Thapar Institute of Engineering & Technology, Patiala
Department of Mathematics

UMA021: Numerical Linear Algebra
Assignment 2
Interpolation and Integration

1. Find the unique polynomial $P(x)$ of degree 2 or less such that

$$P(1) = 1,\ P(3) = 27,\ P(4) = 64$$

using Lagrange interpolation. Evaluate $P(1.05)$.

2. For the given functions $f(x)$, let $x_0 = 1$, $x_1 = 1.25$, and $x_2 = 1.6$. Construct Lagrange interpolation polynomials of degree at most one and at most two to approximate $f(1.4)$, and find the absolute error.

(a) $f(x) = \sin \pi x$
(b) $f(x) = e^{2x} - x$

3. Let $P_3(x)$ be the Lagrange interpolating polynomial for the data $(0, 0)$, $(0.5, y)$, $(1, 3)$ and $(2, 2)$. Find y if the coefficient of x^3 in $P_3(x)$ is 6.

4. Construct the Lagrange interpolating polynomials for the following functions, and find a bound for the absolute error on the interval $[x_0, x_n]$.

(a) $f(x) = \sin x,\ x_0 = 2.0,\ x_1 = 2.4,\ x_2 = 2.6,\ n = 2.$
(b) $f(x) = e^{2x} \cos 3x,\ x_0 = 0,\ x_1 = 0.3,\ x_2 = 0.6,\ n = 2.$

5. Using Newton's divided difference interpolation, construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.

$$f(0.43) \text{ if } f(0) = 1, f(0.25) = 1.64872, f(0.5) = 2.71828, f(0.75) = 4.4816.$$

6. Show that the polynomial interpolating the following data has degree 3.

x	-2	-1	0	1	2	3
$f(x)$	1	4	11	16	13	-4

7. Let $f(x) = e^x$, show that $f[x_0, x_1, \ldots, x_m] > 0$ for all values of m and all distinct equally spaced nodes $\{x_0 < x_1 < \cdots < x_m\}$.

8. The following data are given for a polynomial $P(x)$ of unknown degree.

x	0	1	2	3
$f(x)$	4	9	15	18

Determine the coefficient of x^3 in $P(x)$ if all fourth-order forward differences are 1.

9. Construct the interpolating polynomial that fits the following data using Newton's forward and backward

x	0	0.1	0.2	0.3	0.4	0.5
$f(x)$	-1.5	-1.27	-0.98	-0.63	-0.22	0.25

difference interpolation. Hence find the values of $f(x)$ at $x = 0.15$ and 0.45.

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10. For a function f, the forward-divided differences are given by

$x_0 = 0.0$	$f[x_0]$		
$x_1 = 0.4$	$f[x_1]$	$f[x_0, x_1]$	$f[x_0, x_1, x_2] = \frac{50}{7}$
$x_1 = 0.7$	$f[x_2] = 6$	$f[x_1, x_2] = 10$	

Determine the missing entries in the table.

11. A fourth-degree polynomial $P(x)$ satisfies $\Delta^4 P(0) = 24$, $\Delta^3 P(0) = 6$, and $\Delta^2 P(0) = 0$, where $\Delta P(x) = P(x+1) - P(x)$. Compute $\Delta^2 P(10)$.

12. Given

$$I = \int_0^2 x^2 e^{-x^2}\, dx.$$

Approximate the value of I using trapezoidal and Simpson's one-third method.

13. Approximate the integral

$$\int_1^{1.5} x^2 \ln x \, dx$$

using the (non-composite) trapezoidal rule. Provide a rigorous error bound on this approximation.

14. Approximate the integral

$$\int_0^{0.5} \frac{2}{x-4}\, dx$$

using the (non-composite) Simpson's rule. Provide a rigorous error bound on this approximation.

15. The Trapezoidal rule applied to $\int_0^2 f(x)\,dx$ gives the value 4, and Simpson's rule gives the value 2. What is $f(1)$?

16. Suppose that $f(0) = 1$, $f(0.5) = 2.5$, $f(1) = 2$, and $f(0.25) = f(0.75) = \alpha$. Find α if the composite Trapezoidal rule with $n = 4$ gives the value 1.75 for $\int_0^1 f(x)\,dx$.

17. Calculate $I = \int_0^4 x\,dx$ using the composite Trapezoidal and Simpson's rules with 2 and 4 subintervals.

Answers: $(A1)$ 11.895; $(A2)(a)$ $P_1(x) = -0.6967x + 0.16425$, $P_1(1.4) = -0.8117$; Absolute error: $E_a = 0.1394$; Similarly, $P_2(x) = 3.5524x^2 - 10.8213x + 7.2689$; $P_2(1.4) = -0.91822$; Absolute error $= 0.03284$ (b) Do same as part (a); $(A3)$ $y = 4.25$; $(A4)$ (a) 0.31493, (b) 0.0014084; $(A5)$ $P_1(0.43) = 2.22454$, $P_2(0.43) = 2.34886474$, $P_3(0.43) = 2.3606$; $(A8)$ $-\frac{11}{12}$; $(A11)$ 1140; $(A12)$ $I_T = 0.073263$, $I_S = 0.51493$; $(A13)$ 0.228, $|E| = 0.040$; $(A14)$ -0.26706349, $|E| \approx 10^{-6}$; $(A15)$ $\frac{1}{2}$; $(A16)$ $\frac{3}{2}$; $(A17)$ for each case: $I = 8$