# School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

End-Term Examination, June 2022

B.E. IV Semester
Time Limit: 02 Hours
Instructor(s) : Dr. Meenu Rani, Dr. Munish Kansal, Dr. Paramjeet Singh

UMA007 / UMA011 : Numerical Analysis
Maximum Marks: 35

**Instructions:** You are expected to answer ALL FIVE questions. Arrange your work in a reasonably neat, organized, and coherent way. Mysterious or unsupported answers will not receive full credit. Scientific Calculator is permitted.

**1.** Apply power method to approximate the dominant eigenvalue and the corresponding eigenvector of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

using $x^{(0)} = [1, 1, 1]^t$ as an initial approximation. Use stopping criteria $||x^{(k)} - x^{(k-1)}||_\infty < 0.01$, where $x^{(k)}$ denotes the eigenvector at $k$-th iteration. [7 marks]

**2.** Use the method of least squares to fit the curve of the form $y = a + \frac{b}{x}$ to the following data:

| $x$ | -3 | -2 | -1 | 1 | 2 |
|---|---|---|---|---|---|
| $f(x)$ | -10 | 4 | 6 | -2 | 0 |

[7 marks]

**3.** Let $f \in C^n[a, b]$ and $x_0, x_1, x_2, \cdots, x_n$ are distinct numbers in $[a, b]$. Let $P_n(x)$ be the interpolating polynomial in Newton's form. Then prove that there exists a point $\xi \in (a, b)$ such that

$$f[x_0, x_1, x_2, \cdots, x_n] = \frac{f^{(n)}(\xi)}{n!}.$$

[7 marks]

**4.** Determine the values of subintervals $n$ and step-size $h$ required to approximate

$$\int_1^2 x \ln x \, dx$$

to within 0.01 by using composite trapezoidal rule and hence compute the approximate value of the integral. [7 marks]

**5.** Apply the fourth order Runge-Kutta method to the following initial-value problem to find the approximate solution at $t = 0.2$ with step-size $h = 0.2$.

$$\frac{dy}{dt} = y - t^2 + 1, \quad y(0) = 1.$$

[7 marks]