DEPARTAMENTO DE INFORMÁTICA
UNIVERSIDAD CARLOS III DE MADRID

Grado en Matemática Aplicada y Computación
Grado en Ingeniería Informática

Formal Languages and Automata Theory
Exercises Finite Automata
Unit 3 – Parts 3 and 4

1. The parking with the maximum time in a street in GOTHAM costs 1 euro. The parking meters only admit money until a maximum amount of 1 euro. When the introduced amount is less or equal to 1 euro, it is possible to press the button to emit a ticket, so that this ticket is emitted and the parking meter indicates that the introduced credit is 0. Nevertheless, when a greater amount of money is introduced, i.e. more than 1 euro, the parking meter automatically gives back all the money, without pressing the button to emit ticket, and it indicates that the introduced credit is 0. In the parking meter, the only coins that are allowed are 10 cents, 20 cents, 50 cents, and 1 euro.

   Design the corresponding DFA to control the machine. Would you design an automaton with a λ-transition to control the machine? Justify your answer.

2. The enchanted castle. The problem is the exposed in this letter:

   "Dear friend: soon after buying this old mansion I had the disagreeable surprise of verifying that it is enchanted with two sounds from the beyond that practically make it inhabitable: a picaresque song and a sardonic laughter.

   Nevertheless, I still have certain hope, because the experience has demonstrated to me that their behavior obey certain laws, dark but infallible, and this behavior can be modified by touching an organ and burning incense. Every minute, each sound is present or absent. Their behavior in the following minute depends on which it is happening in the current minute, by the following way:

   > *The song will conserve the current state (present or absent) except if during the current minute the laughter is not heard and I touch the organ, in which case the song takes the opposite state.*
   >
   > *Regarding the laughter, if I do not burn incense, it will be heard or not according to whether the song is present or absent (so that, the laughter imitates the song with a minute of delay).*

However, if I burn incense, then the laughter will exactly make the opposite of what the song did. In this current moment in which I am writing this letter, I am simultaneously listening to the laughter and the song.

I will be very grateful if you could tell me which steps I have to follow using the organ and the incense to definitively restore the calm".

Help (not given in an exam): which would the alphabet be and what would it represent?; which would the set of states be, that is to say, what would the states represent?

3. Consider the following problem:

A man, a wolf, a sheep, and a lettuce are on the left side of a river. There is a boat in which the man can go with only one of the other three elements. The man would want to cross the river but he can only cross only with another element each time. Nevertheless, if the man leaves the wolf and the sheep, the wolf will surely eat the sheep. In addition, if the sheep and the lettuce are left on the same side, the sheep will eat the lettuce. Is it possible to cross the river avoiding the sheep or the lettuce to be eaten?

Resolve this problem by means of a DFA. Consider that the initial state is the one in which the four elements are on the left side and the final state is that one in which the four are on the right side.

It is required:

a) Indicate which alphabet would be defined. What is the meaning of the elements in the alphabet? What would be the meaning of a word? What would be the meaning of a sentence?

b) Indicate the set of states. What is the meaning of the states?

c) How would you reformulate the question "Is it possible to cross the river without the sheep or the lettuce becoming eaten" from the point of view of Languages and Automata.

d) Design the DFA to recognize the words included in the language.

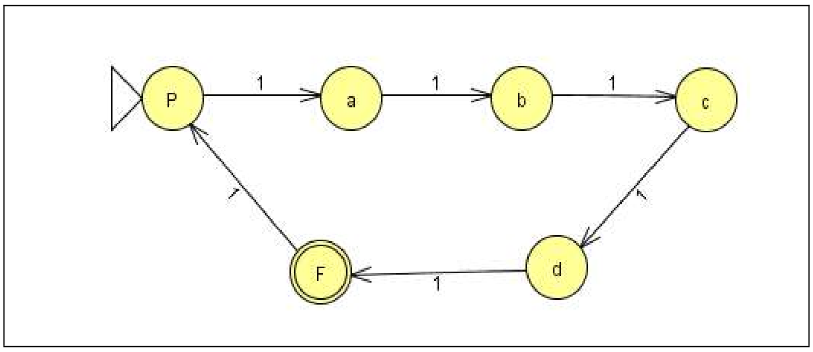
4. Indicate the graph of a NFA, only including the number of indicated states, which recognizes each one of the following languages. The alphabet is always {0.1}.

a) The language $0^m1^n0^p$ $(m \geq 0, n \geq 0, p \geq 1)$ <u>with only 3 states</u>.

b) The language {0} <u>with only 2 states</u>.

c) The language of strings ending in 01, with only 3 states.

5. Given the following FA, indicate which of the following statements are correct:



   a. Q/E0 would consist of the following equivalence classes C1={P, a, b, c, d} and C2={F}
   b. Q/E1 would consist of the following equivalence classes C1={P, a, b, c} and C2={d, F}
   c. Q/E3 would consist of the following equivalence classes C1={P, c, d}, C2={d,F}, and C3={a,b}
   d. Q/E4 would consist of the same equivalence classes indicated in c for Q/E3.
   e. It is necessary to calculate Q/E1, Q/E2, Q/E3 and Q/E4 to determine whether the previous statements are correct.

6. Given the language $(01)^n$ with $n \geq 0$, indicate which of the following finite automata generates this language. In addition, obtain the minimal equivalent DFA for the selected automaton.
   a. FA=[{0,1}, {A,B,C,F}, f, A, {F}]
      f(A,0)=B, f(A, λ)= λ, f(C,0)=B, f(B,1)=C, f(B,1)=λ
   b. FA=[{0,1}, {A,B,C,F}, f, A, {F}]
      f(A,0)=B, f(A, λ)=F, f(C,0)=B, f(B,1)=C, f(B,1)=F
   c. FA=[{0,1}, {A,B,C,F}, f, A,{F}]
      f(A, B)=0, f(A,F)= λ, f(C,B)=0, f(B,C)=1, f(B,F)=1
   d. FA=[{0,1}, {A,B,C,F}, f, A, {F}]
      f(B,0)=A, f(F,λ)=A, f(B,0)=C, f(C,1)=B, f(F,1)=B

7. Indicate which of the following statements are correct. Justify your answer.

   a) Given a NFA, it is always possible to obtain a DFA which recognizes the same language.

   b) Two DFAs are equivalent if their respective minimal FA are isomorphic.

   c) Every NFA can be transformed into an equivalent DFA.

   d) If a FA does not include lambda transitions, then it is deterministic.

   e) A FA, in which with different symbols it makes a transition between two states q0 and q1, then the automaton is nondeterministic.

   f) If in the process to calculate the quotient set of a DFA with 5 states we have obtained Q/E3, then we can affirm that Q/E3= Q/E.

   g) Nondeterministic finite automata necessarily make transitions between states by means of λ.

   h) Nondeterministic finite automata cannot accept any word, therefore they only accept the empty language.

   i) f'(p,111) = s and f'(p,110) =s indicate that the FA is nondeterministic.

   j) f'(p,110) = s and f'(p,110) = q indicate that the FA is nondeterministic.

   k) A DFA is connected if every state is reachable from the initial state.

   l) A DFA is connected if every state is reachable between them.

   m) Given two non-connected DFAs which recognize the same language, they still recognize the same language if the unreachable states are eliminated.

   n) A non-connected DFA cannot recognize any language.

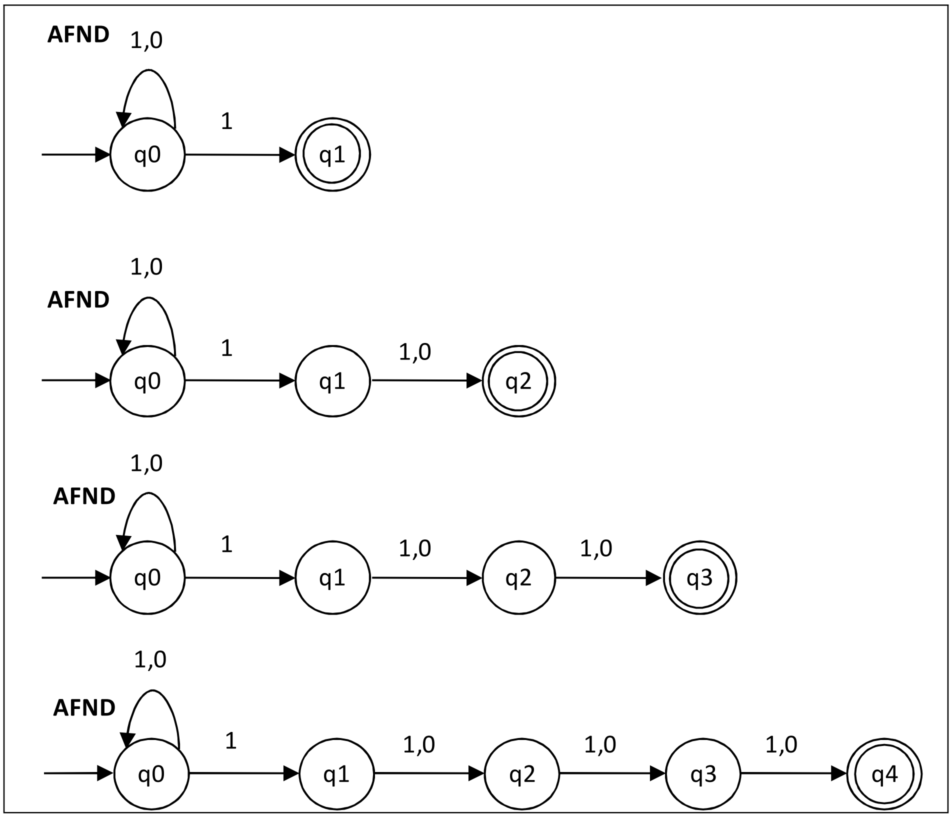
   o) Two non-equivalent DFAs cannot be isomorphic.

   p) If two DFAs are isomorphic, then they recognize the same language.

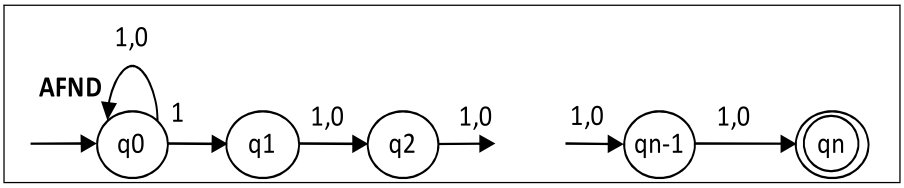
   q) If two DFA are equivalent, then they have to be isomorphic.

   r) Two DFAs are equivalent if the final states in the automata sum are included in the same equivalence class.

8. Indicate the graph of the DFA which corresponds to the NFAs indicated in the following figures.



How many states will the equivalent DFA to the following NFA have?



9. Obtain the minimal equivalent DFA for the following Non-Deterministic Finite Automata. Describe the intermediate transformations: NFA → DFA → Minimal DFA.

a) NFA$_A$=({a,b,c},{Q0,Q1,Q2,Q3},f,Q0,Q3) f(Q0, a) = Q1 ; f(Q0, b) = Q2 ; f(Q0, c) = Q3 f(Q1, a) = Q2 ; f(Q1, b) = Q3 ; f(Q1, c) = Q1 f(Q2, a) = Q3 ; f(Q2, b) = Q1 ; f(Q2, c) = Q3	b) NFA$_B$=({a,b,c},{Q0,Q1,Q2,Q3,Q7},f,Q0,Q3) f(Q0, a) = Q0,Q2,Q3 ; f(Q0, c) = Q1 f(Q1, c) = Q3 ; f(Q2, b) = Q2,Q3 f(Q7, b) = Q2,Q3
c) NFA$_C$=({a,b,c},{Q0,Q1,Q2,Q3,Q4,Q5},f,Q0,Q5) f(Q0, a) = Q1 ; f(Q1, a) = Q1,Q5 f(Q1, b) = Q2 ; f(Q2, b) = Q2,Q3,Q5 f(Q3, b) = Q2,Q3 ; f(Q3, c) = Q4 f(Q4, b) = Q2,Q3 ; f(Q4, c) = Q3	d) NFA$_D$=({c,f,d},{Q0,Q1,Q2,Q3,Q4,Q5,Q6},f,Q0,Q6) f(Q0, c) = Q1,Q4 ; f(Q0, f) = Q2,Q6 ; f(Q1, c) = Q1 f(Q1, f) = Q3 ; f(Q1, d) = Q4 ; f(Q2, c) = Q0 f(Q3, c) = Q3 ; f(Q3, f) = Q3 ; f(Q4, c) = Q4 f(Q4, f) = Q5 ; f(Q4, d) = Q5 ; f(Q5, c) = Q5 f(Q5, f) = Q5

10. Draw the graph of a Deterministic Finite Automaton. The alphabet is always {0, 1}. The problem can be solved by directly designing the DFA, or by starting from the NFAs obtained in problem 4.

    a) The language $0^m1^n0^p$ ($m \geq 0, n \geq 0, p \geq 1$) (the NFA included only three states)
    b) The language {0} (the NFA included only two states)

11. Given the NFA (with lambda transitions) described by the following table, obtain the minimal equivalent DFA.

	a	**b**	**c**	**λ**
→p	p	q		q
q	q	p,r		r
r			s	p
*** s**	s			

12. Indicate the graph of the DFA corresponding to the following NFA: