Occupancy Grid Mapping

(자율주행 핵심기술 SLAM 단기강좌)

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Mapping

- Why maps?
  - Learning maps is one of the fundamental problems in mobile robotics
  - Maps allow robots to efficiently carry out their tasks
    and allow localization
  - Successful robot systems rely on maps for localization, path planning, activity planning etc.
- Mapping is to calculate the most likely map given the sensor data

$$m^* = \underset{m}{\operatorname{argmax}}\, p(m \mid z_{1:t}, u_{1:t})$$

Mapping and Localization

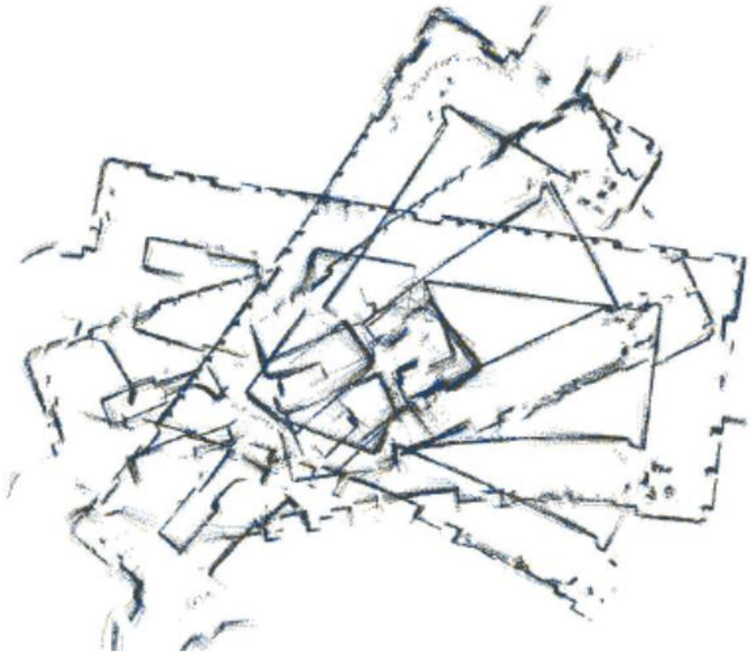
- So far we have learned
  - how to estimate the pose of the vehicle given the control and the measurement, and the map.
- Mapping involves to simultaneously estimate the pose of the vehicle and the map.
- How to calculate a map given we know the pose of the vehicle?

Problems in Mapping

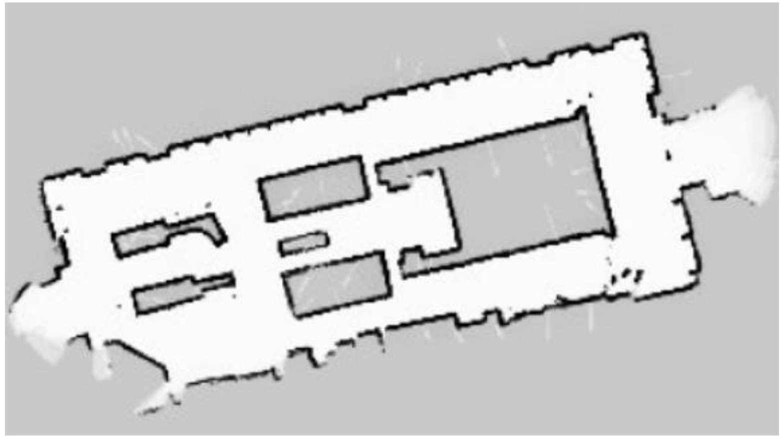
- Sensor interpretation
  - How do we extract relevant information from raw sensor measurement?
  - How do we represent and integrate this information over time?
- Robot locations have to be estimated
  - How can we identify that we are at a previously visited place?
  - This problem is the so-called data association problem.

Problems in Mapping

- Occupancy grid map



(a) Raw range data, position indexed by odometry.

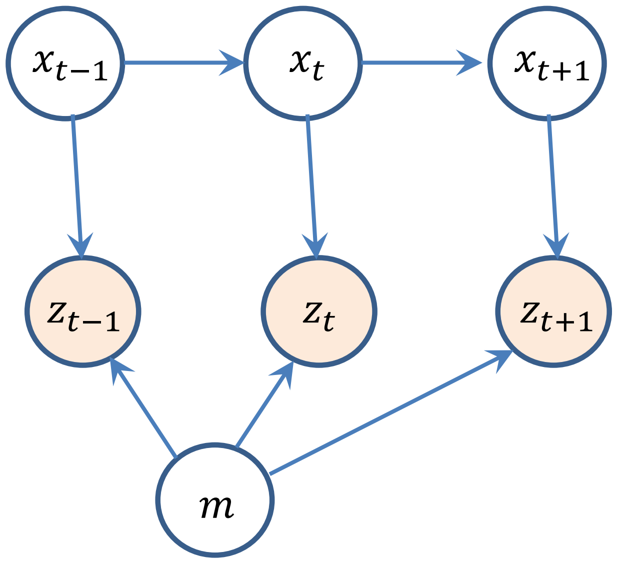


(b) Occupancy grid map.

From Thrun Burgard Fox, Probabilistic Robotics, MIT Press 2006

Mapping with Known Poses

- Graphical model



Graphical model of mapping with known poses. The shaded variables are known. The goal of mapping is to recover the map.

Occupancy Grid Maps

- Introduced by Moravec and Elfes in 1985
- Discretize the world into cells
- Represent environment by grids
- Rigid grid structure
- Each cell is assumed to be occupied or empty
- Non-parametric model
- Require substantial memory resources
- Do not rely on a feature detector
- Represents the map as a field of random variables arranged in an evenly spaced grid

Occupancy Grid Maps

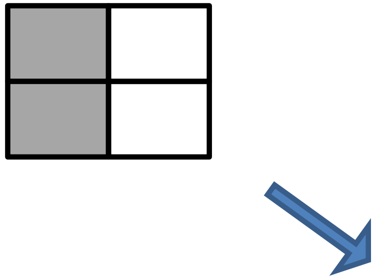
- Estimate the probability that a cell is occupied by an obstacle.
- Each cell is a binary random variable that models the occupancy of the cell
  - Cell is occupied $p(m_i) = 1$
  - Cell is empty $p(m_i) = 0$
  - Not sure $p(m_i) = 1/2$
- Key assumptions
  - Occupancy of individual grid cells is independent
  - Robot positions are known

Occupancy Grid Maps

- The probability distribution of the map

$$p(m) = \prod_i p(m_i)$$

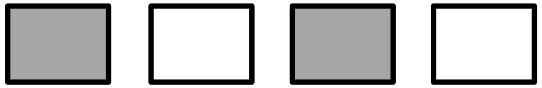
- Example of 4x4 map



The probability of occupancy for each cell

0.7	0.2
0.6	0.1

The probability of this particular map



0.7 $*$ 0.8 $*$ 0.6 $*$ 0.9 = 0.3024

Estimating a Map from Data

- Estimate the map given sensor data and the poses

$$p(m|z_{1:t}, x_{1:t}) = \prod_i p(m_i|z_{1:t}, x_{1:t})$$

Binary random variable

- No control information and only observations are given.
- Static state binary Bayes filter

Binary Bayes Filter

$$p(m_i|z_{1:t}, x_{1:t}) = \frac{p(z_t|m_i, z_{1:t-1}, x_{1:t})\, p(m_i|z_{1:t-1}, x_{1:t})}{p(z_t|z_{1:t-1}, x_{1:t})}$$ (Bayes rule)

$$= \frac{p(z_t|m_i, x_t)\, p(m_i|z_{1:t-1}, x_{1:t-1})}{p(z_t|z_{1:t-1}, x_{1:t})}$$ (Markov)

$$= \frac{p(m_i|z_t, x_t)p(z_t|\, x_t)\, p(m_i|z_{1:t-1}, x_{1:t-1})}{p(m_i|x_t)\, p(z_t|z_{1:t-1}, x_{1:t})}$$ (Bayes rule)

$$= \frac{p(m_i|z_t, x_t)p(z_t|\, x_t)\, p(m_i|z_{1:t-1}, x_{1:t-1})}{p(m_i)\, p(z_t|z_{1:t-1}, x_{1:t})}$$ (Markov)

$$p(z_t|m_i, x_t) = \frac{p(z_t|x_t)\, p(m_i|z_t, x_t)}{p(m_i|x_t)}$$

Binary Bayes Filter

- Do the same for the opposite event

$$p(-m_i|z_{1:t},x_{1:t}) = \frac{p(-m_i|z_t,x_t)p(z_t|\ x_t)\ p(-m_i|z_{1:t-1},x_{1:t-1})}{p(-m_i)\ p(z_t|z_{1:t-1},x_{1:t})}$$

- Compute the ratio between those two

$$\frac{p(m_i|z_{1:t},x_{1:t})}{p(-m_i|z_{1:t},x_{1:t})} = \frac{\dfrac{p(m_i|z_t,x_t)\cancel{p(z_t|\ x_t)}\ p(m_i|z_{1:t-1},x_{1:t-1})}{p(m_i)\ \cancel{p(z_t|z_{1:t-1},x_{1:t})}}}{\dfrac{p(-m_i|z_t,x_t)\cancel{p(z_t|\ x_t)}\ p(-m_i|z_{1:t-1},x_{1:t-1})}{p(-m_i)\ \cancel{p(z_t|z_{1:t-1},x_{1:t})}}}$$

$$= \frac{p(m_i|z_t,x_t)\ p(m_i|z_{1:t-1},x_{1:t-1})p(-m_i)}{p(-m_i|z_t,x_t)\ p(-m_i|z_{1:t-1},x_{1:t-1})p(m_i)}$$

$$= \underbrace{\frac{p(m_i|z_t,x_t)}{1-p(m_i|z_t,x_t)}}_{\text{uses } z_t}\ \underbrace{\frac{p(m_i|z_{1:t-1},x_{1:t-1})}{1-p(m_i|z_{1:t-1},x_{1:t-1})}}_{\text{recursive terms}}\ \underbrace{\frac{1-p(m_i)}{p(m_i)}}_{\text{prior}}$$

Log Odds Notation

- Log odds ratio is defined as

$$l(x) = \log \frac{p(x)}{1 - p(x)}$$

- $p(x)$ is obtained as

$$p(x) = 1 - \frac{1}{1 + \exp l(x)}$$

Occupancy Mapping in Log Odds

- The ratio can be written using log odds form as

$$l(m_i|z_{1:t}, x_{1:t}) = \underbrace{l(m_i|z_t, x_t)}_{\text{inverse sensor model}} + \underbrace{l(m_i|z_{1:t-1}, x_{1:t-1})}_{\text{recursive terms}} - \underbrace{l(m_i)}_{\text{prior}}$$

- In short

$$l_{t,i} = \text{inverse\_sensor\_model}(m_i, z_t, x_t) + l_{t-1,i} - l_0$$

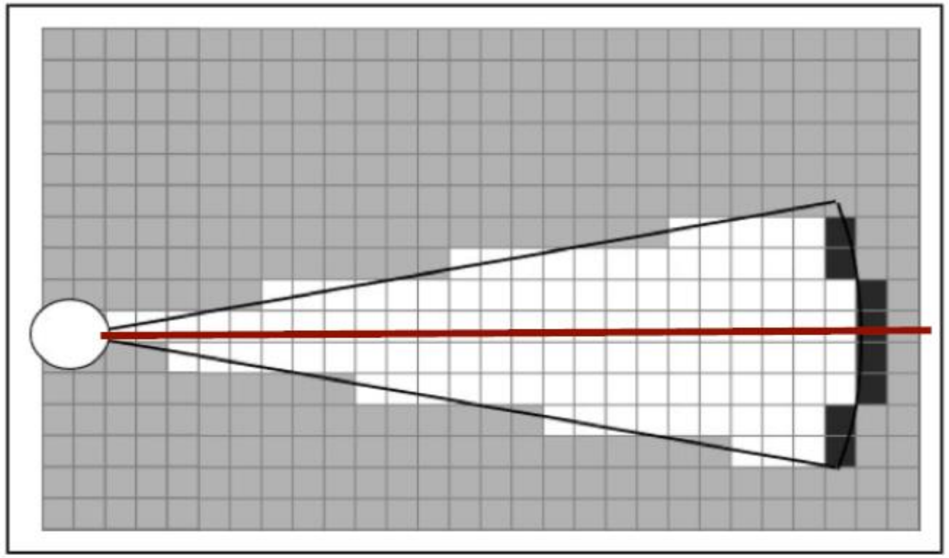
$$\text{inverse\_sensor\_model}(m_i, z_t, x_t) = \log \frac{p(m_i|z_t, x_t)}{1 - p(m_i|z_t, x_t)}$$

Occupancy Mapping Algorithm

Algorithm occupancy_grid_mapping($\{l_{t-1,i}\}$, x_t , z_t)

```
for all cells m_i do
    if m_i in perceptual field of z_t, then
        l_{t,i} = l_{t-1,i} + inverse_sensor_model(m_i, z_t, x_t) - l_0
    else
        l_{t,i} = l_{t-1,i}
    end if
end for
return {l_{t-1,i}}
```

Inverse Sensor Model

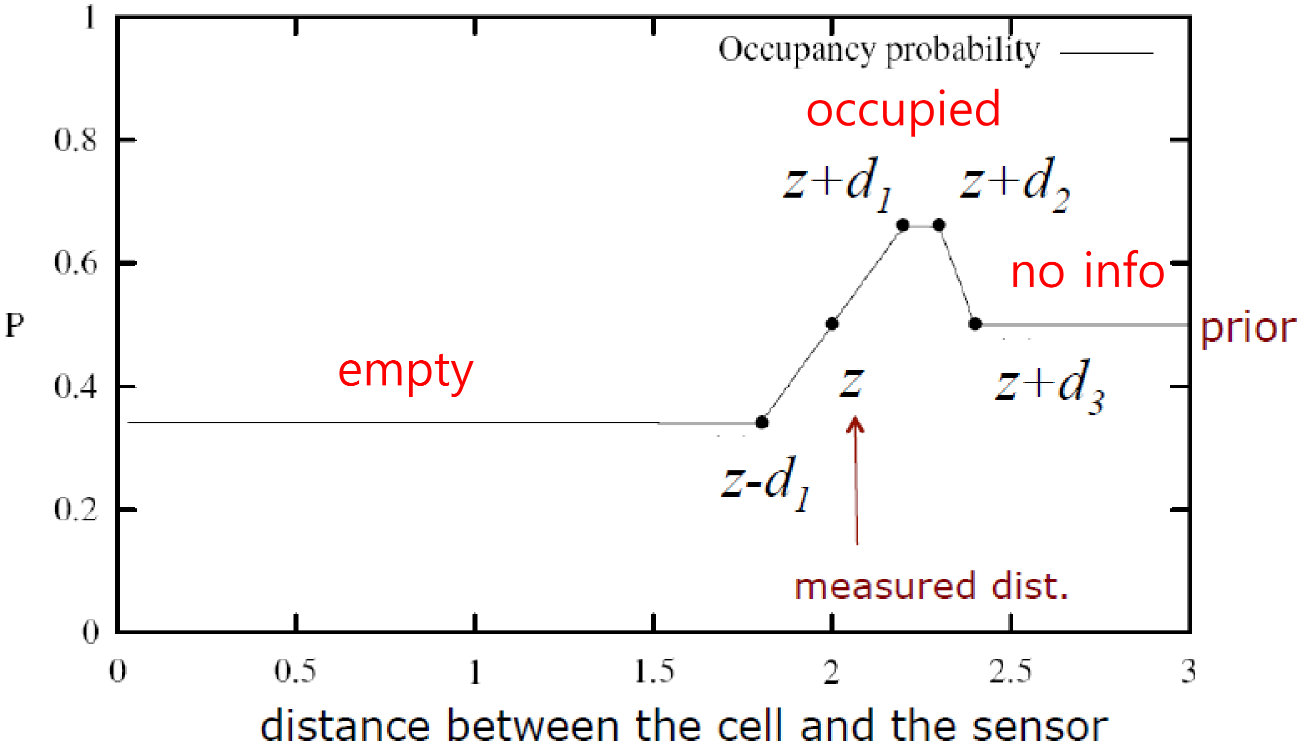


In the following, consider the cells along the optical axis (red line)

From Thrun Burgard Fox, Probabilistic Robotics, MIT Press 2006

Occupancy Value

- For sonar sensors



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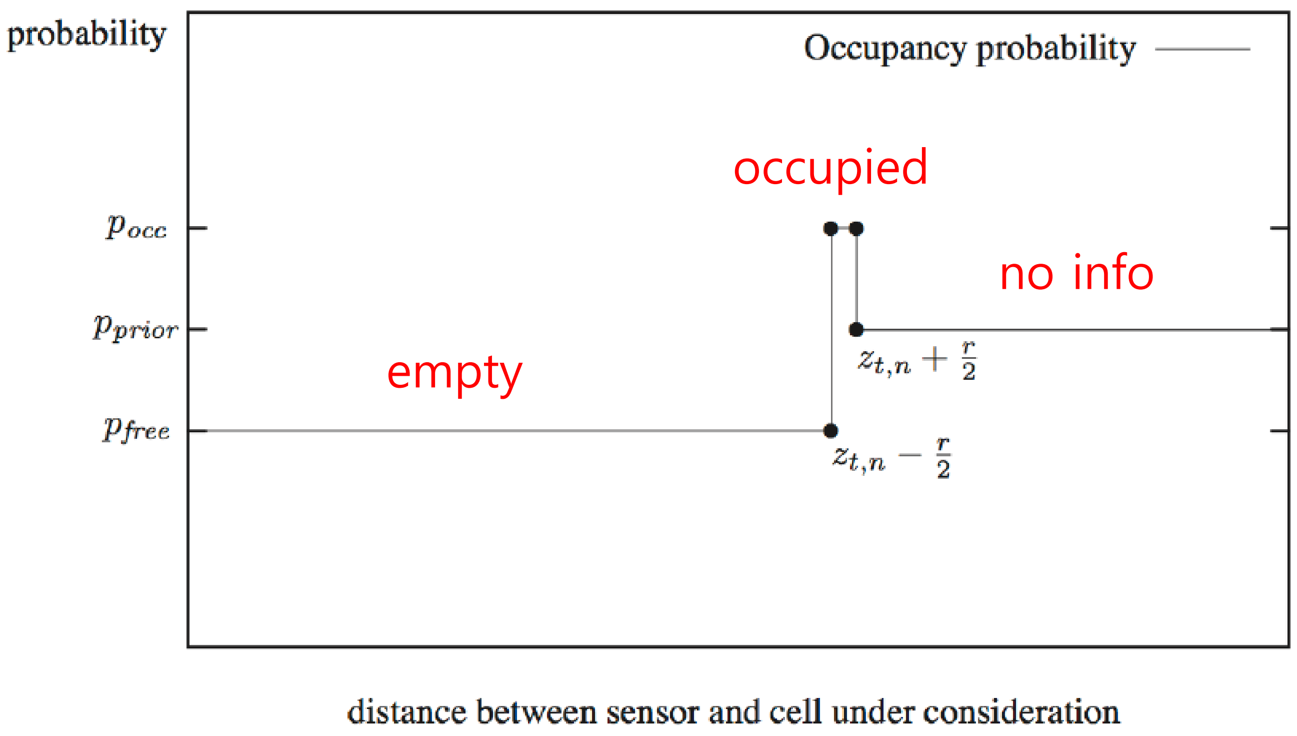
Occupancy Value

- For laser range finders



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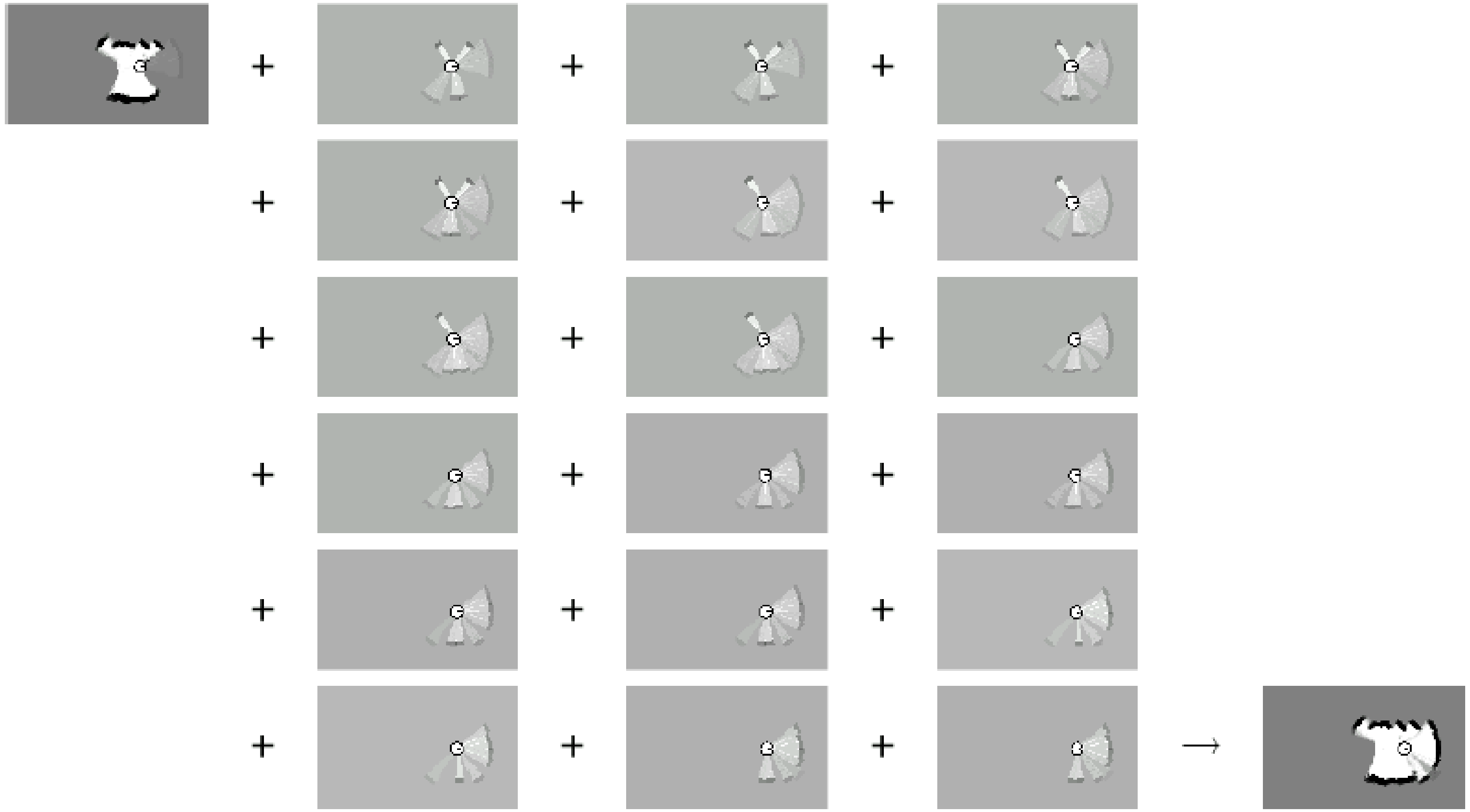
Incremental Update



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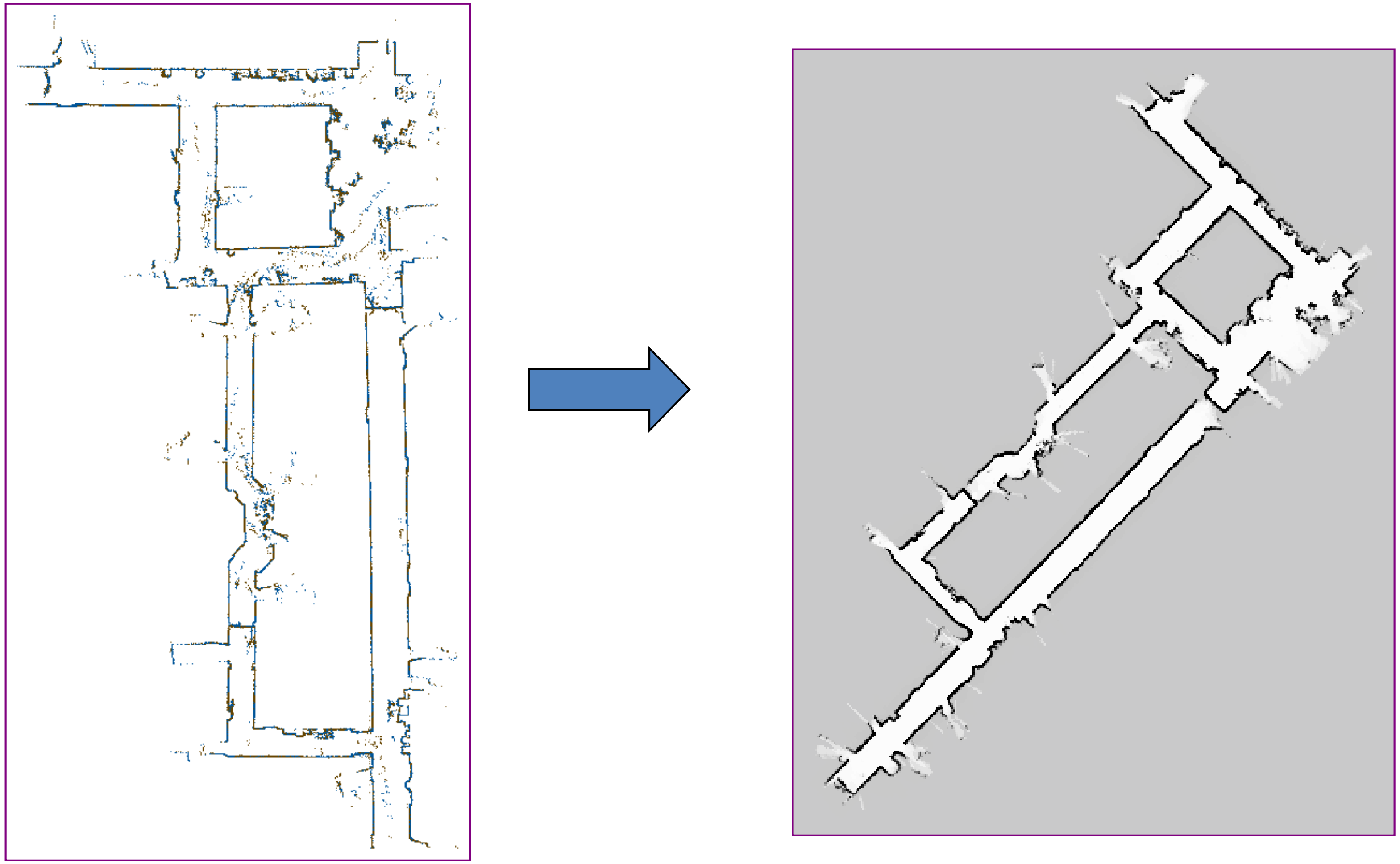
Example



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