



ACM/ICPC Template Manual

Shanghai University

CSL

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0 Include

```
1 #include <bits/stdc++.h>
2 using namespace std;
3 #define clr(a, x) memset(a, x, sizeof(a))
4 #define mp(x, y) make_pair(x, y)
5 #define pb(x) push_back(x)
6 #define X first
7 #define Y second
8 #define fastin \
9     ios_base::sync_with_stdio(0); \
10    cin.tie(0);
11 typedef long long ll;
12 typedef long double ld;
13 typedef pair<int, int> PII;
14 typedef vector<int> VI;
15 const int INF = 0x3f3f3f3f;
16 const int mod = 1e9 + 7;
17 const double eps = 1e-6;
18
19 int main()
20 {
21     #ifndef ONLINE_JUDGE
22         freopen("test.in", "r", stdin);
23         freopen("test.out", "w", stdout);
24     #endif
25
26     return 0;
27 }
```

1 Math

1.1 Prime

1.1.1 Eratosthenes Sieve

$O(n \log \log n)$ 筛出 $\max n$ 内所有素数
 $\text{notprime}[i] = 0/1$ 0 为素数 1 为非素数

```

1  const int maxn = "Edit";
2  bool notprime[maxn] = {1, 1};    // 0 && 1 为非素数
3  void GetPrime()
4  {
5      for (int i = 2; i < maxn; i++)
6          if (!notprime[i] && i <= maxn / i) // 筛到√n为止
7              for (int j = i * i; j < maxn; j += i)
8                  notprime[j] = 1;
9  }
```

1.1.2 Euler Sieve

$O(n)$ 得到欧拉函数 $\phi[i]$ 、素数表 $\text{prime}[]$ 、素数个数 tot
 传入的 n 为函数定义域上界

```

1  const int maxn = "Edit";
2  bool vis[maxn];
3  int tot, phi[maxn], prime[maxn];
4  void CalPhi(int n)
5  {
6      clr(vis, 0);
7      phi[1] = 1;
8      tot = 0;
9      for (int i = 2; i < n; i++)
10     {
11         if (!vis[i])
12             prime[tot++] = i, phi[i] = i - 1;
13         for (int j = 0; j < tot; j++)
14         {
15             if (i * prime[j] > n) break;
16             vis[i * prime[j]] = 1;
17             if (i % prime[j] == 0)
18             {
19                 phi[i * prime[j]] = phi[i] * prime[j];
20                 break;
21             }
22             else
23                 phi[i * prime[j]] = phi[i] * (prime[j] - 1);
24         }
25     }
26 }
```

1.1.3 Prime Factorization

函数返回素因数个数
 数组以 $\text{fact}[i][0]^{\text{fact}[i][1]}$ 的形式保存第 i 个素因数

```

1 ll fact[100][2];
2 int getFactors(ll x)
3 {
4     int cnt = 0;
5     for (int i = 0; prime[i] <= x / prime[i]; i++)
6     {
7         fact[cnt][1] = 0;
8         if (x % prime[i] == 0)
9         {
10             fact[cnt][0] = prime[i];
11             while (x % prime[i] == 0) fact[cnt][1]++, x /= prime[i];
12             cnt++;
13         }
14     }
15     if (x != 1) fact[cnt][0] = x, fact[cnt++][1] = 1;
16     return cnt;
17 }

```

1.1.4 Miller Rabin

$O(s \log n)$ 内判定 2^{63} 内的数是不是素数, s 为测定次数

```

1 bool Miller_Rabin(ll n, int s)
2 {
3     if (n == 2) return 1;
4     if (n < 2 || !(n & 1)) return 0;
5     int t = 0;
6     ll x, y, u = n - 1;
7     while ((u & 1) == 0) t++, u >>= 1;
8     for (int i = 0; i < s; i++)
9     {
10         ll a = rand() % (n - 1) + 1;
11         ll x = Pow(a, u, n);
12         for (int j = 0; j < t; j++)
13         {
14             ll y = Mul(x, x, n);
15             if (y == 1 && x != 1 && x != n - 1) return 0;
16             x = y;
17         }
18         if (x != 1) return 0;
19     }
20     return 1;
21 }

```

1.1.5 Segment Sieve

对区间 $[a, b)$ 内的整数执行筛法。

函数返回区间内素数个数

$\text{is_prime}[i-a]=\text{true}$ 表示 i 是素数

$a < b \leq 10^{12}, b - a \leq 10^6$

```

1 const int maxn = "Edit";
2 bool is_prime_small[maxn], is_prime[maxn];
3 int prime[maxn];
4 int segment_sieve(ll a, ll b)
5 {
6     int tot = 0;

```

```
7   for (ll i = 0; i * i < b; ++i)
8       is_prime_small[i] = true;
9   for (ll i = 0; i < b - a; ++i)
10      is_prime[i] = true;
11   for (ll i = 2; i * i < b; ++i)
12       if (is_prime_small[i])
13       {
14           for (ll j = 2 * i; j * j < b; j += i)
15               is_prime_small[j] = false;
16           for (ll j = max(2LL, (a + i - 1) / i) * i; j < b; j += i)
17               is_prime[j - a] = false;
18       }
19   for (ll i = 0; i < b - a; ++i)
20       if (is_prime[i]) prime[tot++] = i + a;
21   return tot;
22 }
```

1.2 Euler phi

1.2.1 Euler

```
1 ll Euler(ll n)
2 {
3     ll rt = n;
4     for (int i = 2; i * i <= n; i++)
5         if (n % i == 0)
6         {
7             rt -= rt / i;
8             while (n % i == 0) n /= i;
9         }
10    if (n > 1) rt -= rt / n;
11    return rt;
12 }
```

1.2.2 Sieve

```
1 const int N = "Edit";
2 int phi[N] = {0, 1};
3 void CalEuler()
4 {
5     for (int i = 2; i < N; i++)
6         if (!phi[i])
7             for (int j = i; j < N; j += i)
8             {
9                 if (!phi[j]) phi[j] = j;
10                phi[j] = phi[j] / i * (i - 1);
11            }
12 }
```

1.3 Basic Number Theory

1.3.1 Extended Euclidean

```
1 ll exgcd(ll a, ll b, ll &x, ll &y)
2 {
3     ll d = a;
4     if (b) d = exgcd(b, a % b, y, x), y -= x * (a / b);
```



```

5     else x = 1, y = 0;
6     return d;
7 }

```

1.3.2 $ax+by=c$

引用返回通解: $X = x + k * dx, Y = y - k * dy$

引用返回的 x 是最小非负整数解, 方程无解函数返回 0

```

1 #define Mod(a, b) (((a) % (b)) + (b)) % (b)
2 bool solve(ll a, ll b, ll c, ll& x, ll& y, ll& dx, ll& dy)
3 {
4     if (a == 0 && b == 0) return 0;
5     ll x0, y0;
6     ll d = exgcd(a, b, x0, y0);
7     if (c % d != 0) return 0;
8     dx = b / d, dy = a / d;
9     x = Mod(x0 * c / d, dx);
10    y = (c - a * x) / b;
11    // y = Mod(y0 * c / d, dy); x = (c - b * y) / a;
12    return 1;
13 }

```

1.3.3 Multiplicative Inverse Modulo

利用 exgcd 求 a 在模 m 下的逆元, 需要保证 $\gcd(a, m) == 1$.

```

1 ll inv(ll a, ll m)
2 {
3     ll x, y;
4     ll d = exgcd(a, m, x, y);
5     return d == 1 ? (x + m) % m : -1;
6 }

```

$a < p$ 且 p 为素数时, 有以下两种求法

费马小定理

```

1 ll inv(ll a, ll p) { return Pow(a, p - 2, p); }

```

贾志鹏线性筛

```

1 for (int i = 2; i < n; i++) inv[i] = inv[p % i] * (p - p / i) % p;

```

1.4 Modulo Linear Equation

1.4.1 Chinese Remainder Theory

$X = r_i \pmod{m_i}$; 要求 m_i 两两互质

引用返回通解 $X = re + k * mo$

```

1 void crt(ll r[], ll m[], ll n, ll &re, ll &mo)
2 {
3     mo = 1, re = 0;
4     for (int i = 0; i < n; i++) mo *= m[i];
5     for (int i = 0; i < n; i++)
6     {
7         ll x, y, tm = mo / m[i];
8         ll d = exgcd(tm, m[i], x, y);
9         re = (re + tm * x * r[i]) % mo;

```

```

10     }
11     re = (re + mo) % mo;
12 }

```

1.4.2 ExCRT

$X = r_i \pmod{m_i}$; m_i 可以不两两互质

引用返回通解 $X = re + k * mo$; 函数返回是否有解

```

1 bool excrt(ll r[], ll m[], ll n, ll &re, ll &mo)
2 {
3     ll x, y;
4     mo = m[0], re = r[0];
5     for (int i = 1; i < n; i++)
6     {
7         ll d = exgcd(mo, m[i], x, y);
8         if ((r[i] - re) % d != 0) return 0;
9         x = (r[i] - re) / d * x % (m[i] / d);
10        re += x * mo;
11        mo = mo / d * m[i];
12        re %= mo;
13    }
14    re = (re + mo) % mo;
15    return 1;
16 }

```

1.5 Combinatorics

1.5.1 Combination

$0 \leq m \leq n \leq 1000$

```

1 const int maxn = 1010;
2 ll C[maxn][maxn];
3 void CalComb()
4 {
5     C[0][0] = 1;
6     for (int i = 1; i < maxn; i++)
7     {
8         C[i][0] = 1;
9         for (int j = 1; j <= i; j++) C[i][j] = (C[i - 1][j - 1] + C[i - 1][j]) % mod;
10    }
11 }

```

$0 \leq m \leq n \leq 10^5$, 模 p 为素数

```

1 const int maxn = 100010;
2 ll f[maxn];
3 ll inv[maxn]; // 阶乘的逆元
4 void CalFact()
5 {
6     f[0] = 1;
7     for (int i = 1; i < maxn; i++) f[i] = (f[i - 1] * i) % p;
8     inv[maxn - 1] = Pow(f[maxn - 1], p - 2, p);
9     for (int i = maxn - 2; ~i; i--) inv[i] = inv[i + 1] * (i + 1) % p;
10 }
11 ll C(int n, int m) { return f[n] * inv[m] % p * inv[n - m] % p; }

```

1.5.2 Lucas

$1 \leq n, m \leq 1000000000, 1 < p < 100000, p$ 是素数

```

1  const int maxp = 100010;
2  ll f[maxn];
3  ll inv[maxn]; // 阶乘的逆元
4  void CalFact()
5  {
6      f[0] = 1;
7      for (int i = 1; i < maxn; i++) f[i] = (f[i - 1] * i) % p;
8      inv[maxn - 1] = Pow(f[maxn - 1], p - 2, p);
9      for (int i = maxn - 2; ~i; i--) inv[i] = inv[i + 1] * (i + 1) % p;
10 }
11 ll Lucas(ll n, ll m, ll p)
12 {
13     ll ret = 1;
14     while (n && m)
15     {
16         ll a = n % p, b = m % p;
17         if (a < b) return 0;
18         ret = ret * f[a] % p * inv[b] % p * inv[a - b] % p;
19         n /= p, m /= p;
20     }
21     return ret;
22 }
```

1.5.3 Big Combination

$0 \leq n \leq 10^9, 0 \leq m \leq 10^4, 1 \leq k \leq 10^9 + 7$

```

1  vector<int> v;
2  int dp[110];
3  ll Cal(int l, int r, int k, int dis)
4  {
5      ll res = 1;
6      for (int i = l; i <= r; i++)
7      {
8          int t = i;
9          for (int j = 0; j < v.size(); j++)
10             {
11                 int y = v[j];
12                 while (t % y == 0) dp[j] += dis, t /= y;
13             }
14         res = res * (ll)t % k;
15     }
16     return res;
17 }
18 ll Comb(int n, int m, int k)
19 {
20     clr(dp, 0);
21     v.clear();
22     int tmp = k;
23     for (int i = 2; i * i <= tmp; i++)
24         if (tmp % i == 0)
25             {
26                 int num = 0;
27                 while (tmp % i == 0) tmp /= i, num++;

```

```

28         v.pb(i);
29     }
30     if (tmp != 1) v.pb(tmp);
31     ll ans = Cal(n - m + 1, n, k, 1);
32     for (int j = 0; j < v.size(); j++) ans = ans * Pow(v[j], dp[j], k) % k;
33     ans = ans * inv(Cal(2, m, k, -1), k) % k;
34     return ans;
35 }

```

1.5.4 Polya

推论：一共 n 个置换，第 i 个置换的循环节个数为 $gcd(i, n)$

$N * N$ 的正方形格子, $c^{n^2} + 2c^{\frac{n^2+3}{4}} + c^{\frac{n^2+1}{2}} + 2c^{n\frac{n+1}{2}} + 2c^{\frac{n(n+1)}{2}}$
 正六面体, $\frac{m^8+17m^4+6m^2}{24}$ 正四面体, $\frac{m^4+11m^2}{12}$

```

1 // 长度为n的项链串用c种颜色染
2 ll solve(int c, int n)
3 {
4     if (n == 0) return 0;
5     ll ans = 0;
6     for (int i = 1; i <= n; i++) ans += Pow(c, __gcd(i, n));
7     if (n & 1) ans += n * Pow(c, n + 1 >> 1);
8     else ans += n / 2 * (1 + c) * Pow(c, n >> 1);
9     return ans / n / 2;
10 }

```

1.6 Fast Power

```

1 ll Mul(ll a, ll b, ll mod)
2 {
3     ll t = 0;
4     for (; b >= 1, a = (a << 1) % mod)
5         if (b & 1) t = (t + a) % mod;
6     return t;
7 }
8 ll Pow(ll a, ll n, ll mod)
9 {
10     ll t = 1;
11     for (; n; n >= 1, a = (a * a % mod))
12         if (n & 1) t = (t * a % mod);
13     return t;
14 }

```

1.7 Mobius Inversion

1.7.1 Mobius

$$F(n) = \sum_{d|n} f(d) \Rightarrow f(n) = \sum_{d|n} \mu(d) F\left(\frac{n}{d}\right)$$

$$F(n) = \sum_{n|d} f(d) \Rightarrow f(n) = \sum_{n|d} \mu\left(\frac{d}{n}\right) F(d)$$

```

1 ll ans;
2 const int maxn = "Edit";
3 int n, x, prime[maxn], tot, mu[maxn];
4 bool check[maxn];
5 void calmu()

```

```

6 {
7     mu[1] = 1;
8     for (int i = 2; i < maxn; i++)
9     {
10         if (!check[i]) prime[tot++] = i, mu[i] = -1;
11         for (int j = 0; j < tot; j++)
12         {
13             if (i * prime[j] >= maxn) break;
14             check[i * prime[j]] = true;
15             if (i % prime[j] == 0)
16             {
17                 mu[i * prime[j]] = 0;
18                 break;
19             }
20             else mu[i * prime[j]] = -mu[i];
21         }
22     }
23 }

```

1.7.2 Number of Coprime-pair

有 n 个数 ($n \leq 100000$), 问这 n 个数中互质的数的对数

```

1 ll solve()
2 {
3     int b[100005];
4     ll _max, ans = 0;
5     clr(b, 0);
6     for (int i = 0; i < n; i++)
7     {
8         scanf("%d", &x);
9         if (x > _max) _max = x;
10        b[x]++;
11    }
12    for (int i = 1; i <= _max; i++)
13    {
14        int cnt = 0;
15        for (ll j = i; j <= _max; j += i) cnt += b[j];
16        ans += 1LL * mu[i] * cnt * cnt;
17    }
18    return (ans - b[1]) / 2;
19 }

```

1.7.3 VisibleTrees

$\gcd(x, y) = 1$ 的对数, $x \leq n, y \leq m$

```

1 ll solve(int n, int m)
2 {
3     if (n < m) swap(n, m);
4     ll ans = 0;
5     for (int i = 1; i <= m; ++i) ans += (ll)mu[i] * (n / i) * (m / i);
6     return ans;
7 }

```

1.8 Fast Transformation

1.8.1 FFT

```

1  const double PI = acos(-1.0);
2  //复数结构体
3  struct Complex
4  {
5      double x, y; //实部和虚部 x+yi
6      Complex(double _x = 0.0, double _y = 0.0) { x = _x, y = _y; }
7      Complex operator-(const Complex& b) const { return Complex(x - b.x, y - b.y); }
8      Complex operator+(const Complex& b) const { return Complex(x + b.x, y + b.y); }
9      Complex operator*(const Complex& b) const { return Complex(x * b.x - y * b.y, x * b
        .y + y * b.x); }
10 };
11 /*
12  * 进行FFT和IFFT前的反转变换。
13  * 位置i和 (i二进制反转后位置) 互换
14  * len必须取2的幂
15  */
16 void change(Complex y[], int len)
17 {
18     for (int i = 1, j = len / 2; i < len - 1; i++)
19     {
20         if (i < j) swap(y[i], y[j]);
21         //交换互为小标反转的元素, i<j保证交换一次
22         //i做正常的+1, j左反转类型的+1,始终保持i和j是反转的
23         int k = len / 2;
24         while (j >= k) j -= k, k /= 2;
25         if (j < k) j += k;
26     }
27 }
28 /*
29  * 做FFT
30  * len必须为2^k形式,
31  * on==1时是DFT, on==-1时是IDFT
32  */
33 void fft(Complex y[], int len, int on)
34 {
35     change(y, len);
36     for (int h = 2; h <= len; h <= 1)
37     {
38         Complex wn(cos(-on * 2 * PI / h), sin(-on * 2 * PI / h));
39         for (int j = 0; j < len; j += h)
40         {
41             Complex w(1, 0);
42             for (int k = j; k < j + h / 2; k++)
43             {
44                 Complex u = y[k];
45                 Complex t = w * y[k + h / 2];
46                 y[k] = u + t, y[k + h / 2] = u - t;
47                 w = w * wn;
48             }
49         }
50     }
51     if (on == -1)
52         for (int i = 0; i < len; i++) y[i].x /= len;
53 }

```

1.8.2 NTT

模数 P 为费马素数, G 为 P 的原根。 $G^{\frac{P-1}{n}}$ 具有和 $w_n = e^{\frac{2i\pi}{n}}$ 相似的性质。具体的 P 和 G 可参考 1.11

```

1  const int mod = 998244353;
2  const int G = 3;
3  ll wn[20];
4  void getwn()
5  { // 千万不要忘记
6      for (int i = 0; i < 20; i++) wn[i] = Pow(G, (mod - 1) / (1 << i), mod);
7  }
8  void change(ll y[], int len)
9  {
10     for (int i = 1, j = len / 2; i < len - 1; i++)
11     {
12         if (i < j) swap(y[i], y[j]);
13         int k = len / 2;
14         while (j >= k) j -= k, k /= 2;
15         if (j < k) j += k;
16     }
17 }
18 void ntt(ll y[], int len, int on)
19 {
20     change(y, len);
21     for (int h = 2, id = 1; h <= len; h <<= 1, id++)
22     {
23         for (int j = 0; j < len; j += h)
24         {
25             ll w = 1;
26             for (int k = j; k < j + h / 2; k++)
27             {
28                 ll u = y[k] % mod;
29                 ll t = w * (y[k + h / 2] % mod) % mod;
30                 y[k] = (u + t) % mod, y[k + h / 2] = ((u - t) % mod + mod) % mod;
31                 w = w * wn[id] % mod;
32             }
33         }
34     }
35     if (on == -1)
36     {
37         // 原本的除法要用逆元
38         ll inv = Pow(len, mod - 2, mod);
39         for (int i = 1; i < len / 2; i++) swap(y[i], y[len - i]);
40         for (int i = 0; i < len; i++) y[i] = y[i] * inv % mod;
41     }
42 }

```

1.8.3 FWT

```

1  void fwt(int f[], int m)
2  {
3      int n = __builtin_ctz(m);
4      for (int i = 0; i < n; ++i)
5          for (int j = 0; j < m; ++j)
6              if (j & (1 << i))
7              {
8                  int l = f[j ^ (1 << i)], r = f[j];
9                  f[j ^ (1 << i)] = l + r, f[j] = l - r;

```

```

10         // or: f[j] += f[j ^ (1 << i)];
11         // and: f[j ^ (1 << i)] += f[j];
12     }
13 }
14 void ifwt(int f[], int m)
15 {
16     int n = __builtin_ctz(m);
17     for (int i = 0; i < n; ++i)
18         for (int j = 0; j < m; ++j)
19             if (j & (1 << i))
20             {
21                 int l = f[j ^ (1 << i)], r = f[j];
22                 f[j ^ (1 << i)] = (l + r) / 2, f[j] = (l - r) / 2;
23                 // 如果有取模需要使用逆元
24                 // or: f[j] -= f[j ^ (1 << i)];
25                 // and: f[j ^ (1 << i)] -= f[j];
26             }
27 }

```

1.9 Numerical Integration

1.9.1 Adaptive Simpson's Rule

$$\int_a^b f(x)dx \approx \frac{b-a}{6}[f(a) + 4f(\frac{a+b}{2}) + f(b)]$$

$$|S(a, c) + S(c, b) - S(a, b)|/15 < \epsilon$$

```

1 double F(double x) {}
2 double simpson(double a, double b)
3 { // 三点Simpson法
4     double c = a + (b - a) / 2;
5     return (F(a) + 4 * F(c) + F(b)) * (b - a) / 6;
6 }
7 double asr(double a, double b, double eps, double A)
8 { //自适应Simpson公式(递归过程)。已知整个区间[a,b]上的三点Simpson值A
9     double c = a + (b - a) / 2;
10    double L = simpson(a, c), R = simpson(c, b);
11    if (fabs(L + R - A) <= 15 * eps) return L + R + (L + R - A) / 15.0;
12    return asr(a, c, eps / 2, L) + asr(c, b, eps / 2, R);
13 }
14 double asr(double a, double b, double eps) { return asr(a, b, eps, simpson(a, b)); }

```

1.9.2 Berlekamp-Massey

```

1 const int N = 1 << 14;
2 ll res[N], base[N], _c[N], _md[N];
3 vector<int> Md;
4 void mul(ll* a, ll* b, int k)
5 {
6     for (int i = 0; i < k + k; i++) _c[i] = 0;
7     for (int i = 0; i < k; i++)
8         if (a[i])
9             for (int j = 0; j < k; j++) _c[i + j] = (_c[i + j] + a[i] * b[j]) % mod;
10    for (int i = k + k - 1; i >= k; i--)
11        if (_c[i])
12            for (int j = 0; j < Md.size(); j++) _c[i - k + Md[j]] = (_c[i - k + Md[j]]
13            - _c[i] * _md[Md[j]]) % mod;
14    for (int i = 0; i < k; i++) a[i] = _c[i];

```



```

15 int solve(ll n, VI a, VI b)
16 {
17     ll ans = 0, pnt = 0;
18     int k = a.size();
19     assert(a.size() == b.size());
20     for (int i = 0; i < k; i++) _md[k - 1 - i] = -a[i];
21     _md[k] = 1;
22     Md.clear();
23     for (int i = 0; i < k; i++)
24         if (_md[i] != 0) Md.push_back(i);
25     for (int i = 0; i < k; i++) res[i] = base[i] = 0;
26     res[0] = 1;
27     while ((1LL << pnt) <= n) pnt++;
28     for (int p = pnt; p >= 0; p--)
29     {
30         mul(res, res, k);
31         if ((n >> p) & 1)
32         {
33             for (int i = k - 1; i >= 0; i--) res[i + 1] = res[i];
34             res[0] = 0;
35             for (int j = 0; j < Md.size(); j++) res[Md[j]] = (res[Md[j]] - res[k] * _md
[Md[j]]) % mod;
36         }
37     }
38     for (int i = 0; i < k; i++) ans = (ans + res[i] * b[i]) % mod;
39     if (ans < 0) ans += mod;
40     return ans;
41 }
42 VI BM(VI s)
43 {
44     VI C(1, 1), B(1, 1);
45     int L = 0, m = 1, b = 1;
46     for (int n = 0; n < s.size(); n++)
47     {
48         ll d = 0;
49         for (int i = 0; i <= L; i++) d = (d + (ll)C[i] * s[n - i]) % mod;
50         if (d == 0)
51             ++m;
52         else if (2 * L <= n)
53         {
54             VI T = C;
55             ll c = mod - d * Pow(b, mod - 2) % mod;
56             while (C.size() < B.size() + m) C.pb(0);
57             for (int i = 0; i < B.size(); i++) C[i + m] = (C[i + m] + c * B[i]) % mod;
58             L = n + 1 - L, B = T, b = d, m = 1;
59         }
60         else
61         {
62             ll c = mod - d * Pow(b, mod - 2) % mod;
63             while (C.size() < B.size() + m) C.pb(0);
64             for (int i = 0; i < B.size(); i++) C[i + m] = (C[i + m] + c * B[i]) % mod;
65             ++m;
66         }
67     }
68     return C;
69 }
70 int gao(VI a, ll n)
71 {
72     VI c = BM(a);

```

```

73     c.erase(c.begin());
74     for (int i = 0; i < c.size(); i++) c[i] = (mod - c[i]) % mod;
75     return solve(n, c, VI(a.begin(), a.begin() + c.size()));
76 }

```

1.10 Others

约瑟夫问题

n 个人围成一圈，从第一个开始报数，第 m 个将被杀掉

```

1 int josephus(int n, int m)
2 {
3     int r = 0;
4     for (int k = 1; k <= n; ++k) r = (r + m) % k;
5     return r + 1;
6 }

```

n^n 最左边一位数

```

1 int leftmost(int n)
2 {
3     double m = n * log10((double)n);
4     double g = m - (ll)m;
5     return (int)pow(10.0, g);
6 }

```

$n!$ 位数

```

1 int count(ll n)
2 {
3     if (n == 1) return 1;
4     return (int)ceil(0.5 * log10(2 * M_PI * n) + n * log10(n) - n * log10(M_E));
5 }

```

1.11 Formula

1. 约数定理：若 $n = \prod_{i=1}^k p_i^{a_i}$ ，则

(a) 约数个数 $f(n) = \prod_{i=1}^k (a_i + 1)$

(b) 约数和 $g(n) = \prod_{i=1}^k (\sum_{j=0}^{a_i} p_i^j)$

2. 小于 n 且互素的数之和为 $n\varphi(n)/2$

3. 若 $\gcd(n, i) = 1$ ，则 $\gcd(n, n - i) = 1 (1 \leq i \leq n)$

4. 错排公式： $D(n) = (n - 1)(D(n - 2) + D(n - 1)) = \sum_{i=2}^n \frac{(-1)^i n!}{i!} = \lfloor \frac{n!}{e} + 0.5 \rfloor$

5. 威尔逊定理： p is prime $\Rightarrow (p - 1)! \equiv -1 \pmod{p}$

6. 欧拉定理： $\gcd(a, n) = 1 \Rightarrow a^{\varphi(n)} \equiv 1 \pmod{n}$

7. 欧拉定理推广： $\gcd(n, p) = 1 \Rightarrow a^n \equiv a^{n \% \varphi(p)} \pmod{p}$

8. 素数定理：对于不大于 n 的素数个数 $\pi(n)$ ， $\lim_{n \rightarrow \infty} \pi(n) = \frac{n}{\ln n}$

9. 位数公式：正整数 x 的位数 $N = \log_{10}(n) + 1$

10. 斯特灵公式 $n! \approx \sqrt{2\pi n} (\frac{n}{e})^n$

11. 设 $a > 1, m, n > 0$ ，则 $\gcd(a^m - 1, a^n - 1) = a^{\gcd(m, n)} - 1$

12. 设 $a > b, \gcd(a, b) = 1$ ，则 $\gcd(a^m - b^m, a^n - b^n) = a^{\gcd(m, n)} - b^{\gcd(m, n)}$

$$G = \gcd(C_n^1, C_n^2, \dots, C_n^{n-1}) = \begin{cases} n, & n \text{ is prime} \\ 1, & n \text{ has multy prime factors} \\ p, & n \text{ has single prime factor } p \end{cases}$$

$$\gcd(\text{Fib}(m), \text{Fib}(n)) = \text{Fib}(\gcd(m, n))$$

13. 若 $\gcd(m, n) = 1$, 则:

(a) 最大不能组合的数为 $m * n - m - n$

(b) 不能组合数个数 $N = \frac{(m-1)(n-1)}{2}$

14. $(n+1)lcm(C_n^0, C_n^1, \dots, C_n^{n-1}, C_n^n) = lcm(1, 2, \dots, n+1)$

15. 若 p 为素数, 则 $(x + y + \dots + w)^p \equiv x^p + y^p + \dots + w^p \pmod{p}$

16. 卡特兰数: 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012

$$h(0) = h(1) = 1, h(n) = \frac{(4n-2)h(n-1)}{n+1} = \frac{C_{2n}^n}{n+1} = C_{2n}^n - C_{2n}^{n-1}$$

17. FFT 常用素数

$r \cdot 2^k + 1$	r	k	g
3	1	1	2
5	1	2	2
17	1	4	3
97	3	5	5
193	3	6	5
257	1	8	3
7681	15	9	17
12289	3	12	11
40961	5	13	3
65537	1	16	3
786433	3	18	10
5767169	11	19	3
7340033	7	20	3
23068673	11	21	3
104857601	25	22	3
167772161	5	25	3
469762049	7	26	3
998244353	119	23	3
1004535809	479	21	3
2013265921	15	27	31
2281701377	17	27	3
3221225473	3	30	5
75161927681	35	31	3
77309411329	9	33	7
206158430209	3	36	22
2061584302081	15	37	7
2748779069441	5	39	3
6597069766657	3	41	5
39582418599937	9	42	5
79164837199873	9	43	5
263882790666241	15	44	7
1231453023109121	35	45	3
1337006139375617	19	46	3
3799912185593857	27	47	5
4222124650659841	15	48	19
7881299347898369	7	50	6
31525197391593473	7	52	3
180143985094819841	5	55	6
1945555039024054273	27	56	5
4179340454199820289	29	57	3

2 String Processing

2.1 KMP

```

1 // 返回y中x的个数
2 const int N = "Edit";
3 int next[N];
4 void initkmp(char x[], int m)
5 {
6     int i = 0, j = next[0] = -1;
7     while (i < m)
8     {
9         while (j != -1 && x[i] != x[j]) j = next[j];
10        next[++i] = ++j;
11    }
12 }
13 int kmp(char x[], int m, char y[], int n)
14 {
15     int i, j, ans;
16     i = j = ans = 0;
17     initkmp(x, m);
18     while (i < n)
19     {
20         while (j != -1 && y[i] != x[j]) j = next[j];
21         i++, j++;
22         if (j >= m) ans++, j = next[j];
23     }
24     return ans;
25 }

```

2.2 ExtendKMP

```

1 //next[i]:x[i...m-1]与x[0...m-1]的最长公共前缀
2 //extend[i]:y[i...n-1]与x[0...m-1]的最长公共前缀
3 const int N = "Edit";
4 int next[N], extend[N];
5 void pre_ekmp(char x[], int m)
6 {
7     next[0] = m;
8     int j = 0;
9     while (j + 1 < m && x[j] == x[j + 1]) j++;
10    next[1] = j;
11    int k = 1;
12    for (int i = 2; i < m; i++)
13    {
14        int p = next[k] + k - 1;
15        int L = next[i - k];
16        if (i + L < p + 1)
17            next[i] = L;
18        else
19        {
20            j = max(0, p - i + 1);
21            while (i + j < m && x[i + j] == x[j]) j++;
22            next[i] = j;
23            k = i;
24        }
25    }
26 }

```

```

27 void ekmp(char x[], int m, char y[], int n)
28 {
29     pre_ekmp(x, m, next);
30     int j = 0;
31     while (j < n && j < m && x[j] == y[j]) j++;
32     extend[0] = j;
33     int k = 0;
34     for (int i = 1; i < n; i++)
35     {
36         int p = extend[k] + k - 1;
37         int l = next[i - k];
38         if (i + l < p + 1)
39             extend[i] = l;
40         else
41         {
42             j = max(0, p - i + 1);
43             while (i + j < n && j < m && y[i + j] == x[j]) j++;
44             extend[i] = j, k = i;
45         }
46     }
47 }

```

2.3 Manacher

$O(n)$ 求解最长回文子串

```

1  const int N = "Edit";
2  char s[N], str[N << 1];
3  int p[N << 1];
4  void Manacher(char s[], int& n)
5  {
6      str[0] = '$', str[1] = '#';
7      for (int i = 0; i < n; i++) str[(i << 1) + 2] = s[i], str[(i << 1) + 3] = '#';
8      n = 2 * n + 2;
9      str[n] = 0;
10     int mx = 0, id;
11     for (int i = 1; i < n; i++)
12     {
13         p[i] = mx > i ? min(p[2 * id - i], mx - i) : 1;
14         while (str[i - p[i]] == str[i + p[i]]) p[i]++;
15         if (p[i] + i > mx) mx = p[i] + i, id = i;
16     }
17 }
18 int solve(char s[])
19 {
20     int n = strlen(s);
21     Manacher(s, n);
22     return *max_element(p, p + n) - 1;
23 }

```

2.4 Aho-Corasick Automaton

```

1  const int maxn = "Edit";
2  struct Trie
3  {
4      int ch[maxn][26], f[maxn], val[maxn];
5      int sz, rt;

```

```

6   int newnode() { clr(ch[sz], -1), val[sz] = 0; return sz++; }
7   void init() { sz = 0, rt = newnode(); }
8   inline int idx(char c) { return c - 'A'; }
9   void insert(const char* s)
10  {
11      int u = 0, n = strlen(s);
12      for (int i = 0; i < n; i++)
13      {
14          int c = idx(s[i]);
15          if (ch[u][c] == -1) ch[u][c] = newnode();
16          u = ch[u][c];
17      }
18      val[u]++;
19  }
20  void build()
21  {
22      queue<int> q;
23      f[rt] = rt;
24      for (int c = 0; c < 26; c++)
25      {
26          if (~ch[rt][c])
27              f[ch[rt][c]] = rt, q.push(ch[rt][c]);
28          else
29              ch[rt][c] = rt;
30      }
31      while (!q.empty())
32      {
33          int u = q.front();
34          q.pop();
35          // val[u] += val[f[u]];
36          for (int c = 0; c < 26; c++)
37          {
38              if (~ch[u][c])
39                  f[ch[u][c]] = ch[f[u]][c], q.push(ch[u][c]);
40              else
41                  ch[u][c] = ch[f[u]][c];
42          }
43      }
44  }
45  //返回主串中有多少模式串
46  int query(const char* s)
47  {
48      int u = rt, n = strlen(s);
49      int res = 0;
50      for (int i = 0; i < n; i++)
51      {
52          int c = idx(s[i]);
53          u = ch[u][c];
54          int tmp = u;
55          while (tmp != rt)
56          {
57              res += val[tmp];
58              val[tmp] = 0;
59              tmp = f[tmp];
60          }
61      }
62      return res;
63  }
64  };

```

2.5 Suffix Array

```

1 //倍增算法构造后缀数组,复杂度O(nlogn)
2 const int maxn = "Edit";
3 char s[maxn];
4 int sa[maxn], t[maxn], t2[maxn], c[maxn], rank[maxn], height[maxn];
5 //n为字符串的长度,字符集的值0~m-1
6 void build_sa(int m, int n)
7 {
8     n++;
9     int *x = t, *y = t2;
10    //基数排序
11    for (int i = 0; i < m; i++) c[i] = 0;
12    for (int i = 0; i < n; i++) c[x[i] = s[i]]++;
13    for (int i = 1; i < m; i++) c[i] += c[i - 1];
14    for (int i = n - 1; ~i; i--) sa[--c[x[i]]] = i;
15    for (int k = 1; k <= n; k <= 1)
16    {
17        //直接利用sa数组排序第二关键字
18        int p = 0;
19        for (int i = n - k; i < n; i++) y[p++] = i;
20        for (int i = 0; i < n; i++)
21            if (sa[i] >= k) y[p++] = sa[i] - k;
22        //基数排序第一关键字
23        for (int i = 0; i < m; i++) c[i] = 0;
24        for (int i = 0; i < n; i++) c[x[y[i]]]++;
25        for (int i = 0; i < m; i++) c[i] += c[i - 1];
26        for (int i = n - 1; ~i; i--) sa[--c[x[y[i]]]] = y[i];
27        //根据sa和y数组计算新的x数组
28        swap(x, y);
29        p = 1;
30        x[sa[0]] = 0;
31        for (int i = 1; i < n; i++)
32            x[sa[i]] = y[sa[i - 1]] == y[sa[i]] && y[sa[i - 1] + k] == y[sa[i] + k] ? p
33            - 1 : p++;
34        if (p >= n) break; //以后即使继续倍增,sa也不会改变,推出
35        m = p; //下次基数排序的最大值
36    }
37    n--;
38    int k = 0;
39    for (int i = 0; i <= n; i++) rank[sa[i]] = i;
40    for (int i = 0; i < n; i++)
41    {
42        if (k) k--;
43        int j = sa[rank[i] - 1];
44        while (s[i + k] == s[j + k]) k++;
45        height[rank[i]] = k;
46    }
47 }
48 int dp[maxn][30];
49 void initrmq(int n)
50 {
51     for (int i = 1; i <= n; i++)
52         dp[i][0] = height[i];
53     for (int j = 1; (1 << j) <= n; j++)
54         for (int i = 1; i + (1 << j) - 1 <= n; i++)
55             dp[i][j] = min(dp[i][j - 1], dp[i + (1 << (j - 1))][j - 1]);
56 }

```

```

57 int rmq(int l, int r)
58 {
59     int k = 31 - __builtin_clz(r - l + 1);
60     return min(dp[l][k], dp[r - (1 << k) + 1][k]);
61 }
62 int lcp(int a, int b)
63 { // 求两个后缀的最长公共前缀
64     a = rank[a], b = rank[b];
65     if (a > b) swap(a, b);
66     return rmq(a + 1, b);
67 }

```

2.6 Suffix Automation

```

1  const int maxn = "Edit";
2  struct SAM
3  {
4      int len[maxn << 1], link[maxn << 1], ch[maxn << 1][26];
5      int sz, rt, last;
6      int newnode(int x = 0)
7      {
8          len[sz] = x;
9          link[sz] = -1;
10         clr(ch[sz], -1);
11         return sz++;
12     }
13     void init() { sz = last = 0, rt = newnode(); }
14     void extend(int c)
15     {
16         int np = newnode(len[last] + 1);
17         int p;
18         for (p = last; ~p && ch[p][c] == -1; p = link[p]) ch[p][c] = np;
19         if (p == -1)
20             link[np] = rt;
21         else
22         {
23             int q = ch[p][c];
24             if (len[p] + 1 == len[q])
25                 link[np] = q;
26             else
27             {
28                 int nq = newnode(len[p] + 1);
29                 memcpy(ch[nq], ch[q], sizeof(ch[q]));
30                 link[nq] = link[q], link[q] = link[np] = nq;
31                 for (; ~p && ch[p][c] == q; p = link[p]) ch[p][c] = nq;
32             }
33         }
34         last = np;
35     }
36     int topcnt[maxn], topsam[maxn << 1];
37     void sort()
38     { // 加入串后拓扑排序
39         clr(topcnt, 0);
40         for (int i = 0; i < sz; i++) topcnt[len[i]]++;
41         for (int i = 0; i < maxn - 1; i++) topcnt[i + 1] += topcnt[i];
42         for (int i = 0; i < sz; i++) topsam[--topcnt[len[i]]] = i;
43     }
44 };

```


3 Data Structure

3.1 Binary Indexed Tree

$O(\log n)$ 查询和修改数组的前缀和

```

1 // 注意下标应从1开始 n是全局变量
2 const int maxn = "Edit";
3 int bit[N], n;
4 int sum(int x)
5 {
6     int s = 0;
7     for (int i = x; i; i -= i & -i) s += bit[i];
8     return s;
9 }
10 void add(int x, int v)
11 {
12     for (int i = x; i <= n; i += i & -i) bit[i] += v;
13 }
```

3.2 Segment Tree

```

1 #define lson rt << 1          // 左儿子
2 #define rson rt << 1 | 1      // 右儿子
3 #define Lson l, m, lson      // 左子树
4 #define Rson m + 1, r, rson  // 右子树
5 void PushUp(int rt);          // 用lson和rson更新rt
6 void PushDown(int rt[, int m]); // rt的标记下移, m为区间长度 (若与标记有关)
7 void build(int l, int r, int rt); // 以rt为根节点, 对区间[l, r]建立线段树
8 void update(..., int l, int r, int rt) // rt[l, r]内寻找目标并更新
9 int query(int L, int R, int l, int r, int rt) // rt[l, r]内查询[L, R]
```

3.2.1 Single-point Update

```

1 const int maxn = "Edit";
2 int sum[maxn << 2]; // sum[rt]用于维护区间和
3 void PushUp(int rt) { sum[rt] = sum[lson] + sum[rson]; }
4 void build(int l, int r, int rt)
5 {
6     if (l == r)
7     {
8         scanf("%d", &sum[rt]); // 建立的时候直接输入叶节点
9         return;
10    }
11    int m = (l + r) >> 1;
12    build(Lson);
13    build(Rson);
14    PushUp(rt);
15 }
16 void update(int p, int add, int l, int r, int rt)
17 {
18     if (l == r)
19     {
20         sum[rt] += add;
21         return;
22     }
23     int m = (l + r) >> 1;
```

```

24     if (p <= m)
25         update(p, add, Lson);
26     else
27         update(p, add, Rson);
28     PushUp(rt);
29 }
30 int query(int L, int R, int l, int r, int rt)
31 {
32     if (L <= l && r <= R) return sum[rt];
33     int m = (l + r) >> 1, s = 0;
34     if (L <= m) s += query(L, R, Lson);
35     if (m < R) s += query(L, R, Rson);
36     return s;
37 }

```

3.2.2 Interval Update

```

1  const int maxn = "Edit";
2  int seg[maxn << 2], sum[maxn << 2]; // seg[rt] 用于存放懒惰标记, 注意PushDown时标记的传递
3  void PushUp(int rt) { sum[rt] = sum[lson] + sum[rson]; }
4  void PushDown(int rt, int m)
5  {
6      if (seg[rt] == 0) return;
7      seg[lson] += seg[rt];
8      seg[rson] += seg[rt];
9      sum[lson] += seg[rt] * (m - (m >> 1));
10     sum[rson] += seg[rt] * (m >> 1);
11     seg[rt] = 0;
12 }
13 void build(int l, int r, int rt)
14 {
15     seg[rt] = 0;
16     if (l == r)
17     {
18         scanf("%lld", &sum[rt]);
19         return;
20     }
21     int m = (l + r) >> 1;
22     build(Lson);
23     build(Rson);
24     PushUp(rt);
25 }
26 void update(int L, int R, int add, int l, int r, int rt)
27 {
28     if (L <= l && r <= R)
29     {
30         seg[rt] += add;
31         sum[rt] += add * (r - l + 1);
32         return;
33     }
34     PushDown(rt, r - l + 1);
35     int m = (l + r) >> 1;
36     if (L <= m) update(L, R, add, Lson);
37     if (m < R) update(L, R, add, Rson);
38     PushUp(rt);
39 }
40 int query(int L, int R, int l, int r, int rt)
41 {

```

```

42     if (L <= l && r <= R) return sum[rt];
43     PushDown(rt, r - l + 1);
44     int m = (l + r) >> 1, ret = 0;
45     if (L <= m) ret += query(L, R, Lson);
46     if (m < R) ret += query(L, R, Rson);
47     return ret;
48 }

```

3.3 Splay Tree

```

1  #define key_value ch[ch[root][1]][0]
2  const int maxn = "Edit";
3  struct Splay
4  {
5      int a[maxn];
6      int sz[maxn], ch[maxn][2], fa[maxn];
7      int key[maxn], rev[maxn];
8      int root, tot;
9      int stk[maxn], top;
10     void init(int n)
11     {
12         tot = 0, top = 0;
13         root = newnode(0, -1);
14         ch[root][1] = newnode(root, -1);
15         for (int i = 0; i < n; i++) a[i] = i + 1;
16         key_value = build(0, n - 1, ch[root][1]);
17         pushup(ch[root][1]);
18         pushup(root);
19     }
20     int newnode(int p = 0, int k = 0)
21     {
22         int x = top ? stk[top--] : ++tot;
23         fa[x] = p;
24         sz[x] = 1;
25         ch[x][0] = ch[x][1] = 0;
26         key[x] = k;
27         rev[x] = 0;
28         return x;
29     }
30     void pushdown(int x)
31     {
32         if (rev[x])
33         {
34             swap(ch[x][0], ch[x][1]);
35             if (ch[x][0]) rev[ch[x][0]] ^= 1;
36             if (ch[x][1]) rev[ch[x][1]] ^= 1;
37             rev[x] = 0;
38         }
39     }
40     void pushup(int x) { sz[x] = sz[ch[x][0]] + sz[ch[x][1]] + 1; }
41     void rotate(int x, int d)
42     {
43         int y = fa[x];
44         pushdown(y), pushdown(x);
45         ch[y][d ^ 1] = ch[x][d];
46         fa[ch[x][d]] = y;
47         if (fa[y]) ch[fa[y]][ch[fa[y]][1] == y] = x;
48         fa[x] = fa[y];

```

```

49     ch[x][d] = y;
50     fa[y] = x;
51     pushup(y);
52 }
53 void splay(int x, int goal = 0)
54 {
55     pushdown(x);
56     while (fa[x] != goal)
57     {
58         if (fa[fa[x]] == goal)
59             rotate(x, ch[fa[x]][0] == x);
60         else
61         {
62             int y = fa[x];
63             int d = ch[fa[y]][0] == y;
64             ch[y][d] == x ? rotate(x, d ^ 1) : rotate(y, d);
65             rotate(x, d);
66         }
67     }
68     pushup(x);
69     if (goal == 0) root = x;
70 }
71 int kth(int r, int k)
72 {
73     pushdown(r);
74     int t = sz[ch[r][0]] + 1;
75     if (t == k) return r;
76     return t > k ? kth(ch[r][0], k) : kth(ch[r][1], k - t);
77 }
78 int build(int l, int r, int p)
79 {
80     if (l > r) return 0;
81     int mid = l + r >> 1;
82     int x = newnode(p, a[mid]);
83     ch[x][0] = build(l, mid - 1, x);
84     ch[x][1] = build(mid + 1, r, x);
85     pushup(x);
86     return x;
87 }
88 void select(int l, int r)
89 {
90     splay(kth(root, l), 0);
91     splay(kth(ch[root][1], r - l + 2), root);
92 }
93 // 各种操作
94 };

```

3.4 Functional Segment Tree

静态查询区间第 k 小的值
必要时进行离散化

```

1  const int maxn = "Edit";
2  int a[maxn], rt[maxn];
3  int cnt;
4  int lson[maxn << 5], rson[maxn << 5], sum[maxn << 5];
5  #define Lson l, m, lson[x], lson[y]
6  #define Rson m + 1, r, rson[x], rson[y]

```

```

7 void update(int p, int l, int r, int& x, int y)
8 {
9     lson[++cnt] = lson[y], rson[cnt] = rson[y], sum[cnt] = sum[y] + 1, x = cnt;
10    if (l == r) return;
11    int m = (l + r) >> 1;
12    if (p <= m) update(p, Lson);
13    else update(p, Rson);
14 }
15 int query(int l, int r, int x, int y, int k)
16 {
17     if (l == r) return l;
18     int m = (l + r) >> 1;
19     int s = sum[lson[y]] - sum[lson[x]];
20     if (s >= k) return query(Lson, k);
21     else return query(Rson, k - s);
22 }

```

3.5 Sparse Table

```

1 const int maxn = "Edit";
2 int mmax[maxn][30], mmin[maxn][30];
3 int a[maxn], n, k;
4 void init()
5 {
6     for (int i = 1; i <= n; i++) mmax[i][0] = mmin[i][0] = a[i];
7     for (int j = 1; (1 << j) <= n; j++)
8         for (int i = 1; i + (1 << j) - 1 <= n; i++)
9             {
10                mmax[i][j] = max(mmax[i][j - 1], mmax[i + (1 << (j - 1))][j - 1]);
11                mmin[i][j] = min(mmin[i][j - 1], mmin[i + (1 << (j - 1))][j - 1]);
12            }
13 }
14 // op=0/1 返回[l,r]最大/小值
15 int rmq(int l, int r, int op)
16 {
17     int k = 31 - __builtin_clz(r - l + 1);
18     if (op == 0)
19         return max(mmax[l][k], mmax[r - (1 << k) + 1][k]);
20     return min(mmin[l][k], mmin[r - (1 << k) + 1][k]);
21 }

```

二维 RMQ

```

1 void init()
2 {
3     for (int i = 0; (1 << i) <= n; i++)
4         for (int j = 0; (1 << j) <= m; j++)
5             {
6                 if (i == 0 && j == 0) continue;
7                 for (int row = 1; row + (1 << i) - 1 <= n; row++)
8                     for (int col = 1; col + (1 << j) - 1 <= m; col++)
9                         if (i)
10                            dp[row][col][i][j] = max(dp[row][col][i - 1][j],
11                                                        dp[row + (1 << (i - 1))][col][i - 1][j]);
12                         else
13                            dp[row][col][i][j] = max(dp[row][col][i][j - 1],
14                                                        dp[row][col + (1 << (j - 1))][i][j - 1]);
15             }
16 }

```

```

17 int rmq(int x1, int y1, int x2, int y2)
18 {
19     int kx = 31 - __builtin_clz(x2 - x1 + 1);
20     int ky = 31 - __builtin_clz(y2 - y1 + 1);
21     int m1 = dp[x1][y1][kx][ky];
22     int m2 = dp[x2 - (1 << kx) + 1][y1][kx][ky];
23     int m3 = dp[x1][y2 - (1 << ky) + 1][kx][ky];
24     int m4 = dp[x2 - (1 << kx) + 1][y2 - (1 << ky) + 1][kx][ky];
25     return max(max(m1, m2), max(m3, m4));
26 }

```

3.6 Heavy-Light Decomposition

```

1  const int maxn = "Edit";
2  struct HLD
3  {
4      int n, dfs_clock;
5      int sz[maxn], top[maxn], son[maxn], dep[maxn], fa[maxn], id[maxn];
6      vector<int> G[maxn];
7      void init(int n)
8      {
9          this->n = n, clr(son, -1), dfs_clock = 0;
10         for (int i = 0; i < n; i++) G[i].clear();
11     }
12     void add_edge(int u, int v) { G[u].pb(v), G[v].pb(u); }
13     void dfs(int u, int p, int d)
14     {
15         dep[u] = d, fa[u] = p, sz[u] = 1;
16         for (auto& v : G[u])
17         {
18             if (v == p) continue;
19             dfs(v, u, d + 1);
20             sz[u] += sz[v];
21             if (son[u] == -1 || sz[v] > sz[son[u]]) son[u] = v;
22         }
23     }
24     void link(int u, int t)
25     {
26         top[u] = t, id[u] = ++dfs_clock;
27         if (son[u] == -1) return;
28         link(son[u], t);
29         for (auto& v : G[u])
30             if (v != son[u] && v != fa[u]) link(v, v);
31     }
32     // 数据结构相关操作，一般使用线段树或者树状数组
33     int query_path(int u, int v)
34     {
35         int ret = 0;
36         while (top[u] != top[v])
37         {
38             if (dep[top[u]] < dep[top[v]]) swap(u, v);
39             ret += query(id[top[u]], id[u]);
40             u = fa[top[u]];
41         }
42         if (dep[u] > dep[v]) swap(u, v);
43         ret += query(id[u], id[v]);
44     }
45 };

```

3.7 Link-Cut Tree

动态维护一个森林

```

1  const int maxn = "Edit";
2  struct LCT
3  {
4      int val[maxn], sum[maxn]; // 基于点权
5      int rev[maxn], ch[maxn][2], fa[maxn];
6      int stk[maxn];
7      inline void init(int n)
8      { // 初始化点权
9          for (int i = 1; i <= n; i++) scanf("%d", val + i);
10     }
11     inline bool isroot(int x) { return ch[fa[x]][0] != x && ch[fa[x]][1] != x; }
12     inline bool get(int x) { return ch[fa[x]][1] == x; }
13     void pushdown(int x)
14     {
15         if (!rev[x]) return;
16         swap(ch[x][0], ch[x][1]);
17         if (ch[x][0]) rev[ch[x][0]] ^= 1;
18         if (ch[x][1]) rev[ch[x][1]] ^= 1;
19         rev[x] ^= 1;
20     }
21     void pushup(int x) { sum[x] = val[x] + sum[ch[x][0]] + sum[ch[x][1]]; }
22     void rotate(int x)
23     {
24         int y = fa[x], z = fa[fa[x]], d = get(x);
25         if (!isroot(y)) ch[z][get(y)] = x;
26         fa[x] = z;
27         ch[y][d] = ch[x][d ^ 1], fa[ch[y][d]] = y;
28         ch[x][d ^ 1] = y, fa[y] = x;
29         pushup(y), pushup(x);
30     }
31     void splay(int x)
32     {
33         int top = 0;
34         stk[++top] = x;
35         for (int i = x; !isroot(i); i = fa[i]) stk[++top] = fa[i];
36         for (int i = top; i; i--) pushdown(stk[i]);
37         for (int f; !isroot(x); rotate(x))
38             if (!isroot(f = fa[x])) rotate(get(x) == get(f) ? f : x);
39     }
40     void access(int x)
41     {
42         for (int y = 0; x; y = x, x = fa[x]) splay(x), ch[x][1] = y, pushup(x);
43     }
44     int find(int x) { access(x), splay(x); while (ch[x][0]) x = ch[x][0]; return x; }
45     void makeroot(int x) { access(x), splay(x), rev[x] ^= 1; }
46     void link(int x, int y) { makeroot(x), fa[x] = y, splay(x); }
47     void cut(int x, int y) { makeroot(x), access(y), splay(y), fa[x] = ch[y][0] = 0; }
48     void update(int x, int v) { val[x] = v, access(x), splay(x); }
49     int query(int x, int y) { makeroot(y), access(x), splay(x); return sum[x]; }
50 };

```

4 Graph Theory

4.1 Union-Find Set

```

1  const int maxn = "Edit";
2  int n, fa[maxn], ra[maxn];
3  void init()
4  {
5      clr(ra, 0);
6      iota(fa, fa + n, 0);
7  }
8  int find(int x) { return fa[x] != x ? fa[x] = find(fa[x]) : x; }
9  void unite(int x, int y)
10 {
11     x = find(x), y = find(y);
12     if (x == y) return;
13     if (ra[x] < ra[y])
14         fa[x] = y;
15     else
16     {
17         fa[y] = x;
18         if (ra[x] == ra[y]) ra[x]++;
19     }
20 }
21 bool same(int x, int y) { return find(x) == find(y); }
```

4.2 Minimal Spanning Tree

4.2.1 Kruskal

```

1  typedef pair<int, PII> Edge;
2  vector<Edge> G;
3  void add_edge(int u, int v, int d) { G.pb(mp(d, mp(u, v))); }
4  int Kruskal(int n)
5  {
6      init(n); // 并查集初始化
7      sort(G.begin(), G.end());
8      int num = 0, ret = 0;
9      for (auto& e : G)
10     {
11         int x = e.Y.X, y = e.Y.Y;
12         int d = e.X;
13         if (!same(x, y))
14         {
15             unite(x, y);
16             num++;
17             ret += d;
18         }
19         if (num == n - 1) break;
20     }
21     return ret;
22 }
```

4.2.2 Prim

```

1  // 耗费矩阵cost[], 标号从0开始, 0~n-1
2  // 返回最小生成树的权值, 返回-1表示原图不连通
3  const int maxn = "Edit";
```



```

4 bool vis[maxn];
5 int lowc[maxn];
6 int Prim(int cost[][maxn], int n)
7 {
8     int ans = 0;
9     clr(vis, 0);
10    vis[0] = 1;
11    for (int i = 1; i < n; i++)
12        lowc[i] = cost[0][i];
13    for (int i = 1; i < n; i++)
14    {
15        int minc = INF;
16        int p = -1;
17        for (int j = 0; j < n; j++)
18            if (!vis[j] && minc > lowc[j])
19            {
20                minc = lowc[j];
21                p = j;
22            }
23        if (minc == INF) return -1;
24        vis[p] = 1;
25        ans += minc;
26        for (int j = 0; j < n; j++)
27            if (!vis[j] && lowc[j] > cost[p][j])
28                lowc[j] = cost[p][j];
29    }
30    return ans;
31 }

```

4.3 Shortest Path

4.3.1 Dijkstra

```

1 // pair<权值, 点>
2 // 记得初始化
3 const int maxn = "Edit";
4 typedef pair<int, int> PII;
5 typedef vector<PII> VII;
6 VII G[maxn];
7 int vis[maxn], dis[maxn];
8 void init(int n)
9 {
10    for (int i = 0; i < n; i++) G[i].clear();
11 }
12 void add_edge(int u, int v, int w) { G[u].pb(mp(w, v)); }
13 void Dijkstra(int s, int n)
14 {
15    clr(vis, 0), clr(dis, 0x3f);
16    dis[s] = 0;
17    priority_queue<PII, VII, greater<PII> > q;
18    q.push(mp(dis[s], s));
19    while (!q.empty())
20    {
21        PII t = q.top();
22        int x = t.Y;
23        q.pop();
24        if (vis[x]) continue;
25        vis[x] = 1;
26        for (int i = 0; i < G[x].size(); i++)

```

```

27     {
28         int y = G[x][i].Y, w = G[x][i].X;
29         if (!vis[y] && dis[y] > dis[x] + w)
30         {
31             dis[y] = dis[x] + w;
32             q.push(mp(dis[y], y));
33         }
34     }
35 }
36 }

```

4.3.2 Bellman-Ford

```

1  // G[u] = mp(v, w)
2  // BellmanFord()返回0表示存在负环
3  const int maxn = "Edit";
4  vector<PII> G[maxn];
5  bool vis[maxn];
6  int dis[maxn];
7  int inqueue[maxn];
8  void init(int n)
9  {
10     for (int i = 0; i < n; i++) G[i].clear();
11 }
12 void add_edge(int u, int v, int w) { G[u].pb(mp(v, w)); }
13 bool BellmanFord(int s, int n)
14 {
15     clr(vis, 0), clr(dis, 0x3f), clr(inqueue, 0);
16     dis[s] = 0;
17     queue<int> q; // 待优化的节点入队
18     q.push(s);
19     vis[s] = true, ++inqueue[s];
20     while (!q.empty())
21     {
22         int x = q.front();
23         q.pop();
24         vis[x] = false;
25         for (int i = 0; i < G[x].size(); i++)
26         {
27             int y = G[x][i].X, w = G[x][i].Y;
28             if (dis[y] > dis[x] + w)
29             {
30                 dis[y] = dis[x] + w;
31                 if (!vis[y])
32                 {
33                     q.push(y);
34                     vis[y] = true;
35                     if (++inqueue[y] >= n) return 0;
36                 }
37             }
38         }
39     }
40     return 1;
41 }

```

4.3.3 Floyd

$O(n^3)$ 求出任意两点间最短路
邻接矩阵存图需注意判断重边

```

1  const int maxn = "Edit";
2  int G[maxn][maxn];
3  void init(int n)
4  {
5      clr(G, 0x3f);
6      for (int i = 0; i < n; i++) G[i][i] = 0;
7  }
8  void add_edge(int u, int v, int w) { G[u][v] = min(G[u][v], w); }
9  void Floyd(int n)
10 {
11     for (int k = 0; k < n; k++)
12         for (int i = 0; i < n; i++)
13             for (int j = 0; j < n; j++)
14                 G[i][j] = min(G[i][j], G[i][k] + G[k][j]);
15 }

```

4.4 Topo Sort

存图前记得初始化
Ans 排序结果, G 邻接表, deg 入度, map 用于判断重边
排序成功返回 1, 存在环返回 0

```

1  const int maxn = "Edit";
2  int Ans[maxn];
3  vector<int> G[maxn];
4  int deg[maxn];
5  map<PII, bool> S;
6  void init(int n)
7  {
8      S.clear();
9      for (int i = 0; i < n; i++) G[i].clear();
10     clr(deg, 0), clr(Ans, 0);
11 }
12 void add_edge(int u, int v)
13 {
14     if (S[mp(u, v)]) return;
15     G[u].pb(v), S[mp(u, v)] = 1, deg[v]++;
16 }
17 bool Toposort(int n)
18 {
19     int tot = 0;
20     queue<int> q;
21     for (int i = 0; i < n; ++i)
22         if (deg[i] == 0) q.push(i);
23     while (!q.empty())
24     {
25         int u = q.front();
26         q.pop();
27         Ans[tot++] = u;
28         for (auto& v : G[u])
29             if (--deg[v] == 0) q.push(v);
30     }
31     if (tot < n - 1) return false;
32     return true;
33 }

```

4.5 LCA

4.5.1 Tarjan

Tarjan 离线算法

时间复杂度 $O(n + q)$

```

1  const int maxn = "Edit";
2  int par[maxn];           //并查集
3  int ans[maxn];           //存储答案
4  vector<int> G[maxn];      //邻接表
5  vector<PII> query[maxn]; //存储查询信息
6  bool vis[maxn];          //是否被遍历
7  inline void init(int n)
8  {
9      for (int i = 1; i <= n; i++)
10     {
11         G[i].clear(), query[i].clear();
12         par[i] = i, vis[i] = 0;
13     }
14 }
15 inline void add_edge(int u, int v) { G[u].pb(v); }
16 inline void add_query(int id, int u, int v)
17 {
18     query[u].pb(mp(v, id));
19     query[v].pb(mp(u, id));
20 }
21 void tarjan(int u)
22 {
23     vis[u] = 1;
24     for (auto& v : G[u])
25     {
26         if (vis[v]) continue;
27         tarjan(v);
28         unite(u, v);
29     }
30     for (auto& q : query[u])
31     {
32         int &v = q.X, &id = q.Y;
33         if (!vis[v]) continue;
34         ans[id] = find(v);
35     }
36 }

```

4.5.2 DFS+ST

DFS+ST 在线算法

时间复杂度 $O(n \log n + q)$

```

1  const int maxn = "Edit";
2  vector<int> G[maxn], sp;
3  int dep[maxn], dfn[maxn];
4  PII dp[21][maxn << 1];
5  void init(int n)
6  {
7      for (int i = 0; i < n; i++) G[i].clear();
8      sp.clear();
9  }
10 void dfs(int u, int fa)

```

```

11 {
12     dep[u] = dep[fa] + 1;
13     dfn[u] = sp.size();
14     sp.push_back(u);
15     for (auto& v : G[u])
16     {
17         if (v != fa) dfs(v, u);
18         sp.push_back(u);
19     }
20 }
21 void initrmq()
22 {
23     int n = sp.size();
24     for (int i = 0; i < n; i++) dp[0][i] = {dfn[sp[i]], sp[i]};
25     for (int i = 1; (1 << i) <= n; i++)
26         for (int j = 0; j + (1 << i) - 1 < n; j++)
27             dp[i][j] = min(dp[i - 1][j], dp[i - 1][j + (1 << (i - 1))]);
28 }
29 int lca(int u, int v)
30 {
31     int l = dfn[u], r = dfn[v];
32     if (l > r) swap(l, r);
33     int k = 31 - __builtin_clz(r - l + 1);
34     return min(dp[k][l], dp[k][r - (1 << k) + 1]).Y;
35 }

```

4.6 Depth-First Traversal

4.6.1 Biconnected-Component

```

1 //割顶的bccno无意义
2 const int maxn = "Edit";
3 int pre[maxn], iscut[maxn], bccno[maxn], dfs_clock, bcc_cnt;
4 vector<int> G[maxn], bcc[maxn];
5 stack<PII> s;
6 void init(int n)
7 {
8     for (int i = 0; i < n; i++) G[i].clear();
9 }
10 inline void add_edge(int u, int v) { G[u].pb(v), G[v].pb(u); }
11 int dfs(int u, int fa)
12 {
13     int lowu = pre[u] = ++dfs_clock;
14     int child = 0;
15     for (auto& v : G[u])
16     {
17         PII e = mp(u, v);
18         if (!pre[v])
19         {
20             //没有访问过v
21             s.push(e);
22             child++;
23             int lowv = dfs(v, u);
24             lowu = min(lowu, lowv); //用后代的low函数更新自己
25             if (lowv >= pre[u])
26             {
27                 iscut[u] = true;
28                 bcc_cnt++;
29                 bcc[bcc_cnt].clear(); //注意! bcc从1开始编号

```

```

30         for (;;)
31         {
32             PII x = s.top();
33             s.pop();
34             if (bccno[x.X] != bcc_cnt)
35                 bcc[bcc_cnt].pb(x.X), bcc[x.X] = bcc_cnt;
36             if (bccno[x.Y] != bcc_cnt)
37                 bcc[bcc_cnt].pb(x.Y), bcc[x.Y] = bcc_cnt;
38             if (x.X == u && x.Y == v) break;
39         }
40     }
41 }
42 else if (pre[v] < pre[u] && v != fa)
43 {
44     s.push(e);
45     lowu = min(lowu, pre[v]); //用反向边更新自己
46 }
47 }
48 if (fa < 0 && child == 1) iscut[u] = 0;
49 return lowu;
50 }
51 void find_bcc(int n)
52 {
53     //调用结束后s保证为空, 所以不用清空
54     clr(pre, 0), clr(iscut, 0), clr(bccno, 0);
55     dfs_clock = bcc_cnt = 0;
56     for (int i = 0; i < n; i++)
57         if (!pre[i]) dfs(i, -1);
58 }

```

4.6.2 Strongly Connected Component

```

1  const int maxn = "Edit";
2  vector<int> G[maxn];
3  int pre[maxn], lowlink[maxn], sccno[maxn], dfs_clock, scc_cnt;
4  stack<int> S;
5  inline void init(int n)
6  {
7      for (int i = 0; i < n; i++) G[i].clear();
8  }
9  inline void add_edge(int u, int v) { G[u].pb(v); }
10 void dfs(int u)
11 {
12     pre[u] = lowlink[u] = ++dfs_clock;
13     S.push(u);
14     for (auto& v : G[u])
15     {
16         if (!pre[v])
17         {
18             dfs(v);
19             lowlink[u] = min(lowlink[u], lowlink[v]);
20         }
21         else if (!sccno[v])
22             lowlink[u] = min(lowlink[u], pre[v]);
23     }
24     if (lowlink[u] == pre[u])
25     {
26         scc_cnt++;

```

```

27     for (;;)
28     {
29         int x = S.top();
30         S.pop();
31         sccno[x] = scc_cnt;
32         if (x == u) break;
33     }
34 }
35 }
36 void find_scc(int n)
37 {
38     dfs_clock = 0, scc_cnt = 0;
39     clr(sccno, 0), clr(pre, 0);
40     for (int i = 0; i < n; i++)
41         if (!pre[i]) dfs(i);
42 }

```

4.6.3 2-SAT

```

1 struct TwoSAT
2 {
3     int n;
4     vector<int> G[maxn << 1];
5     bool mark[maxn << 1];
6     int S[maxn << 1], c;
7     void init(int n)
8     {
9         this->n = n;
10        for (int i = 0; i < (n << 1); i++) G[i].clear();
11        clr(mark, 0);
12    }
13    bool dfs(int x)
14    {
15        if (mark[x ^ 1]) return false;
16        if (mark[x]) return true;
17        mark[x] = true;
18        S[c++] = x;
19        for (auto& y : G[x])
20            if (!dfs(y)) return false;
21        return true;
22    }
23    //x = xval or y = yval
24    void add_clause(int x, int xval, int y, int yval)
25    {
26        x = (x << 1) + xval;
27        y = (y << 1) + yval;
28        G[x ^ 1].pb(y);
29        G[y ^ 1].pb(x);
30    }
31    bool solve()
32    {
33        for (int i = 0; i < (n << 1); i += 2)
34            if (!mark[i] && !mark[i + 1])
35            {
36                c = 0;
37                if (!dfs(i))
38                {
39                    while (c > 0) mark[S[--c]] = false;

```

```

40         if (!dfs(i + 1)) return false;
41     }
42 }
43 return true;
44 }
45 };

```

4.7 Euler Path

- 基本概念:
 - 欧拉图: 能够没有重复地一次遍历所有边的图。(必须是连通图)
 - 欧拉路: 上述遍历的路径就是欧拉路。
 - 欧拉回路: 若欧拉路是闭合的 (一个圈, 从起点开始遍历最终又回到起点), 则为欧拉回路。
- 无向图 G 有欧拉路径的充要条件
 - G 是连通图
 - G 中奇顶点 (连接边的数量为奇数) 的数量等于 0 或 2。
- 无向图 G 有欧拉回路的充要条件
 - G 是连通图
 - G 中每个顶点都是偶顶点
- 有向图 G 有欧拉路径的充要条件
 - G 是连通图
 - u 的出度比入度大 1, v 的出度比入度小 1, 其他所有点出度和入度相同。(u 为起点, v 为终点)
- 有向图 G 有欧拉回路的充要条件
 - G 是连通图
 - G 中每个顶点的出度等于入度

4.7.1 Fleury

若有两个点的度数是奇数, 则此时这两个点只能作为欧拉路径的起点和终点。

```

1  const int maxn = "Edit";
2  int G[maxn][maxn];
3  int deg[maxn][maxn];
4  vector<int> Ans;
5  inline void init() { clr(G, 0), clr(deg, 0); }
6  inline void AddEdge(int u, int v) { deg[u]++, deg[v]++, G[u][v]++, G[v][u]++; }
7  void Fleury(int s)
8  {
9      for (int i = 0; i < n; i++)
10         if (G[s][i])
11             {
12                 G[s][i]--, G[i][s]--;
13                 Fleury(i);
14             }
15     Ans.pb(s);
16 }

```


4.8 Bipartite Graph Matching

1. 一个二分图中的最大匹配数等于这个图中的最小点覆盖数

2. 最小路径覆盖 = $|G|$ - 最大匹配数

在一个 $N \times N$ 的有向图中, 路径覆盖就是在图中找一些路径, 使之覆盖了图中的所有顶点, 且任何一个顶点有且只有一条路径与之关联;

(如果把这些路径中的每条路径从它的起始点走到它的终点, 那么恰好可以经过图中的每个顶点一次且仅一次); 如果不考虑图中存在回路, 那么每每条路径就是一个弱连通子集.

由上面可以得出:

- (a) 一个单独的顶点是一条路径;
- (b) 如果存在一路径 p_1, p_2, \dots, p_k , 其中 p_1 为起点, p_k 为终点, 那么在覆盖图中, 顶点 p_1, p_2, \dots, p_k 不再与其它顶点之间存在有向边.

最小路径覆盖就是找出最小的路径条数, 使之成为 G 的一个路径覆盖.

路径覆盖与二分图匹配的关系: 最小路径覆盖 = $|G|$ - 最大匹配数;

3. 二分图最大独立集 = 顶点数 - 二分图最大匹配

独立集: 图中任意两个顶点都不相连的顶点集合。

4.8.1 Hungry(Matrix)

时间复杂度: $O(VE)$.

顶点编号从 0 开始

```

1  const int maxn = "Edit";
2  int uN, vN;           //uN是匹配左边的顶点数,vN是匹配右边的顶点数
3  int g[maxn][maxn];   //邻接矩阵g[i][j]表示i->j的有向边就可以了,是左边向右边的匹配
4  int linker[maxn];
5  bool used[maxn];
6  bool dfs(int u)
7  {
8      for (int v = 0; v < vN; v++)
9          if (g[u][v] && !used[v])
10             {
11                 used[v] = true;
12                 if (linker[v] == -1 || dfs(linker[v]))
13                     {
14                         linker[v] = u;
15                         return true;
16                     }
17             }
18     return false;
19 }
20 int hungary()
21 {
22     int res = 0;
23     clr(linker, -1);
24     for (int u = 0; u < uN; u++)
25     {
26         clr(used, 0);
27         if (dfs(u)) res++;
28     }
29     return res;
30 }
```

4.8.2 Hungry(List)

使用前用 `init()` 进行初始化

加边使用函数 `addedge(u,v)`

```

1  const int maxn = "Edit";
2  int n;
3  vector<int> G[maxn];
4  int linker[maxn];
5  bool used[maxn];
6  inline void init(int n)
7  {
8      for (int i = 0; i < n; i++) G[i].clear();
9  }
10 inline void addedge(int u, int v) { G[u].pb(v); }
11 bool dfs(int u)
12 {
13     for (auto& v : G[u])
14     {
15         if (!used[v])
16         {
17             used[v] = true;
18             if (linker[v] == -1 || dfs(linker[v]))
19             {
20                 linker[v] = u;
21                 return true;
22             }
23         }
24     }
25     return false;
26 }
27 int hungary()
28 {
29     int ans = 0;
30     clr(linker, -1);
31     for (int u = 0; u < n; u++)
32     {
33         clr(used, 0);
34         if (dfs(u)) ans++;
35     }
36     return ans;
37 }

```

4.8.3 Hopcroft-Carp

复杂度 $O(\sqrt{n} * E)$

uN 为左端的顶点数, 使用前赋值 (点编号 0 开始)

```

1  const int maxn = "Edit";
2  vector<int> G[maxn];
3  int uN;
4  int Mx[maxn], My[maxn];
5  int dx[maxn], dy[maxn];
6  int dis;
7  bool used[maxn];
8  inline void init(int n)
9  {
10     for (int i = 0; i < n; i++) G[i].clear();

```

```

11 }
12 inline void addedge(int u, int v) { G[u].pb(v); }
13 bool bfs()
14 {
15     queue<int> q;
16     dis = INF;
17     clr(dx, -1), clr(dy, -1);
18     for (int i = 0; i < uN; i++)
19         if (Mx[i] == -1)
20             q.push(i), dx[i] = 0;
21     while (!q.empty())
22     {
23         int u = q.front();
24         q.pop();
25         if (dx[u] > dis) break;
26         for (auto& v : G[u])
27         {
28             if (dy[v] == -1)
29             {
30                 dy[v] = dx[u] + 1;
31                 if (My[v] == -1)
32                     dis = dy[v];
33                 else
34                 {
35                     dx[My[v]] = dy[v] + 1;
36                     q.push(My[v]);
37                 }
38             }
39         }
40     }
41     return dis != INF;
42 }
43 bool dfs(int u)
44 {
45     for (auto& v : G[u])
46     {
47         if (!used[v] && dy[v] == dx[u] + 1)
48         {
49             used[v] = true;
50             if (My[v] != -1 && dy[v] == dis) continue;
51             if (My[v] == -1 || dfs(My[v]))
52             {
53                 My[v] = u, Mx[u] = v;
54                 return true;
55             }
56         }
57     }
58     return false;
59 }
60 int MaxMatch()
61 {
62     int res = 0;
63     clr(Mx, -1), clr(My, -1);
64     while (bfs())
65     {
66         clr(used, false);
67         for (int i = 0; i < uN; i++)
68             if (Mx[i] == -1 && dfs(i)) res++;
69     }

```

```

70     return res;
71 }

```

4.8.4 Hungry(Multiple)

```

1  const int maxn = "Edit";
2  const int maxm = "Edit";
3  int uN, vN;           //u,v的数目,使用前面必须赋值
4  int g[maxn][maxm];    //邻接矩阵
5  int linker[maxm][maxn];
6  bool used[maxm];
7  int num[maxm];        //右边最大的匹配数
8  bool dfs(int u)
9  {
10     for (int v = 0; v < vN; v++)
11         if (g[u][v] && !used[v])
12             {
13                 used[v] = true;
14                 if (linker[v][0] < num[v])
15                     {
16                         linker[v][++linker[v][0]] = u;
17                         return true;
18                     }
19                 for (int i = 1; i <= num[v]; i++)
20                     if (dfs(linker[v][i]))
21                         {
22                             linker[v][i] = u;
23                             return true;
24                         }
25             }
26     return false;
27 }
28 int hungary()
29 {
30     int res = 0;
31     for (int i = 0; i < vN; i++) linker[i][0] = 0;
32     for (int u = 0; u < uN; u++)
33     {
34         clr(used, 0);
35         if (dfs(u)) res++;
36     }
37     return res;
38 }

```

4.8.5 Kuhn-Munkres

```

1  const int maxn = "Edit";
2  int nx, ny;           //两边的点数
3  int g[maxn][maxn];    //二分图描述
4  int linker[maxn], lx[maxn], ly[maxn]; //y中各点匹配状态,x,y中的点标号
5  int slack[N];
6  bool visx[N], visy[N];
7  bool dfs(int x)
8  {
9     visx[x] = true;
10     for (int y = 0; y < ny; y++)
11     {
12         if (visy[y]) continue;

```

```

13     int tmp = lx[x] + ly[y] - g[x][y];
14     if (tmp == 0)
15     {
16         visy[y] = true;
17         if (linker[y] == -1 || dfs(linker[y]))
18         {
19             linker[y] = x;
20             return true;
21         }
22     }
23     else if (slack[y] > tmp)
24         slack[y] = tmp;
25 }
26 return false;
27 }
28 int KM()
29 {
30     clr(linker, -1), clr(ly, 0);
31     for (int i = 0; i < nx; i++)
32     {
33         lx[i] = -INF;
34         for (int j = 0; j < ny; j++)
35             if (g[i][j] > lx[i]) lx[i] = g[i][j];
36     }
37     for (int x = 0; x < nx; x++)
38     {
39         clr(slack, 0x3f);
40         for (;;)
41         {
42             clr(visx, 0), clr(visy, 0);
43             if (dfs(x)) break;
44             int d = INF;
45             for (int i = 0; i < ny; i++)
46                 if (!visy[i] && d > slack[i]) d = slack[i];
47             for (int i = 0; i < nx; i++)
48                 if (visx[i]) lx[i] -= d;
49             for (int i = 0; i < ny; i++)
50                 if (visy[i])
51                     ly[i] += d;
52                 else
53                     slack[i] -= d;
54         }
55     }
56     int res = 0;
57     for (int i = 0; i < ny; i++)
58         if (~linker[i]) res += g[linker[i]][i];
59     return res;
60 }

```

4.9 Network Flow

```

1 struct Edge
2 {
3     int from, to, cap, flow;
4     Edge(int u, int v, int c, int f)
5         : from(u), to(v), cap(c), flow(f) {}
6 };

```

```

1 struct Edge
2 {
3     int from, to, cap, flow, cost;
4     Edge(int u, int v, int c, int f, int w)
5         : from(u), to(v), cap(c), flow(f), cost(w) {}
6 };

```

建模技巧

二分图带权最大独立集。给出一个二分图，每个结点上有一个正权值。要求选出一些点，使得这些点之间没有边相连，且权值和最大。

解：在二分图的基础上添加源点 S 和汇点 T ，然后从 S 向所有 X 集中的点连一条边，所有 Y 集中的点向 T 连一条边，容量均为该点的权值。 X 结点与 Y 结点之间的边的容量均为无穷大。这样，对于图中的任意一个割，将割中的边对应的结点删掉就是一个符合要求的解，权和为所有权减去割的容量。因此，只需要求出最小割，就能求出最大权和。

公平分配问题。把 m 个任务分配给 n 个处理器。其中每个任务有两个候选处理器，可以任选一个分配。要求所有处理器中，任务数最多的那个处理器所分配的任务数尽量少。不同任务的候选处理器集 $\{p_1, p_2\}$ 保证不同。

解：本题有一个比较明显的二分图模型，即 X 结点是任务， Y 结点是处理器。二分答案 x ，然后构图，首先从源点 S 出发向所有的任务结点引一条边，容量等于 1，然后从每个任务结点出发引两条边，分别到达它所能分配到的两个处理器结点，容量为 1，最后从每个处理器结点出发引一条边到汇点 T ，容量为 x ，表示选择该处理器的任务不能超过 x 。这样网络中的每个单位流量都是从 S 流到一个任务结点，再到处理器结点，最后到汇点 T 。只有当网络中的总流量等于 m 时才意味着所有任务都选择了一个处理器。这样，我们通过 $O(\log m)$ 次最大流便算出了答案。

区间 k 覆盖问题。数轴上有一些带权值的左闭右开区间。选出权和尽量大的一些区间，使得任意一个数最多被 k 个区间覆盖。

解：本题可以用最小费用流解决，构图方法是把每个数作为一个结点，然后对于权值为 w 的区间 $[u, v)$ 加边 $u \rightarrow v$ ，容量为 1，费用为 $-w$ 。再对所有相邻的点加边 $i \rightarrow i+1$ ，容量为 k ，费用为 0。最后，求最左点到最右点的最小费用最大流即可，其中每个流量对应一组互不相交的区间。如果数值范围太大，可以先进行离散化。

最大闭合子图。给定带权图 G （权值可正可负），求一个权和最大的点集，使得起点在该点集中的任意弧，终点也在该点集中。

解：新增附加源 s 和附加汇 t ，从 s 向所有正权点引一条边，容量为权值；从所有负权点向汇点引一条边，容量为权值的相反数。求出最小割以后， $S - \{s\}$ 就是最大闭合子图。

4.9.1 EdmondKarp

```

1 const int maxn = "Edit";
2 struct EdmondsKarp //时间复杂度O(V*E*E)
3 {
4     int n, m;
5     vector<Edge> edges; //边数的两倍
6     vector<int> G[maxn]; //邻接表, G[i][j]表示节点i的第j条边在e数组中的序号
7     int a[maxn]; //起点到i的可改进量
8     int p[maxn]; //最短路树上p的入弧编号
9     void init(int n)
10    {
11        for (int i = 0; i < n; i++) G[i].clear();
12        edges.clear();
13    }
14    void AddEdge(int from, int to, int cap)
15    {
16        edges.pb(Edge(from, to, cap, 0));
17        edges.pb(Edge(to, from, 0, 0)); //反向弧
18        m = edges.size();

```

```

19     G[from].pb(m - 2);
20     G[to].pb(m - 1);
21 }
22 int Maxflow(int s, int t)
23 {
24     int flow = 0;
25     for (;;)
26     {
27         clr(a, 0);
28         queue<int> q;
29         q.push(s);
30         a[s] = INF;
31         while (!q.empty())
32         {
33             int x = q.front();
34             q.pop();
35             for (int i = 0; i < G[x].size(); i++)
36             {
37                 Edge& e = edges[G[x][i]];
38                 if (!a[e.to] && e.cap > e.flow)
39                 {
40                     p[e.to] = G[x][i];
41                     a[e.to] = min(a[x], e.cap - e.flow);
42                     q.push(e.to);
43                 }
44             }
45             if (a[t]) break;
46         }
47         if (!a[t]) break;
48         for (int u = t; u != s; u = edges[p[u]].from)
49         {
50             edges[p[u]].flow += a[t];
51             edges[p[u] ^ 1].flow -= a[t];
52         }
53         flow += a[t];
54     }
55     return flow;
56 }
57 };

```

4.9.2 Dinic

```

1  const int maxn = "Edit";
2  struct Dinic
3  {
4      int n, m, s, t;           //结点数, 边数 (包括反向弧), 源点编号和汇点编号
5      vector<Edge> edges;       //边表。edge[e]和edge[e^1]互为反向弧
6      vector<int> G[maxn];     //邻接表, G[i][j]表示节点i的第j条边在e数组中的序号
7      bool vis[maxn];          //BFS使用
8      int d[maxn];             //从起点到i的距离
9      int cur[maxn];           //当前弧下标
10     void init(int n)
11     {
12         this->n = n;
13         for (int i = 0; i < n; i++) G[i].clear();
14         edges.clear();
15     }
16     void AddEdge(int from, int to, int cap)

```

```

17     {
18         edges.pb(Edge(from, to, cap, 0));
19         edges.pb(Edge(to, from, 0, 0));
20         m = edges.size();
21         G[from].pb(m - 2);
22         G[to].pb(m - 1);
23     }
24     bool BFS()
25     {
26         clr(vis, 0);
27         clr(d, 0);
28         queue<int> q;
29         q.push(s);
30         d[s] = 0;
31         vis[s] = 1;
32         while (!q.empty())
33         {
34             int x = q.front();
35             q.pop();
36             for (int i = 0; i < G[x].size(); i++)
37             {
38                 Edge& e = edges[G[x][i]];
39                 if (!vis[e.to] && e.cap > e.flow)
40                 {
41                     vis[e.to] = 1;
42                     d[e.to] = d[x] + 1;
43                     q.push(e.to);
44                 }
45             }
46         }
47         return vis[t];
48     }
49     int DFS(int x, int a)
50     {
51         if (x == t || a == 0) return a;
52         int flow = 0, f;
53         for (int& i = cur[x]; i < G[x].size(); i++)
54         {
55             //从上次考虑的弧
56             Edge& e = edges[G[x][i]];
57             if (d[x] + 1 == d[e.to] && (f = DFS(e.to, min(a, e.cap - e.flow))) > 0)
58             {
59                 e.flow += f;
60                 edges[G[x][i] ^ 1].flow -= f;
61                 flow += f;
62                 a -= f;
63                 if (a == 0) break;
64             }
65         }
66         return flow;
67     }
68     int Maxflow(int s, int t)
69     {
70         this->s = s;
71         this->t = t;
72         int flow = 0;
73         while (BFS())
74         {
75             clr(cur, 0);

```



```

76         flow += DFS(s, INF);
77     }
78     return flow;
79 }
80 };

```

4.9.3 ISAP

```

1  const int maxn = "Edit";
2  struct ISAP
3  {
4      int n, m, s, t;           //结点数, 边数 (包括反向弧), 源点编号和汇点编号
5      vector<Edge> edges;       //边表。edges[e]和edges[e^1]互为反向弧
6      vector<int> G[maxn];      //邻接表, G[i][j]表示结点i的第j条边在e数组中的序号
7      bool vis[maxn];          //BFS使用
8      int d[maxn];             //起点到i的距离
9      int cur[maxn];           //当前弧下标
10     int p[maxn];              //可增广路上的一条弧
11     int num[maxn];            //距离标号计数
12     void init(int n)
13     {
14         this->n = n;
15         for (int i = 0; i < n; i++) G[i].clear();
16         edges.clear();
17     }
18     void AddEdge(int from, int to, int cap)
19     {
20         edges.pb(Edge(from, to, cap, 0));
21         edges.pb(Edge(to, from, 0, 0));
22         int m = edges.size();
23         G[from].pb(m - 2);
24         G[to].pb(m - 1);
25     }
26     int Augument()
27     {
28         int x = t, a = INF;
29         while (x != s)
30         {
31             Edge& e = edges[p[x]];
32             a = min(a, e.cap - e.flow);
33             x = edges[p[x]].from;
34         }
35         x = t;
36         while (x != s)
37         {
38             edges[p[x]].flow += a;
39             edges[p[x] ^ 1].flow -= a;
40             x = edges[p[x]].from;
41         }
42         return a;
43     }
44     void BFS()
45     {
46         clr(vis, 0);
47         clr(d, 0);
48         queue<int> q;
49         q.push(t);
50         d[t] = 0;

```

```

51     vis[t] = 1;
52     while (!q.empty())
53     {
54         int x = q.front();
55         q.pop();
56         int len = G[x].size();
57         for (int i = 0; i < len; i++)
58         {
59             Edge& e = edges[G[x][i]];
60             if (!vis[e.from] && e.cap > e.flow)
61             {
62                 vis[e.from] = 1;
63                 d[e.from] = d[x] + 1;
64                 q.push(e.from);
65             }
66         }
67     }
68 }
69 int Maxflow(int s, int t)
70 {
71     this->s = s;
72     this->t = t;
73     int flow = 0;
74     BFS();
75     clr(num, 0);
76     for (int i = 0; i < n; i++)
77         if (d[i] < INF) num[d[i]]++;
78     int x = s;
79     clr(cur, 0);
80     while (d[s] < n)
81     {
82         if (x == t)
83         {
84             flow += Augument();
85             x = s;
86         }
87         int ok = 0;
88         for (int i = cur[x]; i < G[x].size(); i++)
89         {
90             Edge& e = edges[G[x][i]];
91             if (e.cap > e.flow && d[x] == d[e.to] + 1)
92             {
93                 ok = 1;
94                 p[e.to] = G[x][i];
95                 cur[x] = i;
96                 x = e.to;
97                 break;
98             }
99         }
100         if (!ok) //Retreat
101         {
102             int m = n - 1;
103             for (int i = 0; i < G[x].size(); i++)
104             {
105                 Edge& e = edges[G[x][i]];
106                 if (e.cap > e.flow) m = min(m, d[e.to]);
107             }
108             if (--num[d[x]] == 0) break; //gap优化
109             num[d[x] = m + 1]++;

```

```

110         cur[x] = 0;
111         if (x != s) x = edges[p[x]].from;
112     }
113 }
114 return flow;
115 }
116 };

```

4.9.4 MinCost MaxFlow

```

1  const int maxn = "Edit";
2  struct MCMF
3  {
4      int n, m;
5      vector<Edge> edges;
6      vector<int> G[maxn];
7      int inq[maxn]; //是否在队列中
8      int d[maxn]; //bellmanford
9      int p[maxn]; //上一条弧
10     int a[maxn]; //可改进量
11     void init(int n)
12     {
13         this->n = n;
14         for (int i = 0; i < n; i++) G[i].clear();
15         edges.clear();
16     }
17     void AddEdge(int from, int to, int cap, int cost)
18     {
19         edges.pb(Edge(from, to, cap, 0, cost));
20         edges.pb(Edge(to, from, 0, 0, -cost));
21         m = edges.size();
22         G[from].pb(m - 2);
23         G[to].pb(m - 1);
24     }
25     bool BellmanFord(int s, int t, int& flow, ll& cost)
26     {
27         for (int i = 0; i < n; i++) d[i] = INF;
28         clr(inq, 0);
29         d[s] = 0;
30         inq[s] = 1;
31         p[s] = 0;
32         a[s] = INF;
33         queue<int> q;
34         q.push(s);
35         while (!q.empty())
36         {
37             int u = q.front();
38             q.pop();
39             inq[u] = 0;
40             for (int i = 0; i < G[u].size(); i++)
41             {
42                 Edge& e = edges[G[u][i]];
43                 if (e.cap > e.flow && d[e.to] > d[u] + e.cost)
44                 {
45                     d[e.to] = d[u] + e.cost;
46                     p[e.to] = G[u][i];
47                     a[e.to] = min(a[u], e.cap - e.flow);
48                     if (!inq[e.to])

```

```
49         {
50             q.push(e.to);
51             inq[e.to] = 1;
52         }
53     }
54 }
55 }
56 if (d[t] == INF) return false; // 当没有可增广的路时退出
57 flow += a[t];
58 cost += (ll)d[t] * (ll)a[t];
59 for (int u = t; u != s; u = edges[p[u]].from)
60 {
61     edges[p[u]].flow += a[t];
62     edges[p[u] ^ 1].flow -= a[t];
63 }
64 return true;
65 }
66 int MincostMaxflow(int s, int t, ll& cost)
67 {
68     int flow = 0;
69     cost = 0;
70     while (BellmanFord(s, t, flow, cost));
71     return flow;
72 }
73 };
```

5 Computational Geometry

5.1 Basic Function

```

1 #define zero(x) ((fabs(x) < eps ? 1 : 0))
2 #define sgn(x) (fabs(x) < eps ? 0 : ((x) < 0 ? -1 : 1))
3
4 struct point
5 {
6     double x, y;
7     point(double a = 0, double b = 0) { x = a, y = b; }
8     point operator-(const point& b) const { return point(x - b.x, y - b.y); }
9     point operator+(const point& b) const { return point(x + b.x, y + b.y); }
10    // 两点是否重合
11    bool operator==(point& b) { return zero(x - b.x) && zero(y - b.y); }
12    // 点积(以原点为基准)
13    double operator*(const point& b) const { return x * b.x + y * b.y; }
14    // 叉积(以原点为基准)
15    double operator^(const point& b) const { return x * b.y - y * b.x; }
16    // 绕P点逆时针旋转a弧度后的点
17    point rotate(point b, double a)
18    {
19        double dx, dy;
20        (*this - b).split(dx, dy);
21        double tx = dx * cos(a) - dy * sin(a);
22        double ty = dx * sin(a) + dy * cos(a);
23        return point(tx, ty) + b;
24    }
25    // 点坐标分别赋值到a和b
26    void split(double& a, double& b) { a = x, b = y; }
27 };
28 struct line
29 {
30     point s, e;
31     line() {}
32     line(point ss, point ee) { s = ss, e = ee; }
33 };

```

5.2 Position

5.2.1 Point-Point

```

1 double dist(point a, point b) { return sqrt((a - b) * (a - b)); }

```

5.2.2 Line-Line

```

1 // <0, *> 表示重合; <1, *> 表示平行; <2, P> 表示交点是P;
2 pair<int, point> spoint(line l1, line l2)
3 {
4     point res = l1.s;
5     if (sgn((l1.s - l1.e) ^ (l2.s - l2.e)) == 0)
6         return mp(sgn((l1.s - l2.e) ^ (l2.s - l2.e)) != 0, res);
7     double t = ((l1.s - l2.s) ^ (l2.s - l2.e)) / ((l1.s - l1.e) ^ (l2.s - l2.e));
8     res.x += (l1.e.x - l1.s.x) * t;
9     res.y += (l1.e.y - l1.s.y) * t;
10    return mp(2, res);
11 }

```

5.2.3 Segment-Segment

```
1 bool segxseg(line l1, line l2)
2 {
3     return
4         max(l1.s.x, l1.e.x) >= min(l2.s.x, l2.e.x) &&
5         max(l2.s.x, l2.e.x) >= min(l1.s.x, l1.e.x) &&
6         max(l1.s.y, l1.e.y) >= min(l2.s.y, l2.e.y) &&
7         max(l2.s.y, l2.e.y) >= min(l1.s.y, l1.e.y) &&
8         sgn((l2.s - l1.e) ^ (l1.s - l1.e)) * sgn((l2.e - l1.e) ^ (l1.s - l1.e)) <= 0 &&
9         sgn((l1.s - l2.e) ^ (l2.s - l2.e)) * sgn((l1.e - l2.e) ^ (l2.s - l2.e)) <= 0;
10 }
```

5.2.4 Line-Segment

```
1 //l1是直线,l2是线段
2 bool segxline(line l1, line l2)
3 {
4     return sgn((l2.s - l1.e) ^ (l1.s - l1.e)) * sgn((l2.e - l1.e) ^ (l1.s - l1.e)) <=
5         0;
6 }
```

5.2.5 Point-Line

```
1 double pointtoline(point p, line l)
2 {
3     point res;
4     double t = ((p - l.s) * (l.e - l.s)) / ((l.e - l.s) * (l.e - l.s));
5     res.x = l.s.x + (l.e.x - l.s.x) * t, res.y = l.s.y + (l.e.y - l.s.y) * t;
6     return dist(p, res);
7 }
```

5.2.6 Point-Segment

```
1 double pointtosegment(point p, line l)
2 {
3     point res;
4     double t = ((p - l.s) * (l.e - l.s)) / ((l.e - l.s) * (l.e - l.s));
5     if (t >= 0 && t <= 1)
6         res.x = l.s.x + (l.e.x - l.s.x) * t, res.y = l.s.y + (l.e.y - l.s.y) * t;
7     else
8         res = dist(p, l.s) < dist(p, l.e) ? l.s : l.e;
9     return dist(p, res);
10 }
```

5.2.7 Point on Segment

```
1 bool PointOnSeg(point p, line l)
2 {
3     return
4         sgn((l.s - p) ^ (l.e - p)) == 0 &&
5         sgn((p.x - l.s.x) * (p.x - l.e.x)) <= 0 &&
6         sgn((p.y - l.s.y) * (p.y - l.e.y)) <= 0;
7 }
```

5.3 Polygon

5.3.1 Area

```

1 double area(point p[], int n)
2 {
3     double res = 0;
4     for (int i = 0; i < n; i++) res += (p[i] ^ p[(i + 1) % n]) / 2;
5     return fabs(res);
6 }

```

5.3.2 Point in Convex

```

1 // 点形成一个凸包，而且按逆时针排序(如果是顺时针把里面的<0改为>0)
2 // 点的编号：[0,n)
3 // -1：点在凸多边形外
4 // 0：点在凸多边形边界上
5 // 1：点在凸多边形内
6 int PointInConvex(point a, point p[], int n)
7 {
8     for (int i = 0; i < n; i++)
9         if (sgn((p[i] - a) ^ (p[(i + 1) % n] - a)) < 0)
10             return -1;
11         else if (PointOnSeg(a, line(p[i], p[(i + 1) % n])))
12             return 0;
13     return 1;
14 }

```

5.3.3 Point in Polygon

```

1 // 射线法,poly[]的顶点数要大于等于3,点的编号0~n-1
2 // -1：点在凸多边形外
3 // 0：点在凸多边形边界上
4 // 1：点在凸多边形内
5 int PointInPoly(point p, point poly[], int n)
6 {
7     int cnt;
8     line ray, side;
9     cnt = 0;
10    ray.s = p;
11    ray.e.y = p.y;
12    ray.e.x = -1000000000000.0; // -INF,注意取值防止越界
13    for (int i = 0; i < n; i++)
14    {
15        side.s = poly[i], side.e = poly[(i + 1) % n];
16        if (PointOnSeg(p, side)) return 0;
17        //如果平行轴则不考虑
18        if (sgn(side.s.y - side.e.y) == 0)
19            continue;
20        if (PointOnSeg(side.s, ray))
21            cnt += (sgn(side.s.y - side.e.y) > 0);
22        else if (PointOnSeg(side.e, ray))
23            cnt += (sgn(side.e.y - side.s.y) > 0);
24        else if (segxseg(ray, side))
25            cnt++;
26    }
27    return cnt % 2 == 1 ? 1 : -1;
28 }

```

5.3.4 Judge Convex

```

1 //点可以是顺时针给出也可以是逆时针给出
2 //点的编号1~n-1
3 bool isconvex(point poly[], int n)
4 {
5     bool s[3];
6     clr(s, 0);
7     for (int i = 0; i < n; i++)
8     {
9         s[sgn((poly[(i + 1) % n] - poly[i]) ^ (poly[(i + 2) % n] - poly[i])) + 1] = 1;
10        if (s[0] && s[2]) return 0;
11    }
12    return 1;
13 }

```

5.4 Integer Points

5.4.1 On Segment

```

1 int OnSegment(line l) { return __gcd(fabs(l.s.x - l.e.x), fabs(l.s.y - l.e.y)) + 1; }

```

5.4.2 On Polygon Edge

```

1 int OnEdge(point p[], int n)
2 {
3     int i, ret = 0;
4     for (i = 0; i < n; i++)
5         ret += __gcd(fabs(p[i].x - p[(i + 1) % n].x), fabs(p[i].y - p[(i + 1) % n].y));
6     return ret;
7 }

```

5.4.3 Inside Polygon

```

1 int InSide(point p[], int n)
2 {
3     int i, area = 0;
4     for (i = 0; i < n; i++)
5         area += p[(i + 1) % n].y * (p[i].x - p[(i + 2) % n].x);
6     return (fabs(area) - OnEdge(n, p)) / 2 + 1;
7 }

```

5.5 Circle

5.5.1 Circumcenter

```

1 point waixin(point a, point b, point c)
2 {
3     double a1 = b.x - a.x, b1 = b.y - a.y, c1 = (a1 * a1 + b1 * b1) / 2;
4     double a2 = c.x - a.x, b2 = c.y - a.y, c2 = (a2 * a2 + b2 * b2) / 2;
5     double d = a1 * b2 - a2 * b1;
6     return point(a.x + (c1 * b2 - c2 * b1) / d, a.y + (a1 * c2 - a2 * c1) / d);
7 }

```


6 Dynamic Programming

6.1 Subsequence

6.1.1 Max Sum

```

1 // 传入序列a和长度n, 返回最大子序列和
2 int MaxSeqSum(int a[], int n)
3 {
4     int rt = 0, cur = 0;
5     for (int i = 0; i < n; i++)
6         cur += a[i], rt = max(cur, rt), cur = max(0, cur);
7     return rt;
8 }

```

6.1.2 Longest Increase

```

1 // 序列下标从1开始, LIS()返回长度, 序列存在lis[]中
2 const int N = "Edit";
3 int len, a[N], b[N], f[N];
4 int Find(int p, int l, int r)
5 {
6     while (l <= r)
7     {
8         int mid = (l + r) >> 1;
9         if (a[p] > b[mid])
10             l = mid + 1;
11         else
12             r = mid - 1;
13     }
14     return f[p] = l;
15 }
16 int LIS(int lis[], int n)
17 {
18     int len = 1;
19     f[1] = 1, b[1] = a[1];
20     for (int i = 2; i <= n; i++)
21     {
22         if (a[i] > b[len])
23             b[++len] = a[i], f[i] = len;
24         else
25             b[Find(i, 1, len)] = a[i];
26     }
27     for (int i = n, t = len; i >= 1 && t >= 1; i--)
28         if (f[i] == t) lis[--t] = a[i];
29     return len;
30 }
31
32 // 简单写法(下标从0开始, 只返回长度)
33 int dp[N];
34 int LIS(int a[], int n)
35 {
36     clr(dp, 0x3f);
37     for (int i = 0; i < n; i++) *lower_bound(dp, dp + n, a[i]) = a[i];
38     return lower_bound(dp, dp + n, INF) - dp;
39 }

```

6.1.3 Longest Common Increase

```

1 // 序列下标从1开始
2 int LCIS(int a[], int b[], int n, int m)
3 {
4     clr(dp, 0);
5     for (int i = 1; i <= n; i++)
6     {
7         int ma = 0;
8         for (int j = 1; j <= m; j++)
9         {
10             dp[i][j] = dp[i - 1][j];
11             if (a[i] > b[j]) ma = max(ma, dp[i - 1][j]);
12             if (a[i] == b[j]) dp[i][j] = ma + 1;
13         }
14     }
15     return *max_element(dp[n] + 1, dp[n] + 1 + m);
16 }

```

6.2 Digit Statistics

```

1 int a[20];
2 ll dp[20][state];
3 ll dfs(int pos, /*state变量*/, bool lead /*前导零*/, bool limit /*数位上界变量*/)
4 {
5     //递归边界, 既然是按位枚举, 最低位是0, 那么pos==-1说明这个数枚举完了
6     if (pos == -1) return 1;
7     /*这里一般返回1, 表示枚举的这个数是合法的, 那么这里就需要在枚举时必须每一位都要满足题目条件,
8     也就是说当前枚举到pos位, 一定要保证前面已经枚举的数位是合法的。*/
9     if (!limit && !lead && dp[pos][state] != -1) return dp[pos][state];
10    /*常规写法都是在没有限制的条件记忆化, 这里与下面记录状态是对应*/
11    int up = limit ? a[pos] : 9; //根据limit判断枚举的上界up
12    ll ans = 0;
13    for (int i = 0; i <= up; i++) //枚举, 然后把不同情况的个数加到ans就可以了
14    {
15        if () ...
16        else if () ...
17        ans += dfs(pos - 1, /*状态转移*/, lead && i == 0, limit && i == a[pos])
18        //最后两个变量传参都是这样写的
19        /*当前数位枚举的数是i, 然后根据题目的约束条件分类讨论
20        去计算不同情况下的个数, 还要要根据state变量来保证i的合法性*/
21    }
22    //计算完, 记录状态
23    if (!limit && !lead) dp[pos][state] = ans;
24    /*这里对应上面的记忆化, 在一定条件下时记录, 保证一致性,
25    当然如果约束条件不需要考虑lead, 这里就是lead就完全不用考虑了*/
26    return ans;
27 }
28 ll solve(ll x)
29 {
30     int pos = 0;
31     do //把数位都分解出来
32         a[pos++] = x % 10;
33     while (x /= 10);
34     return dfs(pos - 1 /*从最高位开始枚举*/, /*一系列状态 */, true, true);
35     //刚开始最高位都是有限制并且有前导零的, 显然比最高位还要高的一位视为0
36 }

```

7 Others

7.1 Matrix

7.1.1 Matrix FastPow

```

1 typedef vector<ll> vec;
2 typedef vector<vec> mat;
3 mat mul(mat& A, mat& B)
4 {
5     mat C(A.size(), vec(B[0].size()));
6     for (int i = 0; i < A.size(); i++)
7         for (int k = 0; k < B.size(); k++)
8             if (A[i][k]) // 对稀疏矩阵的优化
9                 for (int j = 0; j < B[0].size(); j++)
10                     C[i][j] = (C[i][j] + A[i][k] * B[k][j]) % mod;
11     return C;
12 }
13 mat Pow(mat A, ll n)
14 {
15     mat B(A.size(), vec(A.size()));
16     for (int i = 0; i < A.size(); i++) B[i][i] = 1;
17     for (; n >= 1, A = mul(A, A))
18         if (n & 1) B = mul(B, A);
19     return B;
20 }
```

7.1.2 Gauss Elimination

```

1 void gauss()
2 {
3     int now = 1, to;
4     double t;
5     for (int i = 1; i <= n; i++, now++)
6     {
7         /*for (to = now; !a[to][i] && to <= n; to++);
8         //做除法时减小误差, 可不写
9         if (to != now)
10             for (int j = 1; j <= n + 1; j++)
11                 swap(a[to][j], a[now][j]);*/
12         t = a[now][i];
13         for (int j = 1; j <= n + 1; j++) a[now][j] /= t;
14         for (int j = 1; j <= n; j++)
15             if (j != now)
16             {
17                 t = a[j][i];
18                 for (int k = 1; k <= n + 1; k++) a[j][k] -= t * a[now][k];
19             }
20     }
21 }
```

7.2 Tricks

7.2.1 Stack-Overflow

```

1 // 解决爆栈问题
2 #pragma comment(linker, "/STACK:1024000000,1024000000")
```

7.2.2 Fast-Scanner

```

1 // 适用于正负整数
2 template <class T>
3 inline bool scan_d(T &ret)
4 {
5     char c;
6     int sgn;
7     if (c = getchar(), c == EOF) return 0; //EOF
8     while (c != '-' && (c < '0' || c > '9')) c = getchar();
9     sgn = (c == '-') ? -1 : 1;
10    ret = (c == '-') ? 0 : (c - '0');
11    while (c = getchar(), c >= '0' && c <= '9') ret = ret * 10 + (c - '0');
12    ret *= sgn;
13    return 1;
14 }
15 inline void out(int x)
16 {
17     if (x > 9) out(x / 10);
18     putchar(x % 10 + '0');
19 }

```

7.2.3 Strok-Sscanf

```

1 // 空格作为分隔输入,读取一行的整数
2 gets(buf);
3 int v;
4 char *p = strtok(buf, " ");
5 while (p)
6 {
7     sscanf(p, "%d", &v);
8     p = strtok(NULL, " ");
9 }

```

7.3 Mo Algorithm

莫队算法, 可以解决一类静态, 离线区间查询问题。分成 \sqrt{x} 块, 分块排序。

```

1 struct query { int L, R, id; };
2 void solve(query node[], int m)
3 {
4     tmp = 0;
5     clr(num, 0);
6     clr(ans, 0);
7     sort(node, node + m, [](query a, query b) { return a.l / unit < b.l / unit || a.l /
8         unit == b.l / unit && a.r < b.r; });
9     int L = 1, R = 0;
10    for (int i = 0; i < m; i++)
11    {
12        while (node[i].L < L) add(a[--L]);
13        while (node[i].L > L) del(a[L++]);
14        while (node[i].R < R) del(a[R--]);
15        while (node[i].R > R) add(a[++R]);
16        ans[node[i].id] = tmp;
17    }
18 }

```

7.4 BigNum

7.4.1 High-precision

```

1 // 加法 乘法 小于号 输出
2 struct bint
3 {
4     int l;
5     short int w[100];
6     bint(int x = 0)
7     {
8         l = x == 0, clr(w, 0);
9         while (x) w[l++] = x % 10, x /= 10;
10    }
11    bool operator<(const bint& x) const
12    {
13        if (l != x.l) return l < x.l;
14        int i = l - 1;
15        while (i >= 0 && w[i] == x.w[i]) i--;
16        return (i >= 0 && w[i] < x.w[i]);
17    }
18    bint operator+(const bint& x) const
19    {
20        bint ans;
21        ans.l = l > x.l ? l : x.l;
22        for (int i = 0; i < ans.l; i++)
23        {
24            ans.w[i] += w[i] + x.w[i];
25            ans.w[i + 1] += ans.w[i] / 10;
26            ans.w[i] = ans.w[i] % 10;
27        }
28        if (ans.w[ans.l] != 0) ans.l++;
29        return ans;
30    }
31    bint operator*(const bint& x) const
32    {
33        bint res;
34        int up, tmp;
35        for (int i = 0; i < l; i++)
36        {
37            up = 0;
38            for (int j = 0; j < x.l; j++)
39            {
40                tmp = w[i] * x.w[j] + res.w[i + j] + up;
41                res.w[i + j] = tmp % 10;
42                up = tmp / 10;
43            }
44            if (up != 0) res.w[i + x.l] = up;
45        }
46        res.l = l + x.l;
47        while (res.w[res.l - 1] == 0 && res.l > 1) res.l--;
48        return res;
49    }
50    void print()
51    {
52        for (int i = l - 1; ~i; i--) printf("%d", w[i]);
53        puts("");
54    }
55 };

```

7.4.2 Complete High-precision

```

1  #define N 10000
2  class bint
3  {
4  private:
5      int a[N]; // 用 N 控制最大位数
6      int len;  // 数字长度
7  public:
8      // 构造函数
9      bint() { len = 1, clr(a, 0); }
10     // int -> bint
11     bint(int n)
12     {
13         len = 0;
14         clr(a, 0);
15         int d = n;
16         while (n)
17             d = n / 10 * 10, a[len++] = n - d, n = d / 10;
18     }
19     // char[] -> int
20     bint(const char s[])
21     {
22         clr(a, 0);
23         len = 0;
24         int l = strlen(s);
25         for (int i = l - 1; ~i; i--) a[len++] = s[i];
26     }
27     // 拷贝构造函数
28     bint(const bint& b)
29     {
30         clr(a, 0);
31         len = b.len;
32         for (int i = 0; i < len; i++) a[i] = b.a[i];
33     }
34     // 重载运算符 bint = bint
35     bint& operator=(const bint& n)
36     {
37         len = n.len;
38         for (int i = 0; i < len; i++) a[i] = n.a[i];
39         return *this;
40     }
41     // 重载运算符 bint + bint
42     bint operator+(const bint& b) const
43     {
44         bint t(*this);
45         int res = b.len > len ? b.len : len;
46         for (int i = 0; i < res; i++)
47         {
48             t.a[i] += b.a[i];
49             if (t.a[i] >= 10) t.a[i + 1]++, t.a[i] -= 10;
50         }
51         t.len = res + a[res] == 0;
52         return t;
53     }
54     // 重载运算符 bint - bint
55     bint operator-(const bint& b) const
56     {
57         bool f = *this > b;

```

```

58     bint t1 = f ? *this : b;
59     bint t2 = f ? b : *this;
60     int res = t1.len, j;
61     for (int i = 0; i < res; i++)
62         if (t1.a[i] < t2.a[i])
63         {
64             j = i + 1;
65             while (t1.a[j] == 0) j++;
66             t1.a[j--]--;
67             while (j > i) t1.a[j--] += 9;
68             t1.a[i] += 10 - t1.a[i];
69         }
70         else
71             t1.a[i] -= t2.a[i];
72     t1.len = res;
73     while (t1.a[t1.len - 1] == 0 && t1.len > 1) t1.len--;
74     if (f) t1.a[t1.len - 1] = 0 - t1.a[t1.len - 1];
75     return t1;
76 }
77 // 重载运算符 bint * bint
78 bint operator*(const bint& b) const
79 {
80     bint t;
81     int i, j, up, tmp, tmp1;
82     for (i = 0; i < len; i++)
83     {
84         up = 0;
85         for (j = 0; j < b.len; j++)
86         {
87             tmp = a[i] * b.a[j] + t.a[i + j] + up;
88             if (tmp > 9)
89                 tmp1 = tmp - tmp / 10 * 10, up = tmp / 10, t.a[i + j] = tmp1;
90             else
91                 up = 0, t.a[i + j] = tmp;
92         }
93         if (up) t.a[i + j] = up;
94     }
95     t.len = i + j;
96     while (t.a[t.len - 1] == 0 && t.len > 1) t.len--;
97     return t;
98 }
99 // 重载运算符 bint / int
100 bint operator/(const int& b) const
101 {
102     bint t;
103     int down = 0;
104     for (int i = len - 1; ~i; i--)
105         t.a[i] = (a[i] + down * 10) / b, down = a[i] + down * 10 - t.a[i] * b;
106     t.len = len;
107     while (t.a[t.len - 1] == 0 && t.len > 1) t.len--;
108     return t;
109 }
110 // 重载运算符 bint ^ n (n次方快速幂, 需保证n非负)
111 bint operator^(const int n) const
112 {
113     bint t(*this), rt(1);
114     if (n == 0) return 1;
115     if (n == 1) return *this;
116     int m = n;

```

```

117         for (; m; m >>= 1, t = t * t)
118             if (m & 1) rt = rt * t;
119         return rt;
120     }
121     // 重载运算符 bint > bint 比较大小
122     bool operator>(const bint& b) const
123     {
124         int p;
125         if (len > b.len) return 1;
126         if (len == b.len)
127         {
128             p = len - 1;
129             while (a[p] == b.a[p] && p >= 0) p--;
130             return p >= 0 && a[p] > b.a[p];
131         }
132         return 0;
133     }
134     // 重载运算符 bint > int 比较大小
135     bool operator>(const int& n) const { return *this > bint(n); }
136     // 输出
137     void out()
138     {
139         for (int i = len - 1; ~i; i--) printf("%d", a[i]);
140         puts("");
141     }
142 };

```

7.5 VIM

```

1 syntax on
2 set cindent
3 set nu
4 set tabstop=4
5 set shiftwidth=4
6 set background=dark
7 set mouse=a
8
9 map<C-A> gg vG"+y
10 map<F5> :call Run()<CR>
11
12 func! Run()
13     exec "w"
14     exec "!g++ -std=c++11 -O2 % -o %<"
15     exec "!time ./%<"
16 endfunc
17
18 autocmd BufNewFile *.cpp 0r ~/include.cpp
19 autocmd BufNewFile *.cpp normal G
20
21 inoremap ( (<Esc>i
22 inoremap [ [<Esc>i
23 inoremap { {<CR><Esc>O
24 inoremap ' ''<Esc>i
25 inoremap " ""<Esc>i
26
27 inoremap ) <c-r>=ClosePair('')<CR>
28 inoremap ] <c-r>=ClosePair('']<CR>
29

```



```
30 func ClosePair(char)
31     if getline(' ')[col('.')-1]==a:char
32         return "\<Right>"
33     else
34         return a:char
35     endif
36 endfunc
```