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School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

End-Term Examination, December 2023

B.E. IV Semester — UMA011 : Numerical Analysis

Time Limit: 03 Hours — Maximum Marks: 35

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Instructions: This question paper has two printed pages. You are expected to answer any FIVE questions. Arrange your work in a reasonably neat, organized, and coherent way. Mysterious or unsupported answers will not receive full credit. Scientific Calculator is permitted.

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1. (a) Perform two iterations of Jacobi method for the following system of equations with initial guess $x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$:

$$\begin{aligned} 4x_1 - x_2 + x_3 &= 7 \\ 4x_1 - 8x_2 + x_3 &= -21 \\ -2x_1 + x_2 + 5x_3 &= 15. \end{aligned}$$

[3.5 marks]

(b) We require to solve the following system of linear equations using LU decomposition.

$$\begin{aligned} x_1 + x_2 - x_3 &= 3 \\ x_1 + 2x_2 - 2x_3 &= 2 \\ -2x_1 + x_2 + x_3 &= 1. \end{aligned}$$

Find the matrices L and U using Gauss elimination. Using those values of L and U, solve the given system of equations. [3.5 marks]

2. (a) Perform one iteration of QR algorithm to find all the eigenvalues of matrix $A = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}$. In the algorithm, use Gram-Schmidt process to decompose matrix A as $A = QR$. [3.5 marks]

(b) Use the method of least squares to fit a curve of the form $y = a + b\sqrt{x}$ to the following data:

x	1	4	9	16
$f(x)$	0	2	3	6

[3.5 marks]

3. (a) Determine the spacing h in a table of equally spaced values of the function $f(x) = \ln x$ between 1 and 2, so that interpolation with a linear polynomial will yield an accuracy of 0.001. [3.5 marks]

(b) Let $f \in C^n[a, b]$ and $x_0, x_1, x_2, \cdots, x_n$ are distinct numbers in $[a, b]$. Let $P_n(x)$ be the interpolating polynomial in Newton's form. Then prove that there exists a point $\xi \in (a, b)$ such that

$$f[x_0, x_1, x_2, \cdots, x_n] = \frac{f^{(n)}(\xi)}{n!}.$$

[3.5 marks]

4. Consider the integral

$$\int_0^{\pi/2} \cos x \, dx.$$

(a) Determine the number of subintervals n and step-size h required to approximate the given integral to within 10^{-3} using composite Simpson's one-third rule. [4 marks]

(b) Use n and h from part (a) to compute the approximate value of the given integral by the composite Simpson's one-third rule. [3 marks]

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5. (a) Let $f \in C^2[a,b]$ and $h = b - a$, then show that

$$\int_a^b f(x)dx = \frac{h}{2}[f(a) + f(b)] - \frac{h^3}{12} f''(c), \text{ where } c \in (a,b).$$

[3.5 marks]

(b) Evaluate the integral

$$I = \int_0^1 x^2\, e^x\, dx$$

by using two point Gauss-Legendre formula. Also compare with the exact value of the integral.

[3.5 marks]

6. Given the initial-value problem

$$\frac{dy}{dt} = y - t^2 + 1, \quad y(0) = 0.5.$$

(a) Use Runge-Kutta method of order four with step-size $h = 0.2$ to compute $y(0.2)$. [4 marks]

(b) Use the values from part (a) and linear Lagrange interpolation to find the approximate value of $y(0.15)$.

[3 marks]