uc3m | Universidad Carlos III de Madrid
Departamento de Estadística

# Bachelor in Computer Science and Engineering

## Statistics Problems

## IV Random Variables

1. A discrete random variable X takes values $x_i = 1,2,..6$ with probability function P(X)=1/6. Calculate using the probability function and the distribution function:

   a) $P(2 < X \leq 4)$
   b) $P(2 \leq X \leq 4)$
   c) $P(3 < X \leq 4{,}3)$

2. Given a continuous random variable X with density function

$$f(x) = \begin{cases} k(x+2) & 0 \leq x \leq 4 \\ 0 & en\ el\ resto \end{cases}$$

   Find:

   a. The value of k so that the function really be a density function
   b. The distribution function
   c. The mean
   d. The variance
   e. P(2<=X<=3)

3. Given a random variable X, whose density function is

$$f(x) = \begin{cases} k(1-x^2) & 0 < x < 1 \\ 0\ en\ el\ resto \end{cases}$$

   Find the value for k, the mean, and the variance for variable Y= 3X-1

4. Fort he following distribution function:

$$F(x) = \begin{cases} 0 & x < 0 \\ x - {x^2}/{4} & 0 < x < 2 \\ 1 & x \geq 2 \end{cases}$$

a. Find de median
b. Calculate the expectation of X
c. Find the probaility that X be less than 1, knowing that X is defined in interval (0,5; 1,5)

5. Given a random variable X with density function:

$$f(x) = \begin{cases} kx^3 \; si \; 0 < x < 1 \\ 0 \; en \; otro \; caso \end{cases}$$

a. Obtain the value of k needed for the function to be a density function
b. Calculate P(X<=2/3 / X>1/3)
c. Now consider the discrete random variable Y with cumulative distribution function:

$$F(x) = \begin{cases} 0 \; si \; y < 3 \\ 0.3 \;\; si \; 3 \leq y < 5 \\ 0.5 \; si \; 5 \leq y < 10 \\ 0.8 \;\; si \; 8 \leq y < 10 \\ 1 \;\; si \; y \geq 10 \end{cases}$$

Determine the probability function for variable Y, and its mean and variance.

6. Given a random variable X and its density function defined as:

$$f(x) = \begin{cases} 0 & para \; x \leq 0 \\ \frac{x}{4} & para \; 0 < x \leq \frac{1}{4} \\ x^2 & para \; \frac{1}{4} < x < a \\ 0 & para \; x > a \end{cases}$$

a. Obtain the value of a needed for the function to be a density function
b. Calculate the corresponding distribution function
c. Calculate the median

7. A bus arrives to a certain bus stop every 8 minutes. The waiting time for a user that arrives to that bus stop may be defined by density function $f_1(t)$, being t in minutes. If the bus is late the waiting time is distributed according to density function $f_2(t)$:

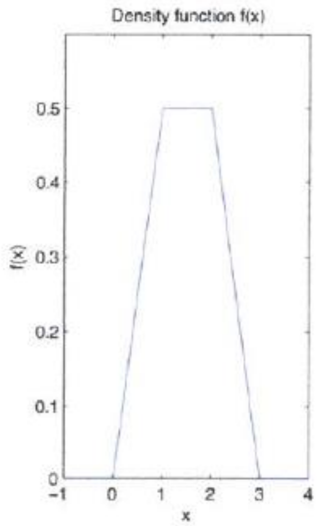
$$f_1(t) = \frac{1}{8} \quad si\ 0 \leq t \leq 8, y\ 0\ en\ caso\ contrario$$

$$f_2(t) = k \cdot e^{-\frac{t}{10}}\ si\ t \geq 0, y\ 0\ en\ caso\ contrario$$

a. Find the value of k for $f_2(t)$ be density function
b. Calculate the median for the waiting time when the bus is not delayed
c. Knowing that a day out of three the bus is delayed, calculate the probability for a user to wait more than 5 minutes.

8. Let X be a continuous random variable defined in interval (0;3) and its density function is given by:

$$f(x) = \begin{cases} \frac{x}{2}\ si\ x \in (0,1) \\ a\ si\ x \in [1,2) \\ \frac{3-x}{2}\ si\ x \in [2,3) \\ 0\ en\ otro\ caso \end{cases}$$




a. Find the value of a for f(x) be a density function
b. Calculate the expectation of X
c. Calculate the probability of X being less than 2,5