Lecture 24: Numerical Linear Algebra (UMA021): Matrix Algebra

Dr. Meenu Rani

Department of Mathematics
TIET, Patiala
Punjab-India

System of linear equations

Example

Apply Gaussian elimination to the system:

$$0.003000x_1 + 59.14x_2 = 59.17$$
$$5.291x_1 - 6.130x_2 = 46.78$$

using four-digit arithmetic with rounding, and compare the results to the exact solution $x_1 = 10.00$ and $x_2 = 1.000$.

System of linear equations

Pivot element

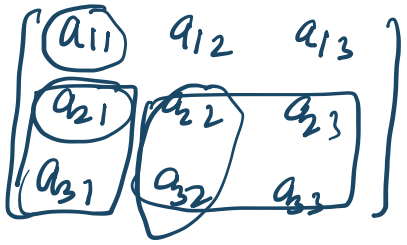
In the elimination process, we divide with diagonal element a_{ii} at each stage and assume that $a_{ii} \neq 0$. These elements are known as pivot element.

Pivot Strategies

1) Partial Pivoting

Partial Pivoting

If at any stage of elimination, one of the pivot becomes small (or zero) then we bring other element as pivot by interchanging the rows. This process is called Gauss elimination with partial pivoting.



$$\max\{|a_{11}|, |a_{21}|, |a_{31}|\} = |a_{21}|$$

$$E_1 \leftrightarrow E_2$$

System of linear equations

Example

Apply Gaussian elimination to the system:

$$0.003000x_1 + 59.14x_2 = 59.17$$
$$5.291x_1 - 6.130x_2 = 46.78$$

using partial pivoting and four-digit arithmetic with rounding, and compare the results to the exact solution $x_1 = 10.00$ and $x_2 = 1.000$.

Solution:

$$[A:b] = \begin{matrix} E_1 \\ E_2 \end{matrix}\begin{bmatrix} 0.003000 & 59.14 & : & 59.17 \\ 5.291 & -6.130 & : & 46.78 \end{bmatrix}$$

Use partial Pivoting

$$\max\{|a_{11}|, |a_{21}|\} = \max\{|0.003000|, |5.291|\}$$
$$= 5.291 = a_{21}$$

$E_1 \leftrightarrow E_2$

System of linear equations

$$[A:b] \sim \begin{matrix} E_1 \\ E_2 \end{matrix} \begin{bmatrix} 5.291 & -6.130 & : & 46.78 \\ 0.003000 & 59.14 & : & 59.17 \end{bmatrix}$$

$$E_2 \rightarrow E_2 - \frac{0.003000}{5.291} E_1$$

$$59.14 - 0.000567 * (-6.130)$$

$$59.14 + 0.003476$$

$$= 59.14$$

$$E_2 - 0.000567\, E_1$$

$$\sim \begin{bmatrix} 5.291 & -6.130 & : & 46.78 \\ 0 & 59.14 & : & 59.14 \end{bmatrix}$$

$$59.17 - 0.000567 * (46.78)$$

$$59.17 - 0.02652$$

use back sub.

$$59.14\, x_2 = 59.14 \Rightarrow x_2 = 1$$

$$5.291 x_1 - 6.130\,(1) = 46.78$$

$$5.291\, x_1 \qquad 52.91$$

$$x_1 = 10$$

Ans

System of linear equations

Example:

Using four-digit arithmetic operations, solve the following system of equations by Gaussian elimination with partial pivoting

$$\begin{aligned} 0.729x_1 + 0.81x_2 + 0.9x_3 &= 0.6867 \\ x_1 + x_2 + x_3 &= 0.8338 \\ 1.331x_1 + 1.21x_2 + 1.1x_3 &= 1.000. \end{aligned}$$

This system has exact solution, rounded to four places $x_1 = 0.2245$, $x_2 = 0.2814$, $x_3 = 0.3279$. Compare your answers!

System of linear equations

$$[A:b] = \left[\begin{array}{ccc:c} 0.729 & 0.81 & 0.9 & 0.6867 \\ 1 & 1 & 1 & 0.8338 \\ 1.331 & 1.21 & 1.1 & 1.000 \end{array}\right]$$

$$\max\{|0.729|, |1|, |1.331|\} = 1.331 = a_{31}$$

$$E_1 \leftrightarrow E_3$$

$$\sim \begin{matrix} E_1 \\ E_2 \\ E_3 \end{matrix} \left[\begin{array}{ccc:c} 1.331 & 1.21 & 1.1 & 1.000 \\ 1 & 1 & 1 & 0.8338 \\ 0.729 & 0.81 & 0.9 & 0.6867 \end{array}\right]$$

$$E_2 \to E_2 - \frac{1}{1.331} E_1 \quad , \quad E_3 \to E_3 - \frac{0.729}{1.331} E_1$$

$$\sim \begin{matrix} E_1 \\ E_2 \\ E_3 \end{matrix} \begin{bmatrix} 1.331 & 1.21 & 1.1 & : & 1.000 \\ 0 & 0.0909 & 0.1736 & : & 0.0825 \\ 0 & 0.1473 & 0.2975 & : & 0.1390 \end{bmatrix}$$

$$\max\{|a_{22}|, |a_{32}|\} = \max\{|0.0909|, |0.1473|\}$$

$$= 0.1473 = a_{32}$$

$$E_2 \leftrightarrow E_3$$

$$\sim \begin{matrix} E_1 \\ E_2 \\ E_3 \end{matrix} \begin{bmatrix} 1.331 & 1.21 & 1.1 & : & 1.000 \\ 0 & 0.1473 & 0.2975 & : & 0.1390 \\ 0 & 0.0909 & 0.1736 & : & 0.0825 \end{bmatrix}$$

$$E_3 \to E_3 - \frac{0.0909}{0.1473} E_2$$

2

$$\left[\begin{array}{ccc:c} 1.331 & 1.21 & 1.1 & 1.000 \\ 0 & 0.1471 & 0.2975 & 0.1390 \\ 0 & 0 & -0.01 & -0.003280 \end{array}\right]$$

use back sub.

$x_3 = 0.3280$

$x_2 = 0.2812$

$x_1 = 0.2246$ Ans

System of linear equations:

Exercise:

1. Use Gaussian elimination with partial pivoting and three-digit chopping arithmetics to solve the following linear system, and compare the approximations with the actual solution $[0, 10, 1/7]^T$.

$$
\begin{aligned}
3.03x_1 - 12.1x_2 + 14x_3 &= -119 \\
-3.03x_1 + 12.1x_2 - 7x_3 &= 120 \\
6.11x_1 - 14.2x_2 + 21x_3 &= -139.
\end{aligned}
$$