DEPARTAMENTO DE INFORMÁTICA
UNIVERSIDAD CARLOS III DE MADRID

Grado en Matemática Aplicada y Computación

Grado en Ingeniería Informática

Formal Languages and Automata Theory
Exercises Languages and Formal Grammars
Unit 4 – Part 1

1. **Create a grammar to generate the following languages:**
   a. { a, aa, aaa }
   b. { a, aa, aaa, aaaa, aaaaa, …)
   c. { λ, a, aa, aaa }
   d. { λ, a, aa, aaa, aaaa, aaaaa, …)

   The notation used to represent each language is:
   a. { a^n | n ∈[1, 3] }
   b. { a^n | n > 0 }
   c. { a^n | n ∈ [0, 3] }
   d. { a^n | n≥ 0 }

2. **Given the grammars G=({c,d}, {S,A,T}, S, Pi} where:**

P_1: S → λ | A A → AA | c	P_2: S → λ | A A → cAd | cd	P_3: S → λ | A A → AcA | c	P_4: S → cA A → d | cA | Td T → Td | d	P_5: S → λ | A A → Ad | cA | cd

   Determine the associated languages.

3. **Create a grammar to generate the following languages:**
   a. { a^n b^n | n > 0 }
   b. { a^n b^m | n > 0, 0 < m < n }
   c. { a^n b^m | n > 0, 0 ≤ m < n }

4. **Determine the type of the following grammars in the Chomsky Hierarchy. Justify your answer.**
    a. G=({a,b}, {A,B,S}, S, P), P={S::=aA, A::=bB, A::=aA, A::=a, B::=λ}
    b. G=({a,b,c}, {A,B,C,S}, S, P), P={S::=aAb, S::=Ba, S::=λ, aAbC::=aAbB, aAbC::=aabC, BCc::=AaCc, BCc::=BaAbc, C::=Ca, C::=a}
    c. G=({house, garden, cat}, {S, CASTLE, FOREST, TIGER}, S, P), P={ S::=TIGER garden, S::=FOREST CASTLE, FOREST::=λ, garden CASTLE TIGER house::=garden FOREST TIGER house, cat CASTLE FOREST ::=cat FOREST house TIGER FOREST, FOREST::=TIGER house, FOREST::=garden}
    d. G=({x,y}, {C,A,B,S}, S, P), P={S::=Cx, S::=Cy, S::=By, S::=Ax, S::=x, S::=y, A::=Ax, A::=Cx, A::=x, B::=By, B::=yA, C::=xA}
    e. G=({a,b,c}, {S,B}, S, P), P={S::=abc, S::=aBSc, Ba::=aB, Bb::=bb}

5. **Given the grammar G,**

G=({a,b,c}, {S,A,B}, S, P), P={S::=λ, S::=aAc, A::=aA, A::=Ac, A::=B, B::=b, B::=Bb}

It is required:
    a. Specify the type of G in the Chomsky Hierarchy. Justify your answer.
    b. Determine the language L generated by the grammar.
    c. Construct two different derivation trees for a word in L(G).
    d. Verify if the following sentential forms are valid in G, and write a derivation chain to generate the valid ones.
        1) aaAcc
        2) ac
        3) ababBcc
        4) abbccc

6. **Obtain the grammar corresponding to the language L={$a^n b^m c^p a^q b^n$, such that q=p+m; n,m≥1; p≥0}**

7. **Obtain a grammar for the language with alphabet {a, b, c, d} that consists of all the strings that can be formed by combining these symbols excluding those that contain the substring“bc”.**

8. **Obtain a grammar for the language L = {$x^n y^m z^k$ | m, n, k ≥0, k=m+n }**

9. **Obtain a grammar for the language {$ab^n a$ / n=0, 1, …}**

10. **Obtain a type-0 grammar for the language L={$a^n b^n c^n$} where n≥1.**

11. **Obtain the language generated by the grammar G=({0,1},{S,A,B,C},S,P), where P:**

S → BAB
BA →BC
CA → AAC
CB → AAB
A→0
B →1

12. **Design a grammar to generate natural numbers.**