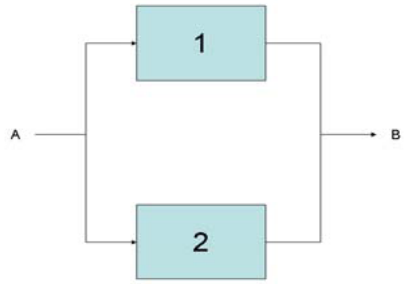
Chapter V: Probability Models

PROBLEMS

Proposed Problems

1. A service station receives in an independent and stable way an average of 10 clients per hour. Compute the probability that in the next half hour 8 clients will arrive to the service station knowing that in the last hour it received 14 clients
   SOLUTION:
   $p = 6.5\%$.
2. The distance D between two successive vehicles in a lane of a motorway follows an exponential distribution with mean 200 meters. What is the probability that in 1 km of the rail there are exactly 5 cars?
   SOLUTION:
   $p = 17.5\%$.
3. The daily number of failures of a machine follows a Poisson distribution of average 0.4 failures. Calculate the probability that there are three successive days without fault.
   SOLUTION:
   $p = 30\%$.
4. A parking lot has two entrances. The cars arrive at the entrance I according to a Poisson with 3 cars per hour and at the entrance II with 4 cars per hour. If the number of cars arriving at each input are independent what is the probability that in an hour 3 car enter the garage? (June 98)
   SOLUTION:
   $p = 5.2\%$.
5. It is known that certain pieces have a duration (time until they break down measured from last repairing time) that is distributed exponentially. We know that the expected duration of this process is 1000 hours. Calculate:
   a) The value of λ.
   b) The variance of the distribution.
   c) The probability that a piece breaks down before 250 hours.
   d) The probability that a piece that has lasted 1000 hours lasts 1250 hours more.
   e) Discuss and compare the answers to the questions c) and d) (Sep 98)
   SOLUTION:
   a) $\lambda = 1/1000$, b) 1000^2, c) 22.12%, d) 77.88%, e).The two results are one the complementary of the other due to the memoryless property of the exponential distribution.
6. The probability of finding a left-handed person is 0.1. In a class of 20 students there are 3 desks for left-handed people. Calculate the probability that there are not enough desks. (June 97)
   SOLUTION:
   $p = 13.32\%$.
7. The process of customer arrivals to a service station evolves in time in a stable and independent way. On average it arrives one customer every minute.
   a) What is the probability that no customers arrive in 3 minutes?
   b) What is the probability that the time interval between two consecutive customers is less than one minute?
   c) The above service shop is kept open for 8 hours a day and it results profitable if it is visited by at least 500 people a day. What proportion of days will it be profitable?
   SOLUTION:
   a) 5%, b) 63%, c) 18%

8. Consider a system consisting of 2 components in parallel as in Figure where the two components have similar characteristics.



9. The failure of a component is independent of the state of another component. The system is functioning if between A and B is possible to find a complete path of operating components
    a) Suppose that the life of a component (time since it is connected up to it breaks) can be modeled with an exponential and that the average duration of a component is 5000 hours. Calculate the probability that the system consisting of two components in parallel works more than 10,000 hours continuously.
    b) Suppose now that the system has been running since 8000 hours without breaking down. What will now the probability of reaching the 10,000 hours without a break down (i.e. operating at least 2000 more hours continuously)?

    SOLUTION:
    a) 25.23%, b) 69.50%
10. Two male friends of same age speak about their respective heights. The shorter of them is 160 cm tall and he says that he feels shorter than most of her generation and that he finds that only 1 in 10 boys is shorter than him. The taller measures 175cm and he says that he feels that his stature does not seem at all extreme; indeed an equal number of people is highest and lowest than him. Assuming that the height of boys of that age are normally distributed, we want to know:
    a) Calculate the mean μ and variance σ^2 of the normal distribution starting from the information these two guys provide
    b) What is the proportion of guys taller than 190cm?
    c) We randomly select 4 boys. What is the probability that most of them measure less than 160 cm?

    SOLUTION:
    a) $\mu = 175;\ \sigma^2 = 137,$ b) 10%, c) 0.37%
11. A server of a web page of a ministry receives an average of 10 hits per hour. It is assumed that users who access that server do so independently of each other. The technician hat manages the computing resources of this ministry notes that nobody has tried to access on the last half an hour. Should s/he worry and check if there is something wrong with the system?

    SOLUTION:
    It is a very unlikely event. The technician will do well to review the system.
12. In the following a set of random variables is described. Specify for each of them which is the corresponding probability model (choose between: binomial B(n,p), discrete uniform Ud(a,b), continuous uniform Uc(a,b), Poisson P(λ), exponential Exp(λ), or any of the above). Specify the value of the parameters that that model has.

    Example: X = number of faces that are obtained by tossing 3 times a coin -> X ~ B (3,0.5)
    a) X = Value obtained by rolling a die; X ~?
    b) To the lobby of a train station an average of 120 passengers arrive in one hour. The arrival of the passenger has a stable mean rate and they arrive in an independent way. The random variables of interest are:
        i. X = Number of passengers arriving in one hour; X ~?
        ii. X = Number of passengers arriving in four hours; X ~?
        iii. X = Minutes passed without arriving passengers, X ~?
    c) A building has two entrances, one at the front and one at the side. In both entrances it is found that the average rate of influx of people is stable. It is assumed that people

come independently to the building. On average, three people per minute enter through the front door, while through the side door it enters on average one person every five minutes. The random variables of interest are:

   i. X = Number of people served by the front door in an hour; X ~?
   ii. X = Number of people accessing every hour through the side door; X ~?
   iii. X = Total number of people entering the building in one hour; X ~?
   iv. X = Time between two consecutive persons who enter through the front door; X ~?
   v. X = Time between two consecutive persons who enter through the side door; X ~?
   vi. X = Time (minutes) elapsed since a person accessing the main gate to the time another one was accessing the side door; X ~?

d) A machine produces on average 1% of defective items; these appear in a fortuitous way and with a constant average rate. The random variables of interest are:
   i. X = number of defective items in a batch of 100 items; X ~?
   ii. X = number of defective items in two batches of 100 items each; X ~?
   iii. X = Number of items to be produced to have one acceptable; X ~?
   iv. X = Number of defective items to be produced to have one acceptable; X ~?

e) 30% of people carry a mobile phone. The random variables of interest are:
   i. X = Number of people who have a mobile in a classroom with 35 people; X ~?
   ii. X = Number of phones in a classroom of 54 people; X ~?

f) A production process has 4 parallel production lines that operate independently. The probability that a line is stopped by some sort of accidental damage is 0.05. The random variables of interest are:
   i. X = Number of lines operating at a given time; X ~?
   ii. X = Number of lines that stop by a breakdown in a given instant; X ~?

g) A telephone cable manufacturer sells a cable with a quality of on average 1 defect per kilometer of cable. The random variables of interest are:
   i. X = Number of defects per meter of cable; X ~?
   ii. X = Number of defects per kilometer of cable; X ~?
   iii. X = Length of cable free of defects; X ~?

Short Problems to solve by computer

Note: To avoid ambiguities in the answer, always use the probabilities with 4 decimals.

13. We have a batch of 20 items from a production system that produces, on average, 10% of defective items. Assuming stability and independence in the appearance of defective articles, we want to know:
    a) The probability of producing 3 defective items
    b) The probability of producing defective items within 3
    c) The probability of producing 2 or more defective items
    d) Calculate the quartiles of the random variable

    SOLUTION:
    a) 0.1901, b) 0.6769, c) 0.6083, d) $Q_1 = 1$, $Q_2 = 2$, $Q_3 = 3$.
14. Keep analyzing the batch of 20 items from the previous question. A technician begins to analyze whether the items in a lot are defective or acceptable. After reviewing only 2 s/he finds that both are defective. What is now the probability that the batch of 20 articles has a total of 3 defective items?

    SOLUTION: 0.3125
15. An Internet server receives an average of 5 accesses per minute during the workday. In this time, users access the server with a steady rate and independently. The server saturates, providing a very slow service, when there are more than 8 hits per minute. What percentage of time will it be saturated?

    SOLUTION: 0.0681

16. During weekends, the server above only receives an average of 2 hits per minute, we assume that the process is a Poisson process. What is the probability that the server is more than one minute without receiving calls?
    SOLUTION: 0.1353
17. The duration of an alkaline battery of brand A is an exponential random variable with average 5000 hours, while the duration for a another battery of brand B, not alkaline, is an exponential with average 2000 hours. What is the probability for each battery to last less than 1000 hours?
    SOLUTION: 0.1813 and 0.3935
18. Continue with the two previous batteries. Assuming that the duration of each battery is independent of the other, what is the probability that both are operating after 1000 hours?
    SOLUTION: 0.0713
19. Continue with the two previous batteries. Assuming that the duration of each battery is independent of the other and that both continue to operate after 1000 hours. What is the probability that both still are working after 1,000 additional hours?
    SOLUTION: 0.0713
20. What is the probability that a continuous uniform variable in the interval [3,5] generates values between 4 and 4.3?
    SOLUTION: 0.1500
21. If the duration of a component follows an exponential with an average of 100 units of time, what is the probability that it is still working after this average duration?
    SOLUTION: 0.3679
22. The duration of a type of component is an exponential random variable whose average is 5000 hours. If we simultaneously activate many components, at what instant of time only 10% of them is still operating?
    SOLUTION: 11512.9 $hours$
23. Continuing with the previous example. At what instant of time only half of them is still operating?
    SOLUTION: 3465.74 $hours$
24. The duration of the execution of some task on a computer can be modeled as a normal random variable with mean 100 seconds and standard deviation 5 seconds. What is the probability that the task lasts over 110 seconds?
    SOLUTION: 0.0228
25. Continuing with the previous example. What is the probability that the task lasts exactly 100 seconds?
    SOLUTION: 0
26. Continuing with the previous example. The task has been running for 100 seconds and it still not over. What is the probability that the task duration is greater than 110 seconds?
    SOLUTION: 0.0455
27. Continuing with the previous example. What percentage of the time that the task is executed it takes less than 90 seconds?
    SOLUTION: 0.0228
28. Continuing with the previous example. 90% of the time the task runs, it takes more than ...
    SOLUTION: 93.59 $seconds$
29. Continuing with the previous example. 90% of the time the task runs, it takes less than ...
    SOLUTION: 106.40 $seconds$
30. Continuing with the previous example. If the task has been running 110 seconds, what is the probability that it ends during the next second?
    SOLUTION: 0.3860
31. The time Y it takes to read a file is a linear function of its size X according to the simple regression model Y = 3 + 2X + e, where e is normal with mean 0 and standard deviation 0.1. How long will it take to read on average a file of size 1?
    SOLUTION: 5 $units\ of\ time$
32. Continuing with the previous example. What is the probability that a file of size 1 lasts more than 5 units of time to be read?
    SOLUTION: 0.5

33. Continuing with the previous example. What is the probability that a file of size 2 lasts more than 5 units of time to be read?

    SOLUTION: 1