CALCULUS

Bachelor in Computer Science and Engineering

Course 2022–2023

Techniques of integration

Problem 10.1. Calculate the following integrals.

$$\int \arctan(3x)\,dx\,.$$
$$\int e^x \sin(x)\,dx\,.$$
$$\int \cos(\ln(x))\,dx\,.$$
$$\int \cos^2(\ln(x))\,dx\,.$$
$$\int \frac{dx}{\sqrt{e^x-4}}\,.$$
$$\int \frac{dx}{x\sqrt{x^2-1}}\,.$$

Problem 10.2. Calculate the following integrals by using an appropriate change of variable.

$$\int_1^2 \frac{\sqrt{t^2-1}}{t}\,dt\,.$$
$$\int_0^{\ln(2)} \sqrt{e^t-1}\,dt\,.$$
$$\int \frac{dx}{(x+2)\sqrt{1+x}}\,.$$
$$\int \frac{dx}{1+\sqrt[3]{1-x}}\,.$$

Problem 10.3. Calculate the following integrals of *rational* functions.

$$\int \frac{dx}{3x^2+4x+2}\,.$$
$$\int \frac{x^5-2x^3}{x^4-2x^2+1}\,dx\,.$$
$$\int \frac{x^2+1}{x^4-x^2}\,dx\,.$$
$$\int \frac{x^3+1}{x^2+4x+13}\,dx\,.$$
$$\int \frac{x^2+6x-1}{x^3-7x^2+15x-9}\,dx\,.$$

Problem 10.4. Calculate the following integrals by using the given hints.

1. $\int \cos^3(x)\,dx$ (change of variable: $u=\sin(x)$)
2. $\int \sin^4(x)\,dx$ (identities: $\cos(2\alpha) = 1-2\sin^2(\alpha) = 2\cos^2(\alpha)-1$)
3. $\int \frac{e^{4x}}{e^{2x}+2e^x+2}\,dx$ (change of variable: $u=e^x$)
4. $\int \frac{\sin^3(x)}{1+\cos^2(x)}\,dx$ (change of variable: $u=\cos(x)$)
5. $\int \frac{dx}{\cos(x)}$ (change of variable: $u=\sin(x)$)
6. $\int \sqrt{a^2-x^2}\,dx,\ a\in\mathbb{R}$ (change of variable: $x=a\sin(u)$)