

1. Using the bisection method, determine the point of intersection of the curves given by  $y = 3x$  and  $y = e^x$  in the interval  $[0, 1]$  with an accuracy 0.1.
2. Use the bisection method to find solution accurate to within  $10^{-3}$  for  $x - 2^{-x} = 0$  for  $0 \leq x \leq 1$ .
3. Find a bound for the number of iterations needed to achieve an approximation of  $\sqrt[3]{25}$  by the bisection method with an accuracy  $10^{-2}$ . Hence find the approximation with given accuracy.
4. Show that  $g(x) = 2^{-x}$  has a unique fixed point on  $[\frac{1}{3}, 1]$ . Use fixed-point iteration to find an approximation to the fixed point accurate to within  $10^{-2}$ .
5. Use the fixed-point iteration method to find smallest and second smallest positive roots of the equation  $\tan x = 4x$ , correct to 4 decimal places.
6. The iterates  $x_{n+1} = 2 - (1 + c)x_n + cx_n^3$  will converge to  $\alpha = 1$  for some values of constant  $c$  (provided that  $x_0$  is sufficiently close to  $\alpha$ ). Find the values of  $c$  for which convergence occurs? For what values of  $c$ , if any, convergence is quadratic.
7. Show that if  $A$  is any positive number, then the sequence defined by

$$x_n = \frac{1}{2}x_{n-1} + \frac{A}{2x_{n-1}}, \quad \text{for } n \geq 1,$$

converges to  $\sqrt{A}$  whenever  $x_0 > 0$ . What happens if  $x_0 < 0$ ?

8. Use Newton's method to find solutions accurate to within  $10^{-3}$  to the following problems
  - (a)  $x - e^{-x} = 0$  for  $x \in [0, 1]$
  - (b)  $2x \cos(2x) - (x - 2)^2 = 0$  for  $x \in [2, 3]$  and  $x \in [3, 4]$
9. The function  $f(x) = \sin(x)$  has a zero on the interval  $(3, 4)$ , namely,  $x = \pi$ . Perform three iterations of Newton's method to approximate this zero, using  $x_0 = 4$ . Determine the absolute error in each of the computed approximations. What is the apparent order of convergence?
10. Use Newton's method to approximate, to within  $10^{-4}$ , the value of  $x$  that produces the point on the graph of  $y = x^2$  that is closest to  $(1, 0)$ .
11. Use Newton's method and the modified Newton's method to find a solution of

$$\cos(x + \sqrt{2}) + x\left(\frac{x}{2} + \sqrt{2}\right) = 0, \quad \text{for } -2 \leq x \leq -1$$

accurate to within  $10^{-3}$ .

12. Apply the Newton's method with  $x_0 = 0.8$  to the equation  $f(x) = x^3 - x^2 - x + 1 = 0$ , and verify that the convergence is only of first-order. Further show that root  $\alpha = 1$  has multiplicity 2 and then apply the modified Newton's method with  $m = 2$  and verify that the convergence is of second-order.
13. Suppose  $\alpha$  is a zero of multiplicity  $m$  of  $f$ , where  $f^{(m)}$  is continuous on an open interval containing  $\alpha$ . Show that the fixed-point method  $x = g(x)$  with the following  $g$  has second-order convergence:

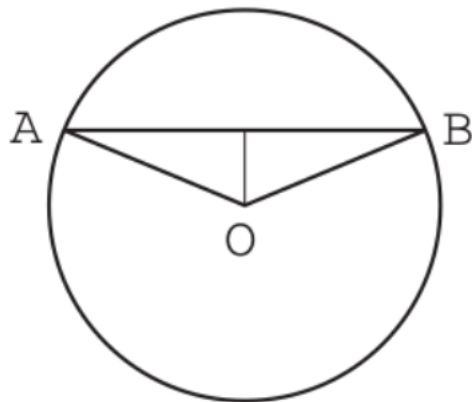
$$g(x) = x - m \frac{f(x)}{f'(x)}.$$

14. It costs a firm  $C(q)$  dollars to produce  $q$  grams per day of a certain chemical, where

$$C(q) = 1000 + 2q + 3q^{2/3}.$$

The firm can sell any amount of the chemical at \$4 a gram. Find the break-even point of the firm, that is, how much it should produce per day in order to have neither a profit nor a loss. Use the Newton's method and give the answer to the nearest gram.

15. The circle below has radius 1, and the longer circular arc joining A and B is twice as long as the chord AB. Find the length of the chord AB, correct to four decimal places. Use Newton's method.



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