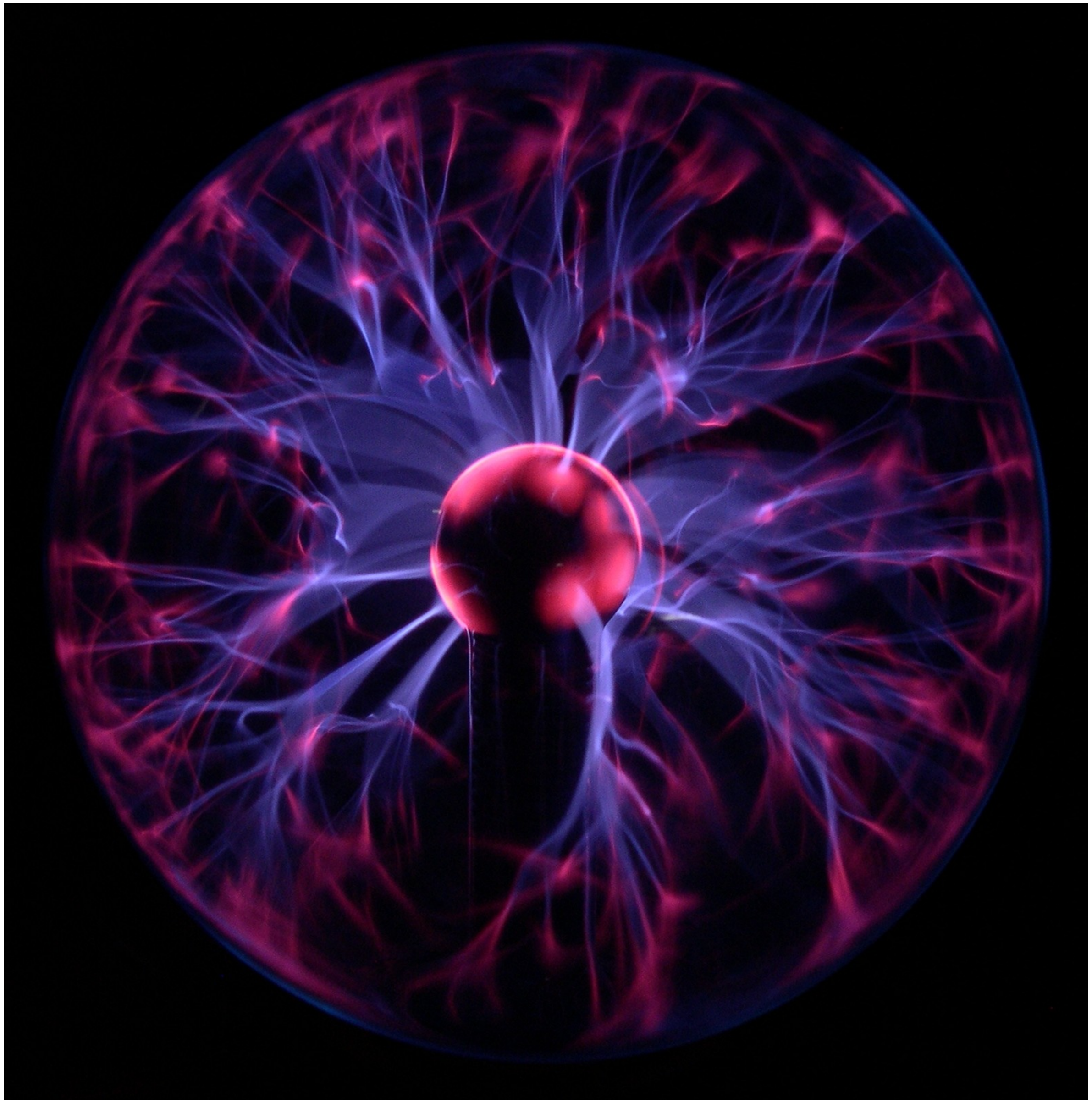
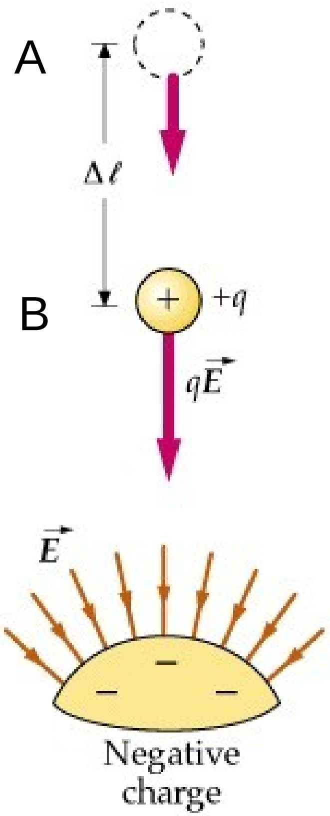
Electric potential



Electrostatic work and potential energy

- The electrostatic force <u>is conservative</u> → It has a potential energy function U associated to it:

$$W_{AB}^{el} = -\Delta U = U_A - U_B$$



- The work done by the electrostatic force is independent of the trajectory.

- The work done by the electrostatic force (W_{el}) decreases U. In order to increase it, we need to do work against the electrostatic force (W_{ext}).

- When $W_{el} > 0$ → spontaneous process
- When $W_{el} < 0$ → non-spontaneous process

Potential energy and electric potential

Change in electric potential energy of a system

$$\Delta U = -\int_A^B \vec{F}_{el} \cdot d\,\vec{l} = -\int_A^B q\,\vec{E} \cdot d\,\vec{l}$$

SI units: J

electric potential

$$\boxed{V = \frac{potential\ energy}{charge} = \frac{U}{q}}$$

SI units: J/C=V Scalar quantity!

Electric field SI units: N/C=V/m

V only depends on the source charge distribution creating the field, not on the test charge.

The electric field lines always point in the direction of decreasing potential V.

electric potential difference

$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\,\vec{l} = -\frac{W_{AB}}{q}$$

for a uniform Electric Field

$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\,\vec{l} = -Ed$$

Obtaining electric field from potential

$$\Delta V = -\int_A^B \vec{E} \cdot d\vec{l} \rightarrow \vec{E} = -\frac{dV}{dl}\vec{u} = -\vec{\nabla} V$$

Gradient:

$$\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

Example:

Find the electric field for the electric potential function given by V(x)=100V - (25V/m)x

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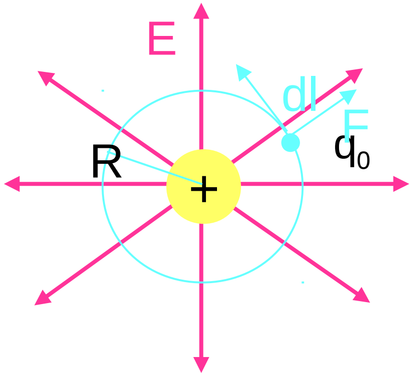
Find the electric field for the electric potential function given by V(x)=100V - (25V/m)x

Answer: E = 25 $\vec{i}$ *(N/C)*

Electric potential

- How does the electric potential vary when we move a test charge along a trajectory?

a) Along a circular trajectory



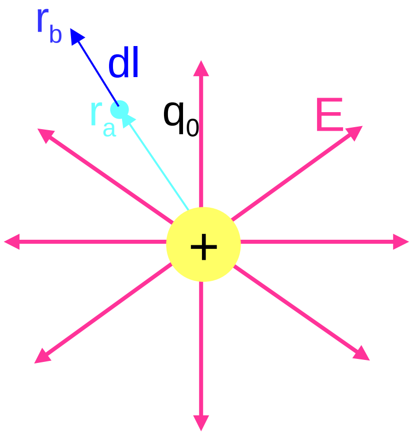
$$\vec{E} \perp d\vec{l} \Rightarrow \vec{E} \cdot d\vec{l} = 0 \Rightarrow \Delta V = 0 \Rightarrow V = const$$

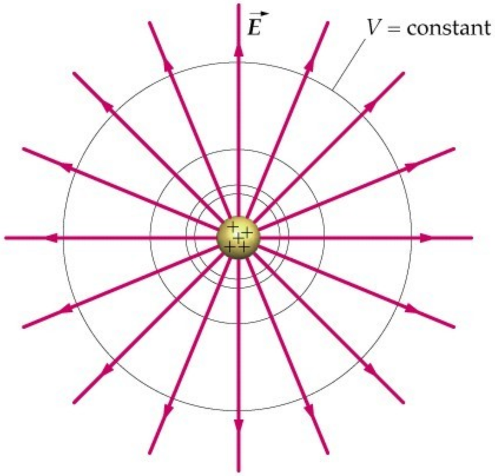
The surfaces in which V=const are called equipotential surfaces.

They are perpendicular to the electric field lines at each point.

b) Along a radial trajectory

$$\vec{E} || d\vec{l} \Rightarrow \Delta V = \text{maximum}$$

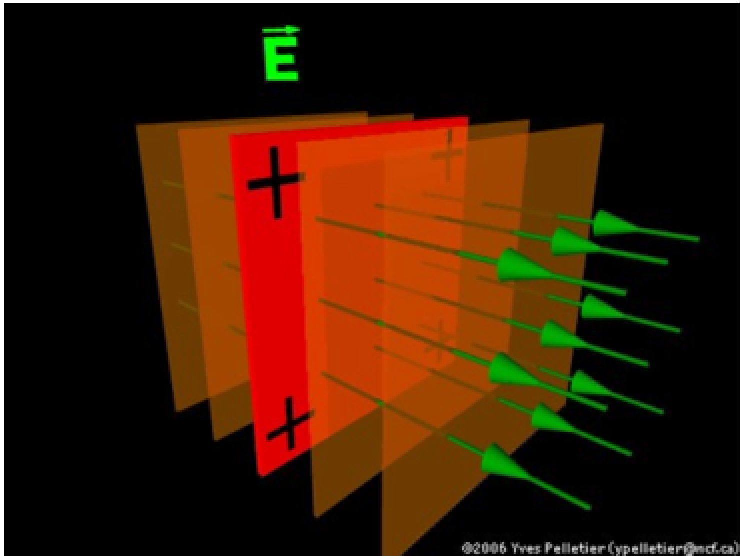


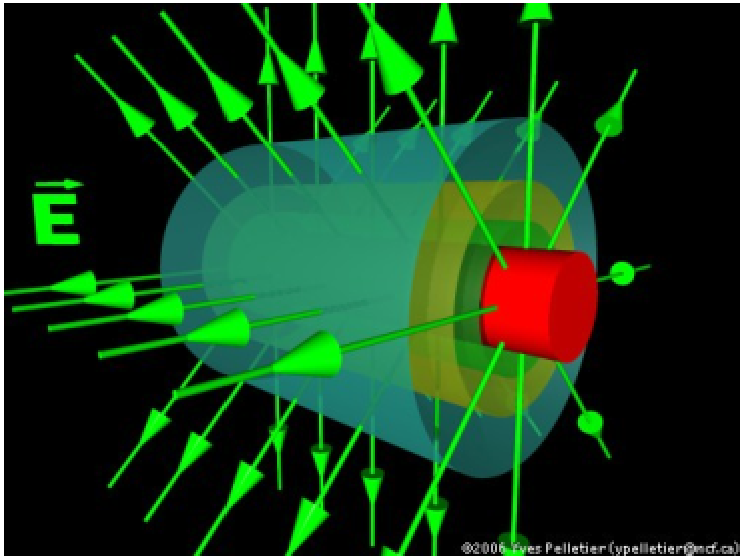


Electric potential

Other equipotential surfaces:

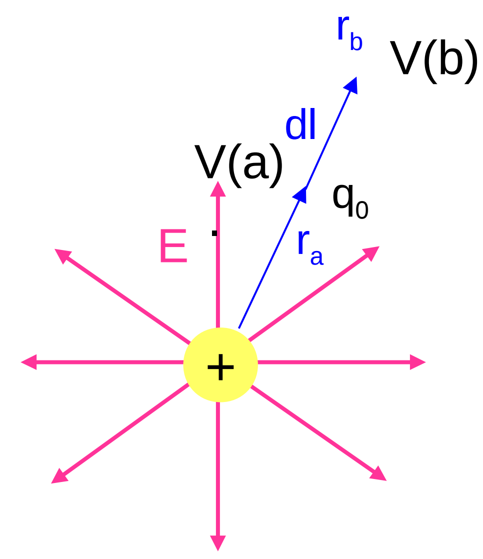
Equipotential surfaces due to an infinite charged plane and an infinite charged cylinder





Electric potential

Electric potential due to a point charge



$$\Delta V = V_b - V_a = \frac{\Delta U_{ba}}{q_0} = \frac{-1}{q_0}\int_a^b \vec{F}\cdot d\vec{l} = -\int_a^b \frac{\vec{F}}{q_0}\cdot d\vec{l} = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\vec{dl} = dr\,\vec{u}_r$$

$$-\int_a^b \vec{E}\cdot d\vec{l} = -\int_a^b E\,dl = -\int_a^b \frac{kQ}{r^2}\,dr = kQ\left[\frac{1}{r_b} - \frac{1}{r_a}\right]$$

In order to define the potential at a distance r_b (V_b), we take the distance r_a as a reference and choose $V_a = 0$ (we choose the origin of potential):

If we choose $V_a = 0$ at $r_a \rightarrow \infty \Rightarrow \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_a} \rightarrow 0$ $\qquad \Delta V = V_b = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_b}\right]$

V due to a point charge:
(origin of potential at r=∞)

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]}$$

Coulomb's potential

Electric potential

Electric potential due to a system of point charges

We apply the superposition principle:

$$\boxed{V_{net} = \sum V_i = \sum \frac{Q_i}{4\pi\varepsilon_0}\left[\frac{1}{r_i}\right]}$$

<u>Example:</u> Two positive point charges of equal magnitude +5 nC are located along the x axis. One is at the origin and the other at x=8cm.

Find the electric potential at a point along the y axis where y=6 cm.

Electric potential

Electric potential due to a system of point charges

We apply the superposition principle:

$$\boxed{V_{net} = \sum V_i = \sum \frac{Q_i}{4\pi\varepsilon_0}\left[\frac{1}{r_i}\right]}$$

Example: Two positive point charges of equal magnitude +5 nC are located along the x axis. One is at the origin and the other at x=8cm.
Find the electric potential at a point along the y axis where y=6 cm.

Answer: 1200 V

Electric potential

Electric potential due to a continuous charge distribution

A METHOD: By integrating the dV created by each dq:

$$V = \int_{V} dV = \int_{Q} K \frac{dq}{r}$$

B METHOD: By integrating the electric field along the trajectory

$$\boxed{\Delta V = -\int_{a}^{b} \vec{E} \cdot d\,\vec{l}}$$

<u>Example</u>: Find the electric potential inside and outside a uniformly charged sphere with charge density ρ.

Electric potential

Electric potential due to a uniformly charged solid sphere with charge density ρ

- Electric charge on sphere: $Q = \rho V = \frac{4\pi}{3}\rho R^3$
- Electric field at $r > R$: $E = \frac{kQ}{r^2}$
- Electric field at $r < R$: $E = \frac{kQ}{R^3}r$
- Electric potential at $r > R$:

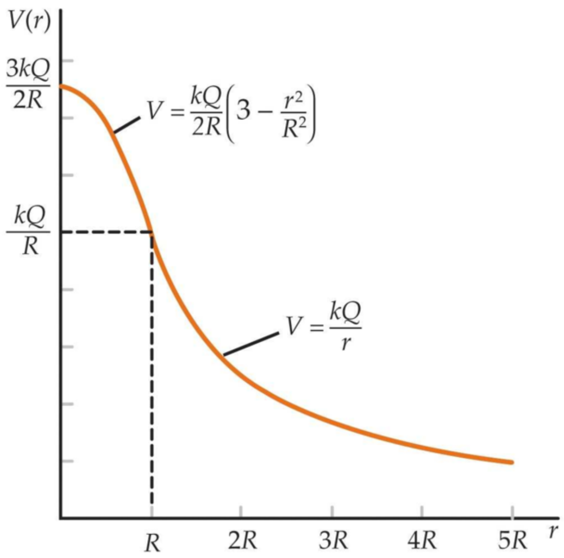
$$V = -\int_{\infty}^{r} \frac{kQ}{r^2}\,dr = \frac{kQ}{r}$$

- Electric potential at $r < R$:

$$V = -\int_{\infty}^{R} \frac{kQ}{r^2}\,dr - \int_{R}^{r} \frac{kQ}{R^3}\,r\,dr$$

$$\Rightarrow V = \frac{kQ}{R} - \frac{kQ}{2R^3}\left(r^2 - R^2\right) = \frac{kQ}{2R}\left(3 - \frac{r^2}{R^2}\right)$$

Choosing V = 0 at r = ∞.

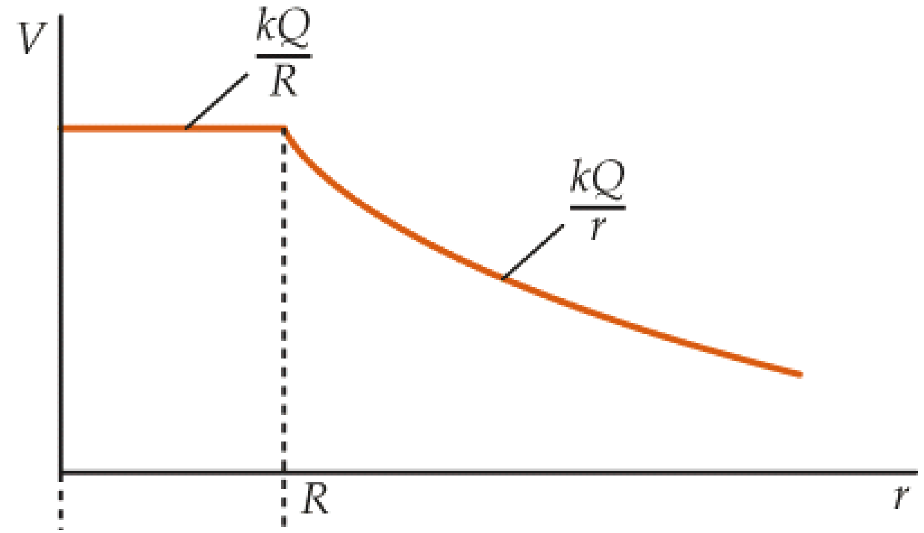


Outside the sphere: $V = \frac{\rho R^3}{3\varepsilon_o r}$ Inside the sphere: $V = \frac{\rho R^2}{2\varepsilon_o} - \frac{\rho r^2}{6\varepsilon_o}$

Electric potential

Electric potential due to a spherical shell of charge density σ

We choose V = 0 at r = ∞.



Outside the sphere: $V = \dfrac{\sigma R^2}{\varepsilon_o r}$

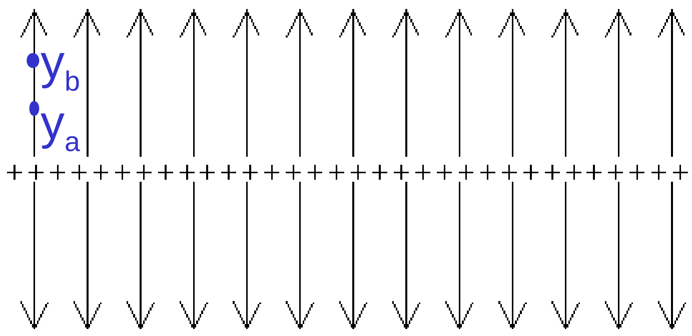
V is continuous

Inside the sphere: $V = \dfrac{\sigma R}{\varepsilon_o}$

V is constant inside the shell (as E=0)

Electric potential

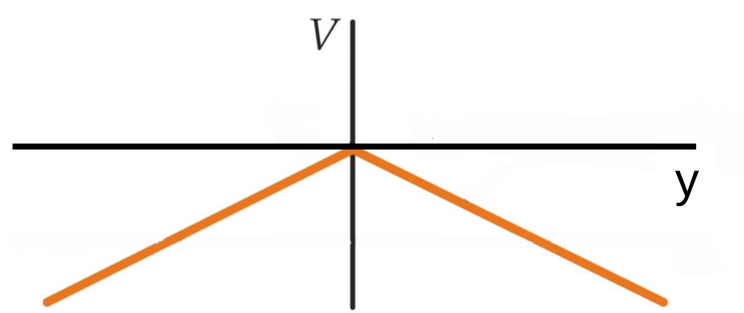
Electric potential due to an infinite charged plane with charge density σ



$$V_b - V_a = -\frac{\sigma}{2\varepsilon_0}\left(|y_b| - |y_a|\right)$$

We may choose V = 0 at any finite point. If we choose y = 0:

$$V = -\frac{\sigma|y|}{2\varepsilon_0}$$



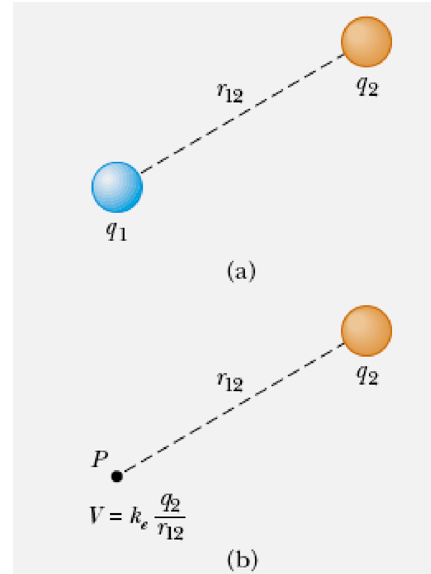
V is continuous

In these examples, we have checked that V is always a continuous function (as opposed to E).

Potential energy of a system of point charges

Electrostatic potential energy of a system of point charges

1) Potential energy of a pair of charges $U_{12} = q_2 V_1 = k \dfrac{q_1 q_2}{r_{12}}$

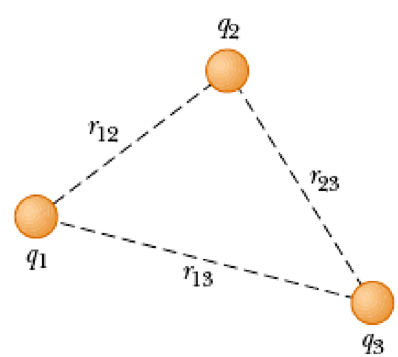


Active Figure 25.10 (a) If two point charges are separated by a distance r_{12}, the potential energy of the pair of charges is given by $k_e q_1 q_2 / r_{12}$. (b) If charge q_1 is removed, a potential $k_e q_2 / r_{12}$ exists at point P due to charge q_2.

2) Total potential energy (U_{net}) of a system of charges

If there are more than two charges, the total U can be obtained by calculating U for every pair of charges and adding the terms algebraically. For example, for three charges:

$$U_{system} = k \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$



$$U_{system} = \frac{1}{2} \sum_{i=1}^{n} q_i V_{net,i}$$

U_{system} : the total work done by an external agent to assemble the configuration, beginning with the charges at infinite separation.