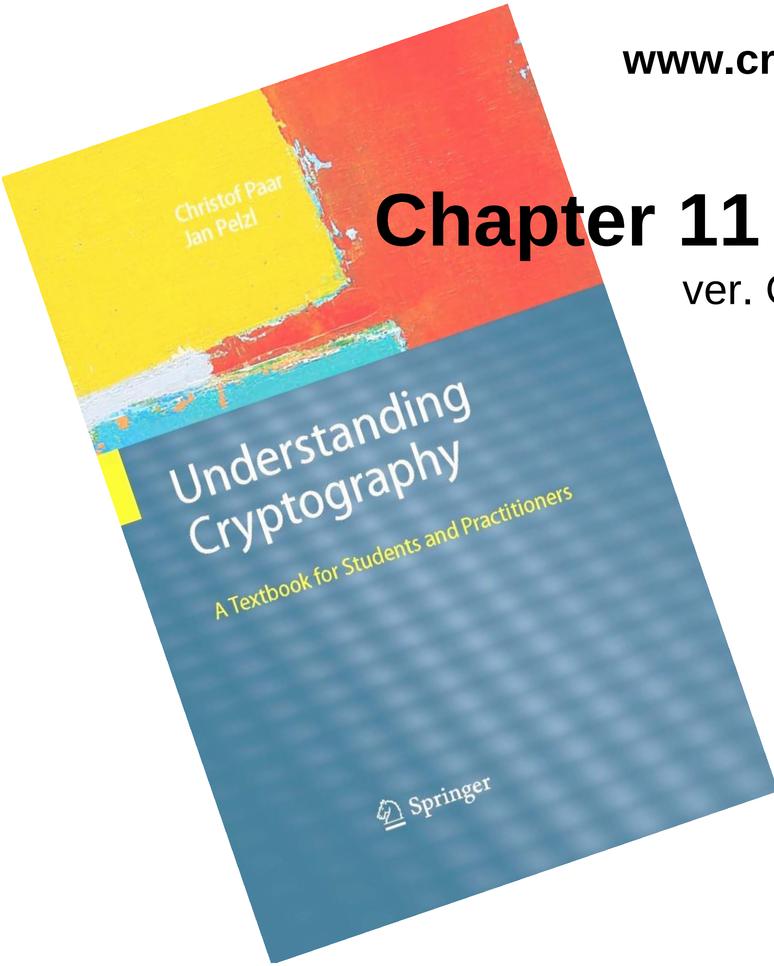
Understanding Cryptography – A Textbook for Students and Practitioners

by Christof Paar and Jan Pelzl

www.crypto-textbook.com



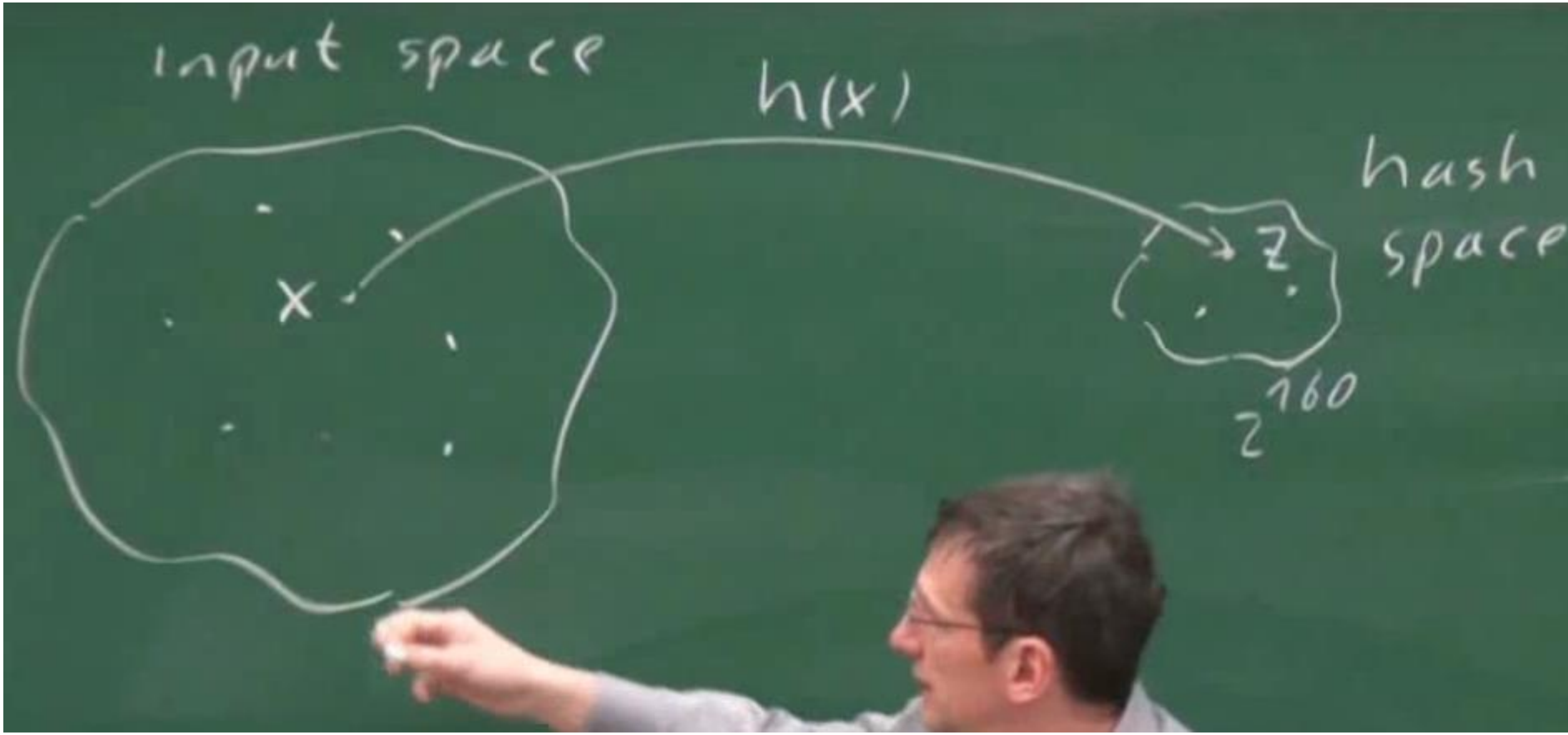
Chapter 11 – Hash Functions

ver. October 29, 2009

These slides were prepared by Stefan Heyse and Christof Paar and Jan Pelzl

Some legal stuff (sorry): Terms of Use

- The slides can used free of charge. All copyrights for the slides remain with Christof Paar and Jan Pelzl.
- The title of the accompanying book “Understanding Cryptography” by Springer and the author’s names must remain on each slide.
- If the slides are modified, appropriate credits to the book authors and the book title must remain within the slides.
- It is not permitted to reproduce parts or all of the slides in printed form whatsoever without written consent by the authors.


input space
h(x)
hash space
x
z
2^{160}

Content of this Chapter

- Why we need hash functions
- How does it work
- Security properties
- Algorithms
- Example: The Secure Hash Algorithm SHA-1

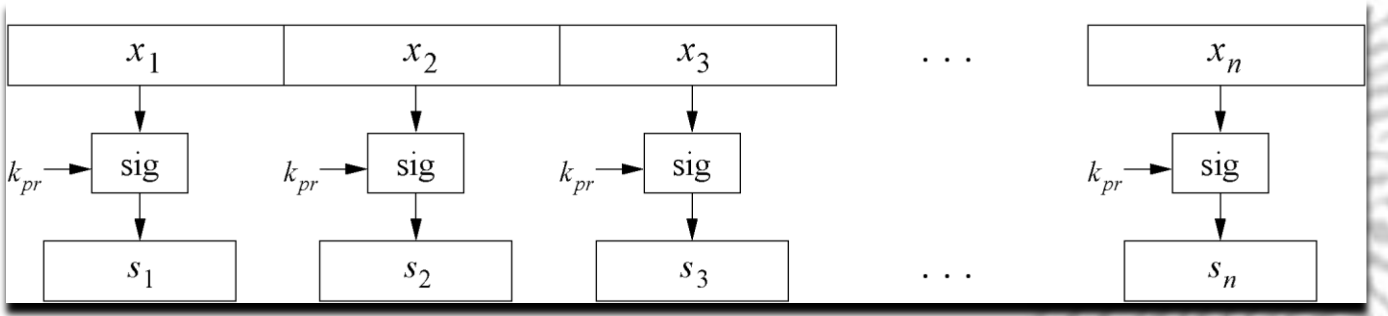
Content of this Chapter

- **Why we need hash functions**
- How does it work
- Security properties
- Algorithms
- Example: The Secure Hash Algorithm SHA-1

Motivation

Problem:

Naive signing of long messages generates a signature of same length.



- Three Problems
- Computational overhead
- Message overhead
- Security limitations
- For more info see Section 11.1 in “*Understanding Cryptography*”.

Solution:

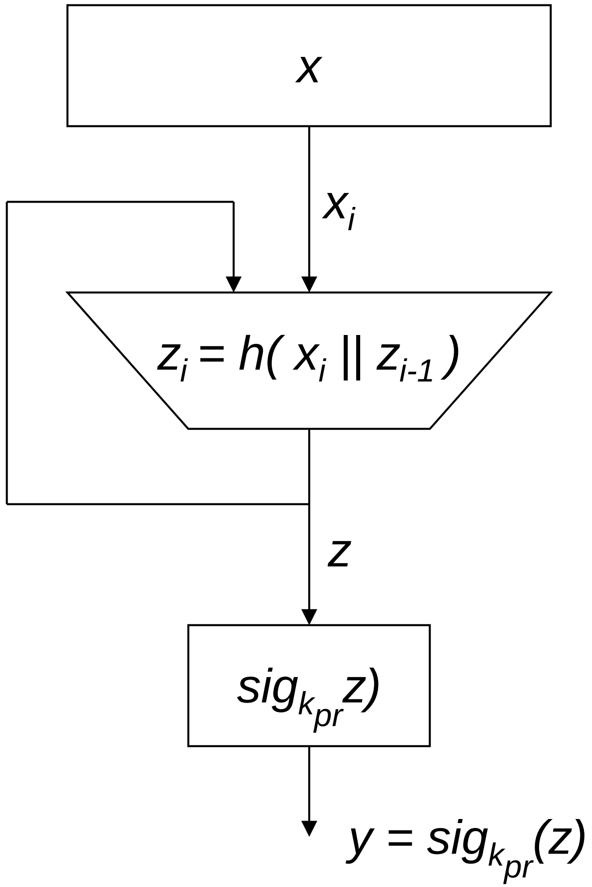
Instead of signing the whole message, sign only a digest (=hash)

Also secure, but much faster

Needed:

Hash Functions

Digital Signature with a Hash Function



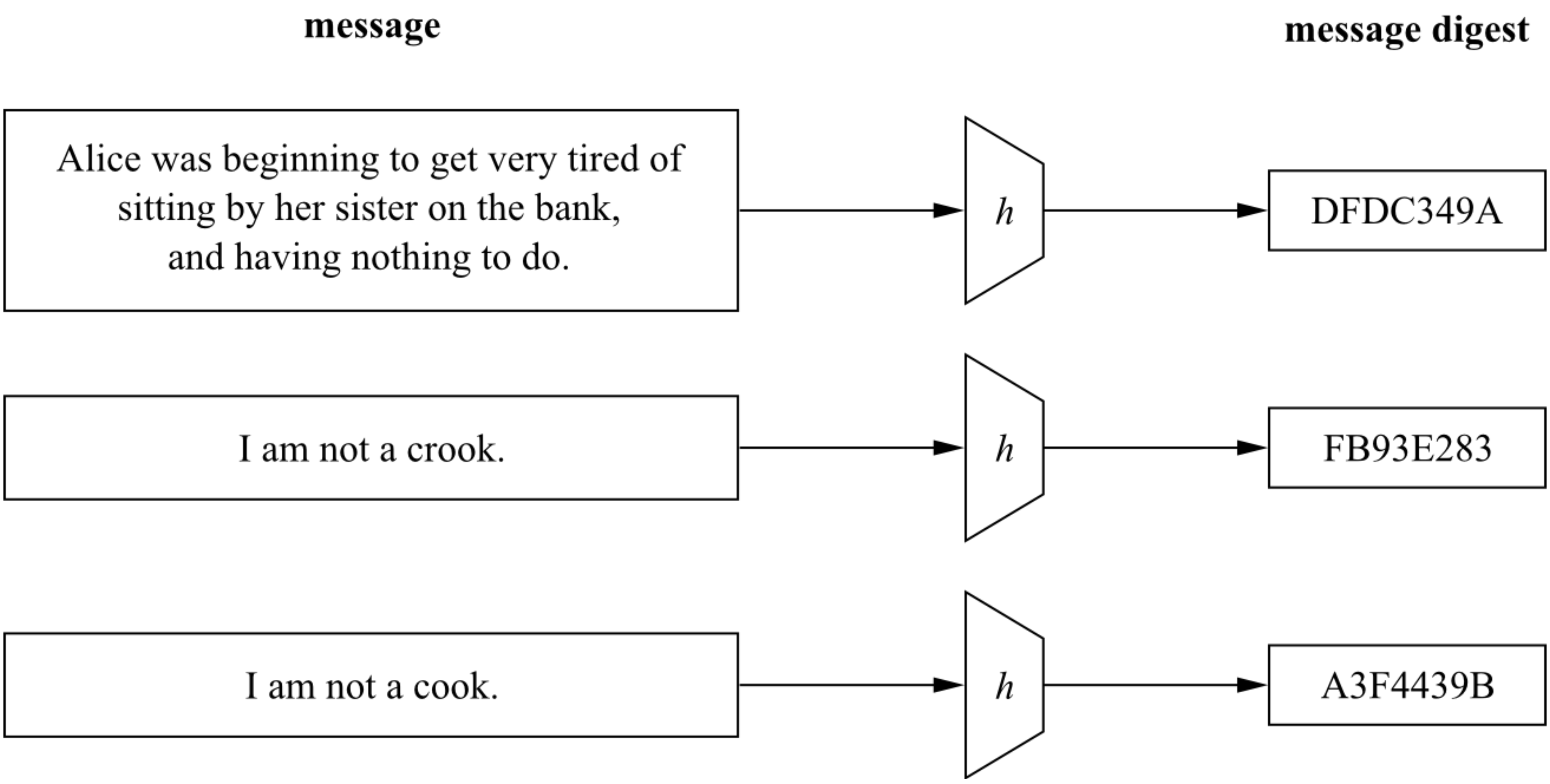
Notes:

- x has fixed length
- z, y have fixed length
- z, x do not have equal length in general
- $h(x)$ does not require a key.
- $h(x)$ is public.

- **Basic Protocol for Digital Signatures with a Hash Function:**

Alice		Bob
	$\xleftarrow{\quad K_{pub} \quad}$	
		$z = h(x)$
		$s = sig_{K_{pr}}(z)$
	$\xleftarrow{\quad (x, s) \quad}$	
$z' = h(x)$		
$ver_{K_{pub}}(s,z')=true/false$		

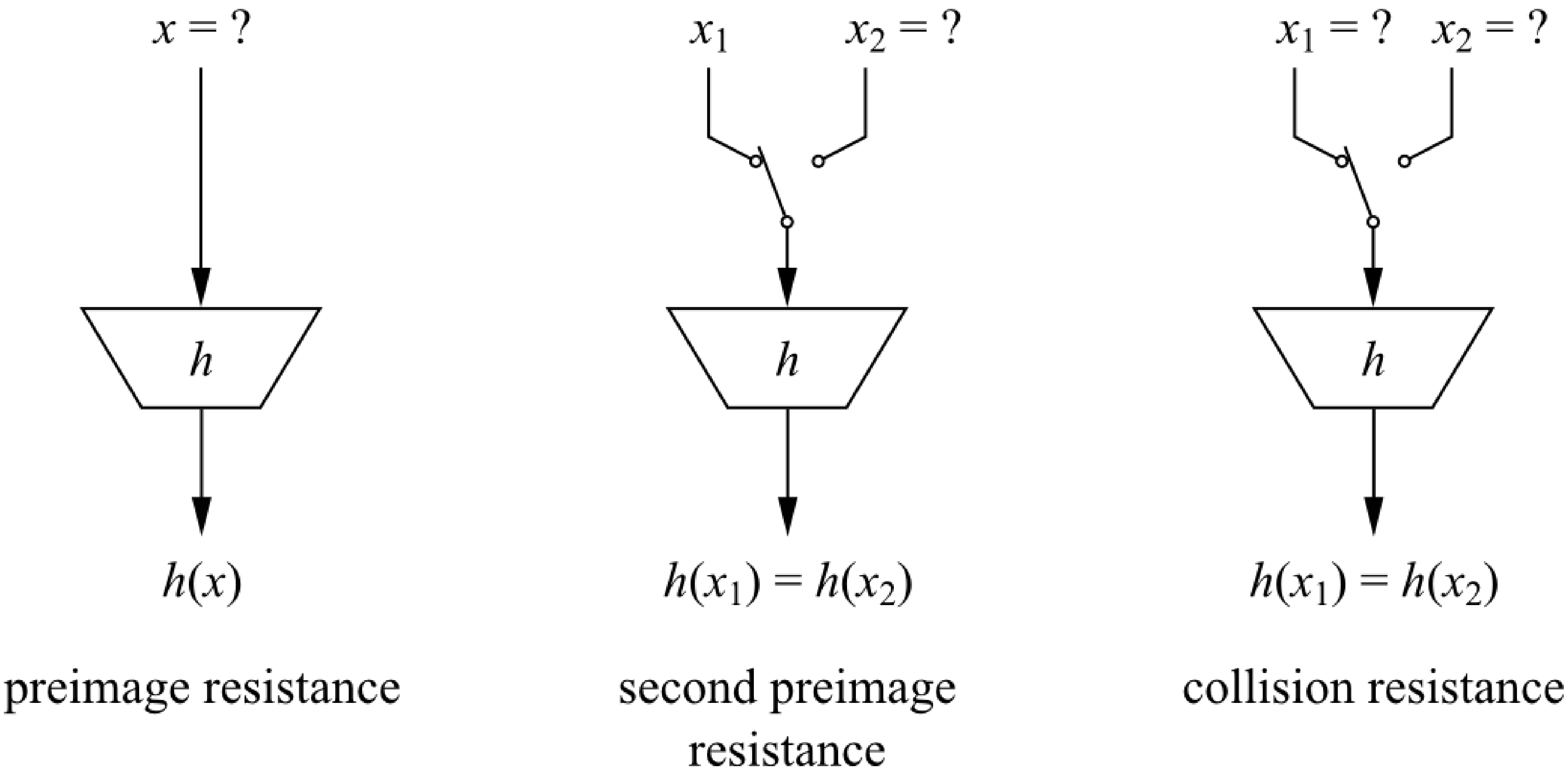
Principal input–output behavior of hash functions



Content of this Chapter

- Why we need hash functions
- How does it work
- **Security properties**
- Algorithms
- Example: The Secure Hash Algorithm SHA-1

The three security properties of hash functions



Hash Funktionen: Security Properties

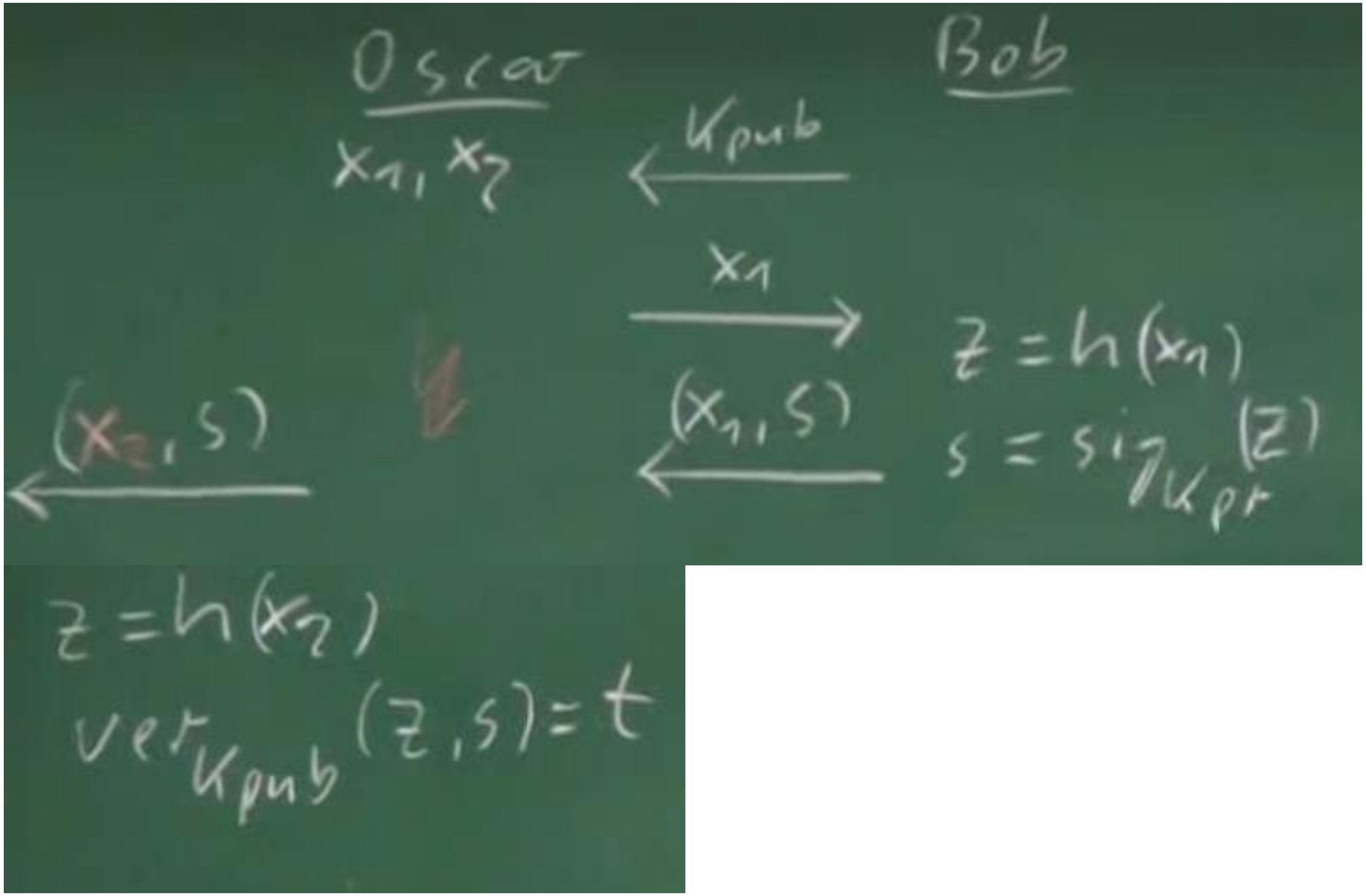
- **Preimage resistance:** For a given output z, it is impossible to find any input x such that h(x) = z, i.e., h(x) is one-way.

- **Second preimage resistance:** Given x_1, and thus $h(x_1)$, it is computationally infeasible to find any x_2 such that $h(x_1) = h(x_2)$. (uniqueness/weak collision resistance)

  what if “transfer $10” vs “transfer $100” have the same hash.

  “transfer $100” can have variations, e.g., “transfer $99”

- **Collision resistance:** It is computationally infeasible to find any pairs $x_1 \neq x_2$ such that $h(x_1) = h(x_2)$.


Oscar
Bob
x_1, x_2
k_{pub}
x_1
$z = h(x_1)$
(x_1, s)
$s = sig_{k_{pr}}(z)$
(x_2, s)
$z = h(x_2)$
$ver_{k_{pub}}(z, s) = t$

Hash Funktionen: Security

It turns out that collision resistance causes most problems

(See additional PPT for details)

- How hard is it to find a collision with a probability of 0.5 ?
- Related Problem: How many people are needed such that two of them have the same birthday with a probability of 0.5 ?
- No! Not 365/2=183. 23 are enough ! This is called the birthday paradoxon (Search takes $\approx\sqrt{2^n}$ steps) .
- For more info see Chapter 11.2.3 in *Understanding Cryptography*.
- To deal with this paradox, hash functions need a output size of at least 160 bits.

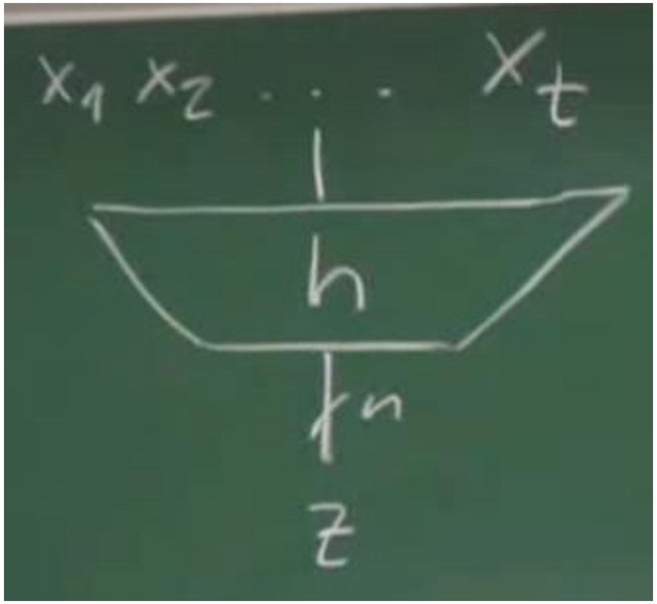
$$P(\text{no coll. among 2 people}) = 1 - \frac{1}{365}$$

$$P(\;\text{..}\quad\text{..}\quad\text{..}\quad 3\;\text{..}\;) = \left(1 - \frac{1}{365}\right)\left(1 - \frac{2}{365}\right)$$

$$\vdots$$

$$P(\quad\text{..}\quad t\;\text{..}\;) = \prod_{i=1}^{t-1}\left(1 - \frac{i}{365}\right)$$

for $t=23$ $\quad p = \prod_{i=1}^{22}\left(1 - \frac{i}{365}\right) = 0{,}507 \approx 50\%$



[see book]

$$\boxed{t = 2^{\frac{n+1}{2}} \sqrt{\ln\left(\frac{1}{1-\lambda}\right)}}$$

probability for at least 1 coll.

Ex. $n = 80$, $\lambda = 0{,}5$

$$t = 2^{81/2} \sqrt{\ln 2} \approx 2^{40{,}2}$$

Content of this Chapter

- Why we need hash functions
- How does it work
- Security properties
- **Algorithms**
- Example: The Secure Hash Algorithm SHA-1

Hash Funktionen: Algorithms

Hash Algorithms

Special Algorithms,
e.g. MD5 - family

based on
block ciphers

- **MD5 -** family
- **SHA-1**: output - 160 Bit; input - 512 bit chunks of message *x;*
  operations - bitwise AND, OR, XOR, complement und cyclic shifts.
- **RIPE-MD 160:** output - 160 Bit; input - 512 bit chunks of message *x*; operations – like in SHA-1, but two in parallel and combinations of them after each round.

Content of this Chapter

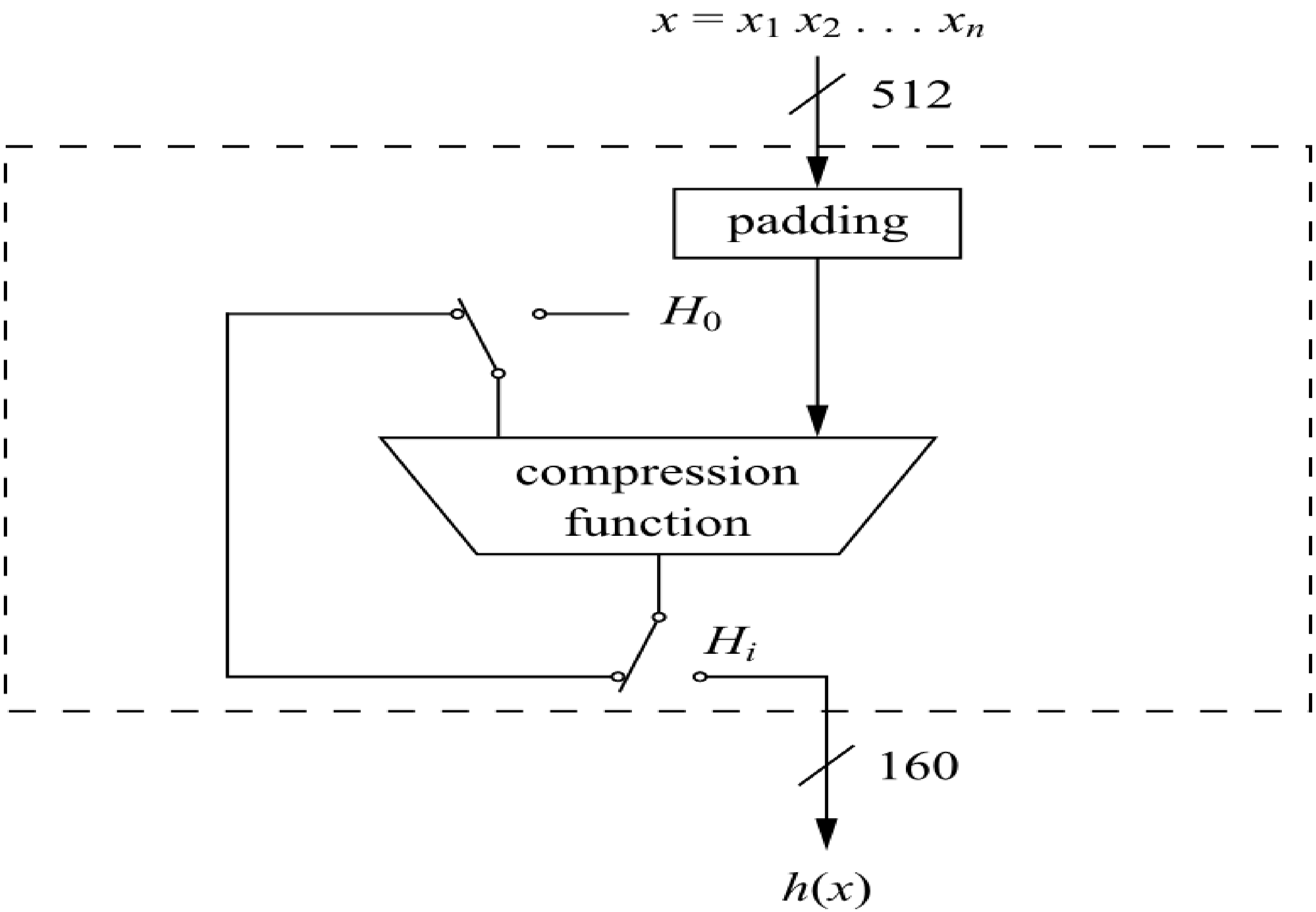
- Why we need hash functions
- How does it work
- Security properties
- Algorithms
- **Example: The Secure Hash Algorithm SHA-1**

SHA-1

- Part of the MD-4 family.
- Based on a Merkle-Dåmgard construction.
- 160-bit output from a message of maximum length 2^{64} bit.
- Widely used (even tough some weaknesses are known)

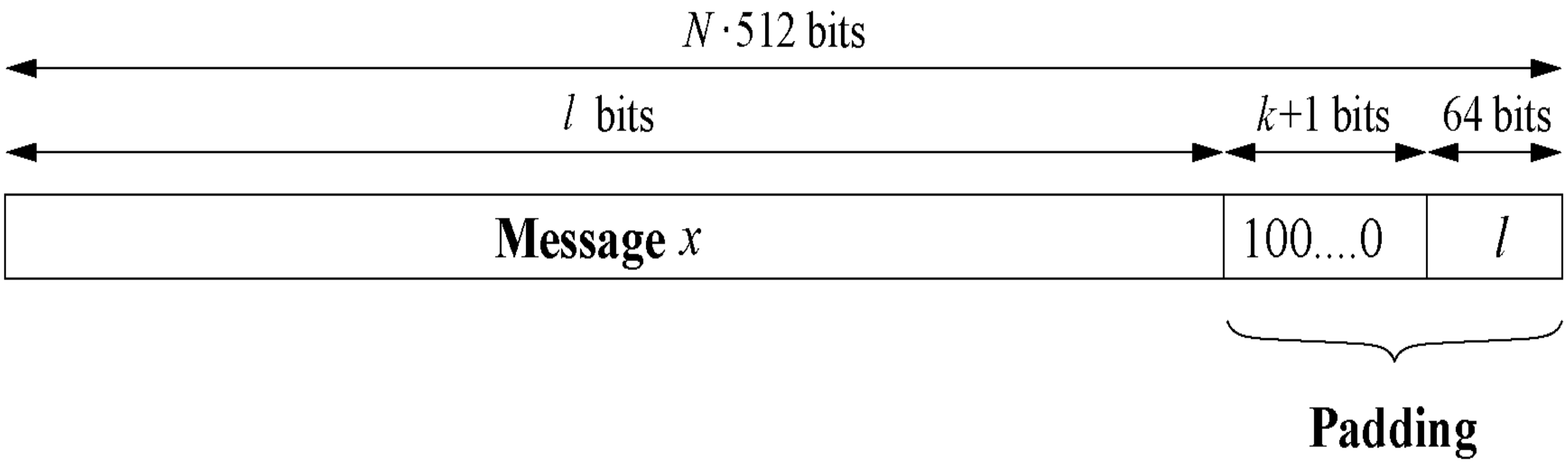
SHA-1 High Level Diagramm

- Compression Function consists of 80 rounds which are divided into four stages of 20 rounds each



SHA-1: Padding

- Message x has to be padded to fit a size of a multiple of 512 bit.
- $k \equiv 512 - 64 - 1 - l = 448 - (l + 1) \bmod 512.$



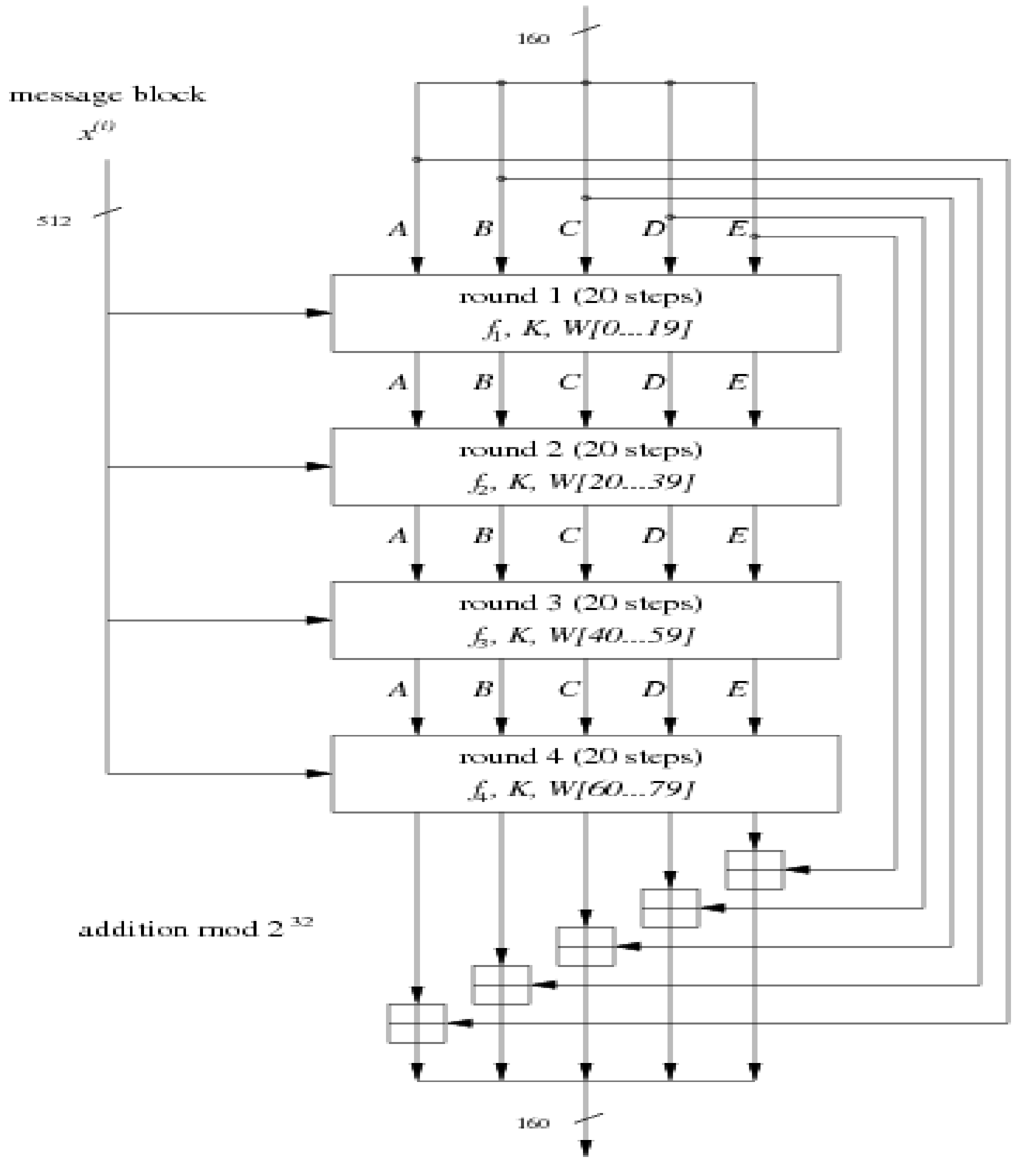
SHA-1: Hash Computation

- Each message block x_i is processed in four stages with 20 rounds each

SHA-1 uses:

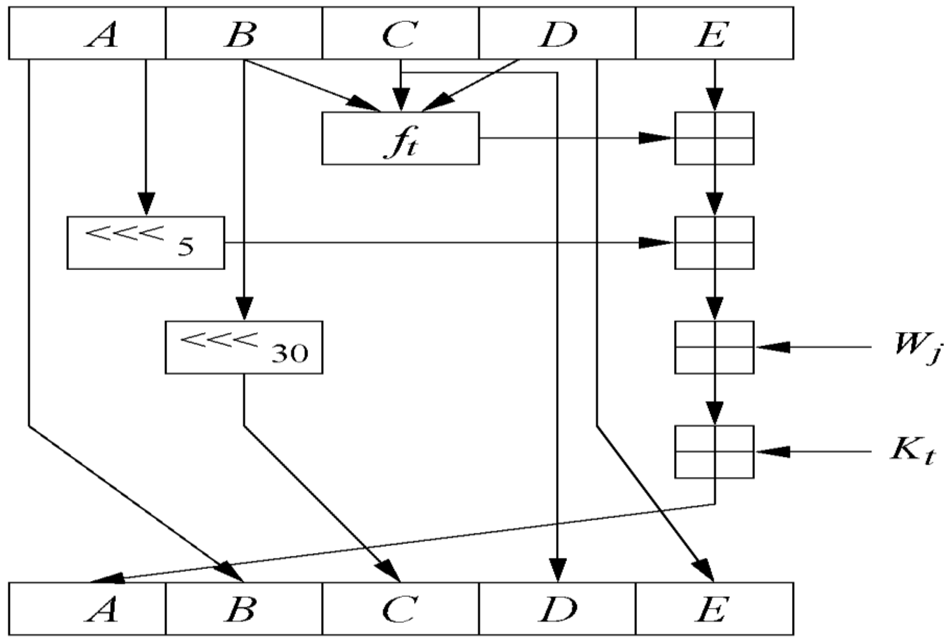
- A message schedule which computes a 32-bit word W0,W1,...,W79 for each of the 80 rounds
- Five working registers of size of 32 bits A,B,C,D,E
- A hash value H_i consisting of five 32-bit words $H_i^{(0)}$, $H_i^{(1)}$, $H_i^{(2)}$, $H_i^{(3)}$, $H_i^{(4)}$
- In the beginning, the hash value holds the initial value H_0, which is replaced by a new hash value after the processing of each single message block.
- The final hash value H_n is equal to the output h(x) of SHA-1.

SHA-1: All four stages



Chapter 11 of *Understanding Cryptography* by Christof Paar and Jan Pelzl

SHA-1: Internals of a Round



Stage t	Round j	Constant K_t	Function f_t
1	00…19	K=5A827999	$f(B,C,D)=(B\wedge C)\vee(\bar{B}\wedge D)$
2	20…39	K=6ED9EBA1	$f(B,C,D)=B\oplus C\oplus D$
3	40…59	K=8F1BBCDC	$f(B,C,D)=(B\oplus C)\vee(B\oplus D)\vee(C\oplus D)$
4	60…79	K=CA62C1D6	$f(B,C,D)=B\oplus C\oplus D$

Lessons Learned**: Hash-Funktionen**

- Hash functions are keyless. The two most important applications of hash functions are their use in digital signatures and in message authentication codes such as HMAC.
- The three security requirements for hash functions are one-wayness, second preimage resistance and collision resistance.
- Hash functions should have at least 160-bit output length in order to withstand collision attacks; 256 bit or more is desirable for long-term security.
- MD5, which was widely used, is insecure. Serious security weaknesses have been found in SHA-1, and the hash function should be phased out. The SHA-2 algorithms all appear to be secure.
- The ongoing SHA-3 competition will result in new standardized hash functions in a few years.

Further Informations: **Hash-Funktionen**

- Overview over many Hash Functions with Spezifications:
  - http://ehash.iaik.tugraz.at/wiki/The_Hash_Function_Zoo
- Birthday Paradox: Wikipedia has a nice explanation
  - http://en.wikipedia.org/wiki/Birthday_problem
- SHA Standards
  - SHA1+2: http://csrc.nist.gov/publications/fips/fips180-2/fips180-2withchangenotice.pdf
  - SHA3 Overview: http://ehash.iaik.tugraz.at/wiki/The_SHA-3_Zoo
- CrypTool is a learning program which also can hash:
  - http://www.cryptool.org/