbols.

13. Indicate which are the correct statements:

    a. Given a movement in a PDA, it is possible to determine the pair (image, anti-image) of the corresponding transition function.
    b. Push-Down Automata recognizing by empty stack cannot be transformed themselves into PDA recognizing by final states.
    c. Push-Down Automata by final states recognize a word when the stack is empty and there is anything to be read in the tape.
    d. Push-Down Automata recognizing by final states are never deterministic.

14. Indicate which are the correct statements:

    a. (p,a,A;p,Z) indicates that only the symbol Z is introduced in the stack.
    b. (p,a,A;p,Z) indicates that the symbol A is extracted from the stack
    c. (p,a,A;p,A) indicates that the stack is not modified after the transition.
    d. (p,a,λ;p,λ) indicates that the stack is not modified after the transition.

15. Indicate which are the correct statements:
    a. $f(q, \lambda, A) = \{(q, \lambda)\}$ is a transition independent of the input.

    b. The instantaneous description (q, λ, λ) in a PDA which recognizes when the stack is empty indicates that the word has been completely read and the stack is empty.

    c. The alphabet of the stack and the input alphabet are disjoint sets.

    d. The transition $f(q,a, A)=\{ (q2, z1),(q1, z1) \}$ indicates that the PDA is non-deterministic.

16. Describe the transition functions which generate the following movements:

    $(p,1001, A) \vdash (p, 001,1A) \vdash (p, 01, 01A) \vdash (q, 1, 1A) \vdash (q, \lambda, A) \vdash (q, \lambda, \lambda)$

17. [Isasi, Martínez, Borrajo; pp. 272-320] Exercises 4.9 to 4.25. Transformations between PDAE, PDAF and G.

18. Obtain the PDA corresponding to the following grammar.

    $G = (\Sigma_T, \Sigma_N, A, P)$, $P= \{A ::= a\ B\ A \mid b\ ,\ B ::= b\ A\ B \mid a\}$