

# CALCULUS

## Bachelor in Computer Science and Engineering

Course 2021–2022

### Real numbers: inequalities, subsets; methods of proof

**Problem 1.1.** Find all values of  $x \in \mathbb{R}$  satisfying the following conditions.

- 1)  $x^2 + \frac{2}{x} > 3.$
- 2)  $|\sqrt{x} - 2| \leq 3.$
- 3)  $-8 \leq |x - 5| - |x - 3| \leq 8.$
- 4)  $|x - 3| \leq 8.$
- 5)  $0 < |x - 2| < \frac{1}{2}.$
- 6)  $x^2 - 5x + 6 \geq 0.$
- 7)  $x^3(x + 3)(x - 5) > 0.$
- 8)  $\frac{2x + 8}{x^2 + 8x + 7} > 0.$
- 9)  $|x - 1| + |x - 2| > 1.$
- 10)  $|x - 1||x + 2| = 3.$
- 11)  $|x^2 - 2x| < 1.$

**Problem 1.2.** Find supremum, infimum, maximum, and minimum (if they exist) of the following subsets of  $\mathbb{R}$ .

- 1)  $A_1 = \{1/n : n \in \mathbb{N}\}.$
- 2)  $A_2 = \{1/n : n \in \mathbb{Z}, n \neq 0\}.$
- 3)  $A_3 = \{x \in \mathbb{Q} : 0 \leq x \leq \sqrt{2}\}.$
- 4)  $A_4 = \{x \in \mathbb{R} : x^2 + x + 1 \geq 0\}.$
- 5)  $A_5 = \{x \in \mathbb{R} : x^2 + x - 1 < 0\}.$
- 6)  $A_6 = \{x \in \mathbb{R} : x < 0, x^2 + x - 1 < 0\}.$
- 7)  $A_7 = \{1/n + (-1)^n : n \in \mathbb{N}\}.$
- 8)  $A_8 = \{x \in \mathbb{R} : 3x^2 - 10x + 3 < 0\}.$
- 9)  $A_9 = \{x \in \mathbb{R} : (x - a)(x - b)(x - c)(x - d) < 0, a, b, c, d \in \mathbb{R}, a < b < c < d\}.$
- 10)  $A_{10} = \{2^{-p} + 5^{-q} : p, q \in \mathbb{N}\}.$

**Problem 1.3.** Prove the following properties by the most appropriate method.

1)  $\sqrt{2}$  is an irrational number .

2)  $\sum_{n=0}^N r^n = \frac{1 - r^{N+1}}{1 - r}, \quad r \in \mathbb{R}, \quad r \neq 1, \quad N \in \mathbb{N}.$

3)  $\sum_{n=1}^N n = \frac{N(N+1)}{2}, \quad N \in \mathbb{N}.$

4)  $0 < x < y \implies x < \sqrt{xy} < \frac{x+y}{2} < y \quad (x, y \in \mathbb{R}).$

5)  $0 < x < y \implies \frac{x}{y} < \frac{x+k}{y+k}, \quad \forall k > 0 \quad (x, y \in \mathbb{R}).$

6)  $|x + y| = |x| + |y| \iff xy \geq 0 \quad (x, y \in \mathbb{R}).$