

School of Mathematics, Thapar Institute of Engineering & Technology, Patiala
End-Term Examination, June 2022

B.E. IV Semester

Time Limit: 02 Hours

Instructor(s) : Dr. Meenu Rani, Dr. Munish Kansal, Dr. Paramjeet Singh

UMA011 / UMA011 : Numerical Analysis
Maximum Marks: 35

Instructions: You are expected to answer ALL FIVE questions. Arrange your work in a reasonably neat, organized, and coherent way. Mysterious or unsupported answers will not receive full credit. Scientific Calculator is permitted.

1. Apply power method to approximate the dominant eigenvalue and the corresponding eigenvector of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

using $x^{(0)} = [1, 1, 1]^t$ as an initial approximation. Use stopping criteria $\|x^{(k)} - x^{(k-1)}\|_{\infty} < 0.01$, where $x^{(k)}$ denotes the eigenvector at k -th iteration. [7 marks]

2. Use the method of least squares to fit the curve of the form $y = a + \frac{b}{x}$ to the following data:

x	-3	-2	-1	1	2
$f(x)$	-10	4	6	-2	0

[7 marks]

3. Let $f \in C^n[a, b]$ and $x_0, x_1, x_2, \dots, x_n$ are distinct numbers in $[a, b]$. Let $P_n(x)$ be the interpolating polynomial in Newton's form. Then prove that there exists a point $\xi \in (a, b)$ such that

$$f[x_0, x_1, x_2, \dots, x_n] = \frac{f^{(n)}(\xi)}{n!}.$$

[7 marks]

4. Determine the values of subintervals n and step-size h required to approximate

$$\int_1^2 x \ln x \, dx$$

to within 0.01 by using composite trapezoidal rule and hence compute the approximate value of the integral.

[7 marks]

5. Apply the fourth order Runge-Kutta method to the following initial-value problem to find the approximate solution at $t = 0.2$ with step-size $h = 0.2$.

$$\frac{dy}{dt} = y - t^2 + 1, \quad y(0) = 1.$$

[7 marks]