School of Mathematics, Thapar Institute of Engineering and Technology, Patiala

Auxiliary Examination, March 2022

Name:________________________ **Roll No.** ________________________

B.E. III Semester — **UMA007/UMA011: Numerical Analysis**

Time Limit: 02 Hours — **Maximum Marks: 50**

Instructor: Dr. Arvind Kumar Lal

Instructions: You are expected to answer **ANY FIVE** questions. Each problem has equal weight (10 marks) Arrange your work in a reasonably neat, organized and coherent way. Mysterious or unsupported answers will not receive full credit. Scientific calculator is allowed.

1. Consider the function $f(x) = \sqrt{x+1} - \sqrt{x}$ and $x = 12345$. Show that this function is well – conditioned but not stable (use six decimal digit rounding arithmetic). **(10.0)**

2. Find a positive root, between 0 and 1, of the equation $x\, e^{x} = 1$ correct to two decimal places using bisection method. **(10.0)**

3. Using Gauss elimination method, determine the LU factorization for matrix A in the linear system $Ax = b$, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix}$. **(10.0)**

4. Let $f \in C^n[a, b]$ and $x_0, x_1, \cdots, x_n$ are distinct numbers in $[a, b]$. Let $P_n(x)$ be the interpolating polynomial in Newton's form. Then prove that there exists a point $\xi \in (a, b)$ such that $f[x_0, x_1, \cdots, x_n] = \frac{f^{(n)}(\xi)}{n!}$. **(10.0)**

5. Use the method of least squares to fit a curve of the form $y = a\, b^{x}$ to the following data. **(10.0)**

x	0		1	2	3
y	10		21	35	59

6 (a) Determine the constants x_0, x_1 and c_1 that will produce a quadrature formula

$$\int_0^1 f(x)\, dx = \frac{1}{2} f(x_0) + c_1 f(x_1)$$

that has the highest possible degree of precision. **(5.0)**

(b) Evaluate the integral $\int_0^1 \frac{dx}{1+x^2}$ using Gauss quadrature two point formula and compare your answer with exact value of given integral. **(5.0)**

7. Apply the fourth –order Runge – Kutta method for the system to find the approximate solution at $x = 0.1$ with step – size $h = 0.1$.

$$\frac{dy}{dx} = y - z + 2, \quad y(0) = -1; \quad \frac{dz}{dx} = -y + z + 4x, z(0) = 0$$ **(10.0)**

END

Name: ________________ Roll No.:____________ Tutorial Group:_______

SCHOOL OF MATHEMATICS, THAPAR INSTITUTE OF ENGINEERING AND TECHNOLOGY, PATIALA

U – Grade Exam. Numerical Analysis (UMA007/011) March 12, 2022

Max. Mark: 35 **Time: 35 Minutes**

Note: (i) Tick the appropriate answer. (ii) Each question carries 1.4 marks.

Q. 1	If $f(x)$ is $(k+1)$ times differentiable on the interval $[a.\,b]$ and $x_0, x_1, \cdots, x_k, x_{k+1}$ are $(k+2)$ distinct points in $[a.\,b]$, then for some $\xi \in (a,\,b)$, $f[x_0, x_1, \cdots, x_{k+1}]$ equals (A) $\dfrac{f^{(k)}(\xi)}{k!}$ (B) $\dfrac{f^{(k+1)}(\xi)}{k!}$ (C) $\dfrac{f^{(k+1)}(\xi)}{k+1!}$ (D) None of these
Q. 2	For certain function $f(x)$, divided differences are given as $f[-1]=2,\ f[-1,1]=1,$. The Lagrange's interpolating polynomial based on the nodal points – 1 and 1 is (A) $x+3$ (B) $2x-1$ (C) x^2+x+1 (D) x^2-3
Q. 3	For which type of polynomials does the Simpson's integration rule give an exact result? (A) Polynomial with degree greater than three. (B) All trigonometric functions. (C) All transcendental functions. (D) All polynomials with degree three or less.
Q. 4	Compute the ∞-norm of a matrix, $\|\|A\|\|_\infty$. Consider $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix}$. (A) $\sqrt{7+\sqrt{7}}$ (B) $\sqrt{7-\sqrt{7}}$ (C) 4 (D) None of these
Q. 5	While solving systems of linear equations using an iteration scheme of the form $\boldsymbol{x}^{(k)} = T\boldsymbol{x}^{(k-1)} + \boldsymbol{c}, \forall k \geq 1$, the iterates converge to a unique solution if and only if the following condition holds: (A) $\rho(T) < 1$ (B) the induced norm $\|\|T\|\| \to \infty$ (C) T is invertible (D) T has at least one zero eigenvalue

Q.6	The dominant eigenvalue of the matrix $A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \end{pmatrix}$ is (A) 5 (B) 0 (C) 3 (D) does not exist.
Q. 7	The Trapezoidal rule and Simpson's one - third rule give the value 4 and 2, respectively on applying to $\int_0^2 f(x)\,dx$. What is $f(1)$? (A) $\frac{1}{4}$ (B) $\frac{1}{5}$ (C) 1 (D) $\frac{1}{2}$
Q. 8	Let $p(x)$ be an interpolating polynomial of degree atmost 3 that passes through the points $(-2,12), (-1,1), (0,2)$ and $(2,-8)$. Then, the coefficient of x^3 in the $p(x)$ is equal to (A) -2 (B) 1 (C) -1 (D) None of these
Q. 9	The value of $y(0.1)$ obtained by solving initial value problem: $\frac{dy}{dx} = x + y$, $y(0) = 1$ by modified Euler's method is equal to (A) 0.20 (B) 0.21 (C) 2.38 (D) 1.64
Q. 10	Let $(x_0, f(x_0)) = (0,-1)$, $(x_1, f(x_1)) = (1, \alpha)$ and $(x_2, f(x_2)) = (2, \beta)$. If the first-order divided differences are $f[x_0, x_1] = 5$ and $f[x_1, x_2] = \gamma$ and the second-order divided difference is $f[x_0, x_1, x_2] = -\frac{3}{2}$, then the values of α, β and γ are, respectively (A) 4, 2, 4 (B) 2, 4, 6 (C) 4, 6, 2 (D) 6, 2, 4
Q. 11	Let $x_0, x_1, \cdots, x_n$ be $n+1$ distinct points. If the Lagrange polynomial for $(n+1)$ points takes the form $l_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{x - x_i}{x_k - x_i}$, $k = 0, 1, \cdots, n$, then $\sum_{k=0}^{n} l_k(x)$ equals to (A) 0 (B) 1 (C) 2 (D) None of these
Q.12	Using Euler method, the value of y at $t = 0.1$ for initial value problem: $\frac{dy}{dt} = y$, $y(0) = 1$, is (A) 1.1 (B) 1.2 (C) 0.1 (D) 1

Q. 13 Which one of the integration method is used to correct the value in modified Euler's method?

(A) Trapezoidal rule. (B) Simpson 1/3 rule.
(C) Simpson 3/8 rule (D) Gauss quadrature two-point rule.

Q. 14 For solving the integral $\int_1^2 \frac{dx}{\sqrt{4-x^2}}$, which of the following method is the most appropriate

(A) Trapezoidal Rule (B) Simpson (1/3rd) Rule
(C) Gauss – two point formula (D) None of these

Q. 15 Runge - Kutta fourth order method to solve initial value problem:
$\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$ with step size h is given by

$$K_1 = h\, f(x_i,\ y_i); \qquad K_2 = hf\left(x_i + \frac{h}{2},\ y_i + \frac{K_1}{2}\right);$$

$$K_3 = hf\left(x_i + \frac{h}{2},\ y_i + \frac{K_2}{2}\right) \text{ and } K_4 = hf(x_i + h,\ y_i + K_3)$$

$$y_{i+1} = y_i + \frac{1}{6}(K_1 + aK_2 + b\, K_3 + c\, K_4); \quad x_{i+1} = x_i + h,\ i = 0, 1, 2, \cdots.$$

The value of $a,\ b,\ and\ c$ in the above formula are respectively

(A) $2, 1,\ and\ 2$ (B) $1, 2,\ and\ 2$ (C) $2, 2,\ and\ 1$ (D) $2, 2,\ and\ 2$

Q. 16 The two - point Gauss quadrature formula for $\int_{-1}^{1} f(x)dx$ is equal to

(A) $f(-0.774)$+ $f(0.5774)$
(B) $f(-0.5774)$+$f(0)$+ $f(0.5774)$
(C) $0.8889f(-0.7746)$+) $0.5556f(0)$+ $0.8889f(0.7746)$
(D) $0.5556f(-0.7746)$+) $0.8889f(0)$+ $0.5556f(0.7746)$

Q. 17 Value of all $\alpha > 0 \quad and \quad \beta > 0$ so that matrix $A = \begin{bmatrix} 3 & 2 & \beta \\ \alpha & 5 & \beta \\ 2 & 1 & \alpha \end{bmatrix}$ is strictly diagonally dominant, are

(A) $0 < \beta < 1$ and $3 < \alpha < 5 - \beta$
(B) $1 < \beta < 2$ and $3 < \alpha < 5 + \beta$
(C) $0 < \beta < 1$ and $4 < \alpha < 6 - \beta$
(D) $1 < \beta < 2$ and $3 < \alpha < 5 - \beta$

Q. 18 Applying Trapezoidal rule to $\int_0^2 e^x\, dx$ gives

(A) 8.999 (B) 8.909 (C) 8.363 (D) 8.389

Q. 19	Suppose $f(0)=1,\ f(0.5)=2.5,\ f(1)=2,$ and $f(0.25)=f(0.75)=\alpha$. The value of α, for which the composite Trapezoidal rule with $N=4$ gives $\int_0^1 f(x)\,dx=1.75$, is (A) 4 (B) 5 (C) 6 (D) None of these
Q. 20	Suppose that is $\bar{X}$ is an approximation to the solution of $AX=b$, A is nonsingular matrix and r is the residual vector for $\bar{X}$. Then for any natural norm, which one of the following is correct? (i) $\\|X-\bar{X}\\| \le \\|A\\|\ \\|r\\|$ (ii) $\frac{\\|X-\bar{X}\\|}{\\|X\\|} \le \frac{\\|A^{-1}\\|}{\\|b\\|}\\|r\\|$ (A) Only (i) (B) only (ii) (C) both (i) and (ii) (D) None of these
Q.21	Consider a sequence of vectors $X^{(k)}=\left(1,\ 2+\frac{1}{k},\ \frac{3}{k^2},\ e^{-k}sink\right)$. For l_∞ norm, this sequence converges to the vector (A) $X=(1,\ 2,\ 1,0)$ (B) $X=(1,\ 0,2,\ 0)$ (C) $X=(1,\ 3,\ 3,\ 0)$ (D) $X=(1,\ 2,0,\ 0)$
Q.22	If a matrix A has eigenvalue 5, 3, 1 then $(A-2\,I)^{-1}$ has eigenvalues (A) $\frac{1}{3},1,-1$ (B) $-\frac{1}{3},1,-1$ (C) $\frac{1}{3},\ 1,\ 1$ (D) $\frac{1}{3},\ 1,\ 0$
Q.23	Which of the method of order two cannot be used to solve the IVP : $y'=y^{1/3}, y(0)=0$ on $[0,1]$ with $h=0.2$? (A) Taylor Series Method of order two (B) Euler method (C) Runge - Kutta second order method (D) Runge - Kutta fourth order method
Q.24	The normal equation obtained for fitting the curve $y=a\sqrt{x}+\frac{b}{x}$ to some data $(x_i,\ y_i), i=1,2,\cdots N$, is $a\sum_{i=1}^{N}x_i+b\sum_{i=1}^{N}\frac{1}{\sqrt{x_i}}=\sum_{i=1}^{N}y_i\sqrt{x_i}$; $a\sum_{i=1}^{N1}\frac{1}{\sqrt{x_i}}+b\,P=Q$, where $P\ and\ Q$ are given as (A) $P=\sum_{i=1}^{N}\frac{1}{x_i^2}, Q=\sum_{i=1}^{N}\frac{y_i}{x_i^2}$ (B) $P=\sum_{i=1}^{N}\frac{y_i}{x_i^2}, Q=\sum_{i=1}^{N}\frac{y_i}{x_i}$ (C) $P=\sum_{i=1}^{N}\frac{1}{x_i^2}, Q=\sum_{i=1}^{N}\frac{y_i}{x_i}$ (D) $P=\sum_{i=1}^{N}\frac{1}{x_i^2}, Q=\sum_{i=1}^{N}\frac{\sqrt{y_i}}{x_i^2}$
Q.25	The condition number $\kappa(A)$ of a matrix A of order n is (A) Exactly equal to one (B) At least one (C) At most one (D) None of these.