Roll Number: ________________

Thapar Institute of Engineering and Technology, Patiala

School of Mathematics

Mid Semester Test

B.E.	Course Code: UMA011	Course Name: Numerical Analysis
Time: 2 Hours	M. Marks: 35	Name of Coordinators: Amit Kumar, Tina Verma

Note: **(1) Attempt all the questions.**
(2) Calculator without graphing mode and alphanumeric memory is permitted.

1. (a) Consider the function $f(x) = \dfrac{e^x - 1}{x}$. Show that the algorithm will be unstable to compute the given function $f(x)$ for very small values of x. [4marks]

   (b) Find the largest eigen value in magnitude correct up to two decimal places and its corresponding eigen vector of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ using power method. Take $(1, 0, 0)$ as initial eigen vector. [5marks]

   (c) Find the multiplicity of the root $x = 1$ for the equation $x^3 - x^2 - x + 1 = 0$. Use modified Newton's method to find approximate root of the given equation by taking initial approximation $x_0 = 0.9$. [4marks]

2. (a) Find a real root of the equation $x^4 - x - 10 = 0$ using bisection method correct up to two decimal places in the interval $[1.75, 2]$. [5marks]

   (b) Let g be a continuous function on $[a, b]$ such that $g(x) \in [a, b]$ for all $x \in [a, b]$. Suppose in addition, g' exists on (a, b) and $g'(x) \neq 0$ for all $x \in [a, b]$. If α is the fixed point of g then, show that the order of convergence of fixed point method defined by $x_{n+1} = g(x_n), n \geq 0$ is linear. [4marks]

3. (a) Use Gauss elimination method to solve the system of equations
   $x_1 + 2x_2 + 3x_3 - x_4 = 10;$
   $2x_1 + 3x_2 - 3x_3 - x_4 = 1;$
   $2x_1 - x_2 + 2x_3 + 3x_4 = 7$
   $3x_1 + 2x_2 - 4x_3 + 3x_4 = 2$ [5marks]

   (b) Consider the following system of equations. Check whether Gauss Seidal method will converge to the solution or not.
   $x_1 - x_3 = 0.2;$
   $-\frac{1}{2}x_1 + x_2 - \frac{1}{4}x_3 = -1.425;$
   $x_1 - \frac{1}{2} + x_3 = 2.$ [5marks]

   (c) Suppose that x^* is an approximation to the solution of the system $Ax = b$, A is non-singular matrix and r is the residual vector for x^*. Then show that for any natural norm $\|x - x^*\| \leq \|r\|.\|A^{-1}\|$. Further, if $x \neq 0$ and $b \neq 0$ then show that $\dfrac{\|x - x^*\|}{\|x\|} \leq \dfrac{\|r\|.\|A\|.\|A^{-1}\|}{\|b\|}$. [3marks]

End of Question Paper