Chapter VII: Large-Sample Inference

PROBLEMS

Proposed Problems

1. A random sample taken from a population with variance parameter $\sigma^2 = 100$, presents a sample mean $\bar{x} = 160$. For n = 144 compute
   a) A 95% confidence interval for the population mean μ
   b) A 90% confidence interval for the population mean μ
   SOLUTION:
   a) [158.36, 161.63]
   b) [158.625, 161.375]
2. In a survey to 10000 High School students the question about their weekly consumption of soft drinks revealed a mean of 5 bottles, with a standard deviation of 2. Find the 95%confidence interval for the average consumption of the entire population of high school students.
   SOLUTION:
   [4.96, 5.04]
3. In order to determine the proportion of people who own a car in a particular province was performed a simple random sampling, so that among the 100 respondents, 30 of them had a car.
   a) Find a confidence interval for the proportion of the population to own a car ($\alpha = 0.05$).
   b) If you wished to estimate the proportion with a precision of 0.02 and 95% confidence, how many people should be surveyed? (sep. 99)
   SOLUTION:
   a) [21.01%, 38.98%]
   b) 2017
4. Let X be the unit consumption of certain material in a production process (milligrams per unit of obtained product). We know that X is normal with mean μ and standard deviation $\sigma = 20$ mg. We take a random sample made of 25 observations and compute that $\bar{x} = 120$ mg.
   a) Based on this sample information, compute an interval estimate with 95% confidence of the average consumption of this material.
   b) What sample size should be taken to have a 95% interval of confidence with 10 mg of amplitude? (interval amplitude = difference between its ends) (June 02)
   SOLUTION:
   a) [112.16, 127.84]
   b) $n \cong 62$
5. A store chain plans to open a new store in a central pedestrian zone. The final decision would depend on the pedestrian traffic moving through these streets. It is known that the store to be successful requires that a pedestrian flow of at least 2000 pedestrians per day goes through the street during business hours. To check whether this condition holds an experiment is performed on two streets in the area. The experiment consists of counting the number of pedestrians who, during business hours, passes in these two main streets. The experiment lasted for one week (7 days). During that week, in the street-1, 12600 pedestrians passed while in the street-2, 12880 pedestrians did. Assuming that the number of people that passes every day a street is a Poisson random variable, compute:
   a) A 95% confidence interval for the parameter of the Poisson distribution "number of daily pedestrian" travelling through the street-1.
   b) A 95% confidence interval for the parameter of the Poisson distribution "number of daily pedestrian" travelling through the street-2. (June 02)
   SOLUTION:
   a) $\lambda_1 \in [1768, 1831]$
   b) $\lambda_2 \in [1808, 1872]$

6. A transistor manufacturer of type BC547B knows that when the production stays in the normal values of quality the current gain (a parameter of the transistor denoted by the adimensional coefficient β) has a distribution with mean 290. A sample of this type of transistors is taken and STATGRAPHICS shows the following results in the Summary Statistics

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Summary Statistics for BC547B

Count = 100
Average = 282,29
Mode = 304,0
Variance = 766,854
Standard deviation = 27,6921
Skewness = 0,324111
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Confidence Intervals for BC547B

95,0% confidence interval for mean:  [276,795;287,785]

95,0% confidence interval for standard deviation: [24,3139;32,1693]
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Answer, justifying the response, whether the following statements are true or false:

a) The confidence interval of the average may not be valid because we do not know if the variable X is normal.
b) The confidence interval of the mean is valid because it contains the population mean.
c) The population standard deviation is σ = 28.24.
d) The "unbiased" sample standard deviation is $\hat{s}_X^2 = 28.24$.
e) The population mean of X cannot be 290.
f) The 90% confidence interval is narrower than the 95% confidence interval.

SOLUTION:

Only the statement f) is true.

7. The file Resistencias1KO.sf3 contains data from a sample of resistors with nominal value 1000Ohms. The measurements belong to two types of resistance: the resistances of golden band and the ones of brown band. The brown band resistances must have a value nearest to the nominal one than the gold band resistances. Compute:

a) A statistical test with significance 5% that the brown band resistances come from a population with nominal value of 1000Ohms.
b) A 95% confidence interval for their mean.
c) Repeat the analyses above for the golden band resistances and compare the two results

In the following it is shown the summary statistics for both kinds of resistances produced by STATGRAPHICS

Summary Statistics

	Golden Band	Brown Band
Count	50	50
Average	989,02	999,14
Median	988,5	999,0
Variance	267,612	48,8576
Standard deviation	16,3588	6,98982
Standard error	2,31349	0,988509
Minimum	946,0	983,0
Maximum	1025,0	1016,0

SOLUTION:

a) We cannot reject with significance 5% the Null Hypothesis
b) [997.20, 1001.08]
c) [984.49, 993.55]