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This article presents methodologies for constructing and examining equity factor portfolios, which are illustrated using the monthly returns and factor exposures on 1,000 large U.S. stocks for the 50 years ending 2016. Specifically, we construct primary and pure portfolios for five well-known factors: value, momentum, small size, low beta, and profitability. We add a sixth *bond beta* factor to examine the dynamic nature of interest rate risk on the equity market and on the various factor portfolios. The intent of this case study is not to argue for another factor in equity market analysis, but rather to illustrate how interest rate exposure can be compared to the better-known factor exposures. Similarly, we employ the value factor, as measured by the inverse price/earnings (P/E), to examine the "cheapness" of the various factor portfolios over time and compared to each other, but we emphasize that value is only one of several possible equity market factors.

The methodological contribution of this article begins with primary factor portfolios constructed using a security-weighting formula that incorporates both market capitalization and standardized factor exposure. The security-weighting formula is consistent with the role of the capitalization-weighted portfolio as the benchmark and the use of Fama and MacBeth [1973] cross-sectional regressions as an empirical investigation tool. When the regressions are multivariate (i.e., security returns are regressed on several exposures simultaneously), the slope coefficients equate to implementable analytic formulas for the security weights of factor portfolios that have market-relative exposures of zero to the other factors. Tying factor portfolio return calculations to analytically derived security weights avoids the black-box nature of backtests based on single and double sorts with quintile cutoffs and other ad hoc portfolio construction criteria. The concept of equating regression slope coefficients with investable portfolios is discussed in Clarke, de Silva, and Thorley [2014], with further extensions in the online Appendix A of this article at www.iijpm.com.

Numerous studies in both academic and practitioner literature document the performance of one or more equity market factors but include uncontrolled and often substantial exposures to secondary factors. Fama and French [1996] established a process for controlling the exposure of a factor portfolio with a second factor: The small size factor portfolio SMB is created by a secondary sort on value exposures, and the value factor portfolio HML is created by a secondary sort on size exposures. Two-way sorting, typically with quintile delimiters that lead to 25 portfolios, is also commonly used, despite the inherent lumpiness of that process and the impracticality of extensions to three (i.e., 125 portfolios) or four (i.e., 625 portfolios) simultaneous factors.

For example, a recent academic paper by Novy-Marx [2016] inaccurately attributes much of the low-beta-factor performance to the profitability factor using a double-sort quintile portfolio methodology. Using less robust methodologies, Greenline Partners [2016] attributes the performance of the low-beta factor to interest rate exposures, whereas Arnott, Beck, and Kalesnik [2016] argue that the value factor explains almost everything. Looking at equity portfolios in terms of valuation levels is common practice—whether to evaluate the overall level of the market, the valuation level of a particular sector, or the attractiveness of an individual portfolio. When evaluating a low-beta strategy, for example, investors often compare the valuation level of the portfolio—measured in terms of earnings yield—to determine whether the strategy is attractive relative to other available opportunities. But value as measured by any simple ratio is only one of several factors that have historically generated positive returns.

In this study, we confirm that, during 2016, the primary factor portfolio for low beta was slightly expensive as measured by its market-relative value factor exposure. However, the wide selection of securities within the large-cap U.S. equity market allows investors seeking exposure to the low-beta factor to construct portfolios that are neutral to the value factor. We show that not only does the long-term performance of the various equity market factors not go away when constructing pure factor portfolios, but factor portfolio information ratios are actually enhanced by eliminating the secondary factor exposures.

The empirical contributions of this study include the documentation of a structural shift in the stock–bond return correlation, from positive values before the turn of the century to persistently negative thereafter. If the current sign of the correlation holds going forward, unanticipated increases in interest rates will be associated with positive rather than negative returns in the stock market. In particular, as of early 2017, the primary low-beta factor portfolio is exposed to interest rate sensitivity that is lower than the benchmark because of a large negative correlation between traditional equity market beta and bond beta. On the other hand, the portfolio construction methodologies in this article show ample opportunity within the large-cap U.S. equity investable set to form pure low-beta portfolios that have the same interest rate sensitivity as the general equity market.

FACTOR PORTFOLIO CONSTRUCTION

We use the term *primary* to designate a factor portfolio with weights that deviate from market weights based on just one factor. The primary factor portfolios have secondary exposures to the other factors under examination, in contrast to the "pure" factor portfolios we describe later. Our primary factor portfolios are constructed using the simple security weight formula

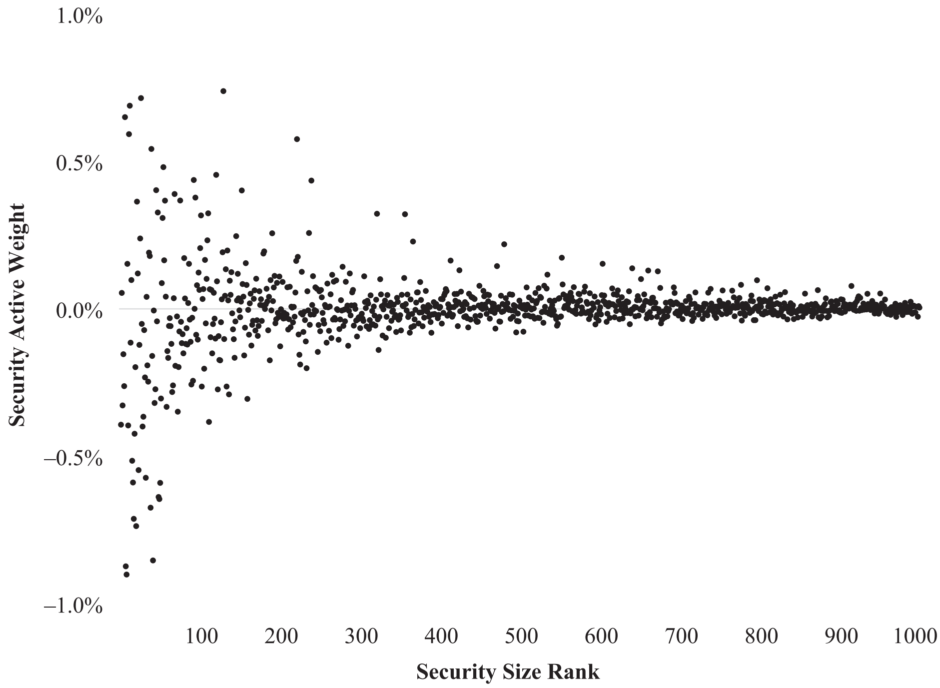
$$w_{Pi} = w_{Mi}\,(1 + b_i), \tag{1}$$

where w_{Pi} and w_{Mi} are the security i factor portfolio and market portfolio weights, respectively, and b_i is the standardized exposure of security i to the factor of interest. As explained in the online Appendix A, raw factor exposures—for example, the inverse P/E for the value factor—are scaled each period to have a cross-sectional capitalization-weighted variance of one and a cross-sectional capitalization-weighted mean of zero.

Exposure standardization ensures that the security weights in Equation 1 deviate from market weights based solely on the factor exposure of interest and that the weights sum to 100%. Specifically, a security with a negative standardized factor exposure, $b_i < 0$, leads to a benchmark "underweight" position in Equation 1, whereas positive factor exposure, $b_i > 0$, leads to a benchmark "overweight" position. Unit-variance scaling of the exposures allows for securities with large negative exposures, $b_i < -1$, to have negative weights, but the associated short-sell positions are generally insignificant in terms of their collective impact on portfolio composition.

More importantly, the *active* weights (i.e., differences between the primary factor portfolio and market portfolio weights) from Equation 1 are larger for larger-cap stocks and smaller for smaller-cap stocks. For example, Exhibit 1 shows the active weights for 1,000 stocks for a typical factor portfolio in 2017, with the stocks sorted by descending market capitalization. Market weights start as high as 3.0% for the largest stock (Apple Computer) but decline to less than 0.1% for the 200th stock. As shown in Exhibit 1, the active portfolio weights are larger for larger stocks and the

EXHIBIT 1
Factor Portfolio Active Weights for 1,000 Stocks, 2017



absolute value of active (i.e., market differential) weights declines with market capitalization. The resulting factor portfolios are inherently more investable than other factor portfolio construction methodologies because trading activity (i.e., rebalancing) corresponds to market capitalization.

The returns on a factor portfolio defined by Equation 1 are consistent with Fama and MacBeth [1973] regression analysis, which has a long history in terms of empirical research on factor performance. Specifically, the direct calculation of the realized portfolio return using security weights, w_{Pi}, and security returns, r_i, is

$$r_P = \sum_{i=1}^{N} w_{Pi}\ r_i. \tag{2}$$

As shown in the online Appendix A, the portfolio return in Equation 2 is equal to the intercept term (i.e., the market return) plus the slope coefficient (i.e., *active* portfolio return) in a weighted regression of security returns on standardized factor exposures, b_i.

Multivariate analysis of security returns on *several* sets of standardized factors, generically labeled as Factor 1, Factor 2, etc., reduces to the regression equation

$$r_i = r_M + (r_1 - r_M)\,b1_i + (r_2 - r_M)\,b2_i + \ldots + \varepsilon_i, \tag{3}$$

where the intercept term is the capitalization-weighted benchmark portfolio return, r_M. As shown in the online Appendix A, each of the slope coefficients in Equation 3 is the one-period return to a pure factor portfolio minus the benchmark return, $r_P - r_M$, where the pure factor portfolios are labeled P = 1, 2, etc., and the factor exposures take the position of the "X" or independent variables in standard regression notation. Note that while regression coefficients are typically interpreted as estimates of some underlying population parameter, the coefficients in Equation 3 are exact active portfolio returns.

HISTORICAL PERFORMANCE OF PRIMARY FACTOR PORTFOLIOS

The first five factors considered in this study are a well-established set of equity market factors in academic research, including some of the most widely followed factors in active portfolio management. We also introduce a sixth factor to capture interest rate sensitivity. The names and specific factor exposure definitions are as follows:

- *Value* is the inverse P/E or *earnings yield* using the beginning-of-month stock price (from the Center for Research in Security Prices) and earnings per share, with a one-quarter lag on the most recent annual income statement (from Compustat).
- *Momentum* is the 11-month stock return, with dividends, lagged by one month, sometimes called Carhart [1997] momentum. The momentum factor in this study refers to the price momentum of the stock, first identified as a factor by Jegadeesh and Titman [1993], not the earnings momentum of the corporation.
- *Small Size* is the log of 1 over beginning-of-month market capitalization (i.e., negative log market capitalization), as used in Fama and French [1992]. This factor measures smallness within the investable set of the largest 1,000 U.S. common stocks (approximately the Russell 1000 Index), not the exposure to a separate small-capitalization index such as the Russell 2000.
- *Low Beta* is −1 times the time-series regression coefficient of the stock return on the S&P 500 return, using the most recent 36 months. Low beta is a simple sign switch on the Sharpe [1964] CAPM beta, but beta calculations are used in this study to measure market return sensitivity, not expected returns associated with any particular equilibrium asset-pricing model. Low beta is a more precise manifestation of the low-volatility anomaly discussed in Clarke, de Silva, and Thorley [2010] and popularized in Frazzini and Pedersen [2014].
- *Profitability* is the gross profit margin, defined as revenues minus cost of goods sold from the most recent annual income statement, divided by total assets from the most recent annual balance sheet. We use the accounting numbers with a one-quarter lag to ensure the data would have been available to investors historically. This factor is often called Novy–Marx [2013] Profitability but is also referred to as a quality factor by some investors.
- *Bond Beta* is the beta coefficient from a time-series regression of monthly stock returns on 10-year Treasury bond returns. Bond Beta measures sensitivity to bond market returns, analogous to the equity market beta. Unlike equity market betas, which consistently average about one across stocks, the market-wide average bond beta may be positive or negative at any point in time depending on the realized correlation of overall stock and bond market returns over the prior 36 months.

We begin in Exhibit 2 by reporting the 50-year (1967 to 2016) performance of the market portfolio and primary factor portfolios, constructed from the largest 1,000 U.S. common stocks. The primary factor portfolios are designed to establish constant exposure to the primary factor of interest but also have secondary tilts to other factors. The factor portfolio performance in Exhibit 2 emphasizes annualized returns and risks that are incremental to the market portfolio. In the first column, the market portfolio excess (net of the risk-free rate) mean return is 6.29%, with a standard deviation of 15.27%, giving a realized Sharpe ratio of 0.41.

EXHIBIT 2
Market-Relative Returns of Primary Factor Portfolios, 1967–2016

		Market Relative					
	Market	**Value**	**Momentum**	**Small Size**	**Low Beta**	**Profitability**	**Bond Beta**
Mean Return	6.29%	1.12%	2.47%	1.28%	0.06%	0.91%	0.03%
Std. Dev.	15.27%	4.57%	6.20%	4.36%	6.44%	3.99%	4.63%
Sharpe Ratio	0.41						
Market Beta	1.00	–0.08	–0.01	0.08	–0.27	–0.01	–0.07
Alpha	0.00%	1.64%	2.52%	0.77%	1.74%	1.00%	0.48%
Active Risk		4.40%	6.21%	4.18%	4.98%	3.99%	4.50%
Info. Ratio		0.37	0.41	0.18	0.35	0.25	0.11

The mean returns in each of the next columns of Exhibit 2 are the average differences in return between the factor portfolios and the market portfolios, with risk measured by the annualized standard deviation of the differential returns. For example, the total excess (of risk-free rate) return on the value portfolio, including the market return, is $6.29 + 1.12 = 7.41\%$, and the total excess return on the momentum portfolio is $6.29 + 2.47 = 8.76\%$. Ignoring the nonzero incremental market betas of the various factor returns (which we discuss next), one can approximate the total risk of the value portfolio as $(0.1527^2 + 0.0457^2)^{1/2} = 15.94\%$, and the total risk of the momentum portfolio as $(0.1527^2 + 0.0620^2)^{1/2} = 16.48\%$.

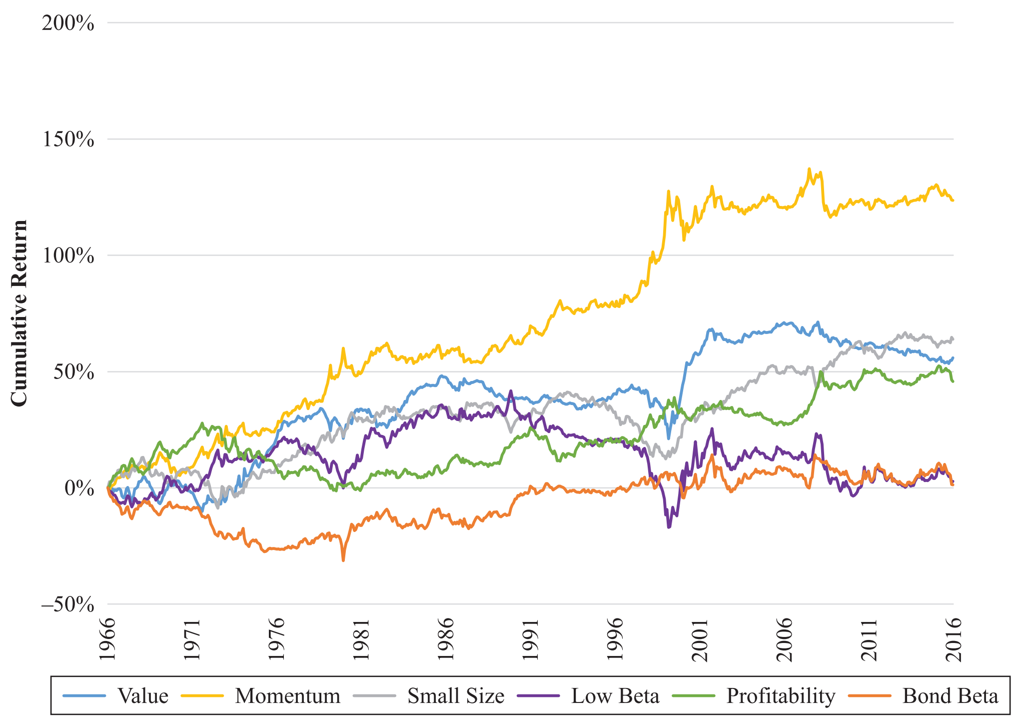
The market betas reported in the lower half of Exhibit 2 are from a time-series regression of the monthly factor portfolio market-relative returns on market returns over the entire 50-year period. For brevity, we do not include the statistical significance numbers associated with the time-series regressions, but the incremental betas are not significantly different from zero, except for the low-beta portfolio at −0.27. The total realized market beta for the low-beta portfolio is thus about $1.00 - 0.27 = 0.73$, whereas the other portfolios have total market betas of about 1.00. The alpha row in Exhibit 2 reports the annualized return associated with the intercept term in the single 50-year time-series regression; active risk has values associated with the regression residuals, and the information ratio is the quotient of alpha over active risk.

From the perspective of the realized information ratios reported at the bottom of Exhibit 2, the momentum portfolio has the highest market-relative performance at 0.41, followed by the value portfolio at 0.37, the low-beta portfolio at 0.35, and profitability at 0.25. We also note that the small size portfolio in this study only captures the advantage of smaller capitalization within what is traditionally defined as the large-capitalization market.

To provide a visual perspective on the track record of the primary factor portfolios, Exhibit 3 plots the cumulative market-differential returns through the end of 2016, with a year-end 1966 starting value of zero. For example, the outstanding performance of the momentum portfolio flattens out after the turn-of-the-century tech bubble, which was simultaneously

EXHIBIT 3

Cumulative Market-Relative Returns of Primary Factor Portfolios, 1967–2016



marked by large drawdowns in the value and low-beta portfolios. Alternatively, the performance of the profitability portfolio appears to be more slow and steady, at least since the 1980s. More to a point made later in this study, Exhibit 3 provides a visual perspective that the market-differential returns of the factor portfolios are at times highly correlated. For example, the cumulative returns for the low-beta and bond beta portfolios jumped up simultaneously during the 2008 financial crisis, while the momentum portfolio cumulative return dropped during 2008.

The visual impression of factor return correlations in Exhibit 3 are manifest in the correlation coefficient matrix reported in Exhibit 4. For example, reading from left to right in the first row, the primary value portfolio's market-relative returns are negatively correlated with momentum returns at −0.43, positively correlated with low-beta returns at 0.45, and negatively correlated with profitability returns at −0.49. Alternatively, reading from top to bottom in the final column of Exhibit 4, the primary factor portfolios with the largest positive correlations to bond returns appear to be low beta, with a correlation coefficient of 0.36, and value, with a correlation coefficient of 0.21. We note again that the plots of cumulative returns in Exhibit 3 and the correlation coefficients in Exhibit 4 are for market-*differential* returns. The total returns of all six factor portfolios (i.e., with the embedded market returns) have large positive correlations to each other.

TIME DYNAMICS OF FACTOR CORRELATIONS AND EXPOSURES

The correlation coefficients over the entire 50-year period shown in Exhibit 4 mask the fact that portfolio return correlations are dynamic over time. For example, over the last 10 years (i.e., 2007 to 2016) the near-zero −0.04 correlation reported in Exhibit 4 between the profitability and low-beta-factor portfolios has been a large positive value of 0.59. Over that decade, the correlation between the profitability and value portfolios. reported as −0.49 in Exhibit 4, was essentially zero. Under the simplifying assumption of a zero null hypothesis, the standard error of a correlation coefficient is the inverse of the square root of the number of observations—in this case 120 months, or 1/SQRT(120) = 0.09. Changes in the realized correlation coefficient on the order of five times the magnitude of the standard error are very unlikely in a statistical sense if the "true" underlying parameter values are in fact stable. The underlying composition of the various portfolios and the correlations of returns between the factor portfolios clearly change over time.

Although this study reports the returns and exposures of several better-known equity market factors, we focus on two issues currently of interest to factor-oriented investors: the "cheap" versus "expensive" perspective on factor portfolios as measured by their inverse P/E (i.e., value factor exposure) and the interest rate sensitivity of the various factor portfolios. First, consider the raw value exposure of the market portfolio over the last 50 years, as plotted in Exhibit 5. Each monthly observation is the capitalization-weighted average of the trailing EPS over share price for the largest 1,000 U.S. stocks. Because of the capitalization weighting, the values plotted in Exhibit 5 are equivalent to the earnings yield of the market as a whole, defined as the sum of the reported net income for the 1,000 largest stocks divided by their collective capitalization.

For example, at an earnings yield of 7%, a rough approximation of the long-term average in Exhibit 5,

EXHIBIT 4

Correlations of Primary Factor Portfolio Market-Relative Returns, 1967–2016

	Correlation Matrix					
	Value	Momentum	Small Size	Low Beta	Profitability	Bond Beta
Value	1.00	–0.43	0.18	0.45	–0.49	0.21
Momentum	–0.43	1.00	–0.13	0.01	0.24	0.07
Small Size	0.18	–0.13	1.00	–0.30	–0.31	–0.11
Low Beta	0.45	0.01	–0.30	1.00	–0.04	0.36
Profitability	–0.49	0.24	–0.31	–0.04	1.00	0.06
Bond Beta	0.21	0.07	–0.11	0.36	0.06	1.00

the marketwide trailing P/E is 1/.07 = 14.3. The market was cheap during most of the 1970s, but at about 4% yield (i.e., P/E of 25.0) the market looks expensive at the end of 2016. Of course, there may be basic valuation explanations behind these secular trends, including the level of interest rates, changes in expected corporate earnings growth, and better tools for measuring long-term earnings (i.e., variants on the Shiller cyclically adjusted price/earnings [CAPE]). We show Exhibit 5 to provide a composite measure of the broad market's value exposure to inform the subsequent examination of differential value exposures of the various factor portfolios.

With a perspective on how the market portfolio's value factor exposure has changed over time, we now examine the market-relative exposure to value of the various factor portfolios. Exhibit 6 plots the standardized value exposure at the beginning of each month for each of the six factor portfolios. Because the factor exposures are market relative and standardized, the units on the vertical axis in Exhibit 6 are "scores" expressed as standard deviation units. For example, the standardized value portfolio exposure is by construction exactly 1.0 over time, and the market portfolio value exposure is exactly 0. Exhibit 6 shows that the profitability portfolio's value exposure was about −0.4 during the 1960s and 1970s, meaning that profitability stocks tended to have lower earnings yield and be more expensive than the market. Alternatively, the small portfolio's value exposure was high, at about 0.4 in the 1960s and 1970s, meaning that small-cap stocks had higher earnings yield and were thus relatively cheap compared to the market.

Note that five of the factor portfolios (i.e., all except for value) dipped slightly below zero during 2016, as shown in the far right end of Exhibit 6, although some came back into positive territory toward the end of the year. All five of these factor portfolios were slightly expensive during 2016 on a relative P/E basis. But based on the long-term perspective in Exhibit 6, the value exposures of the other factor portfolios were relatively close to the market as a whole in 2016. For example, the profitability factor portfolio historically plotted in the lower half of Exhibit 6, whereas other primary portfolios such as low beta have generally plotted in the upper half.

The value factor as discussed in academic literature has often been defined by the book-to-market ratio, following the Fama–French three-factor model.

EXHIBIT 5
Earnings Yield of the Market Portfolio, 1967–2016

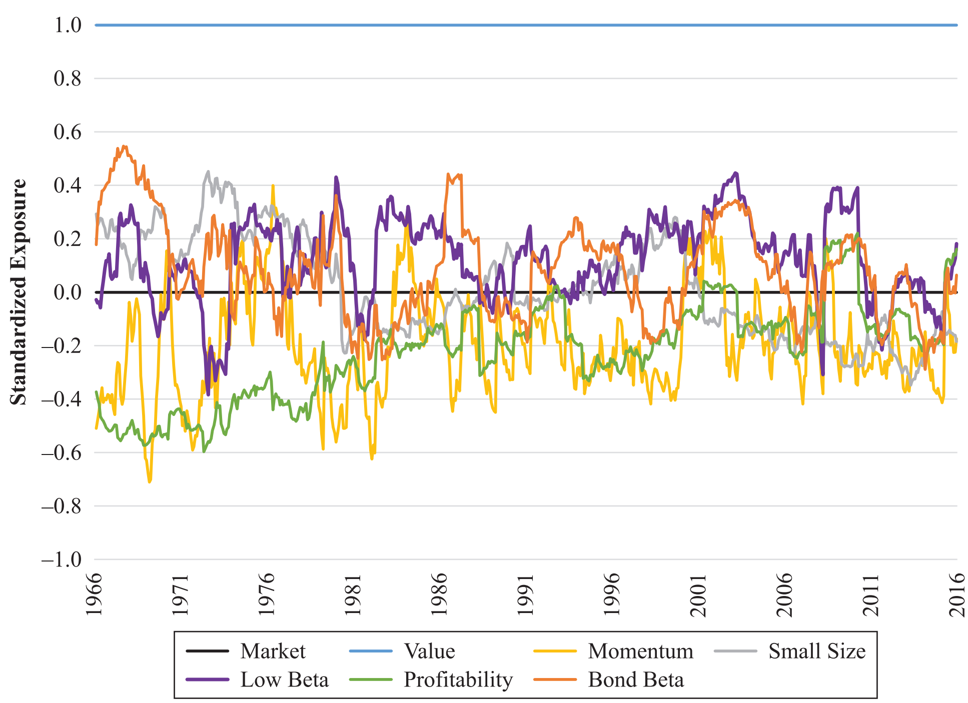
We use earnings yield in this study based on the general perception that this ratio is preferred to, and may in fact have replaced, the book-to-market as the key value indicator in investment management practice. We do not include both measures of value exposure in the multifactor analysis that follows because the two are highly correlated at each point in time, as are the realized active return correlations of the primary factor portfolios over time. For reference purposes, we note the 50-year book-to-market portfolio performance track record: annualized average active return of 0.99% (compared to 1.12% for earnings yield in Exhibit 2), market-differential return standard deviation of 4.85%, and information ratio of 0.23, well below the value factor (i.e., earnings yield) information ratio of 0.37 reported in Exhibit 2.

We next analyze the interest rate sensitivity of stock returns to illustrate how the portfolio construction methodologies in this article can inform factor-based investors. First, consider Exhibit 7, which plots the bond beta of the U.S. equity market portfolio over the last half-century. The values plotted for each month in Exhibit 7 are the capitalization-weighted averages of the 1,000 individual security bond betas, with each calculated by a trailing 36-month regression of security returns on 10-year Treasury bond returns. Because of market-cap weighting each month, the average bond beta plotted in Exhibit 7 is close to a regression of the market return on the returns to the 10-year Treasury bond, although not an exact match because the market-capitalization weights of the individual securities are not constant over the prior 36 months. The key takeaway from Exhibit 7 is that the bond beta of the equity market was generally positive roughly until the turn of the century—and it has been generally negative since then.

For example, in Exhibit 7, the interpretation of a bond beta of 0.5 is that a Treasury bond yield increase over a given month that corresponds with say a −2.0% negative bond return was associated with a −1.0% negative (i.e., same sign) return in the stock market. Note that the relationship between Treasury bond yield changes and holding period returns is based on the financial mathematics of fixed-income securities. Specifically, a Treasury bond yield-to-maturity *increase* of 10 basis points during a given month corresponds with a holding period return of about *negative* 70 basis points, if the effective duration of the 10-year Treasury bond

E X H I B I T 6

Market-Relative Value Exposure of Primary Factor Portfolios, 1967–2016



is 7.0 years. On the other hand, the phrase "associated with" used in the first sentence of this paragraph refers to the empirically observed interest rate sensitivity of equity securities, which changes over time. In contrast to the positive bond beta of about +0.5 before the year 2000, the average value of about −0.5 after the turn of the century in Exhibit 7 indicates that a −2.0% bond return has more recently been associated with a positive (i.e., opposite sign) 1.0% return in the stock market.

A critical issue in portfolio theory and risk measurement is *how* two capital market returns correlate to each other. A conceptual explanation of *why* the correlation has changed over time would be helpful in evaluating the persistence of the historical correlation, but a review of economic theory on the underlying relationship between the stock and bond market returns is outside of the scope of this study. We note that our results on interest rate sensitivity measured by returns on the 10-year Treasury bond are generally similar to results for the 2-year Treasury bond, and when available for the 30-year Treasury bond. Variations in the term structure of interest rates over time (i.e., the shape of the yield curve) do not appear to be the explanation behind the change in the sign of the stock market bond beta plotted in Exhibit 7.

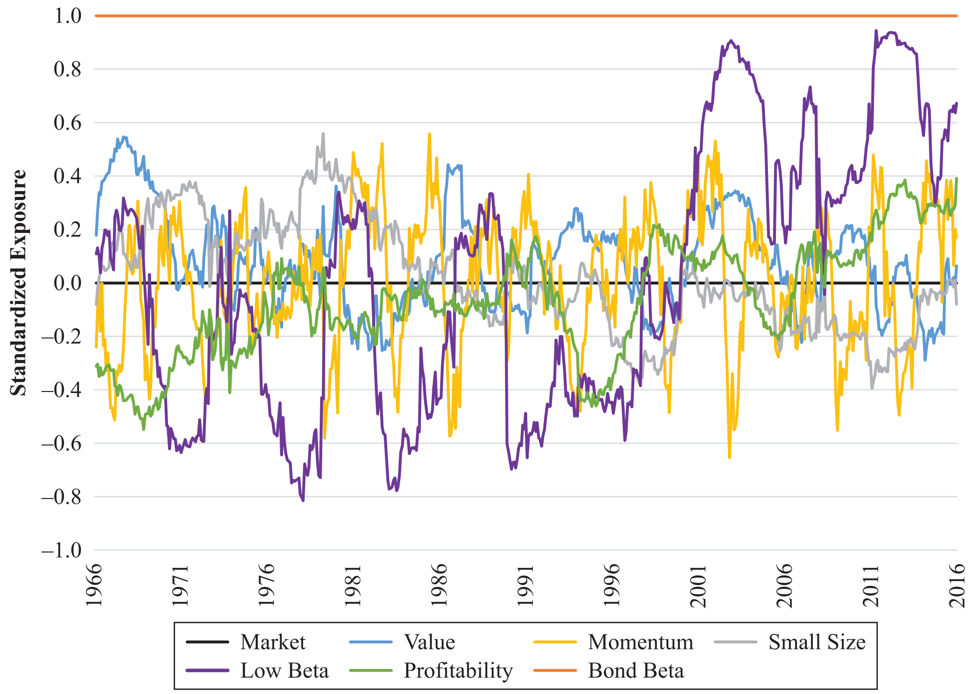
Recognizing that the bond beta for the broad equity market has changed over time, we examine the market-relative interest rate risk exposures of the various primary factor portfolios at each point in time. Exhibit 8 plots the standardized bond beta exposure of the six factor portfolios, similar to the market-relative value factor exposures in Exhibit 6. Like the value factor exposures, the vertical axis unit of measure in Exhibit 8 represents standardized scores, so the bond beta portfolio exposure is exactly 1.0 and the equity market portfolio is exactly 0. The value portfolio's standardized bond beta was about 0.5 in the late 1960s, meaning that value stocks tended to be more correlated with bond returns than the stock market as a whole. Similarly, the standardized bond beta exposure of the profitability portfolio is about 0.3 in recent years, meaning that the negative marketwide exposure shown in Exhibit 7 has been less negative in profitability stocks.

The most dramatic aspect of Exhibit 8 is the shift in bond beta exposures of the primary low-beta portfolio from negative before the turn of the century to positive

EXHIBIT 7
Bond Beta of the Market Portfolio, 1967–2016

EXHIBIT 8

Market-Relative Bond Beta of Primary Factor Portfolios, 1967–2016



over the last 15 years. In fact, in some recent years, the bond beta exposure of the primary low-beta portfolio has been close to 1.0, almost equal to the exposure of the primary bond beta portfolio, which by design is built from stocks with a bond-like reaction to changes in long-term interest rates. But with a 2016 market-relative exposure of about 0.6, the *total* bond beta exposure of the low-beta portfolio is slightly positive. In other words, although the returns of the equity market as a whole are now *negatively* correlated with bond returns, the primary low-beta portfolio still has a *positive* correlation to bond returns.

POINT-IN-TIME PORTFOLIO COMPOSITION AND EXPOSURES

In this section, we compare the composition of factor portfolios at the beginning of 1992 (i.e., 25 years ago) with the composition of factor portfolios at the beginning of 2017. Exhibit 9 reports summary statistics on the security weights and factor exposures of the market portfolio and six factor portfolios in 1992, whereas Exhibit 10 shows the same numbers for the market and six factor portfolios in 2017. We present these numbers to emphasize that a point-in-time analysis of factor *exposures*, rather than the primary factor portfolio *returns*, provides a more timely perspective on factor correlations. The comparisons in this section also illustrate how the factor composition of the broad equity market has changed over time in economically meaningful ways.

For example, the largest security weight (among 1,000 weights) in 1992 as reported in Exhibit 9 was 2.39% (Exxon Corporation), compared to 3.01% in 2017 (Apple Computer) as reported in Exhibit 10. The market portfolio at each point in time is by definition long-only, with 1,000 positive weights. In contrast, the six factor portfolios have some negative weights, in accordance with Equation 1. However, the sum of the negative weights is generally less than 10% of the total notional value of the factor portfolios, which can be described as "110/10" long/short extension portfolios. For example, in 1992 the value portfolio was 108.4% long and 8.4% short. For long-only investors, we note that eliminating the short positions and rescaling the weights to sum to 100% does not materially change the portfolio factor exposures.

The lower sections of Exhibits 9 and 10 report cross-sectional correlation coefficients between the

Exhibit 9
Primary Factor Portfolio Composition in 1992

	Market	Value	Momentum	Small Size	Low Beta	Profitability	Bond Beta
		Factor Portfolios					
Security Weights							
Sum	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Max	2.39%	4.02%	3.92%	0.29%	5.74%	5.04%	3.60%
Min	0.01%	–1.25%	–0.30%	–1.30%	–0.66%	–0.19%	–0.66%
Long	100.0%	108.4%	101.9%	107.0%	109.6%	103.7%	109.0%
Short	0.0%	–8.4%	–1.9%	–7.0%	–9.6%	–3.7%	–9.0%
Factor Exposures							
Market		1.000	1.000	1.000	1.000	1.000	1.000
Value		1.000	–0.281	–0.015	0.199	–0.183	–0.188
Momentum		–0.281	1.000	0.104	–0.341	0.186	0.263
Small Size		–0.015	0.104	1.000	–0.183	–0.254	–0.011
Low Beta		0.199	–0.341	–0.183	1.000	–0.047	–0.654
Profitability		–0.183	0.186	–0.254	–0.047	1.000	0.040
Bond Beta		–0.188	0.263	–0.011	–0.654	0.040	1.000

Exhibit 10
Primary Factor Portfolio Composition in 2017

	Market	Value	Momentum	Small Size	Low Beta	Profitability	Bond Beta
		Factor Portfolios					
Security Weights							
Sum	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Max	3.01%	4.89%	2.16%	0.30%	4.22%	4.75%	4.27%
Min	0.01%	–0.97%	–0.39%	–2.02%	–0.27%	–0.26%	–1.49%
Long	100.0%	109.5%	104.0%	105.7%	107.6%	104.4%	111.5%
Short	0.0%	–9.5%	–4.0%	–5.7%	–7.6%	–4.4%	–11.5%
Factor Exposures							
Market		1.000	1.000	1.000	1.000	1.000	1.000
Value		1.000	–0.289	–0.170	0.179	0.157	0.058
Momentum		–0.289	1.000	0.177	–0.228	–0.221	–0.310
Small Size		–0.170	0.177	1.000	–0.098	–0.036	–0.066
Low Beta		0.179	–0.228	–0.098	1.000	0.205	0.670
Profitability		0.157	–0.221	–0.036	0.205	1.000	0.388
Bond Beta		0.058	–0.310	–0.066	0.670	0.388	1.000

factor exposures in 1992 and 2017, respectively. The diagonal elements of the exposure matrix are by definition one, and the off-diagonal elements measure the exposure of the factor portfolio named at the top of each column to the factor named on the left-hand side of each row. Specifically, the exposure of a primary factor portfolio to a secondary factor is calculated using the security weights in Equation 1, with the primary factor exposures designated by $b1_i$ and secondary exposures designated by $b2_i$. The sum product of portfolio weights and factor exposures simplifies to the capitalization-weighted correlation coefficient

$$\rho = \sum_{i=1}^{N} w_{M\,i}\ b1_i\ b2_i. \tag{4}$$

Equation 4 is symmetric with respect to the primary and secondary factor exposures, so the same calculation applies if the factor exposures are reversed and the elements in Exhibits 9 and 10 are symmetric around the diagonal.

Exhibits 9 and 10 provide several examples of how the factor exposure correlations have changed over time. With respect to momentum, the profitability portfolio had an exposure of 0.186 in 1992 (see Exhibit 9) compared to an exposure of −0.221 in 2017 (see Exhibit 10). The stocks in the profitability portfolio had more momentum than the market as whole in 1992 but less momentum than the market as whole in 2017. We note that momentum exposure changes more quickly than other factors. Specifically, momentum is based on the returns over the prior year, whereas low-beta and bond beta exposures are time-series calculations that incorporate the most recent three years.

The lower sections of Exhibits 9 and 10 indicate that the most dramatic change between 1992 and 2017 is the low-beta portfolio's exposure to the bond beta factor. As plotted back in Exhibit 8, the bond beta exposure of the low-beta portfolio was −0.654 in 1992 (see Exhibit 9), compared to 0.670 in 2017 (see Exhibit 10). Low-beta stocks had less interest rate sensitivity than the market as a whole in 1992, and they still have much less interest rate sensitivity than the market as a whole in 2017, but the sign of the stock–bond correlation has changed. With respect to the value factor exposures, perhaps the most interesting contrast between 2017 and earlier years is the profitability portfolio. The value factor exposure of the profitability portfolio was 0.157 in 2017 (see Exhibit 10), compared to negative exposure of −0.183 in 1992 (see Exhibit 9).

HISTORICAL PERFORMANCE OF PURE FACTOR PORTFOLIOS

In this section, we describe the construction and performance of factor portfolios that are pure with respect to specified secondary factor exposures, in contrast to the primary factor portfolios discussed in the previous section. Specifically, the pure factor portfolios are rebalanced at the beginning of each month to have exposures of exactly zero to all but the primary factor of interest. As detailed in the online Appendix A, the security weights of the pure portfolios are based on multivariate Fama and MacBeth [1973] regressions of security returns on two or more factor exposures in order to control for the influence of the additional factors. Adding or subtracting the set of control factors will change the weights depending of the cross-sectional correlations between factor exposures.

Consider the case of the market plus just two non-market factors, labeled Factors 1 and 2. The security weights for a pure portfolio with market-relative exposure of exactly one to Factor 1 and a market-relative exposure of exactly zero to Factor 2 are

$$w_{Pi} = w_{Mi}\left[1 + b1_i\left(\frac{1}{1-\rho^2}\right) - b2_i\left(\frac{\rho}{1-\rho^2}\right)\right], \qquad (5)$$

where $b1_i$ and $b2_i$ are the security exposures to Factors 1 and 2, respectively, and ρ is the market-weighted correlation coefficient between factor exposures, as defined in Equation 4. If the exposure correlation ρ in Equation 4 happens to be zero, then Equation 5 collapses to Equation 1. In contrast to the portfolio defined by Equation 5, single- and double-sorting methodologies for factor portfolio construction in Fama and French [1992] do not precisely control for secondary factor exposures.

Exhibit 11, presented in the same format as Exhibit 2, reports the performance of the market and six pure factor portfolios over 50 years; each pure factor portfolio has a zero exposure to all five of the other nonmarket factors. For example, the annualized average market-relative return to the pure value portfolio in Exhibit 11 is 1.90%, for a total return of 6.29 + 1.90 = 8.19%. Although the average returns in the first row of Exhibit 11 differ in each case from the same entries in Exhibit 2, the more dramatic contrast is the standard deviation of market-differential returns in the second row. Each pure factor portfolio in Exhibit 11 has return standard deviations that are substantially lower than the corresponding primary factor portfolio in Exhibit 2. For example, the return standard deviation of the pure value portfolio is 2.29%, compared to 4.57% in Exhibit 2. Similarly, the market beta and alpha entries in Exhibit 11 are similar to Exhibit 2, but the active risk numbers are generally lower, leading to larger pure factor portfolio information ratios.

Exhibit 12 plots the cumulative relative returns of the six pure factor portfolios, similar to Exhibit 3 for the six primary portfolios. Although the cumulative returns end up being somewhat different in magnitudes and

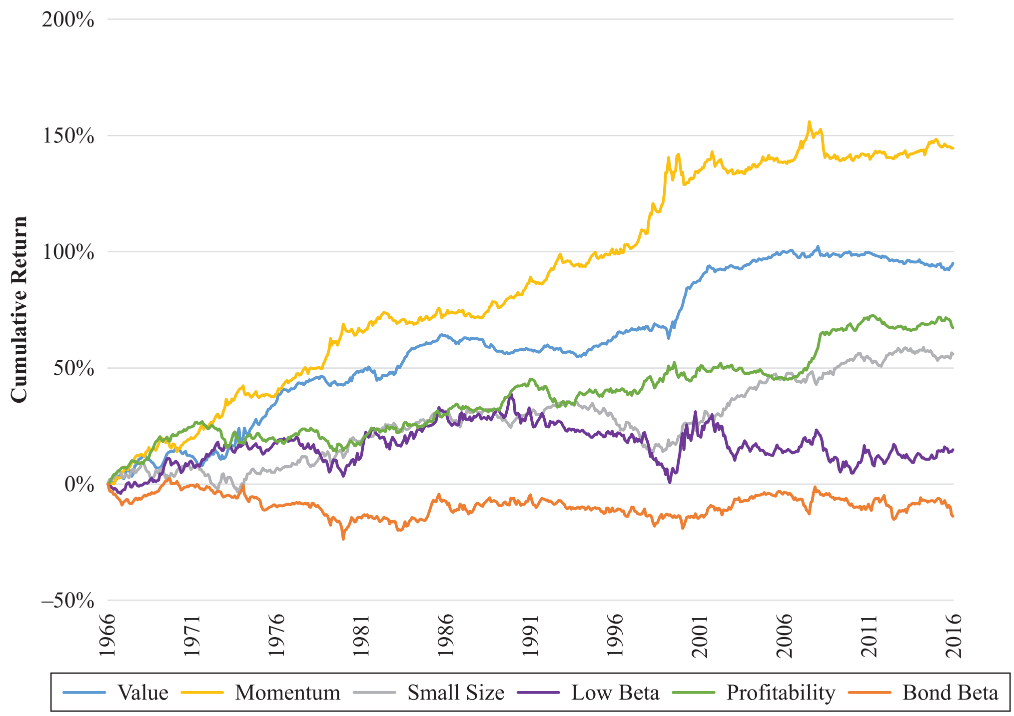
Exhibit 11

Market-Relative Returns of Pure Factor Portfolios, 1967–2016

		Market Relative					
	Market	Value	Momentum	Small Size	Low Beta	Profitability	Bond Beta
Mean Return	6.29%	1.90%	2.89%	1.12%	0.30%	1.35%	–0.28%
Std. Dev.	15.27%	2.29%	4.75%	3.67%	5.56%	3.22%	3.64%
Sharpe Ratio	0.41						
Market Beta	1.000	–0.04	–0.01	0.06	–0.22	–0.01	–0.06
Alpha	0.00%	2.16%	2.96%	0.77%	1.67%	1.41%	0.11%
Active Risk		2.93%	4.75%	3.59%	4.46%	3.22%	3.52%
Info. Ratio		0.74	0.62	0.21	0.37	0.44	0.03

Exhibit 12

Cumulative Market-Relative Returns of Pure Factor Portfolios, 1967–2016



orders, the most dramatic visual impression of Exhibit 12 compared to Exhibit 3 is that the cumulative returns are smoother, consistent with the lower standard deviations reported in Exhibit 11. In addition, an important visual impression from the cumulative returns in Exhibit 12 is that the realized returns are less correlated with each other. For example, the bursting of the turn-of-the-century "technology bubble" is evident in both the value and low-beta plots in Exhibit 3, but it is only visually evident in the low-beta plot in Exhibit 12.

Exhibit 13 reports return correlations over 50 years in the same format as Exhibit 4. For example, the realized time-series correlation between the value and momentum pure portfolio returns is −0.13, compared to a much larger negative correlation of −0.43 in Exhibit 4. In fact, except for the off-diagonal elements in Exhibit 4 that were already close to zero, all the entries in Exhibit 13 are indicative of less correlated returns.

When other factor exposures are neutralized, the highest performing factor in Exhibit 11 is value, with

EXHIBIT 13
Correlation of Pure Factor Portfolio Market-Relative Returns, 1967–2016

	Correlation Matrix					
	Value	**Momentum**	**Small Size**	**Low Beta**	**Profitability**	**Bond Beta**
Value	1.00	–0.13	–0.01	0.20	–0.23	0.00
Momentum	–0.13	1.00	–0.07	0.02	0.03	0.00
Small Size	–0.01	–0.07	1.00	–0.15	0.00	–0.09
Low Beta	0.20	0.02	–0.15	1.00	–0.11	0.28
Profitability	–0.23	0.03	0.00	–0.11	1.00	0.09
Bond Beta	0.00	0.00	–0.09	0.28	0.09	1.00

EXHIBIT 14
Pure Factor Portfolio Composition in 2017

		Factor Portfolios					
	Market	**Value**	**Momentum**	**Small Size**	**Low Beta**	**Profitability**	**Bond Beta**
Security Weights							
Sum	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Max	3.01%	5.28%	3.54%	0.54%	4.24%	4.94%	9.24%
Min	0.01%	–1.04%	–0.33%	–2.07%	–4.24%	–0.29%	–2.60%
Long	100.0%	109.8%	107.4%	106.5%	120.6%	106.4%	124.6%
Short	0.0%	–9.8%	–7.4%	–6.5%	–20.6%	–6.4%	–24.6%
Factor Exposures							
Market		1.000	1.000	1.000	1.000	1.000	1.000
Value		1.000	0.000	0.000	0.000	0.000	0.000
Momentum		0.000	1.000	0.000	0.000	0.000	0.000
Small Size		0.000	0.000	1.000	0.000	0.000	0.000
Low Beta		0.000	0.000	0.000	1.000	0.000	0.000
Profitability		0.000	0.000	0.000	0.000	1.000	0.000
Bond Beta		0.000	0.000	0.000	0.000	0.000	1.000

an information ratio of 0.74, although the cumulative market-relative return plot for the pure value portfolio in Exhibit 12 shows essentially flat performance for the last 15 years. Momentum, with an information ratio of 0.62, ranks second in Exhibit 11, but as shown in Exhibit 12, most of the market-relative performance preceded the popularization of that factor by Carhart [1997] in the late 1990s. The profitability portfolio comes in third with an information ratio of 0.44 in Exhibit 11, followed closely by low beta at 0.37, although that performance is not directly evident in the cumulative return plot in Exhibit 12, which does not account for nonzero relative market beta (i.e., nonunitary total beta). Small size within the large-cap market has an information ratio of 0.21 in Exhibit 11. Although a potentially important risk exposure in equity portfolio management, bond beta shows little promise of positive returns on average with a pure portfolio information ratio of just 0.03.

We end this section with an examination of the 2017 composition of the pure factor portfolios. Specifically, Exhibit 14 is presented in the same format as Exhibit 10 for the primary factor portfolios. As in Exhibit 10, the security weights in the first section of Exhibit 14 are indicative of long–short extension portfolios, although some portfolios are closer to 120/20 instead of 110/10. Consistent with the design of the pure factor portfolios, the nondiagonal exposures in the lower section of Exhibit 14 are identically zero, meaning that each of the pure factor portfolios has exactly the same exposure to the ancillary factors as the benchmark portfolio.

CONCLUSIONS

The primary contribution of this study is methodological in terms of factor portfolio construction and the correspondence between portfolio returns and Fama and MacBeth regression analysis. But the long-term horizon and simultaneous examination of popular equity market factors allows for a number of interesting empirical observations on historical factor performance. Although the 50-year numbers for the pure value factor portfolio are impressive, once purged of secondary factor exposures, pure value portfolio returns in the United States have been flat for over 15 years. Momentum factor performance, as tracked by a *pure* momentum portfolio, has also declined since its popularization in the late 1990s. Pure profitability comes in third and was particularly strong during the 2008 financial crisis.

Low-beta-portfolio returns only slightly exceed the market portfolio but have the third highest long-term information ratio based on the lower realized market beta. Without adjusting for secondary factor exposures, we find that, as of early 2017, portfolios constructed on lower-beta stocks have higher than historically observed standardized bond beta, suggesting they will underperform the equity market benchmark in response to unanticipated increases in interest rates. But pure low-beta portfolios can be constructed ex ante using multivariate regression analysis, and doing so historically would not have reduced ex post performance. In fact, as reported in the online Appendix B to this article, the various factor portfolios can also be made pure with respect to industry group composition, further increasing their information ratios.

Cross-sectional regression analysis, as introduced by Fama and MacBeth [1973], has been and will continue to be a mainstay of empirical research on factor returns. But without benchmark weighting, estimated regression coefficients often have little correspondence with the active returns on implementable factor portfolios. We reconcile regression analysis with pure factor portfolio construction by standardizing exposures using benchmark portfolios weights. Coefficients from multivariate regression analysis equate to the active return on a pure factor portfolio, with the intercept term equal to the benchmark portfolio return. The complete set of security weights in each portfolio is analytically derived, resulting in portfolios that are investable and cleared of secondary exposures.

Factor-based investors often struggle with parsing out the secondary effects of multiple factor exposures, using double quintile sorts and other ad hoc methodologies. Even worse, the reported performance in single-sort backtests often have little bearing on the actual performance of the pure factor being examined. Unless the secondary exposures are controlled, conclusions about the source of returns in single-factor portfolios are often misleading. Multivariate regression analysis has long been the go-to empirical methodology across the social sciences for calculating and analyzing the separate effects of correlated variables. The key to applying multivariate regression analysis to factor portfolios is weighting observations by benchmark (e.g., capitalization) weights.

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