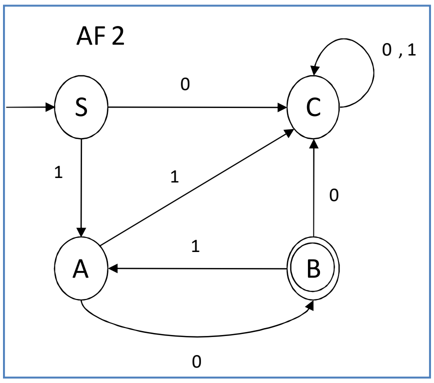
DEPARTAMENTO DE INFORMÁTICA
UNIVERSIDAD CARLOS III DE MADRID

Grado en Matemática Aplicada y Computación
Grado en Ingeniería Informática

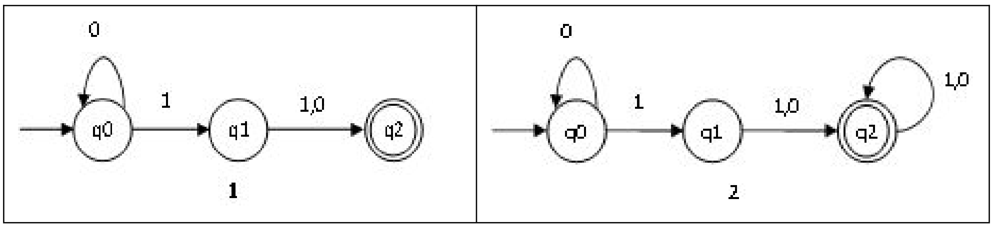
Formal Languages and Automata Theory
Exercises Finite Automata
Unit 3 – Part 2

1. Indicate the graph of a DFA which recognizes each one of the following languages. The alphabet is {0, 1}.
   a) The language {0}
   b) The language $0^m1^n0^p$ ($m \geq 0, n \geq 0, p \geq 1$)

2. Indicate which is the language recognized by the FA2.



3. For each one of the following statements, indicate if they are true or false and give a detailed explanation.
    a) Sentences recognized by a DFA cannot be longer than a specified length.
    b) The transitions required to recognize a word by an automaton can be infinite.
    c) A DFA can recognize the empty word.
    d) A DFA can only recognize a limited number of sentences.
    e) Let n be the number of states of the automaton (|Q|=n). A finite automaton where |Q|=n will only recognize words with length equal or less than n, i.e. x Є *, |x|≤n.
    f) It is possible to have a finite automaton in which every state is a final state.
    g) Given a DFA with 5 states, we have obtained Q/E3, then we can conclude Q/E3= Q/E.
    h) The automata 1 and 2 are equivalent.



    i) The language recognized by a DFA with every state connected and every state is final except the initial state, is Σ^{+}.
    j) The language recognized by a DFA with every state connected and every state is final including the initial state, is Σ^{*}.

4. [**Exam**] We have a door with only one lock. To open it, it is necessary to use three different keys (called *a*, *b*, and *c*), in a predefined order, which is following described:

    - Key *a*, then key *b*, then key *c*, or
    - Key *b*, then key *a*, then key *c*.

    If this order is not followed, then the lock is blocked (for instance, if the key *a* is used and following it is introduced again).

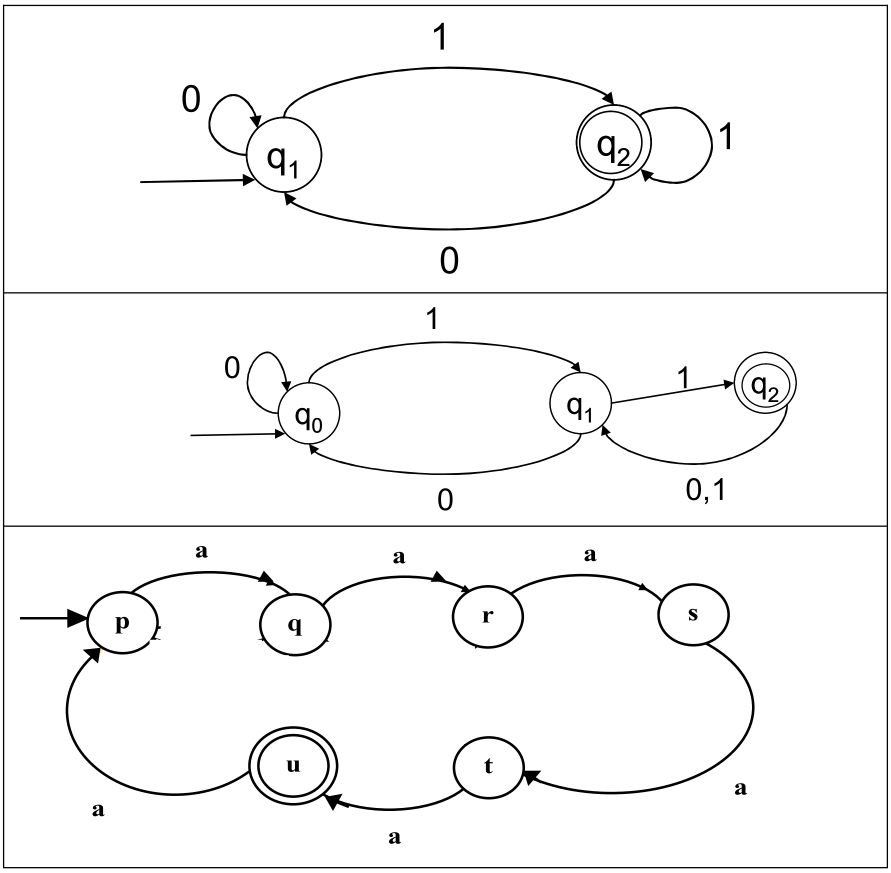
    Once the door is open, the introduction of keys in the lock (in every possible order) does not affect the closing device (i.e. the door remains open).

    Consider that the names of the different keys are symbols of an alphabet, over which a language L whose words are the valid sequences for the opening of the door is defined. For instance, *abcbc* is a word included in the language.

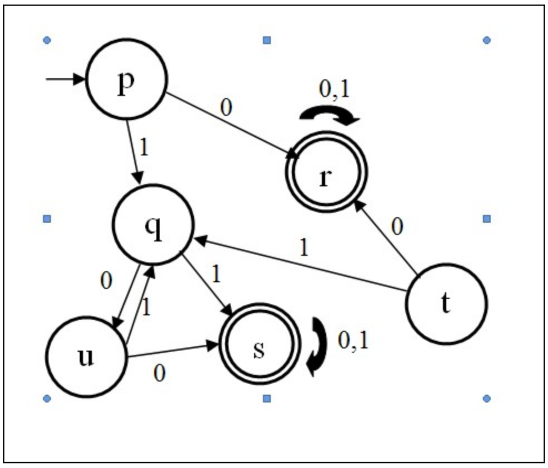
    It is required:

    a) Design a finite automata FA which accepts L.
    b) (**After Unit 5**) Grammar (clean and well-formed) which generates words in L.

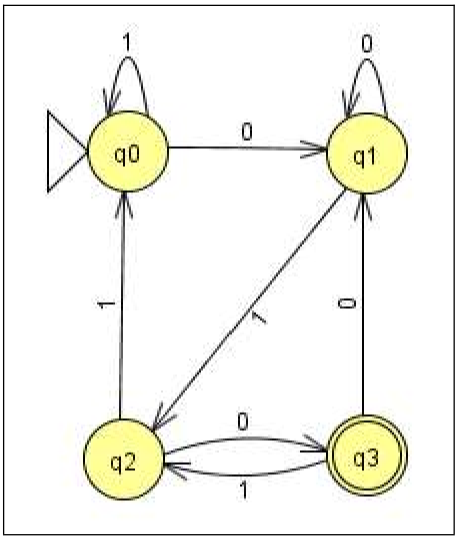
5. Calculate the Q/E of the following automata:

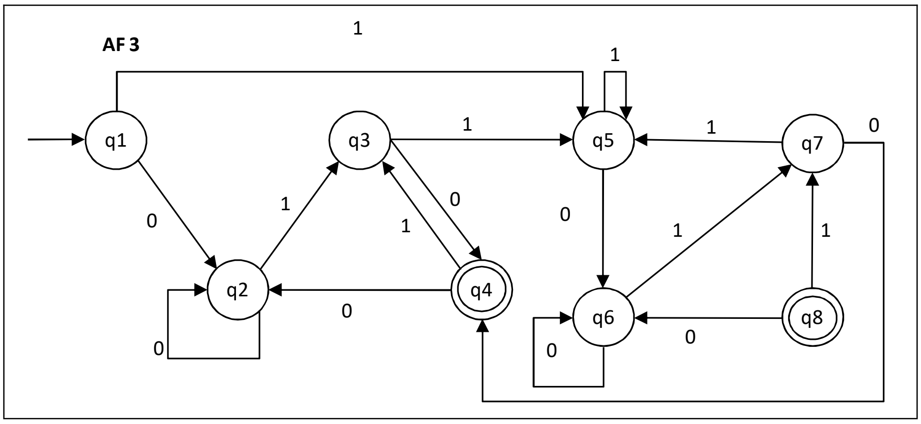


6. Given the following DFA, obtain the minimal equivalent DFA.



7. Verify if the following two DFA are equivalent (by obtaining the minimal DFA for each one of them).





8. Obtain the minimal DFA for the following DFA.

DFA_1=({a,b,c},{Q0,Q1,Q2,Q3,Q4},f,Q0,Q3)

f(Q0, a) = Q1 ; f(Q0, b) = Q2 ; f(Q0, c) = Q3
f(Q1, a) = Q2 ; f(Q1, b) = Q3 ; f(Q1, c) = Q1
f(Q2, a) = Q3 ; f(Q2, b) = Q1 ; f(Q2, c) = Q3
f(Q3, a) = Q4 ; f(Q3, b) = Q4 ; f(Q3, c) = Q4
f(Q4, a) = Q4 ; f(Q4, b) = Q4 ; f(Q4, c) = Q4

DFA_2; =({a,b,c}, {Q0,Q1,Q3,Q4,Q5,Q6,Q8},f,Q0,{Q3,Q4,Q6,Q8})

f(Q0, a) = Q4 ; f(Q0, b) = Q5 ; f(Q0, c) = Q1
f(Q1, a) = Q5 ; f(Q1, b) = Q5 ; f(Q1, c) = Q3
f(Q3, a) = Q5 ; f(Q3, b) = Q5 ; f(Q3, c) = Q5
f(Q4, a) = Q4 ; f(Q4, b) = Q8 ; f(Q4, c) = Q1
f(Q5, a) = Q5 ; f(Q5, b) = Q5 ; f(Q5, c) = Q5
f(Q6, a) = Q5 ; f(Q6, b) = Q8 ; f(Q6, c) = Q5
f(Q8, a) = Q5 ; f(Q8, b) = Q6 ; f(Q8, c) = Q5

DFA_3=({a,b,c},{Q0,Q1,Q2,Q3,Q4,Q6,Q7,Q8,Q9},f,Q0,{Q7,Q8})

f(Q0, a) = Q1 ; f(Q0, b) = Q6 ; f(Q0, c) = Q6
f(Q1, a) = Q7 ; f(Q1, b) = Q2 ; f(Q1, c) = Q6
f(Q7, a) = Q7 ; f(Q7, b) = Q2 ; f(Q7, c) = Q6
f(Q2, a) = Q6 ; f(Q2, b) = Q8 ; f(Q2, c) = Q6
f(Q8, a) = Q6 ; f(Q8, b) = Q8 ; f(Q8, c) = Q4
f(Q4, a) = Q6 ; f(Q4, b) = Q9 ; f(Q4, c) = Q3
f(Q9, a) = Q6 ; f(Q9, b) = Q8 ; f(Q9, c) = Q4
f(Q3, a) = Q6 ; f(Q3, b) = Q9 ; f(Q3, c) = Q4
f(Q6, a) = Q6 ; f(Q6, b) = Q6 ; f(Q6, c) = Q6

DFA_4=({c,f,d},{Q0,Q5,Q8,Q9,Q10,Q11,Q12},f,Q0,Q10)

f(Q0, c) = Q9 ; f(Q0, f) = Q10 ; f(Q0, d) = Q8
f(Q9, c) = Q9 ; f(Q9, f) = Q11 ; f(Q9, d) = Q12
f(Q10, c) = Q0 ; f(Q10, f) = Q8 ; f(Q10, d) = Q8
f(Q11, c) = Q11 ; f(Q11, f) = Q11 ; f(Q11, d) = Q8
f(Q12, c) = Q12 ; f(Q12, f) = Q5 ; f(Q12, d) = Q5
f(Q5, c) = Q5 ; f(Q5, f) = Q5 ; f(Q5, d) = Q8
f(Q8, c) = Q8 ; f(Q8, f) = Q8 ; f(Q8, d) = Q8