School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

End-Term Examination, May 2023

B.E. IV Semester — UMA011 : Numerical Analysis

Time Limit: 03 Hours — Maximum Marks: 45

Instructor(s) : Dr. Arvind K. Lal, Dr. Paramjeet Singh, Dr. Sanjeev Kumar

Instructions: You are expected to answer all the questions. **All question carry equal marks.** Arrange your work in a reasonably neat, organized, and coherent way. Mysterious or unsupported answers will not receive full credit. Scientific Calculator is permitted.

1. (a) Find the multiplicity of the root $\alpha = 3$ for equation $(x-3)^2 \sin x = 0$. Also find the root using modified Newton's method starting with $x_0 = 2.5$. Use stopping criteria $|x_k - x_{k-1}| < 0.1$, where x_k denotes the approximation of the root at k-th iteration.

(b) Perform two iterations of Jacobi method for the following system of equations with initial guess $x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$:

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 5 \\ 4x_1 + 4x_2 - 3x_3 &= 3 \\ -2x_1 + 3x_2 - x_3 &= 1. \end{aligned}$$

2. Let us consider matrix $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$.

(a) Apply Gram-Schmidt process to decompose A as $A = QR$.

(b) Perform two iterations of QR algorithm to find all the eigenvalues of A.

3. (a) Let $f \in C^n[a,b]$ and $x_0, x_1, x_2, \cdots, x_n$ are distinct numbers in $[a,b]$. Let $P_n(x)$ be the interpolating polynomial in Newton's form. Then prove that there exists a point $\xi \in (a,b)$ such that

$$f[x_0, x_1, x_2, \cdots, x_n] = \frac{f^{(n)}(\xi)}{n!}.$$

(b) Use the method of least squares to fit a curve of the form $y = a + b\sqrt{x}$ to the following data:

x	0	1	4	9
$f(x)$	2	3	5	8

4. (a) Determine the number of subintervals n and step-size h required to approximate

$$\int_0^2 \sqrt{x+3}\, dx$$

to within 10^{-2} using composite trapezoidal rule.

(b) Determine constants a, b, c, and d that will produce a quadrature formula

$$\int_{-1}^{1} f(x)dx = af(-1) + bf(1) + cf'(-1) + df'(1)$$

that has degree of precision 3.

5. Given the initial-value problem

$$\frac{dy}{dt} = y\, e^t, \quad y(1) = 1.$$

(a) Use modified Euler's method with step-size $h = 0.1$ to compute $y(1.1)$ and $y(1.2)$.

(b) Use the values from part (a) and linear Lagrange interpolation to find the approximate value of $y(1.15)$.