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### 1 Basic

#### 1.1 Default Code

```
#include <bits/stdc++.h>
using namespace std;
using lld = int64_t;
using llu = uint6\overline{4} t;
using llf = long double;
using PII = pair<int,int>;
using PIL = pair<int, lld>;
using PLI = pair<lld,int>;
using PLL = pair<lld, lld>;
template<typename T>
using maxHeap = priority_queue<T, vector<T>, less<T>>;
template<typename T>
using minHeap = priority queue<T, vector<T>, greater<T>>;
#define FF first
#define SS second
#define SZ(x) (int)((x).size())
#define ALL(x) begin(x), end(x)
#define PB push back
#define WC(x) while ((x) --)
template<typename Iter>
\verb| ostream \& \_out(ostream \&s, Iter b, Iter e) | \{ \\
 s<<"/";
 for (auto it=b;it!=e;it++) s<<(it==b?"":" ") <<*it;</pre>
  s<<"]";
 return s;
template<typename A, typename B>
ostream& operator<<(ostream &s,const pair<A,B>&p)
{return s<<"("<<p.FF<<","<<p.SS<<")";}
template<typename T>
ostream& operator<<(ostream &s,const vector<T>&c)
{return _out(s,ALL(c));}
bool debug = 0;
#define DUMP(x) if(debug)cerr<< PRETTY FUNCTION <<\</pre>
":"<< _LINE__<< " - "<< (#x) << "="<< (x) << '\n' template<typename T>
void DEBUG(const T& x) {if(debug)cerr<<x;}</pre>
template<typename T, typename...Args>
void DEBUG(const T&head,const Args&...tail) {
 if (debug) {cerr<<head; DEBUG(tail...); }</pre>
int main(int argc,char* argv[]) {
  \textbf{if} (\texttt{argc} > 1 \& \& \texttt{string} (\texttt{argv} [1]) == "-D") \texttt{ debug} = 1;
  if(!debug) {ios_base::sync_with_stdio(0);cin.tie(0);}
```

### 1.2 IncreaseStackSize

```
//stack resize(change esp to rsp if 64-bit system)
asm( "mov %0,%%esp\n" ::"g"(mem+10000000) );
// craziest way
static void run_stack_sz(void(*func)(),size_t stsize){
    char *stack, *send;
    stack=(char *) malloc(stsize);
    send=stack+stsize-16;
    send=(char *) ((uintptr_t)send/16*16);
    asm volatile(
        "mov %%rsp, (%0)\n"
        "mov %0, %%rsp\n"
        :
        : "r" (send));
    func();
    asm volatile(
        "mov (%0), %%rsp\n"
        :
        : "r" (send));
    free(stack);
```

# 1.3 Pragma optimization

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popent,abm,mmx,avx,tune=native")
```

### 1.4 Debugger

```
#! /usr/bin/env python3
import subprocess as sp
os_name = __import__('platform').system()
cmd,prefix = [],""
if os_name == 'Windows':
   cmd=["cmd", "/C"]
 cmd = ["bash", "-c"]
  prefix = "./"
def GetTestData(exe):
 myout=sp.check_output(cmd+["%s%s"%(prefix, exe)])
  return myout.decode("utf8")
def Judge(a,b,testdata):
 f = open("test.in", "w+")
 f.write(testdata)
 f.close()
  c=sp.check_output(cmd+["%s%s<test.in"%(prefix, a)])</pre>
  d=sp.check output(cmd+["%s%s<test.in"%(prefix, b)])
  if not c == d:
    print("answer: %s"%c.decode("utf8"), end="")
    print("output: %s"%d.decode("utf8"), end="")
    print("WA!")
    return False
 return True
if __name__ == '__main__':
  cnt = 0
  isOK = True
  while isOK:
    cnt += 1
    print(cnt)
    isOK=Judge("sol", "mysol", GetTestData("gen"))
```

#### 1.5 Quick Random

```
template < class T, T x1, T x2, T x3, int y1, int y2, int y3>
struct PRNG {
 using S = typename std::make signed<T>::type;
  PRNG(T s = 0) : s(s) {}
 T next() {
   T z = (s += x1);
   z = (z ^ (z >> y1)) * x2;

z = (z ^ (z >> y2)) * x3;
   return z ^ (z >> y3);
 T next(T n) { return next() % n; }
 S next(S 1, S r) { return l+next(r-l+1);}
 T operator()() { return next(); }
 T operator()(T n) { return next(n); }
 S operator()(S 1, S r) { return next(1, r); }
 template<class U>
 void shuffle(U first,U last){
    size_t n=last-first;
   for(size t i=0;i<n;i++)
     swap(first[i], first[next(i+1)]);
 }
};
using R32=PRNG<uint32 t,0x9E3779B1,0x85EBCA6B,
0xC2B2AE35,16,13,16>;
R32 r32;
using R64=PRNG<uint64 t,0x9E3779B97F4A7C15,
0xBF58476D1CE4E5B9,0x94D049BB133111EB,30,27,31>;
R64 r64;
```

# 1.6 IO Optimization

```
static inline int gc() {
   static char buf[ 1 << 20 ], *p = buf, *end = buf;
   if ( p == end ) {
      end = buf + fread( buf, 1, 1 << 20, stdin );
      if ( end == buf ) return EOF;
      p = buf;
   }
   return *p++;
}
template < typename T >
static inline bool gn( T &_ ) {
   register int c = gc(); register T __ = 1; _ = 0;
```

```
while(('0'>c||c>'9') && c!=EOF && c!='-') c = gc();
if(c == '-') { _ = -1; c = gc(); }
if(c == EOF) return false;
while('0'<=c&&c<='9') _ = _ * 10 + c - '0', c = gc();
    _ *= _ ;
return true;
}
template < typename T, typename ...Args >
static inline bool gn( T &x, Args &...args )
{ return gn(x) && gn(args...); }
```

# 2 Data Structure

# 2.1 Bigint

```
class BigInt{
    using lld = int fast64 t;
    #define PRINTF ARG PRIdFAST64
    #define LOG_BASE_STR "9"
    static constexpr lld BASE = 1000000000;
    static constexpr int LOG BASE = 9;
    vector<lld> dig;
    bool neg;
    inline int len() const { return (int) dig.size(); }
    inline int cmp_minus(const BigInt& a) const {
      if(len() == \overline{0} \&\& a.len() == 0) return 0;
      if (neg ^ a.neg) return (int) a.neg*2 - 1;
      if(len()!=a.len())
        return neg?a.len()-len():len()-a.len();
      for (int i=len()-1;i>=0;i--) if (dig[i]!=a.dig[i])
        return neg?a.dig[i]-dig[i]:dig[i]-a.dig[i];
      return 0;
    inline void trim() {
      while(!dig.empty()&&!dig.back())dig.pop_back();
      if(dig.empty()) neg = false;
  public:
    BigInt(): dig(vector<lld>()), neg(false){}
    BigInt(lld a): dig(vector<lld>()) {
      neg = a<0; dig.push_back(abs(a));</pre>
      trim();
    BigInt(const string& a): dig(vector<lld>()) {
      assert(!a.empty()); neg = (a[0]=='-');
      for (int i=((int)a.size())-1;i>=neg;i-=LOG BASE) {
        11d cur = 0;
        for(int j=min(LOG BASE-1,i-neg);j>=0;j--)
          cur = cur*10+a[\overline{i}-j]-'0';
        dig.push back(cur);
      } trim();
    inline bool operator<(const BigInt& a)const</pre>
      {return cmp minus(a)<0;}
    inline bool operator<=(const BigInt& a)const</pre>
      {return cmp_minus(a)<=0;}
    inline bool operator==(const BigInt& a)const
      {return cmp minus(a) == 0;}
    inline bool operator!=(const BigInt& a)const
      {return cmp_minus(a)!=0;}
    inline bool operator>(const BigInt& a)const
      {return cmp minus(a)>0;}
    inline bool operator>=(const BigInt& a)const
      {return cmp minus(a)>=0;}
    BigInt operator-() const {
      BigInt ret = *this;
      ret.neg ^= 1;
      return ret;
    BigInt operator+(const BigInt& a) const {
      if (neg) return -(-(*this)+(-a));
      if(a.neg) return (*this)-(-a);
      int n = max(a.len(), len());
      BigInt ret; ret.dig.resize(n);
      11d pro = 0;
      for (int i=0;i<n;i++) {</pre>
        ret.dig[i] = pro;
        if(i < a.len()) ret.dig[i] += a.dig[i];</pre>
        if(i < len()) ret.dig[i] += dig[i];</pre>
        pro = 0;
        if(ret.dig[i] >= BASE) pro = ret.dig[i]/BASE;
```

```
ret.dig[i] -= BASE*pro;
  if(pro != 0) ret.dig.push back(pro);
  return ret;
BigInt operator-(const BigInt& a) const {
  if(neg) return -(-(*this) - (-a));
  if(a.neg) return (*this) + (-a);
  int diff = cmp minus(a);
  if(diff < 0) return -(a - (*this));</pre>
  if(diff == 0) return 0;
  BigInt ret; ret.dig.resize(len(), 0);
  for (int i=0; i<len(); i++) {</pre>
    ret.dig[i] += dig[i];
    if(i < a.len()) ret.dig[i] -= a.dig[i];</pre>
    if(ret.dig[i] < 0){
      ret.dig[i] += BASE;
      ret.dig[i+1]--;
  ret.trim();
  return ret;
BigInt operator*(const BigInt& a) const {
  if(!len()||!a.len()) return 0;
  BigInt ret; ret.dig.resize(len()+a.len()+1);
  ret.neg = neg ^ a.neg;
  for (int i=0; i<len(); i++)</pre>
    for (int j=0; j<a.len(); j++) {</pre>
      ret.dig[i+j] += dig[i] * a.dig[j];
      if(ret.dig[i+j] >= BASE) {
        lld x = ret.dig[i+j] / BASE;
        ret.dig[i+j+1] += x;
        ret.dig[i+j] -= x * BASE;
  ret.trim();
  return ret;
BigInt operator/(const BigInt& a) const {
  assert(a.len());
  if(len() < a.len()) return 0;</pre>
  BigInt ret; ret.dig.resize(len()-a.len()+1);
  ret.neg = a.neg;
  for (int i=len()-a.len();i>=0;i--) {
    11d 1 = 0, r = BASE;
    while (r-1 > 1) {
      lld mid = (l+r) >> 1;
      ret.dig[i] = mid;
      if (ret*a<=(neg?-(*this):(*this))) l = mid;</pre>
      else r = mid;
    ret.dig[i] = 1;
  ret.neg ^= neg; ret.trim();
  return ret;
BigInt operator%(const BigInt& a) const {
  return (*this) - (*this) / a * a;
friend BigInt abs(BigInt a) {
  a.neg = 1; return a;
friend void swap(BigInt& a, BigInt& b) {
  swap(a.dig, b.dig); swap(a.neg, b.neg);
friend istream& operator>>(istream& ss, BigInt& a){
  string s; ss >> s; a = s;
  return ss;
friend ostream& operator<<(ostream& ss, BigInt& a) {</pre>
  if(a.len() == 0) return ss << '0';</pre>
  if(a.neg) ss << '-';
  ss << a.dig.back();
  for (int i=a.len()-2;i>=0;i--)
    ss<<setw(LOG BASE)<<setfill('0')<<a.dig[i];</pre>
  return ss;
inline void print() const {
  if(len() == 0) {putchar('0');return;}
  if(neg) putchar('-');
  printf("%" PRINTF ARG, dig.back());
  for (int i=len()-2;i>=0;i--)
    printf("%0" LOG_BASE_STR PRINTF_ARG, dig[i]);
#undef PRINTF ARG
```

```
#undef LOG_BASE_STR
};
```

# 2.2 Dark Magic

```
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
using __gnu_pbds::pairing_heap_tag;
using __gnu_pbds::binary_heap_tag;
using __gnu_pbds::binomial_heap_tag;
using __gnu_pbds::rc_binomial_heap_tag;
      gnu_pbds::thin_heap_tag;
using
{\tt template}{<}{\tt typename}\  \, \mathbb{T}{>}
using pbds_heap=__gnu_pbds::prioity_queue<T,less<T>, \
                                       pairing heap tag>;
using __gnu_pbds::rb_tree_tag;
using __gnu_pbds::ov_tree_tag;
       _gnu_pbds::splay_tree_tag;
using
template<typename T>
using ordered set =
                     __gnu_pbds::tree<T,\
 gnu pbds::null type,less<T>,rb tree tag,\
  gnu_pbds::tree_order statistics node update>;
template<typename A, typename B>
using hTable1=__gnu_pbds::cc_hash_table<A,B>;
template<typename A, typename B>
using hTable2=__gnu_pbds::gp_hash_table<A,B>;
int main(){
 ordered set<int> ss;
  ss.insert(1); ss.insert(5);
  assert(*ss.find_by_order(0)==1);
  assert(ss.order of key(-1) == 0);
  pbds heap pq1, pq2;
  pq1.push(1); pq2.push(2);
  pq1.join(pq2);
  assert(pq2.size() == 0);
  auto it = pq1.push(87);
  pq1.modify(it, 19);
  return 0:
```

# 2.3 SkewHeap

```
template < typename T, typename cmp = less< T > >
class SkewHeap{
private:
  struct SkewNode {
    T x;
    SkewNode *lc, *rc;
    SkewNode( T a = 0 ) : x(a), lc(0), rc(0) {}
  } *root:
  cmp CMP ;
  size t count;
  SkewNode* Merge ( SkewNode* a, SkewNode* b ) {
   if ( !a or !b ) return a ? a : b;
    if ( CMP_( a->x, b->x ) ) swap( a, b );
    a -> rc = Merge( a->rc, b );
    swap(a \rightarrow lc, a\rightarrow rc);
    return a;
public:
  SkewHeap(): root( 0 ), count( 0 ) {}
  size t size() { return count; }
  bool empty() { return count == 0; }
  T top() { return root->x; }
  void clear() { root = 0; count = 0; }
  void push ( const T& x ) {
    SkewNode* a = new SkewNode(x);
    count += 1; root = Merge( root, a );
  void join( SkewHeap& a ) {
    count += a.count; a.count = 0;
    root = Merge( root, a.root );
  void pop() {
    count--; root = Merge( root->lc, root->rc );
  friend void swap( SkewHeap& a, SkewHeap& b ) {
    swap( a.root, b.root ); swap( a.count, b.count );
};
```

# 2.4 Disjoint Set

```
class DJS{
private:
  vector< int > fa, sz, sv;
  vector< pair< int*, int > > opt;
inline void assign( int *k, int v ) {
     opt.emplace_back( k, *k );
     *k = v;
public:
  inline void init( int n ) {
     fa.resize( n ); iota( fa.begin(), fa.end(), 0 );
     sz.resize( n ); fill( sz.begin(), sz.end(), 1 );
    opt.clear();
  int query( int x ) {
    return ( fa[ x ] == x ) ? x : query( fa[ x ] );
  inline void merge( int a, int b ) {
    int af = query( a ), bf = query( b );
     if( af == bf ) return;
    if( sz[ af ] < sz[ bf ] ) swap( af, bf );</pre>
    assign( &fa[ bf ], fa[ af ] );
assign( &sz[ af ], sz[ af ] + sz[ bf ] );
  inline void save() {sv.push back( (int)opt.size() );}
  inline void undo() {
    int ls = sv.back(); sv.pop back();
     while ( ( int ) opt.size() > ls ) {
       pair< int*, int > cur = opt.back();
*cur.first = cur.second;
       opt.pop_back();
  }
};
```

# 2.5 Treap

```
namespace Treap{
 #define sz(x)((x)?((x)->size):0)
 struct node{
   int size;
   uint32_t pri;
   node *lc, *rc;
   node() : size(0), pri(rand()), lc( 0 ), rc( 0 ) {}
   void pull() {
     size = 1;
      if ( lc ) size += lc->size;
      if ( rc ) size += rc->size;
 } :
 node* merge( node* L, node* R ) {
   if ( not {\tt L} or not {\tt R} ) return {\tt L} ? {\tt L} : R;
    if ( L->pri > R->pri ) {
     L->rc = merge(L->rc, R);
      L->pull();
     return L;
   } else {
     R->lc = merge(L, R->lc);
      R->pull();
      return R:
 void split_by_size( node*rt,int k,node*&L,node*&R ) {
   if ( not rt ) L = R = nullptr;
   else if( sz( rt->lc ) + 1 <= k ) {
     L = rt;
      split by size( rt->rc, k-sz(rt->lc)-1, L->rc, R );
      L->pull();
    } else {
      R = rt;
      split by size( rt->lc, k, L, R->lc );
      R->pull();
   }
  #undef sz
```

# 2.6 SparseTable

```
template < typename T, typename Cmp = less< T > >
class SparseTable {
private:
  vector< vector< T > > tbl;
   vector< int > lg;
  T cv(Ta, Tb) {
    return Cmp_()( a, b ) ? a : b;
public:
  void init( T arr[], int n ) {
     // 0-base
     lg.resize(n + 1);
     lg[0] = -1;
     for( int i=1 ; i<=n ; ++i ) lg[i] = lg[i>>1] + 1;
     tbl.resize(lg[n] + 1);
     tbl[ 0 ].resize( n );
     copy( arr, arr + n, tbl[ 0 ].begin() );
     for ( int i = 1 ; i <= lg[ n ] ; ++ i ) {
  int len = 1 << ( i - 1 ), sz = 1 << i;</pre>
       tbl[i].resize(n - sz + 1);
       for ( int j = 0 ; j <= n - sz ; ++ j )</pre>
         tbl[i][j] = cv(tbl[i-1][j], tbl[i-1][j+len]);
    }
  T query( int 1, int r ) {
     // 0-base [1, r)
     int wh = lg[ r - l ], len = 1 << wh;</pre>
     return cv( tbl[ wh ][ l ], tbl[ wh ][ r - len ] );
};
```

#### 2.7 Linear Basis

```
struct LinearBasis {
private:
  int n, sz;
  vector< llu > B;
  inline llu two( int x ){ return ( ( llu ) 1 ) << x; }</pre>
public:
  void init( int n_ ) {
   n = n_; B.clear();
    B.resize( n ); sz = 0;
  void insert( llu x ) {
    // add x into B
    for ( int i = n-1; i >= 0 ; --i ) if( two(i) & x ) {
      if (B[i]) x ^= B[i];
      else 
        B[i] = x; sz++;
        for ( int j = i - 1 ; j >= 0 ; -- j )
          if(B[j] && (two(j) & B[i]))
           B[ i ] ^= B[ j ];
        for (int j = i + 1 ; j < n ; ++ j )</pre>
          if ( two( i ) & B[ j ] )
            B[ j ] ^= B[ i ];
        break;
      }
    }
  inline int size() { return sz; }
  bool check( llu x ) {
    // is x in span(B) ?
    for ( int i = n-1 ; i >= 0 ; --i ) if( two(i) & x )
      if( B[ i ] ) x ^= B[ i ];
      else return false;
    return true;
  llu kth_small(llu k) {
    /** 1-base would always > 0 **/
    /** should check it **/
    /* if we choose at least one element
      but size(B) (vectors in B) == N(original elements)
       then we can't get 0 */
    llu ret = 0;
    for ( int i = 0 ; i < n ; ++ i ) if( B[ i ] ) {</pre>
     if( k & 1 ) ret ^= B[ i ];
      k >>= 1;
    return ret;
} base;
```

# 3 Graph

# 3.1 BCC Edge

```
class BCC{
private:
 vector< int > low, dfn;
  int cnt;
  vector< bool > bcc;
 vector< vector< PII > > G;
 void dfs( int w, int f ) {
   dfn[w] = cnt++;
   low[w] = dfn[w];
    for ( auto [ u, t ] : G[ w ] ) {
     if ( u == f ) continue;
      if ( dfn[ u ] != 0 ) {
       low[ w ] = min( low[ w ], dfn[ u ] );
      }else{
       dfs( u, w );
       low[ w ] = min( low[ w ], low[ u ] );
       if ( low[ u ] > dfn[ w ] ) bcc[ t ] = true;
   }
public:
 void init( int n, int m ) {
   G.resize(n);
   fill( G.begin(), G.end(), vector< PII >() );
    bcc.clear(); bcc.resize( m );
   low.clear(); low.resize( n );
   dfn.clear(); dfn.resize( n );
   cnt = 0:
 void add edge( int u, int v ) {
    // should check for multiple edge
   G[ u ].emplace_back( v, cnt );
   G[ v ].emplace_back( u, cnt ++ );
 void solve() { cnt = 1; dfs( 0, 0 ); }
  // the id will be same as insert order, 0-base
 bool is bcc( int x ) { return bcc[ x ]; }
} bcc:
```

#### 3.2 BCC Vertex

```
class BCC{
 private:
    int n, ecnt;
    vector< vector< pair< int, int > > > G;
    vector< int > low, dfn, id;
    vector< bool > vis, ap;
    void dfs( int u, int f, int d ) {
      int child = 0;
      dfn[ u ] = low[ u ] = d; vis[ u ] = true;
      for ( auto e : G[ u ] ) if ( e.first != f ) {
        if ( vis[ e.first ] ) {
          low[ u ] = min( low[ u ], dfn[ e.first ] );
        } else {
          dfs( e.first, u, d + 1 ); child ++;
          low[ u ] = min( low[ u ], low[ e.first ] );
          if ( low[ e.first ] >= d ) ap[ u ] = true;
      if ( u == f and child <= 1 ) ap[ u ] = false;</pre>
    void mark( int u, int idd ) {
      // really??????????
      if (ap[u]) return;
      for ( auto e : G[ u ] )
       if( id[ e.second ] != -1 ) {
          id[ e.second ] = idd;
          mark( e.first, idd );
 public:
    void init( int n_ ) {
      ecnt = 0, n = \overline{n};
      G.clear(); G.resize( n );
      low.resize( n ); dfn.resize( n );
      ap.clear(); ap.resize( n );
      vis.clear(); vis.resize( n );
```

```
void add edge( int u, int v ) {
  G[ u ].emplace_back( v, ecnt );
  G[ v ].emplace back( u, ecnt ++ );
void solve() {
  for ( int i = 0 ; i < n ; ++ i )</pre>
    if ( not vis[ i ] ) dfs( i, i, 0 );
  id.resize( ecnt );
  fill( id.begin(), id.end(), -1 );
  ecnt = 0;
  for ( int i = 0 ; i < n ; ++ i )
  if ( ap[ i ] ) for ( auto e : G[ i ] )</pre>
      if( id[ e.second ] != -1 ) {
        id[ e.second ] = ecnt;
         mark( e.first, ecnt ++ );
int get_id( int x ) { return id[ x ]; }
int count() { return ecnt; }
bool is ap( int u ) { return ap[ u ]; }
```

# 3.3 Bipartite Matching

```
class BipartiteMatching{
  private:
    vector<int> X[N], Y[N];
    int fX[N], fY[N], n;
    bitset<N> walked;
    bool dfs(int x) {
      for(auto i:X[x]){
        if (walked[i]) continue;
        walked[i]=1;
        if(fY[i] ==-1||dfs(fY[i])){
          fY[i]=x; fX[x]=i;
          return 1;
        }
      return 0;
  public:
    void init(int n) {
      n=n;
      for (int i=0;i<n;i++) {</pre>
        X[i].clear();
        Y[i].clear();
        fX[i]=fY[i]=-1;
      walked.reset();
    void add edge(int x, int y) {
      X[x].push_back(y);
      Y[y].push back(y);
    int solve(){
      int cnt = 0;
      for (int i=0;i<n;i++) {</pre>
        walked.reset();
        if(dfs(i)) cnt++;
      // return how many pair matched
      return cnt;
};
```

## 3.4 Minimum Cost Maximum Flow

```
class MiniCostMaxiFlow{
   using CapT = int;
   using WeiT = int64_t;
   using PCW = pair<CapT,WeiT>;
   static constexpr CapT INF_CAP = 1 << 30;
   static constexpr WeiT INF_WEI = 1LL<<60;
   private:
   struct Edge{
    int to, back;
    WeiT wei;
    CapT cap;
   Edge() {}
   Edge(int a,int b,WeiT c,CapT d):
        to(a),back(b),wei(c),cap(d)
   {}
}</pre>
```

```
int ori, edd:
  vector<vector<Edge>> G;
  vector<int> fa, wh;
  vector<bool> inq;
  vector<WeiT> dis;
  PCW SPFA() {
    fill(inq.begin(),inq.end(),false);
    fill(dis.begin(), dis.end(), INF WEI);
    queue<int> qq; qq.push(ori);
    dis[ori]=0;
    while(!qq.empty()){
      int u=qq.front();qq.pop();
inq[u] = 0;
      for (int i=0; i < SZ(G[u]); ++i) {</pre>
        Edge e=G[u][i];
        int v=e.to;
        WeiT d=e.wei;
        if (e.cap<=0||dis[v]<=dis[u]+d)
          continue;
        dis[v]=dis[u]+d;
        fa[v]=u, wh[v]=i;
        if(inq[v]) continue;
        qq.push(v);
        inq[v]=1;
      }
    if(dis[edd] == INF WEI)
      return {-1,-1};
    CapT mw=INF CAP;
    for (int i=edd; i!=ori; i=fa[i])
     mw=min(mw,G[fa[i]][wh[i]].cap);
    for (int i=edd;i!=ori;i=fa[i]) {
      auto &eg=G[fa[i]][wh[i]];
      eq.cap-=mw;
      G[eg.to][eg.back].cap+=mw;
    return {mw,dis[edd]};
public:
  void init(int a,int b,int n) {
   ori=a,edd=b;
    G.clear(); G.resize(n);
    fa.resize(n); wh.resize(n);
    inq.resize(n); dis.resize(n);
 void add edge(int st,int ed,WeiT w,CapT c) {
   G[st].emplace_back(ed,SZ(G[ed]),w,c);
    G[ed].emplace_back(st,SZ(G[st])-1,-w,0);
 PCW solve(){
   CapT cc=0; WeiT ww=0;
    while(true){
     PCW ret=SPFA();
      if(ret.first==-1) break;
      cc+=ret.first;
      ww+=ret.second;
    return {cc,ww};
} mcmf;
```

### 3.5 MaximumFlow

```
class Dinic{
private:
 using CapT = int64 t;
 struct Edge{
    int to, rev;
   CapT cap;
 int n, st, ed;
 vector<vector<Edge>> G;
 vector<int> lv;
 bool BFS() {
   fill(lv.begin(), lv.end(), -1);
    queue<int> bfs;
   bfs.push(st);
    lv[st] = 0;
    while(!bfs.empty()){
      int u = bfs.front(); bfs.pop();
      for(auto e: G[u]){
        if(e.cap <= 0 or lv[e.to]!=-1) continue;</pre>
        lv[e.to] = lv[u] + 1;
```

```
bfs.push(e.to);
    return (lv[ed]!=-1);
  CapT DFS(int u, CapT f) {
    if(u == ed) return f;
    CapT ret = 0;
    for(auto& e: G[u]) {
      if(e.cap <= 0 or lv[e.to]!=lv[u]+1) continue;</pre>
      CapT nf = DFS(e.to, min(f, e.cap));
      ret += nf; e.cap -= nf; f -= nf;
      G[e.to][e.rev].cap += nf;
      if(f == 0) return ret;
    if (ret == 0) lv[u] = -1;
    return ret;
public:
  void init(int n , int st , int ed ) {
    n = n_, st = st_, ed = ed_;
    G.resize(n); lv.resize(n);
    fill(G.begin(), G.end(), vector<Edge>());
  void add edge(int u, int v, CapT c) {
    G[u].push_back({v, (int)G[v].size(), c});
G[v].push_back({u, ((int)G[u].size())-1, 0});
  CapT max flow() {
    CapT ret = 0;
    while (BFS()) {
      CapT f = DFS(st, numeric limits<CapT>::max());
      ret += f;
      if(f == 0) break;
    return ret;
} flow;
```

#### 3.6 Kuhn Munkres

```
struct KM{
  static constexpr lld INF = 1LL<<60;</pre>
  lld w[N][N], lx[N], ly[N], slack[N];
  int match[N], n, vx[N], vy[N], step;
  void init(int n_) {
    n=n_,step=0;
    memset(w,0,sizeof(w));
    memset(lx,0,sizeof(lx));
    memset(ly,0,sizeof(ly));
    memset(slack, 0, sizeof(slack));
    memset(match, 0, sizeof(match));
    memset(vx,0,sizeof(vx));
    memset(vy,0,sizeof(vy));
  void add edge(int u,int v,lld w) {w[u][v]=w;}
  bool dfs(int x) {
    vx[x] = step_;
for (int i = 0; i < n; ++i) {</pre>
      if (vy[i]==step_) continue;
if (lx[x] + ly[i] > w[x][i]) {
         slack[i] = min(slack[i], lx[x] + ly[i] - w[x][i]
             1);
         continue;
       vy[i] = step ;
      if (match[i] == -1 || dfs(match[i])) {
        match[i] = x;
         return true;
      }
    return false;
  11d solve() {
    fill n(match, n, -1);
    fill_n(lx, n, -INF);
fill_n(ly, n, 0);
    for (int i = 0; i < n; ++i)</pre>
       for (int j = 0; j < n; ++j)</pre>
        lx[i] = max(lx[i], w[i][j]);
     for (int i = 0; i < n; ++i) {</pre>
      fill n(slack, n, INF);
       while (true) {
         step_++;
```

```
if (dfs(i)) break;
    lld dlt = INF;
    for (int j = 0; j < n; ++j) if (vy[j] != step_)
        dlt = min(dlt, slack[j]);
    for (int j = 0; j < n; ++j) {
        if (vx[j]==step_) lx[j] -= dlt;
        if (vy[j]==step_) ly[j] += dlt;
        else slack[j] -= dlt;
    }
}
lld res = 0;
for (int i = 0; i < n; ++i) res += w[match[i]][i];
    return res;
}
km;</pre>
```

#### 3.7 2-SAT

```
class TwoSat{
 private:
   int n;
    vector<vector<int>> rG,G,sccs;
    vector<int> ord,idx;
    vector<bool> vis,result;
    void dfs(int u) {
      vis[u]=true;
      for(int v:G[u])
        if(!vis[v])
          dfs(v);
      ord.push back(u);
    void rdfs(int u) {
      vis[u]=false;
      idx[u] = sccs.size() -1;
      sccs.back().push back(u);
      for(int v:rG[u])
        if (vis[v])
          rdfs(v);
 public:
    void init(int n_) {
      n=n_;
      G.clear();
      G.resize(n);
      rG.clear();
      rG.resize(n):
      sccs.clear();
      ord.clear();
      idx.resize(n);
      result.resize(n);
    void add edge(int u,int v) {
      G[u].push back(v);
      rG[v].push_back(u);
    void orr(int x,int v) {
      if ((x^y) ==1) return;
      add edge (x^1, y);
      add edge (y^1, x);
    bool solve() {
      vis.clear();
      vis.resize(n);
      for (int i=0; i<n; ++i)</pre>
        if(not vis[i])
          dfs(i);
      reverse(ord.begin(),ord.end());
      for (int u:ord) {
        if(!vis[u])
          continue;
        sccs.push back(vector<int>());
        rdfs(u);
      for (int i=0; i<n; i+=2)</pre>
        if(idx[i]==idx[i+1])
          return false;
      vector<bool> c(sccs.size());
      for(size_t i=0;i<sccs.size();++i){</pre>
        for(size t j=0;j<sccs[i].size();++j){</pre>
          result[sccs[i][j]]=c[i];
          c[idx[sccs[i][j]^1]]=!c[i];
        }
      }
```

```
return true;
}
bool get(int x) {return result[x];}
inline int get_id(int x) {return idx[x];}
inline int count() {return sccs.size();}
} sat2;
```

# 3.8 Lowbit Decomposition

```
class LowbitDecomp{
private:
  int time_, chain_, LOG_N;
  vector< vector< int > > G, fa;
  vector< int > tl, tr, chain, chain st;
  // chain : number of chain
  // tl, \overline{\operatorname{tr}}[u] : subtree interval in the seq. of u
  // chain_st[ u ] : head of the chain contains u
  // chian[ u ] : chain id of the chain u is on
  inline int lowbit( int x ) {
    return x & ( -x );
  void predfs( int u, int f ) {
    chain[u] = 0;
    for ( int v : G[ u ] ) {
      if ( v == f ) continue;
      predfs( v, u );
      if( lowbit( chain[ u ] ) < lowbit( chain[ v ] ) )</pre>
        chain[ u ] = chain[ v ];
    if ( not chain[ u ] )
      chain[ u ] = chain ++;
  void dfschain( int u, int f ) {
    fa[ u ][ 0 ] = f;
    for ( int i = 1 ; i < LOG N ; ++ i )</pre>
      fa[u][i] = fa[fa[\overline{u}][i-1]][i-1];
    tl[u] = time ++;
    if ( not chain_st[ chain[ u ] ] )
      chain st[ chain[ u ] ] = u;
    for ( int v : G[ u ] )
      if ( v != f and chain[ v ] == chain[ u ] )
       dfschain( v, u );
    for ( int v : G[ u ] )
      if ( v != f and chain[ v ] != chain[ u ] )
        dfschain( v, u );
    tr[u] = time;
  inline bool anc( int u, int v ) {
    return tl[ u ] <= tl[ v ]</pre>
      and tr[ v ] <= tr[ u ];</pre>
public:
  inline int lca( int u, int v ) {
    if ( anc( u, v ) ) return u;
    for ( int i = LOG_N - 1 ; i >= 0 ; -- i )
      if ( not anc( fa[ u ][ i ], v ) )
       u = fa[ u ][ i ];
    return fa[ u ][ 0 ];
  void init( int n ) {
    n ++;
    for ( LOG N = 0 ; ( 1 << LOG N ) < n ; ++ LOG_N );</pre>
    fa.clear();
    fa.resize( n, vector< int > ( LOG N ) );
    G.clear(); G.resize( n );
    tl.clear(); tl.resize( n );
    tr.clear(); tr.resize( n );
    chain.clear(); chain.resize( n );
    chain st.clear(); chain st.resize( n );
  void add edge( int u , int v ) {
    // 1-base
    G[ u ].push_back( v );
    G[ v ].push back( u );
  void decompose() {
   chain = 1;
    predfs(1, 1);
    time_ = 0;
    dfschain(1,1);
  PII get inter( int u ) { return {tl[ u ], tr[ u ]}; }
  vector< PII > get_path( int u , int v ) {
  vector< PII > res;
```

```
int g = lca( u, v );
while ( chain[ u ] != chain[ g ] ) {
      int s = chain st[ chain[ u ] ];
      res.emplace back( tl[ s ], tl[ u ] + 1 );
      u = fa[s][0];
    res.emplace\_back(tl[g],tl[u]+1);
    while ( chain[ v ] != chain[ g ] ) {
  int s = chain st[ chain[ v ] ];
      res.emplace back( tl[ s ], tl[ v ] + 1 );
      v = fa[s][0];
    res.emplace back( tl[g] + 1, tl[v] + 1);
    return res;
    /* res : list of intervals from u to v
     * ( note only nodes work, not edge )
     * usage :
     * vector< PII >& path = tree.get_path( u , v )
* for( auto [ l, r ] : path ) {
         0-base [ 1, r )
} tree;
```

# 3.9 MaxClique

```
struct MaxClique {
    int n, deg[maxn], ans;
    bitset<maxn> adj[maxn];
    vector<pair<int, int>> edge;
    void init(int n) {
        n = _n;
        for (int i = 0; i < n; ++i) adj[i].reset();</pre>
        for (int i = 0; i < n; ++i) deg[i] = 0;</pre>
        edge.clear();
    void add edge(int a, int b) {
        edge.emplace_back(a, b);
        ++deg[a]; ++deg[b];
    int solve() {
        vector<int> ord;
        for (int i = 0; i < n; ++i) ord.push_back(i);</pre>
        sort(ord.begin(), ord.end(), [&](const int &a,
             const int &b) { return deg[a] < deg[b]; });</pre>
        vector<int> id(n);
        for (int i = 0; i < n; ++i) id[ord[i]] = i;</pre>
         for (auto e : edge) {
             int u = id[e.first], v = id[e.second];
             adj[u][v] = adj[v][u] = true;
        bitset<maxn> r, p;
        for (int i = 0; i < n; ++i) p[i] = true;</pre>
        ans = 0;
        dfs(r, p);
        return ans;
    void dfs(bitset<maxn> r, bitset<maxn> p) {
        if (p.count() == 0) return ans = max(ans, (int)
             r.count()), void();
        if ((r | p).count() <= ans) return;</pre>
        int now = p._Find_first();
        bitset<maxn> cur = p & ~adj[now];
        for (now = cur. Find first(); now < n; now =</pre>
             cur. Find next(now)) {
             r[now] = \overline{true};
             dfs(r, p & adj[now]);
             r[now] = false;
            p[now] = false;
    }
};
```

# 4 Math

#### 4.1 Prime Table

```
1002939109, 1020288887, 1028798297, 1038684299, \\ 1041211027, 1051762951, 1058585963, 1063020809, \\ 1147930723, 1172520109, 1183835981, 1187659051, \\
```

```
\begin{array}{c} 1241251303, 1247184097, 1255940849, 1272759031, \\ 1287027493, 1288511629, 1294632499, 1312650799, \\ 1868732623, 1884198443, 1884616807, 1885059541, \\ 1909942399, 1914471137, 1923951707, 1925453197, \\ 1979612177, 1980446837, 1989761941, 2007826547, \\ 2008033571, 2011186739, 2039465081, 2039728567, \\ 2093735719, 21116097521, 2123852629, 2140170259, \\ 3148478261, 3153064147, 3176351071, 3187523093, \\ 3196772239, 3201312913, 3203063977, 3204840059, \\ 3210224309, 3213032591, 3217689851, 3218469083, \\ 3219857533, 3231880427, 3235951699, 3273767923, \\ 3276188869, 3277183181, 3282463507, 3285553889, \\ 3319309027, 3327005333, 3327574903, 3341387953, \\ 3373293941, 3380077549, 3380892997, 3381118801 \end{array}
```

# 4.2 $\left| \frac{n}{s} \right|$ Enumeration

```
T_0 = 1, T_{i+1} = \lfloor \frac{n}{\lfloor \frac{n}{T_i + 1} \rfloor} \rfloor
```

# 4.3 ax+by=gcd

```
// ax+ny = 1, ax+ny == ax == 1 (mod n)
void exgcd(lld x,lld y,lld &g,lld &a,lld &b) {
  if (y == 0) g=x,a=1,b=0;
  else
    exgcd(y,x%y,g,b,a),b==(x/y)*a;
}
```

#### 4.4 Pollard Rho

```
// does not work when n is prime
// return any non-trivial factor
llu pollard rho(llu n) {
  static auto f=[&](llu x,llu k){
    return add(k,mul(x,x,n),n);
  if (!(n&1)) return 2;
  mt19937 rnd(120821011);
  while(true){
    llu y=2, yy=y, x=rnd()%n, t=1;
    for(llu sz=2;t==1;sz<<=1) {</pre>
      for(llu i=0;i<sz;++i){</pre>
        if(t!=1)break;
        yy=f(yy,x);
        t=gcd(yy>y?yy-y:y-yy,n);
      }
      y=yy;
    if(t!=1&&t!=n) return t;
```

# 4.5 Pi Count (Linear Sieve)

```
static constexpr int N = 1000000 + 5;
lld pi[N];
vector<int> primes;
bool sieved[N];
lld cube_root(lld x) {
  11d s=cbrt(x-static cast<long double>(0.1));
  while(s*s*s <= x) ++s;
  return s-1;
lld square_root(lld x){
  11d s=sqrt(x-static_cast<long double>(0.1));
  while (s*s <= x) ++s;
  return s-1;
void init() {
  primes.reserve(N);
  primes.push back(1);
  for (int i=2;i<N;i++) {</pre>
    if(!sieved[i]) primes.push_back(i);
    pi[i] = !sieved[i] + pi[i-1];
for(int p: primes) if(p > 1) {
      if (p * i >= N) break;
sieved[p * i] = true;
       if(p % i == 0) break;
```

```
lld phi(lld m, lld n) {
 static constexpr int MM = 80000, NN = 500;
  static lld val[MM][NN];
 if (m<MM&&n<NN&&val[m][n])return val[m][n]-1;</pre>
 if(n == 0) return m;
 if(primes[n] >= m) return 1;
 lld ret = phi(m,n-1)-phi(m/primes[n],n-1);
 if (m<MM&&n<NN) val[m][n] = ret+1;
 return ret;
lld pi count(lld);
lld P2(lld m, lld n) \{
  11d sm = square root(m), ret = 0;
  for(lld i = n+1;primes[i] <= sm; i++)</pre>
   ret+=pi count(m/primes[i])-pi count(primes[i])+1;
  return ret;
lld pi_count(lld m) {
 if (m < N) return pi[m];</pre>
  lld n = pi count(cube_root(m));
 return phi(m, n) + n - 1 - P2(m, n);
```

# 4.6 NloglogN Sieve

```
void Sieve(int n) {
  for(int i=2;i<=n;i++) {
    if(notprime[i]) continue;
    primes.push_back(i);
  for(int j=i*i;j<=n;j+=i) notprime[i]=true;
  }
}</pre>
```

# 4.7 Range Sieve

# 4.8 Miller Rabin

```
bool isprime(llu x) {
  static llu magic[]={2,325,9375,28178,\
                     450775,9780504,1795265022};
  static auto witn=[](llu a,llu u,llu n,int t){
    a = mpow(a,u,n);
    if (!a)return 0;
    while(t--){
      llu a2=mul(a,a,n);
      if(a2==1 && a!=1 && a!=n-1)
        return 1;
      a = a2;
    return a!=1;
  } :
  if(x<2)return 0;</pre>
  if(!(x&1))return x==2;
  llu x1=x-1; int t=0;
  while (!(x1&1))x1>>=1,t++;
  for(llu m:magic)
    if (with (m, x1, x, t))
      return 0;
  return 1;
```

#### 4.9 Inverse Element

```
// x's inverse mod k
long long GetInv(long long x, long long k) {
    // k is prime: euler_(k) = k-1
    return qPow(x, euler_phi(k)-1);
}
// if you need [1, x] (most use: [1, k-1]
void solve(int x, long long k) {
    inv[1] = 1;
    for(int i=2;i<x;i++)
        inv[i] = ((long long)(k - k/i) * inv[k % i]) % k;
}</pre>
```

#### 4.10 Euler Phi Function

```
extended euler:
   a^b mod p
   if gcd(a, p) ==1: a^(b%phi(p))
   elif b < phi(p): a^b mod p
   else a^(b%phi(p) + phi(p))
lld euler_phi(int x) {
  11d r=1;
  for (int i=2;i*i<=x;++i) {</pre>
    if (x%i==0) {
      x/=i:
      r*=(i-1);
      while (x%i==0) {
        x/=i;
        r*=i;
      }
    }
  if (x>1) r*=x-1;
  return r;
vector<int> primes;
bool notprime[N];
lld phi[N];
void euler_sieve(int n) {
  for (int i=2;i<n;i++) {</pre>
    if(!notprime[i]){
      primes.push back(i);
      phi[i] = i - \overline{1};
    for(auto j: primes) {
      if(i*j >= n) break;
      notprime[i*j] = true;
      phi[i*j] = phi[i] * phi[j];
      if(i % j == 0){
        phi[i*j] = phi[i] * j;
        break;
    }
  }
```

#### 4.11 Gauss Elimination

```
typedef long double llf;
const int N = 300;
const llf EPS = 1e-8;
// make m[i][i] = x, m[i][j] = 0
// v is for solving equation:
// for(int i=0;i < n;i++) ans[pos[i]] = val[i]/mtx[i][pos
    [i]];
// for(int i=0;i<n;i++) cout << ans[i] << '\n';
bool Gauss(llf m[N][N], llf v[N], int n, int pos[N]) {
  for (int i=0;i<n;i++) {</pre>
    int x=-1, y=-1; llf e = 0;
    for (int j=i; j<n; j++) for (int k=i; k<n; k++) {</pre>
      if (fabs(m[j][pos[k]])>e) {
        e = fabs(m[j][pos[k]]);
        x = j, y = k;
      }
    if(x==-1 or y==-1) return false;
```

```
swap(m[x], m[i]);
swap(v[x], v[i]);
swap(pos[y], pos[i]);
for(int j=i+1;j<n;j++) {
    llf xi = m[j][pos[i]]/m[i][pos[i]];
    for(int k=0;k<n;k++) m[j][pos[k]] -= xi*m[i][pos[k]];
    v[j] -= xi*v[i];
}

for(int i=n-1;i>=0;i--) {
    for(int j=i-1;j>=0;j--) {
        llf xi = m[j][pos[i]]/m[i][pos[i]];
        for(int k=0;k<n;k++) m[j][pos[k]] -= xi*m[i][pos[k]];
        v[j] -= xi*v[i];
    }

return true;
}</pre>
```

### 4.12 Fast Fourier Transform

```
polynomial multiply:
   DFT(a, len); DFT(b, len);
   for (int i=0; i<len; i++) c[i] = a[i]*b[i];
   iDFT(c, len);
   (len must be 2^k and = 2^m (\max(a, b)))
   Hand written Cplx would be 2x faster
Cplx omega[2][N];
void init omega(int n) {
  static constexpr llf PI=acos(-1);
  const llf arg=(PI+PI)/n;
  for (int i=0; i<n; ++i)</pre>
    omega[0][i]={cos(arg*i),sin(arg*i)};
  for (int i=0;i<n;++i)</pre>
    omega[1][i]=conj(omega[0][i]);
void tran(Cplx arr[],int n,Cplx omg[]) {
  for (int i=0, j=0; i<n; ++i) {</pre>
    if(i>j)swap(arr[i],arr[j]);
    for (int l=n>>1; (j^=1) <1; l>>=1);
  for (int l=2; l<=n; l<<=1) {</pre>
    int m=1>>1;
    for (auto p=arr;p!=arr+n;p+=1) {
      for (int i=0;i<m;++i) {</pre>
        Cplx t=omg[n/l*i]*p[m+i];
        p[m+i]=p[i]-t;
        p[i]+=t;
      }
    }
  }
void DFT(Cplx arr[],int n){
  tran(arr,n,omega[0]);
void iDFT(Cplx arr[],int n){
 tran(arr,n,omega[1]);
  for (int i=0; i < n; ++i) arr[i] /=n;</pre>
```

#### 4.13 Chinese Remainder

```
lld crt(lld ans[], lld pri[], int n) {
    lld M = 1;
    for(int i=0;i<n;i++) M *= pri[i];
    lld ret = 0;
    for(int i=0;i<n;i++) {
        lld inv = (gcd(M/pri[i], pri[i]).first + pri[i]) %
            pri[i];
        ret += (ans[i]*(M/pri[i])%M * inv)%M;
        ret %= M;
    }
    return ret;
}
/*
Another:
x = a1 % m1
x = a2 % m2</pre>
```

```
\begin{array}{ll} g = \gcd(\text{m1, m2}) \\ assert((a1-a2) \, {}^*\!\!\! g == 0) \\ [p, \, q] = \exp(\text{cd}(\text{m2/g, m1/g}) \\ return \, a2 + m2 \, {}^*\!\!\! (p \, {}^*\!\!\! (a1-a2) \, {}^*\!\!\! /g) \\ 0 <= x < 1 \, {}^*\!\!\! cm(\text{m1, m2}) \\ */ \end{array}
```

# 4.14 Berlekamp Massey

```
// x: 1-base, p[]: 0-base
template<size t N>
vector<llf> BM(llf x[N], size t n) {
  size t f[N] = \{0\}, t=0; llf d[\overline{N}];
  vector<llf> p[N];
  for (size t i=1,b=0;i<=n;++i) {</pre>
    for(size_t j=0;j<p[t].size();++j)</pre>
       d[i] += x[i-j-1] *p[t][j];
     if (abs (d[i] -=x[i]) <=EPS) continue;</pre>
    f[t]=i;if(!t) {p[++t].resize(i);continue;}
    vector<llf> cur(i-f[b]-1);
    llf k=-d[i]/d[f[b]]; cur.PB(-k);
    for (size_t j=0;j<p[b].size();j++)</pre>
      cur.PB(p[b][j]*k);
    if(cur.size() < p[t].size()) cur.resize(p[t].size());</pre>
    for(size t j=0;j<p[t].size();j++)cur[j]+=p[t][j];</pre>
    if(i-f[b]+p[b].size()>=p[t].size()) b=t;
    p[++t]=cur;
  return p[t];
```

#### 4.15 NTT

```
Remember coefficient are mod P
/* p=a*2^n+1
      2^n
65536
  n
                                      root
                                 а
                     65537
   16
                                 7
                                      3 */
   20 1048576
                     7340033
// (must be 2^k)
template<LL P, LL root, int MAXN>
struct NTT{
  static LL bigmod(LL a, LL b) {
    LL res = 1;
    for (LL bs = a; b; b >>= 1, bs = (bs * bs) % P)
     if(b&1) res=(res*bs)%P;
    return res;
  static LL inv(LL a, LL b) {
    if(a==1)return 1;
    return (((LL)(a-inv(b%a,a))*b+1)/a)%b;
  LL omega[MAXN+1];
  NTT() {
    omega[0] = 1;
    LL r = bigmod(root, (P-1)/MAXN);
    for (int i=1; i<=MAXN; i++)</pre>
      omega[i] = (omega[i-1]*r)%P;
  // n must be 2^k
  void tran(int n, LL a[], bool inv_ntt=false){
    int basic = MAXN / n , theta = basic;
    for (int m = n; m >= 2; m >>= 1) {
      int mh = m >> 1;
      for (int i = 0; i < mh; i++) {</pre>
        LL w = omega[i*theta%MAXN];
        for (int j = i; j < n; j += m) {</pre>
          int k = j + mh;
          LL x = a[j] - a[k];
          if (x < 0) x += P;
          a[j] += a[k];
          if (a[j] > P) a[j] -= P;
          a[k] = (w * x) % P;
      theta = (theta * 2) % MAXN;
    for (int j = 1; j < n - 1; j++) {
  for (int k = n >> 1; k > (i ^= k); k >>= 1);
      if (j < i) swap(a[i], a[j]);</pre>
    if (inv_ntt) {
```

```
LL ni = inv(n,P);
    reverse( a+1 , a+n );
    for (i = 0; i < n; i++)
        a[i] = (a[i] * ni) % P;
}
};
const LL P=2013265921, root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;
```

#### 4.16 FWT

```
/* xor convolution:
 \begin{array}{l} * \ x = (x0, x1) \ , \ y = (y0, y1) \\ * \ z = (\ x0y0 \ + \ x1y1 \ , \ x0y1 \ + \ x1y0 \ ) \end{array}
 * x' = ( x0+x1 , x0-x1 ) , y' = ( y0+y1 , y0-y1 ) * z' = ( ( x0+x1 )( y0+y1 ) , ( x0-x1 )( y0-y1 ) )
 * z = (1/2) * z''
 * or convolution:
 * x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
 * and convolution:
 * x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
const LL MOD = 1e9+7;
inline void fwt( LL x[ MAXN ] , int N , bool inv=0 ) {
  for( int d = 1 ; d < N ; d <<= 1 ) {</pre>
    int d2 = d << 1;
     for( int s = 0 ; s < N ; s += d2 )</pre>
       for( int i = s , j = s+d ; i < s+d ; i++, j++ ) {</pre>
         LL ta = x[i], tb = x[j];
          x[i] = ta+tb;
         x[ j ] = ta-tb;
if(x[ i ] >= MOD ) x[ i ] -= MOD;
         if( x[ j ] < 0 ) x[ j ] += MOD;
  if( inv )
    for( int i = 0 ; i < N ; i++ ) {</pre>
       x[ i ] *= inv( N, MOD );
       x[ i ] %= MOD;
```

# 4.17 DiscreteLog

```
// Baby-step Giant-step Algorithm
11d BSGS(11d P, 11d B, 11d N) {
  // find B^L = N \mod P
  unordered map<lld, int> R;
 lld sq = \overline{\text{(lld)}} sqrt(P);
  11d t = 1;
 for (int i = 0; i < sq; i++) {</pre>
    if (t == N)
      return i;
    if (!R.count(t))
     R[t] = i;
    t = (t * B) % P;
  lld f = inverse(t, P);
 for (int i=0;i<=sq+1;i++) {</pre>
    if (R.count(N))
     return i * sq + R[N];
   N = (N * f) % P;
 return -1;
```

# 5 Geometry

#### 5.1 Point Class

```
template<typename T>
struct Point{
  typedef long double llf;
  static constexpr llf EPS = 1e-8;
  T x, y;
  Point(T _=0, T __=0): x(_), y(__){}
```

```
template<tvpename T2>
    Point(const Point<T2>& a): x(a.x), y(a.y){}
  inline llf theta() const {
    return atan2((llf)y, (llf)x);
  inline llf dis() const {
    return hypot((llf)x, (llf)y);
  inline llf dis(const Point& o) const {
    return hypot((llf)(x-o.x), (llf)(y-o.y));
  Point operator-(const Point& o) const {
    return Point(x-o.x, y-o.y);
  Point operator -= (const Point& o) {
    x-=0.x, y-=0.y;
    return *this;
  Point operator+(const Point& o) const {
    return Point (x+o.x, v+o.v);
  Point operator+=(const Point& o) {
    x+=0.x, y+=0.y;
     return *this;
  Point operator* (const T& k) const {
    return Point(x*k, y*k);
  Point operator*=(const T& k) {
    x*=k, y*=k;
    return *this;
  Point operator/(const T& k) const {
    return Point(x/k, y/k);
  Point operator/=(const T& k) {
    x/=k, y/=k;
    return *this;
  Point operator-() const {
    return Point(-x, -y);
  Point rot90() const {
    return Point(-y, x);
  template<typename T2>
  bool in(const Circle<T2>& a) const {
    /* Add struct Circle at top */
    return a.o.dis(*this) +EPS <= a.r;</pre>
  bool equal(const Point& o, true type) const {
    return fabs(x-o.x) < EPS and fabs(y-o.y) < EPS;
  bool equal(const Point& o, false type) const {
    return tie(x, y) == tie(o.x, o.y);
  bool operator==(const Point& o) const {
    return equal(o, is_floating_point<T>());
  bool operator!=(const Point& o) const {
    return ! (*this == 0);
  bool operator<(const Point& o) const {</pre>
    return theta() < o.theta();</pre>
     // sort like what pairs did
     // if(is_floating_point<T>()) return fabs(x-o.x)
         EPS?y<o.y:x<o.x;
    // else return tie(x, y) < tie(o.x, o.y);
  friend inline T cross(const Point& a, const Point& b)
    return a.x*b.y - b.x*a.y;
  friend inline T dot(const Point& a, const Point &b) {
     return a.x*b.x + a.y*b.y;
  friend ostream& operator<<(ostream& ss, const Point&</pre>
    ss<<"("<<o.x<<", "<<o.v<<")";
    return ss;
};
```

#### 5.2 Circle Class

### 5.3 Line Class

```
const Point<long double> INF P(-1e20, 1e20);
const Point<long double> NOT EXIST(1e20, 1e-20);
template<typename T>
struct Line{
 static constexpr long double EPS = 1e-8;
  // ax+by+c = 0
 T a, b, c;
 Line(): a(0), b(1), c(0){}
Line(T , T , T ): a(
               _, T
                       _): a(_), b(__), c(___){
    assert(fabs(a)>EPS or fabs(b)>EPS);
 template<typename T2>
   Line(const Line<T2>& x): a(x.a), b(x.b), c(x.c) {}
  typedef Point<long double> Pt;
 bool equal(const Line& o, true type) const {
   return fabs(a-o.a) < EPS and fabs(b-o.b) < EPS and
       fabs(c-o.b) < EPS;
 bool euqal(const Line& o, false_type) const {
   return a==o.a and b==o.b and c==o.c;
 bool operator==(const Line& o) const {
   return euqal(o, is_floating_point<T>());
 bool operator!=(const Line& o) const {
    return ! (*this == 0);
 friend inline bool on_line__(const Point<T>& p, const
      Line& l, true_type){
    return fabs(l.a*p.x + l.b*p.y + l.c) < EPS;</pre>
 friend inline bool on_line__(const Point<T>& p, const
       Line& l, false type) {
    return 1.a*p.x + 1.b*p.y + 1.c == 0;
  friend inline bool on line(const Point<T>&p const
     Line& 1) {
   return on line (p, l, is floating point<T>());
 friend inline bool is_parallel__(const Line& x, const
       Line& y, true_type) {
    return fabs(x.a*y.b - x.b*y.a) < EPS;</pre>
 friend inline bool is_parallel__(const Line& x, const
       Line& y, false_type) {
    return x.a*y.b == x.b*y.a;
 friend inline bool is parallel(const Line& x, const
     Line& v) {
    return is_parallel__(x, y, is_floating_point<T>());
 friend inline Pt get inter(const Line& x, const Line&
      ∨) {
    typedef long double llf;
    if(x==y) return INF P;
    if(is_parallel(x, y)) return NOT_EXIST;
    llf delta = x.a*y.b - x.b*y.a;
    llf delta x = x.b*y.c - x.c*y.b;
    llf delta y = x.c*y.a - x.a*y.c;
    return Pt (delta_x / delta, delta_y / delta);
```

```
friend ostream& operator<<(ostream& ss, const Line& o
    ) {
    ss<<o.a<<"x+"<<o.b<<"y+"<<o.c<<"=0";
    return ss;
}
};
template<typename T>
inline Line<T> get_line(const Point<T>& a, const Point<
    T>& b) {
    return Line<T>(a.y-b.y, b.x-a.x, (b.y-a.y)*a.x-(b.x-a.x)*a.y);
}
```

# 5.4 Triangle Circumcentre

# 5.5 2D Convex Hull

```
template<typename T>
class ConvexHull 2D{
private:
  typedef Point<T> PT;
  vector<PT> dots;
  struct myhash{
    uint64 t operator()(const PT& a) const {
      uint\overline{6}4_t xx=0, yy=0;
      memcpy(&xx, &a.x, sizeof(a.x));
      memcpy(&yy, &a.y, sizeof(a.y));
      uint64 t ret = xx*17+yy*31;
      ret = (\text{ret } ^ (\text{ret } >> 16))*0x9E3779B1;
      ret = (ret ^ (ret >> 13))*0xC2B2AE35;
      ret = ret ^ xx;
      return (ret ^ (ret << 3)) * yy;</pre>
  };
  unordered set<PT, myhash> in hull;
public:
  inline void init() {in hull.clear(); dots.clear();}
  void insert(const PT& x) {dots.PB(x);}
  void solve(){
    sort(ALL(dots), [](const PT& a, const PT& b){
      return tie(a.x, a.y) < tie(b.x, b.y);</pre>
    });
    vector<PT> stk(SZ(dots)<<1);
    int top = 0;
    for(auto p: dots) {
      while(top >= 2 and cross(p-stk[top-2], stk[top
          -1]-stk[top-2]) <= 0)
        top --;
      stk[top++] = p;
    for (int i=SZ(dots)-2, t = top+1;i>=0;i--) {
      while(top >= t and cross(dots[i]-stk[top-2], stk[
          top-1]-stk[top-2]) <= 0)
        top --;
      stk[top++] = dots[i];
    stk.resize(top-1);
    swap(stk, dots);
    for(auto i: stk) in hull.insert(i);
  vector<PT> get() {return dots;}
  inline bool in_it(const PT& x) {
    return in hull.find(x)!=in hull.end();
```

#### 5.6 2D Farthest Pair

#### 5.7 2D Cosest Pair

```
struct Point{
  llf x, y;
  llf dis;
} arr[N];
inline llf get dis(Point a, Point b){
 return hypot(a.x-b.x, a.y-b.y);
llf solve(){
 int cur = rand()%n;
 for(int i=0;i<n;i++) arr[i].dis = get dis(arr[cur],</pre>
      arr[i]);
  sort(arr, arr+n, [](Point a, Point b){return a.dis <</pre>
      b.dis; });
  llf ans = 1e50;
 for (int i=0; i<n; i++) {</pre>
    for (int j=i+1; j<n; j++) {</pre>
      if(arr[j].dis - arr[i].dis > ans) break;
      ans = min(ans, get dis(arr[i], arr[j]));
 return ans;
```

# 5.8 SimulateAnnealing

```
double getY(double);
int main(){
 int rr, 11;
 default random engine rEng(time(NULL));
 uniform real distribution < double > Range (-1,1);
 uniform_real_distribution<double> expR(0,1);
 auto Random=bind(Range, rEng), expRand=bind(expR, rEng)
 int step=0;
 double pace=rr-ll, mini=0.95; // need to search for
      it.
 double x=max(min(Random()*pace+ll, rr), ll), y=getY(x
      );
 while (pace>=1e-7) {
    double newX = max(min(x + Random()*pace, rr), 11);
    double newY = getY(newX);
    if(newY < y \mid \mid expRand() < exp(-step))
     x=newX, y=newY;
    step++;
   pace*=mini;
```

#### 5.9 Ternary Search on Integer

```
int TernarySearch(int 1, int r) {
    // (1, r]
    while (r - 1 > 1) {
        int mid = (1 + r)>>1;
        if (f(mid) > f(mid + 1)) r = mid;
        else 1 = mid;
    }
    return 1+1;
}
```

# 5.10 Minimum Covering Circle

```
template<typename T>
Circle<llf> MinCircleCover(const vector<Point<T>>& pts)
  random shuffle(ALL(pts));
  Circle < llf > c = {pts[0], 0};
  int n = SZ(pts);
  for (int i=0;i<n;i++) {</pre>
    if(pts[i].in(c)) continue;
    c = \{pts[i], 0\};
    for (int j=0; j<i; j++) {</pre>
      if(pts[j].in(c)) continue;
      c.o = (pts[i] + pts[j]) / 2;
      c.r = pts[i].dis(c.o);
      for (int k=0; k<j; k++) {</pre>
        if(pts[k].in(c)) continue;
        c = get circum(pts[i], pts[j], pts[k]);
    }
  return c;
```

# 5.11 KDTree (Nearest Point)

```
const int MXN = 100005;
struct KDTree
  struct Node {
    int x, y, x1, y1, x2, y2;
    int id,f;
    Node *L, *R;
  }tree[MXN];
  int n;
  Node *root;
  LL dis2(int x1, int y1, int x2, int y2) {
    LL dx = x1-x2;
    LL dy = y1-y2;
    return dx*dx+dy*dy;
  static bool cmpx(Node& a, Node& b) { return a.x<b.x; }</pre>
  static bool cmpy(Node& a, Node& b) { return a.y<b.y; }</pre>
  void init(vector<pair<int,int>> ip) {
    n = ip.size();
    for (int i=0; i<n; i++) {</pre>
      tree[i].id = i;
       tree[i].x = ip[i].first;
       tree[i].y = ip[i].second;
    root = build tree(0, n-1, 0);
  Node* build tree(int L, int R, int dep) {
    if (L>R) return nullptr;
    int M = (L+R)/2;
    tree[M].f = dep%2;
    nth_element(tree+L, tree+M, tree+R+1, tree[M].f ?
         cmpy : cmpx);
    tree[M].x1 = tree[M].x2 = tree[M].x;
    tree[M].y1 = tree[M].y2 = tree[M].y;
    tree[M].L = build tree(L, M-1, dep+1);
    if (tree[M].L) {
       tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
       tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
       tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
    tree[M].R = build_tree(M+1, R, dep+1);
    if (tree[M].R) {
       tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
       tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
       \texttt{tree} \, [\texttt{M}] \, . \, \texttt{y1} \, = \, \texttt{min} \, (\texttt{tree} \, [\texttt{M}] \, . \, \texttt{y1} \, , \, \, \, \texttt{tree} \, [\texttt{M}] \, . \, \texttt{R} \text{--} \text{y1}) \, ;
       tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
    return tree+M;
  int touch(Node* r, int x, int y, LL d2){
    LL dis = sqrt(d2)+1;
    if (x<r->x1-dis || x>r->x2+dis ||
         y<r->y1-dis || y>r->y2+dis)
       return 0;
    return 1;
```

```
void nearest(Node* r, int x, int y,
               int &mID, LL &md2) {
    if (!r || !touch(r, x, y, md2)) return;
    LL d2 = dis2(r->x, r->y, x, y);
    if (d2 < md2 \mid | (d2 == md2 \&\& mID < r->id)) {
      mID = r->id;
      md2 = d2:
    // search order depends on split dim
    if ((r->f == 0 \&\& x < r->x) ||
        (r->f == 1 && y < r->y)) {
      nearest(r->L, x, y, mID, md2);
      nearest(r->R, x, y, mID, md2);
     else {
      nearest(r->R, x, y, mID, md2);
      nearest(r->L, x, y, mID, md2);
  int query(int x, int y) {
   int id = 1029384756;
    LL d2 = 102938475612345678LL;
   nearest(root, x, y, id, d2);
    return id;
}tree;
```

# 6 Stringology

#### 6.1 Hash

```
class Hash{
private:
  static const int N = 1000000;
  const int p = 127, q = 1208220623;
  int sz, prefix[N], power[N];
  inline int add(int x, int y) {return x+y>=q?x+y-q:x+y
      ; }
  inline int sub(int x, int y) {return x-y<0?x-y+q:x-y;}</pre>
  inline int mul(int x, int y) {return 1LL*x*y%q;}
public:
  void init(const std::string &x) {
    sz = x.size();
    prefix[0]=0;
    for (int i=1;i<=sz;i++) prefix[i]=add(mul(prefix[i</pre>
         -1], p), x[i-1]);
    power[0]=1;
    for (int i=1;i<=sz;i++) power[i]=mul(power[i-1], p);</pre>
  int query(int 1, int r) {
     // 1-base (1, r]
    return sub(prefix[r], mul(prefix[l], power[r-l]));
1 } ;
```

# 6.2 Suffix Array

```
//help by http://www.geeksforgeeks.org/suffix-array-set
   -2-a-nlognlogn-algorithm/
struct sfx{
  int index;
 int r,nr;
char str[N + 10];
int len;
vector<sfx> srs[N + 10];
int mapping[N + 10];
sfx sa[N + 10];
bool cmp(sfx a,sfx b) {
 if(a.r==b.r){
    return a.nr<b.nr;</pre>
   return a.r<b.r;</pre>
void SA() {
 len = strlen(str);
  for (int i=0; i<len; i++) {</pre>
   sa[i].index = i;
    sa[i].r=str[i];
    sa[i].nr=(i+1>=len)?0:str[i+1];
```

```
//sort(sa,sa+len,cmp);
  radixSort();
  for (int j=2; j<=len; j*=2) {</pre>
    int cnt=1;
    int rr = sa[0].r;
    sa[0].r=cnt;
    mapping[sa[0].index]=0;
    for (int i=1; i<len; i++) {</pre>
      if(sa[i].r == rr && sa[i].nr == sa[i-1].nr) {
        rr=sa[i].r;
         sa[i].r=cnt;
      }else{
        rr=sa[i].r;
        sa[i].r=++cnt;
      mapping[sa[i].index]=i;
    for (int i=0;i<len;i++) {</pre>
      int nn = sa[i].index+j;
      sa[i].nr = (nn>=len)?0:sa[mapping[nn]].r;
    //sort(sa, sa+len, cmp);
    radixSort();
void radixSort() {
  int m = 0;
  for (int i=0;i<len;i++) {</pre>
    srs[sa[i].nr].PB(sa[i]);
    m=max(m,sa[i].nr);
  int cnt=0;
  for (int i=0;i<=m;i++) {</pre>
    if(srs[i].empty())continue;
    for(auto j:srs[i]){
      sa[cnt++] = j;
    srs[i].clear();
  m = 0;
  for (int i=0; i<len; i++) {</pre>
    srs[sa[i].r].PB(sa[i]);
    m=max(m,sa[i].r);
  cnt=0:
  for (int i=0;i<=m;i++) {</pre>
    if(srs[i].empty())continue;
    for(auto j:srs[i]){
      sa[cnt++] = j;
    srs[i].clear();
```

# 6.3 Aho-Corasick Algorithm

```
class AhoCorasick{
  private:
    static constexpr int Z = 26;
    struct node{
     node *nxt[ Z ], *fail;
      vector< int > data;
      node(): fail( nullptr ) {
        memset( nxt, 0, sizeof( nxt ) );
        data.clear();
    } *rt;
    inline int Idx( char c ) { return c - 'a'; }
  public:
    void init() { rt = new node(); }
    void add( const string& s, int d ) {
      node* cur = rt;
      for ( auto c : s ) {
        if ( not cur->nxt[ Idx( c ) ] )
         cur->nxt[ Idx( c ) ] = new node();
        cur = cur->nxt[ Idx( c ) ];
      cur->data.push_back( d );
    void compile() {
     vector< node* > bfs;
      size_t ptr = 0;
      for ( int i = 0 ; i < Z ; ++ i ) {
```

```
if ( not rt->nxt[ i ] )
          continue;
        rt->nxt[ i ]->fail = rt;
        bfs.push back( rt->nxt[ i ] );
      while ( ptr < bfs.size() ) {</pre>
        node* u = bfs[ptr ++];
        for ( int i = 0 ; i < Z ; ++ i ) {
          if ( not u->nxt[ i ] )
            continue;
          node* u_f = u->fail;
          while ( u_f ) {
            if ( not u_f->nxt[ i ] ) {
  u_f = u_f->fail;
            u->nxt[ i ]->fail = u_f->nxt[ i ];
          if ( not u f ) u->nxt[ i ]->fail = rt;
          bfs.push back( u->nxt[ i ] );
     }
    void match( const string& s, vector< int >& ret ) {
      node* u = rt;
      for ( auto c : s ) {
        while ( u != rt and not u->nxt[ Idx( c ) ] )
         u = u - > fail;
        u = u - nxt[Idx(c)];
        if ( not u ) u = rt;
        node* tmp = u;
        while ( tmp != rt ) {
          for ( auto d : tmp->data )
            ret.push back( d );
          tmp = tmp - \overline{\phantom{a}}fail;
      }
} ac;
```

# 6.4 KMP

```
int F[N<<1];</pre>
void KMP(char s1[], char s2[], int n, int m) {
  // make F[] for s1+'\0'+s2;
  char ss[N<<1];</pre>
  int len = n+m+1;
  for(int i=0;i<n;i++) ss[i] = s1[i];</pre>
  ss[n] = ' \setminus 0';
  for(int i=0;i<m;i++) ss[i+1+n] = s2[i];</pre>
  F[0] = F[1] = 0;
  for (int i=1; i<len; i++) {</pre>
    int j = F[i];
    while(j > 0 and ss[i]!=ss[j]) j = F[j];
    F[i+1] = (ss[i] == ss[j]?j+1:0);
  // just find (F[len2+i] == len2)
  // i from 1 to len+1 for matching
 [0, i]是個循環字串,且循環節為i-f[i]:
  if(f[i]>0 and i%(i-f[i])==0)
cout << i << " " << i/(i-f[i]) << '\n';</pre>
```

# 6.5 Z value

```
char s[MAXN];
int len,z[MAXN];
void Z_value() {
   int i,j,left,right;
   left=right=0; z[0]=len;
   for(i=1;i<len;i++) {
      j=max(min(z[i-left],right-i),0);
      for(;i+j<len&&s[i+j]==s[j];j++);
      z[i]=j;
      if(i+z[i]>right) {
        right=i+z[i];
        left=i;
    }
}
```

```
6.6 Lexicographically Smallest Rotation
```

```
string mcp(string s) {
  int n = s.length();
  s += s;
  int i=0, j=1;
  while (i<n && j<n) {
    int k = 0;
    while (k < n && s[i+k] == s[j+k]) k++;
    if (s[i+k] <= s[j+k]) j += k+1;
    else i += k+1;
    if (i == j) j++;
  }
  int ans = i < n ? i : j;
  return s.substr(ans, n);
}</pre>
```

#### 6.7 BWT

}

```
struct BurrowsWheeler{
 #define SIGMA 26
#define BASE 'a'
   vector<int> v[ SIGMA ];
   void BWT(char* ori, char* res) {
   // make ori -> ori + ori
     // then build suffix array
  void iBWT(char* ori, char* res) {
   for( int i = 0 ; i < SIGMA ; i ++ )</pre>
       v[ i ].clear();
     int len = strlen( ori );
     for( int i = 0 ; i < len ; i ++ )</pre>
       v[ ori[i] - BASE ].push_back( i );
     vector<int> a;
     for( int i = 0 , ptr = 0 ; i < SIGMA ; i ++ )</pre>
        for( auto j : v[ i ] ){
          a.push_back( j );
          ori[ ptr ++ ] = BASE + i;
     for( int i = 0 , ptr = 0 ; i < len ; i ++ ){</pre>
       res[ i ] = ori[ a[ ptr ] ];
       ptr = a[ ptr ];
     res[len] = 0;
} bwt;
```

# 7 Misc

### 7.1 Degree Sequence Validity

#### 7.1.1 Erdős-Gallai Theorem

 $d_1 \geq \cdots \geq d_n$  is a valid degree sequence iff

$$\sum_{k=1}^n d_k \equiv 0 \mod 2$$
 
$$\wedge$$
 
$$\forall 1 \leq k \leq n, \sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i,k)$$

### 7.2 Havel-Hakimi algorithm

find the vertex who has greatest degree unused, connect it with other greatest vertex.

### 7.3 MaximumEmptyRect

```
int max_empty_rect(int n, int m, bool blocked[N][N]){
 static int mxu[2][N], me=0,he=1,ans=0;
  for (int i=0;i<m;i++) mxu[he][i]=0;</pre>
 for (int i=0:i<n:i++) {</pre>
    stack<PII, vector<PII>> stk;
    for (int j=0; j<m; ++j) {</pre>
      if(blocked[i][j]) mxu[me][j]=0;
      else mxu[me][j]=mxu[he][j]+1;
      int la = j;
      while(!stk.empty()&&stk.top().FF>mxu[me][j]){
        int x1 = i - stk.top().FF, x2 = i;
int y1 = stk.top().SS, y2 = j;
        la = stk.top().SS; stk.pop();
        ans=\max(ans, (x2-x1)*(y2-y1));
      if(stk.empty()||stk.top().FF<mxu[me][j])</pre>
        stk.push({mxu[me][j],la});
    while(!stk.empty()){
      int x1 = i - stk.top().FF, x2 = i;
      int y1 = stk.top().SS-1, y2 = m-1;
      stk.pop();
      ans=max(ans, (x2-x1)*(y2-y1));
    swap (me, he);
  return ans;
```

# 7.4 DP-opt Condition

#### 7.4.1 totally monotone (concave/convex)

```
\begin{array}{l} \forall i < i', j < j', B[i][j] \leq B[i'][j] \implies B[i][j'] \leq B[i'][j'] \\ \forall i < i', j < j', B[i][j] \geq B[i'][j] \implies B[i][j'] \geq B[i'][j'] \end{array}
```

#### 7.4.2 monge condition (concave/convex)

```
\begin{array}{l} \forall i < i', j < j', B[i][j] + B[i'][j'] \geq B[i][j'] + B[i'][j] \\ \forall i < i', j < j', B[i][j] + B[i'][j'] \leq B[i][j'] + B[i'][j] \end{array}
```

#### 7.5 Convex 1D/1D DP

```
struct segment {
 int i, 1, r;
  segment() {}
 segment(int a, int b, int c): i(a), l(b), r(c) {}
inline long long f(int 1, int r) {
 return dp[l] + w(l + 1, r);
void solve() {
 dp[0] = 011;
  deque<segment> deq; deq.push back(segment(0, 1, n));
 for (int i = 1; i <= n; ++i) {
    dp[i] = f(deq.front().i, i);
    while (deq.size() \&\& deq.front().r < i + 1) deq.
        pop_front();
    deq.front().l = i + 1;
    segment seg = segment(i, i + 1, n);
    while (deq.size() \&\& f(i, deq.back().l) < f(deq.
        back().i, deq.back().l)) deq.pop_back();
    if (deq.size())
      int d = 1048576, c = deq.back().1;
      while (d >>= 1) if (c + d <= deq.back().r) {</pre>
        if (f(i, c + d) > f(deq.back().i, c + d)) c +=
            d;
      deq.back().r = c; seg.l = c + 1;
    if (seg.l <= n) deq.push_back(seg);</pre>
```