Roll Number:

Thapar Institute of Engineering & Technology, Patiala
School of Mathematics

B.E.(Sem. III) MST **UMA011: Numerical Analysis**

Course Instructors: Deepika Singh, Meenu Rani, MD Hasanuzzaman, Parimita Roy, Pankaj Narula, Sanjeev Kumar, Tina Verma, Vivek Sangwan

Time: 02 Hours; M. Marks: 25 **October 1, 2022**

Note: ATTEMPT ALL PARTS OF A QUESTION AT ONE PLACE. All questions are compulsory.

1. (a) Let x be any real number and $fl(x)$ be the approximation to x by chopping. Then what is the largest possible relative error in $fl(x)$ to x? (3 Marks)

   (b) Using four-digit rounding arithmetic and the formula for calculating the roots, find the most accurate approximations to the roots of the quadratic equation

   $$\frac{1}{3}x^2 + \frac{123}{4}x - \frac{1}{6} = 0.$$

   Also compute the relative errors. For calculation of exact roots, take nine digits rounding arithmetic. (3 Marks)

2. (a) Find the multiple root of the equation $f(x) = 27x^5 + 27x^4 + 36x^3 + 28x^2 + 9x + 1 = 0$ using modified Newton's method by taking the initial approximation $x_0 = -1.0$ such that the absolute error should be less than or equal to 10^{-2}. (3 Marks)

   (b) Use secant method to find the root of the equation $\cos(x) = xe^x$ correct to four decimal places in the interval [0,1] by taking initial approximations as $x_0 = 0$ and $x_1 = 1$. (3 Marks)

3. (a) Consider the system of linear equations given by

   $$3x_1 - 0.1x_2 - 0.2x_3 = 7.85,\ 0.1x_1 + 7x_2 - 0.3x_3 = -19.3\ \ \&\ \ 0.3x_1 - 0.2x_2 + 10x_3 = 71.4.$$

   Perform first two iterations of the Gauss Seidel method to approximate the solution starting with $x^{(0)} = (0, 0, 0)^T$ with four digit rounding. Also, find the value of $\frac{||x^{(2)} - x^{(1)}||_\infty}{||x^{(2)}||_\infty}$. (2+1=3 Marks)

   (b) Solve the following system of equations

   $$2x_1 + x_2 + x_3 - 2x_4 = -10,\ 4x_1 + 2x_3 + x_4 = 8,\ 3x_1 + 2x_2 + 2x_3 = 7\ \&\ x_1 + 3x_2 + 2x_3 - x_4 = -5$$

   using Gauss elimination method with partial pivoting. (4 Marks)

4. (a) Given that α is a fixed point of the iteration function $g(x)$ in the fixed point iteration method which is m-times continuously differentiable in some neighborhood of α. If $g'(\alpha) = g''(\alpha) = \ldots = g^{(m-1)}(\alpha) = 0$ and $g^{(m)}(\alpha) \neq 0$, then find the order of convergence of the method. (3 Marks)

   (b) Using LU Decomposition method, solve the following system of linear equations (3 Marks)

   $$\begin{aligned} 4x + 2y - z &= 0 \\ 8x + 14y + z &= -4 \\ 4x - 3y + 7z &= 21. \end{aligned}$$