Roll Number : ....   Branch :............   Group :.........

TIET Patiala
Department of Mathematics
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UG   Numerical Analysis (UMA011/UMA007)
Feb , 2024   Time: 3 Hrs; M. Marks: 100

Note: Total five questions are there. All questions are compulsory.

1. (i) The quadratic formula is used for computing the roots of equation $ax^2 + bx + c = 0$, $a \neq 0$ is given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Use four digit rounding arithmetic to find the most accurate approximation of roots of equation $x^2 + 62.10x + 1 = 0$.   10

   (ii) Find the largest eigen value of the matrix

   $$A = \begin{bmatrix} 0 & 11 & -5 \\ -2 & 17 & -7 \\ -4 & 26 & -10 \end{bmatrix}$$

   Taking initial guess $(1, 1, 1)^t$ 10

2. (i) Find the real root of equation $x^3 - x^2 - x - 3 = 0$, by Bisection method, correct to three decimal places   12

   (ii) Compute first two iterations of secant iteration to solve the equation $xe^x = 3$, using $x_0 = 1$ & $x_1 = 1.1$.   8

3. (i) Apply Gauss-elimination method to solve the following system:
   $2x_1 + 3x_2 - x_3 = 5$, $4x_1 + 4x_2 - 3x_3 = 3$, & $-2x_1 + 3x_2 - x_3 = 1$.   12

   (ii) Suppose that $\bar{x}$ is an approximation to the solution of $Ax = b$, A is a nonsingular matrix, and r is residual vector of $\bar{x}$ then for, any natural matrix norm,

   $$||x - \bar{x}|| \leq ||r||.||A^{-1}||$$

   and if $x \neq 0$ & $b \neq 0$,
   $$\frac{||x - \bar{x}||}{||x||} \leq ||A||.||A^{-1}||\frac{||r||}{||b||}.$$

                                                    8

4. (i) Use Runge-Kutta method of order four to solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$ at $x = 0.2$ & 0.4, take $h = 0.2$.   12

   (ii) Use the numbers $x_0 = 2$, $x_1 = 2.75$, & $x_2 = 4$ to find second degree Lagrange interpolating polynomial for $f(x) = \frac{1}{x}$.   8

.... PTO...................................

5. Evaluate $\int_1^7 \frac{1}{x} \, dx$, by subdividing the interval $[1, 7]$ into six equal parts, by using

   (i) Composite trapezoidal rule

   (ii) Composite Simpson's rule.

   Hence compare the results with the actual value of the integral.   **10+10**