

March 2010

Jose Menchero



#### | March 2010

#### Introduction

At its core, the success of active management rests upon the ability of the portfolio manager to differentiate assets along meaningful dimensions. It is essential for the active manager to distinguish outperforming securities from underperforming ones. It is also crucial to identify and manage the sources of portfolio risk.

For equities, one important way of distinguishing stocks is by country of exposure. The portfolio manager may have views about the relative performance of various countries and use that information to weight the stocks from those countries accordingly. Another common approach for distinguishing stocks is by economic sector. For example, if the portfolio manager believes that the economy is entering a recession, he may choose to overweight stocks in defensive sectors while underweighting those in cyclical sectors. A third major approach for distinguishing stocks is by investment style. In this case, the portfolio manager may have views regarding the relative performance of, say, large stocks versus small stocks, or value versus growth.

One investment approach is to consider a single characteristic, or variable, in isolation. For instance, the portfolio manager may make investment decisions by grouping stocks into countries or sectors. Such an approach, however, may lead to unintentional exposures. A bullish view on Japan, for instance, might lead to an inadvertent overweight of automobile stocks.

One way to avoid this pitfall is to group stocks jointly according to countries *and* sectors. This, however, introduces a different problem: Namely, the number of groups can quickly spiral out of control. For instance, an investment universe consisting of 24 countries and 10 sectors leads to 240 groups of stocks. Even this level of granularity does not eliminate the problem of unintentional exposures, since an explicit decision to overweight say US Health Care may lead to unintended style exposures. Due to such shortcomings, analyzing a portfolio along only a single dimension is of limited practical benefit.

Clearly, there are many meaningful dimensions along which to differentiate stocks. In practice, portfolio managers will often combine several of these views. Factor models are specifically designed for this purpose, as they cleanly disentangle the effects of multiple variables acting in concert. Factor models also provide the active manager with a means of identifying and controlling portfolio exposures so that only intentional bets are placed. Furthermore, they explain how the performance and risk of a portfolio is attributed to the underlying return drivers (e.g., countries, industries, and styles).

The full benefit of the factor approach can be attained only when the meaning and interpretation of the factors themselves is clearly established. While much progress has been made on this front over the years, factor models are still regarded by some as indiscernible "black boxes." This paper aims to develop greater intuition behind factor models by interpreting the factors in terms of easily understood portfolios. The focus of this paper is on global equity factor models, but the underlying concepts are applicable and relevant to any type of factor model.

#### | March 2010

The remainder of this paper is organized as follows. First, we consider the case of a single variable or grouping scheme in isolation; this leads to the notion of *simple* factor portfolios. We then examine multiple variables and grouping schemes acting simultaneously, thus giving rise to the concept of *pure* factor portfolios. We explain how the collinearity between factors drives differences between simple and pure factor portfolios. Next, we introduce several intuitive measures to quantify the extent to which pure factors deviate from their simple counterparts. We also examine the empirical distribution of these measures, which provides greater insight into the nature of global equity factors. Finally, to illustrate these concepts, we analyze the sources of return for a set of global style factors in August 2009.

#### Simple Factor Portfolios

Factor returns typically are estimated by cross-sectional regression. In this framework, each factor can be represented by a portfolio for which the return exactly replicates the payoff to the factor. There are two ways of constructing factor-mimicking portfolios. Simple factor portfolios result from univariate regressions that effectively treat the factor in isolation.

Pure factor portfolios, on the other hand, result from multivariate regressions that consider all factors simultaneously. Simple factor portfolios are important because their holdings are clear and intuitive, and they serve as the foundation for understanding pure factors.

Global equity factor models typically use countries, industries, and styles as explanatory variables. In addition, a World factor is often included to capture the overall effect of the global equity market. Country and industry factors are usually treated as indicator variables. That is, the stock is assigned an exposure of 0 or 1, depending on whether it belongs to the country or industry under consideration. Style exposures, by contrast, typically are distributed in a continuous fashion with mean 0 and standard deviation 1.

To derive the simple factor portfolios for styles, we perform a univariate cross-sectional regression including an intercept term,

$$r_{n} = f_{w}^{S} + X_{ns} f_{s}^{S} + u_{n}^{S} , (1)$$

where  $r_n$  is the local excess return of stock n,  $f_w^S$  is the intercept term,  $f_s^S$  is the return to style factor s,  $X_{ns}$  is the stock exposure to the style factor, and  $u_n^S$  is the specific return of the stock. Every stock has an exposure of 1 to the intercept term, which we identify as the World factor.

Factor returns must be estimated on a universe of stocks. In this paper, we consider the estimation universe to be the set of all stocks contained in a broad world portfolio, such as the MSCI All Country World Investable Market Index (ACWI IMI).

#### | March 2010

In order to reduce estimation error in the factor returns, regression weights are used so that "noisy" stocks (i.e., those with high specific risk) are down-weighted. In practice, regression weights  $v_n$  are often taken as proportional to the square root of market capitalization, although other weighting schemes are possible. We standardize regression weights so that they sum to 1 over the estimation universe.

$$\sum_{n} \nu_n = 1 \tag{2}$$

We have the freedom to define the mean and standard deviation of the style exposures without altering the regression fit. For present purposes, we standardize style factor exposures to be regression-weighted mean zero,

$$\sum_{n} v_n X_{ns} = 0 (3)$$

This implies no collinearity between the style and the World factor. We also set the regression-weighted standard deviation of the style factor to 1,

$$\sum_{n} v_n X_{ns}^2 = 1 . {4}$$

With this standardization convention, the simple style factor return is given by

$$f_s^S = \sum_n \left( v_n X_{ns} \right) r_n \quad , \tag{5}$$

which represents the return of a portfolio with weights  $v_n X_{ns}$ . In other words, the simple style factor portfolio goes long in stocks with positive exposure, and shorts stocks with negative exposure, while taking proportionately larger positions in stocks with greater regression weight. The portfolio is also strictly dollar neutral, since the weights sum to zero by virtue of Equation 3. The portfolio exposure to the factor, given by the sum product of stock weight  $v_n X_{ns}$  and stock exposure  $X_{ns}$ , is equal to 1 by Equation 4.

The return of the simple World factor in Equation 1 is given by

$$f_w^S = \sum_n v_n \, r_n \quad . \tag{6}$$

This is just the regression-weighted return of the estimation universe. In practice, for any reasonable regression weighting scheme, the factor return  $f_w^S$  will be highly correlated with the return of the cap-weighted world portfolio.

Next, we consider simple factor portfolios for indicator variables. These can be used to represent countries, industries, or any other grouping scheme. We assume that every asset belongs to one and only one group.

# Characteristics of Factor Portfolios | March 2010

In the regression to form the simple group factor portfolios, we consider only one grouping scheme at a time. We estimate factor returns using an intercept term to represent the World factor,

$$r_{n} = f_{w}^{S} + \sum_{g} X_{ng} f_{g}^{S} + u_{n}^{S} , \qquad (7)$$

where  $X_{ng}$  is the exposure (0,1) of stock n to group g, and  $f_g^S$  is the simple factor return of the group. Note that although we use the same symbol  $f_w^S$  for the simple World factor return in Equation 7 and Equation 1; they represent slightly different portfolios, as we shall see below.

The factor structure in Equation 7 contains an exact collinearity, meaning that if we sum the columns in the exposure matrix corresponding to groups, we obtain a column of 1s, which corresponds to the World factor. Therefore, we must impose a constraint to obtain a unique regression solution. An intuitive and commonly used constraint is to say that the cap-weighted group factor returns sum to zero,

$$\sum_{g} W_g f_g^S = 0 \tag{8}$$

where  $W_{\scriptscriptstyle\sigma}$  is the capitalization weight of group  $\,g$  .

With this constraint, the return of the simple World factor in Equation 7 is given by

$$f_w^S = \sum_g W_g \sum_{n \in g} \left( \frac{v_n r_n}{V_g} \right) , \tag{9}$$

where  $V_{g}$  is the regression weight of group g .

For the case of indicator variables, therefore, the simple World factor portfolio is long only and fully invested. Each group is market-cap weighted, but the stocks within the groups are regression weighted. Equation 9 should be contrasted with Equation 6, which represents the simple World factor portfolio for the case of a single style factor.

Simple group factor returns are given by

$$f_g^S = \left(\frac{1}{V_g} \sum_{n \in g} v_n r_n\right) - f_w^S , \qquad (10)$$

with  $f_w^S$  defined by Equation 9. In other words, the simple group factor portfolio goes long the regression-weighted group portfolio and goes short the simple World factor portfolio. Note that by adding the World factor  $f_w^S$  to the group factor  $f_g^S$ , one obtains a fully invested regression-weighted portfolio concentrated in a single group.

#### | March 2010

#### **Pure Factor Portfolios**

Pure factor portfolios are formed by multivariate regression. For this study, we adopt the factors from the Barra Global Equity Model (GEM2), as described by Menchero, Morozov, and Shepard (2009). The GEM2 factor structure consists of a World factor, 55 country factors, 34 industry factors (based on the Global Industry Classification Standard, GICS®), and 8 styles,

$$r_{n} = f_{w}^{P} + \sum_{c} X_{nc} f_{c}^{P} + \sum_{i} X_{ni} f_{i}^{P} + \sum_{s} X_{ns} f_{s}^{P} + u_{n}^{P} , \qquad (11)$$

with the superscript P used to denote the *pure* factor. The model uses regression weights proportional to the square root of market capitalization, and an estimation universe based on the MSCI All Country World Investable Market Index (ACWI IMI), which is designed to capture the full breadth of investment opportunities for global equity investors. Each stock has unit exposure to the World factor, and unit exposure to the particular country or industry to which it belongs. The style factors are mean zero and standard deviation 1, in the manner described below. The specific returns  $u_n^P$  are assumed to be mutually uncorrelated and also uncorrelated with the factor returns.

Since every stock belongs to a country and industry, the factor structure in Equation 11 contains *two* exact collinearities. In order to obtain a unique regression solution, we impose two constraints: the cap-weighted country factor returns sum to zero, and the cap-weighted industry factor returns sum to zero. These constraints were also used by Heston and Rouwenhorst (1994), and effectively calibrate the model so that the country and industry factors collectively contribute zero to the return of the cap-weighted world portfolio.

When introducing simple factor portfolios, it proved useful to standardize the style factors to be regression-weighted mean zero, as in Equation 3. This made the style factors orthogonal (i.e., no collinearity) to the World factor, which in turn facilitated interpretation of the style factor return, as in Equation 5. Although such standardization is certainly still possible in the multivariate case, the motivation is lost, since collinearity among style factors precludes simple analytic solutions as in Equation 5. Instead, we adopt the convention that the style exposures are *cap-weighted* mean zero,

$$\sum_{n} w_n X_{ns} = 0 \tag{12}$$

for all styles s. This calibrates the model so that the cap-weighted world portfolio is style neutral.

Factor returns are estimated using weighted and restricted least-squares regression. The general solution, as described in Ruud (2000), can be written as

$$f_k^P = \sum_n \Omega_{nk}^P r_n \quad , \tag{13}$$

where  $\Omega_{nk}^P$  represents the weight of stock n in pure factor portfolio k. The full insight of factor modeling can only be attained when the pure factor portfolios are clearly interpreted.



#### | March 2010

The pure World factor is closely related to the cap-weighted world portfolio. As noted, the constraints on the cap-weighted country and industry factor returns imply that neither countries nor industries contribute to the return of the world portfolio. Styles also do not contribute, due to the standardization convention that the world portfolio be style neutral. Therefore, the return of the world portfolio,  $R_{\rm w}$ , can be attributed using Equation 11,

$$R_{w} = f_{w}^{P} + \sum_{n} w_{n} u_{n}^{P} , \qquad (14)$$

where  $w_n$  is the weight of stock n in the world portfolio (i.e., the estimation universe). The specific contribution to the world portfolio return is extremely small, since specific returns diversify away<sup>1</sup>. Equation 14 therefore implies that the pure World factor  $f_w^P$  essentially represents the cap-weighted world portfolio. Indeed, Menchero, Morozov and Shepard (2009) found that the time series correlation between the pure World factor and the world portfolio was 0.994, thus confirming the validity of this interpretation. The pure World factor is also industry and country neutral, meaning that the weights exactly match those of the cap-weighted world portfolio in every country and industry.

In Equation 10, we saw that simple country factor portfolios go long the regression-weighted country portfolio and go short the World factor. Such a portfolio generally has non-zero exposures to industries and styles. Pure country factor portfolios eliminate these exposures. In other words, pure country factor portfolios are 100 percent long the country and 100 percent short the World factor, but have zero exposure to every industry and style factor. An investor in a pure country factor portfolio thus places a precise bet that the country will outperform the world, without placing incidental bets on industries or styles.

Similarly, pure industry factor portfolios go 100 percent long the particular industry and 100 percent short the World factor. However, they have zero exposure to every country and every style. Therefore, they represent pure bets that the industry will outperform the world, without placing incidental bets on countries or styles.

Similar to the simple style factor portfolios defined by Equation 5, pure style factor portfolios have unit exposure to the style in question. In contrast to simple factors, pure style factor portfolios have zero exposure to all other styles, countries, and industries. Therefore, they represent pure bets on the particular style factor, without incidental exposures to any other factors.

<sup>&</sup>lt;sup>1</sup> Note that the *regression-weighted* specific returns sum to zero by construction.



# Characteristics of Factor Portfolios | March 2010

#### Example 1

In Table 1, we present the weights of several pure factor portfolios in select market segments. The factors are taken from the Barra Global Equity Model, GEM2, for the month of July 2009. The pure World factor is 100 percent *net* long, but includes short positions as well. Its net weights exactly match the cap-weighted world portfolio in any segment corresponding to a factor. For instance, Spanish equities constitute 2.0 percent of the global equity market, and the pure World factor portfolio also has a 2.0 percent weight in Spain. Spanish banks represent 65 bps of the global equity markets, and therefore comprise about one third of the Spanish equity market. Note, however, that since Spanish banks do not comprise a factor in the model, the weights of the pure World factor and the world portfolio differ in this segment.

The pure Spain factor is long/short and dollar neutral, with zero net exposure to the Bank factor and all other industry factors. The pure Spain factor is 100 percent long Spain and 100 percent short the World factor. Since the world itself is 2.0 percent Spain, however, the net weight of the pure factor portfolio in Spain is 98 percent. Note that the pure Spain factor has a short position in UK banks. This is required in order to hedge out the large exposure to banking one finds in the Spanish market.

The pure Banks factor portfolio goes 100 percent long banks, and 100 percent short the World factor portfolio. Since the World factor portfolio has about 10 percent weight in banks, it follows that the pure Banks factor is about 90 percent net long banks and 90 percent net short other industries. It has net zero exposure, however, to all countries (e.g., Spain or UK) and styles.

The pure Value factor portfolio is strictly dollar neutral, with zero weight in every country or industry that corresponds to a model factor. For instance, the portfolio has zero net weight in Spain, UK, and Banks. However, it has nonzero weight in segments that do not correspond to model factors (e.g., Spanish banks or UK banks). The pure Value factor portfolio also has unit exposure to the Value factor and zero exposure to every other style factor.

Finally, note that adding the pure World factor to a country factor (e.g., Spain) creates a pure factor portfolio that is 100 percent net long the particular country (Spain), and has zero weight in every other country (e.g., UK). Such a portfolio is industry neutral, meaning that the industry weights match those of the world portfolio. The portfolio also has zero exposure to every style factor. Similarly, adding the pure World factor to an industry factor (say, Banks) creates a portfolio that is 100 percent net long the particular industry (Banks), has zero exposure to every other industry or style, and is country neutral (i.e., the country weights match those of the world portfolio). In other words, it represents a pure net-long bet on the particular industry.

# Characteristics of Factor Portfolios | March 2010

#### **Measuring Collinearity Effects**

The concepts of simple and pure factor portfolios provide an intuitive framework for understanding and measuring the effects of collinearity in global equity factor models. Simple factor portfolios, by construction, have no collinearity. The effects of collinearity in a factor model can be understood, therefore, by comparing pure factor portfolios to their simple counterparts.

One simple and intuitive measure of collinearity is the *Factor Weight Correlation*, defined as the cross-sectional correlation between the weights of the simple and pure factor portfolios. Let  $\Omega^P_{nk}$  denote the weight of stock n in pure factor portfolio k, and let  $\Omega^S_{nk}$  denote the corresponding weight in the simple factor portfolio. The Factor Weight Correlation is given by

$$\rho_k^{CS} = \frac{\sum_n \Omega_{nk}^P \cdot \Omega_{nk}^S}{\sigma(\Omega_k^P) \sigma(\Omega_k^S)} . \tag{15}$$

Note that since the pure and simple factor portfolios are strictly dollar neutral (i.e., zero exposure to the World factor), their weights are mean zero. The correlation in Equation 15 quantifies the "similarity" in weights between simple and pure factor portfolios.

In Figure 1, we present a histogram of the Factor Weight Correlation for the GEM2 factors as of January 2008. The vast majority of factors have correlations in excess of 0.95, indicating high similarity between the pure and simple factor portfolios. Interestingly, the style factors score lowest by this measure, ranging from 0.70 to about 0.90.

The relatively low Factor Weight Correlation for style factors can be understood as arising from collinearity between industries and styles. Consider, for instance, the Value factor. We know that the simple Value factor portfolio tends to take long positions in Financials stocks, and short positions in Information Technology stocks. The pure factor portfolio, however, must have zero net weight in these sectors. Therefore, the pure factor portfolio must assume short positions in Financials stocks with relatively low (but perhaps still positive) Value exposure, and long positions in Information Technology stocks with relatively high Value exposure. As a result, stocks that have, say, positive weight in the simple factor portfolio may have negative weight in the pure factor portfolio, and vice versa. This serves to reduce the Factor Weight Correlation for these portfolios.

Another intuitive measure of collinearity is the *Factor Return Correlation*, given by the time series correlation of factor returns,

$$\rho_k^{TS} = \frac{\sum_{t} \left( f_{kt}^P - \overline{f}_k^P \right) \left( f_{kt}^S - \overline{f}_k^S \right)}{\sigma \left( f_k^P \right) \sigma \left( f_k^S \right)} . \tag{16}$$



#### | March 2010

In Figure 2, we present the Factor Return Correlation histogram. Results were computed using monthly factor returns over a 13-year period (Jan 1997 to Jan 2010). Of the 12 factors with timeseries correlation below 0.78, half correspond to style factors. However, the two strongest style factors, Volatility and Momentum, have correlations of 0.97 and 0.94, respectively.

Another useful measure of collinearity is the *Factor Leverage Ratio*, defined as the ratio of the leverage of the pure and simple factor portfolios,

$$L_k = \frac{\sum_n \left| \Omega_{nk}^P \right|}{\sum_n \left| \Omega_{nk}^S \right|} \ . \tag{17}$$

Intuitively, we expect the Factor Leverage Ratio to be greater than 1, since the pure factor portfolio must assume additional long/short positions to hedge out exposures to the other factors. In Figure 3, we present a histogram of Factor Leverage Ratios for the GEM2 model for analysis date January 2008. The vast majority are less than 1.3, indicating mild collinearity. The greatest Factor Leverage Ratios correspond to thin countries with concentrated industries.

Consider, for instance, Jordan, which has a Factor Leverage Ratio in excess of 2. Jordan has more than 80 percent of its market capitalization concentrated in a single industry (Banks). The simple Jordan factor portfolio, which goes long Jordan and shorts the World factor, will be greatly overweight the Banks factor and underweight all other industries. In order to hedge out these exposures, the pure Jordan factor must short banking stocks in other countries and go long non-banking stocks. These additional positions serve to increase the Factor Leverage Ratio significantly.

#### **Reducing Collinearity in Factor Structure**

When two factors are highly collinear, they essentially explain the same effect in isolation. In other words, their *simple* factor portfolios are nearly identical. When combined together, it becomes difficult to cleanly disentangle the two competing effects. Econometrically, the effect of high collinearity is to increase the estimation error in the factor returns. In practical terms, the pure factors become less intuitive, which manifests itself through low Factor Weight Correlation, low Factor Return Correlation, and high Factor Leverage Ratio.

One way of dealing with collinearity is to rotate the offending factor so that it is perpendicular (i.e, orthogonal) to the other collinear factors. This has the benefit of removing the collinearity, but may complicate interpretation since the rotated factor now represents something different.

In some instances, however, the rotated factor may actually be *easier* to interpret than the original one. For instance, the GEM2 model incorporates a Non-linear Size factor to capture nonlinearities in the payoff to Size exposure. The Non-linear Size factor is customarily defined as the cube of the Size factor exposure. Unfortunately, the Size-cubed factor is highly collinear with the Size factor, as evident from Figure 4. More specifically, large-cap stocks have positive exposure to both Size and Size-cubed, while small-cap stocks have negative exposure to both factors.

#### | March 2010

Therefore, to a first approximation, the simple factor portfolios for Size and Size-cubed are very similar and describe the same effect (i.e., the performance differential between large-cap and small-cap stocks).

Alternatively, we can construct the Non-linear Size (NLS) factor as a linear combination of the Size and Size-cubed factor:

$$X_n(NLS) = X_n(Size^3) - bX_n(Size)$$
 (18)

The coefficient b is determined so that

$$\sum_{n} v_n X_n(NLS) X_n(Size) = 0 , \qquad (19)$$

where  $v_n$  is the regression weight in stock n. Mathematically, we interpret the Non-linear Size factor as a rotation of the Size-cubed factor such that it is perpendicular to the Size factor.

The question remains how to *interpret* the Non-linear Size factor. In Figure 4, we plot the Non-linear Size exposure versus Size exposure. Large-cap stocks and small-cap stocks both have negative exposure to Non-linear Size, while mid-cap stocks (with Size exposure between –2.5 and 0) have positive exposure. Therefore, the simple factor portfolio goes long mid-cap stocks, and goes short large-cap and small-cap stocks. Roughly speaking, therefore, the Non-linear Size factor captures the performance differential between the mid-cap segment and the rest of the market.

Not only does orthogonalization provide a more intuitive interpretation of the Non-linear Size factor, it also gives a more intuitive interpretation of the *Size* factor as well. By eliminating the collinearity between Size and Non-linear Size, the Factor Leverage Ratio for Size drops from 1.86 to 1.12, while increasing the Factor Weight Correlation from 0.83 to 0.89.

#### **Example 2**

The month of August 2009 saw large moves for several GEM2 style factors. One way to gain insight into these style factor returns is to decompose them by economic sector. In Table 2, we report the return contributions to each GEM2 style factor, segmented according to GICS® economic sector. Since every factor portfolio has net zero weight within each sector, the return contribution is just the absolute weight in the sector multiplied by the return difference between the longs and the shorts,

$$f_{s}^{P} = \sum_{m} W_{ms}^{L} \left( r_{ms}^{L} - r_{ms}^{S} \right) . \tag{20}$$

Here,  $f_s^P$  is the return of pure style factor s,  $W_{ms}^L$  is the long weight of the style factor portfolio in sector m,  $r_{ms}^L$  is the return of the long stocks in the sector, and  $r_{ms}^S$  is the corresponding return for the short stocks.



The two largest style factor returns in August 2009 were Momentum (declining 2.43 percent) and Volatility (gaining 2.02 percent). To a first approximation, the Momentum factor captures the return difference between stocks that have performed well over the last year and those that that have performed poorly, while Volatility captures the return difference between high-beta and low-beta stocks. Interestingly, for both factors, the largest contribution by far came from the Financials sector. Furthermore, the second most important sector was Consumer Discretionary. We interpret these results to mean that beaten-down high-beta stocks performed well in these sectors. These stocks tended to have negative weight in the Momentum factor portfolio, and positive weight in the Volatility factor portfolio.

It is also interesting to compare the sector return contributions for other factors. For instance, the Size factor was down 30 bps for the month, indicating large cap underperformed small cap, all else equal. In the Financials sector, however, the opposite was true. That is, the return contribution was positive, indicating that large-cap financial stocks outperformed their small-cap peers. This example illustrates that style factors do not always move in lock-step across economic sectors.

#### Summary

We present an intuitive interpretation of factor models in terms of portfolios that replicate the payoffs to the factors. In a univariate regression, this gives rise to simple factor portfolios, whereas the multivariate case leads to pure factor portfolios. We introduce several measures based on simple and pure factor portfolios to help understand and quantify the effects of collinearity in global equity factor models. We show that rotating a factor may lead to a more intuitive interpretation of the factor while also reducing the effects of collinearity. Finally, we present several examples to illustrate the concepts contained herein.



#### | March 2010

#### References

Heston, S.L., and K.G. Rouwenhorst, 1994. "Does Industrial Structure Explain the Benefits of International Diversification?" *Journal of Financial Economics*, vol. 36: 3-27.

Menchero, Jose, Andrei Morozov, and Peter Shepard, "Global Equity Risk Modeling," *The Handbook of Portfolio Construction: Contemporary Applications of Markowitz Techniques*, Edited by J. Guerard, (New York: Springer, 2009).

Ruud, Paul, *An Introduction to Classical Econometric Theory*, (New York: Oxford University Press, 2000).

## | March 2010

Table 1
Weights of selected pure factor portfolios in market segments (July 2009).

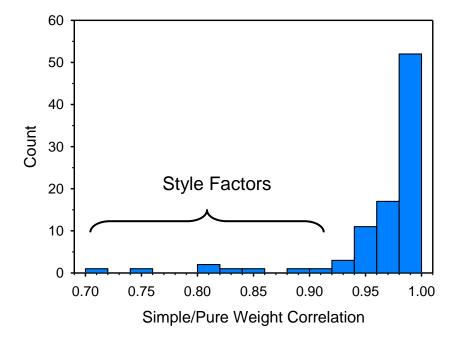
	(ESTU)	Pure	Pure	Pure	Pure	Pure
	World	World	Spain	UK	Banks	Value
Segment	Portfolio	Factor	Factor	Factor	Factor	Factor
World (Net)	100.00	100.00	0.00	0.00	0.00	0.00
Long	100.00	107.02	121.89	95.85	97.88	47.20
Short	0.00	-7.02	-121.89	-95.85	-97.88	-47.20
Spain (Net)	2.00	2.00	98.00	-2.00	0.00	0.00
Long	2.00	2.00	98.00	0.00	2.24	0.65
Short	0.00	0.00	0.00	-2.00	-2.24	-0.65
UK (Net)	7.22	7.22	-7.22	92.78	0.00	0.00
Long	7.22	7.22	0.65	92.78	3.11	2.23
Short	0.00	0.00	-7.87	0.00	-3.11	-2.23
Banks (Net)	10.12	10.12	0.00	0.00	89.88	0.00
Long	10.12	10.35	15.22	4.00	89.88	3.22
Short	0.00	-0.23	-15.22	-4.00	0.00	-3.22
Spain Banks (Net)	0.65	0.43	14.92	-0.23	2.24	0.08
Long	0.65	0.43	14.92	0.00	2.24	0.16
Short	0.00	0.00	0.00	-0.23	0.00	-0.08
UK Banks (Net)	1.10	0.58	-0.52	3.63	2.26	-0.13
Long	1.10	0.58	0.00	3.63	2.26	0.06
Short	0.00	0.00	-0.52	0.00	0.00	-0.19

| March 2010

Figure 1

Histogram of Factor Weight Correlation for the Barra Global Equity Model (GEM2). This gives the cross-sectional correlation between the weights of the simple factor portfolios and the pure factor portfolios. Analysis date is January 2008.

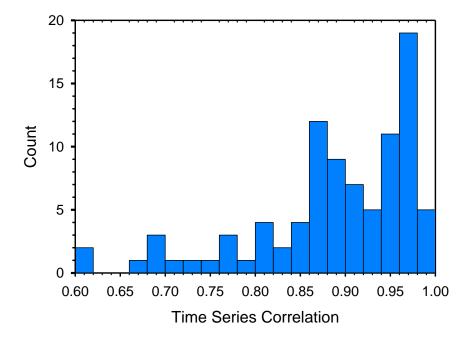
15 of 21



| March 2010

Figure 2

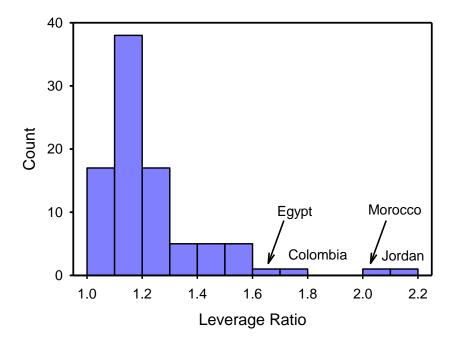
Histogram of Factor Return Correlation for the Barra Global Equity Model (GEM2). This gives the time series correlation between simple and pure factor returns. The period is from January 1997 to January 2010.



| March 2010

Figure 3

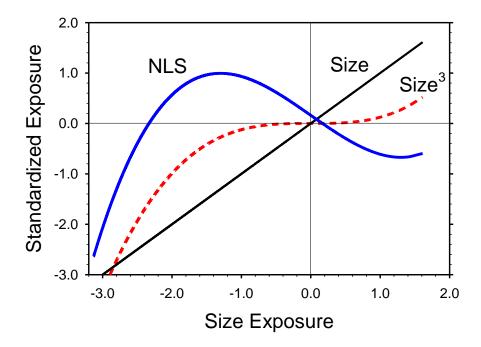
Histogram of Factor Leverage Ratio for the Barra Global Equity Model (GEM2). Analysis date is January 2008.



| March 2010

Figure 4

Size-cubed and Non-linear Size (NLS) factor exposures. The Size-cubed factor is highly collinear with the Size factor. The Non-linear Size factor has the collinearity removed.





### | March 2010

Table 2

Attribution of pure style factor returns according to GICS® economic sector (August 2009).

Sector	Volatility	Moment	Size	Value	Growth	NL Size	Liquidity	Leverage
Energy	-0.07	-0.14	-0.10	-0.04	-0.02	0.00	0.07	0.03
Materials	0.12	-0.28	-0.09	-0.01	0.10	0.02	-0.13	0.13
Industrials	0.13	-0.32	-0.11	0.35	-0.14	-0.09	-0.13	0.04
Consumer Discretionary	0.50	-0.51	-0.11	0.12	0.07	0.06	0.02	0.20
Consumer Staples	0.06	0.01	-0.06	0.11	-0.02	0.01	0.01	-0.02
Health Care	0.10	-0.12	0.02	0.07	-0.05	0.06	-0.07	0.02
Financials	1.12	-0.95	0.13	0.47	-0.13	0.17	0.07	0.33
Information Technology	0.09	-0.06	0.03	-0.01	-0.06	-0.03	-0.07	0.04
Telecommunications	-0.05	0.00	-0.02	0.08	-0.06	-0.06	0.05	0.04
Utilities	0.02	-0.06	0.01	0.08	-0.03	0.03	0.01	-0.04
Total	2.02	-2.43	-0.30	1.20	-0.34	0.16	-0.15	0.77



### | March 2010

#### **Contact Information**

clientservice@mscibarra.com

#### **Americas**

1.888.588.4567 (toll free)
+ 1.404.551.3212
+ 1.617.532.0920
+ 1.312.675.0545
+ 1.514.847.7506
+ 52.81.1253.4020
+ 1.212.804.3901
+ 1.415.836.8800
+ 55.11.3706.1360
+1.203.325.5630
+ 1.416.628.1007

#### Europe, Middle East & Africa

Amsterdam	+ 31.20.462.1382
Cape Town	+ 27.21.673.0100
Frankfurt	+ 49.69.133.859.00
Geneva	+ 41.22.817.9777
London	+ 44.20.7618.2222
Madrid	+ 34.91.700.7275
Milan	+ 39.02.5849.0415
Paris	0800.91.59.17 (toll free)
Zurich	+ 41.44.220.9300

#### **Asia Pacific**

China North	10800.852.1032 (toll free)
China South	10800.152.1032 (toll free)
Hong Kong	+ 852.2844.9333
Seoul	+ 827.0768.88984
Singapore	800.852.3749 (toll free)
Sydney	+ 61.2.9033.9333
Tokyo	+ 81.3.5226.8222

www.mscibarra.com

#### | March 2010

#### **Notice and Disclaimer**

- This document and all of the information contained in it, including without limitation all text, data, graphs, charts (collectively, the "Information") is the property of MSCI Inc. ("MSCI"), Barra, Inc. ("Barra"), or their affiliates (including without limitation Financial Engineering Associates, Inc.) (alone or with one or more of them, "MSCI Barra"), or their direct or indirect suppliers or any third party involved in the making or compiling of the Information (collectively, the "MSCI Barra Parties"), as applicable, and is provided for informational purposes only. The Information may not be reproduced or redisseminated in whole or in part without prior written permission from MSCI or Barra, as applicable.
- The Information may not be used to verify or correct other data, to create indices, risk models or analytics, or in connection with issuing, offering, sponsoring, managing or marketing any securities, portfolios, financial products or other investment vehicles based on, linked to, tracking or otherwise derived from any MSCI or Barra product or data.
- Historical data and analysis should not be taken as an indication or guarantee of any future performance, analysis, forecast or prediction.
- None of the Information constitutes an offer to sell (or a solicitation of an offer to buy), or a promotion or recommendation of, any security, financial product or other investment vehicle or any trading strategy, and none of the MSCI Barra Parties endorses, approves or otherwise expresses any opinion regarding any issuer, securities, financial products or instruments or trading strategies. None of the Information, MSCI Barra indices, models or other products or services is intended to constitute investment advice or a recommendation to make (or refrain from making) any kind of investment decision and may not be relied on as such.
- The user of the Information assumes the entire risk of any use it may make or permit to be made of the Information.
- NONE OF THE MSCI BARRA PARTIES MAKES ANY EXPRESS OR IMPLIED WARRANTIES OR REPRESENTATIONS WITH RESPECT TO THE INFORMATION (OR THE RESULTS TO BE OBTAINED BY THE USE THEREOF), AND TO THE MAXIMUM EXTENT PERMITTED BY LAW, MSCI AND BARRA, EACH ON THEIR BEHALF AND ON THE BEHALF OF EACH MSCI BARRA PARTY, HEREBY EXPRESSLY DISCLAIMS ALL IMPLIED WARRANTIES (INCLUDING, WITHOUT LIMITATION, ANY IMPLIED WARRANTIES OF ORIGINALITY, ACCURACY, TIMELINESS, NON-INFRINGEMENT, COMPLETENESS, MERCHANTABILITY AND FITNESS FOR A PARTICULAR PURPOSE) WITH RESPECT TO ANY OF THE INFORMATION.
- Without limiting any of the foregoing and to the maximum extent permitted by law, in no event shall any of the MSCI Barra Parties have any liability regarding any of the Information for any direct, indirect, special, punitive, consequential (including lost profits) or any other damages even if notified of the possibility of such damages. The foregoing shall not exclude or limit any liability that may not by applicable law be excluded or limited, including without limitation (as applicable), any liability for death or personal injury to the extent that such injury results from the negligence or wilful default of itself, its servants, agents or sub-contractors.
- Any use of or access to products, services or information of MSCI or Barra or their subsidiaries requires a license from MSCI or Barra, or their subsidiaries, as applicable. MSCI, Barra, MSCI Barra, EAFE, Aegis, Cosmos, BarraOne, and all other MSCI and Barra product names are the trademarks, registered trademarks, or service marks of MSCI, Barra or their affiliates, in the United States and other jurisdictions. The Global Industry Classification Standard (GICS) was developed by and is the exclusive property of MSCI and Standard & Poor's. "Global Industry Classification Standard (GICS)" is a service mark of MSCI and Standard & Poor's.

#### © 2010 MSCI Barra. All rights reserved.

#### **About MSCI Barra**

MSCI Barra is a leading provider of investment decision support tools to investment institutions worldwide. MSCI Barra products include indices and portfolio risk and performance analytics for use in managing equity, fixed income and multi-asset class portfolios.

The company's flagship products are the MSCI International Equity Indices, which include over 120,000 indices calculated daily across more than 70 countries, and the Barra risk models and portfolio analytics, which cover 59 equity and 48 fixed income markets. MSCI Barra is headquartered in New York, with research and commercial offices around the world.