DEGREE IN COMPUTER ENGINEERING

PHYSICS **EXERCISES CH 6**

Conductors.

1. A discharged spherical conductor has an inner radius *a* and an outer radius *b*. A positive point charge *q* is placed in the cavity at the centre of the sphere.
   a) Find the charge on the internal and external surfaces of the conductor.
   b) Find the charge densities at each surface.
   c) Find the electric field and the electric potential outside
the sphere, inside the sphere and in the cavity (suppose V=0 when r → ∞).

2. Two conducting spheres of radii 2 and 5 cm are charged respectively with 10 nC and - 15 nC. The spheres are separated by a distance much greater than their radii and joined together by a wire.

a) Find the charge and electric potential of each sphere after connection.

b) What is the potential energy of the system before and after connection?

3. Two metallic spheres of radii R_1 = 5 cm and R_2 = 7 cm are charged with Q_1 = 3 nC and Q_2. They are put in contact and then separated a very large distance. After their separation, the electric potential measured for the sphere of radius R_1 is 150 V. Find Q_2.

4. A conducting sphere of radius 10 cm and charge -1 μC is located inside a hollow metallic sphere of inner radius 20 cm, outer radius 30 cm and charge 2 μC. The common centre of the two spheres is at the origin of a Cartesian reference frame.

a) What are the charge densities on the inner and outer surfaces of the hollow sphere?

b) Find the electric field at (0, 15, 0) cm and (0, 35, 20) cm.

c) Find the electric potential at (0, 0, 5) cm.

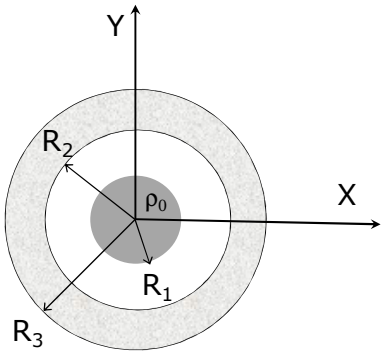
d) What would be the charge densities on the inner and outer surfaces of the hollow sphere if it was grounded? Find the electric potential at (0, 0, 5) cm after the hollow sphere is grounded.

5. A solid sphere of radius R_1 is uniformly charged with a volume charge density ρ_0. A metallic spherical shell of inner radius R_2 and outer radius R_3 concentric with the solid sphere is charged with Q.

   a) Find the charge densities at each surface of the metallic sphere.

   b) Find the electric field in all regions of space.

   c) Find the electric potential in the region inside the solid sphere (suppose V(r=∞)=0).



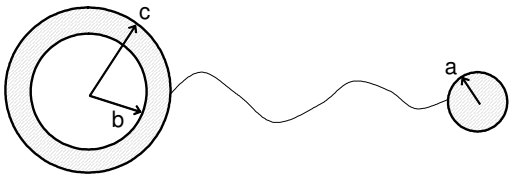
DATA: ρ_0 = - 2 μC/m^3; R_1 = 15 cm; R_2 = 20 cm; R_3 = 40 cm; Q = 40 nC

DEGREE IN COMPUTER ENGINEERING

PHYSICS **EXERCISES CH 6**

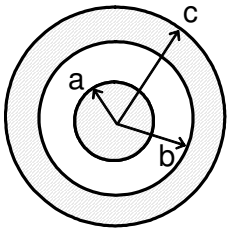
Conductors.

6. A metallic hollow sphere charged with Q has inner and outer radii b and c, respectively. An uncharged solid metallic sphere of radius a (a< b) is located at a very large distance from the hollow sphere.



a) The two spheres are connected by a conducting wire. Find the electric potential of each sphere after reaching electrostatic equilibrium.

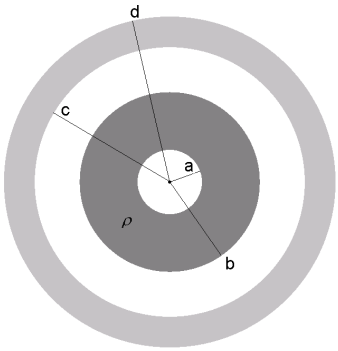
The conducting wire is taken away and the connection between the spheres is broken. Subsequently, the solid sphere of radius a is placed inside the hollow sphere as shown in the figure.



b) Find the charge densities at all metallic surfaces.

c) Find the electric field in all regions of space.

7. A non-conducting spherical shell of inner radius a and outer radius b has a uniform volume charge density ρ. It is surrounded by a hollow uncharged metallic sphere of inner radius c and outer radius d. Find:



a) The total charge of the system.

b) The electric field in all regions of space.

c) The electric potential at any point outside the spheres.

d) The surface charge density on the outer and inner surfaces of the metallic sphere.

e) Graphically represent the electric field due to this charge distribution.

8. Two uncharged spherical conductors have radii R_1=6 cm and R_2=2 cm. They are separated by a distance much greater than 6 cm and connected by a conducting wire. A charge Q=80 nC is placed on the first sphere and the system is allowed to reach electrostatic equilibrium.

a) Find the charge on each sphere.

b) Find the charge density σ on each sphere.

c) What is the magnitude of the electric field at the surface of each sphere?

DEGREE IN COMPUTER ENGINEERING

PHYSICS **EXERCISES CH 6**

Conductors.

ANSWERS

1.

a) $Q_{int} = -q$, $Q_{ext} = q$, b) $\sigma_{int} = \frac{-q}{4\pi a^2}$, $\sigma_{ext} = \frac{q}{4\pi b^2}$ c) $r < a$ $\vec{E} = k\frac{q}{r^2}\vec{u}_r$,

$V = k\frac{q}{r} + kq\left(\frac{1}{b} - \frac{1}{a}\right)$, $a < r < b$ $\vec{E} = 0$ $V = k\frac{q}{b}$, $r > b$ $\vec{E} = k\frac{q}{r^2}\vec{u}_r$, $V = k\frac{q}{r}$

2. $Q_1 = -1.4\times10^{-9}$ C $Q_2 = -3.6\times10^{-9}$ C

$V_1 = V_2 = -642.3$ V $U_i = 4.27\times10^{-5}$ J $U_f = 1.6\times10^{-6}$ J

3. Q= - 1 nC

4. a) $\sigma_{inner} = 1.99\ \mu C/m^2$ $\sigma_{outer} = 0.88\ \mu C/m^2$

b) $\vec{E} = -4\times10^5\vec{j}\ N/C$ (0,15,0)
$\vec{E} = 4.81\times10^4\vec{j} + 2.75\times10^4\vec{k}\ N/C$ (0,35,20)

c) $V = -15\text{x}10^3$ V

d) $\sigma_{inner} = 1.99\ \mu C/m^2$ $\sigma_{outer} = 0$ $V = -45\text{x}10^3$ V

5. a) $\sigma(R_2) = 5.62\times10^{-8}$ C/m^2 $\sigma(R_3) = 5.83\times10^{-9}$ C/m^2

b) $\vec{E} = \frac{105.43}{r^2}\vec{u}_r$ $(r > R_3)$

$\vec{E} = 0$ $(R_2 < r < R_3)$

$\vec{E} = -\frac{254.24}{r^2}\vec{u}_r$ $(R_1 < r < R_2)$

$\vec{E} = -7.53\times10^4\, r\,\vec{u}_r$ $(r < R_1)$

c)

$$V(r < R_1) = \frac{105.4}{R_3} - 254.2\left[\frac{1}{R_1} - \frac{1}{R_2}\right] + 3.8\times10^4\left[r^2 - R_1^2\right]$$

DEGREE IN COMPUTER ENGINEERING

PHYSICS **EXERCISES CH 6**

Conductors.

6. a) $V_{sh} = V_{sm} = \dfrac{Q}{4\pi\varepsilon_0 (c+a)}$

b) $\sigma(a) = \dfrac{Q}{4\pi a (c+a)}$

$\sigma(b) = -\dfrac{aQ}{4\pi b^2 (c+a)}$ $\sigma(c) = \dfrac{Q}{4\pi c^2}$

c) $E(r) = \dfrac{Q}{4\pi\varepsilon_0 r^2}$ $(r > c)$

$E(r) = 0$ $(b < r < c)$

$E(r) = \dfrac{aQ}{4\pi\varepsilon_0 (c+a) r^2}$ $(a < r < b)$

$E(r) = 0$ $(r < a)$

7. a) $Q = \rho \dfrac{4}{3}\pi (b^3 - a^3)$

b) $r < a$ $E(r) = 0$; $a < r < b$ $E(r) = \dfrac{\rho(r^3 - a^3)}{3\varepsilon_0 r^2}$

$b < r < c$ $E(r) = \dfrac{\rho(b^3 - a^3)}{3\varepsilon_0 r^2}$; $c < r < d$ $E(r) = 0$

$r > d$ $E(r) = \dfrac{\rho(b^3 - a^3)}{3\varepsilon_0 r^2}$

c) $r > d$ $V(r) = \dfrac{\rho(b^3 - a^3)}{3\varepsilon_0 r}$

d) $\sigma(c) = -\dfrac{\rho(b^3 - a^3)}{3c^2}$; $\sigma(d) = \dfrac{\rho(b^3 - a^3)}{3d^2}$

8. a) $Q_1 = 60$ nC, $Q_2 = 20$ nC b) $\sigma_1 = 1326$ nC/m^2, $\sigma_2 = 3979$ nC/m^2

c) $E_1 = 149.9$ kN/C, $E_2 = 449.6$ kN/C