

# Summary: Unit 2 Lecture 1 Review L'Hôpital's Rule

## L'Hôpital's Rule Version 1: Indeterminate form $\frac{0}{0}$

If

$$\begin{aligned} f(x) &\rightarrow 0 \\ g(x) &\rightarrow 0 \end{aligned} \quad \text{as } x \rightarrow a,$$

and the functions  $f$  and  $g$  are differentiable near the point  $x = a$ , then limit

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} \quad (1)$$

provided that the right hand limit exists or is  $\pm\infty$ .

## L'Hôpital's Rule Version 2: Indeterminate form $\frac{\infty}{\infty}$

If

$$\begin{aligned} f(x) &\rightarrow \pm\infty \\ g(x) &\rightarrow \pm\infty \end{aligned} \quad \text{as } x \rightarrow a,$$

and the functions  $f$  and  $g$  are differentiable near the point  $x = a$ , then limit

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} \quad (2)$$

provided that the right hand limit exists or is  $\pm\infty$ .

Note that

- We can replace  $a$  with  $a^+$  or  $a^-$  and the results (versions 1 and 2) still hold.
- We can replace  $a$  with  $\pm\infty$ , and the results (versions 1 and 2) still hold.

## Other indeterminate forms

Other indeterminate forms  $0 \cdot \infty$ ,  $\infty - \infty$ ,  $0^0$ ,  $1^\infty$ , and  $\infty^0$  should be rearranged to be of the form  $0/0$  or  $\infty/\infty$  in order to apply l'Hôpital's rule.