16.元

南京邮电大学 2016 / 2017 学年第一学期

《数字信号处理（双语）》期末试卷

院(系)________ 班级________ 学号________ 姓名________

题号	一	二	三	四							总 分
得分											

自觉遵守考试规则，诚信考试，绝不作弊

装订线内不要答题

得 分

一、选择题（10 分，每题 1 分）

语音信号 $f_s = 8kHz$ 反混叠

1. When sampling a speech signal at a rate 8kHz, the cutoff frequency of the antialiasing prefilter should be (C)kHz. $f_s = 8kHz$

A. 16 B. 8 C. 4 D. 2

$H(z) = \frac{1-2z^{-1}}{1-0.5z^{-1}}$

2. Consider a causal and stable system $H(z) = (1-2z^{-1})/(1-0.5z^{-1})$. To make its inverse system $H_{inv}(z)$ stable, the impulse response of $H_{inv}(z)$ should be (B).　B

$H_{inv}(z) = \frac{1}{H(z)}$

A. $2^n u(n) - 0.5 \cdot 2^{n-1} u(n-1)$ [✓ B. $-2^n u(-n-1) + 0.5 \cdot 2^{n-1} u(-n)$]

C. $2^n u(n) - 0.5 \cdot 2^{n-1} u(n)$. D. $-2^n u(-n) + 0.5 \cdot 2^{n-1} u(-n+1)$

3. The system $H(z) = (1+z^{-1})(1+2z^{-1})(1+3z^{-1})$ is a (A) filter.

分子有什么通过分 分子一次 分母0次

A. lowpass B. highpass C. bandpass D. bandstop

w=0
w=π

4. A time-windowing process is performed to $x(n) = Ae^{j2\pi\omega_1 n}$ $(-\infty < n < \infty)$. If the DTFT of the window $w(n)$ is $W(\omega)$, then the DTFT of the windowed signal $x(n) \cdot w(n)$ is (C).

$A\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - 2\pi\omega_1 + 2\pi k)$

A. $2\pi A\delta(\omega - \omega_1)$ B. $A\delta(\omega - \omega_1)$

[✓ C. $AW(\omega - \omega_1)$] D. $A\delta(\omega - \omega_1)W(\omega - \omega_1)$

$\frac{A}{2\pi}\int_{-\pi}^{\pi}$ $2\pi\delta(\theta - 2\pi\omega_1)W(\omega-\theta)$

5. To decrease the number of delay units in a system, we may realize the system by (C).

$\int 2\pi\delta(\theta - 2\pi\omega_1)W(\omega-\theta)$

正准实现形式

~~A. direct form~~ B. cascade form [✓ C. canonical form] D. parallel form

$\int_{-\pi}^{\pi}\delta(\theta)d\theta$

6. The 8-point DFT of x is $[8, 0, -8j, 8, 8, 8, 8j, 0]$. Then the 4-point DFT of $\frac{1}{2} \cdot x$ is (C).

$[8\ 0\ -8j\ 8\ 8\ 8\ 8j\ 0]$

A. $[8, -8j, 8, 8j]$ B. $[0, 8, 8, 0]$ [✓ C. $[4, -4j, 4, 4j]$] D. $[0, 4, 4, 0]$

$[4, 0, -4j, 4, 4, 4, 4j, 0]$

4 4 4j 0

4 0 -4j 4

8 4 0 4 8

7. When the N-point DFT of a length-N signal is implemented by matrix form, the total number of complex multiplications is (C). $N\times L$ 矩阵形式

A. $N\log_2 N$ B. $\frac{N}{2}\log_2 N$ C. N^2 D. $N(N-1)$

8. Compute the linear convolution of a long sequence x and the order-4 FIR filter h using overlap-add method. If the block convolutions use length-16 circular convolution, then the length of the blocks should be (A) $M=4$ 16 overlap-add. 重叠相加法.

$L_{xi}+M=N$ A. 12 B. 16 块长度 C. 8 D. 5

$L_{xi}=16-4=12$.

9. The DTFT $X(\omega)$ of a complex signal $x(n)$ satisfies (C) $X[n]$ $X^*[n]$

$X[n]$ A. $X(\omega)=X(-\omega)$ B. $X(\omega)=X^*(-\omega)$ → x(n)为实数.

C. $X(\omega)=X(2\pi+\omega)$ D. $X(\omega)=X(\pi+\omega)$

10. The bilinear transform maps the left-hand s-plane into the area of (D) on the z-plane to guarantee the stability and causality of the designed digital filter.

A. $|z|>0$ B. $|z|>1$ C. $|z|<0$ D. $|z|<1$

得 分

二、填空题（10 分，每空 1 分）

1. In general, the windowing process has two major effects: ①使频率分辨率下降; and ②引入原信号频域中不存在的高频分量，导致频率泄漏.

In the three realization forms of digital filter (direct, canonical, cascade), the form less sensitive to coefficient quantization errors than the other two is cascade form. 系数量化误差最不敏感的为串联实现

3. To generate a periodic signal $[1,-2,0,3,1,-2,0,3,\ldots]$ with period of 4, the system should be $H(z)=\frac{1-2z^{-1}+3z^{-3}}{1-z^{-4}}$.

$X(z)=\sum_{n=0}^{\infty} g(n)z^{-n}$

$(z^0+z^{-4}+z^{-8}+\cdots)$ $-2(z^{-1}+z^{-5}+z^{-9}+\cdots)$ $3(z^{-3}+z^{-7}+z^{-11}+\cdots)$

$\frac{1}{1-z^{-4}}+\frac{-2z^{-1}}{1-z^{-4}}+\frac{3z^{-3}}{1-z^{-4}}$

4. A 10kHz sinusoidal signal is sampled at 40kHz and 10 periods of the signal are collected. Suppose a 20-point DFT is performed, the DFT indices in the range $0\le k\le 19$ corresponding to the peaks in the spectrum are $k=5$, and $N-k=15$. $\frac{40}{10}=4$, $L=4\times10=40$

$k=N\frac{f_0}{f_s}=20\times\frac{10}{40}=5$ $N-k=20-5=15$

Suppose the DTFT of $[x_0,x_1,x_2,x_3]$ is $X(\omega)$. The DTFT of $[0,0,x_0,x_1,x_2,x_3]$ is $e^{-j2\omega}X(\omega)$. $X(\omega-2)$

6. Consider a causal FIR system $h(n)=[h(0),h(1),\cdots h(M)]$. $y(n)$ is the output signal for the input $x(n)=[x(0),x(1),\cdots x(L-1)]$. If $L<M$, the range of the input-on transient, 稳态的 steady state and input-off transient are ~~[0, M-1]~~, ~~[M, L-1]~~, ~~[L, M+L-1]~~ respectively. $y(n)=\sum_m h(m)x(n-m)$ $[0, L-1)$, 无, $[L-1, M+L-1)$ $(L, M+L-1)$

$L<M$ 无稳态.

$$\begin{cases}0\le m\le M\\0\le n-m\le L-1\end{cases}\Rightarrow \max(0,n-L+1)\quad ① n<L-1<M$$

$n-(L-1)\le m\le n$ $\min(n,M)$ $\sum_{\max[0,n-(L-1)]}^{\min(n,M)}$ $L<M$

$0\le n\le M+L-1$.

[illegible]

$f_s = 4kHz$; $[-2, 2]$

$\therefore f_{1a} = f_1 \bmod f_s = \frac{5}{2}\cancel{kHz} \bmod 4$

$= -\frac{3}{2}kHz$.

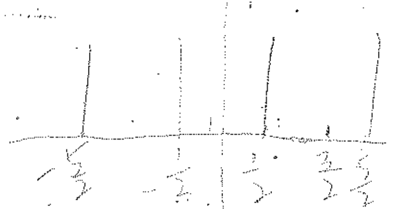
$f_{2a} = f_2 = \frac{1}{2}kHz$.

$\therefore x_a(t) = \cos(3\pi t) + 4\sin(\pi t)$

$\frac{1L \cdot fs \cdot 12}{8} = 12 f_s$

$= 60s \times 4 \times 10^3$

$= 240K$ Bytes



得分

三、计算、分析题(共 60 分)

$\frac{2\pi}{5\pi} = \frac{2}{5}$ $\frac{2\pi}{\pi} = 2$

1. The signal $x(t) = \cos(5\pi t) + 4\sin(\pi t)$, where t is in milliseconds, is sampled at sampling rate $f_s = 4$ kHz.

毫秒

ms.

a) Determine the signal $x_a(t)$ aliased with $x(t)$.

b) If each sampled value is quantized by 8 bits, how many bytes of disk space should be to store 1 minute signal.

(10 分)

2° 解(1) $\because H(z)|_{z=-1}$

$= H(\omega)|_{\omega=\pi} = \frac{b}{1+a}$

$\because b = 1+a$

$\therefore H(z)|_{z=-1} = 1$ ✓

$\therefore s(-1)^n$ is unchanged in the output of the filter.

2. Suppose we use the lowpass filter $H(z) = b/(1-az^{-1})$, where $0<a<1$, $b = 1+a$, to extract the high-frequency signal $x(n) = s(-1)^n + v(n)$, where s is constant and $v(n)$ is zero-mean white noise of variance σ_v^2.

a) Explain why the desired part $s(-1)^n$ is unchanged in the output of the filter.

b) Calculate the NRR and determine if the noise is amplified. 放大

(10 分)

(2) $h(n) = b \cdot a^n u(n)$ $\because 0<a<1$

$NRR = \sum_n h_n^2$

$= (1+a)^2 \frac{1}{1-a^2}$

$= \frac{1+a}{1-a}$

$\because NRR = \frac{\sigma_{yv}^2}{\sigma_v^2} = \frac{1+a}{1-a} > 1$

$\therefore \sigma_{yv}^2 > \sigma_v^2$

$\therefore$ 噪声被放大

3. A signal consisting of four sinusoids of frequencies of 1, 1.5, 2 and 2.25kHz is sampled at a rate of 8kHz.

正弦

a) Determine the minimum number of samples that should be collected for the frequency spectrum to exhibit four distinct peaks at these frequencies.

b) How many samples should be collected if a Hamming window is used for preprocessing?

(10 分)

3° (a) $\Delta f_{min} = 2.25k - 2k$

$= 0.25kHz$.

$\because \Delta f_{min} \geq \frac{f_s}{L}$

$\therefore L \geq \frac{f_s}{\Delta f_{min}} = \frac{8k}{0.25k} = 32$

(b) $L \geq c \cdot \frac{f_s}{\Delta f_{min}} = 64$

4. Considering the signal $x(n) = \cos(\pi n/4) + 2\cos(\pi n/8)$ $(n = 0,1,\ldots,15)$. Determine the 16-point DFT and 8-point DFT spectrum of $x(n)$ without performing any DFT or FFT.

(10 分)

4 解: (a) $x(n) = \cos(\frac{\pi n}{4}) + 2\cos(\frac{\pi n}{8})$ $(n = 0,1\cdots15)$

$= \frac{1}{2}(e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n}) +$

$e^{j\frac{\pi n}{8}} + e^{-j\frac{\pi}{8}n}$

$= \frac{1}{2}e^{j\frac{\pi}{4}n} + \frac{1}{2}e^{j\frac{7}{4}\pi n} + e^{j\frac{\pi n}{8}} + e^{j\frac{15}{8}\pi n}$

$\because L = 16 = N = 16$

$\therefore \hat{x}(n) = x(n)$

$\therefore \hat{x}(n) = \frac{1}{16}\sum_{k=0}^{15} X(k) e^{j\frac{2\pi}{16}kn}$

$X(1) = 16$; $X(2) = 8$;

$X(12) = 8$; $X(15) = 16$

$\therefore X = [0, 16, 8, 0, 0, 0, 0, 0, 0, 0, 0, 0, 8, 0, 0, 16]$

8-point: $[0, 8, 0, 0, 0, 0, 8, 0]$

5. Design a lowpass digital FIR filter with cutoff frequency $\omega_c = 0.4\pi$ using a rectangular window of length $N=21$.

a) Determine the impulse response $h(n)$ of the designed filter.

b) What should we do if we want to decrease the transition width and improve the stopband attenuation simultaneously?

(10 分)

解: $N = 21$, $M = \frac{N-1}{2} = 10$

(a) $d(k) = \frac{\sin(\omega_c k)}{\pi k}$, $-\infty < k < +\infty$

$h(n) = d(n-M)$

$= \frac{\sin[\omega_c(n-10)]}{\pi(n-10)}$

$= \frac{\sin[0.4\pi(n-10)]}{\pi(n-10)}$, $0 \leq n \leq 20$

(b) ~~增大N~~

用汉明窗,并把窗长增大为原来的2倍以上

3

$\therefore w_c = \frac{\;}{f_s} = \frac{\;}{10k} = 0.5\pi$

~~$\Omega_c = \tan(\frac{w_c}{2}) = \tan(0.25\pi)$~~

$= 1$ $\Omega_c = \cot(\frac{w_c}{2}) = 1$

$\therefore H_a(s) = \frac{1}{s^2+\sqrt{2}s+1}$ → 双线性变换

~~$H(\Omega) = \frac{1}{(j\Omega)^2+\sqrt{2}(j\Omega)+1}$~~

$\therefore$ ~~$|H(\Omega)|^2$~~ $=$

6. Using the bilinear transformation and a lowpass analog Butterworth filter

模拟

$$H_a(s) = \frac{1}{(\frac{s}{\Omega_0})^2+\sqrt{2}\frac{s}{\Omega_0}+1}$$

, design a second-order highpass digital filter with 3-dB

二阶

cutoff frequency 2.5 kHz and operating at a sampling rate of 10 kHz. Determine the transfer function of the designed filter.

(10 分)

得 分

四、简答、画图分析题(共 20 分)

课本；

→ 先截断再延时吗？如何写步骤？

1. An anticausal but stable system has impulse response $h(n) = 0.4\delta(n) - 0.6(1.25)^n u(-n-1)$. Describe the standard method for making this system into a causal system.

$n=-D$ 时截断；再整体向右延时 D

$u[-(n-D)-1]$

(5 分)

2. A system is described by its impulse response $h(n) = (-0.8)^n[u(n) - u(n-8)]$

a) Determine the transfer function $H(z)$ of the system.

b) Determine the ROC of the system.

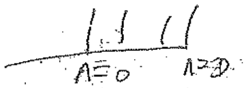
c) Determine the poles and zeros of the system.

d) Draw a rough sketch of the magnitude response $|H(\omega)|$.

(15 分)

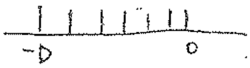
1° 解：1° $h(n) = 0.4\delta(n) - 0.6(1.25)^n u(-n-1)$

~~$h(n) = 0.4\delta(n) - 0.6(1.25)$~~

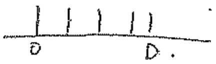


1° 先截断：

$h(n) = 0.4\delta(n) - 0.6(1.25)^n\{u(-n-1) - u[-(n-D)-1]\}$



再延时D个时间



$h(n-D) = 0.4\delta(n-D)$

$-0.6(1.25)^{n-D}[u[-(n-D)-1] -$

$u[-(n-2D)-1]]$.

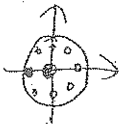
2° (a) $h(n) = (-0.8)^n[u(n) - u(n-8)]$

$H(z) = \frac{1}{1+0.8z^{-1}} - \frac{z^{-8}\cdot(-0.8)^8}{1+0.8z^{-1}}$

$= \frac{1-z^{-8}\cdot(-0.8)^8}{1+0.8z^{-1}} = \frac{1-(0.8)^8 z^{-8}}{1+0.8z^{-1}}$

(b) 有限长，ROC: $|z| \neq 0$

(c) $H(z) = \frac{1-0.8^8 z^{-8}}{1+0.8z^{-1}} = \frac{z^8-(0.8)^8}{z^8+0.8z^7} = \frac{z^8-(0.8)^8}{z^7(z+0.8)}$

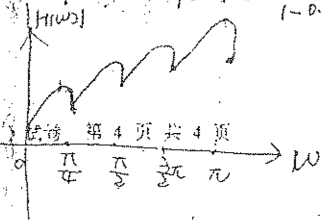


零点：$z^8 = (0.8)^8 \Rightarrow z = 0.8e^{j\frac{2\pi}{8}n}$ $n=0,1\cdots7$.

极点：$z = 0$ (7阶)，$z = -0.8$.

(d) $|H(\omega)|$ $H(\omega)|_{\omega=0} = H(z)|_{z=1} = \frac{1-(0.8)^8}{1.8} = 0.46$

$H(\omega)|_{\omega=\pi} = H(z)|_{z=-1} = \frac{1-(0.8)^8}{1-0.8} = 4.16$



南京邮电大学 2015 / 2016 学年第二学期

《数字信号处理 A（双语）》试卷

院(系)＿＿＿＿＿＿ 班级＿＿＿＿＿ 学号＿＿＿＿＿ 姓名＿＿＿＿＿

题号	一	二	三	四	五	六	七	八	九	十	总 分
得分											

一、填空题

60s / $\frac{1}{8k}$ × 8bit / 8

(1) To record 1 minute of speech signal with sampling rate of 8kHz, and each sampled value is quantized by 8 bits. Then we need ＿480 k＿ bytes of disk space to store the signal.

(2) A wheel, rotating at 8Hz and turning clockwise, is seen in a dark room by means of a strobe light flashing once per 0.1 second. Determine the apparent rotational speed ＿once per 0.5 s＿, and sense of rotation of the wheel ＿wrong＿.

$\frac{1}{0.1}$ = 10Hz

f_a = f mod f_s = -2Hz

(3) A signal $(A_1e^{j\omega_1 n}+A_2e^{j\omega_2 n})u(n)$ is the input of a LTI system whose frequency response is $H(\omega)$, then the steady-state response of this system is ＿$A_1H(\omega_1)e^{j\omega_1 n}+A_2H(\omega_2)e^{j\omega_2 n}$＿

(4) The zero of $H(z)=\frac{1}{1-0.5z^{-1}}$ is ＿$z=0$＿ and the pole is ＿$z=0.5$＿.

$\frac{z}{z-0.5}$

(5) A signal $x(t)$=cos($2\pi f_1 t$)+cos($2\pi f_2 t$) is sampled at 20kHz and 128 samples are collected. Suppose a 64-point DFT is performed, the physical frequency resolution is ＿$\Delta f = \frac{f_s}{L}$＿ and the computational frequency resolution is ＿$\Delta f_{bin} = \frac{f_s}{N}$＿.

f_s = 20kHz

N = 64

L = 128

时分算法

(6) Suppose a 8-point DFT is computed using the decimation-in-time radix-2 (基二) FFT algorithm, the cost of the complex multiplications is ＿$\frac{N}{2}\log_2 N$＿ and the number of the complex addition is ＿$N\log_2 N$＿.

二、选择题

(1) The bilinear transform maps the left-hand s-plane into the area of (D) on z-plane to guarantee the stability and causality of the designed digital filter.

A. $|z|>0$ B. $|z|>1$ C. $|z|<0$ D. $|z|<1$

(2) Consider a FIR filter $\mathbf{h}=[h_0,h_1,\cdots,h_9]$ and an input signal $\mathbf{x}=[x_0,x_1,\cdots,x_6]$. If we want to compute the output of the filter in matrix form $\mathbf{y}=\mathbf{Hx}$, the dimension (维数) of $\mathbf{H}$ should be (C).

M = 9

L = 7

A. 17×6 B. 16×6 C. 16×7 D. 17×7

16×7

5

(3) In order to sample a signal at a rate f_s, the cutoff frequency of the antialiasing prefilter should be (A). 反混叠

A. $f_s/2$ B. f_s C. $f_s/4$ D. $2f_s$

(4) Consider a causal and stable system $H(z)=(1-2z^{-1})/(1-0.5z^{-1})$. To make its inverse system $H_{inv}(z)$ stable, the impulse response of $H_{inv}(z)$ should be (B). ROC包含单位圆

$h(n)$

$H(z) = \frac{1-0.5z^{-1}}{1-2z^{-1}} = \frac{1}{1-2z^{-1}} - \frac{0.5z^{-1}}{1-2z^{-1}}$

A. $2^n u(n) - 0.5\cdot 2^{n-1}u(n-1)$ B. $-2^n u(-n-1) + 0.5\cdot 2^{n-1}u(-n)$

C. $2^n u(n) - 0.5\cdot 2^{n-1}u(n)$ D. $-2^n u(-n) + 0.5\cdot 2^{n-1}u(-n+1)$

(5) The 8-point DFT of x is [8, 0, −8j, 8, 8, 8, 8j, 0]. Then the 4-point DFT of x/2 is (A).

A. $[8,-8j,8,8j]$ B. $[0,8,8,0]$ C. $[4,-4j,4,4j]$ D. $[0,4,4,0]$ 希尔伯特变换

(6) The impulse response of an ideal Hibert transformer is $d(n)=(1-\cos(\pi n))/\pi n$, $-\infty<n<\infty$. The symmetric (对称的) property of the frequency response $D(\omega)$ is (D). 性能

实偶→实偶
实[illegible] 奇→虚奇

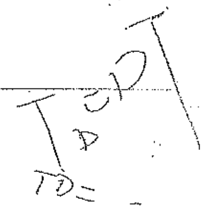
A. real and even B. imaginary and even

C. real and odd D. imaginary and odd

(7) The system $H(z)=(1+z^{-1})(1+2z^{-1})(1+3z^{-1})$ is a (A) filter.

A. lowpass B. highpass C. bandpass D. bandstop

(8) In the several realization forms of LTI system, ~~(B) is more robust to the~~ coefficient quantization error. 鲁棒性 系数量化 →级联



A. direct form B. cascade form C. canonical form D. parallel form

(9) The signal $\sin(n\omega_0)$ ($n=-\infty,\ldots,-1,0,1,\ldots,\infty$) is NOT periodic when $2\pi/\omega_0$ equals to (D) $= \sin[(n+D)\omega_0] = \sin(n\omega_0 + D\omega_0)$

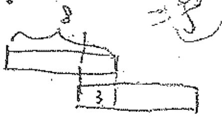
$\omega_0 D = 2\pi$ $\frac{2\pi}{\omega_0} = \frac{5}{2}$

A. 1 B. 5/2 $\frac{2}{5}$ C. 2/3 $\frac{3}{2}$ D. π

$D = \frac{2\pi}{\omega_0}$

$\sin(\frac{8}{5}\pi n + \frac{4}{5}\pi n_0)$ $\frac{2\pi}{\omega_0} = \frac{5}{2}, \omega_0 = \frac{4}{5}\pi$ $D\omega_0 = 2\pi$ $\omega_0 = 2\pi$

(10) To compute the linear convolution of a long sequence x and the order-3 FIR filter h using overlap-add 重叠相加 method. If the block convolutions use length-8 circular convolution, then the length of the blocks should be (B).

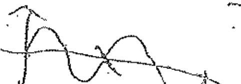


$\frac{2}{3} = 2\pi/\omega_0$

A. 4 B. 8 C. 3 D. 5

$\omega_0 = \frac{4}{3}\pi$

三、计算题



(1) Using partial fractions 部分分式 or power series expansions 幂级数展开, compute the inverse z-transform of $X(z)=\frac{3(1+0.3z^{-1})}{1-0.81z^{-2}}$, sketch the ROCs, and discuss the stability and causality properties.

$\sin(\frac{4}{3}\pi n + \frac{4}{3}\pi n_0)$

$\sin(2n + 2n_0)$

$X(z) = ~~3(1-0.81)~~ \frac{A_1}{1-0.9z^{-1}} + \frac{A_2}{1+0.9z^{-1}}$

$= \frac{2}{1-0.9z^{-1}} + \frac{1}{1+0.9z^{-1}}$

$X(z) = \frac{3(1+0.3z^{-1})}{(1+0.9z^{-1})(1-0.9z^{-1})}$

$= \frac{1}{1+0.9z^{-1}} + \frac{2}{1-0.9z^{-1}}$

$x(n) = 2(0.9)^n u(n) + (-0.9)^n u(n)$ $|z| > 0.9$ 不稳 因果

$-2(0.9)^n u(-n-1) - (-0.9)^n u(-n-1)$ $|z| < 0.9$ 稳定 非因

(1) $x(n) = \sin(\frac{2\pi f}{f_s} n) = \sin(\frac{2\pi 10}{8} n)$

$f_0 = 2kHz$ $f_s = 8kHz$ $[-\frac{f_s}{2}, \frac{f_s}{2}]$ $[-4kHz, 4kHz]$

$= -2kHz$

$N = k\frac{f_s}{f_0} = 4k$ $k=2$ $N=8$ $N-k=6$

$f = 10kHz$

$f_s = 8kHz$.

$\angle = \frac{f}{f_s}$

(2) A sinusoid signal $x(t)=\sin(2\pi ft)$ with f=10kHz is sampled at a rate of $2\pi f_a =$ 8kHz and a finite portion of the signal is collected. $f \bmod f_s = 2kHz$

a) Determine the peak frequencies lying in the Nyquist interval $[-\pi, \pi]$. 存在 频谱 $k = \frac{f}{f_s} N$ 间隔 f_0

b) Suppose an N-point DFT is to be performed. If we expect to see the peaks in the DFT spectrum, determine the possible number of N.

(3) Designing FIR digital filter with window method:

a) Using the inverse DTFT, derive the impulse response of an ideal lowpass digital filter with cutoff frequency ω_c. $h(n) = \frac{\sin \omega_c n}{\pi n}$ ~~$d(k)$~~ $= \frac{\sin \omega_c k}{\pi k}$

b) Using a rectangular window, design a length N=11 lowpass FIR filter of cutoff frequency $\omega_c = 0.3\,\pi$. $k = \frac{N-1}{2} = 5$ $d(k) = \frac{\sin \omega_c k}{\pi k}$ $h(n) = \frac{\sin[\omega_c(n-5)]}{\pi(n-5)}$

(4) Design a resonator filter of the form $H(z)=1/(1+a_1z^{-1}+a_2z^{-1})$ which has a peak at f_0 =600Hz and the effective time constant n_{eff} =300 (samples) corresponding to $\varepsilon = 1\%$, and is operating at the rate of f_s=5kHz. $n_{eff} = \frac{2\ln\varepsilon}{-\Delta\omega}$ $2(1-R) = \Delta\omega$

a) Compute the values of a_1 and a_2. $\omega_0 = \frac{2\pi f_0}{f_s}$ $\Delta\omega = \frac{2\pi \Delta f}{f_s}$ $a_1 = -2R\cos\omega_0$

b) Determine the 3-dB width $\Delta\omega$ of the resonator. $a_2 = R^2$

(1) $R = \varepsilon^{\frac{1}{n_{eff}}}$

$\omega_0 = 2\pi\frac{f_0}{f_s} = 0.24\pi$

$a_1 = -2R\cos\omega_0$

$a_2 = R^2$

$\Delta\omega = 2(1-R)$

(5) Using the bilinear transformation and a lowpass analog Butterworth filter ,design a lowpass digital filter operating at a rate of 40kHz and having the following specifications : A_{pass}=3dB, f_{pass}=10kHz, f_{stop}=15kHz, A_{stop}=35dB.

$\omega_p = \frac{2\pi F_P}{F_T} = \frac{2\pi \times}{}$

$= 0.$

$W_s = \frac{2\pi F_s}{F_T} = \frac{2\pi\times}{40}$

$= 0.$

$-10\log_{10}\frac{1}{1+(\Omega_p/\Omega_0)^{2N}} = 3dB$

$\Omega_p = \tan(\frac{10kHz}{…}) = \tan 0.25\pi = 0.0137086$

$-10\log_{10}\frac{1}{1+(\Omega_s/\Omega_0)^{2N}} = 35dB$

$\Omega_s = \tan(\frac{15}{…}) = \tan = 0.0205457$

(The magnitude response of Butterworth filter is $|H(\Omega)|^2 = \frac{1}{1+(\frac{\Omega}{\Omega_0})^{2N}}$)

N 取3

$$N = \frac{\log_{10}\sqrt{\frac{10^{\frac{35}{10}}-1}{10^{\frac{3}{10}}-1}}}{\log_{10}(\Omega_s/\Omega_p)} \Rightarrow 6 \; ?$$

$\Rightarrow 0.67$

Table: When $\Omega_0 = 1$, the analog Butterworth lowpass filter $H(s) = 1/D(s)$

N	$D(s)$
3	$(1+s)(1+s+s^2)$
4	$(1+0.7654s+s^2)(1+1.8478s+s^2)$
5	$(1+s)(1+0.6180s+s^2)(1+1.6180s+s^2)$
6	$(1+0.5176s+s^2)(1+1.4142s+s^2)(1+1.9319s+s^2)$
7	$(1+s)(1+0.4450s+s^2)(1+1.2470s+s^2)(1+1.8019s+s^2)$

$\uparrow$ $\frac{s}{\Omega_0}$

$\omega_{pass} = \frac{2\pi f_{pass}}{f_s}$

$\Omega_{pass} = \tan\frac{\omega_{pass}}{2}$

$A_{pass} = 10\log_{10}\left[1+\left(\frac{\Omega_{pass}}{\Omega_0}\right)^{2N}\right]$

$$N_{exact} = \frac{\ln\sqrt{\frac{10^{A_{stop}/10}-1}{10^{A_{pass}/10}-1}}}{\ln(\frac{\Omega_{stop}}{\Omega_{pass}})}$$

$$\Omega_0 = \frac{\Omega_{pass}}{(10^{A_{pass}/10}-1)^{\frac{1}{2N}}} = \frac{\Omega_p}{(10^{\frac{3}{10}}-1)^{\frac{1}{6}}} = 0.0137190456$$

四、简答、画图、分析

(1) Define the following transforms and explain how they are related to each other:

(i) Discrete time Fourier transform (DTFT) 离散时间序列 $x(n) \longrightarrow$ 连续周期谱 $X(\omega)$ $[e^{j\omega n}]$

(ii) Discrete Fourier transform (DFT) (有限长) $x(n) \longrightarrow$ (有限长) $X(k)$ $e^{j\frac{2\pi}{N}kn}$

(iii) Fast Fourier transform (FFT). DFT 的快速算法

Explain how the DTFT is related to the analogue Fourier transform.

FT $\underset{连续时间信号}{x(t)} \rightarrow \underset{连续谱}{X(\Omega)}$ $[e^{j\Omega t}]$

(2) Considering the signal $x(n)$=0.5+2sin(0.5πn)+1.5cos(πn), n=0,1,2,……,15

$1e^{j0.5\pi n} - je^{-j0.5\pi n} + 0.75e^{j\pi n} + 0.75e^{-j\pi n}$

a) Sketch(画草图) the spectrum $X(\omega)$ ($0 \le \omega \le 2\pi$) of $x(n)$.

b) Without performing any DFT or FFT computations, determine the 16-point DFT of $x(n)$

$x(n) = 0.5 + je^{-j\frac{\pi}{2}n} - je^{j\frac{3\pi}{2}n} + 1.5e^{j\pi n}$

$W_0 = 0.5 \quad W_4 = j \quad W_{12} = -j \quad W_8 = 1.5$

c) Determine the 8-point DFT of $x(n)$

$X16$ $(8, 0, 0, 0, 16j, 0, 0, 0, 24, 0, 0, 0, -16j, 0,$

(3) A filter is described by the system of difference equations:

$v(n) = x(n) + 0.25v(n-4)$

$y(n) = 2v(n) - 3v(n-1) + 2v(n-2) - 3v(n-3)$

a) Determine the transfer function $H(z)$ of the filter.

b) Draw the cascade form of SOS realization of $H(z)$. Each SOS should be implemented with canonical form. 典型

应用

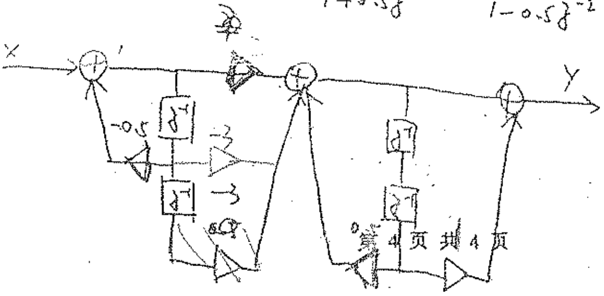
$$V(z) = X(z) + 0.25V(z)z^{-4}$$

$$V(z) = \frac{1}{1-0.25z^{-4}}X(z)$$

$$Y(z) = (2-3z^{-1}+2z^{-2}-3z^{-3})V(z)$$

$$H(z) = \frac{2-3z^{-1}+2z^{-2}-3z^{-3}}{1-0.25z^{-4}} \qquad \frac{(2-3z^{-1})(1+z^{-2})}{(1-0.5z^{-2})(1+0.5z^{-2})}$$

$$= \frac{2-3z^{-1}}{1+0.5z^{-2}} \cdot \frac{1+z^{-2}}{1-0.5z^{-2}} \qquad 2-3z^{-1}\overline{\left)\, 2-3z^{-1}+2z^{-2}-3z^{-3}\right.}\ \ 1+z^{-2}$$



南京邮电大学 2015/2016 学年第 一 学期

16 [illegible]

《数字信号处理A(双语)》期末试卷（A卷）

自觉遵守考试规则，诚信考试，绝不作弊

装订线内不要答题

院(系)____ 班级____ 学号____ 姓名____

题号	一	二	三	四	五	六	七	八	九	十	总分
得分											

得分

一、选择题（每题2分，共10分）…………

$\frac{8}{8000} = \frac{1}{1000}$ 1 一致抽

① A 8 second long segment signal of a continuous-time signal is uniformly sampled without aliasing and generating a finite length sequence containing 8000 samples. The highest frequency component that could be present in the continuous-time signal is (D)

A. 1 kHz B. 2 kHz C. 4 kHz D. 0.5 kHz

$A_1 e^{j\omega_1 n} \to A_1 \sum_{k=-\infty}^{\infty} 2\pi\delta(\omega-\omega_1+2\pi k)$ ω_1, ω_1+

② A signal $A_1 e^{j\omega_1 n}$ $(-\infty < n < +\infty)$ is the input of a LTI system whose frequency response is $H(e^{j\omega})$, then the response of this system is (B) H(ω)以2π为周期，H(ω1)为常数

A. $A_1 e^{j\omega_1 n}$ B. $A_1 H(e^{j\omega_1}) e^{j\omega_1 n}$ C. $2\pi H(e^{j\omega_1}) A_1 \delta(\omega-\omega_1)$ D. $A_1 H(e^{j\omega}) e^{j\omega_1 n}$

3. Consider the length-7 sequence $\{x[n]\}=\{3, -5, 1, 2, 7, -4, -2\}$ with $X(e^{j\omega})$ denoting its DTFT. Let $Y[k]$ denote the 4-point DFT obtained by evaluating

A. $A_1 e^{j\omega_1 n}$ B. $A_1 H(e^{j\omega_1}) e^{j\omega_1 n}$ C. $2\pi H(e^{j\omega_1}) A_1 \delta(\omega-\omega_1)$ D. $A_1 H(e^{j\omega}) e^{j\omega_1 n}$

③ Consider the length-7 sequence $\{x[n]\}=\{3, -5, 1, 2, 7, -4, -2\}$ with $X(e^{j\omega})$ denoting its DTFT. Let $Y[k]$ denote the 4-point DFT obtained by evaluating

$X(e^{j\omega})$ at $\omega = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, then $y[n] = \text{IDFT}\{Y[k]\}$ is (A)

A. {10, -9, -1, 2} B. {3, -5, 1, 2, 7, -4, -2}

C. {3, -5, 1, 2} D. {-1, -7, 1, 2, 7}

4. If a Type 1 linear-phase FIR filter with a transfer function $H_1(z)$ has the

$z_1 = -1-j$ $z_1 = \frac{1}{-1+j} = -0.5-0.5j$ $z_2 = -0.5+0.5j$

following zeros $z_1 = 1, z_2 = -0.6, z_3 = -1+j$, determine which of the following z-plane point is also its zero. (C)

A. $z = 0.6$ B. $z = \frac{5}{3}$ C. $z = \frac{-1-j}{2}$ D. $z = 1+j$

5. The bilinear transform maps the left-hand s-plane into the area of (D) on the z-plane to guarantee the stability and causality of the designed digital filter.

1

A. $|z|>0$ B. $|z|>1$ C. $|z|<0$ D. $|z|<1$

得 分

二、填空题（每空 1 分，共 5 分）

$X_{cs} = \frac{1}{2}[x[n]+x^*[-n]]$

(1) The conjugate-symmetric part of the sequence $x[n]=j\cos(\pi n/2)+\sin(2\pi n/7)$ is ____0____

~~陷~~陷波

(2) The transfer function of a notch filter is $H(z)=1-z^{-1}+z^{-2}$, then its notch frequency is ____$\frac{\pi}{3}$____.

$w\in[0,\pi]$ $2\cos(w_0)=1$ $1-2\cos w_0 z^{-1}+z^{-2}$

7
0 1 2 3 4 5 6 7

(3) Suppose the length-8 real signal $x[n]=\{-3, -4, -1, 3, -4, 3, 6, 7\}$, $0\leqslant n\leqslant 7$, and its DFT is $X[k]$, then $\sum_{k=0}^{7}X[k]=$ ____-24____

$x[0]=\frac{1}{8}[X[0]+\cdots+X[7]]$

(4) Suppose the DTFT of $x[n]$ is $X(e^{j\omega})$, the DTFT of $nx[n]$ is ____$j\frac{dX(e^{jw})}{dw}$____

(5) Write out how to use the Matlab function "freqz" to compute the frequency spectrum of a causal sequence $x[n]$ at the frequency points $\omega=[-\text{pi}:\text{pi}/255:\text{pi}]$: ____freqz (____

三、简答题（5 分）

$h[n]=h[N-n]$ $0\leq n\leq N$ $c=-\frac{N}{2}$

$h[n]=-h[N-n]$ $0\leq n\leq N$ $c=-\frac{N}{2}$

(1)（2 分）What condition should the impulse response $h[n]$ of linear-phase FIR filter satisfy?

(2)（3 分）Explain the relationship between $\delta[n]$ and $\mu[n]$.

$\delta[n]=\mu[n]-\mu[n-1]$

得 分

四、计算、分析与画图（共 55 分）

$-3\ \ 4\ \ -1\ \ 4$
$1\ \ 2\ \ 1\ \ 2$

(1)（15 分）

Let $\{x[n]\}=\{-3\ \ 4\ \ -1\ \ 4\}$ and $\{h[n]\}=\{1\ \ 2\ \ 1\ \ -2\}$ be two length-4 sequences defined for $0\leq n\leq 3$.

a) Compute the linear convolution of $x[n]$ and $h[n]$.

$-3\ \ 4\ \ -1\ \ 4$
$-6\ \ 8\ \ -2\ \ 8$
$-3\ \ 4\ \ -1\ \ 4$
$6\ \ -8\ \ 2\ \ -8$

$-3\ \ -2\ \ 4\ \ 12\ \ -1\ \ 6\ \ -8$

1 · 2 1 -2

-3 4 -1 4

b) Compute the circular convolution $y_c[n] = x[n] ④ h[n]$ in frequency domain

8 -6 8 -2

using the formula $y_c[n] = \mathrm{IDFT}\{\mathrm{DFT}(x[n]) \cdot \mathrm{DFT}(h[n])\}$.

-1 4 -3 4

-8 2 -8 6

(2) (8 分) Determine whether or not the system $y[n] = 2x[3-n]$ is

8

线性 时不变 -4 4 -4 12

a) linear, b) shift-invariant, c) causal, d) stable.

时变 非因果

当 $x_1[n] = x[n-n_0]$ 时

$y_1[n] = 2x[3-n-n_0]$

$y[n-n_0] = 2x[3-(n-n_0)]$

(3) (10 分) The system is described by its impulse response

$$h[n] = (-0.8)^n(\mu[n] - \mu[n-8])$$

a) Determine the transfer function $H(z)$ of the system.

$H(z) = \frac{1}{1+0.8z^{-1}} - \frac{z^{-8}}{1+0.8z^{-1}} = \frac{1-z^{-8}}{1+0.8z^{-1}} = \frac{z-z^{-7}}{z+0.8}$

10 b) Determine the ROC of the system.

$|z| > 0.8$

c) Determine the poles and the zeros of the system.

极点：$z = -0.8$ zeros：$z = 0$, $z = 1$, $z = -1$

(4) (10 分) A filter is described by the I/O difference equation

$$y[n] = 0.64y[n-4] + 2x[n] - 3x[n-1] + 2x[n-2] - 3x[n-3]$$

$Y(z) - 0.64z^{-4}Y(z) = 2X(z) - 3z^{-1}X(z) + 2z^{-2}X(z) - 3z^{-3}X(z)$

$H(z) = \frac{Y(z)}{X(z)} = \frac{2-3z^{-1}+2z^{-2}-3z^{-3}}{1-0.64z^{-4}} = \frac{2-3z^{-1}}{1+0.8z^{-2}} \cdot \frac{1+z^{-2}}{1-0.8z^{-2}}$

10 b) Determine the ROC of the system.

c) Determine the poles and the zeros of the system.

(4) (10 分) A filter is described by the I/O difference equation

$$y[n] = 0.64y[n-4] + 2x[n] - 3x[n-1] + 2x[n-2] - 3x[n-3]$$

a) Determine the transfer function $H(z)$ of the filter

$H(z) = \frac{2-3z^{-1}+2z^{-2}-3z^{-3}}{1-0.64z^{-4}}$

b) Draw the cascade form of second-order section realization of $H(z)$. Each second-order section should be implemented with canonical (direct II) form.

级联 二阶 直接二型

(5) (12 分) A 128-point DFT of a length-112 sequence $x[n]$ is to be computed.

a) How many zero-valued samples should be appended to $x[n]$ prior to the computation of the DFT? 在计算DFT之前 $128-112=16$

MN

$(M-1)N$

b) What are the total number of complex multiplications and additions needed for the direct evaluation of all DFT samples? 128×127 128^2

c) What are the total number of complex multiplications and additions needed if a Decimation-in-Time FFT is used to compute all DFT samples? 3

c) What are the total number of complex multiplications and additions needed if a Decimation-in-Time FFT is used to compute all DFT samples?

得 分

五、综合设计题（25 分）

(1) (10 分) Design a lowpass digital FIR filter meeting the following specifications:
$\omega_p = 0.65\pi, \omega_s = 0.76\pi, \delta_p = 0.002, \delta_s = 0.004$

a) Determine the impulse response $h[n]$ of the designed filter.

b) Would the transition width and the ripple (波纹) size of the stopband be improved if the length of window increases ?

解 (a) $\alpha_s = -20\log_{10}\delta_s = 47.96$ $\omega_c = \frac{\omega_p+\omega_s}{2} = 0.705\pi$ $\Delta\omega = \omega_s - \omega_p = 0.11\pi$

汉明窗: $M = \frac{C}{\Delta\omega} = \frac{3.32\pi}{0.11\pi} = 30.18$ 布莱克曼窗 $M = 50.545$

取 $[2M+1] = [61.36] = 62$ $h_{lp}[n] = \frac{\sin\omega_c n}{\pi n} = \frac{\sin 0.705\pi n}{\pi n}$

$M = \frac{62-1}{2} = 30.5$ $w[n] = 0.54+0.46\cos(\frac{\pi n}{30.5})$ $-30.5\le n\le$

$h[n] = \frac{\sin 0.705\pi(n-30.5)}{\pi(n-30.5)}[0.54+0.46\cos(\frac{\pi(n-}{30.5}$

(b)

Table Properties of some fixed window functions

Type of window (length N=2M+1)	Main Lobe Width	Relative Sidelobe Level(dB)	Minimum Stopband Attenuation(dB)	Transition Bandwidth
Rectangular	$4\pi/(2M+1)$	13.3	20.9	$0.92\pi/M$
Hann	$8\pi/(2M+1)$	31.5	43.9	$3.11\pi/M$
Hamming	$8\pi/(2M+1)$	42.7	54.5	$3.32\pi/M$
Blackman	$12\pi/(2M+1)$	58.1	75.3	$5.56\pi/M$

Window functions:

- Hann: $w[n]=0.5+0.5\cos(\pi n/M)$, $-M\le n\le M$
- Hamming: $w[n]=0.54+0.46\cos(\pi n/M)$, $-M\le n\le M$
- Blackman: $w[n]=0.42+0.5\cos(\pi n/M)+0.08\cos(2\pi n/M)$, $-M\le n\le M$

(2) (15 分)Using the bilinear transformation method and a
prototype, design a lowpass digital filter operating at a rate of 20kHz and having passband and stopband frequencies of 4kHz and 5kHz, respectively. The maximum passband and minimum stopband attenuations are required to be 0.5dB and 10dB respectively. (The magnitude response of Butterworth filter is $|H_a(j\Omega)|^2=\dfrac{1}{1+(\frac{\Omega}{\Omega_c})^{2N}}$)

$-10\log_{10}\frac{1}{1+(\Omega_p/\Omega_c)^{2N}}=0.5$

$-10\log_{10}\frac{1}{1+(\Omega_s/\Omega_c)^{2N}}=10$

$W_p=\frac{2\pi F_p}{F_T}=\frac{2\pi\times 4K}{20K}=0.4\pi$

$W_s=\frac{2\pi F_s}{F_T}=\frac{2\pi\times 5K}{20K}=0.5\pi$

Table: When $\Omega_c=1$, the analog Butterworth lowpass filter $H(s)=1/D(s)$

$\Omega_p=\tan(\frac{W_p}{2})=\tan(0.2\pi)=0.010967$

$\Omega_s=\tan(\frac{W_s}{2})=\tan(0.25\pi)=0.0137$ 0.8642

$N=\dfrac{\log_{10}\sqrt{\frac{10^{\frac{10}{10}}-1}{10^{\frac{0.5}{10}}-1}}}{\log_{10}(\Omega_s/\Omega_p)}=$

N	D(s)
3	$(1+s)(1+s+s^2)$
4	$(1+0.7654s+s^2)(1+1.8478s+s^2)$
5	$(1+s)(1+0.6180s+s^2)(1+1.6180s+s^2)$
6	$(1+0.5176s+s^2)(1+1.4142s+s^2)(1+1.9319s+s^2)$
7	$(1+s)(1+0.4450s+s^2)(1+1.2470s+s^2)(1+1.8019s+s^2)$

$\frac{1}{k}=\frac{\Omega_s}{\Omega_p}=0.7999819$

$20\lg A=10$ $\quad\frac{1}{k_1}=\frac{\sqrt{A^2-1}}{\varepsilon}=\frac{\sqrt{9}}{\sqrt{0.122018}}=8.588323825$

$\lg A=\frac{1}{2}$ $\quad N=\frac{\lg(1/k_1)}{\lg(1/k)}=9.6$ N

$20\log_{10}(\sqrt{1+\varepsilon^2})=0.5$

$\sqrt{1+\varepsilon^2}=10^{0.025}$

$1+\varepsilon^2=10^{0.05}$

$\varepsilon^2=0.122018$

$A=\sqrt{10}$

$A^2=10$

南京邮电大学 2013/2014 学年第 学期

《数字信号处理A(双语)》期末试卷（A 卷）

院（系）____ 班级 ____ 学号 ____ 姓名 ____

题号	一	二	三	四	五	六	七	八	九	十	总 分
得分											

自觉遵 装订线内

得 分

一、填空题（每空 1 分，共 15 分）

$\frac{k}{T}$ $N=\frac{t}{T}=\frac{k}{\frac{1}{8000}}=$

$f_s=8000Hz$

(1) A speech signal is sampled at 8000Hz and the sampling period is 1 second. 1s Every sample is quantized to 8 bits, determine the quantized signal has __64000__ bits 64000

(2) When the length of the input signal x is very long, we can compute the linear convolution y=x*h by the following two methods: __overlap add__ and __overlap save__

(3) A signal $(A_1e^{j\omega_1 n}+A_2e^{j\omega_2 n})\mu[n]$ is the input of a LTI system whose frequency response is $H(e^{j\omega})$, then the steady-state response of this system is __$A_1H(e^{j\omega_1})e^{j\omega_1 n}+A_2H(e^{j\omega_2})e^{j\omega_2 n}$__

5

linear convolution y=x*h by the following two methods: [illegible] and [illegible].

(3) A signal $(A_1e^{j\omega_1 n}+A_2e^{j\omega_2 n})\mu[n]$ is the input of a LTI system whose frequency response is $H(e^{j\omega})$, then the steady-state response of this system is [illegible]

(4) Suppose a 16-point DFT is computed using the decimation-in-time radix-2 (基二) FFT algorithm, the cost of the complex multiplications is [illegible] and the number of the complex additions is [illegible].

(5) Suppose the length-9 real signal $x[n]=\{3, \dot{1}, -5, -11, 0, -5, 3, 3, 8\}$, $-5\leqslant n\leqslant 3$, and its DTFT is $X(e^{j\omega})$, then $X(e^{j0})=$ -3, $X(e^{j\pi})=$ 3,

-5 -4 -3 -2 -1 0

$e^{j\pi}=-1$

$X(e^{j0})=\sum_{n=-\infty}^{\infty}x[n]e^{j0}$

$X[0]=\int_{-\pi}^{\pi}X(e^{j\omega})d\omega=$ -5

$X(e^{j\pi})=\sum_{n=-\infty}^{\infty}-x[n]=$

$x[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\omega})e^{j\omega n}d\omega$

(6) If the transfer function of a *causal and stable* filter is $H(z)=\dfrac{b}{1-az^{-1}}$, the pole a should satisfy [illegible]. $a<1$ $\dfrac{bz}{z-a}$

(7) Suppose you have a causal IIR digital filter with transfer function, you will

因果

得 分

二、判断题（每题 2 分，共 10 分）

线性 时不变

(1) The system $h[n]=2^{-n}\mu[n-1]$ is linear and time-invariant. (×)

(2) When designing the FIR filter with window method, we may increase the window's length N to eliminate the largest passband and stopband ripples. (~~×~~)

[illegible] N扩 波纹变多 但最大波纹值不变

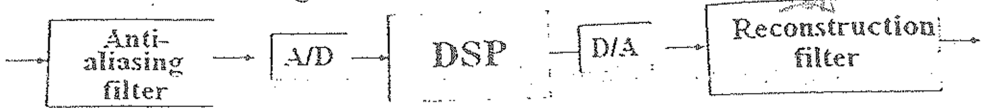
(3) The FFT is a fast implementation of the DFT. (~~×~~) ✓

(4) Suppose the DTFT of $x[n]$ is $X(e^{j\omega})$. If $x[n]$ is real and even, $X(e^{j\omega})$ is also real and even. (✓)

(5) A stable and causal filter can have zeros outside the unit circle. (~~×~~) ✓

得 分 三、问答题（每题 5 分，

(1) (5 分) The complete block-diagram 框图 for digital processing of a continuous-time signal can be shown as below:



Describe:

a) The type of anti-aliasing filter and reconstruction filter, respectively;

b) The purpose of anti-aliasing filter and reconstruction filter, respectively.

(2) (5 分) Define the following transforms and explain how they are related to each other:

a) Discrete time Fourier transform (DTFT) 离散时间傅里叶变换

b) Discrete Fourier transform (DFT) 离散傅里叶变换

c) Z-transform. Z变换 $\sum_{n=-\infty}^{\infty} x[n]z^{-n}$

Explain how the DTFT is related to the analogue Fourier transform. 模拟傅里叶变换

$f(\omega)=\sum x[n]e^{jkn}$

(3) (5 分) What is the condition for linear-phase FIR Filter ? Write down the phase response for the four types of linear-phase FIR Filter.

$h[n]=h[N-n] \quad 0\le n\le N \quad c=-\frac{N}{2}$

$h[n]=-h[N-n] \quad 0\le n\le N \quad c=-\frac{N}{2}$

1,2 $\beta=0$ 或 π

3,4 $\beta=\frac{\pi}{2}$ 或 $\frac{3\pi}{2}$ $\Leftarrow$ $\theta(\omega)=-\frac{N\omega}{2}+\beta$

得 分

四、画图与计算（共 45 分）

(1) (15 分) Consider a LTI *causal* system whose I/O difference equation is

$$y[n]=\frac{5}{2}y[n-1]-y[n-2]+x[n-1]$$

a) Compute the transfer function. $H(z)=\frac{z^{-1}}{1-\frac{5}{2}z^{-1}+z^{-2}}=\frac{z}{z^2-\frac{5}{2}z+1}=\frac{z}{(z-2)(z-\frac{1}{2})}$

b) Determine the corresponding pole/zero pattern and the ROC. $=\frac{z^{-1}}{(1-2z^{-1})(1-\frac{1}{2}z^{-1})}=\frac{\frac{2}{3}}{1-2z^{-1}}+\frac{-\frac{2}{3}}{1-\frac{1}{2}z^{-1}}$

c) Compute the impulse response.

d) It is easy to know this system is not stable. Determine another stal anticausal) system satisfying the same I/O difference equation.

pole：z=0

zeros: z=2, z=$\frac{1}{2}$

ROC：|z|>2 $\Leftarrow$

$h[n]=\frac{2}{3}2^n u[n]-\frac{2}{3}(\frac{1}{2})^n u[n]$

$h[n]=\frac{2}{3}(2)^n u[-n-1]-\frac{2}{3}(\frac{1}{2})^n u[n]$

ROC：$\frac{1}{2}<|z|<2$

(2) (10 分) Design a lowpass digital FIR filter with cutoff frequency ω_c using a rectangular window of length N=11

a) Determine the impulse response $h[n]$ of the designed filter.

南京邮电大学 2014/2015 学年第二学期

《数字信号处理 A（双语）》期末试卷（B 卷）

院(系)______________ 班级__________ 学号__________ 姓名__________

题号	一	二	三	四	五	六	七	八	九	十	总 分
得分											

装订线内不要答题

自觉遵守考试规则，诚信考试，绝不作弊

得 分

1. 选择题（10 分，每题 2 分）

(1) Suppose the DTFT of $x=[x_0, x_1, x_2, x_3]$ is $X(\omega)$. The DTFT of $x=[x_0, x_1, x_2, x_3, 0, 0, 0]$ is (A).

$X(n+3)$

A. $e^{-j3\omega}X(\omega)$ B. $X(\omega+3)$ C. $X(\omega)$ D. $X(\omega-3)$

(2) The inverse DTFT of the $H(e^{j\omega})=[-4+3\cos(\omega)+6\cos(2\omega)]e^{-j2\omega}$ is (A).

$-4+\frac{3(e^{j\omega}+e^{-j\omega})}{2}+\frac{6(e^{j2\omega}+e^{-j2\omega})}{2}$

A. $\{h[n]\}=\{3,\ 1.5,\ -4,\ 1.5,\ 3\}$ B. $\{h[n]\}=\{1.5,\ 3,\ -4,\ 3,\ 1.5\}$

$-4e^{-j2\omega}+\frac{3}{2}e^{-j\omega}+\frac{3}{2}e^{-j3\omega}+\frac{6}{2}+\frac{6}{2}e^{-j4\omega}$

C. $\{h[n]\}=\{6,\ 3,\ -4,\ 6,\ 3\}$ D. $\{h[n]\}=\{3,\ 6,\ -4,\ 3,\ 6\}$

$3\quad \frac{3}{2}\quad -4\quad \frac{3}{2}\quad 3$

$\frac{1}{1-0.3z}$

(3) The ROC of the z-transform of the sequences $x[n]=(0.3)^n\mu[n+1]$ is (B). $\frac{1}{0.3}\cdot 0.3^{n+1}u[n+1]$

A. $0<|z|<0.3$ B. $0.3<|z|<\infty$ C. $|z|>0.3$ D. $|z|<0.3$

$\Leftrightarrow \frac{z\cdot\frac{1}{0.3}}{1-0.3z^{-1}}$

(4) The 3-dB cutoff frequency of the FIR lowpass filter $H(z)=\frac{1}{2}(1+z^{-1})$ is (B).

A. $\pi/4$ B. $\pi/2$ ✓ C. $\pi/8$ D. $3\pi/4$

$H(e^{j\omega})=\frac{1}{2}(1+e^{-j\omega})$

$=\frac{1+e^{-j\omega}}{2}$

$=\frac{e^{-\frac{j\omega}{2}}(e^{-\frac{j\omega}{2}}+e^{\frac{j\omega}{2}})}{2}$

$=e^{-\frac{j\omega}{2}}\cos(\frac{\omega}{2})$

$|H(e^{j\omega})|^2=\frac{1}{2}$

$\cos^2\frac{\omega}{2}=\frac{1}{2}$

$\cos\frac{\omega}{2}=\frac{\sqrt{2}}{2}$

$\frac{\omega}{2}=\frac{\pi}{4}$

(5) $X(e^{j\omega})$ is the DTFT of the length-4 sequence $x=[1,2,2,1]$. Suppose the DFT sequence X is obtained by sampling $X(e^{j\omega})$ at uniform intervals of $\pi/4$ starting from $\omega=0$. Then the IDFT of X is (D).

统一时间间隔

$\frac{2\pi k}{N}$

A. [1,2,2,1,1,2,2,1] B. [2,4,4,2] C. [1,2,2,1] D. [1,2,2,1,0,0,0,0]

$X(e^{j\omega})=\sum_{n=0}^{\infty}x[n]e^{-j\omega n}$

$=\sum_{n=0}^{3}x[n]e^{-j\omega n}$

$=x[0]+x[1]e^{-j\omega}+x[2]e^{-j2\omega}+x[3]e^{-j3\omega}$

1 2 2 1

DFT X : $X(e^{j\omega})$

$x=[1,2,2,1]$

1 2 2 1

1 2 2 1 0 0 0

$\frac{2\pi k}{N}=\frac{\pi}{4}$

1221 N=8

得 分

2．填空题（10 分，每空 1 分）

(1) A 32-point DFT of the sequence x[n] is to be computed. The total number of complex multiplications needed for the direct evaluation of all DFT samples is __(1)__. If a Cooley-Tukey type FFT is used to compute the DFT samples, the total number of complex multiplications needed is __(2)__.

(2) The four samples of a length-6 real sequence x[n] with a real-valued 6-point DFT X[k] are given by x[0]=-4.9, x[1]=6.2, x[3]=8.58, and x[4]=-3.1. The remaining two samples of x[n] are x[2]= __(3)__ and x[5]= __(4)__.

$x[5]=x^*[6-5]$
$=x[1]$

-3.1　6.2

$X[k]=X^*[\langle -k\rangle_N]$
$X[2]=X^*[\langle 6-2\rangle]=X[4]$

(3) A type 2 linear-phase FIR filter with a transfer function $H(z)$ must have the zero z= __(5)__. -1

2型

(4) A continuous-time sinusoidal $x_a(t)=\cos(30t)$ is sampled at $t=\frac{\pi}{36}n$, $-\infty<n<\infty$, generating the discrete-time sequence x[n]= __(6)__. The fundamental period of x[n] is __(7)__.

$30\times\frac{\pi}{36}n$

$W=2\pi f$　$\cos(\frac{5}{6}\pi n)$　$2\pi\times\frac{6}{5\pi}=\frac{12}{5}$

(5) A continuous-time signal $x_a(t)$ is composed of a linear combination of sinusoidal signals of frequencies 300Hz, 500Hz, 2.8 kHz, and 3.4kHz. The signal $x_a(t)$ is sampled at a 3 kHz rate, and the sampled sequence is passed through an ideal lowpass filter with a cutoff frequency of 450Hz, generating a continuous-time signal $y_a(t)$. The frequency components present in the reconstructed signal $y_a(t)$ are __(8)__, __(9)__ and __(10)__.

$f_s=3kHz$
$f_c=450Hz$

300Hz　200Hz　400Hz

$f_s\geq 2f_H$　2.8 K　3K

得 分

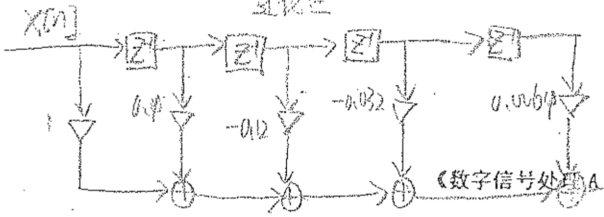
3、画图题（20 分，每题 10 分）

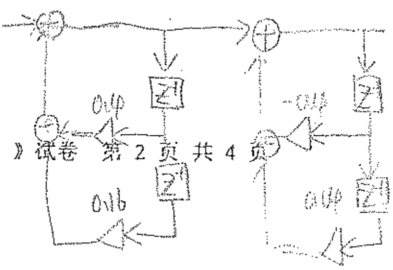
3.1 Draw the flow-graph of the radix-2 DIF FFT algorithm for N=8.

3.2 Realize the FIR transfer function $H(z)=(1+0.4z^{-1})^2(1-0.2z^{-1})^2$ in the following forms (a) direct-form. (b) cascade of two second-order sections.

$=1+0.4z^{-1}-0.12z^{-2}-0.032z^{-3}+0.0064z^{-4}$

直接型





得 分

4、计算题（30 分，每题 10 分）

4.1 The transfer function of a LTI discrete-time system is given by

$$H(z)=\frac{1+0.5z^{-1}}{1+1.5z^{-1}-z^{-2}} = \frac{\frac{5}{3}}{1+2z^{-1}} + \frac{\frac{2}{5}}{1-\frac{1}{2}z^{-1}}$$

因果 $|z|>2$

a. If the system is causal, determine the ROC of $H(z)$ and its corresponding inverse z-transform.

$\frac{5}{3}(-2)^n u[n] + \frac{2}{5}(\frac{1}{2})^n u[n]$.

稳定

b. If the system is stable, determine the ROC of $H(z)$ and its corresponding inverse z-transform.

$\frac{1}{2}<|z|<2$ 包含单位圆 $-2^n u[-n-1]\frac{5}{3} + \frac{2}{5}(\frac{1}{2})^n u[n]$

不因果 不稳定

c. If the system is noncausal and unstable, determine the ROC of $H(z)$ and its corresponding inverse z-transform.

$|z|<\frac{1}{2}$ $-2^n u[-n-1]\frac{5}{3} + \frac{2}{5}\cdot -(\frac{1}{2})^n u[-n-1]$

4.2 The first three samples of the impulses of the FIR filter are given by h[0]=1, h[1]=3, and h[2]=3.5.

1 3 2 0 -2 -3 -1

有对称

a. Determine the remaining impulse response samples of lowest order for type III. 类型 3

$H_3(z) = 1+3z^{-1}+2z^{-2}-2z^{-4}-3z^{-5}-z^{-6}$，收敛域：$|z|>1$ 包含单位圆

b. Determine the transfer function $H(z)$ and the frequency response $H(e^{j\omega})$ of the filter.

$H(e^{jw}) = 1+3e^{-jw}+2e^{-2jw}-2e^{-4jw}-3e^{-5jw}-e^{-6jw}$

c. Determine the values of $H(e^{j\omega})$ at $\omega=0$ and $\omega=\pi$.

$H(e^{j0}) = 0$ $H(e^{j\pi}) = 1-3+2-2+3-1=0$

4.3 {g[n]}={1, 2, 2}, {h[n]}={2, 1, 2, -3, 2, 1, 2, -3}.

0 1 2 3 4 5 6

a. Determine the $y_L[n]$ obtained by a linear convolution of g[n] and h[n].

b. determine the sequence $y_C[n]$ given by 6-point circular convolution of g[n] and h[n] from $y_L[n]$.

2, 1, 2, -3, 2, 1, 2, -3

1, 2, 2

2 1 2 -3 2 1 2 -3

4 2 4 -6 4 2 4 -6

4 2 4 -6 4 2 4 -6

2 5 8 3 0 -1 8 3 -2 -6

$y_C[n] = \sum_{m=0}^{5} g[m]h[\langle n-m\rangle_6]$

$y_C[0] = g[0]h[6]+g[1]h[5]+g[2]h + g[3]h[3]+g[4]h[2]+g$

$= 2+2+4=8$

$y_C[1] = g[0]h[$

11

得 分

5、设计题（30 分，每题 10 分）

5.1 Designing FIR digital filter with window method:

导出

a. Using the inverse DTFT, derive the impulse response $h_{LP}[n]$ of an ideal lowpass digital

$H_{LP}(e^{jw}) = \begin{cases} 1 & |w| \le w_c \\ 0 & w_c < |w| \le \pi \end{cases}$ $h_{LP}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{LP}(e^{jw})e^{jwn}dw = \frac{1}{2\pi}\int_{-w_c}^{w_c} e^{jwn}dw$

filter with cutoff frequency ω_c. $= \frac{1}{2\pi}\frac{e^{jw_cn}-e^{-jw_cn}}{jn} = \frac{1}{2\pi}\cdot\frac{2j\sin w_cn}{jn} = \frac{\sin w_cn}{\pi n}$ $-\infty < n < \infty$

b. Using a rectangular window, design a length N=9 lowpass FIR filter of cutoff frequency

$M = \frac{N-1}{2} = 4$ $\omega_c = 0.5\pi$. $h[n] = \begin{cases} \frac{\sin(0.5\pi(n-4))}{\pi(n-4)} & 0 \le n \le 8 \\ 0 & 其他 \end{cases}$

5.2 Design a FIR notch filter with notch frequency ω_0.

a. Determine the transfer function $H(z)$. $H(z) = 1 - 2\cos w_0 z^{-1} + z^{-2}$

$H(z) = 1 - z^{-1} + z^{-2}$ $h[n] = \delta[n] - \delta[n-1] + \delta[n-2]$

b. Suppose $\omega_0 = \pi/3$, Determine the impulse response h[n] of the notch filter.

c. Compute the output sequence y[n]of the filter, when the sinusoidal sequence $x[n] = \sin(\pi n/3)$ is applied at the input of the filter.

$y[n] = x[n] \circledast h[n] = \sin(\frac{\pi n}{3}) - \sin(\frac{\pi}{3}(n-1)) + \sin(\frac{\pi}{3}(n-2))$

$= \sin\frac{\pi n}{3}$

3阶

5.3 Using the bilinear transformation and a third-order lowpass analog Butterworth prototype filter, design a lowpass digital filter operating at a rate of 40 kHz. The attenuation 3 dB frequency of the Butterworth prototype filter is 10kHz.

$f_c = 40kHz$

a. Determine the transfer function $H_a(s)$ of the analog lowpass Butterworth filter.

b. Determine the transfer function $H(z)$ of the digital lowpass filter.

Butterworth polynomials

N	$D(s)$
1	$(1+s)$
2	$(1+1.4142s+s^2)$
3	$(1+s)(1+s+s^2)$
4	$(1+0.7654s+s^2)(1+1.8478s+s^2)$

自觉遵守考试规则，诚信考试，绝不作弊

装订线内不要答题

南京邮电大学 2013/2014 学年第一学期

《数字信号处理 A（双语）》试卷

院（系）______________ 班级__________ 学号__________ 姓名__________

题号	一	二	三	四	五	六	七	八	九	十	总 分
得分											

得 分

1、选择题（5 分，每题 1 分）

时域抽样 ⟺ 频域延拓

(1) A sinusoid of frequency 10Hz is sampled at a rate of 12Hz. The sampled signal will contain the frequency (C). 10+12N

A. 2Hz　　B. 12Hz　　C. 22Hz　　D. 24Hz

(2) The analog signal $x(t) = \sin(24\pi t) + \sin(34\pi t)$ is sampled at a rate 10 kHz and t is in milliseconds. The signal aliased with $x(t)$ is (A).

A. $\sin(4\pi t) - \sin(6\pi t)$　　B. $2\sin(8\pi t)$　　C. $2\sin(4\pi t)$　　D. $2\sin(14\pi t)$

(3) A signal $x(n) = e^{j\frac{\pi}{4}n}$ $(-\infty < n < +\infty)$ is applied at the input of a LTI system whose frequency response is $H(\omega)$, then the output of this system is (C).

$e^{j\frac{\pi}{4}n} \Longleftrightarrow \sum_{k=-\infty}^{\infty} 2\pi\delta(\omega-\frac{\pi}{4}+2\pi k)$

A. $H(\omega)e^{j\frac{\pi}{4}n}$　　B. $H(\omega)\delta(\omega-\frac{\pi}{4})$　　C. $H(\frac{\pi}{4})e^{j\frac{\pi}{4}n}$　　D. $H(\omega)e^{j\omega n}$

(4) An analog signal $x(t)$ with maximum frequency $f_{\max}$ is sampled at interval T (sampling rate $f_s = 1/T$) and the sampled signal is $\hat{x}(t)$. The approximation of the spectrum $\hat{X}(f)$ to the original spectrum $X(f)$ becomes exact when (B).

A. $T \to \infty$　　B. $T \to 0$　　C. $f_s \geq 2f_{\max}$　　D. $f_s \geq f_{proc}$

(5) Consider a FIR filter $\mathbf{h} = [h_0, h_1, \cdots, h_9]$ and an input signal $\mathbf{x} = [x_0, x_1, \cdots, x_6]$. If we want to compute the output of the filter in matrix form $\mathbf{y} = \mathbf{H}\mathbf{x}$, the dimension (维数) of $\mathbf{H}$ should be (C). 矩阵

A. 17×6　　B. 16×6　　C. 16×7　　D. 17×7

16

得 分

2、填空题（5分，每空1分）

(1) A wheel, rotating at 8Hz and turning clockwise, is seen in a dark room by means of a strobe light flashing once per 0.1 second. Determine the apparent rotational speed ____________, and sense of rotation of the wheel____________.

(2) If the anticausal part of a stable system $h(n)$ is infinite, describe the two standard steps to make it causal: ___截取过程___ , ___平移___ .

(3) The transfer function of a IIR filter is $H(z)=\frac{b}{1-0.9z^{-1}}$. If this filter's frequency response $H(\omega)$ at $\omega=0$ is unity (that is $H(\omega)\big|_{\omega=0}=1$), the parameter b should be ___0.1___ .

一到

$H(\omega) = \frac{b}{1-0.9e^{-j\omega}}$

得 分

3. 判断题（在每小题末尾括号内打√或×，每空1分，共5分）

(1) $y(n)=\begin{cases}x(n/2) & \text{if } n \text{ is even}\\ 0 & \text{if } n \text{ is odd}\end{cases}$ is a time invariant system. 时不变 (√)

(2) The filter $H(z)=\frac{1+0.9z^{-1}}{1+0.9z^{-1}}$ is a lowpass filter. 高通 (×)

-1.6

$1.6z^{-1}$

(3) The inverse filter of the system $H(z)=(1+z^{-1})(1+2z^{-1})$ is stable and causal. (√)

谐振器　　$z=0$为极点

(4) The resonator filter can remove the power frequency interferance in the signal. (×)

(5) The DTFT spectrum $X(\omega)$ of a complex-valued signal $x(n)$ satisfies $X(-\omega)=X(\omega)^*$ 复值 实虚 (×)

5　　12　　-5

$f=5$　　12　　35

得 分
-12

-35

4. Consider the analog signal $x(t)=\sin(10\pi t)+\sin(24\pi t)+\sin(70\pi t)$, where t is in milliseconds. This signal is prefiltered by an analog antialiasing

$x_a(t)=\sin(2\pi\cdot 5t)+\sin(2\pi\cdot 12t)+\sin(2\pi\cdot(-5)\cdot t)$

$f \bmod (f_s)$　　$-\frac{f_s}{2} < f \le \frac{f_s}{2}$

prefilter $H(f)$ and then sampled at $\overset{f_s}{40}$kHz. The resulting samples are immediately reconstructed using an ideal reconstructor. Determine the output $y_a(t)$ of the reconstructor in the following cases:

a) When there is no $\overset{预滤器}{\text{prefilter}}$, that is, $H(f)=1$.

b) When $H(f)$ is an ideal prefilter with cutoff frequency 20kHz.

c) When $H(f)$ is a practical prefilter that has a flat passband up to 20kHz and attenuates at a rate of 40dB/octave beyond 20kHz.

（15 分）

$B=4,\ R=8$

$Q=\frac{R}{2^B}=\frac{8}{16}=\frac{1}{2}$

$y=x+\frac{1}{2}Q=-3.2+\frac{1}{4}=-2.95$

$y<0 \to b_1=1$

test	$b_1b_2b_3b_4$	x_Q	$C=u(y-x_Q)$
2.	1100	-2	
	1010	-3	
4.	1011	-2.5	
	101~~1~~0	~~-2.5~~ -3	

得 分

⑤ Consider a 4-bit $\overset{逐次逼近}{\text{successive approximation}}$ A/D converter with full-scale range of 8 volts. Using the successive approximation algorithm and quantizing by rounding, determine the two's complement representation of the analog input $x=-3.2$ volts. (Write out the steps testing each bit) $x_Q=R(\bar{b}_1 2^{-1}+b_2 2^{-2}+\cdots+b_B 2^{-B}-\frac{1}{2})$ ~~(3 分)~~

得 分

1

0

6. The input signal is nonzero over $\overset{x}{5\le n\le 18}$, $L=14$, the impulse response $\overset{h}{h(n)}$ is nonzero over $4\le n\le 7$. The relationship between filter's input and output is described with LTI form of convolution $y(n)=\sum_m x(m)h(n-m)$. 长 $M=7-4=3$ 4

a) Determine the overall $\overset{指标}{\text{index}}$ range n for the output. $9\le n\le 25$ 长度 $=L+M-1$

$0\le n\le L-1-M$

$0\le n\le 10$ X $9\le n\le 25$

b) Determine the summation range over m.

（10 分）

$$① \begin{cases} 5\le m\le 18 \\ 4\le n-m\le 7 \end{cases} \Rightarrow \max(5,n-7)\le m\le \min(18,n-4)$$

$-n\le -m\le 7-n$

$-7+n\le m\le n-4$

$5\le m\le 18$

得 分

7. Determine the z-transforms and ROCs of the following systems.

a. $x(n)=\delta(n-5)$

$\delta(n)\to 1$

$\delta(n-5)\to z^{-5}$ ROC: $z\ne 0$

$\delta(n+5)\to z^{5}$ ROC: $z\ne\infty$ 有限长序列判断说: $z=0$ 或 $z=\infty$ $z\ne 0$.

b. $x(n)=e^{an}[u(n)-u(n-N)]$ $(N>0)$ $=e^{an}u(n)-[e^{a(n-N)}u(n$

$X(z)=\dfrac{1}{1-e^{a}z^{-1}}-\dfrac{a^{aN}z^{-N}}{1-e^{a}z^{-1}}$ ROC: ~~(|z|> 分)~~ e^{a}

得 分

8. A signal $x(n)=u(n)-u(n-25)$ is applied to the input of the system

$$H(z)=\frac{1}{5}(1-z^{-1})^5=0.2-z^{-1}+2z^{-2}-2z^{-3}+z^{-4}-0.2z^{-5}$$

$0.2-(-1)+2+2-1+0.2$

$0.4+1+1+0.4$

a) Determine the zeros and poles. 零: $\frac{1}{5}(1-z^{-1})^5=0$, $z=1$ 极点: $z=0$

b) Determine the $\overset{直流增益}{\text{DC gain}}$ of the filter. $H(0)=H(z)|_{z=1}=0$

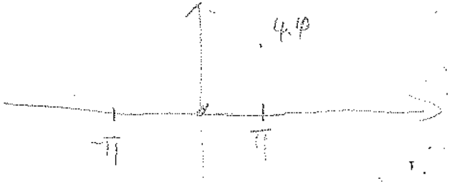
ⓒ Draw a rough sketch of the magnitude response $|H(\omega)|$.

（10 分）

$H(\omega)=0.2-e^{-j\omega}+2e^{-2j\omega}-2e^{-3j\omega}-e^{-4j\omega}-0.2e^{-5j\omega}$

$W=0$ $H(0)=0$

$W=\pi$



15

mediately

onstructor

attenuates

(15 分)

ale range

lizing by

log input

(3 分)

$h(n)$ is

utput is

分)

item

9. Consider a LTI causal system whose I/O difference equation is

$$y(n) = \frac{5}{2}y(n-1) - y(n-2) + x(n-1)$$

a) Compute the transform function $H(z)$ of this system

b) Determine the corresponding pole/zero pattern and the ROC of $H(z)$.

c) Compute the impulse response of this system.

d) It is easy to know this system is not stable. Determine another stable (but anticausal) system satisfying the same I/O difference equation.

(15 分)

10. A resonator filter of the form $H(z) = \dfrac{G}{(1-p_1 z^{-1})(1-p_2 z^{-1})}$ operates at the rate of f_s =5kHz and has a peak at $f_0 = 600Hz$. When the effective time constant corresponding to $\varepsilon = 1\%$ is n_{eff} =300 (samples),

a) Determine the two poles p_1, p_2.

b) Determine the 3-dB width $\Delta\omega$ of the resonator.

(10 分)

⑩

a) $H(z) = \dfrac{G}{(1-p_1z^{-1})(1-p_2z^{-1})} = \dfrac{G}{1-(p_1+p_2)z^{-1}+p_1p_2z^{-2}}$

$$\begin{cases} a_1 = -(p_1+p_2) = -2R\cos w_0 \\ a_2 = p_1p_2 = R^2 \end{cases}$$

$R = \varepsilon^{\frac{1}{n_{eff}}} = 0.01^{\frac{1}{300}} =$

$W_0 = \dfrac{2\pi f_0}{f_s} = \dfrac{2\pi \times 600}{5000} = \dfrac{6}{25}\pi = 0.24\pi$.

b) $\Delta W = 2(1-R)$.

④

$x(t) = \sin(10\pi t) + \sin(24\pi t) + \sin(70\pi t)$

$f_1 = 5kHz$ $f_2 = 12kHz$ $f_3 = 35kHz$ $f_s = 40kHz$ [-20, 20] Nyquist interval

$f_{1a} = 5k$ $f_{2a} = 12k$ $f_{3a} = f_3 \bmod f_s = -5k$.

a) $y_a(t) = x_a(t) = \sin(10\pi t) + \sin(24\pi t) + \sin(-10\pi t) = \sin(24\pi t)$

b) ∵ Nyquist f = cutoff f

∴ no aliasing

∴ $y_a(t) = y(t) = \sin(10\pi t) + \sin(24\pi t)$

c) $|H(f_1)| = |H(f_2)| = 1$

$A = \log_2 \dfrac{f_3}{f_s/2} = \log_2 \dfrac{35}{20} \rightarrow$ 40dB/octave × A octaves = $\underline{x\ dB}$

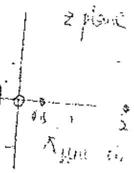
$|H(f_3)| = 10^{-x/20} \times 1$

$y(t) = \sin(10\pi t) + \sin(24\pi t) + |H(f_3)|\sin(70\pi t)$

$y_a(t) = \sin(10\pi t) + \sin(24\pi t) + 10^{-x/20}\sin(-10\pi t)$

⑨ a) $Y(z) = \frac{5}{2}z^{-1}Y(z) - z^{-2}Y(z) + z^{-1}X(z)$

$H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{z^{-1}}{1-\frac{5}{2}z^{-1}+z^{-2}} = \dfrac{z^{-1}}{(1-2z^{-1})(1-\frac{1}{2}z^{-1})}$



b) 零点: $z=0$ 极点: $z=2/0.5$

∵ causal system: 因果

ROC: $R>2$ $2<|z|<\infty$

c) $H(z) = \dfrac{A_1}{(1-2z^{-1})} + \dfrac{A_2}{(1-\frac{1}{2}z^{-1})}$

$A_1 = \dfrac{z^{-1}}{1-\frac{1}{2}z^{-1}}\Big|_{z=2} = \dfrac{2}{3}$ $A_2 = \dfrac{z^{-1}}{1-2z^{-1}}\Big|_{z=\frac{1}{2}} = -\dfrac{2}{3}$

∴ $H(z) = \dfrac{\frac{2}{3}}{1-2z^{-1}} + \dfrac{-\frac{2}{3}}{1-\frac{1}{2}z^{-1}}$

$h(n) = \frac{2}{3}2^n u(n) - \frac{2}{3}(\frac{1}{2})^n u(n)$ ROC: R>2

d) $h(n) = -\frac{2}{3}2^n u(-n-1) + \frac{2}{3}(\frac{1}{2})^n u(-n-1)$ ~~ROC: R<0.5~~

稳定的: ROC 包含单位圆

$\frac{1}{2} < |R| < 2$

$h[n] = -\frac{2}{3}2^n u[-n-1] - \frac{2}{3}(\frac{1}{2})^n u[n]$

南京邮电大学 2009/2010 学年第 二 学期

一、填空题

$\therefore H(z)=\frac{1}{1+0.9z^{-1}}$, $h(n)=(-0.9)^{n}\cdot u(n)$

1. If the I/O difference equations of the LTI systems are y(n)=3x(n)-2x(n-1)+4x(n-3) and y(n)=-0.9y(n-1)+x(n), the 因果 causal impulse h(n) for n≥0 are 3δ(n)-2δ(n-1)+4δ(n-3) and $(-0.9)^n$μ(n) respectively.

$X(z)=1+(0.8)z^{-1}+(0.8)^2z^{-2}+\cdots(0.8)^7z^{-7}$

2. The ROCs of the z-transform of x(n)=$(0.8)^n$[μ(n)-μ(n-8)] and x(n)=δ(n+5) are |z|>0 and |z|<∞ respectively.

$z>0$

z^5 z^{-5} $|z|\neq 0$ $|z|\neq\infty$

3. If the 8-point DFT of x(n) is [X0, X1, X2, X3, X4, X5, X6, X7], the 4-point DFT of x(n) is [X0, X2, X4, X6].

4. When the length of the input signal x is very long, we can compute the convolution y=x*h by the following two methods: overlap-add and overlap-save based on FFT. ~~over lap method~~:

overlap-add
overlap-save

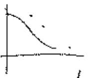
overlap-add
overlap-save

~~over lap~~

5. The DTFT of d(k) is D(w), and if d(k) is real and even, D(w) must be real and even.

d(k)实偶 ⇒ D(w) 实偶 [illegible] 实 实偶/偶

d(k)实奇 ⇒ 虚奇



$H(z)=\frac{b}{1-az^{-1}}$

$H(z)|_{z=1}=\frac{b}{1-a}=1$

$\therefore b=1-a$

6. A noisy measured signal x(n)=s+v(n) is applied to the system H(z)=b/(1-az^{-1}) (0<a<1), s is the desired constant signal and v(n) is white Gaussian noise, if the desired signal comes out unchanged, b should be 1-a

要 H(z)=1

$\frac{b}{1-az^{-1}}$

二、选择题

1. In the following systems, the only causal and stable systems is (B).

$|z|>0.5$

A. h(n)=-0.5^n μ(-n-1) B. h(n)=0.5^n μ(n)

C. h(n)=2^n μ(n) D. h(n)=-2^n μ(-n-1)

$|z|>2$

B

2. The bilinear transform maps the left-hand s-plane into the area of (D) on the z-plane to guarantee the stability and causality of the designed digital filter.

D

A. |z|>0 B. |z|>1 C. z<1 D. |z|<1

$H(z)=\frac{1-2z^{-1}}{1-0.5z^{-1}}=$

3. Consider a causal and stable system H(z)=$(1-2z^{-1})/(1-0.5z^{-1})$, To make its inverse system $H_{inv}(z)$ stable, the impulse response of $H_{inv}(z)$ should be (B).

$H_{inv}(z)=\frac{1-0.5z^{-1}}{1-2z^{-1}}$ $|z|<2$ $\Rightarrow -(2)^n u(-n-1)+0.5\cdot(2)^{n-1}u(-n)$ B.

A. 2^n μ(n)-0.5*2^{n-1} μ(n-1) B. -2^n μ(-n-1)+0.5*2^{n-1} μ(-n)

C. 2^n μ(n)-0.5*2^{n-1} μ(n) D. -2^n μ(-n)+0.5*2^{n-1} μ(-n+1)

4.

Suppose the DTFT of x=[x0, x1, x2, x3] is X(w). The DTFT of x=[0, 0, 0, x0, x1, x2, x3] is (A) A

$x(n-3)$ $e^{-3jw}X(w)$

[✓ A.] $e^{-j3w}X(w)$ B. $e^{j3w}X(w)$ C. X(w+3) D. X(w-3)

5. The system $y(n)=e^{x(n)}$ is (B). B $e^{a_1x_1(n)+a_2x_2(n)}$ $a_1e^{x_1(n)}+a_2e^{x_2(n)}$

A. linear and time-invariant [✓ B.] nonlinear and time-invariant

LTI

C. linear and time-variant D. nonlinear and time-variant

6.

Consider a pair d(n) and D(w)=DTFT[d(n)]. If d(n) is only real, D(w) satisfies (B). B

~~$D^*(w)=D(w)$~~ $D^*(w)=D(-w)$ $X_{re}(e^{jw})=X_{re}(e^{-jw})$

A. D(w)=D(−w) [✓ B.] D(w)=D*(−w) C. D(w)=−D(−w) D. D(w)=D*(w)

N≥L时 7.
$\tilde{x}(n)=x(n)$

Suppose the N-point DFT of a length-L sequence x is X. IDFT(X) equals to x only if (A). A

$\tilde{x}(n)=x(n)$

[✓ A.] N≥L B. N=[Nexact] C. N<L D. N≤L

8. A signal $x(n)=e^{j(pi/4)n}$ (−∞<n<+∞) is applied at the input of a LTI system whose frequency response is H(w), then the output of this system is (D). D

$x(n)=e^{j\frac{\pi}{4}n}$ $H(\frac{\pi}{4})e^{j\frac{\pi}{4}n}$

$y(n)=h(n)*x(n)$ $e^{jw_0t}\xrightarrow{H}H(w_0)e^{jw_0t}$

A. $H(w)\,e^{j(pi/4)n}$ B. $H(w)\,e^{jwn}$ C. H(w) δ (w-pi/4) [✓ D.] $H(pi/4)\,e^{j(pi/4)n}$

9. The system $H(z)=(1+z^{-1})(1+2z^{-1})(1+3z^{-1})$ is a (A) filter.

$\Rightarrow 2, 3, 4$ $=24$ 0

[✓ A.] lowpass B. highpass C. bandpass D. bandstop A

(10) A time-windowing process is performed to $x(n)=Ae^{j2pi w_1 n}$ (−∞<n<+∞). If the DTFT of the window w(n) is W(w), then the DTFT of the windowed signal x(n)*w(n) is (C) C

$Ae^{j2\pi w_1n}\Leftrightarrow$ A

$x(n)=Ae^{j2\pi w_1 n}$ $Ae^{j2\pi w_1 n}\rightarrow A2\pi\delta(w-w_1)$

A. 2pi A δ (w−w1) B. A δ (w−w1) [✓ C.] A W(w−w1) D. A δ (w−w1)W(w−w1)

~~next~~

三. 计算题 ~~repeat~~:

$X(w)*W(w)$ 卷积 $\Leftarrow$ $\frac{A}{2\pi}\int_{-\pi}^{\pi}2\pi\delta(\theta-2\pi w_1)$ $Ae^{j2\pi w_1 n}\Leftrightarrow\sum 2\pi\delta(w-2\pi w_1$

$AW(w-2\pi w_1)$

(1) A causal IIR filter has impulse response h(n)=5, if n=0; $h(n)=6*(0.8)^{n-1}$, if n≥1

1. Working with the convolutional equation, derive the difference equation satisfied by y(n). 要求用卷积方程 $y(n)=\sum h(m)x(n-m)$.

差分 方程 获得

$y(n)=6\times0.8^{n-1}x_n+5\cdot X$.

Solution: if n=0, y(0)=5x(n)

$h(0)=5$

$h(1)=6$

$h(2)=6\times0.8$

$y(n+1)-0.8y(n)=5x(n+1)+6x(n)-4x(n)$

$=5x(n+1)+2x(n)$

$y(n+1)=5x(n+1)+6x(n)+6\times0.8x(n-1)\ldots$

$+6\times0.8^2x(n-2)$

$y(n)=5x(n)+6x(n-1)+6\times0.8x(n-2)\ldots$

~~$y(n+1)=5x(n+1)+6x(n)+6\times0.8x(n-1)+6\times0.8^2x(n-2)$~~

$y(n+1) = 5x(n+1) + \sum_{m=1}^{n+1} 6\times0.8^{m} x(n+1-m)$

解：$y(n) = \sum_{m} h(m)x(n-m)$

$h(0)=5$，$h(1)=6$；

$h(2)=6\cdot0.8$；$h(3)=6\cdot(0.8)^2$

对 $n>0$, $y(n) = h(n) * x(n) = \sum_{m=0}^{n} h(m)x(n-m) = 5x(n) + \sum_{m=1}^{n} h(m)x(n-m)$

$y(n) = \sum_{m=0}^{n} h(m)x(n-m)$

$= 5x(n) + \sum_{m=1}^{n} 6\times0.8^{m}x(n-m)$

$= 5x(n) + 6x(n-1) + 6\cdot(0.8)x(n-2) + 6\cdot(0.8)^2 x(n-3) + \cdots$

$0.8y(n-1) = 4x(n-1) + 6\cdot(0.8)x(n-2) + 6\cdot(0.8)^2 x(n-3) + \cdots$

$h(n) = 6\times0.8^{n-1}$

$\therefore y(n) - 0.8y(n-1) = 5x(n) + 2x(n-1)$ ~~$-0.8y(n) = x(n) - 6x(n+1)$~~

$\therefore y(n) = 0.8y(n-1) + 5x(n) + 2x(n-1)$

2. A 10kHz sinusoidal signal is sampled at 80kHz and 64 samples are collected and used to compute the 64-point DFT of this signal. At what DFT indices, k=0, 1, ⋯, 63 would you expect to see any peaks in the DFT?

Solution:

$y(n) = 5x(n) + \sum 6\times0.8^{m}\cdot x(n-m)$

$0.8y(n-1) = 4x(n-1) + \sum_{m=1}^{n-1} 6\times0.8^{m}\cdot x(n-m)$

$f = 10kHz \qquad f_s = 80kHz \qquad N = 64$

$k = N\frac{f}{f_s} = 64\times\frac{10}{80} = 8$

$N - k = 64 - 8 = 56$

$f = 10kHz$. $y(n) - 0.8y(n-1) = 5x(n) - 4x(n-1)$

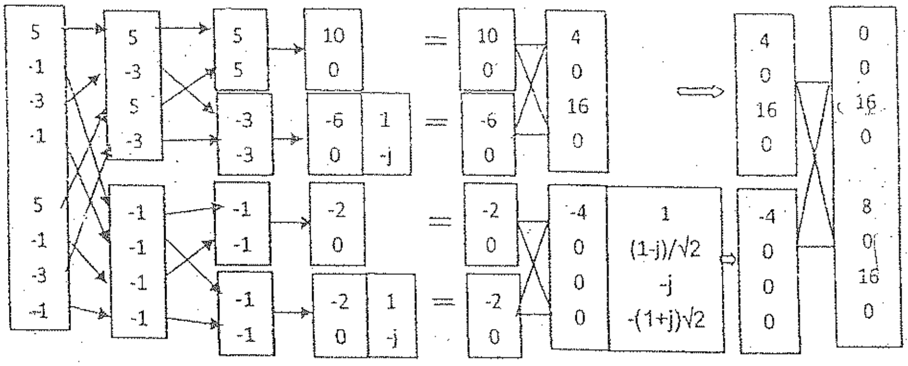
$f_s = 80kHz$; $N = 64$

$k = N\frac{f}{f_s} = 64\times\frac{10k}{80k} = 8$. $N-k = 64-8 = 56$

? 3. Compute the 8-point FFT of the length-8 signal: x=[5, −1, −3, −1, 5, −1, −3, −1]. Nothing that these samples are the first 8 samples of x(n)=4cos(0.5 pi*n) + cos(pi*n) -1, -3, -1FT of the length-8 signal:x= compute the 64-point DFT of this signal, y the fo discuss whether the 8 computed FFT values accurately represent the expected spectrum of x(n). (What FFT indices correspond to the two frequencies of the cosinusoids? 意思?)

Solution:

①



$\frac{2\pi}{8}\cdot2$

$\frac{2\pi}{8}\cdot4$

$\frac{2\pi}{8}\cdot6$

27

$$x(n)=4\times\frac{e^{j0.5\pi n}+e^{-j0.5\pi n}}{2}+\frac{e^{j\pi n}+e^{-j\pi n}}{2}$$

$$=2e^{j0.5\pi n}+2e^{j1.5\pi n}+e^{j\pi n}$$

② $x(n)=4\cos(\frac{\pi n}{2})+\cos(\pi n)$

$=2\cdot e^{j\frac{\pi}{2}n}+2\cdot e^{-j\frac{\pi}{2}n}+e^{j\pi n}$

$=2\cdot e^{j\frac{\pi}{2}n}+2\cdot e^{j\frac{3}{2}\pi n}+e^{j\pi n}$

8-point FFT 计算 x(n) 的频谱：

$\tilde{x}(n)=x(n)=\frac{1}{8}\sum_{k=0}^{7}X(k)e^{j\frac{2\pi}{8}kn}$

∴ k=2时，⇒16

k=4时，⇒8 ∴ [0,0,16,0,8,0,16,0]

k=6时，⇒16

∴8点时FFT计算x(n)时频谱与x(n)的真实频谱相比谱线位置一样，幅度比例关系一致，但幅度不是完全相同；

③ $x(n)=4\cos(\frac{\pi n}{2})+\cos(\pi n)$

$=2e^{j\frac{\pi}{2}n}+2e^{j\frac{3}{2}\pi n}+e^{j\pi n}$

$\omega_{n1}=\frac{\pi}{2}$；$\omega_{n2}=\pi$；$\omega_{n3}=\frac{3}{2}\pi$

$\omega_k=\frac{2\pi}{8}b$，$b=0,1,\cdots 7$

∴在 $\omega_k|_{k=2}$、$\omega_k|_{k=4}$、$\omega_k|_{k=6}$ 处

可看到 x(n) 的频谱

4. Design a length-41 lowpass digital FIR filter of cutoff frequency w_c=0.2*pi using a rectangular window.

$h[n]=\frac{\sin 0.2\pi n}{\pi n}$ $-20\le n\le 20$

a. Determine the impulse response h(n) of the designed filter.

$h_t[n]=\frac{\sin(0.2\pi(n-20))}{\pi(n-20)}$; $0\le n\le 40$

b. Would the transition width and the attenuation of the stopband be improved if the length of the filter is increased? Give the explanation.

Solution:

解：(a) $d(k)=\frac{\sin(\omega_c k)}{\pi k}$, ~~$0\le k\le 40$~~

$=\frac{\sin(0.2\pi k)}{\pi k}$; $-\infty<k<+\infty$

$M=\frac{N-1}{2}=20$

∴ $h(n)=d(n-M)=\frac{\sin[0.2\pi(n-20)]}{\pi(n-20)}$, $0\le n\le 40$

(b) yes.

A) $M=\frac{N-1}{2}=20$

$$d(k)=\frac{\sin(0.2\pi k)}{\pi k},-20\le k\le 20$$

$$h(k)=d(k-20)=\frac{\sin(0.2\pi k)}{\pi k-20\pi},0\le k\le 40$$

B) yes ✓

5. Consider the following length-16 signal: x(n)=0.5+2sin(0.5pi*n)+1.5cos(pi*n), n=0, 1, ⋯, 15

$=0.5+2\sin(\frac{\pi n}{2})+1.5\cos(\pi n)$

a. Determine the DTFT X(w) of this finite sequence, and sketch it roughly 对 versus w in the range 0⩽w⩽2pi. [Hint: Remember that each spectral line gets replaced by the rectangular window's frequency response.]

谱线 粗略

b. Without performing any DFT or FFT computations, determine the 16-point DFT of this sequence. Then, determine the 8-point DFT of the same sequence.

Y[K]的N点

$y[n]=\frac{1}{N}\sum_{n=0}^{N-1}Y[K]$

---

c.) Place the 16-point DFT values on the graph of X(w) of part (a).

$X_1(\omega)=0.5-j[W(\omega-\frac{\pi}{2})-W(\omega-\frac{3}{2}\pi)]+1.5\,W(\omega-\pi)$

Solution:

$$X(\omega)=\sum_{n=0}^{15}x(n)e^{-j\omega n}$$

解：(a) $x(n)=0.5+2\sin(0.5\pi n)+1.5\cos(\pi n)$

$=0.5+2\frac{e^{j\frac{\pi}{2}n}-e^{-j\frac{\pi}{2}n}}{2j}+1.5\frac{e^{j\pi n}+e^{-j\pi n}}{2}$

$\delta[n]\leftrightarrow 1$

$1\leftrightarrow 2\pi\delta(w)$

$e^{j\omega_0 n}\ \ -\infty<n<+\infty \longrightarrow 2\pi\delta(\omega-\omega_0)$

$\therefore X(w)=0.5-j[2\pi\delta(\omega-\frac{\pi}{2})-2\pi\delta(\omega+\frac{\pi}{2})]$

$+0.75[2\pi\delta(\omega-\pi)+2\pi\delta(\omega+\pi)]$

$\because 0\le\omega\le 2\pi$

$\therefore X(\omega)=0.5-j[2\pi\delta(\omega-\frac{\pi}{2})-2\pi\delta(\omega-\frac{3}{2}\pi)]$

$+0.75[2\pi\delta(\omega-\pi)+2\pi\delta(\omega-\pi)]$

$=0.5-j[2\pi\delta(\omega-\frac{\pi}{2})-2\pi\delta(\omega-\frac{3}{2}\pi)]+1.5[2\pi\delta(\omega-\pi)]$

已负移至正去 以2π为周期

?

(b) $x(n)=0.5+2\sin(0.5\pi n)+1.5\cos(\pi n)$

$=0.5+2\frac{e^{j\frac{\pi}{2}n}-e^{-j\frac{\pi}{2}n}}{2j}+1.5e^{j\pi n}$

$=0.5-je^{j\frac{\pi}{2}n}+je^{j\frac{3\pi}{2}n}+1.5e^{j\pi n}$

$\tilde{x}(n)=x(n)\quad\therefore\frac{1}{16}X(0)=0.5\Rightarrow X(0)=8;$

$\frac{1}{16}X(4)=-j\Rightarrow X(4)=-16j;$

$\frac{1}{16}X(8)=1.5\Rightarrow X(8)=24;$

$\frac{1}{16}X(12)=j\Rightarrow X(12)=16j;$

$\therefore$ 16-point DFT: $X=[8,0,0,0,-16j,0,0,0,24,0,0,0,16j,0,0,0]$

$\therefore$ 8-point DFT: $X=[8,0,-16j,0,24,0,16j,0]$

(c)

$N=16$

$W_N^{-kn}=e^{-j\frac{2\pi}{N}}$

$=e^{-j\frac{\pi}{8}}$

$n=0$

$n=$

?

四、简答、画图、分析题

1. Using the lowpass filter $H(z)=b/(1-az^{-1})$, where $0<a<1$, to extract the high-frequency signal $x(n)=s(-1)^n+v(n)$, where v(n) is zero-mean white noise of variance σ 零均值

2. How should you choose b so that the part $s(-1)^n$ comes out unchanged? Show that in this case the noise will be amplified. 被放大 Explain this result by calculating the NRR as well as graphically by sketching the frequency spectra of the signals and filter.

Solution:

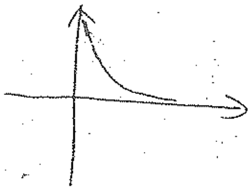
(a) $H(z)\big|_{z=-1}=\frac{b}{1+a}=1\Rightarrow b=1+a;$

$h(n)=b\cdot a^n\cdot u(n)$

$NRR=\sum_{n=0}^{\infty}h_n^2=b^2\sum_{n=0}^{\infty}a^{2n}$

$=\frac{b^2}{1-a^2}=\frac{(1+a)^2}{1-a^2}$

$=\frac{1+a}{1-a}$



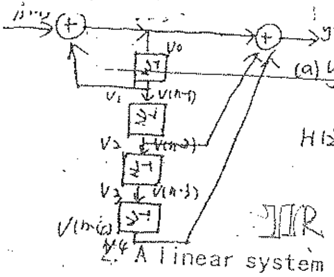
$\because NRR=\frac{\sigma_{yv}^2}{\sigma_v^2}=\frac{1+a}{1-a}\quad\because 0<a<1$

$\therefore NRR>1\quad\therefore\sigma_{yv}^2>\sigma_v^2$

$\therefore$ 噪声被放大.

29

con

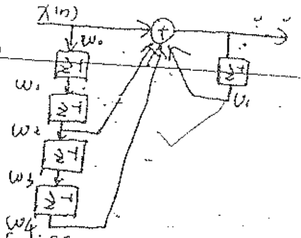


(a) $y(n)=y(n-1)+x(n-2)+x(n-4)+x(n)$

$\Downarrow$

$$H(z)=\frac{1+z^{-2}+z^{-4}}{1-z^{-1}}$$

IIR



x do:

$w_0=x$

$y=w_0+w_2+w_4+v_1$;

$w_4=w_3$

$w_3=w_2$

$w_2=w_1$

$w_1=w_0$

$v_1=y$

2. A linear system is described by the system of difference equations.

v(n)=x(n)+v(n-1) y(n)=v(n)+v(n-2)+v(n-4)

②③ canonical form:

$v_0=x+v_1$

$y=v_0+v_2+v_4$

$v_4=v_3$

$v_3=v_2$

$v_2=v_1$

$v_1=v_0$

③ 3 cascade form:

$$H(z)=\frac{1+z^{-2}+z^{-4}}{1-z^{-1}}$$

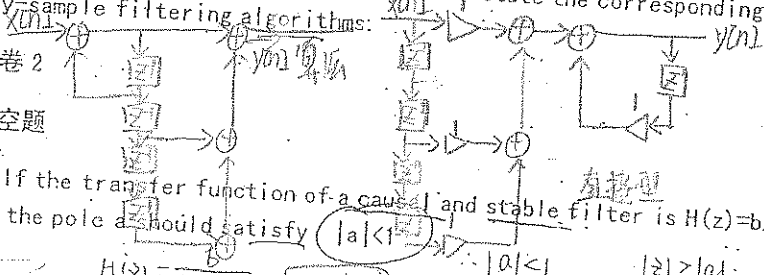
$$=\frac{\boxed{(1+z^{-1}+z^{-2})(1-z^{-1}+z^{-2})}}{1-z^{-1}}$$

a. Determine the transfer function H(z) of the filter.

(b) Draw the direct, the canonical, and the cascade form of SOS realization (with real coefficients). In each case, state the corresponding sample-by-sample filtering algorithms:

典型

级联



试卷 2

填空题

1. If the transfer function of a causal and stable filter is $H(z)=b/(1-az^{-1})$, the pole a should satisfy $|a|<1$.

直接型

$H(z)=\frac{b}{1-az^{-1}}$: $\boxed{|z|>|a|}$ $\therefore |a|<1$ $|z|>|a|$ $|a|<1$.

计算题

2. A noisy measured signal x(n)=s+v(n) is applied to the system H(z)=b/(1-az-1) (0<a<1). S is the desired constant signal and v(n) is white Gaussian noise. If the desired signal comes out unchanged, b should be 1-a.

$H(z)|_{z=1}=\frac{b}{1-a}=1 \Rightarrow b=1-a$.

$H(z)|_{z=1}=\frac{b}{1-az^{-1}}$

$b=1-a$.

3. If the 8-point DFT of x(n) is [X0, X1, X2, X3, X4, X5, X6, X7], the 4-point DFT of x(n) is [X0, X2, X4, X6].

4. When the length of the input signal x is very long, we can compute the convolution y=x*h by the following two methods: overlap-add and overlap-save based on FFT. overlap-add overlap-save

5. The DTFT of d(k) is D(w) and if d(k) is real and even, D(w) must be real and even.

偶 real even

d(k) 实奇 $\Downarrow$ D(w) 虚奇

6. To design a highpass digital filter using a lowpass analog filter, the bilinear transform we should choose is $s=\frac{1+z^{-1}}{1-z^{-1}}$.

d(k) 实 ⇒ D(w) = D(-w).

模拟低通 ⇒ 数字高通

D(w) = D(-w)

$s=\frac{1+z^{-1}}{1-z^{-1}}$

⑦ The ROCs of the z-transform of $x(n)=(0.8)^n[\mu(n)-\mu(n-8)]$ and $x(n)=\delta(n+5)$ are $|z|>0$ and $|z|<\infty$ respectively.

$|z|\neq 0$ $|z|\neq\infty$

$\frac{1+z^{-1}}{1-z^{-1}}$ $\frac{1}{1-0.8z^{-1}}$ $\frac{0.8z}{1-0.8z^{-1}}$

$|z|>0$, $|z|<\infty$.

选择题

1. In order to sample a signal at a rate fs, the cutoff frequency of the antialiasing prefilter should be (A). A

答.

$f_s \geq 2f_H$

$e^{\pm j\theta}=\cos\theta\pm j\sin\theta$

$e^{-j\pi}=\cos\pi$

A. fs/2 B. fs C. fs/4 D. 2 fs

2. Suppose the DTFT of x=[x0, x1, x2, x3] is X(w). The DTFT of x=[0, 0, 0, x0, x1, x2, x3] is (A).

A $e^{-j3w}X(w)$ $x[n-3]$ e^{-j3w}

A. e−j3w X(w) B. ej3w X(w) C. X(w+3) D. X(w−3)

3. The matrix form of 2-point DFT of a length-2 signal is (B).

A. 1 1 B. 1 1 C. 1 −1 D. 1 −1 $\boxed{W_N^{kn}=e^{-j\frac{2\pi}{N}kn}}$ B.

1 −j 1 −1 1 1 1 j

4. The system y(n)=x2(n−2)+x(3n) is (B). D.

$y(n)=x^2(n-2)+x(3n)$

$x(n)=a_1x_1(n)+a_2x_2(n)$

$y(n)=a_1x_1^2(n-2)+a_2x_2(n-2)+a_1x_1(3n)+a_2x_2(3n)$

$a_1y_1(n)+a_2y_2(n)$

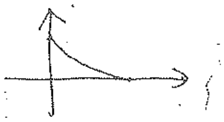
A. linear and time-invariant B. nonlinear and time-invariant

C. linear and time-variant D. nonlinear and time-variant 时变

5. The system H(z)=(1+z−1) (1+2z−1) (1+3z−1) is a (A) filter.

A -1 -2 -3

A. lowpass B. highpass C. bandpass D. bandstop



6. 运算次数的问题：

When N-point DFT of a length-N signal is implemented by decimation-in-time radix-2 FFT and N is a power (幂) of 2, the total number of complex multiplications is (B) B

复加：$N\log_2N$

A. N log2 N B. N/2 log2 N C. N2 D. N(N−1)

$\frac{N}{2}\log_2N$

7.

Consider a pair d(n) and D(w)=DTFT[d(n)]. If d(n) is only real, D(w) satisfies (B) B 实

$D(w)=D^*(-w)$ $D(w)=D^*(-w)$

A. D(w)=D(−w) B. D(w)=D*(−w) C. D(w)=−D(−w) D. D(w)=D*(w)

8.

The 4-point DFT of x=[1, 2, 2, 1, 3, 2, 2, 1] is X. Then the IDFT of X is (B). $\hat{x}$

3 2 2 1

A. [1, 2, 2, 1, 3, 2, 2, 1] B. [4, 4, 4, 2] C. [1, 2, 2, 1] D. [3, 2, 2, 1] B

4 4 4 2

9. B

The binlinear transform maps the left-hand s-plane into the area of (D) on the z-plane to guarantee the stablility and causality of the designed digital filter.

D

A. |z|>0 B. |z|>1 C. z<1 D. |z|<1

$e^{j\frac{\pi}{4}n}$

10. A signal x(n)=ej(pi/4)n (-∞<n<+∞) is applied at the input of a LTI system whose frequency response is H(w), then the output of this system is (D) D

A. H(w) ej(pi/4)n B. H(w) ejwn C. H(w) δ (w-pi/4) D. H(pi/4) ej(pi/4)n

$H(\frac{\pi}{4})e^{j}$

计算题

1. Design a 2-pole resonator filter with peak at f0=500Hz and width Δf=32Hz, operating at the sampling rate of fc=10kHz.

解：$f_0=500Hz$, $f_c=10KHz$; $\Delta f=32Hz$.

$$\omega_0=\frac{2\pi f_0}{f_c}=\frac{2\pi\times 500}{10\times10^3}=0.1\pi \text{ [radians/sample]}$$

$$\Delta\omega=\frac{2\pi\Delta f}{f_c}=\frac{2\pi\times 32}{10\times10^3}=0.02$$

$$2(1-R)=\Delta\omega \Rightarrow R=0.99$$

$$\begin{cases}a_1=-2R\cos\omega_0\\ a_2=R^2\end{cases} \Rightarrow \begin{matrix}a_1=-1.8831\\ a_2=0.9801\end{matrix}$$

$$G=(1-R)\sqrt{1-2R\cos 2\omega_0+R^2}=0.0062$$

$$\therefore H(z)=\frac{G}{1+a_1z^{-1}+a_2z^{-2}}=\frac{0.0062}{1-1.8831z^{-1}+0.9801z^{-2}}$$

$x(n)=2\sin(0.5\pi n)$, $n=0,1,\cdots 7$

$x(n)=2\sin\frac{\pi}{2}n \quad n=0,1,2\cdots 7$

$$x(n)=2\,\frac{e^{j\frac{\pi}{2}n}-e^{-j\frac{\pi}{2}n}}{2j}=je^{j\frac{\pi}{2}n}-je^{-j\frac{\pi}{2}n}$$

$-je^{j\frac{\pi}{2}n}$

$e^{-j\frac{\pi}{2}}$

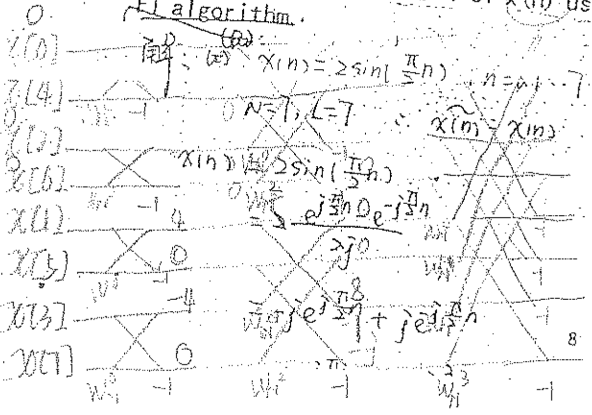
2. Consider the sinusoidal signal x(n)=2sin(0.5pi*n), n=0, 1, …, 7.

a. Without performing any DFT or FFT computations, determine the 8-point DFT of x(n).

$k=2$

$k=6$

b. Compute the 8-point FFT of x(n) using the decimation-in-time radix-2 FFT algorithm.



解：(a) $x(n)=2\sin(\frac{\pi}{2}n)$ $n=0,1\cdots 7$

$N=8$, $L=7$ $\therefore \tilde{x}(n)=x(n)$

$x(n)=2\sin(\frac{\pi}{2}n)$

$=2\,\frac{e^{j\frac{\pi}{2}n}-e^{-j\frac{\pi}{2}n}}{2j}$

$=-je^{j\frac{\pi}{2}n}+je^{-j\frac{\pi}{2}n}$

$$\tilde{x}(n)=\frac{1}{8}\sum_{k=0}^{7}X(k)e^{j\frac{2\pi}{8}kn}=\frac{1}{8}\sum_{k=0}^{7}X(k)e^{j\frac{\pi}{4}kn}$$

$\therefore k=2$时，$X(2)=-8j$； $\frac{1}{8}X[2]=-j$

$k=6$时，$X(6)=8j$ $\frac{1}{8}X[6]=j$

$X=[0,0,-8j,0,0,0,8j,0,0]$

(b) 8-point FFT.

$\Rightarrow [0,0,-8j,0,0,0,8j,0]$.

h\x	2	1	2	-3	2	1	2	-3		
1	2	1	2	-3	2	1	2	-3		
2	4	2	4	-6	4	2	4	-6		
2	4	2	4	-6	4	2	4	-6		
y	2	5	8	3	0	-1	8	3	-2	-6

$$\begin{array}{cccccccccc} & & 2 & 1 & 2 & -3 & 2 & 1 & 2 & -3 \\ & & 1 & 2 & 2 & & & & & \\ \hline & & 2 & 1 & 2 & -3 & 2 & 1 & 2 & -3 \\ & & & 4 & 2 & 4 & -6 & 4 & 2 & 4 & -6 \\ & & & & 4 & 2 & 4 & -6 & 4 & 2 & 4 & -6 \\ \hline & & & & 8 & 3 & 0 & -1 & 8 & 3 & -2 & -6 \end{array}$$

3. h=[1, 2, 2], x=[2, 1, 2, -3, 2, 1, 2, -3].

a. Compute the convolution of the above two signals: y=h*x.

b. Compute the modulo-4 circular convolution y of x and h.

解：a：$y = h*x = [1, 2, 2] * [2, 1, 2, -3, 2, 1, 2, -3]$

$= [2, 5, 8, 3, 0, -1, 8, 3, -2, -6]$

b：modulo-4 circular convolution

~~$\tilde{y} = \widetilde{h*x} = \mathrm{IDFT}(\mathrm{DFT}(h) \cdot \mathrm{DFT}(x))$~~

$y = [2, 5, 8, 3, 0, -1, 8, 3, -2, -6]$

$y = [2, 5, 8, 3]\,[0, -1, 8, 3]\,[-2, -6, 0, 0]$

$\Rightarrow \therefore \tilde{y} = [0, -2, 16, 6]$

$$\begin{array}{cccc} 2 & 1 & 2 & -3 \\ -6 & 4 & 2 & 4 \\ 4 & -6 & 4 & 2 \\ \hline 0 & -1 & 6 & 3 \\ 0 & -1 & 8 & 3 \\ \hline 0 & -2 & 16 & \end{array}$$

4. Design a 长度 length-41 lowpass digital FIR filter of cutoff frequency wc=0.2*pi using a rectangular window.

[illegible]

a. Determine the impulse response h(n) of the designed filter.

$h[n] = \dfrac{\sin 0.2\pi n}{\pi n} \quad -\infty < n < +\infty$

$M = \dfrac{N-1}{2} = 20$

$h[n] = \begin{cases} \dfrac{\sin 0.2\pi(n-20)}{\pi(n-20)} & 0 \le n \le 40 \end{cases}$

b. Would the transition width 过渡 and the attenuation of the stopband be improved if the length of the filter is increased? Give the explanation.

解：a: $d(k)=\frac{\sin(\omega_c k)}{\pi k}$ $-\infty<k<+\infty$, $M=\frac{N-1}{2}=\frac{41-1}{2}=20$.

$h(n)=d(n-M)$.

$=\frac{\sin[0.2\pi(n-20)]}{\pi(n-20)}$, $0\leq n\leq 40$

…………

b. Yes.

5. Given a first order IIR filter $H(z)=\beta/(1-\alpha z^{-1})$.

a. Determine the causal impulse response h(n).

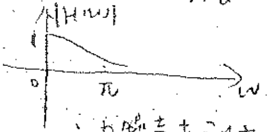
b. Determine the noise reduction ratio NRR.

c. To make the signal x(n)=5 comes out unchanged, determine the parameter β.

d. Suppose 0<α<1 and β is chosen to be the value in c, sketch the magnitude response |H(w)| over 0≤w≤n. Would the input white noise be amplified? 放大.

$H(z)|_{z=1}=\frac{1-\alpha}{1-\alpha}=1$

$H(z)|_{z=-1}=\frac{1-\alpha}{1+\alpha}<1$

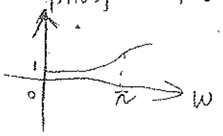


∴白噪声未放大.

e. Suppose -1<α<0 and β is chosen to be the value in c, sketch |H(w)| over 0≤w≤n. Would the input white noise be amplified?

$H(z)|_{z=1}=\frac{1-\alpha}{1-\alpha}=1$, $H(z)|_{z=-1}=\frac{1-\alpha}{1+\alpha}$

∵ $-1<\alpha<0$ ∴ $\frac{1-\alpha}{1+\alpha}>1$



∴噪声被放大

解：$H(z)=\frac{\beta}{1-\alpha z^{-1}}$

a: $h(n)=(\alpha)^n u(n)\cdot\beta$

b: $NRR=\sum_n h_n^2=\sum_n(\alpha)^{2n}u(n)\cdot\beta^2=\frac{\beta^2(1-\alpha^{2n})}{1-\alpha^2}$

c: ∵α(n)≠5 ∴$0<\alpha<1$ ⇒ β≠3

∞ 无失真的通过

∴ $H(z)|_{z=1}=\frac{\beta}{1-\alpha}=1$ ∴ $\beta=1-\alpha$

d:

简答、画图、分析题

1. A filter has transfer function: H(z)=0.5/(1+0.5jz−1)+0.5/(1−0.5jz−1)+2/(1+0.5z−1)+2/(1−0.5z−1).

a. Determine all possible impulse responses h(n) and their ROCs.

b. Draw the direct realization form of H(z).

c. Draw the cascade realization form.

解：a: $H(z)=\frac{0.5}{1+0.5jz^{-1}}+\frac{0.5}{1-0.5jz^{-1}}+\frac{2}{1+0.5z^{-1}}+\frac{2}{1-0.5z^{-1}}$

① $|z|>0.5\in ROC$:

$h(n)=0.5\cdot(-0.5j)^n u(n)+0.5\cdot(0.5j)^n\cdot u(n)+2\cdot(-0.5)^n u(n)+2\cdot(0.5)^n u(n)$

ROC:

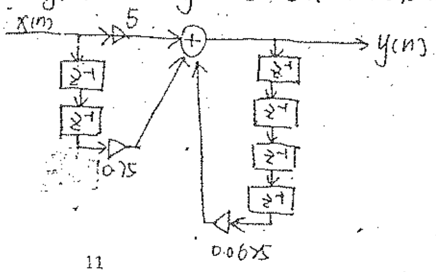
② $|z|<0.5$时.

$h(n)=-0.5\cdot(0.5j)^n\cdot u(-n-1)-0.5\cdot(0.5j)^n u(-n-1)$

b: $H(z)=\frac{1}{1+0.25z^{-2}}+\frac{4}{1-(0.5z^{-1})^2}=\frac{1}{1+0.25z^{-2}}+\frac{4}{1-0.25z^{-2}}$

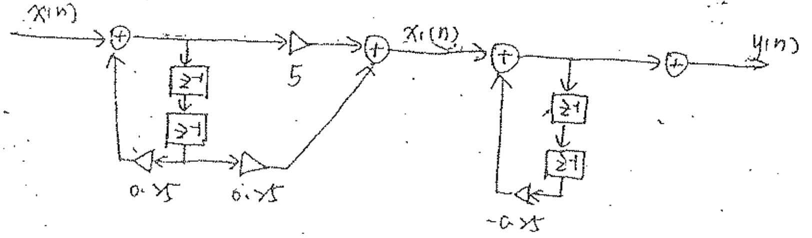
$=\frac{5+0.75z^{-2}}{1-0.0625z^{-4}}$

∴ $y(n)=0.0625y(n-4)+5x(n)+0.75x(n-2)$



11

$H(z)=\frac{5+0.75z^{-2}}{1-0.0625z^{-4}}=\frac{(5+0.75z^{-2})}{(1-0.25z^{-2})(1+0.25z^{-2})}$



2. A LTI system is described by the difference equation: y(n)=0.25y(n-2) +x(n).

a. Determine the transfer function H(z) of this system.

b. If the system is causal and stable, write out the corresponding impulse response h(n).

c. Determine the zero pole pattern of the transfer function on the z-plane.

d. Draw a rough sketch (草图) of its magnitude response |H(w)| over the frequency interval 0≤w≤pi. Write out the value of |H(w)| at frequency w=0 and w=pi.

解: a: $y(n)=0.25y(n-2)+x(n)$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{1}{1-0.25z^{-2}}$$

b: causal and stable:

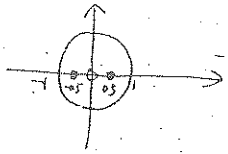
$$H(z)=\frac{0.5}{1-0.5z^{-1}}+\frac{0.5}{1+0.5z^{-1}}$$

ROC: $|z|>0.5$

$h(n)=0.5\cdot(0.5)^n u(n)+0.5\cdot(-0.5)^n u(n)$

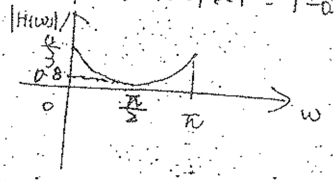
C: $$H(z)=\frac{1}{1-0.25z^{-2}}=\frac{z^2}{z^2-0.25}$$

零点 z = 0 (二阶); 极点: $z_1=0.5$, $z_2=-0.8$



d: $H(\omega)|_{\omega=0}=H(z)|_{z=1}=\frac{1}{1-0.25}=\frac{4}{3}$

$H(\omega)|_{\omega=\pi}=H(z)|_{z=-1}=\frac{1}{1-0.25}=\frac{4}{3}$



$H(\omega)|_{\omega=\frac{\pi}{2}}=0.8$

$Z=e^{jw}$

$H(w)=\frac{1}{1-0.25e^{-j2w}}$

$w=\frac{\pi}{2}$

$H(\frac{\pi}{2})=\frac{1}{1-0.25e^{-j\pi}}$

3. A signal consisting of four sinusoids of frequencies of 1, 1.5, 2 and 2.25kHz is sampled at a rate of 8kHz. Determine the minimum number of samples that should be collected for the frequency spectrum to exhibit four distinct peaks at these frequencies.

解: $f_s=8kHz$.

$\Delta f_{min}=2.25-2=0.25kHz$.

$L\geq\frac{f_s}{\Delta f}=\frac{8K}{0.25K}=32$ samples.

$\Delta f\geq\frac{f_s}{L}$.

得分

1、填空题（每空 1 分，共 10 分）

(1) If the transfer function of a causal and stable filter is $H(z)=\frac{b}{1-az}$, the pole a should satisfy $|a|<1$.

(2) A noisy measured signal $x(n)=s+v(n)$ is applied to the system $H(z)=\frac{b}{1-az^{-1}}$ $(0<a<1)$, s is the desired constant signal and $v(n)$ is white Gaussian noise. If the desired signal comes out unchanged, b should be $1-a$.

(3) If the 8-point DFT of $x(n)$ is $\{X_0, X_1, X_2, X_3, X_4, X_5, X_6, X_7\}$, the 4-point DFT of $x(n)$ is $[X_0, X_2, X_4, X_6]$.

(4) When the length of the input signal x is very long, we can compute the convolution y=x*h by the following two methods: Overlap-Add and Overlap-Save based on FFT.

(5) The DTFT of $d(k)$ is $D(\omega)$ and if $d(k)$ is **real** and even, $D(\omega)$ must be ~~@~~ real and even.

(6) To design a **highpass** digital filter using a lowpass analog filter, the bilinear transform we should choose is $s=$ ~~$\frac{1+z^{-1}}{1-z^{-1}}$~~ $\frac{1+z^{-1}}{1-z^{-1}}$ P374

(7) The ROCs of the z-transform of $x(n)=(0.8)^n[u(n)-u(n-8)]$ and $x(n)=\delta(n+5)$ are $z\neq 0$, $|z|>0$ and $z\in C$, $|z|<\infty$, respectively.

得 分

2、选择题（每题 1 分，共 10 分）

(1) In order to sample a signal at a rate f_s, the cutoff frequency of the antialiasing prefilter should be (A).

A. $f_s/2$　　B. f_s　　C. $f_s/4$　　D. $2f_s$

(2) Suppose the DTFT of $x=[x_0, x_1, x_2, x_3]$ is $X(\omega)$. The DTFT of $x=[0,0,0,x_0,x_1,x_2,x_3]$ is (A).

A. $e^{-j3\omega}X(\omega)$　　B. $e^{j3\omega}X(\omega)$　　C. $X(\omega+3)$　　D. $X(\omega-3)$

(3) The matrix form of 2-point DFT of a length-2 signal is (B).

A. $\begin{bmatrix}1 & 1\\ 1 & -j\end{bmatrix}$　　B. $\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$　　C. $\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}$　　D. $\begin{bmatrix}1 & -1\\ 1 & j\end{bmatrix}$

(4) The system $y(n)=x^2(n-2)+x(3n)$ is (D).

A. linear and time-invariant　　B. nonlinear and time-invariant
C. linear and time-variant　　D. nonlinear and time-variant

(5) The system $H(z)=(1+z^{-1})(1+2z^{-1})(1+3z^{-1})$ is a (A) filter.

A. lowpass　　B. highpass　　C. bandpass　　D. bandstop

(6) When N-point DFT of a length-N signal is implemented by decimation-in-time radix-2 FFT and N is a power (幂) of 2, the total number of complex multiplications is (B).

A. $N\log_2 N$　　B. $\frac{N}{2}\log_2 N$　　C. N^2　　D. $N(N-1)$

(7) Consider a pair $d(n)$ and $D(\omega)=DTFT[d(n)]$. If $d(n)$ is only real, $D(\omega)$ satisfies (B).

A. $D(\omega)=D(-\omega)$　B. $D(\omega)=D^*(-\omega)$　C. $D(\omega)=-D(-\omega)$　D. $D(\omega)=D^*(\omega)$

(8) The 4-point DFT of $x=[1,2,2,1,3,2,2,1]$ is X. Then the IDFT of X is (B).

A. [1,2,2,1,3,2,2,1]　　B. [4,4,4,2]　　C. [1,2,2,1]　　D. [3,2,2,1]

(9) The bilinear transform maps the left-hand s-plane into the area of (D) on the z-plane to guarantee the stablility and causality of the designed digital filter.

A. $|z|>0$　　B. $|z|>1$　　C. $z<1$　　D. $|z|<1$

(10) A signal $x(n)=e^{j\frac{\pi}{4}n}$ $(-\infty<n<+\infty)$ is applied at the input of a LTI system whose frequency response is $H(\omega)$, then the output of this system is (D).

A. $H(\omega)e^{j\frac{\pi}{4}n}$　　B. $H(\omega)e$　　C. $H(\omega)\delta(\omega-\frac{\pi}{4})$　　D. $H(\frac{\pi}{4})e^{j\frac{\pi}{4}n}$

$$\therefore H(z) = \frac{0.0062}{1-1.8831z^{-1}+0.9801z^{-2}}$$

得分

3、计算题（共 55 分）

$w_0 = \frac{2\pi f_0}{f_s} = 0.1\pi$ [radians/sample]

$\Delta w = \frac{2\pi \Delta f}{f_s} = 0.02 = 2(1-R) \Rightarrow R = 0.99$

(1) (10 分) Design a 2-pole resonator filter with peak at f_0 =500Hz and width Δf =32Hz operating at the sampling rate of f_s =10kHz.

$a_1 = -2R\cos w_0$, $a_2 = R^2$ $\Rightarrow \begin{cases} a_1 = -1.8831 \\ a_2 = 0.9801 \end{cases}$

$G = (1-R^2)\sqrt{1-2R\cos(2w_0)+R^2} = 0.0062$

(2) (10 分) Consider the sinusoidal signal $x(n) = 2\sin(\frac{\pi}{2}n)$; $n = 0,1,\ldots,7$.

a. Without performing any DFT or FFT computations, determine the 8-point DFT of $x(n)$..

b. Compute the 8-point FFT of $x(n)$ using the decimation-in-time radix-2 FFT algorithm.

蝶形图

(3) (10 分) h=[1 2 2], x=[2 1 2 −3 2 −1 2 −3]. $x|^h$

a. Compute the convolution of the above two signals: y = h * x. y=[2, 5, 8, 3; 0, −1, 8, 3; −2, −6]

b. Compute the modulo-4 circular convolution $\tilde{y}$ of x and h. $\tilde{y}$ = [0, −2, 16, 6]

(4) (10 分) Design a length-41 lowpass digital FIR filter of cutoff frequency $\omega_c = 0.2\pi$ using a rectangular window.

$M = \frac{L-1}{2} = 20$ $d(k) = \frac{\sin(w_c k)}{\pi k}$ $h(k) = d(k-M) = \frac{\sin[0.2\pi(k-20)]}{\pi(k-20)}$

a. Determine the impulse response $h(n)$ of the designed filter.

$\Delta w = \frac{2\pi}{L}$

b. Would the transition width and the attenuation of the stopband be improved if the length of the filter is increased? Give the explanation.

(5) (15 分) Given a first order IIR filter $H(z) = \frac{\beta}{1-\alpha z^{-1}}$,

a. Determine the causal impulse response $h(n)$.

b. Determine the noise reduction ratio NRR.

c. To make the signal x(n)=5 comes out unchanged, determine the parameter β.

d. Suppose $0 < \alpha < 1$ and β is chosen to be the value in c., sketch the magnitude response $|H(\omega)|$ over $0 \le \omega \le \pi$. Would the input white noise be amplified?

e. Suppose $-1 < \alpha < 0$ and β is chosen to be the value in c., sketch $|H(\omega)|$ over $0 \le \omega \le \pi$. Would the input white noise be amplified?

得 分

4、简答、画图、分析题（共25分）

(1) (10分) A filter has transfer function:

$$H(z)=\frac{0.5}{1+0.5jz^{-1}}+\frac{0.5}{1-0.5jz^{-1}}+\frac{2}{1+0.5z^{-1}}+\frac{2}{1-0.5z^{-1}}$$

a. Determine all possible impulse responses $h(n)$ and their ROCs.

b. Draw the direct realization form of $H(z)$.

c. Draw the cascade realization form.

(2) (10分) A LTI system is described by the difference equation :

$$y(n)=0.25y(n-2)+x(n)$$

a. Determine the transfer function $H(z)$ of this system ;

b. If the system is causal and stable, write out the corresponding impulse response $h(n)$.

c. Determine the zero/pole pattern of the transfer function on the z-plane ;

d. Draw a rough sketch（草图）of its magnitude response $|H(\omega)|$ over the frequency interval $0\leq\omega\leq\pi$. Write out the value of $|H(\omega)|$ at frequency $\omega=0$ and $\omega=\pi$.

(3) (5分) A signal consisting of four sinusoids of frequencies of 1, 1.5, 2 and 2.25kHz is sampled at a rate of 8kHz. Determine the minimum number of samples that should be collected for the frequency spectrum to exhibit four distinct peaks at these frequencies.

$L\geqslant\frac{f_s}{\Delta f}=\frac{8k}{0.25k}=32$

得 分

1、填空题（每空1分，共10分）

(1) If the I/O difference equations of the LTI systems are

$$y(n)=3x(n)-2x(n-1)+4x(n-3) \quad \text{and} \quad y(n)=-0.9y(n-1)+x(n)$$

the causal impulse response $h(n)$ for $n\geq 0$ are $3\delta(n)-2\delta(n-1)+4\delta(n-3)$ and $(-0.9)^n u(n)$ respectively.

(2) The ROCs of the z-transform of $x(n)=(0.8)^n[u(n)-u(n-8)]$ and $x(n)=\delta(n+5)$ are $|z|\geq 0$ and $|z|\leq\infty$ respectively.

(3) If the 8-point DFT of $x(n)$ is $[X_0, X_1, X_2, X_3, X_4, X_5, X_6, X_7]$, the 4-point DFT of $x(n)$ is $[X_0 \; X_2 \; X_4 \; X_6]$

(4) When the length of the input signal x is very long, we can compute the convolution y=x*h by the following two methods: overlap-add and overlap-save based on FFT.

(5) The DTFT of $d(k)$ is $D(\omega)$ and if $d(k)$ is real and even, $D(\omega)$ must be real and even.

(6) A noisy measured signal $x(n)=s+v(n)$ is applied to the system $H(z)=\dfrac{b}{1-az^{-1}}$ $(0<a<1)$. s is the desired constant signal and $v(n)$ is white Gaussian noise. If the desired signal comes out unchanged, b should be $1-a$ $(\,H(z)|_{z=1}=1\,)$

得 分

2、选择题（每题 1 分，共 10 分）

(1) In the following systems, the only causal and stable system is (B).

A. $h(n) = -0.5^n u(-n-1)$ B. $h(n) = 0.5^n u(n)$

C. $h(n) = 2^n u(n)$ D. $h(n) = -2^n u(-n-1)$

(2) The binlinear transform maps the left-hand s-plane into the area of (D) on the z-plane to guarantee the stablilty and causality of the designed digital filter.

A. $|z|>0$ B. $|z|>1$ C. $z<1$ D. $|z|<1$

(3) Consider a causal and stable system $H(z)=(1-2z^{-1})/(1-0.5z^{-1})$. To make its inverse system $H_{Inv}(z)$ stable, the impulse response of $H_{Inv}(z)$ should be (B).

A. $2^n u(n) - 0.5 \cdot 2^{n-1} u(n-1)$ B. $-2^n u(-n-1) + 0.5 \cdot 2^{n-1} u(-n)$

C. $2^n u(n) - 0.5 \cdot 2^{n-1} u(n)$ D. $-2^n u(-n) + 0.5 \cdot 2^{n-1} u(-n+1)$

(4) Suppose the DTFT of $x=[x_0, x_1, x_2, x_3]$ is $X(\omega)$. The DTFT of $x=[0,0,0,x_0,x_1,x_2,x_3]$ is (A).

A. $e^{-j3\omega}X(\omega)$ B. $e^{j3\omega}X(\omega)$ C. $X(\omega+3)$ D. $X(\omega-3)$

(5) The system $y(n)=e^{x(n)}$ is (B).

A. linear and time-invariant B. nonlinear and time-invariant

C. linear and time-variant D. nonlinear and time-variant

(6) Consider a pair $d(n)$ and $D(\omega)=DTFT[d(n)]$. If $d(n)$ is only real, $D(\omega)$ satisfies (B).

A. $D(\omega)=D(-\omega)$ B. $D(\omega)=D^*(-\omega)$ C. $D(\omega)=-D(-\omega)$ D. $D(\omega)=D^*(\omega)$

(7) Suppose the N-point DFT of a length-L sequence x is X. IDFT(X) equals to x only if (A).

A. $N \geq L$ B. $N=[N_{exact}]$. C. $N<L$ D. $N \leq L$

(8) A signal $x(n)=e^{j\frac{\pi}{4}n}$ $(-\infty<n<+\infty)$ is applied at the input of a LTI system whose frequency response is $H(\omega)$, then the output of this system is (D).

A. $H(\omega)e^{j\frac{\pi}{4}n}$ B. $H(\omega)e^{j\omega n}$ C. $H(\omega)\delta(\omega-\frac{\pi}{4})$ D. $H(\frac{\pi}{4})e^{j\frac{\pi}{4}n}$

(9) The system $H(z)=(1+z^{-1})(1+2z^{-1})(1+3z^{-1})$ is a (A) filter.

A. lowpass B. highpass C. bandpass D. ban

(10) A time-windowing process is performed to $x(n) = Ae^{j2\pi\omega_1 n}$ $(-\infty < n < \infty)$. If the DTFT of the window $w(n)$ is $W(\omega)$, then the DTFT of the windowed signal $x(n) \cdot w(n)$ is (D C)

A. $2\pi A\delta(\omega - \omega_1)$　　　B. $A\delta(\omega - \omega_1)$

C. $AW(\omega - \omega_1)$　　　D. $A\delta(\omega - \omega_1)W(\omega - \omega_1)$

得 分

3、计算题（共55分）

(1) (10 分) A causal IIR filter has impulse response

$$h(n) = \begin{cases} 5, & if\ n = 0 \\ 6(0.8)^{n-1}, & if\ n \geq 1 \end{cases}$$

Working with the convolutional equation, derive the difference equation satisfied by $y(n)$.

(2) (10 分) A 10kHz sinusoidal signal is sampled at 80kHz and 64 samples are collected and used to compute the 64-point DFT of this signal. At what DFT indices, $k = 0, 1, \ldots, 63$ would you expect to see any peaks in the DFT?

(3) (10 分) Compute the 8-point FFT of the length-8 signal.

$$x = [5, -1, -3, -1, 5, -1, -3, -1]$$

Noting that these samples are the first 8 samples of

$$x(n) = 4\cos(0.5\pi n) + \cos(\pi n)$$

discuss whether the 8 computed FFT values accurately represent the expected spectrum of $x(n)$. What FFT indices correspond to the two frequencies of the cosinusoids?

(4) (10 分) Design a length-41 lowpass digital FIR filter of cutoff frequency $\omega_c = 0.2\pi$ using a rectangular window.

a. Determine the impulse response $h(n)$ of the designed filter.

b. Would the transition width and the attenuation of the stopband be improved if the length of the filter is increased? Give the explanation.

43 B

(5) (15 分) Consider the following length-16 signal:

$$x(n) = 0.5 + 2\sin(0.5\pi n) + 1.5\cos(\pi n), \quad n = 0, 1, \cdots, 15$$

a. Determine the DTFT $X(\omega)$ of this finite sequence, and sketch it roughly versus ω in the range $0 \le \omega \le 2\pi$. [Hint: Remember that each spectral line gets replaced by the rectangular window's frequency response.]

b. Without performing any DFT or FFT computations, determine the 16-point DFT of this sequence. Then, determine the 8-point DFT of the same sequence.

c. Place the 16-point DFT values on the graph of $X(\omega)$ of part (a).

得 分

4、简答、画图、分析题（共25分）

(1) (10 分) Using the lowpass filter $H(z) = \dfrac{b}{1 - az^{-1}}$, where $0 < a < 1$, to extract the high-frequency signal $x(n) = s(-1)^n + v(n)$, where $v(n)$ is zero-mean white noise of variance σ_v^2. How should you choose b so that the part $s(-1)^n$ comes out unchanged? Show that in this case the noise will be amplified. Explain this result by calculating the NRR as well as graphically by sketching the frequency spectra of the signals and filter.

(2) (15 分) A linear system is described by the system of difference equations:

$$v(n) = x(n) + v(n-1)$$
$$y(n) = v(n) + v(n-2) + v(n-4)$$

a. Determine the transfer function $H(z)$ of the filter.

b. Draw the direct, the canonical, and the cascade form of SOS realization (with real coefficients). In each case, state the corresponding sample-by-sample filtering algorithms.

1.compute the z-transform of the following sequences and determine the corresponding region of convergence

(1)X(n)=*(n+5) $X(z)=z^5 \quad ROC:|z|\neq\infty$

$$x(n)=(-0.5)^n[u(n)-u(n-10)]$$

(2)

$$x(n)=(-0.5)^n u(n)-(-0.5)^{10}(-0.5)^{n-10}u(n-10)$$

$$X(z)=\frac{1}{1+0.5z^{-1}}-(-0.5)^{10}\frac{z^{-10}}{1+0.5z^{-1}}$$

$$=\frac{1-(0.5z^{-1})^{10}}{1+0.5z^{-1}} \qquad ROC:|z|\neq 0$$

(3) $x(n)=2(0.9)^n\cos(\pi n/2)u(n)$

$$x(n)=(0.9)^n[e^{j\frac{n\pi}{2}}+e^{-j\frac{n\pi}{2}}]u(n)$$

$$=[(0.9j)^n+(-0.9j)^n]u(n)$$

$$X(z)=\frac{1}{1-0.9jz^{-1}}+\frac{1}{1+0.9jz^{-1}}$$

$ROC:|z|>0.9$

2.a filter is described by the following sample processing algorithm relating the input and output sample x and y; for each input sample x do:

$w_0=x+0.64w_4 \quad y=w_0+w_3 \quad w_4=w_3 \quad w_3=w_2 \quad w_2=w_1 \quad w_1=w_0$

Determine the transfer function H(z)of this filter .factor H(z)into factors of order up to two (with real-valued coefficients)and draw the corresponding cascade realization State the sample processing algorithm for that realization

过滤器是描述有关下列样品处理算法的输入和输出样本x和y,每一个输入样品的x:确定传递函数的 H(z)的过滤.factor H(z)到秩序两因素(实系数),得出相应的来样加工状态层叠算法实现的实现

(1)

(2) $$H_1(z)=\frac{1+z^{-3}}{1-0.64z^{-4}}$$

(3) cascade realization

(4) sample-by-sample algorithm

$$\begin{cases} w_{00} = x + 0.8w_{02} \\ y_0 = w_{00} + w_{01} \\ w_{02} = w_{01}; \quad w_{01} = w_{00} \\ w_{10} = y_0 - 0.8w_{12} \\ y = w_{10} - w_{11} + w_{12} \\ w_{12} = w_{11}; \quad w_{11} = w_{10} \end{cases}$$

$$H(z) = \frac{b(1+z^{-1})}{1-az^{-1}}$$

$$令 H(1) = \frac{2b}{1-a} = 1 \Rightarrow b = \frac{1-a}{2}$$

$$\therefore H(z) = \frac{(\frac{1-a}{2})(1+z^{-1})}{1-az^{-1}}$$

$$\Rightarrow h(n) = A_0\delta(n) + A_1 a^n u(n) \qquad (A_0 = -\frac{b}{a}, A_1 = \frac{b(1+a)}{a})$$

$$\Rightarrow NRR = \sum_n h_n^2 = \frac{1-a}{2}$$

3 滤波器输入：$x(n) = s + v(n)$

(有用信号$s(n)$为恒值s，所以频谱只含有DC频率，噪声为白噪声)

低通滤波器：$H(z) = \frac{b}{1-az^{-1}}$，$H(\omega) = \frac{b}{1-ae^{-j\omega}}$

我们希望信号无失真通过系统，则要求

$$H(z)\big|_{z=1} = H(\omega)\big|_{\omega=0} = 1 \Rightarrow b = 1-a$$

$$\therefore \text{NRR} = \frac{\sigma_{y_v}^2}{\sigma_v^2} = \sum_n h_n^2 = \frac{1-a}{1+a}$$

4 这里就存在着去噪与系统响应速度的矛盾：

去噪效果好，要求NRR小，即$a \to 1$，但$n_{eff} = \frac{\ln\varepsilon}{\ln a} \to \infty \qquad (a \to 1)$

滤波器输入：$x(n)=(-1)^n s+v(n)$

(有用信号频谱只含有AC频率，噪声为白噪声)

高通滤波器：$H(z)=\dfrac{b}{1+az^{-1}}$，$H(\omega)=\dfrac{b}{1+ae^{-j\omega}}, h(n)=b(-a)^n u(n)$

我们希望信号无失真通过系统，则要求

$H(z)\big|_{z=-1}=H(\omega)\big|_{\omega=\pi}=1\Rightarrow b=1-a$ ……

$$\therefore \mathrm{NRR}=\frac{\sigma_{y_v}^2}{\sigma_v^2}=\sum_n h_n^2=\frac{1-a}{1+a}$$

这里同样存在着去噪与系统响应速度的矛盾：

5. 去噪效果好，要求NRR小，即$a\to 1$，但$n_{eff}=\dfrac{\ln\varepsilon}{\ln a}\to\infty \quad (as\to 1)$

$$\therefore \frac{1-a}{2}<\frac{1-a}{1+a} \qquad (0<a<1\Rightarrow 1<1+a<2)$$

$f_0=10KHz$；$f_s=80KHz$

$$\frac{f_s}{N}k=f_0\Rightarrow k=f_0\Big/\frac{f_s}{N}=8 \qquad N-k=64-8=56$$

6.A 10KHZ sinusoidal signal is sampled at 80 kHZ and 64 samples are collected and used to compute the 64-point DFT of this signal,At what DET indices k=0.1,,,,63would you expect it see any peaks in the DFT?

10 KHZ 正弦信号取样80千赫和64的样品,会被收集并用来计算 DFT 的64点这个信号,在指标 k = 0.1),使用63,你希望看到任何的山峰 DFT 吗?

7.A 5kHZ sinusoidal signal is sampled at 40kHZ and 16 periods of the signal are collected.What is the length N of the collected samples? Suppose an N-point DFT is performed.then,at what DFT indices.k=0.1.....N-1.do you expect to see any peaks in the DFT spectrum?

In general,how is the number of periods contained in the N samples related to the DFT index at which you get a peak?

5千赫正弦信号取样40 kHZ 及16时期的信号进行采集。什么是长度 N 的收集样品吗?假设一N-point DFT 是,在什么 performed.then DFT indices.k = 0.1 ..... N- 1. -你希望看到任何的山峰 DFT 谱吗?

一般来说,怎样的数量是包含在 N 样品时期相关的 DFT 的索引在你所能得到的一个高峰?

$$f_0 = 5KHz;\ \ f_s = 40KHz;\ \ T_L = T_{周} \cdot 16 = \frac{1}{f_0} \cdot 16 = 3.2mes$$

$$1)\ \ L = \frac{T_L}{T_{采}} = T_L \cdot f_s = 3.2 \times 40 = 128$$

$$或\ \frac{T_{周}}{T_{采}} = \frac{f_s}{f_0} = 8(说明一个周期有8个采样) \Longrightarrow L = 8 \times 16 = 128$$

$$2)\ \ N = L = 128;\ \ k = \frac{f_0}{f_s} \cdot N = 16;\ \ N - k = 128 - 16 = 112$$

$$3)\ \ 设\ L个采样含\ \ c个周期的采样 \quad \Rightarrow L = N = c \cdot \frac{T_{周}}{T_{采}} = c \cdot \frac{f_s}{f_0}$$

$$k = \frac{f_0}{f_s} \cdot N = \frac{f_0}{f_s} \cdot c \cdot \frac{f_s}{f_0} = c = 整数$$

8.A dual-tone multi-frequency(DTMF)transmitter(touch-tone phone)encodes each keypress as a sum of two sinusoidal tones.with one frequency taken from group A and One from group B,where:

Group A:697 .770. 852. 941 HZ Group B:1209, 1336, 1477 HZ

A digital DTME receiver computes the spectrum of the received dual-tone signal and Determines the two frequencies that are present, and thus, the key that was pressed What is the smallest number of time samples L that we should collect at a sampling Rate of 8 KHZ, in order for the group-A frequencies to be resolvable from the group-B frequencies?What is L a hamming window is used prior to computing the spectrum?

一个 dual-tone 多频(DTMF)发送(电话)编码基于每一个键盘作为一笔两正弦 tones.with 取自一组频率一个来自 B 组,地点:

计算数字 DTME 接收机接收的 dual-tone 光谱信号

决定两个频率,那是在场的,因此,压的关键

什么是最小的若干时间样品,我们应该收集我在取样吗

8率 KHZ,为了解决赛区的频率是 b 组的频率?什么是我一个海明窗采用计算光谱前吗

$$1)\text{频域采样间隔为}\Delta f_{bin} = \frac{f_s}{N} = \frac{8}{16} = 0.5$$

9.an 18 KHZ sinusoid is sampled at a rate of 8 KHZ and a 16-point DFT of a finite portion of the signal is computed.At what DFT indices in the range 0<k<15 do you expect to see any peaks in the DFT spectrum?Would it matter if first we folded the 18 KHZ frequency to lie within the Nyquist interval and then computed the DFT?Explain

一个18 KHZ 正弦形取样率8千赫和发起反攻,这个有限的一部分 DFT 信号计算。指数在什么范围 DFT 0 < k < 15你希望看到任何的山峰 DFT 谱吗?你会介意如果首先我们把18 KHZ 躺在奈奎斯特频率区间,然后计算 DFT ?解释一下好吗

$$\pm f_m = \Delta f_{bin} \cdot m = 0.5 \cdot m = \pm 18 \Rightarrow m = \pm 36$$

$$k = m \bmod(16) = 4\text{和}12$$

$$2)\ \text{先求进入Nyquist间隔的}f_a = 2\text{和}6$$

$$f_a = \Delta f_{bin} \cdot k \Rightarrow k = 4\text{和}12$$

10.the following analog signal x(t),where t is in msec, is sampled at a rate of 8 kHZ

$$x(t) = \cos(24\pi t) + 2\sin(12\pi t)\cos(8\pi t)$$

A,Determine the signal Xa(t)that is aliased whih x(t)

B,Eight consecutive samples of x(t) are collected .without performing any DFT or FFT operation,determine the 8-point DFT of these 8 samples

下面的模拟信号 x(t),在那里 t,在 msec 采样速率为8 kHZ

a.确定信号 Xa(t),与叠加 x(t)

b.八场 x(t)的样品收集或没有执行任何操作,DFT 频谱履行8点密度泛函确定这8样品

$$\Delta f_{\min} = 1209 - 941 = 268$$

$$\Delta f_{\min} \ge \frac{f_s}{L} \Rightarrow L \ge \frac{f_s}{\Delta f_{\min}} = \frac{8\times10^3}{268} = 29.85 \quad \therefore L = 30$$

$$hamming \quad window: \Delta f_{\min} \ge 2\frac{f_s}{L} \Rightarrow L \ge 2\cdot\frac{f_s}{\Delta f_{\min}} = 59.7 \quad \therefore L = 60$$

$$x(t) = \cos(24\pi t) + 2\sin(12\pi t)\cos(8\pi t) = \cos(24\pi t) + \sin(20\pi t) + \sin(4\pi t)$$

$$\therefore f_1 = 12k, f_2 = 10k, f_3 = 2k, f_s = 8k \Rightarrow f_{1a} = 4k, f_{2a} = 2k, f_{3a} = 2k$$

1) $x_a(t) = \cos(8\pi t) + 2\sin(4\pi t)$

2) $x(t) = \frac{1}{2}e^{j24\pi t} + \frac{1}{2}e^{-j24\pi t} + \frac{1}{2j}e^{j20\pi t} - \frac{1}{2j}e^{-j20\pi t} + \frac{1}{2j}e^{j4\pi t} - \frac{1}{2j}e^{-j4\pi t}$

$$x(nT) = x(n) = \frac{1}{2}e^{\frac{j24\pi n}{8}} + \frac{1}{2}e^{\frac{-j24\pi n}{8}} + \frac{1}{2j}e^{\frac{j20\pi n}{8}} - \frac{1}{2j}e^{\frac{-j20\pi n}{8}} + \frac{1}{2j}e^{\frac{j4\pi n}{8}} - \frac{1}{2j}e^{\frac{-j4\pi n}{8}}$$

$$x(n) = \frac{1}{2}e^{j\pi n} + \frac{1}{2}e^{j\pi n} + \frac{1}{2j}e^{\frac{j\pi n}{2}} - \frac{1}{2j}e^{\frac{j3\pi n}{2}} + \frac{1}{2j}e^{\frac{j\pi n}{2}} - \frac{1}{2j}e^{\frac{j3\pi n}{2}} \qquad 式(1)$$

$\because L = 8, N = 8$

$$\because \tilde{x}(n) = x(n) = \frac{1}{8}\sum_{k=0}^{7} X(k)\cdot e^{j\frac{2\pi}{8}nk} = \frac{1}{8}\sum_{k=0}^{7} X(k)\cdot e^{j\frac{\pi}{4}nk} \qquad 式(2)$$

式(1)和式(2)比较可得：

$X(k) = [0,0,-8j,0,8,0,8j,0]$

1) *overlap – save method*

∵ 要做 *length – 8 circular convolutions* ∴ 每段长度取 8

∵ h = [1 −1 −1 1], 阶数为 3 ∴ 每段间重叠 3，即 $M = 3$

$$\therefore \begin{cases} x_1 = [0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1 \quad 3] \\ x_2 = [1 \quad 1 \quad 3 \quad 3 \quad 3 \quad 3 \quad 1 \quad 1] \\ x_3 = [3 \quad 1 \quad 1 \quad 1 \quad 2 \quad 2 \quad 2 \quad 2] \\ x_4 = [2 \quad 2 \quad 2 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0] \\ x_5 = [1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \end{cases}$$

$\tilde{y}_1 = [-3 \quad 2 \quad 3 \quad 1 \quad 0 \quad -1 \quad 0 \quad 2]$, $\tilde{y}_2 = [2 \quad 0 \quad 2 \quad 0 \quad -2 \quad 0 \quad -2 \quad 0]$,

$\tilde{y}_3 = [1 \quad -2 \quad -1 \quad 2 \quad 1 \quad 0 \quad -1 \quad 0]$, $\tilde{y}_4 = [2 \quad 1 \quad -2 \quad 1 \quad 0 \quad 1 \quad 0 \quad -1]$,

$\tilde{y}_5 = [1 \quad 0 \quad -2 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0]$

y = [1 0 −1 0 2 0 −2 0 −2 0 2 1 0 −1 0 −1
0 1 0 −1 0 1]

y = x * h = [1 0 −1 0 2 0 −2 0 −2 0 2 1 0 −1 0 −1
0 1 0 −1 0 1]

∴每段卷积输出长度≤8

即$N+M\le 8$（N为每子段的长度M为滤波器阶数）∴取$N=5$

$x_1=[1\ \ 1\ \ 1\ \ 1\ \ 3], x_2=[3\ \ 3\ \ 3\ \ 1\ \ 1], x_3=[1\ \ 2\ \ 2\ \ 2\ \ 2]$

$x_4=[1\ \ 1\ \ 1\ \ 1\ \ 0]$

$\widetilde{y}_1=y_1=[1\ \ 0\ \ -1\ \ 0\ \ 2\ \ -3\ \ -2\ \ 3],$

$\widetilde{y}_2=y_2=[3\ \ 0\ \ -3\ \ -2\ \ 0\ \ 1\ \ 0\ \ 1],$

$\widetilde{y}_3=y_3=[1\ \ 1\ \ -1\ \ -1\ \ 0\ \ -2\ \ 0\ \ 2],$

$\widetilde{y}_4=y_4=[1\ \ 0\ \ -1\ \ 0\ \ -1\ \ 0\ \ 1\ \ 0]$

以上y_1到y_4重叠相加得

$y=[1\ \ 0\ \ -1\ \ 0\ \ 2\ \ 0\ \ -2\ \ 0\ \ -2\ \ 0\ \ 2\ \ 1\ \ 0\ \ -1\ \ 0\ \ -1$
$\quad 0\ \ 1\ \ 0\ \ -1\ \ 0\ \ 1]$

12.the parameters N and Ω_0 of an analog Butterworth filter are determined by solving the two specification equations A(Ω_{pass})= A_{pass} ,A(Ω_{stop})= A_{stop} ,The resulting filter order is then rounded up to the next integer-value N.

Using this slightly larger N,show that if Ω_0 is found from the passband specification ,that is ,by solving A(Ω_{pass})= A_{pass} ,then the stopband specification is more than satisfied ,that is A(Ω_{stop})> A_{stop} ,Similarly, show that if we find Ω_0 form the stopband specification A(Ω_{stop})= A_{stop} then the passband specification is more than satisfied ,that is A(Ω_{pass})< A_{pass}

N 的参数的一种模拟滤波器的视界由两个规格方程求解一个()()=,=,产生的滤波顺序则是把到下一个整型值的 N。

使用这个稍大的 N,表明如果是发现的规格,系为求解一个()=,然后多阶规范满足,那是一个()>,

同样表明,如果我们发现形式规格(一阶)=然后频带种类超过满足,那是一个()<

$$令\begin{cases} A(\Omega_{pass})=10\lg[1+(\dfrac{\Omega_{pass}}{\Omega_0})^{2N}]=A_{pass} & (1) \\ A(\Omega_{stop})=10\lg[1+(\dfrac{\Omega_{stop}}{\Omega_0})^{2N}]=A_{stop} & (2) \end{cases}$$

$$N_{exact}=\ln(\sqrt{\frac{10^{A_{stop}/10}-1}{10^{A_{pass}/10}-1}})\Big/\ln(\frac{\Omega_{stop}}{\Omega_{pass}}) \qquad (3)$$

$$N=[N_{exact}]\ge N_{exact} \qquad (4)$$

$$将(4)代入(1)得\Omega_0'=\frac{\Omega_{pass}}{(10^{A_{pass}/10}-1)^{\frac{1}{2N}}} \qquad (5)$$

(5)式与 (4)式一起使 (1)式等式两边准确成立，但不能使 (2)式等式两边成立。

记 $\Omega_{0exact} = \dfrac{\Omega_{pass}}{(10^{A_{pass}/10}-1)^{\frac{1}{2N_{exact}}}}$ (6)

(6)式与 (3)式一起精确满足方程组 (1)式和 (2)式.

将(6)代入 (2)得 $10\lg[1+(\dfrac{\Omega_{stop}}{\Omega_{pass}})^{2N_{exact}}\cdot(10^{A_{pass}/10}-1)] = A_{stop}$

将(5)代入 (2)第二个等号左边得：

$$A(\Omega_{stop}) = 10\lg[1+(\frac{\Omega_{stop}}{\Omega_{pass}})^{2N}\cdot(10^{A_{pass}/10}-1)].$$

$$\because (\frac{\Omega_{stop}}{\Omega_{pass}}) > 1,\quad N \geq N_{exact}$$

$$\therefore A(\Omega_{stop}) = 10\lg[1+(\frac{\Omega_{stop}}{\Omega_{pass}})^{2N}\cdot(10^{A_{pass}/10}-1)]$$

$$> 10\lg[1+(\frac{\Omega_{stop}}{\Omega_{pass}})^{2N_{exact}}\cdot(10^{A_{pass}/10}-1)] = A_{stop}$$

$$\therefore A(\Omega_{stop}) > A_{stop}$$

同理得 $\Omega_0 = \dfrac{\Omega_{stop}}{(10^{A_{stop}/10}-1)^{\frac{1}{2N}}}$, $\Omega_{0exact} = \dfrac{\Omega_{stop}}{(10^{A_{stop}/10}-1)^{\frac{1}{2N_{exact}}}}$

$$10\lg[1+(\frac{\Omega_{pass}}{\Omega_{stop}})^{2N_{exact}}\cdot(10^{A_{stop}/10}-1)] = A_{pass}$$

$$A(\Omega_{pass}) = 10\lg[1+(\frac{\Omega_{pass}}{\Omega_{stop}})^{2N}\cdot(10^{A_{stop}/10}-1)] \xrightarrow{(\frac{\Omega_{pass}}{\Omega_{stop}})<1,\ N\geq N_{exact}}$$

$$A(\Omega_{stop}) < 10\lg[1+(\frac{\Omega_{pass}}{\Omega_{stop}})^{2N_{exact}}\cdot(10^{A_{stop}/10}-1)] = A_{pass}$$

$$\therefore A(\Omega_{pass}) < A_{pass}$$

13.using the bilinear transformation and a lowpass analog Butterworth prototype filter design a lowpass digital filter operating at a rate of 40 kHZ and having the following specifications Fpass=10kHZ,Apass=3dB,Fstop=15kHZ,Astop=35dB.carry out all the design steps by hand Draw the cascade realization form and write the difference equatinons and the corresponding sample processing algorithm inplementing this realization in the time domain,

采用双线性变换和一个低通模拟原型滤波器设计一个视界低通数字滤波器操作速度40 kHZ 及具有以下规格 Fpass = 10 kHZ,Apass = 3分贝,Fstop = 15千赫,Astop = 35 dB.carry 出所有的设计步骤的手

画梯级写这些差异实现形式 equatinons 和相应的样品处理算法实现过程中实施时间域、

$$f_{pass} = 10k\,\mathrm{Hz},\ f_{stop} = 15k\,\mathrm{Hz},\ f_s = 40k\,\mathrm{Hz}$$

$$A_{pass} = 3dB,\ A_{stop} = 35\,dB$$

$$\omega_{pass} = \frac{2\pi\cdot f_{pass}}{f_s} = \frac{2\pi\cdot 10}{40} = \frac{\pi}{2},\ \omega_{stop} = \frac{2\pi\cdot f_{stop}}{f_s} = \frac{2\pi\cdot 15}{40} = \frac{3\pi}{4}$$

$$\Omega_{pass} = tg(\frac{\omega_{pass}}{2}) = tg(\frac{\pi}{4}) = 1,\ \Omega_{stop} = tg(\frac{\omega_{stop}}{2}) = tg(\frac{3\pi}{8}) = 2.4142$$

$$N_{exact} = \ln\left(\sqrt{\frac{10^{A_{stop}/10}-1}{10^{A_{pass}/10}-1}}\right)\Big/\ln\left(\frac{\Omega_{stop}}{\Omega_{pass}}\right) = 4.5744$$

$$\therefore N = 5$$

$$\Omega_0 = \frac{\Omega_{pass}}{(10^{A_{pass}/10}-1)^{\frac{1}{2N}}} = 1.0005 \approx 1$$

$$或\ \Omega_0 = \frac{\Omega_{stop}}{(10^{A_{stop}/10}-1)^{\frac{1}{2N}}} = 1.0784 \approx 1$$

$$H_a(s) = \frac{1}{(1+s)(1+0.618s+s^2)(1+1.618s+s^2)}$$

$$H(z) = H_a(s)\Big|_{s=\frac{1-z^{-1}}{1+z^{-1}}}$$

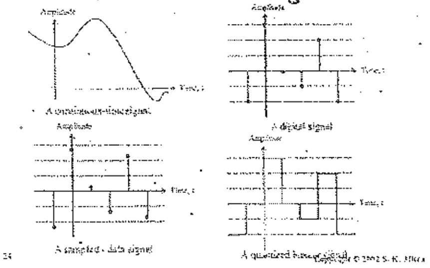
第一章：

1、四种信号的概念及转换过程：

模拟信号－连续幅值的连续时间信号
量化信号－离散幅值的连续时间信号
抽样信号－连续幅值的离散时间信号
数字信号－离散幅值的离散时间信号

模拟信号通过抽样得到抽样信号再通过量化得到数字信号

Characterization and Classification of Signals



A continuous-time signal

A digital signal

A sampled-data signal

A quantized boxcar signal

Copyright © 2002 S. K. Mitra

典型数字信号处理系统的主要构成。
模拟信号数字化处理框图，图中各部分的功能作用。

Complete block-diagram

- Note: Both the anti-aliasing filter and the reconstruction filter are analog lowpass filters

前后两个滤波器的作用分别是：抗混叠、平滑

T=2015： F7=节. 521228F7.   25以式中间分析 22号

二. 时域中的离散时间信号.

1. 时域序列表示：如{x[n]} = {…, -0.2, $\underset{\uparrow}{2.1}$, 1.1, 0.2 …} 注意箭头含义，代表n=0时的取值.

∴上述 x[n]：x[-1]=-0.2, x[0]=2.1, x[1]=1.1, x[2]=0.2 .

对于复序列：$\{x[n]\}=\{x_{re}[n]\}+j\{x_{im}[n]\}$ = 实部+虚部. 共轭(conjugate)：$\{x^*[n]\}=\{x_{re}[n]\}-j\{x_{im}[n]\}$

常用推广式：$\{x[n]\}=x_{cs}[n]+x_{ca}[n]$ = 共轭对称+共轭反对称.(conjugate-symmetric / conjugate-antisymmetric)

$x_{cs}[n]=\frac{1}{2}(x[n]+x^*[-n])$ $x_{ca}[n]=\frac{1}{2}(x[n]-x^*[-n])$

共轭对称序列：$x[n]=x^*[-n]$，实共轭对称序列是偶序列(even sequence).

共轭反对称序列：$x[n]=-x^*[-n]$，实共轭反对称序列是奇序列(odd sequence)，x[0]=0

习题对应：书例2.8，后习题2.21，2.22.

2. 序列长度
- 有限长(finite-length)，对于$N_1\le n\le N_2$，序列长度$N=N_2-N_1+1$ ⇒ 习题中用补0法增长序列.
- 无限长(infinite-length)
  - 右边序列：当$n<N_1$时，x[n]=0 ⇒ 若$N_1\ge 0$，则为因果序列.
  - 左边序列：当$n>N_2$时，x[n]=0 ⇒ 若$N_2\le 0$，则为非因果序列.
  - 双边序列

3. 序列的基本概念.
1) 奇/偶序列 2) 能量信号/功率信号(见本章6点) periodic 3) 绝对可和/平方可和/有界序列

4) 因果/非因果序列(上面提到). 5) 周期/非周期序列. 周期序列：x[n]=x[n+kN] for all n

对于正弦序列：$x[n]=A\cos(\omega_0 n+\phi)$：$\omega_0$：归一化角频率. 对于正弦序列周期性判别：$\frac{2\pi}{\omega_0}=\frac{N}{r}$ → 为整数 → 不整数

则N为周期. 如习题2.4 (书后)，2.5

总体现：连续一定周期 (sin(ωt))
离散不一定周期

4. 序列的基本操作(运算). ① $x[n]\to\boxed{z^{-1}}\to w[n]$ ② $x[n]\to\boxed{z}\to w[n]$ ③ $x[n],\ y[n]\to\otimes\to w[n]$

1) 时移(time-shifting)：右移(delay)=x[n-1]① 左移(advance)=x[n+1]②   ④ $x[n]\xrightarrow{A}w[n]$

调制(modulation)：x[n]y[n]③ 乘法器(multiplier)：Ax[n]④   ⑤ $x[n],\ y[n]\to\oplus\to w[n]$

加法器(adder)：x[n]+y[n]⑤ 节点(pick-off node)⑥   ⑥ $x[n]\to x[n]$, $\downarrow x[n]$

2) 时间反转运算 time-reversal operation (也称折叠运算 folding)：x[-n]，移位运算(左加右减).

运算时，等长序列直接运算；不等长，可补0 zero padding (具体问题参见书例2.3, 2.2) ⇒ 习题2.3

框图题. 如：书后. 式2.19对应框图2.8 ⇒ 习题2.4

综合反转与移位运算 如：x[k] → x[n-k]，可以：$x[k]\xrightarrow[\text{反转}]{}x[-k]\xrightarrow[\text{右移}]{}x[-(k-n)]=x[n-k]$：注意每一步都是针对变量而言.

3) 卷积和 convolution sum $y[n]=x[n]\circledast h[n] \overset{定义}{\Rightarrow} y[n]=\sum_{k=-\infty}^{+\infty} x[k]h[n-k] = \sum_{k=-\infty}^{+\infty} x[n-k]h[k]$.（线性卷积）

注意定义式中的下标和特点：x和h的下标之和＝y的下标。如：$y[3]=x[0]h[3]+x[1]h[2]+x[2]h[1]+x[3]h[0]$

求解卷积和的方法：①定义式 同上（书P48 例题2.5）②不进位乘法.

不进位乘法，如：$x_1[n]=\{\underset{\uparrow}{4},3,2,1\}$，$x_2[n]=\{\underset{\uparrow}{3},2,1\}$，求线性卷积：$y[n]=x_1[n]\circledast x_2[n]=$＿

```
      4  3  2  1
         3  2  1
   ---------------
      4  3  2  1
   8  6  4  2
12 9  6  3
-----------------
12 17 16 10 4  1
```

$\therefore y[n]=\{\underset{\uparrow}{12},17,16,10,4,1\}$ 注意箭头位置.

线性卷积后的序列长度＝$M+N-1$. 如上：$y[n]$长度＝$4+3-1=6$

$r\in[0,N-1]$

$r=\langle m\rangle_N \Rightarrow r=m+lN$：$l$为整数(正或负)

6. 圆周运算 ⇒实际：模运算 modulo 对于模运算实际取余过程，如：

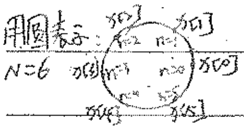
$\langle 25\rangle_7=4 \Rightarrow r=25-7\times3=4$

$\langle -15\rangle_7=6 \Rightarrow r=-15+7\times3=6$

1) 圆周反转：$y[n]=x[\langle -n\rangle_N] \Rightarrow x[\langle n-N\rangle_N]=x[n]$ 应用例题：书P48. 例2.7

2) 圆周时移：$y[n]=x[\langle n-n_0\rangle_N]$ 移n_0位 $\Rightarrow=\begin{cases}x[n-n_0], & n_0\le n\le N-1\\ x[N+n-n_0], & 0\le n<n_0\end{cases}$ ：eg. $x[\langle n-1\rangle_6]=x[\langle n+5\rangle_6]$

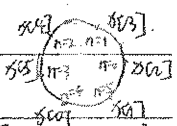
用圆表示：N=6



（逆圆表示）

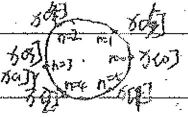
$x[\langle n-4\rangle_6]$ 右移 ⇒ 逆时针转动4格：课例

$x[\langle n+2\rangle_6]$ 左移 ⇒ 顺时针转动2格.



$\{x[\langle n-4\rangle_6]\}=\{x[\langle n+2\rangle_6]\}=\{x[2],x[3],x[4],x[5],x[0],x[1]\}$.

反转：$x[\langle -n\rangle_N]$ ⇒ 中心反折



$\therefore x[\langle -n\rangle_6]=\{x[0],x[5],x[4],x[3],x[2],x[1]\}$

3) 圆周卷积（超过范围的平移，即叠加相加法，重叠保留法，用DFT实现，卷积后长度＝N 见书的第五章）

6. 能量信号：能量为有限值，平均功率为0 →有限长序列；能量公式：$\mathcal{E}_x=\sum_{n=-\infty}^{\infty}|x[n]|^2$

功率信号：平均功率为有限值，能量无限 →周期序列. 平均功率公式：$P_x=\lim_{K\to\infty}\frac{1}{2K+1}\sum_{n=-K}^{K}|x[n]|^2$

绝对可和：$\sum_{n=-\infty}^{\infty}|x[n]|<\infty$ 有界序列：$|x[n]|\le B_x<\infty$

平方可和：$\sum_{n=-\infty}^{\infty}|x[n]|^2<\infty$ ⇒说明能量为有限值

平方可和并不一定绝对可和.（如$h[n]=\frac{\sin\omega_c n}{\pi n}$.

7. 特殊序列：单位冲激序列$\delta[n]$，单位阶跃序列$\mu[n]$ 实正弦序列 $x[n]=A\cos(\omega_0 n+\varphi)$ ⇒在第3点讲过.

$\delta[n]=\begin{cases}1, & n=0\\ 0, & n\ne0\end{cases}$ $\delta[n-k]=\begin{cases}1, & n=k\\ 0, & n\ne k\end{cases}$ $\mu[n]=\begin{cases}1, & n\ge0\\ 0, & n<0\end{cases}$ $\mu[n-k]=\begin{cases}1, & n\ge k\\ 0, & n<k\end{cases}$

$\delta[n]$与$\mu[n]$间关系：$x[n]=\sum_{k=-\infty}^{+\infty}x[k]\delta[n-k]$ $\delta[n]=\mu[n]-\mu[n-1]$ 习题2.26

正弦序列的几种表示方式：$x[n]=A\cos(\omega_0 n+\varphi)$　$x[n]=A\alpha^n=|A|e^{j\varphi}e^{(\sigma_0+j\omega_0)n}=x_{re}[n]+jx_{im}[n]$

$x_{re}[n]=|A|e^{\sigma_0 n}\cos(\omega_0 n+\varphi)$ ∴ $x_{im}[n]=|A|e^{\sigma_0 n}\sin(\omega_0 n+\varphi)$　$\sigma_0=0$时，常振幅．$\sigma_0>0$时，振幅递增

$\sigma_0<0$时，振幅递减

8. 采样

对连续模拟信号 $x(t)=A\cos(2\pi f_0 t+\varphi)=A\cos(\Omega_0 t+\varphi)$ ∵Ω_0为模拟角频率

└→ 采样：$t=nT$，采样周期为T，抽样频率 $F_T=\frac{1}{T}$．

（$R=\frac{F_T'}{F_T}$　F_T'为新的采样的频率，F_T为原序列频率．若$R>1$，则为内插，→$\boxed{\uparrow L}$→上抽样器 f变高
若$R<1$，则为抽取，→$\boxed{\downarrow M}$→下抽样器 f变低，具体R倍
interpolation，decimation

↓

采样后：$x[n]=x(t)|_{t=nT}=A\cos(\Omega_0 nT+\varphi)$

↓ 抽样角频率 $\Omega_T=2\pi F_T=\frac{2\pi}{T}$

$x[n]=A\cos(\frac{2\pi\Omega_0}{\Omega_T}n+\varphi)$

令 $\frac{2\pi\Omega_0}{\Omega_T}=\omega_0 \Rightarrow \omega_0=\Omega_0 T=\frac{\Omega_0}{F_T}=\frac{2\pi\Omega_0}{\Omega_T}$ = 归一化数字角频率 ω_0：$x[n]=A\cos(\omega_0 n+\varphi)$

ω_0单位：radians/sample．Ω_0单位：radians/second．f_0：Hz

9：现有3个信号：$g_1(t)=\cos(6\pi t)$，$g_2(t)=\cos(14\pi t)$，$g_3(t)=\cos(26\pi t)$．用采样频率 $F_T=10\text{Hz}$ 采样，求采样后序列．

解：原 $f_1=3\text{Hz}$，$f_2=7\text{Hz}$，$f_3=13\text{Hz}$．$\Omega_0=2\pi f$　则 $F_T=10\text{Hz}$ ∴ $T=0.1\text{s}$　∴ $\omega_0=\Omega_0 T=0.1\Omega_0$．

∴ 采样后（$x[n]=A\cos(\omega_0 n+\varphi)$）（代入）：$g_1[n]=\cos(0.6\pi n)$，$g_2[n]=\cos(1.4\pi n)$，$g_3[n]=\cos(2.6\pi n)$．

采样过程，抽样定理 ┌ 时域采样 $\xrightarrow{\text{不混叠(失真) aliasing}}$ 奈奎斯特抽样率：$\Omega_T\geq 2\Omega_m$．（DTFT中使用），

└ 频域采样 → 序列长度 $N\geq M$．（DFT中使用）

1.傅里叶变换的4种形式及相互关系、 CTFT: [illegible]

非周期.连续 $x(t)$ $\xrightarrow{CTFT}$ $X(j\Omega)$ 连续,非周期

周期.连续 $x(t)$ $\xrightarrow{CTFS}$ $X(jk\Omega)$ 离散,非周期

离散.非周期 $x[n]$ $\xrightarrow{DTFT}$ $X(e^{j\omega})$ 周期.连续.

离散.周期 $x[n]$ $\xrightarrow{DFS}$ $\tilde{X}(k\omega)$ 周期.离散.

时域 频域

非周期 ⟷ 连续

周期 ⟷ 离散 (周期延拓).

相乘 ⟷ 卷积

$$\begin{tikzcd} \text{DTFT} \arrow[r, "\text{频域离散}", "\text{频域采样}"'] & \text{DFT} \arrow[r, "\text{实现工具}"] \arrow[d, "\text{周期}"] & \text{FFT(快速傅里叶变换)} \\ & \text{DFS} & \end{tikzcd}$$

$$\begin{tikzcd} N\text{点 } x[n] \arrow[r, shift left, "DTFT"] \arrow[d, shift left, "DFT"] & X(e^{j\omega}) \arrow[l, shift left, "IDTFT"] \arrow[r, "z=e^{j\omega}"'] & Z(\omega) \\ X(k) \arrow[u, shift left, "IDFT"] & & \end{tikzcd}$$

2. CTFT 正变换: $X_a(j\Omega)=\int_{-\infty}^{\infty} x_a(t) e^{-j\Omega t} dt$ 逆变换: $x_a(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X_a(j\Omega) e^{j\Omega t} d\Omega$

$x_a(t)$ 需满足狄利克雷条件 才能保证 CTFT 的 $X_a(j\Omega)$ 存在.

狄利克雷条件: 1. 在任一有限区间内,有有限个最大最小值、 2). 满足绝对可积 (absolutely integrable). $\int_{-\infty}^{\infty}|x_a(t)|dt<\infty$.

3. 能量谱密度: $\varepsilon_x=\int_{-\infty}^{\infty}|x_a(t)|^2dt=\int_{-\infty}^{\infty}x_a(t)x_a^*(t)dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}|X_a(j\Omega)|^2d\Omega$. (Parseval's theorem).

4. DTFT 正变换: $X(e^{j\omega})=\sum_{n=-\infty}^{\infty}x[n]e^{-j\omega n}$ 逆变换: $x[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\omega})e^{j\omega n}d\omega$. (如习题 3.16, 3.21.

题: 已知: $x=\{3,1,-5,-11,0,\underset{\uparrow}{-5},3,3,2\}$, $n=[-5,3]$. 求 $X(e^{j0})=$ ____ , $X(e^{j\pi})=$ ____ . $\int_{-\pi}^{\pi}X(e^{j\omega})d\omega=$ ____

由上式可得: $X(e^{j0})\overset{\omega=0}{=\!=\!=}\sum_{n=-\infty}^{\infty}x[n]e^{0}=\sum_{n=-\infty}^{\infty}x[n]=3+1+(-5)+(-11)+0+(-5)+3+3+2=-9$

$X(e^{j\pi})\overset{\omega=\pi}{=\!=\!=}\sum_{n=-\infty}^{\infty}x[n]e^{-j\pi n}$ 当 n 为奇数时, $e^{-j\pi n}=-1$; n 为偶数, $e^{-j\pi n}=1$ ∴ $X(e^{j\pi})=-3+1-(-5)+(-11)-0+(-5)-3+3-2=-15$

$\int_{-\pi}^{\pi}X(e^{j\omega})d\omega=x[0]\cdot 2\pi=-5\times 2\pi=-10\pi$.

5. DTFT 性质 ①幅度谱为偶函数. $|X(e^{j\omega})|=|X(e^{-j\omega})|$, 相位谱为奇函数. $\theta(\omega)=-\theta(-\omega)$. ∵ $\theta(\omega)$ 不能唯一确定. 解答主值: $-\pi\leq\theta(\omega)<$

②是周期为 2π 的周期函数. $X(e^{j(\omega+2\pi k)})=X(e^{j\omega})$ …… 如习题 3.24. ~~(实奇[illegible])~~ 习题 3.30. 习题 3.31.

③对于复数序列: 实偶虚奇.

$x(n)$ 为实偶 $\xrightarrow{DTFT}$ 实偶. $x(n)$ 为虚奇 $\xrightarrow{DTFT}$ 虚奇

6. 表 3.1, 3.2 P90; P97 表 3.3. P100 表 3.4. ☆☆(考的).

7. DTFT 存在条件: $x[n]$ 绝对可和是 DTFT 存在的充要条件. DTFT 的能量谱密度: $\varepsilon_g=\frac{1}{2\pi}\int_{-\pi}^{\pi}|G(e^{j\omega})|^2d\omega$

均方收敛 → DTFT 不存在. (收敛: convergence condition)

低通滤波器 LP. 有限能量, 但不绝对可和. ⇒ 吉布斯(Gibbs)现象. 书 P85 图 3.6. K↑, 振荡数率↑, 过渡带斜率变陡.

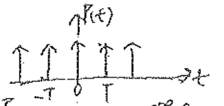
8. 时域采样 → 频域周期延拓. Anti-aliasing filter 抗混叠滤波器 ① S/H: 抽样保持器: sample and hold

对连续时间信号的离散化处理过程: P108 框图. A/D: 模/数 转换器. Digital processor. 数字处理器 Reconstruction filter 重构滤波器. ②

① ② 均为低通模拟滤波器

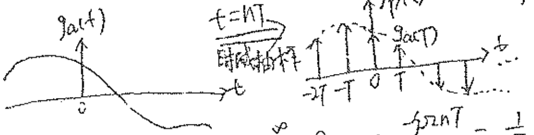
基带 $-\frac{\Omega_T}{2}\leq\Omega\leq\frac{\Omega_T}{2}$ (奈奎斯特带) ⇒ 实际 $\frac{\Omega_T}{2}=\Omega_m$

周期采样信号 $p(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT)$



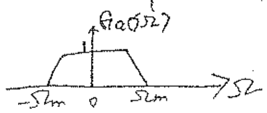
连续时间信号 $g_a(t)$ 非周期 $\xrightarrow{CTFT}$ $G_a(j\Omega)=\int_{-\infty}^{\infty}g_a(t)e^{-j\Omega t}dt$

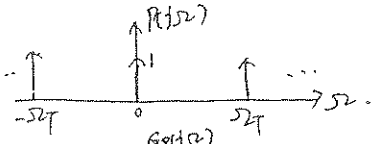
采样后信号 $g_p(t)=g_a(t)p(t)=\sum_{n=-\infty}^{\infty}g_a(nT)\delta(t-nT)$

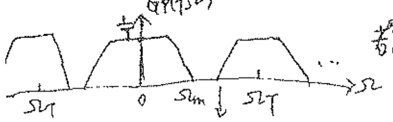


$g_p(t)\xrightarrow{DTFT}G_p(j\Omega)=\sum_{n=-\infty}^{\infty}g_a(nT)e^{-j\Omega nT}=\frac{1}{T}\sum_{k=-\infty}^{\infty}G_a(j(\Omega-k\Omega_T))$ ∴ $G_p(j\Omega)$ 是 $G_a(j\Omega)$ 的平移, 并幅度变为 $\frac{1}{T}$

Ω_T 为平移长度, 也为采样频率. $T=\frac{2\pi}{\Omega_T}$



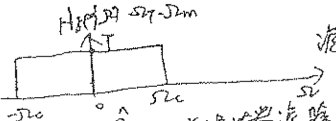




(抽样定理)

若不发生混叠, 则应满足 $\Omega_T\geq 2\Omega_m$. ($2\Omega_m$ 为奈奎斯特频率)

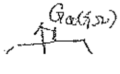
采样:
- 过抽样 (oversampling) $\Omega_T>2\Omega_m$
- 欠抽样 (undersampling) $\Omega_T<2\Omega_m$
- 临界抽样 (critical sampling) $\Omega_T=2\Omega_m$



滤波器带宽 $\Omega_m<\Omega_c<(\Omega_T-\Omega_m)$

模拟角频率与数字角频率关系: $\omega=\Omega T$. $\Omega=\frac{\omega}{T}$.

$\hat{G}_a(j\Omega)$ 为滤波器滤除后恢复的原图形



采样习题: 3.60, 3.61, 3.65. 书 P115 例题 3.17.

第四章：

4.1 从差分方程了解几种简单滤波器的功能：

1、累加器

$$y[n]=\sum_{\ell=-\infty}^{-1}x[\ell]+\sum_{\ell=0}^{n}x[\ell]$$
$$=y[-1]+\sum_{\ell=0}^{n}x[\ell],\ n\geq 0$$

2、滑动平均滤波器：

$$y[n]=\frac{1}{M}\sum_{k=0}^{M-1}x[n-k]$$

3、线性内插器；

4、中值滤波器：从小到大排列，中间的值即为中值，常用于去除加性随机冲激噪声；

4.2 判断一般离散时间系统的性质：**Problem 4.3**

1、线性系统：

对任意的 α,β，

当输入 $x[n]=\alpha x_1[n]+\beta x_2[n]$

其响应 $y[n]=\alpha y_1[n]+\beta y_2[n]$

2、时不变系统：

当输入 $x[n]=x_1[n-n_0]$

输出 $y[n]=y_1[n-n_0]$

3、因果系统：输出不能超前输入

例如下列系统为因果系统：

$$y[n]=\alpha_1 x[n]+\alpha_2 x[n-1]+\alpha_3 x[n-2]+\alpha_4 x[n-3]$$

$$y[n]=b_0 x[n]+b_1 x[n-1]+b_2 x[n-2]$$
$$+a_1 y[n-1]+a_2 y[n-2]$$

$$y[n]=y[n-1]+x[n]$$

下列系统为非因果系统

$$y[n] = x_u[n] + \frac{1}{2}(x_u[n-1] + x_u[n+1])$$

(A causal implementation: $y[n] = x_u[n-1] + \frac{1}{2}(x_u[n-2] + x_u[n])$)

4、稳定系统：输入和输出都是有界的

$|x[n]| < B_x$, $|y[n]| < B_y$

B_x 和 B_y 都是有限正常量.

5、无源系统：输出序列 y[n]的能量不能超过输入 x[n]的能量

$$\sum_{n=-\infty}^{\infty} |y[n]|^2 \le \sum_{n=-\infty}^{\infty} |x[n]|^2 < \infty$$

6、无损系统：输出序列 y[n]的能量等于输入 x[n]的能量

$$\sum_{n=-\infty}^{\infty} |y[n]|^2 = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

4.3 冲激响应和阶跃响应的定义：

① 当输入 $x[n] = \delta[n]$，输出就是冲激响应

② 当输入 $x[n] = u[n]$，输出就是阶跃响应.

4.4 LTI 系统的 I/O 卷积关系　　卷积和的计算：Problems 4.20, 4.23

LTI 系统的稳定性、因果性条件（时域判据。z 域判据见 6.7 节）

① 用输入信号与 LTI 系统的冲激响应进行卷积，即可得到输出.

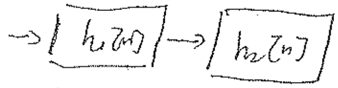
② 列表法计算卷积和（书上例题 4.14）

③ 稳定性：当且仅当 LTI 数字滤波器的冲激响应序列 $\{h[n]\}$ 绝对可和时，即 $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$，该系统是稳定的

④ 因果性：当且仅当 LTI 离散时间系统的冲激响应序列 $\{h[n]\}$ 为因果序列时，该系统才是因果的.

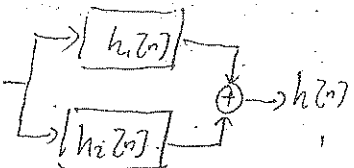
4.5 系统互联 Problem 4.30

① 级联：



$h[n]=h_1[n] \circledast h_2[n]$

② 并联：



$h[n]=h_1[n]+h_2[n]$

4.6 通常研究的 LTI 系统的差分方程：

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

4.7 FIR/IIR 系统、递归/非递归系统的定义

① 当 $h[n]$ 具有有限长时，则它是一个有限冲激响应(FIR)离散时间系

② 当 $h[n]$ 无限长，则它是一个无限冲激响应(IIR)离散时间系

③ 若仅仅知道当前和过去时刻的输入样本就可以计算出输出样本，该系统称为非递归离散时间系统。

④ 若计算输出时除了需要知道当前和过去时刻的输入样本外，还需要知道过去时刻的输出样本，则称为递

4.8（可与 6.7 节、Ch7 一起复习）

重点：频率响应的概念：Problem 4.58 4.63

第五章：

5.2 DFT 和 IDFT 的定义：求和式、含因子的求和式、矩阵形式

DFT 和 IDFT 的计算：例题，Problems 5.9，5.10，5.20，5.55（矩阵形式）

DFT 和 IDFT 的意义

5.3 DFT 与 DTFT 的关系：Problem 5.25

L 点 $x[n]$_ N 点 $Y[k]=DFT(x[n])$ N 点 $y[n]$

（$0 \leq n \leq N-1$），$N>=L$ 时，$y[n]＝x[n]$。

5.4 圆周卷积的定义和计算（用于 5.7 节）：Problem 5.2 或例题

5.5 圆周共轭对称/反对称的定义（用于 5.6 节）

5.6 DFT 对称性：Problems 5.43, 5.45

5.7 DFT 定理：Problem 5.55

5.10 用 DFT 求线性卷积 圆周卷积和线性卷积的关系：N 点序列 x 和 M 点序列 h，其线性卷积 $yL[n]$长 $L=N+M-1$，L 点圆周卷积 $yC[n]＝yL[n]$。（$P<L$ 点圆周卷积为 $yL[n]$以 P 为周期延拓后的$[0,P-1]$段，Problem5.28）

圆周卷积用 DFT 实现（利用 5.7 节 DFT 的圆周卷积定理）。

长序列的重叠相加法、重叠保留法的做法。

第五章

5.2 ① DFT. $X[k]=\sum_{n=0}^{N-1}x[n]\cdot e^{j2\pi kn/N}$. , $0\le k\le N-1$

当 $W_N=e^{-j2\pi/N}$ 时，上式也可写为 $X[k]=\sum_{n=0}^{N-1}x[n]\cdot W_N^{kn}$, $0\le k\le N-1$

IDFT: $x[n]=\frac{1}{N}\sum_{k=0}^{N-1}X[k]\cdot W_N^{-kn}$, $0\le n\le N-1$

② 矩阵形式：

DFT: $X=D_N\cdot x$,

$$\begin{bmatrix} X[0] \\ X[1] \\ \vdots \\ X[N-1] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N^{1} & W_N^{2} & \cdots & W_N^{N-1} \\ 1 & W_N^{2} & W_N^{4} & \cdots & W_N^{2(N-1)} \\ \vdots & & & & \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix}$$

IDFT: $x=D_N^{-1}\cdot X$

DFT是DTFT的频域抽样： DTFT $\underset{内插}{\overset{抽样}{\rightleftharpoons}}$ DFT

DTFT的变换域是z平面的单位圆

DFT的变换域是z平面上单位圆上的N个离散点.

5.4 圆周卷积的定义和计算： problem 5.2 书例题

步骤：① 和线性卷积一样，用到表相乘法；

② 卷积后序列的长度还是N，所以下标对N取余.

5.5 长度为N的序列 $x[n]$ 表示为

$$x[n]=x_{cs}[n]+x_{ca}[n],\quad 0\le n\le N-1$$

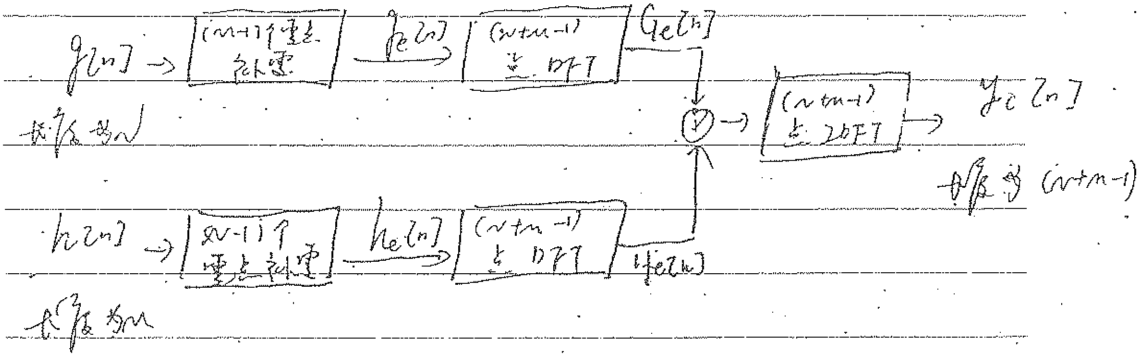
· 周期共轭对称部分：$x_{cs}[n]=\frac{1}{2}\left(x[n]+x^{*}[\langle -n\rangle_N]\right)$, $0\le n\le N-1$

周期共轭反对称部分：$x_{ca}[n]=\frac{1}{2}\left(x[n]-x^{*}[\langle -n\rangle_N]\right)$, $0\le n\le N-1$

DFT的对称性. 书上表5.1, 5.2

DFT的一些定理. 书上表5.3

5.10. 用DFT实现线性卷积.



第六章

1. z变换定义

- For a given sequence $g[n]$, its z-transform $G(z)$ is defined as

$$G(z) = \sum_{n=-\infty}^{\infty} g[n] z^{-n}$$

where $z = \mathcal{R}e(z) + j\mathcal{I}m(z)$ is a complex variable

2.ROC（region of convergence）收敛域：

（大家好好看一下下面的例子，并做课后习题 Problems 6.2，6.5，6.7，6.8，6.16）

例一、

- Example - Determine the z-transform $X(z)$ of the causal sequence $x[n] = \alpha^n \mu[n]$ and its ROC
- Now $X(z) = \sum_{n=-\infty}^{\infty} \alpha^n \mu[n] z^{-n} = \sum_{n=0}^{\infty} \alpha^n z^{-n}$
- The above power series converges to

$$X(z) = \frac{1}{1-\alpha z^{-1}}, \quad \text{for } \left|\alpha z^{-1}\right| < 1$$

- ROC is the annular region $|z| > |\alpha|$

例二、

- Example - Consider the anti-causal sequence $y[n] = -\alpha^n \mu[-n-1]$
- Its z-transform is given by

$$Y(z) = \sum_{n=-\infty}^{-1} -\alpha^n z^{-n} = -\sum_{m=1}^{\infty} \alpha^{-m} z^m$$

$$= -\alpha^{-1} z \sum_{m=0}^{\infty} \alpha^{-m} z^m = -\frac{\alpha^{-1} z}{1-\alpha^{-1} z}$$

$$= \frac{1}{1-\alpha z^{-1}}, \quad \text{for } \left|\alpha^{-1} z\right| < 1$$

- ROC is the annular region $|z| < |\alpha|$

Table 6.1: Commonly Used z-Transform Pairs

Sequence	z-Transform	ROC
$\delta[n]$	1	All values of z
$\mu[n]$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$\alpha^n\mu[n]$	$\dfrac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert > \lvert\alpha\rvert$
$(r^n \cos\omega_o n)\mu[n]$	$\dfrac{1-(r\cos\omega_o)z^{-1}}{1-(2r\cos\omega_o)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
$(r^n \sin\omega_o n)\mu[n]$	$\dfrac{(r\sin\omega_o)z^{-1}}{1-(2r\cos\omega_o)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$

Theorems	Sequence	z-Transform	ROC
	$g[n]$ $h[n]$	$G(z)$ $H(z)$	$\mathcal{R}_g$ $\mathcal{R}_h$
Conjugation	$g^*[n]$	$G^*(z^*)$	$\mathcal{R}_g$
Time-reversal	$g[-n]$	$G(1/z)$	$1/\mathcal{R}_g$
Linearity	$\alpha g[n]+\beta h[n]$	$\alpha G(z)+\beta H(z)$	Includes $\mathcal{R}_g \cap \mathcal{R}_h$
Time-shifting	$g[n-n_o]$	$z^{-n_o}G(z)$	$\mathcal{R}_g$, except possibly the point $z=0$ or ∞
Multiplication by an exponential sequence	$\alpha^n g[n]$	$G(z/\alpha)$	$\lvert\alpha\rvert\mathcal{R}_g$
Differentiation of $G(z)$	$ng[n]$	$-z\dfrac{dG(z)}{dz}$	$\mathcal{R}_g$, except possibly the point $z=0$ or ∞
Convolution	$g[n] \circledast h[n]$	$G(z)H(z)$	Includes $\mathcal{R}_g \cap \mathcal{R}_h$
Modulation	$g[n]h[n]$	$\frac{1}{2\pi j}\oint_C G(v)H(z/v)v^{-1}\,dv$	Includes $\mathcal{R}_g\mathcal{R}_h$
Parseval's relation	$\sum_{n=-\infty}^{\infty} g[n]h^*[n] = \frac{1}{2\pi j}\oint_C G(v)H^*(1/v^*)v^{-1}\,dv$		

Note: If $\mathcal{R}_g$ denotes the region $R_{g-} < \lvert z\rvert < R_{g+}$ and $\mathcal{R}_h$ denotes the region $R_{h-} < \lvert z\rvert < R_{h+}$, then $1/\mathcal{R}_g$ denotes the region $1/R_{g+} < \lvert z\rvert < 1/R_{g-}$ and $\mathcal{R}_g\mathcal{R}_h$ denotes the region $R_{g-}R_{h-} < \lvert z\rvert < R_{g+}R_{h+}$.

4.有理 Z 变换（rational Z-transform）

The ROC of a rational Z-transform cannot contain any poles and is bounded by the poles.

5. - Example - Consider

$$G(z)=\frac{2+0.8z^{-1}+0.5z^{-2}+0.3z^{-3}}{1+0.8z^{-1}+0.2z^{-2}}$$

- By long division in reverse order we arrive at

$$G(z)=-3.5+1.5z^{-1}+\underbrace{\frac{5.5+2.1z^{-1}}{1+0.8z^{-1}+0.2z^{-2}}}_{\text{Proper fraction}}$$

（ - Example - Let the z-transform $H(z)$ of a causal sequence $h[n]$ be given by

；变换）

$$H(z)=\frac{z(z+2)}{(z-0.2)(z+0.6)}=\frac{1+2z^{-1}}{(1-0.2z^{-1})(1+0.6z^{-1})}$$

- A partial-fraction expansion of $H(z)$ is then of the form

$$H(z)=\frac{\rho_1}{1-0.2z^{-1}}+\frac{\rho_2}{1+0.6z^{-1}}$$

Example: Determine the z-transform and corresponding

7.（收敛域：右边序列在圆外，左边序列在圆内）

1) $x[n]=(0.8)^n\mu[n]+(1.25)^n\mu[n]$

2) $x[n]=(0.8)^n\mu[n]-(1.25)^n\mu[-n-1]$

3) $x[n]=-(0.8)^n\mu[-n-1]-(1.25)^n\mu[-n-1]$

4) $x[n]=-(0.8)^n\mu[-n-1]+(1.25)^n\mu[n]$

$$X(z)=\frac{1}{1-0.8z^{-1}}+\frac{1}{1-1.25z^{-1}}$$

- Let $\{x[n]\}, 0 \le n \le L,$ denote a finite-length sequence of length $L+1$
- Let $\{h[n]\}, 0 \le n \le M,$ denote a finite-length sequence of length $M+1$

- We shall evaluate $y[n] = x[n] \circledast h[\mathrm{n}]$ using z-transform
- Note: $\{y[n]\}$ is a sequence of length $L+M+1$

- Let $\{x[n]\}$ and $\{h[n]\}$ be two length-N sequences defined for $0 \le n \le N-1$ with $X(z)$ and $H(z)$ denoting their z-transforms
- Let $y_C[n] = x[n] \textcircled{N} h[n]$ denote the N-point circular convolution of $x[n]$ and $h[n]$
- Let $y_L[n] = x[n] \circledast h[n]$ denote the linear convolution of $x[n]$ and $h[n]$
- Let $Y_C(z)$ and $Y_L(z)$ denote the z-transforms of $y_C[n]$ and $y_L[n]$
- It can be shown that

$$Y_C(z) = \langle Y_L(z) \rangle_{(z^{-N}-1)}$$

第七章

1.四种线性相位 FIR 滤波器的 h[n]特点、特点、零点分布情况。

- For a real impulse response, the magnitude response $|H(e^{j\omega})|$ is an even function of ω, i.e.,

$$|H(e^{j\omega})| = |H(e^{-j\omega})|$$

- Since $|H(e^{j\omega})| = |\breve{H}(\omega)|$, the amplitude response is then either an even function or an odd function of ω, i.e.

$$\breve{H}(-\omega) = \pm\breve{H}(\omega)$$

- The frequency response satisfies the relation

$$H(e^{j\omega}) = H^*(e^{-j\omega})$$

or, equivalently, the relation

$$e^{j(c\omega+\beta)}\breve{H}(\omega) = e^{-j(-c\omega+\beta)}\breve{H}(-\omega)$$

$$h[n] = h[N-n], \qquad 0 \le n \le N \qquad (c = -N/2)$$

- Thus, the FIR filter with an even amplitude response will have a linear phase if it has a symmetric impulse response

- If $\breve{H}(\omega)$ is an odd function of ω, then

$$h[n] = -h[N-n], \quad 0 \le n \le N \qquad (c = -N/2)$$

- Therefore, a FIR filter with an odd amplitude response will have linear-phase response if it has an antisymmetric impulse response

Type 1: Symmetric Impulse Response with Odd Length

- In this case, the degree N is even
- The frequency response is of the form

$$H(e^{j\omega}) = e^{-jN\omega/2}\breve{H}(\omega)$$

where the amplitude response $\breve{H}(\omega)$ is of the form

$$\breve{H}(\omega) = h[\tfrac{N}{2}] + 2\sum_{n=1}^{N/2} h[\tfrac{N}{2} - n]\cos(\omega n)$$

Type 2: Symmetric Impulse Response with Even Length

- In this case, the degree N is odd

- The expression for the frequency response in the general case for Type 2 FIR filters is of the form

$$H(e^{j\omega}) = e^{-jN\omega/2}\breve{H}(\omega)$$

where the amplitude response is given by

$$\breve{H}(\omega) = 2\sum_{n=1}^{(N+1)/2} h[\tfrac{N+1}{2} - n]\cos(\omega(n - \tfrac{1}{2}))$$

Type 3: Antiymmetric Impulse Response with Odd Length

- In this case, the degree N is even

- In the general case

$$H(e^{j\omega}) = je^{-jN\omega/2}\breve{H}(\omega)$$

where the amplitude response is of the form

$$\breve{H}(\omega) = 2\sum_{n=1}^{N/2} h[\tfrac{N}{2} - n]\sin(\omega n)$$

Type 4: Antiymmetric Impulse Response with Even Length

- In this case, the degree N is even

- In the general case we have

$$H(e^{j\omega}) = je^{-jN\omega/2}\breve{H}(\omega)$$

where now the amplitude response is of the form

$$\breve{H}(\omega) = 2\sum_{n=1}^{(N+1)/2} h[\tfrac{N+1}{2} - n]\sin(\omega(n - \tfrac{1}{2}))$$

Zero Locations of Linear-Phase FIR Transfer Functions

Type 1	Type 2	Type 3	Type 4
No restriction Can design any type	Cannot design highpass and bandstop Zero at $\omega = \pi$	Cannot design lowpass, highpass, and bandstop Zero at $\omega = 0$ and $\omega = \pi$	Cannot design lowpass, and bandstop Zero at $\omega = 0$

2.几种简单滤波器的零极点分布和频响的联系。Problem 7.55

注：未注明的章节不要求掌握

Chapter 8. Digital Filter Structures

8.1.1 Block Diagram Representation 框图表示

如：

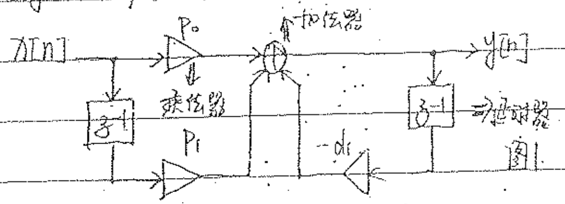


图1

$y[n] = -d_1 y[n-1] + p_0 x[n] + p_1 x[n-1]$ （一阶）

8.1.4 Canonic Structures 正准型 —— 延时数量等于差分方程的阶数

Noncanonic Structures 非正准型 —— otherwise

图一所示为非正准型，延时器数量为2，差分方程阶数为1.

8.2 Equivalent Structures 等效结构 —— 滤波器有相同传输函数

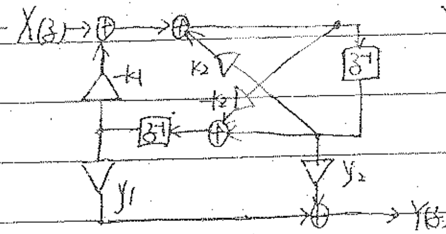
等效结构的实现方式：transpose operation（转置运算法则）

(I) 倒转所有路径，箭头倒置

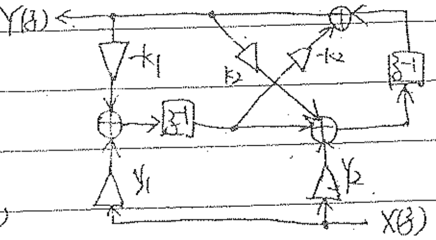
(II) 把网络节点转换成加法器，把加法器换成网络节点

(III) 交换输入节点和输出节点

Example –



Original structure.



Transposed structure.

8.3. Basic FIR Digital Filter Structures （FIR滤波器）

$$\begin{cases} H(z) = \sum_{k=0}^{N} h[k] z^{-k} & \text{（N阶因果FIR）} \\ y[n] = \sum_{k=0}^{N} h[k] x[n-k] \end{cases}$$

8.3.1. Direct-form Structures 直接型

如：$y[n] = h[0]x[n] + h[1]x[n-1] + h[2]x[n-2] + h[3]x[n-3] + h[4]x[n-4]$ （四阶即 $N=4$）.

注意：$N+1$ 个乘数乘法器，N 个双输入加法器

直接型分为 I型, II型 框图见 P376 图(a)、(b) 分别为 I, II型.

8.3.2 Cascade-Form Structures 级联型

$$H(z)=\sum_{k=0}^{N} h[k]z^{-k}$$

因式分解 → 表示为: $H(z)=h[0]\prod_{k=1}^{K}(1+\beta_{1k}z^{-1}+\beta_{2k}z^{-2})$

其中 若N为偶数 $K=\frac{N}{2}$, 若N为奇数 $K=\frac{N+1}{2}$, $\beta_{2K}=0$

也就是说, 例如N=6时可由____个2阶FIR部分组成的级联实现?

答案是3个, 因为 $K=\frac{6}{2}=3$. (理解到这一步就够了).

8.3.4 Linear-Phase FIR Structures 线性相位FIR.

N阶线性相位FIR滤波器可以用对称冲激响应

$$h[n]=h[N-n]$$

或反对称冲激响应

$$h[n]=-h[N-n]$$ 来描述

见 ch7 P323
N为阶数

因此在直接型实现中: $H(z)=h[0]+h[1]z^{-1}+h[2]z^{-2}+h[3]z^{-3}+h[4]z^{-4}+h[5]z^{-5}+h[6]z^{-6}$ (N=6时)

$=h[0]+h[1]z^{-1}+h[2]z^{-2}+h[3]z^{-3}+h[2]z^{-4}+h[1]z^{-5}+h[0]z^{-6}$

$=h[0](1+z^{-6})+h[1](z^{-1}+z^{-5})+h[2](z^{-2}+z^{-4})+h[3]z^{-3}$

可以看出利用此性质可以减少近一半的乘法器.

注: 直接型FIR是特殊的抽头延时线 → Tapped delay line.

8.4 Basic IIR Digital Filter Structures — IIR滤波器的基本结构

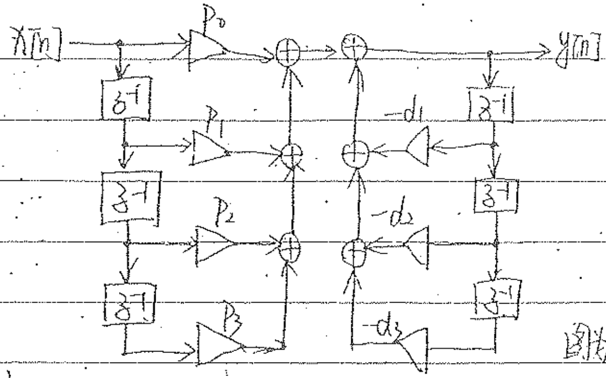
8.4.1 Direct-Form Structures 直接型

$$H(z)=\frac{P(z)}{D(z)}=\frac{p_0+p_1z^{-1}+p_2z^{-2}+p_3z^{-3}}{1+d_1z^{-1}+d_2z^{-2}+d_3z^{-3}}\xrightarrow{\text{时域}} y[n]+d_1y[n-1]+d_2y[n-2]+d_3y[n-3]$$

$$=xp_0+p_1x[n-1]+p_2x[n-2]+p_3x[n-3]$$

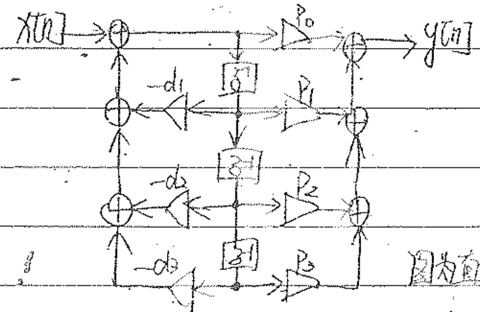
$y[n]=xp_0+p_1x[n-1]+p_2x[n-2]+p_3x[n-3]-d_1y[n-1]-d_2y[n-2]-d_3y[n-3]$

★ 直接I型: 根据以上差分方程得到.



因为直接I型，其等效结构可由转置运算生成

直接II型：正准型形式，根图需要最少记忆 而损依据等



因为直接II型，其等效结构可由转置运算生成

8.4.2 Cascade Realization 级联型

$$H(z)=\frac{P_1(z)P_2(z)P_3(z)}{D_1(z)D_2(z)D_3(z)}$$ P383

掌握总共有36种组合规则

通常因式分解可表示为1阶和2阶的级联：

$$H(z)=P_0\prod_k\left(\frac{1+\beta_{1k}z^{-1}+\beta_{2k}z^{-2}}{1+\alpha_{1k}z^{-1}+\alpha_{2k}z^{-2}}\right)$$ （和FIR比较）

对一阶而言，$\alpha_{2k}=\beta_{2k}=0$

对三阶 可表示成 $H(z)=P_0\left(\frac{1+\beta_{11}z^{-1}}{1+\alpha_{11}z^{-1}}\right)\left(\frac{1+\beta_{12}z^{-1}+\beta_{22}z^{-2}}{1+\alpha_{12}z^{-1}+\alpha_{22}z^{-2}}\right)$

8.4.3 Parallel Realizations 并联型

部分分式展开 以并联形式来实现

$$H(z)=\gamma_0+\sum_k\left(\frac{\gamma_{0k}+\gamma_{1k}z^{-1}}{1+\alpha_{1k}z^{-1}+\alpha_{2k}z^{-2}}\right)$$ 在上式中对于实极点，有 $\alpha_{2k}=\gamma_{1k}=0$

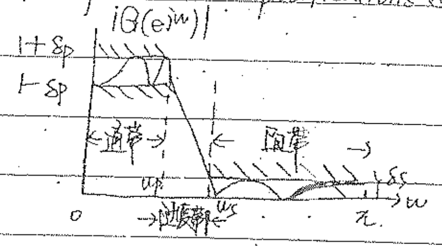
并联I型 分子阶数比分母阶数小一阶

并联II型 $H(z)=\delta_0+\sum_k\left(\frac{\delta_{1k}z^{-1}+\delta_{2k}z^{-2}}{1+d_{1k}z^{-1}+d_{2k}z^{-2}}\right)$ 分子、分母阶数一致

并联II型只作了解

Chapter 9 IIR Digital Filter Design

9.1.1 Digital Filter Specifications 数字滤波器的指标



$1-\delta_p\le|G(e^{j\omega})|\le1+\delta_p$ for $|\omega|\le\omega_p$.

$|G(e^{j\omega})|\le\delta_s$ for $\omega_s\le|\omega|\le\pi$

数字滤波器的频率响应 $G(e^{j\omega})$ 是 ω 的周期函数

而实系数数字滤波器的幅度响应是 ω 的偶函数.

ω 范围为 $-\pi\sim\pi$，因此只需关注 $0\sim\pi$ 范围即可.

指标通常由单位为dB的损耗函数给出，即 $A(\omega)=-20\lg|G(e^{j\omega})|$.

其中，峰值通带波纹 α_p 和最小阻带衰减 α_s 的单位为dB，则损耗指标为

$$\alpha_p=-20\lg(1-\delta_p)\,dB$$
$$\alpha_s=-20\lg(\delta_s)\,dB$$

归一化形式：假设通带幅度响应的最大值为1，最小值为 $\frac{1}{\sqrt{1+\varepsilon^2}}$，最大阻带幅度为 $\frac{1}{A}$

则 $\alpha_p=-20\lg\frac{1}{\sqrt{1+\varepsilon^2}}$

$\alpha_s=-20\lg\frac{1}{A}$

因此，最大通带衰减为 $\alpha_{max}=20\lg(\sqrt{1+\varepsilon^2})\,dB$.

对于 $\delta_p\ll1$，$\alpha_{max}\cong2\alpha_p$

注：归一化角频率 $\omega_p=\frac{\Omega_p}{F_T}=\frac{2\pi F_p}{F_T}=2\pi F_pT$

$\omega_s=\frac{\Omega_s}{F_T}=\frac{2\pi F_s}{F_T}=2\pi F_sT$

9.1.2

FIR 优点：线性相位、稳定

缺点：计算复杂度高

9.1.3 数字滤波器设计的基本方法

$H_a(s)=\frac{P_a(s)}{D_a(s)}$ 模拟滤波器 $\longrightarrow$ $G(z)=\frac{P(z)}{D(z)}$ 数字滤波器

由 s 域映射到 z 域

条件满足：(a) s 平面的虚轴 ($j\Omega$) 必须映射到 z 平面的单位圆上

(b) 稳定的模拟传输函数能变换为稳定的数字传输函数

9.2 Bilinear Transformation Method of IIR Filter Design.

双线性变换法

$S=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$ 反变换 $z=\frac{1+\frac{T}{2}s}{1-\frac{T}{2}s}$ 通常取 $T=2$

for $S=\sigma_0+j\Omega_0$

$\sigma_0<0$ $|z|<1$

$\sigma_0>0$ $|z|>1$ 注意平面域的变换

$\sigma_0=0$ $|z|=1$

模拟角频率 $\Omega=\frac{2}{T}\tan\left(\frac{\omega}{2}\right)$ ω 为数字角频率

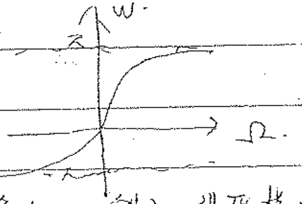
$\omega=2\tan^{-1}\left(\frac{\Omega T}{2}\right)$

☆ 双线性变换法的三个步骤

(1) 通过 (ω_p、ω_s) 预先加以畸变找到它们的等效模拟频率 Ω_p 和 Ω_s.

(2) 设计模拟滤波器 $H_a(s)$

(3) 对 $H_a(s)$ 进行双线性变换，得到数字滤波器传输函数 $G(z)$.



你会设计吗 非常重要!!!

双线性变换法：例3：设采样周期 $T=250\mu s$ ($f_s=4kHz$) ($f_c=1kHz$)，采用双线性变换法，设计一三阶巴特沃思低通滤波器。

解：首先 $W_c = 2\pi f_c T = 0.5\pi$

预畸变的模拟滤波器临界频率 $\Omega_c = \frac{2}{T}\tan(\frac{W_c}{2}) = \frac{2}{T}$ rad/s.

归一化的三阶巴特沃思滤波器写为 $H_a' = \frac{1}{1+2s+2s^2+s^3}$

$$H_a(s) = \frac{1}{1+2(\frac{s}{\Omega_c})+2(\frac{s}{\Omega_c})^2+(s/\Omega_c)^3}$$

代入 $\Omega_c = \frac{2}{T}$ 得

$$H_a(s) = \frac{1}{1+2(s/\frac{2}{T})+2(s/\frac{2}{T})^2+(s/\frac{2}{T})^3}$$

$$H(z) = H_a(s)\Big|_{s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}}$$

整理 $= \frac{1}{2}\cdot\frac{(1+z^{-1})^3}{3+z^{-2}}$

Chapter 10. FIR Digital Filter Design

10.1.1 Basic Approaches to FIR Digital Filter Design

(1) 加窗傅里叶级数法

(2) 频率抽样法

(3) 基于计算机的数字滤波器设计方法

10.1.2 滤波器阶数估计

Kaiser等方程只需了解，不需要掌握

10.2 基于加窗傅里叶级数的FIR滤波器设计

10.2.1 Least Integral-Squared Error Design of FIR Filters

最小积分平方误差设计

一个长度为2M+1的有限长冲激响应序列 $\{h_t[n]\}$，其DTFT $H_t(e^{j\omega})$ 在某种程度上逼近于所求的DTFT $H_d(e^{j\omega})$，一种常用的逼近准则就是最小积分平方误差

推导过程（略）

结论：若 $-M\le n\le M$ 时，$h_t[n]=h_d[n]$，则积分平方误差最小；换言之，在均方误差准则下，理想无限长冲激响应的最佳和最简单的有限长逼近是通过截短来得到的。

冲激响应为 $h[n]$ 的因果FIR滤波器可以通过将 $h_t[n]$ 延时M个样本后得到，即

$$h[n]=h_t[n-M]$$

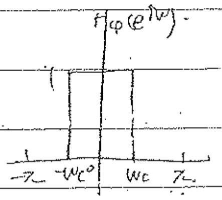
注意：因果滤波器 $h[n]$ 和非因果滤波器 $h_t[n]$ 具有相同的幅度响应，且它的相位响应相对于非因果滤波器有一个斜率为 ωM 的线性相移

10.2.2 理想滤波器的冲激响应 → 重点掌握

(1) ideal lowpass filter — $h_{LP}[n]=\frac{\sin\omega_c n}{\pi n}$ $\quad -\infty<n<\infty$

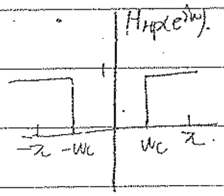
$N=2M+1$

$$\hat{h}_{LP}[n]=\begin{cases}\frac{\sin(\omega_c(n-M))}{\pi(n-M)} & 0\le n<N\\ 0 & \text{otherwise}\end{cases}\quad H_{LP}(e^{j\omega})=\begin{cases}1, & |\omega|\le\omega_c\\ 0, & \omega_c<|\omega|\le\pi\end{cases}$$

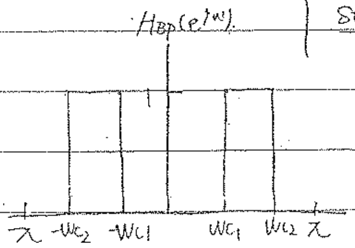


(2) Ideal highpass filter — $h_{HP}[n]=\begin{cases}1-\frac{\omega_c}{\pi} & n=0\\ -\frac{\sin(\omega_c n)}{\pi n} & n\ne0\end{cases}$

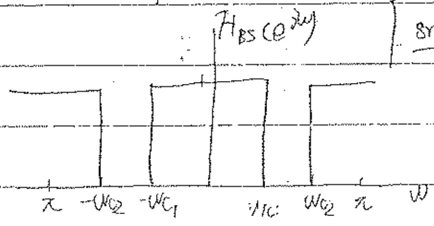
$$H_{HP}(e^{j\omega})=\begin{cases}0, & |\omega|<\omega_c\\ 1, & \omega_c\le|\omega|\le\pi\end{cases}$$



(3) ideal bandpass filter — $h_{BP}[n]=\begin{cases}\frac{\omega_{c2}}{\pi}-\frac{\omega_{c1}}{\pi} & (n=0)\\ \frac{\sin(\omega_{c2}n)}{\pi n}-\frac{\sin(\omega_{c1}n)}{\pi n} & (n\ne0)\end{cases}$



(4) ideal bandstop filter — $h_{BS}[n]=\begin{cases}1-\left(\frac{\omega_{c2}-\omega_{c1}}{\pi}\right) & (n=0)\\ \frac{\sin(\omega_{c1}n)}{\pi n}-\frac{\sin(\omega_{c2}n)}{\pi n} & (n\ne0)\end{cases}$



10.2.3 Gibbs Phenomenon 吉布斯现象

对于给定的理想滤波器的冲激响应系数进行简单截短，可得到因果FIR滤波器，这些截短滤波器的幅度响应呈现振动的现象，通常称为吉布斯现象

截短运算可以认为是将无限长冲激响应系数与一个有限长窗序列 $w[n]$ 相乘的结果，产生吉布斯现象的原因可以在频域中研究该加窗过程而得到解释，

$$h_t[n] = h_d[n]\cdot w[n]$$

$$H_t(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\varphi})\,\Psi(e^{j(\omega-\varphi)})\,d\varphi$$

where $H_t(e^{j\omega})$ and $\Psi(e^{j\omega})$ are the DTFTs of $h_t[n]$ and $w[n]$, respectively.

10.2.4 Fixed Window Functions 固定窗函数

P471 见图10.8. $\Delta\omega$ 为过渡带带宽. $\Delta\omega = \omega_s - \omega_p < \Delta_{ML}$

Δ_{ML} 为主瓣宽.

首先需通过 $\omega_c = (\omega_p+\omega_s)/2$ 来确定截止频率 ω_c，其中 ω_s 和 ω_p 分别为通带和阻带截止频率，然后利用 $\Delta\omega \approx \frac{c}{M}$ 估算出 M，其中，对于选取的窗函数，常数 c 的值可以从表 10.2 中得到，最后得到的具有最小的滤波器长度 $[2M+1]$。

影响性能的因素：窗长，窗类型 → 窗谱参数（主瓣宽度，相对旁瓣电平）→ 滤波器过渡带宽，阻带最小衰减

Chapter 11 DSP Algorithm Implementation

11.3. DFT的计算

$$X[k] = X(e^{j\omega})\Big|_{\omega=\frac{2\pi k}{N}} = \sum_{n=0}^{N-1} x[n]\,e^{-j\frac{2\pi kn}{N}} \qquad 0\le k\le N-1$$

$$X[k] = \sum_{n=0}^{N-1} x[n]\,W_N^{kn} \qquad 0\le k\le N-1.$$

$$W_N = e^{-j2\pi/N}$$

N点DFT序列的计算需要 N^2 次复数相乘和 $N(N-1)$ 次复数相加，在一个序列的长度为 N 的

情况下，可以看出，其N点DFT序列的计算需要$4N^2$次实数相乘和$(4N-2)N$次实数相加。对于较大的N值，复数相乘和相加的数目近似等于N^2

11.3.2 Cooley-Tukey FFT Algorithms
{ 基于时间抽取 图11.24 P542
{ 基于频率抽取 图11.28 P547.