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Assignment - II

Q1

Box A: 2 White, 4 Black
Box B: 5 white, 7 Black

E_1: White ball transferred from bag A to B.
E_2: Black ball transferred from bag A to B.

E: White ball drawn from bag B.

$$P(E) = P(E_1 \cap E) + P(E_2 \cap E)$$
$$= P(E_1) \times P(E/E_1) + P(E_2) \times P(E/E_2)$$
$$= \frac{2}{6} \times \frac{6}{13} + \frac{4}{6} \times \frac{5}{13}$$
$$= \frac{12+20}{13 \times 6} = \frac{\cancel{32}\,16}{\cancel{78}\,39} = \frac{16}{39} = 0.410$$

Q2.

E_1: Toy manufactured from machine A $\quad P(E_1) = 0.25$
E_2: " " " " B $\quad P(E_2) = 0.35$
E_3: " " " " C $\quad P(E_3) = 0.40$

E: Toy is defective.
$P(E/E_1) = 0.05$, $\quad P(E/E_2) = 0.04$, $\quad P(E/E_3) = 0.02$

$$P(E) = P(E \cap E_1) + P(E \cap E_2) + P(E \cap E_3)$$
$$= P(E_1) \times P(E/E_1) + P(E_2) \times P(E/E_2) + P(E_3) \times P(E/E_3)$$
$$= 0.25 \times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02$$
$$= 0.0125 + 0.0140 + 0.008$$
$$= 0.0345$$

PTO.

$$P(E_1/E) = \frac{P(E_1) \times P(E/E_1)}{P(E)} = \frac{0.25 \times 0.05}{0.0345} = \underline{0.362}$$

Q3

$$p(x) = \begin{cases} \frac{kx}{20} & , x = 1,2,3,4 \\ 0 & , \text{otherwise.} \end{cases}$$

$$\sum_{x=1}^{\infty} p(x) = 1 \qquad \text{(Property of Probability function)}$$

$$\therefore \left[p(x=1) + p(x=2) + p(x=3) + p(x=4) + 0 + 0 \ldots \right] = 1$$

$$= \frac{k}{20} + \frac{2k}{20} + \frac{3k}{20} + \frac{4k}{20} = 1$$

$$10k = 20$$

$$\underline{\underline{k = 2}}$$

Q4

$$f(x) = \begin{cases} kx & ; 0 < x < 5 \\ k(10-x) & ; 5 \leq x < 10 \\ 0 & ; \text{otherwise.} \end{cases}$$

(i) $$\int_{-\infty}^{\infty} f(x)\,dx = 1 \qquad \text{(Property)}$$

$$\Rightarrow \int_0^5 f(x)\,dx + \int_5^{10} f(x)\,dx = 1$$

$$\int_0^5 kx\,dx + \int_5^{10} k(10-x)\,dx = 1 \Rightarrow k\left[\left[\frac{x^2}{2}\right]_0^5 + \left[10x - \frac{x^2}{2}\right]_5^{10} \right] = 1$$

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$$k\left[\frac{25}{2} + \left[\left(100 - \frac{100}{2}\right) - \left(50 - \frac{25}{2}\right)\right]\right] = 1$$

$$k\left[\frac{25}{2} + \cancel{100} - \cancel{50} - \cancel{50} + \frac{25}{2}\right] = 1 \quad \Rightarrow \quad k = \frac{1}{25}$$

(ii) $$E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_0^5 kx^2\,dx + \int_5^{10} k(10x - x^2)\,dx = \frac{1}{25}\left[\left[\frac{x^3}{3}\right]_0^5 + \left[\frac{10x^2}{2} - \frac{x^3}{3}\right]_5^{10}\right]$$

$$= \frac{1}{25}\left[\frac{125}{3} + \left(\frac{1000}{2} - \frac{1000}{3}\right) - \left(\frac{250}{2} - \frac{125}{3}\right)\right]$$

$$= \frac{1}{25}\left[\frac{-750}{3} + \frac{750}{2}\right] = \frac{30}{6} = 5$$

(iii) $$P(5 < x \le 12) = \int_5^{12} f(x)\,dx$$

$$= \int_5^{10} f(x)\,dx + \int_{10}^{12} 0\,dx \Rightarrow k\int_5^{10} (10 - x)\,dx = k\left[10x - \frac{x^2}{2}\right]_5^{10}$$

$$= k\left[\left(100 - \frac{100}{2}\right) - \left(50 - \frac{25}{2}\right)\right] = \frac{1}{25} \times \frac{25}{2} = \frac{1}{2}$$

Q5

$a = 4$

$\mu_1' = 1$

$\mu_2' = 4$

$\mu_3' = 10$

$\mu_4' = 45$

$\rightarrow$ Mean: $\mu_1' + a = 1 + 4 = 5$

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$\mu_1 = 0$

$\mu_2 = \mu_2' - \mu_1'^2 = 4 - (1)^2 = 3$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 = 10 - 3(4)(1) + 2(1)^3 = 0$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4 = 45 - 4(10)(1) + 6(4)\ldots$

$= 26$

Skewness	Kurtosis
$\beta_1 = \frac{\mu_3^2}{\mu_2^2} = 0$	$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{26}{9} = 2.88$

Here, the distribution is symmetrical and platykurtic.

Q6

$p(x) = \frac{1}{2^x}, \quad x = 1, 2, 3, \ldots$

$M_0(t) = \sum_x e^{tx} \times \frac{1}{2^x}, \quad x = 1, 2, 3, \ldots$

$= \sum_x \left(\frac{e^t}{2}\right)^x = \frac{e^t}{2} + \left(\frac{e^t}{2}\right)^2 + \left(\frac{e^t}{2}\right)^3 + \ldots$

$= \frac{\frac{e^t}{2}}{1 - \frac{e^t}{2}} \Rightarrow M_0(t) = \frac{e^t}{2 - e^t}; \quad |e^t| < 0$

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Q7

$f(x) = \frac{1}{c} e^{-x/c}$, $0 \le x < \infty$, $c > 0$

$$M_0(t) = \int_0^\infty e^{tx} \times \frac{1}{c} e^{-x/c}\, dx = \frac{1}{c}\int_0^\infty e^{x(t-\frac{1}{c})}\, dx$$

$$= \frac{1}{c} \times \frac{c}{(ct-1)} \left[e^{x(t-1/c)} \right]_0^\infty = \frac{1}{(ct-1)}(0-1) = \frac{1}{1-ct}$$

$M_0(t) = (1-ct)^{-1}$; $ct < 1$

$= 1 + ct + c^2t^2 + c^3t^3 + \cdots$

$\mu_1' =$ Coeff. of t in the expansion of $M_0(t) = c$

$\mu_2' =$ Coeff. of $t^2/2!$ in the expansion of $M_0(t) = 2c^2$

Mean $= \mu_1' + a = c + 0 = \underline{\underline{c}}$

$(S.D)^2 = \mu_2 = \mu_2' - \mu_1'^2 = 2c^2 - c^2 = c^2$

$\underline{S.D = c}$

Q8

E_1: Bomb dropped will hit the plane

$P(E_1) = 0.2 = p$

$q = 1 - p = 0.8$

$n = 6$

(i) $P(x=2) = {}^6C_2 (p)^2 (q)^4 = \frac{6!}{4!\,2!} \times (0.2)^2 \times (0.8)^4 = 15 \times 0.0163$

$= 0.246$

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(ii) $P(x \geq 2) = 1 - P(x < 2)$

$= 1 - [P(x=0) + P(x=1)]$ ~~reject~~

$= 1 - [{}^6C_0 (p)^0 (q)^6 + {}^6C_1 (p)^1 (q)^5]$

$= 1 - [(0.8)^6 + 6 \times 0.2 \times (0.8)^5]$

$= 1 - (0.262 + 0.393)$

$= 1 - 0.655$

$= 0.345$

Q9. For Binomial distribution,

$$P(x=r) = {}^nC_r (p)^r (q)^{n-r}$$

$$= \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!} \times p^r \cdot (1-p)^{n-r}$$

$$= \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!} \times \left(\frac{\lambda}{n}\right)^r \times \left(1-\frac{\lambda}{n}\right)^{n-r}$$

$$= \frac{\lambda^r}{r!} \frac{n(n-1)(n-2)\ldots(n-r+1)}{n^r} \times \left(1-\frac{\lambda}{n}\right)^{n-r}$$

$$= \frac{\lambda^r}{r!} \underbrace{\left(\frac{n}{n}\right)\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\left(1-\frac{(r-1)}{n}\right)}_{\text{Tending to 1 as } n \to \infty} \times \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^r}$$

$= \cancel{\frac{\lambda^r}{r!}}$ As $n \to \infty$,

$\left(1-\frac{1}{n}\right), \left(1-\frac{2}{n}\right) \ldots \left(1-\frac{(r-1)}{n}\right)$ tends to 1,

Also $\left(1-\frac{\lambda}{n}\right)^r \to 1$

$$= \frac{\lambda^r}{r!} \left[\left(1-\frac{\lambda}{n}\right)^{-n/\lambda}\right]^{-\lambda} \Rightarrow \frac{\lambda^r e^{-\lambda}}{r!}$$

$$P(x=r) = \frac{\lambda^r e^{-\lambda}}{r!} \quad (r = 0, 1, 2, 3, \ldots)$$

(b)

$p = 0.001$

$n = 2000$

$\lambda = np = \underline{\underline{2}}$

$P(X>2) = 1 - P(X \le 2)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - \left[\frac{e^{-2} \times 2^0}{0!} + \frac{e^{-2} \times (2)^1}{1!} + \frac{e^{-2} \times (2)^2}{2!}\right]$

$= 1 - e^{-2}(1 + 2 + 2) = 1 - 5e^{-2}$

$= 1 - 0.676$

$= 0.323$

Q10

$P(X<35) = 0.07$

$P(X<63) = 0.89$

Given: $P(0 \le Z \le 0.18) = 0.07$

$P(0 \le Z \le 1.48) = 0.43$

$P(0 \le Z \le 1.23) = 0.39$

$Z = \frac{X-\mu}{\sigma}$

→ Pf $Z_1 = \frac{X_1 - \mu}{\sigma}$ $\qquad$ $Z_2 = \frac{X_2 - \mu}{\sigma}$

(Where, $Z = Z_1$ at $X = 35$) $\qquad$ (where, $Z = Z_2$ at $X = 63$)

$Z_1 = \frac{35-\mu}{\sigma}$ $\qquad$ $Z_2 = \frac{63-\mu}{\sigma}$

$P(X<35) = P(Z<Z_1)$ $\qquad$ $P(X>63) = P(Z>Z_2)$

$P(Z<Z_1) = 0.07$ $\qquad$ $P(Z>Z_2) = 1-0.89 = 0.11$

$P(Z > -Z_1) = 0.07$

$0.5 - P(0 \le Z \le -Z_1) = 0.07$

$P(0 \le Z \le Z_1) = 0.5 - 0.07 = 0.43$

$-Z_1 = 1.48$

$P(Z > +Z_2)$

$P(X > 63) = \cancel{0.89}\ 0.11$

$0.5 - P(0 \le Z \le +Z_2) = \cancel{0.89}\ 0.11$

$P(0 \le Z \le +Z_2) = 0.5 - \cancel{0.89}\ 0.11$

$Z_2 = 1.23$

$Z_1 = \dfrac{x - \mu}{\sigma}$

$-1.48 = \dfrac{35 - \mu}{\sigma}$

$\mu - 1.48\sigma = 35$ — (1)

$Z_2 = \dfrac{x - \mu}{\sigma}$

$1.23 = \dfrac{63 - \mu}{\sigma}$

$\mu + 1.23\sigma = 63$ — (2)

(2) − (1),

$2.71\sigma = 28 \Rightarrow \sigma = \underline{10.33}$

$\mu = 63 - 12.706 = \underline{\underline{50.294}}$