Roll Number:

Thapar Institute of Engineering and Technology, Patiala
School of Mathematics

BE-(III Semester) EST **UMA011: Numerical Analysis**
7 December 2022, 16:30-19:30 **Time: 03 Hours; MM: 40**

Name of the Instructors: Dr. Deepika Singh, Dr. Hasanuzzaman, Dr. Meenu Rani, Dr. Pankaj Narula, Dr. Parimita Roy, Dr. Sanjeev Kumar, Dr. Tina Verma, Dr. Vivek Sangwan.

Note: (1) Use of calculator is allowed.
(2) Attempt all the questions in given order.
(3) Attempt all parts of each question at one place.

1. (a) Show that the expression $f(x) = \sqrt{x+1} - \sqrt{x}$ is not stable at $x = 246801357$. Find an equivalent expression and hence discuss the stability. [3 Marks]

   (b) Find the number of iterations required by the Bisection method to get an approximate root of the equation $x^3 + 2x^2 - 3x - 1 = 0$ in $[1, 1.4]$ with an accuracy of $10^{-1.5}$ and hence find the root. [3 Marks]

   (c) Perform five iterations to find the smallest eigen-value of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ using Power method by taking initial guess $x^{(0)} = (1, 1, 1)^T$. Use at least 4-digit arithmetic rounding. [4 Marks]

2. (a) Find the missing term in the following table using Lagrange interpolation [3 Marks]

x	0	1	2	3	4
y	1	3	9	...	81

   (b) Using Newton's divided difference interpolation for the data set (0, 18), (1, 10), (3, -18) and (6, 90), find the slope of the approximate curve at $x = 2$. [4 Marks]

   (c) Find the curve of best fit of the type $y = ae^{bx}$ to the following data by the method of least squares: [3 Marks]

x :	1	5	7	9	12
y :	10	15	12	15	21

P.T.O.

3. (a) Use composite Simpson's rule to determine $\int_3^5 \frac{1}{\sqrt{1-x^2}}$ by considering 8 subintervals using 4 digit arithmatic rounding. [3 marks]

(b) Find C_0, C_1 and x_1 such that the quadrature formula $\int_0^1 f(x)dx = C_0 f(0) + C_1 f(x_1)$ is exact for highest degree of precision. [3 Marks]

(c) Derive two-point Gauss quadrature formula for integration. [4 marks]

4. (a) Solve the following initial value problem using the modified Euler's method for $x = 0.2$ by taking step size $h = 0.1$: [4 Marks]

$$y' + 2y = x^3 e^{-2x}, \quad y(0) = 1.$$

(b) Using Runge-Kutta fourth-order method, solve the following system of initial value problems

$$y' = z, \quad y(0) = 3,$$
$$z' = -5y - 4z, \quad z(0) = -5$$

for $x = 0.1$ by taking step size $h = 0.1$. [6 Marks]