SHANGHAI JIAOTONG UNIVERSITY
X071571: OPTIMIZATION METHODS

Here the exercises refer to the textbook on Convex Optimization by Boyd and Vandenberghe.

Problem Set 3

Problem 1. Consider the minimization problem

$$\begin{cases} \min f_0(x) \\ \text{s.t. } Ax = b, \end{cases}$$

where f_0 continuously differentiable, A is an $p \times n$ matrix and $b \in \mathbb{R}^p$. Show that $x^\star \in \operatorname{dom} f_0$ is optimal if and only if there is $\lambda \in \mathbb{R}^p$ such that

$$\nabla f_0(x^\star) + A^T \lambda = 0.$$

Problem 2. Exercise 4.5, page 190.

Problem 3. Exercise 4.12, page 193.

Problem 4. Exercise 4.21, page 196.

Problem 5. Exercise 4.25, page 197.

Problem 6. Consider an optimization problem

$$\begin{cases} \min & f_0(x) & \\ \text{s.t.} & f_j(x) \leq 0, & j = 1, \dots, m \\ & h_j(x) = 0, & j = 1, \dots, p \end{cases}$$

and the set

$$\mathcal{A} = \{(u, v, t) : \text{ there exists } x \text{ s.t. } f_j(x) \leq u_j,\ h_j(x) = v_j,\ f_0(x) \leq t\} \subset \mathbb{R}^m \times \mathbb{R}^p \times \mathbb{R}.$$

Show that if the optimization problem is convex, then $\mathcal{A}$ is convex.

Problem 7. Exercise 5.1, page 273.

Problem 8. Exercise 5.4, page 273.

Problem 9. Exercise 5.11, page 276.

Problem 10. Exercise 5.21,(a)(b)(c) page 280.

Problem 11. Let f be a continuously differentiable function with a Lipschitz gradient, i.e there is a positive constant $L \geq 0$ such that

$$\|\nabla f(x) - \nabla f(y)\| \leq L\|x - y\|.$$

Show that for any x, y, one has

$$f(y) \leq f(x) + (\nabla f(x))^T (y - x) + \frac{L}{2}\|x - y\|^2.$$