Contents

1	Basi	ic																1
	1.1	vimrc							 								 	1
	1.2	IncreaseStackSize							 								 	1
	1.3	Pragma optimization							 								 	1
	1.4	Debugger					_		 								 	1
	1.5	Quick Random																1
	1.6	IO Optimization																2
		•																
2	Data	a Structure																2
	2.1	Bigint							 								 	2
	2.2	Dark Magic							 								 	3
	2.3	SkewHeap							 								 	3
	2.4	Disjoint Set							 								 	3
	2.5	Treap							 								 	3
	2.6	SparseTable							 								 	4
	2.7	Linear Basis							 								 	4
	_																	
3	Grap	•																4
	3.1	BCC Edge																4
	3.2	BCC Vertex																5
	3.3	Bipartite Matching																
	3.4	Minimum Cost Maximum I																5
	3.5	General Graph Matching																6
	3.6	MaximumFlow																6
	3.7	Kuhn Munkres																6
	3.8	2-SAT																7
	3.9	Lowbit Decomposition																
	3.10	MaxClique			٠		٠	٠	 	٠	٠	 ٠	٠	٠		٠	 	8
4	Math	h																8
-	4.1	n Prime Table																8
	4.2	$\lfloor \frac{n}{i} \rfloor$ Enumeration																8
	4.3	ax+by=gcd																8
	4.4	Pollard Rho																8
	4.5	Pi Count (Linear Sieve)																9
	4.6	Range Sieve																9
	4.7	Miller Rabin																9
	4.8	Inverse Element																9
	4.9	Euler Phi Function																9
	4.10	Gauss Elimination							 								 	9
	4.11	Fast Fourier Transform .							 								 	10
	4.12	Chinese Remainder							 								 	10
	4.13	Berlekamp Massey							 								 	10
		NTT																10
	4.15	FWT							 								 	11
	4.16	DiscreteLog							 								 	11
_	_																	
5		metry																11
	5.1	Point Class																11
	5.2	Circle Class																12
	5.3	Line Class																12
	5.4	Triangle Circumcentre																12
	5.5	2D Convex Hull																12
	5.6	2D Farthest Pair																13
	5.7	2D Cosest Pair																13
	5.8	SimulateAnnealing Ternary Search on Integer					٠	•	 	•	٠	 ٠	٠	•	•	٠	 	13
	5.9						٠	•	 • •	٠	٠	 ٠	•	•	•	•	 	13
	5.10	Minimum Covering Circle KDTree (Nearest Point) .	•		•		٠	•	 • •	٠	•	 ٠	•	•	•	•	 	13 13
	5.11	KDTree (Nearest Point) .	•		٠		٠	•	 • •	٠	•	 ٠	٠	•	•	•	 	13
6	Strir	ngology																14
	6.1	Hash							 								 	14
		Suffix Array							 								 	14
	6.2																	14
	6.2 6.3								 									4.5
		Aho-Corasick Algorithm . KMP																15
	6.3	Aho-Corasick Algorithm . KMP			:	 			 									15
	6.3 6.4	Aho-Corasick Algorithm .		 		 			 								 	
	6.3 6.4 6.5	Aho-Corasick Algorithm . KMP Z value	t Ro	 ota	tior	 			 								 	15
	6.3 6.4 6.5 6.6 6.7	Aho-Corasick Algorithm KMP Z value Lexicographically Smalles	t Ro	 ota	tior	 			 								 	15 15 15
7	6.3 6.4 6.5 6.6 6.7	Aho-Corasick Algorithm . KMP Z value	t Ro	ota	tior	 n .			 			 					 	15 15 15
7	6.3 6.4 6.5 6.6 6.7	Aho-Corasick Algorithm . KMP Z value Lexicographically Smalles BWT	t Ro	ota	tior	 n .			 			 					 	15 15 15 15
7	6.3 6.4 6.5 6.6 6.7	Aho-Corasick Algorithm . KMP	t Ro	ota	tion	 n . 			 			 					 	15 15 15 15 15
7	6.3 6.4 6.5 6.6 6.7 Misc 7.1	Aho-Corasick Algorithm KMP	t Ro		tion				 			 					 	15 15 15 15 15 15
7	6.3 6.4 6.5 6.6 6.7 Miso 7.1	Aho-Corasick Algorithm KMP Z value Lexicographically Smalles BWT Degree Sequence Validity 7.1.1 Erdős-Gallai Theo 7.1.2 Havel-Hakimi algo MaximumEmptyRect	t Ro		tion				 			 					 	15 15 15 15 15 15 16 16
7	6.3 6.4 6.5 6.6 6.7 Misc 7.1	Aho-Corasick Algorithm KMP Z value Lexicographically Smalles BWT Degree Sequence Validity 7.1.1 Erdős-Gallai Theo 7.1.2 Havel-Hakimi algo MaximumEmptyRect DP-opt Condition DP-opt Condition DP-opt Condition C Z Value Res	t Ro	ota	tion							 					 · ·	15 15 15 15 15 15 16 16 16
7	6.3 6.4 6.5 6.6 6.7 Miso 7.1	Aho-Corasick Algorithm KMP	t Ro	ota	tion		ve		 			 						15 15 15 15 15 15 16 16 16
7	6.3 6.4 6.5 6.6 6.7 Misc 7.1 7.2 7.3	Aho-Corasick Algorithm KMP	t Ro	ota	tion	n .	ve											15 15 15 15 15 16 16 16 16
7	6.3 6.4 6.5 6.6 6.7 Miso 7.1	Aho-Corasick Algorithm KMP	t Ro	ota	tion		· · · · · · · · · · · · · · · · · · ·											15 15 15 15 15 15 16 16 16

1 Basic

1.1 vimrc

se nu rnu bs=2 ru mouse=a cin et ts=4 sw=4 sts=4 syn on filetype indent on inoremap <CR> <CR><Esc>O

1.2 IncreaseStackSize

```
//stack resize(change esp to rsp if 64-bit system)
asm( "mov %0, %%esp\n" :: "g"(mem+10000000));
// craziest way
static void run_stack_sz(void(*func)(),size_t stsize){
 char *stack, *send;
stack=(char *) malloc(stsize);
  send=stack+stsize-16:
  send=(char *)((uintptr t)send/16*16);
  asm volatile(
    "mov %%rsp, (%0)\n"
    "mov %0, %%rsp\n"
    :
: "r" (send));
  func();
  asm volatile(
    "mov (%0), %%rsp\n"
    : "r" (send));
  free(stack);
```

1.3 Pragma optimization

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,sse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,tune=native")
```

1.4 Debugger

```
| #! /usr/bin/env python3
import subprocess as sp
os_name = __import__('platform').system()
cmd,prefix = [],""
if os name == 'Windows':
 cmd=["cmd", "/C"]
else:
 cmd = ["bash", "-c"]
  prefix = "./"
def GetTestData(exe):
  myout=sp.check_output(cmd+["%s%s"%(prefix, exe)])
  return myout.decode("utf8")
def Judge(a,b,testdata):
  f = open("test.in", "w+")
  f.write(testdata)
  f.close()
  c=sp.check_output(cmd+["%s%s<test.in"%(prefix, a)])</pre>
  d=sp.check_output(cmd+["%s%s<test.in"%(prefix, b)])
  if not c == d:
    print("answer: %s"%c.decode("utf8"),end="")
    print("output: %s"%d.decode("utf8"),end="")
    print("WA!")
    return False
  return True
if __name__ == '__main__':
  cnt = 0
  isOK = True
   while isOK:
    cnt += 1
    print(cnt)
    isOK=Judge("sol", "mysol", GetTestData("gen"))
```

1.5 Quick Random

```
template < class T,T x1,T x2,T x3,int y1,int y2,int y3>
struct PRNG {
   using S = typename std::make_signed < T > :: type;
```

```
T s;
  PRNG(T s = 0) : s(_s) {}
  T next() {
    Tz = (s += x1);
    z = (z ^ (z >> y1)) * x2;
    z = (z ^ (z >> y2)) * x3;
return z ^ (z >> y3);
  T next(T n) { return next() % n; }
  S next(S 1, S r) {return l+next(r-l+1);}
  T operator()() { return next(); }
  T operator()(T n) { return next(n); }
  S operator()(S 1, S r) { return next(1, r); }
  static T gen(T s) { return PRNG(s)(); }
  template<class U>
  void shuffle(U first,U last){
    size t n=last-first;
    for (size_t i=0;i<n;i++)</pre>
      swap(first[i], first[next(i+1)]);
};
using R32=PRNG<uint32 t,0x9E3779B1,0x85EBCA6B,
0xC2B2AE35,16,13,16>;
R32 r32;
using R64=PRNG<uint64 t,0x9E3779B97F4A7C15,
0xBF58476D1CE4E5B9,0x94D049BB133111EB,30,27,31>;
R64 r64;
```

1.6 IO Optimization

```
static inline int qc() {
  static char buf[ 1 << 20 ], *p = buf, *end = buf;</pre>
  if ( p == end ) {
    end = buf + fread( buf, 1, 1 << 20, stdin );
    if ( end == buf ) return EOF;
   p = buf;
 return *p++;
template < typename T >
static inline bool gn( T &_ ) {
                                         = 1;
 register int c = gc(); register T
                                                = 0:
  while(('0'>c||c>'9') && c!=EOF && \overline{c!}='-') \overline{c} = gc();
  if(c == '-') { __ = -1; c = gc(); }
  if(c == EOF) return false;
 while ('0' \le c\&c \le '9') = -*10 + c - '0', c = gc();
 return true;
template < typename T, typename ...Args >
static inline bool gn( T &x, Args &...args )
{ return gn(x) && gn(args...); }
```

2 Data Structure

2.1 Bigint

```
class BigInt{
 private:
    using lld = int fast64 t;
    #define PRINTF ARG PRIdFAST64
    #define LOG BASE STR "9"
    static constexpr lld BASE = 1000000000;
    static constexpr int LOG_BASE = 9;
    vector<lld> dig;
   bool neg;
    inline int len() const { return (int) dig.size(); }
    inline int cmp_minus(const BigInt& a) const {
     if(len() == 0 \&\& a.len() == 0) return 0;
     if(neg ^ a.neg)return (int)a.neg*2 - 1;
     if(len()!=a.len())
       return neg?a.len()-len():len()-a.len();
      for(int i=len()-1;i>=0;i--) if(dig[i]!=a.dig[i])
       return neg?a.dig[i]-dig[i]:dig[i]-a.dig[i];
     return 0;
    inline void trim(){
     while(!dig.empty()&&!dig.back())dig.pop back();
     if(dig.empty()) neg = false;
```

```
public:
  BigInt(): dig(vector<lld>()), neg(false){}
  BigInt(lld a): dig(vector<lld>()) {
    neg = a<0; dig.push back(abs(a));
    trim();
  BigInt(const string& a): dig(vector<lld>()){
    assert(!a.empty()); neg = (a[0] == '-');
    for (int i=((int)a.size())-1;i>=neg;i-=LOG BASE) {
      11d cur = 0;
      for (int j=min(LOG BASE-1,i-neg);j>=0;j--)
        cur = cur*10+a[\bar{i}-j]-'0';
      dig.push_back(cur);
    } trim();
  inline bool operator<(const BigInt& a)const</pre>
    {return cmp minus(a)<0;}
  inline bool operator<=(const BigInt& a)const</pre>
    {return cmp_minus(a)<=0;}
  inline bool operator==(const BigInt& a)const
    {return cmp minus(a) == 0;}
  inline bool operator!=(const BigInt& a) const
    {return cmp_minus(a)!=0;}
  inline bool operator>(const BigInt& a)const
    {return cmp minus(a)>0;}
  inline bool operator>=(const BigInt& a)const
    {return cmp minus(a)>=0;}
  BigInt operator-() const {
    BigInt ret = *this;
    ret.neg ^= 1;
    return ret;
  BigInt operator+(const BigInt& a) const {
    if (neg) return -(-(*this)+(-a));
    if(a.neg) return (*this)-(-a);
    int n = max(a.len(), len());
    BiqInt ret; ret.dig.resize(n);
    11d pro = 0;
    for (int i=0;i<n;i++) {</pre>
      ret.dig[i] = pro;
      if(i < a.len()) ret.dig[i] += a.dig[i];</pre>
      if(i < len()) ret.dig[i] += dig[i];</pre>
      pro = 0;
      if(ret.dig[i] >= BASE) pro = ret.dig[i]/BASE;
      ret.dig[i] -= BASE*pro;
    if (pro != 0) ret.dig.push_back(pro);
    return ret;
  BigInt operator-(const BigInt& a) const {
    if (neg) return -(-(*this) - (-a));
    if(a.neg) return (*this) + (-a);
    int diff = cmp_minus(a);
    if (diff < 0) return - (a - (*this));</pre>
    if(diff == 0) return 0;
    BigInt ret; ret.dig.resize(len(), 0);
    for(int i=0;i<len();i++) {</pre>
      ret.dig[i] += dig[i];
      if(i < a.len()) ret.dig[i] -= a.dig[i];</pre>
      if(ret.dig[i] < 0){
        ret.dig[i] += BASE;
        ret.dig[i+1]--;
      }
    ret.trim();
    return ret;
  BigInt operator*(const BigInt& a) const {
    if(!len()||!a.len()) return 0;
    BigInt ret; ret.dig.resize(len()+a.len()+1);
    ret.neg = neg ^ a.neg;
    for (int i=0;i<len();i++)</pre>
      for (int j=0; j<a.len(); j++) {</pre>
        ret.dig[i+j] += dig[i] * a.dig[j];
        if(ret.dig[i+j] >= BASE) {
          lld x = ret.dig[i+j] / BASE;
          ret.dig[i+j+1] += x;
          ret.dig[i+j] -= x * BASE;
        }
      1
    ret.trim();
    return ret;
  BigInt operator/(const BigInt& a) const {
    assert(a.len());
    if(len() < a.len()) return 0;</pre>
```

```
BigInt ret; ret.dig.resize(len()-a.len()+1);
  ret.neg = a.neg;
  for(int i=len()-a.len();i>=0;i--){
    lld l = 0, r = BASE;
    while(r-1 > 1) {
      11d \ mid = (1+r) >> 1;
      ret.dig[i] = mid;
      if (ret*a<=(neg?-(*this):(*this))) l = mid;</pre>
      else r = mid;
    ret.dig[i] = 1;
  ret.neg ^= neg; ret.trim();
  return ret;
BigInt operator%(const BigInt& a) const {
  return (*this) - (*this) / a * a;
friend BigInt abs(BigInt a) {
  a.neg = 1; return a;
friend void swap(BigInt& a, BigInt& b) {
  swap(a.dig, b.dig); swap(a.neg, b.neg);
friend istream& operator>>(istream& ss, BigInt& a) {
  string s; ss >> s; a = s;
  return ss;
friend ostream& operator<<(ostream& ss, BigInt& a) {</pre>
  if(a.len() == 0) return ss << '0';
if(a.neg) ss << '-';</pre>
  ss << a.dig.back();
  for (int i=a.len()-2;i>=0;i--)
    ss << setw(LOG_BASE) << setfill('0') << a.dig[i];
  return ss;
inline void print() const {
  if(len() == 0) {putchar('0');return;}
 if(neg) putchar('-');
printf("%" PRINTF_ARG, dig.back());
  for (int i=len()-2;i>=0;i--)
    printf("%0" LOG BASE STR PRINTF ARG, dig[i]);
#undef PRINTF ARG
#undef LOG BASE STR
```

2.2 Dark Magic

```
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb ds/priority queue.hpp>
using __gnu_pbds::pairing_heap_tag;
using __gnu_pbds::binary_heap_tag;
using __gnu_pbds::binomial_heap_tag;
using __gnu_pbds::rc_binomial_heap tag;
using __gnu_pbds::thin_heap_tag;
template<typename T>
using pbds_heap=__gnu_pbds::prioity_queue<T,less<T>, \
                                      pairing heap tag>;
using __gnu_pbds::rb_tree_tag;
using __gnu_pbds::ov_tree_tag;
       __gnu_pbds::splay_tree_tag;
using
template<typename T>
using ordered_set = __gnu_pbds::tree<T,\</pre>
__gnu_pbds::null_type,less<T>,rb_tree_tag,\
 gnu pbds::tree order statistics node update>;
template<typename A, typename B>
using hTable1=__gnu_pbds::cc_hash_table<A,B>;
template<typename A, typename B>
using hTable2= gnu pbds::gp hash table<A,B>;
int main(){
 ordered set<int> ss;
 ss.insert(1); ss.insert(5);
 assert(*ss.find_by_order(0)==1);
 assert(ss.order of key(-1) == 0);
 pbds heap pg1, pg2;
 pq1.push(1); pq2.push(2);
 pq1.join(pq2);
 assert(pq2.size()==0);
 auto it = pq1.push(87);
 pq1.modify(it, 19);
 return 0;
```

2.3 SkewHeap

```
template < typename T, typename cmp = less< T > >
class SkewHeap{
private:
  struct SkewNode{
    T x;
    SkewNode *lc, *rc;
    SkewNode(T a = 0): x(a), lc(0), rc(0) {}
  cmp CMP ;
  size t count;
  SkewNode* Merge ( SkewNode* a, SkewNode* b ) {
    if ( !a or !b ) return a ? a : b;
    if ( CMP ( a->x, b->x ) ) swap( a, b );
    a -> rc = Merge( a->rc, b);
    swap( a -> lc, a->rc );
    return a:
public:
  SkewHeap(): root( 0 ), count( 0 ) {}
  size_t size() { return count; }
bool empty() { return count == 0; }
  T top() { return root->x; }
  void clear() { root = 0; count = 0; }
  void push ( const T& x ) {
    SkewNode* a = new SkewNode( x );
    count += 1; root = Merge( root, a );
  void join( SkewHeap& a ) {
    count += a.count; a.count = 0;
    root = Merge( root, a.root );
  void pop() {
   count--; root = Merge( root->lc, root->rc );
  friend void swap( SkewHeap& a, SkewHeap& b ) {
    swap(a.root, b.root); swap(a.count, b.count);
};
```

2.4 Disjoint Set

```
class DJS{
private:
  vector< int > fa, sz, sv;
  vector< pair< int*, int > > opt;
  inline void assign( int *k, int v ) {
    opt.emplace_back( k, *k );
    *k = v;
public:
  inline void init( int n ) {
    fa.resize( n ); iota( fa.begin(), fa.end(), 0 );
    sz.resize( n ); fill( sz.begin(), sz.end(), 1 );
    opt.clear();
  int query( int x ) {
    return ( fa[ x ] == x ) ? x : query( fa[ x ] );
  inline void merge( int a, int b ) {
    int af = query( a ), bf = query( b );
    if( af == bf ) return;
    if( sz[ af ] < sz[ bf ] ) swap( af, bf );</pre>
    assign( &fa[ bf ], fa[ af ] );
    assign( &sz[ af ], sz[ af ] + sz[ bf ] );
  inline void save() {sv.push back( (int)opt.size() );}
  inline void undo() {
    int ls = sv.back(); sv.pop_back();
    while ( ( int ) opt.size() > ls ) {
      pair< int*, int > cur = opt.back();
      *cur.first = cur.second;
      opt.pop_back();
```

2.5 Treap

```
namespace Treap{
 #define sz( x ) ( ( x ) ? ( ( x )->size ) : 0 )
  struct node{
    int size;
   uint32 t pri;
   node *lc, *rc;
   node() : size(0), pri(rand()), lc(0), rc(0) {}
    void pull() {
     size = 1;
     if ( lc ) size += lc->size;
     if ( rc ) size += rc->size;
   }
 };
 node* merge( node* L, node* R ) {
   if ( not L or not R ) return L ? L : R;
   if ( L->pri > R->pri ) {
     L->rc = merge(L->rc, R);
     L->pull();
     return L;
    } else {
     R->lc = merge(L, R->lc);
     R->pull();
     return R;
 void split by size( node*rt,int k, node*&L, node*&R ) {
   if ( not rt ) L = R = nullptr;
    else if( sz( rt->lc ) + 1 <= k )
     L = rt;
     split by size( rt->rc, k-sz(rt->lc)-1, L->rc, R );
     L->pull();
    } else {
     R = rt;
     split by size( rt->lc, k, L, R->lc );
     R->pull();
   }
  #undef sz
```

```
n = n : B.clear():
    B.resize( n ); sz = 0;
  void insert( llu x ) {
    // add x into B
    for ( int i = n-1; i >= 0 ; --i ) if( two(i) & x ) {
      if (B[i]) x ^= B[i];
      else {
        B[i] = x; sz++;
        for ( int j = i - 1 ; j >= 0 ; -- j )
          if( B[ j ] && ( two( j ) & B[ i ] ) )
B[ i ] ^= B[ j ];
        for (int j = i + 1; j < n; ++ j)
if ( two(i) & B[ j ] )</pre>
            B[ j ] ^= B[ i ];
        break;
    }
  inline int size() { return sz; }
  bool check( llu x ) {
    // is x in span(B) ?
    for ( int i = n-1 ; i >= 0 ; --i ) if( two(i) & x )
      if( B[ i ] ) x ^= B[ i ];
      else return false;
    return true;
  llu kth_small(llu k) {
    /** 1-base would always > 0 **/
    /** should check it **/
    /* if we choose at least one element
       but size(B)(vectors in B) == N(original elements)
       then we can't get 0 */
    llu ret = 0;
    for ( int i = 0 ; i < n ; ++ i ) if( B[ i ] ) {</pre>
      if( k & 1 ) ret ^= B[ i ];
      k >>= 1;
    return ret;
} base;
```

2.6 SparseTable

```
template < typename T, typename Cmp_ = less< T > >
class SparseTable {
private:
  vector< vector< T > > tbl;
  vector< int > la;
  T cv(Ta, Tb) {
    return Cmp_()( a, b ) ? a : b;
public:
  void init( T arr[], int n ) {
    // 0-base
    lg.resize(n + 1);
    lg[0] = -1;
    for( int i=1 ; i<=n ; ++i ) lg[i] = lg[i>>1] + 1;
    tbl.resize(lg[n] + 1);
    tbl[ 0 ].resize( n );
    copy( arr, arr + n, tbl[ 0 ].begin() );
    for ( int i = 1 ; i <= lg[ n ] ; ++ i ) {</pre>
      int len = 1 << ( i - 1 ), sz = 1 << i;</pre>
       tbl[ i ].resize( n - sz + 1 );
for ( int j = 0 ; j <= n - sz ; ++ j )</pre>
         tbl[i][j] = cv(tbl[i-1][j], tbl[i-1][j+len]);
    }
  T query( int 1, int r ) {
     // 0-base [1, r)
    int wh = lg[ r - l ], len = 1 << wh;</pre>
    return cv( tbl[ wh ][ l ], tbl[ wh ][ r - len ] );
1 } ;
```

2.7 Linear Basis

```
struct LinearBasis {
private:
   int n, sz;
   vector< llu > B;
   inline llu two( int x ) { return ( ( llu ) 1 ) << x; }
public:
   void init( int n_ ) {</pre>
```

3 Graph

3.1 BCC Edge

```
class BCC{
private:
  vector< int > low, dfn;
  int cnt;
  vector< bool > bridge;
  vector< vector< PII > > G;
  void dfs( int w, int f ) {
    dfn[w] = cnt++;
    low[ w ] = dfn[ w ];
          \textbf{for} \ (\ \textbf{auto}\ [\ \textbf{u},\ \textbf{t}\ ]\ :\ \textbf{G}[\ \textbf{w}\ ]\ )\ \{ \\
      if ( u == f ) continue;
      if ( dfn[ u ] != 0 ) {
        low[ w ] = min( low[ w ], dfn[ u ] );
      }else{
         dfs(u, w);
        low[ w ] = min( low[ w ], low[ u ] );
        if ( low[ u ] > dfn[ w ] ) bridge[ t ] = true;
      }
    }
public:
  void init( int n, int m ) {
    G.resize( n );
    fill( G.begin(), G.end(), vector< PII >() );
    bridge.clear(); bridge.resize( m );
    low.clear(); low.resize( n );
    dfn.clear(); dfn.resize( n );
    cnt = 0;
  void add_edge( int u, int v ) {
    // should check for multiple edge
    G[ u ].emplace_back( v, cnt );
    G[ v ].emplace back( u, cnt ++ );
  void solve() { cnt = 1; dfs(0,0); }
  // the id will be same as insert order, 0-base
```

```
bool is_bridge( int x ) { return bridge[ x ]; }
} bcc;
```

3.2 BCC Vertex

```
class BCC(
 private:
    int n, ecnt;
    vector< vector< pair< int, int > > > G;
    vector< int > low, dfn, id;
    vector< bool > vis, ap;
    void dfs( int u, int f, int d ) {
      int child = 0;
      dfn[ u ] = low[ u ] = d; vis[ u ] = true;
      for ( auto e : G[ u ] ) if ( e.first != f ) {
        if ( vis[ e.first ] ) {
          low[u] = min(low[u], dfn[e.first]);
        } else {
          dfs( e.first, u, d + 1 ); child ++;
low[ u ] = min( low[ u ], low[ e.first ] );
if ( low[ e.first ] >= d ) ap[ u ] = true;
      if ( u == f and child <= 1 ) ap[ u ] = false;</pre>
    void mark( int u, int idd ) {
       // really??????????
      if (ap[u]) return;
      for ( auto e : G[ u ] )
        if( id[ e.second ] != -1 ) {
  id[ e.second ] = idd;
          mark( e.first, idd );
  public:
    void init( int n_ ) {
      ecnt = 0, n = n;
      G.clear(); G.resize( n );
      low.resize( n ); dfn.resize( n );
      ap.clear(); ap.resize( n );
      vis.clear(); vis.resize( n );
    void add_edge( int u, int v ) {
      G[ u ].emplace back( v, ecnt );
      G[ v ].emplace_back( u, ecnt ++ );
    void solve() {
      for ( int i = 0 ; i < n ; ++ i )</pre>
        if ( not vis[ i ] ) dfs( i, i, 0 );
      id.resize( ecnt );
      fill( id.begin(), id.end(), -1 );
      ecnt = 0;
      for ( int i = 0 ; i < n ; ++ i )</pre>
        if (ap[i]) for (auto e : G[i])
          if( id[ e.second ] != -1 ) {
            id[ e.second ] = ecnt;
            mark( e.first, ecnt ++ );
    int get id( int x ) { return id[ x ]; }
    int count() { return ecnt; }
    bool is_ap( int u ) { return ap[ u ]; }
} bcc:
```

3.3 Bipartite Matching

```
class BipartiteMatching{
  private:
    vector<int> X[N], Y[N];
  int fX[N], fY[N], n;
  bitset<N> walked;
  bool dfs(int x) {
    for(auto i:X[x]) {
      if(walked[i]) continue;
      walked[i]=1;
      if(fY[i]==-1||dfs(fY[i])) {
         fY[i]=x;fX[x]=i;
         return 1;
      }
    }
    return 0;
}
```

```
public:
    void init(int n){
      n=_n;
      for (int i=0;i<n;i++) {</pre>
        X[i].clear();
         Y[i].clear();
        fX[i] = fY[i] = -1;
      walked.reset();
    void add edge(int x, int y) {
      X[x].push_back(y);
      Y[y].push back(y);
    int solve(){
      int cnt = 0;
      for (int i=0;i<n;i++) {</pre>
        walked.reset();
         if(dfs(i)) cnt++;
      // return how many pair matched
      return cnt;
};
```

3.4 Minimum Cost Maximum Flow

```
class MiniCostMaxiFlow{
  using CapT = int;
using WeiT = int64_t;
  using PCW = pair<CapT, WeiT>;
  static constexpr CapT INF CAP = 1 << 30;</pre>
  static constexpr WeiT INF_WEI = 1LL<<60;</pre>
private:
  struct Edge{
    int to, back;
    WeiT wei;
    CapT cap;
    Edge() {}
    Edge(int a,int b,WeiT c,CapT d):
      to(a),back(b),wei(c),cap(d)
    {}
  };
  int ori, edd;
  vector<vector<Edge>> G;
  vector<int> fa, wh;
  vector<bool> inq;
  vector<WeiT> dis;
  PCW SPFA() {
    fill(inq.begin(),inq.end(),false);
    fill(dis.begin(),dis.end(),INF WEI);
    queue<int> qq; qq.push(ori);
    dis[ori]=0;
    while(!qq.empty()){
      int u=qq.front();qq.pop();
      inq[u] = 0;
      for (int i=0; i < SZ(G[u]); ++i) {</pre>
        Edge e=G[u][i];
        int v=e.to;
        WeiT d=e.wei;
        if(e.cap<=0||dis[v]<=dis[u]+d)
          continue;
        dis[v]=dis[u]+d;
        fa[v]=u, wh[v]=i;
        if(inq[v]) continue;
        qq.push(v);
        inq[v]=1;
      }
    if(dis[edd] == INF WEI)
      return {-1,-1};
    CapT mw=INF CAP;
    for(int i=edd;i!=ori;i=fa[i])
      mw=min(mw,G[fa[i]][wh[i]].cap);
    for (int i=edd;i!=ori;i=fa[i]) {
      auto &eg=G[fa[i]][wh[i]];
      eq.cap-=mw;
      G[eg.to][eg.back].cap+=mw;
    return {mw,dis[edd]};
public:
  void init(int a,int b,int n) {
    ori=a,edd=b;
```

```
G.clear(); G.resize(n);
    fa.resize(n); wh.resize(n);
    inq.resize(n); dis.resize(n);
 void add edge(int st,int ed,WeiT w,CapT c) {
   G[st].emplace back(ed,SZ(G[ed]),w,c);
    G[ed].emplace back(st,SZ(G[st])-1,-w,0);
 PCW solve(){
   /* might modify to
    cc += ret.first * ret.second
    or
    ww += ret.first * ret.second
    CapT cc=0; WeiT ww=0;
   while(true) {
     PCW ret=SPFA();
      if(ret.first==-1) break;
      cc+=ret.first;
      ww+=ret.second;
   return {cc,ww};
} mcmf;
```

3.5 General Graph Matching

```
const int N = 514, E = (2e5) * 2;
struct Graph{
  int to[E],bro[E],head[N],e;
  int lnk[N], vis[N], stp, n;
  void init( int _n ) {
  stp = 0; e = 1; n =
                           n;
    for( int i = 1 ; i <= n ; i ++ )</pre>
      lnk[i] = vis[i] = 0;
  void add edge(int u,int v) {
    to[e]=v,bro[e]=head[u],head[u]=e++;
    to[e]=u,bro[e]=head[v],head[v]=e++;
 bool dfs(int x){
    vis[x]=stp;
    for (int i=head[x];i;i=bro[i]) {
      int v=to[i];
      if(!lnk[v]){
        lnk[x]=v, lnk[v]=x;
        return true;
       }else if(vis[lnk[v]]<stp){</pre>
        int w=lnk[v];
        lnk[x]=v, lnk[v]=x, lnk[w]=0;
        if (dfs(w)) {
           return true;
        lnk[w]=v, lnk[v]=w, lnk[x]=0;
      }
    return false:
  int solve() {
    int ans = 0;
    for (int i=1;i<=n;i++)</pre>
      if(!lnk[i]){
        stp++; ans += dfs(i);
    return ans;
} graph;
```

3.6 MaximumFlow

```
class Dinic{
private:
    using CapT = int64_t;
    struct Edge{
        int to, rev;
        CapT cap;
    };
    int n, st, ed;
    vector<vector<Edge>> G;
    vector<int> lv;
    bool BFS() {
        fill(lv.begin(), lv.end(), -1);
```

```
queue<int> bfs;
    bfs.push(st);
    lv[st] = 0;
    while(!bfs.empty()){
      int u = bfs.front(); bfs.pop();
      for(auto e: G[u]) {
        if(e.cap <= 0 or lv[e.to]!=-1) continue;</pre>
        lv[e.to] = lv[u] + 1;
        bfs.push(e.to);
      }
    }
    return (lv[ed]!=-1);
  CapT DFS(int u, CapT f) {
    if(u == ed) return f;
    CapT ret = 0;
    for (auto& e: G[u]) {
      if(e.cap <= 0 or lv[e.to]!=lv[u]+1) continue;</pre>
      CapT nf = DFS(e.to, min(f, e.cap));
      ret += nf; e.cap -= nf; f -= nf;
      G[e.to][e.rev].cap += nf;
      if(f == 0) return ret;
    if (ret == 0) lv[u] = -1;
    return ret;
public:
  void init(int n_, int st_, int ed_){
    n = n_, st = st_, ed = ed_;
    G.resize(n); lv.resize(n);
    fill(G.begin(), G.end(), vector<Edge>());
  void add_edge(int u, int v, CapT c){
    G[u].push\_back({v, (int)G[v].size(), c});
    G[v].push back({u, ((int)G[u].size())-1, 0});
  CapT max flow() {
    CapT ret = 0;
    while (BFS()) {
      CapT f = DFS(st, numeric limits<CapT>::max());
      ret += f;
      if(f == 0) break;
    return ret;
} flow;
```

3.7 Kuhn Munkres

```
struct KM{
  static constexpr lld INF = 1LL<<60;</pre>
  lld w[N][N], lx[N], ly[N], slack[N];
  int match[N], n, vx[N], vy[N], step;
  void init(int n_) {
    n=n_, step=0;
    memset(w,0,sizeof(w));
    memset(lx,0,sizeof(lx));
    memset(ly,0,sizeof(ly));
    memset(slack, 0, sizeof(slack));
    memset(match, 0, sizeof(match));
    memset(vx,0,sizeof(vx));
    memset(vy,0,sizeof(vy));
  void add edge(int u,int v,lld w) {w[u][v]=w;}
  bool dfs(int x) {
    vx[x] = step_;
for (int i = 0; i < n; ++i) {</pre>
      if (vy[i]==step_) continue;
if (lx[x] + ly[i] > w[x][i]) {
        slack[i] = min(slack[i], lx[x] + ly[i] - w[x][i]
             ]);
        continue;
      }
      vy[i] = step_;
      if (match[i] == -1 || dfs(match[i])) {
        match[i] = x;
        return true;
      }
    return false;
  lld solve() {
    fill_n(match, n, -1);
    fill_n(lx, n, -INF);
```

```
fill_n(ly, n, 0);
    for (int i = 0; i < n; ++i)</pre>
      for (int j = 0; j < n; ++j)</pre>
        lx[i] = max(lx[i], w[i][j]);
    for (int i = 0; i < n; ++i) {</pre>
      fill n(slack, n, INF);
      while (true) {
        step ++;
        if (dfs(i)) break;
        lld dlt = INF;
        for (int j = 0; j < n; ++j) if (vy[j] != step )</pre>
          dlt = min(dlt, slack[j]);
         for (int j = 0; j < n; ++j) {</pre>
          if (vx[j]==step_) lx[j] -= dlt;
          if (vy[j] == step_) ly[j] += dlt;
          else slack[j] -= dlt;
      }
    11d res = 0;
    for (int i = 0; i < n; ++i) res += w[match[i]][i];</pre>
    return res;
} km;
```

3.8 2-SAT

```
class TwoSat{
 private:
    vector<vector<int>> rG,G,sccs;
    vector<int> ord,idx;
    vector<bool> vis,result;
    void dfs(int u) {
      vis[u]=true;
      for (int v:G[u])
        if(!vis[v])
          dfs(v);
      ord.push back(u);
    void rdfs(int u) {
     vis[u]=false:
      idx[u]=sccs.size()-1;
      sccs.back().push back(u);
      for(int v:rG[u])
        if (vis[v])
          rdfs(v):
 public:
    void init(int n ) {
     n=n;
      G.clear():
      G.resize(n);
      rG.clear();
      rG.resize(n);
      sccs.clear();
      ord.clear();
      idx.resize(n);
      result.resize(n);
    void add edge(int u,int v) {
      G[u].push_back(v);
      rG[v].push_back(u);
    void orr(int x,int y) {
      if ((x^y) ==1) return;
      add edge(x^1, y);
      add_edge(y^1,x);
    bool solve() {
      vis.clear();
      vis.resize(n);
      for (int i=0;i<n;++i)</pre>
        if(not vis[i])
          dfs(i);
      reverse (ord.begin(), ord.end());
      for (int u:ord) {
        if(!vis[u])
          continue;
        sccs.push back(vector<int>());
        rdfs(u);
      for(int i=0;i<n;i+=2)</pre>
        if(idx[i] == idx[i+1])
```

```
return false;
vector<bool> c(sccs.size());
for(size_t i=0;i<sccs.size();++i){
   for(size_t j=0;j<sccs[i].size();++j){
      result[sccs[i][j]]=c[i];
      c[idx[sccs[i][j]]*c[i];
   }
   }
   return true;
}
bool get(int x){return result[x];}
inline int get_id(int x){return idx[x];}
inline int count(){return sccs.size();}
} sat2;</pre>
```

3.9 Lowbit Decomposition

```
class LowbitDecomp{
  int time , chain , LOG N;
  vector< \frac{-}{\text{vector}} < \frac{-}{\text{int}} > \frac{-}{\text{s}} G, fa;
  vector< int > tl, tr, chain, chain_st;
  // chain_ : number of chain
  // tl, \overline{\operatorname{tr}}[ u ] : subtree interval in the seq. of u
  // chain st[u] : head of the chain contains u
  // chian[ u ] : chain id of the chain u is on
  inline int lowbit( int x ) {
    return x & ( -x );
  void predfs( int u, int f ) {
    chain[u] = 0;
    for ( int v : G[ u ] ) {
      if ( v == f ) continue;
      predfs( v, u );
      if( lowbit( chain[ u ] ) < lowbit( chain[ v ] ) )</pre>
        chain[ u ] = chain[ v ];
    if ( not chain[ u ] )
      chain[ u ] = chain_ ++;
  void dfschain( int u, int f ) {
    fa[u][0] = f;
    for ( int i = 1 ; i < LOG_N ; ++ i )</pre>
      fa[u][i] = fa[fa[\overline{u}][i-1]][i-1];
    tl[u] = time ++;
    if ( not chain st[ chain[ u ] ] )
      chain st[ chain[ u ] ] = u;
    for ( int v : G[ u ] )
      if ( v != f and chain[ v ] == chain[ u ] )
        dfschain( v, u );
    for ( int v : G[ u ] )
      {f if} ( v != f {f and} chain[ v ] != chain[ u ] )
        dfschain( v, u );
    tr[u] = time;
  inline bool anc( int u, int v ) {
    return tl[ u ] <= tl[ v ] \</pre>
      and tr[ v ] <= tr[ u ];</pre>
public:
  inline int lca( int u, int v ) {
    if ( anc( u, v ) ) return u;
for ( int i = LOG_N - 1 ; i >= 0 ; -- i )
      if ( not anc( fa[ u ][ i ], v ) )
        u = fa[ u ][ i ];
    return fa[ u ][ 0 ];
  void init( int n ) {
    for ( LOG N = 0 ; ( 1 << LOG N ) < n ; ++ LOG N );
    fa.clear();
    fa.resize( n, vector< int >( LOG N ) );
    G.clear(); G.resize( n );
    tl.clear(); tl.resize( n );
    tr.clear(); tr.resize( n );
    chain.clear(); chain.resize( n );
    chain_st.clear(); chain_st.resize( n );
  void add_edge( int u , int v ) {
     // 1-base
    G[ u ].push back( v );
    G[ v ].push back( u );
  void decompose() {
```

```
chain = 1;
    predfs(1, 1);
    time = 0;
    dfschain(1,1);
  PII get_inter( int u ) { return {tl[ u ], tr[ u ]}; }
  vector< PII > get_path( int u , int v ){
    vector< PII > res;
    int g = lca( u, v );
    while ( chain[ u ] != chain[ g ] ) {
      int s = chain st[ chain[ u ] ];
      res.emplace_back( tl[ s ], tl[ u ] + 1 );
      u = fa[s][0];
    res.emplace back( tl[g], tl[u] + 1);
    while ( chain[ v ] != chain[ g ] ) {
  int s = chain_st[ chain[ v ] ];
      res.emplace_back( tl[ s ], tl[ v ] + 1 );
      v = fa[s][0];
    res.emplace back( tl[g] + 1, tl[v] + 1);
    return res;
    /* res : list of intervals from u to v
     * ( note only nodes work, not edge )
     * vector< PII >& path = tree.get path( u , v )
     * for( auto [ 1, r ] : path ) {
        0-base [ 1, r )
} tree;
```

3.10 MaxClique

```
#define N 111
struct MaxClique{ // 0-base
 typedef bitset< N > Int;
  Int linkto[ N ] , v[ N ];
 int n;
 void init( int _n ){
   n = _n;
    for( int i = 0 ; i < n ; i ++ ) {</pre>
     linkto[ i ].reset();
      v[ i ].reset();
   }
 void add_edge( int a , int b ){
   v[a][b] = v[b][a] = 1;
 int popcount (const Int& val)
  { return val.count(); }
 int lowbit(const Int& val)
  { return val. Find first(); }
 int ans , stk[ N ];
 int id[ N ] , di[ N ] , deg[ N ];
 Int cans;
 void maxclique(int elem_num, Int candi){
    if(elem num > ans){
      ans = elem num;
      cans.reset();
      for( int i = 0 ; i < elem num ; i ++ )</pre>
        cans[ id[ stk[ i ] ] ] = 1;
    int potential = elem num + popcount(candi);
    if(potential <= ans) return;</pre>
    int pivot = lowbit(candi);
    Int smaller_candi = candi & (~linkto[pivot]);
    while (smaller candi.count() && potential>ans) {
      int next = lowbit(smaller candi);
      candi[ next ] = !candi[ next ];
      smaller candi[next] = !smaller_candi[next];
      potential --;
      if (next!=pivot
        &&!(smaller candi&linkto[next]).count())
          continue;
      stk[elem num] = next;
      maxclique(elem_num+1, candi&linkto[next]);
  int solve(){
   for( int i = 0 ; i < n ; i ++ ) {</pre>
     id[ i ] = i;
      deg[i] = v[i].count();
```

```
sort( id , id + n , [&](int id1, int id2){
           return deg[id1] > deg[id2]; } );
     for( int i = 0 ; i < n ; i ++ )</pre>
      di[ id[ i ] ] = i;
    for( int i = 0 ; i < n ; i ++ )</pre>
      for( int j = 0 ; j < n ; j ++ )</pre>
         if( v[ i ][ j ] )
           linkto[ di[ i ] ][ di[ j ] ] = 1;
    Int cand; cand.reset();
    for( int i = 0 ; i < n ; i ++ )</pre>
      cand[i] = 1;
    ans = 1;
    cans.reset(); cans[ 0 ] = 1;
    maxclique(0, cand);
    return ans;
} solver;
```

4 Math

4.1 Prime Table

```
\begin{array}{c} 1002939109, 1020288887, 1028798297, 1038684299, \\ 1041211027, 1051762951, 1058585963, 1063020809, \\ 1147930723, 1172520109, 1183835981, 1187659051, \\ 1241251303, 1247184097, 1255940849, 1272759031, \\ 1287027493, 1288511629, 1294632499, 1312650799, \\ 1868732623, 1884198443, 1884616807, 1885059541, \\ 1909942399, 1914471137, 1923951707, 1925453197, \\ 1979612177, 1980446837, 1989761941, 2007826547, \\ 2008033571, 2011186739, 2039465081, 2039728567, \\ 2093735719, 2116097521, 2123852629, 2140170259, \\ 3148478261, 3153064147, 3176351071, 3187523093, \\ 3196772239, 3201312913, 3203063977, 3204840059, \\ 3210224309, 3213032591, 3217689851, 3218469083, \\ 3219857533, 3231880427, 3235951699, 3273767923, \\ 3276188869, 3277183181, 3282463507, 3285553889, \\ 3319309027, 3327005333, 3327574903, 3341387953, \\ 3373293941, 3380077549, 3380892997, 3381118801 \\ \end{array}
```

4.2 $\lfloor \frac{n}{i} \rfloor$ Enumeration

```
T_0 = 1, T_{i+1} = \lfloor \frac{n}{\lfloor \frac{n}{T \cdot + 1} \rfloor} \rfloor
```

4.3 ax+by=gcd

```
// ax+ny = 1, ax+ny == ax == 1 (mod n)
void exgcd(lld x,lld y,lld &g,lld &a,lld &b) {
  if (y == 0) g=x,a=1,b=0;
  else
    exgcd(y,x%y,g,b,a),b==(x/y)*a;
}
```

4.4 Pollard Rho

```
// does not work when n is prime
// return any non-trivial factor
llu pollard rho(llu n){
  static auto f=[](llu x,llu k,llu m){
   return add(k, mul(x, x, m), m);
  };
  if (!(n&1)) return 2;
  mt19937 rnd(120821011);
  while(true) {
    llu y=2, yy=y, x=rnd()%n, t=1;
    for(llu sz=2;t==1;sz<<=1) {</pre>
      for(llu i=0;i<sz;++i) {</pre>
        if (t!=1)break;
        yy=f(yy,x,n);
         t=gcd(yy>y?yy-y:y-yy,n);
      y=yy;
    if(t!=1&&t!=n) return t;
```

4.5 Pi Count (Linear Sieve)

```
static constexpr int N = 1000000 + 5;
lld pi[N];
vector<int> primes;
bool sieved[N]:
lld cube root(lld x) {
 11d s=cbrt(x-static_cast<long double>(0.1));
 while (s*s*s \le x) + s;
 return s-1;
lld square root(lld x) {
 11d s=sqrt(x-static_cast<long double>(0.1));
  while (s*s <= x) ++s;
 return s-1;
void init(){
 primes.reserve(N);
 primes.push back(1);
 for (int i=2;i<N;i++) {</pre>
   if(!sieved[i]) primes.push back(i);
    pi[i] = !sieved[i] + pi[i-1];
    for(int p: primes) if(p > 1) {
     if(p * i >= N) break;
      sieved[p * i] = true;
      if(p % i == 0) break;
 }
lld phi(lld m, lld n) {
 static constexpr int MM = 80000, NN = 500;
 static lld val[MM][NN];
 if (m<MM&&n<NN&&val[m][n]) return val[m][n]-1;</pre>
 if(n == 0) return m;
 if(primes[n] >= m) return 1;
 lld ret = phi(m,n-1)-phi(m/primes[n],n-1);
 if(m<MM&&n<NN) val[m][n] = ret+1;
 return ret;
lld pi count(lld);
11d P2(11d m, 11d n) {
 11d sm = square_root(m), ret = 0;
 for(lld i = n+1;primes[i] <= sm; i++)</pre>
    ret+=pi count(m/primes[i])-pi count(primes[i])+1;
 return ret;
lld pi_count(lld m) {
 if(m < N) return pi[m];</pre>
 lld n = pi_count(cube_root(m));
 return phi(m, n) + n - 1 - P2(m, n);
```

4.6 Range Sieve

```
const int MAX_SQRT_B = 50000;
const int MAX_L = 200000 + 5;

bool is_prime_small[MAX_SQRT_B];
bool is_prime[MAX_L];

void sieve(lld l, lld r){
    // [l, r)
    for(lld i=2;i*i<r;i++) is_prime_small[i] = true;
    for(lld i=1;i<r;i++) is_prime[i-1] = true;
    if(l==1) is_prime[0] = false;
    for(lld i=2;i*i<r;i++){
        if(!is_prime_small[i]) continue;
        for(lld j=i*i;j*j<r;j+=i) is_prime_small[j]=false;
    for(lld j=std::max(2LL, (l+i-1)/i)*i;j<r;j+=i)
        is_prime[j-l]=false;
    }
}</pre>
```

4.7 Miller Rabin

```
while(t--) {
    llu a2=mul(a,a,n);
    if(a2==1 && a!=1 && a!=n-1)
        return 1;
    a = a2;
}
    return a!=1;
};
if(x<2) return 0;
if(!(x&1)) return x==2;
llu x1=x-1;int t=0;
while(!(x1&1)) x1>>=1,t++;
for(llu m:magic)
    if(witn(m,x1,x,t))
    return 0;
return 1;
}
```

4.8 Inverse Element

```
// x's inverse mod k
long long GetInv(long long x, long long k) {
    // k is prime: euler_(k)=k-1
    return qPow(x, euler_phi(k)-1);
}
// if you need [1, x] (most use: [1, k-1]
void solve(int x, long long k) {
    inv[1] = 1;
    for(int i=2;i<x;i++)
        inv[i] = ((long long)(k - k/i) * inv[k % i]) % k;
}</pre>
```

4.9 Euler Phi Function

```
extended euler:
   a^b mod p
   if gcd(a, p) ==1: a^(b%phi(p))
   elif b < phi(p): a^b \mod p
   else a^(b%phi(p) + phi(p))
lld euler phi(int x){
  11d r=1;
  for (int i=2;i*i<=x;++i) {</pre>
    if(x%i==0){
      x/=i;
      r*=(i-1);
      while (x%i==0) {
        x/=i:
         r*=i;
    }
  if (x>1) r*=x-1;
  return r;
vector<int> primes;
bool notprime[N];
lld phi[N];
void euler_sieve(int n) {
  for (int i=2;i<n;i++) {</pre>
    if(!notprime[i]){
      primes.push back(i);
      phi[i] = i-1;
    for(auto j: primes) {
  if(i*j >= n) break;
      notprime[i*j] = true;
      phi[i*j] = phi[i] * phi[j];
      if(i % j == 0){
        phi[i*j] = phi[i] * j;
         break;
      }
    }
  }
```

4.10 Gauss Elimination

```
typedef long double llf;
const int N = 300;
const llf EPS = 1e-8;
// make m[i][i] = x, m[i][j] = 0
// v is for solving equation:
// for(int i=0;i<n;i++) ans[pos[i]] = val[i]/mtx[i][pos</pre>
    [i]];
// for(int i=0;i<n;i++) cout << ans[i] << '\n';
bool Gauss(llf m[N][N], llf v[N], int n, int pos[N]){
 for (int i=0;i<n;i++) {</pre>
    int x=-1, y=-1; llf e = 0;
    for (int j=i;j<n;j++) for (int k=i;k<n;k++) {</pre>
      if(fabs(m[j][pos[k]])>e){
        e = fabs(m[j][pos[k]]);
        x = j, y = k;
    if(x==-1 or y==-1) return false;
    swap(m[x], m[i]);
    swap(v[x], v[i]);
    swap(pos[y], pos[i]);
    for (int j=i+1; j<n; j++) {</pre>
      llf xi = m[j][pos[i]]/m[i][pos[i]];
      for (int k=0; k<n; k++) m[j] [pos[k]] -= xi*m[i] [pos[</pre>
          k]];
      v[i] -= xi*v[i];
    }
  for (int i=n-1;i>=0;i--) {
    for (int j=i-1; j>=0; j--) {
      llf xi = m[j][pos[i]]/m[i][pos[i]];
      for(int k=0;k<n;k++) m[j][pos[k]] -= xi*m[i][pos[</pre>
          k]];
      v[j] = xi*v[i];
  return true;
```

4.11 Fast Fourier Transform

```
polynomial multiply:
   DFT(a, len); DFT(b, len);
   for(int i=0;i<len;i++) c[i] = a[i]*b[i];</pre>
   iDFT(c, len);
   (len must be 2^k and = 2^k (max(a, b)))
  Hand written Cplx would be 2x faster
Cplx omega[2][N];
void init omega(int n) {
 static constexpr llf PI=acos(-1);
  const llf arg=(PI+PI)/n;
  for (int i=0;i<n;++i)</pre>
    omega[0][i]={cos(arg*i),sin(arg*i)};
 for (int i=0;i<n;++i)</pre>
    omega[1][i]=conj(omega[0][i]);
void tran(Cplx arr[], int n, Cplx omg[]) {
 for (int i=0, j=0; i<n; ++i) {</pre>
    if(i>j)swap(arr[i],arr[j]);
    for (int l=n>>1; (i^=1)<1; l>>=1);
 for (int 1=2;1<=n;1<<=1) {</pre>
    int m=1>>1;
    for (auto p=arr;p!=arr+n;p+=1) {
      for (int i=0; i<m; ++i) {</pre>
        Cplx t=omg[n/l*i]*p[m+i];
        p[m+i]=p[i]-t;
        p[i]+=t;
   }
 }
void DFT(Cplx arr[],int n){
  tran(arr,n,omega[0]);
void iDFT(Cplx arr[],int n){
 tran(arr,n,omega[1]);
  for(int i=0;i<n;++i)arr[i]/=n;</pre>
```

4.12 Chinese Remainder

```
lld crt(lld ans[], lld pri[], int n) {
  11d M = 1;
  for (int i=0;i<n;i++) M *= pri[i];</pre>
  11d ret = 0:
  for (int i=0;i<n;i++) {</pre>
    lld inv = (gcd(M/pri[i], pri[i]).first + pri[i])%
        pri[i];
    ret += (ans[i] * (M/pri[i]) %M * inv) %M;
   ret %= M;
  return ret;
/*
Another:
x = a1 % m1
x = a2 % m2
g = gcd(m1, m2)
assert((a1-a2)%g==0)
[p, q] = exgcd(m2/g, m1/g)
return a2+m2*(p*(a1-a2)/g)
0 <= x < 1cm(m1, m2)
```

4.13 Berlekamp Massey

```
// x: 1-base, p[]: 0-base
template < size t N>
vector<llf> BM(llf x[N], size t n) {
  size t f[N] = \{0\}, t=0; llf d[\overline{N}];
  vector<llf> p[N];
  for(size_t i=1,b=0;i<=n;++i) {</pre>
    for(size t j=0;j<p[t].size();++j)</pre>
       d[i] += x[i-j-1] *p[t][j];
    if (abs (d[i] -=x[i]) <=EPS) continue;</pre>
    f[t]=i;if(!t) {p[++t].resize(i);continue;}
    vector<llf> cur(i-f[b]-1);
    llf k=-d[i]/d[f[b]]; cur.PB(-k);
    for(size t j=0;j<p[b].size();j++)</pre>
       cur.PB(p[b][j]*k);
    if(cur.size()<p[t].size())cur.resize(p[t].size());</pre>
    for (size_t j=0;j<p[t].size();j++)cur[j]+=p[t][j];</pre>
    if(i-f[b]+p[b].size()>=p[t].size()) b=t;
    p[++t]=cur;
  return p[t];
```

4.14 NTT

```
// Remember coefficient are mod P
/* p=a*2^n+1
      2^n
  n
                                а
                                      root
  16 65536
20 1048576
                     65537
                                7
                                      3 */
                    7340033
// (must be 2^k)
template<LL P, LL root, int MAXN>
struct NTT{
 static LL bigmod(LL a, LL b) {
    LL res = 1:
    for (LL bs = a; b; b >>= 1, bs = (bs * bs) % P)
     if(b&1) res=(res*bs)%P;
   return res;
 static LL inv(LL a, LL b) {
    if(a==1)return 1;
    return (((LL) (a-inv(b%a,a))*b+1)/a)%b;
 LL omega[MAXN+1];
 NTT() {
    omega[0] = 1;
    LL r = bigmod(root, (P-1)/MAXN);
    for (int i=1; i<=MAXN; i++)</pre>
      omega[i] = (omega[i-1]*r)%P;
  // n must be 2^k
  void tran(int n, LL a[], bool inv ntt=false) {
    int basic = MAXN / n , theta = \overline{b}asic;
    for (int m = n; m >= 2; m >>= 1) {
      int mh = m >> 1;
```

```
for (int i = 0; i < mh; i++) {</pre>
         LL w = omega[i*theta%MAXN];
         for (int j = i; j < n; j += m) {
  int k = j + mh;</pre>
           LL x = a[j] - a[k];
           if (x < 0) x += P;
           a[j] += a[k];
           if (a[j] > P) a[j] -= P;
           a[k] = (w * x) % P;
      theta = (theta * 2) % MAXN;
    int i = 0;
    for (int j = 1; j < n - 1; j++) {</pre>
      for (int k = n >> 1; k > (i ^= k); k >>= 1);
      if (j < i) swap(a[i], a[j]);</pre>
    if (inv_ntt) {
      LL ni = inv(n, P);
      reverse( a+1 , a+n );

for (i = 0; i < n; i++)
        a[i] = (a[i] * ni) % P;
 }
};
const LL P=2013265921, root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;
```

4.15 FWT

```
/* xor convolution:
* x = (x0, x1) , y = (y0, y1)
* z = (x0y0 + x1y1 , x0y1 + x1y0 )
 * x' = (x0+x1, x0-x1), y' = (y0+y1, y0-y1)
 * z' = ((x0+x1)(y0+y1), (x0-x1)(y0-y1))
 * z = (1/2) * z''
 * or convolution:
 * x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
 * and convolution:
 * x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
const LL MOD = 1e9+7;
inline void fwt( LL x[ MAXN ] , int N , bool inv=0 ) {
 for ( int d = 1 ; d < N ; d <<= 1 ) {
    int d2 = d << 1;
    for( int s = 0 ; s < N ; s += d2 )</pre>
      for( int i = s , j = s+d ; i < s+d ; i++, j++ ){
  LL ta = x[ i ] , tb = x[ j ];</pre>
        x[i] = ta+tb;
        x[ j ] = ta-tb;
        if(x[i] >= MOD) x[i] -= MOD;
        if( x[j] < 0 ) x[j] += MOD;
  if (inv)
    for( int i = 0 ; i < N ; i++ ) {</pre>
      x[ i ] *= inv( N, MOD );
      x[ i ] %= MOD;
```

4.16 DiscreteLog

```
//\ {\it Baby-step\ Giant-step\ Algorithm}
11d BSGS(11d P, 11d B, 11d N) {
 // find B^L = N \mod P
 unordered map<lld, int> R;
 lld sq = (lld) sqrt(P);
 11d t = 1;
 for (int i = 0; i < sq; i++) {</pre>
   if (t == N)
      return i;
    if (!R.count(t))
     R[t] = i;
    t = (t * B) % P;
  lld f = inverse(t, P);
 for (int i=0; i<=sq+1; i++) {</pre>
   if (R.count(N))
      return i * sq + R[N];
```

```
N = (N * f) % P;
}
return -1;
}
```

5 Geometry

5.1 Point Class

```
template<typename T>
struct Point{
  typedef long double llf;
  static constexpr llf EPS = 1e-8;
  T \times, y;
  Point(T =0, T __=0): x(_), y(__){}
  template<typename T2>
    Point(const Point<T2>& a): x(a.x), y(a.y){}
  inline llf theta() const {
   return atan2((llf)y, (llf)x);
  inline llf dis() const {
    return hypot((llf)x, (llf)y);
  inline llf dis(const Point& o) const {
   return hypot((llf)(x-o.x), (llf)(y-o.y));
  Point operator-(const Point& o) const {
   return Point(x-o.x, y-o.y);
  Point operator = (const Point& o) {
   x-=o.x, y-=o.y;
    return *this;
  Point operator+(const Point& o) const {
   return Point(x+o.x, v+o.v);
  Point operator+=(const Point& o){
   x+=0.x, y+=0.y;
   return *this;
  Point operator*(const T& k) const {
    return Point(x*k, y*k);
  Point operator*=(const T& k) {
   x*=k, y*=k;
    return *this;
  Point operator/(const T& k) const {
   return Point(x/k, y/k);
  Point operator/=(const T& k) {
   x/=k, y/=k;
    return *this;
  Point operator-() const {
   return Point(-x, -y);
  Point rot90() const {
   return Point(-y, x);
  template<typename T2>
  bool in(const Circle<T2>& a) const {
    /* Add struct Circle at top */
    return a.o.dis(*this) +EPS <= a.r;</pre>
 bool equal(const Point& o, true_type) const {
    return fabs(x-o.x) < EPS and fabs(y-o.y) < EPS;</pre>
  bool equal(const Point& o, false type) const {
   return tie(x, y) == tie(o.x, o.y);
 bool operator==(const Point& o) const {
   return equal(o, is floating point<T>());
  bool operator!=(const Point& o) const {
   return ! (*this == 0);
 bool operator<(const Point& o) const {</pre>
   return theta() < o.theta();</pre>
    // sort like what pairs did
    // if(is_floating_point<T>()) return fabs(x-o.x)<
        EPS?y<o.y:x<o.x;
```

```
// else return tie(x, y) < tie(o.x, o.y);
}
friend inline T cross(const Point& a, const Point& b)
    {
    return a.x*b.y - b.x*a.y;
}
friend inline T dot(const Point& a, const Point &b){
    return a.x*b.x + a.y*b.y;
}
friend ostream& operator<<(ostream& ss, const Point&
    o){
    ss<<"("<<o.x<<", "<<o.y<<")";
    return ss;
}
};</pre>
```

5.2 Circle Class

```
template<typename T>
struct Circle{
  static constexpr llf EPS = 1e-8;
  Point<T> o;
  Tr;
  vector<Point<llf>> operator&(const Circle& aa)const{
    // https://www.cnblogs.com/wangzming/p/8338142.html
    llf d=o.dis(aa.o);
    if(d > r+aa.r+EPS or d < fabs(r-aa.r)-EPS) return</pre>
        { };
    11f dt = (r*r - aa.r*aa.r)/d, d1 = (d+dt)/2;
    Point<llf> dir = (aa.o-o); dir /= d;
    Point<llf> pcrs = dir*d1 + o;
    dt=sqrt(max(0.0L, r*r - d1*d1)), dir=dir.rot90();
    return {pcrs + dir*dt, pcrs - dir*dt};
 }
};
```

5.3 Line Class

```
const Point<long double> INF P(-1e20, 1e20);
const Point<long double> NOT EXIST(1e20, 1e-20);
template<typename T>
struct Line{
 static constexpr long double EPS = 1e-8;
  // ax+by+c = 0
 T a, b, c;
 Line(): a(0), b(1), c(0){}
Line(T _, T __, T ___): a(_), b(__), c(___){
   assert(fabs(a)>EPS or fabs(b)>EPS);
 template<typename T2>
   Line(const Line\langle T2 \rangle \& x): a(x.a), b(x.b), c(x.c){}
  typedef Point<long double> Pt;
 bool equal(const Line& o, true type) const {
   return fabs(a-o.a) < EPS and fabs(b-o.b) < EPS and
        fabs(c-o.b) < EPS;
 bool euqal(const Line& o, false type) const {
   return a==o.a and b==o.b and c==o.c;
 bool operator==(const Line& o) const {
   return euqal(o, is_floating_point<T>());
 bool operator!=(const Line& o) const {
   return ! (*this == 0);
 friend inline bool on_line__(const Point<T>& p, const
      Line& 1, true_type) {
   return fabs(l.a*p.x + l.b*p.y + l.c) < EPS;
 friend inline bool on_line__(const Point<T>& p, const
       Line& l, false_type) {
    return 1.a*p.x + 1.b*p.y + 1.c == 0;
 friend inline bool on_line(const Point<T>&p const
      Line& 1) {
    return on_line__(p, l, is_floating_point<T>());
 friend inline bool is parallel (const Line& x, const
      Line& y, true type) {
   return fabs(x.a*y.b - x.b*y.a) < EPS;</pre>
```

```
friend inline bool is parallel__(const Line& x, const
       Line& y, false_type) {
    return x.a*y.b == x.b*y.a;
  friend inline bool is parallel(const Line& x, const
      Line& v) {
    return is_parallel__(x, y, is_floating_point<T>());
  friend inline Pt get inter(const Line& x, const Line&
       у) {
    typedef long double llf;
    if(x==y) return INF P;
    if(is_parallel(x, y)) return NOT_EXIST;
    llf \overline{delta} = x.a*y.b - x.b*y.a;
    llf delta x = x.b*y.c - x.c*y.b;
    llf delta y = x.c*y.a - x.a*y.c;
    return Pt(delta_x / delta, delta_y / delta);
  friend ostream& operator<<(ostream& ss, const Line& o</pre>
    ss<<o.a<<"x+"<<o.b<<"y+"<<o.c<<"=0";
    return ss;
 }
template<typename T>
inline Line<T> get line(const Point<T>& a, const Point<</pre>
    T>& b) {
  return Line<T>(a.y-b.y, b.x-a.x, (b.y-a.y)*a.x-(b.x-a
      .x) *a.y);
```

5.4 Triangle Circumcentre

5.5 2D Convex Hull

```
template<typename T>
class ConvexHull 2D{
private:
  typedef Point<T> PT;
  vector<PT> dots;
  struct myhash{
    uint64 t operator()(const PT& a) const {
      uint\overline{6}4 t xx=0, yy=0;
      memcpy(&xx, &a.x, sizeof(a.x));
      memcpy(&yy, &a.y, sizeof(a.y));
      uint64 t ret = xx*17+yy*31;
      ret = (\text{ret } ^ (\text{ret } >> 16))*0x9E3779B1;
      ret = (ret ^ (ret >> 13)) *0xC2B2AE35;
      ret = ret ^ xx;
      return (ret ^ (ret << 3)) * yy;</pre>
  };
  unordered set<PT, myhash> in hull;
public:
  inline void init() {in hull.clear(); dots.clear();}
  void insert(const PT& x) {dots.PB(x);}
  void solve(){
    sort(ALL(dots), [](const PT& a, const PT& b){
      return tie(a.x, a.y) < tie(b.x, b.y);</pre>
    });
    vector<PT> stk(SZ(dots)<<1);
    int top = 0;
    for(auto p: dots) {
      while(top >= 2 and cross(p-stk[top-2], stk[top
           -1]-stk[top-2]) <= 0)
```

5.9 Ternary Search on Integer

x=newX, y=newY;

step++; pace*=mini;

```
int TernarySearch(int 1, int r) {
    // (1, r)
    while (r - 1 > 1) {
        int mid = (1 + r) >> 1;
        if (f(mid) > f(mid + 1)) r = mid;
        else 1 = mid;
    }
    return 1+1;
}
```

5.6 2D Farthest Pair

5.7 2D Cosest Pair

```
struct Point{
 llf x, y;
  llf dis;
} arr[N];
inline llf get dis (Point a, Point b) {
 return hypot(a.x-b.x, a.y-b.y);
llf solve(){
  int cur = rand()%n;
  for(int i=0;i<n;i++) arr[i].dis = get_dis(arr[cur],</pre>
      arr[i]);
  sort(arr, arr+n, [](Point a, Point b){return a.dis <
      b.dis; });
  llf ans = 1e50;
 for (int i=0; i<n; i++) {</pre>
    for (int j=i+1; j<n; j++) {</pre>
      if(arr[j].dis - arr[i].dis > ans) break;
      ans = min(ans, get dis(arr[i], arr[j]));
   }
  return ans;
```

5.8 SimulateAnnealing

```
double getY(double);
int main(){
 int rr, ll;
 default random engine rEng(time(NULL));
 uniform_real_distribution<double> Range(-1,1);
 uniform_real_distribution<double> expR(0,1);
 auto Random=bind(Range, rEng), expRand=bind(expR, rEng)
 int step=0;
 double pace=rr-ll, mini=0.95; // need to search for
      it.
 double x=max(min(Random()*pace+ll, rr), ll), y=getY(x
      );
 while (pace>=1e-7) {
   double newX = max(min(x + Random()*pace, rr), ll);
    double newY = getY(newX);
   if(newY < y || expRand() < exp(-step))</pre>
```

5.10 Minimum Covering Circle

```
template<typename T>
Circle<llf> MinCircleCover(const vector<Point<T>>& pts)
  random shuffle (ALL (pts));
  Circle<llf> c = \{pts[0], 0\};
  int n = SZ(pts);
  for (int i=0;i<n;i++) {</pre>
    if(pts[i].in(c)) continue;
    c = \{pts[i], 0\};
    for (int j=0; j<i; j++) {</pre>
      if(pts[j].in(c)) continue;
      c.o = (pts[i] + pts[j]) / 2;
      c.r = pts[i].dis(c.o);
      for (int k=0; k<j; k++) {</pre>
        if(pts[k].in(c)) continue;
        c = get_circum(pts[i], pts[j], pts[k]);
    }
  return c;
```

5.11 KDTree (Nearest Point)

```
const int MXN = 100005;
struct KDTree {
  struct Node {
    int x,y,x1,y1,x2,y2;
    int id,f;
    Node *L, *R;
  }tree[MXN];
  int n;
  Node *root:
  LL dis2(int x1, int y1, int x2, int y2) {
    LL dx = x1-x2;
    LL dy = y1-y2;
    return dx*dx+dy*dy;
  static bool cmpx(Node& a, Node& b) { return a.x<b.x; }</pre>
  static bool cmpy(Node& a, Node& b) { return a.y<b.y; }</pre>
  void init(vector<pair<int,int>> ip) {
   n = ip.size();
    for (int i=0; i<n; i++) {</pre>
      tree[i].id = i;
      tree[i].x = ip[i].first;
      tree[i].y = ip[i].second;
    root = build tree(0, n-1, 0);
  Node* build tree(int L, int R, int dep) {
    if (L>R) return nullptr;
    int M = (L+R)/2;
    tree[M].f = dep%2;
    nth_element(tree+L, tree+M, tree+R+1, tree[M].f ?
        cmpy : cmpx);
    tree[M].x1 = tree[M].x2 = tree[M].x;
    tree[M].y1 = tree[M].y2 = tree[M].y;
    tree[M].L = build_tree(L, M-1, dep+1);
```

```
if (tree[M].L) {
      tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
      tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
      tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
      tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
    tree[M].R = build_tree(M+1, R, dep+1);
    if (tree[M].R) {
      tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
      tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
      tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
    return tree+M;
  int touch(Node* r, int x, int y, LL d2) {
    LL dis = sqrt(d2)+1;
    if (x<r->x1-dis || x>r->x2+dis ||
        y < r -> y1 - dis \mid \mid y > r -> y2 + dis)
      return 0;
    return 1;
  void nearest(Node* r, int x, int y,
                int &mID, LL &md2) {
    if (!r || !touch(r, x, y, md2)) return;
    LL d2 = dis2(r->x, r->y, x, y);
    if (d2 < md2 || (d2 == md2 && mID < r->id)) {
      mID = r->id;
      md2 = d2;
       search order depends on split dim
    if ((r->f == 0 && x < r->x) ||
         (r->f == 1 && y < r->y))
      nearest(r->L, x, y, mID, md2);
      nearest(r->R, x, y, mID, md2);
    } else {
      nearest(r->R, x, y, mID, md2);
      nearest(r->L, x, y, mID, md2);
  int query(int x, int y) {
    int id = 1029384756;
    LL d2 = 102938475612345678LL;
    nearest(root, x, y, id, d2);
    return id;
}tree;
```

6 Stringology

6.1 Hash

```
class Hash{
private:
  static const int N = 1000000;
  const int p = 127, q = 1208220623;
  int sz, prefix[N], power[N];
  inline int add(int x, int y){return x+y>=q?x+y-q:x+y
  inline int sub(int x, int y) {return x-y<0?x-y+q:x-y;}</pre>
  inline int mul(int x, int y) {return 1LL*x*y%q;}
  void init(const std::string &x) {
    sz = x.size();
    prefix[0]=0;
    for (int i=1;i<=sz;i++) prefix[i]=add(mul(prefix[i])</pre>
        -1], p), x[i-1]);
    power[0]=1;
    for (int i=1;i<=sz;i++) power[i]=mul(power[i-1], p);</pre>
  int query(int 1, int r) {
       1-base (1, r
    return sub(prefix[r], mul(prefix[l], power[r-l]));
};
```

6.2 Suffix Array

```
char str[N + 10];
int len;
vector<sfx> srs[N + 10];
int mapping[N + 10];
sfx sa[N + 10];
bool cmp(sfx a,sfx b) {
  if(a.r==b.r){
    return a.nr<b.nr;</pre>
  }else{
    return a.r<b.r;</pre>
void SA(){
  len = strlen(str);
  for (int i=0;i<len;i++) {</pre>
    sa[i].index = i;
    sa[i].r=str[i];
    sa[i].nr=(i+1>=len)?0:str[i+1];
  //sort(sa,sa+len,cmp);
  radixSort();
  for (int j=2; j<=len; j*=2) {</pre>
    int cnt=1;
    int rr = sa[0].r;
    sa[0].r=cnt;
    mapping[sa[0].index]=0;
    for (int i=1; i<len; i++) {</pre>
      if(sa[i].r == rr && sa[i].nr == sa[i-1].nr) {
        rr=sa[i].r;
         sa[i].r=cnt;
      }else{
        rr=sa[i].r;
        sa[i].r=++cnt;
      mapping[sa[i].index]=i;
    for (int i=0; i<len; i++) {</pre>
      int nn = sa[i].index+j;
      sa[i].nr = (nn>=len)?0:sa[mapping[nn]].r;
    //sort(sa, sa+len, cmp);
    radixSort();
void radixSort() {
  int m = 0;
  for (int i=0;i<len;i++) {</pre>
    srs[sa[i].nr].PB(sa[i]);
    m=max(m,sa[i].nr);
  int cnt=0;
  for (int i=0;i<=m;i++) {</pre>
    if(srs[i].empty())continue;
    for(auto j:srs[i]){
      sa[cnt++] = i;
    srs[i].clear();
  m = 0:
  for (int i=0;i<len;i++) {</pre>
    srs[sa[i].r].PB(sa[i]);
    m=max(m,sa[i].r);
  cnt=0:
  for (int i=0;i<=m;i++) {</pre>
    if(srs[i].empty())continue;
    for(auto j:srs[i]){
      sa[cnt++] = j;
    srs[i].clear();
6.3 Aho-Corasick Algorithm
```

struct sfx{

int index:

int r,nr;

```
class AhoCorasick{
  private:
    static constexpr int Z = 26;
    struct node{
    node *nxt[ Z ], *fail;
```

```
vector< int > data;
      node(): fail( nullptr ) {
        memset( nxt, 0, sizeof( nxt ) );
         data.clear();
    } *rt;
    inline int Idx( char c ) { return c - 'a'; }
  public:
     void init() { rt = new node(); }
    void add( const string& s, int d ) {
      node* cur = rt;
      for ( auto c : s ) {
        if ( not cur->nxt[ Idx( c ) ] )
          cur->nxt[ Idx( c ) ] = new node();
         cur = cur->nxt[ Idx( c ) ];
      cur->data.push back( d );
    void compile() {
      vector< node* > bfs;
      size t ptr = 0;
      for ( int i = 0 ; i < Z ; ++ i ) {</pre>
        if ( not rt->nxt[ i ] )
          continue;
         rt->nxt[ i ]->fail = rt;
        bfs.push back( rt->nxt[ i ] );
      while ( ptr < bfs.size() ) {</pre>
         node* u = bfs[ ptr ++ ];
         for ( int i = 0 ; i < Z ; ++ i ) {</pre>
          if ( not u->nxt[ i ] )
            continue;
           node* u_f = u->fail;
           while (uf) {
             if ( not u f->nxt[ i ] ) {
              u f = u \overline{f} - > fail;
               continue;
             u->nxt[ i ]->fail = u_f->nxt[ i ];
           if ( not u f ) u->nxt[ i ]->fail = rt;
          bfs.push back( u->nxt[ i ] );
      }
    void match( const string& s, vector< int >& ret ) {
      node* u = rt;
      for ( auto c : s ) {
         while ( u != rt and not u->nxt[ Idx( c ) ] )
          u = u \rightarrow fail;
        u = u - nxt[Idx(c)];
        if ( not u ) u = rt;
node* tmp = u;
         while ( tmp != rt ) {
          for ( auto d : tmp->data )
            ret.push back( d );
          tmp = tmp->fail;
      }
| } ac;
```

6.4 KMP

```
int F[N<<1];</pre>
void KMP(char s1[], char s2[], int n, int m) {
  // make F[] for s1+'\0'+s2;
 char ss[N<<1];</pre>
 int len = n+m+1;
 for(int i=0;i<n;i++) ss[i] = s1[i];</pre>
 ss[n] = ' \setminus 0';
 for(int i=0;i<m;i++) ss[i+1+n] = s2[i];</pre>
 F[0] = F[1] = 0;
 for(int i=1;i<len;i++) {</pre>
   int j = F[i];
    while (j > 0 \text{ and } ss[i]!=ss[j]) j = F[j];
    F[i+1] = (ss[i] == ss[j]?j+1:0);
  // just find (F[len2+i] == len2)
 // i from 1 to len+1 for matching
 [0, i]是個循環字串,且循環節為i-f[i]:
```

```
if(f[i]>0 and i%(i-f[i])==0)
cout << i << " " << i/(i-f[i]) << '\n';
*/</pre>
```

6.5 Z value

```
char s[MAXN];
int len,z[MAXN];
void Z_value() {
   int i,j,left,right;
   left=right=0; z[0]=len;
   for(i=1;i<len;i++) {
      j=max(min(z[i-left],right-i),0);
      for(;i+j<len&&s[i+j]==s[j];j++);
      z[i]=j;
      if(i+z[i]>right) {
        right=i+z[i];
        left=i;
      }
}
```

6.6 Lexicographically Smallest Rotation

```
string mcp(string s) {
  int n = s.length();
  s += s;
  int i=0, j=1;
  while (i<n && j<n) {
    int k = 0;
    while (k < n && s[i+k] == s[j+k]) k++;
    if (s[i+k] <= s[j+k]) j += k+1;
    else i += k+1;
    if (i == j) j++;
  }
  int ans = i < n ? i : j;
  return s.substr(ans, n);
}</pre>
```

6.7 BWT

```
struct BurrowsWheeler{
 #define SIGMA 26
 #define BASE 'a'
   vector<int> v[ SIGMA ];
   void BWT(char* ori, char* res){
     // make ori -> ori + ori
     // then build suffix array
   void iBWT(char* ori, char* res) {
     for( int i = 0 ; i < SIGMA ; i ++ )</pre>
       v[ i ].clear();
     int len = strlen( ori );
     for( int i = 0 ; i < len ; i ++ )</pre>
       v[ ori[i] - BASE ].push_back( i );
     vector<int> a;
     for( int i = 0 , ptr = 0 ; i < SIGMA ; i ++ )
for( auto j : v[ i ] ){</pre>
         a.push_back( j );
         ori[ ptr ++ ] = BASE + i;
     for( int i = 0 , ptr = 0 ; i < len ; i ++ ){</pre>
      res[ i ] = ori[ a[ ptr ] ];
       ptr = a[ ptr ];
     res[len] = 0;
} bwt;
```

7 Misc

7.1 Degree Sequence Validity

7.1.1 Erdős-Gallai Theorem

```
d_1 \geq \cdots \geq d_n is a valid degree sequence iff \sum_{k=1}^n d_k \equiv 0 \mod 2
```

```
\stackrel{\wedge}{\forall} 1 \leq k \leq n, \sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i,k)
```

7.1.2 Havel-Hakimi algorithm

find the vertex who has greatest degree unused, connect it with other greatest vertex.

7.2 MaximumEmptyRect

```
int max_empty_rect(int n, int m, bool blocked[N][N]){
  static int mxu[2][N], me=0, he=1, ans=0;
  for(int i=0;i<m;i++) mxu[he][i]=0;</pre>
 for (int i=0; i<n; i++) {</pre>
    stack<PII, vector<PII>> stk;
    for (int j=0; j<m; ++j) {</pre>
      if(blocked[i][j]) mxu[me][j]=0;
      else mxu[me][j]=mxu[he][j]+1;
      int la = j;
      while(!stk.empty()&&stk.top().FF>mxu[me][j]){
        int x1 = i - stk.top().FF, x2 = i;
int y1 = stk.top().SS, y2 = j;
        la = stk.top().SS; stk.pop();
        ans=\max(ans, (x2-x1)*(y2-y1));
      if(stk.empty()||stk.top().FF<mxu[me][j])</pre>
        stk.push({mxu[me][j],la});
    while(!stk.empty()){
      int x1 = i - stk.top().FF, x2 = i;
      int y1 = stk.top().SS-1, y2 = m-1;
      stk.pop();
      ans=max(ans, (x2-x1)*(y2-y1));
    swap (me, he);
  return ans;
```

7.3 DP-opt Condition

7.3.1 totally monotone (concave/convex)

```
\begin{array}{ll} \forall i < i', j < j', B[i][j] \leq B[i'][j] \implies B[i][j'] \leq B[i'][j'] \\ \forall i < i', j < j', B[i][j] \geq B[i'][j] \implies B[i][j'] \geq B[i'][j'] \end{array}
```

7.3.2 monge condition (concave/convex)

```
\begin{array}{l} \forall i < i', j < j', B[i][j] + B[i'][j'] \geq B[i][j'] + B[i'][j] \\ \forall i < i', j < j', B[i][j] + B[i'][j'] \leq B[i][j'] + B[i'][j] \end{array}
```

7.4 Convex 1D/1D DP

```
struct seament {
 int i, l, r;
  segment() {}
 segment(int a, int b, int c): i(a), l(b), r(c) {}
inline long long f(int 1, int r) {
 return dp[l] + w(l + 1, r);
void solve() {
 dp[0] = 011;
 deque<segment> deq; deq.push_back(segment(0, 1, n));
 for (int i = 1; i <= n; ++i) {
   dp[i] = f(deq.front().i, i);
    while (deq.size() && deq.front().r < i + 1) deq.</pre>
        pop_front();
    deq.front().l = i + 1;
    segment seg = segment(i, i + 1, n);
    while (deq.size() && f(i, deq.back().l) < f(deq.</pre>
        back().i, deq.back().l)) deq.pop back();
    if (deq.size())
      int d = 1048576, c = deq.back().1;
      while (d >>= 1) if (c + d <= deq.back().r) {</pre>
        if (f(i, c + d) > f(deq.back().i, c + d)) c +=
            d;
      deq.back().r = c; seg.l = c + 1;
```

```
if (seg.1 <= n) deq.push_back(seg);
}</pre>
```

7.5 Josephus Problem

```
// n people kill m for each turn
int f(int n, int m) {
   int s = 0;
   for (int i = 2; i <= n; i++)
        s = (s + m) % i;
   return s;
}
// died at kth
int kth(int n, int m, int k) {
   if (m == 1) return n-1;
   for (k = k*m+m-1; k >= n; k = k-n+(k-n)/(m-1));
   return k;
}
```