Peter Smith, *Introduction to Formal Logic* (CUP, 2nd edition)

Exercises 8: Negation and other connectives

(a) *Usually, we can ambiguously negate a statement by prefixing it with 'It is not the case that'. Can you think of any exceptions?*

There are, trivially, examples which use a different sense of 'case' (as in *luggage*, or *box of wine*): thus compare the pair

1. I usually pack for a weekend.
2. It is not the case that I usually pack for a weekend.

and likewise

3. I ordered from the wine shop yesterday.
4. It is not the case that I ordered from the wine shop yesterday.

There's a natural reading of (2) in which it isn't the negation of (1), and a natural reading of (4) in which it isn't the negation of (3).

Still, it might be said that those examples are cheats! – the intended question was surely: does 'It is not the case that' *still used as meaning 'it is not true that'*, when prefixed to a sentence, always unambiguously express the negation of what the original sentence says. (To be sure, that wasn't originally spelt out. But equally, to use an example of Wittgenstein's, if I say "show the children a game" and you teach them to gamble at poker, I might reasonably enough complain "I didn't mean *that* sort of game" even if I didn't actually have that exclusion in mind when I made my request.)

So let's set aside those examples. What about

5. This sentence has less than ten words.
6. It is not the case that this sentence has less than ten words.

At least in (6) 'it is not the case that' is being used with the intended meaning! However, this time, prefixing those words changes the topic of the whole claim by changing the reference of 'this sentence'. Not so much of a cheat, perhaps, but still an anomalous case, we might reasonably think: we were thinking of prefixing 'it is not the case that' in its usual sense, while also not changing the content of what follows.

So what about examples where 'it is not the case that' retains its equivalence to 'it is not true that', and the topic of the sentence it is applied to stays the same? How can prefixing 'it is not the case that' then fail to unambiguously negate what it is applied to?

One way is by it not being clear how much of what follows the prefix *does* apply to. Thus in *IFL2*, p. 65, we considered the pair:

7. Jack loves Jill and it is not the case that Jill loves Jack.
8. It is not the case that Jack loves Jill and it is not the case that Jill loves Jack.

In (8), on the more natural reading, the first 'it is not the case that' applies to just the clause 'Jack loves Jill'. So both (7) and (8) are *false* if Jill *does* love Jack. What is happening here is that 'it is not the case' hasn't changed its meaning, isn't changing the topic of what follows, but its scope isn't (or at least, isn't unambiguously) the whole of the sentence it is being applied to.

What about this sort of pair?

9. Oedipus, who killed his father at the crossroads, was guilty of murder.
10. It is not the case that Oedipus, who killed his father at the crossroads, was guilty of murder.

It might be said that both imply that Oedipus killed his father at the crossroads: so if Oedipus hadn't killed his father, both would have been false (but is this right?). However, if (9) and (10) can be therefore be false together, one is not the negation of the other. Again, this is like a scope phenomenon – the clause 'who killed his father at the crossroads' is insulated from the negation in (10).

We've mentioned reductio arguments. Sometimes we show that some assumption A has to be false by first arguing that *if A then C*, and then arguing that we also have *if A then it* isn't *the case that C*. And since C and its negation can't be true together, we conclude that A is false. (For example, the Exercises 4 proof in effect shows that if $\sqrt{2} = m/n$, a fraction in lowest terms, then m is even, and if $\sqrt{2} = m/n$ then m is not even, and concludes that $\sqrt{2}$ isn't a fraction.)

Only slightly re-arranging, we have propositions of the form

11. *C, if A*
12. *It is not that case that C, if A*

both true together. So one isn't the negation of the other. Again, this is a scope phenomenon – there is a natural reading on which the initial 'it is not the case' doesn't apply past the comma.

Are there examples where prefixing 'it is not the case that' (with the usual meaning) doesn't unambiguously negate what follows which *aren't* to explained as arising from scope phenomena? Well, what about this pair?

13. Anyone can run a mile in four minutes.
14. It is not the case that anyone can run in a mile in four minutes.

(13) is false: most of us are far too slow! But on one natural reading, (14) is also false – any elite middle distance runner can run a sub four minute mile these days.

Note though that, like (8), (14) is ambiguous; especially if the 'anyone' is stressed, it can also be construed as the simple negation of (13). And as we will see later in *IFL2*, the ambiguity here can also be thought of as of scope-ambiguity. We can represent the two readings of (14) like this:

(14′) (Anyone is such that)(it is not the case that) they can run a mile in four minutes.
(14″) (It is not the case that)(anyone is such that) they can run a mile in four minutes.

(14′) represents the reading of (14) which is not the negation of (13). And thought of like that, we can regard the superficially prefixed 'it is not the case that' in (14) as not really governing everything that follows it.

(b) *Give negations of the following in natural English:*

(1) *It is not the case that both Jack and Jill went up the hill.*
Jack and Jill went up the hill.
(2) *Neither Jack nor Jill went up the hill.* Jack or Jill (or both) went up the hill.
(3) *No one loves Jack.* Someone loves Jack.
(4) *Only tall men love Jill.* Someone who isn't tall man loves Jill.
(5) *Everyone who loves Jack admires Jill.* Someone loves Jack but doesn't admire Jill.
(6) *Someone loves both Jack and Jill.* No one loves both Jack and Jill.
(7) *Some who love Jill are not themselves loveable.*
Everyone who loves Jill is loveable.
(8) *Jill always arrives on time.* Jill sometimes does not arrive on time.
(9) *Whoever did that ought to pay for the damage.*
There's someone who did that but who needn't pay for the damage.
(10) *Whenever it rains, it pours.* Sometimes it rains without pouring.
(11) *No one may smoke.* At least one person may smoke.

(c) *Two propositions are* contraries *if they cannot be true together; they are* contradictories *if one is true exactly when the other is false. (Example: 'All philosophers are wise' and 'No philosophers are wise' are contraries – they can't both be true. But maybe they are both false, so they are not contradictories.) Give examples of propositions which are contraries but not contradictories of the propositions in (b).*

Evidently there are going to many possible answers. Here are just some suggestions:

(1)	*It is not the case that both Jack and Jill went up the hill.*	Jack, Jill and Jo went up the hill.
(2)	*Neither Jack nor Jill went up the hill.*	Jack went up the hill.
(3)	*No one loves Jack.*	Jill loves Jack.
(4)	*Only tall men love Jill.*	Only short men love Jill.
(5)	*Everyone who loves Jack admires Jill.*	Jack loves himself but doesn't admire Jill.
(6)	*Someone loves both Jack and Jill.*	No woman loves both Jack and Jill.
(7)	*Some who love Jill are not themselves loveable.*	Everyone who loves Jill is a loveable man.
(8)	*Jill always arrives on time.*	Jill never arrives on time.
(9)	*Whoever did that ought to pay for the damage.*	No one need pay for the damage.
(10)	*Whenever it rains, it pours.*	It never pours when it rains.
(11)	*No one may smoke.*	Everyone may smoke.

(d) *Render the following as best you can into the* PL *language with the following glossary:*

P: *Jack loves Jill.*
Q: *Jill loves Jack.*
R: *Jo loves Jill.*
S: *Jack is wise.*

(1)	*Jack is unwise and loves Jill.*	$(\neg S \land P)$
(2)	*Jack and Jo both love Jill.*	$(P \land R)$
(3)	*It isn't true that Jack doesn't love Jill.*	$\neg\neg P$
(4)	*Jack loves Jill but Jo doesn't.*	$(P \land \neg R)$
(5)	*Jack doesn't love Jill, neither is he wise.*	$(\neg P \land \neg S)$
(6)	*Either Jack loves Jill or Jill loves Jack.*	$(P \lor Q)$
(7)	*Either Jack loves Jill or Jill loves Jack, but not both.*	$((P \lor Q) \land \neg(P \land Q))$
(8)	*Either Jack is unwise or he loves Jill and Jo loves Jill.*	Ambiguous! Either $((\neg S \lor Q) \land R)$ or $(\neg S \lor (Q \land R))$