Importance Sampling

Abstract

The paper propose and evaluate sampling-based methods which require network passes only for the observed "**positive example**" and a few sampled **negative example word**s.

Introduction

- Words can be **efficiently** represented by artificial neural network, where efficiently refers here to the statistical sense, meaning that **generalize well** and have **low perplexity**.
- However they are computationally **much more expensive** than n-grams.
- Goal : Speed up.

Neural Architecture for Representing High-Dimensional Distributions

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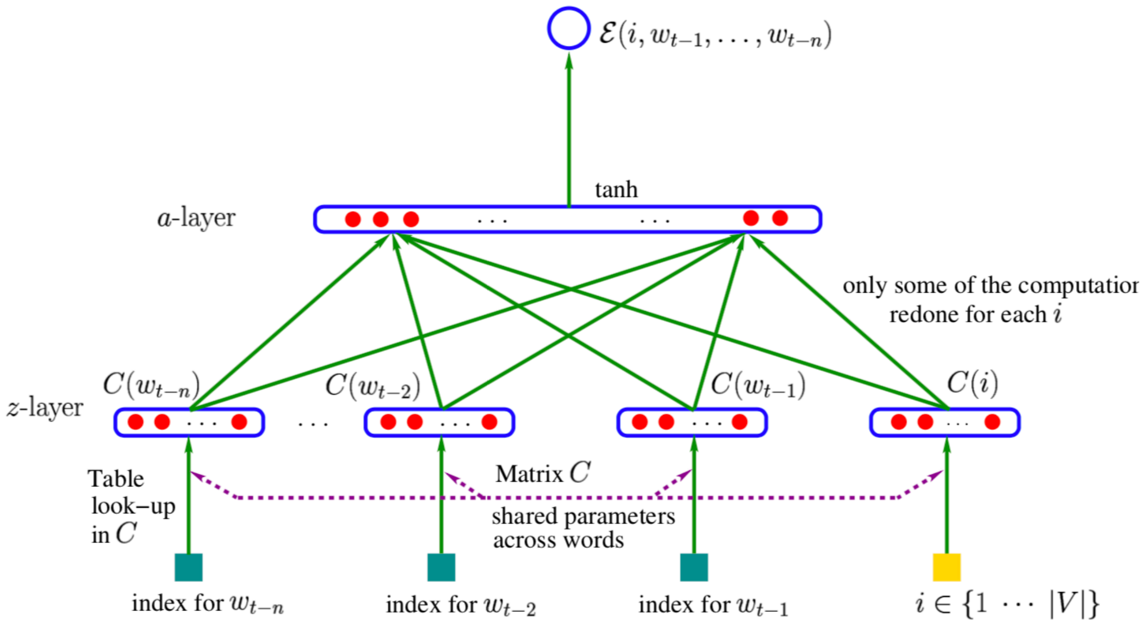
$$P(w_t = i \mid context) = e^{-\mathcal{E}(i,w_{t-1},\ldots,w_{t-n})} / \sum_j e^{-\mathcal{E}(j,w_{t-1},\ldots,w_{t-n})}$$
$$= \mathrm{softmax}(-\mathcal{E}(.,w_{t-1},\ldots,w_{t-n}))$$



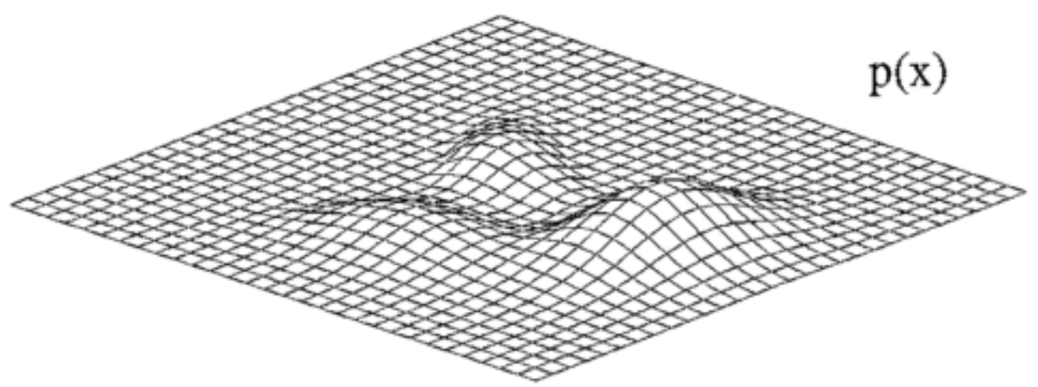
- The output of the neural network depends on **the next word** w_t and **the previous words** $h_t = (w_{t-1}, w_{t-2}, \ldots, w_{t-n})$ as follows.
- In the features layer, one maps each word w_{t-i} in $(w_{t-1}, w_{t-2}, \ldots, w_{t-n})$ to a lower-dimensional continuous subspace z_i: $z_i = C_{w_{t-i}}, i \in 0, 1, \ldots, n, z = (z_0, z_1, \ldots, z_n)$ C_j: the j-th column of the **word features** matrix of free parameters. z: the input vector for the the hidden layer. (the **concatenation** of the projections z_i)

- The hidden layer: $a = tanh(d + Wz)$ d: hidden units biases W: hidden layer weights a: a vector of hidden units activations.
- Finally, the output is a scalar energy function: $\varepsilon(w_t, h_t) = b_{w_t} + V_{w_t} \cdot a$ $\varepsilon()$ is a parametrized function which is **low** for plausible configurations of (w_t, h_t), and **high** for improbable ones.
- To obtain conditional probabilities, we **normalize** the exponentiated engergies:
  $P(w_t|h_t) = \frac{e^{-\varepsilon(w_t,h_t)}}{\sum_{w'} e^{-\varepsilon(w',h_t)}}$,
  $P(Y = y|X = x) = \frac{e^{-\varepsilon(y,x)}}{\sum_{y'} e^{-\varepsilon(y',x)}}$
- The difficulty with these energy models is in learning the parameters of the engergy function, without an explict computation of the partition function (normalizing denominator, 归一化分母).
- CD算法 (Contrastive Divergence, Hinton 2002) Contrastive Divergence is based on a sampling approximation of the log-likelihood gradient, $\frac{\partial logP(Y=y)}{\partial\theta}$. More generally, the gradient can be decomposed in two parts: positive reinforcement for $Y = y$ (the observed value) and negative reinforcement for every y', weighted by $P(Y = y')$, as follows.
  $\frac{\partial logP(Y=y)}{\partial\theta} = -\frac{\partial\varepsilon(y)}{\partial\theta} + \sum_{y'} P(y')\frac{\partial\varepsilon(y')}{\partial\theta}$
  Speed up: Replace the right-hand above weighted average by **Monte-Carlo samples**. How to sample: **Importance Sampling**

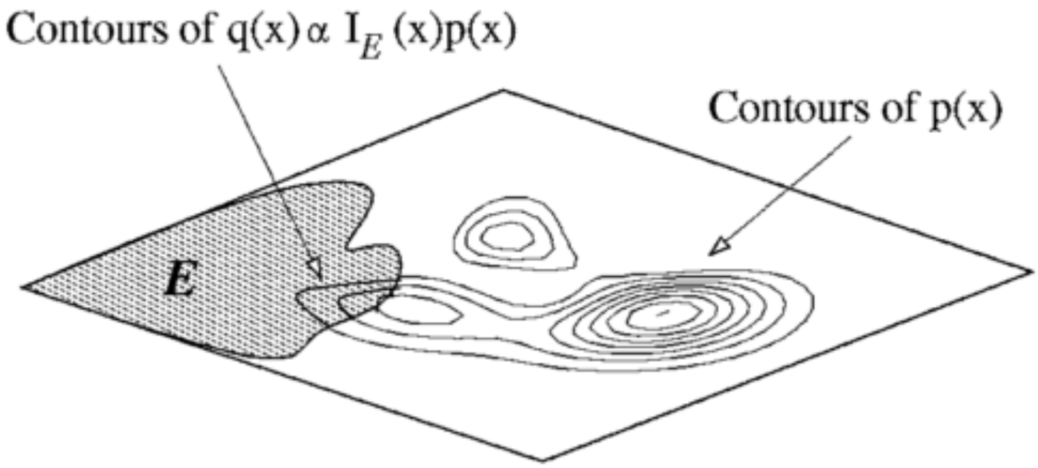
Importance Sampling Approximation

引用1:

> Importance Sampling 也是借助了容易抽样的分布q（proposal distribution）来解决原始分布p难以找到的问题。 $E[f] = \int f(z)p(z)\mathrm{d}z = \int f(z)\frac{p(z)}{q(z)}q(z)\mathrm{d}z \simeq \frac{1}{L}\sum_{l=1}^{L}\frac{p(z^{(l)})}{q(z^{(l)})}f(z^l)$ 其中 $\frac{p(z)}{q(z)}$ 可以看作importance weight。要引入什么样的分布q才会让采样的效果变得更好呢？直观的感觉是，样本的方差越小期望收敛速率越快。比如一次采样是 0, 一次采样是 1000, 平均值是 500,这样采样效果很差，如果一次采样是 499, 一次采样是 501, 你说期望是 500,可信度还比较高。在上式中，我们目标是 $p \times f/q$ 方差越小越好，所以 $|p \times f|$ 大的地方，proposal distribution $q(z)$ 也应该大。举个稍微极端的例子：



第一个图



> 表示 p 分布， 第二个图的阴影区域 $f=1$，非阴影区域 $f=0$, 那么一个良好的 q 分布应该在左边箭头所指的区域有很高的分布概率，因为在其他区域的采样计算实际上都是无效的。这表明 Importance Sampling 有可能比用原来的p 分布抽样更加有效。

引用2:

> 让我们回顾一下期望的求法 $E(f(x))=\int(p(x)*f(x))\mathrm{d}x$。那么，现在我们引入另一个概率分布s(x)，相比于p(x)，s(x)是非常简单能找到cdf的。那么我们变形一下E(f(x)) = sum(p(x) * f(x) / s(x) * s(x)) dx ， 再仔细看看，这个求f(x)的期望变成了，求在s(x)分布下，p(x)*f(x)/s(x)的期望。 重要性采样的关键就在这里，把对f(x)不好求的期望，变成了一个在另一个分布下相对好求的期望。

详见论文

参考文献

https://www.cnblogs.com/xbinworld/p/4266146.html
http://blog.csdn.net/tudouniurou/article/details/6277526
https://zhuanlan.zhihu.com/p/29934206 http://www.jianshu.com/p/22fb279aa16b