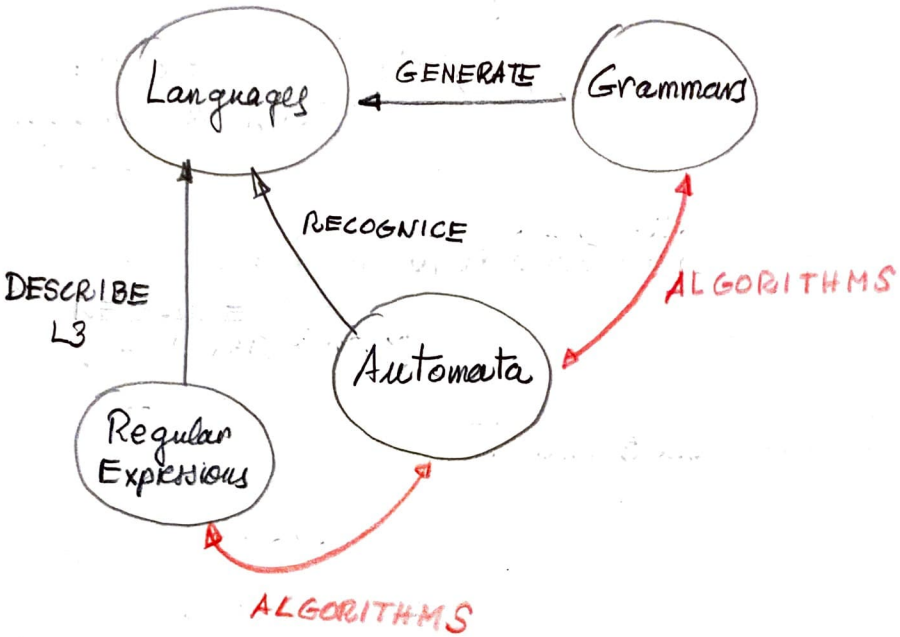
1



2

More examples

$\Sigma = \{a, AB, b, 12\}$

$AB \in \Sigma$ AB is a symbol of the alphabet

Some words over Σ

a, $\underbrace{aAB}_{2}$, $\underbrace{ABa}_{2}$, $\underbrace{ab12}_{3}$, $\underbrace{abAB}_{3}$ ~~$abBA$~~

Example of $W(\Sigma)$

$\Sigma = \{a, b\}$ $W(\Sigma) = \{\lambda, \overbrace{a, b}^{\Sigma}, aa, ab, ba, bb, aaa, \ldots\}$

λ included in every Universal Language)

3

More examples

$\Sigma = \{a, b\}$

$x = aba \quad y = bab$

$x, y \in W(\Sigma)$

Neutral element

$x\lambda = x$

$xy = ababab$

$yx = bababa$

} Non-Commutative

$xy, yx \in W(\Sigma)$

$(xy)z = x(yz)$ Associative

$x^2 = abaaba \quad y^3 = babbab$

$x^{-1} = aba \quad z = aab \quad z^{-1} = baa$

4

$W(\Sigma) = \{\lambda, a, b, aa, ab, ba, bb, aaa, \ldots\}$

A language L_2

$L_2 = \{\lambda\}$

A language L_1

$L_1 = \{aa, ba, aaa\}$

Another language $L_3 = \{\lambda, a, aa\}$

ALL are SUBSETS OF $W(\Sigma)$

ALL are languages OVER Σ

$\phi \subset W(\Sigma) \quad \phi = \{\}$

← No words

Examples of operations with languages

$L_1 = \{ab, a\}$ $L_2 = \{bbb, ba\}$

$L_1 \cup L_2 = \{ab, a, bbb, ba\}$

$L_1 \cdot L_2 = \{abbbb, abba, abbb, aba\}$

$L_1^2 = L_1 \cdot L_1 = \{abab, aba, aab, aa\}$

$L_1^+ = L_1^1 \cup L_1^2 \cup L_1^3 \cup \dots$

If a Language $\rightarrow$ $\Sigma = \{a, b\}$ $\quad W(\Sigma) = \{\lambda, a, b, aa, ab, ba, bb, \dots\}$

$\Sigma^1 = \Sigma$ $\quad \Sigma^2 = \{aa, ab, ba, bb\}$

$\Sigma^+ = \{a,b\} \cup \{aa, ab, ba, bb\} \cup \dots = W(\Sigma) - \lambda$

λ is not a symbol

$L_1^* = L_1^0 \cup L_1^1 \cup L_1^2 \cup \dots =$

$= \lambda \cup \{ab, a\} \cup \{abab, aba, aab, aa\} \cup \dots$

$L_1^* = L^+ \cup \lambda$

$\Sigma^* = W(\Sigma) \equiv$ ALL the words over Σ including λ

$L_1 = \{ab, a\}$ $\quad L_1^{-1} = \{ba, a\}$

Reflected