Thapar Institute of Engineering and Technology

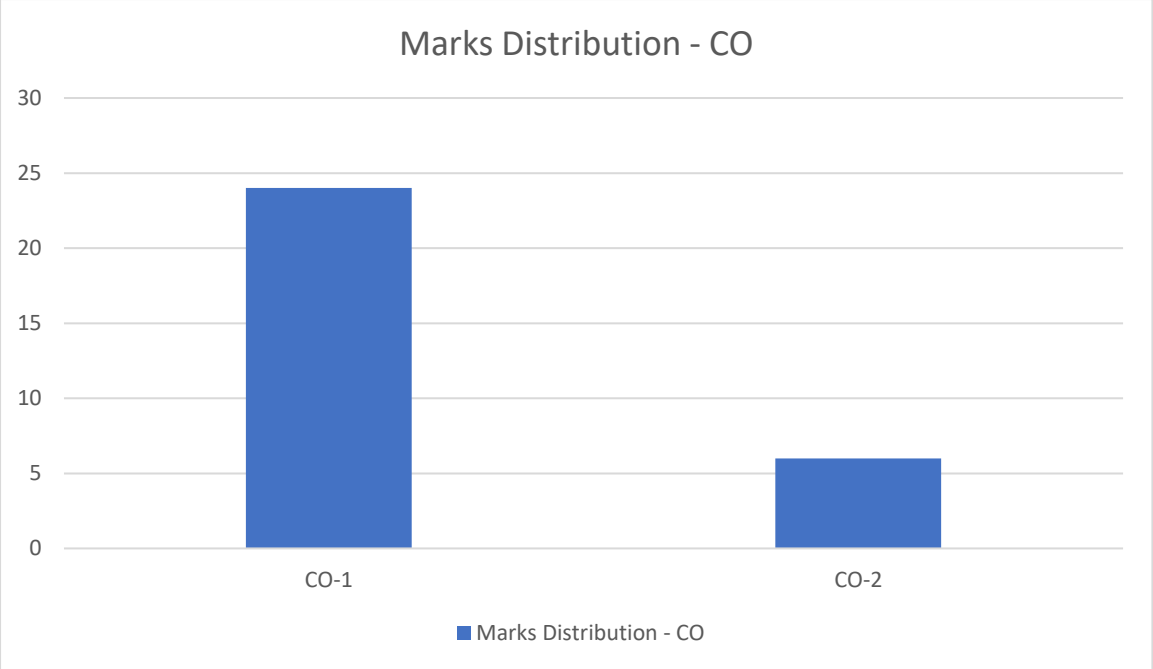
Department of Mathematics

MST Question Paper (Max Marks 30; Total Duration 2 Hrs)

Course Name: Numerical Analysis

Course Code: UMA011

Q1. (a) Proof that the maximum relative error obtained in rounding off any decimal number N correct up to k significant digits is 5×10^{-k}.	4 Marks	CO1	L2
(b) Give an example where overflow and underflow errors occur in computing the determinant of a 2×2 matrix. $(4+2)$	2 Marks	CO1	L5
Q2. (a) Mr Mike wants to apply the bisection method on the function $f(x) = (x-0.5)(x+1)^3(x-2)$. It is known that the function has roots at $x = 0.5$, $x = -1$ and $x = 2$. Determine to which root the bisection method will converge when the following intervals are considered as starting intervals: (i) $(-3, 3)$, (ii) $(-2, 3)$, (iii) $(-1.5, 3)$, (iv) $(1, 3)$.	4 Marks	CO1	L3
(b) What difficulties Mr Mike may face while applying the bisection method on the function $g(x) = (x-0.5)(x+1)^2(x-2)$ to find the root $x = -1$? Suggest a possible way, if exists, to apply the bisection method on the function $g(x)$ to find the root near $x = -1$. $(4+2)$	2 Marks	CO1	L6
Q3. Consider the function $f(x) = 1 - x + \ln(x)$. It is known that $f(1) = 0$. The equation $f(x) = 0$ is solved by Newton Raphson (NR) method with initial guess $x_0 = 2$ by the given algorithm(s). (a) Perform 4 iterations to solve by *Algorithm-1:* $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$.	2 Marks	CO1	L2
(b) Perform 4 iterations to solve by *Algorithm-2:* $x_{k+1} = x_k - \frac{\mu(x_k)}{\mu'(x_k)}$, where $\mu(x) = \frac{f(x)}{f'(x)}$.	2 Marks	CO1	L3
(c) From *Algorithm-1,2*, which algorithm is the best. State proper reason based on the convergence by computing the errors in each iteration and multiplicity of the root $x = 1$. $(2+2+2)$	2 Marks	CO1	L4
Q4. The equation $x^2 - 3 = 0$ is solved by the fixed point iteration method with initial guess $x_0 = 2$ applying two given algorithms. *Algorithm-A:* $x_{k+1} = \frac{1}{2}\left(x_k + \frac{\sqrt{3}}{x_k}\right)$; and *Algorithm-B:* $x_{k+1} = \frac{\sqrt{3}}{x_k}$. (a) Perform 4 iterations to solve the equation by *Algorithm-A* and *Algorithm-B*.	4 Marks	CO1	L2
(b) Comment on the convergence of each algorithm by stating a proper reason.	2 Marks	CO1	L4
Q5. Given the system of linear equations $Ax = b$ as $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$. (a) Solve the system $Ax = b$ by Gauss elimination method.	4 Marks	CO2	L2
(b) Find the LU decomposition of matrix A by the Doolittle method (where all the diagonal entries of the lower triangular matrix are 1). $(3+3)$	2 Marks	CO2	L3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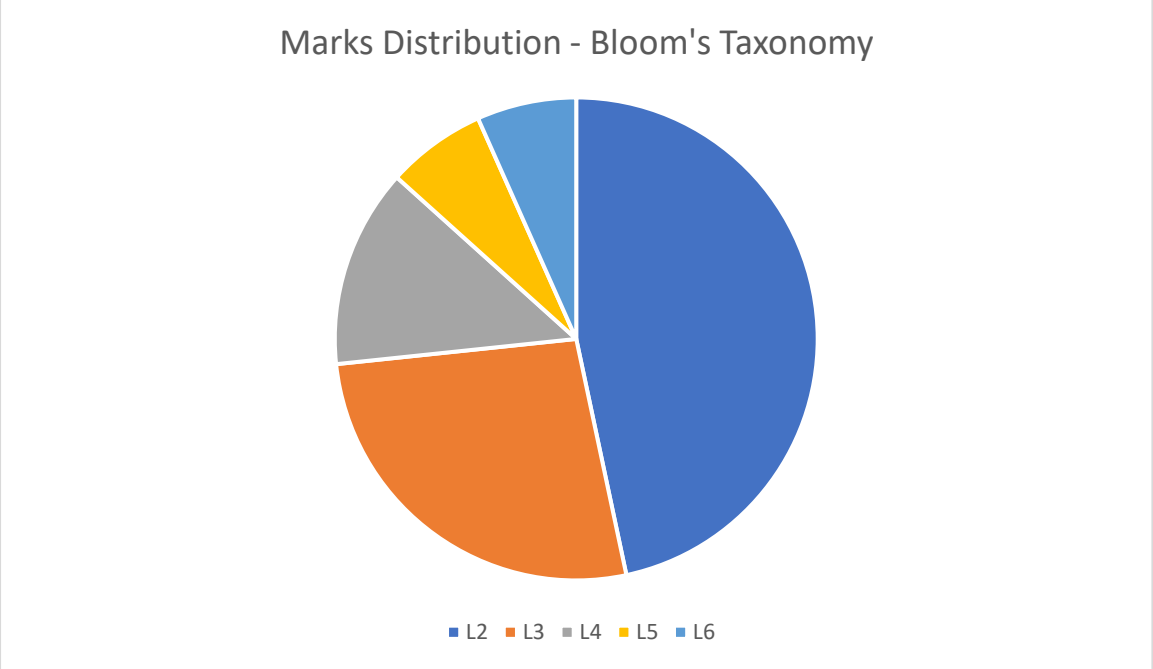
Marks Distribution - Bloom's Taxonomy
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