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### 1 Basic

#### 1.1 Default Code

```
#include <bits/stdc++.h>
using namespace std;
using lld = int64 t;
using llu = uint6\overline{4} t;
using llf = long double;
using PII = pair<int,int>;
using PIL = pair<int, lld>;
using PLI = pair<lld,int>;
using PLL = pair<lld, lld>;
template<typename T>
using maxHeap = priority_queue<T, vector<T>, less<T>>;
template<typename T>
using minHeap = priority_queue<T,vector<T>,greater<T>>;
#define FF first
#define SS second
#define SZ(x) (int)((x).size())
#define ALL(x) begin(x), end(x)
#define PB push_back
#define WC(x) \overline{\text{while}}((x)--)
template<typename Iter>
ostream& _out(ostream &s, Iter b, Iter e) {
 s<<"[";
  for (auto it=b;it!=e;it++) s<<(it==b?"": " ") <<*it;</pre>
 s<<"]";
  return s;
template<typename A, typename B>
ostream& operator<<(ostream &s,const pair<A,B>&p)
{return s<<"("<<p.FF<<","<<p.SS<<")";}
template<typename T>
ostream& operator<<(ostream &s,const vector<T>&c)
{return _out(s,ALL(c));}
bool debug = 0;
#define DUMP(x) if(debug)cerr<< PRETTY FUNCTION <<":"</pre>
<< LINE <<" - "<<(#x) << "=""<<(x) << \(^1 \) \n"

template< typename T>
void DEBUG(const T& x) {if(debug)cerr<<x;}</pre>
template<typename T, typename...Args>
void DEBUG(const T&head,const Args&...tail) {
  if (debug) {cerr<<head; DEBUG(tail...); }</pre>
int main(int argc,char* argv[]){
  if(argc>1&&string(argv[1]) == "-D") debug=1;
  if(!debug) {ios_base::sync_with_stdio(0);cin.tie(0);}
  return 0;
```

### 1.2 IncreaseStackSize

```
//stack resize(change esp to rsp if 64-bit system)
asm( "mov %0, %%esp\n" :: "g"(mem+10000000) );
// craziest way
static void run_with_stack_size(void(*func)(),size_t
    stsize) {
  char *stack, *send;
  stack=(char *)malloc(stsize);
  send=stack+stsize-16;
  send=(char *)((uintptr_t) send/16*16);
  asm volatile(
    "mov %%rsp, (%0)\n"
    "mov %0, %%rsp\n'
    :
: "r" (send));
  func();
  asm volatile(
    "mov (%0), %%rsp\n"
    : "r" (send));
  free (stack);
```

## 1.3 Pragma optimization

#pragma GCC optimize("Ofast,no-stack-protector,no-matherrno,unroll-loops")

```
#pragma GCC target("sse,sse2,sse3,ssse3,sse4,popcnt,abm
    ,mmx,avx,tune=native")
```

## 1.4 Debugger

```
#! /usr/bin/env python3
import subprocess, platform
os_name = platform.system()
cmd, prefix = [], ""
if os name == 'Windows':
 cmd=["cmd", "/C"]
else:
 \texttt{cmd} = ["bash", "-c"]
 prefix = "./"
def GetTestData(exe):
 myout=subprocess.check output(cmd+["%s%s"%(prefix,
     exe)])
 return myout.decode("utf8")
def Judge(a,b,testdata):
 f = open("test.in", "w+")
 f.write(testdata)
 f.close()
 c=subprocess.check_output(cmd+["%s%s < test.in"%(</pre>
      prefix, a)])
 d=subprocess.check output(cmd+["%s%s < test.in"%(
     prefix, b)])
 if not c == d:
   print("answer: %s"%c.decode("utf8"),end="")
    print("output: %s"%d.decode("utf8"), end="")
   print("WA!")
    return False
 return True
    __name__ == '__main__':
if
 cnt = 0
  isOK = True
 while isOK:
   cnt. += 1
    print(cnt)
    isOK = Judge("1234.exe", "test.exe", GetTestData("
        gen.exe"))
```

#### 1.5 Quick Random

```
template<class T,T x1,T x2,T x3,int y1,int y2,int y3>
struct PRNG {
       using S = typename std::make signed<T>::type;
        T s;
       PRNG(T _s = 0) : s(_s) {}
       T next() {
               T z = (s += x1);
               z = (z ^ (z >> y1)) * x2;
               z = (z ^ (z >> y2)) * x3;
               return z ^ (z >> y3);
       T next(T n) { return next() % n; }
       S next(S 1, S r) {return l+next(r-l+1);}
       T operator()() { return next(); }
        \begin{tabular}{lll} $\mathbb{T}$ & {\bf operator}\,()\,(\begin{tabular}{lll} $\mathbb{T}$ & {\bf next}\,(\begin{tabular}{lll} $\mathbb{T}$ & {\bf next}\,(\begin{tabular}) & {\bf next}\,(\begin{tabular}{lll} $\mathbb{T}$ & {\bf next}\,(\begin{
       S operator()(S 1, S r) { return next(1, r); }
       static T gen(T s) { return PRNG(s)(); }
         template<class U>
       void shuffle(U first,U last){
                 size_t n=last-first;
                 for (size t i=0;i<n;i++) swap (first[i], first[next(i</pre>
       }
};
using R32 =PRNG<uint32_t,0x9E3779B1,0x85EBCA6B,0</pre>
                 xC2B2AE35,16,13,16>;
R32 r32;
using R64=PRNG<uint64 t,0x9E3779B97F4A7C15,
0xBF58476D1CE4E5B9,0x94D049BB133111EB,30,27,31>;
R64 r64;
```

### 1.6 IO Optimization

```
static inline int gc() {
   static char buf[1 << 20], *p = buf, *end = buf;
   if (p == end) {</pre>
```

```
if ((end = buf + fread(buf, 1, 1 << 20, stdin)) ==</pre>
         buf) return EOF;
    p = buf;
  return *p++;
template<typename T>
static inline bool gn(T & ){
                                              = 1;
  register int c = gc(); register T
  while(!isdigit(c) and c!=EOF and c!='-') c = gc();
  if(c == '-') { __ = -1; c = gc(); }
  if(c == EOF) return false;
  while(isdigit(c)) _ = _ * 10 + c - '0', c = gc();
  return true;
template <typename T, typename ...Args>
\textbf{static inline bool} \ \texttt{gn} \ ( \texttt{T \&x, Args\& ...args} ) \ \{ \textbf{return} \ \texttt{gn} \ ( \texttt{x} ) \\
      and gn(args...);}
```

## 2 Data Structure

## 2.1 Bigint

```
class BigInt{
 private:
    using lld = int fast64 t;
    #define PRINTF ARG PRIdFAST64
    #define LOG BASE STR "9"
    static constexpr lld BASE = 1000000000;
    static constexpr int LOG_BASE = 9;
    vector<lld> dig;
    bool neg;
    inline int len()const{return (int)dig.size();}
    inline int cmp_minus(const BigInt& a) const {
      if(len() == 0 and a.len() == 0) return 0;
      if(neg ^ a.neg) return (int)a.neg*2 - 1;
      if(len() != a.len()) return neg?a.len()-len():len
          ()-a.len();
      for (int i=len()-1;i>=0;i--) if (dig[i] != a.dig[i
          1) {
        return neg?a.dig[i]-dig[i]:dig[i]-a.dig[i];
      return 0;
    inline void trim(){
      while(!dig.empty() and dig.back()==0) dig.
          pop back();
      if(dig.empty()) neg = false;
  public:
    BigInt(): dig(vector<lld>()), neg(false){}
    BigInt(lld a): dig(vector<lld>()){
      neg = a<0; dig.push back(abs(a));</pre>
      trim();
    BigInt(const string& a): dig(vector<lld>()) {
      assert(!a.empty()); neg = (a[0]=='-');
      for(int i=((int)(a.size()))-1;i>=neg;i-=LOG BASE)
        11d cur = 0;
        for(int j=min(LOG BASE-1, i-neg);j>=0;j--) cur
            = cur*10+a[i-j]-'0';
        dig.push back(cur);
      } trim();
    inline bool operator<(const BigInt& a)const{return</pre>
        cmp minus(a)<0;}</pre>
    inline bool operator<=(const BigInt& a)const{return</pre>
         cmp minus(a) <=0;}</pre>
    inline bool operator==(const BigInt& a)const{return
         cmp minus(a) ==0;}
    inline bool operator!=(const BigInt& a)const{return
         cmp minus(a)!=0;}
    inline bool operator>(const BigInt& a)const{return
        cmp_minus(a)>0;}
    inline bool operator>=(const BigInt& a)const{return
         cmp minus(a)>=0;}
    BigInt operator-() const {
     BigInt ret = *this;
      ret.neg ^= 1;
      return ret;
```

```
BigInt operator+(const BigInt& a) const {
  if (neg) return -(-(*this)+(-a));
  if(a.neg) return (*this)-(-a);
  int n = max(a.len(), len());
  BigInt ret; ret.dig.resize(n);
  11d pro = 0;
  for(int i=0;i<n;i++) {</pre>
    ret.dig[i] = pro;
    if(i < a.len()) ret.dig[i] += a.dig[i];</pre>
    if(i < len()) ret.dig[i] += dig[i];</pre>
    pro = 0;
    if(ret.dig[i] >= BASE) pro = ret.dig[i]/BASE;
    ret.dig[i] -= BASE*pro;
  if(pro != 0) ret.dig.push back(pro);
  return ret;
BigInt operator-(const BigInt& a) const {
  if(neg) return -(-(*this) - (-a));
  if(a.neg) return (*this) + (-a);
  int diff = cmp_minus(a);
  if(diff < 0) return -(a - (*this));
if(diff == 0) return 0;</pre>
  BigInt ret; ret.dig.resize(len(), 0);
  for(int i=0;i<len();i++) {</pre>
    ret.dig[i] += dig[i];
    if(i < a.len()) ret.dig[i] -= a.dig[i];</pre>
    if(ret.dig[i] < 0){
      ret.dig[i] += BASE;
      ret.dig[i+1]--;
    }
  }
  ret.trim();
  return ret;
BigInt operator*(const BigInt& a) const {
  if(len() == 0 or a.len() == 0) return 0;
  BigInt ret; ret.dig.resize(len()+a.len()+1);
  ret.neg = neg ^ a.neg;
  for (int i=0;i<len();i++) for (int j=0;j<a.len();j</pre>
      ++) {
    ret.dig[i+j] += dig[i] * a.dig[j];
if(ret.dig[i+j] >= BASE) {
      lld x = ret.dig[i+j] / BASE;
      ret.dig[i+j+1] += x;
      ret.dig[i+j] -= x * BASE;
    }
  ret.trim();
  return ret;
BigInt operator/(const BigInt& a) const {
  assert(a.len());
  if(len() < a.len()) return 0;</pre>
  BigInt ret; ret.dig.resize(len()-a.len()+1);
  ret.neg = a.neg;
  for(int i=len()-a.len();i>=0;i--){
    11d 1 = 0, r = BASE;
    while(r-1 > 1) {
      11d \ mid = (1+r) >> 1;
      ret.dig[i] = mid;
      if(ret*a <= (neg?-(*this):(*this))) l = mid;</pre>
      else r = mid;
    ret.dig[i] = 1;
  ret.neg ^= neg; ret.trim();
  return ret;
BigInt operator%(const BigInt& a) const {
  return (*this) - (*this) / a * a;
friend BigInt abs(BigInt a) {
  a.neg = 1; return a;
friend void swap(BigInt& a, BigInt& b){
  swap(a.dig, b.dig); swap(a.neg, b.neg);
friend istream& operator>>(istream& ss, BigInt& a){
  string s; ss >> s;
  a = s;
  return ss;
friend ostream& operator<<(ostream& ss, const</pre>
    BigInt& a) {
```

```
if(a.len() == 0) return ss << '0';
    if(a.neg) ss << '-';
    ss << a.dig.back();
    for(int i=a.len()-2;i>=0;i--) ss << setw(LOG_BASE
        ) << setfill('0') << a.dig[i];
    return ss;
}
inline void print() const {
    if(len() == 0) {putchar('0'); return;}
    if(neg) putchar('-');
    printf("%" PRINTF_ARG, dig.back());
    for(int i=len()-2;i>=0;i--) printf("%0"
        LOG_BASE_STR PRINTF_ARG, dig[i]);
}
#undef PRINTF_ARG
#undef LOG_BASE_STR
};
```

## 2.2 unordered\_map

```
#include <ext/pb_ds/assoc_container.hpp>
using __gnu_pbds::cc_hash_table;
using __gnu_pbds::gp_hash_table;
template<typename A, typename B>
using hTable1=cc_hash_table<A,B>;
template<typename A, typename B>
using hTable2=gp_hash_table<A,B>;
```

### 2.3 extc\_balance\_tree

```
#include <ext/pb_ds/assoc_container.hpp>
using __gnu_pbds::tree;
using __gnu_pbds::rb_tree_tag;
using __gnu_pbds::ov_tree_tag;
using __gnu_pbds::splay_tree_tag;
using __gnu_pbds::null_type;
using
        gnu pbds::tree order statistics node update;
template<typename T>
using ordered set = tree<T, null type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;
template<typename A, B>
using ordered map = tree<A, B, less<A>, rb tree tag,
    tree order statistics node update>;
int main(){
  ordered set<int> ss;
  ordered map<int,int> mm;
  ss.insert(1);
  ss.insert(5);
  assert(*ss.find_by_order(0)==1);
  assert(ss.order of key(-1)==0);
  assert(ss.order_of_key(87) == 2);
  return 0;
```

## 2.4 extc\_heap

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb ds/priority queue.hpp>
using __gnu_pbds::priority_queue;
using __gnu_pbds::pairing_heap_tag;
using __gnu_pbds::binary_heap_tag;
using __gnu_pbds::binomial_heap_tag;
using __gnu_pbds::rc_binomial_heap_tag;
using __gnu_pbds::thin_heap_tag;
int main(){
  priority_queue<int,less<int>,pairing_heap_tag> pq1,
      pq2;
  pq1.push(1);
  pq2.push(2);
  pq1.join(pq2);
  assert(pq2.size()==0);
  auto it = pq1.push(87);
  pq1.modify(it, 19);
  while(!pq1.empty()) {
    pq1.top();
    pq1.pop();
  return 0;
```

## 2.5 SkewHeap

```
template<typename T, typename cmp=less<T> >
class SkewHeap{
private:
  struct SkewNode{
    T x;
    SkewNode *lc, *rc;
    SkewNode(T a=0):x(a), lc(nullptr), rc(nullptr){}
  } *root;
  cmp CMP ;
  size t count;
  SkewNode* Merge (SkewNode* a, SkewNode* b) {
    if(!a or !b) return a?a:b;
    if(CMP_(a->x, b->x)) swap(a, b);
    a->rc = Merge(a->rc, b);
    swap(a->lc, a->rc);
    return a;
  void clear(SkewNode*& a) {
    if(!a) return;
    clear(a->lc); clear(a->rc);
    delete a; a = nullptr;
public:
  SkewHeap(): root(nullptr), count(0){}
  bool empty() {return count==0;}
  size t size() {return count;}
  T top() {return root->x;}
  void clear() {clear(root);count = 0;}
  void push(const T& x) {
    SkewNode* a = new SkewNode(x);
    count += 1;
    root = Merge(root, a);
  void join(SkewHeap& a){
    count += a.count; a.count = 0;
    root = Merge(root, a.root);
  void pop(){
    count -= 1;
SkewNode* rt = Merge(root->lc, root->rc);
    delete root; root = rt;
  friend void swap(SkewHeap& a, SkewHeap& b) {
    swap(a.root, b.root);
};
```

## 2.6 Disjoint Set

```
class DJS{
private:
  vector<int> fa, sz, sv;
  vector<pair<int*, int>> opt;
  inline void assign(int *k, int v) {
    opt.emplace_back(k, *k);
    *k = v;
public:
 inline void init(int n){
    fa.resize(n); iota(fa.begin(), fa.end(), 0);
    sz.resize(n); fill(sz.begin(), sz.end(), 1);
    opt.clear();
 int query(int x) {
    if(fa[x] == x) return x;
    return query(fa[x]);
 inline void merge(int a, int b) {
   int af = query(a), bf = query(b);
    if(af == bf) return;
    if(sz[af] < sz[bf]) swap(af, bf);</pre>
    assign(&fa[bf], fa[af]);
   assign(&sz[af], sz[af]+sz[bf]);
  inline void save() {sv.push_back((int)opt.size());}
 inline void undo() {
    int ls = sv.back(); sv.pop_back();
    while((int)opt.size() > ls){
     pair<int*, int> cur=opt.back();
*cur.first = cur.second;
      opt.pop_back();
```

```
}
| };
```

## 2.7 Treap

```
namespace Treap{
  \#define sz(x)((x)?((x)->size):0)
  \#define sm(x) ((x)? ((x)->sum):0)
  struct node{
    int size, cnt, sum;
    uint32_t pri;
    node *lc, *rc;
    node():size(0),cnt(0),sum(0),pri(rand()),
          lc(nullptr),rc(nullptr){}
    node(int x):size(1),cnt(x),sum(x),pri(rand()),
                lc(nullptr),rc(nullptr){}
    void pull()
     sum = cnt;
     if ( lc ) sum += lc->sum;
     if ( rc ) sum += rc->sum;
     size = 1;
      if ( lc ) size += lc->size;
      if ( rc ) size += rc->size;
    }
  };
  node* merge( node* L, node* R ) {
    if ( not L or not R ) return L ? L : R;
    if ( L->pri > R->pri ) {
     L->rc = merge(L->rc, R);
     L->pull();
     return L;
    } else {
      R->lc = merge(L, R->lc);
     R->pull();
     return R;
  void split_by_size(node*rt,int k,node*&L,node*&R) {
   if ( not rt ) L = R = nullptr;
    else if( sz( rt->lc ) + 1 <= k ) {
     L = rt;
      split by size(rt->rc, k-sz(rt->lc)-1, L->rc, R);
     L->pull();
    } else {
     R = rt;
      split by size( rt->lc, k, L, R->lc );
     R->pull();
  void split_by_sum(node*rt,int k,node*&L,node*&R) {
    if ( not rt ) L = R = nullptr;
    else if( sm( rt->lc ) + rt->cnt <= k ) {
     L = rt;
      split by sum(rt->rc,k-sm(rt->lc)-rt->cnt,L->rc,R)
     L->pull();
   } else {
     R = rt;
      split by sum( rt->lc, k, L, R->lc);
      R->pull();
    }
  #undef sz
  #undef sm
```

### 2.8 SparseTable

```
template<typename T, typename Cmp_=less<T>>
class SparseTable{
private:
   vector<vector<T>> table;
   vector<int> lg;
   T cmp_(T a, T b){
      return Cmp_() (a, b)?a:b;
   }
public:
   void init(T arr[], int n){
      // 0-base
   lg.resize(n+1);
   lg[0] = -1, lg[1] = 0;
```

```
for (int i=2;i<=n;i++) lg[i] = lg[i>>1]+1;
    table.resize(lg[n]+1);
    table[0].resize(n);
    for (int i=0;i<n;i++) table[0][i] = arr[i];</pre>
    for (int i=1;i<=lg[n];i++) {</pre>
      int len = 1 << (i-1), sz = 1 << i;
       table[i].resize(n-sz+1);
      for (int j=0; j<=n-sz; j++) {</pre>
         table[i][j] = cmp (table[i-1][j], table[i-1][j+
             len]);
      }
    }
  T query(int 1, int r) {
    // 0-base [1, r)
    int wh = lg[r-l], len=1<<wh;</pre>
    return cmp_(table[wh][1], table[wh][r-len]);
};
```

## 2.9 Linear Basis

```
struct LinearBasis{
private:
  int n, sz;
  vector<llu> B;
  inline llu two(int x) {return ((llu)1) << x;}</pre>
public:
  void init(int n ) {
    n = n ; B.clear();
    B.resize(n); sz = 0;
  void insert(llu x){
    // add x into B
    for (int i=n-1;i>=0;i--) if (two(i) & x) {
      if(B[i]) x ^= B[i];
      else{
        B[i] = x; sz++;
        for (int j=i-1; j>=0; j--)
          if(B[j] and two(j) & B[i])
            B[i] ^= B[j];
        for (int j=i+1; j<n; j++)</pre>
          if(two(i) & B[j])
            B[j] ^= B[i];
      }
    }
  inline int size() {return sz;}
  bool check(llu x) {
    // is x in span(B) ?
    for (int i=n-1;i>=0;i--) if (two(i) & x) {
      if(B[i]) x ^= B[i];
      else return false;
    return true;
  llu kth_small(llu k) {
    /** 1-base would always > 0 **/
    /** should check it **/
    /* if we choose at least one element
       but size(B) (vectors in B) == N(original elements)
       then we can't get 0 */
    llu ret = 0;
    for (int i=0;i<n;i++) if (B[i]) {</pre>
      if(k & 1) ret ^= B[i];
      k >>= 1;
    return ret;
} base;
```

# 3 Graph

## 3.1 BCC Edge

```
class BCC{
private:
  vector<int> low, dfn;
  int cnt;
```

```
vector<bool> bcc;
  vector<vector<PII>> G:
  void dfs(int w, int f) {
    dfn[w] = cnt++;
    low[w] = dfn[w];
    for(auto i: G[w]){
      int u = i.FF, t = i.SS;
      if(u == f) continue;
      if (dfn[u]!=0) {
        low[w] = min(low[w], dfn[u]);
      }else{
        dfs(u, w);
        low[w] = min(low[w], low[u]);
        if(low[u] > dfn[w]) bcc[t] = true;
    }
public:
  void init(int n, int m) {
    G.resize(n);
    fill(G.begin(), G.end(), vector<PII>());
    bcc.clear(); bcc.resize(m);
    low.clear(); low.resize(n);
    dfn.clear(); dfn.resize(n);
   cnt = 0;
  void add edge(int u, int v) {
    // should check for multiple edge
    G[u].PB({v, cnt});
    G[v].PB({u, cnt});
    cnt++;
  void solve() {cnt = 1;dfs(0, 0);}
  // the id will be same as insert order, 0-base
  bool is bcc(int x) {return bcc[x];}
```

#### 3.2 BCC Vertex

```
class BCC{
  private:
    int n, ecnt;
    vector< vector< pair< int, int > > > G;
    vector< int > low, id;
    vector< bool > vis, ap;
    void dfs( int u, int f, int dfn ) {
      low[ u ] = dfn; vis[ u ] = true;
      for ( auto e : G[ u ] ) if ( e.first != f ) {
        if ( vis[ e.first ] ) {
          low[ u ] = min( low[ u ], low[ e.first ] );
         } else {
          dfs(e.first, u, dfn + 1);
          if ( low[ e.first ] >= dfn ) ap[ u ] = true;
      }
    void mark( int u, int idd ) {
       // really??????????
      if (ap[u]) return;
       for ( auto e : G[ u ] )
        if( id[ e.second ] != -1 ) {
        id[ e.second ] = idd;
        mark( e.first, idd );
  public:
    void init( int n_ ) {
      ecnt = 0, n = n_;
      G.clear();
      G.resize(n);
      low.clear();
      low.resize( n );
      ap.clear();
      ap.resize( n );
       vis.clear();
      vis.resize( n );
    void add_edge( int u, int v ) {
      G[ u ].emplace_back( v, ecnt );
      G[ v ].emplace back( u, ecnt ++ );
    void solve() {
      for ( int i = 0 ; i < n ; ++ i )</pre>
        if ( not vis[ i ] ) dfs( i, i, 0 );
```

```
id.resize( ecnt );
  fill( id.begin(), id.end(), -1 );
  ecnt = 0;
  for ( int i = 0 ; i < n ; ++ i )
    if ( ap[ i ] ) for ( auto e : G[ i ] )
        if( id[ e.second ] != -1 ) {
        id[ e.second ] = ecnt;
            mark( e.first, ecnt ++ );
        }
  int get_id( int x ) { return id[ x ]; }
  int count() { return ecnt; }
  bool is_ap( int u ) { return ap[ u ]; }
} bcc;</pre>
```

## 3.3 Bipartite Matching

```
class BipartiteMatching{
  private:
    vector<int> X[N], Y[N];
    int fX[N], fY[N], n;
    bitset<N> walked;
    bool dfs(int x) {
      for(auto i:X[x]){
        if (walked[i]) continue;
        walked[i]=1;
        if (fY[i] ==-1||dfs(fY[i])) {
           fY[i]=x;fX[x]=i;
           return 1;
        }
      return 0;
  public:
    void init(int _n) {
      n=n;
      for (int i=0; i<n; i++) {</pre>
        X[i].clear();
        Y[i].clear();
        fX[i] = fY[i] = -1;
      walked.reset();
    void add edge(int x, int y) {
      X[x].push back(v);
      Y[y].push_back(y);
    int solve(){
      int cnt = 0;
      for (int i=0;i<n;i++) {</pre>
        walked.reset();
        if(dfs(i)) cnt++;
       // return how many pair matched
      return cnt;
};
```

### 3.4 Minimum Cost Maximum Flow

```
class MiniCostMaxiFlow{
 using CapT = int;
using WeiT = int64_t;
  using PCW = pair<CapT, WeiT>;
 static constexpr CapT INF CAP = 1 << 30;</pre>
 static constexpr WeiT INF_WEI = 1LL<<60;</pre>
private:
 struct Edge{
    int to, back;
    WeiT wei;
    CapT cap;
    Edge() {}
    Edge(int a,int b, WeiT c, CapT d):
      to(a),back(b),wei(c),cap(d)
    { }
  };
  int ori, edd;
  vector<vector<Edge>> G;
  vector<int> fa, wh;
 vector<bool> ing;
 vector<WeiT> dis;
 PCW SPFA() {
```

```
fill(inq.begin(),inq.end(),false);
    fill(dis.begin(), dis.end(), INF WEI);
    queue<int> qq; qq.push(ori);
    dis[ori]=0;
    while(!qq.empty()){
      int u=qq.front();qq.pop();
      inq[u] = 0;
      for(int i=0;i<SZ(G[u]);++i){</pre>
        Edge e=G[u][i];
        int v=e.to;
        WeiT d=e.wei;
        if(e.cap<=0||dis[v]<=dis[u]+d)
          continue;
        dis[v]=dis[u]+d;
        fa[v]=u, wh[v]=i;
        if(inq[v]) continue;
        qq.push(v);
        inq[v]=1;
    if (dis[edd] == INF WEI)
     return {-1,-1};
    CapT mw=INF_CAP;
    for(int i=edd;i!=ori;i=fa[i])
      mw=min(mw,G[fa[i]][wh[i]].cap);
    for (int i=edd;i!=ori;i=fa[i]) {
     auto &eg=G[fa[i]][wh[i]];
      eg.cap-=mw;
      G[eg.to][eg.back].cap+=mw;
   return {mw,dis[edd]};
public:
  void init(int a,int b,int n) {
    ori=a,edd=b;
    G.clear(); G.resize(n);
    fa.resize(n); wh.resize(n);
    inq.resize(n); dis.resize(n);
  void add edge(int st,int ed,WeiT w,CapT c) {
    G[st].emplace back(ed,SZ(G[ed]),w,c);
    G[ed].emplace back(st,SZ(G[st])-1,-w,0);
  PCW solve() {
    CapT cc=0; WeiT ww=0;
    while(true) {
      PCW ret=SPFA();
      if(ret.first==-1) break;
      cc+=ret.first;
      ww+=ret.second;
    return {cc,ww};
} mcmf;
```

#### 3.5 MaximumFlow

```
class Dinic{
private:
  using CapT = int64 t;
  struct Edge{
    int to, rev;
    CapT cap;
  int n, st, ed;
  vector<vector<Edge>> G;
  vector<int> lv;
  bool BFS() {
    fill(lv.begin(), lv.end(), -1);
    queue<int> bfs;
    bfs.push(st);
    lv[st] = 0;
    while(!bfs.empty()){
      int u = bfs.front(); bfs.pop();
      for(auto e: G[u]){
        if(e.cap <= 0 or lv[e.to]!=-1) continue;</pre>
        lv[e.to] = lv[u] + 1;
        bfs.push(e.to);
    return (lv[ed]!=-1);
  CapT DFS(int u, CapT f) {
    if(u == ed) return f;
```

```
CapT ret = 0;
    for(auto& e: G[u]) {
      if(e.cap <= 0 or lv[e.to]!=lv[u]+1) continue;</pre>
      CapT nf = DFS(e.to, min(f, e.cap));
      ret += nf; e.cap -= nf; f -= nf;
      G[e.to][e.rev].cap += nf;
      if(f == 0) return ret;
    if (ret == 0) lv[u] = -1;
   return ret;
public:
 void init(int n_, int st_, int ed_) {
    n = n_, st = st_, ed = ed_;
    G.resize(n); lv.resize(n);
    fill(G.begin(), G.end(), vector<Edge>());
  void add_edge(int u, int v, CapT c){
    G[u].push_back({v, (int)(G[v].size()), c});
    G[v].push back({u, (int)(G[u].size())-1, 0});
 CapT max flow() {
    CapT ret = 0;
    while (BFS()) {
      CapT f = DFS(st, numeric limits<CapT>::max());
      ret += f;
      if(f == 0) break;
    return ret;
} flow;
```

#### 3.6 Kuhn Munkres

```
struct KM{
  static constexpr 1ld INF = 1LL<<60;</pre>
  lld w[N][N], lx[N], ly[N], slack[N];
  int match[N], n, vx[N], vy[N], step;
  void init(int n_){
    n=n , step=0;
    memset(w,0,sizeof(w));
    memset(lx,0,sizeof(lx));
    memset(ly,0,sizeof(ly));
    memset(slack, 0, sizeof(slack));
    memset (match, 0, sizeof (match));
    memset(vx,0,sizeof(vx));
    memset(vy,0,sizeof(vy));
  void add edge(int u,int v,lld w) {w[u][v]=w;}
 bool dfs(int x) {
    vx[x] = step_{:}
    for (int i = 0; i < n; ++i) {</pre>
      if (vy[i]==step_) continue;
if (lx[x] + ly[i] > w[x][i]) {
         slack[i] = min(slack[i], lx[x] + ly[i] - w[x][i]
             ]);
        continue;
      vy[i] = step_;
      if (match[i] == -1 || dfs(match[i])) {
        match[i] = x;
        return true;
      }
    return false;
  lld solve() {
    fill_n(match, n, -1);
    fill n(lx, n, -INF);
    fill_n(ly, n, 0);
    for (int i = 0; i < n; ++i)</pre>
      for (int j = 0; j < n; ++j)</pre>
        lx[i] = max(lx[i], w[i][j]);
    for (int i = 0; i < n; ++i) {</pre>
      fill n(slack, n, INF);
      while (true) {
        step ++;
        if (dfs(i)) break;
        11d dlt = INF;
        for (int j = 0; j < n; ++j) if (vy[j] != step_)</pre>
        dlt = min(dlt, slack[j]);
for (int j = 0; j < n; ++j) {
          if (vx[j] == step_) lx[j] -= dlt;
           if (vy[j]==step_) ly[j] += dlt;
```

```
else slack[j] -= dlt;

}

}

lld res = 0;
   for (int i = 0; i < n; ++i) res += w[match[i]][i];
   return res;
}

km;</pre>
```

#### 3.7 2-SAT

```
class TwoSat{
  private:
     int n;
     vector<vector<int>> rG,G,sccs;
     vector<int> ord,idx;
     vector<bool> vis, result;
     void dfs(int u) {
       vis[u]=true;
       for(int v:G[u])
         if(!vis[v])
           dfs(v);
       ord.push back(u);
     void rdfs(int u){
       vis[u]=false;
       idx[u]=sccs.size()-1;
       sccs.back().push back(u);
       for(int v:rG[u])
         if(vis[v])
           rdfs(v);
  public:
     void init(int n ) {
      n=n;
       G.clear():
       G.resize(n);
       rG.clear();
      rG.resize(n);
       sccs.clear();
       ord.clear():
       idx.resize(n);
       result.resize(n);
     void add edge(int u,int v) {
      G[u].push_back(v);
       rG[v].push back(u);
     void orr(int x,int y) {
       if ((x^v)==1) return;
       add edge (x^1, y);
       add edge(y^1, x);
     bool solve() {
       vis.clear();
       vis.resize(n);
       for (int i=0;i<n;++i)</pre>
         if(not vis[i])
           dfs(i);
       reverse(ord.begin(),ord.end());
       for (int u:ord) {
         if(!vis[u])
           continue
         sccs.push back(vector<int>());
         rdfs(u);
       for (int i=0;i<n;i+=2)</pre>
         if(idx[i] == idx[i+1])
           return false;
       vector<bool> c(sccs.size());
       for(size t i=0;i<sccs.size();++i){</pre>
         for (size_t j=0; j<sccs[i].size();++j){</pre>
           result[sccs[i][j]]=c[i];
           c[idx[sccs[i][j]^1]]=!c[i];
         }
       }
       return true;
     bool get(int x) {return result[x];}
     inline int get_id(int x){return idx[x];}
     inline int count(){return sccs.size();}
} sat2;
```

## 3.8 Heavy Light Decomposition

```
#define REP(i, s, e) for(int i = (s); i <= (e); i++)
#define REPD(i, s, e) for(int i = (s); i \ge (e); i--)
const int MAXN = 100010;
const int LOG = 19;
struct HLD{
 int n;
 vector<int> g[MAXN];
 int sz[MAXN], dep[MAXN];
 int ts, tid[MAXN], tdi[MAXN], tl[MAXN], tr[MAXN];
 // ts : timestamp , useless after yutruli
     tid[u]: pos. of node u in the seq.
tdi[i]: node at pos i of the seq.
  // tl , tr[ u ] : subtree interval in the seq. of
 int prt[MAXN][LOG], head[MAXN];
  // head[ u ] : head of the chain contains u
 void dfssz(int u, int p) {
    dep[u] = dep[p] + 1;
    prt[u][0] = p; sz[u] = 1; head[u] = u;
    for (int& v:g[u]) if (v != p) {
      dep[v] = dep[u] + 1;
      dfssz(v, u);
      sz[u] += sz[v];
 void dfshl(int u){
    ts++:
    tid[u] = tl[u] = tr[u] = ts;
    tdi[tid[u]] = u;
    sort(ALL(g[u]),
         [&] (int a, int b) {return sz[a] > sz[b];});
   bool flag = 1;
    for(int& v:g[u]) if(v != prt[u][0]){
      if(flag) head[v] = head[u], flag = 0;
      dfshl(v);
      tr[u] = tr[v];
  inline int lca(int a, int b) {
    if(dep[a] > dep[b]) swap(a, b);
   int diff = dep[b] - dep[a];
REPD(k, LOG-1, 0) if(diff & (1<<k)){</pre>
      b = prt[b][k];
    if(a == b) return a;
    REPD(k, LOG-1, 0) if(prt[a][k] != prt[b][k]) {
      a = prt[a][k]; b = prt[b][k];
    return prt[a][0];
 void init( int _n ) {
   n = _n; REP( i , 1 , n ) g[ i ].clear();
  void addEdge( int u , int v ){
   g[ u ].push_back( v );
    g[ v ].push back( u );
  void yutruli(){
    dfssz(1, 0);
    ts = 0;
    dfshl(1);
    REP(k, 1, LOG-1) REP(i, 1, n)
      prt[i][k] = prt[prt[i][k-1]][k-1];
 vector< PII > getPath( int u , int v ){
  vector< PII > res;
    while( tid[ u ] < tid[ head[ v ] ] ){</pre>
      res.push_back( PII(tid[ head[ v ] ] , tid[ v ]) )
      v = prt[ head[ v ] ][ 0 ];
    res.push\_back( \ PII( \ tid[ \ u \ ] \ , \ tid[ \ v \ ] \ ) \ );
    reverse( ALL( res ) );
    return res;
    /* res : list of intervals from u to v
     * u must be ancestor of v
     * usage :
     * vector< PII >& path = tree.getPath( u , v )
     * for( PII tp : path ) {
        int l, r; tie(l, r) = tp;
         upd(1,r);
         uu = tree.tdi[ l ] , vv = tree.tdi[ r ];
         uu ~> vv is a heavy path on tree
```

```
*/
}
} tree;
```

## 3.9 MaxClique

```
struct MaxClique {
     int n, deg[maxn], ans;
     bitset<maxn> adj[maxn];
     vector<pair<int, int>> edge;
     void init(int _n) {
         n = _n;
         for (int i = 0; i < n; ++i) adj[i].reset();
for (int i = 0; i < n; ++i) deg[i] = 0;</pre>
         edge.clear();
     void add edge(int a, int b) {
         edge.emplace_back(a, b);
          ++deg[a]; ++deg[b];
     int solve() {
         vector<int> ord;
         for (int i = 0; i < n; ++i) ord.push_back(i);</pre>
         sort(ord.begin(), ord.end(), [&](const int &a,
              const int &b) { return deg[a] < deg[b]; });</pre>
         vector<int> id(n);
         for (int i = 0; i < n; ++i) id[ord[i]] = i;</pre>
         for (auto e : edge) {
              int u = id[e.first], v = id[e.second];
              adj[u][v] = adj[v][u] = true;
         bitset<maxn> r, p;
for (int i = 0; i < n; ++i) p[i] = true;
         ans = 0;
         dfs(r, p);
         return ans;
     void dfs(bitset<maxn> r, bitset<maxn> p) {
         if (p.count() == 0) return ans = max(ans, (int)
              r.count()), void();
         if ((r | p).count() <= ans) return;</pre>
         int now = p._Find_first();
         bitset<maxn> cur = p & ~adj[now];
         for (now = cur._Find_first(); now < n; now =</pre>
              cur. Find next(now)) {
              r[now] = true;
              dfs(r, p & adj[now]);
              r[now] = false;
              p[now] = false;
         }
     }
};
```

## 4 Math

## 4.1 Prime Table

```
\begin{array}{c} 1002939109, 1020288887, 1028798297, 1038684299, \\ 1041211027, 1051762951, 1058585963, 1063020809, \\ 1147930723, 1172520109, 1183835981, 1187659051, \\ 1241251303, 1247184097, 1255940849, 1272759031, \\ 1287027493, 1288511629, 1294632499, 1312650799, \\ 1868732623, 1884198443, 1884616807, 1885059541, \\ 1909942399, 1914471137, 1923951707, 1925453197, \\ 1979612177, 1980446837, 1989761941, 2007826547, \\ 2008033571, 2011186739, 2039465081, 2039728567, \\ 2093735719, 2116097521, 2123852629, 2140170259, \\ 3148478261, 3153064147, 3176351071, 3187523093, \\ 3196772239, 3201312913, 3203063977, 3204840059, \\ 3210224309, 3213032591, 3217689851, 3218469083, \\ 3219857533, 3231880427, 3235951699, 3273767923, \\ 3276188869, 3277183181, 3282463507, 3285553889, \\ 3319309027, 3327005333, 3327574903, 3341387953, \\ 3373293941, 3380077549, 3380892997, 3381118801 \\ \end{array}
```

# 4.2 $\left|\frac{n}{i}\right|$ Enumeration

$$T_0 = 1, T_{i+1} = \lfloor \frac{n}{\lfloor \frac{n}{T_i + 1} \rfloor} \rfloor$$

### 4.3 ax+by=gcd

```
// ax+ny = 1, ax+ny == ax == 1 (mod n)
void exgcd(lld x,lld y,lld &g,lld &a,lld &b) {
   if (y == 0) g=x,a=1,b=0;
   else
      exgcd(y,x%y,g,b,a),b==(x/y)*a;
}
```

#### 4.4 Pollard Rho

```
// does not work when n is prime
// return any non-trivial factor
llu pollard rho(llu n){
 static auto f=[](llu x,llu k,llu m){
   return add(k, mul(x, x, m), m);
 if (!(n&1)) return 2;
 mt19937 rnd(120821011);
  while(true) {
    llu y=2, yy=y, x=rnd()%n, t=1;
    for(llu sz=2;t==1;sz<<=1) {</pre>
      for(llu i=0;i<sz;++i){</pre>
        if (t!=1)break;
        yy=f(yy,x,n);
        t=gcd(yy>y?yy-y:y-yy,n);
      y=yy;
    if(t!=1&&t!=n) return t;
```

## 4.5 Pi Count (Linear Sieve)

```
static constexpr int N = 1000000 + 5;
lld pi[N];
vector<int> primes;
bool sieved[N]:
lld cube root(lld x) {
  1ld s=cbrt(x-static_cast<long double>(0.1));
  while(s*s*s <= x) ++s;</pre>
 return s-1;
lld square root(lld x) {
 11d s=sqrt(x-static cast<long double>(0.1));
  while (s*s <= x) ++s;
 return s-1;
void init(){
 primes.reserve(N);
 primes.push_back(1);
 for (int i=2; i<N; i++) {</pre>
   if(!sieved[i]) primes.push back(i);
    pi[i] = !sieved[i] + pi[i-1];
    for(int p: primes) if(p > 1) {
      if(p * i >= N) break;
      sieved[p * i] = true;
      if(p % i == 0) break;
   }
lld phi(lld m, lld n) {
 static constexpr int MM = 80000, NN = 500;
  static lld val[MM][NN];
 if (m<MM&&n<NN&&val[m][n])return val[m][n]-1;</pre>
 if(n == 0) return m;
 if(primes[n] >= m) return 1;
 lld ret = phi(m,n-1)-phi(m/primes[n],n-1);
 if(m<MM\&\&n<NN) val[m][n] = ret+1;
 return ret;
lld pi count(lld);
lld P2(lld m, lld n) {
 11d sm = square_root(m), ret = 0;
  for(lld i = n+1;primes[i] <= sm; i++)</pre>
    ret+=pi count(m/primes[i])-pi count(primes[i])+1;
  return ret;
lld pi_count(lld m) {
 if(m < N) return pi[m];</pre>
```

```
lld n = pi_count(cube_root(m));
return phi(m, n) + n - 1 - P2(m, n);
}
```

## 4.6 NloglogN Sieve

```
void Sieve(int n) {
  for(int i=2;i<=n;i++) {
    if(notprime[i]) continue;
    primes.push_back(i);
  for(int j=i*i;j<=n;j+=i) notprime[i]=true;
  }
}</pre>
```

## 4.7 Range Sieve

#### 4.8 Miller Rabin

```
bool isprime(llu x){
  static llu magic[]={2,325,9375,28178,\
                     450775,9780504,1795265022};
  static auto witn=[](llu a,llu u,llu n,int t){
    a = mpow(a,u,n);
    if (!a)return 0;
    while(t--){
      11u a2=mul(a,a,n);
      if(a2==1 && a!=1 && a!=n-1)
        return 1;
      a = a2;
    return a!=1;
  };
  if(x<2)return 0;</pre>
  if(!(x&1))return x==2;
  llu x1=x-1; int t=0;
  while (!(x1&1))x1>>=1,t++;
  for(llu m:magic)
    if(witn(m,x1,x,t))
      return 0;
  return 1:
```

#### 4.9 Inverse Element

```
// x's inverse mod k
long long GetInv(long long x, long long k) {
    // k is prime: euler_(k)=k-1
    return qPow(x, euler_phi(k)-1);
}
// if you need [1, x] (most use: [1, k-1]
void solve(int x, long long k) {
    inv[1] = 1;
    for(int i=2;i<x;i++)
        inv[i] = ((long long)(k - k/i) * inv[k % i]) % k;
}</pre>
```

#### 4.10 Euler Phi Function

```
extended euler:
   a^b mod p
   if gcd(a, p) == 1: a^(b%phi(p))
   elif b < phi(p): a^b mod p
   else a^(b%phi(p) + phi(p))
lld euler_phi(int x) {
  11d r=1;
  for (int i=2;i*i<=x;++i) {</pre>
    if (x%i==0) {
      x/=i;
       r*=(i-1):
       while (x%i==0) {
        x/=i;
         r*=i;
    }
  if(x>1) r*=x-1;
  return r;
vector<int> primes;
bool notprime[N];
lld phi[N];
void euler_sieve(int n) {
  for(int i=2;i<n;i++) {</pre>
    if(!notprime[i]){
      primes.push back(i);
      phi[i] = i - \overline{1};
    for (auto j: primes) {
      if(i*j >= n) break;
       notprime[i*j] = true;
       phi[i*j] = phi[i] * phi[j];
       if(i % j == 0) {
  phi[i*j] = phi[i] * j;
         break;
  }
```

### 4.11 Gauss Elimination

```
typedef long double llf;
const int N = 300;
const llf EPS = 1e-8;
// make m[i][i] = x, m[i][j] = 0
// v is for solving equation:
// for (int i=0; i< n; i++) ans [pos[i]] = val[i]/mtx[i][pos
// for(int i=0;i<n;i++) cout << ans[i] << '\n';
bool Gauss(llf m[N][N], llf v[N], int n, int pos[N]){</pre>
  for (int i=0; i < n; i++) {</pre>
    int x=-1, y=-1; llf e = 0;
    for (int j=i;j<n;j++) for (int k=i;k<n;k++) {</pre>
      if(fabs(m[j][pos[k]])>e){
         e = fabs(m[j][pos[k]]);
         x = j, y = k;
      }
    if(x==-1 or y==-1) return false;
    swap(m[x], m[i]);
    swap(v[x], v[i]);
    swap(pos[y], pos[i]);
    for (int j=i+1; j<n; j++) {</pre>
      llf xi = m[j][pos[i]]/m[i][pos[i]];
      for (int k=0; k<n; k++) m[j][pos[k]] -= xi*m[i][pos[</pre>
           k]];
      v[j] -= xi*v[i];
  for(int i=n-1;i>=0;i--) {
    for (int j=i-1; j>=0; j--) {
      llf xi = m[j][pos[i]]/m[i][pos[i]];
       for (int k=0; k<n; k++) m[j][pos[k]] -= xi*m[i][pos[</pre>
           k]];
      v[j] = xi*v[i];
```

```
}
}
return true;
}
```

#### 4.12 Fast Fourier Transform

```
polynomial multiply:
   DFT(a, len); DFT(b, len);
   for(int i=0;i<len;i++) c[i] = a[i]*b[i];</pre>
   iDFT(c, len);
   (len must be 2^k and = 2^k (max(a, b)))
   Hand written Cplx would be 2x faster
Cplx omega[2][N];
void init_omega(int n) {
  static constexpr llf PI=acos(-1);
  const llf arg=(PI+PI)/n;
  for (int i=0; i<n; ++i)</pre>
    omega[0][i]={cos(arg*i),sin(arg*i)};
  for (int i=0;i<n;++i)</pre>
    omega[1][i]=conj(omega[0][i]);
void tran(Cplx arr[],int n,Cplx omg[]) {
  for (int i=0, j=0; i<n; ++i) {</pre>
    if(i>j)swap(arr[i],arr[j]);
    for (int l=n>>1; (j^=1)<1; l>>=1);
  for (int 1=2;1<=n;1<<=1) {</pre>
    int m=1>>1;
    for(auto p=arr;p!=arr+n;p+=1) {
      for (int i=0;i<m;++i) {</pre>
        Cplx t=omg[n/l*i]*p[m+i];
        p[m+i]=p[i]-t;
        p[i]+=t;
      }
    }
 }
void DFT(Cplx arr[],int n){
  tran(arr,n,omega[0]);
void iDFT(Cplx arr[],int n) {
  tran(arr,n,omega[1]);
  for (int i=0;i<n;++i) arr[i]/=n;</pre>
```

#### 4.13 Chinese Remainder

```
lld crt(lld ans[], lld pri[], int n) {
  11d M = 1;
  for (int i=0;i<n;i++) M *= pri[i];</pre>
  11d ret = 0;
  for (int i=0;i<n;i++) {</pre>
    lld inv = (gcd(M/pri[i], pri[i]).first + pri[i])%
         pri[i];
    ret += (ans[i] * (M/pri[i]) %M * inv) %M;
    ret %= M;
  return ret;
}
Another:
x = a1 % m1
x = a2 \% m2
g = gcd(m1, m2)
assert\left(\left(a1\text{-}a2\right)\%g\text{==0}\right)
[p, q] = exgcd(m2/g, m1/g)
return a2+m2*(p*(a1-a2)/g)
0 <= x < 1cm(m1, m2)
```

## 4.14 Berlekamp Massey

```
// x: 1-base, p[]: 0-base
template<size_t N>
vector<llf> BM(llf x[N], size_t n) {
    size_t f[N]={0}, t=0; llf d[N];
```

```
vector<llf> p[N];
for(size_t i=1,b=0;i<=n;++i) {
    for(size_t j=0;j<p[t].size();++j)
        d[i]+=x[i-j-1]*p[t][j];
    if(abs(d[i]-=x[i])<=EPS) continue;
    f[t]=i;if(!t) {p[++t].resize(i); continue;}
    vector<llf> cur(i-f[b]-1);
    llf k=-d[i]/d[f[b]]; cur.PB(-k);
    for(size_t j=0;j<p[b].size();j++)
        cur.PB(p[b][j]*k);
    if(cur.size()<p[t].size())cur.resize(p[t].size());
    for(size_t j=0;j<p[t].size();j++)cur[j]+=p[t][j];
    if(i-f[b]+p[b].size())>=p[t].size()) b=t;
    p[++t]=cur;
}
return p[t];
```

### 4.15 NTT

```
Remember coefficient are mod P
/* p=a*2^n+1
   n 2^n
16 65536
20 1048576
                                       root.
                                  а
                      65537
                                  7
                     7340033
                                        3 */
// (must be 2^k)
template<LL P, LL root, int MAXN>
struct NTT {
  static LL bigmod(LL a, LL b) {
    LL res = 1;
    for (LL bs = a; b; b >>= 1, bs = (bs * bs) % P)
      if(b&1) res=(res*bs)%P;
    return res;
  static LL inv(LL a, LL b) {
    if(a==1)return 1;
    return (((LL) (a-inv(b%a,a))*b+1)/a)%b;
  LL omega[MAXN+1];
  NTT() {
    omega[0] = 1;
    LL r = bigmod(root, (P-1)/MAXN);
    for (int i=1; i<=MAXN; i++)</pre>
      omega[i] = (omega[i-1]*r)%P;
  // n must be 2^k
  void tran(int n, LL a[], bool inv_ntt=false) {
    int basic = MAXN / n , theta = \overline{b}asic;
    for (int m = n; m >= 2; m >>= 1) {
      int mh = m >> 1;
      for (int i = 0; i < mh; i++) {</pre>
        LL w = omega[i*theta%MAXN];
        for (int j = i; j < n; j += m) {
  int k = j + mh;</pre>
           LL x = a[j] - a[k];
           if (x < 0) x += P;
           a[j] += a[k];
           if (a[j] > P) a[j] -= P;
           a[k] = (w * x) % P;
        }
      theta = (theta * 2) % MAXN;
    int i = 0;
    for (int j = 1; j < n - 1; j++) {</pre>
      for (int k = n >> 1; k > (i ^= k); k >>= 1);
      if (j < i) swap(a[i], a[j]);</pre>
    if (inv_ntt) {
      LL ni = inv(n, P);
      reverse( a+1 , a+n );
      for (i = 0; i < n; i++)</pre>
        a[i] = (a[i] * ni) % P;
  }
const LL P=2013265921, root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;
```

## 4.16 DiscreteLog

```
/ Baby-step Giant-step Algorithm
11d BSGS(11d P, 11d B, 11d N) {
  // find B^L = N \mod P
  unordered map<lld, int> R;
  lld sq = (lld) sqrt(P);
  11d t = 1;
  for (int i = 0; i < sq; i++) {</pre>
    if (t == N)
     return i;
    if (!R.count(t))
     R[t] = i;
    t = (t * B) % P;
  ild f = inverse(t, P);
  for (int i=0;i<=sq+1;i++) {</pre>
    if (R.count(N))
      return i * sq + R[N];
    N = (N * f) % P;
  return -1;
```

## 5 Geometry

#### 5.1 Point Class

```
template<typename T>
struct Point{
 typedef long double llf;
  static constexpr llf EPS = 1e-8;
 T x, y;
          _=0, T __=0): x(_), y(__){}
  Point(T
  template<typename T2>
    Point(const Point<T2>& a): x(a.x), y(a.y){}
  inline llf theta() const {
   return atan2((llf)y, (llf)x);
  inline llf dis() const {
   return hypot((llf)x, (llf)y);
 inline llf dis(const Point& o) const {
    return hypot((llf)(x-o.x), (llf)(y-o.y));
  Point operator-(const Point& o) const {
   return Point(x-o.x, y-o.y);
  Point operator = (const Point& o) {
   x-=o.x, y-=o.y;
   return *this;
 Point operator+(const Point& o) const {
    return Point(x+o.x, y+o.y);
  Point operator+=(const Point& o) {
   x+=0.x, y+=0.y;
    return *this;
  Point operator*(const T& k) const {
   return Point(x*k, y*k);
  Point operator*=(const T& k) {
    x*=k, y*=k;
    return *this;
  Point operator/(const T& k) const {
    return Point(x/k, y/k);
  Point operator/=(const T& k) {
   x/=k, y/=k;
    return *this;
  Point operator-() const {
   return Point(-x, -y);
  Point rot90() const {
   return Point(-y, x);
  template<typename T2>
 bool in(const Circle<T2>& a) const {
   /* Add struct Circle at top */
    return a.o.dis(*this) +EPS <= a.r;</pre>
```

```
bool equal(const Point& o, true type) const {
    return fabs(x-o.x) < EPS and fabs(y-o.y) < EPS;
  bool equal(const Point& o, false_type) const {
    return tie(x, y) == tie(o.x, o.y);
  bool operator==(const Point& o) const {
    return equal(o, is floating point<T>());
  bool operator!=(const Point& o) const {
    return ! (*this == 0);
  bool operator<(const Point& o) const {</pre>
    return theta() < o.theta();</pre>
    // sort like what pairs did
    // if(is floating point<T>()) return fabs(x-o.x)
        EPS?y<o.y:x<o.x;
    // else return tie(x, y) < tie(o.x, o.y);
  friend inline T cross(const Point& a, const Point& b)
    return a.x*b.y - b.x*a.y;
  friend inline T dot(const Point& a, const Point &b) {
    return a.x*b.x + a.y*b.y;
  friend ostream& operator<<(ostream& ss, const Point&</pre>
    ss<<"("<<o.x<<", "<<o.y<<")";
    return ss;
};
```

## 5.2 Circle Class

### 5.3 Line Class

```
const Point<long double> INF_P(-1e20, 1e20);
const Point<long double> NOT EXIST(1e20, 1e-20);
template<typename T>
struct Line{
 static constexpr long double EPS = 1e-8;
  // ax+by+c = 0
 T a, b, c;
 Line(): a(0), b(1), c(0) {}
Line(T_, T__, T__): a(
   ine(T _, T __, T ___): a(_), b(__), (
    assert(fabs(a)>EPS or fabs(b)>EPS);
                                       ), c(___
  template<typename T2>
    Line(const Line<T2>& x): a(x.a), b(x.b), c(x.c){}
  typedef Point<long double> Pt;
 bool equal(const Line& o, true type) const {
    return fabs(a-o.a) < EPS and fabs(b-o.b) < EPS and
        fabs(c-o.b) < EPS;</pre>
 bool eugal(const Line& o, false type) const {
    return a==o.a and b==o.b and c==o.c;
 bool operator==(const Line& o) const {
    return euqal(o, is floating point<T>());
 bool operator!=(const Line& o) const {
    return ! (*this == 0);
```

```
friend inline bool on_line__(const Point<T>& p, const
       Line& l, true_type){
    return fabs(1.a*p.x + 1.b*p.y + 1.c) < EPS;
  friend inline bool on_line__(const Point<T>& p, const
       Line& l, false_type) {
    return 1.a*p.x + \overline{1}.b*p.y + 1.c == 0;
  friend inline bool on line(const Point<T>&p const
      Line& 1) {
    return on line (p, l, is floating point<T>());
  friend inline bool is parallel (const Line& x, const
       Line& y, true_type) {
    return fabs(x.a*y.b - x.b*y.a) < EPS;</pre>
  friend inline bool is_parallel__(const Line& x, const
       Line& y, false_type) {
    return x.a*y.b == x.b*y.a;
  friend inline bool is parallel(const Line& x, const
      Line& v) {
    return is_parallel__(x, y, is_floating_point<T>());
  friend inline Pt get inter(const Line& x, const Line&
       A) {
    typedef long double llf;
    if(x==y) return INF_P;
    if(is parallel(x, y)) return NOT EXIST;
    llf delta = x.a*y.b - x.b*y.a;
    llf delta_x = x.b*y.c - x.c*y.b;
llf delta_y = x.c*y.a - x.a*y.c;
    return Pt(delta_x / delta, delta_y / delta);
  friend ostream& operator<<(ostream& ss, const Line& o</pre>
    ss<<o.a<<"x+"<<o.b<<"y+"<<o.c<<"=0";
    return ss;
};
template<typename T>
inline Line<T> get line(const Point<T>& a, const Point<</pre>
  return Line<T>(a.y-b.y, b.x-a.x, (b.y-a.y) *a.x-(b.x-a
      .x) *a.y);
```

## 5.4 Triangle Circumcentre

#### 5.5 2D Convex Hull

```
template<typename T>
class ConvexHull_2D{
private:
    typedef Point<T> PT;
    vector<PT> dots;
    struct myhash{
        uint64_t operator() (const PT& a) const {
            uint64_t xx=0, yy=0;
            memcpy(&xx, &a.x, sizeof(a.x));
            memcpy(&yy, &a.y, sizeof(a.y));
            uint64_t ret = xx*17+yy*31;
            ret = (ret ^ (ret >> 16))*0x9E3779B1;
            ret = (ret ^ (ret >> 13))*0xC2B2AE35;
            ret = ret ^ xx;
```

```
return (ret ^ (ret << 3)) * yy;</pre>
  } :
  unordered set<PT, myhash> in hull;
  inline void init() {in hull.clear(); dots.clear();}
  void insert(const PT& x) {dots.PB(x);}
  void solve() {
    sort(ALL(dots), [](const PT& a, const PT& b){
      return tie(a.x, a.y) < tie(b.x, b.y);</pre>
    });
    vector<PT> stk(SZ(dots)<<1);
    int top = 0;
    for (auto p: dots) {
      while(top >= 2 and cross(p-stk[top-2], stk[top
          -1]-stk[top-2]) <= 0)
        top --;
      stk[top++] = p;
    for (int i=SZ(dots)-2, t = top+1;i>=0;i--) {
      while(top >= t and cross(dots[i]-stk[top-2], stk[
          top-1]-stk[top-2]) <= 0)
        top --;
      stk[top++] = dots[i];
    stk.resize(top-1);
    swap(stk, dots);
    for(auto i: stk) in_hull.insert(i);
  vector<PT> get() {return dots;}
 inline bool in it(const PT& x) {
    return in hull.find(x)!=in hull.end();
};
```

### 5.6 2D Farthest Pair

### 5.7 2D Cosest Pair

```
struct Point{
  11f x, y;
 llf dis;
} arr[N];
inline llf get dis(Point a, Point b){
 return hypot(a.x-b.x, a.y-b.y);
llf solve(){
 int cur = rand()%n;
 for(int i=0;i<n;i++) arr[i].dis = get_dis(arr[cur],</pre>
      arr[i]);
 sort(arr, arr+n, [](Point a, Point b){return a.dis <</pre>
      b.dis; });
 llf ans = 1e50;
  for (int i=0; i < n; i++) {</pre>
    for (int j=i+1; j<n; j++) {</pre>
      if(arr[j].dis - arr[i].dis > ans) break;
      ans = min(ans, get dis(arr[i], arr[j]));
    }
  return ans;
```

## 5.8 SimulateAnnealing

```
double getY(double);
int main(){
  int rr, ll;
  default random engine rEng(time(NULL));
  uniform_real_distribution<double> Range(-1,1);
  uniform_real_distribution<double> expR(0,1);
  auto Random=bind(Range, rEng), expRand=bind(expR, rEng)
  int step=0;
  double pace=rr-ll, mini=0.95; // need to search for
      it
  double x=max(min(Random()*pace+ll, rr), ll), y=getY(x
  while (pace>=1e-7) {
    double newX = max(min(x + Random()*pace, rr), 11);
    double newY = getY(newX);
    if(newY < y \mid \mid expRand() < exp(-step))
     x=newX, y=newY;
    step++;
   pace*=mini;
```

## 5.9 Ternary Search on Integer

```
int TernarySearch(int 1, int r) {
    // (1, r]
    while (r - 1 > 1) {
        int mid = (1 + r) >> 1;
        if (f(mid) > f(mid + 1)) r = mid;
        else 1 = mid;
    }
    return 1+1;
}
```

## 5.10 Minimum Covering Circle

```
template<typename T>
Circle<llf> MinCircleCover(const vector<Point<T>>& pts)
  random shuffle (ALL (pts));
  Circle<llf> c = \{pts[0], 0\};
  int n = SZ(pts);
  for (int i=0;i<n;i++) {</pre>
    if(pts[i].in(c)) continue;
    c = \{pts[i], 0\};
    for (int j=0; j<i; j++) {</pre>
      if(pts[j].in(c)) continue;
      c.o = (pts[i] + pts[j]) / 2;
      c.r = pts[i].dis(c.o);
      for (int k=0; k<j; k++) {</pre>
        if(pts[k].in(c)) continue;
        c = get_circum(pts[i], pts[j], pts[k]);
  return c;
```

### 5.11 KDTree (Nearest Point)

```
const int MXN = 100005;
struct KDTree {
  struct Node {
    int x,y,x1,y1,x2,y2;
    int id,f;
    Node *L, *R;
  }tree[MXN];
  int n;
  Node *root;
  LL dis2(int x1, int y1, int x2, int y2) {
    LL dx = x1-x2;
    LL dy = y1-y2;
    return dx*dx+dy*dy;
  static bool cmpx(Node& a, Node& b) { return a.x<b.x; }</pre>
  static bool cmpy(Node& a, Node& b) { return a.y<b.y; }</pre>
  void init(vector<pair<int,int>> ip) {
    n = ip.size();
```

```
for (int i=0; i<n; i++) {</pre>
       tree[i].id = i;
       tree[i].x = ip[i].first;
       tree[i].y = ip[i].second;
    root = build tree(0, n-1, 0);
  Node* build tree(int L, int R, int dep) {
     if (L>R) return nullptr;
     int M = (L+R)/2;
     tree[M].f = dep%2;
    nth_element(tree+L, tree+M, tree+R+1, tree[M].f ?
     cmpy : cmpx);
tree[M].x1 = tree[M].x2 = tree[M].x;
    tree[M].y1 = tree[M].y2 = tree[M].y;
     tree[M].L = build tree(L, M-1, dep+1);
    if (tree[M].L) {
       tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
       tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
       tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
     tree[M].R = build_tree(M+1, R, dep+1);
    if (tree[M].R) {
       tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
       tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
       \texttt{tree} \, [\texttt{M}] \, . \, \texttt{y1} \, = \, \texttt{min} \, (\texttt{tree} \, [\texttt{M}] \, . \, \texttt{y1} \, , \, \, \, \texttt{tree} \, [\texttt{M}] \, . \, \texttt{R} \text{--} \text{y1}) \, ;
       tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
    return tree+M;
  int touch(Node* r, int x, int y, LL d2){
    LL dis = sqrt(d2)+1;
     if (x<r->x1-dis || x>r->x2+dis ||
         y<r->y1-dis || y>r->y2+dis)
       return 0;
    return 1;
  void nearest(Node* r, int x, int y,
                  int &mID, LL &md2) {
     if (!r || !touch(r, x, y, md2)) return;
    LL d2 = dis2(r->x, r->y, x, y);
     if (d2 < md2 \mid | (d2 == md2 && mID < r->id)) {
       mID = r->id;
       md2 = d2;
     // search order depends on split dim
     if ((r->f == 0 \&\& x < r->x) | |
         (r->f == 1 && y < r->y))
       nearest(r->L, x, y, mID, md2);
nearest(r->R, x, y, mID, md2);
     } else {
       nearest(r->R, x, y, mID, md2);
       nearest(r->L, x, y, mID, md2);
  int query(int x, int y) {
    int id = 1029384756;
    LL d2 = 102938475612345678LL;
    nearest(root, x, y, id, d2);
    return id;
  }
}tree;
```

# 6 Stringology

### 6.1 Hash

```
class Hash{
private:
    static const int N = 1000000;
    const int p = 127, q = 1208220623;
    int sz, prefix[N], power[N];
    inline int add(int x, int y) {return x+y>=q?x+y-q:x+y
        ;}
    inline int sub(int x, int y) {return x-y<0?x-y+q:x-y;}
    inline int mul(int x, int y) {return 1LL*x*y%q;}

public:
    void init(const std::string &x) {
        sz = x.size();
        prefix[0]=0;</pre>
```

## 6.2 Suffix Array

```
//help by http://www.geeksforgeeks.org/suffix-array-set
    -2-a-nlognlogn-algorithm/
struct sfx{
  int index:
  int r,nr;
char str[N + 10];
int len;
vector<sfx> srs[N + 10];
int mapping[N + 10];
sfx sa[N + 10];
bool cmp(sfx a, sfx b) {
  if(a.r==b.r){
    return a.nr<b.nr;</pre>
  }else{
    return a.r<b.r;</pre>
void SA() {
  len = strlen(str);
  for (int i=0;i<len;i++) {</pre>
    sa[i].index = i;
    sa[i].r=str[i];
    sa[i].nr=(i+1>=len)?0:str[i+1];
  //sort(sa,sa+len,cmp);
  radixSort();
  for (int j=2; j<=len; j*=2) {</pre>
    int cnt=1;
    int rr = sa[0].r;
    sa[0].r=cnt;
    mapping[sa[0].index]=0;
    for(int i=1;i<len;i++) {</pre>
      if(sa[i].r == rr && sa[i].nr == sa[i-1].nr) {
        rr=sa[i].r;
         sa[i].r=cnt;
      }else{
        rr=sa[i].r;
         sa[i].r=++cnt;
      mapping[sa[i].index]=i;
    for(int i=0;i<len;i++) {</pre>
      int nn = sa[i].index+j;
      sa[i].nr = (nn>=len)?0:sa[mapping[nn]].r;
    //sort(sa, sa+len, cmp);
    radixSort();
void radixSort() {
  int m = 0;
  for (int i=0;i<len;i++) {</pre>
    srs[sa[i].nr].PB(sa[i]);
    m=max(m,sa[i].nr);
  int cnt=0;
  for (int i=0;i<=m;i++) {</pre>
    if(srs[i].empty())continue;
    for(auto j:srs[i]){
      sa[cnt++] = j;
    srs[i].clear();
  }
  m = 0;
  for (int i=0;i<len;i++) {</pre>
    srs[sa[i].r].PB(sa[i]);
    m=max(m,sa[i].r);
  cnt=0;
  for (int i=0;i<=m;i++) {</pre>
```

```
if(srs[i].empty()) continue;
for(auto j:srs[i]) {
    sa[cnt++] = j;
    }
    srs[i].clear();
}
```

#### 6.3 KMP

```
int F[N<<1];</pre>
void KMP(char s1[], char s2[], int n, int m) {
  // make F[] for s1+'\0'+s2;
  char ss[N<<1];</pre>
  int len = n+m+1:
  for(int i=0;i<n;i++) ss[i] = s1[i];</pre>
  ss[n] = ' \setminus 0';
  for(int i=0;i<m;i++) ss[i+1+n] = s2[i];</pre>
  F[0] = F[1] = 0;
  for (int i=1; i < len; i++) {</pre>
    int j = F[i];
    while (j > 0 \text{ and } ss[i]!=ss[j]) j = F[j];
    F[i+1] = (ss[i]==ss[j]?j+1:0);
  // just find (F[len2+i] == len2), i from 1 to len+1
       for matching
  [0, i]是個循環字串,且循環節為i-f[i]:
  if(f[i]>0 \text{ and } i\%(i-f[i])==0) \text{ cout } << i << " " << i/(i
       -f[i]) << '\n';
```

#### 6.4 Z value

```
char s[MAXN];
int len,z[MAXN];
void Z_value() {
   int i,j,left,right;
   left=right=0; z[0]=len;
   for(i=1;i<len;i++) {
      j=max(min(z[i-left],right-i),0);
      for(;i+j<len&&s[i+j]==s[j];j++);
      z[i]=j;
      if(i+z[i]>right) {
        right=i+z[i];
        left=i;
      }
}
```

## 6.5 Lexicographically Smallest Rotation

```
string mcp(string s) {
  int n = s.length();
  s += s;
  int i=0, j=1;
  while (i<n && j<n) {
    int k = 0;
    while (k < n && s[i+k] == s[j+k]) k++;
    if (s[i+k] <= s[j+k]) j += k+1;
    else i += k+1;
    if (i == j) j++;
  }
  int ans = i < n ? i : j;
  return s.substr(ans, n);
}</pre>
```

#### 6.6 BWT

```
struct BurrowsWheeler{
#define SIGMA 26
#define BASE 'a'
  vector<int> v[ SIGMA ];
  void BWT(char* ori, char* res){
    // make ori -> ori + ori
```

```
// then build suffix array
  void iBWT(char* ori, char* res) {
    for( int i = 0 ; i < SIGMA ; i ++ )</pre>
      v[ i ].clear();
    int len = strlen( ori );
    for( int i = 0 ; i < len ; i ++ )</pre>
      v[ ori[i] - BASE ].push back( i );
    vector<int> a;
    for( int i = 0 , ptr = 0 ; i < SIGMA ; i ++ )</pre>
      for( auto j : v[ i ] ){
        a.push_back( j );
        ori[ ptr ++ ] = BASE + i;
    for( int i = 0 , ptr = 0 ; i < len ; i ++ ){</pre>
      res[ i ] = ori[ a[ ptr ] ];
      ptr = a[ ptr ];
    res[len] = 0;
} bwt;
```

## 7 Misc

## 7.1 MaximumEmptyRect

```
int max_empty_rect(int n, int m, bool blocked[N][N]){
  static int mxu[2][N], me=0,he=1,ans=0;
  for (int i=0;i<m;i++) mxu[he][i]=0;</pre>
  for (int i=0;i<n;i++) {</pre>
    stack<PII, vector<PII>> stk;
    for(int j=0;j<m;++j) {</pre>
      if(blocked[i][j]) mxu[me][j]=0;
      else mxu[me][j]=mxu[he][j]+1;
      int la = j;
      while(!stk.empty()&&stk.top().FF>mxu[me][j]){
        int x1 = i - stk.top().FF, x2 = i;
int y1 = stk.top().SS, y2 = j;
         la = stk.top().SS; stk.pop();
         ans=max(ans, (x2-x1)*(y2-y1));
      if(stk.emptv()||stk.top().FF<mxu[me][j])</pre>
         stk.push({mxu[me][j],la});
    while(!stk.empty()){
      int x1 = i - stk.top().FF, x2 = i;
      int y1 = stk.top().SS-1, y2 = m-1;
      stk.pop();
      ans=\max(ans, (x2-x1)*(y2-y1));
    swap (me, he);
  return ans;
```

#### 7.2 DP-opt Condition

## 7.2.1 totally monotone (concave/convex)

```
\begin{array}{l} \forall i < i', j < j', B[i][j] \leq B[i'][j] \implies B[i][j'] \leq B[i'][j'] \\ \forall i < i', j < j', B[i][j] \geq B[i'][j] \implies B[i][j'] \geq B[i'][j'] \end{array}
```

## 7.2.2 monge condition (concave/convex)

```
\begin{array}{l} \forall i < i', j < j', B[i][j] + B[i'][j'] \geq B[i][j'] + B[i'][j] \\ \forall i < i', j < j', B[i][j] + B[i'][j'] \leq B[i][j'] + B[i'][j] \end{array}
```

### 7.3 Convex 1D/1D DP

```
struct segment {
   int i, 1, r;
   segment() {}
   segment(int a, int b, int c): i(a), l(b), r(c) {}
};

inline long long f(int l, int r) {
   return dp[l] + w(l + 1, r);
}
```

```
void solve() {
 dp[0] = 011;
  deque<segment> deq; deq.push back(segment(0, 1, n));
  for (int i = 1; i <= n; ++i) {</pre>
    dp[i] = f(deq.front().i, i);
    while (deq.size() && deq.front().r < i + 1) deq.</pre>
        pop_front();
    deq.front().l = i + 1;
    segment seg = segment(i, i + 1, n);
    while (deq.size() && f(i, deq.back().1) < f(deq.
    back().i, deq.back().1)) deq.pop_back();</pre>
    if (deq.size()) {
  int d = 1048576, c = deq.back().1;
      while (d >>= 1) if (c + d <= deq.back().r) {</pre>
         if (f(i, c + d) > f(deq.back().i, c + d)) c +=
             d;
      deq.back().r = c; seg.l = c + 1;
    if (seg.1 <= n) deq.push back(seg);</pre>
```