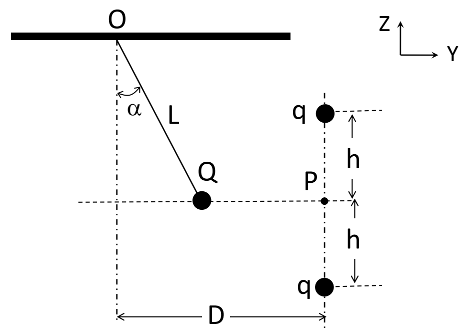
PROBLEMS:

P1. ***(2p)*** A particle of mass M and charge Q is suspended from point O by a thread of negligible mass and length L. In addition, there are two point particles of charge q, which are fixed in the positions indicated in the figure. When the pendulum and the vertical axis form an angle α, the particle of the pendulum is in equilibrium. Knowing that in this configuration the magnitude of the rope's tension is T = 0.179 N

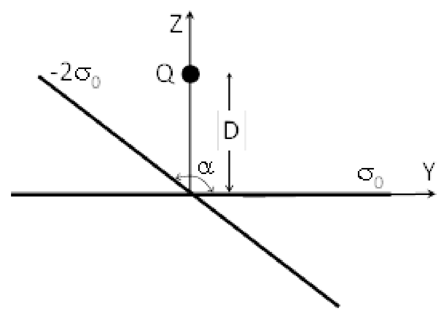


a) Calculate the value of Q

b) Calculate the work needed to bring a particle of charge q' from infinity to the point P indicated in the figure.

DATA: q= -1.25 μC; M= 15 g; L= 2.4 m; α= 35°; D= 4 m; h= 1.2 m; q'= 8 μC; g=9.8 m/s^2

P2. ***(2p)*** The electrostatic system shown in the figure consists of:



- An uniformly charged infinite plane, that coincides with the XY plane and has a charge density σ_0.
- A second, uniformly charged infinite plane with a charge density $-2\sigma_0$, that passes through the origin and forms an angle α with the XY plane (see figure).
- A point charge Q located at (0, 0, D).

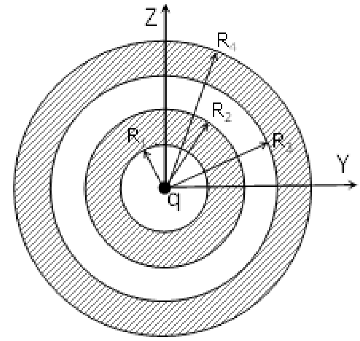
a) Deduce the general expression for the electric field created by an infinite, uniformly charged plane with a surface charge density σ

b) Calculate the electric field vector $\vec{E}$ at point P (0, D/2, D/2).

c) For which value of Q, the electric field at P is equal to $\vec{E}(P) = E\vec{k}$

DATA: σ_0 = 1.4×10^{-6} C/m^2; α= 140°; Q= 4.5×10^{-5} C; D= 6 m

P3. ***(2p)*** Consider two hollow conductive spheres placed concentrically, as indicated in the figure, with the center located at the origin of coordinates. The sphere of internal radius R_1 and external radius R_2 has charge Q_1, while the sphere of internal radius R_3 and external radius R_4 has charge Q_2. In addition, a point charge q is placed in the center of the spheres. Given the points A (0, 0, 2) and B(0, 0, 7) and that the ratio of the magnitudes of the electric field at these points is $\frac{E(A)}{E(B)} = 37.69$



a) Calculate the value of Q_1.

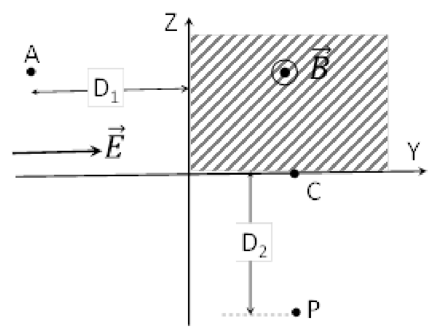
b) Calculate the charge densities on all conductive surfaces.

c) Given the points P1 (0, 0, 15) and P2 (0, 0, 20) calculate the potential difference $\left(V(P2) - V(P1)\right)$

DATA: R_1 = 4 cm; R_2 = 6 cm; R_3 = 8 cm;R_4 = 12 cm; q= 8 μC; Q_2= 3 μC

NOTE: All coordinates are expressed in cm

P4. ***(2p)*** An α particle (a nucleus of He, consisting of 2 protons and 2 neutrons) is placed initially at rest, at point A, see figure. In the region of space defined by y<0, a uniform electric field is applied $\vec{E} = E_0\vec{j}$. In addition, in the region of space defined by y>0 and z>0 (shaded region of the figure) a uniform magnetic field is applied $\vec{B} = B_0\vec{i}$. Knowing that the total time it takes the α particle to go from point A to point P is
t_{total} = 0.145 ms



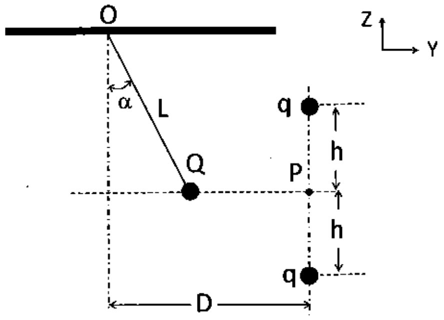
a) Calculate the value of B_0

b) Calculate the Cartesian coordinates of point C of the figure (point where the α particle trajectory intersects with the Y axis)

DATA: E_0= 2400 N/C; D_1= 450 m; D_2= 350 m

Neglect gravity interaction

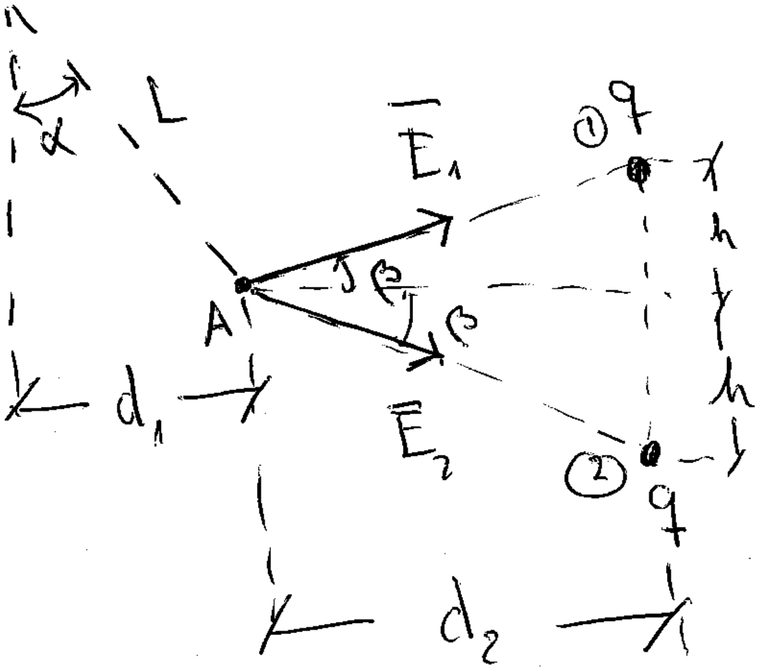
P1. ***(2p)*** A particle of mass M and charge Q is suspended from point O by a thread of negligible mass and length L. In addition, there are two point particles of charge q, which are fixed in the positions indicated in the figure. When the pendulum and the vertical axis form an angle α, the particle of the pendulum is in equilibrium. Knowing that in this configuration the magnitude of the rope's tension is T = 0.179 N



a) Calculate the value of Q

b) Calculate the work needed to bring a particle of charge q' from infinity to the point P indicated in the figure.

DATA: q= -1.25 μC; M= 15 g; L= 2.4 m; α= 35°; D= 4 m; h= 1.2 m; q'= 8 μC; g=9.8 m/s^2



From the equilibrium (see figure)

$$\boxed{Q > 0}$$

$$\vec{F}_e = Q\,\vec{E}(A)$$

Due to the symmetry of this exercise

$$\vec{E}(A) = \vec{E}_1(A) + \vec{E}_2(A) = E_y\,\vec{j}$$

$$E_y = 2E_{1y}$$

$$E_{1y} = E_1 \cos\beta$$

$$d_1 = L \sin\alpha$$

$$d_2 = D - L\sin\alpha$$

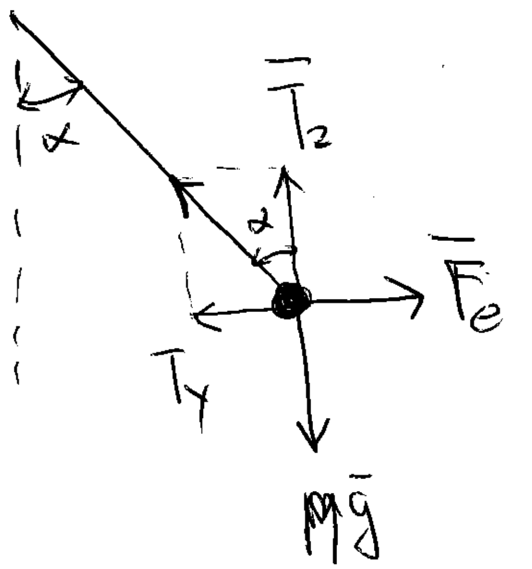
$$\cos\beta = \frac{d_2}{(h^2 + d_2^2)^{1/2}}$$

$$E_1 = \frac{|q|}{4\pi \varepsilon_0 (h^2 + d_2^2)}$$

$$E_{1y} = E_1 \cos\beta = \frac{|q|\, d_2}{4\pi \varepsilon_0 (h^2 + d_2^2)^{3/2}}$$

$$\vec{E}(A) = \frac{|q|\, d_2}{2\pi \varepsilon_0 (h^2 + d_2^2)^{3/2}}\, \vec{j}$$

$$\vec{F}_e(A) = \frac{|q|\, Q\, (D - L \sin\alpha)}{2\pi \varepsilon_0 \left[h^2 + (D - L \sin\alpha)^2\right]^{3/2}}\, \vec{j}$$



$$\sum \vec{F} = 0$$

$$\vec{T} + \vec{F}_e + M\vec{g} = 0$$

$$\boxed{\begin{aligned} T_y &= F_e \\ T_z &= Mg \end{aligned}}$$

$$T_y = T \sin\alpha = \frac{|q| \, Q \, (D - L\sin\alpha)}{2\pi\varepsilon_0 \left[h^2 + (D - L\sin\alpha)^2\right]^{3/2}}$$

$$\boxed{Q = \frac{2\pi\varepsilon_0 T \left[h^2 + (D - L\sin\alpha)^2\right]^{3/2} \sin\alpha}{|q| \, (D - L\sin\alpha)}}$$

$$\boxed{Q = 4.18 \cdot 10^{-5} \, C}$$

b) $W = q' \left[V(P) - V(\infty)\right] \qquad V(\infty) = 0$

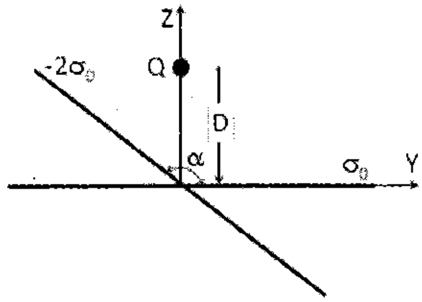
$$V(P) = 2 \frac{q}{4\pi\varepsilon_0 h} + \frac{Q}{4\pi\varepsilon_0 (D - L\sin\alpha)}$$

$$\boxed{W = \frac{q'}{2\pi\varepsilon_0} \left[\frac{q}{h} + \frac{Q}{2(D - L\sin\alpha)}\right]}$$

$$\boxed{W = 0.996 \, J}$$

P2. ***(2p)*** The electrostatic system shown in the figure consists of:

- An uniformly charged infinite plane, that coincides with the XY plane and has a charge density σ_0.
- A second, uniformly charged infinite plane with a charge density $-2\sigma_0$, that passes through the origin and forms an angle α with the XY plane (see figure).
- A point charge Q located at (0, 0, D).



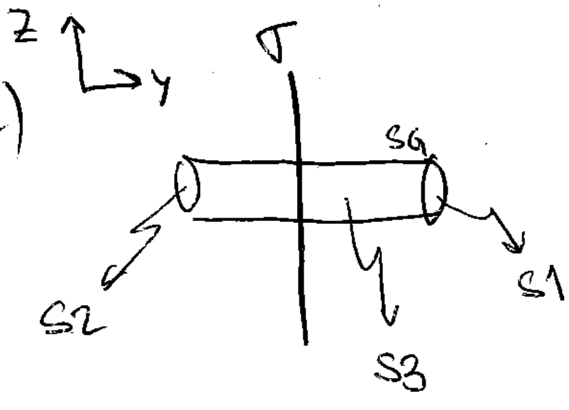
a) Deduce the general expression for the electric field created by an infinite, uniformly charged plane with a surface charge density σ

b) Calculate the electric field vector $E^{\rightarrow}$ at point P (0, D/2, D/2).

c) For which value of Q, the electric field at P is equal to $\vec{E}(P) = E\vec{k}$

DATA: σ_0 = 1.4×10^{-6} C/m^2; α= 140°; Q= 4.5×10^{-5} C; D= 6 m

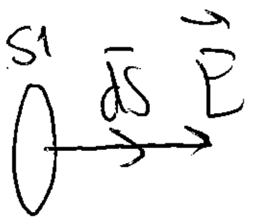
a)



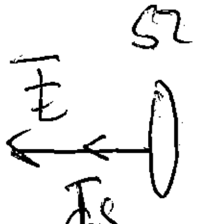
$$\oint\!\!\!\oint_{SG} \vec{E}\cdot d\vec{S} = \iint_{S1} \vec{E}\cdot d\vec{S} + \iint_{S2} \vec{E}\cdot d\vec{S} + \iint_{S3} \vec{E}\cdot d\vec{S}$$

Simmetry $\vec{E} = (\pm) E\vec{j}$

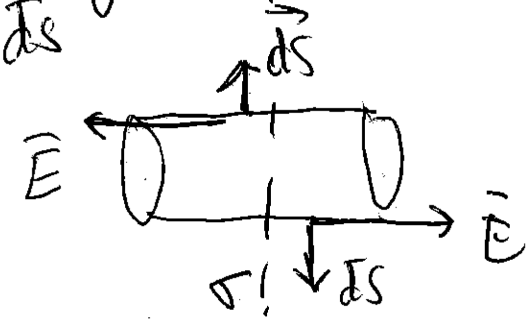
$(\sigma > 0)$



$$\iint_{S1} \vec{E}\cdot d\vec{S} = ES_1$$



$$\iint_{S2} \vec{E}\cdot d\vec{S} = ES_2$$



$$\iint_{S3} \vec{E}\cdot d\vec{S} = 0$$

$$\oiint \vec{E}\cdot d\vec{S} = ES_1 + ES_2 = 2ES$$ P2.2

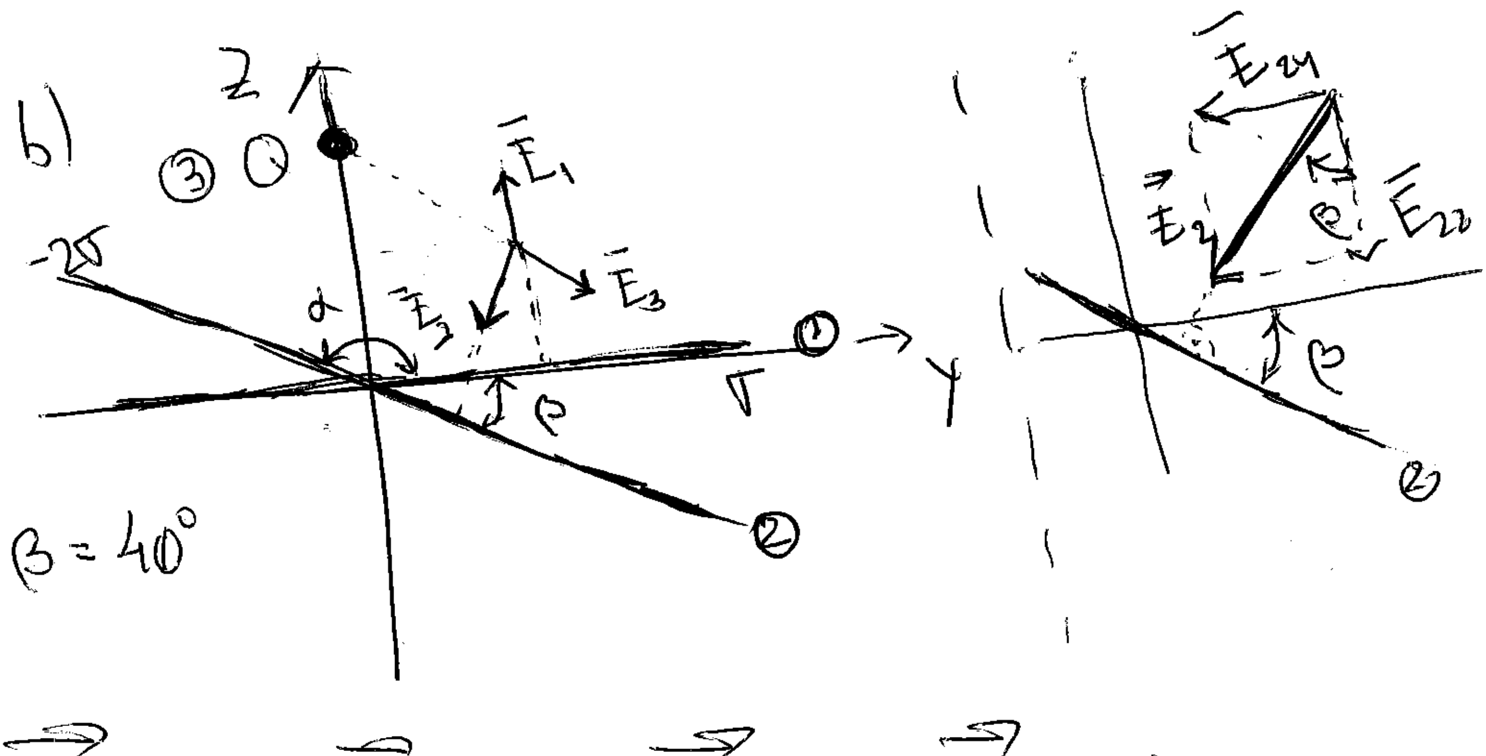
$(S_1 = S_2 = S)$

$$\oiint \vec{E}\cdot d\vec{S} = \frac{q_{in}}{\varepsilon_0} = \frac{1}{\varepsilon_0}(\sigma S)$$

$$2ES = \frac{\sigma S}{\varepsilon_0} \Longrightarrow E = \frac{\sigma}{2\varepsilon_0}$$

$$\vec{E} = (\pm)\frac{\sigma}{2\varepsilon_0}\vec{j}$$

b)



$\beta = 40°$

$$\vec{E}(P) = \vec{E}_1(P) + \vec{E}_2(P) + \vec{E}_3(P)$$

P2.3

$$E_1(P) = \frac{\sigma}{2\varepsilon_0} \qquad \vec{E}_1(P) = \frac{\sigma}{2\varepsilon_0}\vec{k}$$

$$E_2(P) = \frac{|-2\sigma|}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} \qquad E_{21} = \frac{\sigma}{\varepsilon_0}\sin\beta$$

$$E_{22} = \frac{\sigma}{\varepsilon_0}\cos\beta$$

$$\vec{E}_2(P) = -\frac{\sigma}{\varepsilon_0}\sin\beta\,\vec{j} - \frac{\sigma}{\varepsilon_0}\cos\beta\,\vec{k}$$

$$\vec{E}_3(P) = \frac{Q}{4\pi\varepsilon_0|\vec{r}-\vec{r}'|^3}(\vec{r}-\vec{r}')$$

$$\left.\begin{aligned}\vec{r} &= \frac{D}{2}\vec{j} + \frac{D}{2}\vec{k}\\ \vec{r}' &= D\vec{k}\end{aligned}\right\} \qquad \begin{aligned}\vec{r}-\vec{r}' &= \frac{D}{2}\vec{j} - \frac{D}{2}\vec{k}\\ |\vec{r}-\vec{r}'| &= \left(\frac{D^2}{4} + \frac{D^2}{4}\right)^{1/2} =\end{aligned}$$

$$= \frac{\sqrt{2}}{2}D$$

$$\vec{E}_3(P) = \frac{\sqrt{2}\,Q}{4\pi\varepsilon_0 D^2}(\vec{j}-\vec{k})$$

P2.4

$$\vec{E}(P) = \left[\frac{\sqrt{2}Q}{4\pi\varepsilon_0 D^2} - \frac{\sigma}{\varepsilon_0}\sin\beta\right]\vec{j} +$$

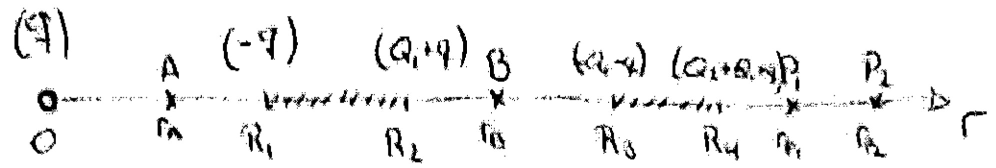
$$+ \left[\frac{\sigma}{2\varepsilon_0} - \frac{\sigma\cos\beta}{\varepsilon_0} - \frac{\sqrt{2}Q}{4\pi\varepsilon_0 D^2}\right]\vec{k}$$

$$\boxed{\vec{E}(P) = -8.58\cdot 10^4\,\vec{j} - 5.8\cdot 10^4\,\vec{k}\ (N/C)}$$

c) $E_y(P) = \phi \Rightarrow \dfrac{\sqrt{2}Q}{4\pi\varepsilon_0 D^2} = \dfrac{\sigma}{\varepsilon_0}\sin\beta$

$$\boxed{\begin{array}{l} Q = 2\sqrt{2}\pi D^2 \sigma \sin\beta \\ Q = 2.88\cdot 10^{-4}\ C \end{array}}$$

3)



a) $E(A) = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q}{r_A^2}$ $\qquad \dfrac{E(A)}{E(B)} = \dfrac{q\, r_B^2}{r_A^2\,(q+Q_1)}$

$E(B) = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q+Q_1}{r_B^2}$ $\qquad Q_1 = q\left(\dfrac{r_B^2}{r_A^2} \dfrac{E(B)}{E(A)} - 1\right) = -5'4 \cdot 10^{-6}\, C$

b) $\sigma_{R_1} = \dfrac{-q}{4\pi R_1^2} = -3'98 \cdot 10^{-4}\, C/m^2$

$\sigma_{R_2} = \dfrac{q+Q_1}{4\pi R_2^2} = 5'75 \cdot 10^{-5}\, C/m^2$

$\sigma_{R_3} = \dfrac{-q-Q_1}{4\pi R_3^2} = -3'23 \cdot 10^{-5}\, C/m^2$

$\sigma_{R_4} = \dfrac{Q_2+Q_1+q}{4\pi R_4^2} = 3'09 \cdot 10^{5}\, C/m^2$

c) $\vec{E}(r) = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q+Q_1+Q_2}{r^2}\, \vec{u}_r \quad \forall\, r > R_4$

$$V(P_2) - V(P_1) = -\int_{P_1}^{P_2} \vec{E} \cdot d\vec{r} = -\int_{r_{P_1}}^{r_{P_2}} \frac{Q_2+Q_1+q}{4\pi\varepsilon_0} \left(\frac{1}{r^2}\right) dr =$$

$$= \frac{Q_2+Q_1+q}{4\pi\varepsilon_0} \left(\frac{1}{r_{P_2}} - \frac{1}{r_{P_1}}\right) = -8'39 \cdot 10^{4}\, V$$

P4.

Newton's second law (region 2) implies that

$$|q|vB_o = m\frac{v^2}{R} \Rightarrow |q|B_o = m\frac{v}{R} \Rightarrow |q|B_o = m\omega \Rightarrow |q|B_o = m\frac{2\pi}{T}$$

Solving for the magnetic field, we have

$$B_o = \frac{2\pi m}{|q|T}$$

Since $m = 4m_p$ and $|q| = 2e$, the former equation is given by

$$B_o = \frac{4\pi m_p}{e\,T}$$

On the other hand, we are given the total time t of the movement

$$t = t_1 + t_2 + t_3$$

Since $t_2 = T/4$, being T the period of the particle due to the magnetic field (region 2), we can write

$$t = t_1 + \frac{T}{4} + t_3 \Rightarrow T = 4(t - t_1 - t_3)$$

The acceleration of the particle into region 1 is

$$a = \frac{|q|E_o}{m}$$

On the other hand, we can obtain the time taken for the particle to travel a distance D_1

$$D_1 = \frac{1}{2}a\,t_1^2 \Rightarrow t_1 = \sqrt{\frac{2D_1}{a}} \Rightarrow t_1 = \sqrt{\frac{2mD_1}{|q|E}} = t_1 = \sqrt{\frac{4m_pD_1}{e\,E_o}}$$

Inserting numerical values

$$t_1 = 8.85 \cdot 10^{-5} s$$

The speed of the particle into region 2 (the same one into region 3) is given by

$$v = at_1 = \sqrt{\frac{2|q| D_1 E_o}{m}} = \sqrt{\frac{e D_1 E_o}{m_p}}$$

, Taking numerical values, we can obtain

$$v = 1.017 \cdot 10^7 \ m/s$$

The time for the particle to travel from P to C is given by

$$t_3 = \frac{D_2}{v} = 3.44 \cdot 10^{-5} \ m/s$$

Substituting into our period formula

$$T = 4(t - t_1 - t_3) = 8.85 \cdot 10^{-5} s$$

Finally, inserting numerical values at the magnetic field expression

$$B_o = \frac{4\pi m_p}{e\,T} = 1.48 \cdot 10^{-3} \text{ T}$$

b)

Firstly, we obtain the radius of the circular motion (region 2) by using Newton's second law

$$|q| v B_o = m\frac{v^2}{R} \Rightarrow |q| B_o = m\frac{v}{R} \Rightarrow R = \frac{mv}{|q| B_o} \Rightarrow R = \frac{2 m_p v}{e B_o}$$

Taking numerical values, the radius is

$$R = 143.5 \text{ m}$$

Finally, the coordinates of C can be written as

$$C(0, R, 0) = C(0, 143.5, 0) \text{ m}$$