Contents

Pref	ace	vii
1 1 1 1 1	What is deductive logic? 1.1 What is an argument? 1.2 Kinds of evaluation 1.3 Deduction vs. induction 1.4 Just a few more examples 1.5 Generalizing 1.6 Summary 1.6 Exercises 1	1 1 2 4 5 7
2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	Validity and soundness 2.1 Validity defined 2.2 Consistency and equivalence 2.3 Validity, truth, and the invalidity principle 2.4 Inferences and arguments 2.5 'Valid' vs 'true' 2.6 What's the use of deduction? 2.7 An illuminating circle? 2.8 Summary Exercises 2	9 9 11 12 14 15 16 17 18
3 3	Forms of inference 3.1 More forms of inference 3.2 Four basic points about the use of schemas 3.3 Arguments can instantiate many patterns 3.4 Summary Exercises 3	20 20 22 24 26 26
4 4 4 4	Proofs 4.1 Proofs: first examples 4.2 Fully annotated proofs 4.3 Glimpsing an ideal 4.4 Deductively cogent multi-step arguments 4.5 Indirect arguments 4.6 Summary	28 28 30 32 33 35

vi		Contents
	Exercises 4	38
5	The counterexample method	39
	5.1 'But you might as well argue'	39
	5.2 The counterexample method, more carefully	40
	5.3 A 'quantifier shift' fallacy	41
	5.4 Summary	43
	Exercises 5	43
6	Logical validity	44
	6.1 Topic-neutrality	44
	6.2 Logical validity, at last	45
	6.3 Logical necessity	47
	6.4 The boundaries of logical validity?	47
	6.5 Definitions of validity as rational reconstructions	49
	6.6 Summary	50
	Exercises 6	51
7	Propositions and forms	52
	7.1 Types vs tokens	52
	7.2 Sense vs tone	52
	7.3 Are propositions sentences?	53
	7.4 Are propositions truth-relevant contents?	55
	7.5 Why we can be indecisive	55
	7.6 Forms of inference again	56
	7.7 Summary	58
	Exercises 7	58
In	terlude: From informal to formal logic	59
8	Three connectives	61
	8.1 Two simple arguments	61
	8.2 'And'	62
	8.3 'Or'	63
	8.4 'Not'	64
	8.5 Scope	65
	8.6 Formalization	66
	8.7 The design brief for PL languages	67
	8.8 One PL language	69
	8.9 Summary	70
	Exercises 8	71
9	,	
	9.1 Syntactic rules for PL languages	72
	9.2 Construction histories, parse trees	74
	9.3 Wffs have unique parse trees!	76
	9.4 Main connectives, subformulas, scope	77
	9.5 Bracketing styles	79

		Contents	
	9.6 Summary	79	
	Exercises 9	80	
10	PL semantics	81	
10	10.1 Interpreting wffs	81	
	10.2 Languages and translation	83	
	10.3 Atomic wffs are true or false	84	
	10.4 Truth values	85	
	10.5 Truth tables for the connectives	86	
	10.6 Evaluating molecular wffs: two examples	87	
	10.7 Uniqueness and bivalence	88	
	10.8 Short working	90	
	10.9 Summary	92	
	Exercises 10	92	
11	'P's, 'Q's, ' α 's, ' β 's – and form again	94	
	11.1 Styles of variable: object languages and metalar	_	
	11.2 Quotation marks, use and mention	95	
	11.3 To Quine-quote or not to Quine-quote?	98	
	11.4 How strict about quotation do we want to be?	99	
	11.5 Why Greek-letter variables?	100	
	11.6 The idea of form, again	101	
	11.7 Summary	102	
	Exercises 11	103	
12	Truth functions	104	
	12.1 Truth-functional vs other connectives	104	
	12.2 Functions and truth functions	105	
	12.3 Truth tables for wffs	107	
	12.4 'Possible valuations'	110	
	12.5 Summary	112	
	Exercises 12	112	
13	Expressive adequacy 113		
	13.1 Conjunction and disjunction interrelated	113	
	13.2 Exclusive disjunction	113	
	13.3 Another example: expressing the dollar truth fu	nction 114	
	13.4 Expressive adequacy defined	116	
	13.5 Some more adequacy results	117	
	13.6 Summary	118	
	Exercises 13	119	
14	Tautologies	120	
	14.1 Tautologies and contradictions	120	
	14.2 Generalizing examples of tautologies	122	
	14.3 Tautologies, necessity, and form	123	
	14.4 Tautologies as analytically true	125	

vii	i	Contents
	14.5 Summary Exercises 14	126 126
15	Tautological entailment 15.1 Three introductory examples 15.2 Tautological entailment defined 15.3 Brute-force truth-table testing 15.4 More examples 15.5 Ordinary-language arguments again 15.6 Tautological consistency and tautological validity 15.7 Summary Exercises 15	127 127 129 130 131 134 135 135
16	More about tautological entailment 16.1 Extending the notion of tautological entailment 16.2 Can there be a more efficient test? 16.3 Truth-table testing and the counterexample method 16.4 '\(\delta\'\) and '\(\text{.'}\'\) 16.5 Generalizing examples of tautological entailment 16.6 Tautological entailment and form 16.7 Tautological equivalence as two-way entailment 16.8 Summary Exercises 16	137 137 138 139 139 140 142 142 143
17	Explosion and absurdity 17.1 Explosion! 17.2 The falsum as an absurdity sign 17.3 Adding the falsum to PL languages 17.4 Summary Exercises 17	145 145 146 147 147
18	The truth-functional conditional 18.1 Some arguments involving conditionals 18.2 Four basic principles 18.3 Introducing the truth-functional conditional 18.4 Ways in which '→' is conditional-like 18.5 'Only if' 18.6 The biconditional 18.7 Extended PL syntax and semantics, officially 18.8 Contrasting '∴' and '⊨' and '→' 18.9 Summary Exercises 18	148 148 149 150 151 155 156 157 159 160
19	'lf's and '→'s 19.1 Types of conditional 19.2 Simple conditionals as truth-functional: for 19.3 Another kind of case where 'if' is truth-functional	162 162 164 165

		Contents
	19.4 Simple conditionals as truth-functional: against	166
	19.5 Three responses	167
	19.6 Adopting the material conditional	169
	19.7 Summary	170
	Exercises 19	171
Int	rerlude: Why natural deduction?	172
20	PL proofs: conjunction and negation	174
	20.1 Rules for conjunction	174
	20.2 Rules for negation	176
	20.3 A double negation rule	180
	20.4 A more complex proof: thinking strategically	182
	20.5 Understanding proofs, discovering proofs	183
	20.6 'Given'	183
	20.7 'We can derive'	185
	20.8 Putting things together	186
	20.9 Explosion and absurdity again	187
	20.10 Summary	189
	Exercises 20	190
21	PL proofs: disjunction	191
	21.1 The iteration rule	191
	21.2 Introducing and eliminating disjunctions	192
	21.3 The disjunction rules, a diagrammatic summary	196
	21.4 Two more proofs	197
	21.5 Disjunctive syllogisms	198
	21.6 Summary	202
	Exercises 21	202
22	PL proofs: conditionals	203
	22.1 Rules for the conditional	203
	22.2 More proofs with conditionals	206
	22.3 The material conditional again	208
	22.4 Summary	209
	Exercises 22	209
23	PL proofs: theorems	211
	23.1 Theorems	211
	23.2 Derived rules	213
	23.3 Excluded middle and double negation	214
	23.4 Summary	215
	Exercises 23	215
24	PL proofs: metatheory	216
	24.1 Metatheory	216
	24.2 Putting everything together	216
	24.3 Vacuous discharge	219

X			Contents
	24.4	Generalizing PL proofs	221
	24.5	'⊨' and '⊢'	221
	24.6	Soundness	222
	24.7	Completeness	224
	24.8	Excluded middle again	225
	24.9	Summary	227
Int	erlude	: Formalizing general propositions	228
25		es and predicates	230
		Names, and other 'terms'	230
		Predicates and their 'arity'	231
	25.3	Predicates, properties and relations	233
	25.4	Predicates: sense vs extension	234
		Sets	235
		Names: sense vs reference	236
	25.7	Reference, extension, and truth	237
	25.8	Summary	239
26		tifiers in ordinary language	240
		Which quantifiers?	240
		Every/any/all/each	241
		Quantifiers and scope	243
		Fixing domains	247
		Summary	248
	Exer	cises 26	248
27		tifier-variable notation	249
		Quantifier prefixes and 'variables' as pronouns	249
		Unary vs binary quantifiers	251
		Domains	252
		Quantifier symbols	253
		Unnamed objects	255
		A variant notation	256
	27.7	Summary	257
28	QL la	inguages	258
	28.1	QL languages – a glimpse ahead	258
	28.2	Names, predicates and atomic wffs in QL: syntax	259
	28.3	Names, predicates, and atomic wffs in QL: interpretation	260
	28.4	One example: introducing QL_1	261
	28.5	Adding the connectives	262
	28.6	Syntax for the quantifiers	262
	28.7	An aside on scope again	265
	28.8	Interpreting the quantifiers	266
	28.9	Quantifier equivalences	268
	28.10	Summary	270

_		Contents
	Exercises 28	270
29	Simple translations	271
	29.1 Restricted quantifiers revisited	271
	29.2 Existential import	273
	29.3 'No'	274
	29.4 Translating via Loglish	275
	29.5 Translations into QL ₂	276
	29.6 Moving quantifiers	279
	29.7 Summary	280
	Exercises 29	281
30	More on translations	282
	30.1 More translations into QL ₂	282
	30.2 Translations from QL ₂	285
	30.3 Choosing a domain	285
	30.4 'Translation' and 'logical form'	286
	30.5 Summary	288
	Exercises 30	289
Int	erlude: Arguing in QL	290
31	Informal quantifier rules	293
	31.1 Arguing with universal quantifiers	293
	31.2 Arguing with existential quantifiers	295
	31.3 Summary	297
	Exercises 31	298
32	QL proofs	299
	32.1 Dummy names in QL languages	299
	32.2 Schematic notation, and instances of quantified wffs	301
	32.3 Inference rules for ' \forall '	302
	32.4 Inference rules for '∃'	304
	32.5 Quantifier equivalences	308
	32.6 QL theorems	311
	32.7 Summary	313
	Exercises 32	314
33	More QL proofs	315
	33.1 The QL rules again	315
	33.2 How to misuse the QL rules	316
	33.3 Old and new logic: three proofs	319
	33.4 Five more QL proofs	322
	33.5 Summary	327
	Exercises 33	327
34	Empty domains?	329
	34.1 Dummy names and empty domains	329

xii		Contents
	34.2 Preserving standard logic 34.3 Summary	331 332
35	Q-valuations 35.1 QL syntax again 35.2 Q-parse trees 35.3 Evaluating quantified wffs 35.4 The official valuational semantics 35.5 Toy examples 35.6 Uniqueness of values 35.7 The structure of valuations 35.8 Summary Exercises 35	333 333 334 336 339 340 342 343 344
36	Q-validity 36.1 Q-validity defined 36.2 'All q-valuations' 36.3 Establishing q-validity/q-invalidity: the headlines 36.4 We can mechanically test for q-validity in simple cases 36.5 'Working backwards' 36.6 The Entscheidungsproblem 36.7 Generalizing again 36.8 Summary Exercises 36	346 346 347 348 349 350 351 352 353
37	QL proofs: metatheory 37.1 The QL proof system reviewed 37.2 Generalizing QL proofs 37.3 Two turnstiles again 37.4 Soundness 37.5 Completeness 37.6 Summary	354 354 355 356 356 357 358
Int	rerlude: Extending QL	359
38	Identity 38.1 Numerical vs qualitative identity 38.2 Equivalence relations 38.3 Identity as the smallest equivalence relation 38.4 Leibniz's Law 38.5 Summary Exercises 38	361 361 363 364 365 366
39	QL ⁼ languages 39.1 '=' as the identity predicate 39.2 Translating into QL ⁼ 39.3 Numerical quantifiers 39.4 Existence claims	367 367 368 370 372

		Contents
	39.5 Summary	373
	Exercises 39	374
40	Definite descriptions	375
	40.1 The project	375
	40.2 Russell's Theory of Descriptions	375
	40.3 Descriptions and existence	377
	40.4 Descriptions and scope	378
	40.5 More translations	380
	40.6 Summary	381
	Exercises 40	381
41	QL= proofs	382
	41.1 Two derivation rules for identity	382
	41.2 More examples	384
	41.3 One and one makes two	387
	41.4 Metatheoretical headlines	388
	41.5 Summary	389
	Exercises 41	390
42	Functions	391
-	42.1 Functions, informally again	391
	42.2 Function symbols, syntax	392
	42.3 Function symbols, semantics	394
	42.4 Functions, functional relations, and definite descriptions aga	
	42.5 Proofs involving functions	397
	42.6 ω -incompleteness!	400
	42.7 And where now?	400
	42.8 Summary	401
	Exercises 42	401
Δn	pendix: Soundness and completeness	402
, ιρ	A1 Soundness for PL	402
	A2 Soundness for QL	404
	A3 Completeness: what we want to prove	405
	A4 PL completeness proved	406
	A5 QL completeness proved	407
	A6 A squeezing argument	411
Th	e Greek alphabet	412
	rther reading	413
Inc	lex	415