Robot Motion Model

(자율주행 핵심기술 SLAM 단기강좌)

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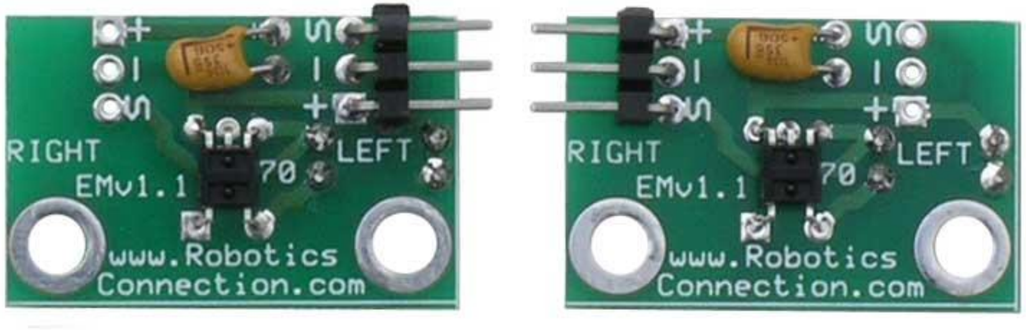
Motion Model

- In practice, one often finds two types of motion models:
  - Odometry-based
  - Velocity-based (dead reckoning)
- Odometry-based models
  - When systems are equipped with wheel encoders.
  - Odometry information is only available after executing a motion.
  - More accurate than velocity-based models
- Velocity-based models
  - When no wheel encoders are given.
  - Velocity control can come before motion.
  - Common in non-land robots (for example, UAVs)

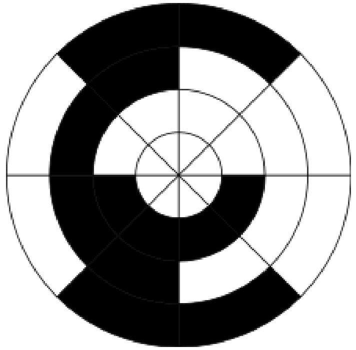
Motion Model

- Wheel encoders

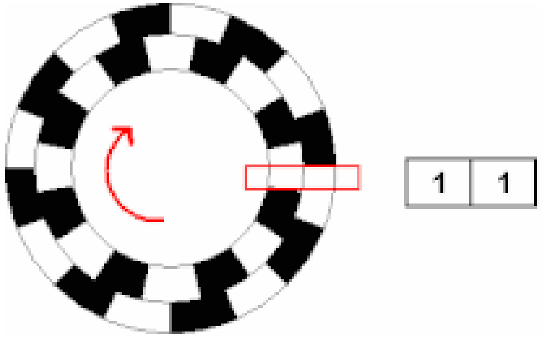
  These modules require +5V and GND to power them. They provide +5V output when they "see" white, and 0V output when they "see“ black.



Rotary disk with high quality laminated colors



Rotary encoder with 3-bit gray coding



Conceptual drawing of a rotary encoder

Source: http://www.active-robots.com/, https://en.wikipedia.org/wiki/Rotary_encoder

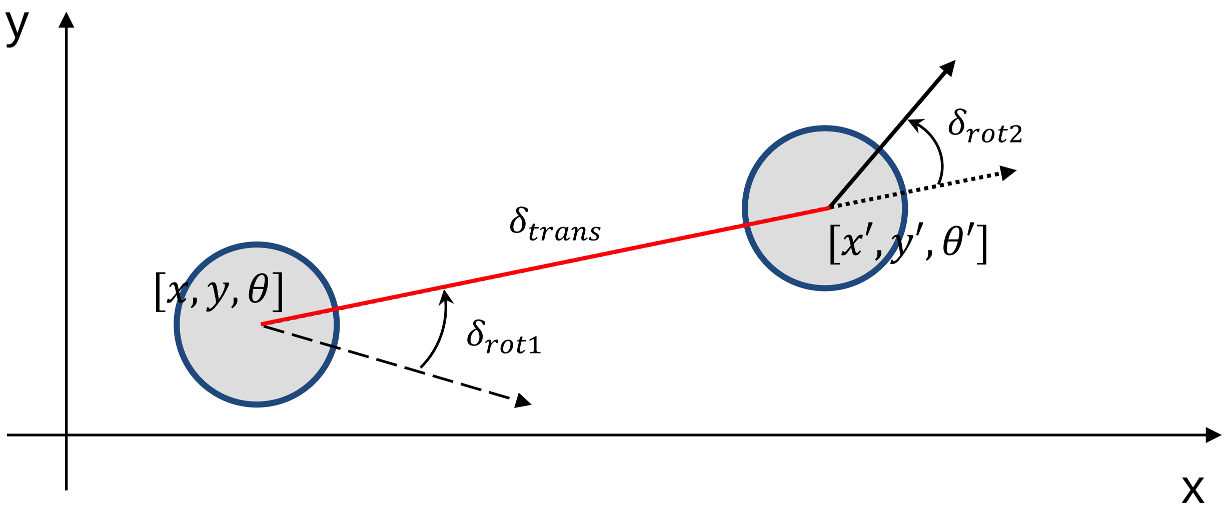
Odometry Model

- Robot moves from $[x, y, \theta]$ to $[x', y', \theta']$
- Odometry information $[\delta_{trans}, \delta_{rot1}, \delta_{rot2}]$

$$\delta_{trans} = \sqrt{(x' - x)^2 + (y' - y)^2}$$

$$\delta_{rot1} = \text{atan2}(y' - y, x' - x) - \theta$$

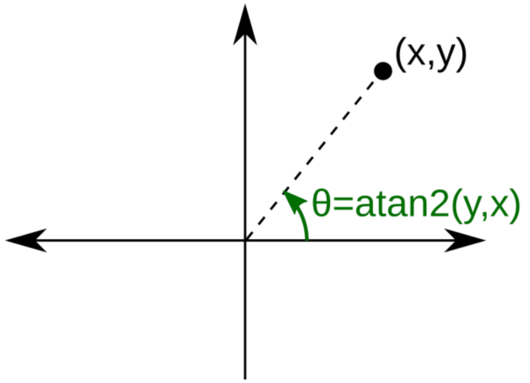
$$\delta_{rot2} = \theta' - \theta - \delta_{rot1}$$



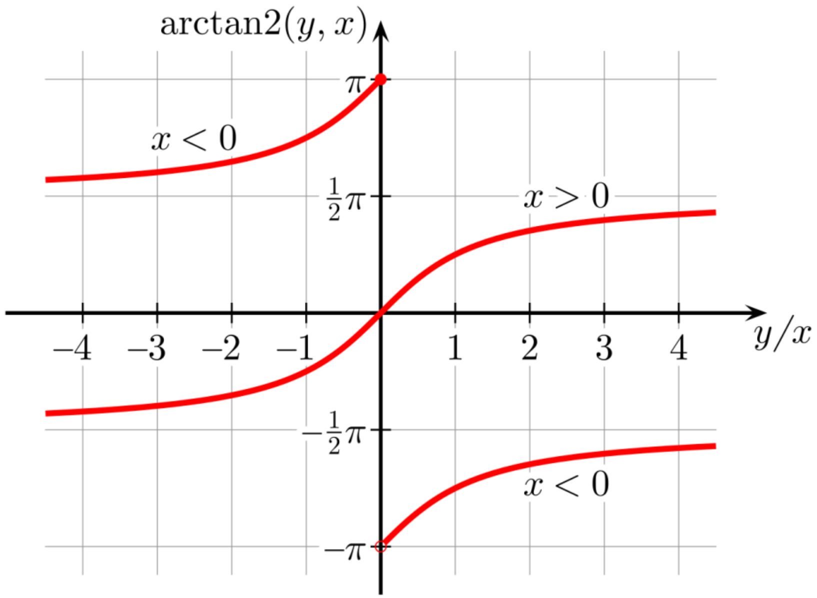
Odometry Model

- atan2 function

$$\operatorname{atan2}(y,x)=\begin{cases}\operatorname{atan}(y/x) & \text{if } x>0\\ \operatorname{sign}(y)\left(\pi-\operatorname{atan}\left(\left|\frac{y}{x}\right|\right)\right) & \text{if } x<0\\ 0 & \text{if } x=y=0\\ \operatorname{sign}(y)\pi/2 & \text{if } x=0, y\neq 0\end{cases}$$



$\operatorname{atan2}(y,x)$ returns the angle θ between the ray to the point (x,y) and the positive x-axis, confined to $(-\pi,\pi]$.



https://en.wikipedia.org/wiki/Atan2

Odometry Model

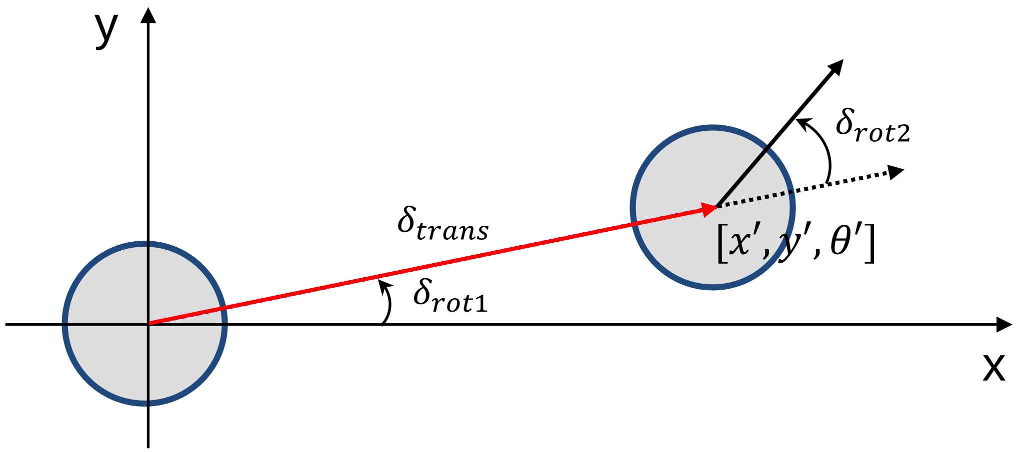
- Assuming the prior robot pose is [0,0,0]

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos\hat{\delta}_{rot1} & 0 & 0 \\ \sin\hat{\delta}_{rot1} & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \hat{\delta}_{trans} \\ \hat{\delta}_{rot1} \\ \hat{\delta}_{rot2} \end{bmatrix}$$

$$\hat{\delta}_{trans} = \delta_{trans} + \epsilon_{trans}$$

$$\hat{\delta}_{rot1} = \delta_{rot1} + \epsilon_{rot1}$$

$$\hat{\delta}_{rot2} = \delta_{rot2} + \epsilon_{rot2}$$



$$\epsilon_{trans} \sim \mathbb{N}(0, \sigma^2_{trans})$$

$$\epsilon_{rot1} \sim \mathbb{N}(0, \sigma^2_{rot1})$$

$$\epsilon_{rot2} \sim \mathbb{N}(0, \sigma^2_{rot2})$$

Odometry Model

- If the prior robot pose is $[x, y, \theta]$

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \cos(\theta + \hat{\delta}_{rot1}) & 0 & 0 \\ \sin(\theta + \hat{\delta}_{rot1}) & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \hat{\delta}_{trans} \\ \hat{\delta}_{rot1} \\ \hat{\delta}_{rot2} \end{bmatrix}$$

$$\hat{\delta}_{trans} = \delta_{trans} + \epsilon_{trans}$$

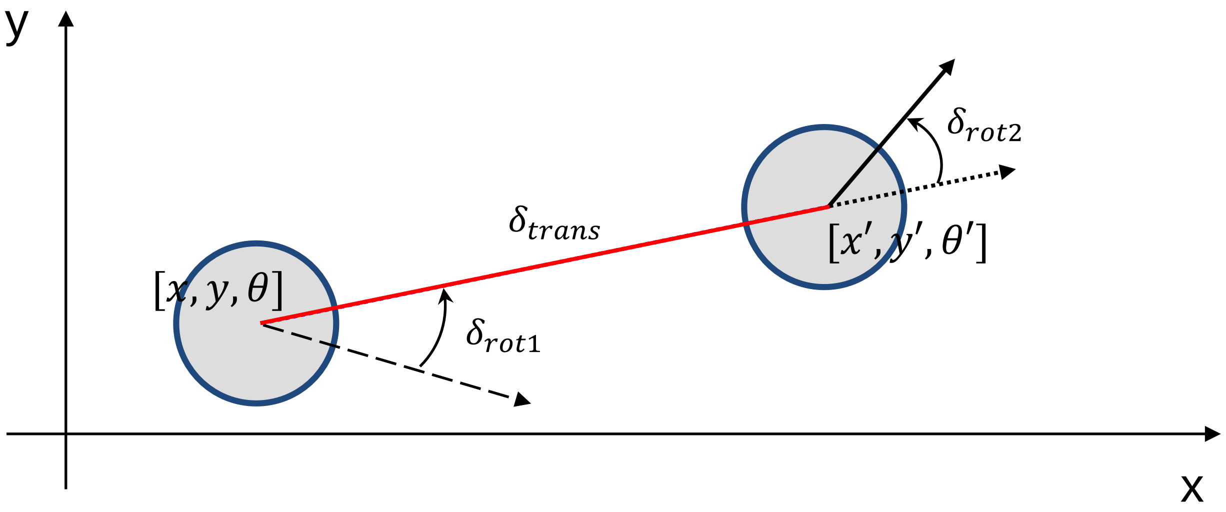
$$\hat{\delta}_{rot1} = \delta_{rot1} + \epsilon_{rot1}$$

$$\hat{\delta}_{rot2} = \delta_{rot2} + \epsilon_{rot2}$$

$$\epsilon_{trans} \sim \mathbb{N}(0, \sigma^2_{trans})$$

$$\epsilon_{rot1} \sim \mathbb{N}(0, \sigma^2_{rot1})$$

$$\epsilon_{rot2} \sim \mathbb{N}(0, \sigma^2_{rot2})$$



Odometry Model

- If the prior robot pose is $[x, y, \theta]$

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \cos(\theta + \hat{\delta}_{rot1}) & 0 & 0 \\ \sin(\theta + \hat{\delta}_{rot1}) & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \hat{\delta}_{trans} \\ \hat{\delta}_{rot1} \\ \hat{\delta}_{rot2} \end{bmatrix}$$

$$\hat{\delta}_{trans} = \delta_{trans} + \epsilon_{trans}$$

$$\hat{\delta}_{rot1} = \delta_{rot1} + \epsilon_{rot1}$$

$$\hat{\delta}_{rot2} = \delta_{rot2} + \epsilon_{rot2}$$

$$\sigma_{rot1} = \alpha_1 \, |\delta_{rot1}| + \alpha_2 \, \delta_{trans}$$

$$\sigma_{rot2} = \alpha_1 \, |\delta_{rot2}| + \alpha_2 \, \delta_{trans}$$

$$\sigma_{trans} = \alpha_3 \, \delta_{trans} + \alpha_4(|\delta_{rot1}| + |\delta_{rot2}|)$$

$$\epsilon_{trans} \sim \mathbb{N}(0, \sigma_{trans}^2)$$

$$\epsilon_{rot1} \sim \mathbb{N}(0, \sigma_{rot1}^2)$$

$$\epsilon_{rot2} \sim \mathbb{N}(0, \sigma_{rot2}^2)$$

Odometry Model

- Algorithm

Algorithm motion_model_odometry(x_t , u_t , x_{t-1})

$$\left.\begin{aligned} \delta_{rot1} &= \mathrm{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta} \\ \delta_{trans} &= \sqrt{(\bar{x}' - \bar{x})^2 + (\bar{y}' - \bar{y})^2} \\ \delta_{rot2} &= \bar{\theta}' - \bar{\theta} - \delta_{rot1} \end{aligned}\right\} \text{odometry values } (u_t)$$

$$\left.\begin{aligned} \hat{\delta}_{rot1} &= \mathrm{atan2}(y' - y, x' - x) - \theta \\ \hat{\delta}_{trans} &= \sqrt{(x' - x)^2 + (y' - y)^2} \\ \hat{\delta}_{rot2} &= \theta' - \theta - \hat{\delta}_{rot1} \end{aligned}\right\} \text{values of interest } (x_t, x_{t-1})$$

$$p_1 = \mathrm{prob}\left(\delta_{rot1} - \hat{\delta}_{rot1}, \alpha_1 \left|\hat{\delta}_{rot1}\right| + \alpha_2\, \hat{\delta}_{trans}\right)$$

$$p_2 = \mathrm{prob}\left(\delta_{trans} - \hat{\delta}_{trans}, \alpha_3\, \delta_{trans} + \alpha_4(|\delta_{rot1}| + |\delta_{rot2}|)\right)$$

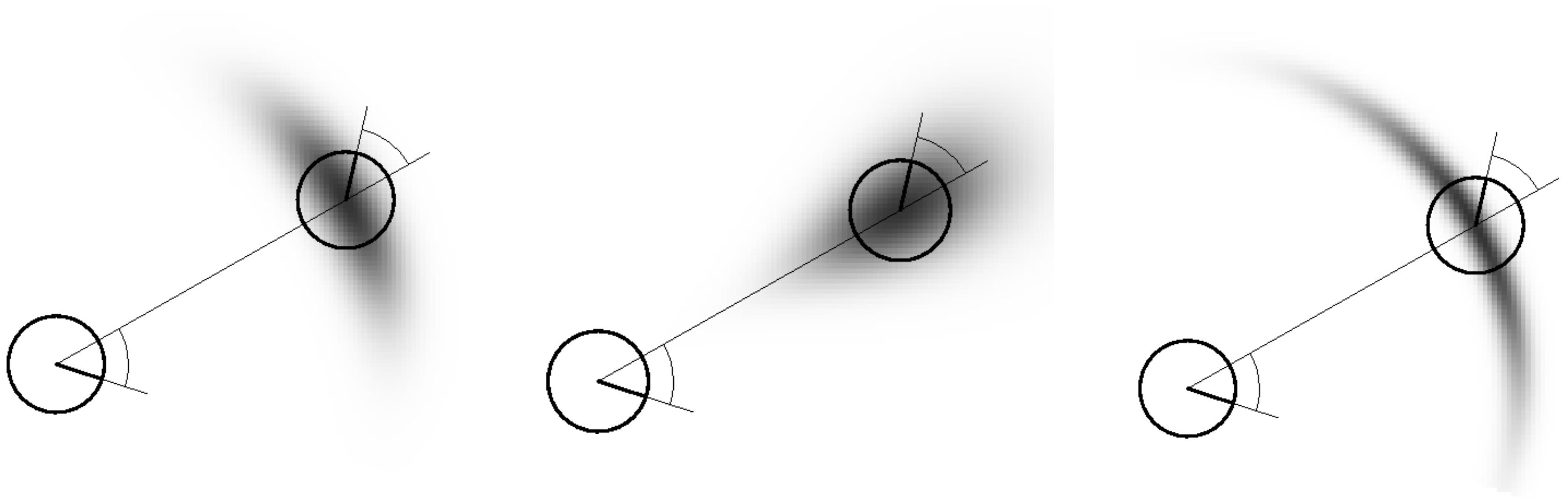
$$p_3 = \mathrm{prob}\left(\delta_{rot2} - \hat{\delta}_{rot2}, \alpha_1 \left|\hat{\delta}_{rot2}\right| + \alpha_2\, \hat{\delta}_{trans}\right)$$

return $p_1 \cdot p_2 \cdot p_3$

$$u_t = (\bar{x}_t, \bar{x}_{t-1})$$
$$\bar{x}_{t-1} = (\bar{x}, \bar{y}, \bar{\theta})$$
$$\bar{x}_t = (\bar{x}', \bar{y}', \bar{\theta}')$$

Odometry Model

- Examples



Odometry motion model for different noise parameter settings.

Velocity Motion Model

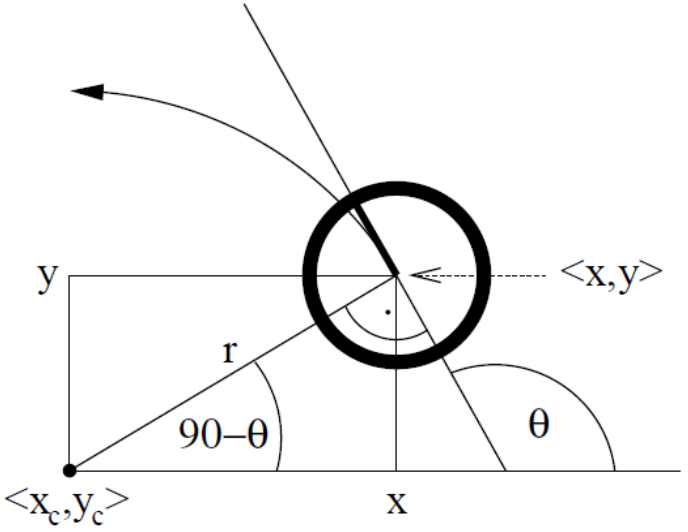
- Control a robot through two velocities
  - Translational velocity v
  - Rotational velocity w

Velocity Motion Model

- Ideal case

$$u_t = \begin{bmatrix} v \\ w \end{bmatrix} \qquad r = \left|\frac{v}{w}\right|$$

$$v = w \cdot r$$



$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} x_c + \frac{v}{w}\sin(\theta + w\Delta t) \\ y_c - \frac{v}{w}\cos(\theta + w\Delta t) \\ \theta + w\Delta t \end{bmatrix}$$

$$= \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} -\frac{v}{w}\sin\theta + \frac{v}{w}\sin(\theta + w\Delta t) \\ \frac{v}{w}\cos\theta - \frac{v}{w}\cos(\theta + w\Delta t) \\ w\Delta t \end{bmatrix}$$

$$x_c = x - \frac{v}{w}\sin\theta$$

$$y_c = x + \frac{v}{w}\cos\theta$$

Velocity Motion Model

- Realistic case
  - The robot performs a rotation $\hat{\gamma}$ when it arrives at its final pose.

$$\theta' = \theta + \hat{w}\Delta t + \hat{\gamma}\Delta t$$

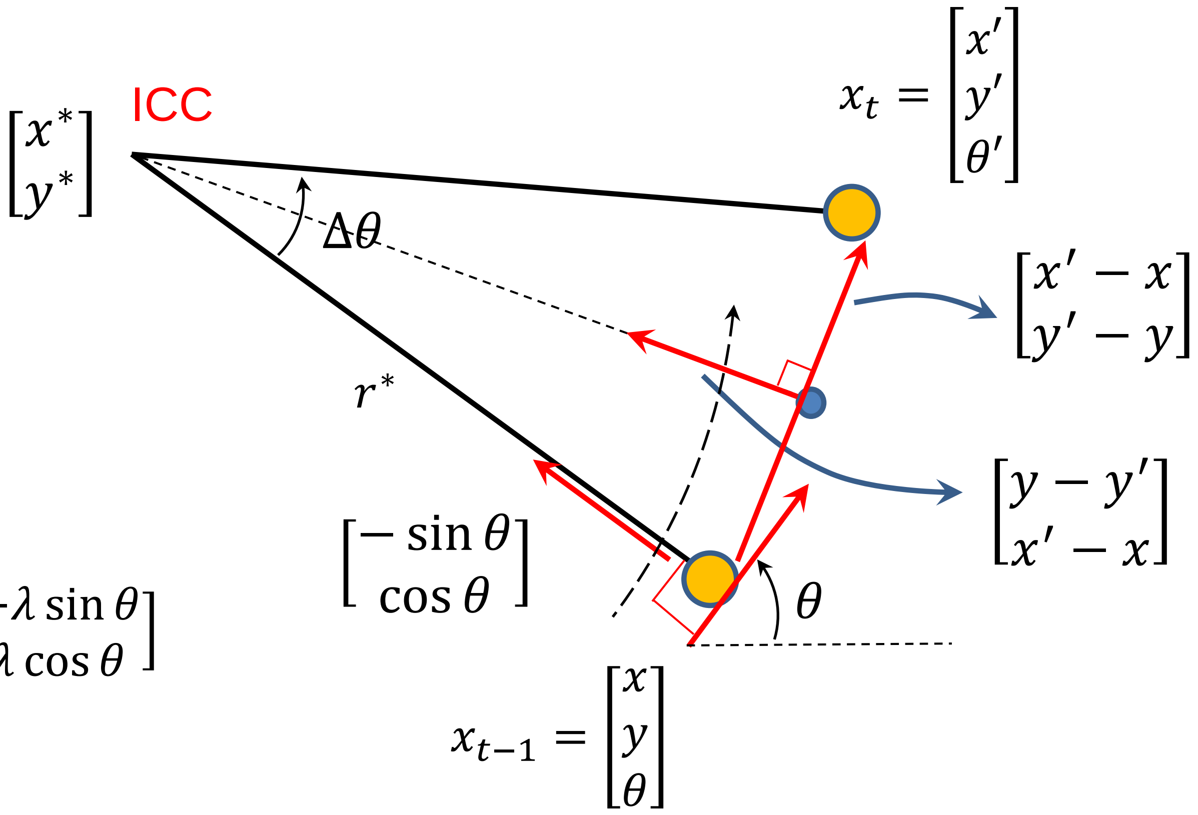
  - The actual velocity with noise

$$\begin{bmatrix} v' \\ w' \end{bmatrix} = \begin{bmatrix} v \\ w \end{bmatrix} + \begin{bmatrix} v_{err} \\ w_{err} \end{bmatrix} = \begin{bmatrix} v \\ w \end{bmatrix} + \begin{bmatrix} \epsilon_{\alpha_1|v|+\alpha_2|w|} \\ \epsilon_{\alpha_3|v|+\alpha_4|w|} \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} -\frac{\hat{v}}{\hat{w}}\sin\theta + \frac{\hat{v}}{\hat{w}}\sin(\theta + \hat{w}\Delta t) \\ \frac{\hat{v}}{\hat{w}}\cos\theta - \frac{\hat{v}}{\hat{w}}\cos(\theta + \hat{w}\Delta t) \\ \hat{w}\Delta t + \hat{\gamma}\Delta t \end{bmatrix}$$

Velocity Motion Model

- Realistic case
  - Rotation of $\Delta\theta$ about (x^*, y^*) from (x, y) to (x', y') in time Δt



$$\begin{bmatrix} x^* \\ y^* \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\lambda\sin\theta \\ \lambda\cos\theta \end{bmatrix}$$

Velocity Motion Model

- Center of a circle

Average of $\begin{bmatrix} x \\ y \end{bmatrix}$ and $\begin{bmatrix} x' \\ y' \end{bmatrix}$

$$\begin{bmatrix} x^* \\ y^* \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\lambda \sin\theta \\ \lambda \cos\theta \end{bmatrix} = \begin{bmatrix} \frac{x+x'}{2} + \mu(y-y') \\ \frac{y+y'}{2} + \mu(x'-x) \end{bmatrix}$$

Vector pointing towards the ICC

$$\mu = \frac{1}{2} \frac{(x-x')\cos\theta + (y-y')\sin\theta}{(y-y')\cos\theta - (x-x')\sin\theta}$$

- Two sets of equations
- Two unknowns (μ, λ)

ICC (instantaneous center of curvature)

Velocity Motion Model

$$\begin{bmatrix} x^* \\ y^* \end{bmatrix} = \begin{bmatrix} \frac{x + x'}{2} + \mu(y - y') \\ \frac{y + y'}{2} + \mu(x' - x) \end{bmatrix}$$

$$\mu = \frac{1}{2} \frac{(x - x')\cos\theta + (y - y')\sin\theta}{(y - y')\cos\theta - (x - x')\sin\theta}$$

$$r^* = \sqrt{(x - x^*)^2 + (y - y^*)^2} = \sqrt{(x' - x^*)^2 + (y' - y^*)^2}$$

$$\Delta\theta = \text{atan2}(y' - y^*, x' - x^*) - \text{atan2}(y - y^*, x - x^*)$$

- Given $\Delta\theta$ and Δdist we can compute the velocities needed to generate the motion

$$\Delta\text{dist} = r^* \cdot \Delta\theta$$

$$\hat{u}_t = \begin{bmatrix} v' \\ w' \end{bmatrix} = \Delta t^{-1} \begin{bmatrix} \Delta\text{dist} \\ \Delta\theta \end{bmatrix}$$

$$\hat{\gamma} = \Delta t^{-1}(\theta' - \theta) - \hat{w}$$

Velocity Motion Model

$$v_{err} = v - \hat{v} = v - \frac{\Delta\text{dist}}{\Delta t}$$

$$w_{err} = w - \hat{w} = w - \frac{\Delta\theta}{\Delta t}$$

$$\gamma_{err} = \gamma - \hat{\gamma}$$

$$v_{err} \sim \mathbb{N}(0, \sigma^2_{verr})$$

$$w_{err} \sim \mathbb{N}(0, \sigma^2_{w_{err}})$$

$$\gamma_{err} \sim \mathbb{N}(0, \sigma^2_{cerr})$$

$$\sigma_{verr} = \alpha_1 |v| + \alpha_2 |w|$$

$$\sigma_{werr} = \alpha_3 |v| + \alpha_4 |w|$$

$$\sigma_{cerr} = \alpha_5 |v| + \alpha_6 |w|$$

– Assume that the robot has independent control over its controlled linear and angular velocities

$$p(x_t | u_t, x_{t-1}) = p(v_{err}) \cdot p(w_{err}) \cdot p(\gamma_{err})$$