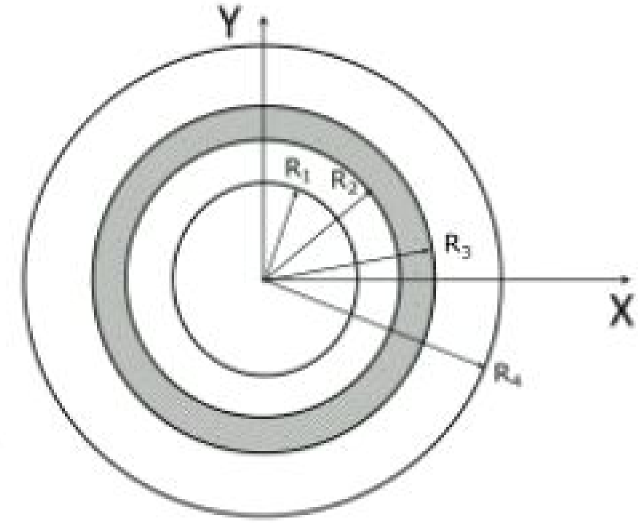
P1. ***(2.5 p)*** Consider the following system:

- A uniformly charged spherical surface of radius R_1 and charge density σ_1.
-A hollow conducting sphere with charge Q, inner radius R_2 and outer radius R_3.
-A uniformly charged spherical surface of radius R_4 and charge density σ_4.
The above charge distributions are all concentric and centred at the origin of the Cartesian coordinate system.



a) If the charge density on the surface of radius R_2 is $\sigma_2 = 6\times10^{-9}$ C/m^2, calculate the charge on the surface of radius R_1.

b) Calculate the general expression of the electric field vector for any point on the X-axis (with x>0). Use as many regions as necessary.

c) If the electric potential of any point on the spherical surface of radius R_4 is 758.6 V, calculate the electric potential of the conducting sphere.

DATA: R_1= 20 cm; R_2= 40 cm; R_3= 70 cm; R_4= 200 cm; $\sigma_4 = 3\times10^{-9}$ C/m^2 ; $Q=3\times10^{-8}$ C

a) If Q_1 is the charge on the surface of radius R_1, then the charge on the inner surface of the conductor with radius R_2 is $Q_2 = -Q_1$ so that the electric field inside the conductor ($R_1 < r < R_2$) is 0.
Therefore $Q_1 = -Q_2$

$$\sim \boxed{Q_1 = -\sigma_2 4\pi R_2^2 = -12.064\ nC}$$

b) We consider points along the x-axis and divide space in 5 areas;
area I with $0 < x < R_1$, area II with $R_1 \leq x \leq R_2$,
area III with $R_2 < x < R_3$, area IV with $R_3 \leq x < R_4$
and area IV with $x \geq R_4$

I: $\boxed{\vec{E}_I = 0}$ because if we consider any Gaussian surface within area I it encloses zero charge.

II: We consider a spherical Gaussian surface with a centre at the coordinates origin and radius $r < R_1$. From Gauss' law the net electric flux through the Gaussian surface $\Phi_{II} = \frac{Q_{ins}}{\varepsilon_0} = \frac{Q_1}{\varepsilon_0}$ (1)

We can calculate Φ_{II} from the definition of the flux: $\Phi_{II} = \oint \vec{E}_{II} d\vec{S}_{II}$

Because at every point of the Gaussian surface S_1 the electric field vector $\vec{E}_{II}$ is parallel to the surface vector $d\vec{S}_{II}$ we obtain $\Phi_{II} = \oint E_{II} dS_{II} = E_{II} \oint dS_{II} =$

$= E_{II} \cdot 4\pi r^2$ (2) where we made use of the fact that the magnitude of the electric field E_{II} is constant at every point of the spherical Gaussian surface.

(1), (2) $\rightarrow$ $\boxed{\vec{E}_{II} = \frac{Q_1}{4\pi\varepsilon_0 x^2}\vec{r}}$, with Q_1

obtained at part a)

$\underline{III}$: Area $\underline{III}$ includes points inside the conductor therefore $\boxed{\vec{E}_{\underline{III}} = 0}$

$\underline{IV}$: We consider a spherical Gaussian surface centred at origin and with radius $R_3 \leq r < R_4$. From Gauss' law:

$\phi_{\underline{IV}} = \frac{Q_{in}}{\varepsilon_0} = \frac{Q + Q_1}{\varepsilon_0}$ (3), where Q the total charge of the conductor.

We compute $\phi_{\underline{IV}} = \oint \vec{E}_{\underline{IV}}\, d\vec{S}_{\underline{IV}} = \oint E_{\underline{IV}}\, dS_{\underline{IV}}$

$\vec{E}_{\underline{IV}} \parallel d\vec{S}_{\underline{IV}}$

$E_{\underline{IV}}$ is constant along the Gaussian surface $S_{\underline{IV}}$

$= E_{\underline{IV}} \oint dS_{\underline{IV}} = E_{\underline{IV}} \cdot 4\pi r^2$ (4)

(3), (4) $\rightarrow$ $\boxed{\vec{E}_{\underline{IV}} = \frac{Q + Q_1}{4\pi\varepsilon_0 x^2}\vec{r}}$

$\underline{V}$: We consider a spherical Gaussian surface $S_{\underline{V}}$ centered at origin and with radius $r \geq R_4$. From Gauss' law $\phi_{\underline{V}} = \frac{Q_{ins\,\underline{V}}}{\varepsilon_0} = \frac{Q + Q_1 + \sigma_4 4\pi R_4^2}{\varepsilon_0}$ (5)

The electric field flux $\phi_{\underline{V}} = \oint \vec{E}_{\underline{V}} d\vec{S}_{\underline{V}}$

$\vec{E}_{\underline{V}} \parallel d\vec{S}_{\underline{V}}$

$$= \oint E_{\underline{V}} dS_{\underline{V}} = E_{\underline{V}} \oint dS_{\underline{V}} = E_{\underline{V}} 4\pi r^2 \quad (6)$$

$E_{\underline{V}}$ is constant along $S_{\underline{V}}$

$$(5), (6) \rightsquigarrow \boxed{\vec{E}_{\underline{V}} = \frac{Q + Q_1 + \sigma_4 4\pi R_4^2}{4\pi\varepsilon_0 x^2} \cdot \vec{r}}$$

c) To calculate the potential of the conducting sphere we just have to calculate the potential at its outer surface with radius R_3, V_{R_3}.

We know the potential on the spherical surface with radius R_4, V_{R_4}

The potential difference

$\vec{E}_{IV} \parallel d\vec{r}$

$$V_{R_3} - V_{R_4} = -\int_{R_4}^{R_3} \vec{E}_{IV}\, d\vec{r} =$$

$$= -\int_{R_4}^{R_3} \frac{Q+q_1}{4\pi\varepsilon_0} \frac{1}{r^2}\, dr = \frac{Q+q_1}{4\pi\varepsilon_0} \left[\frac{1}{r}\right]_{R_4}^{R_3} =$$

$$= \frac{Q+q_1}{4\pi\varepsilon_0} \left(\frac{1}{R_3} - \frac{1}{R_4}\right)$$

$$\rightarrow \boxed{V_{R_3} = V_{R_4} + \frac{Q+q_1}{4\pi\varepsilon_0}\left(\frac{1}{R_3} - \frac{1}{R_4}\right) = 908.36\,V}$$