FEKETE
High Order Interpolation and Quadrature in Triangles

FEKETE is a C++ library which can return information defining any of seven Fekete rules for high order interpolation and quadrature in a triangle.

Fekete points can be defined for any region OMEGA. To define the Fekete points for a given region, let Poly(N) be some finite dimensional vector space of polynomials, such as all polynomials of degree less than L, or all polynomials whose monomial terms have total degree less than some value L.

Let P(1:M) be any basis for Poly(N). For this basis, the Fekete points are defined as those points Z(1:M) which maximize the determinant of the corresponding Vandermonde matrix:

```
V = [ P1(Z1)  P1(Z2)  ... P1(ZM) ]
    [ P2(Z1)  P2(Z2)  ... P2(ZM) ]
    ...
    [ PM(ZM)  P2(ZM)  ... PM(ZM) ]
```

The seven rules have the following orders and precisions:

Rule	Order	Precision
1	10	3
2	28	6
3	55	9
4	91	12
5	91	12
6	136	15
7	190	18

On the triangle, it is known that some Fekete points will lie on the boundary, and that on each side of the triangle, these points will correspond to a set of Gauss-Lobatto points.

Licensing:

Languages:

FEKETE is available in a C++ version and a FORTRAN90 version and a MATLAB version.

Related Data and Programs:

DUNAVANT is a C++ library which defines Dunavant rules for quadrature on a triangle.

FELIPPA is a C++ library which defines quadrature rules for lines, triangles, quadrilaterals, pyramids, wedges, tetrahedrons and hexahedrons.

GM_RULES is a C++ library which defines a Grundmann-Moeller rule for quadrature over a triangle, tetrahedron, or general M-dimensional simplex.

KEAST is a C++ library which defines a number of quadrature rules for a tetrahedron.

LYNESS_RULE is a C++ library which returns Lyness-Jespersen quadrature rules for the triangle.

NCC_TRIANGLE is a C++ library which defines Newton-Cotes closed quadrature rules on a triangle.

NCO_TRIANGLE is a C++ library which defines Newton-Cotes open quadrature rules on a triangle.

STROUD is a C++ library which contains quadrature rules for a variety of unusual areas, surfaces and volumes in 2D, 3D and M-dimensions.

TEST_TRI_INT is a FORTRAN90 library which tests algorithms for quadrature over a triangle.

TOMS612 is a FORTRAN77 library which can estimate the integral of a function over a triangle.

TOMS706 is a FORTRAN77 library which estimates the integral of a function over a triangulated region.

TRIANGLE_MONTE_CARLO, a C++ program which uses the Monte Carlo method to estimate integrals over a triangle.

WANDZURA is a C++ library of routines for defining Wandzura rules for quadrature on a triangle.

Reference:

1. SF Bockman,
   Generalizing the Formula for Areas of Polygons to Moments,
   American Mathematical Society Monthly,
   Volume 96, Number 2, February 1989, pages 131-132.
2. Hermann Engels,
   Numerical Quadrature and Cubature,
   Academic Press, 1980,
   ISBN: 012238850X,
   LC: QA299.3E5.
3. Arthur Stroud,
   Approximate Calculation of Multiple Integrals,
   Prentice Hall, 1971,
   ISBN: 0130438936,
   LC: QA311.S85.
4. Mark Taylor, Beth Wingate, Rachel Vincent,
   An Algorithm for Computing Fekete Points in the Triangle,
   SIAM Journal on Numerical Analysis,
   Volume 38, Number 5, 2000, pages 1707-1720.
5. Stephen Wandzura, Hong Xiao,
   Symmetric Quadrature Rules on a Triangle,
   Computers and Mathematics with Applications,
   Volume 45, 2003, pages 1829-1840.

Source Code:

- fekete.C, the source code.
- fekete.H, the include code.
- fekete.csh, commands to compile the source code.

Examples and Tests:

- fekete_prb.C, a sample calling program.
- fekete_prb.csh, commands to compile and run the sample program.
- fekete_prb_output.txt, the output file.

One of the tests in the sample calling program creates EPS files of the abscissas in the unit triangle. These have been converted to PNG files for display here.

- fekete_rule_1.png, a plot of rule 1.
- fekete_rule_2.png, a plot of rule 2.
- fekete_rule_3.png, a plot of rule 3.
- fekete_rule_4.png, a plot of rule 4.
- fekete_rule_5.png, a plot of rule 5.
- fekete_rule_6.png, a plot of rule 6.
- fekete_rule_7.png, a plot of rule 7.

List of Routines:

- **FEKETE_DEGREE** returns the degree of a given Fekete rule for the triangle.
- **FEKETE_ORDER_NUM** returns the order of a given Fekete rule for the triangle.
- **FEKETE_RULE** returns the points and weights of a Fekete rule.
- **FEKETE_RULE_NUM** returns the number of Fekete rules available.
- **FEKETE_SUBORDER** returns the suborders for a Fekete rule.
- **FEKETE_SUBORDER_NUM** returns the number of suborders for a Fekete rule.
- **FEKETE_SUBRULE** returns a compressed Fekete rule.
- **FEKETE_SUBRULE_1** returns a compressed Fekete rule 1.
- **FEKETE_SUBRULE_2** returns a compressed Fekete rule 2.
- **FEKETE_SUBRULE_3** returns a compressed Fekete rule 3.
- **FEKETE_SUBRULE_4** returns a compressed Fekete rule 4.
- **FEKETE_SUBRULE_5** returns a compressed Fekete rule 5.
- **FEKETE_SUBRULE_6** returns a compressed Fekete rule 6.
- **FEKETE_SUBRULE_7** returns a compressed Fekete rule 7.
- **FILE_NAME_INC** increments a partially numeric file name.
- **I4_MAX** returns the maximum of two integers.
- **I4_MIN** returns the smaller of two integers.
- **I4_MODP** returns the nonnegative remainder of integer division.
- **I4_WRAP** forces an integer to lie between given limits by wrapping.
- **R8_HUGE** returns a "huge" R8.
- **R8_NINT** returns the nearest integer to an R8.
- **REFERENCE_TO_PHYSICAL_T3** maps T3 reference points to physical points.
- **S_LEN_TRIM** returns the length of a string to the last nonblank.
- **TIMESTAMP** prints the current YMDHMS date as a time stamp.
- **TIMESTRING** returns the current YMDHMS date as a string.
- **TRIANGLE_AREA** computes the area of a triangle.
- **TRIANGLE_POINTS_PLOT** plots a triangle and some points.

You can go up one level to the C++ source codes.

Last revised on 12 December 2006.