Lecture 1: Numerical Linear Algebra (UMA021)

Dr. Meenu Rani

Department of Mathematics
TIET, Patiala
Punjab-India

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4 credit

General Information

Lecture-1

Text Books:

1. Richard L. Burden, J. Douglas Faires, and Annette M. Burden, Numerical Analysis, 10th edition, 2015.
2. Gilbert Strang, Linear Algebra and its Applications, Cengage Learning, 4th edition, 2005.
3. J. Desmond Higham and Nicholas J. Higham, MATLAB Guide, Third Edition, Society for Industrial and Applied Mathematics, 2016.

Section 1: Root finding for non-linear equations.
Section 2: Interpolation and Integration.
Section 3: Matrix Algebra
Section 4: Matrix Computations.

Errors

Rounding and Chopping

Floating point Representation.

exact ← $x = 0.a_1 a_2 a_3 \cdots a_n a_{n+1} \cdots \times 10^e$ → real

$x = 3$

$x = 0.3 \times 10^1$

$\boxed{\frac{7}{3}} = 2.3333\cdots$

$0.2333\cdots \times 10^1$

app. ← $x^* = fl(x) = 0.a_1 a_2 \cdots a_n \times 10^e$

Approximate x up to n decimal with chopping

$x^* = 0.a_1 a_2 \cdots a_n \times 10^e$

for e.g. $x = 10.325\,6792$

app. up to 3 decimal

$x^* = 10.325$

10.3255792

10.326

if $a_{n+1} \geq 5$ = $0.a_1 a_2 \cdots a_{n+1}$

$a_{n+1} < 5$ = $0.a_1 a_2 \cdots a_n$

$a_{n+1} = 5$ = If $a_{n+2} \neq 0$ then add 1, If $a_{n+2} = 0$ then chop.

by Rounding $x^{**} = 10.326$

Introduction

In chopping, the max error bound is $\leq 10^{1-n}$

In rounding " " " $\leq \frac{10^{1-n}}{2}$

Absolute error and Relative error

$x \rightarrow$ Exact number

$x^* \rightarrow$ App. number

$$A.E. = |x - x^*|$$

$$R.E. = \frac{|x - x^*|}{|x|} = \frac{A.E.}{|x|}$$

max error in rounding $\quad \frac{|x - x^*|}{|x|} \leq \frac{10^{1-n}}{2}$

in chopping $\quad \frac{|x - x^*|}{|x|} \leq 10^{1-n}$

Examples:

1. Compute the absolute error and relative error in approximations of $\sqrt{2}$ by 1.414.

$x = \sqrt{2} = 1.414213$

$x^* = 1.414$

A.E. $= |x - x^*| = |1.414213 - 1.414| =$

R.E $= \dfrac{(\quad)}{|1.414213|}$

Introduction

Let $x = 1234.567$ → 7 significant digits

$y = 1234.237$ → significant digits

take $x - y = 0.330$

0.33×10^{0} only 2 S.D.

Exercise:

1. Compute the absolute error and relative error in approximations of x by x^*, where $x = \pi$ and $x^* = 22/7$.

loss of significance

Roots of Non-Linear Equations

Roots of non-linear equations

↓

(1) Higher degree eq^{n}s.

(2) Transcendental eq^{n}. which involves trigonometric functions, logarithmic, exp. fun's

$f(x) = \sin x - e^{x} = 0$

Methods for root-finding problem:

To find a solution of an equation $f(x) = 0$, we discuss the following three methods:

1. Bisection method
2. Fixed point Iteration
3. Newton method

Iterative methods

$f(x) = 0$

(1) Take initial guess x_0

(2) Apply method to find 1st iteration x_1

(3) $|x_1 - x_0|$ & tolerance $= 10^{-1}$ (given)

(4) Apply method to find 2nd iteration x_2

(5) $|x_2 - x_1| <$ tol. $= 10^{-1}$

$x_2 \rightarrow$ root of $f(x) = 0$