Review of Kinematics and Dynamics

Classical Mechanics (Newtonian)

Kinematics
(κίνησης=motion)

description of motion

Dynamics
(δύναμης=force)

cause of motion

Physical quantities

- **Scalar**

  described by a **single number**

  Calculations using

  ordinary arithmetics

- **Vector**

  has both a **magnitude** and a **direction** in space

  Calculations using

  linear algebra

Physical quantities

- **Scalars:**

  Distance

  Speed

  Time

  Mass

  Energy

  Electric potential

- **Vectors:**

  Displacement

  Velocity

  Acceleration

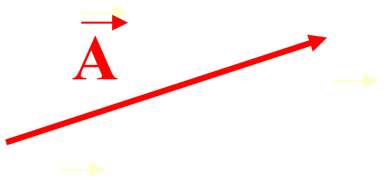
  Force

  Momentum

  Electric field

Vector

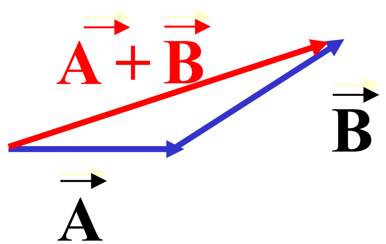
A quantity with a size and direction. Arrows are used to represent vectors.



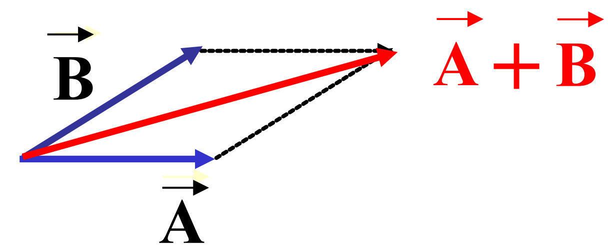
Graphical vector operations

- Addition

head to tail:

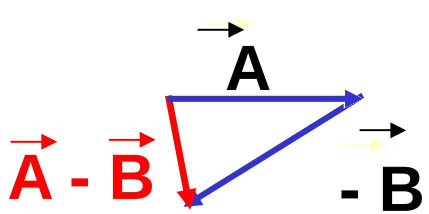


Parallelogram:

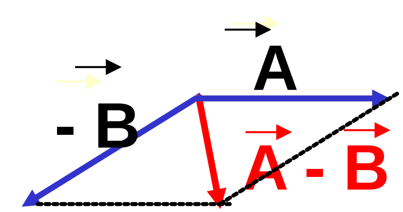


- Subtraction

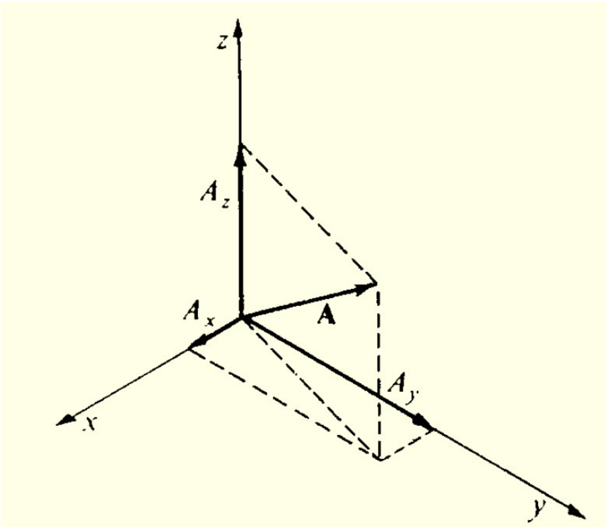
head to tail:



Parallelogram:



Component and unit Vectors



$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$$

MAGNITUDE OF A VECTOR

$$|\vec{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

UNIT VECTOR

$$\vec{u}_{\vec{A}} = \frac{\vec{A}}{|\vec{A}|} = \frac{(A_x, A_y, A_z)}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

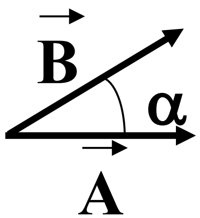
The 3 independent unit vectors in the system

$$\vec{u}_x = \vec{i} = (1,0,0)$$
$$\vec{u}_y = \vec{j} = (0,1,0)$$
$$\vec{u}_z = \vec{k} = (0,0,1)$$

Products of Vectors

- Scalar(Dot) product → scalar

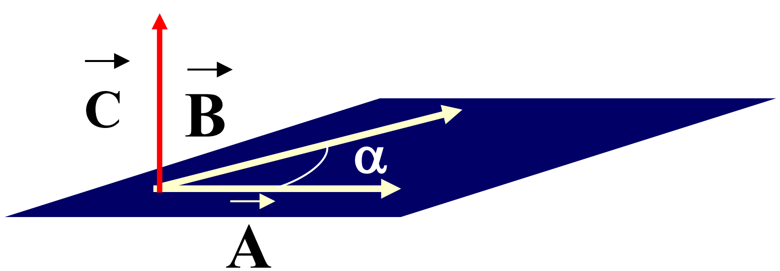
$$\vec{A}\cdot\vec{B}=AB\cos\alpha$$



$$\vec{A}\cdot\vec{B}=\left(a_x,a_y,a_z\right)\cdot\left(b_x,b_y,b_z\right)=\left(a_xb_x+a_yb_y+a_zb_z\right)$$

- Vector(cross) product → a new vector

$$\vec{C}=\vec{A}\times\vec{B}$$



$$\left|\vec{A}\times\vec{B}\right|=AB\sin\alpha$$

$$\vec{A}\times\vec{B}=\begin{vmatrix}\vec{u}_x & \vec{u}_y & \vec{u}_z\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}=\left(A_yB_z-A_zB_y\right)\vec{u}_x+\left(A_zB_x-A_xB_z\right)\vec{u}_y+\left(A_xB_y-A_yB_x\right)\vec{u}_z$$

Kinematics: study of motion

“Τα πάντα ρει, μηδέποτε κατά τ' αυτό μένειν”

“Everything changes and nothing remains still”

Heraclitus of Ephesus (c. 535 – c. 475 BC)

Quantities:

- Position **x**
- Distance d
- Displacement Δ**x**
- Speed
- Velocity **υ**
- Time t
- Acceleration **γ**

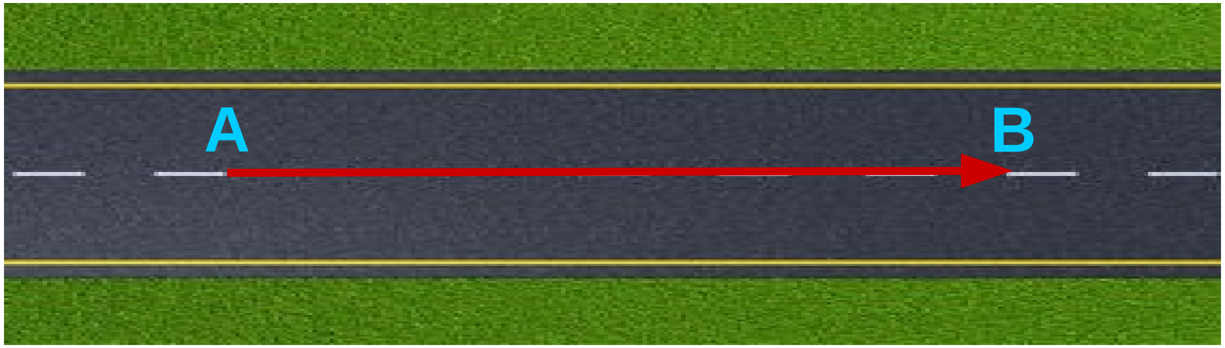
Distance vs displacement

- Distance: the complete length of the path travelled by a moving object

scalar

- Displacement: the length of the straight-line path from a moving object's origin to final position

vector



Average velocity

$$\boxed{\vec{v}_{av} = \frac{resultant\ displacement}{time\ interval} = \frac{\Delta \vec{r}}{\Delta t}}$$

SI units: m/s

- Velocity has direction (vector) and is relative (reference frame).
- Average speed (scalar) $\boxed{s_{av} = \frac{total\ distance}{time\ interval} = \frac{d}{\Delta t}}$

  is NOT the magnitude of average velocity!

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Example: Usain Bolt runs *100 m* in 9.58 s. What is his average speed?

10.44 m/s

Instantaneous velocity

reduce the size of the time interval $\Delta t \rightarrow 0$

$$\boxed{\vec{\upsilon} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = |\vec{\upsilon}|\vec{u}_{\upsilon}}$$

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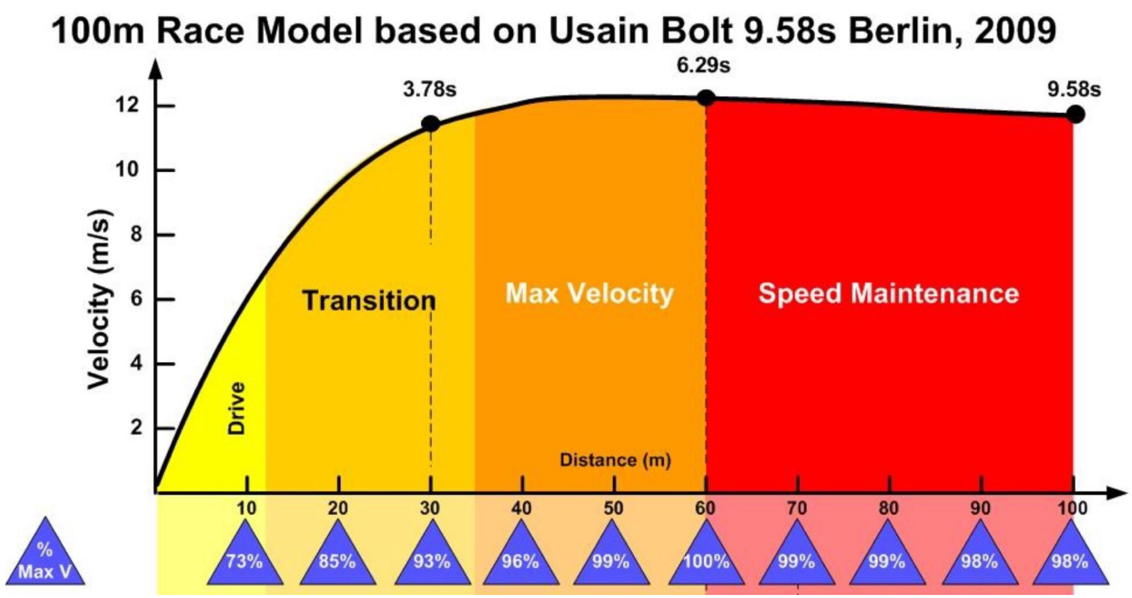
- Example: Usain Bolt's world record race



Acceleration

- average

$$\boxed{\vec{\gamma}_{av} = \frac{\Delta\vec{\upsilon}}{\Delta t}}$$

- instantaneous

$$\boxed{\vec{\gamma} = \lim_{\Delta t \to 0}\frac{\Delta\vec{\upsilon}}{\Delta t} = \frac{d\vec{\upsilon}}{dt}}$$

(vector) SI units: m/s^2

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(vector) SI units: m/s^2

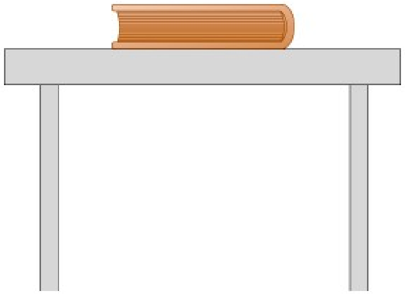
Example: A supercar accelerates from rest to

27.77 m/s in 2.1 s. What is the average acceleration magnitude?

Answer: 13.22 m/s^2

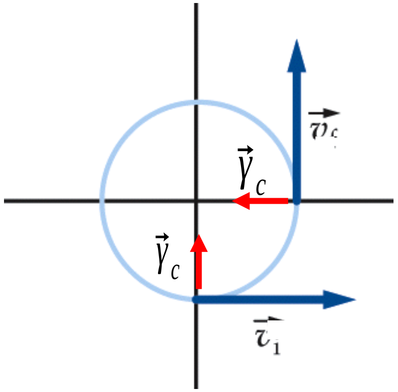
Uniform motion

- An object at rest



velocity is constant (magnitude and direction)

- Uniform circular motion and centripetal acceleration



Speed is constant, but velocity is NOT! Velocity direction changes → acceleration (perpendicular to υ)

$$\boxed{|\vec{\gamma}| = \frac{|\vec{\upsilon}|^2}{R} = \frac{\left(\frac{2\pi R}{T}\right)^2}{R} = \frac{4\pi^2 R}{T^2}}$$

Motion with Constant Acceleration

- Constant acceleration → constant total force

  Example: Free falling object

All bodies in free fall near the earth's surface have the same downward acceleration of

$g = 9.8$ m/s ^{2}

$$\Delta \vec{r} = \frac{1}{2} \vec{g}\, t^2$$

$$\vec{v} = \vec{g}\, t$$

Feather and hammer experiment

- Astronaut David Scott (1971)

  https://www.youtube.com/watch?v=KDp1tiUsZw8

Motion with Constant Acceleration

$$\Delta \vec{r} = \vec{v}_o t + \frac{1}{2} \vec{\gamma} t^2$$
$$\vec{v} = \vec{v}_o + \vec{\gamma} t$$
$$\vec{v}^2 = \vec{v}_o^2 + 2 \vec{\gamma} \Delta \vec{r}$$

- Kinematic equations:

  DERIVE from velocity and acceleration formulas

Dynamics: study of the cause of motion

“Άνευ αιτίου ουδέν εστιν”

“Nothing happens without a cause”

Aristotle (384 – 322 BC)

Quantities:

- Force **F**: any influence that can change the velocity of a body
- Mass m
- Acceleration **γ**
- Time t

Examples of forces

- Contact Forces

  arise from physical

  contact

- Action-at-a-distance forces

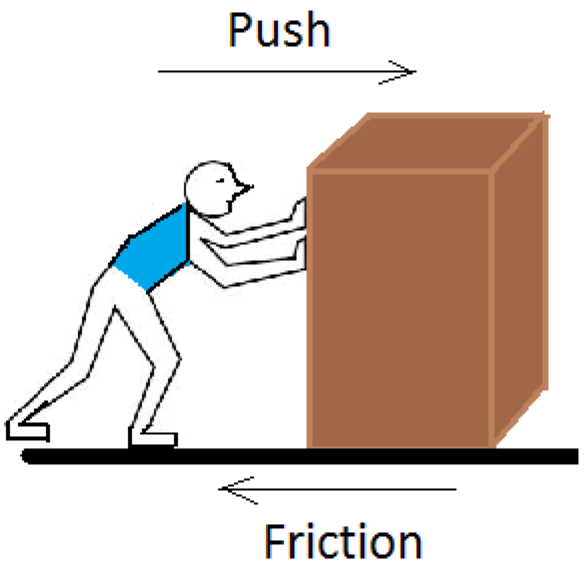
  do not require contact

  (gravity, electromagnetic)

Examples of contact forces

- Frictional forces act to oppose relative motion between surfaces that are in contact.

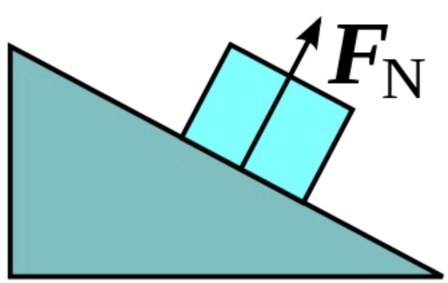
  Such forces act parallel to the surfaces.



static friction
Kinetic friction

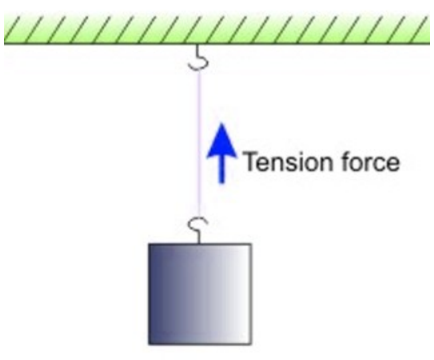
Examples of contact forces

- The normal force is one component of the force that a surface exerts on an object with which it is in contact → perpendicular to the surface



Examples of contact forces

- Cables and ropes transmit forces through tension. The tension T in a cable is the magnitude of the force that any part of the cable exerts on the adjoining part



Newton's First Law of Motion
"law of inertia"

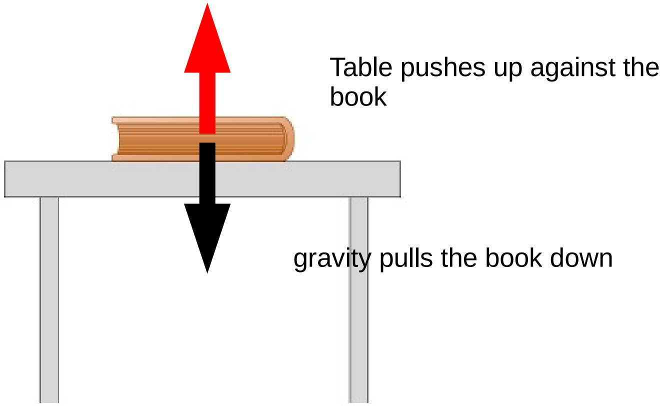
inertia=resistance to change

"A body at rest will remain at rest unless acted upon by a net force.

A body in motion will continue moving at constant velocity unless acted upon by a net force"

NET FORCE

$$\vec{F}_{net} = \sum \vec{F}_i$$



$$\vec{F}_{net} = 0$$

Newton's Second Law of Motion

"The acceleration of an object is directly proportional to the net force acting on the object and inversely proportional to the mass of the object "

$$\boxed{\vec{F}_{net} = \sum \vec{F}_i = \sum m \vec{\gamma}_i}$$

SI units: N=Kg*m/s^2

Mass is an intrinsic property of an object! It measures its resistance to acceleration ie inertia.

Mass is not weight!

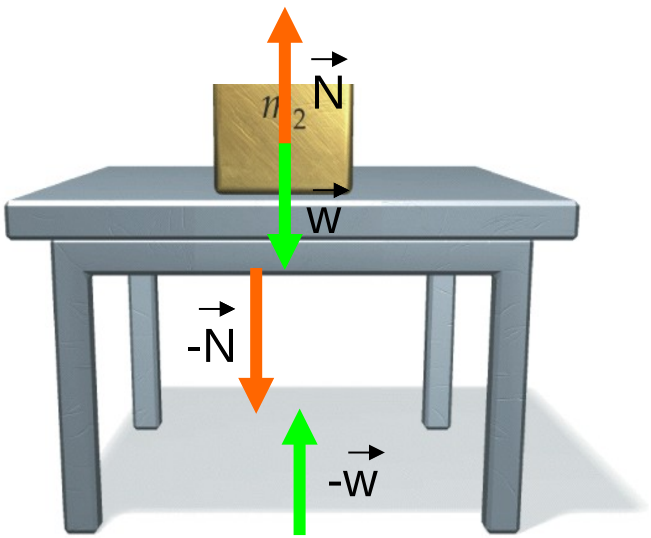
Newton's Third Law of Motion

"For every action, there is an equal and opposite reaction"

$$\boxed{\vec{F}_{AB} = -\vec{F}_{BA}}$$

SI units: N=Kg*m/s^2

EXAMPLE:

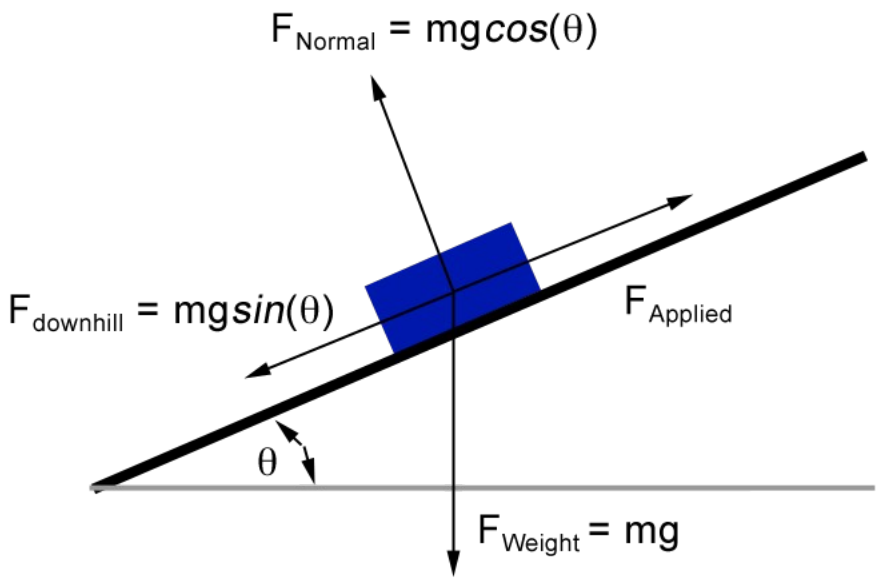


Action and reaction forces can NEVER balance each other, as they act on different objects!

From Newton's 2nd law to kinematics

FREE BODY DIAGRAM

A vector diagram that shows all of the forces that act on the body



Calculate net Force → obtain acceleration →

$$\vec{\gamma} = \frac{d\vec{\upsilon}}{dt} \Longrightarrow \vec{\upsilon}(t) = \vec{\upsilon}_0 + \int_0^t \vec{\gamma}(t)\,dt$$

$$\vec{\upsilon} = \frac{d\vec{r}}{dt} \Longrightarrow \vec{r}(t) = \vec{r}_0 + \int_0^t \vec{\upsilon}(t)\,dt$$