AUTOMATA THEORY AND FORMAL LANGUAGES 2022-23

UNIT 2: AUTOMATA THEORY

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Theory of Formal Languages. Bibliography

(AAM)

Enrique Alfonseca Cubero, Manuel Alfonseca Cubero, Roberto Moriyón Salomón.

Teoría de autómatas y lenguajes formales. McGraw-Hill (2007).

(IMB)

Pedro Isasi, Paloma Martínez y Daniel Borrajo.

Lenguajes, Gramáticas y Autómatas. Un enfoque práctico. Addison-Wesley, (1997)

(HMU)

John E. Hopcroft, Rajeev Motwani, Jeffrey D.Ullman.

Introduction to Automata Theory, Languages and Computation (3rd edition). Ed. Pearson Addison Wesley

(AAM)

Manuel Alfonseca, Justo Sancho, Miguel Martínez Orga.

Teoría de lenguajes, gramáticas y autómatas. Publicaciones R.A.E.C. 1997.

Students Work

- Read webpage with the information of the subject
- Read Chapter 1 (AAM)
- Read Chapter 2 (ASM)
- Read Chapter 1(HMU)
- Study and review the different concepts

OUTLINE

- **Introduction to Automata Theory**

- Formal Languages: Introduction and Definitions
  - Definitions
    - Symbols (of a Language)
    - Alphabet (of a Language)
    - Word (or string); Length of a Word; Empty Word
    - Universe of an Alphabet

  - Operations with Words. Operations with Languages

Introduction and definitions

- In this subject we are interested in knowing whether a problem is computable or not.

  In addition…
  time required,
  memory required, and
  computation model to be used.

Introduction and definitions

- Which is the meaning of **computation**?

- Automata Theory is focused on the computation itself, not in the detailed definition of input and devices.

Mathematical Model of an Automaton

Oxford English Dictionary

Automaton.

(Latin, self-operating machine, from Greek, from neuter of automatos, self-acting; see automatic.).

1. Instrument or device which includes an internal mechanism to facilitate an automatic movement.

2. Machine made in imitation of a human being (appearance and movements).

3. A person who acts mechanically or leads a routine monotonous life.

Mathematical Model of an Automaton

Automaton:

Mathematical model defined for computation.

Automata:

Abstraction of any type of computer and/or programming language.

Set of basic elements (Inputs, States, Transitions, Outputs and auxiliary elements)

Types of Automata

Finite Automata (and sequential machines)

Push-Down Automata

Linear-Bounded Automata

Turing Machine

Cellular Automata

Artificial Neural Networks

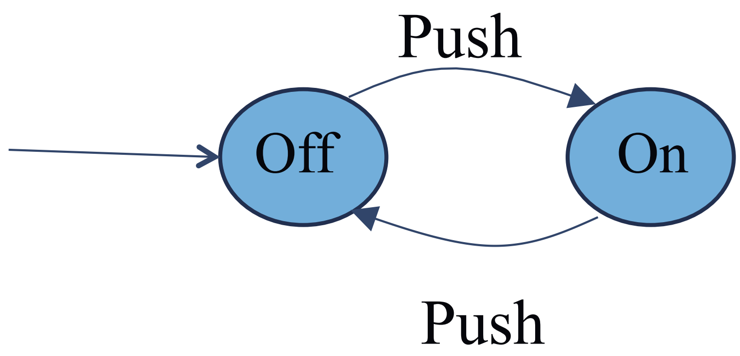
…

capacity of calculation

Automata and Algorithms

Every automaton can be transformed into an algorithm and viceversa.

Finite Automaton:



Discrete, Continuous and Hybrid Automata

Criteria: Number of inputs.

Usually DISCRETE:

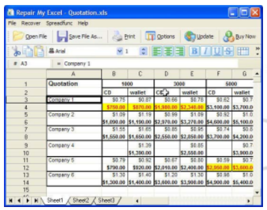
- Finite Automata (and sequential machines)
- Push-Down Automata
- Turing Machines

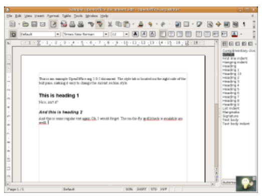
DISCRETE, CONTINUOUS AND/OR HYBRIDS:

- Cellular Automata
- Artificial Neural Networks

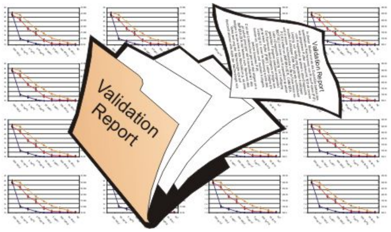
Applications of Automata

Grammars	Languages	Machines	Problems





$$A = \{a^m b^m c^n \mid m, n \geq 0\}$$



Type 3 Chomsky

Type 2 Chomsky

Type 1 Chomsky

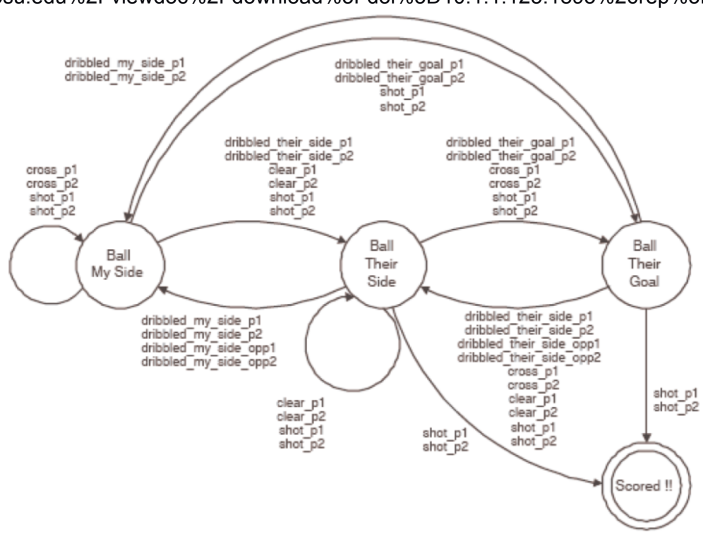
Type 0 Chomsky

Applications of Automata

ROBOTS BEHAVIOUR IN RoboSoccer

- Finite state automata applied to robot soccer (Peter van de Ven)

http://www.google.es/rdr?sa=t&source=web&cd=9&ved=0CEwQFjAI&url=http%3A%2F%2Fciteseerx.ist.psu.edu%2Fviewdoc%2Fdownload%3Fdoi%3D10.1.1.125.1893%26rep%3Drep1%26type%3Dpdf&rct=j&q=robotics%20finite%20automata&ei=BxOvTJafA(jt



GAME OF THE LIFE

- Example of a cellular automaton, designed by the British mathematician John Horton Conway in 1970.
- Transitions depend on the number of alive neighboring cells:
  - A dead cell that has exactly 3 alive neighboring cells will be alive in the following turn.
  - An alive cell with 2 or 3 alive neighboring cells follows alive, in another case it dies or remains dead (by “isolation” or “overpopulation”).
- http://www.youtube.com/watch?v=XcuBvj0pw-E
- http://www.youtube.com/watch?v=FdMzngWchDk
- http://www.youtube.com/watch?v=k2IZ1qsx4CM&NR=1

OUTLINE

- Introduction to Automata Theory

- **Formal Languages: Introduction and Definitions**
  - **Definitions**
    - **Symbols (of a Language)**
    - **Alphabet (of a Language)**
    - **Word (or string); Length of a Word; Empty Word**
    - **Universe of an Alphabet**

  - Operations with Words. Operations with Languages

Introduction and Definitions

- **Symbol:** an abstract entity with indivisible nature.
  - No formal definition (“point” in geometry).
  - Letters, digits, characters, etc.
  - It is possible to find symbols that consist of several characters, e.g.: IF, THEN, ELSE, ...
- **Alphabet (Σ):** Finite set not empty of symbols.
  - Notation: Let “*a*” be a **symbol** and let Σ be an **alphabet**:

    a belongs to this alphabet $\Rightarrow a \in \Sigma$
  - Examples:

    $\Sigma_1 = \{A, B, C, ..., Z\}$

    $\Sigma_2 = \{0, 1\}$

    $\Sigma_3 = \{IF, THEN, ELSE, BEGIN, END\}$

Introduction and Definitions

- **Word, string:** Finite sequence of symbols drawn from an alphabet (sentence and string are also use as synonyms).
  - Examples:

    Words over Σ_1 → HOME, BALL, etc.

    Words over Σ_2 → 00011101

    Words over Σ_3 → IFTHENELSEEND

Note: Words are represented by <u>lowercase</u> by the end of the alphabet (x, y, z), e.g., x= HOME, y= IFTHENELSEEND

Introduction and Definitions

- **Length of a word:** Number of symbols composing the word.
  - The length of a word *x* is represented by $|x|$

  Examples:

  $|x| = |\text{HOME}| = 4$

  $|y| = |\text{IFTHENELSEEND}| = 13$

  FALSE, due to the definition of the alphabet, the length is 4

- **Empty word λ:** Word of length 0.
  - It is represented by λ, $|\lambda| = 0$
  - In every alphabet, it is possible to construct λ
  - Utility: it is the neutral element in many operations with words and languages.

Introduction and Definitions

- **Universe of an alphabet, W(Σ):** All the words that can be created using the symbols of the alphabet Σ

  - It is also called Universal Language of Σ.
  - It is represented by W(Σ).
  - It is an infinite set.
  - E.g., given the alphabet $\mathbf{\Sigma_4}$ = {A},

    $W(\Sigma_4) = \{\lambda, A, AA, AAA, ...\}$ with a $\propto$ number of words

COROLLARY:

$\forall\ \Sigma,\ \ \lambda \in W(\Sigma) \Rightarrow$ The empty word is included in every Universal Language of every alphabet.

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  - **Operations with Words. Operations with Languages**

Formal Languages. Operations with Words

- **Operations with words: on a given speech universe words.**
  1. Words concatenation.
  2. Word power.
  3. Word reflection.

Formal Languages. Operations with Words

- **Operations with words:**
  1. **Word Concatenation:** given two words x, y that fulfill $x \in W(\Sigma)$, $y \in W(\Sigma)$, and $|x| = i = |x_1x_2...x_i|$ and $|y| = j = |y_1y_2...y_j|$,

     The concatenation of x with y is:

     $x \cdot y = xy = x_1x_2...x_i\, y_1y_2...y_i = \mathbf{z}$, where $z \in W(\Sigma)$

     (symbols of x followed by symbols of y)

Properties:
- Closed operation.
- Asociative.
- With neutral element.
- Not conmutative.

Definitions:
- Head.
- Cue.
- Lenght of the word.

Mathematical concepts (Algebra)

- Definition of *closed operation (closure)*
- Definition of *associative property of an operation*
- Definition of *neutral element* of an operation
- Definition of *non-conmutative operation*

Formal Languages. Operations with Words

- **Operations with words:**

  2. Powers of an Alphabet: reduction of the concatenation to cases relating to the same word.

  - *i-th power* of a word is the result of the concatenation of this word with itself *i* times
  - Associate property $\Rightarrow$ the order is not necessary
  - $x^i = x \,.\, x \,.\, x \,. \ldots .x$ $\quad i$ times
  - $|x^i| = i \,.\, |x|$
  - It fulfills:

    $x^1 = x$

    $x^{1+i} = x \,.\, x^i = x^i \,.\, x \quad (i>0)$

    $x^{j+i} = x^j \,.\, x^i = x^i \,.\, x^j \quad (i, j>0)$
  - Due to its definition, $x^0 = \lambda$
    - (i, j>=0)

Formal Languages. Operations with Words

- **Operations with words:**

  3. Word reflection: Be $x = A_1A_2A_3...A_n$ a word, the reflected word of x is

$$x^{-1} = A_n... A_3A_2A_1$$

  - Word with the same symbols in inverse order.
  - $|x^{-1}| = |x|$

Formal Languages. Language

- **Language, L:** A language over the alphabet Σ is:
  - Every subset of the universal language of Σ, $L \subset W(\Sigma)$
  - Every subset of words over a specific Σ (generated from the alphabet of Σ)

Formal Languages. Language

- **Language, L:**
  - Special languages:
    - ϕ = Empty language, $\phi \subset W(\Sigma)$
    - **$\{\lambda\}$** = Language of the empty word.

      they are differenced due to the number of words (cardinality):

      $$C(\phi) = 0 \text{ and } C(\{\lambda\})=1$$

      ϕ y $\{\lambda\}$ are languages generated over every alphabet.
    - An alphabet is one of the languages generated by itself:

      $$\boldsymbol{\Sigma \subset W(\Sigma)}, \text{ e.g. Chinese}$$
    - There are infinite languages associated to an alphabet.

Formal Languages. Operations with Languages

- **Operations with languages:** over a given alphabet
  1. Union of languages.
  2. Languages concatenation.
  3. Power of a language.
  4. Positive closure of a language.
  5. Iteration or closure of a language.
  6. Reflection of languages

Formal Languages. Operations with Languages

- **Operations with languages:** over a given alphabet
  1. **Union of languages**
     - Let L_1 y L_2 be two languages defined using the same alphabet, $L_1, L_2 \subset W(\Sigma)$, the union of these two languages, L_1, L_2 is represented by $L_1 \cup L_2$ and defined by

       $$\mathbf{L_1 \cup L_2 = \{x \mid x \in L_1 \text{ OR } x \in L_2\}}$$

     - Set of words from each one of the languages (equivalent to the plus operation).

Formal Languages. Operations with Languages

- **Operations with languages:**

1. **Union of languages:**

   Properties:

   - Closed operation
   - Associative property $(L_1 + L_2) + L_3 = L_1 + (L_2 + L_3)$
   - With neutral element $\phi + L = L$
   - Conmutative $L_1 + L_2 = L_2 + L_1$
   - Idempotent $L + L = L$

Formal Languages. Operations with Languages

- **Operations with languages:** over a given alphabet

  2. Concatenation of languages

  - Let L_1 and L_2 be two languages defined given the same alphabet, $L_1, L_2 \subset W(\Sigma)$, the **concatenation or product** of two languages, L_1 and L_2 is represented by $L_1 . L_2$ and defined by the language:

    $$\mathbf{L_1.L_2 = \{xy \mid x \in L_1 \text{ AND } y \in L_2\}}$$

  - Set of words that consists of the concatenation of the words of L_1 with the words of L_2
  - Valid definition for languages with almost one element.
  - In the case of the empty language: $\phi \,.\, L = L .\, \phi = \phi$

Formal Languages. Operations with Languages

- **Operations with languages:**

  2. Concatenation of languages

  Properties:

  - Closed operation
  - Associative property
  - With a neutral element
  - Distributive property with regard the union

Formal Languages. Operations with Languages

- **Operations with languages:**

  3. Powers of a language

  - Particular case of the concatenation for only one language.
  - *i-th* power of a language = result of concatenating this language with itself *i* times.
  - Associative property $\Rightarrow$ it is not needed to specify the order
  - $L^i = L \,.\, L \,.\, L \,.\, \ldots\, L$ *i* times
  - Given that $L^1 = L$,

    then:

    $$L^{1+i} = L.L^i = L^i.L \ (i>0)$$

    $$L^{j+i} = L^i.L^j \ (i, j > 0)$$

- Defining $L^0 = \{\lambda\}$

$(i \geq 0)$

$(i, j \geq 0)$

Formal Languages. Operations with Languages

- **Operations with languages:** over a given alphabet

  4. Positive Closure of a language

  - It is denoted by L^+ and it is the language that consists of joining a language L with all its powers except L^0

$$L^+ = \bigcup_{i=1}^{\infty} L^i$$

  - λ is not included in the positive closure if $\lambda \notin L$
  - As Σ is a language over Σ, the positive closure of Σ is:

$$\Sigma^+ = \bigcup_{i=1}^{\infty} \Sigma^i = W(\Sigma) - \{\lambda\}$$

Formal Languages. Operations with Languages

- **Operations with languages:**

  5. Iteration, closure of a language

  - It is defined by L^* and it is the language that consists of joining the language L and all its possible powers.
  - *** is the Kleene unitary operator**

$$L^* = \bigcup_{i=0}^{\infty} L^i$$

  λ Is included in every closure,

  - Properties:
    - $\mathbf{L^* = L^+ \cup \{\lambda\}}$
    - $\mathbf{L^+ = L^* \cdot L = L \cdot L^*}$
  - Given that Σ is a language over Σ, we get the universal language: $\mathbf{\Sigma^* = W(\Sigma)}$ $\longrightarrow$ $\boxed{\textbf{The universal language is } \Sigma^*}$

Formal Languages. Operations with Languages

- **Operations with languages:**

  6. Reflection of languages

  - The reflected or inverse language of L is represented by L^{-1} and defined by the language:

    $$\mathbf{L^{-1} = \{\, x^{-1} \mid x \in L \}}$$

  - It is the language that consists of every reflected word of L.

Formal Languages. Exercises

1. Given the alphabet $\Sigma = \{0,1\}$, write the language that consists of the strings palindromes.
2. Define two languages L1 and L2 with cardinality 3 and then show the result of the following operations:
   - $L_1 \cup L_2$
   - $L_1 . L_2$
   - L_1^2
   - L_1^*
   - L_2^+
   - $(L_1.L_2)^{-1}$
3. Exercises 1, 2 and 3 (Alfonseca).
4. Exercises 2.1, 2.2 and 2.3 (Isasi, Martínez, Borrajo).