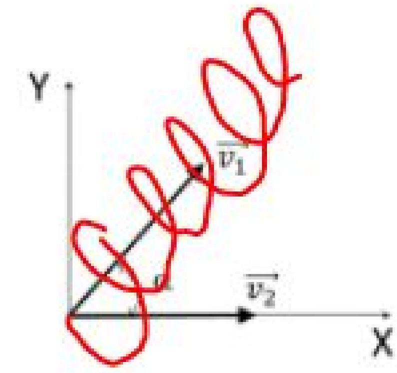
P3. *(2.5 p)* A uniform magnetic field $\vec{B}$ is established in a region of space. We know that when a proton is launched from the origin with a velocity $\vec{v_1}$ that forms an angle α with the X-axis, it does not experience any force that deflects its trajectory. However, if the proton is launched with a velocity $\vec{v_2}$ of the same magnitude as above but directed in the positive direction of the X-axis it experiences a force $\vec{F} = F\vec{k}$.



a) Calculate $\vec{B}$.

b) Calculate the radius and pitch of the helix that the proton would describe in the second case.

DATA: $v_1 = v_2 = 2\times10^5$ m/s; $\alpha = 35°$; $F = 5\times10^{-15}$ N

a) When the proton is launched with $\vec{v}_1$ then $\vec{F}_1 = q\,\vec{v}_1 \times \vec{B} = 0$

$\to \vec{v}_1 \times \vec{B} = 0 \to \vec{v}_1 \uparrow\uparrow \vec{B}$ $\boxed{\text{or}}$ $\vec{v}_1 \uparrow\downarrow \vec{B}$

When the proton is launched with $\vec{v}_2$ then $\vec{F}_2 = q\,\vec{v}_2 \times \vec{B} = F \cdot \vec{k}$

$$\to q \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_2 & 0 & 0 \\ B_x & B_y & 0 \end{vmatrix} = F\vec{k} \to q v_2 B_y \vec{k} = F\vec{k}$$

$$\to B_y = \frac{F}{q v_2} > 0 \to B\sin\theta > 0 \to$$

$\sin\theta > 0$, where θ is the angle between $\vec{v}_2$ and $\vec{B}$ that can be either $\theta = \alpha = 35°$ or $\theta = 180° + \alpha = 215°$ as we have shown above. Since $\sin\theta > 0$ we find that $\theta = a = 35°$ and therefore the magnetic field $\vec{B}$ is parallel to $\vec{V}_1$: $\vec{B} = B\cos a\,\vec{i} + B\sin a\,\vec{j}$

$$F_2 = q V_2 B \sin a \rightarrow B = \frac{F}{q V_2 \sin a}$$

and the magnetic field

$$\boxed{\vec{B} = \frac{F}{q V_2 \sin a}\left(\cos a\,\vec{i} + \sin a\,\vec{j}\right) = 0.223\,\vec{i} + 0.156\,\vec{j} \quad T}$$

b) If the proton is launched with $\vec{V}_2$ in the presense of $\vec{B}$, it will describe a helical trajectory with its axis along $\vec{B}$ (ie $\vec{V}_1$) and radius $R = \frac{m V_\perp}{qB}$ where $V_\perp$ is the component of the speed that is perpendicular to $\vec{B}$ ie $V_\perp = V_2 \sin a$

and $$\boxed{R = \frac{m v_2 \sin a}{qB} = 4.4 \text{ mm}}$$

The pitch $L_p = V_{\parallel} \cdot T$

$$\rightarrow \boxed{L_p = V_2 \cos a \cdot \frac{2\pi m}{qB} = 3.94 \text{ cm}}$$

since along the helix axis (ie along $\vec{B}$) the proton moves with a constant speed $V_{\parallel} = V_2 \cos a$

T is the period of the circular motion