**P1.**

a)Initially, at t=0, the capacitance $C_{in} = \epsilon_0 A/d_0 = Q/V_0$. Therefore the charge stored on the capacitor $Q = \epsilon_0 A V_0/d_0$. Once the negative plate starts moving away from the positive plate with a constant speed, the capacitance will change(decrease) with time $C(t)$, as the geometry of the capacitor, ie the distance between the plates d(t), changes: $C(t) = \epsilon_0 A/d(t) = \epsilon_0 A/(d_0 + v_0 t)$. The charge Q remains constant and the potential difference V(t) increases. The energy stored on the capacitor increases with time:

$$U_e(t) = \frac{Q^2}{2C(t)} = \frac{\frac{\epsilon_0^2 A^2}{d_0^2} V_0^2}{2\frac{\epsilon_0 A}{d(t)}} = \frac{\epsilon_0 A V_0^2}{2d_0^2} d(t) = \frac{\epsilon_0 A V_0^2}{2d_0^2}(d_0 + v_0 t) \tag{1}$$

Henceforth the rate at which the energy stored changes with time

$$\frac{dU_e(t)}{dt} = \frac{\epsilon_0 A V_0^2}{2d_0^2} v_0 \tag{2}$$

From equation 2 we can obtain the speed

$$v_0 = \frac{2d_0^2}{\epsilon_0 A V_0^2} \frac{dU_e(t)}{dt} = 5.1 \times 10^{-3} \quad [m/s] \tag{3}$$

b)The electric field inside the capacitor at t=0, $E(t = 0) = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0} = \frac{C_{in} V_0}{A\epsilon_0} = \frac{V_0}{d} = 7.5 \times 10^5 \quad [V/m]$

Since the surface charge density $\sigma$ does not change with time, the electric field at $t = 3$ s, $E(t = 3s) = \frac{\sigma}{\epsilon_0} = E(t = 0) = 7.5 \times 10^5 \quad [V/m]$

**P2.**

a)Point A is outside the sphere since the magnitude of the position vector $r_A = \sqrt{x_A^2 + y_A^2 + z_A^2} = 0.6708 \ \ [m] > R$. Since the two charge distributions, sphere and shell, are concentric and have radial symmetry, in order to calculate the net electric field at point A, $\vec{E}(A)$, we will consider a spherical imaginary Gaussian surface $S_A$ of radius $r_A$ that passes from point A and is centred at origin. The electric field flux:

$$\Phi = \oint_{S_A} \vec{E} d\vec{s} \stackrel{*}{=} \oint_{S_A} E ds \stackrel{**}{=} E \oint_{S_A} ds = E 4\pi r_A^2 \tag{4}$$

where above we made use of the fact that (*) $\vec{E}//d\vec{s}$ for any point along $S_A$ and that (**) the magnitude of the electric field is constant along the Gaussian surface. Using Gauss's law:

$$\Phi = \frac{Q_{enc}}{\epsilon_0} = \frac{Q_\rho + Q_\sigma}{\epsilon_0} = \frac{\rho \frac{4}{3}\pi R^3 + \sigma 4\pi R^2}{\epsilon_0} = \frac{4\pi R^2}{\epsilon_0}\left(\frac{\rho}{3}R + \sigma\right) \tag{5}$$

where the total charge enclosed by the Gaussian surface $Q_{enc}$ is the sum of the charge of the sphere $Q_\rho$ and the charge of the surface $Q_\sigma$. From equations 4 and 5, we obtain the time dependence of the magnitude of the electric field at A:

$$E(A) = \frac{R^2}{\epsilon_0}\left(\frac{\rho}{3}R + \sigma\right)\frac{1}{r_A^2} = \frac{R^2}{\epsilon_0}\left(\frac{R}{3}\rho_0(1+at) + \sigma\right)\frac{1}{r_A^2} \tag{6}$$

From the above equation we can find the time at which $E(A) = 0$:

$$\frac{R}{3}\rho_0(1+at) + \sigma = 0 \Rightarrow 1 + at = -\frac{3\sigma}{\rho_0 R} \Rightarrow t = \frac{1}{a}\left(-\frac{3\sigma}{\rho_0 R} - 1\right) = 105.14 \quad [s] \tag{7}$$

b) By setting the electric potential at infinity equal to zero we obtain for the electric potential at A:

$$V_A - V_\infty = V_A = -\int_\infty^A \vec{E} \mathrm{d}\vec{r} \stackrel{\vec{E}//d\vec{r}}{=} -\int_\infty^A E dr = -\int_\infty^A \frac{R^2}{\epsilon_0}\left(\frac{R}{3}\rho_0(1+at) + \sigma\right)\frac{1}{r^2} dr =$$
$$= -\frac{R^2}{\epsilon_0}\left(\frac{R}{3}\rho_0(1+at) + \sigma\right)\int_\infty^A \frac{1}{r^2} dr = \frac{R^2}{\epsilon_0 r_A}\left[\frac{R}{3}\rho_0(1+at) + \sigma\right] \tag{8}$$

c) The electric force acting on the nucleus at time $t_2$:

$$\vec{F} = q\vec{E}(t_2) = m\vec{\gamma} \Rightarrow \vec{\gamma} = \frac{q}{m}\vec{E}_A(t_2) = \frac{2|e|}{4m_p}\vec{E}_A(t_2) = \frac{|e|}{2m_p}\vec{E}_A(t_2) \tag{9}$$

where we made use of the fact that the charge of the nucleus $q = 2|e|$ since it contains 2 protons and the mass $m = 2m_p + 2m_n = 4m_p$. The magnitude of the electric field at point A was calculated above, see equation 6 and the electric field vector:

$$\vec{E}_A(t_2) = \frac{R^2}{\epsilon_0}\left(\frac{R}{3}\rho_0(1 + at_2) + \sigma\right)\frac{1}{r_A^2}\vec{u}_{r_A} = \frac{R^2}{\epsilon_0}\left(\frac{R}{3}\rho_0(1 + at_2) + \sigma\right)\frac{1}{r_A^2}\left(\frac{y_A}{r_A}\vec{j} + \frac{z_A}{r_A}\vec{k}\right) \tag{10}$$

with $\vec{u}_{r_A} = \vec{r}_A/r_A$ the unit vector of $\vec{r}_A$. From equations 9 and 10:

$$\vec{\gamma} = -4.78 \times 10^{12}\vec{j} + 2.39 \times 10^{12}\vec{k} \quad [m/s^2] \tag{11}$$

**P3.**

a)Let us first calculate the magnetic field $B_1$ created by the surface current $I_1$ for $r < R$: If we consider a circular path of radius r along the xy plane that is concentric to the cylinder, then through the surface of this circle there is no current passing. Therefore the magnetic field $\vec{B}_1 = 0$ for $r < R$.
For $r \geq R$ we consider a circular path $C_1$ of radius r along the xy plane that is concentric to the cylinder. According to Ampère's law:

$$\oint_{C_1} \vec{B}_1 d\vec{l} = \mu_0 I_1 \tag{12}$$

since the current that passes through the circular surface defined by $C_1$ is $I_1$. The line integral

$$\oint_{C_1} \vec{B}_1 d\vec{l} \stackrel{\text{a}}{=} \oint_{C_1} B_1 dl \stackrel{\text{b}}{=} B_1 \oint_{C_1} dl = B_1 2\pi r \tag{13}$$

where we made use of the fact that (a) $\vec{B}_1 // d\vec{l}$ along the path $C_1$ and (b) the magnitude of the magnetic field is constant along the path. From equations 12 and 13 we obtain

$$B_1 = \frac{\mu_0 I_1}{2\pi r} \tag{14}$$

Considering points along the positive y-axis, the magnetic field $\vec{B}_1$ will be perpendicular to the vector product of the current direction ($\vec{k}$) and the position vector $\vec{y}$ direction ($\vec{j}$), ie $\vec{k} \times \vec{j} = -\vec{i}$

$$\vec{B}_1 = -\frac{\mu_0 I_1}{2\pi y}\vec{i} \tag{15}$$

Now we derive the expression for the magnetic field $B_2$ created by current $I_2$. We consider a circular path $C_2$ of radius $r_2$ along the xy plane that is concentric to the line. According to Ampère's law:

$$\oint_{C_2} \vec{B}_2 d\vec{l} = \mu_0 I_2 \tag{16}$$

since the current that passes through the circular surface defined by $C_2$ is $I_2$. The line integral

$$\oint_{C_2} \vec{B}_2 d\vec{l} \stackrel{\text{c}}{=} \oint_{C_2} B_2 dl \stackrel{\text{d}}{=} B_2 \oint_{C_2} dl = B_2 2\pi r_2 \tag{17}$$

where we made use of the fact that (c) $\vec{B}_2 // d\vec{l}$ along the path $C_2$ and (d) the magnitude of the magnetic field is constant along the path. From equations 16 and 17 we obtain

$$B_2 = \frac{\mu_0 I_2}{2\pi r_2} \tag{18}$$

Note that $r_2$ is the distance from the line. For points along the positive y-axis, with $y < D$:

$$\vec{B}_2 = -\frac{\mu_0 I_2}{2\pi(D-y)}\vec{i} \tag{19}$$

since the magnetic field $\vec{B}_2$ will be perpendicular to the vector product of the current direction $(-\vec{k})$ and the direction of the vector $\vec{r}_2$ from the line to the point $(-\vec{j})$, ie $-\vec{k} \times (-\vec{j}) = -\vec{i}$.

For points along the positive y-axis, with $y > D$:

$$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi(y-D)}\vec{i} \tag{20}$$

since the magnetic field $\vec{B}_2$ will be perpendicular to the vector product of the current direction $(-\vec{k})$ and the direction of the vector $\vec{r}_2$ from the line to the point $(\vec{j})$, ie $-\vec{k} \times \vec{j} = \vec{i}$.

In order to deduce the general expression of the magnetic field for points along the y-axis: $\vec{B} = \vec{B}_1 + \vec{B}_2$ we have to divide it into the following 3 areas:

**(I)** $0 < y < D$:

$$\vec{B} = -\frac{\mu_0 I_2}{2\pi(D-y)}\vec{i} \tag{21}$$

**(II)** $D < y < R$:

$$\vec{B} = \frac{\mu_0 I_2}{2\pi(y-D)}\vec{i} \tag{22}$$

**(III)** $y \geq R$:

$$\vec{B} = -\frac{\mu_0 I_1}{2\pi y}\vec{i} + \frac{\mu_0 I_2}{2\pi(y-D)}\vec{i} \tag{23}$$

b) In areas (I) and (II) the magnetic field cannot be zero since only current $I_2$ contributes. For area (III), setting $\vec{B} = 0$ in equation 23:

$$\frac{\mu_0 I_1}{2\pi y} = \frac{\mu_0 I_2}{2\pi(y-D)} \Rightarrow I_1 y - I_1 D = I_2 y \Rightarrow y = \frac{I_1}{I_1 - I_2} D < 0 \tag{24}$$

Therefore, it is not possible to find a point with $y > 0$ where $\vec{B} = 0$.