Department of Mathematics, Thapar Institute of Engineering & Technology, Patiala
End-Term Examination, May 2024

B.E. IV Semester	UMA011 : Numerical Analysis
Time Limit: 03 Hours	Maximum Marks: 40
Instructor(s) : Dr. Paramjeet Singh, Dr. Vivek Sangwan	

Instructions: This question paper has two printed pages. You are expected to answer any FIVE questions. Arrange your work in a reasonably neat, organized, and coherent way. Mysterious or unsupported answers will not receive full credit. Use of Calculator is permitted.

1. (a) Use modified Newton's method starting with $x_0 = 1$ to find the root of the equation $x^3 - 3x^2 + 4 = 0$ with multiplicity $m = 2$. Use stopping criteria $|x_k - x_{k-1}| < 0.1$, where x_k denotes the approximation of the root at k-th iteration. **[Marks: 4, (CLO1, L3)]**

(b) Solve the following system of equations using Gauss elimination:

$$\begin{aligned} 4x_1 + 2x_2 - x_3 &= -3 \\ x_1 - 4x_2 - 2x_3 &= 3 \\ 2x_1 + x_2 + 4x_3 &= 12. \end{aligned}$$

[Marks: 4, (CLO2, L3)]

2. Let us consider matrix $A = \begin{bmatrix} 4 & -3 & 5 \\ 1 & -4 & 2 \\ 3 & 5 & 2 \end{bmatrix}$.

(a) Apply Gram-Schmidt process to decompose A as $A = QR$. **[Marks: 6, (CLO2, L3)]**

(b) Perform one iterations of QR algorithm to find all the eigenvalues of A. **[Marks: 2, (CLO2, L3)]**

3. (a) Suppose that $x_0, x_1, \cdots, x_n$ are distinct numbers in $[a, b]$ and $f \in C^{n+1}[a, b]$. Let $P_n(x)$ be the interpolating polynomial of degree $\leq n$ then prove that for all $x \in [a, b]$, there exists a point $c \in (a, b)$ such that

$$f(x) - P_n(x) = \frac{(x - x_0)(x - x_1) \cdots (x - x_n)}{(n+1)!} f^{(n+1)}(c).$$

[Marks: 4, (CLO3, L4)]

(b) Find the Lagrange interpolating polynomial of degree 2 or less for given data points:

x	1	3	4
$f(x)$	1	27	64

Hence evaluate $f(2)$. **[Marks: 4, (CLO3, L2)]**

4. (a) The following data are given for a polynomial $P(x)$ of unknown degree:

x	0	1	2	3
$f(x)$	4	9	15	18

Determine the coefficient of x^3 in $P(x)$ if all fourth-order forward differences are 1. **[Marks: 4, (CLO3, L2)]**

(b) Use the method of least squares to fit a curve of the form $y = a + \dfrac{b}{x}$ to the following data:

x	1	2	5
$f(x)$	10	6	2

[Marks: 4, (CLO3, L3)]

5. (a) Determine the value of the integral

$$\int_0^2 \frac{1}{x+3}\, dx$$

using composite Simpson's rule with step-size $h = \frac{1}{3}$. **[Marks: 4, (CLO4, L2)]**

(b) Evaluate the integral

$$\int_{-1}^{1} e^x \cos x \; dx$$

using the Gauss-Legendre one and two point formulae. **[Marks: 4, (CLO4, L1)]**

6. (a) Find the constants c_0, c_1, and x_1 so that the quadrature formula

$$\int_0^1 f(x)dx = c_0 f(0) + c_1 f(x_1)$$

has the highest possible degree of precision. **[Marks: 4, (CLO4, L2)]**

(b) Consider the initial-value problem

$$\frac{dy}{dt} = y - t^2, \quad y(0) = 1.$$

Compute $y(0.2)$ using fourth order Runge-Kutta method with step-size $h = 0.2$.
[Marks: 4, (CLO4, L3)]

Marks Distribution