Universidad Carlos III de Madrid

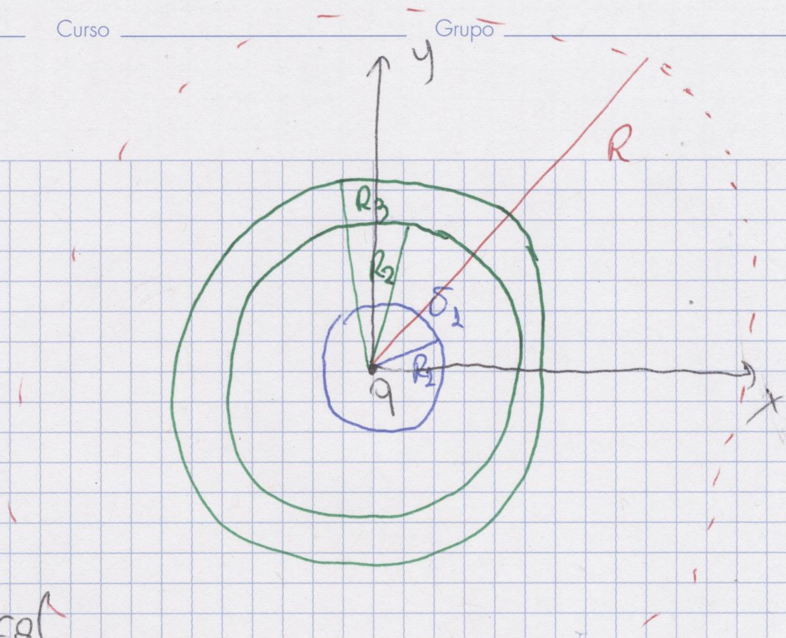
Escuela Politécnica Superior

Asignatura ______

Nombre y Apellidos ______

Fecha ______ Curso ______ Grupo ______

P1



a) The imaginary spherical surface with radius $R = 3R_3$ encloses the point charge q, the spherical shell with charge $Q_{shell} = \sigma_1 \cdot 4\pi R_1^2$ and the charged, conducting hollow sphere that has a total charge Q. Therefore, from Gauss's law: $\phi_0 = \frac{Q_{enc}}{\varepsilon_0} = \frac{q + \sigma_1 4\pi R_1^2 + Q}{\varepsilon_0}$

$$\rightarrow \boxed{Q = \varepsilon_0 \phi_0 - q - \sigma_1 \cdot 4\pi R_1^2} \quad (1)$$

b) The hollow conducting sphere of inner radius R_2 and outer radius R_3 has a net charge Q, calculated above, with $Q = Q_{R_2} + Q_{R_3}$ where Q_{R_2} is the charge on the inner surface and Q_{R_3} the charge on the outer surface. The electric field inside the conducting hollow sphere $E_{sph} = 0$ $(R_2 < r < R_3)$ as the conductor is at electrostatic equilibrium. If we consider a spherical imaginary Gaussian surface concentric to the charge distributions with radius

$R_2 < r < R_3$ the electric flux $\phi_G = \oint_{sph} \vec{E}\, d\vec{S} = 0$ through the Gaussian surface. From Gauss's law:

$$\phi_G = \frac{Q_{enc}(G)}{\varepsilon_0} = \frac{q + \sigma_1 \cdot 4\pi R_1^2 + Q_{R_2}}{\varepsilon_0} = 0 \quad (2)$$

The charge on the inner surface of the hollow sphere $Q_{R_2} = \sigma_2 \cdot 4\pi R_2^2$, where σ_2 the surface charge density.

From equation 2: $\boxed{\sigma_2 = \frac{-q - \sigma_1 \cdot 4\pi R_1^2}{4\pi R_2^2}}$ (3)

The charge density on the outer surface of the hollow sphere $\sigma_3 = \frac{Q_{R_3}}{4\pi R_3^2} = \frac{Q - Q_{R_2}}{4\pi R_3^2} = \frac{Q + q + \sigma_1 4\pi R_1^2}{4\pi R_3^2}$

$\overset{(1)}{\rightsquigarrow}$ $\boxed{\sigma_3 = \frac{\varepsilon_0 \phi_0}{4\pi R_3^2}}$ (4)

c) The Cu made conducting hollow sphere is at electrostatic equillibrium and therefore the electric potential for any point of the sphere is constant and equal to V_{Cu}. We will calculate the electric potential on the outer surface of the hollow sphere $V_{R_3} = V_{Cu}$: $V_{R_3} - \cancel{V_\infty}^{0} = -\int_\infty^{R_3} \vec{E}\, d\vec{r}$, where in the above expression we consider that the electric potential at infinity $V_\infty = 0$

so that $V_{R_3} = -\int_\infty^{R_3} \vec{E}\, d\vec{r}$ (5) where $\vec{E}$ is the net electric field for $r \geq R_3$.

We can obtain the general expression of $\vec{E}$ through Gauss's law. We consider a spherical Gaussian surface, concentric to the hollow sphere with radius r, where $r \geq R_3$. From Gauss's law the electric flux through this surface

$$\phi_G = \frac{Q_{enc}}{\varepsilon_0} = \frac{q + \sigma_1 4\pi R_1^2 + Q}{\varepsilon_0} = \phi_0 \quad (6)$$

From the definition of the electric field flux:

$$\phi_G = \oint \vec{E}\, d\vec{S} \underset{\vec{E} \parallel d\vec{S}}{=} \oint E\, dS \underset{\substack{E = \text{const} \\ \text{along the Gaussian} \\ \text{surface}}}{=} E \cdot \oint dS = E \cdot 4\pi r^2 \quad (7)$$

where we made use of the fact that the electric field is parallel to the surface vector $d\vec{S}$ and has a constant magnitude along the Gaussian surface.

From Eq. (6) and (7) $\rightarrow$ $E = \frac{\phi_0}{4\pi r^2}$ and the vector

$\vec{E} = \frac{\phi_0}{4\pi r^2}\hat{u}_r$ with $\hat{u}_r$ a radial unit vector.

From Eq. (5): $V_{R_3} = -\int_{\infty}^{R_3} \vec{E}\, d\vec{r} \underset{\vec{E} \parallel d\vec{r}}{=} -\int_{\infty}^{R_3} E \cdot dr =$

$$= -\int_{\infty}^{R_3} \frac{\phi_0}{4\pi r^2}\, dr = -\frac{\phi_0}{4\pi} \int_{\infty}^{R_3} \frac{1}{r^2}\, dr =$$

$$= -\frac{\phi_0}{4\pi} \left[-\frac{1}{r} \right]_{\infty}^{R_3} = \frac{\phi_0}{4\pi R_3}$$

So that the electric potential at any point of the Cu hollow sphere: $\boxed{V_{Cu} = \frac{\phi_0}{4\pi R_3}}$