Robot Perception Model

(자율주행 핵심기술 SLAM 단기강좌)

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Measurement Model

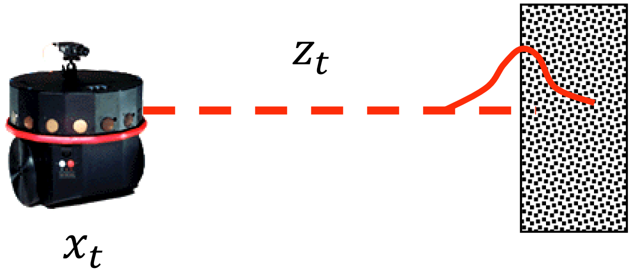
- Measurement model

$$p(z_t|x_t)$$

Assume:

- x_t : robot pose at time t

Due to sensor noise, z_t is available only through a probability density function.



Measurement Model

- Measurement model

$$p(z_t|x_t)$$

Assume:

- x_t : robot pose at time t

$$z_t = \{z_t^1, z_t^2, \cdots, z_t^K\}$$

K : number of measurements

z_t^k : individual measurement

The probability is obtained as the product of the individual measurement likelihoods:

$$p(z_t|\, x_t, m) = \prod_{k=1}^{K} p(z_t^k|\, x_t, m)$$

Maps

- A map m is
  - a list of objects in the environment and their locations

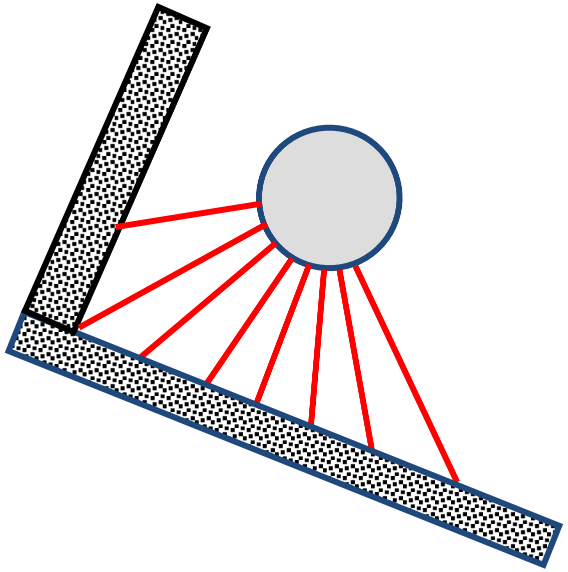
$$m = \{m_1, m_2, m_3, \cdots, m_N\}$$

N : number of objects

- Maps are indexed in one of two ways
  - Feature-based
    - The value of m_n is the Cartesian location of the feature
  - Location-based
    - The index n is a specific location. The element is often written as $m_{x,y}$

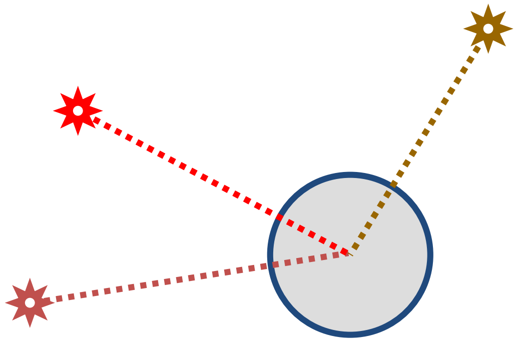
Measurement Model

- Two typical settings



Beam model of range finders.

The environment is known and is represented by line segments (e.g. the walls). The robot senses the distance from them at some given directions.



Landmark measurements.

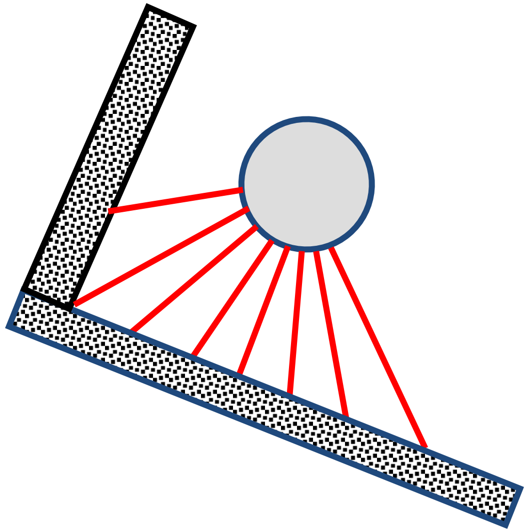
The environment is characterized by features (artificial or natural landmarks), whose position is known and whose identity can be known (e.g. RFID tags) or must be estimated (data association problem). The robot measures the distance and/or the bearing to these fixed points.

Beam Model

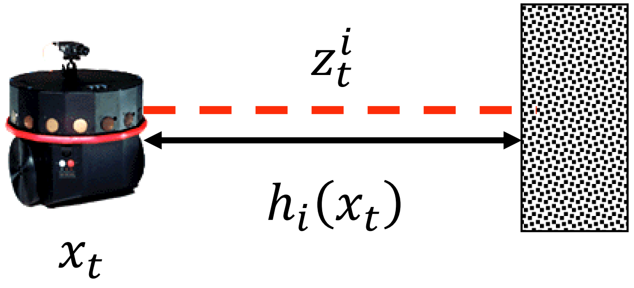
- Beam model of range finders

$$z_t = \{z_t^1, z_t^2, \cdots, z_t^K\}$$

Each measurement z_t^i is a range measurement of the beam (e.g. of a sonar or a laser) generated by a sensor and reflected by an obstacle.



$h_i(x_t)$ Distance of sensor i from the reflecting obstacle (ideal reading of the sensor)

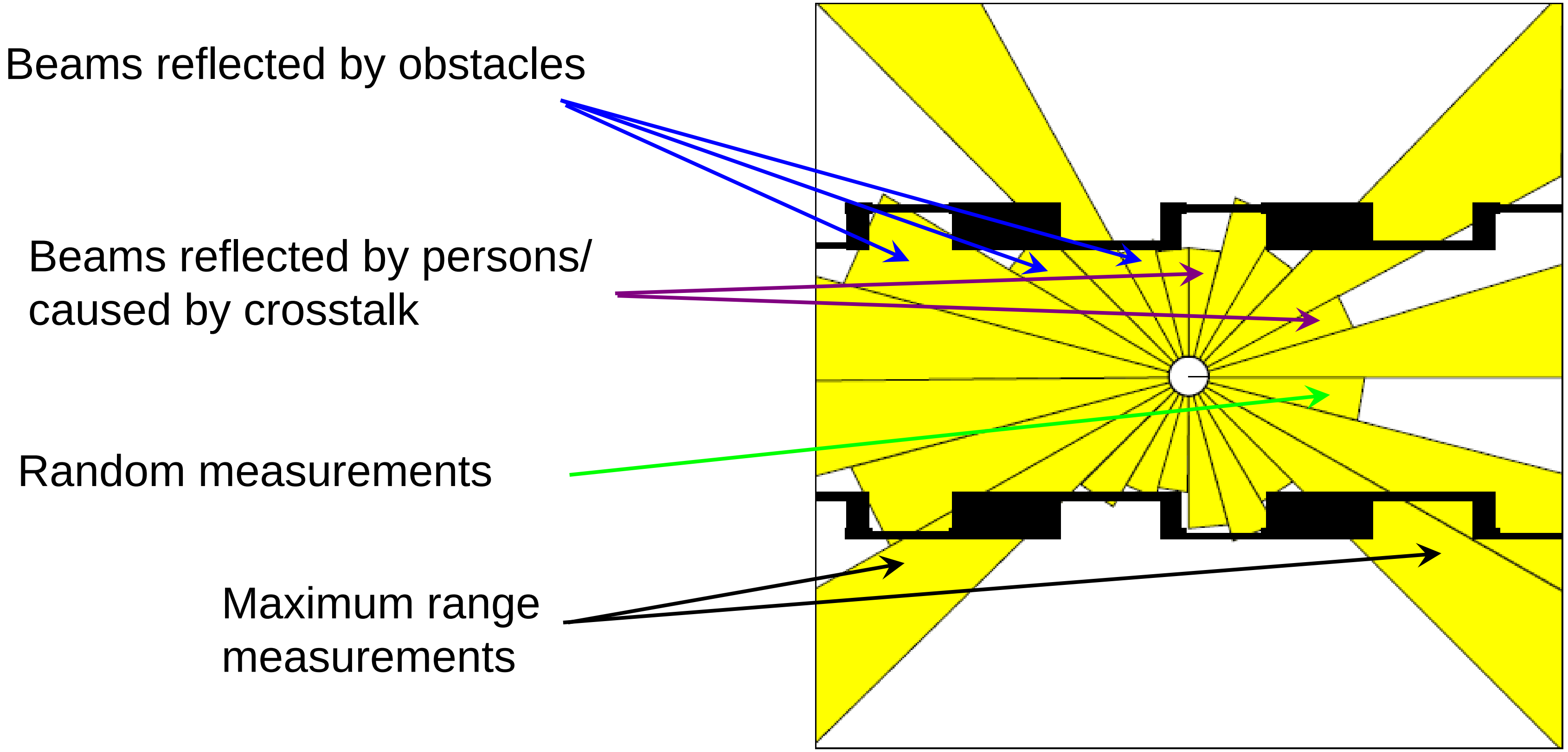


$$z_t^i = h_i(x_t) + \epsilon_t^i$$

$$\epsilon_t^i \sim \mathbb{N}(0, \sigma_r^2)$$

Beam Model

- Typical measurement errors of range measurements



Beam Model

- Measurement errors
  - Correct range with local measurement noise

$$p_{hit}\left(z_t^k \middle| x_t, m\right) = \begin{cases} \eta\ \mathbb{N}(z_t^{k*}, \sigma_{hit}^2), & \text{if } 0 \le z_t^k \le z_{max} \\ 0, & \text{otherwise} \end{cases}$$

  - Unexpected objects

$$p_{short}\left(z_t^k \middle| x_t, m\right) = \begin{cases} \eta \lambda_{short} e^{-\lambda_{short} z_t^k}, & \text{if } 0 \le z_t^k \le z_t^{k*} \\ 0, & \text{otherwise} \end{cases}$$

  - Failures

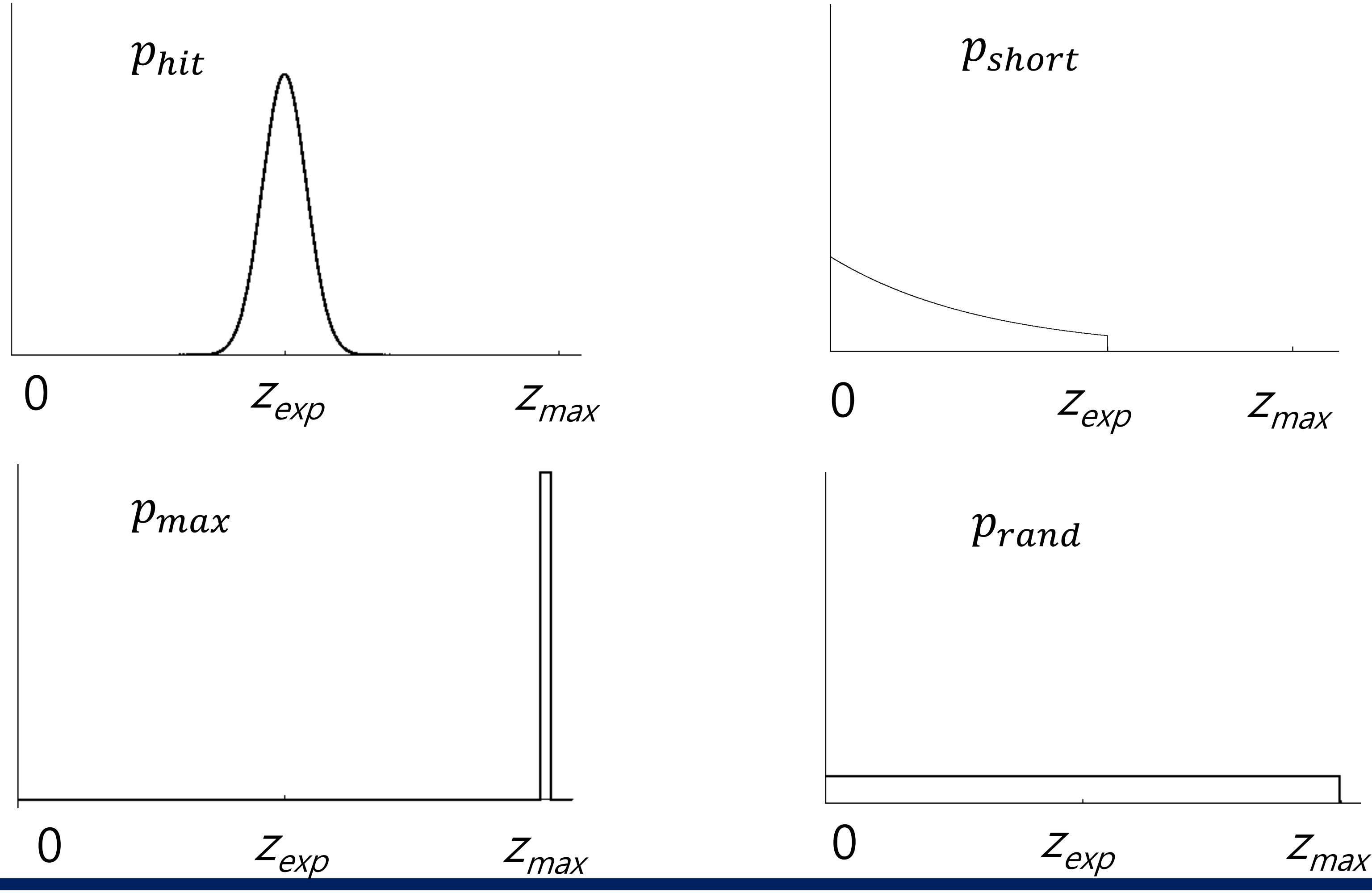
$$p_{max}\left(z_t^k \middle| x_t, m\right) = \begin{cases} 1, & \text{if } z = z_{max} \\ 0, & \text{otherwise} \end{cases}$$

  - Random measurements

$$p_{rand}\left(z_t^k \middle| x_t, m\right) = \begin{cases} \dfrac{1}{z_{max}}, & \text{if } 0 \le z_t^k \le z_{max} \\ 0, & \text{otherwise} \end{cases}$$

Beam Model

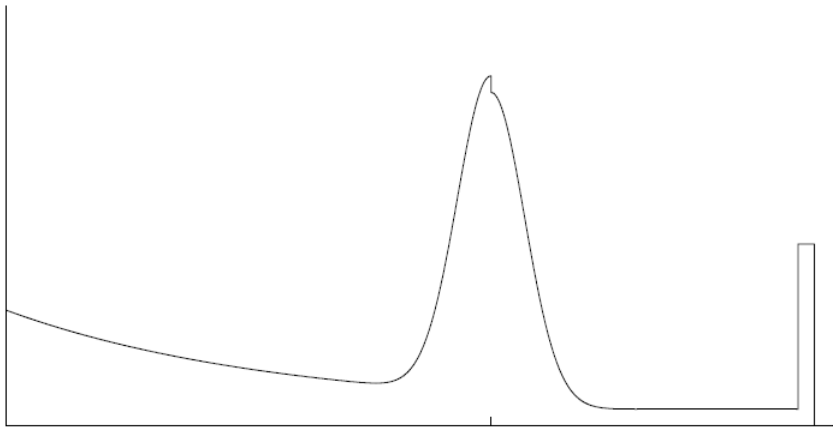
- Components of sensor model



From Thrun Burgard Fox, Probabilistic Robotics, MIT Press 2006

Beam Model

- Resulting mixture density



$$p(z_t^k|x_t,m) = \begin{bmatrix} \alpha_{hit} \\ \alpha_{short} \\ \alpha_{max} \\ \alpha_{rand} \end{bmatrix}^T \begin{bmatrix} p_{hit}(z_t^k|x_t,m) \\ p_{short}(z_t^k|x_t,m) \\ p_{max}(z_t^k|x_t,m) \\ p_{rand}(z_t^k|x_t,m) \end{bmatrix}$$

From Thrun Burgard Fox, Probabilistic Robotics, MIT Press 2006

Beam Model

- Algorithm

Algorithm beam_range_finder_model z_t x_t m):

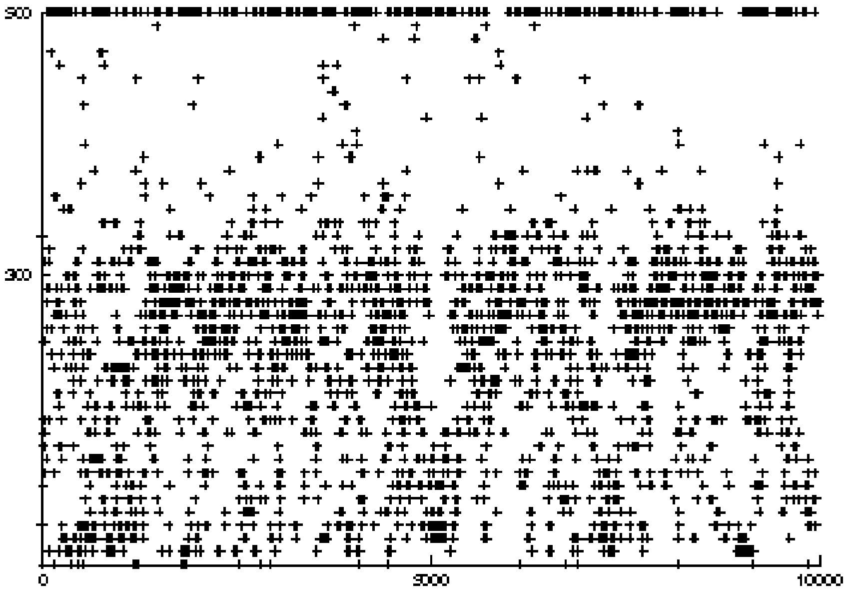
$q = 1$
for all $k = 1$ to K do
    compute z_t^{k*} for the measurement z_t^k using ray casting
    $p = z_{\text{hit}} \cdot p_{\text{hit}}(z_t^k | x_t, m) + z_{\text{short}} \cdot p_{\text{short}}(z_t^k | x_t, m) + z_{\max} \cdot p_{\max}(z_t^k | x_t, m) + z_{\text{rand}} \cdot p_{\text{rand}}(z_t^k | x_t, m)$
    $q = q \cdot p$
return q

- Limitation of beam model
  - Lack of smoothness
  - Computationally expensive

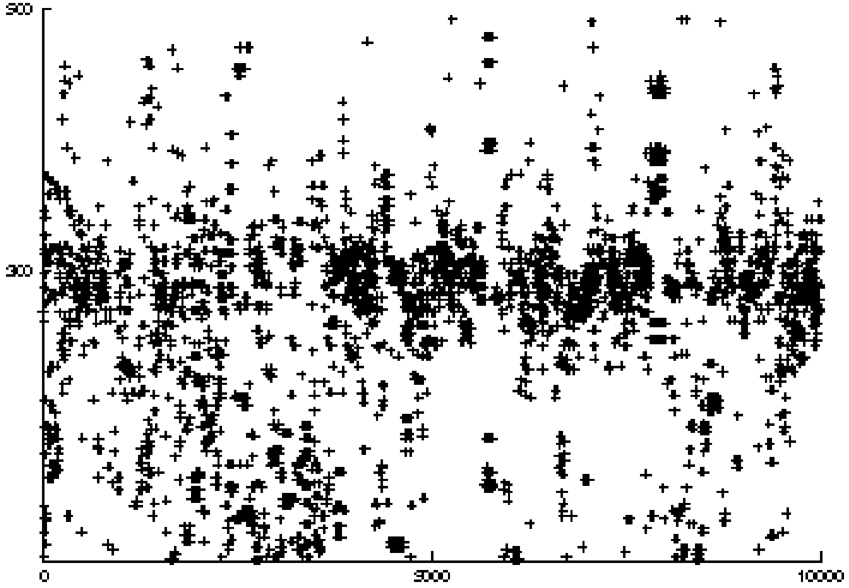
Beam Model

- Raw sensor data

Measured distances for expected distance of 300 cm.



Sonar



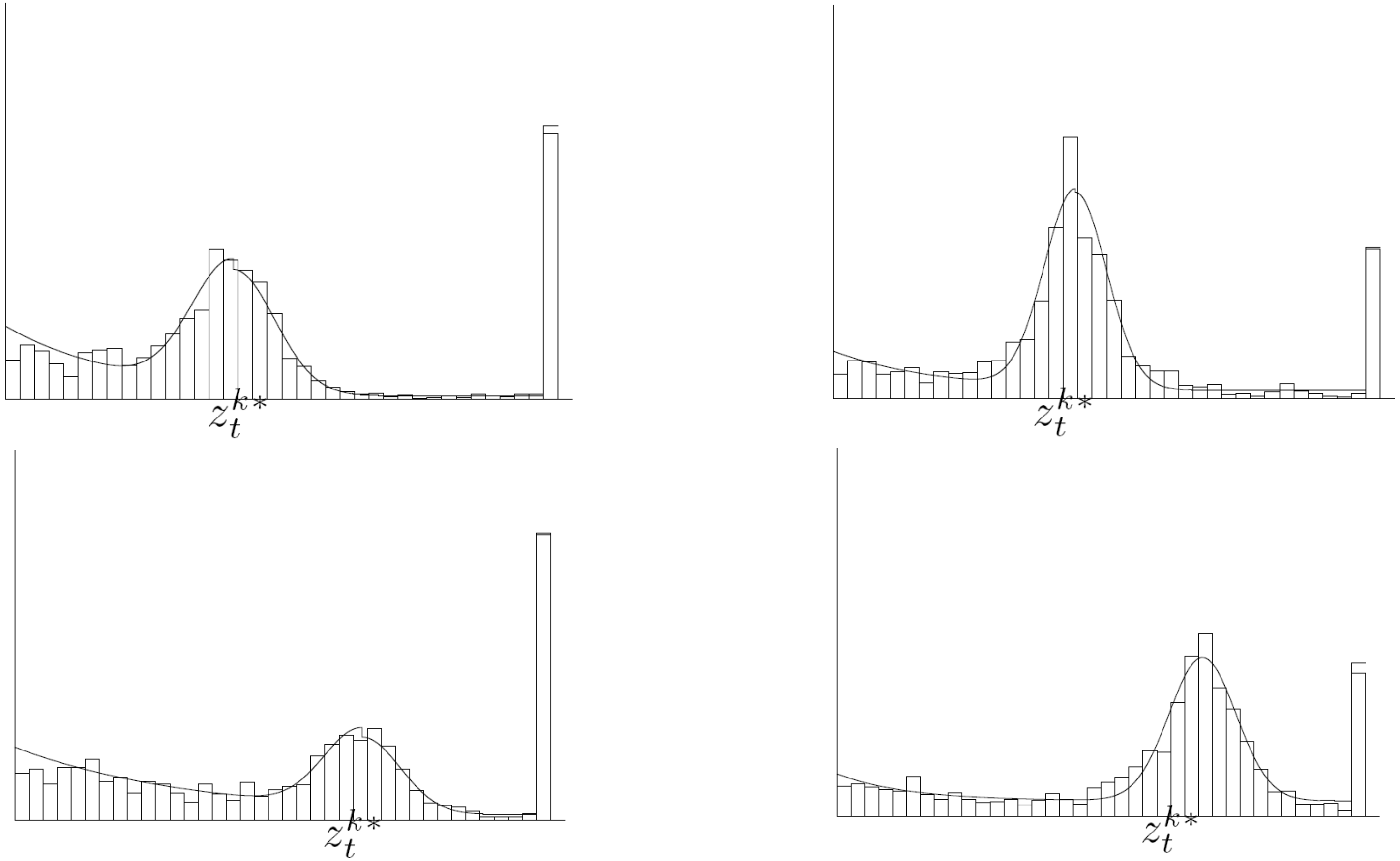
Laser

Beam Model

- Approximation results

300cm

400cm



Sonar

Laser

From Thrun Burgard Fox, Probabilistic Robotics, MIT Press 2006

Likelihood Fields

- Scan-based model
  - Instead of following along the beam, just check the end point.
- Measurement errors
  - Measurement noise
    - Probability of sensor measurement with sensor noise

$$p_{hit}(z_t|x_t, m) = \epsilon_{\sigma_{hit}}(dist)$$

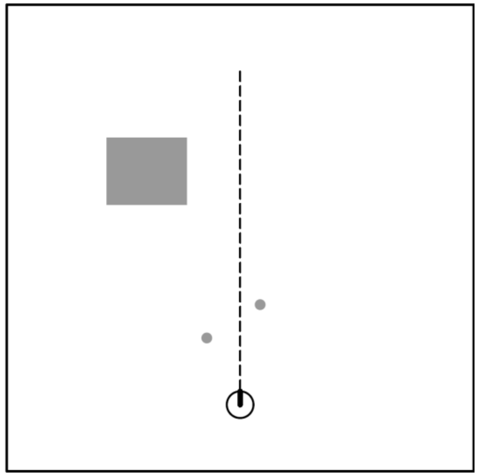
  - Failures : a uniform distribution
  - Unexpected random measurements : a uniform distribution
- Probability $p(z_t|x_t, m)$ combines all three distributions

$$z_{hit} \cdot p_{hit} + z_{rand} \cdot p_{rand} + z_{max} \cdot p_{max}$$

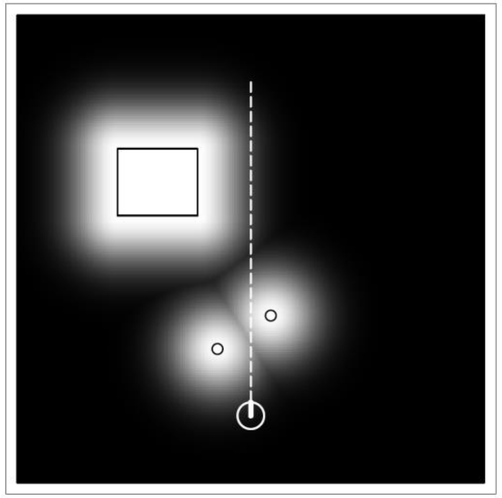
Maxing weights: z_{hit}, z_{rand}, z_{max}

Likelihood Fields

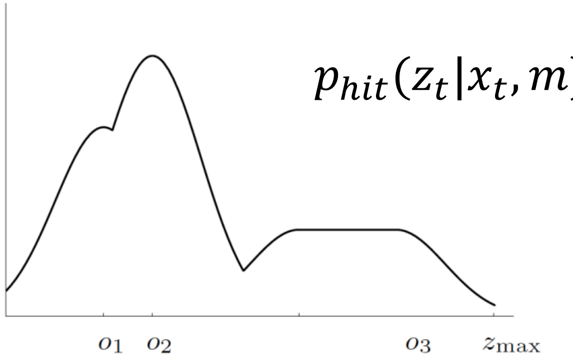
- Likelihood fields



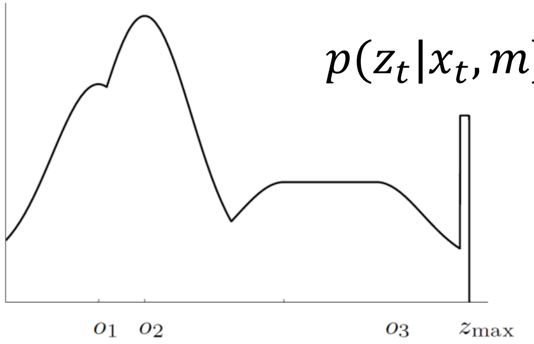
Example environment



Likelihood field



$p_{hit}(z_t|x_t, m)$



$p(z_t|x_t, m)$

From Thrun Burgard Fox, Probabilistic Robotics, MIT Press 2006

Likelihood Fields

- Likelihood fields

Algorithm likelihood_field_range_finder_model(z_t , x_t, m):

$q = 1$

for all k do

  if $z_t^k \neq z_{\max}$

    $x_{z_t^k} = x + x_{k,sens} \cos\theta - y_{k,sens} \sin\theta + z_t^k \cos(\theta + \theta_{k,sens})$

    $y_{z_t^k} = y + y_{k,sens} \cos\theta + x_{k,sens} \sin\theta + z_t^k \sin(\theta + \theta_{k,sens})$

    $dist^2 = \min_{x',y'} \left\{ \left(x_{z_t^k} - x'\right)^2 + \left(y_{z_t^k} - y'\right)^2 \middle| \langle x', y' \rangle \text{ occupied in } m \right\}$

    $q = q \cdot \left(z_{\text{hit}} \cdot \text{prob}\left(dist^2, \sigma_{\text{hit}}^2\right) + \dfrac{z_{\text{random}}}{z_{\max}} \right)$

return q

Landmark Measurement

- Landmark measurement
  - Sensors can measure range and bearing
  - Feature extractor can be used to generate a signature (numerical value)

Feature vector is

$$f(z_t) = \{f_t^1, f_t^2, \cdots\} = \left\{\begin{bmatrix} r_t^1 \\ \phi_t^1 \end{bmatrix}, \begin{bmatrix} r_t^2 \\ \phi_t^1 \end{bmatrix}, \cdots\right\}$$

r: Range

ϕ: Bearing

Assuming conditional independence between features.

$$p(f(z_t) \mid x_t, m) = \prod_i p(r_t^i, \phi_t^i \mid x_t, m)$$

r: Range

ϕ: Bearing

s: Signature

If a signature is used,

$$f(z_t) = \{f_t^1, f_t^2, \cdots\} = \left\{\begin{bmatrix} r_t^1 \\ \phi_t^1 \\ s_t^1 \end{bmatrix}, \begin{bmatrix} r_t^2 \\ \phi_t^1 \\ s_t^2 \end{bmatrix}, \cdots\right\}$$

$$p(f(z_t) \mid x_t, m) = \prod_i p(r_t^i, \phi_t^i, s_t^i \mid x_t, m)$$

Landmark Measurement

- Landmark measurement
  - Each feature has a signature and a location coordinate.
  - The location of a feature

$$\begin{bmatrix} r_t^i \\ \phi_t^i \end{bmatrix} = \begin{bmatrix} \sqrt{(m_{j,x}-x)^2+(m_{j,y}-y)^2} \\ \operatorname{atan2}(m_{j,y}-y, m_{j,x}-x)-\theta \end{bmatrix} + \begin{bmatrix} \epsilon_{\sigma_r^2} \\ \epsilon_{\sigma_\phi^2} \end{bmatrix}$$

$$\epsilon_{\sigma_r^2} \sim \mathbb{N}(0, \sigma_r^2)$$

$$\epsilon_{\sigma_\phi^2} \sim \mathbb{N}(0, \sigma_\phi^2)$$

$$\epsilon_{\sigma_s^2} \sim \mathbb{N}(0, \sigma_s^2)$$

If a signature is used,

$$\begin{bmatrix} r_t^i \\ \phi_t^i \\ s_t^i \end{bmatrix} = \begin{bmatrix} \sqrt{(m_{j,x}-x)^2+(m_{j,y}-y)^2} \\ \operatorname{atan2}(m_{j,y}-y, m_{j,x}-x)-\theta \\ s_j \end{bmatrix} + \begin{bmatrix} \epsilon_{\sigma_r^2} \\ \epsilon_{\sigma_\phi^2} \\ \epsilon_{\sigma_s^2} \end{bmatrix}$$

Landmark Measurement

- Landmark measurement
  - Data association

Correspondence variable between the feature f_t^i and the landmark m_j in the map.

$$c_t^i \in \{1, \cdots, N+1\}$$

N is the number of landmarks in the map.

The i th feature observed at time t corresponds to the j th landmark in the map.

$$c_t^i = j \leq N$$

c_t^i is the true identity of an observed feature.

If the observation does not correspond to any feature in the map,

$$c_t^i = N+1$$

Landmark Measurement

- Algorithm for computing the likelihood of a landmark measurement

Algorithm landmark_model_known_correspondence(f_t^i, c_t^i, x_t, m):

1: $j = c_t^i$

2: $\hat{r} = \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2}$

3: $\hat{\phi} = \mathrm{atan2}\left(m_{j,y} - y, m_{j,x} - x\right)$

4: $q = \mathrm{prob}(r_t^i - \hat{r}, \sigma_r^2) \cdot \mathrm{prob}(\phi_t^i - \hat{\phi}, \sigma_\phi^2)$

5: return q

If a signature is used,

$\vdots$

4: $q = \mathrm{prob}(r_t^i - \hat{r}, \sigma_r^2) \cdot \mathrm{prob}(\phi_t^i - \hat{\phi}, \sigma_\phi^2) \cdot \mathrm{prob}(s_t^i - s_j, \sigma_s^2)$

$\vdots$