Roll Number:__________________

Thapar Institute of Engineering and Technology, Patiala

School of Mathematics

AUXILIARY EXAMINATION

B.E. (Second Year)

Course Code: UMA007

Feb 28, 2020

Course Name: Numerical analysis

Time: 3 Hours, M. Marks: 100

Name of Faculty : Dr. Sapna Sharma, Meenu Rani

Note: ***Attempt all the problems. Please attempt all the parts of a problem at one place and start next problem from a new page. Calculator without graphing mode and alphanumeric memory is permitted***

1. a. Let floating point representation of a real number is $x = (0.a_1a_2a_3........a_na_{n+1}....)_\beta \times \beta_e,\ \ a_1 \neq 0.$ Let $fl(x)$ be its machine approximation with n significant digits by chopping then obtain a bound for absolute relative error $\left|\dfrac{x - fl(x)}{x}\right|$.

b. Consider the stability (by calculating the condition number) of $\sqrt{1+x} - 1$ when x is near zero. Rewrite the expression to rid it of subtractive cancellation **[8+7]**

2.a. Find the first four iterations obtained by the Secant method applied to the given equation $x - \cos x = 0,\ x \in [0, \pi/2]$.

b. Show that $g(x) = 2^{-x}$ has unique fixed point in $\left[\frac{1}{3}, 1\right]$. Use fixed point iteration to find an approximation to the fixed point accurate to within 10^{-2} and initial guess $x_0 = 0.5$. **[7+8]**

3.a Starting with $X^{(0)} = [0, 0, 0]^T$ and working with four decimal digits rounding arithmetic, Perform three iterations of Gauss-Seidel method for the following system of linear equation:

$$4x + y - z = 3$$

$$2x + 7y + z = 19$$

$$x - 3y + 12z = 31.$$

b. Find the largest eigen value correct to three significant digits for the matrix

$$\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$$

using power method by taking initial approximation $X^{(0)} = [1, 0, 0]^T$. Also find the corresponding eigenvector. **[10+8]**

4.a Determine a constant a, b, c that will produce a quadrature formula

$$\int_0^2 f(x)\,dx = af(0) + bf(1) + cf(2),$$

with degree of precision 2.

b. Approximate the given integral using the trapezoidal rule $\int_{-0.25}^{0.25} \sin^2 x\,dx$ with 4 subintervals. Hence find a bound for the error using the error formula..

[8+8]

5.a Suppose that $x_0, x_1, x_2 \ldots\ldots\ldots x_n$ are distinct number in $[a,b]$ and $f \in C^{n+1}[a,b]$. Let $P_n(x)$ be the unique polynomial of degree $\leq n$ that passes through $n+1$ nodal points then prove that $\forall x \in [a,b],\ \exists\ \xi(x) \in (a,b)$ such that

$$f(x) - P_n(x) = \frac{(x-x_0)\ldots\ldots(x-x_n)}{(n+1)!} f^{n+1}(\xi)$$

b. Find the second degree polynomial by least square method for the following data

x	-3.0	-1.0	1.0	3.0
y	15.0	5.0	1.0	5.0

[10+8]

6.a. Solve the initial value problem by the forth-order Runge Kutta method for $x = 0.2$ by taking step size h=0.2

$$\frac{dy}{dx} = 2xy^2, \qquad y(0) = 1,$$

b. Show that the initial value problem $\frac{dy}{dx} = y\cos x,\ \ 0 \leq x \leq 1\ \ y(0) = 1,$ has a unique solution

[10+8]

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