Hints on Q2 and Q3 of Deep Learning Quiz

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It came to our attention that many students had trouble with Q2 and Q3 of Deep Learning Quiz. So we decided to provide some helpful hints.

*** Q2**: (paraphrasing a forum post posted 2 months ago)

For this one, you want to visualize a unit cube where each vertex represents a possible value of (x1,x2,x3). For instance, say we want a linear classifier for x1 AND x2 AND NOT x3.

First we list all combinations of (x1,x2,x3) and corresponding outputs:

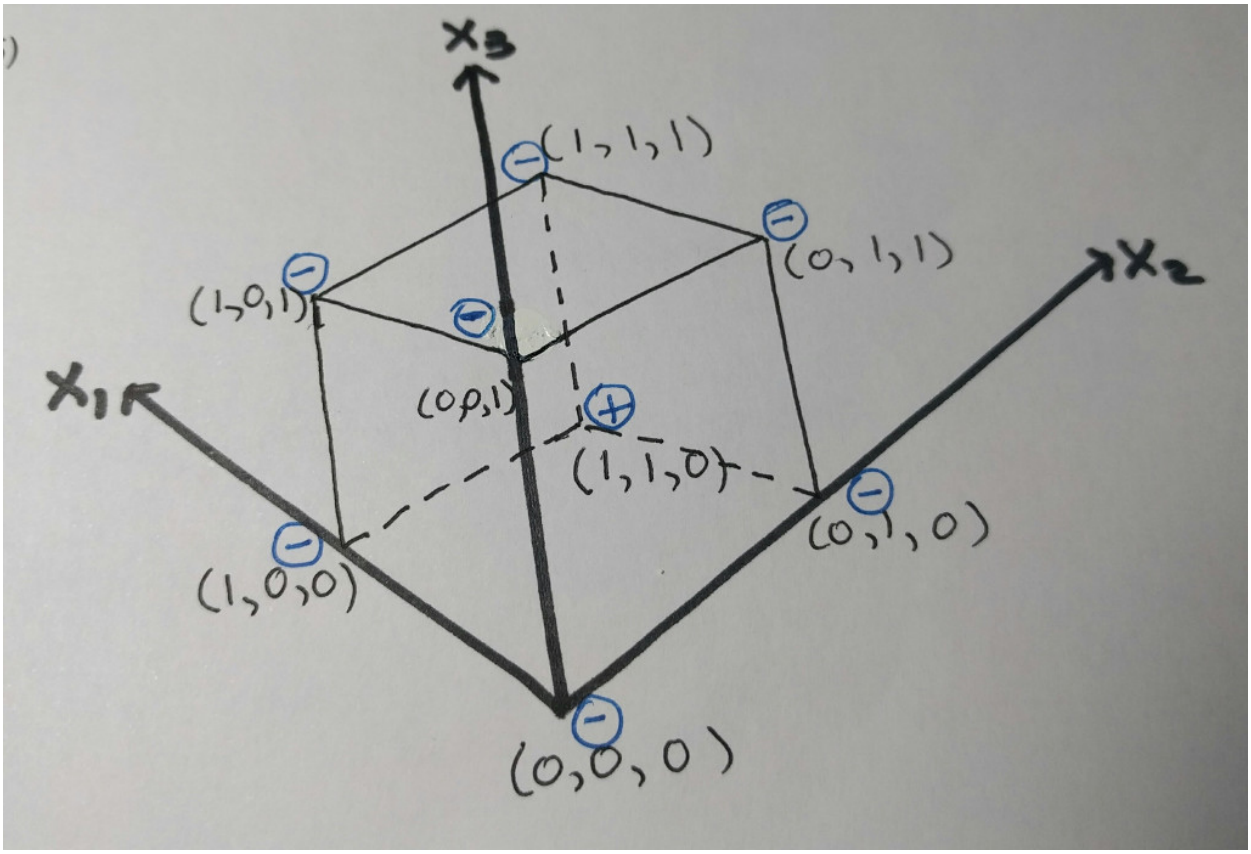
```
x1    x2    x3    | x1 AND x2 AND (NOT x3)

0     0     0     | 0

0     0     1     | 0

0     1     0     | 0

0     1     1     | 0

1     0     0     | 0

1     0     1     | 0

1     1     0     | 1

1     1     1     | 0
```

Now think of (x1,x2,x3) as coordinates in 3D space, and think of x1 AND x2 AND x3 as +/- label:

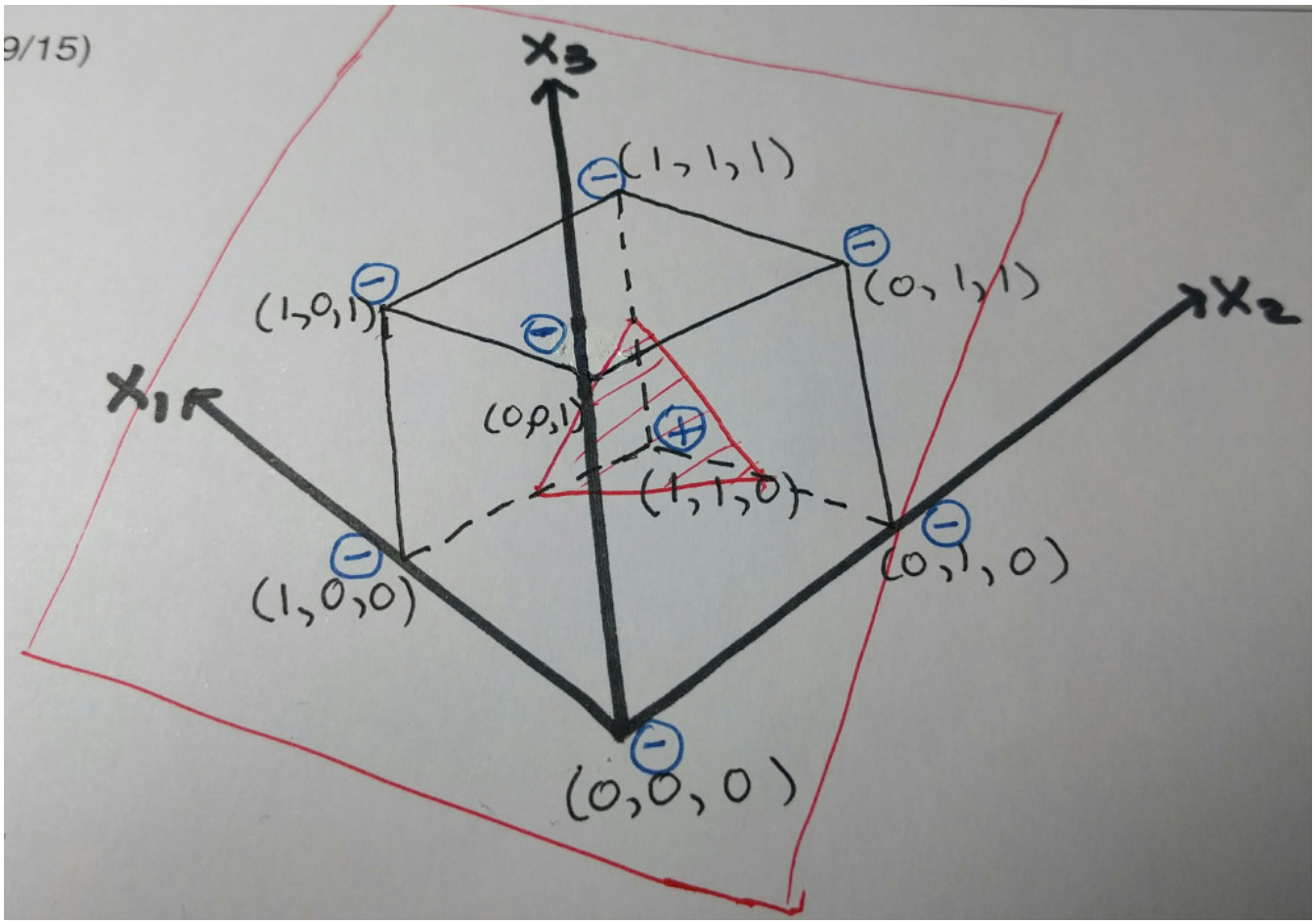
```
(x1,x2,x3) | label

(0,0,0)    | -

(0,0,1)    | -

(0,1,0)    | -

(0,1,1)    | -

(1,0,0)    | -

(1,0,1)    | -
```

```
(1,1,0)    | +

(1,1,1)    | -
```

Now visualize the 3D space with x1,x2,x3 axis. The 8 points would be vertices of a unit cube. Let us mark +/- near each vertex to indicate its label:



Now we ask: **"can we make a plane that separates (+)'s from (-)'s?"** That plane represents the linear classifier. It turns out we can:



This plane is obtained by connecting midpoints between (1,0,0), (1,1,0), and (0,1,0).

Repeat this analysis for all choices in Q2.

*** Q3:** First break down the Boolean expression into two subexpressions:

```
(x1 AND x2) OR (NOT x1 AND NOT x2) = z1 OR z2
```

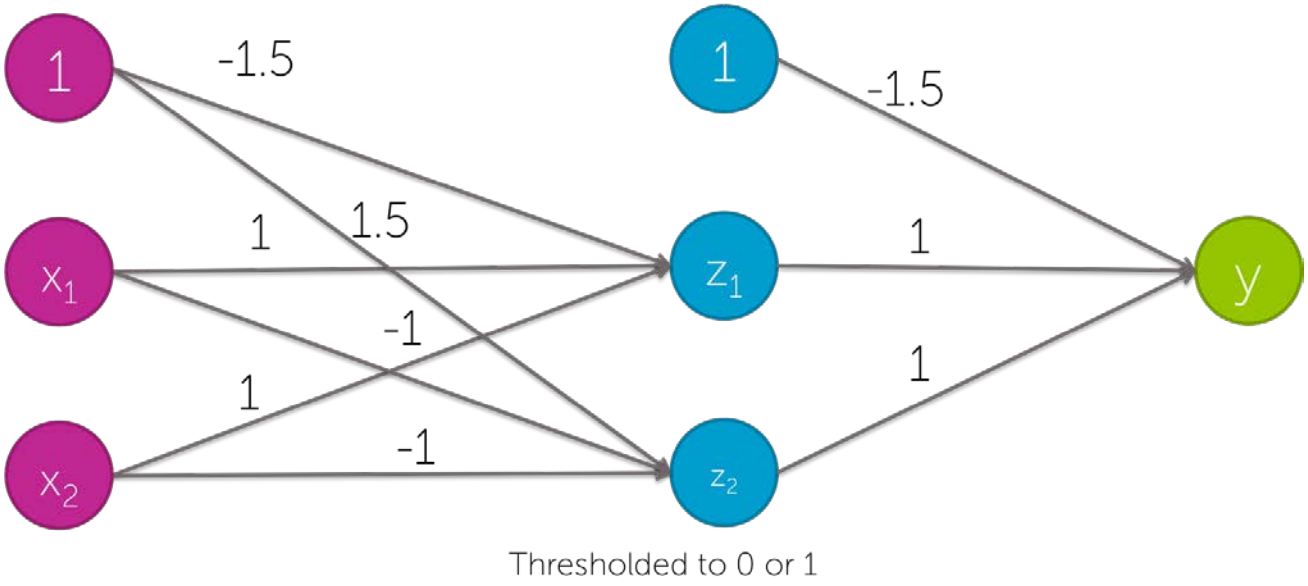
where

```
z1 = x1 AND x2

z2 = NOT x1 AND NOT x2
```

Then check two things:

- **Are z1 and z2 correctly implemented by the units in the middle layer?**
- **Does the last layer implement (z1 OR z2)?**

For instance, let us look at this two-layer neural network:



Truth table for z1:

x1	x2	z1	z1 thresholded	(x1 AND x2)
0	0	-1.5	0	0
0	1	-0.5	0	0
1	0	-0.5	0	0
1	1	0.5	1	1

So yes, first unit in the middle layer correctly implements z1.

How about z2?

x1	x2	z2	z2 thresholded	NOT x1 AND NOT x2
0	0	1.5	1	1
0	1	0.5	1	0
1	0	0.5	1	0
1	1	-0.5	0	0

No, second unit in the middle layer does not implement z2.

How about the second layer? Does it implement (z1 OR z2)?

z1	z2	y	y thresholded	(z1 OR z2)

```
0     0      -1.5   0              | 0

0     1      -0.5   0              | 1

1     0      -0.5   0              | 1

1     1      0.5    1              | 1
```

So no, second layer does not implement (z1 OR z2).

Now repeat this line of reasoning for all other choices of Q3.