§ 9.3　常见的简谐振动
单摆和复摆

一、简谐振动的判断

满足下列条件之一的振动即为简谐振动:

- ▲ $x = A\cos(\omega t + \varphi)$
- ▲ $F = -kx$
- ▲ $\ddot{x} + \omega^2 x = 0$

F 为合外力，x 为离开平衡位置的位移，k 为常数。

二、常见的简谐振动

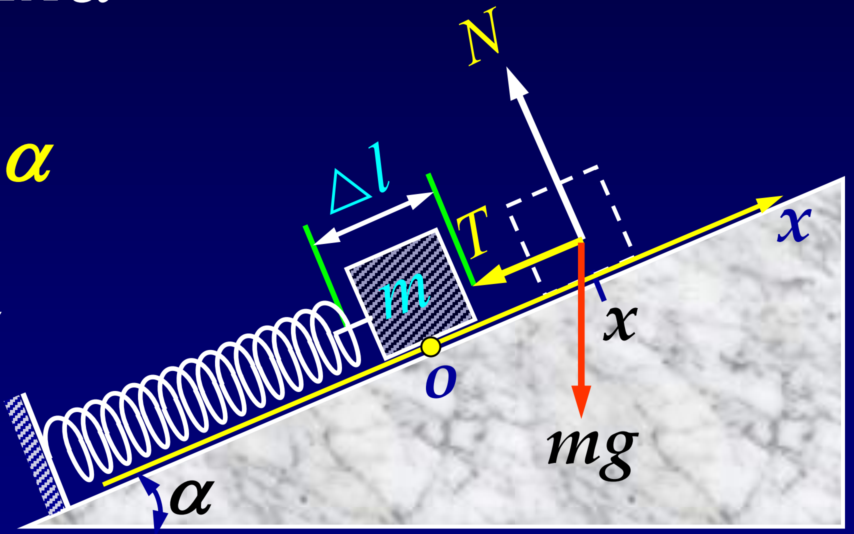
1. 光滑斜面上的弹簧振子：

平衡位置 o 处： $k\cdot\Delta l = mg\cdot\sin\alpha$

在 x 处：合力 $F = T - mg\cdot\sin\alpha$

$$T = -k(x-\Delta l) = -kx + mg\cdot\sin\alpha$$

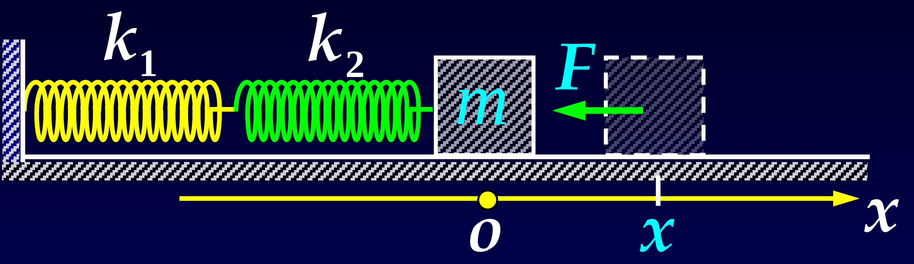
$$\therefore\quad F = -kx$$



即作简谐运动： $\omega = \sqrt{\dfrac{k}{m}}$， $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{k}}$

2. 光滑平面上的复合弹簧:

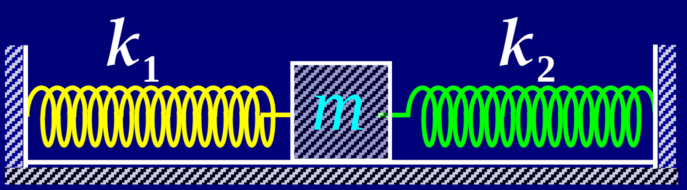
$$\begin{cases} x_1 + x_2 = x \\ k_1 x_1 = k_2 x_2 \end{cases}$$



$$F = -k_2 x_2 = -\frac{k_1 k_2}{k_1 + k_2} x = -kx \qquad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_1 k_2}{m(k_1 + k_2)}}$$

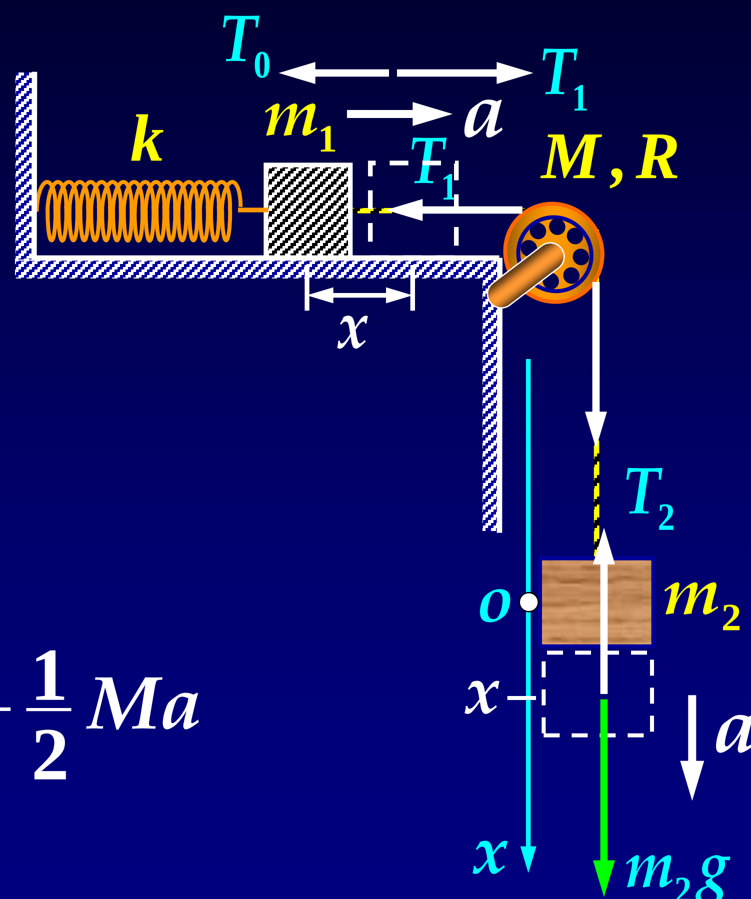
试证明下图中系统角频率为:

$$\omega = \sqrt{\frac{k_1 + k_2}{m}} \quad ?$$



例 如图，光滑桌面，已知：m_1、m_2、M 及 k，轻绳轻弹簧，证明系统运动为简谐振动，并求角频率。

解 考察 m_2 的运动规律。



平衡时：$k\Delta l = m_2 g$

位于 x 处：m_2所受的合力

$$F = m_2 g - T_2 = m_2 a$$

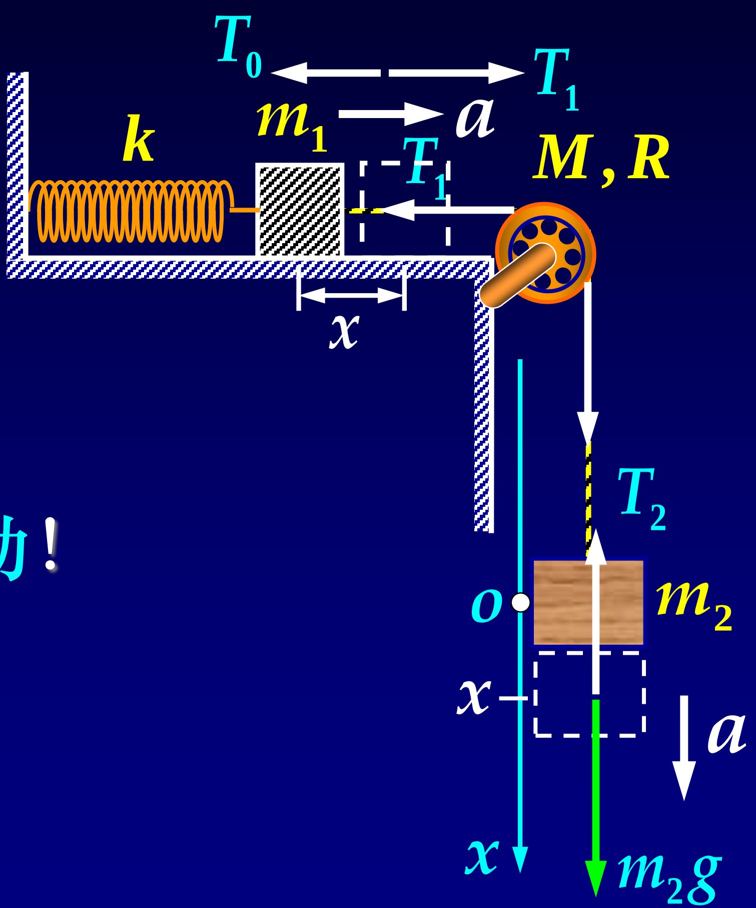
$$J = \frac{1}{2} MR^2$$

$$T_2 R - T_1 R = J\alpha = J\frac{a}{R}, \quad T_2 = T_1 + \frac{1}{2} Ma$$

$$T_1 - T_0 = m_1 a, \quad T_0 = k(x + \Delta l)$$

$$F = m_2 g - k(x + \Delta l) - m_1 a - \frac{1}{2} M a = m_2 a$$

代入 $k\Delta l = m_2 g$



$$a + \frac{k}{m_1 + m_2 + \frac{M}{2}} x = 0$$

$\ddot{x} + \omega^2 x = 0$ 即系统作简谐振动！

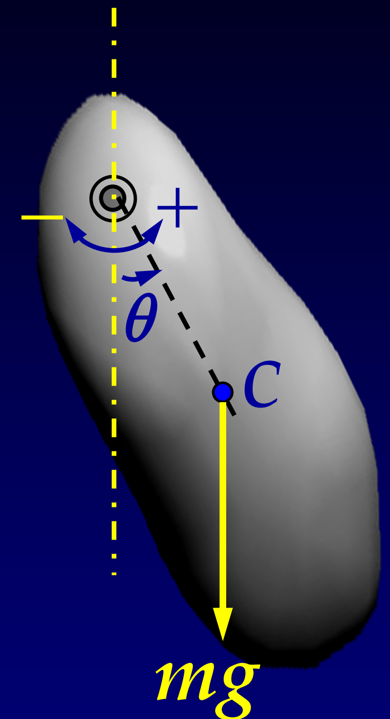
$$\omega = \sqrt{k \Big/ \left(m_1 + m_2 + \frac{M}{2}\right)}$$

(the end)

3. 复摆与单摆:

设 复摆作小角度摆动。

$$M = J\alpha = J\frac{d^2\theta}{dt^2} = J\ddot{\theta}$$（M 为重力矩）



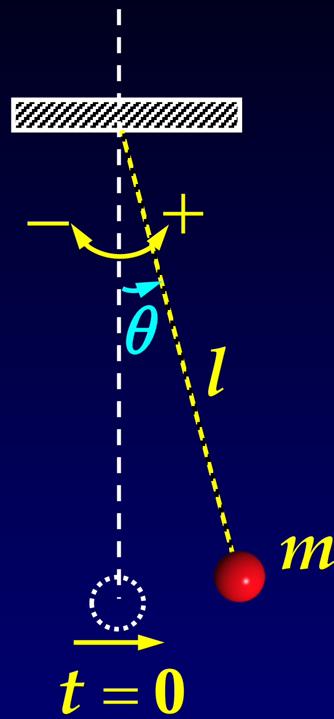
$$M = -mgl \cdot \sin\theta \approx -mgl\theta$$

$$\longrightarrow \quad \ddot{\theta} + \frac{mgl}{J}\theta = 0 \qquad \ddot{\theta} + \omega^2\theta = 0$$

$$\theta = \theta_0 \cos(\omega t + \varphi)$$

$$\boxed{\omega = \sqrt{\frac{mgl}{J}}, T = 2\pi\sqrt{\frac{J}{mgl}}}$$

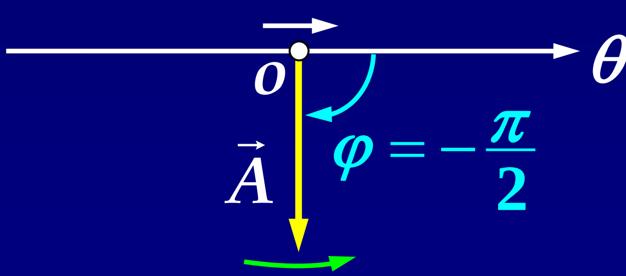
对作**小角度**摆动的单摆： $J = ml^2$



$$\omega = \sqrt{\frac{g}{l}}, \quad T = 2\pi\sqrt{\frac{l}{g}}$$

$$\theta = \theta_0 \cos(\omega t + \varphi)$$

例 若单摆最大摆角为5°，起始状态如图，则由旋转矢量图可知：



$$\varphi = -\frac{\pi}{2}, \quad \theta = \frac{\pi}{36}\cos\left(\sqrt{\frac{g}{l}}\,t - \frac{\pi}{2}\right)$$

归纳

1. 简谐振动满足：

▲ $x = A\cos(\omega t + \varphi)$

▲ $F = -kx$

▲ $\ddot{x} + \omega^2 x = 0$

*2. 阻尼振动的三种状态：

阻尼、临界阻尼、过阻尼

（请看录像）