Chapter IV: Random Variables

PROBLEMS

Proposed Problems

1. Let X be a random variable that assumes the values X = {a, 1, 2, 3} and whose probability function is p(x) = x/10. How much is the value of *a*?
   SOLUTION:
   $a = 4$.
2. Let Y be a continuous random variable with density function $\mathrm{f(y) = y/18}$ with $0 \le \mathrm{y} \le \mathrm{b}$. Find the value of b.
   SOLUTION:
   $b = 6$.
3. Can the density function be of a continuous random variable assume values bigger than one in some points?
   SOLUTION:
   Yes. It can also be unbounded.
4. The density function of the random variable X has the following expression
   $$f(x) = \begin{cases} 12x^2(1-x) & 0 < x < 1 \\ 0 & otherwise \end{cases}$$
   Find:
   a) The variation coefficient of X
   b) The mode
   c) The median
   SOLUTION:
   a) CV = 1/3
   b) Mode = 2/3
   c) Median = 0.6
5. Given the random variable X with density function
   $$f(x) = \begin{cases} k(1-x^2) & 0 < x < 1 \\ 0 & otherwise \end{cases}$$
   Find the value of k together with the mean and the variance of the random variable $Y = 3X - 2$.
   SOLUTION:
   k=3/2, E[Y]=1/8 and Var[Y]=171/320.
6. The density function of the random variable X has the following expression
   $$f(x) = \begin{cases} k\,x^2(1-x) & 0 < x < 1 \\ 0 & otherwise \end{cases}$$
   Find:
   a) The value k
   b) The distribution function F(x)
   c) The values of E[X] and Var[X]
   SOLUTION:
   a) k=12
   b) $F(x) = \begin{cases} 0 & x \le 0 \\ 12\left(\frac{x^3}{3} - \frac{x^4}{4}\right) & 0 < x \le 1 \\ 1 & 1 < x \end{cases}$
   c) E[X] = 3/5 and Var[X] = 1/25

7. The length of some mechanical item is distributed according to the following density function

$$f(x) = \begin{cases} k\,(x-1)(3-x) & 1 < x < 3 \\ 0 & otherwise \end{cases}$$

where the unit measure is expressed in centimeters. The acceptable items are the ones that have length between 1.7 and 2.4 cm. Find:

   a) The value k
   b) The probability that an item is acceptable (Sept 98)

   SOLUTION:
   a) k=3/4
   b) $\Pr(acceptable) = 1/2$

8. An ice cream dealer normally gains 100 euro in a sunny day and 25 euro in a rainy day. Find the expected daily gain for the seller if the probability to be rainy is ¼.

   SOLUTION:
   $E[gain] = 81.25 euro.$

9. We want to insure a car of 12000 euro. The probability that a car is involved in an accident during a year is 0.15 in which case the amount of damage is

   a) 20% of its value with probability 0.8
   b) 60% of its value with probability 0.12
   c) 100% of its value with probability 0.08

   Find the first annual premium the insurer must charge to have the expected cost of the company equal to 0.

   SOLUTION:
   First annual premium $= 561.6 euro.$

10. Say if the following sentences are true or false

   a) Let X be a continuous random variable with density function defined between 0 and 1. Then X cannot take negative values
   b) Let X be a discrete random variable that can take the values X = (0,1,3,6) according to the probability function p(x). Then the expected value (population mean) will be μ = (0 + 1 + 3 + 6)/4 = 2.5
   c) Let the random variable X be as above. Then the population mean can not be non-integer
   d) Let the random variable X be as above. Then its average shall not be greater than 6
   e) Let the random variable X be as above and be $x_1, x_2, \cdots, x_n$ a set of data (realizations) generated by it. Then the average of these n data will be equal to E[X]
   f) Let X be a random variable with variance Var (X) = 5 and let Y be another random variable with Var(Y) = 2 and independent of X. Then Var (X + Y) = 3.5
   g) Let X and Y be two random variables as above.
   Then Var (X - Y) = 3, Var (-X - Y) = −7; Var(2X − Y) = 8

11. Let X be a discrete random variable that assumes the values $x \in \{1,2,3,4,5\}$ and let c be a given constant. Which of the following functions can be a probability mass function for X?

   a) $p(x) = \frac{c}{x-2}$
   b) $p(x) = c(x+1)$
   c) $p(x) = x^2 - 3$
   d) $p(x) = c - x$

   SOLUTION:
   Only the b)

12. Let X be a continuous random variable defined in the interval [1, 2]. Compute its density function assuming that its distribution function grows linearly in this interval.

   SOLUTION:
   X is uniform distributed in [1,2]

13. A random variable X that measure the duration in minutes of pone call received at a call center has distribution function given by

$$F(x) = \begin{cases} 0 & x \le 0 \\ \dfrac{x^2}{25} & 0 < x \le 5 \\ 1 & 5 < x \end{cases}$$

Compute:

a) The probability that a call takes to terminate more than 3 minutes.
b) The density function of X
c) The expected duration of a phone call
d) The probability that a phone call that is active since last 3 minutes has duration of less than 4 minutes.

SOLUTION:

a) 0.64
b) $f(x) = \frac{2x}{25}$ if $x \in [0,5]$
c) 3.33
d) 0.44