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School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

End-Term Examination

Odd Semester 2020-21

B.E. III Semester | UMA011 : Numerical Analysis

Time Limit: 02 Hours | Maximum Marks: 50

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Instructions: This question paper has two printed pages. You are expected to answer ANY FIVE questions. Organize your work in a reasonably neat, organized, and coherent way. Mysterious or unsupported answers will not receive full credit. Scientific Calculator is permitted.

1. Find the multiplicity of the root $\alpha = 1$ for equation $(x-1)^2 \ln(x) = 0$. Also perform the three iterations of modified Newton's method starting with initial guess $x_0 = 0.5$. [10 marks]

2. Apply Gaussian elimination to the system

$$\begin{aligned} 0.003000x_1 + 59.14x_2 &= 59.17 \\ 5.291x_1 - 6.130x_2 &= 46.78, \end{aligned}$$

using four-digit arithmetic with rounding. Also, solve the system with partial pivoting and four-digit arithmetic with rounding. Compare the results to the exact solution $x_1 = 10.00$ and $x_2 = 1.000$. [10 marks]

3. Suppose that $x_0, x_1, \cdots, x_n$ are distinct numbers in $[a, b]$ and $f \in C^{n+1}[a, b]$. Let $P_n(x)$ be the unique polynomial of degree $\leq n$ that passes through $n+1$ given points. Prove that for every $x \in [a, b]$, there exists $\xi = \xi(x) \in (a, b)$ such that

$$f(x) - P_n(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_n)}{(n+1)!} f^{(n+1)}(\xi).$$

[10 marks]

4. (a) The following data are given for a polynomial $P(x)$ of unknown degree.

x	0	1	2
$P(x)$	2	-1	4

Determine the coefficient of x^2 in $P(x)$, if all third-order forward differences are 1. [5 marks]

(b) Use the method of least squares to fit a curve of the form $y = ax^b$ to the following data.

x	2	3	4	5
y	27.8	62.1	110	161

[5 marks]

5. The area A inside the closed curve $y^2 + x^2 = \cos x$ is given by

$$A = 4\int_0^{\alpha} (\cos x - x^2)^{1/2} dx$$

where α is the positive root of the equation $\cos x = x^2$.

(a) Compute α with three correct decimals by using Newton's method.

(b) Use trapezoidal rule to compute the area A by taking two subintervals. [10 marks]

6. Let $f \in C^2[a, b]$ and $h = b - a$, then prove that

$$\int_a^b f(x)dx = \frac{h}{2}[f(a) + f(b)] - \frac{h^3}{12} f''(\xi), \text{ where } \xi \in (a, b).$$

[10 marks]

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7. Given the initial-value problem

$$\frac{dy}{dt} = \frac{2}{t}y + t^2 e^t, \quad y(1) = 0.$$

Use Euler's method with $h = 0.1$ to compute $y(1.1)$ and $y(1.2)$. Further use these values to approximate y at $t = 1.04$.

[10 marks]