Work and Energy

- Closely related...
- Work is a measure of the amount of change that a force produces when it acts on a body

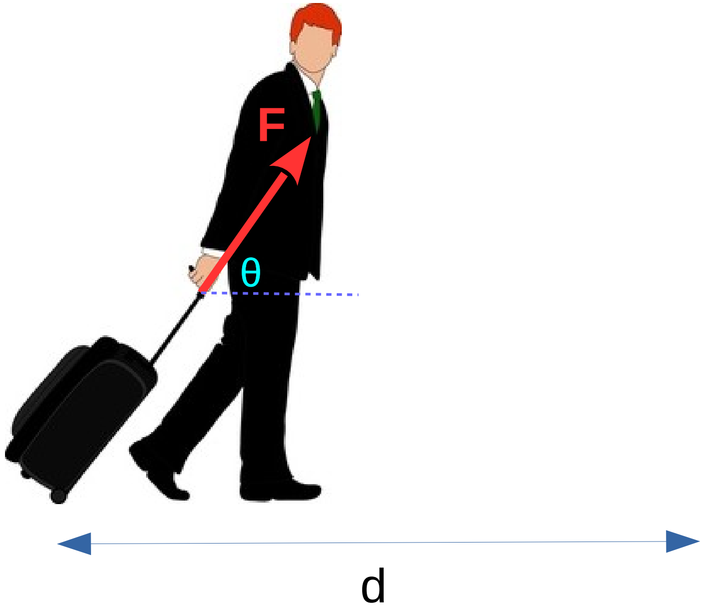
Work by done by constant F

- Work = displacement * force responsible for the displacement. $\boxed{W=\vec{F}\cdot\Delta\vec{r}=F\Delta r\cos\theta}$ SI unit: Joule = N*m

  W is a scalar!

  if the displacement is zero, there is no work done

- Example: pulling a suitcase. Find the work done if F=50 N, θ=60, d=10 m.



Answer: 250 J

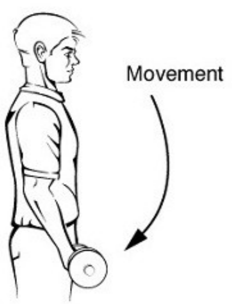
- Example: bicep workout. How much work is done on the barbell?

B

positive work.

A

$A \rightarrow B \rightarrow A$ no work done!



B

negative work

A

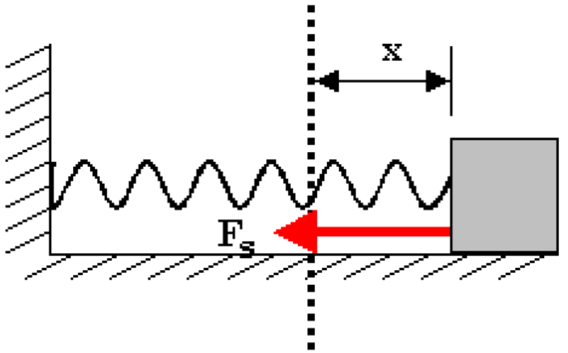
Keep track of who is doing the work!
Work done on a particular body by a specific force!

Total Work

- Total Work done by several Forces acting on a body $\boxed{W_{tot}=\sum W_i=\sum \vec{F}_i\cdot\Delta\vec{r}=\vec{F}_{tot}\cdot\Delta\vec{r}}$

- Work done by non constant force: $\boxed{W=\int_A^B \vec{F}\cdot d\vec{r}}$

- Example: Spring exerts a force F=-kx on an object. What is the work done from the spring to the object as it moves from x=a to x=b?



Work and Energy

- Energy is the ability to do work
- Work is the process of transferring energy

Mechanical Energy

Kinetic Energy

energy of motion

Potential Energy

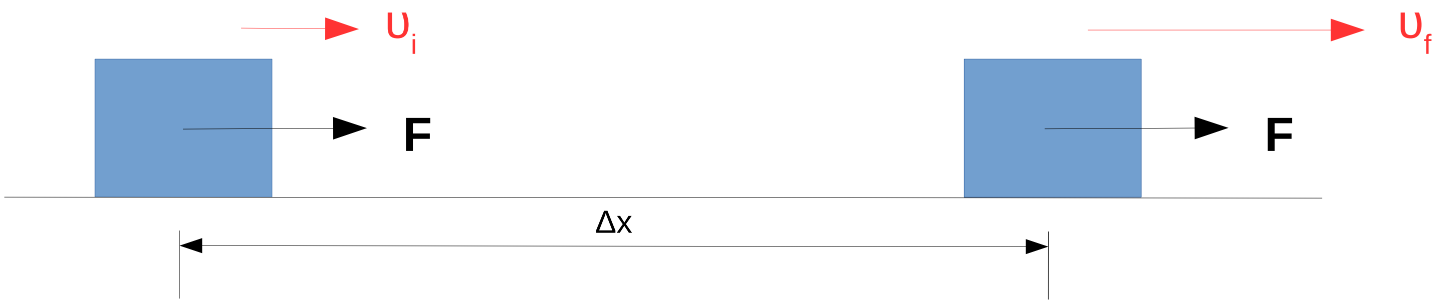
stored energy of position

Kinetic energy

- The amount of kinetic energy a moving object has is given by $\boxed{K=\frac{1}{2}m\upsilon^2}$

Why?

Consider an object moving in 1D under a constant force F. W=?



Work-Kinetic Energy theorem

- The amount of kinetic energy a moving object has is given by $\boxed{K = \frac{1}{2}mv^2}$

- WORK-KINETIC ENERGY THEOREM:

  When a net external force does work on an object, the kinetic energy of the object changes according to $\boxed{W_{total} = \Delta K = K_f - K_i}$

  K of a particle is equal to the total work that was done to accelerate it from rest to its present speed

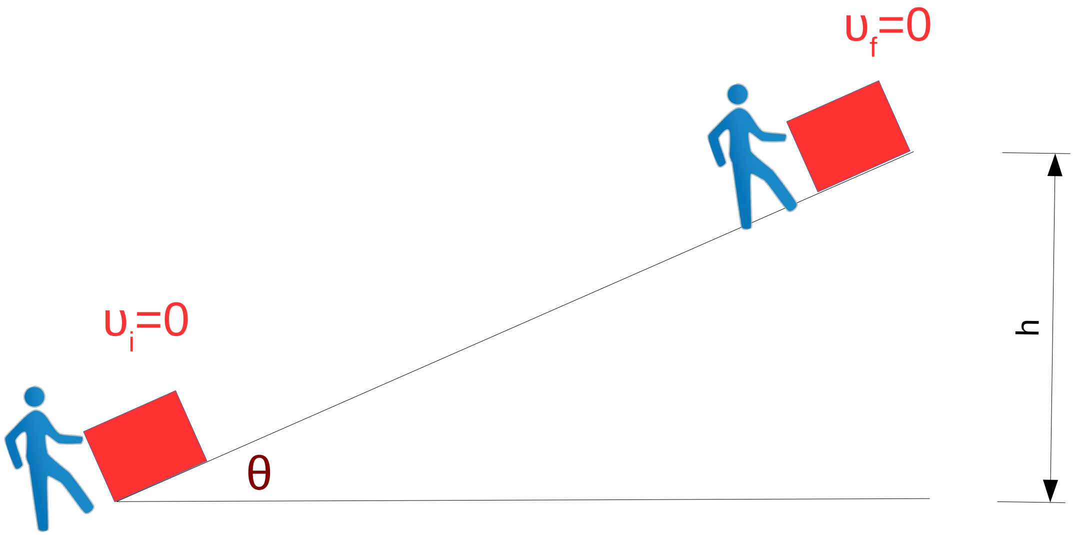
Work-Kinetic Energy theorem

- Example: A gymnast leaves the trampoline at an initial height of 1.20 m and reaches a maximum height of 4.80 m before falling back down. What was the initial speed of the gymnast?

Answer: 8.40 m/s

Work-Kinetic Energy theorem

- What is the work required to push a box along an inclined ramp at height h? Does it depend on the angle θ?



Potential energy

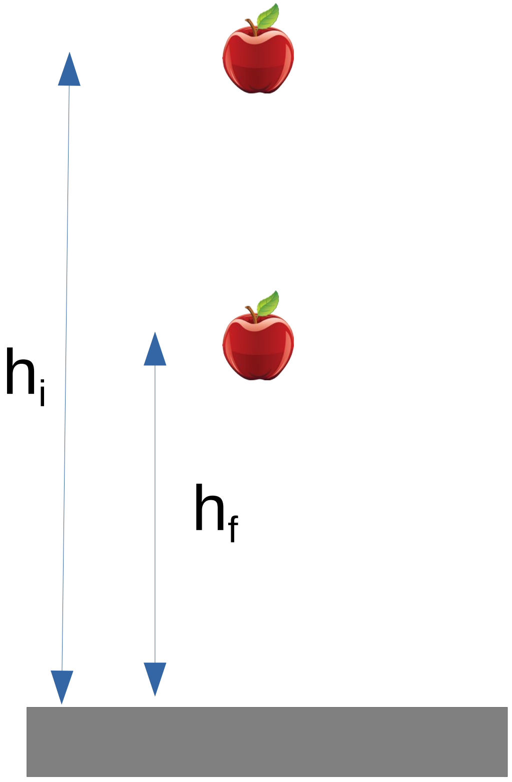
- The potential energy that an object in a certain position acquires is equal to the amount of work it takes to put the object in that position.
- Potential energy: gravitational, elastic, electromagnetic, chemical…

  Gravitational potential energy $U = mgh$

  Electrostatic potential energy $U=qV$

Potential energy

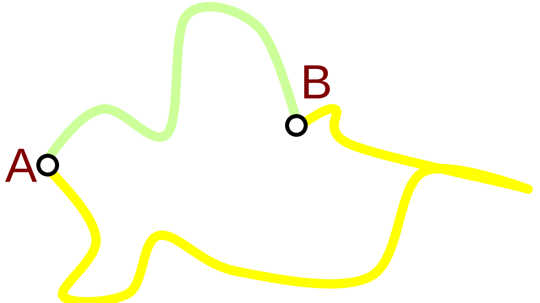
Example: Find the work done by gravity in accelerating an apple from h_i to h_f



$$\boxed{W_g = mg(h_i - h_f) = -\Delta U}$$

Conservative Forces

- v1: A force is conservative when the work it does on a moving object is independent of the path between the object's initial and final positions.
- v2: A force is conservative when it does no work on an object moving around a closed path, starting and finishing at the same point.



$$W_{AB} + W_{BA} = 0$$

Conservative vs Nonconservative Forces

- Conservative F:

  Gravitational

  Electrostatic (ch 3)

  Elastic spring

- Nonconservative F:

  Friction

  Air resistance

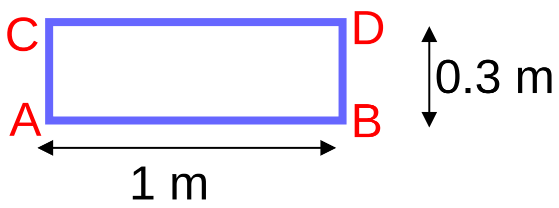
  Push, pull

  Tension

  Normal

Nonconservative Forces

- Example: Two identical boxes of mass 2 kg are moved from A to B along two different trajectories on a table that has a coefficient of kinetic friction of 0.2. The first box is moved along a straight line from A to B, while the second box is moved following the path A→C→D→B (see figure). Find the work done by the frictional force in the two cases, thus demonstrating that friction is a non-conservative force.



Answer: First box → W_f = -3.9 J
Second box → W_f = -6.3 J

Conservation of Mechanical Energy

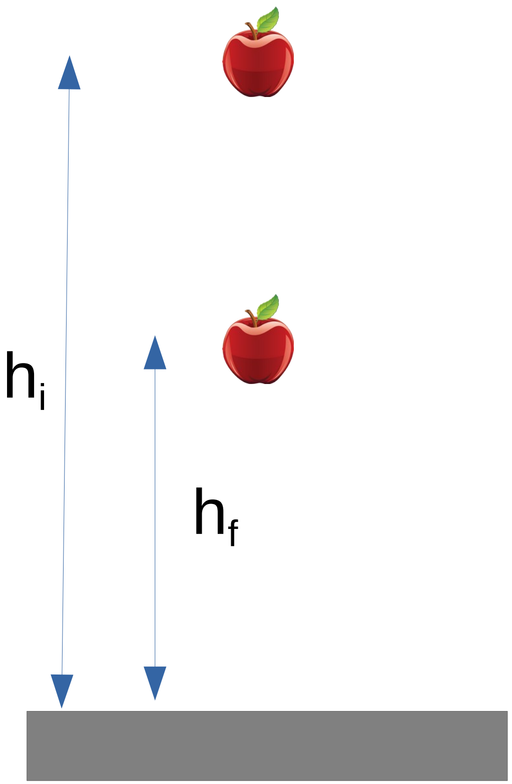
Example: An apple is released from h_i

Work-KE theorem: $\boxed{W_{total} = \Delta K = W_g}$

$$\boxed{W_g = mg\left(h_i - h_f\right) = -\Delta U}$$

$$\boxed{\Delta K + \Delta U = 0}$$

$$\boxed{K_i + U_i = K_f + U_f}$$



Conservation of Mechanical Energy

The total mechanical energy $\boxed{E = K + U}$

of an object is conserved when only conservative forces work on the system.

Example: A body of mass 2 kg is released from a height of 3 m over the floor and free falls.

(a) Apply the work-energy theorem to find the speed of the body at a height of 1 m and when it reaches the floor.

(b) Determine U, K and E in both positions, and check that the mechanical energy E is conserved.

Answer: (a) v (1 m) = 6.3 m/s; v (0 m) = 7.7 m/s
(b) U (1 m) = 19.62 J; U (0 m) = 0 J; K (1 m) = 39.24 J;
K (0 m) = 58.86 J; E = 58.86 J

Conservation of Energy

Energy cannot be created or destroyed – it can only be changed from one form to another.

Assume both conservative and nonconservative forces act simultaneously on an object

$$\boxed{W = W_C + W_{NC}}$$

$$\boxed{W = K_f - K_i = \Delta K} \qquad \boxed{W_C = U_i - U_f = -\Delta U}$$

so that $\boxed{\Delta K = -\Delta U + W_{NC}}$ $\rightarrow$

$$\boxed{W_{NC} = \Delta K + \Delta U = K_f - K_i + U_f - U_i = \left(K_f + U_f\right) - \left(K_i + U_i\right) = E_f - E_i = \Delta E}$$

If the net work on an object by nonconservative forces is zero, then its energy does not change

Power

Power is the time rate at which work is done

- Average power: $\boxed{P_{av} = \frac{Work}{time} = \frac{W}{t}}$ SI units: Watt=J/s

- instantaneous mechanical power generated by a Force $\boxed{P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} = \vec{F} \cdot \vec{v}}$

Example: A small motor is used to raise a load of bricks weighting 800 N to a height of 10 m in 20 s. What is the minimum power the motor must produce?

Answer: $P_{min} = 400\ W$