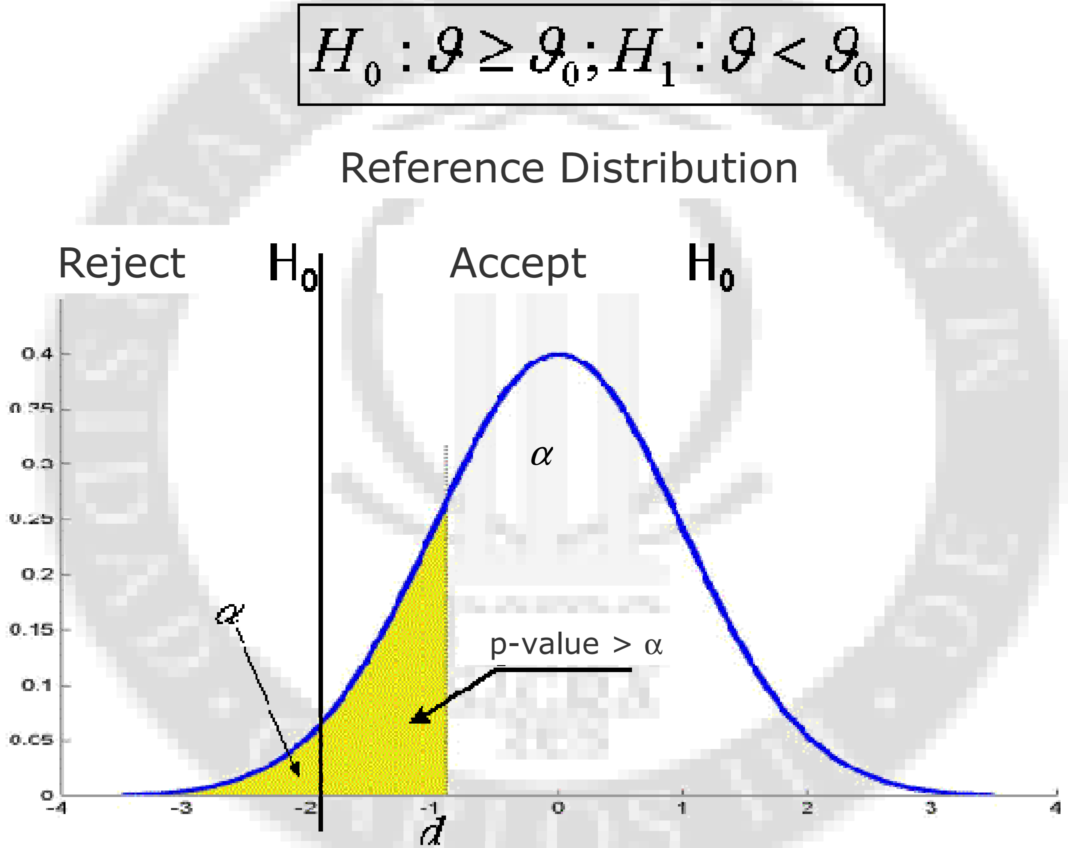
VII. Large Sample Inference



Chapter 7: Large-Sample Inference

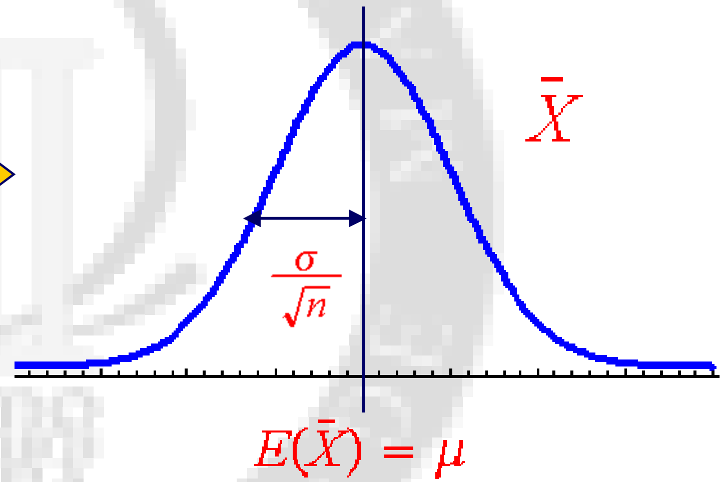
1. Confidence intervals for μ with large samples

Let X be the random variable under analysis with **general** distribution and with

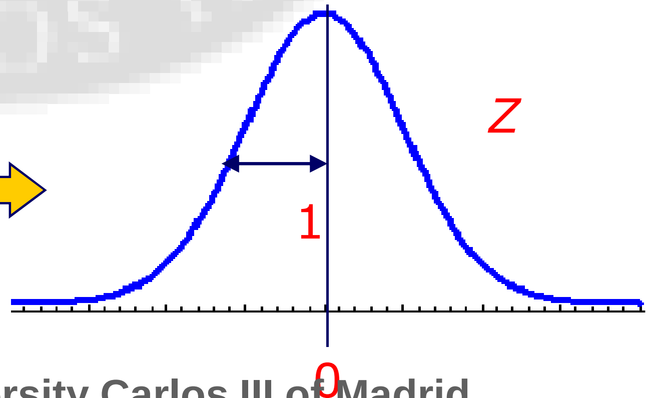
$$E(X) = \mu \qquad \mathrm{Var}(X) = \sigma^2$$

In the previous chapter we saw that when **n** is large (n>30)

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$



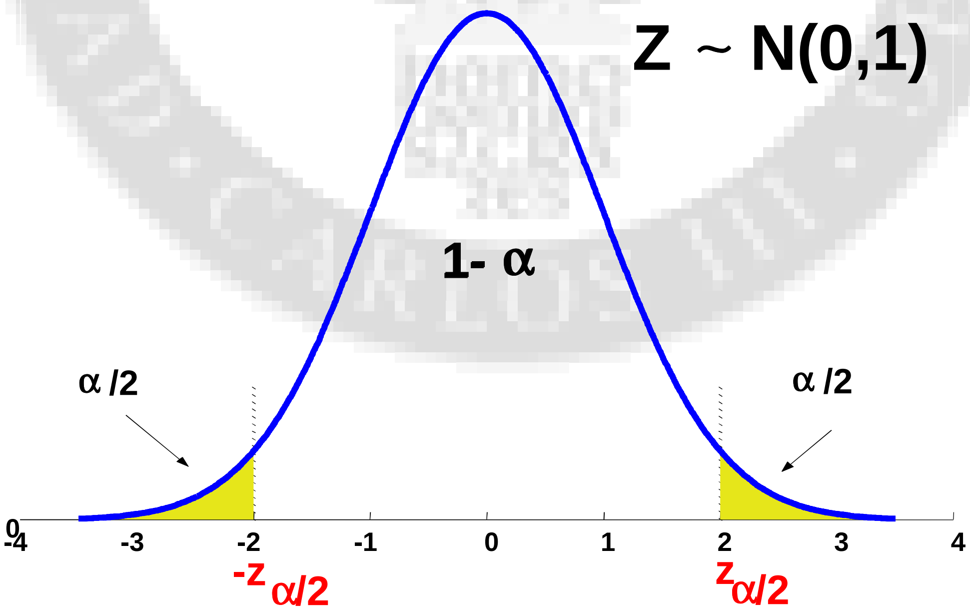
$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$$



$$Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1) \longrightarrow P(-z_{\alpha/2} < Z < z_{\alpha/2}) = (1-\alpha)$$

$$P\left(-z_{\alpha/2} < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < z_{\alpha/2}\right) = (1-\alpha)$$

$$\boxed{P\left(\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right) = (1-\alpha)}$$

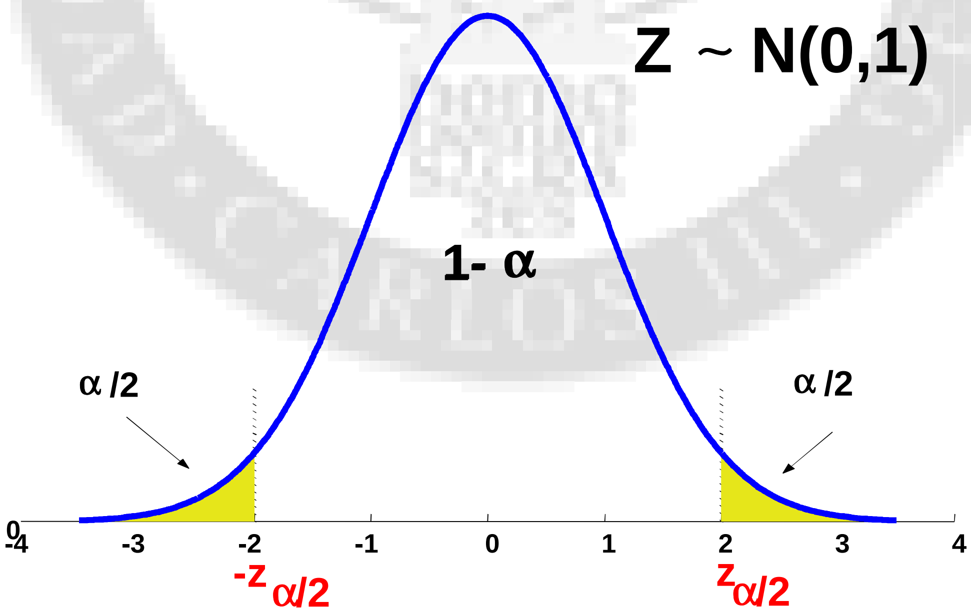


$$P\left(\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right) = (1-\alpha)$$

If we take infinite samples and with each of them we compute the interval

$$\bar{x} \pm z_{a/2}\frac{s}{\sqrt{n}}$$

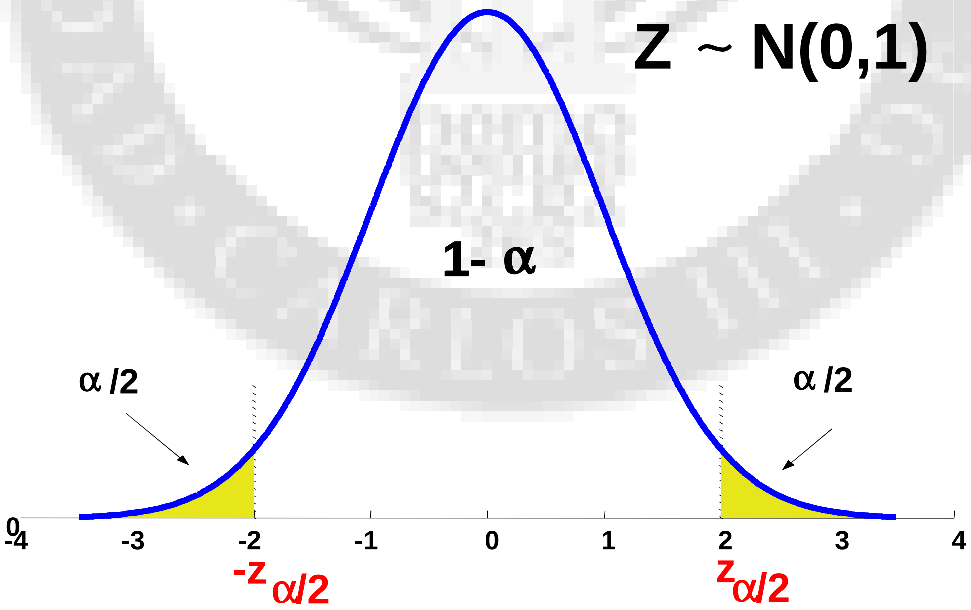
Then 100(1-α)% of them will contain the population mean μ



$$P\left(\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right) = (1-\alpha)$$

In practice:

- ✓We measure only one sample
- ✓We compute only one interval
- ✓The interval DO or DO NOT contain μ
- ✓The uncertainty about if it contains or not μ is called **confidence level**



Confidence interval for μ with confidence level $100\times(1-\alpha)\%$

$$IC(1-\alpha): \mu \in \left\{ \bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \right\}$$

Example

A random sample of size $n=144$ extracted from a population with variance $\sigma^2=100$ has a sample mean = 160. Compute:

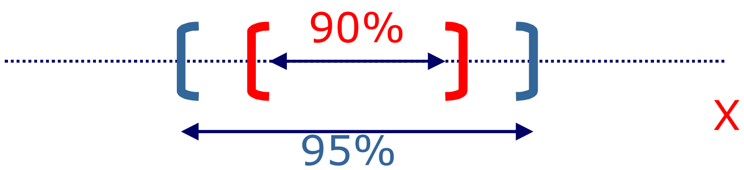
(a) The 95% confidence interval for μ.

(b) The 90% confidence interval for μ.

(a) $z_{\alpha/2} = z_{0.025} = 1.96 \longrightarrow IC(95\%) : \mu \in \left\{ 160 \pm 1.96 \frac{10}{\sqrt{144}} \right\} \longrightarrow \mu \in [158.36, 161.63]$

(b) $z_{\alpha/2} = z_{0.05} = 1.65 \longrightarrow IC(90\%) : \mu \in \left\{ 160 \pm 1.65 \frac{10}{\sqrt{144}} \right\} \longrightarrow \mu \in [158.625, 161.375]$

Greater the confidence = larger the interval



Questions

$$IC(1-\alpha): \mu \in \left\{ \bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \right\}$$

Answer with True, False, Not Definable.

- The confidence interval tells the interval of values that μ may assume from one sample to another
- It is impossible that μ does not belong to the interval
- The computed confidence interval is valid only if the sample mean $\bar{X}$ is normal
- The computed confidence interval is valid only if the random variable X is normal
- It would be better to construct an interval with confidence level of 100%; in such a way we could eliminate the uncertainty
- The confidence interval says the interval of values where we could find the population mean with a predefined confidence level
- In presence of few data the interval may be not valid

$$\mathrm{IC}(1-\alpha): \mu \in \left\{ \bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \right\}$$

This is a parameter as well, and as such it will be not known.

We substitute it by an estimator

$$\mathrm{IC}(1-\alpha): \mu \in \left\{ \bar{x} \pm z_{\alpha/2} \frac{\hat{s}}{\sqrt{n}} \right\}$$

What estimator do we use for σ^2?

What estimator do we use for σ²?

Method of Moments: sample variance

$$\hat{\sigma}^2 = S^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n}$$

We can prove that this estimator is BIASED

$$E(S^2) = \sigma^2 \frac{(n-1)}{n} \neq \sigma^2,$$

$$\text{Bias}(S^2) = \sigma^2 \frac{(n-1)}{n} - \sigma^2 = -\frac{\sigma^2}{n}$$

It subestimates the true variance

What estimator do we use for σ^2?

$$\hat{\sigma}^2 = S^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n}$$

It has BIAS

$$E(S^2) = \sigma^2 \frac{(n-1)}{n} \neq \sigma^2$$

$$E\Bigg(\underbrace{S^2 \frac{n}{n-1}}_{\text{We correct the bias}}\Bigg) = \sigma^2,$$

$$S^2 \frac{n}{n-1} = \left(\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n}\right)\frac{n}{n-1} = \overbrace{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}^{\hat{S}^2}$$

The "official" estimator for the variance will be the unbiased estimator

$$\hat{\sigma}^2 = \hat{S}^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$$

Both estimators are known as "sample variance"

In Spanish you it is know as:

Cuasivarianza, Pseudo varianza, Varianza corregida, Varianza corregida por grados de libertad

Confidence interval for μ with confidence level $100\times(1-\alpha)\%$

$$\mathrm{IC}(1-\alpha): \mu \in \left\{ \bar{x} \pm z_{\alpha/2} \frac{\hat{s}}{\sqrt{n}} \right\}$$

Example

We measure the lifetime of 200 electronic components. The sample mean is equal to 300 hours and the sample variance is 10.000 (hours^2).

Compute the 95% confidence interval for μ

$\bar{X} = 1300$

$\hat{S}^2 = 10.000$

$n = 200$

$a = 0.05$

$z_{0.025} = 1.96$

$$\mu \in \left\{ 1300 \pm 1.96 \frac{\sqrt{10000}}{\sqrt{200}} \right\} \longrightarrow \mu \in [1286{,}1314]$$

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2. Determining the sample size
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7. Relation between the hypothesis test and the confidence intervals

We have seen that...

Confidence interval for μ with confidence level $100\times(1-\alpha)\%$

$$IC(1-\alpha): \mu \in \left\{ \bar{x} \pm z_{\alpha/2} \frac{\hat{s}}{\sqrt{n}} \right\}$$

$$\mu \in \{\bar{x} \pm L\}$$

What size n is needed to get a given value of L?

We can estimate σ with a pilot sample

$$L = z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \Rightarrow n = \left(\frac{z_{\alpha/2}\sigma}{L} \right)^2$$

Example

Let X the amount of impurity contained in some material produced by a manufacturing process (milligrams of impurity per kilogram of product).

We take a sample made of 200 observations e compute a sample mean of 120 mg/Kg and a sample dtandar deviation of 20 mg/Kg.

$$\bar{X} = 120$$

$$\hat{S} = 20$$

$$n_0 = 200$$

Estimate a 95% confidence interval for the average amount of impurity

$$\mu \in \left[\bar{X} \pm z_{\alpha/2} \frac{\hat{\sigma}}{\sqrt{n}} \right] = \left[120 \pm 1.96 \times \frac{20}{\sqrt{200}} \right] = [120 \pm 2.77]$$

How many observations do we need to have the half interval L=1 mg?

$$n = \left(\frac{z_{\alpha/2} \hat{\sigma}}{L} \right)^2 = \left(\frac{1.96 \times 20}{1} \right)^2 \approx 1537$$

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Confidence Interval for a proportion p

If np(1-p)>5, the confidence interval is:

$$p \in \left\{ \hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right\}$$

Confidence Interval for the parameter λ of a Poisson

If n is large:

$$IC(1-\alpha) : \lambda \in \left\{ \hat{\lambda} \pm z_{\alpha/2} \sqrt{\frac{\hat{\lambda}}{n}} \right\}$$

Confidence intervals for normal populations valid for large and small sample sizes

$$IC(1-\alpha) : \mu \in \left\{ \bar{x} \pm t_{n-1;\alpha/2} \sqrt{\frac{\hat{s}^2}{n}} \right\}$$

$$IC(1-\alpha) : \sigma^2 \in \left(\frac{(n-1)\hat{s}^2}{\chi^2_{n-1;\alpha/2}}, \frac{(n-1)\hat{s}^2}{\chi^2_{n-1;1-\alpha/2}} \right)$$

Note: we will see confidence interval for the variance only for the case of normal populations
(Statgraphics uses only this kind of intervals valid only for normal populations)

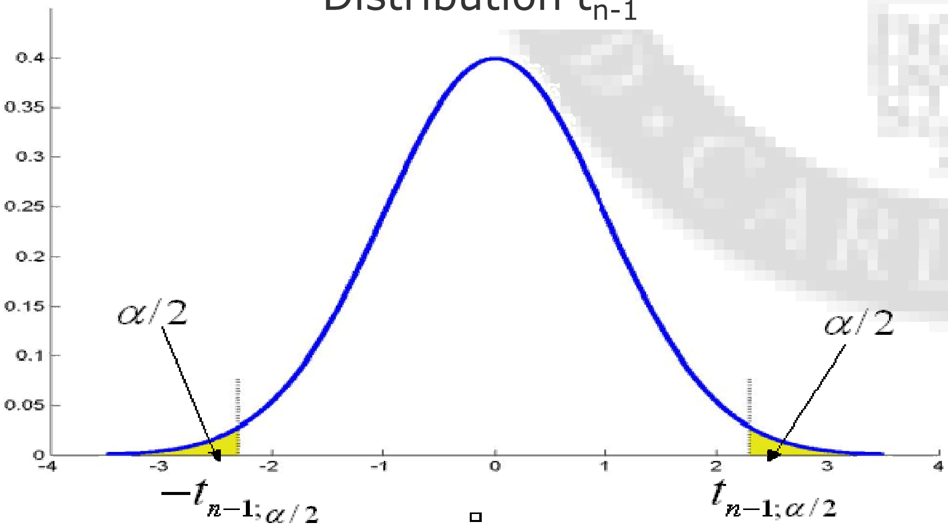
ONLY IF THE POPULATION IS NORMAL:

$$IC(1-\alpha):\mu \in \left\{ \bar{X} \pm \overbrace{t_{n-1;\alpha/2}}^{\text{Instead of } z_{\alpha/2}} \frac{\hat{S}}{\sqrt{n}} \right\}$$

- The Student's t-distribution is a continuous random variable. It is symmetric with zero mean and with a shape similar to the standard normal distribution.
- It has only one parameter, n, that is called degrees-of-freedom. The usual notation is t_n

$$t_{n-1} \rightarrow N(0,1)$$

Distribution t_{n-1}

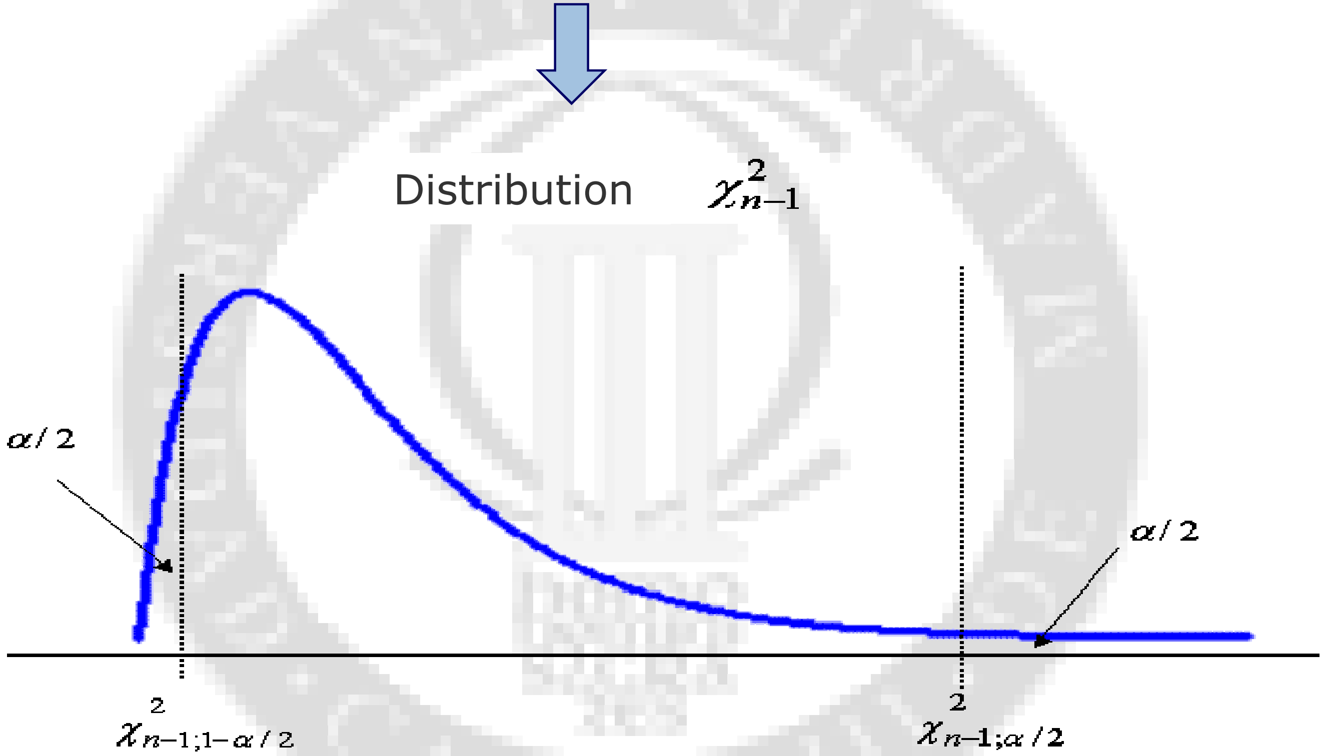


When n is large the interval constructed starting from a t-distribution are equal to the ones obtained with the Normal N(0,1)

Note: these are the intervals that Statgraphics uses. That means that for large-samples we use them for any kind of populations while for small-samples we use them only for normal populations.

ONLY IF THE POPULATION IS NORMAL:

$$IC(1-\alpha) : \sigma^2 \in \left(\frac{(n-1)\hat{s}^2}{\chi^2_{n-1;\alpha/2}}, \frac{(n-1)\hat{s}^2}{\chi^2_{n-1;1-\alpha/2}} \right)$$



Note: we only work with confidence interval for the variance for the case of normal populations
(Statgraphics only uses these kind of intervals, so we can use them only for the case of normal populations)

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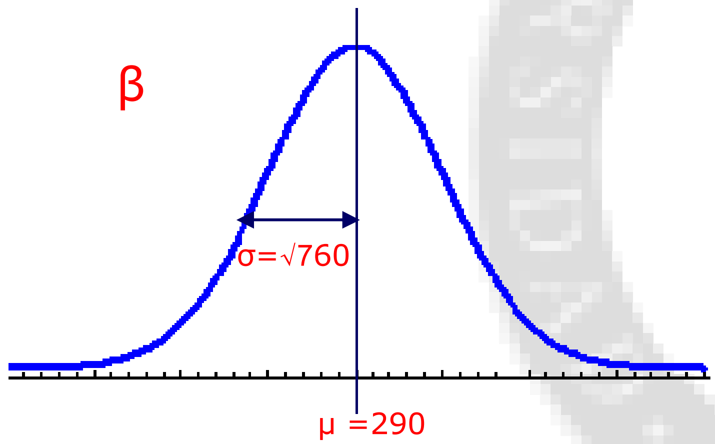
Let us see the idea of hypothesis Test by an example

Example

A transistor manufacturer of type BC547B knows that when the production stays in the normal values of quality the current gain (a parameter of the transistor denoted by the adimensional coefficient β) is normal distributed with mean 290 and variance 760.



$\mu = 290$

$\sigma^2 = 760$

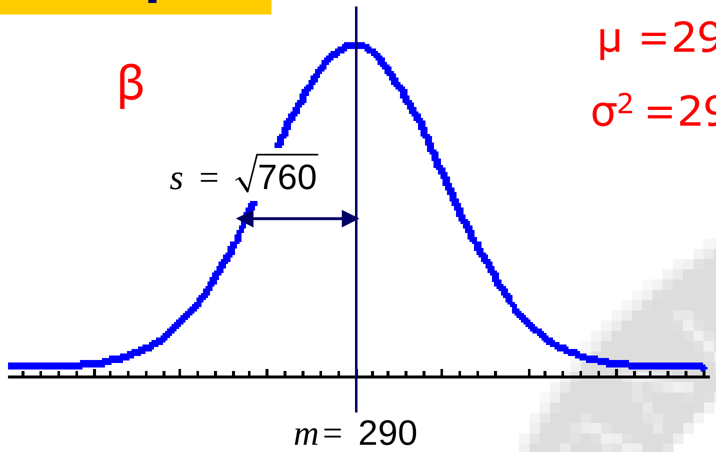
These values are estimations obtained by measuring very large historical samples datasets. We consider them as parameters of the population.

How can we know if the process is keeping constant the parameters?

Is the mean unchanged?

Is the variance unchanged?

Example



$\mu = 290$

$\sigma^2 = 290$

How can we know if the process is keeping constant the parameters?

- Is the mean unchanged?
- Is the variance increased?

These are hypothesis that we want to verify

How can we proceed?

- We take a sample of observations
- Looking at the data we verify if we support or not our hypothesis

(now the objective is to validate and not to estimate)

If $\bar{x} >> 290$ $\longrightarrow$ It seems probable that the mean **DID** change

If $\bar{x} \approx 290$ $\longrightarrow$ It seems probable that the mean **DID NOT** change

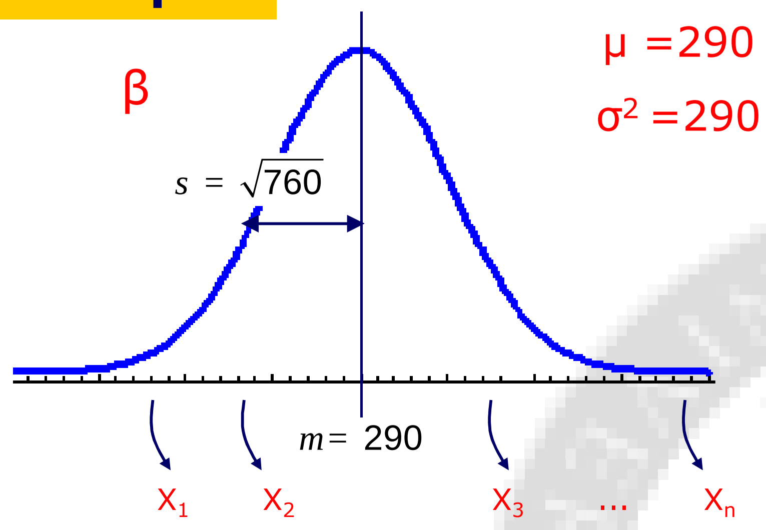
Looking at data we take the most probable decision

(we will never be 100% sure of our decision)

How can we use the statistical inference?

Example

Statistical Method



$\bar{X}, \hat{S}^2$

Objective: Validate an hypothesis by a dataset

↓

Hypothesis Test

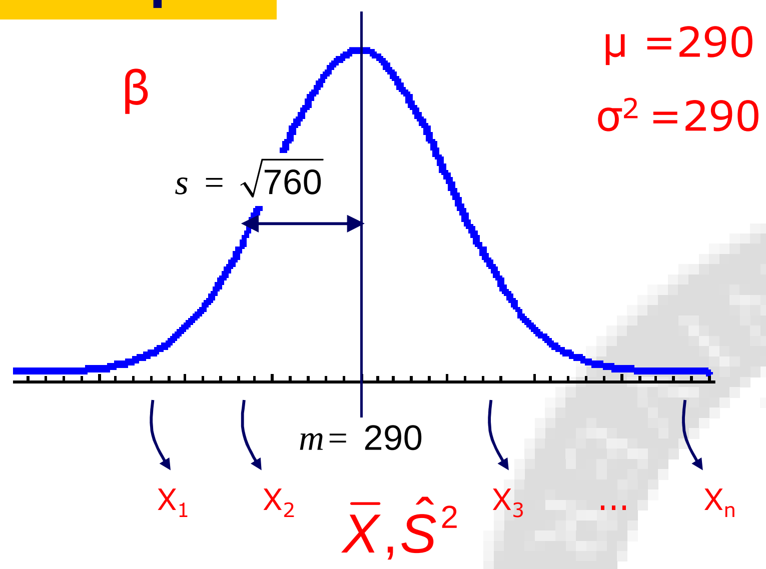
↓

The hypotheses are expressed by limitations on the values of the parameters

	Null Hypothesis H0		**Alternative Hypothesis** H1	
Is the mean unchanged? →	$\mu=290$	or	$\mu\neq290$	**Bilateral alternative**
Is the variance increased? →	$\sigma^2\leq290$	or	$\sigma^2>290$	**Unilateral alternative**

- The union of H0 and H1 contains all the possible values admitted by the parameter
- H0 always contains the sign =
- We accept H0 in any case except when there is high evidence to not doing so

Example



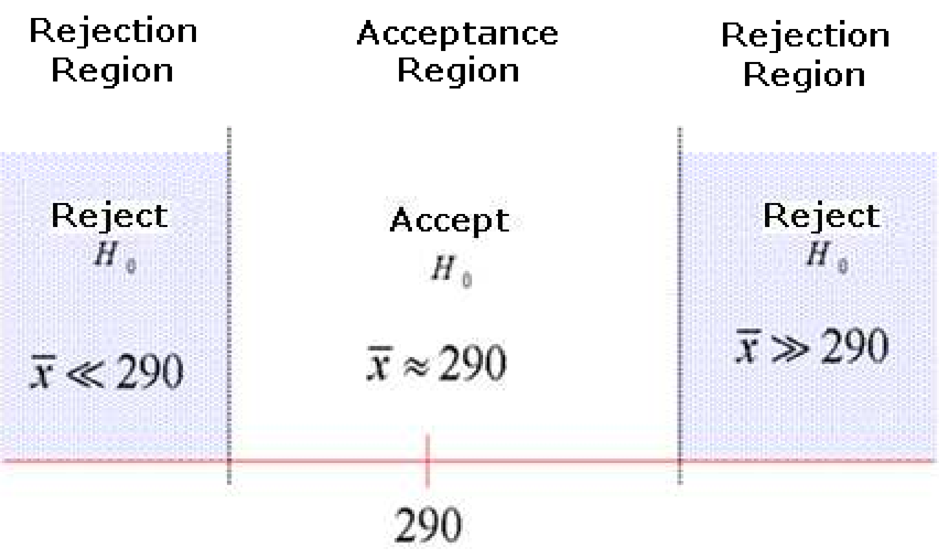
$\mu = 290$

$\sigma^2 = 290$

H_0	H_1
$\mu = 290$	$\mu \neq 290$
$\sigma^2 \leq 290$	$\sigma^2 > 290$

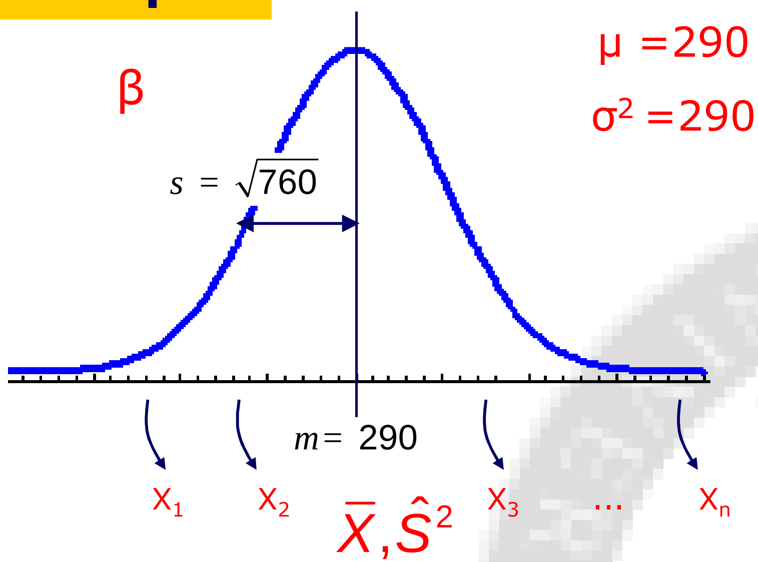
Result of the test:
Accept of Reject H_0

$$H_0 : \mu = 290; H_1 : \mu \neq 290$$



We reject H_0 **only in the case** there is high evidence against it. It means when data confirm with high probability the hypothesis H_1

Example



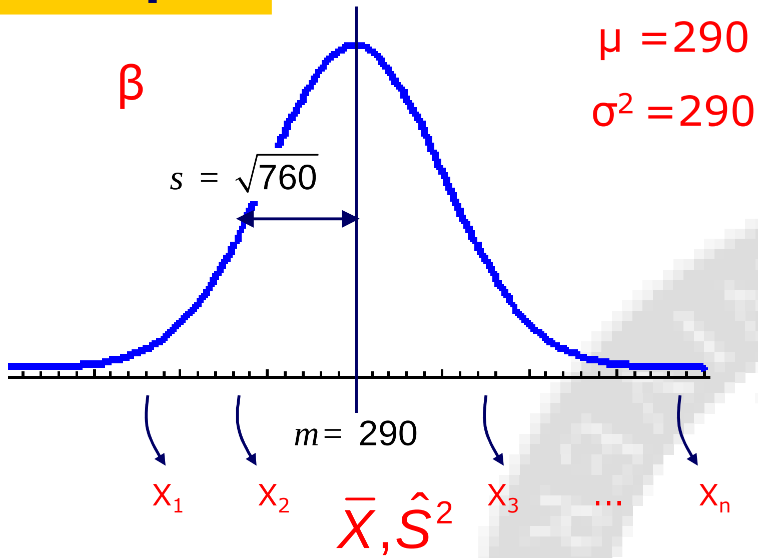
H_0	H_1
$\mu=290$	$\mu\neq290$
$\sigma^2\leq290$	$\sigma^2>290$

The result of a test is summarized by a number called **p-value**
(we will see later how to compute it starting from a data set)

The p-value is a number between 0 and 1
Such that higher is its value the most we are willing to accept H_0

Since we always accept H_0 but when there is high evidence against it,
we will not accept H_0 only when the p-value is very small.
Usually we reject H_0 when p-values<0.05

Example



$\mu = 290$

$\sigma^2 = 290$

H_0	H_1
$\mu = 290$	$\mu \neq 290$
$\sigma^2 \leq 290$	$\sigma^2 > 290$

With 100 data (Statgraphics):

```
Hypothesis Tests for BC547B

Sample mean = 282,29
Sample median = 282,0


t-test
------
Null hypothesis: mean = 290,0
Alternative: not equal

Computed t statistic = -2,78418
P-Value = 0,00642966

Reject the null hypothesis for alpha = 0,05.
```

The p-value is very small.
Therefore le sample mean is very far from the value 290.
This difference is not explicable by only the randomness of the sample .

"We reject Ho with p-value=0.006"

The data are "in favor" of H1

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5. Hypothesis test for the mean μ with large samples

To valid the hypothesis about the mean μ we do the following steps:

STEP1: We specify the null hypothesis. We choose among one of the following, where μ_0 is a fixed numerical value

$H_0 : \mu = \mu_0$ $H_1 : \mu \neq \mu_0$	$H_0 : \mu \leq \mu_0$ $H_1 : \mu > \mu_0$	$H_0 : \mu \geq \mu_0$ $H_1 : \mu < \mu_0$

Example Continuing with the transistors' example. We want to know if the population of the transistors is keeping the mean value $\mu_0 = 290$

H_0	H_1
$\mu = 290$	$\mu \neq 290$

STEP 2: We look for a measure of the difference between the data and H_0

If the difference is big we reject H_0

The measure we use is called **test statistic**

How do we find the **test statistic** that resumes the relevant information of a statistical test? → We use the **properties of the estimators** and we add the information about H_0

We know that for a large sample

$$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

$$\frac{\bar{X}-\mu}{\hat{S}/\sqrt{n}} \sim N(0,1)$$

Test statistic

$$Z_0 = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$$

$$T_0 = \frac{\bar{X}-\mu_0}{\hat{S}/\sqrt{n}}$$

Example

Continuing with the transistors' example. We want to know if the population of the transistors is keeping the mean value $\mu_0 = 290$

H_0	H_1
$\mu=290$	$\mu\neq 290$

With100 observations:

$$\bar{x} = 282.3; \hat{s} = 27.69;$$

$$t_0 = \frac{\bar{x}-\mu_0}{\hat{s}/\sqrt{n}} = \frac{282.3-290}{27.69/10} = -2.78$$

This number resumes the information to choose between H_0 and H_1

STEP 3:

To validate the test statistic we need a reference distribution that can tell us if the obtained value is large or small

The reference distribution is the same distribution of the test statistic obtained when $\mu=\mu_0$ → $N(0,1)$

STEP 4: We localize on the distribution in which region we reject H_0.

We reject H_0 if data are evidently in favor of the hypothesis H_1.

Case (a)

STEP 1:

$$H_0 : \mu = 290; H_1 : \mu \neq 290$$

STEP 2:

$$T_0 = \frac{\bar{X} - 290}{\hat{S} / \sqrt{n}}$$

STEP 3:

$T_0 \sim N(0,1)$

Reject H_0 if

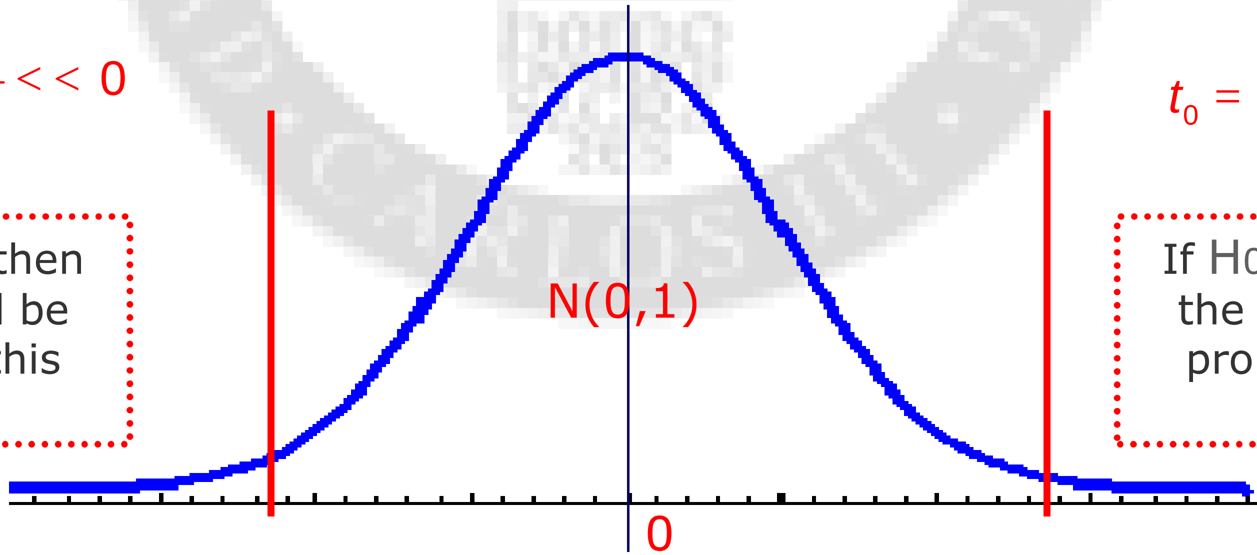
$\bar{x} << 290$

$\bar{x} >> 290$

$$t_0 = \frac{\bar{x} - 290}{\hat{s} / \sqrt{n}} << 0$$

$$t_0 = \frac{\bar{x} - 290}{\hat{s} / \sqrt{n}} >> 0$$

If H_0 is false then the value will be probably in this region



If H_0 is false then the value will be probably in this region

STEP 4: We localize on the distribution in which region we reject H0.
We reject H0 if data are evidently in favor of the hypothesis H1.

Case (b)

STEP 1:

$$H_0 : \mu \leq 290; H_1 : \mu > 290$$

STEP 2:

$$T_0 = \frac{\bar{X} - 290}{\hat{S} / \sqrt{n}}$$

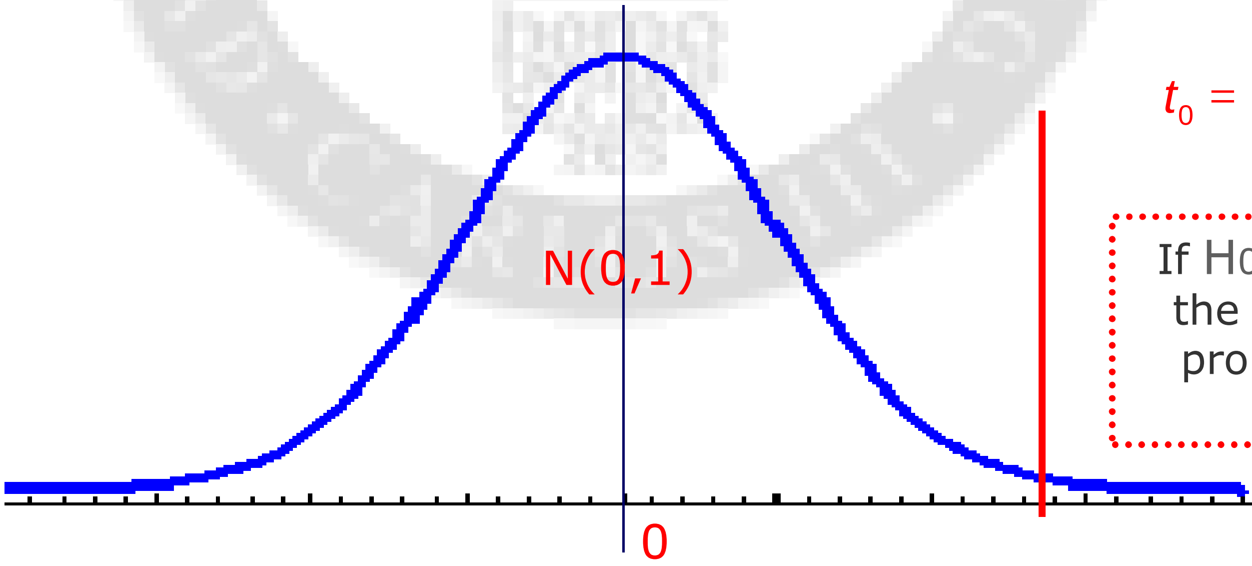
STEP 3:

$T_0 \sim N(0,1)$

Reject H0 if

$$\bar{x} >> 290$$

$$t_0 = \frac{\bar{x} - 290}{\hat{s} / \sqrt{n}} >> 0$$



If H0 is false then the value will be probably in this region

STEP 4: We localize on the distribution in which region we reject H0.

We reject H0 if data are evidently in favor of the hypothesis H1.

Case (c)

STEP 1:

$$H_0 : \mu \geq 290; H_1 : \mu < 290$$

STEP 2:

$$T_0 = \frac{\bar{X} - 290}{\hat{S} / \sqrt{n}}$$

STEP 3:

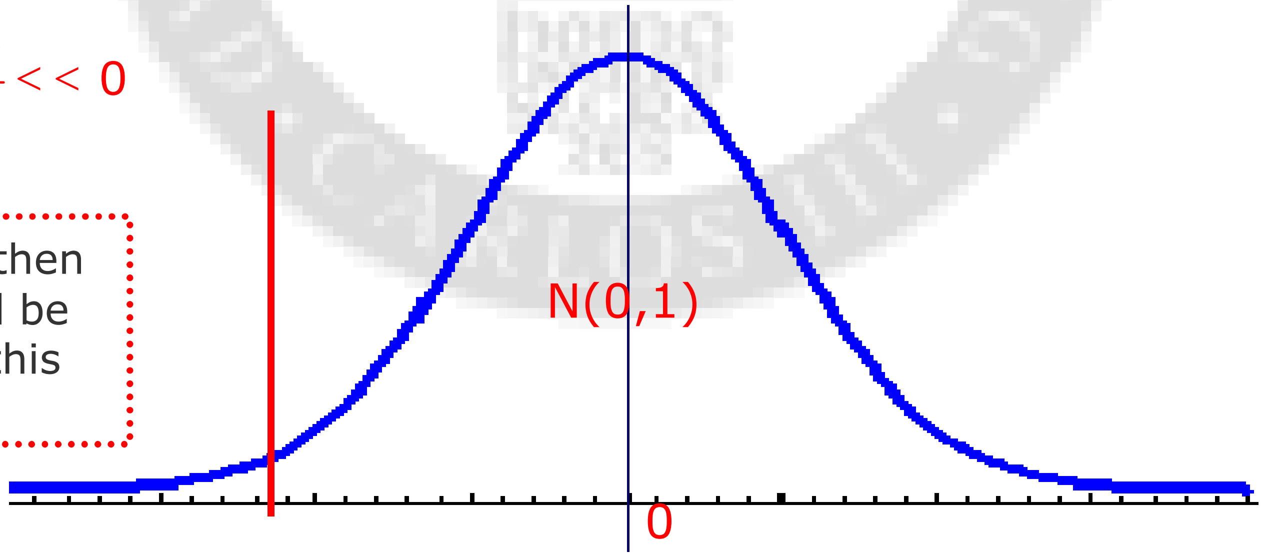
$T_0 \sim N(0,1)$

Reject H0 if

$$\bar{x} << 290$$

$$t_0 = \frac{\bar{x} - 290}{\hat{s} / \sqrt{n}} << 0$$

If H0 is false then the value will be probably in this region



STEP 1:

$$H_0 : \mu = \mu_0 ; H_1 : \mu \neq \mu_0$$

(a)

$$H_0 : \mu \leq \mu_0 ; H_1 : \mu > \mu_0$$

(b)

$$H_0 : \mu \geq \mu_0 ; H_1 : \mu < \mu_0$$

(c)

STEP 2:

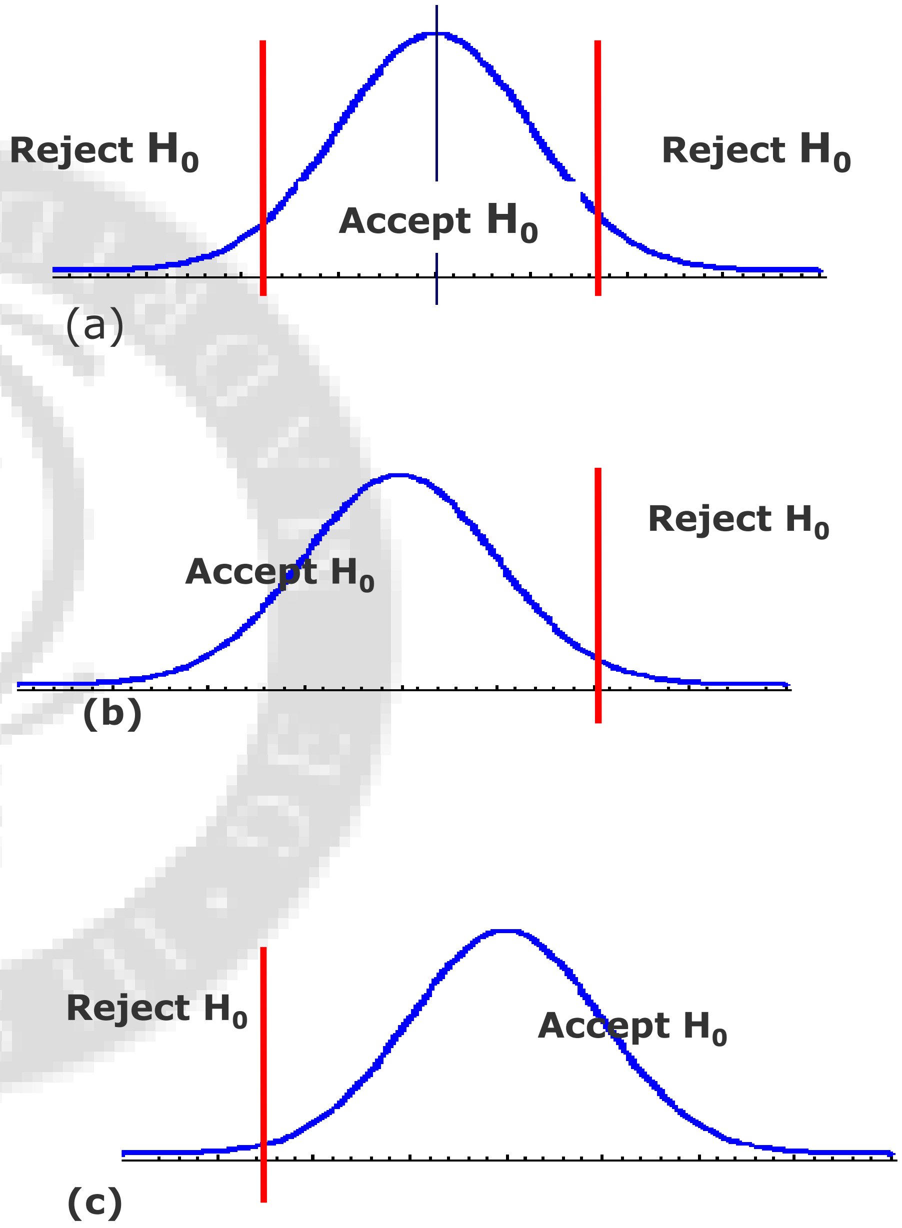
$$Z_0 = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

$$T_0 = \frac{\bar{X} - \mu_0}{\hat{S} / \sqrt{n}}$$

STEP 3:

N(0,1)

STEP 4:



General methodology to make a Hypothesis Test

STEP 1: Specify the Null and the Alternative Hypotheses.

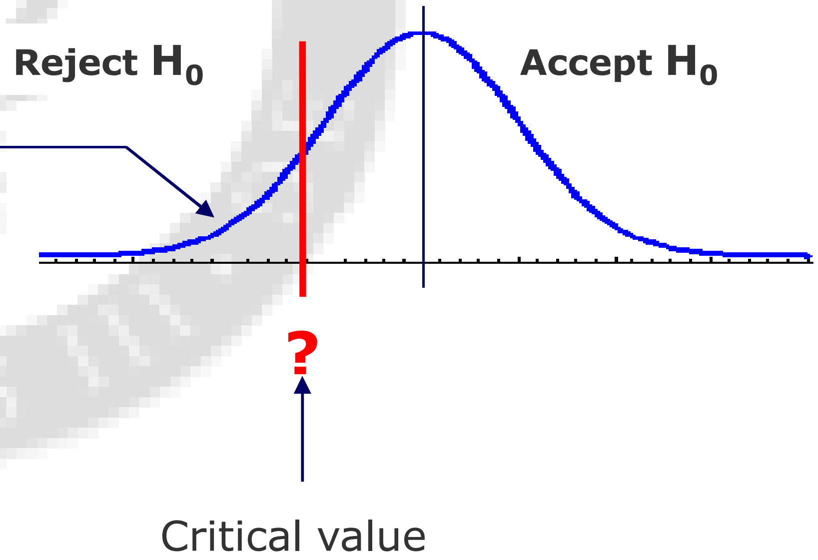
STEP 2: Identify the test statistic

STEP 3: Find the reference distribution

STEP 4: Localize the rejection and the acceptance regions

How much is the area of the rejection regions?

- The rejection region has a small area
- The area is denoted by α = **significance level**
- It value is chosen by the analyst
- Usual values for α are 0.05, 0.10, 0.01



Example

Continuing with the transistors' example. We want to know if the population of the transistors is keeping the mean value $\mu_0 = 290$

H_0	H_1
$\mu = 290$	$\mu \neq 290$

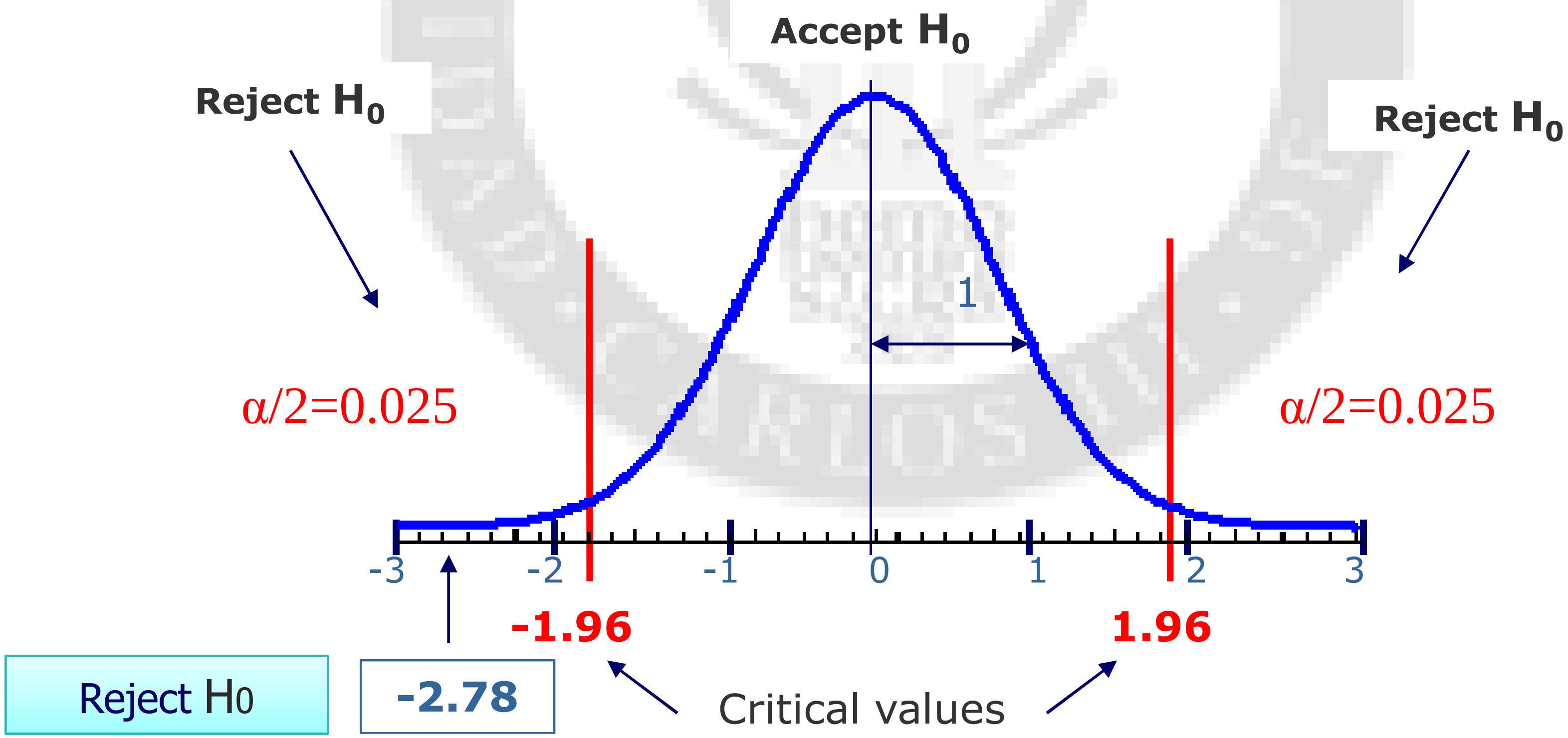
With 100 observations:

$$\bar{x} = 282.3; \quad \hat{s} = 27.57;$$

$$t_0 = \frac{\bar{x} - \mu_0}{\hat{s}/\sqrt{n}} = \frac{282.3 - 290}{27.69/10} = \underline{-2.78}$$

$T_0 \sim N(0,1)$

Significance level $\alpha = 0.05$



Example

Continuing with the transistors' example. We want to know if the population of the transistors is keeping the mean value $\mu_0 = 290$

H_0	H_1
$\mu = 290$	$\mu \neq 290$

With 100 observations:

$$\bar{x} = 282.3; \quad \hat{s} = 27.57;$$

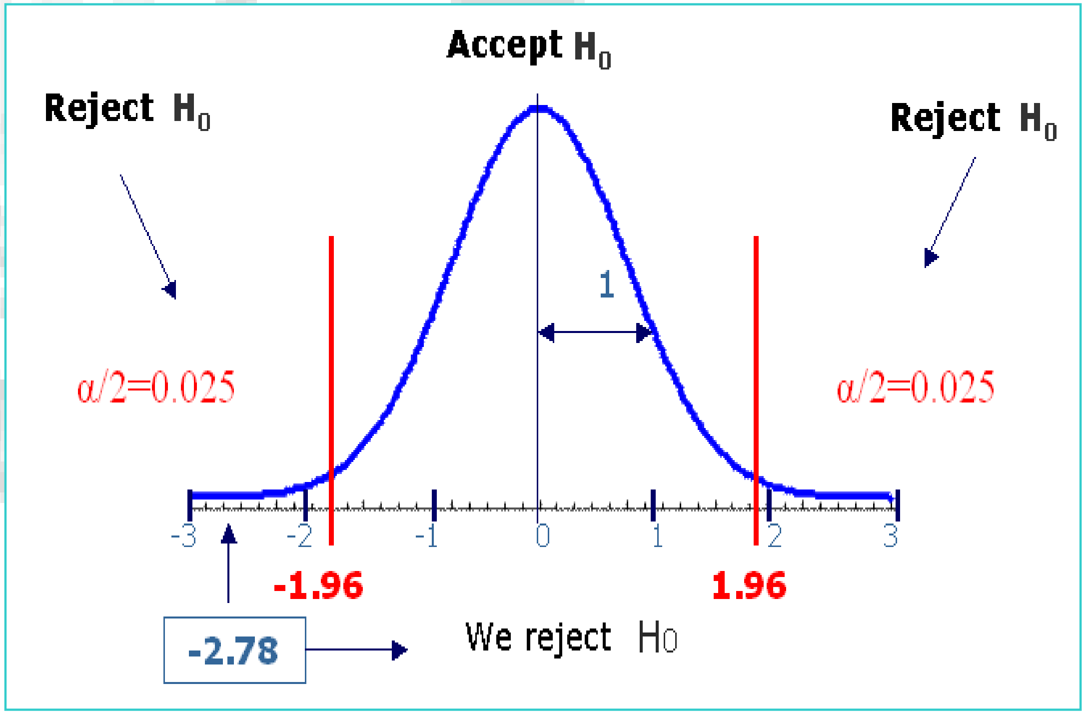
$$t_0 = \frac{\bar{x} - \mu_0}{\hat{s}/\sqrt{n}} = \frac{282.3 - 290}{27.69/10} = \underline{-2.78}$$

$T_0 \sim N(0,1)$

Significance level $\alpha = 0.05$

<u>The difference</u> between the sample mean (282.3) and the hypothesis (290) **<u>is statistically significant</u>** (at level 5%)

We conclude that with a significance level of <u>5%</u> that the population mean is changed



Questions

Answer with True, False, Not Definable.

- By a hypothesis test we look for a confirmation of an hypothesis about a parameter by analyzing a dataset
- If we reject the hypothesis that $\mu=100$ with $\alpha=0.05$, the conclusion is that it is impossible to have $\mu=100$
- We want to test the hypothesis that $\mu=100$ with $\alpha=0.05$. Taking some data we compute $\bar{x} = 104.3$ and the test advice to accept H_0. Therefore it means that with significance value 0.05 the population mean is $\mu=104.3$
- We want to test the hypothesis that $\mu=100$ with $\alpha=0.05$. Taking some data we compute $\bar{x} = 104.3$ and the test advice to accept H_0. Therefore it means that with significance value 0.05 we have $\bar{x} = 100$
- If we have few data the test may be not valid
- An analyst may accept a null hypothesis at $\alpha=0.05$, but s/he can reject it at $\alpha=0.01$

Example

According to some anthropometric studies, young Spanish people that are from 18 to 25 years old have an average height of $\mu_0 = 177$ cm.

We measure the heights of 50 guys from Madrid whose age belongs to the given interval and it results that

$$\bar{x} = 175.9cm \qquad \hat{s} = 5.93cm$$

There is enough statistical evidence that young people of Madrid are on average **less tall** that the national mean?

STEP 1: Specify the Null and the Alternative Hypotheses

Two options

- The mean height is less than the national one → $\mu < 177$
- The mean height is NOT less than the national one → $\mu \geq 177$

$$H_0 : \mu \geq 177$$

$$H_1 : \mu < 177$$

Example

According to some anthropometric studies, young Spanish people that are from 18 to 25 years old have an average height of $\mu_0 = 177$ cm.

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There is enough statistical evidence that young people of Madrid are on average **less tall** that the national mean?

$$H_0 : \mu \geq 177$$
$$H_1 : \mu < 177$$

STEP 2: Identify the test statistic → $t_0 = \dfrac{\bar{x} - \mu_0}{\hat{s}/\sqrt{n}} = \dfrac{175.9 - 177}{5.93/\sqrt{50}} = -1.31$

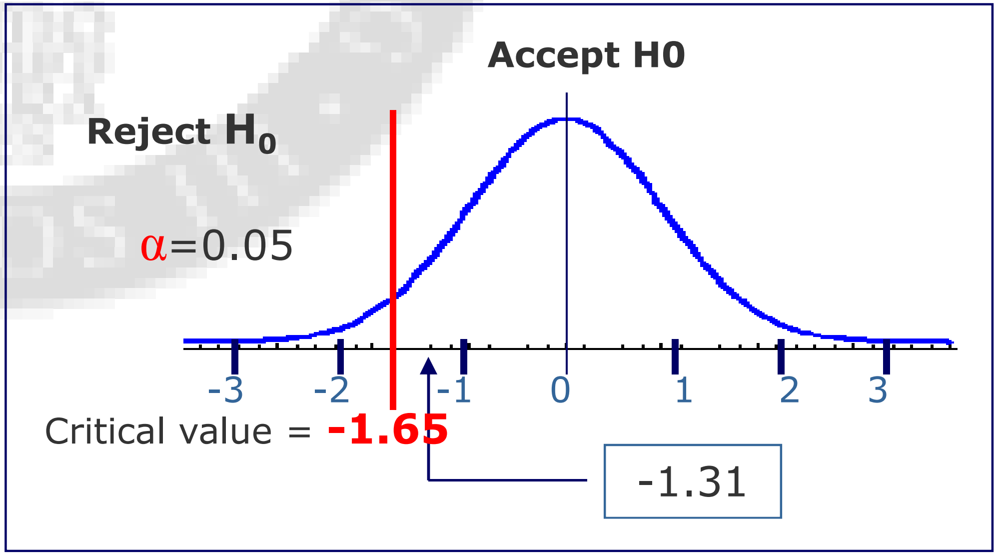
STEP 3: Find the reference distribution → **N(0,1)**

STEP 4: Localize the rejection and the acceptance regions

<u>The difference</u> between the sample mean (175.9) and the null hypotesis **<u>is NOT statistically significant</u>** (at level 5%)

↓

The difference, with a significance level of 5% is due only to the randomness of the sample and not to actual difference



The result of the statistical test (with a finite data set)

The Reality (that we cannot know by analyzing only a finite set of data)

	H_0 is TRUE (H1 is false)	H_0 is FALSE (H1 is true)
Accept H_0 (Reject H1)	**GUESSED!!**	ERROR TIPO II Done with a probability that depends on the experiment
Reject H_0 (Accept H1)	ERROR of Type I **Done with probability** α	**GUESSED!!**

When we report the results of a statistical test we always have to say at which significance level it was done, in order to give a measure of its precision

Chapter 7: Large-Sample Inference

1. Confidence intervals for μ with large samples
2. Determining the sample size
3. Other confidence intervals
4. Introduction to the Hypothesis Testing
5. Hypothesis test for the mean μ with large samples
6. Interpreting the test using the p-value
7. Relation between the hypothesis test and the confidence intervals

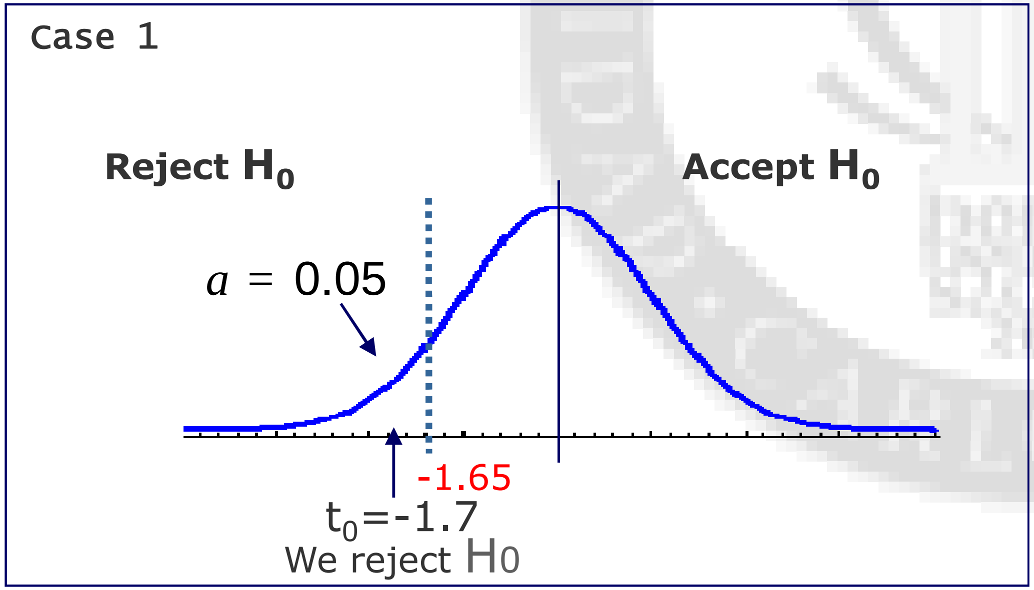
6. Interpreting the test using the p-value

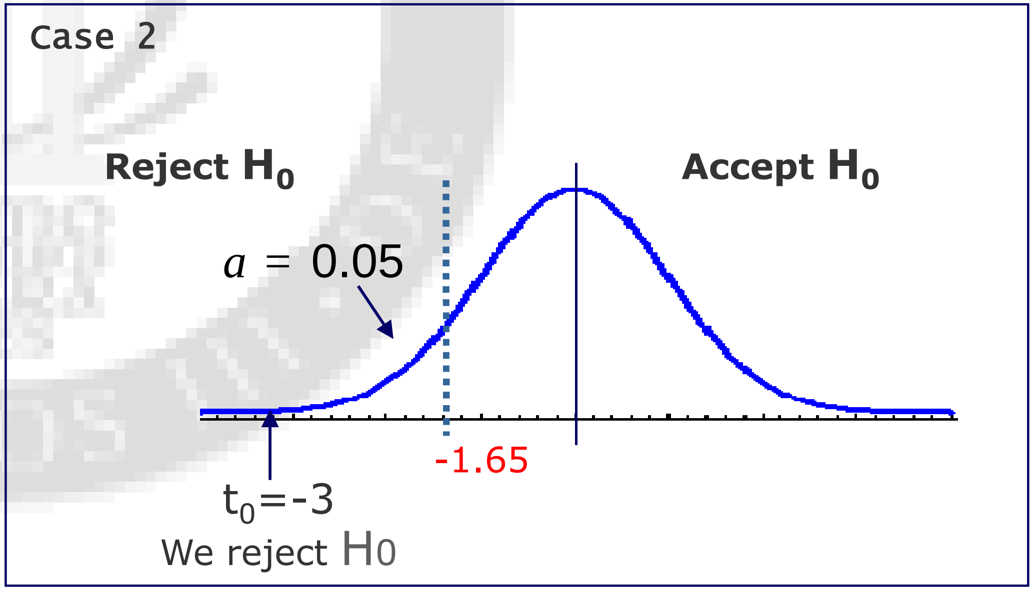
The result of a statistical test is made of **two** elements:

1. We accept or reject H0 → Result of the test
2. The significance level α → Measure of uncertainty

The significance level is low precise measure of the uncertainty

Example We do the hypothesis test $H_0 : \mu \geq \mu_0 ; H_1 : \mu < \mu_0$ with $\alpha=0.05$





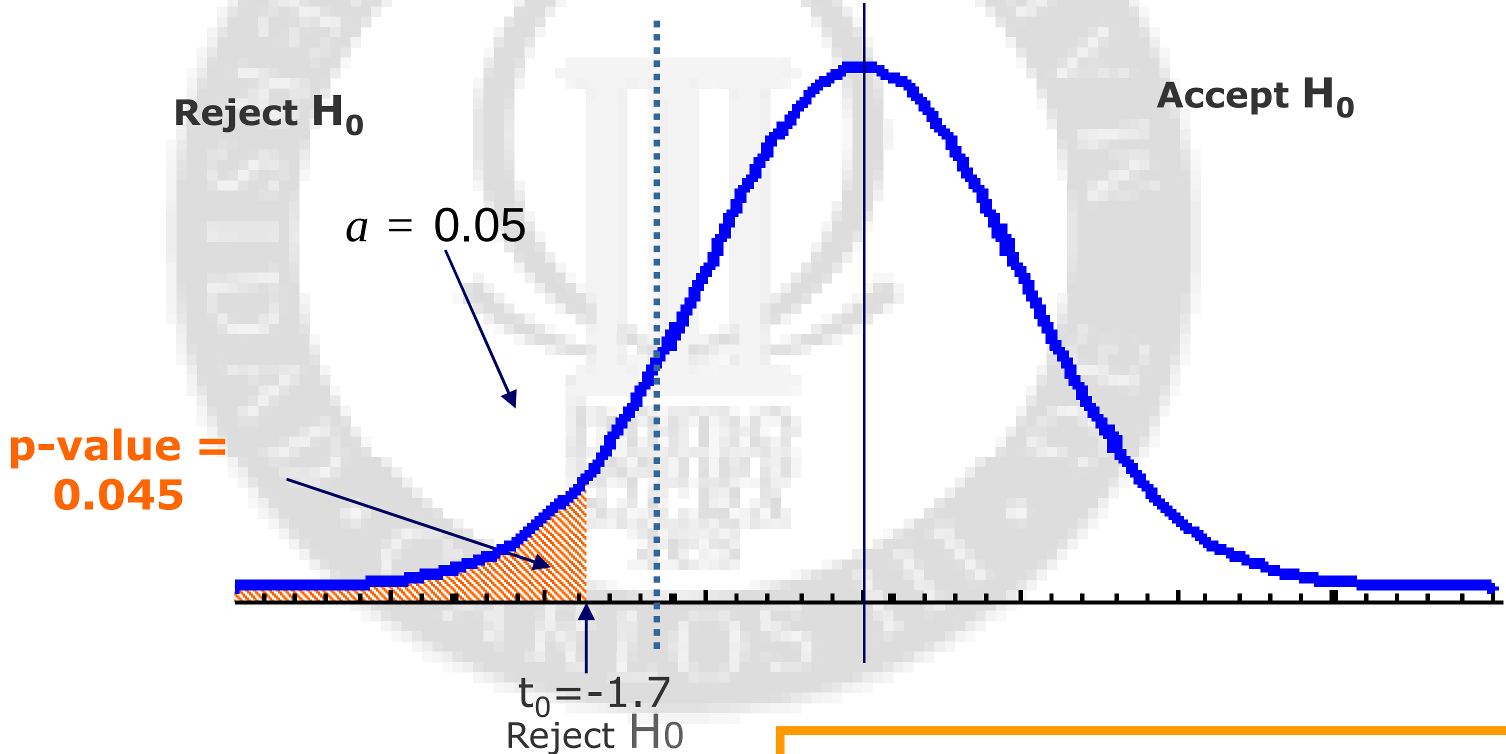
In both cases the conclusion is the same:
We reject the null hypothesis with significance level $\alpha=0.05$

However in the case 2 we are more sure of our decision. **How could we express this?**

We could use a different measure of the result of the statistical test

The **p-value** is the significance level that leaves the value of the test statistics exactly on the **border** of the rejection region.

Case 1



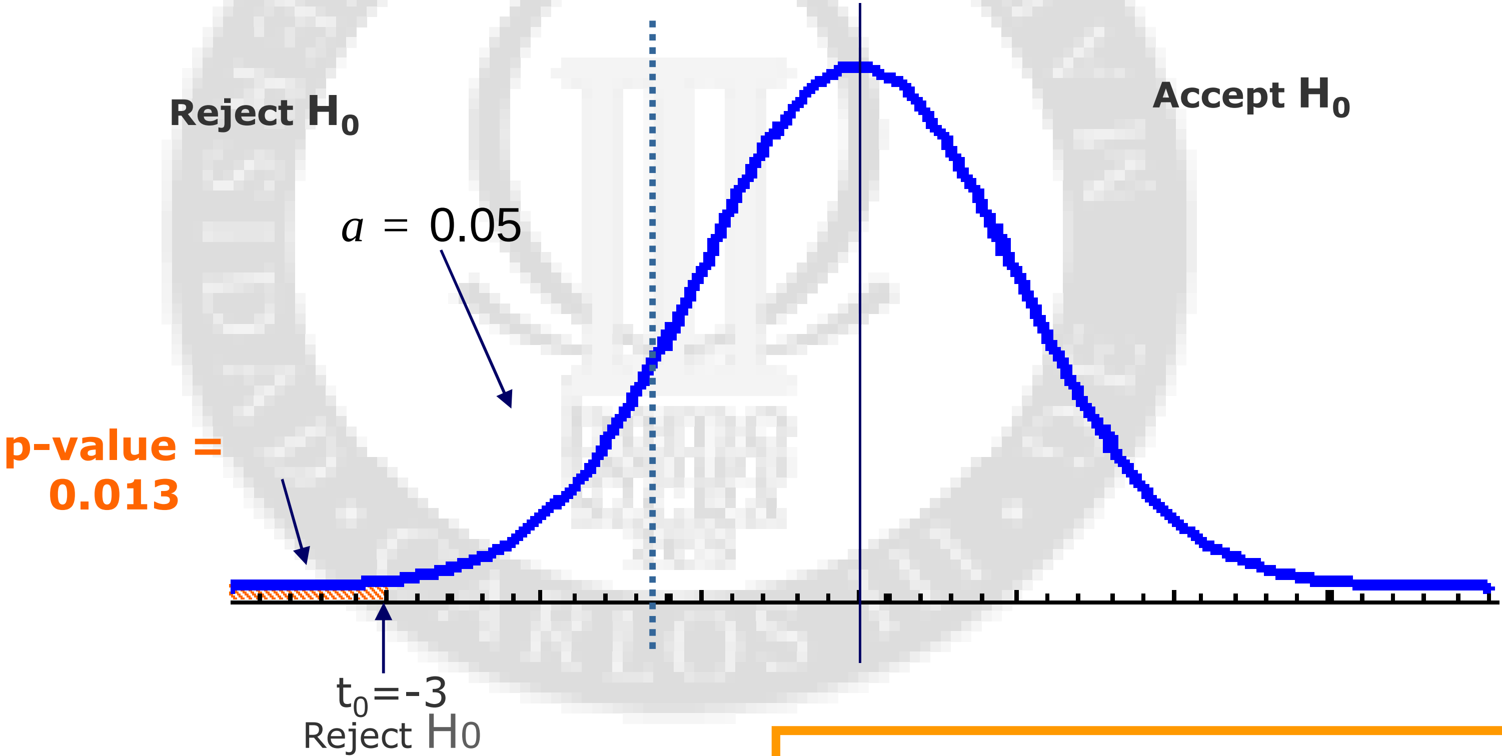
Since p-value<α → we reject H0

The p-value brings more information than the significance level

We could use a different measure of the result of the statistical test

The **p-value** is the significance level that leaves the value of the test statistics exactly on the **border** of the rejection region.

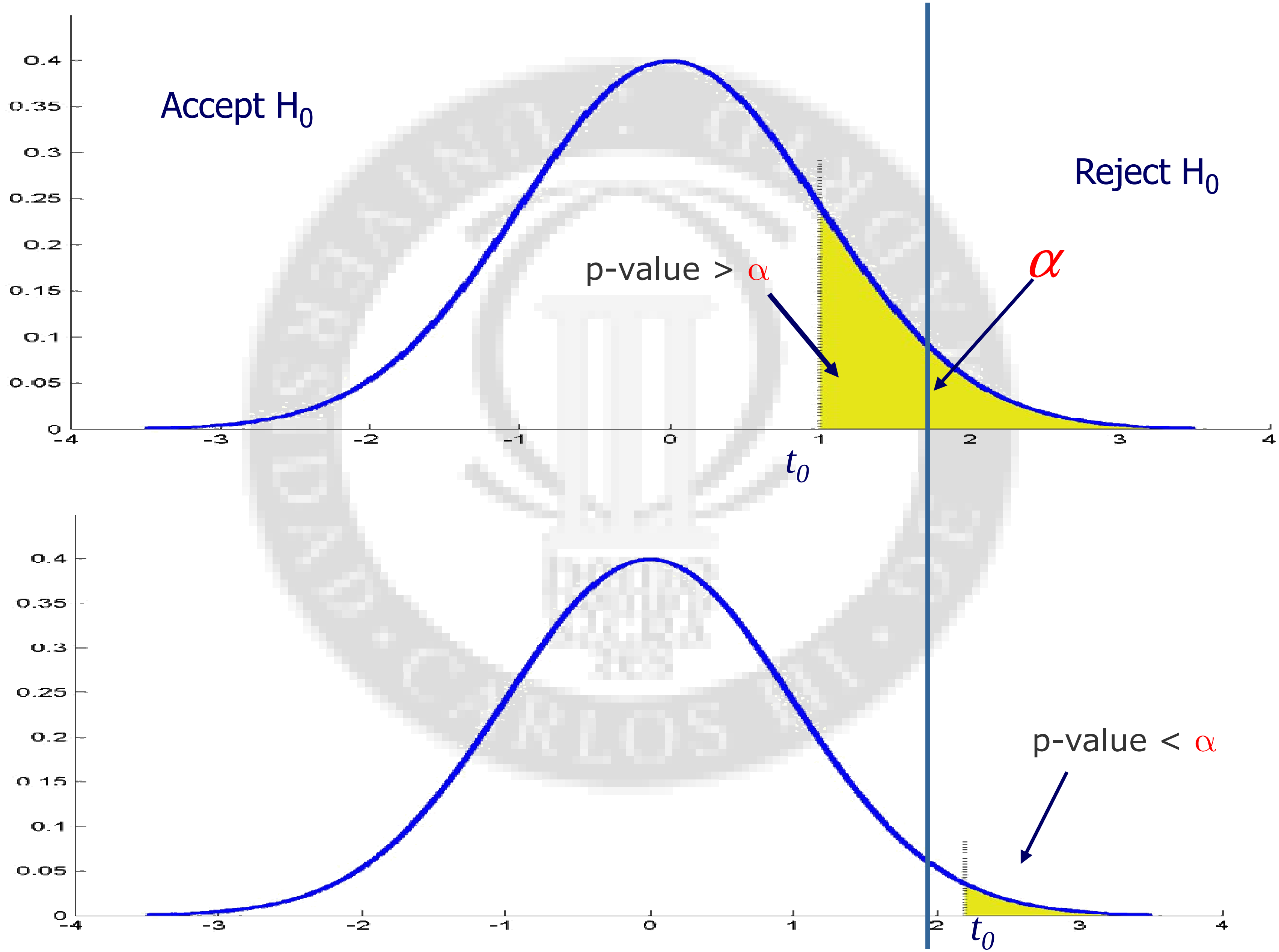
Case 2



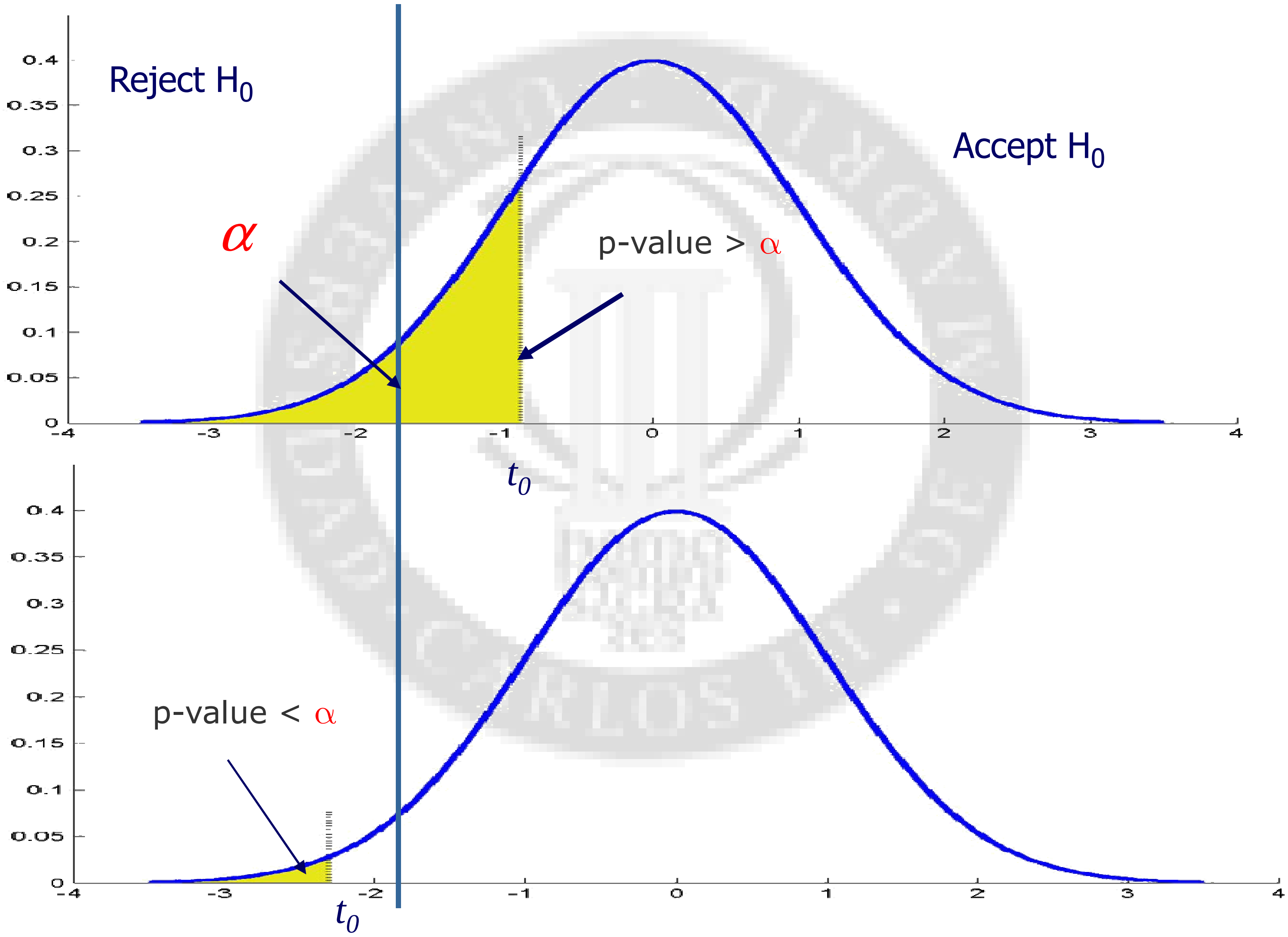
Since p-value<<α → we reject H0

In this case the p-value is very small.
We are very sure about our decision

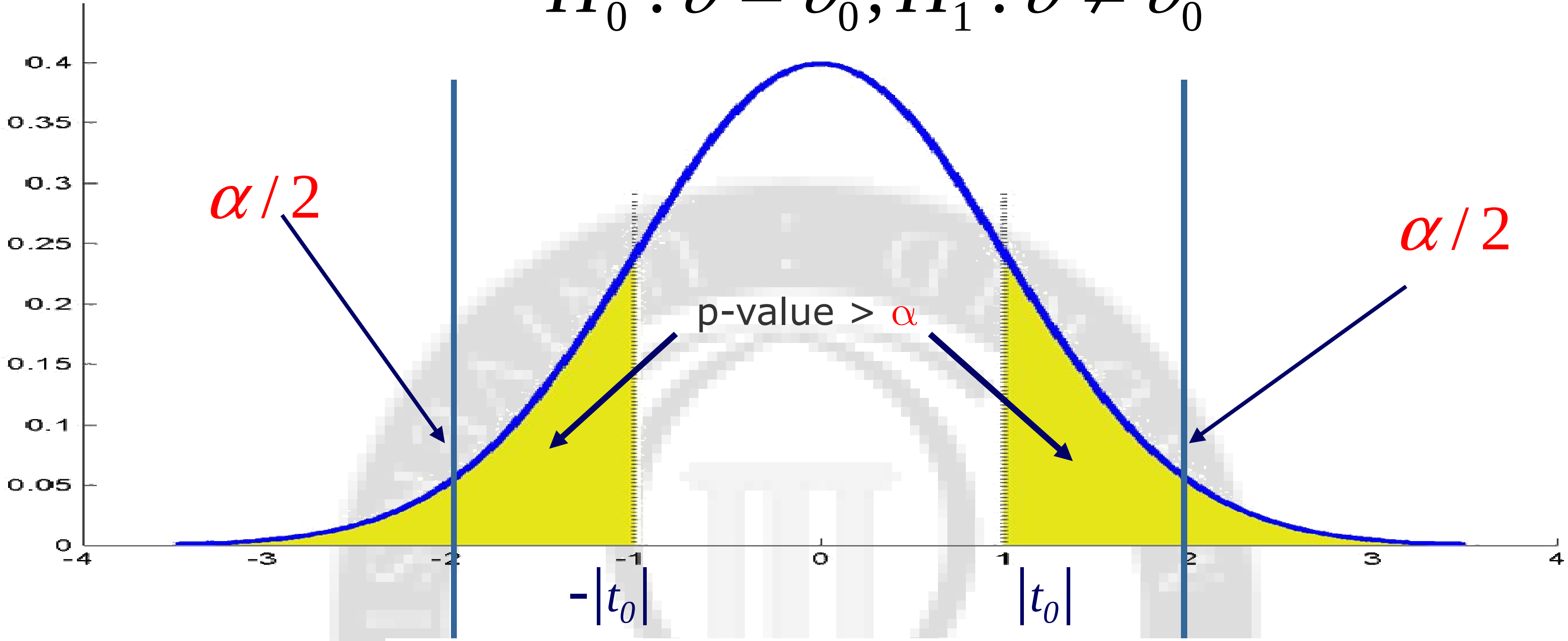
$$H_0 : \vartheta \leq \vartheta_0 ; H_1 : \vartheta > \vartheta_0$$



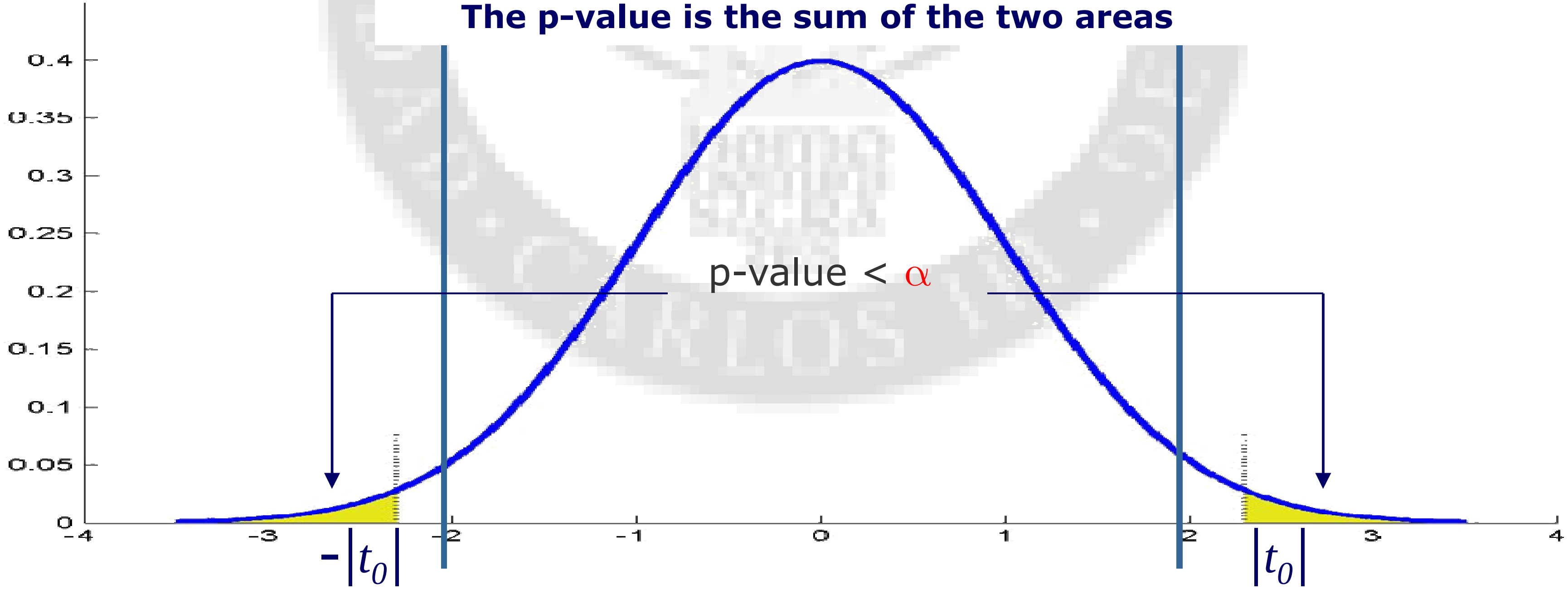
$$H_0 : \vartheta \geq \vartheta_0 ; H_1 : \vartheta < \vartheta_0$$



$$H_0 : \vartheta = \vartheta_0 ; H_1 : \vartheta \neq \vartheta_0$$



The p-value is the sum of the two areas



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7. **Relation between the hypothesis test and the confidence intervals**

7. Relation between the hypothesis test and the confidence intervals

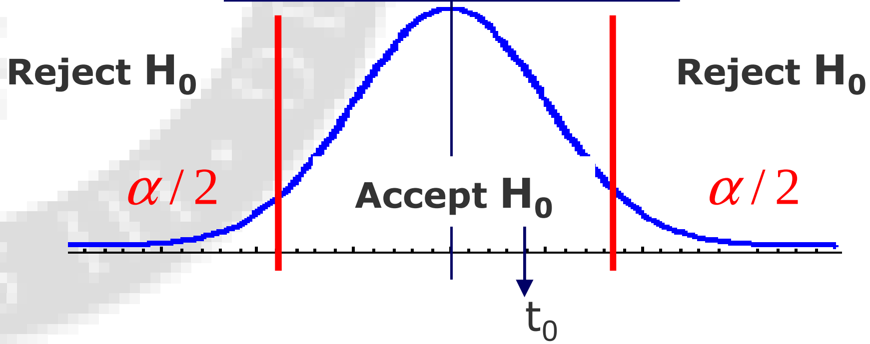
Confidence interval for the mean and the hypothesis test share the same information

$$T = \frac{\bar{X} - \mu}{\hat{S} / \sqrt{n}}$$

$$\mu \in \left\{ \bar{X} \pm \underbrace{z_{\alpha/2}}_{N(0,1)} \frac{\hat{S}}{\sqrt{n}} \right\}$$

$$T_0 = \frac{\bar{X} - \mu_0}{\hat{S} / \sqrt{n}} \sim N(0,1)$$

$$\boxed{H_0 : \mu = \mu_0 ; H_1 : \mu \neq \mu_0}$$



It is possible to prove that to make a **bilateral hypothesis test**

$$\boxed{H_0 : \mu = \mu_0 ; H_1 : \mu \neq \mu_0}$$

with significance level α is the same as calculating a $(1-\alpha)$ **confidence interval** and verifying that the value μ_0 DO or DO NOT belong to this interval.

Example

Continuing with the transistors' example. We want to know if the population of the transistors is keeping the mean value $\mu_0 = 290$

With 100 observations:

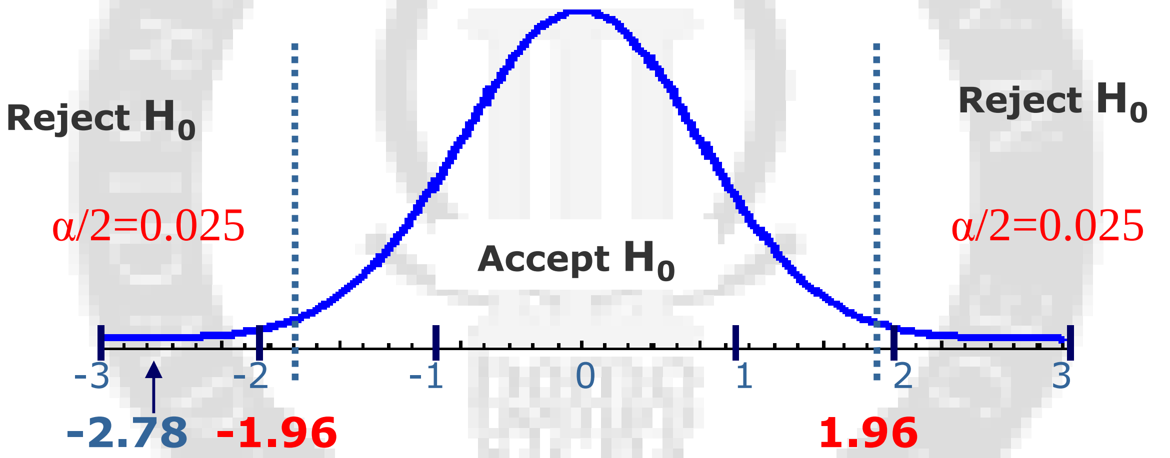
$H_0 \quad \mu = 290$

$H_1 \quad \mu \neq 290$

$$\bar{x} = 282.3; \hat{s} = 27.69;$$

$$t_0 = \frac{\bar{x} - \mu_0}{\hat{s}/\sqrt{n}} = \frac{282.3 - 290}{27.69/10} = -2.78$$

Hypothesis test



We reject H_0: $\mu = 290$

(1- α) Confidence interval

$$\mu \in \left\{ \bar{x} \pm z_{\alpha/2} \frac{\hat{s}}{\sqrt{n}} \right\} = 282.3 \pm 1.96 \frac{27.69}{10} = [276.9; 287.7]$$

It DOES NOT contain the value $\mu_0 = 290$