

SHEET WITH FORMULAS FOR STATISTICS

PROBABILITY MODELS				
Name	Notation	$p(x) = P(X = x); F(x) = P(X \leq x); f(x) = \frac{\partial F(x)}{\partial x}$	Mean μ	Variance σ^2
Bernoulli	$B(p)$	$p(x) = p^x q^{1-x}; x = 0, 1$	p	pq
Binomial	$B(n, p)$	$p(x) = \binom{n}{x} p^x q^{n-x}; x = 0, 1, \dots, n$	np	npq
Geometric	$G(p)$	$p(x) = pq^{x-1}; x = 1, 2, \dots$	$\frac{1}{p}$	$\frac{q}{p^2}$
Poisson	$P(\lambda)$	$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}; x = 0, 1, \dots$	λ	λ
Uniform in (a, b) (continous)	$U(a, b)$	$f(x) = 1/(b - a); a \leq x \leq b$	$\frac{a + b}{2}$	$\frac{(b-a)^2}{12}$
Exponential	$\text{Exp}(\lambda)$	$f(x) = \lambda e^{-\lambda x}; x > 0$ $F(x) = 1 - e^{-\lambda x}; x > 0$ $\frac{(x - \mu)^2}{2\sigma^2}$	$1/\lambda$	$1/\lambda^2$
Normal	$N(\mu, \sigma^2)$	$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x - \mu)^2}{2\sigma^2}}; x \in \mathbb{R}$	μ	σ^2

SIMPLE LINEAR REGRESSION	
$\hat{y} = a + bx$	
$a = \bar{y} - b\bar{x}$	
$b = \frac{s_{x,y}}{s_x^2} = \frac{\hat{s}_{x,y}}{\hat{s}_x^2}$	

INFERENCE FOR ONE POPULATION				
Population	Hypothesis test	Test statistic	Rejection region ($p\text{-value} < \alpha$)	Confidence interval $\text{CI}_{1-\alpha}$
Any r.v. X with $E[X] = \mu, \text{Var}[X] = \sigma^2$ and $n \rightarrow \infty$	(1) $H_0 : \mu = \mu_0; H_1 : \mu \neq \mu_0$ (2) $H_0 : \mu \geq \mu_0; H_1 : \mu < \mu_0$ (3) $H_0 : \mu \leq \mu_0; H_1 : \mu > \mu_0$	(a) $z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ (b) $t_0 = \frac{\bar{x} - \mu_0}{\hat{s}/\sqrt{n}}$	(1a) $ z_0 > z_{\alpha/2}$ (1b) $ t_0 > z_{\alpha/2}$ (2a) $z_0 < -z_{\alpha}$ (2b) $t_0 < -z_{\alpha}$ (3a) $z_0 > z_{\alpha}$ (3b) $t_0 > z_{\alpha}$	$\mu \in (\bar{x} \mp z_{\alpha/2}\sigma/\sqrt{n})$ $\mu \in (\bar{x} \mp z_{\alpha/2}\hat{s}/\sqrt{n})$
Normal $N(\mu, \sigma^2)$	(1) $H_0 : \mu = \mu_0; H_1 : \mu \neq \mu_0$ (2) $H_0 : \mu \geq \mu_0; H_1 : \mu < \mu_0$ (3) $H_0 : \mu \leq \mu_0; H_1 : \mu > \mu_0$	(a) $z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ (b) $t_0 = \frac{\bar{x} - \mu_0}{\hat{s}/\sqrt{n}}$	(1a) $ z_0 > z_{\alpha/2}$ (1b) $ t_0 > t_{n-1;\alpha/2}$ (2a) $z_0 < -z_{\alpha}$ (2b) $t_0 < -t_{n-1;\alpha}$ (3a) $z_0 > z_{\alpha}$ (3b) $t_0 > t_{n-1;\alpha}$	$\mu \in (\bar{x} \mp z_{\alpha/2}\sigma/\sqrt{n})$ $\mu \in (\bar{x} \mp t_{n-1;\alpha/2}\hat{s}/\sqrt{n})$
Bernoulli $B(p)$ with $n \rightarrow \infty$	(1) $H_0 : p = p_0; H_1 : p \neq p_0$ (2) $H_0 : p \geq p_0; H_1 : p < p_0$ (3) $H_0 : p \leq p_0; H_1 : p > p_0$	$z_0 = \frac{\hat{p} - p_0}{\sqrt{p_0 q_0/n}}$	(1) $ z_0 > z_{\alpha/2}$ (2) $z_0 < -z_{\alpha}$ (3) $z_0 > z_{\alpha}$	$p \in \left(\hat{p} \mp z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} \right)$
Normal $N(\mu, \sigma^2)$	(1) $H_0 : \sigma^2 = \sigma_0^2; H_1 : \sigma^2 \neq \sigma_0^2$ (2) $H_0 : \sigma^2 \geq \sigma_0^2; H_1 : \sigma^2 < \sigma_0^2$ (3) $H_0 : \sigma^2 \leq \sigma_0^2; H_1 : \sigma^2 > \sigma_0^2$	$\chi_0^2 = \frac{(n-1)\hat{s}^2}{\sigma_0^2} = \frac{ns^2}{\sigma_0^2}$	(1) $\chi_0^2 > \chi_{n-1;\alpha/2}^2$ or $\chi_0^2 < \chi_{n-1;1-\alpha/2}^2$ (2) $\chi_0^2 < \chi_{n-1;1-\alpha}^2$ (3) $\chi_0^2 > \chi_{n-1;\alpha}^2$	$\sigma^2 \in \left(\frac{(n-1)\hat{s}^2}{\chi_{n-1;\alpha/2}^2}; \frac{(n-1)\hat{s}^2}{\chi_{n-1;1-\alpha/2}^2} \right)$
Any r.v. X with $\hat{\theta}_{MV}$ and $n \rightarrow \infty$	(1) $H_0 : \theta = \theta_0; H_1 : \theta \neq \theta_0$ (2) $H_0 : \theta \geq \theta_0; H_1 : \theta < \theta_0$ (3) $H_0 : \theta \leq \theta_0; H_1 : \theta > \theta_0$	$t_0 = \frac{\hat{\theta}_{MV} - \theta_0}{\sqrt{\widehat{\text{Var}}(\hat{\theta}_{MV})}}$ $\widehat{\text{Var}}(\hat{\theta}_{MV}) = - \left(\frac{\partial^2 L(\hat{\theta}_{MV})}{\partial \theta^2} \right)^{-1}$	(1) $ t_0 > z_{\alpha/2}$ (2) $t_0 < -z_{\alpha}$ (3) $t_0 > z_{\alpha}$	$\theta \in \left(\hat{\theta}_{MV} \mp z_{\alpha/2} \sqrt{\widehat{\text{Var}}(\hat{\theta}_{MV})} \right)$

INFERENCE FOR TWO POPULATIONS

Two populations or r.v. X_1, X_2	Hypothesis test	Test statistic	Rejection region (p -value $< \alpha$)
Any with $E(X_1) = \mu_1, E(X_2) = \mu_2$ $\text{Var}(X_1) = \sigma_1^2$ $\text{Var}(X_2) = \sigma_2^2$	(1) $H_0 : \mu_1 = \mu_2; H_1 : \mu_1 \neq \mu_2$ (2) $H_0 : \mu_1 \geq \mu_2; H_1 : \mu_1 < \mu_2$ (3) $H_0 : \mu_1 \leq \mu_2; H_1 : \mu_1 > \mu_2$	(a) $z_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$ (b) $t_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\hat{s}_1^2}{n_1} + \frac{\hat{s}_2^2}{n_2}}}$	(1a) $ z_0 > z_{\alpha/2}$ (1b) $ t_0 > z_{\alpha/2}$ (1b) $ t_0 > t_{\nu; \alpha/2}$ (2a) $z_0 < -z_{\alpha}$ (2b) $t_0 < -z_{\alpha}$ (2b) $t_0 < -t_{\nu; \alpha}$ (3a) $z_0 > z_{\alpha}$ (3b) $t_0 > z_{\alpha}$ (3b) $t_0 > t_{\nu; \alpha}$ Under normality If $n_1, n_2 \rightarrow \infty$ Under normality or if $n_1, n_2 \rightarrow \infty$
Paired data $D = X_1 - X_2$	(1) $H_0 : \mu_D = 0; H_1 : \mu_D \neq 0$ (2) $H_0 : \mu_D \geq 0; H_1 : \mu_D < 0$ (3) $H_0 : \mu_D \leq 0; H_1 : \mu_D > 0$	(a) $z_0 = \frac{\bar{d}}{\sigma_D / \sqrt{n}}$ (b) $t_0 = \frac{\bar{d}}{\hat{s}_D / \sqrt{n}}$	(1a) $ z_0 > z_{\alpha/2}$ (1b) $ t_0 > z_{\alpha/2}$ (1b) $ t_0 > t_{n-1; \alpha/2}$ (2a) $z_0 < -z_{\alpha}$ (2b) $t_0 < -z_{\alpha}$ (2b) $t_0 < -t_{n-1; \alpha}$ (3a) $z_0 > z_{\alpha}$ (3b) $t_0 > z_{\alpha}$ (3b) $t_0 > t_{n-1; \alpha}$ Under normality If $n \rightarrow \infty$ Under normality or if $n \rightarrow \infty$
Any with $E(X_1) = \mu_1, E(X_2) = \mu_2$ $\text{Var}(X_1) = \text{Var}(X_2) = \sigma^2$	(1) $H_0 : \mu_1 = \mu_2; H_1 : \mu_1 \neq \mu_2$ (2) $H_0 : \mu_1 \geq \mu_2; H_1 : \mu_1 < \mu_2$ (3) $H_0 : \mu_1 \leq \mu_2; H_1 : \mu_1 > \mu_2$	(a) $z_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$ (b) $t_0 = \frac{\bar{x}_1 - \bar{x}_2}{\hat{s}_T \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$ with $\hat{s}_T^2 = \frac{(n_1-1)\hat{s}_1^2 + (n_2-1)\hat{s}_2^2}{n_1+n_2-2}$	(1a) $ z_0 > z_{\alpha/2}$ (1b) $ t_0 > z_{\alpha/2}$ (1b) $ t_0 > t_{n_1+n_2-2; \alpha/2}$ (2a) $z_0 < -z_{\alpha}$ (2b) $t_0 < -z_{\alpha}$ (2b) $t_0 < -t_{n_1+n_2-2; \alpha}$ (3a) $z_0 > z_{\alpha}$ (3b) $t_0 > z_{\alpha}$ (3b) $t_0 > t_{n_1+n_2-2; \alpha}$ Under normality If $n_1, n_2 \rightarrow \infty$ Under normality or if $n_1, n_2 \rightarrow \infty$
Bernoulli r.v. $X_1 \sim B(p_1), X_2 \sim B(p_2)$	(1) $H_0 : p_1 = p_2; H_1 : p_1 \neq p_2$ (2) $H_0 : p_1 \geq p_2; H_1 : p_1 < p_2$ (3) $H_0 : p_1 \leq p_2; H_1 : p_1 > p_2$	$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_0 \hat{q}_0 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$ with $\hat{p}_0 = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2}$	(1) $ z_0 > z_{\alpha/2}$ (2) $z_0 < -z_{\alpha}$ (3) $z_0 > z_{\alpha}$ If $n_1, n_2 \rightarrow \infty$
Normal r.v. $X_1 \sim N(\mu_1, \sigma_1^2)$ $X_2 \sim N(\mu_2, \sigma_2^2)$	(1) $H_0 : \sigma_1^2 = \sigma_2^2; H_1 : \sigma_1^2 \neq \sigma_2^2$ (2) $H_0 : \sigma_1^2 \geq \sigma_2^2; H_1 : \sigma_1^2 < \sigma_2^2$ (3) $H_0 : \sigma_1^2 \leq \sigma_2^2; H_1 : \sigma_1^2 > \sigma_2^2$	$F_0 = \frac{\hat{s}_1^2}{\hat{s}_2^2}$	(1) $F_0 > F_{n_1-1; n_2-1; \alpha/2}$ or $F_0 < F_{n_1-1; n_2-1; 1-\alpha/2}$ where $F_{n_1-1; n_2-1; 1-\alpha/2} = 1 / F_{n_2-1; n_1-1; \alpha/2}$ (2) $F_0 < F_{n_1-1; n_2-1; 1-\alpha}$ (3) $F_0 > F_{n_1-1; n_2-1; \alpha}$

Two populations or r.v. X_1, X_2	Parameter	Confidence interval $CI_{1-\alpha}$
Any with $E(X_1) = \mu_1, E(X_2) = \mu_2$ $\text{Var}(X_1) = \sigma_1^2$ $\text{Var}(X_2) = \sigma_2^2$	$\mu_1 - \mu_2$	$\mu_1 - \mu_2 \in \left(\bar{x}_1 - \bar{x}_2 \mp z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \right) \text{ Under normality or if } n_1, n_2 \rightarrow \infty$ $\mu_1 - \mu_2 \in \left(\bar{x}_1 - \bar{x}_2 \mp z_{\alpha/2} \sqrt{\frac{\hat{s}_1^2}{n_1} + \frac{\hat{s}_2^2}{n_2}} \right) \text{ If } n_1, n_2 \rightarrow \infty$ $\mu_1 - \mu_2 \in \left(\bar{x}_1 - \bar{x}_2 \mp t_{\nu; \alpha/2} \sqrt{\frac{\hat{s}_1^2}{n_1} + \frac{\hat{s}_2^2}{n_2}} \right) \text{ with } \nu \approx \frac{\left(\frac{\hat{s}_1^2}{n_1} + \frac{\hat{s}_2^2}{n_2} \right)^2}{\frac{1}{n_1-1} \left(\frac{\hat{s}_1^2}{n_1} \right)^2 + \frac{1}{n_2-1} \left(\frac{\hat{s}_2^2}{n_2} \right)^2} \text{ Under normality}$
Any with $E(X_1) = \mu_1, E(X_2) = \mu_2$ $\text{Var}(X_1) = \text{Var}(X_2) = \sigma^2$	$\mu_1 - \mu_2$	$\mu_1 - \mu_2 \in \left(\bar{x}_1 - \bar{x}_2 \mp z_{\alpha/2} \sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \right) \text{ Under normality or if } n_1, n_2 \rightarrow \infty$ $\mu_1 - \mu_2 \in \left(\bar{x}_1 - \bar{x}_2 \mp z_{\alpha/2} \hat{s}_T \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \right) \text{ If } n_1, n_2 \rightarrow \infty$ $\mu_1 - \mu_2 \in \left(\bar{x}_1 - \bar{x}_2 \mp t_{n_1+n_2-2; \alpha/2} \hat{s}_T \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \right) \text{ Under normality}$
Bernoulli r.v. $X_1 \sim B(p_1), X_2 \sim B(p_2)$	$p_1 - p_2$	$p_1 - p_2 \in \left(\hat{p}_1 - \hat{p}_2 \mp z_{\alpha/2} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} \right) \text{ If } n_1, n_2 \rightarrow \infty$
Normal r.v. $X_1 \sim N(\mu_1, \sigma_1^2)$ $X_2 \sim N(\mu_2, \sigma_2^2)$	$\frac{\sigma_1^2}{\sigma_2^2}$	$\frac{\sigma_1^2}{\sigma_2^2} \in \left(\frac{\hat{s}_1^2}{\hat{s}_2^2} F_{n_2-1; n_1-1; 1-\alpha/2}, \frac{\hat{s}_1^2}{\hat{s}_2^2} F_{n_2-1; n_1-1; \alpha/2} \right) \text{ where } F_{n_2-1; n_1-1; 1-\alpha/2} = 1 / F_{n_1-1; n_2-1; \alpha/2}$