

DEGREE IN COMPUTER ENGINEERING

PHYSICS

EXERCISES CH 7

Capacitors. Dielectrics.

1. A parallel-plate capacitor with plates of area A and distance between the plates d is charged by connecting it to a battery which maintains a constant potential difference V_0 .

a) Find the capacitance, the charge and the potential of the capacitor supposing that it is disconnected from the battery after being charged and the distance between the plates is decreased in $d/2$.

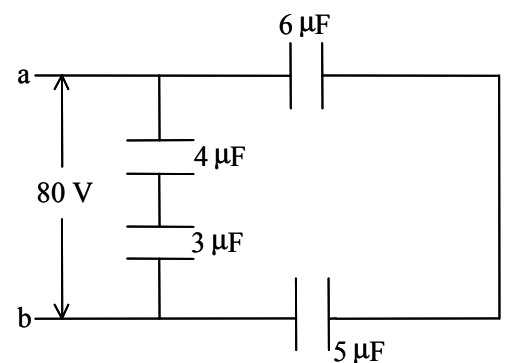
b) Find the capacitance, the charge and the potential of the capacitor supposing that the capacitor is still connected to the battery and the distance between the plates is decreased in $d/2$.

2. A system of four capacitors is formed by combining the capacitors as shown. Find:

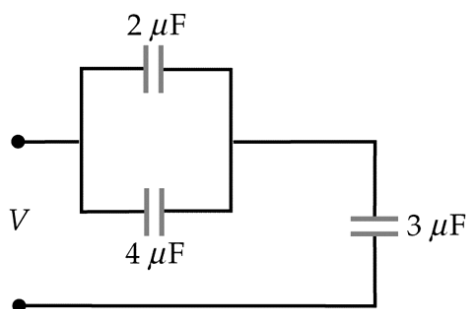
a) The equivalent capacitance of the system.

b) The charge stored on each capacitor.

c) The net energy stored in the system.



3. Three capacitors are connected as shown in the figure. (a) Find the equivalent capacitance of the three-capacitor combination. (b) The capacitors are initially uncharged. The combination is then connected to a 6 V battery. Find the potential difference across each capacitor and the charge on each capacitor after the battery is connected and the charges have stopped flowing. (c) Find the net energy stored in the system.

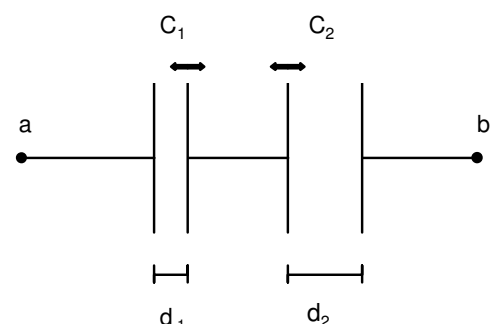


4. Two capacitors with plates of the same area are connected in series. Their capacitances can be varied by moving the inner plates in the direction indicated in the figure. The distances between plates are initially $d_1 = 1\ \text{mm}$ and $d_2 = 9\ \text{mm}$, and the capacitance of the first capacitor is $C_1 = 9 \times 10^{-9}\ \text{F}$. The system is charged by connecting it to a battery of voltage 10 V. Once charged, the system is disconnected from the battery and the plates are moved until $d_1' = 0.5\ \text{mm}$ and $d_2' = 0.5\ \text{mm}$. Find:

a) The total capacitance of the two-capacitor system on the initial and final states.

b) The final potential difference between points a and b.

c) The energy stored in the system on the initial and final states.



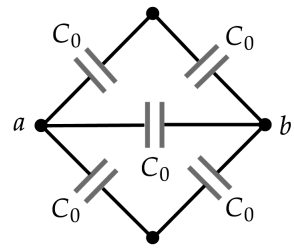
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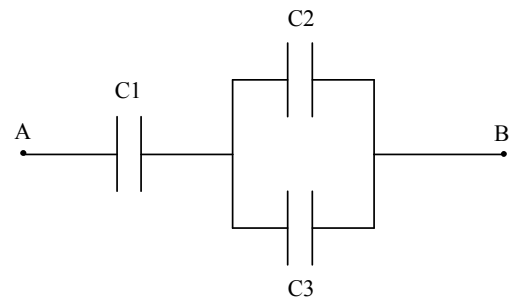
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5. Five identical capacitors of capacitance $C_0 = 1\mu\text{F}$ are connected as shown. What is the equivalent capacitance between points a and b ?



6. Three parallel-plate capacitors of capacitances $C_1 = 10\text{ nF}$, $C_2 = 10\text{ nF}$ y $C_3 = 30\text{ nF}$ are connected as shown in the figure. The system is connected to a generator that maintains a constant potential difference of 100 V between points A and B .

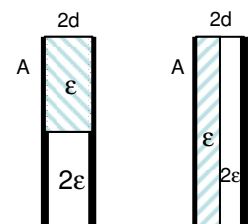
- Find the total charge and energy stored in the system.
- Find the charge and potential of each capacitor.
- The generator is disconnected, and a dielectric of dielectric constant $\epsilon_r = 1.5$ (also called k) is introduced in the 3rd capacitor (C_3). Find the charge stored on each capacitor and the total energy of the system.
- Find the maximum potential difference that can be applied between points A and B if the dielectric strength of air in C_1 and C_2 is $3 \times 10^6\text{ V/m}$ and the dielectric strength of the dielectric in C_3 is 10^7 V/m . The distance between plates is 0.5 mm for the three capacitors.



7. Three capacitors of equal capacitance $C_0 = 5\text{ nF}$ are filled with dielectrics of relative permittivities $\epsilon_{r1} = 2$, $\epsilon_{r2} = 3$ y $\epsilon_{r3} = 6$ and connected to a 200 V battery.

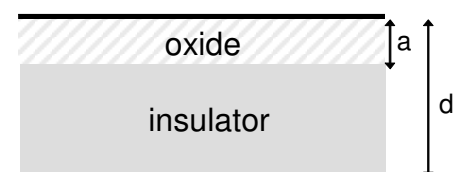
- How must they be connected in order to store the maximum charge? Find the charge and energy stored in the system for that configuration.
- Suppose that the breakdown voltages of the three dielectrics are respectively $V_1 = 150\text{ V}$, $V_2 = 150\text{ V}$ and $V_3 = 300\text{ V}$. In this case, could the above configuration be maintained? How would they have to be connected in order to store the maximum possible charge? Find the charge stored for that configuration.
- The battery is substituted for a variable voltage source V_0 . Find the maximum value that V_0 can have so that there is no dielectric breakdown on any of the capacitors. Suppose the same configuration as in b).

8. Two parallel-plate capacitors of area A and distance between plates $2d$ are half-filled with two dielectrics of permittivities ϵ y 2ϵ . Find the equivalent capacitance of each combination.



9. A parallel-plate capacitor is fabricated by depositing a $1\mu\text{m}$ thick layer of certain insulator on top of a metallic surface of area $A = 0.5\text{ m}^2$. The insulator has a relative permittivity $\epsilon_{r1} = 10^3$ and a dielectric strength $E_{r1} = 10^6\text{ V/m}$. When the top metallic surface is added to form the capacitor, oxidation of the insulator takes place. The depth of the oxide layer is $a = 0.01\mu\text{m}$.

- Find the breakdown voltage of the capacitor, knowing that the oxide layer has a relative permittivity $\epsilon_{r2} = 10^2$ and a dielectric strength $E_{r2} = 10^6\text{ V/m}$.
- Find its charge and energy at breakdown.
- Compare these values with the ones obtained if the oxidation had not taken place.



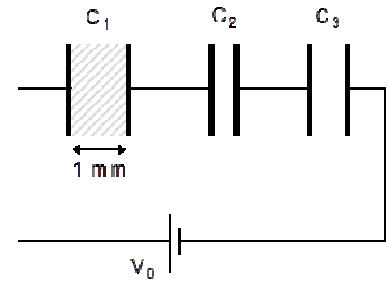
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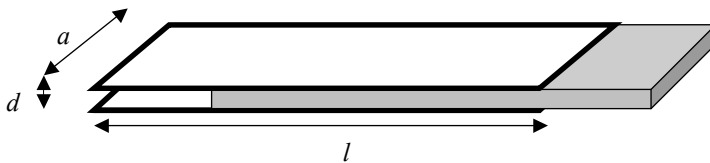
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10. During a laboratory session, students have to find the dielectric strength of certain dielectric. In order to do so, they build a parallel-plate capacitor of capacitance C_1 with distance between plates 1 mm. They completely fill the space between the plates with the dielectric. After inserting the dielectric, the capacitance of the capacitor is $4 \mu\text{F}$. This capacitor is connected in series with two other capacitors of capacitances $C_2 = 10 \mu\text{F}$ and $C_3 = 5 \mu\text{F}$. The system is connected to a generator and the voltage V_0 is slowly raised. The breakdown of the 1st capacitor is observed for $V_0 = 100 \text{ V}$. Which is the dielectric strength of the dielectric?

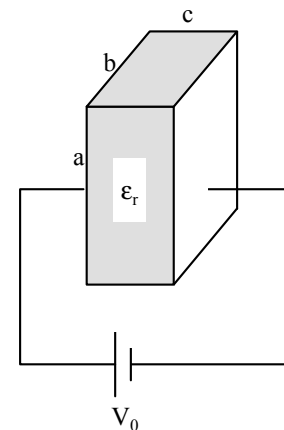


11. A capacitor can be used to tune a radio. In the circuit of a radio tuner, the tuning frequency f is related to the resistance R and the capacitance C of the circuit by the following equation: $f = \frac{1}{2\pi RC}$ where f is expressed in Hz when R is in ohms (Ω) and C in farads (F). A

tuning circuit is fabricated using a resistance of 70Ω and a parallel-plate capacitor of length $l = 10 \text{ cm}$, width $a = 5 \text{ cm}$ and distance between the plates $d = 5 \text{ mm}$. The capacitor is filled with a dielectric of relative permittivity $\epsilon_r = 3$ that can be displaced horizontally by moving the dial wheel. How much would we have to displace the dielectric to tune to 93.2 MHz ?

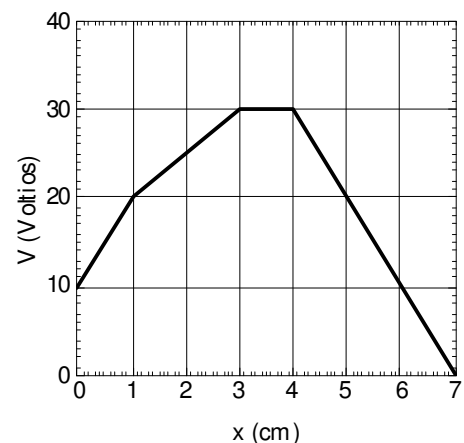


12. A container of dimensions $a \times b \times c$ is filled with a liquid of dielectric constant ϵ_r . The two sides of dimensions $a \times b$ are metallic and are connected to a battery of potential difference V_0 . The battery is disconnected after charging. Find the volume of liquid that should be evacuated from the container in order to increase the potential difference by 25%. The distance between plates is kept constant during the process.



DATA: $a=30 \text{ cm}$, $b=20 \text{ cm}$, $c=0.5 \text{ cm}$, $V_0 = 100 \text{ V}$, $\epsilon_r = 4$

13. Two metallic plates with a very large surface area are respectively located at $x=0$ y $x=7 \text{ cm}$. The space between the plates is divided into four parallel regions of different permittivities, which are parallel to the plates. The attached graphic indicates how does the potential change as a function of x . Find the electric field on each of the four regions between the plates.



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ANSWERS

1. a) $C = \frac{2\epsilon_0 A}{d}$ $Q = \frac{\epsilon_0 A V_0}{d}$ $V = \frac{V_0}{2}$

b) $C = \frac{2\epsilon_0 A}{d}$ $Q = \frac{2\epsilon_0 A V_0}{d}$ $V = V_0$

2. (a) $C_{eq} = 4.4\mu F$

(b)

C (μF)	Q (C)
6	2.2×10^{-4}
4	1.4×10^{-4}
3	1.4×10^{-4}
5	2.2×10^{-4}

(c) $U_e = 14 \text{ mJ}$

3. (a) $C_{eq} = 2 \mu F$; (b) $V_3 = 4 \text{ V}$, $V_{2,4} = 2 \text{ V}$, $Q_2 = 4 \mu C$, $Q_3 = 12 \mu C$, $Q_4 = 8 \mu C$; (c) $U = 36 \mu J$.

4. (a) $C_{eq}^i = 9 \times 10^{-10} F$ $C_{eq}^f = 9 \times 10^{-9} F$

(b) $V_f = 1 \text{ V}$

(c) $U_i = 4.5 \times 10^{-8} J$ $U_f = 4.5 \times 10^{-9} J$

5. $C_{eq} = 2 \mu F$

6. (a) $Q_T = 0.8 \mu C$ (b) $Q_1 = 8 \times 10^{-9} C$ $Q_2 = 2 \times 10^{-9} C$
 $U_T = 40 \mu J$ $Q_3 = 6 \times 10^{-9} C$ $V_1 = 80 V$ $V_2 = V_3 = 20 V$

(c) $Q_1 = 8 \times 10^{-7} C$ $Q_2 = 1.45 \times 10^{-7} C$ (d) $V_{AB}^{\max} = 1.77 \times 10^3 V$
 $Q_3 = 6.55 \times 10^{-7} C$ $U_i = 38 \mu J$

7. a) The maximum charge will be stored when the three capacitors are connected in parallel. $Q = 1,1 \times 10^{-5} C$ $U = 1,1 \times 10^{-3} J$

b) The same structure is not possible. The third capacitor must be connected in parallel to the other two, and these two in series with each other. $Q = 7,2 \times 10^{-6} C$

c) $V_0^{\max} = 250 V$

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8. a) $C = \frac{3\epsilon A}{4d}$ b) $C = \frac{2\epsilon A}{3d}$
9. a) $V_R = 0.11 \text{ V}$ $Q_R = 4.47 \times 10^{-4} \text{ C}$ $U_R = 2.46 \times 10^{-5} \text{ J}$
b) $V_{R0} = 1 \text{ V}$ $Q_{R0} = 4.43 \times 10^{-3} \text{ C}$ $U_{R0} = 2.2 \times 10^{-3} \text{ J}$
10. $E_R = 4.55 \times 10^4 \text{ V/m}$
11. $x = 1.2 \text{ cm}$
12. 80 cm^3
13. $0 < x < 1 \text{ cm}$ $\vec{E} = -1000 \vec{i} \text{ N/C}$
 $1 \text{ cm} < x < 3 \text{ cm}$ $\vec{E} = -500 \vec{i} \text{ N/C}$
 $3 \text{ cm} < x < 4 \text{ cm}$ $\vec{E} = 0$
 $4 \text{ cm} < x < 7 \text{ cm}$ $\vec{E} = 1000 \vec{i} \text{ N/C}$