#### **Contents**

## 1 Basic

#### 1.1 vimrc

```
se nu rnu bs=2 ru mouse=a cin et ts=4 sw=4 sts=4 syn on filetype indent on inoremap {<CR> {<CR>}<Esc>0
```

#### 1.2 IncreaseStackSize

```
//stack resize(change esp to rsp if 64-bit system)
asm( "mov %0,%%esp\n" ::"g"(mem+10000000) );
// craziest way
static void run_stack_sz(void(*func)(),size_t stsize){
    char *stack, *send;
    stack=(char *)malloc(stsize);
    send=stack+stsize-16;
    send=(char *)((uintptr_t)send/16*16);
    asm volatile(
        "mov %%rsp, (%0)\n"
        "mov %0, %%rsp\n"
        :
        : "r" (send));
    func();
    asm volatile(
        "mov (%0), %%rsp\n"
        :
        : "r" (send));
    free(stack);
}
```

# 1.3 Pragma optimization

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,tune=native")
```

# 1.4 Debugger

```
#! /usr/bin/env python3
import subprocess as sp
os_name = __import__('platform').system()
cmd,prefix = [],""
if os_name == 'Windows':
    cmd=["cmd", "/C"]
else:
  cmd = ["bash", "-c"]
prefix = "./"
def GetTestData(exe):
   myout=sp.check_output(cmd+["%s%s"%(prefix, exe)])
   return myout.decode("utf8")
def Judge(a,b,testdata):
    f = open("test.in", "w")
  f.write(testdata)
  f.close()
   c=sp.check_output(cmd+["%s%s<test.in"%(prefix, a)])</pre>
   d=sp.check_output(cmd+["%s%s<test.in"%(prefix, b)])</pre>
  if not c == d:
     print("answer: %s"%c.decode("utf8"),end="")
print("output: %s"%d.decode("utf8"),end="")
print("WA!")
      return False
   return True
      _name__ == '__main__':
  cnt = 0
```

```
isOK = True
while isOK:
  cnt += 1
  print(cnt)
  isOK=Judge("sol", "mysol", GetTestData("gen"))
```

## 1.5 Quick Random

```
template < class T,T x1,T x2,T x3,int y1,int y2,int y3>
struct PRNG {
  using S = typename std::make_signed<T>::type;
  PRNG(T
           _s = 0) : s(_s) {}
  T next() {
    T z = (s += x1);
    z = (z^{(x)}(z)) * x2;

z = (z^{(x)}(z)) * x3;
     return z ^ (z >> y3);
  T next(T n) { return next() % n; }
  S next(S l, S r){return l+next(r-l+1);}
  T operator()() { return next();
  T operator()(T n) { return next(n); }
S operator()(S 1, S r) { return next(1, r); }
static T gen(T s) { return PRNG(s)(); }
  template < class U>
  void shuffle(U first,U last){
     size_t n=last-first;
     for(size_t i=0;i<n;i++)</pre>
       swap(first[i],first[next(i+1)]);
  }
};
using R32=PRNG<uint32_t,0x9E3779B1,0x85EBCA6B,</pre>
0xC2B2AE35,16,13,16>;
R32 r32:
using R64=PRNG<uint64_t,0x9E3779B97F4A7C15</pre>
0xBF58476D1CE4E5B9,0x94D049BB133111EB,30,27,31>;
R64 r64:
```

# 1.6 IO Optimization

```
| static inline int gc() {
    static char buf[ 1 << 20 ], *p = buf, *end = buf;
    if ( p == end ) {
        end = buf + fread( buf, 1, 1 << 20, stdin );
        if ( end == buf ) return EOF;
        p = buf;
    }
    return *p++;
}

template < typename T >
static inline bool gn( T &_ ) {
    register int c = gc(); register T __ = 1; _ = 0;
    while(('0'>c||c>'9') && c!=EOF && c!='-') c = gc();
    if(c == '-') { __ = -1; c = gc(); }
    if(c == EOF) return false;
    while('0'<=c&&c<='9') _ = _ * 10 + c - '0', c = gc();
    _ *= __;
    return true;
}

template < typename T, typename ...Args >
static inline bool gn( T &x, Args &...args )
{ return gn(x) && gn(args...); }
```

# 2 Data Structure

# 2.1 Bigint

```
class BigInt{
  private:
    using lld = int_fast64_t;
    #define PRINTF_ARG PRIDEAST64
  #define LOG_BASE_STR "9"
```

```
static constexpr lld BASE = 1000000000;
  static constexpr int LOG_BASE = 9;
  vector<lld> dig;
  bool neg;
  inline int len() const { return (int) dig.size(); }
inline int cmp_minus(const BigInt& a) const {
    if(len() == 0 && a.len() == 0) return 0;
    if(neg ^ a.neg)return (int)a.neg*2 - 1;
    if(len()!=a.len())
    return neg?a.len()-len():len()-a.len();
for(int i=len()-1;i>=0;i--) if(dig[i]!=a.dig[i])
      return neg?a.dig[i]-dig[i]:dig[i]-a.dig[i];
    return 0;
  inline void trim(){
    while(!dig.empty()&&!dig.back())dig.pop_back();
    if(dig.empty()) neg = false;
public:
 BigInt(): dig(vector<lld>()), neg(false){}
BigInt(lld a): dig(vector<lld>()){
    neg = a<0; dig.push_back(abs(a));</pre>
    trim();
  BigInt(const string& a): dig(vector<lld>()){
    11d cur = 0;
      for(int j=min(LOG_BASE-1,i-neg);j>=0;j--)
    cur = cur*10+a[i-j]-'0';
      dig.push_back(cur);
    } trim();
  inline bool operator<(const BigInt& a)const</pre>
    {return cmp_minus(a)<0;}
  inline bool operator<=(const BigInt& a)const</pre>
    {return cmp_minus(a)<=0;}
  inline bool operator==(const BigInt& a)const
    {return cmp_minus(a)==0;}
  inline bool operator!=(const BigInt& a)const
    {return cmp_minus(a)!=0;}
  inline bool operator>(const BigInt& a)const
    {return cmp_minus(a)>0;}
  inline bool operator>=(const BigInt& a)const
    {return cmp_minus(a)>=0;}
  BigInt operator-() const {
    BigInt ret = *this;
    ret.neg ^= 1;
    return ret;
  BigInt operator+(const BigInt& a) const {
    if(neg) return -(-(*this)+(-a));
    if(a.neg) return (*this)-(-a);
    int n = max(a.len(), len());
    BigInt ret; ret.dig.resize(n);
    11d pro = 0;
    for(int i=0;i<n;i++) {</pre>
      ret.dig[i] = pro;
      if(i < a.len()) ret.dig[i] += a.dig[i];</pre>
      if(i < len()) ret.dig[i] += dig[i];</pre>
      if(ret.dig[i] >= BASE) pro = ret.dig[i]/BASE;
      ret.dig[i] -= BASE*pro;
    if(pro != 0) ret.dig.push_back(pro);
    return ret;
  BigInt operator-(const BigInt& a) const {
    if(neg) return -(-(*this) - (-a));
    if(a.neg) return (*this) + (-a);
int diff = cmp_minus(a);
    if(diff < 0) return -(a - (*this));</pre>
    if(diff == 0) return 0;
    BigInt ret; ret.dig.resize(len(), 0);
    for(int i=0;i<len();i++) {</pre>
      ret.dig[i] += dig[i];
      if(i < a.len()) ret.dig[i] -= a.dig[i];</pre>
      if(ret.dig[i] < 0){</pre>
        ret.dig[i] += BASE;
        ret.dig[i+1]--;
      }
    ret.trim();
    return ret;
  BigInt operator*(const BigInt& a) const {
```

```
if(!len()||!a.len()) return 0;
  BigInt ret; ret.dig.resize(len()+a.len()+1);
  ret.neg = neg ^ a.neg;
for(int i=0;i<len();i++)</pre>
     for(int j=0;j<a.len();j++){
  ret.dig[i+j] += dig[i] * a.dig[j];</pre>
       if(ret.dig[i+j] >= BASE) {
         lld x = ret.dig[i+j] / BASE;
         ret.dig[i+j+1] += x;
ret.dig[i+j] -= x * BASE;
  ret.trim();
  return ret;
BigInt operator/(const BigInt& a) const {
  assert(a.len());
  if(len() < a.len()) return 0;</pre>
  BigInt ret; ret.dig.resize(len()-a.len()+1);
  ret.neg = a.neg;
  for(int i=len()-a.len();i>=0;i--){
     11d 1 = 0, r = BASE;
    while (r-l > 1){
       11d \ mid = (1+r)>>1;
       ret.dig[i] = mid;
       if(ret*a<=(neg?-(*this):(*this))) 1 = mid;</pre>
       else r = mid;
    ret.dig[i] = 1;
  ret.neg ^= neg; ret.trim();
  return ret;
BigInt operator%(const BigInt& a) const {
  return (*this) - (*this) / a * a;
friend BigInt abs(BigInt a){
  a.neg = 1; return a;
friend void swap(BigInt& a, BigInt& b){
  swap(a.dig, b.dig); swap(a.neg, b.neg);
friend istream& operator>>(istream& ss, BigInt& a){
  string s; ss >> s; a = s;
  return ss;
friend ostream& operator<<(ostream& ss, BigInt& a){</pre>
  if(a.len() == 0) return ss << '0';
if(a.neg) ss << '-';</pre>
  ss << a.dig.back();</pre>
  for(int i=a.len()-2;i>=0;i--)
    ss<<setw(LOG_BASE)<<setfill('0')<<a.dig[i];</pre>
  return ss;
inline void print() const {
  if(len() == 0){putchar('0');return;}
  if(neg) putchar('-');
  printf("%" PRINTF_ARG, dig.back());
  for(int i=len()-2;i>=0;i--)
  printf("%0" LOG_BASE_STR PRINTF_ARG, dig[i]);
#undef PRINTF_ARG
#undef LOG_BASE_STR
```

# 2.2 Dark Magic

};

```
_gnu_pbds::tree_order_statistics_node_update>;
template<typename A, typename B>
using hTable1=__gnu_pbds::cc_hash_table<A,B>;
template<typename A, typename B>
using hTable2=__gnu_pbds::gp_hash_table<A,B>;
int main(){
  ordered_set<int> ss;
 ss.insert(1); ss.insert(5);
assert(*ss.find_by_order(0)==1);
  assert(ss.order_of_key(-1)==0);
 pbds_heap pq1, pq2;
 pq1.push(1); pq2.push(2);
 pq1.join(pq2);
  assert(pq2.size()==0);
  auto it = pq1.push(87);
  pq1.modify(it, 19);
  return 0;
```

## 2.3 SkewHeap

```
template < typename T, typename cmp = less< T > >
class SkewHeap{
private:
  struct SkewNode{
    Tx;
    SkewNode *lc, *rc;
    SkewNode( T a = 0 ) : x(a), lc(0), rc(0) {}
  cmp CMP_;
  size_t count;
  SkewNode* Merge( SkewNode* a, SkewNode* b ) {
    if ( !a or !b ) return a ? a : b;
    if ( CMP_( a->x, b->x ) ) swap( a, b );
a -> rc = Merge( a->rc, b );
    swap( a -> lc, a->rc );
    return a;
public:
 SkewHeap(): root( 0 ), count( 0 ) {}
size_t size() { return count; }
  bool empty() { return count == 0; }
  T top() { return root->x; }
  void clear(){ root = 0; count = 0; }
 void push ( const T& x ) {
   SkewNode* a = new SkewNode( x );
    count += 1; root = Merge( root, a );
  void join( SkewHeap& a ) {
    count += a.count; a.count = 0;
    root = Merge( root, a.root );
  void pop() {
    count--; root = Merge( root->lc, root->rc );
  friend void swap( SkewHeap& a, SkewHeap& b ) {
    swap( a.root, b.root ); swap( a.count, b.count );
```

# 2.4 Disjoint Set

```
class DJS{
private:
    vector< int > fa, sz, sv;
    vector< pair< int*, int > opt;
    inline void assign( int *k, int v ) {
        opt.emplace_back( k, *k );
        *k = v;
    }
public:
    inline void init( int n ) {
        fa.resize( n ); iota( fa.begin(), fa.end(), 0 );
        sz.resize( n ); fill( sz.begin(), sz.end(), 1 );
        opt.clear();
    }
    int query( int x ) {
        return ( fa[ x ] == x ) ? x : query( fa[ x ] );
    }
}
```

```
inline void merge( int a, int b ) {
   int af = query( a ), bf = query( b );
   if( af == bf ) return;
   if( sz[ af ] < sz[ bf ] ) swap( af, bf );
   assign( &fa[ bf ], fa[ af ] );
   assign( &sz[ af ], sz[ af ] + sz[ bf ] );
}
inline void save() {sv.push_back( (int)opt.size() );}
inline void undo() {
   int ls = sv.back(); sv.pop_back();
   while ( ( int ) opt.size() > ls ) {
      pair< int*, int > cur = opt.back();
      *cur.first = cur.second;
      opt.pop_back();
   }
}
};
```

#### 2.5 Link-Cut Tree

```
struct Node{
  Node *par, *ch[2];
 int xor_sum,v;
bool is_rev;
  Node(int _v){
    v=xor_sum=_v;
    par=nullptr;
    ch[0]=ch[1]=nullptr;
    is_rev=false;
  inline void set_rev(){
    is rev^=1;
    swap(ch[0],ch[1]);
  inline void down(){
    if(is_rev){
      if(ch[0]!=nullptr) ch[0]->set_rev();
      if(ch[1]!=nullptr) ch[1]->set_rev();
      is_rev=false;
    }
  inline void up(){
    xor sum=v;
    if(ch[0]!=nullptr){
      xor_sum^=ch[0]->xor_sum;
      ch[0]->par=this;
    if(ch[1]!=nullptr){
      xor_sum^=ch[1]->xor_sum;
      ch[1]->par=this;
  inline bool is_root(){
    return par==nullptr ||\
      (par->ch[0]!=this && par->ch[1]!=this);
  inline bool is_rch(){
    return !is_root() && par->ch[1]==this;
} *node[maxn],*stk[maxn];
int top;
void to_child(Node* p,Node* c,bool dir){
 p->ch[dir]=c;
  p->up();
inline void rotate(Node* node){
 Node* par=node->par;
  Node* par_par=par->par
  bool dir=node->is_rch();
  bool par_dir=par->is_rch();
  to_child(par, node -> ch[!dir], dir);
  to_child(node,par,!dir);
  if(par_par!=nullptr && par_par->ch[par_dir]==par)
    to_child(par_par,node,par_dir);
  else node->par=par_par;
inline void splay(Node* node){
  Node* tmp=node;
  stk[top++]=node;
  while(!tmp->is_root()){
    tmp=tmp->par;
    stk[top++]=tmp;
```

```
while(top) stk[--top]->down();
  for(Node *fa=node->par;
   !node->is_root();
   rotate(node),fa=node->par)
    if(!fa->is_root())
      rotate(fa->is_rch()==node->is_rch()?fa:node);
inline void access(Node* node){
  Node* last=nullptr;
  while(node!=nullptr){
    splay(node);
    to_child(node,last,true);
    last=node;
    node=node->par;
  }
inline void change_root(Node* node){
 access(node);
  splay(node);
  node->set_rev();
inline void link(Node* x,Node* y){
  change_root(x);
  splay(x);
  x->par=y;
inline void split(Node* x, Node* y){
  change_root(x);
  access(y);
  splay(x);
  to_child(x,nullptr,true);
 y->par=nullptr;
inline void change_val(Node* node,int v){
  access(node);
  splay(node);
  node->v=v;
  node->up();
inline int query(Node* x,Node* y){
  change_root(x);
  access(y);
  splay(y);
  return y->xor_sum;
inline Node* find_root(Node* node){
  access(node);
  splay(node);
  Node* last=nullptr;
  while(node!=nullptr){
    node->down();
    last=node;
    node=node->ch[0];
  return last;
set<pii> dic;
inline void add_edge(int u,int v){
  if(u>v) swap(u,v);
  if(find_root(node[u])==find_root(node[v])) return;
  dic.insert(pii(u,v));
  link(node[u],node[v]);
inline void del_edge(int u,int v){
  if(u>v) swap(u,v);
  if(dic.find(pii(u,v))==dic.end()) return;
  dic.erase(pii(u,v));
  split(node[u],node[v]);
```

# 2.6 LiChao Segment Tree

```
struct Line{
  int m, k, id;
  Line() : id( -1 ) {}
  Line( int a, int b, int c )
      : m( a ), k( b ), id( c ) {}
  int at( int x ) { return m * x + k; }
};
class LiChao {
  private:
   int n; vector< Line > nodes;
  inline int lc( int x ) { return 2 * x + 1; }
```

```
inline int rc( int x ) { return 2 * x + 2;
    void insert( int 1, int r, int id, Line ln ) {
      int m = (1 + r) >> 1;
      if ( nodes[ id ].id == -1 ) {
        nodes[ id ] = ln;
        return;
      bool atLeft = nodes[ id ].at( l ) < ln.at( l );</pre>
      if ( nodes[ id ].at( m ) < ln.at( m ) ) {</pre>
        atLeft ^= 1;
        swap( nodes[ id ], ln );
      if ( r - l == 1 ) return;
      if ( atLeft ) insert( l, m, lc( id ), ln );
      else insert( m, r, rc( id ), ln );
    int query( int 1, int r, int id, int x ) {
      int ret = 0;
      if ( nodes[ id ].id != -1 )
        ret = nodes[ id ].at( x );
      int m = (1 + r) >> 1;
      if ( r - l == 1 ) return ret;
      else if ( x < m )</pre>
        return max( ret, query( 1, m, lc( id ), x ) );
      else
        return max( ret, query( m, r, rc( id ), x ) );
  public:
    void build( int n_ ) {
      n = n_; nodes.clear();
      nodes.resize( n << 2, Line() );</pre>
    void insert( Line ln ) {
      insert( 0, n, 0, ln );
    int query( int x ) {
      return query( 0, n, 0, x );
} lichao;
```

## 2.7 Treap

```
namespace Treap{
   #define sz(x)((x)?((x)-size):0)
   struct node{
     int size;
     uint32_t pri;
     node *lc, *rc;
node() : size(0), pri(rand()), lc(0), rc(0) {}
     void pull() {
       size = 1;
        if ( lc ) size += lc->size;
       if ( rc ) size += rc->size;
     }
   };
   node* merge( node* L, node* R ) {
     if ( not L or not R ) return L ? L : R;
     if ( L->pri > R->pri ) {
       L \rightarrow rc = merge(L \rightarrow rc, R);
       L->pull();
       return L;
     } else {
       R \rightarrow lc = merge(L, R \rightarrow lc);
       R->pull();
       return R;
  void split_by_size( node*rt,int k,node*&L,node*&R ) {
   if ( not rt ) L = R = nullptr;
   else if( sz( rt->lc ) + 1 <= k ) {</pre>
       L = rt;
        split_by_size( rt->rc,k-sz(rt->lc)-1,L->rc,R );
       L->pull();
     } else {
       R = rt;
        split_by_size( rt->lc, k, L, R->lc );
       R->pull();
   #undef sz
}
```

## 2.8 SparseTable

```
template < typename T, typename Cmp_ = less< T > >
class SparseTable {
private:
  vector< vector< T > > tbl;
  vector< int > lg;
  T cv( T a, T b ) {
     return Cmp_()( a, b ) ? a : b;
public:
  void init( T arr[], int n ) {
     // 0-base
     lg.resize( n + 1 );
lg[ 0 ] = -1;
     for( int i=1 ; i<=n ; ++i ) lg[i] = lg[i>>1] + 1;
     tbl.resize( lg[n] + 1 );
     tbl[ 0 ].resize( n );
     copy( arr, arr + n, tbl[ 0 ].begin() );
for ( int i = 1 ; i <= lg[ n ] ; ++ i ) {</pre>
       int len = 1 << ( i - 1 ), sz = 1 << i;</pre>
       tbl[ i ].resize( n - sz + 1 );
for ( int j = 0 ; j <= n - sz ; ++ j )
          tbl[i][j] = cv(tbl[i-1][j], tbl[i-1][j+len]);
  T query( int 1, int r ) {
    // 0-base [l, r)
int wh = lg[ r - l ], len = 1 << wh;
return cv( tbl[ wh ][ l ], tbl[ wh ][ r - len ] );
};
```

## 2.9 Linear Basis

```
struct LinearBasis {
private:
  int n, sz;
  vector< llu > B;
  inline llu two( int x ){ return ( ( llu ) 1 ) << x; }</pre>
public:
  void init( int n_ ) {
    n = n_; B.clear();
    B.resize( n ); sz = 0;
  void insert( llu x ) {
     // add x into B
     for ( int i = n-1; i >= 0; --i ) if( two(i) & x ){
       if ( B[ i ] ) x ^= B[ i ];
       else
         B[ i ] = x; sz++;
          for ( int j = i - 1 ; j >= 0 ; -- j
         if( B[ j ] && ( two( j ) & B[ i ] ) )
   B[ i ] ^= B[ j ];
for (int j = i + 1; j < n; ++ j )
   if ( two( i ) & B[ j ] )
   B[ j ] ^= B[ i ];
heak:</pre>
         break:
    }
  inline int size() { return sz; }
bool check( llu x ) {
    // is x in span(B) ?
    for ( int i = n-1 ; i >= 0 ; --i ) if( two(i) & x )
  if( B[ i ] ) x ^= B[ i ];
       else return false;
    return true;
  llu kth small(llu k) {
    /** 1-base would always > 0 **/
     /** should check it **/
    /* if we choose at least one element
        but size(B)(vectors in B)==N(original elements)
        then we can't get 0 */
    llu ret = 0;
     for ( int i = 0 ; i < n ; ++ i ) if( B[ i ] ) {</pre>
       if( k & 1 ) ret ^= B[ i ];
    return ret;
```

} base;

# 3 Graph

## 3.1 BCC Edge

```
class BCC{
private:
  vector< int > low, dfn;
  int cnt;
  vector< bool > bridge;
  vector< vector< PII > > G;
  void dfs( int w, int f ) {
    dfn[ w ] = cnt++;
low[ w ] = dfn[ w ];
    for ( auto [ u, t ] : G[ w ] ) {
  if ( u == f ) continue;
       if ( dfn[ u ] != 0 )
         low[ w ] = min( low[ w ], dfn[ u ] );
       }else{
         dfs( u, w );
low[ w ] = min( low[ w ], low[ u ] );
         if ( low[ u ] > dfn[ w ] ) bridge[ t ] = true;
public:
  void init( int n, int m ) {
    G.resize( n );
    fill( G.begin(), G.end(), vector< PII >() );
bridge.clear(); bridge.resize( m );
    low.clear(); low.resize( n );
    dfn.clear(); dfn.resize( n );
    cnt = 0;
  void add_edge( int u, int v ) {
    // should check for multiple edge
    G[ u ].emplace_back( v, cnt );
    G[ v ].emplace_back( u, cnt ++ );
  void solve(){ cnt = 1; dfs( 0, 0 ); }
  // the id will be same as insert order, 0-base
  bool is_bridge( int x ) { return bridge[ x ]; }
} bcc;
```

#### 3.2 BCC Vertex

```
class BCC{
  private:
    int n, ecnt;
    vector< vector< pair< int, int > > > G;
    vector< int > low, dfn, id;
    vector< bool > vis, ap;
    void dfs( int u, int f, int d ) {
       int child = 0;
       dfn[ u ] = low[ u ] = d; vis[ u ] = true;
       for ( auto e : G[ u ] ) if ( e.first != f ) {
   if ( vis[ e.first ] ) {
           low[ u ] = min( low[ u ], dfn[ e.first ] );
         } else {
           dfs( e.first, u, d + 1 ); child ++;
           low[ u ] = min( low[ u ], low[ e.first ] );
           if ( low[ e.first ] >= d ) ap[ u ] = true;
       if ( u == f and child <= 1 ) ap[ u ] = false;</pre>
     void mark( int u, int idd ) {
       // really?????????
       if ( ap[ u ] ) return;
for ( auto e : G[ u ] )
         if( id[ e.second ] != -1 ) {
  id[ e.second ] = idd;
           mark( e.first, idd );
  public:
```

```
void init( int n_ ) {
   ecnt = 0, n = n_;
  G.clear(); G.resize( n );
  low.resize( n ); dfn.resize( n );
   ap.clear(); ap.resize( n );
   vis.clear(); vis.resize( n );
void add_edge( int u, int v ) {
   G[ u ].emplace_back( v, ecnt );
  G[ v ].emplace_back( u, ecnt ++ );
void solve() {
  for ( int i = 0 ; i < n ; ++ i )
  if ( not vis[ i ] ) dfs( i, i, 0 );
   id.resize( ecnt );
  fill( id.begin(), id.end(), -1 );
  for ( int i = 0 ; i < n ; ++ i )
  if ( ap[ i ] ) for ( auto e : G[ i ] )</pre>
       if( id[ e.second ] != -1 ) {
          id[ e.second ] = ecnt;
          mark( e.first, ecnt ++ );
int get_id( int x ) { return id[ x ]; }
int count() { return ecnt; }
bool is_ap( int u ) { return ap[ u ]; }
```

## 3.3 Bipartite Matching

```
class BipartiteMatching{
  private:
    vector<int> X[N], Y[N];
    int fX[N], fY[N], n;
    bitset<N> walked;
    bool dfs(int x){
      for(auto i:X[x]){
        if(walked[i])continue;
        walked[i]=1;
        if(fY[i]==-1||dfs(fY[i])){
           fY[i]=x;fX[x]=i;
           return 1;
        }
      return 0;
  public:
    void init(int _n){
      n = n;
       for(int i=0;i<n;i++){</pre>
        X[i].clear();
        Y[i].clear();
         fX[i]=fY[i]=-1;
      walked.reset();
    void add_edge(int x, int y){
      X[x].push_back(y);
      Y[y].push_back(y);
    int solve(){
      int cnt = 0;
       for(int i=0;i<n;i++){</pre>
        walked.reset();
        if(dfs(i)) cnt++;
       // return how many pair matched
      return cnt;
};
```

#### 3.4 Minimum Cost Maximum Flow

```
class MiniCostMaxiFlow{
  using CapT = int;
  using WeiT = int64_t;
  using PCW = pair<CapT,WeiT>;
  static constexpr CapT INF_CAP = 1 << 30;</pre>
```

```
static constexpr WeiT INF_WEI = 1LL<<60;</pre>
private:
  struct Edge{
    int to, back;
    WeiT wei;
    CapT cap;
    Edge() {}
    Edge(int a,int b,WeiT c,CapT d):
      to(a),back(b),wei(c),cap(d)
    {}
  int ori, edd;
  vector<vector<Edge>> G;
  vector<int> fa, wh;
  vector<bool> inq;
  vector<WeiT> dis;
  PCW SPFA(){
    fill(inq.begin(),inq.end(),false);
    fill(dis.begin(),dis.end(),INF_WEI);
    queue<int> qq; qq.push(ori);
    dis[ori]=0;
    while(!qq.empty()){
      int u=qq.front();qq.pop();
      inq[u] = 0;
      for(int i=0;i<SZ(G[u]);++i){</pre>
        Edge e=G[u][i];
        int v=e.to;
        WeiT d=e.wei;
        if(e.cap<=0||dis[v]<=dis[u]+d)</pre>
          continue;
        dis[v]=dis[u]+d;
        fa[v]=u,wh[v]=i;
        if(inq[v]) continue;
        qq.push(v);
        inq[v]=1;
      }
    if(dis[edd]==INF_WEI)
      return {-1,-1};
    CapT mw=INF_CAP;
    for(int i=edd;i!=ori;i=fa[i])
      mw=min(mw,G[fa[i]][wh[i]].cap);
    for (int i=edd;i!=ori;i=fa[i]){
      auto &eg=G[fa[i]][wh[i]];
      eg.cap-=mw;
      G[eg.to][eg.back].cap+=mw;
    return {mw,dis[edd]};
public:
  void init(int a,int b,int n){
    ori=a,edd=b;
    G.clear();G.resize(n);
    fa.resize(n);wh.resize(n);
    inq.resize(n); dis.resize(n);
  void add_edge(int st,int ed,WeiT w,CapT c){
    G[st].emplace_back(ed,SZ(G[ed]),w,c);
    G[ed].emplace_back(st,SZ(G[st])-1,-w,0);
  PCW solve(){
    /* might modify to
    cc += ret.first * ret.second
    or
    ww += ret.first * ret.second
    CapT cc=0; WeiT ww=0;
    while(true){
      PCW ret=SPFA();
      if(ret.first==-1) break;
      cc+=ret.first:
      ww+=ret.second;
    return {cc,ww};
} mcmf;
```

# 3.5 General Graph Matching

```
const int N = 514, E = (2e5) * 2;
struct Graph{
  int to[E],bro[E],head[N],e;
  int lnk[N],vis[N],stp,n;
```

```
void init( int _n ){
    stp = 0; e = 1; n = _n;
for( int i = 1 ; i <= n ; i ++ )
      lnk[i] = vis[i] = 0;
  void add_edge(int u,int v){
    to[e]=v,bro[e]=head[u],head[u]=e++;
    to[e]=u,bro[e]=head[v],head[v]=e++;
  bool dfs(int x){
    vis[x]=stp;
    for(int i=head[x];i;i=bro[i]){
       int v=to[i];
      if(!lnk[v]){
        lnk[x]=v, lnk[v]=x;
         return true;
       }else if(vis[lnk[v]]<stp){</pre>
        int w=lnk[v];
        lnk[x]=v, lnk[v]=x, lnk[w]=0;
        if(dfs(w)){
           return true;
        lnk[w]=v, lnk[v]=w, lnk[x]=0;
      }
    return false;
  int solve(){
    int ans = 0;
    for(int i=1;i<=n;i++)</pre>
      if(!lnk[i]){
        stp++; ans += dfs(i);
    return ans;
} graph;
```

#### 3.6 Dinic

```
class Dinic{
private:
  using CapT = int64_t;
  struct Edge{
    int to, rev;
    CapT cap;
  int n, st, ed;
  vector<vector<Edge>> G;
  vector<int> lv;
  bool BFS(){
    fill(lv.begin(), lv.end(), -1);
    queue<int> bfs;
    bfs.push(st);
    lv[st] = 0;
    while(!bfs.empty()){
       int u = bfs.front(); bfs.pop();
       for(auto e: G[u]){
         if(e.cap <= 0 or lv[e.to]!=-1) continue;
lv[e.to] = lv[u] + 1;</pre>
         bfs.push(e.to);
      }
    }
    return (lv[ed]!=-1);
  CapT DFS(int u, CapT f){
    if(u == ed) return f;
    CapT ret = 0;
    for(auto& e: G[u]){
       if(e.cap <= 0 or lv[e.to]!=lv[u]+1) continue;</pre>
      CapT nf = DFS(e.to, min(f, e.cap));
ret += nf; e.cap -= nf; f -= nf;
      G[e.to][e.rev].cap += nf;
      if(f == 0) return ret;
    if(ret == 0) lv[u] = -1;
    return ret;
public:
  void init(int n_, int st_, int ed_){
    n = n_, st = st_, ed = ed_;
G.resize(n); lv.resize(n);
    fill(G.begin(), G.end(), vector<Edge>());
```

```
void add_edge(int u, int v, CapT c){
    G[u].push_back({v, (int)G[v].size(), c});
    G[v].push_back({u, ((int)G[u].size())-1, 0});
}
CapT max_flow(){
    CapT ret = 0;
    while(BFS()){
        CapT f = DFS(st, numeric_limits<CapT>::max());
        ret += f;
        if(f == 0) break;
    }
    return ret;
}
flow;
```

#### 3.7 Kuhn Munkres

```
struct KM{
  static constexpr lld INF = 1LL<<60;</pre>
  11d w[N][N], 1x[N], 1y[N], slack[N];
int match[N], n, vx[N], vy[N], step_;
  void init(int n_){
     n=n_,step_=0;
     memset(w,0,sizeof(w));
     memset(lx,0,sizeof(lx));
     memset(ly,0,sizeof(ly));
     memset(slack,0,sizeof(slack));
     memset(match,0,sizeof(match));
     memset(vx,0,sizeof(vx));
     memset(vy,0,sizeof(vy));
  void add_edge(int u,int v,lld c){w[u][v]=c;}
  bool dfs(int x) {
     vx[x] = step_;
     for (int i = 0; i < n; ++i) {</pre>
       if (vy[i]==step_) continue;
if (lx[x] + ly[i] > w[x][i])
          slack[i] = min(slack[i], lx[x] + ly[i] - w[x][i]
               ]);
          continue;
        vy[i] = step_;
       if (match[i] == -1 || dfs(match[i])) {
          match[i] = x;
          return true;
       }
     }
     return false;
  11d solve() {
     fill_n(match, n, -1);
     fill_n(lx, n, -INF);
     fill_n(ly, n, 0);
for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)</pre>
     lx[i] = max(lx[i], w[i][j]);
for (int i = 0; i < n; ++i) {
  fill_n(slack, n, INF);</pre>
        while (true) {
          step_++
          if (dfs(i)) break;
          11d dlt = INF;
          for (int j = 0; j < n; ++j) if (vy[j] != step_)</pre>
          dlt = min(dlt, slack[j]);
for (int j = 0; j < n; ++j) {
   if (vx[j]==step_) lx[j] -= dlt;</pre>
             if (vy[j]==step_) ly[j] += dlt;
             else slack[j] -= dlt;
          }
       }
     11d res = 0;
     for (int i = 0; i < n; ++i) res += w[match[i]][i];</pre>
     return res;
} km;
```

#### 3.8 Flow Models

 $\bullet$  Maximum/Minimum flow with lower/upper bound from s to t

- 1. Construct super source  $\boldsymbol{S}$  and sink  $\boldsymbol{T}$
- 2. For each edge (x,y,l,u), connect  $x \to y$  with capacity u-l
- 3. For each vertex v, denote in(v) as the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds
- 4. If in(v)>0, connect  $S\to v$  with capacity in(v), otherwise, connect  $v\to T$  with capacity -in(v)
  - To maximize, connect  $t \to s$  with capacity  $\infty$ , and let f be the maximum flow from S to T. If  $f \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise, the maximum flow from s to t is the answer.
  - To minimize, let f be the maximum flow from S to T. Connect  $t \to s$  with capacity  $\infty$  and let the flow from S to T be f'. If  $f+f' \neq \sum_{v \in V, in(v)>0} in(v)$ , there's no solution. Otherwise, f' is the answer.
- 5. The solution of each edge e is  $l_e+f_e$ , where  $f_e$  corresponds to the flow on the graph
- Construct minimum vertex cover from maximum matching  ${\cal M}$  on bipartite graph (X,Y)
  - 1. Redirect every edge ( $y \to x$  if  $(x,y) \in M$ ,  $x \to y$  otherwise)
  - 2. DFS from unmatched vertices in  $\boldsymbol{X}$
  - 3.  $x \in X$  is chosen iff x is unvisited
  - 4.  $y \in Y$  is chosen iff y is visited
- Minimum cost cyclic flow
  - 1. Consruct super source  $\boldsymbol{S}$  and sink  $\boldsymbol{T}$
  - 2. For each edge (x,y,c), connect  $x \to y$  with (cost,cap)=(c,1) if c>0, otherwise connect  $y \to x$  with (cost,cap)=(-c,1)
  - 3. For each edge with c<0 , sum these cost as K , then increase d(y) by 1, decrease d(x) by 1
  - 4. For each vertex v with d(v)>0, connect  $S\to v$  with (cost,cap)=(0,d(v))
  - 5. For each vertex v with d(v)<0 , connect  $v\to T$  with (cost,cap)=(0,-d(v))
  - 6. Flow from S to T, the answer is the cost of the flow C+K
- Maximum density induced subgraph
  - 1. Binary search on answer, suppose we're checking answer  ${\cal T}$
  - 2. Construct a max flow model, let K be the sum of  $\big|$  } sat2; all weights
  - 3. Connect source  $s \to v$ ,  $v \in G$  with capacity K
  - 4. For each edge (u,v,w) in G, connect  $u \to v$  and  $v \to u$  3.10 with capacity w
  - 5. For  $v \in G$ , connect it with sink  $v \to t$  with capacity  $K+2T-(\sum_{e \in E(v)} w(e))-2w(v)$
  - 6. T is a valid answer if the maximum flow f < K|V|

#### 3.9 2-SAT

```
class TwoSat{
  private:
    int n;
    vector<vector<int>> rG,G,sccs;
    vector<int>> ord,idx;
    vector<bool> vis,result;
    void dfs(int u){
      vis[u]=true;
      for(int v:G[u])
         if(!vis[v])
```

```
dfs(v);
    ord.push_back(u);
  void rdfs(int u){
    vis[u]=false;
    idx[u]=sccs.size()-1;
    sccs.back().push_back(u);
    for(int v:rG[u])
      if(vis[v])
        rdfs(v);
public:
  void init(int n_){
    n=n :
    G.clear();
    G.resize(n);
    rG.clear();
    rG.resize(n);
    sccs.clear();
    ord.clear();
    idx.resize(n);
    result.resize(n);
  void add_edge(int u,int v){
    G[u].push_back(v);
    rG[v].push_back(u);
  void orr(int x,int y){
    if ((x^y)==1)return;
    add_edge(x^1,y);
    add_edge(y^1,x);
  bool solve(){
    vis.clear();
    vis.resize(n);
    for(int i=0;i<n;++i)</pre>
      if(not vis[i])
        dfs(i);
    reverse(ord.begin(),ord.end());
    for (int u:ord){
      if(!vis[u])
        continue;
      sccs.push_back(vector<int>());
      rdfs(u);
    for(int i=0;i<n;i+=2)</pre>
      if(idx[i]==idx[i+1])
        return false;
    vector<bool> c(sccs.size());
    for(size_t i=0;i<sccs.size();++i){</pre>
      for(size_t j=0;j<sccs[i].size();++j){</pre>
        result[sccs[i][j]]=c[i];
        c[idx[sccs[i][j]^1]]=!c[i];
      }
    return true;
  bool get(int x){return result[x];}
  inline int get_id(int x){return idx[x];}
  inline int count(){return sccs.size();}
```

## 3.10 Lowbit Decomposition

```
#define N 111
     if ( not chain[ u ] )
                                                                        struct MaxClique{ // 0-base
                                                                          typedef bitset< N > Int;
       chain[ u ] = chain_ ++;
                                                                          Int linkto[ N ] , v[ N ];
                                                                          int n;
  void dfschain( int u, int f ) {
     fa[ u ][ 0 ] = f;
                                                                          void init( int _n ){
                                                                            n = _n;
for( int i = 0 ; i < n ; i ++ ){
    linkto[ i ].reset();
}</pre>
     for ( int i = 1 ; i < LOG_N ; ++ i )</pre>
       fa[u][i] = fa[fa[\overline{u}][i-1]][i-1];
     tl[ u ] = time_++;
     if ( not chain_st[ chain[ u ] ] )
  chain_st[ chain[ u ] ] = u;
for ( int v : G[ u ] )
                                                                          void add_edge( int a , int b ){
  v[ a ][ b ] = v[ b ][ a ] = 1;
       if
           ( v != f and chain[v] == chain[u])
         dfschain( v, u );
     for ( int v : G[ u ] )
  if ( v != f and chain[ v ] != chain[ u ] )
                                                                          int popcount(const Int& val)
         dfschain( v, u );
                                                                          { return val.count(); }
    tr[ u ] = time_;
                                                                          int lowbit(const Int& val)
                                                                          { return val._Find_first(); }
                                                                          int ans , stk[ N ];
int id[ N ] , di[ N ] , deg[ N ];
  inline bool anc( int u, int v ) {
  return tl[ u ] <= tl[ v ] \</pre>
       and tr[ v ] <= tr[ u ];</pre>
                                                                          Int cans;
                                                                          void maxclique(int elem_num, Int candi){
public:
                                                                             if(elem_num > ans){
  inline int lca( int u, int v ) {
                                                                               ans = elem_num;
     if ( anc( u, v ) ) return u;
for ( int i = LOG_N - 1 ; i >= 0 ; -- i )
                                                                               cans.reset();
for( int i = 0 ; i < elem_num ; i ++ )</pre>
       if ( not anc( fa[ u ][ i ], v ) )
  u = fa[ u ][ i ];
                                                                                  cans[ id[ stk[ i ] ] = 1;
     return fa[ u ][ 0 ];
                                                                             int potential = elem_num + popcount(candi);
                                                                             if(potential <= ans) return;</pre>
  void init( int n ) {
                                                                             int pivot = lowbit(candi);
    n ++; for ( LOG_N = 0 ; ( 1 << LOG_N ) < n ; ++ LOG_N );
                                                                             Int smaller_candi = candi & (~linkto[pivot]);
                                                                             while(smaller_candi.count() && potential>ans){
     fa.clear();
                                                                               int next = lowbit(smaller_candi);
     fa.resize( n, vector< int >( LOG_N ) );
                                                                               candi[ next ] = !candi[ next ];
     G.clear(); G.resize( n );
                                                                               smaller_candi[next] = !smaller_candi[next];
    t1.clear(); t1.resize( n );
tr.clear(); tr.resize( n );
                                                                               potential -
                                                                               if(next!=pivot
                                                                                  &&!(smaller_candi&linkto[next]).count())
     chain.clear(); chain.resize( n );
     chain_st.clear(); chain_st.resize( n );
                                                                                    continue;
                                                                               stk[elem_num] = next;
  void add_edge( int u , int v ) {
                                                                               maxclique(elem_num+1, candi&linkto[next]);
    // 1-base
    G[ u ].push_back( v );
     G[ v ].push_back( u );
                                                                          int solve(){
                                                                             for( int i = 0 ; i < n ; i ++ ){</pre>
                                                                               id[ i ] = i;
deg[ i ] = v[ i ].count();
  void decompose(){
    chain_ = 1;
     predfs( 1, 1 );
     time_{-} = 0;
                                                                             sort( id , id + n , [&](int id1, int i
    return deg[id1] > deg[id2]; } );
                                                                                                     [&](int id1, int id2){
     dfschain(1,1);
                                                                             for( int i = 0 ; i < n ; i ++ )</pre>
                                                                             di[ id[ i ] ] = i;
for( int i = 0; i < n; i ++ )
    for( int j = 0; j < n; j ++ )</pre>
  PII get_inter( int u ) { return {tl[ u ], tr[ u ]}; }
  vector< PII > get_path( int u , int v ){
  vector< PII > res;
                                                                                  if( v[ i ][ j ] )
  linkto[ di[ i ] ][ di[ j ] ] = 1;
     int g = lca( u, v );
    while ( chain[ u ] != chain[ g ] ) {
  int s = chain_st[ chain[ u ] ];
                                                                             Int cand; cand.reset();
                                                                             for( int i = 0 ; i < n ; i ++ )</pre>
       res.emplace_back( tl[ s ], tl[ u ] + 1 );
       u = fa[ s ][ 0 ];
                                                                              cand[ i ] = 1;
                                                                             ans = 1;
    res.emplace_back( tl[ g ], tl[ u ] + 1 );
while ( chain[ v ] != chain[ g ] ) {
                                                                             cans.reset(); cans[ 0 ] = 1;
                                                                             maxclique(0, cand);
       int s = chain_st[ chain[ v ] ];
res.emplace_back( tl[ s ], tl[ v ] + 1 );
                                                                             return ans;
       v = fa[ s ][ 0 ];
                                                                       } solver;
    res.emplace_back( tl[ g ] + 1, tl[ v ] + 1 );
    return res;
     /* res : list of intervals from u to v
* ( note only nodes work, not edge )
                                                                                   Min-Cut
                                                                        3.12
      * vector< PII >& path = tree.get_path( u , v )
                                                                        const int maxn = 500 + 5;
      * for( auto [ l, r ] : path ) {
* 0-base [ l, r )
                                                                        int w[maxn][maxn], g[maxn];
                                                                        bool v[maxn], del[maxn];
                                                                        void add_edge(int x, int y, int c) {
                                                                             w[x][y] += c;
w[y][x] += c;
} tree;
```

pair<int, int> phase(int n) {

int s = -1, t = -1;

memset(v, false, sizeof(v));
memset(g, 0, sizeof(g));

## 3.11 MaxClique

```
while (true) {
         int c = -1;
         for (int i = 0; i < n; ++i) {</pre>
              if (del[i] || v[i]) continue;
              if (c == -1 || g[i] > g[c]) c = i;
         if (c == -1) break;
         v[c] = true;
         s = t, t = c;
for (int i = 0; i < n; ++i) {
              if (del[i] || v[i]) continue;
              g[i] += w[c][i];
         }
    return make_pair(s, t);
}
int mincut(int n) {
    int cut = 1e9;
    memset(del, false, sizeof(del));
    for (int i = 0; i < n - 1; ++i) {</pre>
         int s, t; tie(s, t) = phase(n);
del[t] = true;
         cut = min(cut, g[t]);
         for (int j = 0; j < n; ++j) {
    w[s][j] += w[t][j];</pre>
              w[j][s] += w[j][t];
         }
    return cut;
```

#### 3.13 Vitural Tree

```
inline bool cmp(const int &i, const int &j) {
  return dfn[i] < dfn[j];</pre>
void build(int vectrices[], int k) {
  static int stk[MAX_N];
  sort(vectrices, vectrices + k, cmp);
  stk[sz++] = 0;
  for (int i = 0; i < k; ++i) {
  int u = vectrices[i], lca = LCA(u, stk[sz - 1]);</pre>
    if (lca == stk[sz - 1]) stk[sz++] = u;
      while (sz \ge 2 \&\& dep[stk[sz - 2]] \ge dep[lca]) {
         addEdge(stk[sz - 2], stk[sz - 1]);
         SZ--;
      if (stk[sz - 1] != lca) {
         addEdge(lca, stk[--sz]);
stk[sz++] = lca, vectrices[cnt++] = lca;
      stk[sz++] = u;
    }
  for (int i = 0; i < sz - 1; ++i)
    addEdge(stk[i], stk[i + 1]);
```

# 4 Math

#### 4.1 Prime Table

```
1002939109, 1020288887, 1028798297, 1038684299, \\ 1041211027, 1051762951, 1058585963, 1063020809, \\ 1147930723, 1172520109, 1183835981, 1187659051, \\ 1241251303, 1247184097, 1255940849, 1272759031, \\ 1287027493, 1288511629, 1294632499, 1312650799, \\ 1868732623, 1884198443, 1884616807, 1885059541, \\ 1909942399, 1914471137, 1923951707, 1925453197, \\ 1979612177, 1980446837, 1989761941, 2007826547, \\ 2008033571, 2011186739, 2039465081, 2039728567, \\ 2093735719, 2116097521, 2123852629, 2140170259, \\ 3148478261, 3153064147, 3176351071, 3187523093, \\ 3196772239, 3201312913, 3203063977, 3204840059, \\ 3210224309, 3213032591, 3217689851, 3218469083, \\ \end{aligned}
```

```
\begin{array}{c} 3219857533, 3231880427, 3235951699, 3273767923, \\ 3276188869, 3277183181, 3282463507, 3285553889, \\ 3319309027, 3327005333, 3327574903, 3341387953, \\ 3373293941, 3380077549, 3380892997, 3381118801 \end{array}
```

# **4.2** $\left| \frac{n}{i} \right|$ Enumeration

```
T_0 = 1, T_{i+1} = \lfloor \frac{n}{\lfloor \frac{n}{T_i + 1} \rfloor} \rfloor
```

## 4.3 ax+by=gcd

```
// ax+ny = 1, ax+ny == ax == 1 (mod n)
void exgcd(lld x,lld y,lld &g,lld &a,lld &b) {
  if (y == 0) g=x,a=1,b=0;
  else
    exgcd(y,x%y,g,b,a),b-=(x/y)*a;
}
```

#### 4.4 Pollard Rho

```
// does not work when n is prime
// return any non-trivial factor
llu pollard_rho(llu n){
    static auto f=[](llu x,llu k,llu m){
        return add(k,mul(x,x,m),m);
    };
    if (!(n&1)) return 2;
    mt19937 rnd(120821011);
    while(true){
        llu y=2,yy=y,x=rnd()%n,t=1;
        for(llu sz=2;t==1;sz<<<=1) {
            for(llu i=0;i<sz;++i){
                if(t!=1)break;
                yy=f(yy,x,n);
                t=gcd(yy>y?yy-y:y-yy,n);
            }
            y=yy;
        }
        if(t!=1&&t!=n) return t;
    }
}
```

# 4.5 Pi Count (Linear Sieve)

```
static constexpr int N = 1000000 + 5;
11d pi[N];
vector<int> primes;
bool sieved[N];
11d cube_root(11d x){
  1ld s=cbrt(x-static_cast<long double>(0.1));
  while(s*s*s <= x) ++s;
  return s-1;
11d square_root(11d x){
  1ld s=sqrt(x-static_cast<long double>(0.1));
  while(s*s <= x) ++s;
  return s-1;
void init(){
  primes.reserve(N);
  primes.push_back(1);
  for(int i=2;i<N;i++) {</pre>
    if(!sieved[i]) primes.push_back(i);
    pi[i] = !sieved[i] + pi[i-1];
for(int p: primes) if(p > 1) {
       if(p * i >= N) break;
sieved[p * i] = true;
       if(p % i == 0) break;
  }
ild phi(lld m, lld n) {
   static constexpr int MM = 80000, NN = 500;
  static lld val[MM][NN];
```

```
if(m<MM&&n<NN&&val[m][n])return val[m][n]-1;</pre>
  if(n == 0) return m;
  if(primes[n] >= m) return 1;
  lld ret = phi(m,n-1)-phi(m/primes[n],n-1);
  if(m < MM\&n < NN) val[m][n] = ret+1;
  return ret;
11d pi_count(11d);
11d P2(11d m, 11d n) {
  11d sm = square_root(m), ret = 0;
  for(lld i = n+1;primes[i]<=sm;i++)</pre>
    ret+=pi_count(m/primes[i])-pi_count(primes[i])+1;
  return ret;
11d pi_count(11d m) {
  if(m < N) return pi[m];</pre>
  11d n = pi_count(cube_root(m));
  return phi(m, n) + n - 1 - P2(m, n);
```

## 4.6 Range Sieve

## 4.7 Miller Rabin

```
bool isprime(llu x){
  static llu magic[]={2,325,9375,28178,\
                     450775,9780504,1795265022};
  static auto witn=[](llu a,llu u,llu n,int t){
    a = mpow(a,u,n);
    if (!a)return 0;
    while(t--){
      1lu a2=mul(a,a,n);
       if(a2==1 && a!=1 && a!=n-1)
        return 1:
      a = a2;
    }
    return a!=1;
  if(x<2)return 0;</pre>
  if(!(x&1))return x==2;
  llu x1=x-1; int t=0;
  while(!(x1&1))x1>>=1,t++;
  for(llu m:magic)
    if(witn(m,x1,x,t))
      return 0;
  return 1;
}
```

## 4.8 Inverse Element

```
// x's inverse mod k
long long GetInv(long long x, long long k){
   // k is prime: euler_(k)=k-1
   return qPow(x, euler_phi(k)-1);
}
// if you need [1, x] (most use: [1, k-1]
void solve(int x, long long k){
```

```
inv[1] = 1;
for(int i=2;i<x;i++)
   inv[i] = ((long long)(k - k/i) * inv[k % i]) % k;
}</pre>
```

#### 4.9 Euler Phi Function

```
extended euler:
    a^b \mod p
    if gcd(a, p)==1: a^(b\%phi(p))
    elif b < phi(p): a^b mod p
    else a^(b\%phi(p) + phi(p))
lld euler_phi(int x){
   11d r=1;
   for(int i=2;i*i<=x;++i){</pre>
     if(x\%i==0){
        x/=i;
        r*=(i-1);
        while(x%i==0){
          x/=i;
          r*=i;
       }
     }
   if(x>1) r*=x-1;
   return r;
vector<int> primes;
bool notprime[N];
11d phi[N];
void euler_sieve(int n){
  for(int i=2;i<n;i++){</pre>
     if(!notprime[i]){
        primes.push_back(i);
        phi[i] = i-1;
     for(auto j: primes){
  if(i*j >= n) break;
        notprime[i*j] = true;
        phi[i*j] = phi[i] * phi[j];
if(i % j == 0){
   phi[i*j] = phi[i] * j;
          break:
       }
     }
  }
}
```

#### 4.10 Gauss Elimination

```
void gauss(vector<vector<double>> &d) {
   int n = d.size(), m = d[0].size();
   for (int i = 0; i < m; ++i) {
      int p = -1;
      for (int j = i; j < n; ++j) {
        if (fabs(d[j][i]) < eps) continue;
        if (p == -1 || fabs(d[j][i])>fabs(d[p][i])) p=j;
      }
      if (p == -1) continue;
      for (int j = 0; j < m; ++j) swap(d[p][j], d[i][j]);
      for (int j = 0; j < n; ++j) {
        if (i == j) continue;
        double z = d[j][i] / d[i][i];
      for (int k = 0; k < m; ++k) d[j][k] -= z*d[i][k];
      }
    }
}</pre>
```

#### 4.11 Fast Fourier Transform

```
/*
polynomial multiply:
DFT(a, len); DFT(b, len);
```

```
for(int i=0;i<len;i++) c[i] = a[i]*b[i];
   iDFT(c, len);
   (len must be 2^k and = 2^k (max(a, b)))
   Hand written Cplx would be 2x faster
Cplx omega[2][N];
void init_omega(int n) {
  static constexpr llf PI=acos(-1);
  const llf arg=(PI+PI)/n;
  for(int i=0;i<n;++i)</pre>
    omega[0][i]={cos(arg*i),sin(arg*i)};
  for(int i=0;i<n;++i)</pre>
    omega[1][i]=conj(omega[0][i]);
void tran(Cplx arr[],int n,Cplx omg[]) {
  for(int i=0,j=0;i<n;++i){</pre>
    if(i>j)swap(arr[i],arr[j]);
    for(int l=n>>1;(j^=l)<l;l>>=1);
  for (int l=2;l<=n;l<<=1){</pre>
    int m=1>>1;
    for(auto p=arr;p!=arr+n;p+=1){
      for(int i=0;i<m;++i){</pre>
        Cplx t=omg[n/1*i]*p[m+i];
        p[m+i]=p[i]-t;
        p[i]+=t;
    }
  }
void DFT(Cplx arr[],int n){
  tran(arr,n,omega[0]);
void iDFT(Cplx arr[],int n){
  tran(arr,n,omega[1]);
  for(int i=0;i<n;++i)arr[i]/=n;</pre>
```

# 4.12 High Speed Linear Recurrence

```
#define mod 998244353
const int N=1000010;
int n,k,m,f[N],h[N],a[N],b[N],ib[N];
int pw(int x,int y){
  int re=1;
  if(y<0)y=mod-1;
  while(y){
    if(y&1)re=(11)re*x%mod;
    y>>=1; x=(11)x*x%mod;
  return re;
void inc(int&x,int y){x+=y;if(x>=mod)x-=mod;}
namespace poly{
  const int G=3;
  int rev[N],L;
  void ntt(int*A,int len,int f){
    for(L=0;(1<<L)<len;++L);</pre>
    for(int i=0;i<len;++i){</pre>
      rev[i]=(rev[i>>1]>>1)|((i&1)<<(L-1));
      if(i<rev[i])swap(A[i],A[rev[i]]);</pre>
    for(int i=1;i<len;i<<=1){</pre>
      int wn=pw(G,f*(mod-1)/(i<<1));</pre>
      for(int j=0;j<len;j+=i<<1){</pre>
        int w=1;
        for(int k=0;k<i;++k,w=(11)w*wn%mod){</pre>
          int x=A[j+k],y=(11)w*A[j+k+i]%mod;
          A[j+k]=(x+y) \mod A[j+k+i]=(x-y+mod) \mod 3
        }
      }
    if(!~f){
      int iv=pw(len,mod-2);
      for(int i=0;i<len;++i)A[i]=(11)A[i]*iv%mod;</pre>
    }
  void cls(int*A,int l,int r){
    for(int i=1;i<r;++i)A[i]=0;}</pre>
  void cpy(int*A,int*B,int 1){
    for(int i=0;i<1;++i)A[i]=B[i];}</pre>
  void inv(int*A,int*B,int 1){
    if(l==1){B[0]=pw(A[0],mod-2);return;}
```

```
static int t[N];
    int len=l<<1;</pre>
    inv(A,B,l>>1);
    cpy(t,A,1);cls(t,1,len);
    ntt(t,len,1);ntt(B,len,1);
    for(int i=0;i<len;++i)</pre>
      B[i]=(11)B[i]*(2-(11)t[i]*B[i]%mod+mod)%mod;
    ntt(B,len,-1);cls(B,l,len);
  void pmod(int*A){
    static int t[N];
    int l=k+1,len=1;while(len<=(k<<1))len<<=1;</pre>
    cpy(t,A,(k<<1)+1);
    reverse(t,t+(k<<1)+1);
    cls(t,1,len);
    ntt(t,len,1);
    for(int i=0;i<len;++i)t[i]=(l1)t[i]*ib[i]%mod;</pre>
    ntt(t,len,-1);
    cls(t,1,len);
    reverse(t,t+1);
    ntt(t,len,1);
    for(int i=0;i<len;++i)t[i]=(11)t[i]*b[i]%mod;</pre>
    ntt(t,len,-1);
    cls(t,1,len);
    for(int i=0;i<k;++i)A[i]=(A[i]-t[i]+mod)%mod;</pre>
    cls(A,k,len);
  void pow(int*A,int n){
    if(n==1){cls(A,0,k+1);A[1]=1;return;}
    pow(A,n>>1);
    int len=1; while(len<=(k<<1))len<<=1;</pre>
    ntt(A,len,1);
    for(int i=0;i<len;++i)A[i]=(11)A[i]*A[i]%mod;</pre>
    ntt(A,len,-1);
    pmod(A);
    if(n&1){
      for(int i=k;i;--i)A[i]=A[i-1];A[0]=0;
      pmod(A);
  }
int main(){
  n=rd();k=rd();
  for(int i=1;i<=k;++i)f[i]=(mod+rd())%mod;</pre>
  for(int i=0;i<k;++i)h[i]=(mod+rd())%mod;</pre>
  for(int i=a[k]=b[k]=1;i<=k;++i)</pre>
    a[k-i]=b[k-i]=(mod-f[i])%mod;
  int len=1; while(len<=(k<<1))len<<=1;</pre>
  reverse(a.a+k+1):
  poly::inv(a,ib,len);
  poly::cls(ib,k+1,len);
  poly::ntt(b,len,1);
  poly::ntt(ib,len,1);
  poly::pow(a,n);
  int ans=0;
  for(int i=0;i<k;++i)inc(ans,(ll)a[i]*h[i]%mod);</pre>
  printf("%d\n",ans);
  return 0;
```

#### 4.13 Chinese Remainder

```
1ld crt(lld ans[], lld pri[], int n){
  11d M = 1;
  for(int i=0;i<n;i++) M *= pri[i];</pre>
  11d ret = 0;
  for(int i=0;i<n;i++){</pre>
     lld inv = (gcd(M/pri[i], pri[i]).first + pri[i])%
          pri[i];
     ret += (ans[i]*(M/pri[i])%M * inv)%M;
    ret %= M;
  return ret;
}
Another:
x = a1 \% m1
x = a2 \% m2
g = gcd(m1, m2)
assert((a1-a2)%g==0)
[p, q] = exgcd(m2/g, m1/g)
return a2+m2*(p*(a1-a2)/g)
\theta \leftarrow x \leftarrow lcm(m1, m2)
```

#### | \*/

## 4.14 Berlekamp Massey

```
// x: 1-base, p[]: 0-base
template < size t N>
vector<llf> BM(llf x[N], size_t n){
  size_t f[N]={0},t=0;llf d[N];
  vector<llf> p[N];
  for(size_t i=1,b=0;i<=n;++i) {</pre>
    for(size_t j=0;j<p[t].size();++j)</pre>
      d[i]+=x[i-j-1]*p[t][j];
    if(abs(d[i]-=x[i])<=EPS)continue;</pre>
    f[t]=i;if(!t){p[++t].resize(i);continue;}
    vector<llf> cur(i-f[b]-1);
    11f k=-d[i]/d[f[b]];cur.PB(-k);
    for(size_t j=0;j<p[b].size();j++)</pre>
      cur.PB(p[b][j]*k);
    if(cur.size()<p[t].size())cur.resize(p[t].size());</pre>
    for(size_t j=0;j<p[t].size();j++)cur[j]+=p[t][j];</pre>
    if(i-f[b]+p[b].size()>=p[t].size()) b=t;
   p[++t]=cur;
  return p[t];
```

#### 4.15 NTT

```
// Remember coefficient are mod P
/* p=a*2^n+1
   n
         2^n
                                          root
         65536
                       65537
   16
        1048576
                                          3 */
                       7340033
   20
// (must be 2^k)
template<LL P, LL root, int MAXN>
struct NTT{
  static LL bigmod(LL a, LL b) {
    LL res = 1;
    for (LL bs = a; b; b >>= 1, bs = (bs * bs) % P)
      if(b&1) res=(res*bs)%P;
    return res;
  static LL inv(LL a, LL b) {
    if(a==1)return 1;
    return (((LL)(a-inv(b%a,a))*b+1)/a)%b;
  LL omega[MAXN+1];
  NTT() {
    omega[0] = 1;
    LL r = bigmod(root, (P-1)/MAXN);
    for (int i=1; i<=MAXN; i++)</pre>
      omega[i] = (omega[i-1]*r)%P;
  // n must be 2^k
  void tran(int n, LL a[], bool inv_ntt=false){
    int basic = MAXN / n , theta = basic;
for (int m = n; m >= 2; m >>= 1) {
       int mh = m >> 1;
       for (int i = 0; i < mh; i++) {
         LL w = omega[i*theta%MAXN];
         for (int j = i; j < n; j += m) {
  int k = j + mh;
  LL x = a[j] - a[k];</pre>
           if (x < 0) x += P;
           a[j] += a[k];
           if (a[j] > P) a[j] -= P;
a[k] = (w * x) % P;
         }
       theta = (theta * 2) % MAXN;
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
  for (int k = n >> 1; k > (i ^= k); k >>= 1);
       if (j < i) swap(a[i], a[j]);</pre>
    if (inv_ntt) {
      LL ni = inv(n,P);
       reverse( a+1 , a+n );
```

## 4.16 Polynomial Sqrt

```
const int mod = (119 << 23) + 1;</pre>
int inv_temp[400010];
void poly_inv(int *f, int *inv, int len) {
  int *inv_t = inv_temp, *g = inv;
  g[0] = get_inv(f[0]);
   for (int l = 2; l <= len; l <<= 1, swap(g, inv_t)) {
     for (int i = 0; i < 1; i++) {</pre>
        inv_t[i] = f[i];
        g[i + 1] = inv_t[i + 1] = 0;
     exec_ntt(inv_t, 1 << 1, 1);
     exec_ntt(g, 1 << 1, 1);
for (int i = 0; i < 2 * 1; i++)
  inv_t[i] = (ll)inv_t[i] * g[i] % mod;
for (int i = 0; i < 2 * 1; i++) {
  if (inv t[i])</pre>
        if (inv_t[i])
           inv_t[i] = mod - inv_t[i];
        inv_t[i] += 2, inv_t[i] %= mod;
     for (int i = 0; i < 2 * 1; i++)
  inv_t[i] = (ll)inv_t[i] * g[i] % mod;</pre>
     exec_ntt(inv_t, 1 << 1, -1);
     for (int i = 0; i < 1; i++)
        inv_t[i + 1] = 0;
  for (int i = 0; i < len; i++)</pre>
     inv[i] = g[i];
int sqrt_temp[400010], inv_t[400010];
void poly_sqrt(int *f, int *sqrt_pol, int len) {
   int *g = sqrt_pol, *t = sqrt_temp, inv2 = get_inv(2);
   for (int 1 = 2; 1 <= len; 1 <<= 1, swap(g, t)) {</pre>
     for (int i = 0; i < 1; i++)</pre>
       t[i] = f[i], t[i + 1] = g[i + 1] = inv_t[i] = 0;
     poly_inv(g, inv_t, 1);
for (int i = 1; i < 2 * 1; i++)</pre>
        inv_t[i] = 0;
     exec_ntt(g, 1 << 1, 1);
     exec_ntt(inv_t, 1 << 1, 1);
     exec_ntt(t, 1 << 1, 1);
     for (int i = 0; i < (1 << 1); i++)</pre>
     t[i]=(ll)inv2*(g[i]+(ll)t[i]*inv_t[i] % mod)%mod;
exec_ntt(t, 1 << 1, -1);
     for (int i = 0; i < 1; i++)
        t[i + 1] = 0;
  for (int i = 0; i < len; i++)</pre>
     sqrt_pol[i] = g[i];
int c[400010], inv[400010], sqrt_pol[400010];
int main(){
  int n, m, x;
scanf("%d%d", &n, &m);
for (int i = 0; i < n; i++)</pre>
     scanf("%d", &x);
     if (x <= m)
        c[x] = mod - 4;
  c[0]++, c[0] \% = mod;
  int len = 1;
  while (len <= m)len <<= 1;</pre>
  poly_sqrt(c, sqrt_pol, len);
sqrt_pol[0]++, sqrt_pol[0] %= mod;
  poly_inv(sqrt_pol, inv, len);
  for (int i = 1; i <= m; i++)
printf("%d\n", (inv[i] + inv[i]) % mod);</pre>
  puts("");
   return 0;
```

## 4.17 Polynomial Division

```
VI inverse(const VI &v, int n) {
  VI q(1, fpow(v[0], mod - 2));
for (int i = 2; i <= n; i <<= 1) {
     VI fv(v.begin(), v.begin() + i);
    VI fq(q.begin(), q.end());

fv.resize(2 * i), fq.resize(2 * i);

ntt(fq, 2 * i), ntt(fv, 2 * i);

for (int j = 0; j < 2 * i; ++j)

fv[j] = fv[j]*111*fq[j]%mod*fq[j]%mod;
     intt(fv, 2 * i);
     VI res(i);
     for (int j = 0; j < i; ++j) {
  res[j] = mod - fv[j];</pre>
       if (j < (i>>1)) (res[j] += 2*q[j]%mod) %= mod;
     q = res;
  return q;
VI divide(const VI &a, const VI &b) {
  // Leading zero should be trimmed
  int n = (int)a.size(), m = (int)b.size();
  int k = 2;
  while (k < n - m + 1) k <<= 1;
  VI ra(k), rb(k);
  for (int i = 0; i < min(n, k); ++i) ra[i] = a[n-i-1];</pre>
  for (int i = 0; i < min(m, k); ++i) rb[i] = b[m-i-1];</pre>
  VI rbi = inverse(rb, k);
  VI res = convolution(rbi, ra);
  res.resize(n - m + 1);
  reverse(res.begin(), res.end());
  return res;
```

#### 4.18 FWT

```
/* xor convolution:
 * x = (x0, x1) , y = (y0, y1)
  * z = (x0y0 + x1y1 , x0y1 + x1y0 )
 * x' = (x0+x1, x0-x1), y' = (y0+y1, y0-y1)
* z' = ((x0+x1)(y0+y1), (x0-x1)(y0-y1))
* z = (1/2) * z''
  * or convolution:
  * x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
  * and convolution:
* x = (x\theta+x1, x1), inv = (x\theta-x1, x1) w/o final div */const LL MOD = 1e9+7;
inline void fwt( LL x[ MAXN ] , int N , bool inv=0 ) \{
   for( int d = 1 ; d < N ; d <<= 1 ) {</pre>
      int d2 = d<<1;</pre>
      for( int s = 0 ; s < N ; s += d2 )
  for( int i = s , j = s+d ; i < s+d ; i++, j++ ){</pre>
           LL ta = x[ i ] , tb = x[ j ];
           x[i] = ta+tb;
           x[ j ] = ta-tb;
if( x[ i ] >= MOD ) x[ i ] -= MOD;
if( x[ j ] < 0 ) x[ j ] += MOD;
        }
   if( inv )
      for( int i = 0 ; i < N ; i++ ) {
  x[ i ] *= inv( N, MOD );</pre>
        x[ i ] %= MOD;
}
```

# 4.19 DiscreteLog

```
// Baby-step Giant-step Algorithm
lld BSGS(lld P, lld B, lld N) {
    // find B^L = N mod P
    unordered_map<lld, int> R;
    lld sq = (lld)sqrt(P);
    lld t = 1;
    for (int i = 0; i < sq; i++) {</pre>
```

```
if (t == N)
    return i;
if (!R.count(t))
    R[t] = i;
    t = (t * B) % P;
}
lld f = inverse(t, P);
for(int i=0;i<=sq+1;i++) {
    if (R.count(N))
        return i * sq + R[N];
    N = (N * f) % P;
}
return -1;
}</pre>
```

## 4.20 Quadratic residue

```
struct Status{
 11 x,y;
};
11 w;
Status mult(const Status& a,const Status& b,ll mod){
  Status res;
  res.x=(a.x*b.x+a.y*b.y%mod*w)%mod;
  res.y=(a.x*b.y+a.y*b.x)%mod;
  return res;
inline Status qpow(Status _base,ll _pow,ll _mod){
  Status res;
  res.x=1, res.y=0;
  while(_pow>0){
    if(_pow&1) res=mult(res,_base,_mod);
    _base=mult(_base,_base,_mod);
    _pow>>=1;
  return res;
inline 11 check(11 x,11 p){
  return qpow_mod(x,(p-1)>>1,p);
inline ll get_root(ll n,ll p){
  if(p==2) return 1;
  if(check(n,p)==p-1) return -1;
  while(true){
    a=rand()%p;
    w=((a*a-n)%p+p)%p;
    if(check(w,p)==p-1) break;
  Status res;
  res.x=a;
  res.y=1;
  res=qpow(res,(p+1)>>1,p);
  return res.x;
```

## 4.21 De-Bruijn

```
int res[maxn], aux[maxn], sz;
void db(int t, int p, int n, int k) {
  if (t > n) {
    if (n % p == 0)
for (int i = 1; i <= p; ++i)
        res[sz++] = aux[i];
  } else +
    aux[t] = aux[t - p];
    db(t + 1, p, n, k);
for (int i = aux[t - p] + 1; i < k; ++i) {
      aux[t] = i;
      db(t + 1, t, n, k);
  }
int de_bruijn(int k, int n) {
  // return cyclic string of len k^n s.t. every string
  // of len n using k char appears as a substring.
  if (k == 1) {
    res[0] = 0;
    return 1;
```

```
}
for (int i = 0; i < k * n; i++) aux[i] = 0;
sz = 0;
db(1, 1, n, k);
return sz;
}</pre>
```

## 4.22 Simplex Construction

Standard form: maximize  $\sum_{1\leq i\leq n}c_ix_i$  such that for all  $1\leq j\leq m$ ,  $\sum_{1\leq i\leq n}A_{ji}x_i\leq b_j$  and  $x_i\geq 0$  for all  $1\leq i\leq n$ .

- 1. In case of minimization, let  $c_i^\prime = -c_i$
- 2.  $\sum_{1 \le i \le n} A_{ji} x_i \ge b_j \to \sum_{1 \le i \le n} -A_{ji} x_i \le -b_j$
- 3.  $\sum_{1 \le i \le n} A_{ji} x_i = b_j$ 
  - $\sum_{1 \le i \le n} A_{ji} x_i \le b_j$
  - $\sum_{1 \le i \le n} A_{ji} x_i \ge b_j$
- 4. If  $x_i$  has no lower bound, replace  $x_i$  with  $x_i x_i'$

## 4.23 Simplex

```
namespace simplex {
// maximize c^Tx under Ax <= B
// return vector<double>(n, -inf) if the solution doesn
     't exist
// return vector<double>(n, +inf) if the solution is
    unbounded
using VD = vector<double>;
using VVD = vector<vector<double>>;
const double eps = 1e-9;
const double inf = 1e+9;
int n, m;
VVD d;
vector<int> p, q;
void pivot(int r, int s) {
  double inv = 1.0 / d[r][s];
  for (int i = 0; i < m + 2; ++i) {
    for (int j = 0; j < n + 2; ++j) {
  if (i != r && j != s)</pre>
         d[i][j] -= d[r][j] * d[i][s] * inv;
    }
  for(int i=0;i<m+2;++i) if (i != r) d[i][s] *= -inv;
for(int j=0;j<n+2;++j) if (j != s) d[r][j] *= +inv;
d[r][s] = inv;</pre>
  swap(p[r], q[s]);
bool phase(int z) {
  int x = m + z;
  while (true) {
    int s = -1;
for (int i = 0; i <= n; ++i) {</pre>
       if (!z && q[i] == -1) continue;
      if (s == -1 || d[x][i] < d[x][s]) s = i;
    if (d[x][s] > -eps) return true;
    for (int i = 0; i < m; ++i) {</pre>
      if (d[i][s] < eps) continue;</pre>
       if (r == -1 || \
         d[i][n+1]/d[i][s] < d[r][n+1]/d[r][s]) r = i;
    if (r == -1) return false;
    pivot(r, s);
VD solve(const VVD &a, const VD &b, const VD &c) {
 m = b.size(), n = c.size();
  d = VVD(m + 2, VD(n + 2));

for (int i = 0; i < m; ++i) {
    for (int j = 0; j < n; ++j) d[i][j] = a[i][j];</pre>
  p.resize(m), q.resize(n + 1);
  for (int i = 0; i < m; ++i)</pre>
```

```
p[i] = n + i, d[i][n] = -1, d[i][n + 1] = b[i];
   for (int i = 0; i < n; ++i) q[i] = i,d[m][i] = -c[i];</pre>
   q[n] = -1, d[m + 1][n] = 1;
   for (int i = 1; i < m; ++i)</pre>
     if (d[i][n + 1] < d[r][n + 1]) r = i;</pre>
   if (d[r][n + 1] < -eps) {</pre>
     pivot(r, n);
     if (!phase(1) || d[m + 1][n + 1] < -eps)
     return VD(n, -inf);

for (int i = 0; i < m; ++i) if (p[i] == -1) {

   int s = min_element(d[i].begin(), d[i].end() - 1)
                  - d[i].begin();
        pivot(i, s);
   if (!phase(0)) return VD(n, inf);
   VD x(n);
   for (int i = 0; i < m; ++i)</pre>
     if (p[i] < n) x[p[i]] = d[i][n + 1];</pre>
   return x;
}}
```

# 5 Geometry

#### 5.1 Point Class

```
template<typename T>
struct Point{
  typedef long double llf;
  static constexpr llf EPS = 1e-8;
  T x, y;
Point(T _=0, T __=0): x(_), y(__){}
template<typename T2>
    Point(const Point<T2>& a): x(a.x), y(a.y){}
  inline llf theta() const {
    return atan2((llf)y, (llf)x);
  inline llf dis() const {
    return hypot((llf)x, (llf)y);
  inline llf dis(const Point& o) const {
    return hypot((11f)(x-o.x), (11f)(y-o.y));
  Point operator-(const Point& o) const {
    return Point(x-o.x, y-o.y);
  Point operator -= (const Point& o){
    x-=o.x, y-=o.y;
return *this;
  Point operator+(const Point& o) const {
    return Point(x+o.x, y+o.y);
  Point operator+=(const Point& o){
    x+=0.x, y+=0.y;
return *this;
  Point operator*(const T& k) const {
    return Point(x*k, y*k);
  Point operator*=(const T& k){
    x^*=k, y^*=k;
    return *this;
  Point operator/(const T& k) const {
    return Point(x/k, y/k);
  Point operator/=(const T& k){
    x/=k, y/=k;
return *this;
  Point operator-() const {
   return Point(-x, -y);
  Point rot90() const {
    return Point(-y, x);
  template<typename T2>
  bool in(const Circle<T2>& a) const {
```

```
/* Add struct Circle at top */
    return a.o.dis(*this)+EPS <= a.r;</pre>
  bool equal(const Point& o, true_type) const {
    return fabs(x-o.x) < EPS and fabs(y-o.y) < EPS;</pre>
  bool equal(const Point& o, false_type) const {
    return tie(x, y) == tie(o.x, o.y);
  bool operator==(const Point& o) const {
    return equal(o, is_floating_point<T>());
  bool operator!=(const Point& o) const {
    return !(*this == o);
  bool operator<(const Point& o) const {</pre>
    return theta() < o.theta();</pre>
    // sort like what pairs did
    // if(is_floating_point<T>()) return fabs(x-o.x)
         EPS?y<o.y:x<o.x;
    // else return tie(x, y) < tie(o.x, o.y);</pre>
  friend inline T cross(const Point& a, const Point& b)
    return a.x*b.y - b.x*a.y;
  friend inline T dot(const Point& a, const Point &b){
    return a.x*b.x + a.y*b.y;
  friend ostream& operator<<(ostream& ss, const Point&</pre>
    o){
ss<<"("<<o.x<<", "<<o.y<<")";
    return ss;
};
```

## 5.2 Circle Class

## 5.3 Line Class

```
const Point<long double> INF_P(-1e20, 1e20);
const Point<long double> NOT_EXIST(1e20, 1e-20);
template<typename T>
struct Line{
 static constexpr long double EPS = 1e-8;
  // ax+by+c = 0
 T a, b, c;
 Line(): a(0), b(1), c(0){}
           `T´
                Ţ
 Line(T
                       ): a( ), b(
                                     _), c(___){
    assert(fabs(a)>EPS or fabs(b)>EPS);
  template<typename T2>
    Line(const Line\langle T2 \rangle \& x): a(x.a), b(x.b), c(x.c){}
  typedef Point<long double> Pt;
  bool equal(const Line& o, true_type) const {
    return fabs(a-o.a) < EPS and fabs(b-o.b) < EPS and
        fabs(c-o.b) < EPS;
  bool euqal(const Line& o, false_type) const {
   return a==o.a and b==o.b and c==o.c;
```

```
bool operator==(const Line& o) const {
    return euqal(o, is_floating_point<T>());
  bool operator!=(const Line& o) const {
    return !(*this == 0);
  friend inline bool on_line__(const Point<T>& p, const
       Line& 1, true_type){
    return fabs(1.a*p.x + 1.b*p.y + 1.c) < EPS;</pre>
  friend inline bool on_line__(const Point<T>& p, const
       Line& 1, false_type){
    return 1.a*p.x + 1.b*p.y + 1.c == 0;
  friend inline bool on line(const Point<T>&p const
      Line& 1){
    return on_line__(p, l, is_floating_point<T>());
  friend inline bool is_parallel__(const Line& x, const
       Line& y, true_type){
    return fabs(x.a*y.b - x.b*y.a) < EPS;</pre>
  friend inline bool is_parallel__(const Line& x, const
       Line& y, false_type){
    return x.a*y.b == x.b*y.a;
  friend inline bool is_parallel(const Line& x, const
      Line& y){
    return is_parallel__(x, y, is_floating_point<T>());
  friend inline Pt get_inter(const Line& x, const Line&
       y){
    typedef long double llf;
    if(x==y) return INF_P;
    if(is_parallel(x, y)) return NOT_EXIST;
    llf delta = x.a*y.b - x.b*y.a;
    llf delta_x = x.b*y.c - x.c*y.b;
    llf delta_y = x.c*y.a - x.a*y.c;
return Pt(delta_x / delta, delta_y / delta);
  friend ostream& operator<<(ostream& ss, const Line& o</pre>
    ss<<o.a<<"x+"<<o.b<<"y+"<<o.c<<"=0";
    return ss:
  }
template<typename T>
inline Line<T> get_line(const Point<T>& a, const Point<</pre>
    T>& b){
  return Line<T>(a.y-b.y, b.x-a.x, (b.y-a.y)*a.x-(b.x-a
       .x)*a.y);
}
```

# 5.4 Triangle Circumcentre

#### 5.5 2D Convex Hull

```
template < typename T>
class ConvexHull_2D{
private:
   typedef Point < T > PT;
   vector < PT > dots;
```

```
struct myhash{
    uint64_t operator()(const PT& a) const {
      uint\overline{64}_t xx=0, yy=0;
      memcpy(&xx, &a.x, sizeof(a.x));
      memcpy(&yy, &a.y, sizeof(a.y));
      uint64_t ret = xx*17+yy*31;
      ret = (ret ^ (ret >> 16))*0x9E3779B1;
      ret = (ret ^ (ret >> 13))*0xC2B2AE35;
      ret = ret ^ xx;
      return (ret ^ (ret << 3)) * yy;</pre>
  };
  unordered_set<PT, myhash> in_hull;
public:
  inline void init(){in_hull.clear();dots.clear();}
  void insert(const PT& x){dots.PB(x);}
  void solve(){
    sort(ALL(dots), [](const PT& a, const PT& b){
     return tie(a.x, a.y) < tie(b.x, b.y);</pre>
    });
    vector<PT> stk(SZ(dots)<<1);</pre>
    int top = 0;
    for(auto p: dots){
      while(top >= 2 and cross(p-stk[top-2], stk[top
          -1]-stk[top-2]) <= 0)
        top --;
      stk[top++] = p;
    for(int i=SZ(dots)-2, t = top+1;i>=0;i--){
      while(top >= t and cross(dots[i]-stk[top-2], stk[
          top-1]-stk[top-2]) <= 0)
      stk[top++] = dots[i];
    stk.resize(top-1);
    swap(stk, dots);
    for(auto i: stk) in_hull.insert(i);
  vector<PT> get(){return dots;}
 inline bool in_it(const PT& x){
    return in_hull.find(x)!=in_hull.end();
};
```

#### 5.6 2D Farthest Pair

#### 5.7 2D Cosest Pair

```
struct Point{
  11f x, y;
  llf dis;
} arr[N];
inline llf get_dis(Point a, Point b){
  return hypot(a.x-b.x, a.y-b.y);
11f solve(){
  int cur = rand()%n;
  for(int i=0;i<n;i++) arr[i].dis = get_dis(arr[cur],</pre>
      arr[i]);
  sort(arr, arr+n, [](Point a, Point b){return a.dis <</pre>
      b.dis;});
  11f ans = 1e50;
  for(int i=0;i<n;i++){</pre>
    for(int j=i+1;j<n;j++){</pre>
      if(arr[j].dis - arr[i].dis > ans) break;
```

```
ans = min(ans, get_dis(arr[i], arr[j]));
}
return ans;
}
```

## 5.8 SimulateAnnealing

```
double getY(double);
int main(){
  int rr, 11;
  default_random_engine rEng(time(NULL));
  uniform_real_distribution<double> Range(-1,1);
  uniform_real_distribution < double > expR(0,1);
  auto Random=bind(Range,rEng), expRand=bind(expR,rEng)
  int step=0;
  double pace=rr-ll, mini=0.95; // need to search for
      it
  double x=max(min(Random()*pace+ll, rr), ll), y=getY(x
      );
  while(pace>=1e-7){
    double newX = max(min(x + Random()*pace, rr), 11);
    double newY = getY(newX);
    if(newY < y || expRand() < exp(-step))</pre>
      x=newX, y=newY;
    step++;
    pace*=mini;
}
```

## 5.9 Ternary Search on Integer

```
int TernarySearch(int 1, int r) {
    // (L, r]
    while (r - 1 > 1){
        int mid = (1 + r)>>1;
        if (f(mid) > f(mid + 1)) r = mid;
        else 1 = mid;
    }
    return 1+1;
}
```

# 5.10 Minimum Covering Circle

```
template<typename T>
Circle<llf> MinCircleCover(const vector<Point<T>>& pts)
  random_shuffle(ALL(pts));
  Circle<llf> c = \{pts[0], 0\};
  int n = SZ(pts);
  for(int i=0;i<n;i++){</pre>
    if(pts[i].in(c)) continue;
    c = \{pts[i], 0\};
    for(int j=0;j<i;j++){</pre>
      if(pts[j].in(c)) continue;
      c.o = (pts[i] + pts[j]) / 2;
      c.r = pts[i].dis(c.o);
      for(int k=0;k<j;k++){</pre>
        if(pts[k].in(c)) continue;
        c = get_circum(pts[i], pts[j], pts[k]);
      }
    }
  }
  return c;
```

# 5.11 KDTree (Nearest Point)

```
const int MXN = 100005;
struct KDTree {
   struct Node {
```

```
int x,y,x1,y1,x2,y2;
     int id,f;
Node *L, *R;
  }tree[MXN];
  int n;
Node *root;
  LL dis2(int x1, int y1, int x2, int y2) {
     LL dx = x1-x2;
     LL dy = y1-y2;
     return dx*dx+dy*dy;
  static bool cmpx(Node& a, Node& b){ return a.x<b.x; }
static bool cmpy(Node& a, Node& b){ return a.y<b.y; }
void init(vector<pair<int,int>> ip) {
     n = ip.size();
     for (int i=0; i<n; i++) {</pre>
       tree[i].id = i;
       tree[i].x = ip[i].first;
tree[i].y = ip[i].second;
     root = build_tree(0, n-1, 0);
  Node* build_tree(int L, int R, int dep) {
     if (L>R) return nullptr;
     int M = (L+R)/2;
     tree[M].f = dep%2;
     nth_element(tree+L, tree+M, tree+R+1, tree[M].f ?
          cmpy : cmpx);
     tree[M].x1 = tree[M].x2 = tree[M].x;
     tree[M].y1 = tree[M].y2 = tree[M].y;
     tree[M].L = build_tree(L, M-1, dep+1);
     if (tree[M].L) {
       tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
       tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
       tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
     tree[M].R = build_tree(M+1, R, dep+1);
     if (tree[M].R) {
       tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
       tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
     return tree+M;
  int touch(Node* r, int x, int y, LL d2){
     LL dis = sqrt(d2)+1;
     if (x<r->x1-dis || x>r->x2+dis ||
         y<r->y1-dis || y>r->y2+dis)
       return 0;
     return 1;
  void nearest(Node* r, int x, int y,
                  int &mID, LL &md2){
     if (!r || !touch(r, x, y, md2)) return;
LL d2 = dis2(r->x, r->y, x, y);
     if (d2 < md2 || (d2 == md2 && mID < r->id)) {
       mID = r \rightarrow id;
       md2 = d2;
     // search order depends on split dim
     if ((r->f == 0 \&\& x < r->x) ||
          (r->f == 1 && y < r->y))
       nearest(r->L, x, y, mID, md2);
       nearest(r->R, x, y, mID, md2);
       nearest(r->R, x, y, mID, md2);
       nearest(r->L, x, y, mID, md2);
     }
  int query(int x, int y) {
     int id = 1029384756;
     LL d2 = 102938475612345678LL;
     nearest(root, x, y, id, d2);
     return id;
}tree;
```

# 6 Stringology

#### 6.1 Hash

```
class Hash{
private:
   static const int N = 1000000;
   const int p = 127, q = 1208220623;
   int sz, prefix[N], power[N];
inline int add(int x, int y){return x+y>=q?x+y-q:x+y
   inline int sub(int x, int y){return x-y<0?x-y+q:x-y;}
inline int mul(int x, int y){return 1LL*x*y%q;}</pre>
public:
   void init(const std::string &x){
     sz = x.size();
     prefix[0]=0;
     for(int i=1;i<=sz;i++) prefix[i]=add(mul(prefix[i</pre>
          -1], p), x[i-1]);
     power[0]=1;
     for(int i=1;i<=sz;i++) power[i]=mul(power[i-1], p);</pre>
   int query(int 1, int r){
     // 1-base (l, r]
     return sub(prefix[r], mul(prefix[l], power[r-l]));
};
```

## 6.2 Suffix Array

```
namespace sfxarray {
bool t[maxn * 2];
int hi[maxn], rev[maxn];
int _s[maxn * 2], sa[maxn * 2], c[maxn * 2];
int x[maxn], p[maxn], q[maxn * 2];
// sa[i]: sa[i]-th suffix is the \

// i-th lexigraphically smallest suffix.
// hi[i]: longest common prefix \
// of suffix sa[i] and suffix sa[i - 1].
void pre(int *sa, int *c, int n, int z) {
  memset(sa, 0, sizeof(int) * n);
   memcpy(x, c, sizeof(int) * z);
void induce(int *sa,int *c,int *s,bool *t,int n,int z){
  memcpy(x + 1, c, sizeof(int) * (z - 1));
for (int i = 0; i < n; ++i)
     if (sa[i] && !t[sa[i] - 1])
       sa[x[s[sa[i] - 1]]++] = sa[i] - 1;
  memcpy(x, c, sizeof(int) * z);
for (int i = n - 1; i >= 0; --i)
   if (sa[i] && t[sa[i] - 1])
        sa[--x[s[sa[i] - 1]]] = sa[i] - 1;
void sais(int *s, int *sa, int *p, int *q,
 bool *t, int *c, int n, int z) {
  bool uniq = t[n - 1] = true;
  int nn=0, nmxz=-1, *nsa = sa+n, *ns=s+n, last=-1;
memset(c, 0, sizeof(int) * z);
   for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;</pre>
   for (int i = 0; i < z - 1; ++i) c[i + 1] += c[i];
   if (uniq) {
     for (int i = 0; i < n; ++i) sa[--c[s[i]]] = i;</pre>
     return;
  for (int i = n - 2; i >= 0; --i)
t[i] = (s[i]==s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
   pre(sa, c, n, z);
for (int i = 1; i <= n - 1; ++i)</pre>
     if (t[i] && !t[i - 1])
        sa[--x[s[i]]] = p[q[i] = nn++] = i;
   induce(sa, c, s, t, n, z);
for (int i = 0; i < n; ++i) {
   if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {</pre>
     bool neq = last < 0 || \</pre>
      memcmp(s + sa[i], s + last,
  (p[q[sa[i]] + 1] - sa[i]) * sizeof(int));
     ns[q[last = sa[i]]] = nmxz += neq;
   sais(ns, nsa, p+nn, q+n, t+n, c+z, nn, nmxz+1);
   pre(sa, c, n, z);
```

```
for (int i = nn - 1; i >= 0; --i)
     sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
  induce(sa, c, s, t, n, z);
void build(const string &s) {
  for (int i = 0; i < (int)s.size(); ++i) _s[i] = s[i];</pre>
   _s[(int)s.size()] = 0; // s shouldn't contain 0
  sais(_s, sa, p, q, t, c, (int)s.size() + 1, 256);
for(int i = 0; i < (int)s.size(); ++i) sa[i]=sa[i+1];</pre>
  for(int i = 0; i < (int)s.size(); ++i) rev[sa[i]]=i;
int ind = 0; hi[0] = 0;</pre>
  for (int i = 0; i < (int)s.size(); ++i) {</pre>
     if (!rev[i]) {
       ind = 0:
       continue;
     while (i + ind < (int)s.size() && \</pre>
      s[i + ind] == s[sa[rev[i] - 1] + ind]) ++ind;
     hi[rev[i]] = ind ? ind -- : 0;
}}
```

## 6.3 Aho-Corasick Algorithm

```
class AhoCorasick{
  private:
    static constexpr int Z = 26;
    struct node{
      node *nxt[ Z ], *fail;
      vector< int > data;
      node(): fail( nullptr ) {
         memset( nxt, 0, sizeof( nxt ) );
         data.clear();
      *rt;
    inline int Idx( char c ) { return c - 'a'; }
  public:
    void init() { rt = new node(); }
    void add( const string& s, int d ) {
      node* cur = rt;
      for ( auto c : s ) {
   if ( not cur->nxt[ Idx( c ) ] )
           cur->nxt[ Idx( c ) ] = new node();
         cur = cur->nxt[ Idx( c ) ];
      cur->data.push_back( d );
    void compile() {
  vector< node* > bfs;
       size_t ptr = 0;
      for ( int i = 0 ; i < Z ; ++ i ) {
  if ( not rt->nxt[ i ] )
         continue;
rt->nxt[ i ]->fail = rt;
         bfs.push_back( rt->nxt[ i ] );
      while ( ptr < bfs.size() ) {</pre>
         node* u = bfs[ ptr ++ ];
for ( int i = 0 ; i < Z ; ++ i ) {
           if ( not u->nxt[ i ] )
             continue;
           node* u_f = u->fail;
           while ( u_f ) {
   if ( not u_f->nxt[ i ] ) {
               u_f = u_f->fail;
                continue;
             u->nxt[ i ]->fail = u_f->nxt[ i ];
           if ( not u_f ) u->nxt[ i ]->fail = rt;
           bfs.push_back( u->nxt[ i ] );
      }
    void match( const string& s, vector< int >& ret ) {
      node* u = rt;
      for ( auto c : s ) {
         while ( u != rt and not u->nxt[ Idx( c ) ] )
           u = u->fail;
         u = u->nxt[ Idx( c ) ];
if ( not u ) u = rt;
         node* tmp = u;
```

```
while ( tmp != rt ) {
    for ( auto d : tmp->data )
        ret.push_back( d );
    tmp = tmp->fail;
    }
}
ac;
```

#### 6.4 Suffix Automaton

```
struct Node{
  Node *green, *edge[26];
  int max_len;
  Node(const int _max_len)
    : green(NULL), max_len(_max_len){
    memset(edge,0,sizeof(edge));
} *ROOT, *LAST;
void Extend(const int c) {
  Node *cursor = LAST;
  LAST = new Node((LAST->max_len) + 1);
  for(;cursor&&!cursor->edge[c]; cursor=cursor->green)
    cursor->edge[c] = LAST;
  if (!cursor)
    LAST->green = ROOT;
  else {
    Node *potential_green = cursor->edge[c];
    if((potential_green->max_len)==(cursor->max_len+1))
      LAST->green = potential_green;
    else {
//assert(potential_green->max_len>(cursor->max_len+1));
      Node *wish = new Node((cursor->max_len) + 1);
      for(;cursor && cursor->edge[c]==potential_green;
            cursor = cursor->green)
         cursor->edge[c] = wish;
      for (int i = 0; i < 26; i++)
wish->edge[i] = potential_green->edge[i];
      wish->green = potential_green->green;
      potential_green->green = wish;
      LAST->green = wish;
  }
char S[10000001], A[10000001];
int N:
int main(){
  scanf("%d%s", &N, S);
  ROOT = LAST = new Node(0);
  for (int i = 0; S[i]; i++)
  Extend(S[i] - 'a');
  while (N--){
  scanf("%s", A);
    Node *cursor = ROOT;
    bool ans = true;
    for (int i = 0; A[i]; i++){
      cursor = cursor->edge[A[i] - 'a'];
      if (!cursor) {
         ans = false;
        break;
      }
    }
    puts(ans ? "Yes" : "No");
  return 0;
}
```

## 6.5 KMP

```
int F[N<<1];
void KMP(char s1[], char s2[], int n, int m){
    // make F[] for s1+'\0'+s2;
    char ss[N<<1];
    int len = n+m+1;
    for(int i=0;i<n;i++) ss[i] = s1[i];
    ss[n] = '\0';
    for(int i=0;i<m;i++) ss[i+1+n] = s2[i];
    F[0] = F[1] = 0;
    for(int i=1;i<len;i++){</pre>
```

```
int j = F[i];
while(j > 0 and ss[i]!=ss[j]) j = F[j];
F[i+1] = (ss[i]==ss[j]?j+1:0);
}
// just find (F[len2+i] == len2)
// i from 1 to len+1 for matching
}
/*

[0, i]是個循環字串,且循環節為i-f[i]:
if(f[i]>0 and i%(i-f[i])==0)
cout << i << " " << i/(i-f[i]) << '\n';
*/</pre>
```

#### 6.6 Z value

```
char s[MAXN];
int len,z[MAXN];
void Z_value() {
   int i,j,left,right;
   left=right=0; z[0]=len;
   for(i=1;i<len;i++) {
      j=max(min(z[i-left],right-i),0);
      for(;i+j<len&&s[i+j]==s[j];j++);
      z[i]=j;
      if(i+z[i]>right) {
        right=i+z[i];
        left=i;
      }
   }
}
```

# 6.7 Lexicographically Smallest Rotation

```
string mcp(string s){
  int n = s.length();
  s += s;
  int i=0, j=1;
  while (i<n && j<n){
    int k = 0;
    while (k < n && s[i+k] == s[j+k]) k++;
    if (s[i+k] <= s[j+k]) j += k+1;
    else i += k+1;
    if (i == j) j++;
}
int ans = i < n ? i : j;
  return s.substr(ans, n);
}</pre>
```

#### 6.8 BWT

```
struct BurrowsWheeler{
#define SIGMA 26
#define BASE 'a'
   vector<int> v[ SIGMA ];
   void BWT(char* ori, char* res){
      // make ori -> ori + ori
      // then build suffix array
   }
   void iBWT(char* ori, char* res){
      for( int i = 0 ; i < SIGMA ; i ++ )
        v[ i ].clear();
      int len = strlen( ori );
      for( int i = 0 ; i < len ; i ++ )
        v[ ori[i] - BASE ].push_back( i );
      vector<int> a;
      for( int i = 0 , ptr = 0 ; i < SIGMA ; i ++ )
        for( auto j : v[ i ] ){
            a.push_back( j );
            ori[ ptr ++ ] = BASE + i;
      }
      for( int i = 0 , ptr = 0 ; i < len ; i ++ ){
            res[ i ] = ori[ a[ ptr ] ];
            ptr = a[ ptr ];
      }
}</pre>
```

```
res[ len ] = 0;
}
} bwt;
```

#### 6.9 Palindromic Tree

```
struct palindromic_tree{
  struct node{
    int next[26],fail,len;
    int cnt,num,st,ed;
    node(int l=0):fail(0),len(1),cnt(0),num(0){
      for(int i=0;i<26;++i)next[i]=0;</pre>
  }:
  vector<node> state;
  vector<char> s;
  int last,n;
  void init(){
    state.clear();
    s.clear();
    last=1;
    n=0;
    state.push_back(0);
    state.push_back(-1);
    state[0].fail=1;
    s.push_back(-1);
  int get_fail(int x){
  while(s[n-state[x].len-1]!=s[n])x=state[x].fail;
    return x;
  void add(int c){
    s.push_back(c-='a');
     ++n;
    int cur=get_fail(last);
    if(!state[cur].next[c]){
      int now=state.size();
      state.push_back(state[cur].len+2);
      state[now].fail=state[get_fail(state[cur].fail)].
           next[c];
       state[cur].next[c]=now;
       state[now].num=state[state[now].fail].num+1;
    last=state[cur].next[c];
    ++state[last].cnt;
  int size(){
    return state.size()-2;
}pt;
int main() {
  string s;
  cin >> s;
  pt.init();
  for (int i=0; i<SZ(s); i++) {</pre>
    int prvsz = pt.size();
    pt.add(s[i]);
     if (prvsz != pt.size()) {
      int r = i;
      }
  }
  return 0;
}
```

## 7 Misc

#### 7.1 Theorems

#### 7.1.1 Kirchhoff's Theorem

Denote L be a  $n \times n$  matrix as the Laplacian matrix of graph G, where  $L_{ii}=d(i)$ ,  $L_{ij}=-c$  where c is the number of edge (i,j) in

G.

- The number of undirected spanning in G is  $|\det(\tilde{L}_{11})|$ .
- The number of directed spanning tree rooted at r in G is  $|\det(\tilde{L}_{rr})|$ .

#### 7.1.2 Tutte's Matrix

Let D be a  $n \times n$  matrix, where  $d_{ij} = x_{ij}$  ( $x_{ij}$  is chosen uniform randomly) if i < j and  $(i,j) \in E$ , otherwise  $d_{ij} = -d_{ji}$ .  $\frac{rank(D)}{2}$  is the maximum matching on G.

#### 7.1.3 Cayley's Formula

- Given a degree sequence  $d_1,d_2,\ldots,d_n$  for each labeled vertices, there're  $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!}$  spanning trees.
- Let  $T_{n,k}$  be the number of labeled forests on n vertices with k components, such that vertex  $1,2,\ldots,k$  belong to different components. Then  $T_{n,k}=kn^{n-k-1}$ .

#### 7.1.4 Erdős-Gallai theorem

A sequence of non-negative integers  $d_1 \geq d_2 \geq \ldots \geq d_n$  can be represented as the degree sequence of a finite simple graph on n vertices if and only if  $d_1+d_2+\ldots+d_n$  is even and

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i,k)$$

holds for all  $1 \leq k \leq n$ .

#### 7.1.5 Havel-Hakimi algorithm

find the vertex who has greatest degree unused, connect it with other greatest vertex.

# 7.2 MaximumEmptyRect

```
int max_empty_rect(int n, int m, bool blocked[N][N]){
  static int mxu[2][N], me=0,he=1,ans=0;
  for(int i=0;i<m;i++) mxu[he][i]=0;
for(int i=0;i<n;i++){</pre>
     stack<PII,vector<PII>> stk;
     for(int j=0;j<m;++j){
   if(blocked[i][j]) mxu[me][j]=0;</pre>
        else mxu[me][j]=mxu[he][j]+1;
        int la = j;
       while(!stk.empty()&&stk.top().FF>mxu[me][j]){
  int x1 = i - stk.top().FF, x2 = i;
  int y1 = stk.top().SS, y2 = j;
  int y1 = stk.top().SS, y2 = j;
          la = stk.top().SS; stk.pop();
          ans=\max(ans,(x2-x1)*(y2-y1));
        if(stk.empty()||stk.top().FF<mxu[me][j])</pre>
          stk.push({mxu[me][j],la});
     while(!stk.empty()){
  int x1 = i - stk.top().FF, x2 = i;
        int y1 = stk.top().SS-1, y2 = m-1;
        stk.pop();
        ans=max(ans,(x2-x1)*(y2-y1));
     swap(me,he);
  return ans;
```

## 7.3 DP-opt Condition

#### 7.3.1 totally monotone (concave/convex)

```
\begin{array}{l} \forall i < i', j < j' \text{, } B[i][j] \leq B[i'][j] \implies B[i][j'] \leq B[i'][j'] \\ \forall i < i', j < j' \text{, } B[i][j] \geq B[i'][j] \implies B[i][j'] \geq B[i'][j'] \end{array}
```

#### 7.3.2 monge condition (concave/convex)

```
 \forall i < i', j < j', \ B[i][j] + B[i'][j'] \ge B[i][j'] + B[i'][j]   \forall i < i', j < j', \ B[i][j] + B[i'][j'] \le B[i][j'] + B[i'][j]
```

## 7.4 Convex 1D/1D DP

```
struct segment {
  int i, l, r;
  segment() {}
  segment(int a, int b, int c): i(a), l(b), r(c) {}
inline long long f(int 1, int r) {
  return dp[1] + w(1 + 1, r);
void solve() {
  dp[0] = 011;
  deque<segment> deq; deq.push_back(segment(0, 1, n));
  for (int i = 1; i <= n; ++i) {
  dp[i] = f(deq.front().i, i);</pre>
     while (deq.size() && deq.front().r < i + 1) deq.</pre>
         pop_front();
     deq.front().l = i + 1;
     segment seg = segment(i, i + 1, n);
     while (deq.size() && f(i, deq.back().1) < f(deq.</pre>
          back().i, deq.back().1)) deq.pop_back();
     if (deq.size()) {
       int d = 1048576, c = deq.back().1;
while (d >>= 1) if (c + d <= deq.back().r) {
  if (f(i, c + d) > f(deq.back().i, c + d)) c +=
       deq.back().r = c; seg.l = c + 1;
     if (seg.l <= n) deq.push_back(seg);</pre>
```

# 7.5 Josephus Problem

```
// n people kill m for each turn
int f(int n, int m) {
  int s = 0;
  for (int i = 2; i <= n; i++)
      s = (s + m) % i;
  return s;
}
// died at kth
int kth(int n, int m, int k){
  if (m == 1) return n-1;
  for (k = k*m+m-1; k >= n; k = k-n+(k-n)/(m-1));
  return k;
}
```

# 7.6 Cactus Matching

```
const int maxn=200010;
vector<int> init_g[maxn],g[maxn*2];
int dfn[maxn],low[maxn],par[maxn],dfs_idx,bcc_id;
int n;
void tarjan(int u){
   dfn[u]=low[u]=++dfs_idx;
   for(int i=0;i<(int)init_g[u].size();i++){
      int v=init_g[u][i];
      if(v==par[u]) continue;</pre>
```

if(!dfn[v]){

par[v]=u;

```
tarjan(v);
       low[u]=min(low[u],low[v]);
       if(dfn[u]<low[v]){</pre>
         g[u].push_back(v);
                                                                 7.7
                                                                          DLX
         g[v].push_back(u);
    }else{
                                                                 struct DLX {
      low[u]=min(low[u],dfn[v]);
                                                                      const static int maxn=210;
       if(dfn[v]<dfn[u]){</pre>
                                                                      const static int maxm=210;
         int temp_v=u;
                                                                      const static int maxnode=210*210;
         bcc_id++;
                                                                      int n, m, size;
         while(temp_v!=v){
                                                                      int row[maxnode], col[maxnode];
                                                                      int U[maxnode], D[maxnode], L[maxnode], R[maxnode];
int H[maxn], S[maxm];
           g[bcc_id+n].push_back(temp_v);
           g[temp_v].push_back(bcc_id+n);
           temp_v=par[temp_v];
                                                                      int ansd, ans[maxn];
void init(int _n, int _m) {
    n = _n, m = _m;
}
         g[bcc_id+n].push_back(v);
         g[v].push_back(bcc_id+n);
                                                                          for(\overline{int} i = \overline{0}; i <= m; ++i) {
         reverse(g[bcc_id+n].begin(),g[bcc_id+n].end());
                                                                               S[i] = 0;
      }
                                                                               U[i] = D[i] = i;
    }
                                                                               L[i] = i-1, R[i] = i+1;
  }
                                                                          L[0] = m, R[m] = 0;
int dp[maxn][2],min_dp[2][2],tmp[2][2],tp[2];
void dfs(int u,int fa){
                                                                          size = m;
                                                                          for(int i = 1; i <= n; ++i) H[i] = -1;</pre>
  if(u<=n){</pre>
    for(int i=0;i<(int)g[u].size();i++){</pre>
                                                                      void Link(int r, int c) {
      int v=g[u][i];
                                                                          ++S[col[++size] = c];
       if(v==fa) continue;
                                                                          row[size] = r; D[size] = D[c];
      dfs(v,u);
                                                                          U[D[c]] = size; U[size] = c; D[c] = size;
       memset(tp,0x8f,sizeof tp);
                                                                          if(H[r] < 0) H[r] = L[size] = R[size] = size;
       if(v<=n){
                                                                          else {
         tp[0]=dp[u][0]+max(dp[v][0],dp[v][1]);
                                                                               R[size] = R[H[r]];
         tp[1]=max(
                                                                               L[R[H[r]]] = size;
           dp[u][0]+dp[v][0]+1,
                                                                               L[size] = H[r];
           dp[u][1]+max(dp[v][0],dp[v][1])
                                                                               R[H[r]] = size;
      }else{
         tp[0]=dp[u][0]+dp[v][0];
                                                                      void remove(int c) {
         tp[1]=max(dp[u][0]+dp[v][1],dp[u][1]+dp[v][0]);
                                                                          L[R[c]] = L[c]; R[L[c]] = R[c];
for(int i = D[c]; i != c; i = D[i])
      dp[u][0]=tp[0],dp[u][1]=tp[1];
                                                                               for(int j = R[i]; j != i; j = R[j]) {
    }
                                                                                   U[D[j]] = U[j];
D[U[j]] = D[j];
  }else{
    for(int i=0;i<(int)g[u].size();i++){</pre>
                                                                                   --S[col[j]];
      int v=g[u][i];
                                                                               }
       if(v==fa) continue;
      dfs(v,u);
                                                                      void resume(int c) {
                                                                          L[R[c]] = c; R[L[c]] = c;
for(int i = U[c]; i != c; i = U[i])
    min_dp[0][0]=0;
    min_dp[1][1]=1;
                                                                               for(int j = L[i]; j != i; j = L[j]) {
    min_dp[0][1]=min_dp[1][0]=-0x3f3f3f3f;
                                                                                   U[D[j]] = j;
    for(int i=0;i<(int)g[u].size();i++){</pre>
                                                                                   D[U[j]] = j;
      int v=g[u][i];
                                                                                   ++S[col[j]];
       if(v==fa) continue;
                                                                          }
       memset(tmp,0x8f,sizeof tmp);
       tmp[0][0]=max(
                                                                      void dance(int d) {
         min_dp[0][0]+max(dp[v][0],dp[v][1]),
                                                                          if(d>=ansd) return;
         min_dp[0][1]+dp[v][0]
                                                                          if(R[0] == 0) {
                                                                               ansd = d;
       tmp[0][1]=min_dp[0][0]+dp[v][0]+1;
                                                                               return:
       tmp[1][0]=max(
         min_dp[1][0]+max(dp[v][0],dp[v][1]),
                                                                          int c = R[0];
         min_dp[1][1]+dp[v][0]
                                                                          for(int i = R[0]; i; i = R[i])
                                                                               if(S[i] < S[c]) c = i;
       tmp[1][1]=min_dp[1][0]+dp[v][0]+1;
                                                                          remove(c);
for(int i = D[c]; i != c; i = D[i]) {
      memcpy(min_dp,tmp,sizeof tmp);
                                                                               ans[d] = row[i];
    dp[u][1]=max(min_dp[0][1],min_dp[1][0]);
                                                                               for(int j = R[i]; j != i; j = R[j])
    dp[u][0]=min_dp[0][0];
                                                                                   remove(col[j]);
                                                                               dance(d+1);
                                                                               for(int j = L[i]; j != i; j = L[j])
int main(){
                                                                                   resume(col[j]);
  int m,a,b;
scanf("%d%d",&n,&m);
                                                                          resume(c);
  for(int i=0;i<m;i++){</pre>
    scanf("%d%d",&a,&b);
                                                                 } sol;
    init_g[a].push_back(b);
    init_g[b].push_back(a);
  par[1]=-1;
  tarjan(1);
  dfs(1,-1);
  printf("%d\n", max(dp[1][0], dp[1][1]));
```

return 0;

}