School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

Auxiliary Examination, August 2022

B.E. IV Semester
Time Limit: 02 Hours
Instructor(s) : Dr. Meenu Rani, Dr. Munish Kansal, Dr. Paramjeet Singh

UMA011 : Numerical Analysis
Maximum Marks: 50

Instructions: You are expected to answer ALL FIVE questions. Arrange your work in a reasonably neat, organized, and coherent way. Mysterious or unsupported answers will not receive full credit. Scientific Calculator is permitted.

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1. Use Newton's method to find the approximate solution of the equation $3x - e^x = 0$ in the interval $[1, 2]$ accurate within 0.01. [10 marks]

2. Perform three iterations of Gauss-Seidel method to find the approximate solution of the following system:

$$\begin{aligned} 4x_1 + x_2 - x_3 &= 3 \\ 2x_1 + 7x_2 + x_3 &= 19 \\ x_1 - 3x_2 + 12x_3 &= 31. \end{aligned}$$

Use initial guess $x^{(0)} = [0, 0, 0]^t$. [10 marks]

3. Let $f(x) = \sqrt{x - x^2}$ and $P_2(x)$ be the Lagrange interpolating polynomial based on the points $x_0 = 0$, x_1 and $x_2 = 1$. Find the largest value of x_1 in interval $(0, 1)$ for which $f(0.5) - P_2(0.5) = -0.25$. [10 marks]

4. Determine the constants x_0, x_1, and c_1 that will produce a quadrature formula

$$\int_0^1 f(x)\, dx = \frac{1}{2} f(x_0) + c_1 f(x_1)$$

that has the highest possible degree of precision. [10 marks]

5. Apply the fourth order Runge-Kutta method to the following initial-value problem to find the approximate solution at $t = 0.1$ with step-size $h = 0.1$.

$$\frac{dy}{dt} = y^2 - t^2, \quad y(0) = 1.$$

[10 marks]