1. **Design a PDA for the following languages:**
   a. **$L = \{ a^n \bullet b^n \mid n \geq 0 \}$**
   b. **$L = \{ a^n \bullet b^{2n} \mid n > 0 \}$**
   c. **$L = \{ a^{2n} \bullet b^n \mid n \geq 0 \}$**
   d. **$L = \{ a^{2n} \bullet b^n \mid n > 0 \}$**

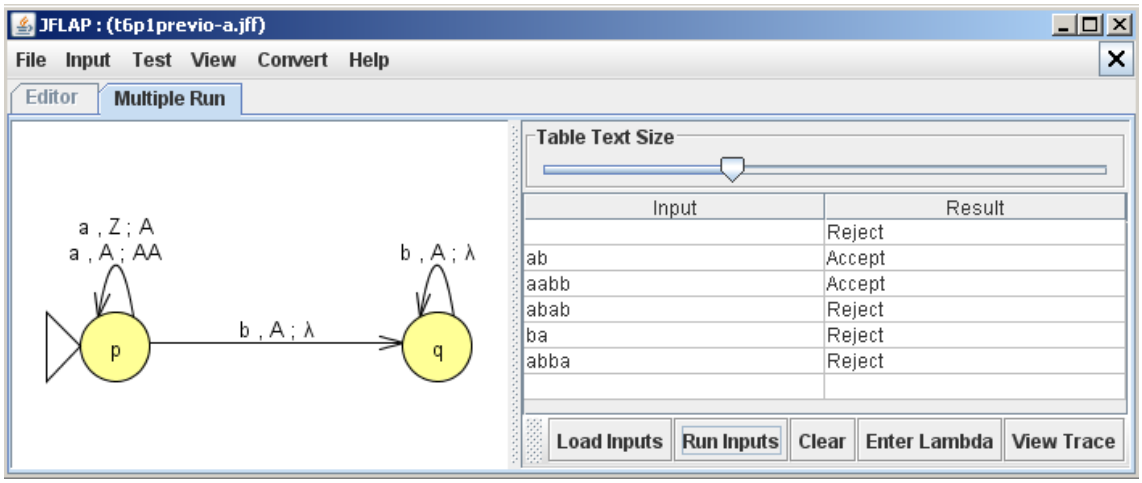
Solution:

   a. **$L = \{ a^n \bullet b^n \mid n \geq 0 \}$**

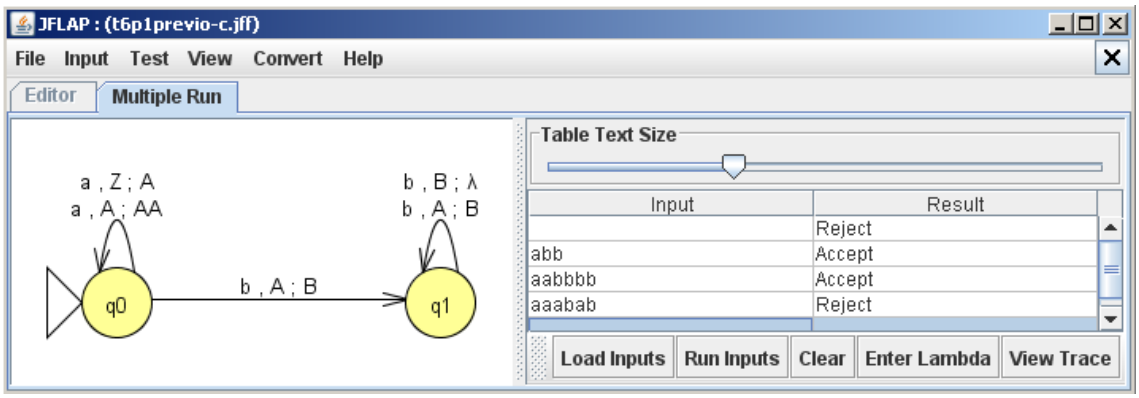
$f(p, a, Z) = (p, A)$ Start APv,
$f(p, a, A) = (p, AA)$ Put in the stack as many A's as a's read

$f(p, b, A) = (q, \lambda)$ Change states, when the first b is read
$f(q, b, A) = (q, \lambda)$ Pop as many A's as b's are read

$f(p, \lambda, Z) = (q, \lambda)$ In the case n=0

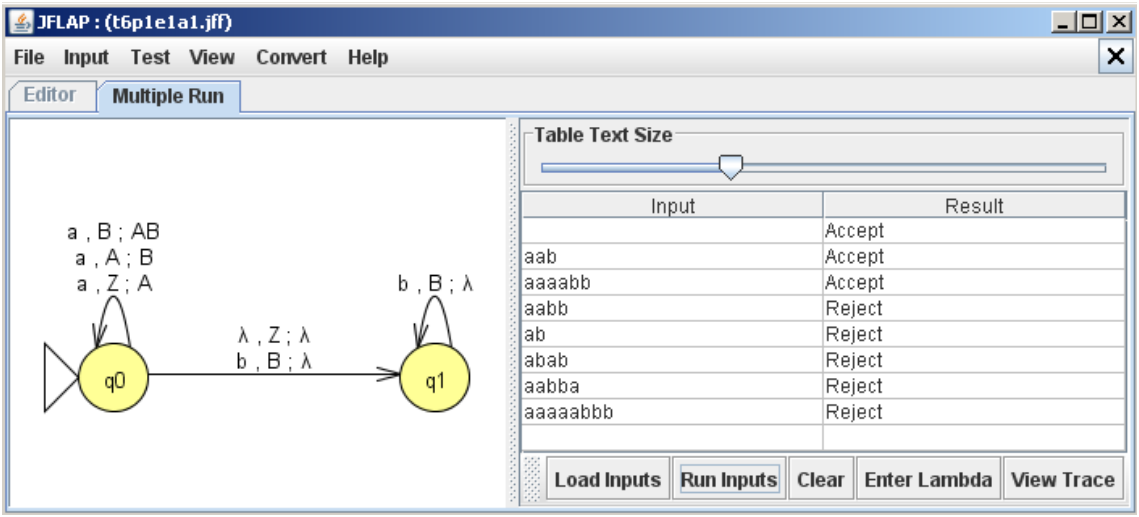


b. $L = \{ a^n \bullet b^{2n} \mid n > 0 \}$



Example:
(p, aabbbb, Z) |- (p, abbbb, A) |- (p, bbbb, AA) |- (q, bbb, BA) |- (q, bb, A) |- (q, b, B) |- (q, λ, λ)

c. $L = \{ a^{2n} \bullet b^n \mid n \geq 0 \}$
Solution
Taking the solution of the last exercise, each "odd" a is signified by an A, and each "even" *a* will substitute the A on the top of the stack by a B.



EXAMPLE:

(p, aaaabb, Z) ⊢ (p, aaabb, A) ⊢ (p, aabb, B) ⊢ (p, abb, AB) ⊢ (p, bb, BB) ⊢ (q, b, B) ⊢ (q, λ, λ)

d. L = { $a^{2n} \bullet b^n$ | n > 0}