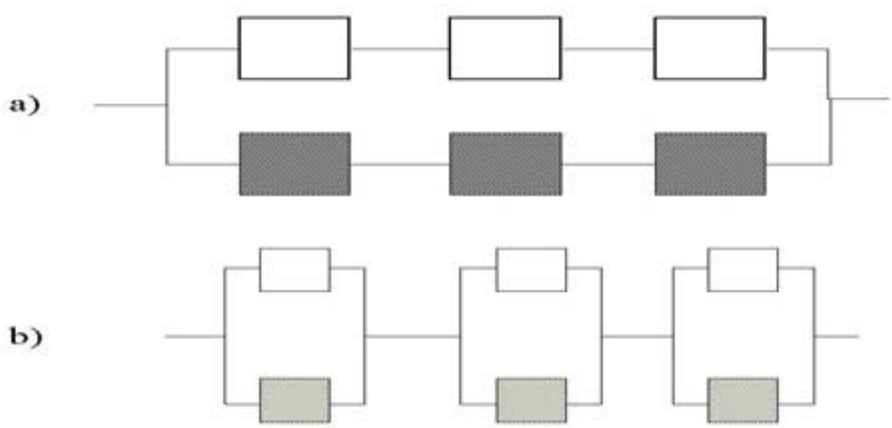
Chapter III: Probability

PROBLEMS

Proposed Problems

1. The quality department of a factory producing some mechanical items realized that a certain type of produced metal anchors may be defective due to the following causes: defects in the thread and defects in dimensions. It has been estimated that 6% of the anchors they produce defects in the thread, while 9% have defects in dimensions. However, 90% of the anchors do not have any type of defect. What is the probability that an anchor has both types of defects?
   SOLUTION:
   $P = 0.05.$

2. A machine consists of three components in series, each of which has a failure probability of 0.01. For security reasons it was decided to place three components in parallel with the first, to reduce the risk of damage to the machine. Assuming that all components act independently, which of the two alternatives presented in the figure is preferable, taking into account that, for economic reasons, the safety components are of inferior quality and have failure probability of 0.05?



   SOLUTION:
   $P(failure\ of\ configuration\ a) = 4.236{\times}10^{-3}.$
   $P(failure\ of\ configuration\ b) = 1.499{\times}10^{-3}.$
   It is therefore preferable to the alternative b) the configuration a).
3. The proportions of defective parts manufactured by two machines M_1 and M_2 are 0.04 and 0.01 respectively. You take a piece at random and it turns to be acceptable. Knowing that the probability of choosing a piece of either machine is 0.5, calculate the probability that it comes from M_1.
   SOLUTION:
   $P = 0.492.$
4. The probability that a component fails in a given time period is 0.01. His state (damaged, working) is checked by a test. This test with probability 0.05 reports that a working component is damaged, and always identifies damaged components. If the test indicates that the component is broken, what is the probability that it is really damaged?
   SOLUTION:
   $P = 0.168.$
5. A drug company wants to bring to market a test to detect a disease. When patient is infected, the test indicates a 95% of the time that s/he is so. However, sometimes the test is positive even if the person does not have the disease. This happens 1% of the time. If the 0.5% of the population has the disease what is the probability that a person is infected when the test is positive? (June 97)

SOLUTION:
$P = 0.323.$

6. In a system protected by an alarm, the probability of occurrence of a situation of danger is 0.1. If this occurs, the probability that the alarm works is 0.95. The probability that the alarm rings without being any danger is 0.03. Find the probability that the alarm sounds without having any danger.

   SOLUTION:
   $P = 0.2213.$

7. Three machines A, B and C produce items with a proportion of defectives of 5%, 3% and 2% respectively. We have a set of 200 items, 100 of them made by A, 50 by B and the rest by C. We take randomly an item
   a) Calculate the probability that the chosen item is defective.
   b) If the item is defective, calculate the probability that it was made by A.

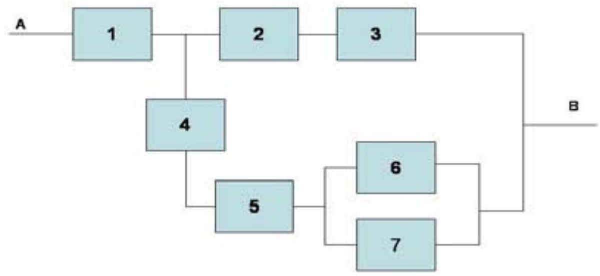
   SOLUTION:
   a) $P = 0.0375.$
   b) $P = 0.66.$

8. A public transportation company operates three peripheral lines of a big city, so that 60% of bus service covers the first line, 30% covers the service of the second line and 10% covers the service of the the third line. It is known that the probability that, daily, a bus breaks down is:
   a) 2% in the first line
   b) 4% in the second line
   c) 1% in the third line.

   Calculate:
   a) The probability that a bus one day is damaged.
   b) Knowing that a bus has suffered a breakdown on any given day, what is the probability that it was serving on the first line?

   SOLUTION:
   a) $P = 0.025.$
   b) $P = 0.48.$

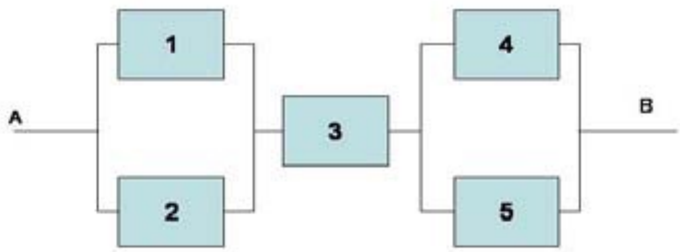
9. We have a system of connected components according to the following figure:



All components are of similar reliability and have a probability of failure of 0.01. The failures of one component are independent of the state of other components. The system is functioning if you can find between A and B a path of components operating. What is the probability that the system works?

SOLUTION:
$P = 0.9896.$

10. Calculate the probability that the following system works, assuming that all components have the same characteristics of the ones in the previous problem



SOLUTION:
$P = 0.9898.$

11. Let A, B and C events whatsoever. Prove the following results (June 05):
    a) Show that $P(A \cup B \cup C)$ is equal to
    $P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$
    (hint: define and use the event $D = B \cup C$)
    b) Show that if A, B and C are mutually exclusive, then
    $$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$
    c) Show that $P(A \cap B \cap C) = P(A|B \cap C)P(B|C)P(C)$.
    d) Let $A = B \cap C$. Show that $P(B|A) = 1$.
    e) Let $A = B \cup C$. Show that $P(B|A) = P(B)/P(A)$.