$2^3 = 8$ Saurabh Kumar (1)

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L-1

~~Prob~~ Probability and distribution

Random variable → Continous Random Variable

$f(x) \geq 0$ Probability Density function

$\int_{-\infty}^{\infty} f(x) = 1$

~~Discrete Random~~ Variable

mean $E(x) = \int_{-\infty}^{\infty} x f(x) dx$

Probability mass function

(I) $P(x) \geq 0$

(II) $\sum P(x) = 1$

Continous: ~~Ex~~ exponetial, Normal, uniform

mean $E(x) = \sum x P(x)$

Binomial Distribution — Number में गाते हैं

Posion Distribution

L-2

Random variable:- Random variable is a real valued function which asign a real number to each sample point in the sample space

Ex-1 Toosing a fair coin thrice then ~~Spa~~ sample space

S {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

$X(s_1) = 3$ तीन बार Head

$X(s_2) = X(s_3) = X(s_4) = 2$ दो बार Head

$X(s_5) = X(s_6) = X(s_7) = 1$ एक बार Head

$X(s_8) = 0$ एक भी बार नहीं

PMF $P(x) \geq 0$
$\sum P(x) = 1$

Prabality distribution

| X (No. Head) | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P | 1/8 | 3/8 | 3/8 | 1/8 |

Discrete Random variable:-

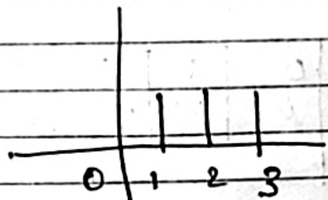
A Random variable which takes finite or as most countable number of values is called discrete random variable

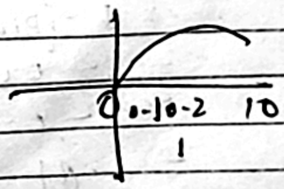
Exp-1 No. of Head obtained when two coin are tossed

No. of defective items in a Lat

Prability Distribution function

$$F(x) = \begin{cases} 1/8 & x \leq 0 \\ 4/8 & x \leq 1 \\ 7/8 & x \leq 2 \\ 1 & x \leq 3 \end{cases}$$



Actual value - disc.

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Q→ A Random variable $x$ has the following Probability distribution

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P(x)$ | 0 | $k$ | $2k$ | $2k$ | $3k$ | $k^2$ | $2k^2$ | $7k^2+k$ |

(i) $k$

(ii) $P(x<6)$ , $P(x \geq 6)$ , $P(0<x<5)$

(iii) Distribution function

(iv) if $P(x \leq c) > 1/2$ find the minimum value of $x$

Solution

(i) if $P(x)$ is PMF

$\Sigma P(x) = 1$

$10k^2 + 9k = 1$

$10k^2 + 9k - 1 = 0$

$10k^2 + 10k - k - 1 = 0$

$10k(k+1) - 1(k+1)$

$(k+1)(10k-1) = 0$

$\cancel{k = -1}$ , $k = 1/10$

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P(x)$ | 0 | 0.1 | 0.2 | 0.2 | 0.3 | 0.01 | 0.02 | 0.17 |

(ii) $P(x<6) = 1 - P(x \geq 6)$

$= 1 - [P(6) + P(7)]$

$= 1 - 0.02 - 0.17$

$= 1 - 0.19$

$= .81$

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$P(x \geq 6) = P(6) + P(7) = 0.02 + 0.17 = 0.19$

$P(0 < x < 5) = P(1) + P(2) + P(3) + P(4)$

$= 0.1 + 0.2 + 0.2 + 0.3$

$= 0.8$

(iii) $F(x) = \begin{cases} 0 & x \leq 0 \\ 0.1 & x \leq 1 \\ 0.3 & x \leq 2 \\ 0.5 & x \leq 3 \\ 0.8 & x \leq 4 \\ 0.81 & x \leq 5 \\ 0.83 & x \leq 6 \\ 1 & x \leq 7 \end{cases}$

(iv) $P(x \leq 0) = 0$

$P(x \leq 1) = 0.1$

$P(x \leq 2) = 0.3$

$P(x \leq 3) = 0.5$

$P(x \leq 4) = 0.8 > 1/2$

$c = 4$

L-3

## Continuous Random variable:-

A Radom variable which can take infinite number of values in an interval is known as continuous ~~Rad~~ Random variable

Exp-) the weight of a group of individuals
weight of group of individuals
Price of House

Probability density function (Pdf)
A function $f(x)$ is pdf

if ① $f(x) \geq 0$, $-\infty < x < \infty$
②
$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

it is also known as density function

~~Exp~~ Q-)

if $x$ is a continuous Random variable with the following Pdf

$$f(x) = \begin{cases} \alpha(2x - x^2) & 0 < x < 2 \\ 0 & o/w \end{cases}$$

(i) $\alpha$
(ii) $P(x > 1)$

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Sol:-

By defination Pdf

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx + \int_{2}^{\infty} f(x)\,dx = 1$$

$$0 + \int_{0}^{2} \alpha(2x - x^2)\,dx + 0 = 1$$

$$\alpha\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2 = 1$$

$$\alpha\left[4 - \frac{8}{3}\right] = 1$$

$$\boxed{\alpha = 3/4}$$

(ii) $P(x>1) = \int_{1}^{\infty} f(x)\,dx$

$$= \int_{1}^{2} 3/4\,(2x - x^2)\,dx$$

$$= 3/4\left(x^2 - \frac{x^3}{3}\right)_1^2$$

$$= 3/4\left[(4-1) - (8/3 - 1/3)\right]$$

$$= 3/4\left[3 - 7/3\right]$$

$$= 3/4 \times \frac{2}{3} = 1/2 \quad \text{Ans}$$

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Q-> A Random variable x has density function

$$f(x) = \begin{cases} kx^2 & -3 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

find k, $P(1 \le x \le 2)$, $P(x \le 2)$, $P(x > 1)$

Solution:-

By defination of Pdf

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-3}^{3} kx^2\,dx = 1$$

$$2x\int_{0}^{3} x^2\,dx = 1$$

$$2k\left[\frac{x^3}{3}\right]_0^3 = 1$$

$$2k\left[\frac{27}{3}\right] = 1$$

$$18k = 1$$

$$\boxed{k = 1/18}$$

Alternate method

$$k\left[\frac{x^3}{3}\right]_{-3}^{3} = 1$$

$$k\left[\frac{27}{3} + \frac{27}{3}\right] = 1$$

$$k\left[\frac{54}{3}\right] = 1$$

$$k\,18 = 1$$

$$\boxed{k = 1/18}$$

$$P(1 \le x \le 2) = \int_1^2 f(x)\,dx$$

$$= \frac{1}{18}\int_1^2 x^2\,dx$$

$$= \frac{1}{18}\left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{1}{18}\left(\frac{8}{3} - \frac{1}{3}\right)$$

$$= \frac{1}{18}\left(\frac{7}{3}\right) = \frac{7}{54}$$

$$P(x \le 2) = \int_{-\infty}^{2} f(x)\,dx$$

$$= \frac{1}{18}\int_{-3}^{2} x^2\,dx$$

$$= \frac{1}{18}\left[\frac{x^3}{3}\right]_{-3}^{2} dx$$

$$= \frac{1}{18}\left[\frac{8}{3} + \frac{27}{3}\right]$$

$$= \frac{35}{54} \text{ Ans}$$

$$p(x>1) = \int_1^{\infty} f(x)\,dx$$

$$= \frac{1}{18}\int_1^3 x^2\,dx$$

$$= \frac{1}{18}\left[\frac{x^3}{3}\right]_1^3$$

$$= \frac{1}{18}\left[\frac{27}{3}-\frac{1}{3}\right]$$

$$= \frac{26}{54}$$

## L-4 Mathematical Expectation

Let $x$ be any random variable & $\phi(x)$ be any function of $x$ then Expection of $\phi(x)$ is doneted by $E\phi(x)$ & is defined by

$E(\phi(x))$

Discrete Random variable: $\sum_x \phi(x)P(x)$ → PMF

Continuous Random variable: $\int_{-\infty}^{\infty} \phi(x) f(x)\,dx$ → PDF

$f(n)$
$\phi(n)$

if $\phi(x) = x$

$\bar{x}$ = mean = $E(x)$ =

DRV: $E(x) = \sum_x x P(x)$

CRV: $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

Variance $= E(x-\bar{x})^2$
$= E(x^2) - (E(x))^2$

Question:-

find mean & variance of the Probability distribution, given by the following table

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $P(x)$ | 0.2 | 0.35 | 0.25 | 0.15 | 0.05 |

$E(x) = \sum x P(x)$

$= 0.2 + 0.70 + 0.75 + 0.6 + 0.25$

$\boxed{\text{mean} = 2.5}$

$E(x^2) = \sum x^2 P(x)$

$= 0.2 + 1.4 + 2.25 + 2.4 + 1.25$

$= 7.5$

~~var~~ Varionce $= \sigma_x^2 = E(x^2) - (E(x))^2$

$= 7.5 - 6.25$

$= 1.25$

Q-) therteen cards are drawn simuntaniasly from a Pack of 52 cards if ace count 1 face cards 10 and other accolrding to there donomination find the expectation of total score in 13 cards

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 10 | 10 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $p$ | 1/13 | 1/13 | 1/13 | 1/13 | 1/13 | 1/13 | $\frac{1}{13}$ | $\frac{1}{13}$ | $\frac{1}{13}$ | $\frac{1}{13}$ | $\frac{1}{13}$ | $\frac{1}{13}$ | $\frac{1}{13}$ |

$E(x) = \frac{1}{13}[1+2+3+4+5+6+7+8+9+10+10+10+10]$

$= 85/13$

PMF
PDF. 3'. $\not{2}^{2+1}$ (10)

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Q-> A continuous Random variable x has density function given by

$$f(x) = \begin{cases} 2e^{-2x} & x>0 \\ 0 & o/w \end{cases}$$

mean

find expected value & variance of x

Soln

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx = \int_{0}^{\infty} x [2e^{-2x}] dx$$

$$= 2\int_{0}^{\infty} x^1 e^{-2x} dx \qquad \left[ \because \int_0^\infty x^{n-1} e^{-ax} dx = \frac{\sqrt{n}}{a^n} \right]$$

$$\text{mean} = 2\left(\frac{\sqrt{2}}{2^2}\right) = 1/2$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx = \int_0^\infty x^2 (2e^{-2x}) dx$$

$$= 2\int_0^\infty x^2 e^{-2x} dx$$

use Define Theorm

$$= 2\frac{\sqrt{3}}{2^3} = \frac{2\times(2\times1)}{8} = \frac{1}{2}$$

$$\text{variance} = \sigma_x^2 = E(x^2) - (E(x))^2$$

$$= \frac{1}{2} - \left(\frac{1}{2}\right)^2$$

$$= \frac{1}{2} - \frac{1}{4}$$

$$= 1/4 \text{ Az}$$

## L-5

## Binomial Distribution :-

MGF
mean
variance

(1) All the trials are independent
(2) Number n of trial is finite
(3) the Probality of success is same for each trial

$$P(x) = {}^nC_x \, p^x q^{n-x} \qquad p+q=1, \quad q = 1-p$$

$${}^nC_r \, p^r q^{n-r}$$

$n's$ $(p^x$ or$)$

### Moment generating function

$$f(x) = {}^nC_x \, p^x q^{n-x}$$

$\because$ MGF
$M(t) = \sum f(x) e^{tx}$
$\underline{L \text{ dist}}$

(var

$$m(t) = \sum f(x) e^{tx}$$

$$= \sum {}^nC_x \, p^x q^{n-x} e^{tx}$$

$$= \sum {}^nC_x (pe^t)^x \cdot q^{n-x} \quad \text{— formula.}$$

$$\boxed{m(t) = (pe^t + q)^n}$$

### Mean of Binomial distribution

$$\text{mean} = \left[\frac{d}{dt} m(t)\right]_{t=0}$$

$$= \left[n(pe^t+q)^{n-1} \times pe^t\right]_{t=0}$$

$$\boxed{\text{mean} = np}$$

## Variance of Binomial distribution

$$v_2 = \left[\frac{d^2}{dt^2} M(t)\right]_{t=0}$$

$$= np\left[(pe^t+q)^{n-1} \times e^t + e^t\left[(n-1)(pe^t+q)^{n-2} \cdot pe^t\right]\right]_{t=0}$$

$$= np[1 + p(n-1)]$$

$$= np[1 + pn - p]$$

$$= np[1 - p + pn]$$

$$= np[Q + np]$$

$$v_2 = Qpn + n^2p^2$$

(Variance) $\mu_2 = v_2 - v_1^2$

$$= Qpn + n^2p^2 - n^2p^2$$

$$\boxed{\mu_2 = qpn}$$

$$\boxed{(p+q)^n = {}^nC_r\, p^n q^{n-r}}$$

Problem Based on Bio D

Q→ The Probability that man aged 60 will live up to 70 is 0.65 out of 10 men now aged 60 find probability

① at least 7 will live 48 to 70
② Exactly 9 will live 40 to 70
③ At most 9 will live 48 to 70

Soln

$n = 10$ , $P = 0.65$ , $q = 1 - P$
$q = 0.35$

$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$= {}^{10}C_{x}\, (0.65)^{x} (0.35)^{10-x}$

① $P(x \geq 7) = P(7) + P(8) + P(9) + P(10)$
$= {}^{10}C_{7} (0.65)^{7} (0.35)^{3} + {}^{10}C_{8} (0.65)^{8} (0.35)^{2} + {}^{10}C_{9} (0.65)^{9} (0.35)^{1} + {}^{10}C_{10} (0.65)^{10} (0.35)^{0}$

$\boxed{P(x \geq 7) = 0.5139}$

(ii) $P(9) = {}^{10}C_{9} (0.65)^{9} (0.35)^{1} = 0.0725$

(iii) $P(x \leq 9) = 1 - P(x > 9)$
$= 1 - P(10)$
$= 1 - {}^{10}C_{10} (0.65)^{10} (0.35)^{0}$
$= 0.9865$

7

7

Q→ out of 800 families with 5 children each How many families would be expected to have

① 3 boys
② 5 girls
③ either 2 or 3 boys
④ at least 2 girls

$N = 800$
$p, q$
$p = \frac{1}{2}, q = \frac{1}{2}$
$n = 5$

$N = 800 \quad , n = 5 \quad , p = 1/2 \quad , q = 1/2$

$P(x) = {}^{n}C_{x} \, p^{x} q^{n-x}$

$= {}^{5}C_{x} \cdot (1/2)^{x} (1/2)^{5-x}$

$= {}^{5}C_{x} (1/2)^{5}$

$= {}^{5}C_{x} \times \frac{1}{32}$

(i) $P(3) = {}^{5}C_{3} \times \frac{1}{32} = \frac{5 \times 4}{1 \times 2} \times \frac{1}{32} = \frac{5}{16}$

No of families $= 800 \times \frac{5}{16} = 250$

(ii) $P(0) = {}^{5}C_{0} \frac{1}{32} = \frac{1}{32}$

No of families $= 800 \times \frac{1}{32} = 25$

(iii) $P(2) + P(3)$

${}^{5}C_{2} \frac{1}{32} + {}^{5}C_{3} \frac{1}{32}$

$\frac{10}{32} + \frac{10}{32} = \frac{20}{32}$

No of families $= \frac{800 \times 20}{32}$

$= 500$

(15) Atleast 0, 1, 2, 3, 4.

(iv) 4 2 3 4 5
B 3 2 1 0

$$P(x \le 3) = P(0) + P(1) + P(2) + P(3)$$
$$= {}^5C_0 \frac{1}{32} + {}^5C_1 \frac{1}{32} + {}^5C_2 \frac{1}{32} + {}^5C_3 \frac{1}{32}$$
$$= 0.5$$

No of families $= 800 \times 0.5 = 400$

Q→ 4 coins are tossed 100 times & following were obtained fit a Bionomial distribution for Data & calcute theortical frequency

$P(x) = {}^nC_x \, p^x \, q^{n-x}$

| No Head ($x$) | frequency ($f$) | $xf$ | $P(x) = {}^4C_x (0.49)^x (0.51)^{4-x}$ | $100\,P(x)$ |
|---|---|---|---|---|
| 0 | 5 | 0 | $P(0) = {}^4C_0 (0.49)^0 (0.51)^{4-0} = 0.0676$ | $6.76 \simeq 7$ |
| 1 | 29 | 29 | $P(1) = {}^4C_1 (0.49)^1 (0.51)^{4-1} = 0.2599$ | $25.99 \simeq 26$ |
| 2 | 36 | 72 | $P(2) = {}^4C_2 (0.49)^2 (0.51)^{4-2} = 0.3747$ | $37.47 \simeq 37$ |
| 3 | 25 | 75 | $P(3) = {}^4C_3 (0.49)^3 (0.51)^{4-3} = 0.2400$ | $24.00 \simeq 24$ |
| 4 | 5 | 20 | $P(4) = {}^4C_4 (0.49)^4 (0.51)^{4-4} = 0.05765$ | $5.76 = 6$ |
| | $\Sigma f = 100$ | $\Sigma fx = 196$ | | 100 |

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{196}{100} = 1.96$$

$np = 1.96$

$4p = 1.96$

$\boxed{p = 0.49}$

$q = 1 - 0.49$

$\boxed{q = 0.51}$

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L-7

**Poisson distribution:-** A Discrete Random variable $x$ which has following Probability mean function

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad x = 0, 1, 2 \dots \infty$$

is called Poisson variate & its distribution is called Poisson Distribution

* only use for DRV
* जब n बहुत ज्यादा हो
* Probability बहुत कम हो

**Moment generating function:-**

$$m(t) = \Sigma p(x) e^{tx}$$

$$= \Sigma \frac{\lambda^x e^{-\lambda}}{x!} e^{tx}$$

$$= e^{-\lambda} \Sigma \frac{\lambda^x e^{tx}}{x!}$$

$$= e^{-\lambda} \Sigma \frac{(\lambda e^t)^x}{x!} \qquad \left[\because \Sigma \frac{t^x}{x!} = e^t\right.$$

$$= e^{-\lambda} e^{\lambda e^t}$$

$$\boxed{m(t) = e^{\lambda[e^t - 1]}}$$

10 of
at least 5
5, 6, 7, 8, 9, 10

(17)

Mean of Poisson Distribution

mean $E(X) = [\frac{d}{dt} M(t)]_{t=0}$

$= \frac{d}{dt}[e^{-\lambda} e^{\lambda e^t}]_{t=0}$

$= [e^{\lambda(e^t-1)} \cdot \lambda e^t]_{t=0}$

$\boxed{E(x) = \lambda}$ mean

Variance of Poisson Distribution

Variance $= E(x^2) - [E(x)]^2$

$E(x^2) = [\frac{d^2}{dt^2} M(t)]_{t=0}$

$= [e^{\lambda(e^t-1)} \cdot \lambda e^t + \lambda e^t e^{\lambda(e^t-1)} \lambda e^t]_{t=0}$

$= \lambda [e^{\lambda(e^t-1)} \cdot e^t + \lambda e^{2t} \cdot e^{\lambda(e^t-1)}]_{t=0}$

$= \lambda[1+\lambda]$

$E(x^2) = \lambda + \lambda^2$

$E(x^2) - [E(x)]^2$

Variance $= \lambda + \lambda^2 - \lambda^2$

$\boxed{\text{Variance} = \lambda}$

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Question :-
Given that 2% of the fuses manufactured by a firm are defective find probability that a box contains 200 fuses has

1. At least 1 defective fuses
2. 3 or more defective fuses
3. No defective fuses

Soln

$n = 200 \quad , p = 0.02$

$\lambda = np$

$= 200 \times 0.02$

$\boxed{\lambda = 4}$

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= \frac{(4)^x e^{-4}}{x!}$$

1. $P(x \geq 1) = 1 - P(x<1)$

$= 1 - P(0)$

$= 1 - \frac{4^0 e^{-4}}{0!}$

$= 1 - e^{-4}$

2. $P(x \geq 3) = \cancel{1-P(x}$

$= 1 - P(x<3)$

$= 1 - [P(0) + P(1) + P(2)]$

$= 1 - \left[\frac{4^0 e^{-4}}{0!} + \frac{4^1 e^{-4}}{1!} + \frac{4^2 e^{-4}}{2!}\right]$

$= 1 - e^{-4}[1 + 4 + 8]$

$\boxed{P(x \geq 3) = 1 - 13e^{-4}}$

3. $P(0) = \frac{4^0 e^{-4}}{0!} = e^{-4}$

(19)

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Q-> in certain factory tunning out blads there is small chance to 0.002 for any blade to be defective the blades are supplied in packts of 10 using poission distribution find approximate number of packets containg

(1) No defective blades
(2) one defective blades
in consignment of 10,000 packets

$n = 10, \quad p = 0.002, \quad N = 10000$

$\lambda = np$

$= 10 \times 0.002$

$\lambda = 0.02$

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(0.02)^x e^{-0.02}}{x!}$$

$$P(0) = \frac{(0.02)^0 e^{-0.02}}{0!} = 0.9802$$

No ~~defective~~ of packets containg zero defective blad $= 10000\, P(0) = 9802$ packets

$$P(1) = \frac{(0.02)^1 e^{-0.02}}{1!}$$

$P(1) = 0.02 \times 0.9802$

No of packts containing one defective blad

$= 10000\, P(1) = 10000 \times 0.02 \times 0.9802$

$= 186.04$

Q-) if Probability of a bad ~~reacher~~ reaction from a contain infection is 0·01 find the chance that out of 200 individuals more than two will get bad reaction

$p = 0{\cdot}01 \quad , \quad n = 200$

$\lambda = 2$

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{2^x e^{-2}}{x!}$$

$$P(x>2) = 1 - P(x \le 2)$$

$$= 1 - [P(0) + P(1) + P(2)]$$

$$= 1 - \left[\frac{2^0 e^{-2}}{0!} + \frac{2^1 e^{-2}}{1!} + \frac{2^2 e^{-2}}{2!}\right]$$

$$= 1 - e^{-2}\,[1 + 2 + 2]$$

$$\boxed{P(x>2) = 1 - 5e^{-2}}$$

(21)

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Q→

A skilled typist on routine work kept a record of mistake made per day during 300 working days

| mistake/day | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| No of days | 143 | 90 | 42 | 12 | 9 | 3 | 1 |

| $x$ mistake day | f | $xf$ | $P(x) = \frac{(0.89)^x e^{-0.89}}{x!}$ | 300 P(x) |
|---|---|---|---|---|
| | | | | |
| 0 | 143 | 0 | $P(0) = (0.89)^0 \cdot e^{-0.89} / 0! = 0.411$ | $123.5 \simeq 123$ |
| 1 | 90 | 90 | $P(1) = (0.89)^1 \cdot e^{-0.89} / 1! = 0.365$ | $109.5 \simeq 110$ |
| 2 | 42 | 84 | $P(2) = (0.89)^2 \cdot e^{-0.89} / 2! = 0.163$ | $48.9 \simeq 49$ |
| 3 | 12 | 36 | $P(3) = (0.89)^3 \cdot e^{-0.89} / 3! = 0.048$ | $14.4 \simeq 14$ |
| 4 | 9 | 36 | $P(4) = (0.89)^4 \cdot e^{-.89} / 4! = 0.011$ | $3.3 \simeq 3$ |
| 5 | 3 | 15 | $P(5) = (0.89)^5 \cdot e^{-0.89} / 5! = 0.002$ | $0.6 \simeq 1$ |
| 6 | 1 | 6 | $P(6) = (0.89)^6 \cdot e^{-0.89} / 6! = 0.003$ | $0.09 \simeq 0$ |
| | 300 | 267 | | $\Sigma f = 300$ |

$$\bar{x} = \frac{\Sigma xf}{\Sigma f}$$

$$\lambda = \frac{267}{300}$$

$$\boxed{\lambda = 0.89}$$

record of ...

6

| 300 P(x) |
|---|
| 123.5 ≃ 123 |
| 109.5 ≃ 110 |
| 48.9 ≃ 49 |
| 14.4 ≃ 14 |
| 3.3 ≃ 3 |
| 0.6 ≃ 1 |
| 0.09 ≃ 0 |
| $\Sigma f = 300$ |



## Normal Distribution :-

A continuous Random variable which has the following Pdf

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

is called Normal variate & its distribution is called Normal distribution & is denoted by

$$\boxed{X \sim N(\mu, \sigma)}$$

$\mu$ = mean $\quad -\infty < \mu < 0$

$\sigma$ = Standard daviation $\quad \sigma > 0$

$x$

Mean of Poisson Distribution

$$\boxed{m_x(t) = e^{\mu t + \sigma^2 t^2/2}}$$

MGF

mean $E(x) = \left[\frac{d}{dt} m_x(t)\right]_{t=0}$

$$= \left[\frac{d}{dt} e^{\mu t + \frac{\sigma^2 t^2}{2}}\right]_{t=0}$$

$$= e^{\mu t + \frac{\sigma^2 t^2}{2}} (\mu + \sigma^2 t)$$

$$= e^0 \cdot \mu$$

$$\boxed{Ex = \mu}$$

mean

$$E(x^2) = \frac{d^2}{dt^2}(m_x(t))_{t=0}$$

$$= \frac{d}{dt}\left[e^{\mu t + \frac{\sigma^2 t^2}{2}} (\mu + \sigma^2 t)\right]_{t=0}$$

$$= e^{\mu t + \frac{\sigma^2 t^2}{2}} (\mu + \sigma^2 t)^2 + e^{\mu t + \frac{\sigma^2 t^2}{2}} \times \sigma^2$$

$$E(x^2) = \mu^2 + \sigma^2$$

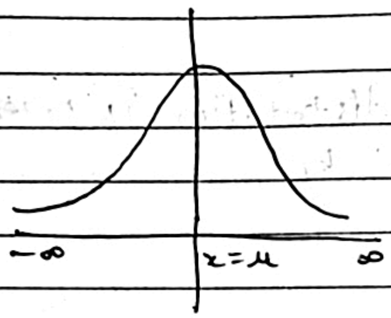
Variane $= E(x^2) - (E(x))^2$

$= \mu^2 + \sigma^2 - \mu^2$

$$\boxed{\text{Variane} = \sigma^2}$$

## Area under the curve

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$



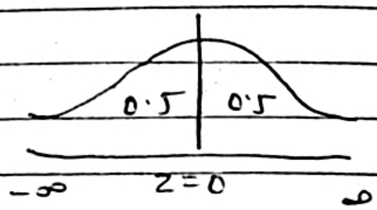

$\mu = 0$

$\sigma = 1$

$$z = \frac{x-\mu}{\sigma}$$

$$f(z) = \frac{1}{\sqrt{2\pi}}\, e^{-z^2/2}$$

$-\infty < z < \infty$




Q → If the Height of 300 students are normally distributed with mean 64.5 inches & standard deviation 3.3 inches. How many ~~stand~~ students have height
(i) Less then 5 feet
(ii) b/w 5 feet & 5 feet 9 inches

Sol$^n$ given $\mu = 64.5$
$\sigma = 3.3$

$$z = \frac{x-\mu}{\sigma}$$

$$z = \frac{x-64.5}{3.3}$$

(i) $P(x<60) = P\left(z < \frac{60-64.5}{3.3}\right)$

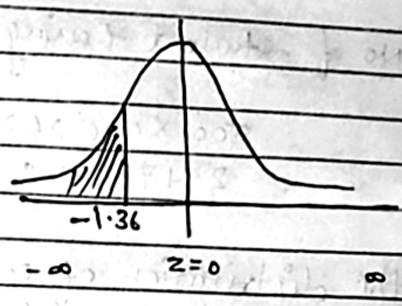
$= P(z < -1.36)$

$= 0.5 - P(-1.36 < z < 0)$

$= 0.5 - P(0 < z < 1.36)$

$= 0.5 - 0.4131$

$P(x<60) = 0.0869$




| | 0.00 | .01 | .02 | 0.03 | 0.06 |
|---|---|---|---|---|---|
| 0 | | | | | |
| 0.1 | | | | | |
| 0.2 | | | | | |
| 0.3 | | | | | |
| ⋮ | | | | | |
| 1.3 | - | - | - | - | 0.4131 |

No of student Heving hiegut Lees then 5 feet

$= 300 P(x<60)$

$= 300 \times 0.0869$

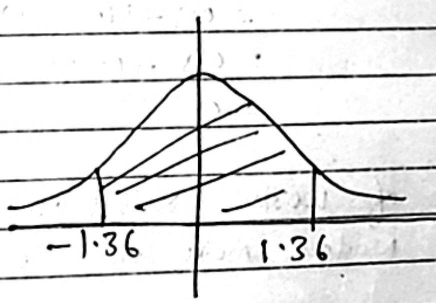
$= 26.07 \simeq 26$ student

(ii)

$P(60 < x < 69)$

$z = \frac{60-64.5}{3.3}$

$z = \frac{69-64.5}{3.3}$




$P(-1.36 < z < 1.36)$

$P(-1.36 < z < 0) + P(0 < z < 1.36)$

$= P(0 < z < 1.36) + P(0 < z < 1.36)$

$= 2P(0 < z < 1.36)$

$= 2 \times 0.4131$

(28)

$P(60 < x < 69) = 0.8262$

No of student having height b/w 5 feet & 5 feet 9 inches

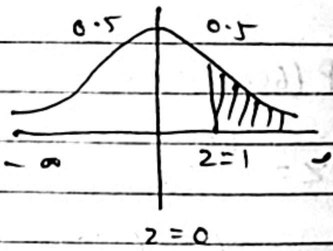
$= 300 \times 0.8262$
$= 247.86 \approx 248$ student.

the distribution of 500 workers in a factory is approximately Normal with mean & standard deviation Rs 75 and Rs 15 respectivily find the Number of worker who recives weekly wedges (I) more than 90 (II) less than 45

$\mu = 75, \sigma = 15$ $\quad z = \frac{x-\mu}{\sigma}$

$z = \frac{x-75}{15}$

(I) $P(x > 90) = P(z > 1)$
$= 0.5 - P(0 < z < 1)$
$= 0.5 - 0.3413$
$= 0.1587$




No of worker recives weekly wadges more than 90

$= 500 \times 0.1587$
$= 79$

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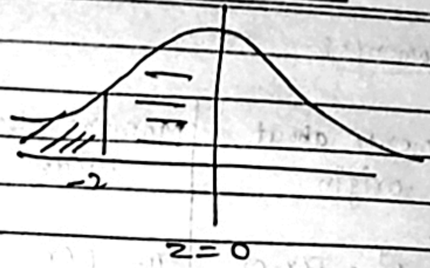
(ii) $P(x<45)$

$P(z<-2)$




$= 0.5 - P(-2<z<0)$

$= 0.5 - P(0<z<2)$

$= 0.5 - 0.4772$

$= 0.0228$

No of worker whose weekly wadge is less then 45

$= 500\, P(x<45)$

$= 500 \times 0.0228$

$= 11.4 \simeq 11$ worker.

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## Moments:-

| Moments about origin | Moment about Mean | Moment about any other Point |
|---|---|---|
| $\mu_r' = E(x-0)^r$ | $\mu_r = E(x-\bar{x})^r$ | $\mu_r'' = E(x-A)^r$ |
| $\mu_0' = E(x-0)^0 = 1$ | $\mu_0 = 1$ | $\mu_1'' = E(x-A)$ |
| $\mu_1' = E(x) = \text{mean} = \bar{x}$ | $\mu_1 = E(x-\bar{x})$ | $\mu_2'' = E(x-A)^2$ |
| $\mu_2' = E(x^2)$ | $= E(x) - E(\bar{x})$ | $\mu_3'' = E(x-A)^3$ |
| $\mu_3' = E(x^3)$ | $= \bar{x} - \bar{x}$ | $\mu_4'' = E(x-A)^4$ |
| $\mu_4' = E(x^4)$ | $\boxed{\mu_1 = 0}$ | |

$$\text{variance} = \mu_2 = E(x^2) - [E(x)]^2$$
$\sigma^2$

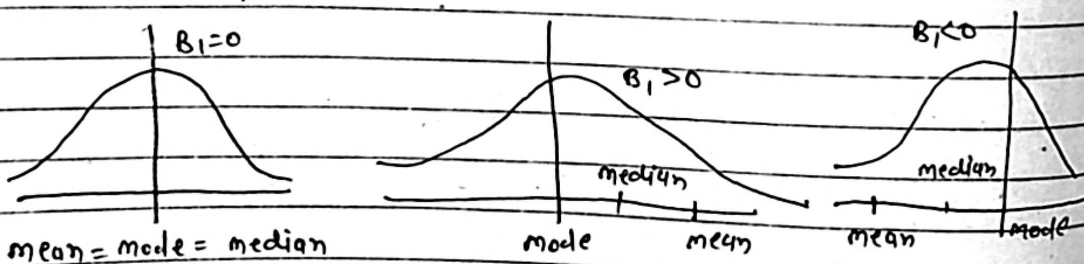
$$\boxed{\mu_2 = \mu_2' - (\mu_1')^2}$$

$$\boxed{\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3}$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$$

**Skeaness:-** mean lack of symmetry

$$\boxed{B_1 = \frac{\mu_3^2}{\mu_2^3}}$$




Symmetrical

Positivaly Skewed

Negetivly Skewed

<u>Kurtosis:</u> the expect to which a distribution is Peaked or Flat called kurtosis

$$\boxed{B_2 = \frac{\mu_4}{\mu_2^2}}$$