

School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

Mid-Term Examination

Even Semester 2021-22

B.E. IV Semester

Time Limit: 02 Hours

Instructor(s) : Dr. Meenu Rani, Dr. Munish Kansal, Dr. Paramjeet Singh

UMA011 : Numerical Analysis

Maximum Marks: 35

Instructions: You are expected to answer ANY FIVE questions out of seven. Arrange your work in a reasonably neat, organized, and coherent way. Mysterious or unsupported answers will not receive full credit. Scientific Calculator is permitted.

1. Write an algorithm to calculate the expression $\sqrt{1+x} - 1$ when $x = 0.0001$. By considering the condition number κ of the subproblem of evaluating the function, show that such a function evaluation is not stable. Rewrite the stable expression. [7 marks]

2. Use four-digit rounding arithmetic and the formula to find the most accurate approximations to the roots of the following quadratic equation:

$$\frac{1}{3}x^2 - \frac{123}{4}x + \frac{1}{6} = 0.$$

Compute the relative errors by taking exact values of the roots with at least eight digits. [7 marks]

3. Using the bisection method, determine the point of intersection of the curves $y = 3x$ and $y = e^x$ in the interval $[a, b] = [0, 1]$. Use stopping criteria $(b - a) < 0.1$. [7 marks]

4. Find the multiplicity of the root $\alpha = 1$ for equation $(x - 1)^2 \ln(x) = 0$. Also perform the three iterations of modified Newton's method starting with $x_0 = 0.5$. [7 marks]

5. To find the value of $2^{1/3}$, we consider the equation $f(x) = x^3 - 2 = 0$ and this can be written in the fixed-point form $x = g(x)$ in two ways:

(a) $x = g_1(x) = x^3 + x - 2$.

(b) $x = g_2(x) = \frac{2 + 5x - x^3}{5}$.

Show the construction of both of $g(x)$ from $f(x)$. Compute first four approximations from $x_{n+1} = g_i(x_n)$, $i = 1, 2$ by using $x_0 = 1.2$. Show that which approximation converges to $2^{1/3}$ and why? [7 marks]

6. Solve the following system of equations using the Gaussian elimination with partial pivoting:

$$x_1 + x_2 + x_3 = 1$$

$$2x_1 + 3x_2 + 4x_3 = 3$$

$$4x_1 + 9x_2 + 16x_3 = 11.$$

[7 marks]

7. Check whether the Gauss-Seidel method converges or not when applied to solve the following linear system:

$$2x_1 - x_2 + x_3 = -1$$

$$x_1 + x_2 + x_3 = 2$$

$$-x_1 - x_2 - 2x_3 = -5.$$

[7 marks]