Department of Mathematics, Thapar Institute of Engineering & Technology, Patiala

UMA021: Numerical Linear Algebra
Assignment 1
Roots of Non-linear Equations

1. Using the bisection method, determine the point of intersection of the curves given by $y = 3x$ and $y = e^x$ in the interval $[0, 1]$ with an accuracy 0.1.

2. Use the bisection method to find solution accurate to within 10^{-3} for $x - 2^{-x} = 0$ for $0 \leq x \leq 1$.

3. Find a bound for the number of iterations needed to achieve an approximation of $\sqrt[3]{25}$ by the bisection method with an accuracy 10^{-2}. Hence find the approximation with given accuracy.

4. Show that $g(x) = 2^{-x}$ has a unique fixed point on $[\frac{1}{3}, 1]$. Use fixed-point iteration to find an approximation to the fixed point accurate to within 10^{-2}.

5. Use the fixed-point iteration method to find smallest and second smallest positive roots of the equation $\tan x = 4x$, correct to 4 decimal places.

6. The iterates $x_{n+1} = 2 - (1 + c)x_n + cx_n^3$ will converge to $\alpha = 1$ for some values of constant c (provided that x_0 is sufficiently close to α). Find the values of c for which convergence occurs? For what values of c, if any, convergence is quadratic.

7. Show that if A is any positive number, then the sequence defined by

$$x_n = \frac{1}{2}x_{n-1} + \frac{A}{2x_{n-1}}, \quad \text{for } n \geq 1,$$

converges to $\sqrt{A}$ whenever $x_0 > 0$. What happens if $x_0 < 0$?

8. Use Newtons method to find solutions accurate to within 10^{-3} to the following problems

(a) $x - e^{-x} = 0$ for $x \in [0, 1]$

(b) $2x\cos(2x) - (x - 2)^2 = 0$ for $x \in [2, 3]$ and $x \in [3, 4]$

9. The function $f(x) = \sin(x)$ has a zero on the interval $(3, 4)$, namely, $x = \pi$. Perform three iterations of Newtons method to approximate this zero, using $x_0 = 4$. Determine the absolute error in each of the computed approximations. What is the apparent order of convergence?

10. Use Newton's method to approximate, to within 10^{-4}, the value of x that produces the point on the graph of $y = x^2$ that is closest to $(1, 0)$.

11. Use Newtons method and the modified Newtons method to find a solution of

$$\cos(x + \sqrt{2}) + x(\frac{x}{2} + \sqrt{2}) = 0, \text{ for } -2 \leq x \leq -1$$

accurate to within 10^{-3}.

12. Apply the Newton's method with $x_0 = 0.8$ to the equation $f(x) = x^3 - x^2 - x + 1 = 0$, and verify that the convergence is only of first-order. Further show that root $\alpha = 1$ has multiplicity 2 and then apply the modified Newton's method with $m = 2$ and verify that the convergence is of second-order.

13. Suppose α is a zero of multiplicity m of f, where $f^{(m)}$ is continuous on an open interval containing α. Show that the fixed-point method $x = g(x)$ with the following g has second-order convergence:

$$g(x) = x - m\frac{f(x)}{f'(x)}.$$

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14. It costs a firm $C(q)$ dollars to produce q grams per day of a certain chemical, where

$$C(q) = 1000 + 2q + 3q^{2/3}.$$

The firm can sell any amount of the chemical at \$4 a gram. Find the break-even point of the firm, that is, how much it should produce per day in order to have neither a profit nor a loss. Use the Newton's method and give the answer to the nearest gram.

15. The circle below has radius 1, and the longer circular arc joining A and B is twice as long as the chord AB. Find the length of the chord AB, correct to four decimal places. Use Newtons method.