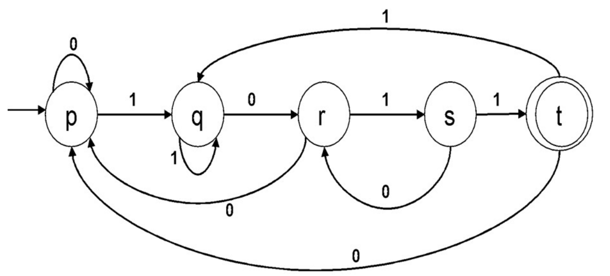
DEPARTAMENTO DE INFORMÁTICA
UNIVERSIDAD CARLOS III DE MADRID

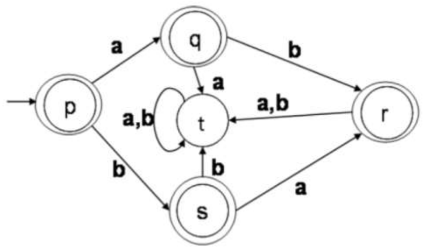
Grados en Matemáticas Aplicadas y Computación
Ciencia e Ingeniería de Datos
Ingeniería Informática

Formal Languages and Automata Theory
Exercises Regular Languages
Unit 5 – Part 1

1. Given the following automata, obtain the corresponding grammars.



(a)



(b)

2. Given the following grammar:

   G =({0,1}, {A,B,C}, A, P),
   P={A::=0B, A::= λ ,B::=1C , B::=1 , C::=0B)}

   Select the automaton that recognizes the generated language:

   a. AG=[{0,1}, {A,B,C,F}, f, A, {F}]
      f(A,0)=B, f(A, λ)= λ, f(C,0)=B, f(B,1)=C, f(B,1)= λ
   b. AG=[{0,1}, {A,B,C,F}, f, A, {F}]
      f(A,0)=B, f(A, λ)=F, f(C,0)=B, f(B,1)=C, f(B,1)=F
   c. AG=[{0,1}, {A,B,C,F}, f, A,{F}]
      f(A, B)=0, f(A,F)= λ, f(C,B)=0, f(B,C)=1, f(B,F)=1
   d. AG=[{0,1}, {A,B,C,F}, f, A, {F}]
      f(B,0)=A, f(F, λ)=A, f(B,0)=C, f(C,1)=B, f(F,1)=B

3. Obtain the Finite Automata that corresponds to the following grammars:

a) $G_1 = (\{a,b\}, \{S,A,B\}, S, P_1)$ $P_1 = \{$ S ::= aA | bA A ::= bB A ::= a B ::= aA | bA }	b) $G_2 = (\{a,b\}, \{S,A,B\}, S, P_2)$ $P_2 = \{$ S ::= aA A ::= aA | bB A ::= a | b B ::= aA | bA }
c) $G_3 = (\{0,1\}, \{S,A,B\}, S, P_3)$ $P_3 = \{$ S ::= 0A | 1A A ::= 0A | 1A A ::= 0B | 1B B ::= 0B | 1B B ::= 0 | 1 }	d) $G_4 = (\{0,1\}, \{S,D,E\}, S, P_4)$ $P_4 = \{$ S ::= 1D | 0S | 0 | λ D ::= 0E | 1S | 1 E ::= 1E | 0D }

4. Obtain the minimum DFA equivalent to each one of the following grammars describing the intermediate steps: G → NFA→ DFA → minimal DFA.

a) $G_A = (\{a,b,c\}, \{S,A,B\}, S, P_A)$ $P_A = \{$ S ::= aA | bB | c A ::= aB | b | cA B ::= a | bA | c }	b) $G_B = (\{a,b,c\}, \{S,B,C,E\}, S, P_B)$ $P_B = \{$ S ::= a | aS | aB | cC C ::= c B ::= bE | b E ::= bB | b }
c) $G_C = (\{a,b,c\}, \{S,A,B,C,D\}, S, P_C)$ $P_C = \{$ S ::= aA A ::= aA | bB | a B ::= bB | bC | b C ::= bC | cD | bB D ::= bC | bB |cC }	d) $G_D = (\{c,f,d\}, \{A,B,C,D,E,F\}, A, P_D)$ $P_D = \{$ A ::= cB | cE | f | fC B ::= cB | fD | dE C ::= cA D ::= cD | fD E ::= cE | fF | dF F ::= cF | fF }

5. (Continuation of Exercise 2, Unit 3, Part 1) Given the alphabet {a,b}, design a DFA which recognizes 3-elements strings of the universal language. <u>Obtain the G3 corresponding to the automaton</u>.

6. (Continuation of Exercise 4, Unit 3, Part 2) [Exam] We have a door with only one lock. To open it, it is necessary to use three different keys (called *a*, *b*, and *c*), in a predefined order, which is following described:

   - Key *a*, then key *b*, then key *c*, or
   - Key *b*, then key *a*, then key *c*.

   If this order is not followed, then the lock is blocked (for instance, if the key *a* is used and following it is introduced again).

   Once the door is open, the introduction of keys in the lock (in every possible order) does not affect the closing device (i.e. the door remains open).

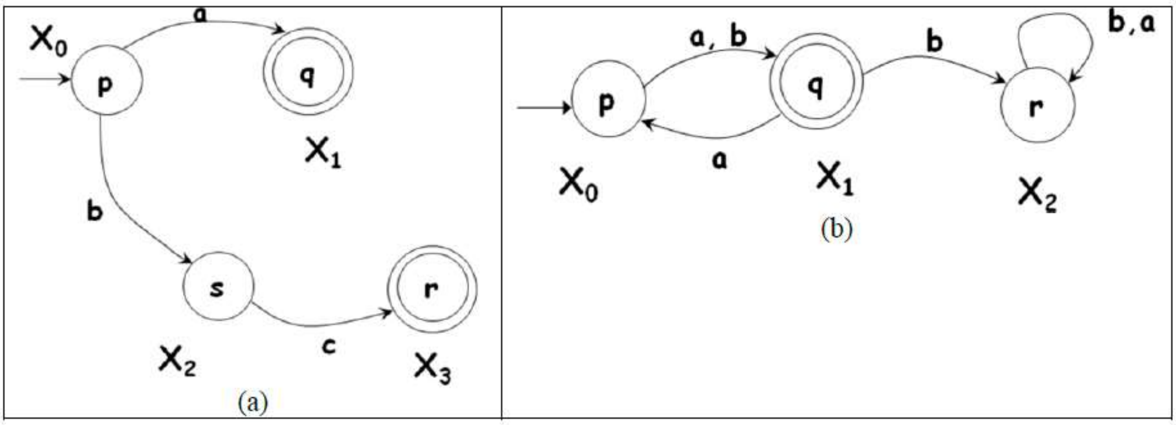
   Consider that the names of the different keys are symbols of an alphabet, over which a language L whose words are the valid sequences for the opening of the door is defined. For instance, *abcbc* is a word included in the language.

   It is required:

   a) Design a finite automata FA which accepts L.
   b) Well-formed Grammar which generates words in L.

7. Given $\Sigma = \{a,b,c\}$ and $\alpha 1 = a\ (a+b+c)^*$ and $\alpha 2 = a+bc+b2a$. Which are $L(\alpha 1)$ and $L(\alpha 2)$?

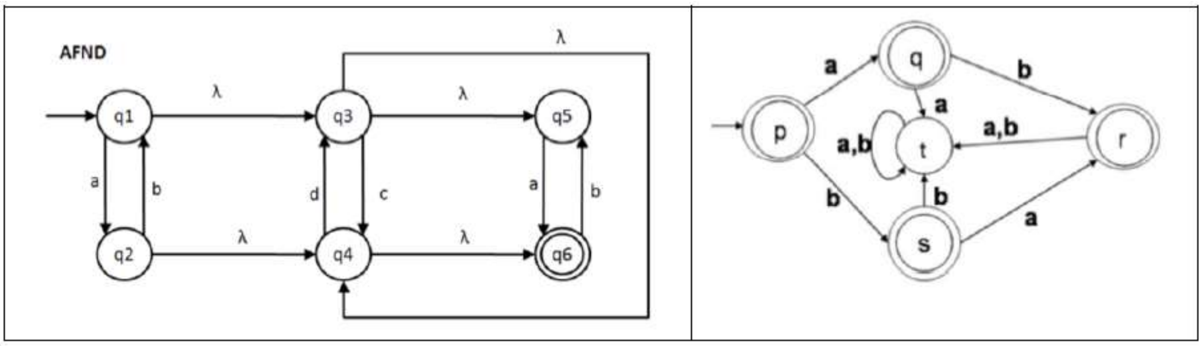
8. Obtain the characteristic formula of the FA defined by the following transition diagrams.



9. Given the RE=(b•a*)*, which represents a regular language, design a FA that accepts this regular language.

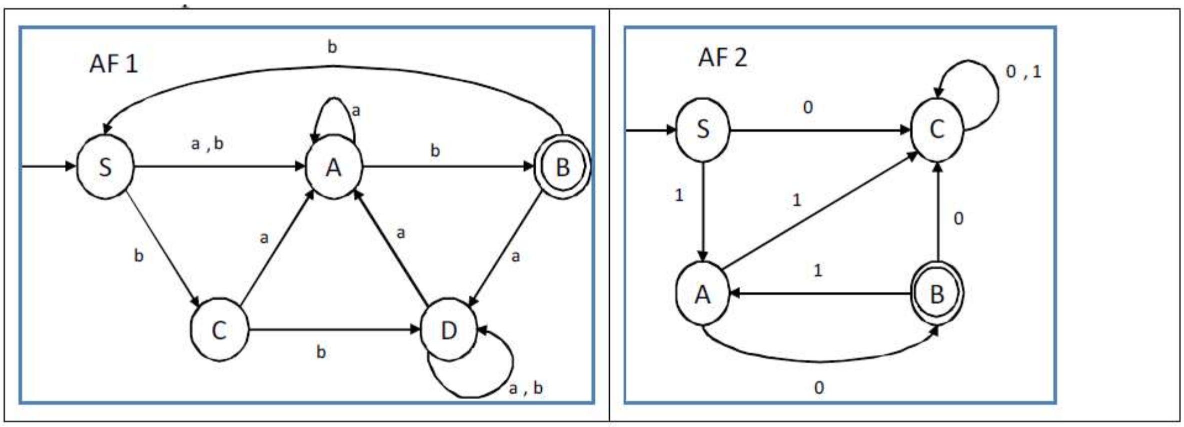
10. Given the RE=(b•a*)* obtain a G3 right-linear grammar that generates the language defined by this RE. Verify whether this G3 right-linear grammar would be the same that the one obtained from the FA designed in the previous problem. That is to say, exercise 4 (RE→G3RL$_1$), exercise 3 (RE→FA), verify whether (FA→G3RL$_2$) these 2 grammars are exactly equal or simply equivalent.

11. Given the FA represented in the following figures, obtain the REs which define the languages recognized by each FA.



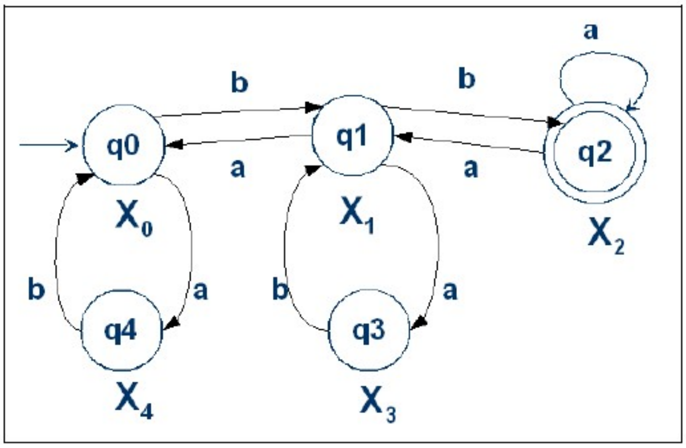
12. Obtain the RE that represents the language accepted for the FA that provides a solution of the enchanted castle exercise (Unit 3 – Parts 3 and 4. Exercise 2).

13. Indicate the REs corresponding to the languages recognized by the following Finite Automata.



14. Given the following grammar: G =({0,1}, {A,B,C}, A, P), P={A::=0B, A::=λ ,B::=1C , B::=1 , C::=0B)}. Obtain the RE which represents the same language generated by the grammar.

15. Determine the language recognized by the following automaton. To do this, use the characteristic formula.



16. Given the following right-linear grammar, G= ({0,1},{S,A,B,C},S,P), where P={ S::= 1A | 1B, A ::= 0A | 0C | 1C | 1 , B ::= 1A | 1C | 1 , C ::= 1}. Calculate formally the RE of the language associated to this grammar.

17. Given the grammar: G=({0,1},{S,A,B,C,D},S,P) where P={ S ::= ABCD , A ::= 0A | 1A | λ , B ::= 0 ,C ::= 0 | 1 , D ::= 0 | 1 }. Formally build, from the regular expression corresponding to the language that is generated, an equivalent regular grammar. It is not required to formally generate the RE of the language generated by the grammar specified in the exercise.

18. What does the Analysis Theorem indicate?

a. Every language accepted by a FA is a regular language.
b. Every regular language is the language accepted by a FA.
c. a) and b)
d. None of them.

19. The Synthesis Theorem states that:

a. "Given a FA, A, find the RE describing L(A)".
b. "Given a RE which represents a regular language, construct a FA which accepts that regular language".
c. "Given a FA, A, find the grammar that describes that FA".
d. None of them.

20. Find a RE equivalent to the following grammar:

$$\begin{array}{|ll|}\hline S \rightarrow & AS \\ S \rightarrow & \lambda \\ A \rightarrow & xB \\ B \rightarrow & xB \\ B \rightarrow & yB \\ A \rightarrow & x \\ \hline\end{array}$$

21. Given the following language, generated by a context-free grammar,

$$\begin{array}{|ll|}\hline S \rightarrow & xSy \\ S \rightarrow & ySx \\ S \rightarrow & \lambda \\ \hline\end{array}$$

and the language L_{RE} corresponding to the regular expression $(x|y)^*$.

Indicate which of the following relationships between L_G and L_{RE} are true; justify formally the answer.

1. $L_G \subset L_{RE}$
2. $L_{RE} \subset L_G$
3. $L_G = L_{RE}$

22. Given a language L which consists of every string that starts with zero or more x's followed by at least one, which is/are (at the same time) also followed by an even number of z's. For instance, xxyyzz, xyyzzzz, and yzzzz are included in L.

a. Write the regular expression whose language is L.

b. Calculate the minimal automaton whose language is L.

23. Indicate the results for the following operations: δ(a), δ(a*), δ(aa*), δ(a*|a) δ(a*b), δ(a*b*), δ(a*b*|b*a*), δ(a*b*|b*a*|a).

24. Calculate the following derivatives:

a. Da(aa*bb*)

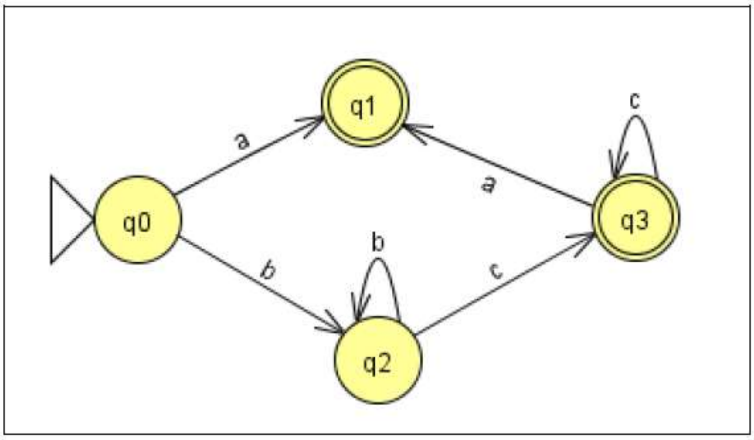
b. Da(a*abb*)

c. Da(abb*)

25. Simplify the following regular expression: *α = a | a(b | aa)(b*aa)*b* | a(aa | b)** by using the equivalence properties of the regular expressions.

26. Calculate the derivative $D_{ab}(\alpha)$ where $\alpha = a^*ab$, using the definitions of the derivatives of regular expressions.

27. Given the following automaton, calculate the associated language by using the algorithm of the characteristic equations.



28. Obtain the grammar for the regular expression a(aa | b)*.

29. Given the following regular expression a*c*(a|b) (cb)*, construct formally an equivalent regular grammar.

30. Given the language denoted by the RE *(ab)*(ba)*(abba)**, obtain the DFA which recognizes the same language. Follow these two processes:

a. Design a Finite Automaton to recognize the sentences in the language. If the Finite Automaton is a NFA, transform it into an equivalent DFA.
b. Obtain the corresponding grammar from the RE (using the concept of derivatives). Obtain the minimal DFA from the grammar.