Comparison of populations

***Problem* 1.** A vote is to be taken among the residents of two towns (A and B) to determine the location of a new nuclear plant. To determine if there is a significant difference in the proportions of town A and town B voters favoring the proposal, i.e., the construction of the nuclear plant at their corresponding town, a poll is taken. If 120 of 200 voters from town A favor the proposal and 240 of 500 voters from town B favor it:

a) (1 point) Would you agree that the proportion of town A voters favoring the proposal is different than the proportion of town B voters favoring it? Use an $\alpha = 0.05$ level of significance.

b) (0.5 points) Build a 95% confidence interval on the difference of the proportions.

c) (0.5 points) By analyzing the confidence interval in b), can we derive the same conclusion than the one in a)? Justify your answer.

Solution: a) Yes , b) [0.0392, 0.2008] , c) Since the interval computed in b) does not include the zero value, then we can conclude to reject the null hypothesis

***Problem* 2.** An experiment was performed to compare the abrasive wear of two different laminated materials. Twelve pieces of material 1 were tested by exposing each piece to a machine measuring wear. Twelve pieces of material 2 were similarly tested. In each case, the depth of wear was observed. The samples of material 1 gave an average (coded) wear of $\bar{x}_1 = 85$ units with a known standard deviation of $\sigma_1 = 4$, while the samples of material 2 gave an average of $\bar{x}_2 = 81$ with a known standard deviation of $\sigma_2 = 2$. Assume each population is normally distributed.

a) (0.75 points) Can we conclude at the 0.05 level of significance that the abrasive wear of material 1 is different than material 2? Use $\alpha = 0.05$

b) (0.5 points) What is the p-value for this test?

c) (0.75 points)Find a 95% Confidence Interval on the difference of the abrasive wear between the two materials. Does it lead to any conclusion about part a)?

d) (0.5 points) What sample size n^* would be needed to reduce the width of the 95% CI computed in c) by half? Assume that the new sample size for sample 1 is n_1^*, the new sample size for sample 2 is n_2^* and that $n^* = n_1^* = n_2^*$.

Solution: a) Yes , b) $p - value \approx 0.002$ *c) [1.47, 6.53] , d)* $n^* = 49$.

***Problem* 3.** Two types of plastic are suitable for use by an electronics component manufacturer. The breaking strength of this plastic is important. It is known that $\sigma_1 = \sigma_2 = 1.0$ psi. From two random samples of size $n_1 = 10$ and $n_2 = 12$, we obtain $\overline{x}_1 = 162.5$ and $\overline{x}_2 = 155.0$. The company is now using plastic 2 and will not adopt plastic 1 unless its mean breaking strength exceeds that of plastic 2 by at least 10 psi. Based on the sample information, should it use plastic 1? Assume that the data are drawn from normal populations and use $\alpha = 0.05$ in reaching a decision.

Solution: No

***Problem* 4.** Two different types of injection-molding machines are used to form plastic parts. A part is considered defective if it has excessive shrinkage or is discolored. Two random samples, each of size 300, are selected, and 15 defective parts are found in the sample from machine 1 while 8 defective parts are found in the sample from machine 2.

a) Is it reasonable to conclude that both machines produce the same fraction of defective parts? Use $\alpha = 0.01$

b) Find the p-value for this test.

c) Find an approximate 95% Confidence Interval on the difference in the proportions. If the sample sizes are kept identical to each other, what sample sizes would be needed to reduce the width of the CI by half?

Solution: a) Yes , b) 0,1362 , c) [-0.0073, 0.054], 1200

Problem 5. A random sample of 500 adult residents of Maricopa County found that 385 were in favor of increasing the highway speed limit to 75mph, while another sample of 400 adult residents of Pima County found that 267 were in favor of the increased speed limit (75mph).

a) Do these data indicate that there is a difference in the support for increasing the speed limit between the residents of the two counties? Use $\alpha = 0.05$.

b) What is the p-value for this test?

c) Find an approximate 99% Confidence Interval on the difference in the proportions.

Solution: a) Yes , b) 0.0006 , c) [0.0434, 0.1616]

Problem 6. The diameter of steel rods manufactured on two different extrusion machines is being investigated. Two random samples of sizes $n_1 = 15$ and $n_2 = 17$ are selected, and the sample means and sample variances are $\overline{x}_1 = 8.73$, $s_1^2 = 0.35$, $\overline{x}_2 = 8.68$, and $s_2^2 = 0.4$ respectively. Assume that the data are drawn from two normal populations with equal variances.

a) Is there evidence to support the claim that the two machines produce rods with different mean diameters? Use $\alpha = 0.05$ in arriving at this conclusion.

b) Find the p-value for the statistic you calculated in part a).

c) Construct a 95% confidence interval for the difference in mean rod diameter. Interpret this interval.

Solution: a) No , b) 0.82 , c) [-0.394, 0.494];

Problem 7. Two companies manufacture a rubber material intended for use in a automotive application. The part will be subjected to abrasive wear, so we decide to compare the material produced by each company in a test. Twenty-five samples of material from each company are tested in an abrasion test, and the amount of wear after 1000 cycles is observed. For company 1, the sample mean and standard deviation of wear are $\overline{x}_1 = 20$ milligrams/1000 cycles and, $s_1 = 2$ milligrams/1000 cycles, while for company 2 we obtain $\overline{x}_2 = 15$ milligrams/1000 cycles and, $s_2 = 8$ milligrams/1000 cycles.

a) Do the data support the claim that the two companies produce material with different mean wear? Use $\alpha = 0.05$, and assume each population is normally distributed but that their variances are not equal.

b) What is the p-value for this test?

c) Do the data support a claim that the material from company 1 has higher mean wear than the material from company 2? Use the same assumptions as in part a).

Solution: a) Yes , b) 0:0024 , c) Yes;

Problem 8. The manager of a fleet of automobiles is testing two brands of radial tires and assigns one tire of each brand at random to the two rear wheels of eight cars and runs the cars until the tires wear out. The data (in kilometers) follow. Find a 99% confidence interval on the difference in mean life. Which brand would you prefer, based on this calculation?

Car	Brand 1	Brand 2
1	36925	34318
2	45300	42280
3	36240	35500
4	32100	31950
5	37210	38015
6	48360	47800
7	38200	37810
8	33500	33215

Solution: We cannot conclude that any of the brands is better than the other;

Problem 9. Oxide layers on semiconductor wafers are etched in a mixture of gases to achieve the proper thickness. The variability in the thickness of these oxide layers is a critical characteristic of the wafer, and low variability is desirable for subsequent processing steps. Two different mixtures of gases are being studied to determine whether one is superior in reducing the variability of the oxide thickness. Twenty wafers are etched in each gas. The sample standard deviations of oxide thickness are $s_1 = 1.96$ angstroms and $s_2 = 2.13$, respectively. Is there any evidence to indicate that the first mixture of gases is preferable? Use $\alpha = 0.05$.

Solution: No