Roll Number:

Thapar Institute of Engineering and Technology, Patiala
School of Mathematics

BE-(III Semester) EST — UMA011/NM3402: Numerical Analysis
March 01, 2023, — Time: 03 Hours; MM: 100
Course Coordinator: Dr. Vivek Sangwan.

Instructions: This question paper has two printed pages. You are expected to answer all the questions. **ATTEMPT ALL PARTS OF A QUESTION AT ONE PLACE.** Organize your work in a reasonably neat, organized, and coherent way. Mysterious or unsupported answers will not receive full credit. Calculator without graphing mode is permitted.

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1. (a) Find the largest interval in which $fl(x)$ must lie to approximate $\sqrt{3}$ with relative error at most 10^{-4}. (10 marks)

   (b) Use four-digit rounding arithmetic and the formula for the roots of a quadratic equation, to find the most accurate approximations to the roots of the following quadratic equation. Compute the absolute and relative errors. [10 marks]

$$1.002x^2 + 11.01x + 0.01265 = 0.$$

2. (a) Perform four iterations of modified Newton-Raphson method to find the approximate root of the equation $f(x) = (x+2) * (x-1)^4 = 0$. Take initial guess as $x_0 = 3$. [10 marks]

   (b) Find first five iterations obtained using power method to approximate the dominant eigenvalue and eigen-vector for the following matrix with $x^{(0)} = [1, 0, 1]^t$.

$$\begin{bmatrix} 4 & 2 & -1 \\ 2 & 0 & 2 \\ -1 & 2 & 0 \end{bmatrix}.$$

[10 marks]

3. (a) Suppose that $x_0, x_1, \cdots, x_n$ are distinct numbers in $[a, b]$ and $f \in C^{n+1}[a, b]$. Let $P_n(x)$ be the unique polynomial of degree $\leq n$ that passes through given points. Then prove that for every $x \in [a, b]$, there exists $\xi = \xi(x) \in (a, b)$ such that [10 marks]

$$f(x) - P_n(x) = \frac{(x - x_0)(x - x_1) \cdots (x - x_n)}{(n+1)!} f^{(n+1)}(\xi).$$

(b) Find the curve of best fit of the type $y = ae^{bx}$ to the following data by the method of least squares:

x :	1	5	7	9	12
y :	10	15	12	15	21

[10 marks]

4. (a) Solve the following systems using LU decomposition method [10 marks]

$$\begin{aligned} 3x_1 + 2x_2 - x_3 &= 1 \\ x_1 - 3x_2 + 2x_3 &= 2 \\ 2x_1 - x_2 + 4x_3 &= 3. \end{aligned}$$

(b) Using three digit rounding arithmetic, perform four iterations of Gauss Seidel method for the system of equations

$$5x - 2y + 3z = -1, \quad -3x + 9y + z = 2, \quad 2x - y - 7z = 3$$

by taking initial guess as $[0, 0, 0]^T$, where T denotes the transpose. Also find relative error for the last two approximations. (10 marks)

5. (a) Determine constants c_0, c_1 and x_1 that will produce a quadrature formula

$$\int_0^1 x\, f(x)\, dx = c_0\, f(0) + c_1\, f(x_1)$$

that has the highest degree of precision. [10 marks]

(b) Use Runge-Kutta method of fourth order to find the solution at $t = 0.2$ with step-size $h = 0.2$ for the initial-value problem [10 marks]

$$\frac{dy}{dt} = -2y + t, \; y(0.0) = 1.0.$$