Magnetic forces and fields

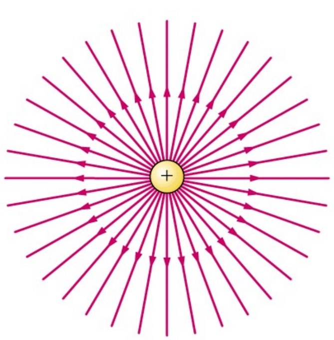
Sources of magnetic field

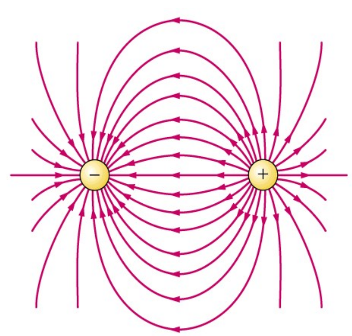
Electric field E

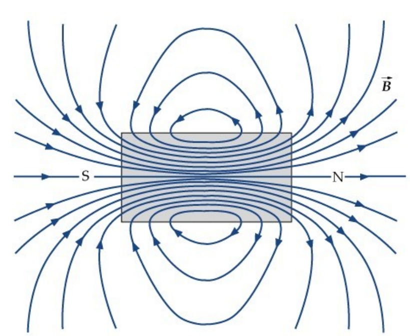
- Created by electric charges
- Felt by electric charges.
- Force F due to E: $\mathbf{F}_e = q\mathbf{E}$
- The field lines of E are tangent to F
- The E lines begin on positive charges and end on negative charges (or at infinity).
- E proportional to the density of lines.

Magnetic field B

- Created by electric charges <u>in motion</u> (point charges,electric currents, magnets)
- Felt by electric charges <u>in motion</u>.
- Force F due to B: $\mathbf{F}_m = q(\boldsymbol{\upsilon} \times \mathbf{B})$
- The field lines of B are perpendicular to F.
- The B lines close upon themselves. In a magnet the B lines go out of the N pole into the S pole
- B proportional to the density of lines.







(b)

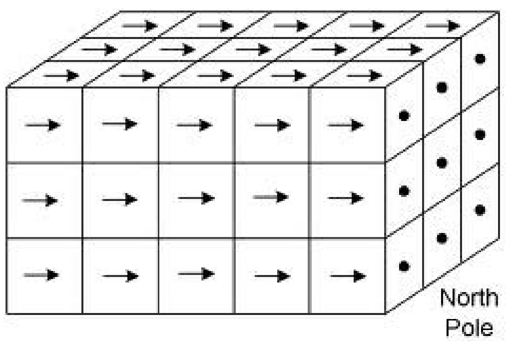
SI UNIT: Tesla (T)

$$1T = 1\frac{N/C}{m/s} = \frac{Ns}{Cm} = \frac{N}{Am}$$

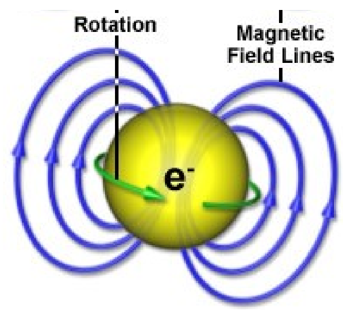
1 T=10^4 G (gauss)

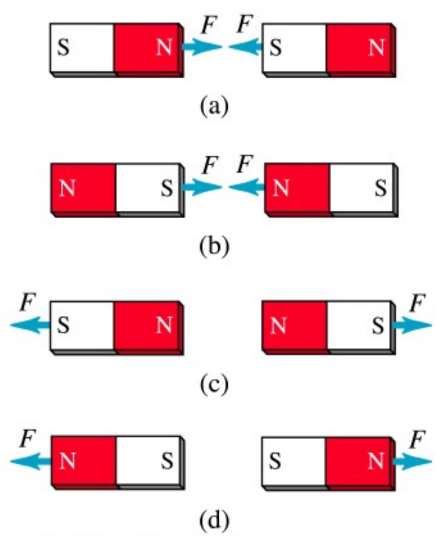
Magnets

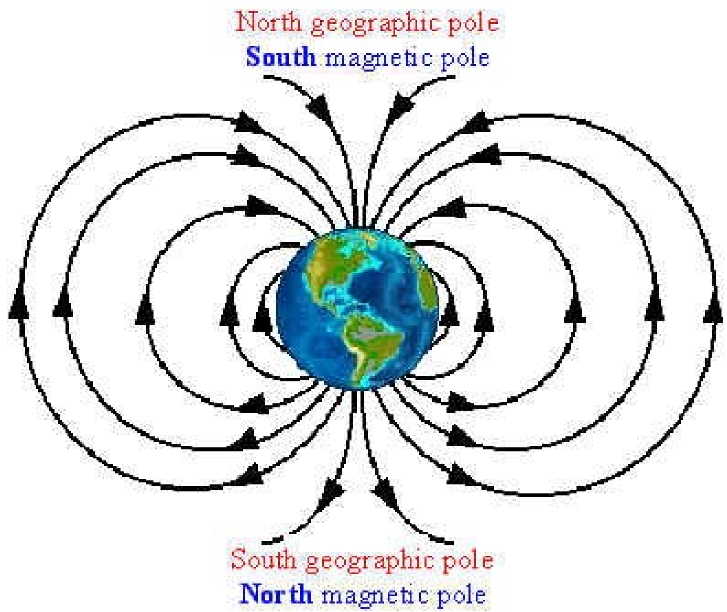
Magnets are made from materials (ferromagnetic) in which the orientation of electrons (charges in motion!) can be such that the magnetic field created by them adds up in a constructive way. Examples of these materials are Fe, Co and Ni.



A lodestone attracting iron filings and nails.







$B_{\text{Earth's surface}} \sim 0.5\ G$

$B_{\text{fridge magnet}} \sim 10\ G$

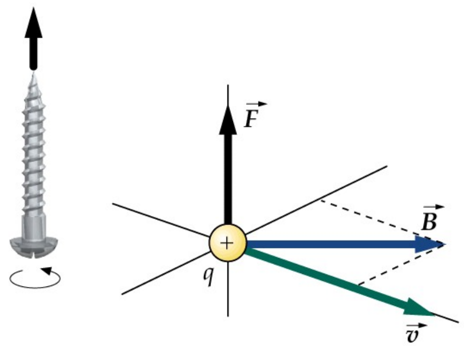
$B_{\text{most powerful magnet}} \sim 1.5\ T$

More about magnets: *http://www.magnet.fsu.edu/education/tutorials/magnetacademy/magnets/fullarticle.html*

More about the magnet Earth: *http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/magearth.html*

Magnetic force on a point charge

The force on a point charge due to a B is: $\vec{F}_{mag.} = q\,\vec{\upsilon} \times \vec{B}$



F_m increases as q, υ and B increase. The direction depends on the sign of the charge q.

$|\vec{F}| = q\upsilon B \sin\theta$

<u>F_m IS PERPENDICULAR TO B and υ</u> (so to the plane formed by B AND υ).

F_m is maximum when the charge moves perpendicular to the field lines, and is zero when the charge moves parallel to the field lines.

B does not exert any force over charges that are at rest.

$$F_m = 0 \quad \text{if} \begin{cases} \vec{\upsilon} = 0 \\ \vec{B} = 0 \\ \vec{\upsilon} \,||\, \vec{B} \end{cases}$$

F_m is perpendicular to υ so it modifies the trajectory of the particle (which changes the direction of the force!).

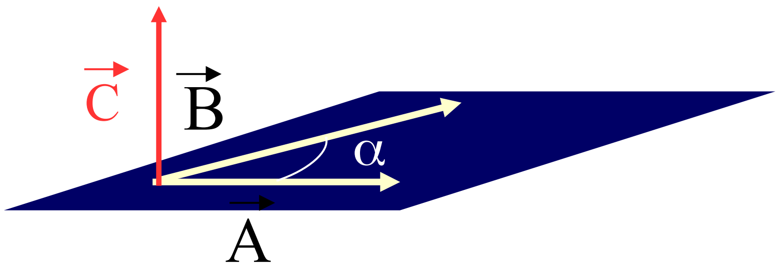
F_m is perpendicular to υ so it does not change the magnitude of υ.

$$F \perp v\,,\ F \,||\, a \Rightarrow\ a \perp v$$

F_m is perpendicular to the trajectory so <u>it does no work on the particle</u> (ΔK = 0).

Vector cross product

$\vec{C} = \vec{A} \times \vec{B}$



THIS OPERATION RETURNS A VECTOR!!

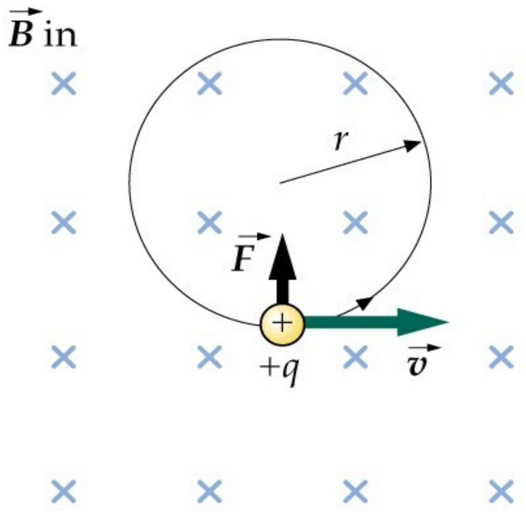
- Its magnitude is given by: $\left|\vec{A} \times \vec{B}\right| = AB \sin\alpha$
- Its direction is: $\vec{A} \times \vec{B} \begin{Bmatrix} \perp \vec{A} \\ \perp \vec{B} \end{Bmatrix}$
- Its orientation is described by the right hand rule (cork screw rule).
- It is calculated by solving the determinant:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \left(A_y B_z - A_z B_y\right)\vec{u}_x + \left(A_z B_x - A_x B_z\right)\vec{u}_y + \left(A_x B_y - A_y B_x\right)\vec{u}_z$$

$$\vec{A} \times \vec{B} = 0 \quad \text{if} \quad \begin{Bmatrix} \vec{A} = 0 \; ; \quad \vec{B} = 0 \\ \vec{A} \,||\, \vec{B} \end{Bmatrix}$$

Charged particles in a magnetic field

Uniform B, E=0, υ perpendicular to B



UNIFORM CIRCULAR MOTION

The charged particle moves in a circle whose plane is perpendicular to B.

F_m acts as a centripetal force.

$$\left.\begin{aligned}\vec{F}_{mag} &= m\vec{a} \\ |\vec{F}|_{mag} &= q|\vec{\upsilon}|B\sin(90^0)\end{aligned}\right\} \quad a_n = \frac{q|\vec{\upsilon}|B}{m}$$

centripetal acceleration: $a_n = |\vec{a}| = \dfrac{|\vec{\upsilon}|^2}{r}$

$$r = \frac{m|\vec{\upsilon}|}{qB}$$

r is proportional to m and υ

r is inversely proportional to q and B

Radius of the circle

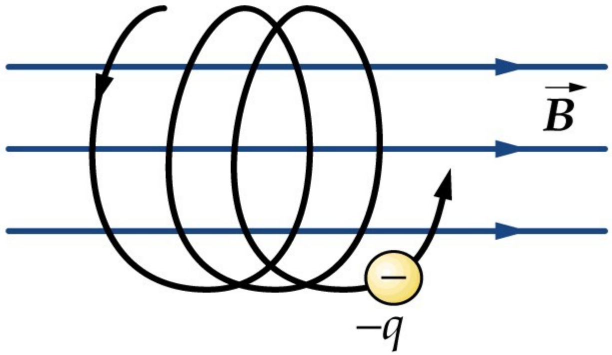
(called cyclotron radius or Larmor radius)

Period of the motion: $T = \dfrac{2\pi r}{|\vec{\upsilon}|} = \dfrac{2\pi m}{qB}$

Angular frequency: $\omega = \dfrac{\upsilon}{r} = \dfrac{qB}{m}$

Charged particles in a magnetic field

Uniform B, E=0, υ NOT perpendicular to B



υ can be seen as having two components: parallel to B and perpendicular to B. The parallel component does not contribute to the magnetic force.

The motion due to $\upsilon_\perp$ is a uniform circular motion.
The motion due to $\upsilon_\parallel$ is a uniform linear motion.
The result is a helix with its axis parallel to B.

BEWARE!! The expressions for *a, R, T*, etc, may NOT be the same as before, as only $\upsilon_\perp$ contributes to the circular motion.

Charged particles in a magnetic field

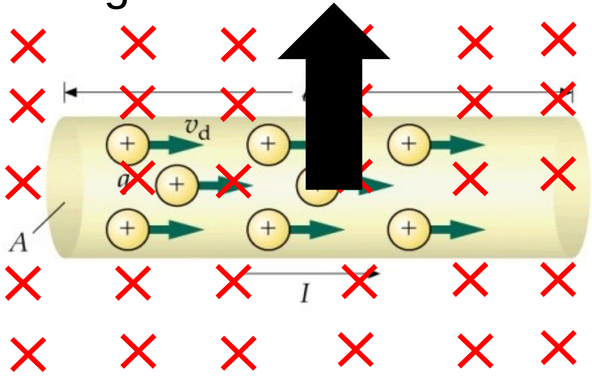
Uniform B and E; v, B and E perpendicular to each other

$$\vec{F} = \vec{F}_{el.} + \vec{F}_{mag.} = q\vec{E} + q\vec{v} \times \vec{B} \quad \Rightarrow \quad \begin{matrix} \vec{F}_{el.} \parallel \vec{E} \\ \vec{F}_{magn.} \perp \vec{B} \end{matrix}$$

Lorentz force

Magnetic force on electric currents

An electric current is made of moving charges. If a current-carrying wire is in a region in which there is a uniform B, there will be a force on the wire.



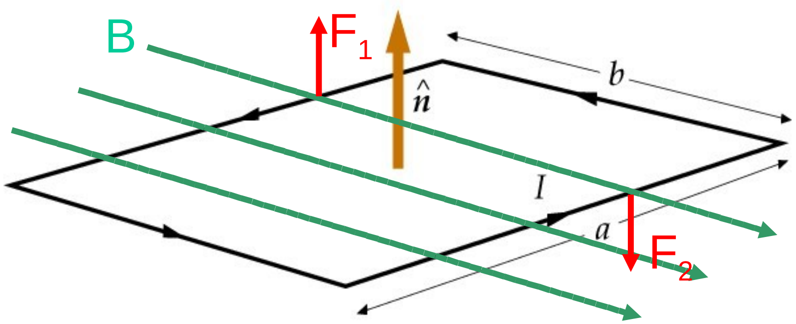
The magnetic force acting on a straight segment of wire carrying a current I is:

$$\vec{F}=\left(q\vec{v}\times\vec{B}\right)N=\left(q\vec{v}\times\vec{B}\right)nAl=I\,\vec{l}\times\vec{B}$$

Total number of charges on that segment

Magnetic force on electric currents

TORQUE ON A CURRENT LOOP IN A UNIFORM MAGNETIC FIELD



The force acting on each straight segment is:

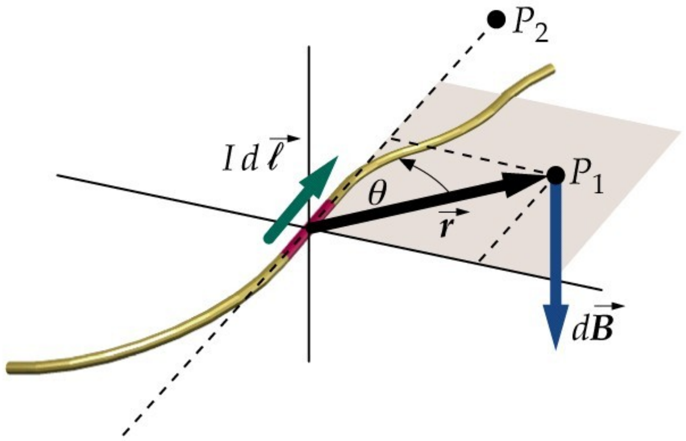
$$\vec{F} = I\,\vec{l} \times \vec{B}$$

- F_1 and F_2 are equal and act on opposite directions → force couple
- Net force acting on the loop: $\mathbf{F_{net} = F_1 + F_2 = 0}$

However, the loop rotates until it is perpendicular to B (independently on the loop shape)

Magnetic field created by an electric current

MAGNETIC FIELD CREATED BY AN ELECTRIC CURRENT



$$\vec{B} = \int d\vec{B} = \int_L \frac{\mu_0}{4\pi} \frac{I\,d\vec{l} \times \vec{r}}{r^3}$$

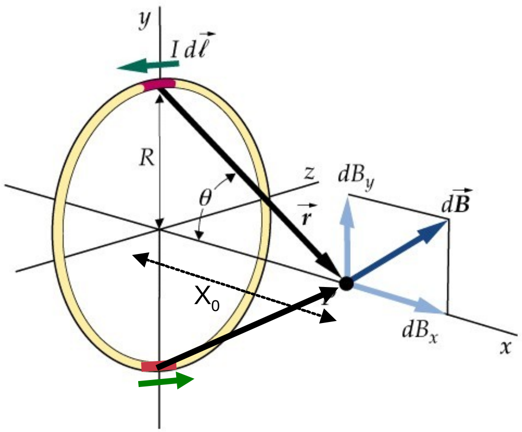
Biot-Savart's Law

μ_0: permeability of free space

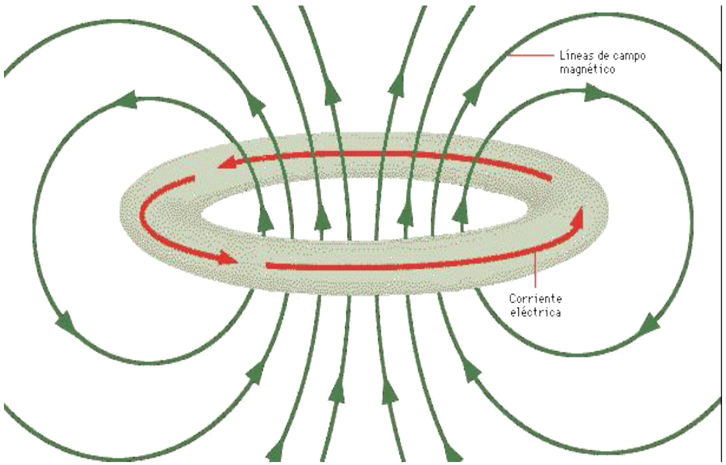
$$\mu_0 = 4\pi \cdot 10^{-7}\, T \cdot m / A$$

Magnetic field created by an electric current

B created by a current loop of radius R at a point of its axis:

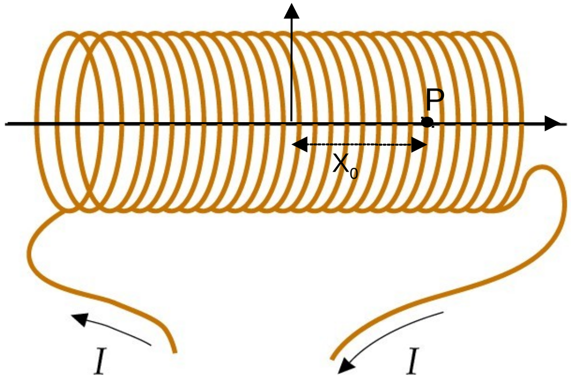


$$\vec{B} = \frac{\mu_0 I R^2}{2\left(x_0^2 + R^2\right)^{3/2}}\,\vec{i}$$



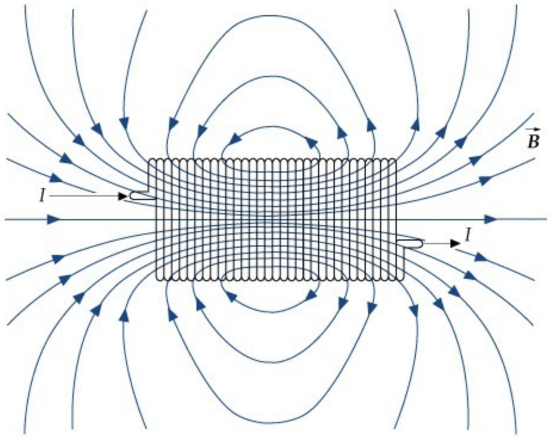
Magnetic field created by an electric current

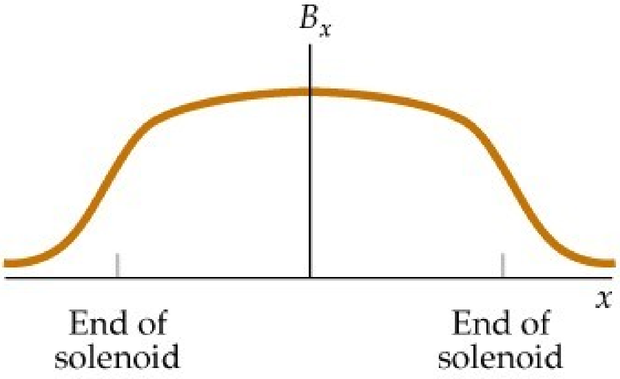
B inside a solenoid of lenght L, at a point of its axis



$$\vec{B}=\frac{\mu_0 In}{2}\left[\frac{(L/2-x_0)}{((L/2-x_0)^2+R^2)^{1/2}}+\frac{(L/2+x_0)}{((L/2+x_0)^2+R^2)^{1/2}}\right]\vec{i}$$

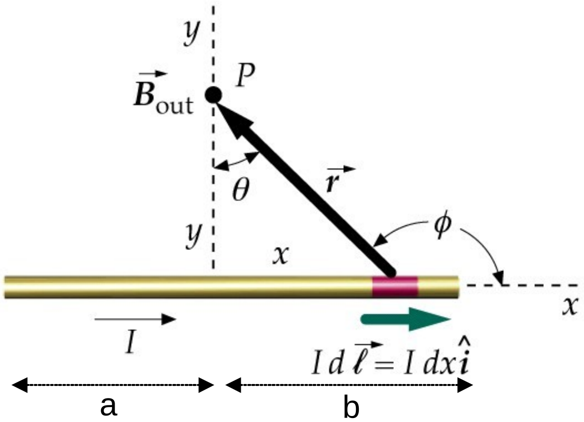
When the solenoid is very long, L >>R → $\vec{B}=\mu_0 nI\,\vec{i}\,(T)$ (far from the edges)





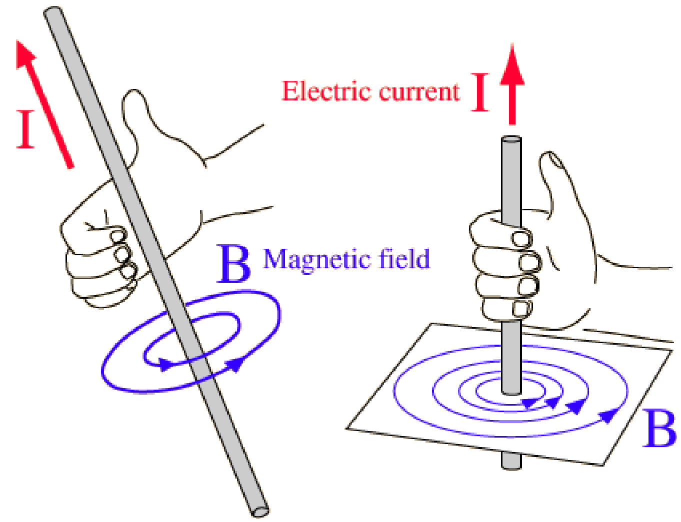
Magnetic field created by an electric current

B due to a straigth wire carrying a current along the X axis on any point of the XY plane:



$$\vec{B} = \frac{\mu_0 I}{4\pi y_0}\left[\frac{a}{\left(a^2 + y_0^2\right)^{1/2}} + \frac{b}{\left(b^2 + y_0^2\right)^{1/2}}\right]\vec{k}$$

When the wire is very long, a, b >>y_0 · $\left|\vec{B}\right| = \frac{\mu_0 I}{2\pi y_0}$



There is another way of calculating B for some of these distributions: Ampère's Law

Ampere's law

When a current-carrying conductor has a high symmetry, we can calculate the magnetic field created by that current using Ampère's Law:

The line integral of B along any closed path (C) is proportional to the current passing through the surface enclosed by C:

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I$$

METHOD:
a) Define C
b) Calculate: $\oint_C \vec{B} \cdot d\vec{l}$
c) Calculate I_c
d) Apply Ampère's Law
e) Isolate |B|

Ampère's Law applies to currents which are steady and continuous in space.

BEWARE!! <u>DO NOT CONFUSE WITH GAUSS' LAW</u>

The flux of the magnetic field through any closed surface is zero, as the B lines always close upon themselves:

$$\Phi_m = \oint \vec{B} \cdot d\vec{S} = 0$$

Ampere's law

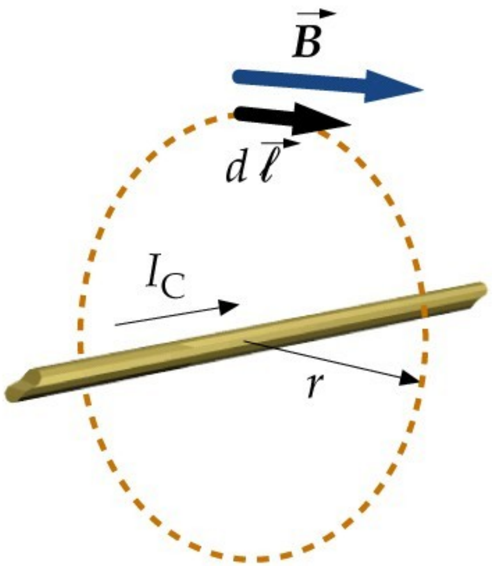
MAGNETIC FIELD DUE TO AN INFINITE CURRENT-CARRYING 1-D WIRE

$$\oint_C \vec{B}\cdot d\vec{l} = \oint_C |\vec{B}||d\vec{l}|\cos\alpha = \oint_C |\vec{B}||d\vec{l}| = |\vec{B}|\oint_C |d\vec{l}| = |\vec{B}|2\pi r$$

$$I_{\substack{\text{through}\\ \text{the surface}\\ \text{enclosed by C}}} = I_C$$

$$B2\pi r = \mu_0 I_C \qquad \vec{B} = \frac{\mu_0 I_C}{2\pi r}\hat{u}_\theta$$

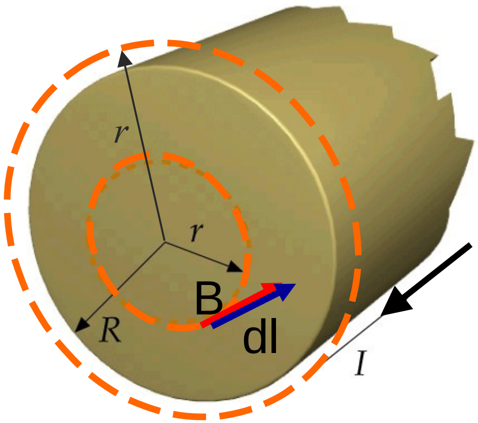
B is NOT radial, it is tangential. We use a unit vector representing the tangential (angular) direction.



Ampere's law

MAGNETIC FIELD DUE TO AN INFINITE SOLID WIRE OF RADIUS R CARRYING A UNIFORM CURRENT I

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_C$$



A) B inside

$|B|$ const along L

$$\oint_C \vec{B}\cdot d\vec{l} = \oint_C |\vec{B}||d\vec{l}|\cos\alpha = \oint_C |\vec{B}||d\vec{l}| = |\vec{B}|\oint_C |d\vec{l}| = |\vec{B}|2\pi r$$

$B \parallel dl$

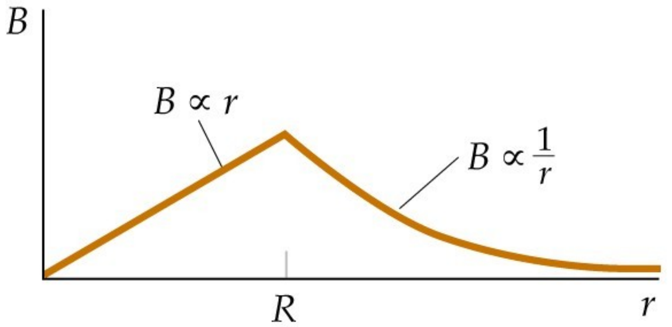
$$I_C = \frac{I}{\pi R^2}\pi r^2 = \frac{I}{R^2}r^2$$

$$\vec{B}_{ins} = \frac{\mu_0 I}{2\pi R^2} r\,\hat{u}_\theta \,(\mathrm{T})$$

B) B outside

Total current intensity going through the solid wire

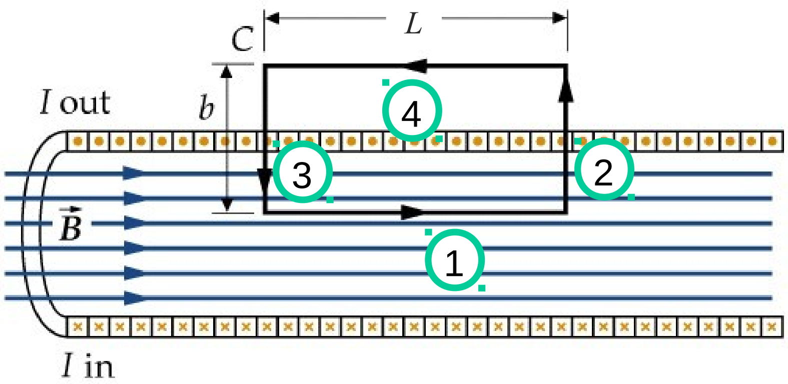
$$\vec{B}_{out} = \frac{\mu_0 I}{2\pi r}\hat{u}_\theta (T)$$



Ampere's law

MAGNETIC FIELD DUE TO AN INFINITE CURRENT-CARRYING SOLENOID

- B inside the solenoid is constant and parallel to its axis.
- B outside the solenoid is zero.



$$\boxed{\oint_C \vec{B}\cdot d\vec{l} = \int_1 \vec{B}\cdot d\vec{l} + \ldots + \int_4 \vec{B}\cdot d\vec{l}}$$

1 $B \parallel dl$ $\quad \oint_{C1} |\vec{B}||d\vec{l}| = |\vec{B}|\oint_{C1} |d\vec{l}| = |\vec{B}|L$

2, 3 $B \perp dl$ $\quad \oint_{C2,3} \vec{B}\cdot d\vec{l} = 0$

4 $B = 0$ $\quad \oint_{C4} |\vec{B}||d\vec{l}| = 0$

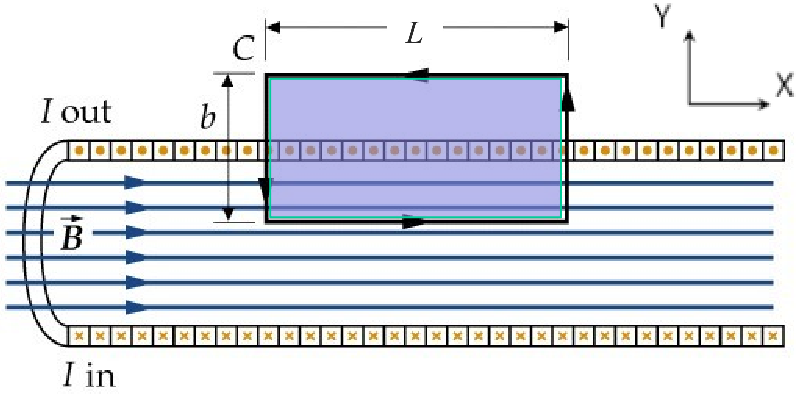
$$\boxed{\oint_C \vec{B}\cdot d\vec{l} = BL}$$

Ampere's law

N: number of turns going through the surface defined by C

n: number of turns per unit length

$$\boxed{\mu_0 I_C = \mu_0 N I = \mu_0 n L I}$$

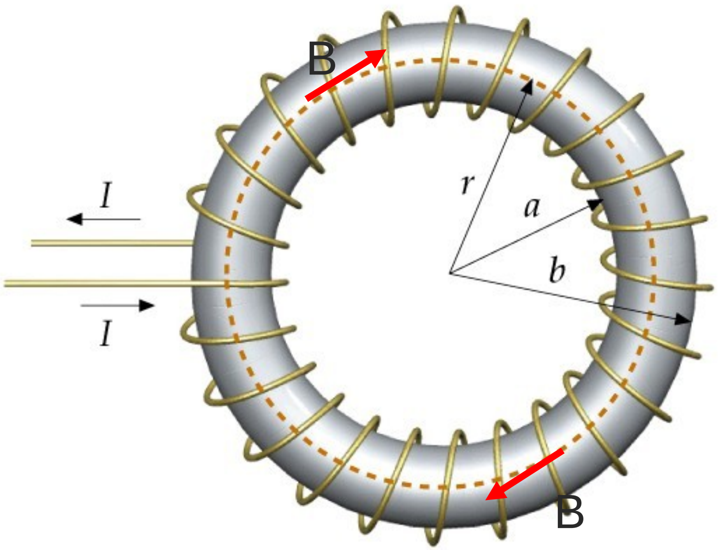


$$\vec{B}_{ins} = \mu_0 n I \vec{i}$$

Ampere's law

MAGNETIC FIELD DUE TO A CURRENT-CARRYING TOROID

- B inside the toroid is tangent to its axis.
- B is ~ const. If (b-a)<<r.
- B outside the toroid is zero.



$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_C$$

$$\vec{B} \| d\vec{l}$$

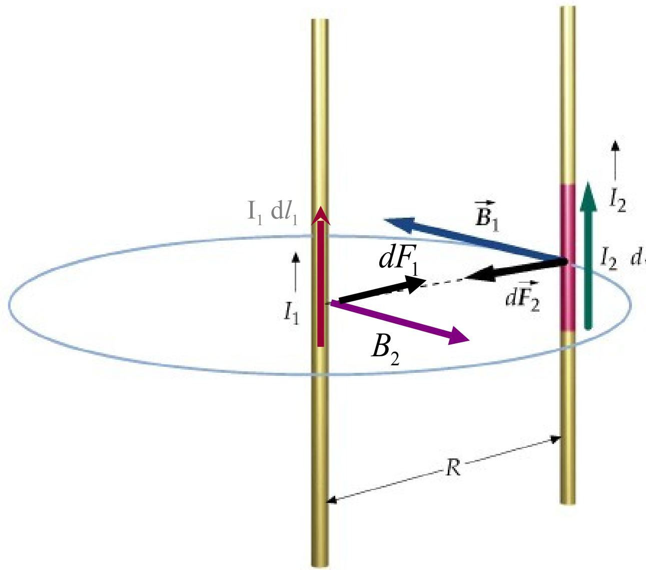
$$\boxed{\oint_C |\vec{B}||d\vec{l}| = |\vec{B}|\oint_C |d\vec{l}| = |\vec{B}| 2\pi r}$$

$$\boxed{\mu_0 I_C = \mu_0 NI}$$

$$|\vec{B}| 2\pi r = \mu_0 NI \Rightarrow |\vec{B}| = \frac{\mu_0 NI}{2\pi r}$$

Ampere's law

MAGNETIC FORCE BETWEEN VERY LONG PARALLEL WIRES



I_1 creates B_1 around it: $$B_1 = \frac{\mu_0 I_1}{2\pi r}$$

At I_2, that is at a distance R from I_1: $$B_1 = \frac{\mu_0 I_1}{2\pi R}$$

Current I_2 experiments a force F_{12} due to B_1

$$\vec{F}_{12} = I_2 \vec{l} \times \vec{B}_1 \Rightarrow |\vec{F}_{12}| = I_2 l B_1 = I_2 l \frac{\mu_0 I_1}{2\pi R}$$

I_2 creates B_2 around it $$B_2 = \frac{\mu_0 I_2}{2\pi R}$$

I_1 experiments F_{21} due to B_2 $$|\vec{F}_{21}| = I_1 l \frac{\mu_0 I_2}{2\pi R}$$

$$|\vec{F}_{21}| = |\vec{F}_{12}| = \frac{\mu_0 I_1 I_2}{2\pi R} l$$

The magnitude of these forces is equal

Currents circulating along the same direction attract each other

Currents circulating along opposite directions repel each other