Goals of analysis

1) reference population
estimate parameters of population

parameters		estimates		Data
μ_{sit}	$\leftarrow$	$\hat{\mu}_{sit}$	$\leftarrow$	$\bar{y}_{sit}$
μ_{stand}	$\leftarrow$	$\hat{\mu}_{stand}$	$\leftarrow$	$\bar{y}_{stand}$

$\delta_{posture} = \mu_{stand} - \mu_{sit} \leftarrow \hat{\delta} = \hat{\mu}_{stand} - \hat{\mu}_{sit}$

2) Describe quality of estimates

error $|\hat{\mu}_{sit} - \mu_{sit}|$ or $|\hat{\delta} - \delta|$

problem: Don't know μ_{sit} or δ

2b) How big a typical error might be given this experimental design?

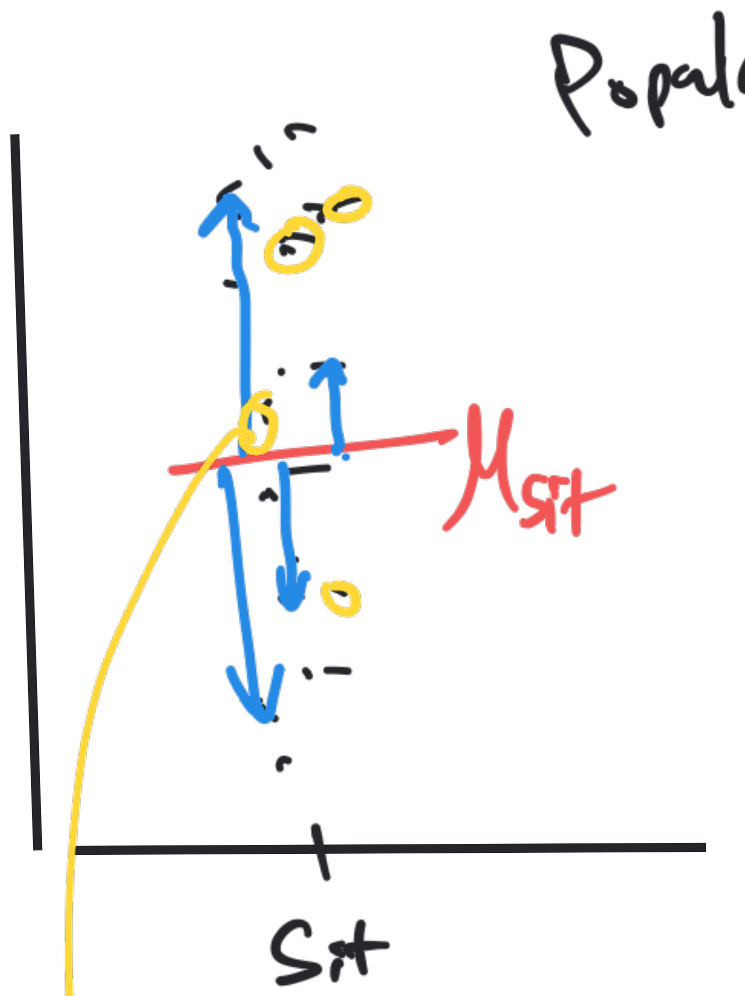
1) Analytical: Theoretical Standard Error

$SE \begin{cases} SEM \\ SED \end{cases}$

2) Estimated Standard Error

$\widehat{SEM}$ or $\widehat{SED}$

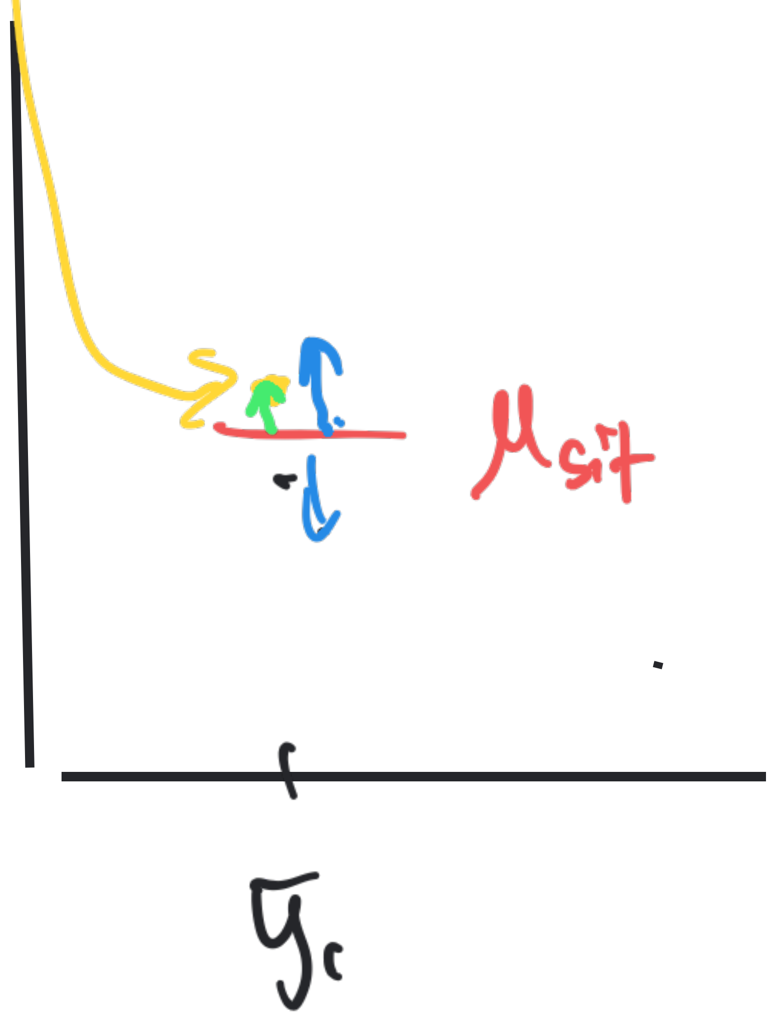
3) Confidence Intervals



$$\sigma^2_{Y_1} = \frac{\sum (Y_{1i} - \mu_1)^2}{N_1}$$

= Ave Deviations^2

σ_Y = ≈ Ave |deviation|



$$\frac{\sum (\bar{y}_{1j} - \mu_1)^2}{R} = \boxed{\sigma^2_{\bar{y}} = \frac{\sigma^2_{Y_1}}{n_1}}$$

= ave error^2

$\sigma_{\bar{y}}$ = ≈ ave |error|

= Standard Error = SEM

$$\delta = \mu_{stand} - \mu_{sit}$$

$$\hat{\delta} = \bar{y}_{stand} - \bar{y}_{sit}$$

$$\boxed{\sigma^2_{\hat{\delta}} = \frac{\sigma^2_{Y_2}}{n_2} + \frac{\sigma^2_{Y_1}}{n_1}}$$

- as long as the estimates are <u>independent</u>

$$\sigma_{\hat{\delta}} = \sqrt{\frac{\sigma^2_{Y_2}}{n_2} + \frac{\sigma^2_{Y_1}}{n_1}}$$ (units bpm)

≈ ave |error| in trt effect.

Standard Error of difference

SED

what is $\sigma^2_{y_1}$?

- estimate from data.
  - ave (deviation)^2 around μ

estimate ave (deviation)^2 arnd $\hat{\mu}$

$$\frac{\sum (y_{1i} - \bar{y}_1)^2}{n_1 - 1} = s_1^2 \Rightarrow \sigma^2_{y_1}$$

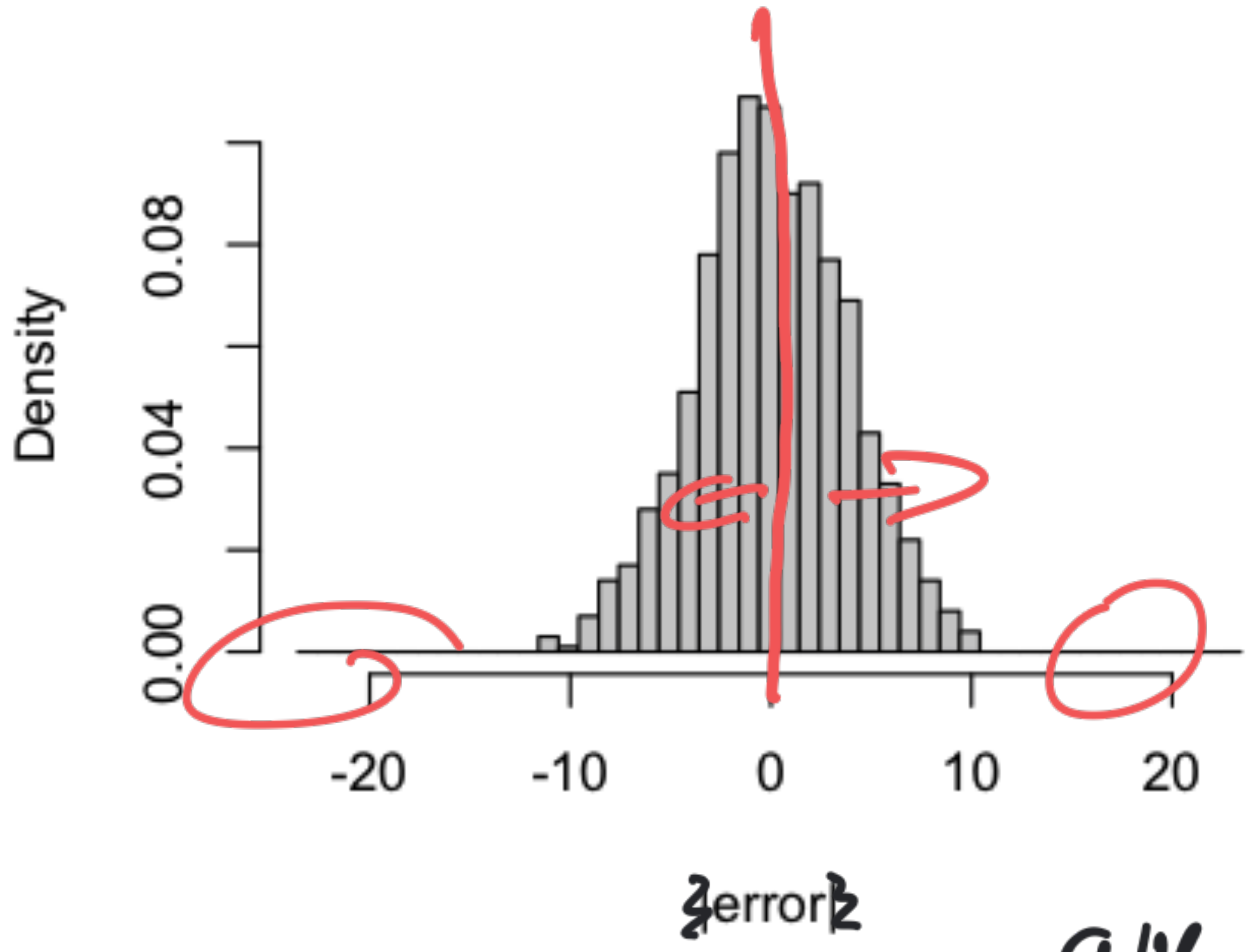
$n_1 - 1$ — # independ deviations

$$\widehat{SEM} = \sqrt{\frac{s_1^2}{n_1}} = \frac{s_1}{\sqrt{n_1}}$$

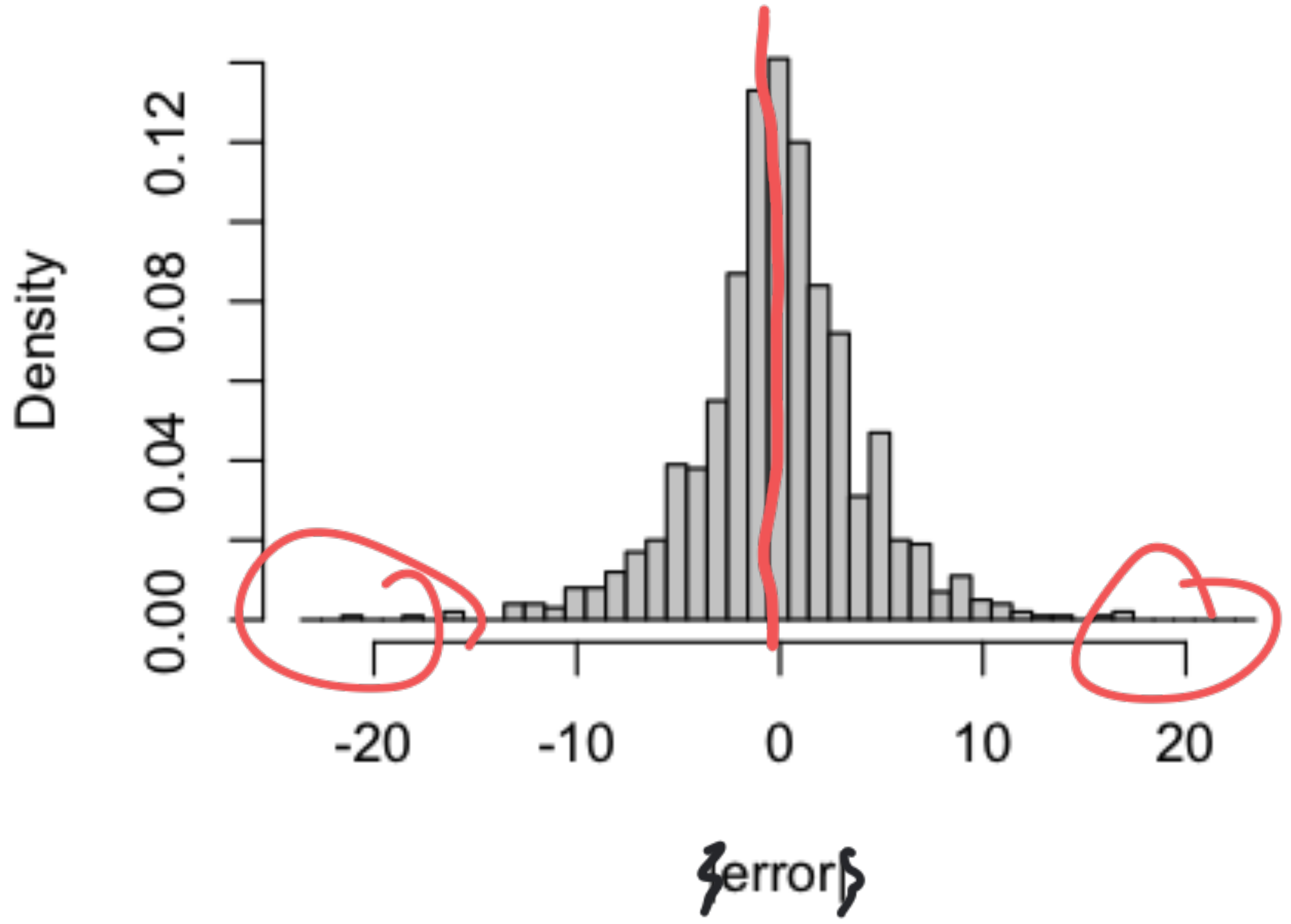
$$\widehat{SED} = \sqrt{\frac{s_2^2}{n_2} + \frac{s_1^2}{n_1}}$$

Estimated Standard Errors — actually a number

Averages can be misleading



ave |error| = 3bpm



Can we put bounds on our possible errors?

If Y_{1i} follow a Normal Distribution,

then so does our error $(\bar{y}_1 - \mu_1)$ and $(\hat{\delta} - \delta)$

If $Y_{1i} \sim N(\mu_1, \sigma^2_{Y_1})$ then

$$\boxed{\frac{\hat{\delta} - \delta}{SED} \sim N(0, 1)}$$

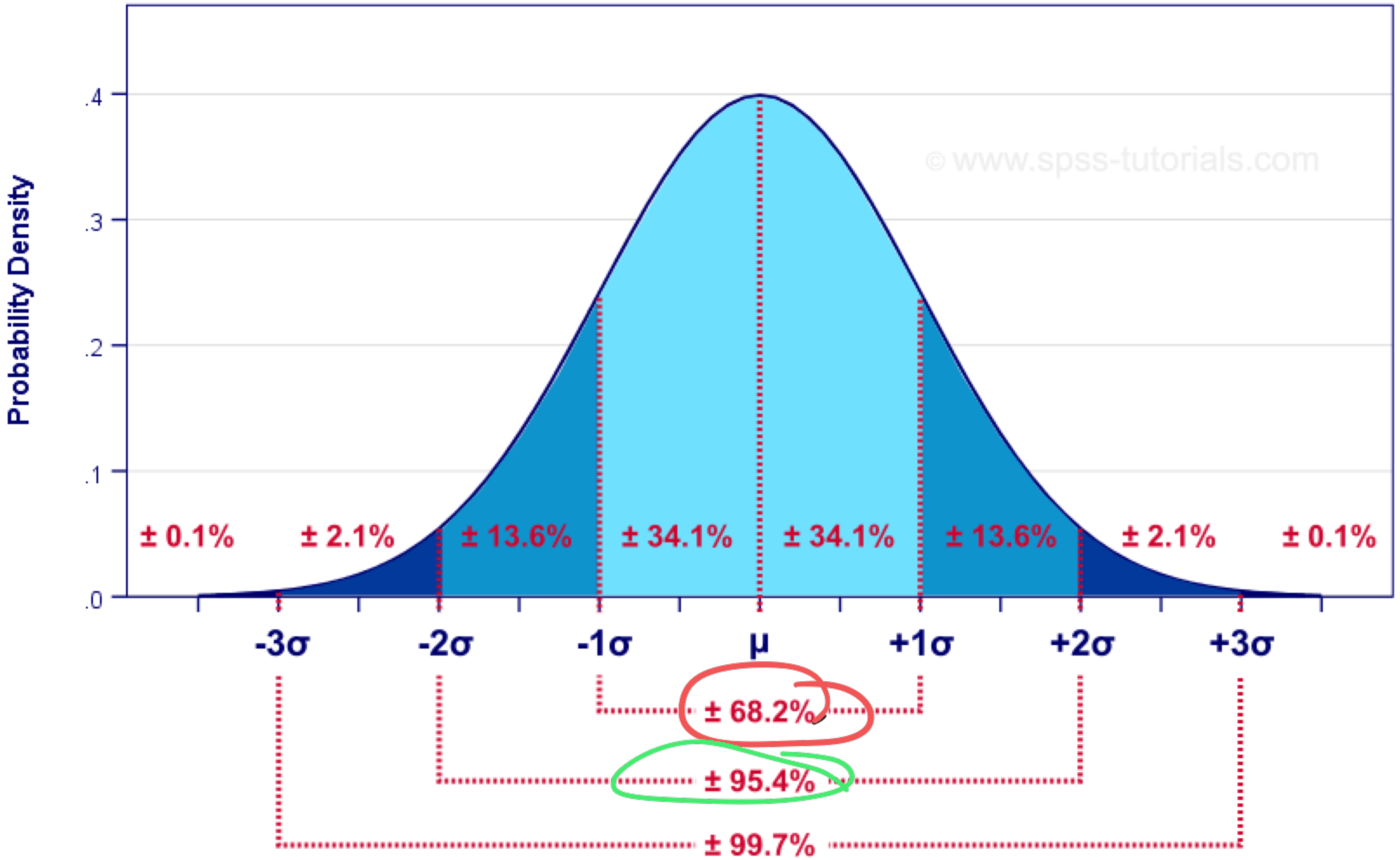
Bayesian Inference

Normal Distribution

population $Y_{ij} \sim N(\mu, \sigma_Y^2)$

SD = $\sqrt{\text{variance}}$

- symmetric around μ
- unimodal
- few outliers



$\sim N(\mu, \sigma^2)$

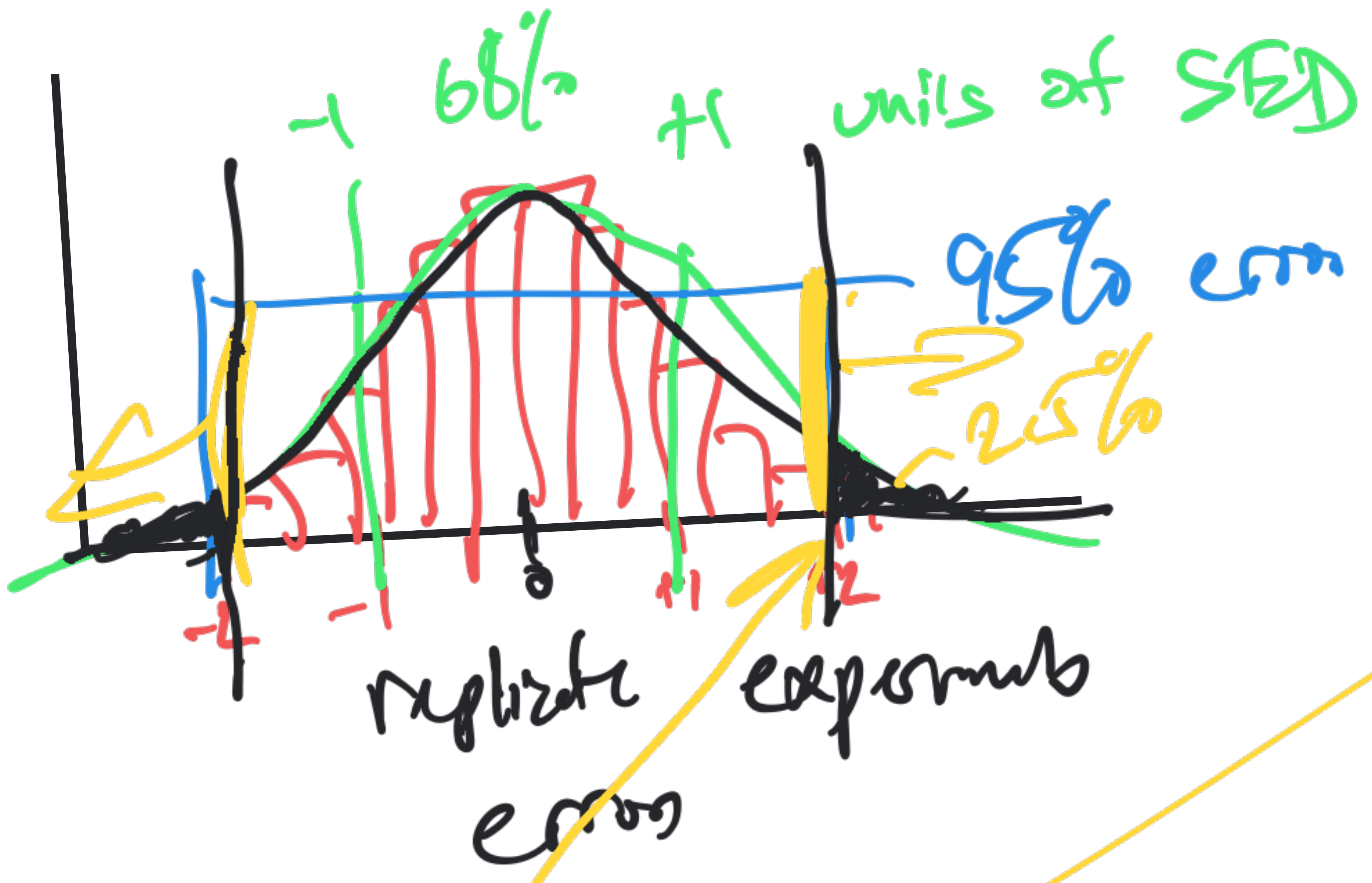
mean

variance

* R dnorm(x, mean, sd)

pnorm

$$\frac{\hat{\delta} - \delta}{SED} \sim N(0,1)$$



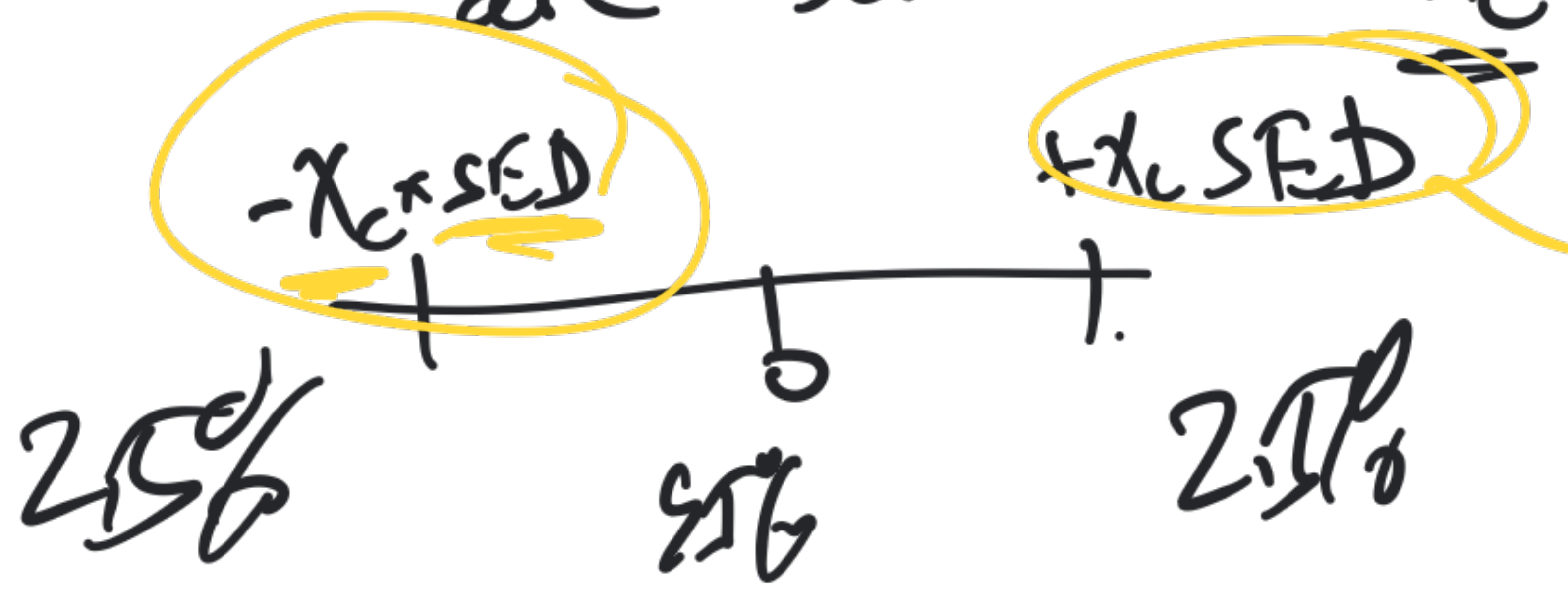
- chance $|\hat{\delta} - \delta| < \underline{2 \times SED}$

~95%.

pnorm(2, lower.tail=F) +
pnorm(-2, lower.tail=T) ≈ 0.05

what threshold bounds 95%
of normalized errors?

x_c such that 95% errors
are between $-x_c \times SED$, $+x_c \times SED$



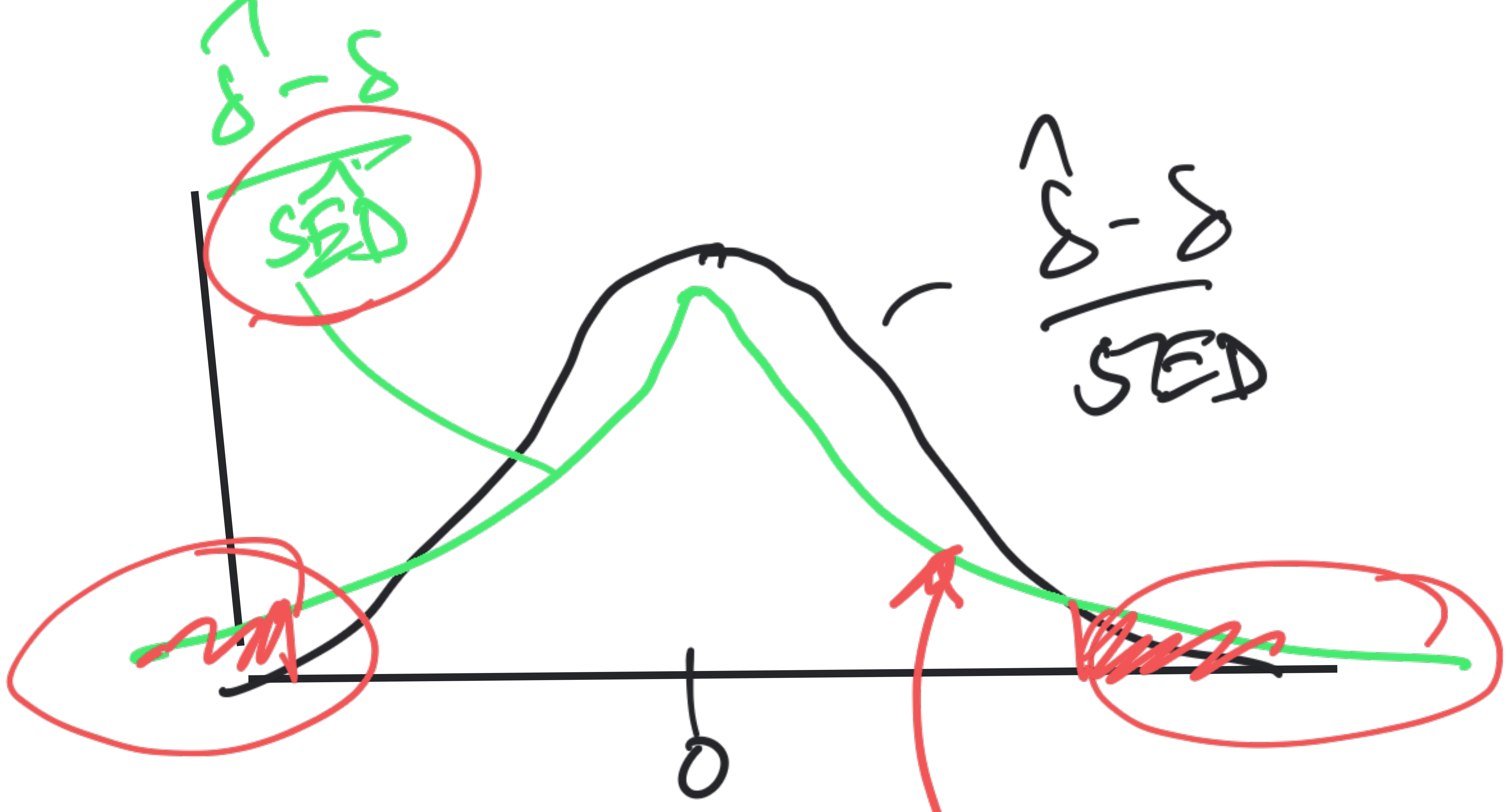
we don't know SED

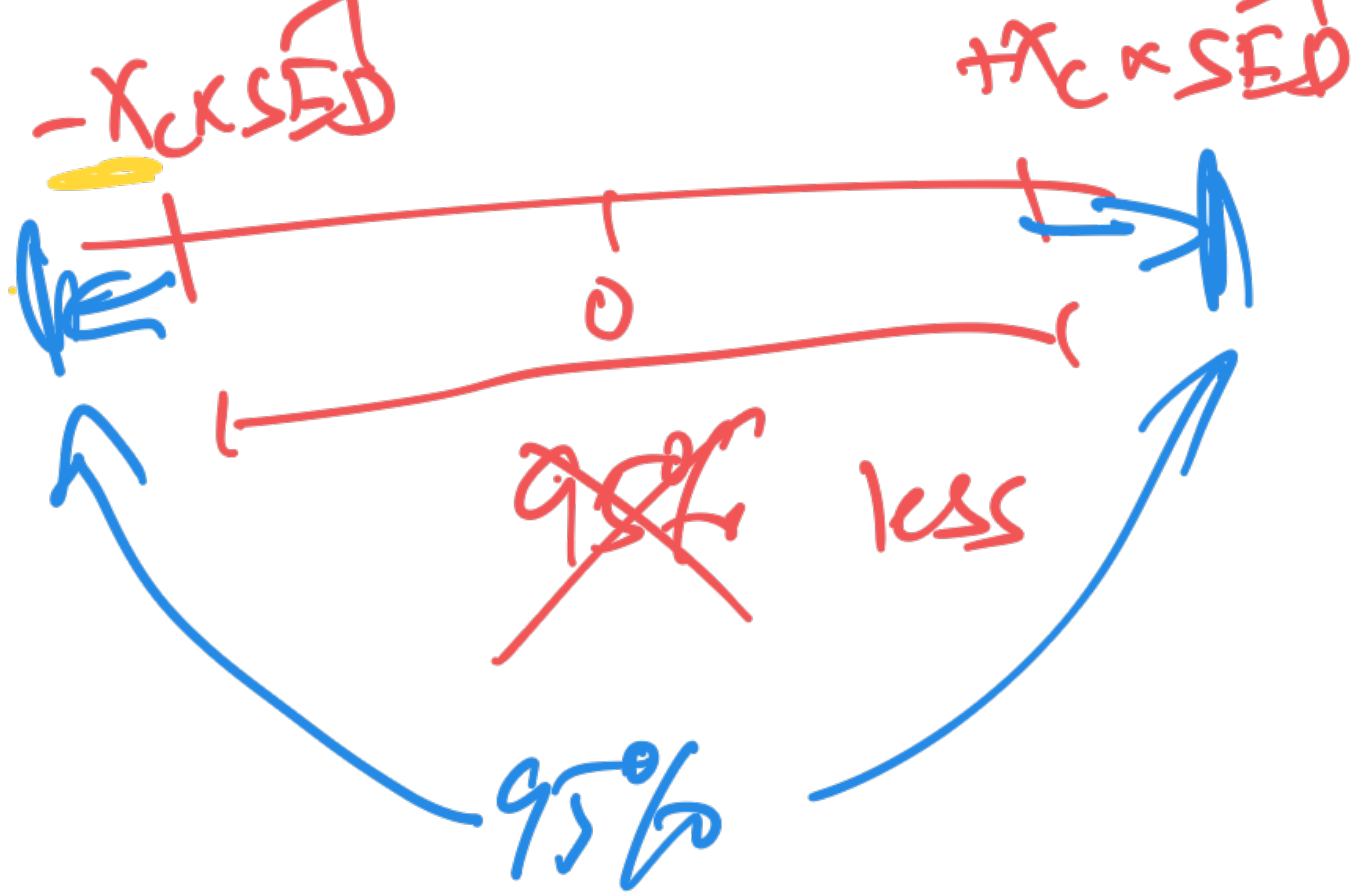
$\hat{SED}$

$-x_c =$ qnorm(0.025, lower.tail=F)

$+x_c =$ qnorm(0.025, lower.tail=T)

(95% bounds





t-distribution

$qnorm\left(\frac{0.05}{2}\right) = x_c$

$qt\left(\frac{0.05}{2}, df, lower.tail=F\right) = t_c$

prob of actual error being outside our bounds

independent obs in estimate of σ_y^2

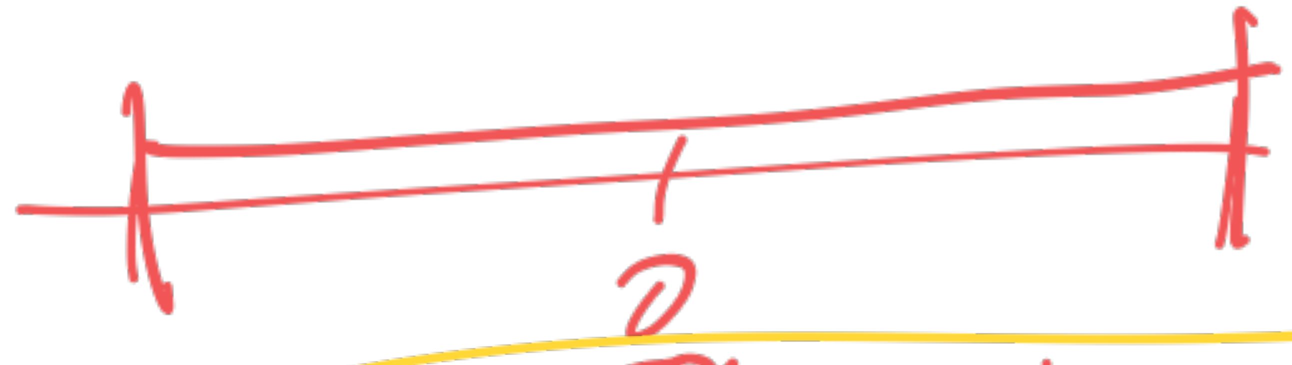
$t_{\frac{0.05}{2}, df}$

$\hat{\sigma}^2 \leftarrow s^2 = \frac{\Sigma(y_i - \bar{y})^2}{n_1 - 1}$ — df

Confidence Interval

estimate ± error bounds. (w/ probability)

$\bar{y} \pm t_c \times \widehat{SEM}$ $\qquad$ $\hat{\delta} \pm t_c \times \widehat{SED}$



95% — $1-\alpha$

experimental design → shrink this interval