Coulomb’s law – Electric field

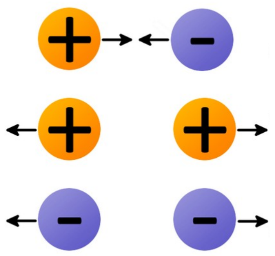
Electric charge

There are two kinds of charge:

1) positive charge → carried by protons
2) negative charge → carried by electrons

Charges of opposite sign attract each other

Charges of the same sign repel each other



unit of charge: Coulomb (C)

$q_p = 1.6\times10^{-19}$ C = +e

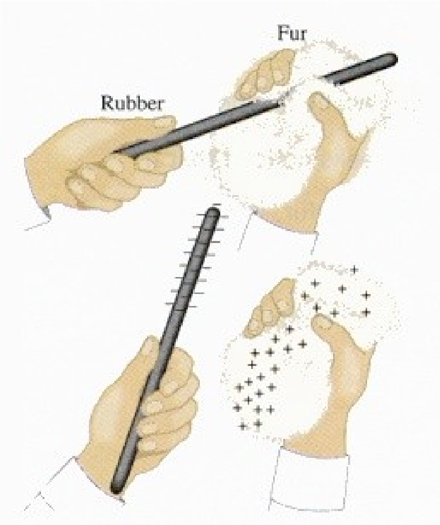
$q_e = 1.6\times10^{-19}$ C = -e

conservation of charge: the net electric charge in an isolated system always remains constant.

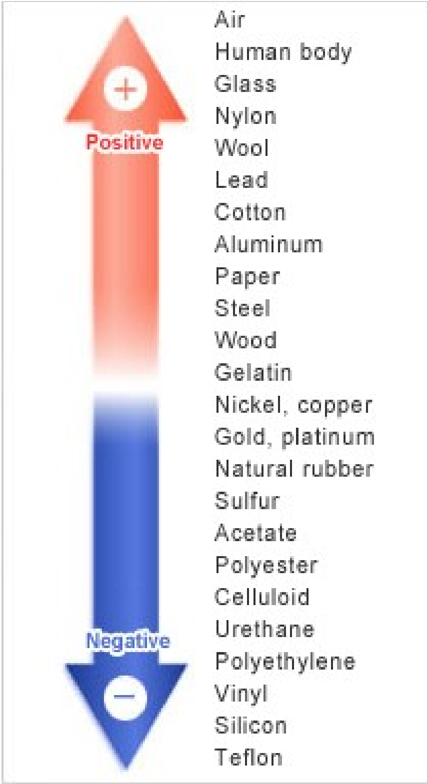
Electric charge: charging

There are 3 ways to induce charge in an object

1. Charging by friction

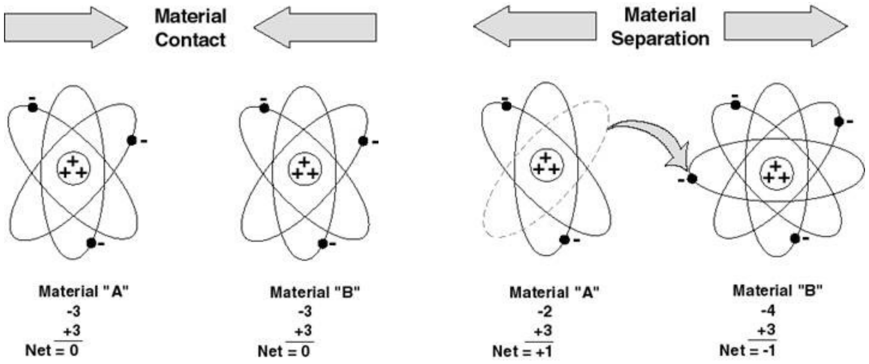


electron donating materials (+)



electron accepting materials (-)

Positively charged nuclei of one of the objects pulls the electrons from the other object



Insulators can be easily charged by friction

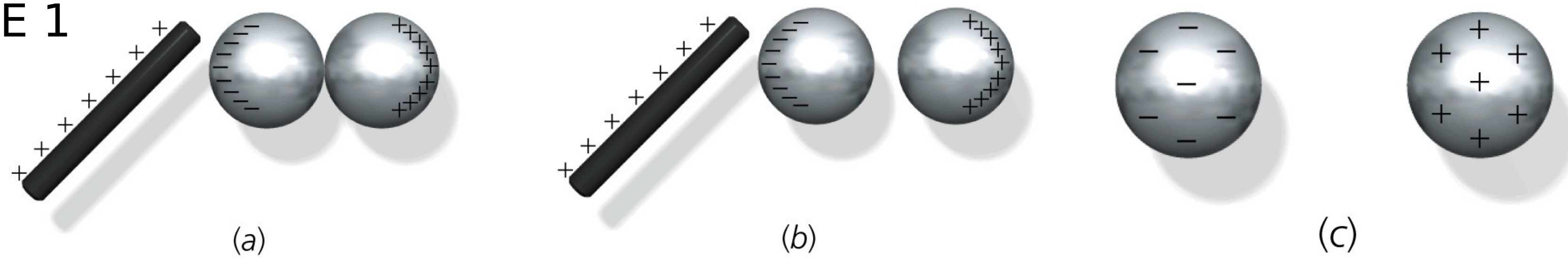
Electric charge: charging

There are 3 ways to induce charge in an object

2. Charging by induction

inducing electrons to move from one object to the other: only for metals

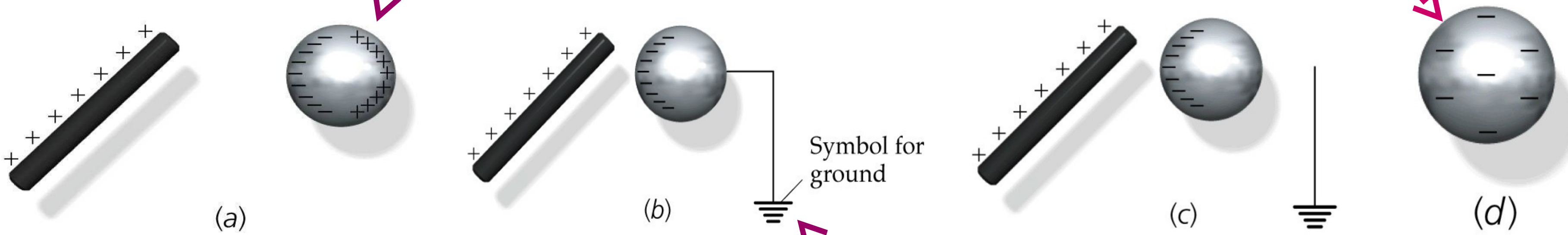
EXAMPLE 1



The sphere is polarized, but not charged

The sphere is charged

EXAMPLE 2



To ground: To put in contact with ground (Earth).

Earth is an enormous conductor that can gain or loose charge and remain unperturbed.

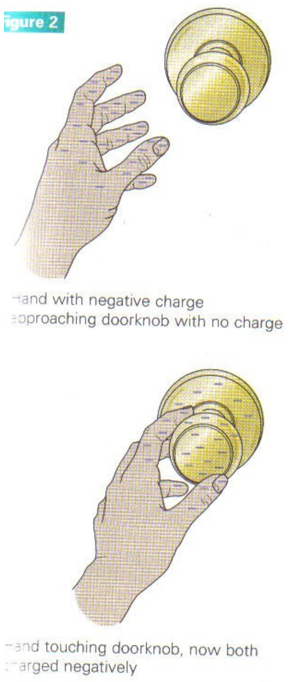
charging of one object by another <u>without direct contact</u>

Electric charge: charging

There are 3 ways to induce charge in an object

3. Charging by conduction

Electrons transfer from one object to the other

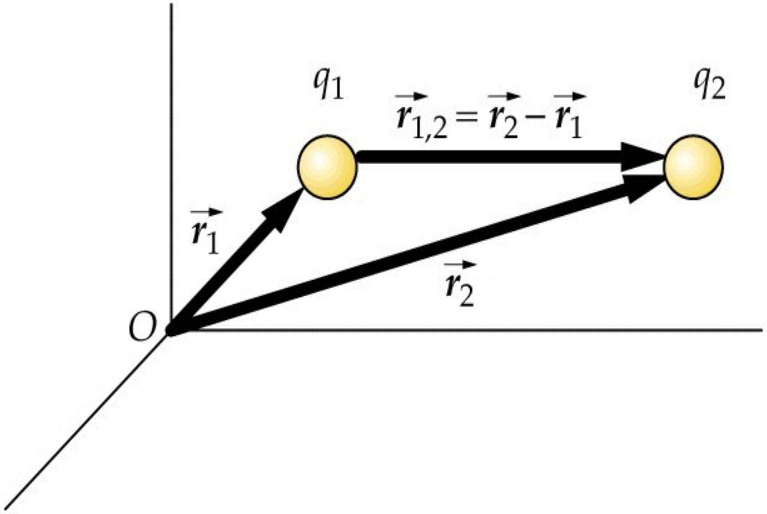


<u>direct contact</u> of a charged object to a neutral object

Coulomb's law

The force one charge exerts on another is given by *Coulomb's law*:

$$\vec{F}_{1,2}=k\frac{q_1 q_2}{r_{1,2}^2}\vec{u}_{1,2}$$



Coulomb's constant: $k=8.99\times10^{9}$ N.m^2/C^2

Also $k=1/(4\pi\varepsilon_0)$

Permittivity of free space $\varepsilon_0=8.854\times10^{-12}$ C^2/N.m^2

Following Newton's third law, the force exerted by q_1 on q_2 is the negative of the force exerted by q_2 on q_1:

$$\vec{F}_{1,2}=-\vec{F}_{2,1}$$

Coulomb's law

EXAMPLE:

1 C is a very large amount of charge!

1. A point charge of q_1=1C is positioned at $\vec{r}_1 = (-1,1,3)$ (m).

(a) What force will it exert on a second charge of q_2=-2C located at $\vec{r}_2 = (2,-1,0)$ (m) ?

(b) Find the magnitude of this force.

Coulomb's law

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Answer: (a) $F_{1,2} = (-5.2, 3.5, 5.2) \times 10^8\, N$ *(b) F = 8.2 x* 10^8 *N*

Electric field

An electric field is a region of space in which a charge would be acted upon by an electric force. An electric field may be produced by one or more charges, and it may be uniform or it may vary in magnitude and/or direction from place to place.

The electric field created by a charge at a point P is the force exerted by the charge on a test charge q_0, divided by the test charge:

$$\vec{E}_{q,P} = \frac{\vec{F}}{q_o} = k\frac{q}{r_{q,P}^2}\vec{u}_{q,P}$$

vector quantity

SI units: N/C

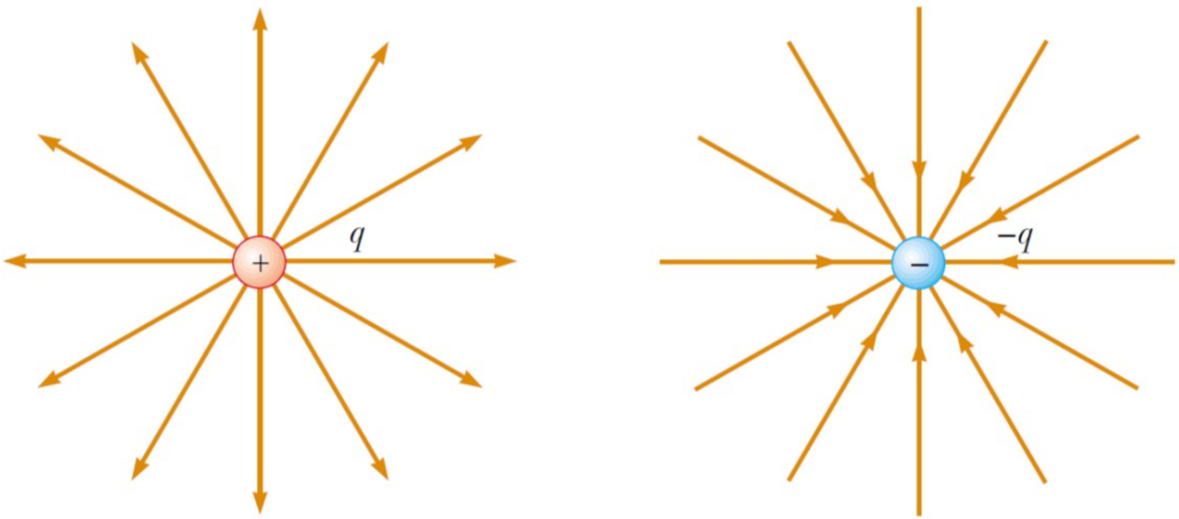
Note: This expression is valid at all points except the one occupied by the charge. The charge does not create an E on itself.

The advantage of knowing the electric field at some point is that we can at once establish the force on any charge q placed there: $\vec{F} = q\vec{E}$

Electric field lines

We can picture E by drawing lines to indicate its direction.

At any point, E will be tangent to the lines.

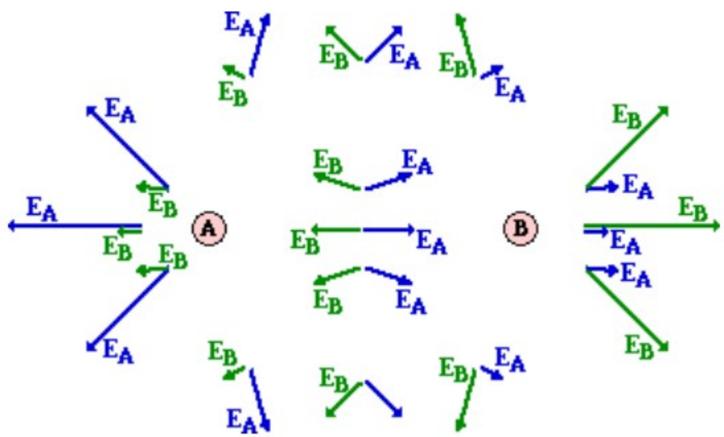


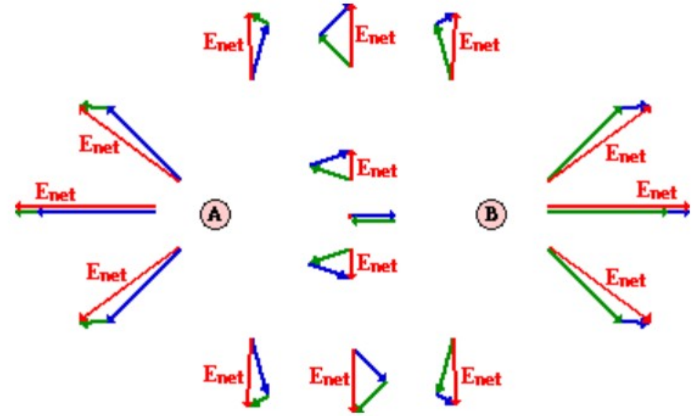
- The electric field lines begin on positive charges (or at infinity) and end on negative charges (or at infinity).
- They are drawn symmetrically entering or leaving an isolated charge.
- The density of lines at any point is proportional to the magnitude of the field at that point.
- Field lines never cross each other.

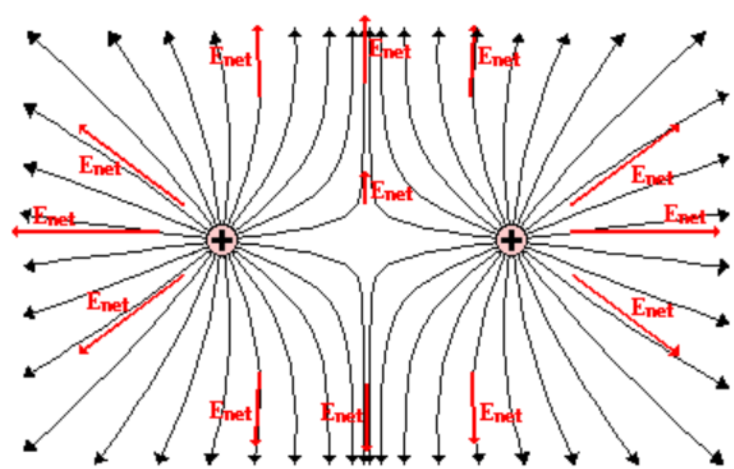
Electric field lines

What happens if we have several charges? The lines will represent the net field.

Example: Two positive equal charges (A, B)



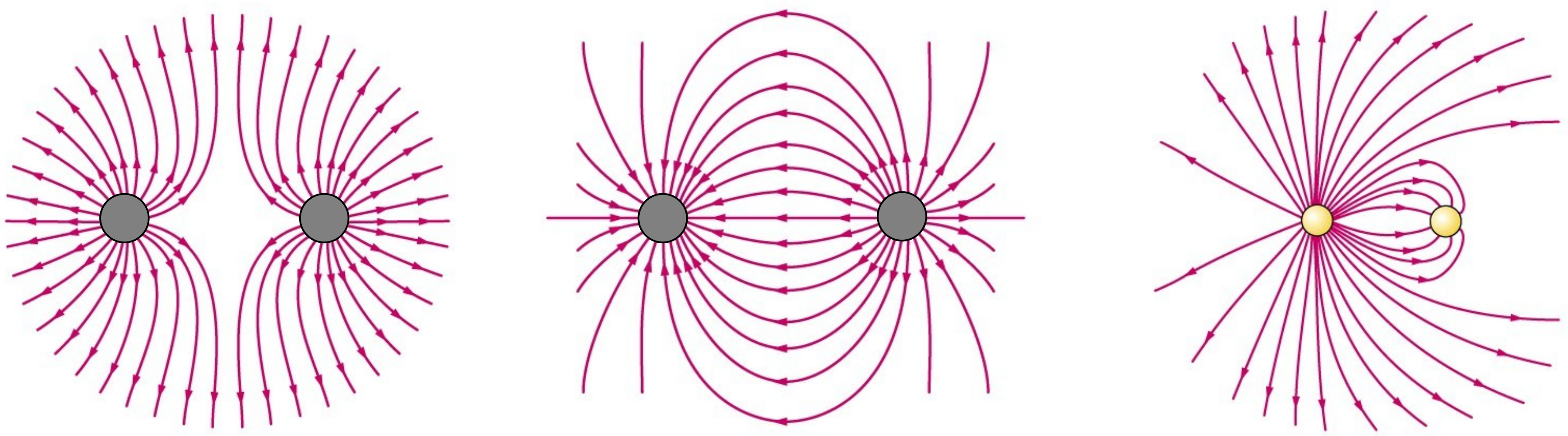




Electric field lines

EXAMPLE:

Excercise 9: The electric field lines for different systems formed by two point charges are shown in the figures below. What is the sign of the charges on each system, and what are the relative magnitudes of the charges?



Electric field lines

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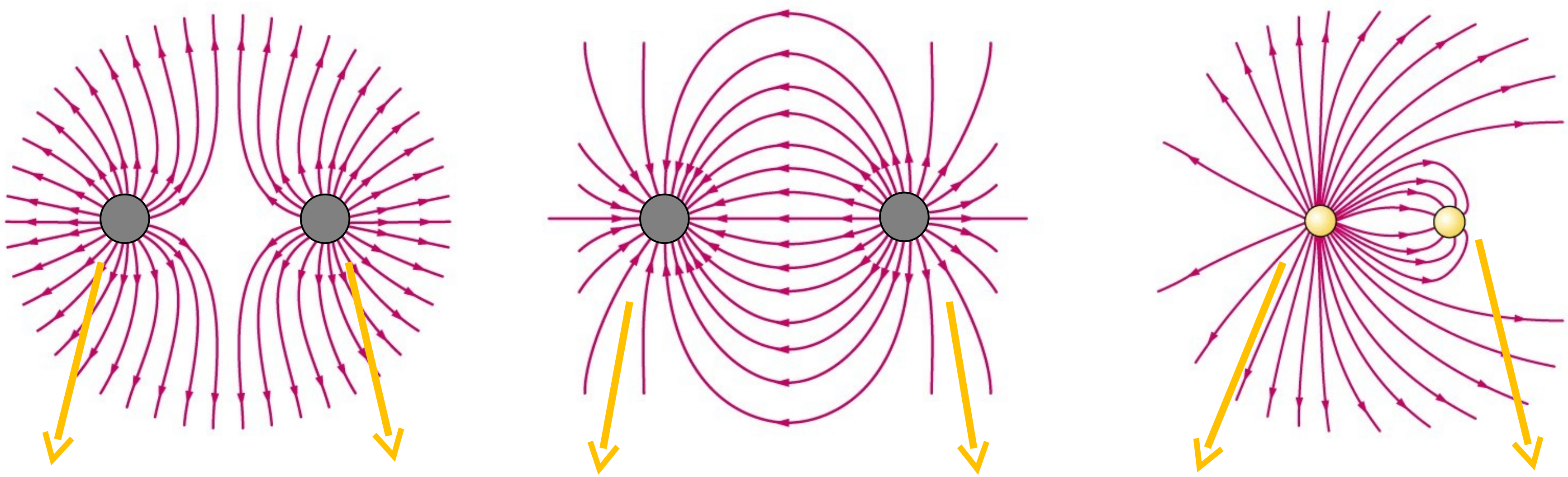
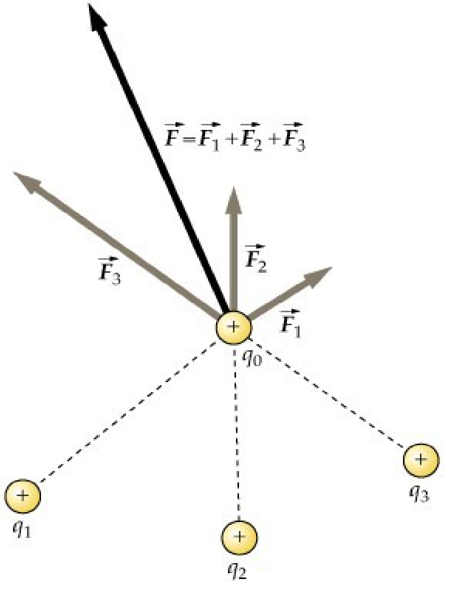


The superposition principle

In a system formed by several point charges, the net force over each charge is found by adding up the forces exerted by each of the other charges over it.

$$\vec{F}_{net} = \sum_i k \frac{q_i q_0}{r_{i,0}^2} \vec{u}_{i,0}$$



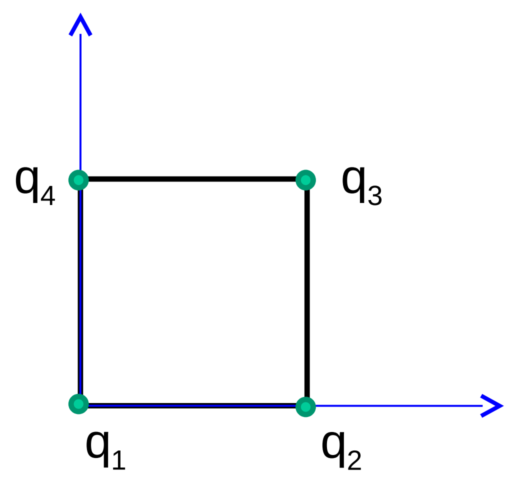
The superposition principle also holds for the electric field:

$$\vec{E}_{net}(p) = \sum_i \vec{E}_i(p) = \sum_i \frac{\vec{F}_i(p)}{q_0} = \sum_i k \frac{q_i}{r_{i,0}^2} \vec{u}_{i,0}$$

The superposition principle

EXAMPLE:

Excercise 6: Four charges of 1nC each are located at the corners of a square (see figure). The length of the sides is 2m, and one corner is taken as the origin of the reference frame. (a) Find the electric field due to q_1 at the centre of the square. (b) What would be the net force exerted on a - 1nC charge located at the centre? (c) Calculate the net electric field acting at (0, 2) m.



The superposition principle

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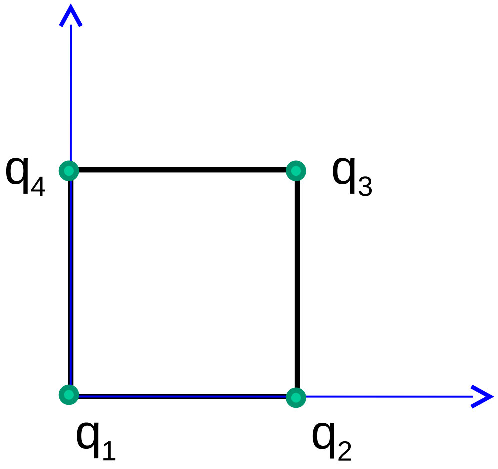


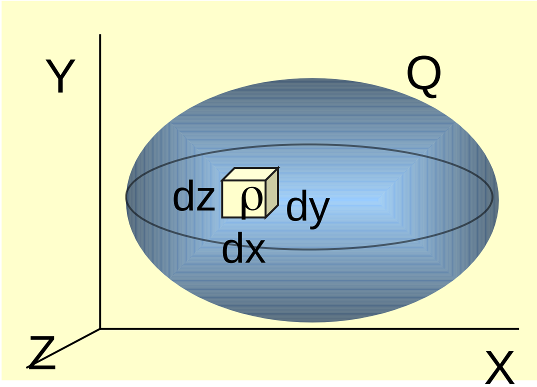
Answer: (a) $E_{1,C} = (3.18, 3.18)\ N/C$ (b) F=0 (c) $E_{net,4} = (-3.04, 3.04)\ N/C$

Continuous charge distibution

Charge densities

VOLUME CHARGE DENSITY

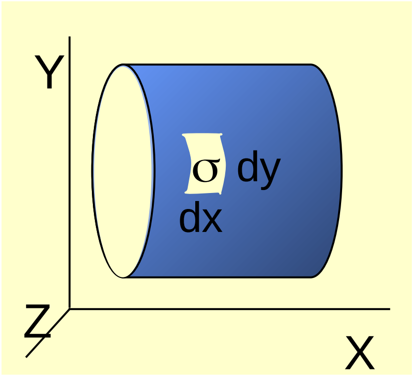
$$\rho = \lim_{\Delta V \to 0} \frac{\Delta Q}{\Delta V} = \frac{dQ}{dV}$$



$$Q = \int_V dQ = \int_V \rho \, dV$$

SURFACE CHARGE DENSITY

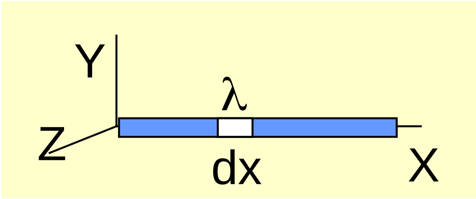
$$\sigma = \lim_{\Delta S \to 0} \frac{\Delta Q}{\Delta S} = \frac{dQ}{dS}$$



$$Q = \int_S dQ = \int_S \sigma \, dS$$

LINEAR CHARGE DENSITY

$$\lambda = \lim_{\Delta L \to 0} \frac{\Delta Q}{\Delta L} = \frac{dQ}{dL}$$



$$Q = \int_L dQ = \int_L \lambda \, dl$$

Continuous charge distribution

EXAMPLE 1:

Find the total charge of a sphere of radius 1 cm knowing that the volume charge density depends on the distance to the centre of the sphere "r" as $\rho = 5r$ (C/m^3 when "r" is given in meters). Note that for a sphere $dV = 4\pi r^2 dr$.

Volume of a sphere: $(4/3)\ \pi R^3$ Surface of a sphere: $4\pi R^2$ Perimeter of a circle: $2\pi R$

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ANSWER: $Q = 1.57x10^{-7}$ *C*

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EXAMPLE 2:

12. A charge of Q = 5 μC is uniformly distributed throughout the volume of a sphere of radius R= 20 cm.

a) Find the charge density.

b) Find the charge density if the charge is uniformly distributed on the surface of the sphere.

c) Find the charge density if the charge is uniformly distributed along a line coinciding with the equator of the sphere.

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EXAMPLE 2: **Exercise 12**

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a) Find the charge density.

b) Find the charge density if the charge is uniformly distributed on the surface of the sphere.

c) Find the charge density if the charge is uniformly distributed along a line coinciding with the equator of the sphere.

ANSWER: a) $\rho = 1.5 \times 10^{-4}\ C/m^3$ b) $\sigma = 1 \times 10^{-5}\ C/m^2$ c) $\lambda = 4 \times 10^{-6}\ C/m$

Volume of a sphere: $(4/3)\ \pi R^3$ Surface of a sphere: $4\pi R^2$ Perimeter of a circle: $2\pi R$

Continuous charge distribution

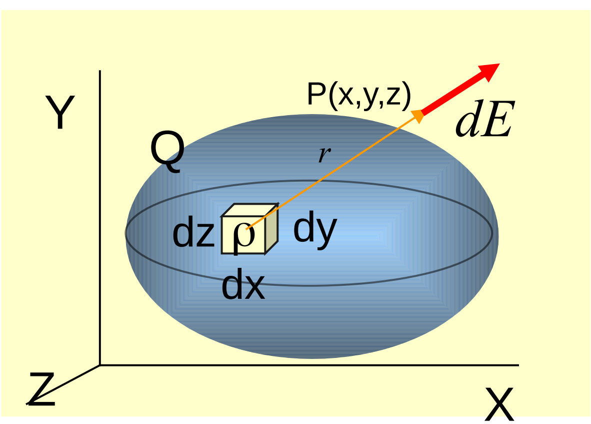
ELECTRIC FIELD DUE TO A "CONTINUOUS" CHARGE DISTRIBUTION

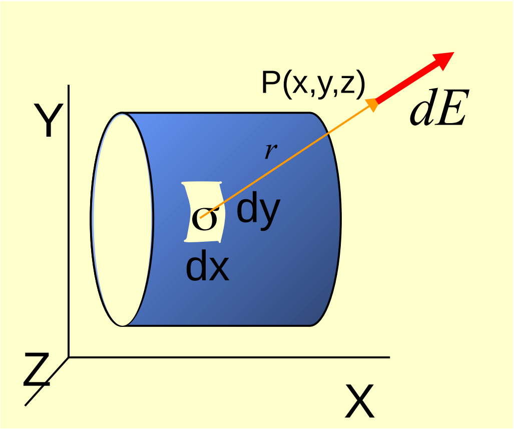
Each element of the volume with a charge dQ could be considered as a point charge creating an electric field dE at a point of space, so dE would be:

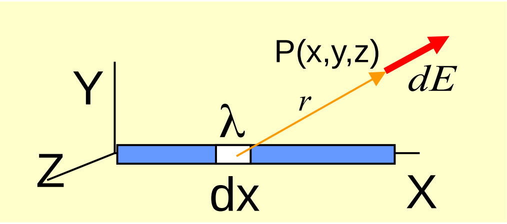
$$d\vec{E} = k\frac{dQ}{r^2}\vec{u}$$

The total electric field can be calculating by "adding" all those dE. This continuous sum is an integral.

$$\vec{E} = \int d\vec{E} = \int k\frac{dQ}{r^2}\vec{u}$$







$$\vec{E} = \int_V d\vec{E} = \int_V k\frac{\rho\, dV}{r^2}\vec{u}$$

$$\vec{E} = \int_S d\vec{E} = \int_S k\frac{\sigma\, dS}{r^2}\vec{u}$$

$$\vec{E} = \int_L d\vec{E} = \int_L k\frac{\lambda\, dl}{r^2}\vec{u}$$