

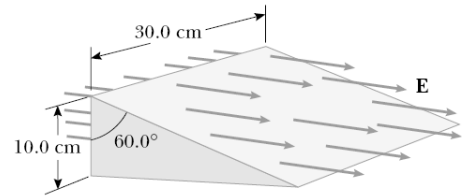
DEGREE IN COMPUTER ENGINEERING

PHYSICS

EXERCISES CH 4

Gauss' Law.

1. Find the electric flux going through each side of the closed triangular prism in the figure, and the net electric flux, knowing that $\vec{E} = 150\vec{i}$ N/C.



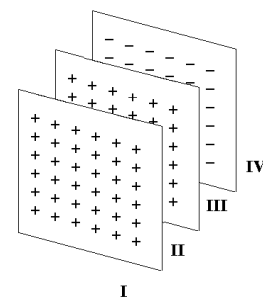
2. Calculate the electric flux produced by a positive Gadolinium ion Gd^+ through one of the faces of a cubic closed surface of $l=1$ m centred on the ion.

3. An electric field is $\vec{E} = 200\vec{i}$ N/C for $x > 0$ and $\vec{E} = -200\vec{i}$ N/C for $x < 0$. A closed cylinder of length 20 cm and radius $R=5$ cm has its centre at the origin and its axis along the x axis, so that one end is at $x=10$ cm and the other at $x=-10$ cm.

- (a) What is the flux through each end?
- (b) What is the flux through the curved surface of the cylinder?
- (c) What is the net outward flux through the entire closed surface?
- (d) What is the net charge enclosed in the cylinder?

4. (a) Use Gauss' law to find the electric field due to a line of charge with linear charge density λ in all regions of space. (b) An infinite charged line with linear charge density $\lambda = -1.5 \mu\text{C/m}$ is parallel to the Y axis on $x = -2$ m, $z = 0$ m. A point charge of $1.3 \mu\text{C}$ is located at $(1, 2, 0)$ m. Find the net electric field at $(2, 1.5, 0)$ m.

5. Three infinite parallel plates have surface charge densities of $+\sigma$, $+\sigma$ and $-\sigma$, respectively. Find the magnitude and direction of the electric field on each of the four regions indicated in the figure.



6. A charged cylindrical foil of length 24 cm and radius 6 cm is centred at the origin of a Cartesian reference frame with its axis coinciding with the Y axis. It has a surface charge density of $\sigma=9 \text{ nC/m}^2$.

- (a) What is the total charge of the cylinder?
- (b) Use Gauss' law to find the electric field due to an infinite cylindrical foil with surface charge density σ in all regions of space.
- (c) At which points of space could we use Gauss' Law to find the electric field due to the charged cylindrical foil of length 24 cm and radius 6 cm?
- (d) Find the electric field at: $(2, 0, 5)$ cm and $(10, 0, 0)$ cm.
- (e) Find the electric force acting on an electron located at $(4, 0, -2)$ cm.

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7. (a) Find the electric field in all regions of space due to the following spherical charge distribution of radius R :

$$\begin{aligned}\rho(r) &= 0 & 0 < r < R/2 \\ \rho(r) &= \rho_0 & R/2 < r < R \quad (\rho_0 \text{ constant})\end{aligned}$$

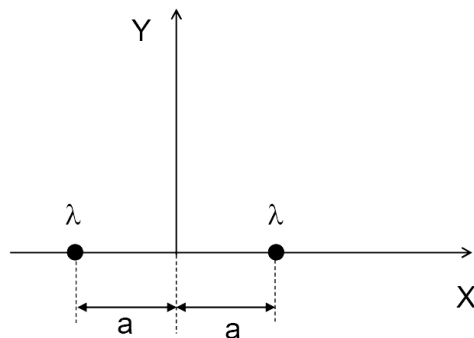
(b) Graphically represent E as a function of the distance r for this charge distribution.

(c) Considering $R = 3 \text{ m}$ and $\rho_0 = 8.85 \times 10^{-12} \text{ C/m}^3$, find the electric field at $(2, -4, 0) \text{ m}$ in rectangular coordinates if the spherical charge distribution is centred at the origin of the reference frame.

8. A solid sphere of radius 20 cm is uniformly charged with a net charge of 10 nC . Deduce the electric field 5 cm away from its centre. Use Gauss' Law to find the electric field.

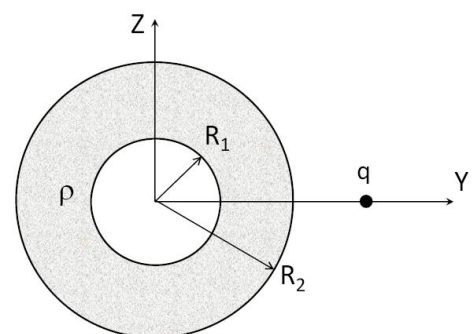
9. A solid sphere of radius R_1 has its centre on the X axis at $x = R_1$. It is uniformly charged with a volume charge density ρ_0 . A spherical shell concentric with the solid sphere has a radius of $R_2 = 2R_1$ and a uniform surface charge density of σ_0 . Find the magnitude of the electric field at $(R_1/2, 0, 0)$, $(5R_1/2, 0, 0)$ and $(2R_2, R_2, 0)$.

10. Two infinite lines with linear charge density $\lambda > 0$ perpendicular to the XY plane are located at $x = a$ and $x = -a$, as shown in the attached figure. Find the expression of the electric field for any point along the Y axis having $y > 0$.



11. A hollow sphere with radii $R_1 = 2 \text{ cm}$ and $R_2 = 4 \text{ cm}$ (see figure) has a volume density $\rho = -3 \times 10^{-6} \text{ C/m}^3$. Also, there is a point charge of 4 μC located at $(0, 6, 0) \text{ cm}$. Find:

- (a) The total charge contained in the hollow sphere.
- (b) The electric force on an electron located at $(0, -3, 1) \text{ cm}$.
- (c) The electric force on an electron located at $(0, 0, 0) \text{ cm}$.



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ANSWERS

1. Back area : $\Phi = -4.5 \frac{\text{N.m}^2}{\text{C}}$; Bottom area = Lateral areas : $\Phi = 0$; Top area : $\Phi = 4.5 \frac{\text{N.m}^2}{\text{C}}$
 $\Phi_{\text{net}} = 0$
2. $\Phi = 3.02 \times 10^{-9} \text{ Nm}^2 \text{ C}^{-1}$
3. (a) $\Phi_{\text{base1}} = \Phi_{\text{base2}} = 0.5\pi \text{ N m}^2/\text{C}$ (b) $\Phi_{\text{lateral}} = 0$ (c) $\Phi_{\text{net}} = \pi \text{ N m}^2/\text{C}$ (d) $Q_{\text{inside}} = 2.78 \cdot 10^{-11} \text{ C}$
4. (a) $\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r} \vec{u}_r (\text{N/C})$ (b) $\vec{E} = 1.7 \times 10^3 \vec{i} - 4.2 \times 10^3 \vec{j} \text{ N/C}$
5. (Y axis: perpendicular to the plates; positive direction: from region I to region IV)
 $\vec{E}_I = -\frac{\sigma}{2\epsilon_0} \vec{j}$; $\vec{E}_{II} = \frac{\sigma}{2\epsilon_0} \vec{j}$; $\vec{E}_{III} = \frac{3\sigma}{2\epsilon_0} \vec{j}$; $\vec{E}_{IV} = \frac{\sigma}{2\epsilon_0} \vec{j}$
6. (a) $Q = 0.814 \text{ nC}$ (b) $\vec{E}_{\text{inside}} = 0$; $\vec{E}_{\text{outside}} = \frac{\sigma R}{\epsilon_0 r} \vec{u}_r (\text{N/C})$ (d) $\vec{E}(2, 0, 5) = 0$
 $\vec{E}(10, 0, 0) = 610 \vec{i} \text{ N/C}$
 (e) $F_e = 0$ (c) We can use GL at points (x, 0, z)
7. (a) $\vec{E}(r < R/2) = 0$; $\vec{E}(R/2 < r < R) = \left(\frac{\rho_o r}{3\epsilon_0} - \frac{\rho_o R^3}{24\epsilon_0 r^2} \right) \vec{u}_r$; $\vec{E}(r > R) = \frac{7\rho_o R^3}{24\epsilon_0 r^2} \vec{u}_r$
 (b) $\vec{E} = 0.18 \vec{i} - 0.35 \vec{j} (\text{N/C})$
8. $\vec{E} = 562 \vec{u}_r (\text{N/C})$
9.

Positions	E
$(R_1/2, 0, 0)$	$\frac{\rho_0 R_1}{6\epsilon_0}$
$(5R_1/2, 0, 0)$	$\frac{4\rho_0 R_1}{27\epsilon_0}$
$(4R_1, 2R_1, 0)$	$\frac{1}{13\epsilon_0} \left[\frac{\rho_0 R_1}{3} + 4\sigma_0 \right]$
10. $\vec{E}(y) = \frac{\lambda y}{\pi \epsilon_0 (a^2 + y^2)} \vec{j}$
11.
 - a) $Q_{\text{esf}} = -7.04 \times 10^{-10} \text{ C}$
 - b) $\vec{F} = 6.98 \times 10^{-13} \vec{j} - 7.71 \times 10^{-14} \vec{k} \text{ [N]}$
 - c) $\vec{F} = 1.6 \times 10^{-12} \vec{j} \text{ [N]}$