Bayes Filter

(자율주행 핵심기술 SLAM 단기강좌)

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What is Localization?

Dictionary

Search for a word

lo·cal·i·za·tion

/ˌlōkələˈzāSHən, ˌlōkəˌlīˈzāSHən/

noun

the process of making something local in character or restricting it to a particular place.
"the tug-of-war between the forces of globalization and localization"

- MEDICINE
  the fact of being or becoming located or fixed in a particular place.
  "differences in localization of growth control molecules in carcinoma"

What is Localization?

- GSM localization
  - determining the location of an active cell phone or wireless transceiver
- Robot localization
  - figuring out robot's position in an environment
- Indoor positioning system
  - a network of devices used to locate objects or people inside a building
- Navigation
  - determining one's position accurately on the surface of earth
- Radiolocation
  - finding the location of something via radio waves
- Satellite navigation
  - a positioning and navigation technique aided by satellites

Categories of Localization Techniques

- Cooperative localization techniques
  - Wireless signal based techniques (WiFi, cellular, UWB)
  - Device-to-device localization (or V2V)
  - Device-to-infrastructure localization (or V2I)
- Sensor-based localization techniques
  - GPS/IMU based techniques
  - Camera based techniques
  - Radar based techniques
  - Lidar based techniques
  - Ultrasonic based techniques

Sensor-based Localization

Technique	Sensors	Accuracy	Advantages	Disadvantages
Pure GPS	GPS	~10m	Low out	Low accuracy (poor signal availability)
GPS/IMU in ECEF coordinates	GPS & IMU	7.2m (RMSE)	Low cost	Low accuracy, cumulative errors
Two-stage vision-based SLAM	Camera	0.75m	Low cost	Susceptible to illumination and observation angle
Stereovision odometry	Camera	20.5m	Low cost	Low accuracy, cumulative errors
Aerial image-based localization	Camera, GPS, IMU	80% with 1m	Low cost	High errors
Microwave-Radar SLAM	Microwave Radar	10.5m	Low cost, Low power	Low accuracy

[REF] S. Kuutti, and et al, "A Survey of the State-of-the-art Localization Techniques and Their potentials for Autonomous Vehicle Applications," *IEEE Internet of Things Journal*, vol. 5, no. 2, Apr. 2018

Sensor-based Localization

Technique	Sensors	Accuracy	Advantages	Disadvantages
Short range Radar SLAM	Radar, GPS, IMU	0.07m lat. 0.38m long.	Low out, Low power, High accuracy	Low robustness to dynamic environments
Lidar SLAM	Lidar, GPS, IMU	0.017m lat. 0.033m long.	High accuracy, robust to changes in environment	High cost, high power and processing, sensitive to weather
Camera localization with Lidar map	Camera, IMU	0.14m lat. 0.19m long.	High accuracy, Low cost	Requires a Lidar map
LRF based localization	LRF, GPS, IMU	3.098m	Low cost	High errors
Ultrasonic SLAM	Ultrasonic		Low cost, Low power	Low accuracy, Long processing time

[REF] S. Kuutti, and et al, "A Survey of the State-of-the-art Localization Techniques and Their potentials for Autonomous Vehicle Applications," *IEEE Internet of Things Journal*, vol. 5, no. 2, Apr. 2018

Localization as an Estimation Problem

- Objective
  - Determination of the pose (= position + orientation) of a mobile robot in a known environment
- The robot must infer its pose from available data

Data (noisy):

- **Motion information:**
  - → **Proprioceptive sensors (e.g. encoders, accelerometers, etc.)**
- **Environment Measurements**
  - → **Exteroceptive sensors (e.g. laser, sonar, IR, GPS, camera, RFID, etc.)**

A filtering approach is required to fuse all information

Probability Theory

- Uncertainty
  - Our main tool is the probability theory, which assigns to each sentence numerical degree of belief between 0 and 1
- Variables
  - Boolean random variables: true or false
  - Discrete random variables: weather might be sunny, rainy, cloudy, snow
    - P(Weather=sunny)
    - P(Weather=rainy)
    - P(Weather=cloudy)
    - P(Weather=snow)
  - Continuous random variables: the temperature has continuous values

Probability Theory

- Before the evidence is obtained; prior probability
  - P(a) the prior probability that the proposition is true
  - P(dice)=1/6
- After the evidence is obtained; posterior probability
  - P(a|b)
  - The probability of a given that all we know is b
  - P(prize|dice)=0.8

Probability Theory

- Theorem of total probability
  - If events $a_1, a_2, \cdots, a_n$ are mutually exclusive and $\sum_{i=1}^{n} p(a_i) = 1$
  - Then

$$p(b) = \sum_{i=1}^{n} p(b|a_i)\, p(a_i) \quad \text{and} \quad p(b) = \sum_{i=1}^{n} p(b, a_i)$$

- Bayes' Rule
  - He set down his findings on probability in "Essay Towards Solving a Problem in the Doctrine of Chances" (1763), published posthumously in the Philosophical Transactions of the Royal Society of London.

$$p(b|a) = \frac{p(a|b)\, p(b)}{p(a)}$$

Bayes Theorem

- Bayes' Theorem
  - shows the relationship between a conditional probability and its inverse.
- It allows us to make an inference from
  - the probability of a hypothesis given the evidence to the probability of that evidence given the hypothesis and vice versa
- Bayes rule

$$p(h|D) = \frac{p(D|h)\,p(h)}{p(D)} = \frac{p(D|h)\,p(h)}{\sum_{h'} p(D|h')\,p(h')} = \eta\,p(D|h)\,p(h)$$

$p(h)$ = prior probability of hypothesis h

$p(D)$ = prior probability of training data D

$p(h|D)$ = probability of h given D -> posterior probability distribution

$p(D|h)$ = probability of D given h -> generative model

Diagnosis

- What is the probability of meningitis in the patient with stiff neck?
  - A doctor knows that the disease meningitis causes the patient to have a stiff neck in 50% of the time -> $p(s|m)$
  - Prior Probabilities:
    - That the patient has meningitis is 1/50.000 -> $p(m)$
    - That the patient has a stiff neck is 1/20 -> $p(s)$

$$p(m|s) = \frac{p(s|m)\,p(m)}{p(s)} = \frac{0.5 \times 0.00002}{0.05} = 0.0002$$

Hypothesis

- Generally want the most probable hypothesis given the training data
- Maximum a posteriori hypothesis h_{MAP}:

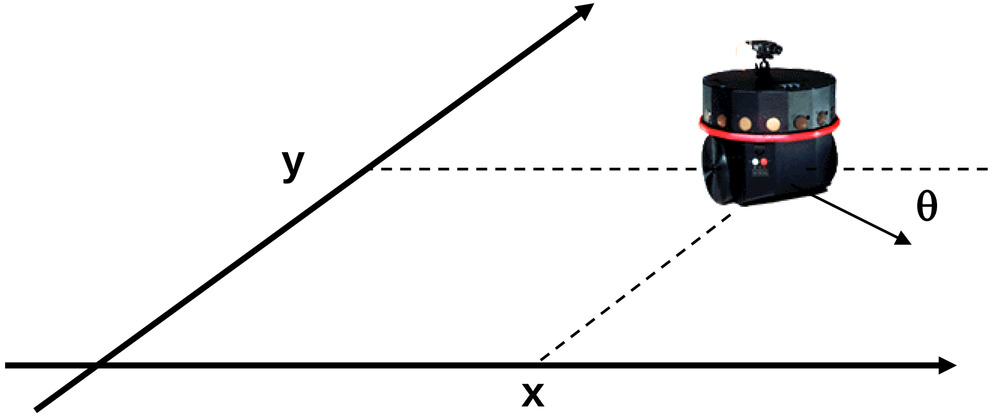
$$h_{MAP} = \underset{h \in H}{\arg\max}\, p(h|D) = \underset{h \in H}{\arg\max} \frac{p(D|h)\, p(h)}{p(D)} = \underset{h \in H}{\arg\max}\, p(D|h)\, p(h)$$

- If assume $p(h_i) = p(h_j)$ for all h_i and h_j, then can further simplify, and choose the Maximum likelihood (ML) hypothesis

$$h_{ML} = \underset{h_i \in H}{\arg\max}\, p(D|h_i)$$

Notation

$x_r = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$ Robot pose



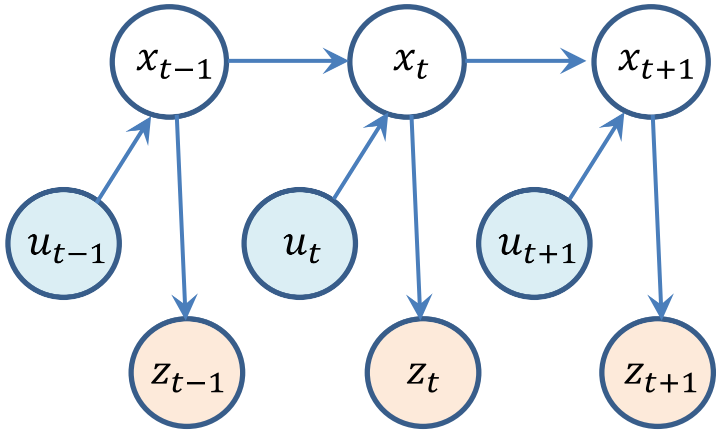
$x_{r,0:t} = \{x_{r,0}, x_{r,1}, \cdots, x_{r,t}\}$ Robot poses from time 0 to time t

$z_{1:t} = \{z_1, z_2, \cdots, z_t\}$ Robot exteroceptive measurements from time 1 to time t

$u_{0:t} = \{u_0, u_1, \cdots, u_t\}$ Motion commands (or proprioceptive measurements) from time 0 to time t

Probabilistic Generative Laws

- The evolution of state and measurements



The dynamic Bayes network (or hidden Markov model) that characterizes the evolution of controls, states, and measurements.

State transition probability:

$$p(x_t|x_{0:t-1}, z_{1:t-1}, u_{1:t}) = p(x_t|x_{t-1}, u_t)$$

The state at time t is stochastically dependent on the state at time $t-1$ and the control u_t.

Measurement probability:

$$p(z_t|x_{0:t}, z_{1:t-1}, u_{1:t}) = p(z_t|x_t)$$

The measurement z_t depends statistically on the state at time t.

Bayes Filters

- The concept of a belief
  - The robot's internal knowledge about the state of the environment.
- Prior belief of the robot at time t
  - Probability density function (pdf) before acquiring the last measurement z_t:

$$\overline{bel}_t(x_r) = p(x_{r,t} = x_r | z_{1:t-1}, u_{0:t})$$

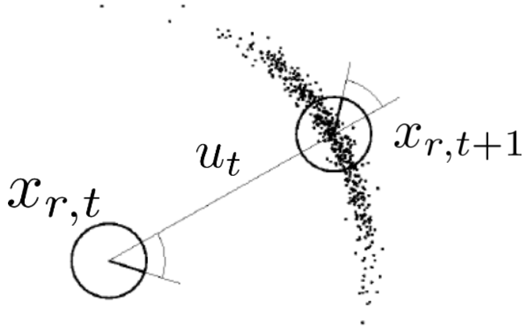
- Belief of the robot at time t
  - Pdf describing the information the robot has regarding its pose at time t, based on all available data (measurements and motion):

$$bel_t(x_r) = p(x_{r,t} = x_r | z_{1:t}, u_{0:t})$$

Bayes Filters

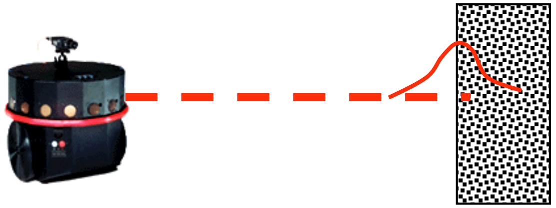
- Robot motion model
  - the pdf of the robot pose at time $t+1$ given the robot pose and the motion action at time t. It takes into account the noise from the proprioceptive sensors:

$$p(x_{r,t+1} | x_{r,t}, u_t)$$



- Measurement model
  - describes the probability of observing a given measurement z_t at time t when the robot pose is $x_{r,t}$. It takes into account the noise from the exteroceptive sensors

$$p(z_t | x_{r,t})$$



Bayes Filters

Prediction

$$\overline{bel}_t(x_r) = \int_{\underbrace{\boxed{\Omega}}_{\text{Robot pose space}}} \underbrace{\boxed{p\left(x_r \middle| x_{r,t-1} = y, u_t\right)}}_{\text{Motion model}} bel_{t-1}(y)dy$$

Based on the total probability theorem:

$$p(A) = \sum_i p(A|B_i)\, p(B_i) \qquad \text{(discrete case)}$$

where B_i, i=1,2,... is a partition of Ω. In the continuous case:

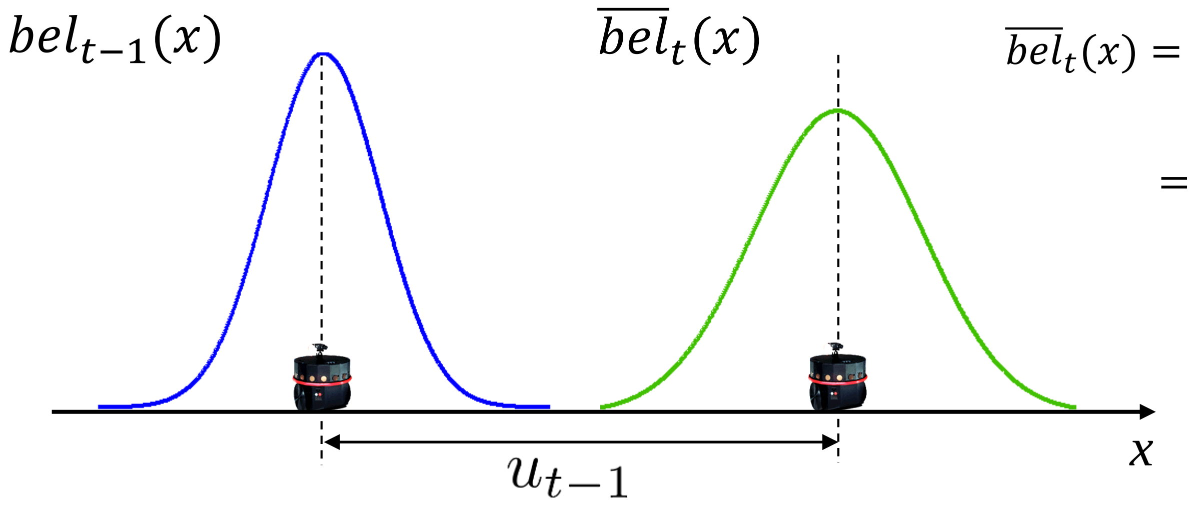
$$p(x) = \int p(x|y)\, p(y)dy$$

Illustration of the Prediction Step

- A uni-dimensional space with a realistic motion model

$$x_t = x_{t-1} + u_t + n_t \qquad \text{where the noise } n_t \sim p_n(n_t)$$

$$\longrightarrow \; p(x_t = x | x_{t-1} = y, u_t) = p_n(x - y - u_t)$$



$$\overline{bel}_t(x) = \int_\Omega p(x | x_{t-1} = y, u_t)\, bel_{t-1}(y) dy$$
$$= \int_\Omega p_n(x - y - u_t)\, bel_{t-1}(y) dy$$

Illustration of the Prediction Step

Correction or Measurement Update

$$bel_t(x_r) = p\left(x_{r,t} = x_r \middle| z_{1:t}, u_{0:t}\right)$$

$$= \underbrace{\boxed{\eta}}_{\text{Normalizing factor}} \underbrace{\boxed{p\left(z_t \middle| x_{r,t} = x_r\right)}}_{\text{Measurement model}} \overline{bel}_t(x_r)$$

Based on the Bayes Rule: $p(A|B,C) = \dfrac{p(B|A,C)\,p(A|C)}{p(B|C)}$

Taking: $A = \{x_{r,t} = x_r\}$ $B = \{z_t\}$ $C = \{z_{1:t-1}, u_{0:t}\}$

We have: $bel_t(x_r) = p\left(x_{r,t} = x_r \middle|\, z_{1:t}, u_{0:t}\right)$

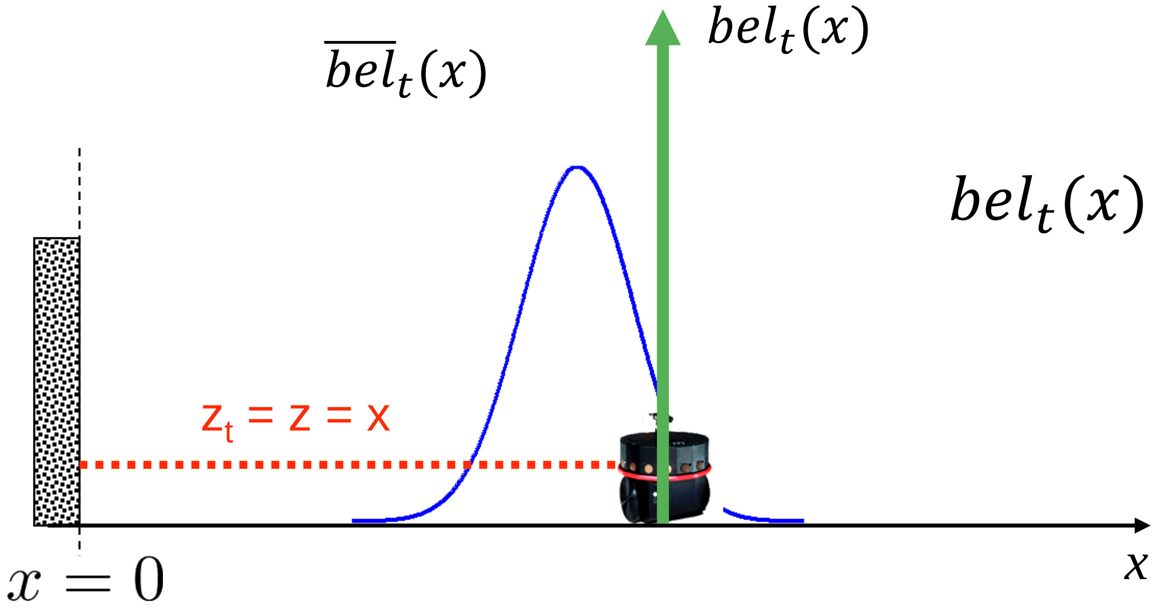
$$= \underbrace{\boxed{p\left(z_t \middle| x_{r,t} = x_r, z_{1:t-1}, u_{0:t}\right)}}_{p(z_t|x_{r,t} = x_r)} \frac{\overbrace{\boxed{p\left(x_{r,t} = x_r \middle|\, z_{1:t-1}, u_{0:t}\right)}}^{\overline{bel}_t(x_r)}}{\underbrace{\boxed{p\left(z_t \middle|\, z_{1:t-1}, u_{0:t}\right)}}_{1/\eta}}$$

Illustration of the Correction Step

- A uni-dimensional space under an ideal measurement model

$$z_t = x_t$$

$$\longrightarrow \quad p(z_t = z | x_t = x) = \delta(x - z)$$



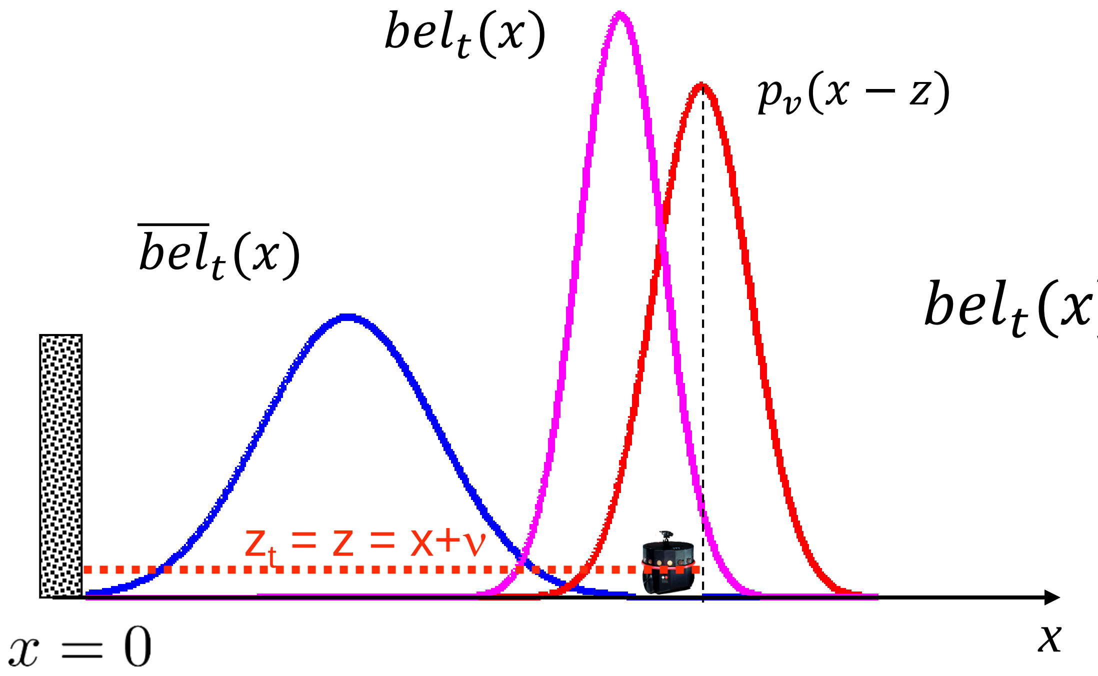
$$
\begin{aligned}
bel_t(x) &= \eta \, p(z_t | x_t = x) \, \overline{bel}_t(x) \\
&= \eta \, \delta(x - z) \, \overline{bel}_t(x) \\
&= \delta(x - z)
\end{aligned}
$$

Illustration of the Correction Step

- A uni-dimensional space under an realistic measurement model

$$z_t = x_t + v_t \qquad \text{where the noise } v_t \sim p_v(v_t)$$

$$\longrightarrow \quad p(z_t = z | x_t = x) = p_v(x - z)$$



$$bel_t(x) = \eta\, p(z_t | x_t = x)\, \overline{bel}_t(x)$$
$$= \eta\, p_v(x - z)\, \overline{bel}_t(x)$$

Bayes Filters

- A recursive algorithm.
- The new belief $bel_t(x_r)$ is determined at time t, given the belief $bel_{t-1}(x_r)$ at time t – 1, the last motion control u_{t-1} and the last measurement z_t.

For all $x_r \in \Omega$ do the following:

$$\overline{bel}_t(x_r) = \int_\Omega \overbrace{p\left(x_r \middle| x_{r,t-1} = y, u_t\right)}^{\text{Motion model}} bel_{t-1}(y)dy$$

$$bel_t(x_r) = \eta \overbrace{p\left(z_t \middle| x_{r,t} = x_r\right)}^{\text{Measurement model}} \overline{bel}_t(x_r)$$