Multi-Criteria Decision Making

TOPSIS METHOD

TOPSIS METHOD

- Technique of Order Preference by Similarity to Ideal Solution
- This method considers three types of attributes or criteria

  - Qualitative benefit attributes/criteria
  - Quantitative benefit attributes
  - Cost attributes or criteria

TOPSIS METHOD

- In this method two artificial alternatives are hypothesized:

- Ideal alternative: the one which has the best level for all attributes considered.
- Negative ideal alternative: the one which has the worst attribute values.

- TOPSIS selects the alternative that is the closest to the ideal solution and farthest from negative ideal alternative.

Input to TOPSIS

- TOPSIS assumes that we have m alternatives (options) and n attributes/criteria and we have the score of each option with respect to each criterion.

- Let x_{ij} score of option i with respect to criterion j
  We have a matrix $X = (x_{ij})$ $m \times n$ matrix.
- Let J be the set of benefit attributes or criteria (more is better)
- Let J' be the set of negative attributes or criteria (less is better)

Steps of TOPSIS

- Step 1: Construct normalized decision matrix.
- This step transforms various attribute dimensions into non-dimensional attributes, which allows comparisons across criteria.
- Normalize scores or data as follows:

$$r_{ij} = x_{ij} / (\sum_{i} x^2_{ij}) \quad \text{for } i = 1, \ldots, m;\ j = 1, \ldots, n$$

Steps of TOPSIS

- Step 2: Construct the weighted normalized decision matrix.
- Assume we have a set of weights for each criteria w_j for $j = 1,\ldots n$.
- Multiply each column of the normalized decision matrix by its associated weight.
- An element of the new matrix is:

$$v_{ij} = w_j \, r_{ij}$$

Steps of TOPSIS

- Step 3: Determine the ideal and negative ideal solutions.

- Ideal solution.

  $A^* = \{ v_1^*, \ldots, v_n^* \}$, where

  $v_j^* = \{ \max_i (v_{ij}) \text{ if } j \in J ; \ \min_i (v_{ij}) \text{ if } j \in J' \}$

- Negative ideal solution.

  $A' = \{ v_1', \ldots, v_n' \}$, where

  $v' = \{ \min_i (v_{ij}) \text{ if } j \in J ; \ \max_i (v_{ij}) \text{ if } j \in J' \}$

Steps of TOPSIS

- Step 4: Calculate the separation measures for each alternative.

- The separation from the ideal alternative is:

$$S_i^* = \left[\sum_j (v_j^* - v_{ij})^2 \right]^{½} \qquad i = 1, \ldots, m$$

- Similarly, the separation from the negative ideal alternative is:

$$S'_i = \left[\sum_j (v_j' - v_{ij})^2 \right]^{½} \qquad i = 1, \ldots, m$$

Steps of TOPSIS

- Step 5: Calculate the relative closeness to the ideal solution C_i^*

$$C_i^* = S'_i / (S_i^* + S'_i) \;, \qquad 0 < C_i^* < 1$$

Select the option with C_i^* closest to 1.

WHY ?

Applying TOPSIS Method to Example

	Style	Reliability	Fuel Eco.	Cost
Weight	0.1	0.4	0.3	0.2
Civic	7	9	9	8
Saturn	8	7	8	7
Ford	9	6	8	9
Mazda	6	7	8	6

Applying TOPSIS to Example

- m = 4 alternatives (car models)
- n = 4 attributes/criteria

- x_{ij} = score of option i with respect to criterion j
  X = {x_{ij}} 4×4 score matrix.
- J = set of benefit attributes: style, reliability, fuel economy (more is better)
- J' = set of negative attributes: cost (less is better)

Steps of TOPSIS

- Step 1(a): calculate $(\Sigma x^2_{ij})^{1/2}$ for each column

	Style	Rel.	Fuel	Cost
Civic	49	81	81	64
Saturn	64	49	64	49
Ford	81	36	64	81
Mazda	36	49	64	36
$\sum_i x_{ij}^2$	230	215	273	230
$(\Sigma x^2)^{1/2}$	15.17	14.66	16.52	15.17

Steps of TOPSIS

- Step 1 (b): divide each column by $(\Sigma x^2_{ij})^{1/2}$ to get $\mathbf{r}_{ij}$

	Style	Rel.	Fuel	Cost
Civic	**0.46**	**0.61**	**0.54**	**0.53**
Saturn	**0.53**	**0.48**	**0.48**	**0.46**
Ford	**0.59**	**0.41**	**0.48**	**0.59**
Mazda	**0.48**	**0.48**	**0.40**	**0.40**

Steps of TOPSIS

- Step 2 (b): multiply each column by w_j to get v_{ij}.

	Style	Rel.	Fuel	Cost
Civic	0.046	0.244	0.162	0.106
Saturn	0.053	0.192	0.144	0.092
Ford	0.059	0.164	0.144	0.118
Mazda	0.040	0.192	0.144	0.080

Steps of TOPSIS

- Step 3 (a): determine ideal solution A*.

$A^* = \{0.059, 0.244, 0.162, 0.080\}$

	Style	Rel.	Fuel	Cost
Civic	**0.046**	**0.244**	**0.162**	**0.106**
Saturn	**0.053**	**0.192**	**0.144**	**0.092**
Ford	**0.059**	**0.164**	**0.144**	**0.118**
Mazda	**0.040**	**0.192**	**0.144**	**0.080**

Steps of TOPSIS

- Step 3 (a): find negative ideal solution A'.

  A' = {0.040, 0.164, 0.144, 0.118}

	Style	Rel.	Fuel	Cost
Civic	0.046	0.244	0.162	0.106
Saturn	0.053	0.192	0.144	0.092
Ford	0.059	0.164	0.144	0.118
Mazda	0.040	0.192	0.144	0.080

Steps of TOPSIS

- Step 4 (a): determine separation from ideal solution A* = {0.059, 0.244, 0.162, 0.080}

$S_i^* = [\sum_j (v_j^* - v_{ij})^2]^{½}$ for each row

	Style	Rel.	Fuel	Cost
Civic	$(.046-.059)^2$	$(.244-.244)^2$	$(0)^2$	$(.026)^2$
Saturn	$(.053-.059)^2$	$(.192-.244)^2$	$(-.018)^2$	$(.012)^2$
Ford	$(.053-.059)^2$	$(.164-.244)^2$	$(-.018)^2$	$(.038)^2$
Mazda	$(.053-.059)^2$	$(.192-.244)^2$	$(-.018)^2$	$(.0)^2$

Steps of TOPSIS

- Step 4 (a): determine separation from ideal solution S_i^*

	$\Sigma(v_j^* - v_{ij})^2$	$S_i^* = [\, \Sigma\, (v_j^* - v_{ij})^2\,]^{½}$
Civic	**0.000845**	**0.029**
Saturn	**0.003208**	**0.057**
Ford	**0.008186**	**0.090**
Mazda	**0.003389**	**0.058**

Steps of TOPSIS

- Step 4 (b): find separation from negative ideal solution $A' = \{0.040, 0.164, 0.144, 0.118\}$

$S_i' = [\sum_j (v_j' - v_{ij})^2]^{1/2}$ for each row

	Style	Rel.	Fuel	Cost
Civic	$(.046-.040)^2$	$(.244-.164)^2$	$(.018)^2$	$(-.012)^2$
Saturn	$(.053-.040)^2$	$(.192-.164)^2$	$(0)^2$	$(-.026)^2$
Ford	$(.053-.040)^2$	$(.164-.164)^2$	$(0)^2$	$(0)^2$
Mazda	$(.053-.040)^2$	$(.192-.164)^2$	$(0)^2$	$(-.038)^2$

Steps of TOPSIS

- Step 4 (b): determine separation from negative ideal solution S_i'

	$\Sigma(v_j' - v_{ij})^2$	$S_i' = [\, \Sigma\, (v_j' - v_{ij})^2\,]^{½}$
Civic	**0.006904**	**0.083**
Saturn	**0.001629**	**0.040**
Ford	**0.000361**	**0.019**
Mazda	**0.002228**	**0.047**

Steps of TOPSIS

- Step 5: Calculate the relative closeness to the ideal solution $C_i^* = S'_i / (S_i^* + S'_i)$

	$S'_i / (S_i^* + S'_i)$	C_i^*		
Civic	**0.083/0.112**	**0.74**	←	**BEST**
Saturn	**0.040/0.097**	**0.41**		
Ford	**0.019/0.109**	**0.17**		
Mazda	**0.047/0.105**	**0.45**		