Assignment 2

Divide and Conquer Approach

1. Given an array of n objects, you need to decide if there is an object which is present more than n/2 times. The only operation by which you can access the objects is a function f, which given two indices i and j, outputs whether the objects at positions i and j in the array are identical or not. Give an O(n log n)-time algorithm and code for this (where each call to f is counted as 1 operation).

2. You are given an infinite array A[] in which the first n cells contain integers in sorted order and the rest of the cells are filled with ∞. You are not given the value of n. Describe an algorithm that takes an integer x as input and finds a position in the array containing x, if such a position exists, in O(log n) time.

3. We are interested in analyzing some hard to obtain data from two databases. Each database contains n numerical values (so there are 2n values in total). Assume that these values are distinct. We would like to determine the median of these 2n values, which we define as the n th smallest value. However, the only way to access these values is through queries to the databases. In a single query, we specify a value k to one of the two databases, and the chosen database returns the k th smallest value that it contains. Give an algorithm which finds the median value using O(logn) queries only.

4. Suppose we are given an array of n integers representing stock prices on a single day. We want to find a pair (buyDay, sellDay), with buyDay $\leq$ sellDay, such that if we bought the stock on buyDay and sold it on sellDay, we would maximize our profit. Give an O(n log n)-time algorithm and code for this.

5. Given an array of 2n elements in the following format { a1, a2, a3, a4, ….., an, b1, b2, b3, b4, …., bn }. The task is shuffle the array to {a1, b1, a2, b2, a3, b3, ……, an, bn } without using extra space. Give an O(n log n)-time algorithm and code for this.