School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

Mid-Semester Examination, March-2020

B.E. IV Semester   UMA007 : Numerical Analysis

Time Limit: 02 Hours   Maximum Marks: 25

Instructor(s) (Dr.) : Meenu Rani, Munish Kansal, Paramjeet Singh, Sanjeev Kumar, Sapna Sharma

Instructions: This question paper has one printed page. You are expected to answer all the questions. Organize your work in a reasonably neat, organized, and coherent way. Mysterious or unsupported answers will not receive full credit.

---

1. (a) Evaluate $f(x) = x^3 - 2.34x^2 + 5.42x + 1.57$ at $x = 3.51$ in nested form by using three digit rounding arithmetic and hence find the relative error. Take five digits for computing the exact value of the polynomial.

   [3 marks]

   (b) Discuss the stability of the algorithm for calculating $f(x) = \sqrt{1+x} - 1$ at $x = 0.001$ Suggest a modification if the procedure is unstable. Also verify the stability of the modified function.

   [4 marks]

2. (a) Let a function f is continuous on $[a, b]$ and $f(a) * f(b) < 0$. Then show that bisection method generates a sequence $\{x_n\}_{n=1}^{\infty}$ approximating a zero α of f with $|x_n - \alpha| \leq \dfrac{b-a}{2^n}$, when $n \geq 1$.

   [3 marks]

   (b) Show that $g(x) = \dfrac{x^2 - 1}{3}$ has a unique fixed point on $[-1, 1]$.

   [3 marks]

3. (a) Show that the equation $f(x) = x^3 - x^2 - x + 1 = 0$ has a root $\alpha = 1$ with multiplicity 2 and then apply modified Newton's method by using 4 decimals digits while calculations with $x_0 = 0.8$, $m = 2$ and tolerance 0.001.

   [3 marks]

   (b) Using secant method, determine the point of intersection of the curves given by $y = ln(x)$ and $y = \dfrac{x^2}{8} - 2$, with an accuracy 0.01. Take initial guess $x_0 = 5$ and $x_1 = 6$.

   [3 marks]

4. (a) Given the linear system

   $$x_1 - x_2 + \alpha x_3 = -2$$
   $$-x_1 + 2x_2 - \alpha x_3 = 3$$
   $$\alpha x_1 + x_2 + x_3 = 2.$$

   (i) Find value(s) of α for which the system has no solution and infinite number of solutions.

   (ii) Assuming a unique solution exists for a given α, find the solution.

   [3 marks]

   (b) Convert the coefficient matrix A in the following linear system into LU factorization and hence solve it.

   $$2x_1 - x_2 + x_3 = -1$$
   $$3x_1 + 3x_2 + 9x_3 = 0$$
   $$3x_1 + 3x_2 + 5x_3 = 4.$$

   [3 marks]