SHANGHAI JIAOTONG UNIVERSITY
X071571: OPTIMIZATION METHODS

Here the exercises refer to the textbook on Convex Optimization by Boyd and Vandenberghe.

Problem Set 4

Problem 1. Exercise 9.10, page 515.

Problem 2. Consider the strictly convex quadratic function

$$\varphi(x) = \frac{1}{2}x^T Ax - b^T x,$$

where $A \in S^n_{++}$. Given an invertible $n \times n$ matrix C, we perform the change of variable $\hat{x} = Cx$ so that φ is transformed as

$$\hat{\varphi}(\hat{x}) = \frac{1}{2}\hat{x}^T (C^{-T} A C^{-1})x - (C^{-T}b)^T \hat{x}.$$

Derive the preconditioned Conjugate Gradient Method by applying the standard CG method in the variables $\hat{x}$ and then transforming back into the original variables.

Problem 3. Exercise 10.1, page 557.

For the following numerical problems, feel free to use any language you are comfortable with. Provide your code with explanations.

Problem 4. Exercise 10.15, page 560.

Problem 5. Exercise 10.16, page 560.

Problem 6. Implement Newton's method (with backtracking search), nonlinear conjugate gradient methods, BFGS method for solving a problem in the form:

$$\min_{x \in \mathbb{R}^n} tc^T x - \sum_{i=1}^{m} \log(b_i - a_i^T x), \quad t > 0$$

by generating a random set of inequalities constraint $a_i^T x \leq b_i$, and a vector c with $m = 500$, $n = 100$. Compare the performance of the three methods on your example.