uc3m Universidad Carlos III de Madrid

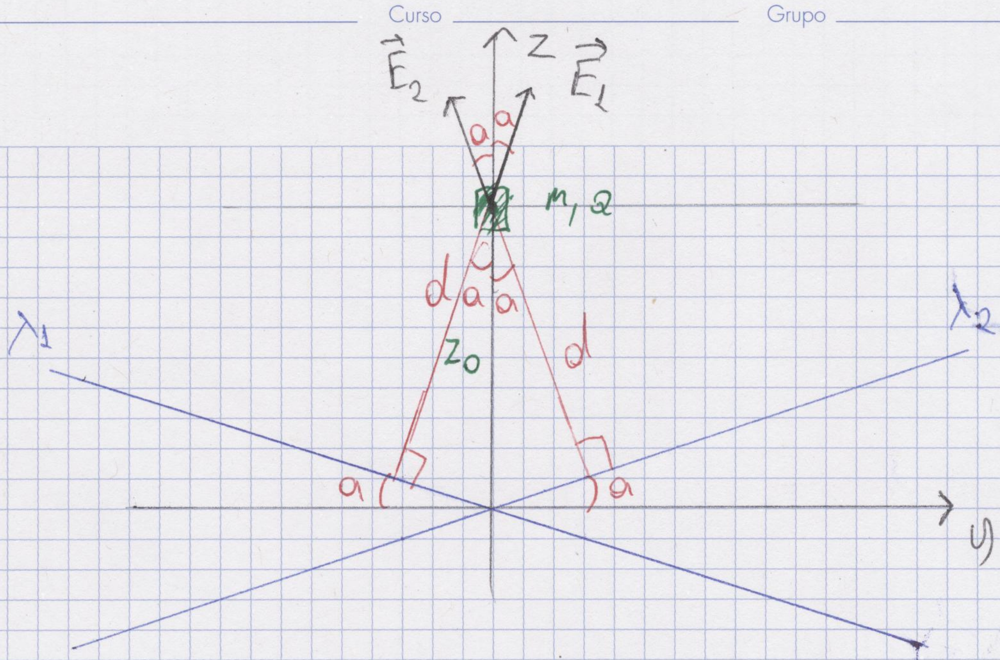
Escuela Politécnica Superior

Asignatura

Nombre y Apellidos

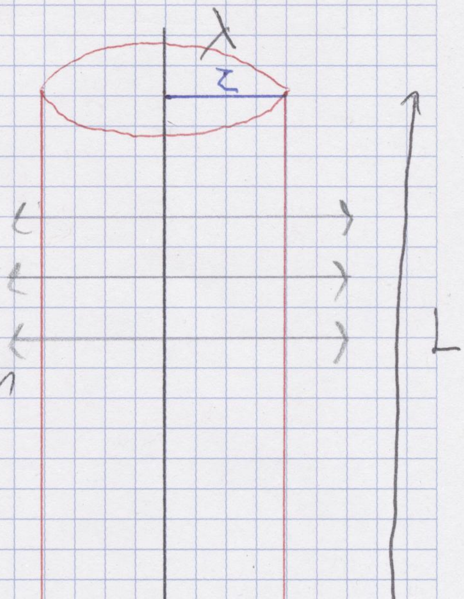
Fecha Curso Grupo

P2



a) We first derive the general expression of the electric field created by a uniformly charged infinite line with charge density λ:

We consider a cylindrical imaginary Gaussian surface concentric to the line and with radius r. From Gauss's law, the electric field flux through the Gaussian surface $\phi_G = \frac{Q_{enc}}{\varepsilon_0}$ (1), with Q_{enc} the charge enclosed by the Gaussian surface with radius r and length L.



$Q_{enc} = \lambda \cdot L$ and (1) $\rightsquigarrow$ $\phi_G = \frac{\lambda \cdot L}{\varepsilon_0}$ (2)

We can also compute ϕ_G from

$$\phi_G = \oint \vec{E} \cdot d\vec{S} \underset{\vec{E} \parallel d\vec{S}}{=} \oint E \cdot dS \underset{\substack{E = \text{const along} \\ \text{the Gaussian surface}}}{=} E \cdot \oint dS = E \cdot 2\pi r \cdot L \quad (3)$$

(2) and (3) $\rightarrow$ $E = \dfrac{\lambda}{2\pi\varepsilon_0 \cdot r}$ and the vector $\boxed{\vec{E} = \dfrac{\lambda}{2\pi\varepsilon_0 r}\,\hat{u}_r}$

with $\hat{u}_r$ the radial to the line unit vector.

b) The radial distance between both lines and point P where the object is located is

$d = z_0 \cdot \cos a$, as indicated in the sketch above.

The electric field created by line λ_1 at point P, $\vec{E}_1 = \dfrac{\lambda}{2\pi\varepsilon_0 d}(\sin a\,\vec{j} + \cos a\,\vec{k}) = \dfrac{\lambda}{2\pi\varepsilon_0 z_0 \cos a}(\sin a\,\vec{j} + \cos a\,\vec{k})$ Eq (4)

$$\rightarrow \boxed{\vec{E}_1 = \frac{\lambda}{2\pi\varepsilon_0 z_0}(\tan a\,\vec{j} + \vec{k}) = 1.174 \cdot 10^5\,\vec{j} + 2.518 \cdot 10^5\,\vec{k}\left(\frac{N}{C}\right)}$$

The electric field created by line λ_2 at point P:

$\vec{E}_2 = \dfrac{\lambda}{2\pi\varepsilon_0 d}(-\sin a\,\vec{j} + \cos a\,\vec{k}) = \dfrac{\lambda}{2\pi\varepsilon_0 z_0 \cos a}(-\sin a\,\vec{j} + \cos a\,\vec{k})$

$$\rightarrow \boxed{\vec{E}_2 = \frac{\lambda}{2\pi\varepsilon_0 z_0}(-\tan a\,\vec{j} + \vec{k}) = -1.174 \cdot 10^5\,\vec{j} + 2.518 \cdot 10^5\,\vec{k}\left(\frac{N}{C}\right)}$$

Eq (5)

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c) The object is at equilibrium at point P

$\rightarrow \sum \vec{F} = 0 \rightarrow \vec{F}_e + m\vec{g} = 0$ (6) with $\vec{F}_e$ the net electric force acting on the charged object because of the two lines: $\vec{F}_e = Q\vec{E}$ (7) with $\vec{E}$ the net electric field at point P: $\vec{E} = \vec{E}_1 + \vec{E}_2$

$$\xrightarrow{(4),(5)} \vec{E} = \frac{\lambda}{\pi \varepsilon_0 z_0} \vec{k} \quad (8)$$

and from Eq (6) and (7):

$$\boxed{m = \frac{Q \cdot \lambda}{\pi \varepsilon_0 z_0 \cdot g} = 0.0164 \text{ kg}} \quad \text{Eq (9)}$$

d) If the object was placed at $P'(0, 0, 2z_0)$

the net force $\vec{F}_{p'} = \vec{F}_e(P') + m\vec{g}$

From 2nd Newton's law $\vec{F}_{p'} = m \cdot \vec{\gamma}$

where $\vec{\gamma}$ the acceleration vector of the object

$$\rightarrow \vec{\gamma} = \frac{\vec{F}_e(P')}{m} + \vec{g} = \frac{Q\vec{E}(P')}{m} + \vec{g} \quad (10)$$

where the net electric field at point P':

$$\xrightarrow{(8)} \vec{E}(P') = \vec{E}_1(P') + \vec{E}_2(P') = \frac{\lambda}{\pi \varepsilon_0 2 z_0} \vec{k}$$

$$\text{Eq (10)} \rightarrow \vec{\gamma} = \left(\frac{\lambda Q}{2\pi \varepsilon_0 z_0 m} - g\right)\vec{k} \xrightarrow{(9)} \boxed{\vec{\gamma} = -\frac{g}{2} \cdot \vec{k} = -4.9\,\vec{k} \left(\frac{m}{s^2}\right)}$$