Peter Smith, *Introduction to Formal Logic* (CUP, 2nd edition)

Exercises 29: Simple QL translations

In the language QL₁, the *proper names* are just

      m: Socrates,
      n: Plato,
      o: Aristotle;

and the *predicates* are just

      F: ① is a philosopher,
      G: ① is a logician,
      H: ① is wise,
      L: ① loves ②,
      M: ① is a pupil of ②,
      R: ① prefers ② to ③.

The *domain of quantification* for QL₁: people, past and present.

(a)   Translate the following into QL₁ as best you can:

  (1)   Aristotle is a pupil of Socrates and Plato.
  (2)   Plato taught Aristotle only if Socrates taught Plato.
  (3)   If Plato is a pupil of someone, he is a pupil of Socrates.
  (4)   Some philosophers are not wise.
  (5)   Not all philosophers are wise.
  (6)   Any logician loves Aristotle.
  (7)   No one who is a wise philosopher prefers Plato to Aristotle.
  (8)   Whoever is a pupil of Plato is wise.
  (9)   Not every wise logician is a pupil of Aristotle.
 (10)   Any logician is a wise philosopher.
 (11)   Aristotle prefers no philosopher to Plato.
 (12)   Some wise people aren't philosophers, and some aren't logicians.
 (13)   Only philosophers love Aristotle.
 (14)   Not only philosophers love Socrates.
 (15)   Socrates is a philosopher whom everyone wise loves.
 (16)   Only some logicians love Plato.
 (17)   No philosophers or logicians are wise.
 (18)   All philosophers and logicians are wise.

(b)   Which of these pairs of wffs are equivalent, which not, and why?

  (1) $(\exists x Fx \lor \exists x Gx), \exists x(Fx \lor Gx)$
  (2) $(\exists x Fx \land \exists x Gx), \exists x(Fx \land Gx)$
  (3) $(\exists x Fx \land \exists x Gx), \exists x(Fx \land \exists y Gy)$
  (4) $(\exists x Fx \land \exists x Gx), \exists x \exists y(Fx \land Gy)$
  (5) $\exists x \forall y(Fx \land Gy), \forall y \exists x(Fx \land Gy)$
  (6) $\exists x \forall y(Fx \lor Gy), \forall x \exists y(Fy \lor Gx)$
  (7) $(\forall x Fx \rightarrow \forall x Gx), \exists x(Fx \rightarrow \forall y Gy)$
  (8) $(\forall x Fx \rightarrow \forall x Gx), \exists x \forall y(Fx \rightarrow Gy)$
  (9) $(\forall x Fx \rightarrow \forall x Gx), \forall y \exists x(Fx \rightarrow Gy)$

(c*)   Use equivalences you now know about to outline a proof that every wff is equivalent to one in *prenex* form, where all quantifiers are at the beginning of the wff.

(d*)   We can render 'Plato and Aristotle are philosophers' by e.g. (Fn ∧ Fo). Why can't we render

'Plato and Aristotle are classmates' by e.g. $(\mathsf{Cn} \wedge \mathsf{Co})$? What does this sort of case tell us about some expressive limitations of QL languages?