Modern Control Theory

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Course Goals

This course focuses on linear system theory of the conventional approaches of state-space equations and the polynomial fraction method of transfer matrices. The course deals with

- **State-space representations and solutions**
- **Controllability and observability, feedback, pole placement, observers**
- **Analysis of Lyapunov stability**
- **Optimal control and linear quadratic control**

A better understanding of the basic concepts and analysis methods for linear multivariable control systems, and firm grasp of design methods for linear time-invariant systems.

Course Requirement

Prerequisites:

- Linear Algebra
- Differential Equations
- Theory of Matrices
- Principles of Automatic Control

Book: **Modern Control Theory and Engineering**

Reference book: **Automatic Control Theory**

（自动控制原理第六版：第9章和第10章）

Course Requirement

Course duration: 40 hours (2.5 credits)
Lab Experiment: Simulation twice (reports included)
Assignments: once every two weeks

Evaluation includes:

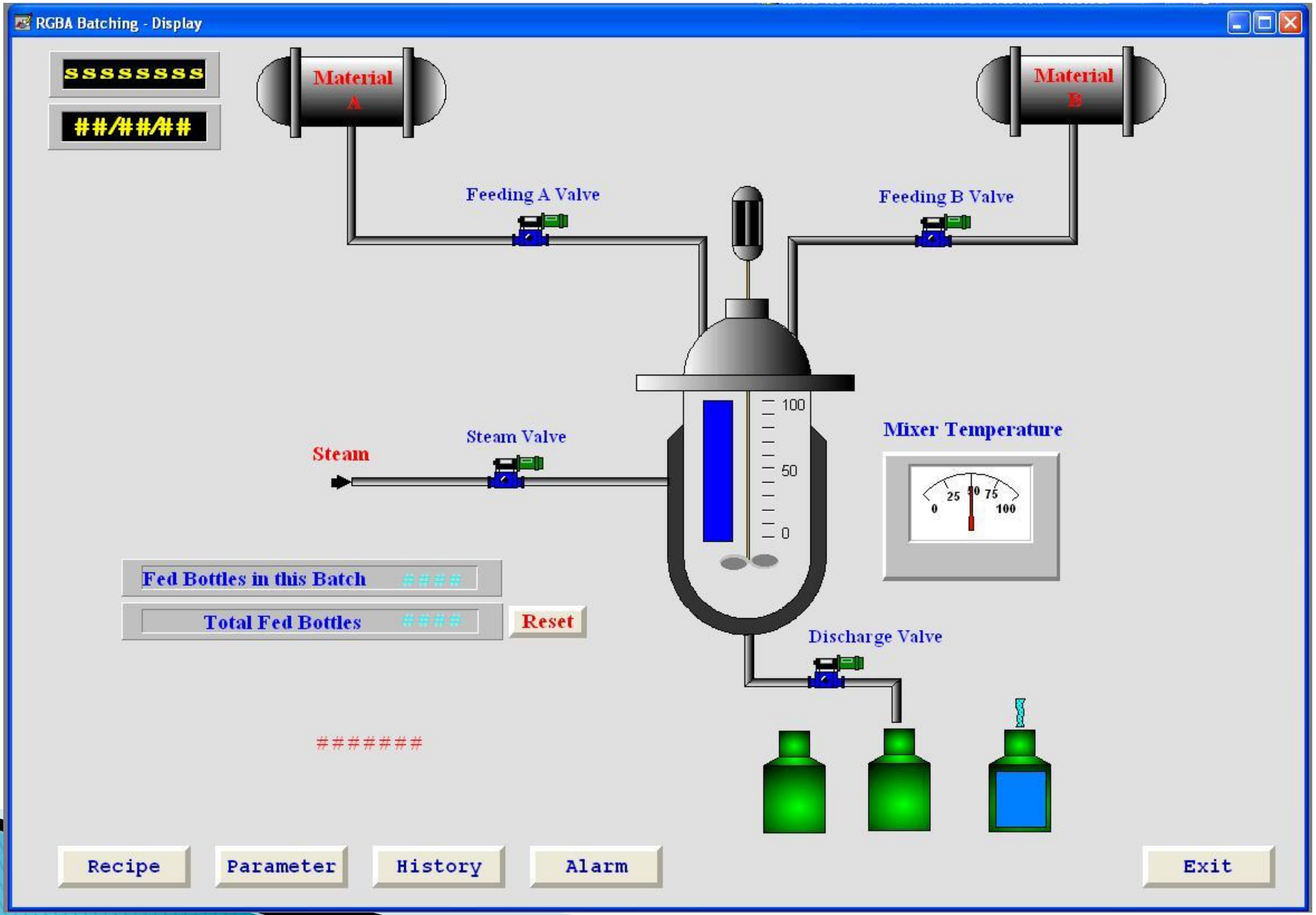
Attendance (10%) +Assignment (20%)+Exam (70%)

Introduction

- Problem Formulation
- History & Development of Control Theory
- Basic concepts and methods of Modern Control Theory
- Preliminaries

Problem Formulation

Industrial process control: velocity, temperature, flow, pressure, etc.



Problem Formulation

Robot control
Network control
Aerospace control

Rocket control system:
Navigator
Attitude control
…

SpaceX launches Falcon Heavy, the world's most powerful rocket

Falcon Heavy took flight Tuesday around 3:45 pm ET from Kennedy Space Center in Florida

Problem Formulation

Controlling Unmanned vehicles can make them in
Consensus
Coordination
Positioning
……

Controlling the Electric power vehicles can make them work in best condition.

But how?

Problem Formulation

Two important concepts in control

- **Feedback control**

  -- Closed loop (error input)

  -- Meet the requirements with uncertainty
- **Optimal control**

  --Minimize or maximize the cost function over a period

  --Pre-regulation and open loop control

Connection: Optimal control can be integrated into a closed loop

History of Control Theory

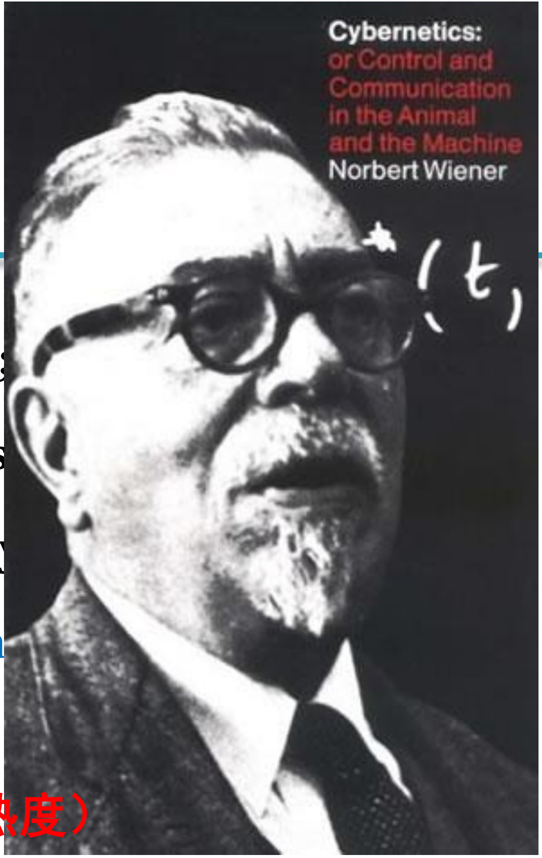
1769, James Watt's steam engine and governor-Flying Ball;

1875, 1895, Routh and Hurwitz developed stability analysis

1932, Nyquist developed a method for system stability analy

1948, Norbert Wiener-`Cybernetics: or Control and Comm

Animal and the Machine'



（1954, 钱学森发表《工业控制论》，形成了国内控制研究热度）

1957, Richard Ernest Bellman, 1920-1984, published 'Dynamic Programming'

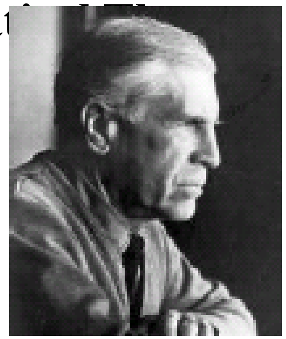
1960, Rudolf Emil Kalman, 1930-2016, On the General Theory of Control Systems

2, Lev Pontryagin propo n Principle in 'The Mathemat

ptimal Processes'

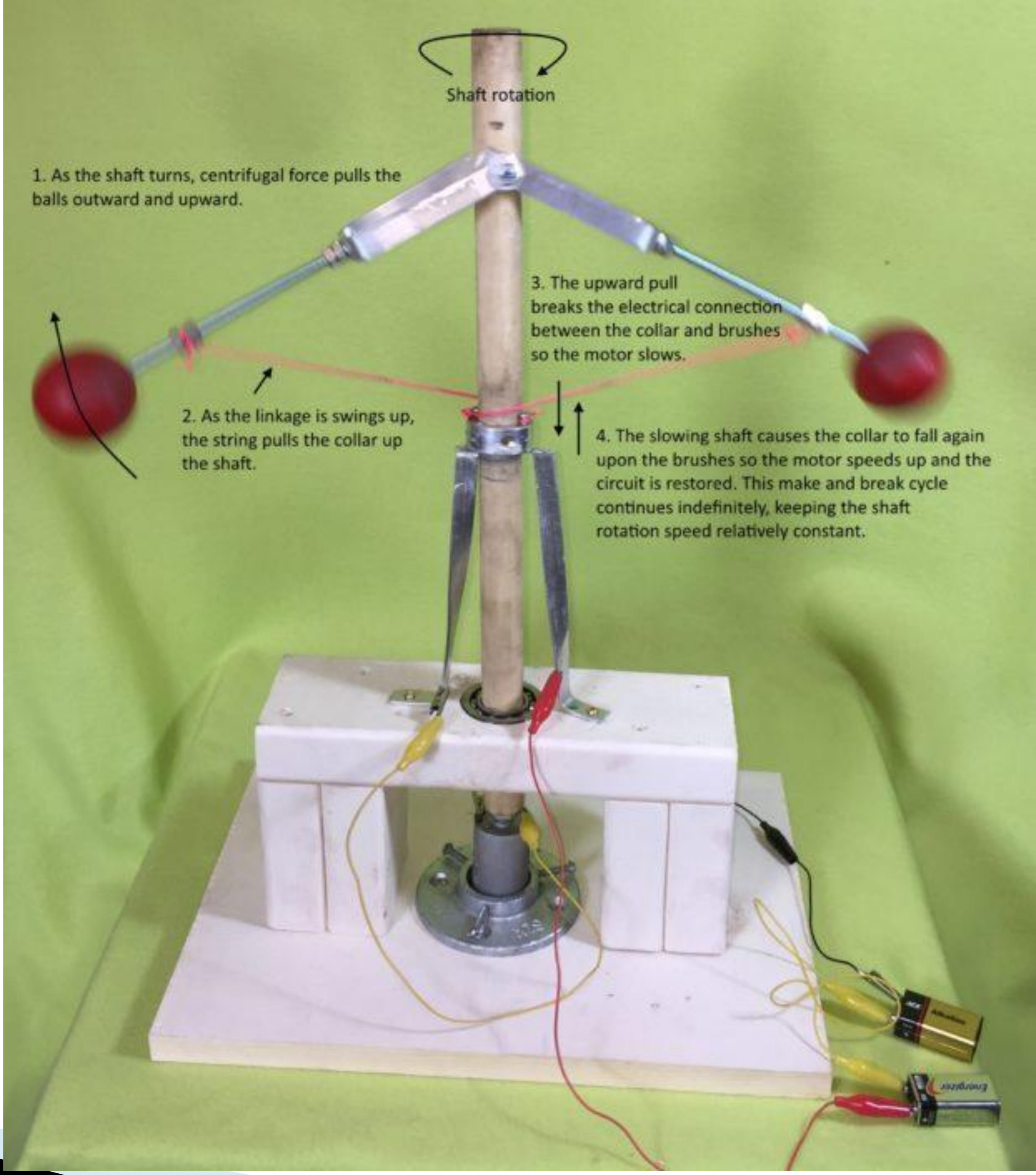
贝尔曼, R.

Rudolf Kalman



How to build Your Flyball Governor

https://makezine.com/projects/remaking-history-james-watt-and-the-flyball-governor/



History of Control Theory

Three Development Periods of Control Theory

Stage 1 Classical Control Theory (From 1868 to 1960's)

--Mathematical Methods: differential equations, Laplace transform, transfer function

--Analysis: time and frequency domain, root locus method

--Main problem: stability and accuracy

Limitations

--Linear time invariant system—LTI:

--Single input-output systems

- **Ignore the internal characteristics of systems**

--Suboptimal solution to control system design

- **The solutions depend on engineers' experience**
- **PID control**

History of Control Theory

Three Development Periods of Control Theory

Stage 2 Modern Control Theory (1960's)

--Mathematical Methods: first order differential equations, theory of matrices

--Analysis: State-Space Model

--Main problem: Stability, Controllability and Observability

Advantages

- **Multiple Input-Output Systems**
- **Time Variant Systems**
- **Time Domain: State-space Model**
- **Optimal Design of Control System**

History of Control Theory

Three Development Periods of Control Theory

Stage 3 Large System (complex network) Control Theory (1970’s--present)

--Analysis: Fuzzy control, artificial neuro network control, etc.

--Main problem: Stability, Controllability and Observability

Basic methods of Modern Control Theory

1. Linear system theory

Mathematical model — linear system state-space model

Core content— Controllability and Observability

Application— State feedback and observer design

Analysis— Lyapunov stability

2. Optimal control theory

3. System identification

4. Adaptive control

5. Intelligent control

Schedule of the course

Chapter 1. Introduction

Chapter 2. State-space Model of Linear System

Chapter 3. Controllability and Observability

Chapter 4. Analysis of Lyapunov Stability

Chapter 5. Feedback and State Observer

Chapter 6. Optimal Control Theory

Preliminaries

Matrix:

- Determinant of A: **det(A)**

$\det[\boldsymbol{A}] \neq 0$: $\Rightarrow$

- A is **Nonsingular Matrix**
- A is invertible
- A is full rank, **rank[A] = n**
- For Homogeneous equation $Ax=0$, $x=0$
- For equation $Ax=b$, $x= A^{-1} b$

Preliminaries

- **Transpose matrix** A^T
- **Inverse matrix** A^{-1}**,** $AB = BA = I_n$ $B = A^{-1}$

$$[\boldsymbol{A} \mid \boldsymbol{I}] \qquad [\boldsymbol{I} \mid \boldsymbol{A}^{-1}]$$

$$AX = B \qquad X = A^{-1}B$$

$$[\boldsymbol{A}, \boldsymbol{B}] \qquad [\boldsymbol{I}_n, \boldsymbol{A}^{-1}\boldsymbol{B}]$$

- **Adjoint matrix adj(A)**

$$\boldsymbol{A}^{-1} = \frac{\mathrm{adj}[\boldsymbol{A}]}{\det[\boldsymbol{A}]},$$

Preliminaries

- **Diagonal matrix diag(A)**

$$\boldsymbol{\Lambda} = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix}.$$

- **Eigenvalues of matrix eig(A)**

$$\boldsymbol{A}\boldsymbol{x} = \lambda \boldsymbol{x}$$

λ is one of the eigenvalues, x is eigenvector according to λ

$|\lambda I - A| = 0$

Left side of the equation is called eigen-polynomial

$$\mathrm{tr}[\boldsymbol{A}] := \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + \cdots + a_{nn} = \lambda_1[\boldsymbol{A}] + \lambda_2[\boldsymbol{A}] + \cdots + \lambda_n[\boldsymbol{A}]$$

Preliminaries

- **Symmetric Matrix** $\boldsymbol{A} = [a_{ij}] \in \mathbb{F}^{n \times n}$

$$a_{ij} = a_{ji},\ i = 1, 2, \cdots, n,\ j = 1, 2, \cdots, n.$$

Obviously, $\boldsymbol{A}^{\mathrm{T}} = \boldsymbol{A}$.

- **Positive Definite Matrix:**

For symmetric matrix A $\boldsymbol{A} = [a_{ij}] \in \mathbb{R}^{n \times n}$

eig(A)>0 **or** $\boldsymbol{x}^{\mathrm{T}} \boldsymbol{A} \boldsymbol{x} > 0$

Positive definite Matrix

- **Orthogonal Matrix:** $\boldsymbol{Q}^{-1} = \boldsymbol{Q}^{\mathrm{T}}, \quad \boldsymbol{Q}\boldsymbol{Q}^{\mathrm{T}} = \boldsymbol{I}$

$$\det[\boldsymbol{Q}] = \pm 1$$