EKF SLAM

(자율주행 핵심기술 SLAM 단기강좌)

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Simultaneous Localization and Mapping

- Online SLAM

$$p(x_t, m \mid z_{1:t}, u_{1:t})$$

Assuming that there are correspondence variables

$$p(x_t, m, c_t \mid z_{1:t}, u_{1:t})$$

- Full SLAM

$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t})$$

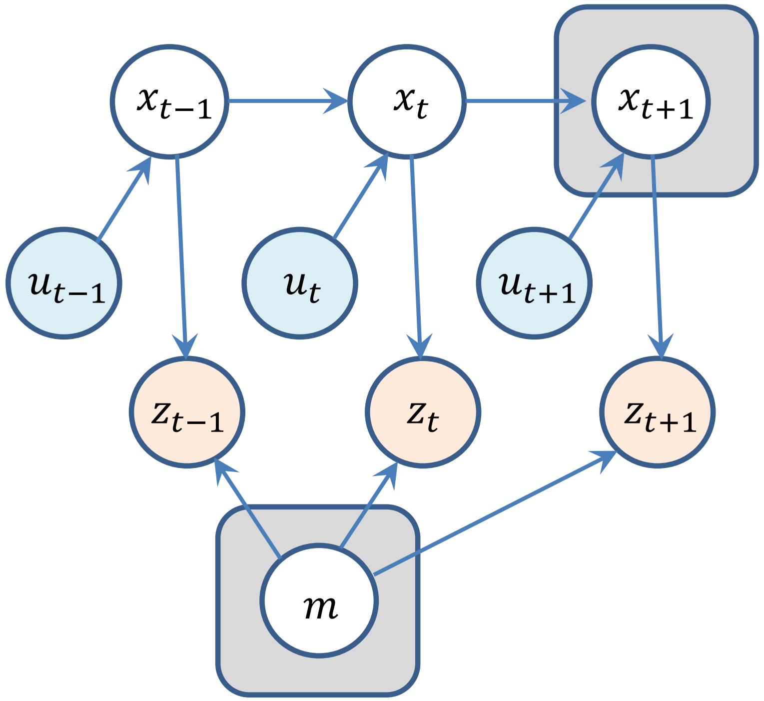
$$p(x_t, m \mid z_{1:t}, u_{1:t}) = \int\int \cdots \int p(x_{1:t}, m \mid z_{1:t}, u_{1:t})\ dx_1 dx_2 \cdots dx_{t-1}$$

Assuming that there are correspondence variables

$$p(x_{1:t}, m, c_{1:t} \mid z_{1:t}, u_{1:t})$$

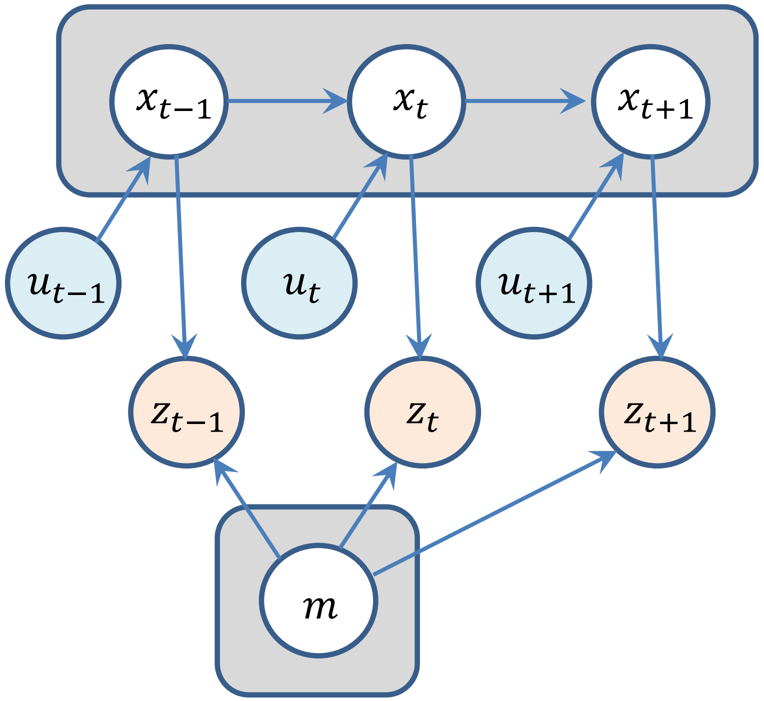
SLAM

- Graphical model
  - Online SLAM



Estimate a posterior over the current robot pose along with the map.

  - Full SLAM



Compute a joint posterior over the whole path of the robot and the map.

SLAM Algorithm Cycle

1. State prediction
2. Measurement prediction
3. Real measurement
4. Data association
5. Update

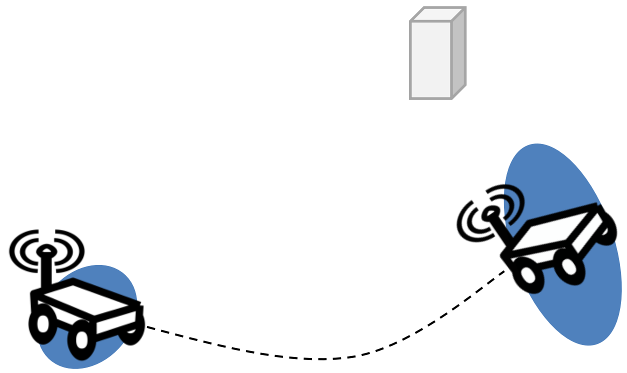
SLAM with Extended Kalman Filter

- State representation
  - Map with N landmarks: (3+2N) dimensional Gaussian
  - Belief is represented by

$$
\mu = \begin{bmatrix} x \\ y \\ \theta \\ m_{1,x} \\ m_{1,y} \\ \vdots \\ m_{N,x} \\ m_{N,y} \end{bmatrix}
\qquad
\Sigma = \begin{bmatrix}
\sigma_{xx} & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xm_{1,x}} & \sigma_{xm_{1,y}} & \cdots & \sigma_{xm_{N,x}} & \sigma_{xm_{N,y}} \\
\sigma_{yx} & \sigma_{yy} & \sigma_{y\theta} & \sigma_{ym_{1,x}} & \sigma_{ym_{1,y}} & \cdots & \sigma_{ym_{N,x}} & \sigma_{ym_{N,y}} \\
\sigma_{\theta x} & \sigma_{\theta y} & \sigma_{\theta\theta} & \sigma_{\theta m_{1,x}} & \sigma_{\theta m_{1,y}} & \cdots & \sigma_{\theta m_{N,x}} & \sigma_{\theta m_{N,y}} \\
\sigma_{m_{1,x}x} & \sigma_{m_{1,x}y} & \sigma_{m_{1,x}\theta} & \sigma_{m_{1,x}m_{1,x}} & \sigma_{m_{1,x}m_{1,y}} & \cdots & \sigma_{m_{1,x}m_{N,x}} & \sigma_{m_{1,x}m_{N,y}} \\
\sigma_{m_{1,y}x} & \sigma_{m_{1,y}y} & \sigma_{m_{1,y}\theta} & \sigma_{m_{1,y}m_{1,x}} & \sigma_{m_{1,y}m_{1,y}} & \cdots & \sigma_{m_{1,y}m_{N,x}} & \sigma_{m_{1,y}m_{N,y}} \\
 & & \vdots & & & \ddots & \vdots & \\
\sigma_{m_{N,x}x} & \sigma_{m_{N,x}y} & \sigma_{m_{N,x}\theta} & \sigma_{m_{N,x}m_{1,x}} & \sigma_{m_{N,x}m_{1,y}} & \cdots & \sigma_{m_{N,x}m_{N,x}} & \sigma_{m_{N,x}m_{N,y}} \\
\sigma_{m_{N,y}x} & \sigma_{m_{N,y}y} & \sigma_{m_{N,y}\theta} & \sigma_{m_{N,y}m_{1,x}} & \sigma_{m_{N,y}m_{1,y}} & \cdots & \sigma_{m_{N,y}m_{N,x}} & \sigma_{m_{N,y}m_{N,y}}
\end{bmatrix}
$$

SLAM with Extended Kalman Filter

- State prediction

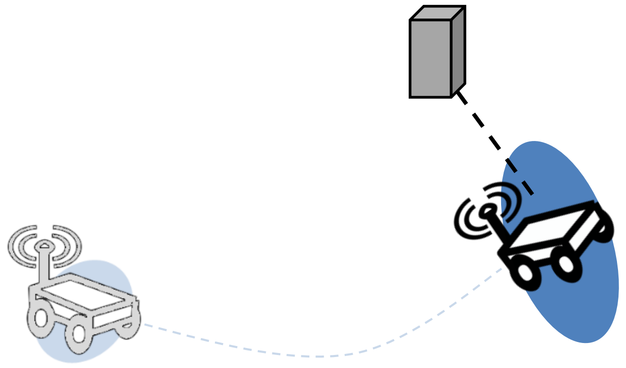


$$\mu = \begin{bmatrix} x_R \\ m_1 \\ \vdots \\ m_N \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_N} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_N} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_N x_R} & \Sigma_{m_N m_1} & \cdots & \Sigma_{m_N m_N} \end{bmatrix}$$

The complexity of the prediction step is linear to the number of landmarks.

SLAM with Extended Kalman Filter

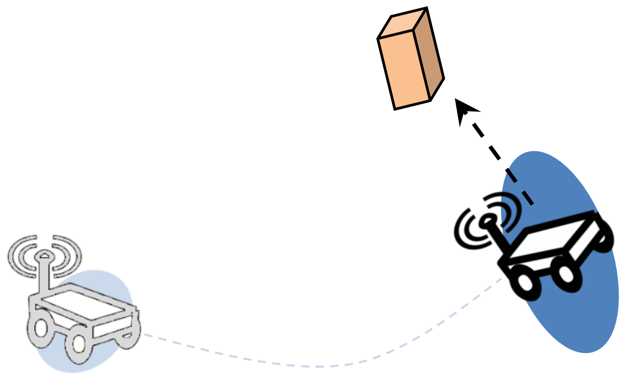
- Measurement prediction



$$\mu = \begin{bmatrix} x_R \\ m_1 \\ \vdots \\ m_N \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_N} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_N} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_N x_R} & \Sigma_{m_N m_1} & \cdots & \Sigma_{m_N m_N} \end{bmatrix}$$

SLAM with Extended Kalman Filter

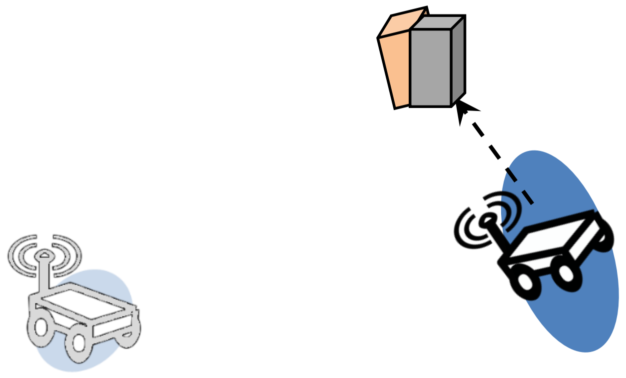
- Real measurement



$$\mu = \begin{bmatrix} x_R \\ m_1 \\ \vdots \\ m_N \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_N} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_N} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_N x_R} & \Sigma_{m_N m_1} & \cdots & \Sigma_{m_N m_N} \end{bmatrix}$$

SLAM with Extended Kalman Filter

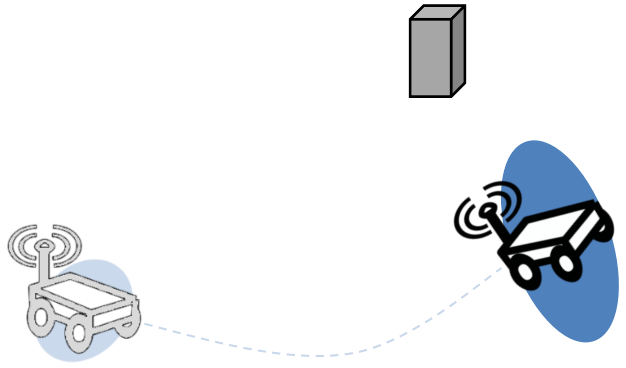
- Data association and difference between h(x) and z



$$\mu = \begin{bmatrix} x_R \\ m_1 \\ \vdots \\ m_N \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_N} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_N} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_N x_R} & \Sigma_{m_N m_1} & \cdots & \Sigma_{m_N m_N} \end{bmatrix}$$

SLAM with Extended Kalman Filter

- Update step



$$\mu = \begin{bmatrix} x_R \\ m_1 \\ \vdots \\ m_N \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_N} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_N} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_N x_R} & \Sigma_{m_N m_1} & \cdots & \Sigma_{m_N m_N} \end{bmatrix}$$

SLAM with Extended Kalman Filter

- Setup and assumption
  - Online SLAM
  - SLAM algorithm based on extended Kalman filter (EKF)
  - Assuming velocity-based motion model
  - Using maximum likelihood data association
  - Using a feature-based map

SLAM with Extended Kalman Filter

- Combined state vector
  - Composed of the robot pose and the landmarks on the map

$$y_t = \{x_t, m\} = \{\underbrace{x, y, \theta,}_{\text{Robot's pose}} \; \underbrace{m_{1,x}, m_{1,y},}_{\text{Landmark 1}} \; \underbrace{m_{2,x}, m_{2,y},}_{\text{Landmark 2}} \cdots, \; \underbrace{m_{N,x}, m_{N,y}}_{\text{Landmark N}}\}^T$$

- Similar to EKF localization
  - EKF localization calculates the posterior

$$p(x_t \mid z_{1:t}, u_{1:t})$$

  - EKF SLAM calculate the online posterior

$$p(y_t \mid z_{1:t}, u_{1:t})$$

SLAM with Extended Kalman Filter

- Initialization
  - Robot starts in its own frame
  - All landmarks are unknown
  - 2N+3 dimensions for N landmarks

$$\mu_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad \Sigma_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \infty & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \infty \end{bmatrix}$$

Prediction Step

- Update state vector based on the robot's motion

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \underbrace{\begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} -\frac{v_t}{w_t}\sin\theta + \frac{v_t}{w_t}\sin(\theta + w_t\Delta t) \\ \frac{v_t}{w_t}\cos\theta - \frac{v_t}{w_t}\cos(\theta + w_t\Delta t) \\ w_t\Delta t \end{bmatrix}}_{g_{x_R}(u_t, x_{t-1})}$$

- Need to map this to the 2N+3 dimensional state vector

$$\begin{bmatrix} x' \\ y' \\ \theta' \\ \vdots \end{bmatrix} = \underbrace{\begin{bmatrix} x \\ y \\ \theta \\ \vdots \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 & 0 & \overbrace{0 & \cdots & 0}^{2N} \\ 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}^T}_{F_x} \begin{bmatrix} -\frac{v_t}{w_t}\sin\theta + \frac{v_t}{w_t}\sin(\theta + w_t\Delta t) \\ \frac{v_t}{w_t}\cos\theta - \frac{v_t}{w_t}\cos(\theta + w_t\Delta t) \\ w_t\Delta t \end{bmatrix}}_{g(u_t, x_{t-1})}$$

Prediction Step

- Jacobian of the motion

$$G_{t,x_R} = \frac{\partial}{\partial(x,y,\theta)^T} g_{x_R}(u_t, x_{t-1})$$

$$= \frac{\partial}{\partial(x,y,\theta)^T}\left(\begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} -\frac{v_t}{w_t}\sin\theta + \frac{v_t}{w_t}\sin(\theta + w_t\Delta t) \\ \frac{v_t}{w_t}\cos\theta - \frac{v_t}{w_t}\cos(\theta + w_t\Delta t) \\ w_t\Delta t \end{bmatrix}\right)$$

$$= I + \frac{\partial}{\partial(x,y,\theta)^T}\begin{bmatrix} -\frac{v_t}{w_t}\sin\theta + \frac{v_t}{w_t}\sin(\theta + w_t\Delta t) \\ \frac{v_t}{w_t}\cos\theta - \frac{v_t}{w_t}\cos(\theta + w_t\Delta t) \\ w_t\Delta t \end{bmatrix}$$

$$= I + \begin{bmatrix} 0 & 0 & -\frac{v_t}{w_t}\cos\theta + \frac{v_t}{w_t}\cos(\theta + w_t\Delta t) \\ 0 & 0 & -\frac{v_t}{w_t}\sin\theta + \frac{v_t}{w_t}\sin(\theta + w_t\Delta t) \\ 0 & 0 & 0 \end{bmatrix}$$

Jacobian (3 x 3)

$$G_t = \begin{bmatrix} G_{t,x_R} & 0 \\ 0 & I \end{bmatrix}$$

Identity (2N x 2N)

EKF SLAM: Prediction Step

Algorithm EKF_SLAM_known_correspondences($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t, R_t$)

1. $F_x = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \underbrace{0 & \cdots & 0}_{2N} \end{bmatrix}$

$$\mu_{t-1} = \begin{bmatrix} \mu_{t-1,x} \\ \mu_{t-1,y} \\ \mu_{t-1,\theta} \end{bmatrix}$$

2. $\bar{\mu}_t = \mu_{t-1} + F_x^T \begin{bmatrix} -\frac{v_t}{w_t}\sin\mu_{t-1,\theta} + \frac{v_t}{w_t}\sin(\mu_{t-1,\theta} + w_t\Delta t) \\ \frac{v_t}{w_t}\cos\mu_{t-1,\theta} - \frac{v_t}{w_t}\cos(\mu_{t-1,\theta} + w_t\Delta t) \\ w_t\Delta t \end{bmatrix}$

3. $G_t = I + F_x^T g_t F_x \qquad g_t = \begin{bmatrix} 0 & 0 & -\frac{v_t}{w_t}\cos\mu_{t-1,\theta} + \frac{v_t}{w_t}\cos(\mu_{t-1,\theta} + w_t\Delta t) \\ 0 & 0 & -\frac{v_t}{w_t}\sin\mu_{t-1,\theta} + \frac{v_t}{w_t}\sin(\mu_{t-1,\theta} + w_t\Delta t) \\ 0 & 0 & 0 \end{bmatrix}$

4. $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + F_x^T R_t F_x$

Correction Step

- Known data association
- $j = c_t^i$: i-th measurement at time t observes the landmark with index j
- Initialize landmark if never been observed
- Compute the expected observation
- Compute the Jacobian of $h(x)$
- Proceed with computing the Kalman gain

Observation of Landmarks

- Range-Bearing observation

$$z_t^i = \begin{bmatrix} r_t^i \\ \phi_t^i \end{bmatrix}$$

- If landmark has not been observed

$$\begin{bmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \end{bmatrix} = \begin{bmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \end{bmatrix} + \begin{bmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \end{bmatrix}$$

Observed location of landmark j

Estimated robot's location

Relative measurement

Expected Observation

- Compute expected observation according to the current estimate
  - Difference between the position of the landmark and the position of the robot

$$\delta = \begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix} = \begin{bmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{bmatrix}$$

  - Euclidean distance from the robot to the landmark

$$q = \delta^T \delta = (\bar{\mu}_{j,x} - \bar{\mu}_{t,x})^2 + (\bar{\mu}_{j,y} - \bar{\mu}_{t,y})^2$$

  - Expected (predicted) observation is

$$\hat{z}_t^i = \begin{bmatrix} r_t^i \\ \phi_t^i \end{bmatrix} = \underbrace{\begin{bmatrix} \sqrt{q} \\ \operatorname{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \end{bmatrix}}_{h(\bar{\mu}_t)}$$

$\bar{\mu}_t$: current pose of the robot

Jacobian for the Observation

- Compute the Jacobian of $h(x)$

$$H_{t,low}^{i} = \frac{\partial h(\bar{\mu}_t)}{\partial \bar{\mu}_t}$$

low-dimensional space (2 x 5)
$(x, y, \theta, m_{j,x}, m_{j,y})$

$$= \begin{bmatrix} \frac{\partial \sqrt{q}}{\partial x} & \frac{\partial \sqrt{q}}{\partial y} & \cdots \\ \frac{\partial \operatorname{atan2}(\cdots)}{\partial x} & \frac{\partial \operatorname{atan2}(\cdots)}{\partial y} & \cdots \end{bmatrix}$$

$$= \frac{1}{q} \begin{bmatrix} -\sqrt{q}\delta_x & -\sqrt{q}\delta_y & 0 & \sqrt{q}\delta_x & \sqrt{q}\delta_y \\ \delta_y & -\delta_x & -q & -\delta_y & \delta_x \end{bmatrix}$$

- Map it to the full dimensional space

$$H_t^i = H_{t,low}^i F_{x,j}$$

$$F_{x,j} = \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 \end{bmatrix} \\ \begin{array}{ccccccccccc} & & & \underbrace{}_{2j-2} & & & \underbrace{}_{2N-2j} \end{array} \end{array}$$

EKF SLAM: Correction Step

5. $Q_t = \begin{bmatrix} \sigma_r & 0 \\ 0 & \sigma_\phi \end{bmatrix}$

6. for all observed features $z_t^i = \begin{bmatrix} r_t^i \\ \phi_t^i \end{bmatrix}$

7. $j = c_t^i$

8. if landmark j never seen before

9. $\begin{bmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \end{bmatrix} = \begin{bmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \end{bmatrix} + \begin{bmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \end{bmatrix}$

10. end if

11. $\delta = \begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix} = \begin{bmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{bmatrix}$

12. $q = \delta^T \delta = (\bar{\mu}_{j,x} - \bar{\mu}_{t,x})^2 + (\bar{\mu}_{j,y} - \bar{\mu}_{t,y})^2$

EKF SLAM: Correction Step

13. $z_t^i = \begin{bmatrix} r_t^i \\ \phi_t^i \end{bmatrix} = \begin{bmatrix} \sqrt{q} \\ \text{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \end{bmatrix}$

14. $F_{x,j} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \underbrace{0 & \cdots & 0}_{2j-2} & 1 & 0 & \underbrace{0 & \cdots & 0}_{2N-2j} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}$

15. $H_t^i = \frac{1}{q} \begin{bmatrix} \sqrt{q}\delta_x & -\sqrt{q}\delta_y & 0 & -\sqrt{q}\delta_x & \sqrt{q}\delta_y \\ \delta_y & \delta_x & -1 & -\delta_y & -\delta_x \end{bmatrix} F_{x,j}$

16. $K_t^i = \bar{\Sigma}_t H_t^{i\,T} (H_t^i \bar{\Sigma}_t H_t^{i\,T} + Q_t)^{-1}$

17. $\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$

18. $\bar{\Sigma}_t = (I - K_t^i H_t^i) \bar{\Sigma}_t$

EKF SLAM

19. end for
20. $\mu_t = \bar{\mu}_t$
21. $\Sigma_t = \bar{\Sigma}_t$
22. return μ_t, Σ_t

Loop Closing

- Loop closing means revisiting an already mapped area
- Look closing reduces the uncertainty in robot and landmark estimates
- This can be used in exploring an environment to get more accurate maps
- Wrong loop closure may result in a divergence

EKF SLAM

Algorithm EKF_SLAM ($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, N_{t-1}$)

1. $N_t = N_{t-1}$
2. $F_x = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \underbrace{0 & \cdots & 0}_{2N} \end{bmatrix}$
3. $\bar{\mu}_t = \mu_{t-1} + F_x^T \begin{bmatrix} -\frac{v_t}{w_t}\sin\mu_{t-1,\theta} + \frac{v_t}{w_t}\sin(\mu_{t-1,\theta} + w_t\Delta t) \\ \frac{v_t}{w_t}\cos\mu_{t-1,\theta} - \frac{v_t}{w_t}\cos(\mu_{t-1,\theta} + w_t\Delta t) \\ w_t\Delta t \end{bmatrix}$
4. $G_t = I + F_x^T g_t F_x \qquad g_t = \begin{bmatrix} 0 & 0 & -\frac{v_t}{w_t}\cos\mu_{t-1,\theta} + \frac{v_t}{w_t}\cos(\mu_{t-1,\theta} + w_t\Delta t) \\ 0 & 0 & -\frac{v_t}{w_t}\sin\mu_{t-1,\theta} + \frac{v_t}{w_t}\sin(\mu_{t-1,\theta} + w_t\Delta t) \\ 0 & 0 & 0 \end{bmatrix}$

EKF SLAM

5. $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + F_x^T R_t F_x$

6. $Q_t = \begin{bmatrix} \sigma_r & 0 \\ 0 & \sigma_\phi \end{bmatrix}$

7. for all observed features $z_t^i = \begin{bmatrix} r_t^i \\ \phi_t^i \end{bmatrix}$

8. $\begin{bmatrix} \bar{\mu}_{N_t+1,x} \\ \bar{\mu}_{N_t+1,y} \end{bmatrix} = \begin{bmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \end{bmatrix} + \begin{bmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \end{bmatrix}$

9. for $k = 1$ to $N + 1$

10. $\delta_k = \begin{bmatrix} \delta_{k,x} \\ \delta_{k,y} \end{bmatrix} = \begin{bmatrix} \bar{\mu}_{k,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{k,y} - \bar{\mu}_{t,y} \end{bmatrix}$

11. $q_k = {\delta_k}^T \delta_k$

12. $\hat{z}_t^k = \begin{bmatrix} \sqrt{q_k} \\ \mathrm{atan2}(\delta_{k,y}, \delta_{k,x}) - \bar{\mu}_{t,\theta} \end{bmatrix}$

EKF SLAM

13. $$F_{x,k} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}$$

14. $$H_t^k = \frac{1}{q_k} \begin{bmatrix} \sqrt{q_k}\delta_{k,x} & -\sqrt{q_k}\delta_{k,y} & 0 & -\sqrt{q_k}\delta_{k,x} & \sqrt{q_k}\delta_{k,y} \\ \delta_{k,y} & \delta_{k,x} & -1 & -\delta_{k,y} & -\delta_{k,x} \end{bmatrix} F_{x,k}$$

15. $$\Psi_k = H_t^k \bar{\Sigma}_t H_t^{k\,T} + Q_t$$

16. $$\pi_k = (z_t^i - \hat{z}_t^k)^T \Psi_k^{-1} (z_t^i - \hat{z}_t^k)$$

17. end for

18. $\pi_{N_t+1} = \alpha$

19. $j(i) = \underset{k}{\operatorname{argmin}}\, \pi_k$

20. $N_t = \max\{N_t, j(i)\}$

EKF SLAM

$K_t^i = \bar{\Sigma}_t (H_t^{j(i)})^T H_t^{-1}$

$\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$

$\bar{\Sigma}_t = (I - K_t^i H_t^i) \bar{\Sigma}_t$

end for

$\mu_t = \bar{\mu}_t$

$\Sigma_t = \bar{\Sigma}_t$

return μ_t, Σ_t