School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

Auxiliary Examination, August 2023

B.E. IV Semester | UMA011 : Numerical Analysis
Time Limit: 03 Hours | Maximum Marks: 100
Instructor(s) : Dr. Arvind K. Lal, Dr. Paramjeet Singh, Dr. Sanjeev Kumar

Instructions: You are expected to answer all the questions. **All questions carry equal marks.** Arrange your work in a reasonably neat, organized, and coherent way. Mysterious or unsupported answers will not receive full credit. Scientific Calculator is permitted.

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1. (a) Perform four iterations of the bisection method to find the $\sqrt{3}$ from the equation $x^2 - 3 = 0$.

(b) Using Newton's method starting with $x_0 = 1.5$, find the positive root of the equation

$$x - 2\sin x = 0$$

correct to three decimals.

2. (a) Solve the following system of linear equations using LU factorization:

$$\begin{aligned} 2x_1 - x_2 + x_3 &= 4 \\ 3x_1 + 3x_2 + 9x_3 &= 8 \\ 3x_1 + 3x_2 + 5x_3 &= 10. \end{aligned}$$

Use Gauss elimination to find matrices L and U.

(b) Perform two iterations of Gauss-Seidel method for the following system starting with initial guess $x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$:

$$\begin{aligned} 4x_1 + x_2 - x_3 &= 3 \\ 2x_1 + 7x_2 + x_3 &= 19 \\ x_1 - 3x_2 + 12x_3 &= 31. \end{aligned}$$

3. (a) Let $f(x) = \sqrt{x - x^2}$ and $P_2(x)$ be the Lagrange interpolating polynomial on the points $x_0 = 0$, x_1 and $x_2 = 1$. Find the largest value of x_1 in $(0, 1)$ for which $f(0.5) - P_2(0.5) = -0.25$.

(b) Find the Newton divided difference interpolating polynomial which interpolate the following data:

x	-2	-1	0	1	2	3
$f(x)$	1	4	11	16	13	-4

4. (a) Determine the values of subintervals n and step-size h required to approximate the integral

$$\int_0^2 \frac{1}{x+4}\, dx$$

to within 10^{-5} using composite Simpson's rule.

(b) Given the initial-value problem

$$\frac{dy}{dt} = \frac{y^2 - t^2}{y^2 + t^2}, \quad y(0) = 1.$$

Use Runge-Kutta method of order four, find the solution y at $t = 0.2$ with step-size $h = 0.2$.