# Contents



# 1 Basic

## 1.1 vimrc

```
se is nu rnu bs=2 ru mouse=a encoding=utf-8
se cin et sw=4 sts=4 t_Co=256 tgc sc hls ls=2
syn on
colorscheme desert
filetype indent on
inoremap {<CR> {<CR>}<ESC>O
map <F8> <ESC>:w<CR>:!g++ "%" -o "%<" -O2 -std=c++17 -
    DKISEKI -Wall -Wextra -Wshadow -Wfatal-errors -
    Wconversion -fsanitize=address -fsanitize=undefined
     -g && echo success<CR>
map <F9> <ESC>:w<CR>:!g++ "%" -o "%<" -O2 -std=c++17 -
    DKISEKI && echo success<CR>
map <F10> <ESC>:!./"%<"<CR>
```

## 1.2 Increase Stack

```
const int size = 256 << 20;
register long rsp asm("rsp");
char *p = (char*)malloc(size)+size, *bak = (char*)rsp;
__asm__("movq %0, %%rsp\n"::"r"(p));
// main
__asm__("movq %0, %%rsp\n"::"r"(bak));
```

## 1.3 Pragma Optimization

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,tune=native")
```

## 1.4 IO Optimization

```
static inline int gc() {
 static char buf[ 1 << 20 ], *p = buf, *end = buf;
 if ( p == end ) {
  end = buf + fread( buf, 1, 1 << 20, stdin );
  if ( end == buf ) return EOF;
  p = buf;
 }
 return *p++;
}
template < typename T >
static inline bool gn( T &_ ) {
 register int c = gc(); register T __ = 1; _ = 0;
 while(('0'>c||c>'9') && c!=EOF && c!='-') c = gc();
 if(c == '-') { __ = -1; c = gc(); }
 if(c == EOF) return false;
 while('0'<=c&&c<='9') _ = _ * 10 + c - '0', c = gc();
 _ *= __;
 return true;
}
```

```
template < typename T, typename ...Args >
static inline bool gn( T &x, Args &...args )
{ return gn(x) && gn(args...); }
```

# 2 Data Structure

## 2.1 Bigint

```
class BigInt{
 private:
 using lld = int_fast64_t;
 #define PRINTF_ARG PRIdFAST64
 #define LOG_BASE_STR "9"
 static constexpr lld BASE = 1000000000;
 static constexpr int LOG_BASE = 9;
 vector<lld> dig; bool neg;
 inline int len() const { return (int) dig.size(); }
 inline int cmp_minus(const BigInt& a) const {
  if(len() == 0 && a.len() == 0) return 0;
  if(neg ^ a.neg)return a.neg ^ 1;
  if(len()!=a.len())
   return neg?a.len()-len():len()-a.len();
  for(int i=len()-1;i>=0;i--) if(dig[i]!=a.dig[i])
   return neg?a.dig[i]-dig[i]:dig[i]-a.dig[i];
  return 0;
 }
 inline void trim(){
  while(!dig.empty()&&!dig.back())dig.pop_back();
  if(dig.empty()) neg = false;
 }
 public:
 BigInt(): dig(vector<lld>()), neg(false){}
 BigInt(lld a): dig(vector<lld>()){
  neg = a<0; dig.push_back(abs(a));
  trim();
 }
 BigInt(const string& a): dig(vector<lld>()){
  assert(!a.empty()); neg = (a[0]=='-');
  for(int i=((int)a.size())-1;i>=neg;i-=LOG_BASE){
   lld cur = 0;
   for(int j=min(LOG_BASE-1,i-neg);j>=0;j--)
    cur = cur*10+a[i-j]-'0';
   dig.push_back(cur);
  } trim();
 }
 inline bool operator<(const BigInt& a)const
  {return cmp_minus(a)<0;}
 inline bool operator<=(const BigInt& a)const
  {return cmp_minus(a)<=0;}
 inline bool operator==(const BigInt& a)const
  {return cmp_minus(a)==0;}
 inline bool operator!=(const BigInt& a)const
  {return cmp_minus(a)!=0;}
 inline bool operator>(const BigInt& a)const
  {return cmp_minus(a)>0;}
 inline bool operator>=(const BigInt& a)const
  {return cmp_minus(a)>=0;}
 BigInt operator-() const {
  BigInt ret = *this;
  ret.neg ^= 1; return ret;
 }
 BigInt operator+(const BigInt& a) const {
  if(neg) return -(-(*this)+(-a));
  if(a.neg) return (*this)-(-a);
  int n = max(a.len(), len());
  BigInt ret; ret.dig.resize(n);
  lld pro = 0;
  for(int i=0;i<n;i++) {
   ret.dig[i] = pro;
   if(i < a.len()) ret.dig[i] += a.dig[i];
   if(i < len()) ret.dig[i] += dig[i];
   pro = 0;
   if(ret.dig[i] >= BASE) pro = ret.dig[i]/BASE;
   ret.dig[i] -= BASE*pro;
  }
  if(pro != 0) ret.dig.push_back(pro);
  return ret;
 }
 BigInt operator-(const BigInt& a) const {
  if(neg) return -(-(*this) - (-a));
  if(a.neg) return (*this) + (-a);
  int diff = cmp_minus(a);
  if(diff < 0) return -(a - (*this));
  if(diff == 0) return 0;
  BigInt ret; ret.dig.resize(len(), 0);
  for(int i=0;i<len();i++) {
   ret.dig[i] += dig[i];
   if(i < a.len()) ret.dig[i] -= a.dig[i];
   if(ret.dig[i] < 0){
    ret.dig[i] += BASE;
    ret.dig[i+1]--;
   }
  }
  ret.trim(); return ret;
 }
 BigInt operator*(const BigInt& a) const {
  if(!len()||!a.len()) return 0;
  BigInt ret; ret.dig.resize(len()+a.len()+1);
  ret.neg = neg ^ a.neg;
  for(int i=0;i<len();i++)
   for(int j=0;j<a.len();j++){
    ret.dig[i+j] += dig[i] * a.dig[j];
    if(ret.dig[i+j] >= BASE) {
     lld x = ret.dig[i+j] / BASE;
     ret.dig[i+j+1] += x;
     ret.dig[i+j] -= x * BASE;
    }
   }
  ret.trim(); return ret;
 }
 BigInt operator/(const BigInt& a) const {
  assert(a.len());
  if(len() < a.len()) return 0;
  BigInt ret; ret.dig.resize(len()-a.len()+1);
  ret.neg = a.neg;
  for(int i=len()-a.len();i>=0;i--){
   lld l = 0, r = BASE;
   while(r-l > 1){
    lld mid = (l+r)>>1;
    ret.dig[i] = mid;
    if(ret*a<=(neg?-(*this):(*this))) l = mid;
    else r = mid;
   }
   ret.dig[i] = l;
  }
  ret.neg ^= neg; ret.trim();
  return ret;
 }
 BigInt operator%(const BigInt& a) const {
  return (*this) - (*this) / a * a;
 }
 friend BigInt abs(BigInt a) { a.neg = 0; return a; }
 friend void swap(BigInt& a, BigInt& b){
  swap(a.dig, b.dig); swap(a.neg, b.neg);
 }
 friend istream& operator>>(istream& ss, BigInt& a){
  string s; ss >> s; a = s; return ss;
 }
 friend ostream&operator<<(ostream&o, const BigInt&a){
  if(a.len() == 0) return o << '0';
  if(a.neg) o << '-';
  o << a.dig.back();
  for(int i=a.len()-2;i>=0;i--)
   o<<setw(LOG_BASE)<<setfill('0')<<a.dig[i];
  return o;
 }
 inline void print() const {
  if(len() == 0){putchar('0');return;}
  if(neg) putchar('-');
  printf("%" PRINTF_ARG, dig.back());
  for(int i=len()-2;i>=0;i--)
   printf("%0" LOG_BASE_STR PRINTF_ARG, dig[i]);
 }
 #undef PRINTF_ARG
 #undef LOG_BASE_STR
};
```

## 2.2 Dark Magic

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
using __gnu_pbds::pairing_heap_tag;
using __gnu_pbds::binary_heap_tag;
using __gnu_pbds::binomial_heap_tag;
using __gnu_pbds::rc_binomial_heap_tag;
using __gnu_pbds::thin_heap_tag;
```

```
template<typename T>
using pbds_heap=__gnu_pbds::prioity_queue<T,less<T>,\
                  pairing_heap_tag>;
// __gnu_pbds::priority_queue<T,less<T>>::
    point_iterator
// x = pq.push(10); pq.modify(x, 87); a.join(b);
using __gnu_pbds::rb_tree_tag;
using __gnu_pbds::ov_tree_tag;
using __gnu_pbds::splay_tree_tag;
template<typename T>
using ordered_set = __gnu_pbds::tree<T,\
__gnu_pbds::null_type,less<T>,rb_tree_tag,\
__gnu_pbds::tree_order_statistics_node_update>;
// find_by_order, order_of_key
template<typename A,typename B>
using hTable1=__gnu_pbds::cc_hash_table<A,B>;
template<typename A,typename B>
using hTable2=__gnu_pbds::gp_hash_table<A,B>;
```

## 2.3 Disjoint Set

```
class DJS {
private:
 vector< int > fa, sz, sv;
 vector< pair< int*, int > > opt;
 void assign( int *k, int v ) {
  opt.emplace_back( k, *k );
  *k = v;
 }
public:
 void init( int n ) {
  fa.resize( n ); iota( fa.begin(), fa.end(), 0 );
  sz.resize( n ); fill( sz.begin(), sz.end(), 1 );
  opt.clear();
 }
 int query(int x) {return fa[x] == x?x:query(fa[x]);}
 void merge( int a, int b ) {
  int af = query( a ), bf = query( b );
  if( af == bf ) return;
  if( sz[ af ] < sz[ bf ] ) swap( af, bf );
  assign( &fa[ bf ], fa[ af ] );
  assign( &sz[ af ], sz[ af ] + sz[ bf ] );
 }
 void save() { sv.push_back( (int) opt.size() ); }
 void undo() {
  int ls = sv.back(); sv.pop_back();
  while ( ( int ) opt.size() > ls ) {
   pair< int*, int > cur = opt.back();
   *cur.first = cur.second;
   opt.pop_back();
  }
 }
};
```

## 2.4 Link-Cut Tree

```
struct Node{
 Node *par,*ch[2];
 int xor_sum,v;
 bool is_rev;
 Node(int _v){
  v=xor_sum=_v;is_rev=false;
  par=ch[0]=ch[1]=nullptr;
 }
 inline void set_rev(){is_rev^=1;swap(ch[0],ch[1]);}
 inline void down(){
  if(is_rev){
   if(ch[0]!=nullptr) ch[0]->set_rev();
   if(ch[1]!=nullptr) ch[1]->set_rev();
   is_rev=false;
  }
 }
 inline void up(){
  xor_sum=v;
  if(ch[0]!=nullptr){
   xor_sum^=ch[0]->xor_sum;
   ch[0]->par=this;
  }
  if(ch[1]!=nullptr){
   xor_sum^=ch[1]->xor_sum;
   ch[1]->par=this;
  }
 }
 inline bool is_root(){
  return par==nullptr ||\
   (par->ch[0]!=this && par->ch[1]!=this);
 }
 bool is_rch(){return !is_root() && par->ch[1]==this;}
} *node[maxn],*stk[maxn];
int top;
void to_child(Node* p,Node* c,bool dir){
 p->ch[dir]=c;
 p->up();
}
inline void rotate(Node* node){
 Node* par=node->par;
 Node* par_par=par->par;
 bool dir=node->is_rch();
 bool par_dir=par->is_rch();
 to_child(par,node->ch[!dir],dir);
 to_child(node,par,!dir);
 if(par_par!=nullptr && par_par->ch[par_dir]==par)
  to_child(par_par,node,par_dir);
 else node->par=par_par;
}
inline void splay(Node* node){
 Node* tmp=node;
 stk[top++]=node;
 while(!tmp->is_root()){
  tmp=tmp->par;
  stk[top++]=tmp;
 }
 while(top) stk[--top]->down();
 for(Node *fa=node->par;
  !node->is_root();
  rotate(node),fa=node->par)
  if(!fa->is_root())
   rotate(fa->is_rch()==node->is_rch()?fa:node);
}
inline void access(Node* node){
 Node* last=nullptr;
 while(node!=nullptr){
  splay(node);
  to_child(node,last,true);
  last=node;
  node=node->par;
 }
}
inline void change_root(Node* node){
 access(node);splay(node);node->set_rev();
}
inline void link(Node* x,Node* y){
 change_root(x);splay(x);x->par=y;
}
inline void split(Node* x,Node* y){
 change_root(x);access(y);splay(x);
 to_child(x,nullptr,true);y->par=nullptr;
}
inline void change_val(Node* node,int v){
 access(node);splay(node);node->v=v;node->up();
}
inline int query(Node* x,Node* y){
 change_root(x);access(y);splay(y);
 return y->xor_sum;
}
inline Node* find_root(Node* node){
 access(node);splay(node);
 Node* last=nullptr;
 while(node!=nullptr){
  node->down();last=node;node=node->ch[0];
 }
 return last;
}
set<pii> dic;
inline void add_edge(int u,int v){
 if(u>v) swap(u,v);
 if(find_root(node[u])==find_root(node[v])) return;
 dic.insert(pii(u,v));
 link(node[u],node[v]);
}
inline void del_edge(int u,int v){
 if(u>v) swap(u,v);
 if(dic.find(pii(u,v))==dic.end()) return;
 dic.erase(pii(u,v));
 split(node[u],node[v]);
}
```

## 2.5 LiChao Segment Tree

```
struct Line{
 int m, k, id;
 Line() : id( -1 ) {}
 Line( int a, int b, int c )
  : m( a ), k( b ), id( c ) {}
 int at( int x ) { return m * x + k; }
};
class LiChao {
 private:
  int n; vector< Line > nodes;
  inline int lc( int x ) { return 2 * x + 1; }
  inline int rc( int x ) { return 2 * x + 2; }
  void insert( int l, int r, int id, Line ln ) {
   int m = ( l + r ) >> 1;
   if ( nodes[ id ].id == -1 ) {
    nodes[ id ] = ln;
    return;
   }
   bool atLeft = nodes[ id ].at( l ) < ln.at( l );
   if ( nodes[ id ].at( m ) < ln.at( m ) ) {
    atLeft ^= 1; swap( nodes[ id ], ln );
   }
   if ( r - l == 1 ) return;
   if ( atLeft ) insert( l, m, lc( id ), ln );
   else insert( m, r, rc( id ), ln );
  }
  int query( int l, int r, int id, int x ) {
   int ret = 0;
   if ( nodes[ id ].id != -1 )
    ret = nodes[ id ].at( x );
   int m = ( l + r ) >> 1;
   if ( r - l == 1 ) return ret;
   else if ( x < m )
    return max( ret, query( l, m, lc( id ), x ) );
   else
    return max( ret, query( m, r, rc( id ), x ) );
  }
 public:
  void build( int n_ ) {
   n = n_; nodes.clear();
   nodes.resize( n << 2, Line() );
  }
  void insert( Line ln ) { insert( 0, n, 0, ln ); }
  int query( int x ) { return query( 0, n, 0, x ); }
} lichao;
```

## 2.6 Treap

```
namespace Treap{
 #define sz( x ) ( ( x ) ? ( ( x )->size ) : 0 )
 struct node{
  int size;
  uint32_t pri;
  node *lc, *rc;
  node() : size(0), pri(rand()), lc( 0 ), rc( 0 ) {}
  void pull() {
   size = 1;
   if ( lc ) size += lc->size;
   if ( rc ) size += rc->size;
  }
 };
 node* merge( node* L, node* R ) {
  if ( not L or not R ) return L ? L : R;
  if ( L->pri > R->pri ) {
   L->rc = merge( L->rc, R ); L->pull();
   return L;
  } else {
   R->lc = merge( L, R->lc ); R->pull();
   return R;
  }
 }
 void split_by_size( node*rt,int k,node*&L,node*&R ) {
  if ( not rt ) L = R = nullptr;
  else if( sz( rt->lc ) + 1 <= k ) {
   L = rt;
   split_by_size( rt->rc,k-sz(rt->lc)-1,L->rc,R );
   L->pull();
  } else {
   R = rt;
   split_by_size( rt->lc, k, L, R->lc );
   R->pull();
  }
 }
 #undef sz
}
```

## 2.7 Sparse Table

```
template < typename T, typename Cmp_ = less< T > >
class SparseTable {
private:
 vector< vector< T > > tbl;
 vector< int > lg;
 T cv( T a, T b ) {
  return Cmp_()( a, b ) ? a : b;
 }
public:
 void init( T arr[], int n ) {
  // 0-base
  lg.resize( n + 1 );
  lg[ 0 ] = -1;
  for( int i=1 ; i<=n ; ++i ) lg[i] = lg[i>>1] + 1;
  tbl.resize( lg[n] + 1 );
  tbl[ 0 ].resize( n );
  copy( arr, arr + n, tbl[ 0 ].begin() );
  for ( int i = 1 ; i <= lg[ n ] ; ++ i ) {
   int len = 1 << ( i - 1 ), sz = 1 << i;
   tbl[ i ].resize( n - sz + 1 );
   for ( int j = 0 ; j <= n - sz ; ++ j )
    tbl[i][j] = cv(tbl[i-1][j], tbl[i-1][j+len]);
  }
 }
 T query( int l, int r ) {
  // 0-base [l, r)
  int wh = lg[ r - l ], len = 1 << wh;
  return cv( tbl[ wh ][ l ], tbl[ wh ][ r - len ] );
 }
};
```

## 2.8 Linear Basis

```
struct LinearBasis {
private:
 int n, sz;
 vector< llu > B;
 inline llu two( int x ){ return ( ( llu ) 1 ) << x; }
public:
 void init( int n_ ) {
  n = n_; B.clear(); B.resize( n ); sz = 0;
 }
 void insert( llu x ) {
  // add x into B
  for ( int i = n-1; i >= 0 ; --i ) if( two(i) & x ){
   if ( B[ i ] ) x ^= B[ i ];
   else {
    B[ i ] = x; sz++;
    for ( int j = i - 1 ; j >= 0 ; -- j )
     if( B[ j ] && ( two( j ) & B[ i ] ) )
      B[ i ] ^= B[ j ];
    for (int j = i + 1 ; j < n ; ++ j )
     if ( two( i ) & B[ j ] )
      B[ j ] ^= B[ i ];
    break;
   }
  }
 }
 inline int size() { return sz; }
 bool check( llu x ) {
  // is x in span(B) ?
  for ( int i = n-1 ; i >= 0 ; --i ) if( two(i) & x )
   if( B[ i ] ) x ^= B[ i ];
   else return false;
  return true;
 }
 llu kth_small(llu k) {
  /** 1-base would always > 0 **/
  /** should check it **/
  /* if we choose at least one element
     but size(B)(vectors in B)==N(original elements)
     then we can't get 0 */
  llu ret = 0;
  for ( int i = 0 ; i < n ; ++ i ) if( B[ i ] ) {
   if( k & 1 ) ret ^= B[ i ];
   k >>= 1;
  }
```

```
  return ret;
 }
} base;
```

# 3 Graph

## 3.1 Euler Circuit

```
bool vis[ N ]; size_t la[ K ];
void dfs( int u, vector< int >& vec ) {
 while ( la[ u ] < G[ u ].size() ) {
  if( vis[ G[ u ][ la[ u ] ].second ] ) {
   ++ la[ u ];
   continue;
  }
  int v = G[ u ][ la[ u ] ].first;
  vis[ G[ u ][ la[ u ] ].second ] = true;
  ++ la[ u ]; dfs( v, vec );
  vec.push_back( v );
 }
}
```

## 3.2 BCC Edge

```
class BCC_Bridge {
 private:
  int n, ecnt;
  vector<vector<pair<int,int>>> G;
  vector<int> dfn, low;
  vector<bool> bridge;
  void dfs(int u, int f) {
   dfn[u] = low[u] = dfn[f] + 1;
   for (auto [v, t]: G[u]) {
    if (v == f) continue;
    if (dfn[v]) {
     low[u] = min(low[u], dfn[v]);
     continue;
    }
    dfs(v, u);
    low[u] = min(low[u], low[v]);
    if (low[v] > dfn[u]) bridge[t] = true;
   }
  }
 public:
  void init(int n_) {
   G.clear(); G.resize(n = n_);
   low.assign(n, ecnt = 0);
   dfn.assign(n, 0);
  }
  void add_edge(int u, int v) {
   G[u].emplace_back(v, ecnt);
   G[v].emplace_back(u, ecnt++);
  }
  void solve() {
   bridge.assign(ecnt, false);
   for (int i = 0; i < n; ++i)
    if (not dfn[i]) dfs(i, i);
  }
  bool is_bridge(int x) { return bridge[x]; }
} bcc_bridge;
```

## 3.3 BCC Vertex

```
class BCC_AP {
 private:
  int n, ecnt;
  vector<vector<pair<int,int>>> G;
  vector<int> bcc, dfn, low, st;
  vector<bool> ap, ins;
  void dfs(int u, int f) {
   dfn[u] = low[u] = dfn[f] + 1;
   int ch = 0;
   for (auto [v, t]: G[u]) if (v != f) {
    if (not ins[t]) {
     st.push_back(t);
     ins[t] = true;
    }
    if (dfn[v]) {
     low[u] = min(low[u], dfn[v]);
     continue;
    } ++ch; dfs(v, u);
    low[u] = min(low[u], low[v]);
    if (low[v] >= dfn[u]) {
     ap[u] = true;
     while (true) {
      int eid = st.back(); st.pop_back();
      bcc[eid] = ecnt;
      if (eid == t) break;
     }
     ecnt++;
    }
   }
   if (ch == 1 and u == f) ap[u] = false;
  }
 public:
  void init(int n_) {
   G.clear(); G.resize(n = n_);
   ecnt = 0; ap.assign(n, false);
   low.assign(n, 0); dfn.assign(n, 0);
  }
  void add_edge(int u, int v) {
   G[u].emplace_back(v, ecnt);
   G[v].emplace_back(u, ecnt++);
  }
  void solve() {
   ins.assign(ecnt, false);
   bcc.resize(ecnt); ecnt = 0;
   for (int i = 0; i < n; ++i)
    if (not dfn[i]) dfs(i, i);
  }
  int get_id(int x) { return bcc[x]; }
  int count() { return ecnt; }
  bool is_ap(int x) { return ap[x]; }
} bcc_ap;
```

## 3.4 2-SAT (SCC)

```
class TwoSat{
 private:
  int n;
  vector<vector<int>> rG,G,sccs;
  vector<int> ord,idx;
  vector<bool> vis,result;
  void dfs(int u){
   vis[u]=true;
   for(int v:G[u])
    if(!vis[v]) dfs(v);
   ord.push_back(u);
  }
  void rdfs(int u){
   vis[u]=false;idx[u]=sccs.size()-1;
   sccs.back().push_back(u);
   for(int v:rG[u])
    if(vis[v])rdfs(v);
  }
 public:
  void init(int n_){
   n=n_;G.clear();G.resize(n);
   rG.clear();rG.resize(n);
   sccs.clear();ord.clear();
   idx.resize(n);result.resize(n);
  }
  void add_edge(int u,int v){
   G[u].push_back(v);rG[v].push_back(u);
  }
  void orr(int x,int y){
   if ((x^y)==1)return;
   add_edge(x^1,y); add_edge(y^1,x);
  }
  bool solve(){
   vis.clear();vis.resize(n);
   for(int i=0;i<n;++i)
    if(not vis[i])dfs(i);
   reverse(ord.begin(),ord.end());
   for (int u:ord){
    if(!vis[u])continue;
    sccs.push_back(vector<int>());
    rdfs(u);
   }
   for(int i=0;i<n;i+=2)
    if(idx[i]==idx[i+1])
     return false;
   vector<bool> c(sccs.size());
   for(size_t i=0;i<sccs.size();++i){
    for(size_t j=0;j<sccs[i].size();++j){
     result[sccs[i][j]]=c[i];
     c[idx[sccs[i][j]^1]]=!c[i];
```

```
    }
   }
   return true;
  }
  bool get(int x){return result[x];}
  inline int get_id(int x){return idx[x];}
  inline int count(){return sccs.size();}
} sat2;
```

## 3.5 Lowbit Decomposition

```
class LowbitDecomp{
private:
 int time_, chain_, LOG_N;
 vector< vector< int > > G, fa;
 vector< int > tl, tr, chain, chain_st;
 // chain_ : number of chain
 // tl, tr[ u ] : subtree interval in the seq. of u
 // chain_st[ u ] : head of the chain contains u
 // chian[ u ] : chain id of the chain u is on
 void predfs( int u, int f ) {
  chain[ u ] = 0;
  for ( int v : G[ u ] ) {
   if ( v == f ) continue;
   predfs( v, u );
   if( lowbit( chain[ u ] ) < lowbit( chain[ v ] ) )
    chain[ u ] = chain[ v ];
  }
  if ( not chain[ u ] )
   chain[ u ] = chain_ ++;
 }
 void dfschain( int u, int f ) {
  fa[ u ][ 0 ] = f;
  for ( int i = 1 ; i < LOG_N ; ++ i )
   fa[ u ][ i ] = fa[ fa[ u ][ i - 1 ] ][ i - 1 ];
  tl[ u ] = time_++;
  if ( not chain_st[ chain[ u ] ] )
   chain_st[ chain[ u ] ] = u;
  for ( int v : G[ u ] )
   if ( v != f and chain[ v ] == chain[ u ] )
    dfschain( v, u );
  for ( int v : G[ u ] )
   if ( v != f and chain[ v ] != chain[ u ] )
    dfschain( v, u );
  tr[ u ] = time_;
 }
 bool anc( int u, int v ) {
  return tl[ u ] <= tl[ v ] and tr[ v ] <= tr[ u ];
 }
public:
 int lca( int u, int v ) {
  if ( anc( u, v ) ) return u;
  for ( int i = LOG_N - 1 ; i >= 0 ; -- i )
   if ( not anc( fa[ u ][ i ], v ) )
    u = fa[ u ][ i ];
  return fa[ u ][ 0 ];
 }
 void init( int n ) {
  fa.assign( ++n, vector< int >( LOG_N ) );
  for ( LOG_N = 0 ; ( 1 << LOG_N ) < n ; ++ LOG_N );
  G.clear(); G.resize( n );
  tl.assign( n, 0 ); tr.assign( n, 0 );
  chain.assig( n, 0 ); chain_st.assign( n, 0 );
 }
 void add_edge( int u , int v ) {
  // 1-base
  G[ u ].push_back( v );
  G[ v ].push_back( u );
 }
 void decompose(){
  chain_ = 1;
  predfs( 1, 1 );
  time_ = 0;
  dfschain( 1, 1 );
 }
 PII get_subtree(int u) { return {tl[ u ],tr[ u ] }; }
 vector< PII > get_path( int u , int v ){
  vector< PII > res;
  int g = lca( u, v );
  while ( chain[ u ] != chain[ g ] ) {
   int s = chain_st[ chain[ u ] ];
   res.emplace_back( tl[ s ], tl[ u ] + 1 );
   u = fa[ s ][ 0 ];
  }
  res.emplace_back( tl[ g ], tl[ u ] + 1 );
  while ( chain[ v ] != chain[ g ] ) {
   int s = chain_st[ chain[ v ] ];
   res.emplace_back( tl[ s ], tl[ v ] + 1 );
   v = fa[ s ][ 0 ];
  }
  res.emplace_back( tl[ g ] + 1, tl[ v ] + 1 );
  return res;
  /* res : list of intervals from u to v
   * ( note only nodes work, not edge )
   * usage :
   * vector< PII >& path = tree.get_path( u , v )
   * for( auto [ l, r ] : path ) {
   *  0-base [ l, r )
   * }
   */
 }
} tree;
```

## 3.6 MaxClique

```
// contain a self loop u to u, than u won't in clique
template < size_t MAXN >
class MaxClique{
private:
 using bits = bitset< MAXN >;
 bits popped, G[ MAXN ], ans;
 size_t deg[ MAXN ], deo[ MAXN ], n;
 void sort_by_degree() {
  popped.reset();
  for ( size_t i = 0 ; i < n ; ++ i )
    deg[ i ] = G[ i ].count();
  for ( size_t i = 0 ; i < n ; ++ i ) {
    size_t mi = MAXN, id = 0;
    for ( size_t j = 0 ; j < n ; ++ j )
      if ( not popped[ j ] and deg[ j ] < mi )
        mi = deg[ id = j ];
    popped[ deo[ i ] = id ] = 1;
    for( size_t u = G[ i ]._Find_first() ;
     u < n ; u = G[ i ]._Find_next( u ) )
      -- deg[ u ];
  }
 }
 void BK( bits R, bits P, bits X ) {
  if (R.count()+P.count() <= ans.count()) return;
  if ( not P.count() and not X.count() ) {
   if ( R.count() > ans.count() ) ans = R;
   return;
  }
  /* greedily chosse max degree as pivot
  bits cur = P | X; size_t pivot = 0, sz = 0;
  for ( size_t u = cur._Find_first() ;
   u < n ; u = cur._Find_next( u ) )
    if ( deg[ u ] > sz ) sz = deg[ pivot = u ];
  cur = P & ( ~G[ pivot ] );
  */ // or simply choose first
  bits cur = P & (~G[ ( P | X )._Find_first() ]);
  for ( size_t u = cur._Find_first() ;
   u < n ; u = cur._Find_next( u ) ) {
   if ( R[ u ] ) continue;
   R[ u ] = 1;
   BK( R, P & G[ u ], X & G[ u ] );
   R[ u ] = P[ u ] = 0, X[ u ] = 1;
  }
 }
public:
 void init( size_t n_ ) {
  n = n_;
  for ( size_t i = 0 ; i < n ; ++ i )
   G[ i ].reset();
  ans.reset();
 }
 void add_edges( int u, bits S ) { G[ u ] = S; }
 void add_edge( int u, int v ) {
  G[ u ][ v ] = G[ v ][ u ] = 1;
 }
 int solve() {
  sort_by_degree(); // or simply iota( deo... )
  for ( size_t i = 0 ; i < n ; ++ i )
   deg[ i ] = G[ i ].count();
  bits pob, nob = 0; pob.set();
  for (size_t i=n; i<MAXN; ++i) pob[i] = 0;
```

```
  for ( size_t i = 0 ; i < n ; ++ i ) {
   size_t v = deo[ i ];
   bits tmp; tmp[ v ] = 1;
   BK( tmp, pob & G[ v ], nob & G[ v ] );
   pob[ v ] = 0, nob[ v ] = 1;
  }
  return static_cast< int >( ans.count() );
 }
};
```

## 3.7 MaxCliqueDyn

```
constexpr int kN = 150;
struct MaxClique { // Maximum Clique
 bitset<kN> a[kN], cs[kN];
 int ans, sol[kN], q, cur[kN], d[kN], n;
 void init(int _n) {
  n = _n; for (int i = 0; i < n; i++) a[i].reset();
 }
 void addEdge(int u, int v) { a[u][v] = a[v][u] = 1; }
 void csort(vector<int> &r, vector<int> &c) {
  int mx = 1, km = max(ans - q + 1, 1), t = 0,
    m = int(r.size());
  cs[1].reset(); cs[2].reset();
  for (int i = 0; i < m; i++) {
   int p = r[i], k = 1;
   while ((cs[k] & a[p]).count()) k++;
   if (k > mx) cs[++mx + 1].reset();
   cs[k][p] = 1;
   if (k < km) r[t++] = p;
  }
  c.resize(m);
  if (t) c[t - 1] = 0;
  for (int k = km; k <= mx; k++) {
   for (int p = int(cs[k]._Find_first());
      p < kN; p = int(cs[k]._Find_next(p))) {
    r[t] = p; c[t++] = k;
   }
  }
 }
 void dfs(vector<int> &r, vector<int> &c, int l,
  bitset<kN> mask) {
  while (!r.empty()) {
   int p = r.back(); r.pop_back();
   mask[p] = 0;
   if (q + c.back() <= ans) return;
   cur[q++] = p;
   vector<int> nr, nc;
   bitset<kN> nmask = mask & a[p];
   for (int i : r)
    if (a[p][i]) nr.push_back(i);
   if (!nr.empty()) {
    if (l < 4) {
     for (int i : nr)
      d[i] = int((a[i] & nmask).count());
     sort(nr.begin(), nr.end(),
      [&](int x, int y) {
       return d[x] > d[y];
      });
    }
    csort(nr, nc); dfs(nr, nc, l + 1, nmask);
   } else if (q > ans) {
    ans = q; copy(cur, cur + q, sol);
   }
   c.pop_back(); q--;
  }
 }
 int solve(bitset<kN> mask) { // vertex mask
  vector<int> r, c;
  for (int i = 0; i < n; i++)
   if (mask[i]) r.push_back(i);
  for (int i = 0; i < n; i++)
   d[i] = int((a[i] & mask).count());
  sort(r.begin(), r.end(),
   [&](int i, int j) { return d[i] > d[j]; });
  csort(r, c);
  dfs(r, c, 1, mask);
  return ans; // sol[0 ~ ans-1]
 }
} graph;
```

## 3.8 Virtural Tree

```
inline bool cmp(const int &i, const int &j) {
 return dfn[i] < dfn[j];
}
void build(int vectrices[], int k) {
 static int stk[MAX_N];
 sort(vectrices, vectrices + k, cmp);
 stk[sz++] = 0;
 for (int i = 0; i < k; ++i) {
  int u = vectrices[i], lca = LCA(u, stk[sz - 1]);
  if (lca == stk[sz - 1]) stk[sz++] = u;
  else {
   while (sz >= 2 && dep[stk[sz - 2]] >= dep[lca]) {
    addEdge(stk[sz - 2], stk[sz - 1]);
    sz--;
   }
   if (stk[sz - 1] != lca) {
    addEdge(lca, stk[--sz]);
    stk[sz++] = lca, vectrices[cnt++] = lca;
   }
   stk[sz++] = u;
  }
 }
 for (int i = 0; i < sz - 1; ++i)
  addEdge(stk[i], stk[i + 1]);
}
```

## 3.9 Centroid Decomposition

```
struct Centroid {
 vector<vector<int64_t>> Dist;
 vector<int> Parent, Depth;
 vector<int64_t> Sub, Sub2;
 vector<int> Sz, Sz2;
 Centroid(vector<vector<pair<int, int>>> g) {
  int N = g.size();
  vector<bool> Vis(N);
  vector<int> sz(N), mx(N);
  vector<int> Path;
  Dist.resize(N);
  Parent.resize(N);
  Depth.resize(N);
  auto DfsSz = [&](auto dfs, int x) -> void {
   Vis[x] = true; sz[x] = 1; mx[x] = 0;
   for (auto [u, w] : g[x]) {
    if (Vis[u]) continue;
    dfs(dfs, u);
    sz[x] += sz[u];
    mx[x] = max(mx[x], sz[u]);
   }
   Path.push_back(x);
  };
  auto DfsDist = [&](auto dfs, int x, int64_t D = 0)
   -> void {
   Dist[x].push_back(D);Vis[x] = true;
   for (auto [u, w] : g[x]) {
    if (Vis[u]) continue;
    dfs(dfs, u, D + w);
   }
  };
  auto Dfs = [&]
   (auto dfs, int x, int D = 0, int p = -1)->void {
   Path.clear(); DfsSz(DfsSz, x);
   int M = Path.size();
   int C = -1;
   for (int u : Path) {
    if (max(M - sz[u], mx[u]) * 2 <= M) C = u;
    Vis[u] = false;
   }
   DfsDist(DfsDist, C);
   for (int u : Path) Vis[u] = false;
   Parent[C] = p; Vis[C] = true;
   Depth[C] = D;
   for (auto [u, w] : g[C]) {
    if (Vis[u]) continue;
    dfs(dfs, u, D + 1, C);
   }
  };
  Dfs(Dfs, 0); Sub.resize(N); Sub2.resize(N);
  Sz.resize(N); Sz2.resize(N);
 }
 void Mark(int v) {
  int x = v, z = -1;
  for (int i = Depth[v]; i >= 0; --i) {
```

```
   Sub[x] += Dist[v][i]; Sz[x]++;
   if (z != -1) {
    Sub2[z] += Dist[v][i];
    Sz2[z]++;
   }
   z = x; x = Parent[x];
  }
 }
 int64_t Query(int v) {
  int64_t res = 0;
  int x = v, z = -1;
  for (int i = Depth[v]; i >= 0; --i) {
   res += Sub[x] + 1LL * Sz[x] * Dist[v][i];
   if (z != -1) res-=Sub2[z]+1LL*Sz2[z]*Dist[v][i];
   z = x; x = Parent[x];
  }
  return res;
 }
};
```

## 3.10 Tree Hashing

```
uint64_t hsah(int u, int f) {
 uint64_t r = 127;
 for (int v : G[ u ]) if (v != f) {
  uint64_t hh = hsah(v, u);
  r=(r+(hh*hh)%1010101333)%1011820613;
 }
 return r;
}
```

## 3.11 Minimum Mean Cycle

```
/* minimum mean cycle O(VE) */
struct MMC{
#define FZ(n) memset((n),0,sizeof(n))
#define E 101010
#define V 1021
#define inf 1e9
 struct Edge { int v,u; double c; };
 int n, m, prv[V][V], prve[V][V], vst[V];
 Edge e[E];
 vector<int> edgeID, cycle, rho;
 double d[V][V];
 void init( int _n ) { n = _n; m = 0; }
 // WARNING: TYPE matters
 void add_edge( int vi , int ui , double ci )
 { e[ m ++ ] = { vi , ui , ci }; }
 void bellman_ford() {
  for(int i=0; i<n; i++) d[0][i]=0;
  for(int i=0; i<n; i++) {
   fill(d[i+1], d[i+1]+n, inf);
   for(int j=0; j<m; j++) {
    int v = e[j].v, u = e[j].u;
    if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
     d[i+1][u] = d[i][v]+e[j].c;
     prv[i+1][u] = v;
     prve[i+1][u] = j;
    }
   }
  }
 }
 double solve(){
  // returns inf if no cycle, mmc otherwise
  double mmc=inf;
  int st = -1;
  bellman_ford();
  for(int i=0; i<n; i++) {
   double avg=-inf;
   for(int k=0; k<n; k++) {
    if(d[n][i]<inf-eps)
     avg=max(avg,(d[n][i]-d[k][i])/(n-k));
    else avg=max(avg,inf);
   }
   if (avg < mmc) tie(mmc, st) = tie(avg, i);
  }
  FZ(vst);edgeID.clear();cycle.clear();rho.clear();
  for (int i=n; !vst[st]; st=prv[i--][st]) {
   vst[st]++;
   edgeID.PB(prve[i][st]);
   rho.PB(st);
  }
  while (vst[st] != 2) {
   int v = rho.back(); rho.pop_back();
   cycle.PB(v);
   vst[v]++;
  }
  reverse(ALL(edgeID));
  edgeID.resize(SZ(cycle));
  return mmc;
 }
} mmc;
```

## 3.12 Mo's Algorithm on Tree

```
int q; vector< int > G[N];
struct Que{
 int u, v, id;
} que[ N ];
int dfn[N], dfn_, block_id[N], block_, stk[N], stk_;
void dfs( int u, int f ) {
 dfn[ u ] = dfn_++; int saved_rbp = stk_;
 for ( int v : G[ u ] ) {
  if ( v == f ) continue;
  dfs( v, u );
  if ( stk_ - saved_rbp < SQRT_N ) continue;
  for ( ++ block_ ; stk_ != saved_rbp ; )
    block_id[ stk[ -- stk_ ] ] = block_;
 }
 stk[ stk_ ++ ] = u;
}
bool inPath[ N ];
void Diff( int u ) {
 if ( inPath[ u ] ^= 1 ) { /*remove this edge*/ }
 else { /*add this edge*/ }
}
void traverse( int& origin_u, int u ) {
 for ( int g = lca( origin_u, u ) ;
  origin_u != g ; origin_u = parent_of[ origin_u ] )
   Diff( origin_u );
 for (int v = u; v != origin_u; v = parent_of[v])
  Diff( v );
 origin_u = u;
}
void solve() {
 dfs( 1, 1 );
 while ( stk_ ) block_id[ stk[ -- stk_ ] ] = block_;
 sort( que, que + q, [](const Que& x, const Que& y) {
  return tie( block_id[ x.u ], dfn[ x.v ] )
      < tie( block_id[ y.u ], dfn[ y.v ] );
 } );
 int U = 1, V = 1;
 for ( int i = 0 ; i < q ; ++ i ) {
  pass( U, que[ i ].u );
  pass( V, que[ i ].v );
  // we could get our answer of que[ i ].id
 }
}
/*
Method 2:
dfs u:
 push u
 iterate subtree
 push u
Let P = LCA(u, v), and St(u)<=St(v)
if (P == u) query[St(u), St(v)]
else query[Ed(u), St(v)], query[St(P), St(P)]
*/
```

## 3.13 Minimum Steiner Tree

```
// Minimum Steiner Tree
// O(V 3^T + V^2 2^T)
struct SteinerTree{
#define V 33
#define T 8
#define INF 1023456789
 int n , dst[V][V] , dp[1 << T][V] , tdst[V];
 void init( int _n ){
  n = _n;
  for( int i = 0 ; i < n ; i ++ ){
   for( int j = 0 ; j < n ; j ++ )
    dst[ i ][ j ] = INF;
   dst[ i ][ i ] = 0;
  }
 }
 void add_edge( int ui , int vi , int wi ){
  dst[ ui ][ vi ] = min( dst[ ui ][ vi ] , wi );
```

```
  dst[ vi ][ ui ] = min( dst[ vi ][ ui ] , wi );
 }
 void shortest_path(){
  for( int k = 0 ; k < n ; k ++ )
   for( int i = 0 ; i < n ; i ++ )
    for( int j = 0 ; j < n ; j ++ )
     dst[ i ][ j ] = min( dst[ i ][ j ],
        dst[ i ][ k ] + dst[ k ][ j ] );
 }
 int solve( const vector<int>& ter ){
  int t = (int)ter.size();
  for( int i = 0 ; i < ( 1 << t ) ; i ++ )
   for( int j = 0 ; j < n ; j ++ )
    dp[ i ][ j ] = INF;
  for( int i = 0 ; i < n ; i ++ )
   dp[ 0 ][ i ] = 0;
  for( int msk = 1 ; msk < ( 1 << t ) ; msk ++ ){
   if( msk == ( msk & (-msk) ) ){
    int who = __lg( msk );
    for( int i = 0 ; i < n ; i ++ )
     dp[ msk ][ i ] = dst[ ter[ who ] ][ i ];
    continue;
   }
   for( int i = 0 ; i < n ; i ++ )
    for( int submsk = ( msk - 1 ) & msk ; submsk ;
         submsk = ( submsk - 1 ) & msk )
      dp[ msk ][ i ] = min( dp[ msk ][ i ],
              dp[ submsk ][ i ] +
              dp[ msk ^ submsk ][ i ] );
   for( int i = 0 ; i < n ; i ++ ){
    tdst[ i ] = INF;
    for( int j = 0 ; j < n ; j ++ )
     tdst[ i ] = min( tdst[ i ],
           dp[ msk ][ j ] + dst[ j ][ i ] );
   }
   for( int i = 0 ; i < n ; i ++ )
    dp[ msk ][ i ] = tdst[ i ];
  }
  int ans = INF;
  for( int i = 0 ; i < n ; i ++ )
   ans = min( ans , dp[ ( 1 << t ) - 1 ][ i ] );
  return ans;
 }
} solver;
```

### 3.14 Directed Minimum Spanning Tree

```
template <typename T> struct DMST {
 T g[maxn][maxn], fw[maxn];
 int n, fr[maxn];
 bool vis[maxn], inc[maxn];
 void clear() {
  for(int i = 0; i < maxn; ++i) {
   for(int j = 0; j < maxn; ++j) g[i][j] = inf;
   vis[i] = inc[i] = false;
  }
 }
 void addEdge(int u,int v,T w){g[u][v]=min(g[u][v],w);}
 T operator()(int root, int _n) {
  n = _n; T ans = 0;
  if (dfs(root) != n) return -1;
  while (true) {
   for(int i = 1;i <= n;++i) fw[i] = inf, fr[i] = i;
   for (int i = 1; i <= n; ++i) if (!inc[i]) {
    for (int j = 1; j <= n; ++j) {
     if (!inc[j] && i != j && g[j][i] < fw[i]) {
      fw[i] = g[j][i]; fr[i] = j;
     }
    }
   }
   int x = -1;
   for(int i = 1;i <= n;++i)if(i != root && !inc[i]){
    int j = i, c = 0;
    while(j!=root && fr[j]!=i && c<=n) ++c, j=fr[j];
    if (j == root || c > n) continue;
    else { x = i; break; }
   }
   if (!~x) {
    for (int i = 1; i <= n; ++i)
     if (i != root && !inc[i]) ans += fw[i];
    return ans;
   }
   int y = x;
   for (int i = 1; i <= n; ++i) vis[i] = false;
   do {
    ans += fw[y]; y = fr[y]; vis[y] = inc[y] = true;
   } while (y != x);
   inc[x] = false;
   for (int k = 1; k <= n; ++k) if (vis[k]) {
    for (int j = 1; j <= n; ++j) if (!vis[j]) {
     if (g[x][j] > g[k][j]) g[x][j] = g[k][j];
     if (g[j][k] < inf && g[j][k]-fw[k] < g[j][x])
      g[j][x] = g[j][k] - fw[k];
    }
   }
  }
  return ans;
 }
 int dfs(int now) {
  int r = 1; vis[now] = true;
  for (int i = 1; i <= n; ++i)
   if (g[now][i] < inf && !vis[i]) r += dfs(i);
  return r;
 }
};
```

### 3.15 Dominator Tree

```
namespace dominator {
vector<int> g[maxn], r[maxn], rdom[maxn];
int dfn[maxn], rev[maxn], fa[maxn], sdom[maxn];
int dom[maxn], val[maxn], rp[maxn], tk;
void init(int n) {
 // vertices are numbered from 0 to n - 1
 fill(dfn, dfn + n, -1);fill(rev, rev + n, -1);
 fill(fa, fa + n, -1);fill(val, val + n, -1);
 fill(sdom, sdom + n, -1);fill(rp, rp + n, -1);
 fill(dom, dom + n, -1); tk = 0;
 for (int i = 0; i < n; ++i) {
  g[i].clear(); r[i].clear(); rdom[i].clear();
 }
}
void add_edge(int x, int y) { g[x].push_back(y); }
void dfs(int x) {
 rev[dfn[x] = tk] = x;
 fa[tk] = sdom[tk] = val[tk] = tk; tk ++;
 for (int u : g[x]) {
  if (dfn[u] == -1) dfs(u), rp[dfn[u]] = dfn[x];
  r[dfn[u]].push_back(dfn[x]);
 }
}
void merge(int x, int y) { fa[x] = y; }
int find(int x, int c = 0) {
 if (fa[x] == x) return c ? -1 : x;
 int p = find(fa[x], 1);
 if (p == -1) return c ? fa[x] : val[x];
 if (sdom[val[x]]>sdom[val[fa[x]]]) val[x]=val[fa[x]];
 fa[x] = p;
 return c ? p : val[x];
}
vector<int> build(int s, int n) {
// return the father of each node in the dominator tree
// p[i] = -2 if i is unreachable from s
 dfs(s);
 for (int i = tk - 1; i >= 0; --i) {
  for (int u:r[i]) sdom[i]=min(sdom[i],sdom[find(u)]);
  if (i) rdom[sdom[i]].push_back(i);
  for (int &u : rdom[i]) {
   int p = find(u);
   if (sdom[p] == i) dom[u] = i;
   else dom[u] = p;
  }
  if (i) merge(i, rp[i]);
 }
 vector<int> p(n, -2); p[s] = -1;
 for (int i = 1; i < tk; ++i)
  if (sdom[i] != dom[i]) dom[i] = dom[dom[i]];
 for (int i = 1; i < tk; ++i) p[rev[i]] = rev[dom[i]];
 return p;
}}
```

# 4 Matching & Flow

### 4.1 Kuhn Munkres

```
class KM {
```

```
private:
 static constexpr lld INF = 1LL << 60;
 vector<lld> hl,hr,slk;
 vector<int> fl,fr,pre,qu;
 vector<vector<lld>> w;
 vector<bool> vl,vr;
 int n, ql, qr;
 bool check(int x) {
  if (vl[x] = true, fl[x] != -1)
   return vr[qu[qr++] = fl[x]] = true;
  while (x != -1) swap(x, fr[fl[x] = pre[x]]);
  return false;
 }
 void bfs(int s) {
  fill(slk.begin(), slk.end(), INF);
  fill(vl.begin(), vl.end(), false);
  fill(vr.begin(), vr.end(), false);
  ql = qr = 0;
  qu[qr++] = s;
  vr[s] = true;
  while (true) {
   lld d;
   while (ql < qr) {
    for (int x = 0, y = qu[ql++]; x < n; ++x) {
     if(!vl[x]&&slk[x]>=(d=hl[x]+hr[y]-w[x][y])){
      if (pre[x] = y, d) slk[x] = d;
      else if (!check(x)) return;
     }
    }
   }
   d = INF;
   for (int x = 0; x < n; ++x)
    if (!vl[x] && d > slk[x]) d = slk[x];
   for (int x = 0; x < n; ++x) {
    if (vl[x]) hl[x] += d;
    else slk[x] -= d;
    if (vr[x]) hr[x] -= d;
   }
   for (int x = 0; x < n; ++x)
    if (!vl[x] && !slk[x] && !check(x)) return;
  }
 }
public:
 void init( int n_ ) {
  n = n_; qu.resize(n);
  fl.clear(); fl.resize(n, -1);
  fr.clear(); fr.resize(n, -1);
  hr.clear(); hr.resize(n); hl.resize(n);
  w.clear(); w.resize(n, vector<lld>(n));
  slk.resize(n); pre.resize(n);
  vl.resize(n); vr.resize(n);
 }
 void set_edge( int u, int v, lld x ) {w[u][v] = x;}
 lld solve() {
  for (int i = 0; i < n; ++i)
   hl[i] = *max_element(w[i].begin(), w[i].end());
  for (int i = 0; i < n; ++i) bfs(i);
  lld res = 0;
  for (int i = 0; i < n; ++i) res += w[i][fl[i]];
  return res;
 }
} km;
```

## 4.2 Bipartite Matching

```
class BipartiteMatching{
private:
 vector<int> X[N], Y[N];
 int fX[N], fY[N], n;
 bitset<N> walked;
 bool dfs(int x){
  for(auto i:X[x]){
   if(walked[i])continue;
   walked[i]=1;
   if(fY[i]==-1||dfs(fY[i])){
    fY[i]=x;fX[x]=i;
    return 1;
   }
  }
  return 0;
 }
public:
 void init(int _n){
  n=_n; walked.reset();
  for(int i=0;i<n;i++){
   X[i].clear();Y[i].clear();
   fX[i]=fY[i]=-1;
  }
 }
 void add_edge(int x, int y){
  X[x].push_back(y); Y[y].push_back(y);
 }
 int solve(){
  int cnt = 0;
  for(int i=0;i<n;i++){
   walked.reset();
   if(dfs(i)) cnt++;
  }
  // return how many pair matched
  return cnt;
 }
};
```

## 4.3 General Graph Matching

```
const int N = 514, E = (2e5) * 2;
struct Graph{
 int to[E],bro[E],head[N],e;
 int lnk[N],vis[N],stp,n;
 void init( int _n ){
  stp = 0; e = 1; n = _n;
  for( int i = 0 ; i <= n ; i ++ )
   head[i] = lnk[i] = vis[i] = 0;
 }
 void add_edge(int u,int v){
  // 1-base
  to[e]=v,bro[e]=head[u],head[u]=e++;
  to[e]=u,bro[e]=head[v],head[v]=e++;
 }
 bool dfs(int x){
  vis[x]=stp;
  for(int i=head[x];i;i=bro[i]){
   int v=to[i];
   if(!lnk[v]){
    lnk[x]=v,lnk[v]=x;
    return true;
   }else if(vis[lnk[v]]<stp){
    int w=lnk[v];
    lnk[x]=v,lnk[v]=x,lnk[w]=0;
    if(dfs(w)) return true;
    lnk[w]=v,lnk[v]=w,lnk[x]=0;
   }
  }
  return false;
 }
 int solve(){
  int ans = 0;
  for(int i=1;i<=n;i++)
   if(not lnk[i]){
    stp++; ans += dfs(i);
   }
  return ans;
 }
} graph;
```

## 4.4 Minimum Weight Matching (Clique version)

```
struct Graph {
 // 0-base (Perfect Match)
 int n, edge[MXN][MXN];
 int match[MXN],dis[MXN],onstk[MXN];
 vector<int> stk;
 void init(int _n) {
  n = _n;
  for (int i=0; i<n; i++)
   for (int j=0; j<n; j++)
    edge[i][j] = 0;
 }
 void set_edge(int u, int v, int w) {
  edge[u][v] = edge[v][u] = w;
 }
 bool SPFA(int u){
  if (onstk[u]) return true;
  stk.PB(u);
  onstk[u] = 1;
  for (int v=0; v<n; v++){
   if (u != v && match[u] != v && !onstk[v]){
```

```
    int m = match[v];
    if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
     dis[m] = dis[u] - edge[v][m] + edge[u][v];
     onstk[v] = 1;
     stk.PB(v);
     if (SPFA(m)) return true;
     stk.pop_back();
     onstk[v] = 0;
    }
   }
  }
  onstk[u] = 0;
  stk.pop_back();
  return false;
 }

 int solve() {
  // find a match
  for (int i=0; i<n; i+=2){
   match[i] = i+1;
   match[i+1] = i;
  }
  while (true){
   int found = 0;
   for (int i=0; i<n; i++)
    dis[i] = onstk[i] = 0;
   for (int i=0; i<n; i++){
    stk.clear();
    if (!onstk[i] && SPFA(i)){
     found = 1;
     while (SZ(stk)>=2){
      int u = stk.back(); stk.pop_back();
      int v = stk.back(); stk.pop_back();
      match[u] = v;
      match[v] = u;
     }
    }
   }
   if (!found) break;
  }
  int ret = 0;
  for (int i=0; i<n; i++)
   ret += edge[i][match[i]];
  return ret>>1;
 }
} graph;
```

## 4.5 Minimum Cost Circulation

```
struct Edge { int to, cap, rev, cost; };
vector<Edge> g[kN];
int dist[kN], pv[kN], ed[kN];
bool mark[kN];
int NegativeCycle(int n) {
 memset(mark, false, sizeof(mark));
 memset(dist, 0, sizeof(dist));
 int upd = -1;
 for (int i = 0; i <= n; ++i) {
  for (int j = 0; j < n; ++j) {
   int idx = 0;
   for (auto &e : g[j]) {
    if(e.cap > 0 && dist[e.to] > dist[j] + e.cost){
     dist[e.to] = dist[j] + e.cost;
     pv[e.to] = j, ed[e.to] = idx;
     if (i == n) {
      upd = j;
      while(!mark[upd])mark[upd]=1,upd=pv[upd];
      return upd;
     }
    }
    idx++;
   }
  }
 }
 return -1;
}
int Solve(int n) {
 int rt = -1, ans = 0;
 while ((rt = NegativeCycle(n)) >= 0) {
  memset(mark, false, sizeof(mark));
  vector<pair<int, int>> cyc;
  while (!mark[rt]) {
   cyc.emplace_back(pv[rt], ed[rt]);
   mark[rt] = true;
   rt = pv[rt];
  }
  reverse(cyc.begin(), cyc.end());
  int cap = kInf;
  for (auto &i : cyc) {
   auto &e = g[i.first][i.second];
   cap = min(cap, e.cap);
  }
  for (auto &i : cyc) {
   auto &e = g[i.first][i.second];
   e.cap -= cap;
   g[e.to][e.rev].cap += cap;
   ans += e.cost * cap;
  }
 }
 return ans;
}
```

## 4.6 Flow Models

- Maximum/Minimum flow with lower bound / Circulation problem
  1. Construct super source $S$ and sink $T$.
  2. For each edge $(x, y, l, u)$, connect $x \to y$ with capacity $u - l$.
  3. For each vertex $v$, denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  4. If $in(v) > 0$, connect $S \to v$ with capacity $in(v)$, otherwise, connect $v \to T$ with capacity $-in(v)$.
     - To maximize, connect $t \to s$ with capacity $\infty$ (skip this in circulation problem), and let $f$ be the maximum flow from $S$ to $T$. If $f \neq \sum_{v \in V, in(v)>0} in(v)$, there's no solution. Otherwise, the maximum flow from $s$ to $t$ is the answer.
     - To minimize, let $f$ be the maximum flow from $S$ to $T$. Connect $t \to s$ with capacity $\infty$ and let the flow from $S$ to $T$ be $f'$. If $f + f' \neq \sum_{v \in V, in(v)>0} in(v)$, there's no solution. Otherwise, $f'$ is the answer.
  5. The solution of each edge $e$ is $l_e + f_e$, where $f_e$ corresponds to the flow of edge $e$ on the graph.

- Construct minimum vertex cover from maximum matching $M$ on bipartite graph $(X, Y)$
  1. Redirect every edge: $y \to x$ if $(x, y) \in M$, $x \to y$ otherwise.
  2. DFS from unmatched vertices in $X$.
  3. $x \in X$ is chosen iff $x$ is unvisited.
  4. $y \in Y$ is chosen iff $y$ is visited.

- Minimum cost cyclic flow
  1. Consruct super source $S$ and sink $T$
  2. For each edge $(x, y, c)$, connect $x \to y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \to x$ with $(cost, cap) = (-c, 1)$
  3. For each edge with $c < 0$, sum these cost as $K$, then increase $d(y)$ by 1, decrease $d(x)$ by 1
  4. For each vertex $v$ with $d(v) > 0$, connect $S \to v$ with $(cost, cap) = (0, d(v))$
  5. For each vertex $v$ with $d(v) < 0$, connect $v \to T$ with $(cost, cap) = (0, -d(v))$
  6. Flow from $S$ to $T$, the answer is the cost of the flow $C + K$

- Maximum density induced subgraph
  1. Binary search on answer, suppose we're checking answer $T$
  2. Construct a max flow model, let $K$ be the sum of all weights
  3. Connect source $s \to v$, $v \in G$ with capacity $K$
  4. For each edge $(u, v, w)$ in $G$, connect $u \to v$ and $v \to u$ with capacity $w$
  5. For $v \in G$, connect it with sink $v \to t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
  6. $T$ is a valid answer if the maximum flow $f < K|V|$

- Minimum weight edge cover
  1. For each $v \in V$ create a copy $v'$, and connect $u' \to v'$ with weight $w(u, v)$.
  2. Connect $v \to v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to $v$.
  3. Find the minimum weight perfect matching on $G'$.

- Project selection problem
  1. If $p_v > 0$, create edge $(s, v)$ with capacity $p_v$; otherwise, create edge $(v, t)$ with capacity $-p_v$.
  2. Create edge $(u, v)$ with capacity $w$ with $w$ being the cost of choosing $u$ without choosing $v$.
  3. The mincut is equivalent to the maximum profit of a subset of projects.

- 0/1 quadratic programming

  $$\sum_x c_x x + \sum_y c_y \bar{y} + \sum_{xy} c_{xy} x \bar{y} + \sum_{xyx'y'} c_{xyx'y'} (x\bar{y} + x'\bar{y}')$$

  can be minimized by the mincut of the following graph:

  1. Create edge $(x, t)$ with capacity $c_x$ and create edge $(s, y)$ with capacity $c_y$.
  2. Create edge $(x, y)$ with capacity $c_{xy}$.
  3. Create edge $(x, y)$ and edge $(x', y')$ with capacity $c_{xyx'y'}$.

## 4.7 Dinic

```
template <typename Cap = int64_t>
class Dinic{
private:
 struct Edge{
  int to, rev;
  Cap cap;
 };
 int n, st, ed;
 vector<vector<Edge>> G;
 vector<int> lv, idx;
 bool BFS(){
  fill(lv.begin(), lv.end(), -1);
  queue<int> bfs;
  bfs.push(st); lv[st] = 0;
  while(!bfs.empty()){
   int u = bfs.front(); bfs.pop();
   for(auto e: G[u]){
    if(e.cap <= 0 or lv[e.to]!=-1) continue;
    bfs.push(e.to); lv[e.to] = lv[u] + 1;
   }
  }
  return (lv[ed]!=-1);
 }
 Cap DFS(int u, Cap f){
  if(u == ed) return f;
  Cap ret = 0;
  for(int &i = idx[u]; i < (int)G[u].size(); ++i){
   auto &e = G[u][i];
   if(e.cap <= 0 or lv[e.to]!=lv[u]+1) continue;
   Cap nf = DFS(e.to, min(f, e.cap));
   ret += nf; e.cap -= nf; f -= nf;
   G[e.to][e.rev].cap += nf;
   if(f == 0) return ret;
  }
  if(ret == 0) lv[u] = -1;
  return ret;
 }
public:
 void init(int n_, int st_, int ed_){
  n = n_, st = st_, ed = ed_;
  G.resize(n); lv.resize(n);
  fill(G.begin(), G.end(), vector<Edge>());
 }
 void add_edge(int u, int v, Cap c){
  G[u].push_back({v, (int)G[v].size(), c});
  G[v].push_back({u, ((int)G[u].size())-1, 0});
 }
 Cap max_flow(){
  Cap ret = 0;
  while(BFS()){
   idx.assign(n, 0);
   Cap f = DFS(st, numeric_limits<Cap>::max());
   ret += f;
   if(f == 0) break;
  }
  return ret;
 }
};
```

## 4.8 Minimum Cost Maximum Flow

```
class MiniCostMaxiFlow{
 using Cap = int; using Wei = int64_t;
 using PCW = pair<Cap,Wei>;
 static constexpr Cap INF_CAP = 1 << 30;
 static constexpr Wei INF_WEI = 1LL<<60;
private:
 struct Edge{
  int to, back;
  Cap cap; Wei wei;
  Edge() {}
  Edge(int a,int b, Cap c, Wei d):
   to(a),back(b),cap(c),wei(d)
  {}
 };
 int ori, edd;
 vector<vector<Edge>> G;
 vector<int> fa, wh;
 vector<bool> inq;
 vector<Wei> dis;
 PCW SPFA(){
  fill(inq.begin(),inq.end(),false);
  fill(dis.begin(),dis.end(),INF_WEI);
  queue<int> qq; qq.push(ori);
  dis[ori]=0;
  while(!qq.empty()){
   int u=qq.front();qq.pop();
   inq[u] = 0;
   for(int i=0;i<SZ(G[u]);++i){
    Edge e=G[u][i];
    int v=e.to;
    Wei d=e.wei;
    if(e.cap<=0||dis[v]<=dis[u]+d)
     continue;
    dis[v]=dis[u]+d;
    fa[v]=u,wh[v]=i;
    if(inq[v]) continue;
    qq.push(v);
    inq[v]=1;
   }
  }
  if(dis[edd]==INF_WEI) return {-1, -1};
  Cap mw=INF_CAP;
  for(int i=edd;i!=ori;i=fa[i])
   mw=min(mw,G[fa[i]][wh[i]].cap);
  for (int i=edd;i!=ori;i=fa[i]){
   auto &eg=G[fa[i]][wh[i]];
   eg.cap-=mw;
   G[eg.to][eg.back].cap+=mw;
  }
  return {mw,dis[edd]};
 }
public:
 void init(int a,int b,int n){
  ori=a,edd=b;
  G.clear();G.resize(n);
  fa.resize(n);wh.resize(n);
  inq.resize(n); dis.resize(n);
 }
 void add_edge(int st, int ed, Cap c, Wei w){
  G[st].emplace_back(ed,SZ(G[ed]),c,w);
  G[ed].emplace_back(st,SZ(G[st])-1,0,-w);
 }
 PCW solve(){
  /* might modify to
  cc += ret.first * ret.second
  or
  ww += ret.first * ret.second
  */
  Cap cc=0; Wei ww=0;
  while(true){
   PCW ret=SPFA();
   if(ret.first==-1) break;
   cc+=ret.first;
   ww+=ret.second;
  }
  return {cc,ww};
 }
} mcmf;
```

## 4.9 Global Min-Cut

```
const int maxn = 500 + 5;
int w[maxn][maxn], g[maxn];
bool v[maxn], del[maxn];
void add_edge(int x, int y, int c) {
 w[x][y] += c; w[y][x] += c;
}
pair<int, int> phase(int n) {
 memset(v, false, sizeof(v));
 memset(g, 0, sizeof(g));
 int s = -1, t = -1;
 while (true) {
  int c = -1;
  for (int i = 0; i < n; ++i) {
   if (del[i] || v[i]) continue;
   if (c == -1 || g[i] > g[c]) c = i;
  }
  if (c == -1) break;
  v[s = t, t = c] = true;
  for (int i = 0; i < n; ++i) {
   if (del[i] || v[i]) continue;
   g[i] += w[c][i];
  }
```

```
 }
 return make_pair(s, t);
}
int mincut(int n) {
 int cut = 1e9;
 memset(del, false, sizeof(del));
 for (int i = 0; i < n - 1; ++i) {
  int s, t; tie(s, t) = phase(n);
  del[t] = true; cut = min(cut, g[t]);
  for (int j = 0; j < n; ++j) {
   w[s][j] += w[t][j]; w[j][s] += w[j][t];
  }
 }
 return cut;
}
```

# 5 Math

## 5.1 Prime Table

1002939109, 1020288887, 1028798297, 1038684299, 1041211027, 1051762951, 1058585963, 1063020809, 1147930723, 1172520109, 1183835981, 1187659051, 1241251303, 1247184097, 1255940849, 1272759031, 1287027493, 1288511629, 1294632499, 1312650799, 1868732623, 1884198443, 1884616807, 1885059541, 1909942399, 1914471137, 1923951707, 1925453197, 1979612177, 1980446837, 1989761941, 2007826547, 2008033571, 2011186739, 2039465081, 2039728567, 2093735719, 2116097521, 2123852629, 2140170259, 3148478261, 3153064147, 3176351071, 3187523093, 3196772239, 3201312913, 3203063977, 3204840059, 3210224309, 3213032591, 3217689851, 3218469083, 3219857533, 3231880427, 3235951699, 3273767923, 3276188869, 3277183181, 3282463507, 3285553889, 3319309027, 3327005333, 3327574903, 3341387953, 3373293941, 3380077549, 3380892997, 3381118801

## 5.2 $\lfloor \frac{n}{i} \rfloor$ Enumeration

$T_0 = 1, T_{i+1} = \lfloor \frac{n}{\lfloor \frac{n}{T_i+1} \rfloor} \rfloor$

## 5.3 ax+by=gcd

```
// ax+ny = 1, ax+ny == ax == 1 (mod n)
void exgcd(lld x,lld y,lld &g,lld &a,lld &b) {
 if (y == 0) g=x,a=1,b=0;
 else exgcd(y,x%y,g,b,a),b-=(x/y)*a;
}
```

## 5.4 Pollard Rho

```
// does not work when n is prime
// return any non-trivial factor
llu pollard_rho(llu n){
 static auto f=[](llu x,llu k,llu m){
  return add(k,mul(x,x,m),m);
 };
 if (!(n&1)) return 2;
 mt19937 rnd(120821011);
 while(true){
  llu y=2,yy=y,x=rnd()%n,t=1;
  for(llu sz=2;t==1;sz<<=1) {
   for(llu i=0;i<sz;++i){
    if(t!=1)break;
    yy=f(yy,x,n);
    t=gcd(yy>y?yy-y:y-yy,n);
   }
   y=yy;
  }
  if(t!=1&&t!=n) return t;
 }
}
```

## 5.5 Pi Count (Linear Sieve)

```
static constexpr int N = 1000000 + 5;
lld pi[N];
vector<int> primes;
bool sieved[N];
lld cube_root(lld x){
 lld s=cbrt(x-static_cast<long double>(0.1));
 while(s*s*s <= x) ++s;
 return s-1;
}
lld square_root(lld x){
 lld s=sqrt(x-static_cast<long double>(0.1));
 while(s*s <= x) ++s;
 return s-1;
}
void init(){
 primes.reserve(N);
 primes.push_back(1);
 for(int i=2;i<N;i++) {
  if(!sieved[i]) primes.push_back(i);
  pi[i] = !sieved[i] + pi[i-1];
  for(int p: primes) if(p > 1) {
   if(p * i >= N) break;
   sieved[p * i] = true;
   if(p % i == 0) break;
  }
 }
}
lld phi(lld m, lld n) {
 static constexpr int MM = 80000, NN = 500;
 static lld val[MM][NN];
 if(m<MM&&n<NN&&val[m][n])return val[m][n]-1;
 if(n == 0) return m;
 if(primes[n] >= m) return 1;
 lld ret = phi(m,n-1)-phi(m/primes[n],n-1);
 if(m<MM&&n<NN) val[m][n] = ret+1;
 return ret;
}
lld pi_count(lld);
lld P2(lld m, lld n) {
 lld sm = square_root(m), ret = 0;
 for(lld i = n+1;primes[i]<=sm;i++)
  ret+=pi_count(m/primes[i])-pi_count(primes[i])+1;
 return ret;
}
lld pi_count(lld m) {
 if(m < N) return pi[m];
 lld n = pi_count(cube_root(m));
 return phi(m, n) + n - 1 - P2(m, n);
}
```

## 5.6 Range Sieve

```
const int MAX_SQRT_B = 50000;
const int MAX_L = 200000 + 5;

bool is_prime_small[MAX_SQRT_B];
bool is_prime[MAX_L];

void sieve(lld l, lld r){
 // [l, r)
 for(lld i=2;i*i<r;i++) is_prime_small[i] = true;
 for(lld i=l;i<r;i++) is_prime[i-l] = true;
 if(l==1) is_prime[0] = false;
 for(lld i=2;i*i<r;i++){
  if(!is_prime_small[i]) continue;
  for(lld j=i*i;j*j<r;j+=i) is_prime_small[j]=false;
  for(lld j=std::max(2LL, (l+i-1)/i)*i;j<r;j+=i)
    is_prime[j-l]=false;
 }
}
```

## 5.7 Miller Rabin

```
bool isprime(llu x){
 static llu magic[]={2,325,9375,28178,\
          450775,9780504,1795265022};
 static auto witn=[](llu a,llu u,llu n,int t)
 ->bool{
  if (!(a = mpow(a%n,u,n)))return 0;
  while(t--){
   llu a2=mul(a,a,n);
   if(a2==1 && a!=1 && a!=n-1)
    return 1;
   a = a2;
  }
  return a!=1;
 };
 if(x<2)return 0;
 if(!(x&1))return x==2;
 llu x1=x-1;int t=0;
 while(!(x1&1))x1>>=1,t++;
 for(llu m:magic)if(witn(m,x1,x,t))return 0;
 return 1;
}
```

## 5.8 Inverse Element

```cpp
// x's inverse mod k
long long GetInv(long long x, long long k){
 // k is prime: euler_(k)=k-1
 return qPow(x, euler_phi(k)-1);
}
// if you need [1, x] (most use: [1, k-1]
void solve(int x, long long k){
 inv[1] = 1;
 for(int i=2;i<x;i++)
  inv[i] = ((long long)(k - k/i) * inv[k % i]) % k;
}
```

## 5.9 Euler Phi Function

```cpp
/*
  extended euler:
  a^b mod p
  if gcd(a, p)==1: a^(b%phi(p))
  elif b < phi(p): a^b mod p
  else a^(b%phi(p) + phi(p))
 */
lld euler_phi(int x){
 lld r=1;
 for(int i=2;i*i<=x;++i){
  if(x%i==0){
   x/=i; r*=(i-1);
   while(x%i==0){
    x/=i; r*=i;
   }
  }
 }
 if(x>1) r*=x-1;
 return r;
}
vector<int> primes;
bool notprime[N];
lld phi[N];
void euler_sieve(int n){
 for(int i=2;i<n;i++){
  if(!notprime[i]){
   primes.push_back(i); phi[i] = i-1;
  }
  for(auto j: primes){
   if(i*j >= n) break;
   notprime[i*j] = true;
   phi[i*j] = phi[i] * phi[j];
   if(i % j == 0){
    phi[i*j] = phi[i] * j;
    break;
   }
  }
 }
}
```

## 5.10 Gauss Elimination

```cpp
void gauss(vector<vector<double>> &d) {
 int n = d.size(), m = d[0].size();
 for (int i = 0; i < m; ++i) {
  int p = -1;
  for (int j = i; j < n; ++j) {
   if (fabs(d[j][i]) < eps) continue;
   if (p == -1 || fabs(d[j][i])>fabs(d[p][i])) p=j;
  }
  if (p == -1) continue;
  for (int j = 0; j < m; ++j) swap(d[p][j], d[i][j]);
  for (int j = 0; j < n; ++j) {
   if (i == j) continue;
   double z = d[j][i] / d[i][i];
   for (int k = 0; k < m; ++k) d[j][k] -= z*d[i][k];
  }
 }
}
```

## 5.11 Fast Fourier Transform

```cpp
namespace fft {
using VI = vector<int>;
using VL = vector<long long>;
const double pi = acos(-1);
cplx omega[maxn + 1];
void prefft() {
 generate_n(omega, maxn + 1, [i=0]()mutable{
  auto j = i++;
  return cplx(cos(2*pi*j/maxn), sin(2*pi*j/maxn));
 });
}
void fft(vector<cplx> &v, int n) {
 int z = __builtin_ctz(n) - 1;
 for (int i = 0; i < n; ++i) {
  int x = 0, j = 0;
  for (;(1 << j) < n;++j) x^=(i >> j & 1)<<(z - j);
  if (x > i) swap(v[x], v[i]);
 }
 for (int s = 2; s <= n; s <<= 1) {
  int z = s >> 1;
  for (int i = 0; i < n; i += s) {
   for (int k = 0; k < z; ++k) {
    cplx x = v[i + z + k] * omega[maxn / s * k];
    v[i + z + k] = v[i + k] - x;
    v[i + k] = v[i + k] + x;
   }
  }
 }
}
void ifft(vector<cplx> &v, int n) {
 fft(v, n);
 reverse(v.begin() + 1, v.end());
 for (int i=0;i<n;++i) v[i] = v[i] * cplx(1. / n, 0);
}
VL convolution(const VI &a, const VI &b) {
 // Should be able to handle N <= 10^5, C <= 10^4
 int sz = 1;
 while (sz < a.size() + b.size() - 1) sz <<= 1;
 vector<cplx> v(sz);
 for (int i = 0; i < sz; ++i) {
  double re = i < a.size() ? a[i] : 0;
  double im = i < b.size() ? b[i] : 0;
  v[i] = cplx(re, im);
 }
 fft(v, sz);
 for (int i = 0; i <= sz / 2; ++i) {
  int j = (sz - i) & (sz - 1);
  cplx x = (v[i] + v[j].conj()) * (v[i] - v[j].conj())
     * cplx(0, -0.25);
  if (j != i) v[j] = (v[j] + v[i].conj()) * (v[j] - v[i
      ].conj()) * cplx(0, -0.25);
  v[i] = x;
 }
 ifft(v, sz);
 VL c(sz);
 for (int i = 0; i < sz; ++i) c[i] = round(v[i].re);
 return c;
}
VI convolution_mod(const VI &a, const VI &b, int p) {
 int sz = 1;
 while (sz + 1 < a.size() + b.size()) sz <<= 1;
 vector<cplx> fa(sz), fb(sz);
 for (int i = 0; i < (int)a.size(); ++i)
  fa[i] = cplx(a[i] & ((1 << 15) - 1), a[i] >> 15);
 for (int i = 0; i < (int)b.size(); ++i)
  fb[i] = cplx(b[i] & ((1 << 15) - 1), b[i] >> 15);
 fft(fa, sz), fft(fb, sz);
 double r = 0.25 / sz;
 cplx r2(0, -1), r3(r, 0), r4(0, -r), r5(0, 1);
 for (int i = 0; i <= (sz >> 1); ++i) {
  int j = (sz - i) & (sz - 1);
  cplx a1 = (fa[i] + fa[j].conj());
  cplx a2 = (fa[i] - fa[j].conj()) * r2;
  cplx b1 = (fb[i] + fb[j].conj()) * r3;
  cplx b2 = (fb[i] - fb[j].conj()) * r4;
  if (i != j) {
   cplx c1 = (fa[j] + fa[i].conj());
   cplx c2 = (fa[j] - fa[i].conj()) * r2;
   cplx d1 = (fb[j] + fb[i].conj()) * r3;
   cplx d2 = (fb[j] - fb[i].conj()) * r4;
   fa[i] = c1 * d1 + c2 * d2 * r5;
   fb[i] = c1 * d2 + c2 * d1;
  }
  fa[j] = a1 * b1 + a2 * b2 * r5;
  fb[j] = a1 * b2 + a2 * b1;
 }
 fft(fa, sz), fft(fb, sz);
 vector<int> res(sz);
 for (int i = 0; i < sz; ++i) {
  long long a = round(fa[i].re), b = round(fb[i].re),
```

```
      c = round(fa[i].im);
   res[i] = (a+((b % p) << 15)+((c % p) << 30)) % p;
  }
  return res;
}}
```

## 5.12 Chinese Remainder

```
lld crt(lld ans[], lld pri[], int n){
 lld M = 1, ret = 0;
 for(int i=0;i<n;i++) M *= pri[i];
 for(int i=0;i<n;i++){
  lld iv = (gcd(M/pri[i],pri[i]).FF+pri[i])%pri[i];
  ret += (ans[i]*(M/pri[i])%M * iv)%M;
  ret %= M;
 }
 return ret;
}
/*
Another:
x = a1 % m1
x = a2 % m2
g = gcd(m1, m2)
assert((a1-a2)%g==0)
[p, q] = exgcd(m2/g, m1/g)
return a2+m2*(p*(a1-a2)/g)
0 <= x < lcm(m1, m2)
*/
```

## 5.13 Berlekamp Massey

```
// x: 1-base, p[]: 0-base
template<size_t N>
vector<llf> BM(llf x[N],size_t n){
 size_t f[N]={0},t=0;llf d[N];
 vector<llf> p[N];
 for(size_t i=1,b=0;i<=n;++i) {
  for(size_t j=0;j<p[t].size();++j)
   d[i]+=x[i-j-1]*p[t][j];
  if(abs(d[i]-=x[i])<=EPS)continue;
  f[t]=i;if(!t){p[++t].resize(i);continue;}
  vector<llf> cur(i-f[b]-1);
  llf k=-d[i]/d[f[b]];cur.PB(-k);
  for(size_t j=0;j<p[b].size();j++)
   cur.PB(p[b][j]*k);
  if(cur.size()<p[t].size())cur.resize(p[t].size());
  for(size_t j=0;j<p[t].size();j++)cur[j]+=p[t][j];
  if(i-f[b]+p[b].size()>=p[t].size()) b=t;
  p[++t]=cur;
 }
 return p[t];
}
```

## 5.14 NTT

```
template <int mod, int G, int maxn>
struct NTT {
 static_assert (maxn == (maxn & -maxn));
 int roots[maxn];
 NTT () {
  int r = modpow(G, (mod - 1) / maxn);
  for (int i = maxn >> 1; i; i >>= 1) {
   roots[i] = 1;
   for (int j = 1; j < i; j++)
    roots[i + j] = modmul(roots[i + j - 1], r);
   r = modmul(r, r);
  }
 }
 // n must be 2^k, and 0 <= F[i] < mod
 void inplace_ntt(int n, int F[], bool inv = false) {
  for (int i = 0, j = 0; i < n; i++) {
   if (i < j) swap(F[i], F[j]);
   for (int k = n>>1; (j^=k) < k; k>>=1);
  }
  for (int s = 1; s < n; s *= 2) {
   for (int i = 0; i < n; i += s * 2) {
    for (int j = 0; j < s; j++) {
     int a = F[i+j];
     int b = modmul(F[i+j+s], roots[s+j]);
     F[i+j] = modadd(a, b);  // a + b
     F[i+j+s] = modsub(a, b); // a - b
    }
   }
  }
  if (inv) {
   int invn = modinv(n);
   for (int i = 0; i < n; i++)
    F[i] = modmul(F[i], invn);
   reverse(F + 1, F + n);
  }
 }
};
const int P=2013265921,root=31;
const int MAXN=1<<20;
NTT<P, root, MAXN> ntt;
```

## 5.15 Polynomial Operations

```
using VL = vector<LL>;
#define fi(s, n) for (int i=int(s); i<int(n); ++i)
#define Fi(s, n) for (int i=int(n); i>int(s); --i)
int n2k(int n) {
 int sz = 1; while (sz < n) sz <<= 1;
 return sz;
}
template<int MAXN, LL P, LL RT> // MAXN = 2^k
struct Poly { // coefficients in [0, P)
 static NTT<MAXN, P, RT> ntt;
 VL coef;
 int n() const { return coef.size(); } // n()>=1
 LL *data() { return coef.data(); }
 const LL *data() const { return coef.data(); }
 LL &operator[](size_t i) { return coef[i]; }
 const LL &operator[](size_t i)const{return coef[i];}
 Poly(initializer_list<LL> a) : coef(a) { }
 explicit Poly(int _n = 1) : coef(_n) { }
 Poly(const LL *arr, int _n) : coef(arr, arr + _n) {}
 Poly(const Poly &p, int _n) : coef(_n) {
  copy_n(p.data(), min(p.n(), _n), data());
 }
 Poly& irev(){return reverse(data(),data()+n()),*this;}
 Poly& isz(int _n) { return coef.resize(_n), *this; }
 Poly& iadd(const Poly &rhs) { // n() == rhs.n()
  fi(0, n()) if ((coef[i]+=rhs[i]) >= P)coef[i]-=P;
  return *this;
 }
 Poly& imul(LL k) {
  fi(0, n()) coef[i] = coef[i] * k % P;
  return *this;
 }
 Poly Mul(const Poly &rhs) const {
  const int _n = n2k(n() + rhs.n() - 1);
  Poly X(*this, _n), Y(rhs, _n);
  ntt(X.data(), _n), ntt(Y.data(), _n);
  fi(0, _n) X[i] = X[i] * Y[i] % P;
  ntt(X.data(), _n, true);
  return X.isz(n() + rhs.n() - 1);
 }
 Poly Inv() const { // coef[0] != 0
  if (n() == 1) return {ntt.minv(coef[0])};
  const int _n = n2k(n() * 2);
  Poly Xi = Poly(*this, (n() + 1)/2).Inv().isz(_n);
  Poly Y(*this, _n);
  ntt(Xi.data(), _n), ntt(Y.data(), _n);
  fi(0, _n) {
   Xi[i] *= (2 - Xi[i] * Y[i]) % P;
   if ((Xi[i] %= P) < 0) Xi[i] += P;
  }
  ntt(Xi.data(), _n, true);
  return Xi.isz(n());
 }
 Poly Sqrt() const { // Jacobi(coef[0], P) = 1
  if (n()==1) return {QuadraticResidue(coef[0], P)};
  Poly X = Poly(*this, (n()+1) / 2).Sqrt().isz(n());
  return X.iadd(Mul(X.Inv()).isz(n())).imul(P/2+1);
 }
 pair<Poly, Poly> DivMod(const Poly &rhs) const {
  // (rhs.)back() != 0
  if (n() < rhs.n()) return {{0}, *this};
  const int _n = n() - rhs.n() + 1;
  Poly X(rhs); X.irev().isz(_n);
  Poly Y(*this); Y.irev().isz(_n);
  Poly Q = Y.Mul(X.Inv()).isz(_n).irev();
  X = rhs.Mul(Q), Y = *this;
  fi(0, n()) if ((Y[i] -= X[i]) < 0) Y[i] += P;
  return {Q, Y.isz(max(1, rhs.n() - 1))};
 }
```

```
Poly Dx() const {
 Poly ret(n() - 1);
 fi(0, ret.n()) ret[i] = (i + 1) * coef[i + 1] % P;
 return ret.isz(max(1, ret.n()));
}
Poly Sx() const {
 Poly ret(n() + 1);
 fi(0, n()) ret[i + 1]=ntt.minv(i + 1)*coef[i] % P;
 return ret;
}
Poly _tmul(int nn, const Poly &rhs) const {
 Poly Y = Mul(rhs).isz(n() + nn - 1);
 return Poly(Y.data() + n() - 1, nn);
}
VL _eval(const VL &x, const auto up)const{
 const int _n = (int)x.size();
 if (!_n) return {};
 vector<Poly> down(_n * 2);
 down[1] = DivMod(up[1]).second;
 fi(2,_n*2) down[i]=down[i/2].DivMod(up[i]).second;
 /* down[1] = Poly(up[1]).irev().isz(n()).Inv().irev()
   ._tmul(_n, *this);
 fi(2, _n * 2) down[i] = up[i ^ 1]._tmul(up[i].n() -
   1, down[i / 2]); */
 VL y(_n);
 fi(0, _n) y[i] = down[_n + i][0];
 return y;
}
static vector<Poly> _tree1(const VL &x) {
 const int _n = (int)x.size();
 vector<Poly> up(_n * 2);
 fi(0, _n) up[_n + i] = {(x[i] ? P - x[i] : 0), 1};
 Fi(0, _n-1) up[i] = up[i * 2].Mul(up[i * 2 + 1]);
 return up;
}
VL Eval(const VL&x)const{return _eval(x,_tree1(x));}
static Poly Interpolate(const VL &x, const VL &y) {
 const int _n = (int)x.size();
 vector<Poly> up = _tree1(x), down(_n * 2);
 VL z = up[1].Dx()._eval(x, up);
 fi(0, _n) z[i] = y[i] * ntt.minv(z[i]) % P;
 fi(0, _n) down[_n + i] = {z[i]};
 Fi(0, _n-1) down[i]=down[i * 2].Mul(up[i * 2 + 1])
  .iadd(down[i * 2 + 1].Mul(up[i * 2]));
 return down[1];
}
Poly Ln() const { // coef[0] == 1
 return Dx().Mul(Inv()).Sx().isz(n());
}
Poly Exp() const { // coef[0] == 0
 if (n() == 1) return {1};
 Poly X = Poly(*this, (n() + 1)/2).Exp().isz(n());
 Poly Y = X.Ln(); Y[0] = P - 1;
 fi(0, n()) if((Y[i] = coef[i] - Y[i]) < 0)Y[i]+=P;
 return X.Mul(Y).isz(n());
}
Poly Pow(const string &K) const {
 int nz = 0;
 while (nz < n() && !coef[nz]) ++nz;
 LL nk = 0, nk2 = 0;
 for (char c : K) {
  nk = (nk * 10 + c - '0') % P;
  nk2 = nk2 * 10 + c - '0';
  if (nk2 * nz >= n()) return Poly(n());
  nk2 %= P - 1;
 }
 if (!nk && !nk2) return Poly({1}, n());
 Poly X(data() + nz, n() - nz * nk2);
 LL x0 = X[0];
 return X.imul(ntt.minv(x0)).Ln().imul(nk).Exp()
  .imul(ntt.mpow(x0, nk2)).irev().isz(n()).irev();
}
Poly InvMod(int L) { // (to evaluate linear recursion)
 Poly R{1, 0}; // *this * R mod x^L = 1 (*this[0] ==
   1)
 for (int level = 0; (1 << level) < L; ++level) {
  Poly O = R.Mul(Poly(data(), min(2 << level, n())));
  Poly Q(2 << level); Q[0] = 1;
  for (int j = (1 << level); j < (2 << level); ++j)
   Q[j] = (P - O[j]) % P;
  R = R.Mul(Q).isz(4 << level);
 }
 return R.isz(L);
}
static LL LinearRecursion(const VL&a,const VL&c,LL n){
 // a_n = \sum c_j a_(n-j)
 const int k = (int)a.size();
 assert((int)c.size() == k + 1);
 Poly C(k + 1), W({1}, k), M = {0, 1};
 fi(1, k + 1) C[k - i] = c[i] ? P - c[i] : 0;
 C[k] = 1;
 while (n) {
  if (n % 2) W = W.Mul(M).DivMod(C).second;
  n /= 2, M = M.Mul(M).DivMod(C).second;
 }
 LL ret = 0;
 fi(0, k) ret = (ret + W[i] * a[i]) % P;
 return ret;
}
};
#undef fi
#undef Fi
using Poly_t = Poly<131072 * 2, 998244353, 3>;
template<> decltype(Poly_t::ntt) Poly_t::ntt = {};
```

## 5.16 FWT

```
/* xor convolution:
 * x = (x0,x1) , y = (y0,y1)
 * z = ( x0y0 + x1y1 , x0y1 + x1y0 )
 * =>
 * x' = ( x0+x1 , x0-x1 ) , y' = ( y0+y1 , y0-y1 )
 * z' = ( ( x0+x1 )( y0+y1 ) , ( x0-x1 )( y0-y1 ) )
 * z = (1/2) * z''
 * or convolution:
 * x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
 * and convolution:
 * x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
const LL MOD = 1e9+7;
inline void fwt( LL x[ MAXN ] , int N , bool inv=0 ) {
 for( int d = 1 ; d < N ; d <<= 1 ) {
  int d2 = d<<1;
  for( int s = 0 ; s < N ; s += d2 )
   for( int i = s , j = s+d ; i < s+d ; i++, j++ ){
    LL ta = x[ i ] , tb = x[ j ];
    x[ i ] = ta+tb;
    x[ j ] = ta-tb;
    if( x[ i ] >= MOD ) x[ i ] -= MOD;
    if( x[ j ] < 0 ) x[ j ] += MOD;
   }
 }
 if( inv )
  for( int i = 0 ; i < N ; i++ ) {
   x[ i ] *= inv( N, MOD );
   x[ i ] %= MOD;
  }
}
```

## 5.17 DiscreteLog

```
lld BSGS(lld P, lld B, lld N) {
 // find B^L = N mod P
 unordered_map<lld, int> R;
 lld sq = (lld)sqrt(P);
 lld t = 1;
 for (int i = 0; i < sq; i++) {
  if (t == N) return i;
  if (!R.count(t)) R[t] = i;
  t = (t * B) % P;
 }
 lld f = inverse(t, P);
 for(int i=0;i<=sq+1;i++) {
  if (R.count(N))
   return i * sq + R[N];
  N = (N * f) % P;
 }
 return -1;
}
```

## 5.18 FloorSum

```
// @param n `n < 2^32`
// @param m `1 <= m < 2^32`
// @return sum_{i=0}^{n-1} floor((ai + b)/m) mod 2^64
llu floor_sum_unsigned(llu n, llu m, llu a, llu b) {
 llu ans = 0;
```

```
 while (true) {
  if (a >= m) {
   ans += n * (n - 1) / 2 * (a / m); a %= m;
  }
  if (b >= m) {
   ans += n * (b / m); b %= m;
  }
  llu y_max = a * n + b;
  if (y_max < m) break;
  // y_max < m * (n + 1)
  // floor(y_max / m) <= n
  n = (llu)(y_max / m), b = (llu)(y_max % m);
  swap(m, a);
 }
 return ans;
}
lld floor_sum(lld n, lld m, lld a, lld b) {
 assert(0 <= n && n < (1LL << 32));
 assert(1 <= m && m < (1LL << 32));
 llu ans = 0;
 if (a < 0) {
  llu a2 = (a % m + m) % m;
  ans -= 1ULL * n * (n - 1) / 2 * ((a2 - a) / m);
  a = a2;
 }
 if (b < 0) {
  llu b2 = (b % m + m) % m;
  ans -= 1ULL * n * ((b2 - b) / m);
  b = b2;
 }
 return ans + floor_sum_unsigned(n, m, a, b);
}
```

## 5.19 Quadratic residue

```
struct Status{
 ll x,y;
};
ll w;
Status mult(const Status& a,const Status& b,ll mod){
 Status res;
 res.x=(a.x*b.x+a.y*b.y%mod*w)%mod;
 res.y=(a.x*b.y+a.y*b.x)%mod;
 return res;
}
inline Status qpow(Status _base,ll _pow,ll _mod){
 Status res = {1, 0};
 while(_pow>0){
  if(_pow&1) res=mult(res,_base,_mod);
  _base=mult(_base,_base,_mod);
  _pow>>=1;
 }
 return res;
}
inline ll check(ll x,ll p){
 return qpow_mod(x,(p-1)>>1,p);
}
inline ll get_root(ll n,ll p){
 if(p==2) return 1;
 if(check(n,p)==p-1) return -1;
 ll a;
 while(true){
  a=rand()%p;
  w=((a*a-n)%p+p)%p;
  if(check(w,p)==p-1) break;
 }
 Status res = {a, 1}
 res=qpow(res,(p+1)>>1,p);
 return res.x;
}
```

## 5.20 De-Bruijn

```
int res[maxn], aux[maxn], sz;
void db(int t, int p, int n, int k) {
 if (t > n) {
  if (n % p == 0)
   for (int i = 1; i <= p; ++i)
    res[sz++] = aux[i];
 } else {
  aux[t] = aux[t - p];
  db(t + 1, p, n, k);
  for (int i = aux[t - p] + 1; i < k; ++i) {
   aux[t] = i;
   db(t + 1, t, n, k);
  }
 }
}
int de_bruijn(int k, int n) {
 // return cyclic string of len k^n s.t. every string
 // of len n using k char appears as a substring.
 if (k == 1) {
  res[0] = 0;
  return 1;
 }
 for (int i = 0; i < k * n; i++) aux[i] = 0;
 sz = 0;
 db(1, 1, n, k);
 return sz;
}
```

## 5.21 Simplex Construction

Standard form: maximize $\sum_{1\le i\le n} c_i x_i$ such that for all $1 \le j \le m$, $\sum_{1\le i\le n} A_{ji}x_i \le b_j$.and $x_i \ge 0$ for all $1 \le i \le n$.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1\le i\le n} A_{ji}x_i \ge b_j \rightarrow \sum_{1\le i\le n} -A_{ji}x_i \le -b_j$
3. $\sum_{1\le i\le n} A_{ji}x_i = b_j$
   - $\sum_{1\le i\le n} A_{ji}x_i \le b_j$
   - $\sum_{1\le i\le n} A_{ji}x_i \ge b_j$
4. If $x_i$ has no lower bound, replace $x_i$ with $x_i - x'_i$

## 5.22 Simplex

```
namespace simplex {
// maximize c^Tx under Ax <= B
// return VD(n, -inf) if the solution doesn't exist
// return VD(n, +inf) if the solution is unbounded
using VD = vector<double>;
using VVD = vector<vector<double>>;
const double eps = 1e-9;
const double inf = 1e+9;
int n, m;
VVD d;
vector<int> p, q;
void pivot(int r, int s) {
 double inv = 1.0 / d[r][s];
 for (int i = 0; i < m + 2; ++i)
  for (int j = 0; j < n + 2; ++j)
   if (i != r && j != s)
    d[i][j] -= d[r][j] * d[i][s] * inv;
 for(int i=0;i<m+2;++i) if (i != r) d[i][s] *= -inv;
 for(int j=0;j<n+2;++j) if (j != s) d[r][j] *= +inv;
 d[r][s] = inv; swap(p[r], q[s]);
}
bool phase(int z) {
 int x = m + z;
 while (true) {
  int s = -1;
  for (int i = 0; i <= n; ++i) {
   if (!z && q[i] == -1) continue;
   if (s == -1 || d[x][i] < d[x][s]) s = i;
  }
  if (d[x][s] > -eps) return true;
  int r = -1;
  for (int i = 0; i < m; ++i) {
   if (d[i][s] < eps) continue;
   if (r == -1 || \
    d[i][n+1]/d[i][s] < d[r][n+1]/d[r][s]) r = i;
  }
  if (r == -1) return false;
  pivot(r, s);
 }
}
VD solve(const VVD &a, const VD &b, const VD &c) {
 m = b.size(), n = c.size();
 d = VVD(m + 2, VD(n + 2));
 for (int i = 0; i < m; ++i)
  for (int j = 0; j < n; ++j) d[i][j] = a[i][j];
 p.resize(m), q.resize(n + 1);
 for (int i = 0; i < m; ++i)
  p[i] = n + i, d[i][n] = -1, d[i][n + 1] = b[i];
 for (int i = 0; i < n; ++i) q[i] = i,d[m][i] = -c[i];
```

```
 q[n] = -1, d[m + 1][n] = 1;
 int r = 0;
 for (int i = 1; i < m; ++i)
  if (d[i][n + 1] < d[r][n + 1]) r = i;
 if (d[r][n + 1] < -eps) {
  pivot(r, n);
  if (!phase(1) || d[m + 1][n + 1] < -eps)
   return VD(n, -inf);
  for (int i = 0; i < m; ++i) if (p[i] == -1) {
   int s = min_element(d[i].begin(), d[i].end() - 1)
       - d[i].begin();
   pivot(i, s);
  }
 }
 if (!phase(0)) return VD(n, inf);
 VD x(n);
 for (int i = 0; i < m; ++i)
  if (p[i] < n) x[p[i]] = d[i][n + 1];
 return x;
}}
```

# 6 Geometry

## 6.1 Basic Geometry

```
using coord_t = int;
using Real = double;
using Point = std::complex<coord_t>;
int sgn(coord_t x) {
 return (x > 0) - (x < 0);
}
coord_t dot(Point a, Point b) {
 return real(conj(a) * b);
}
coord_t cross(Point a, Point b) {
 return imag(conj(a) * b);
}
int ori(Point a, Point b, Point c) {
 return sgn(cross(b - a, c - a));
}
bool operator<(const Point &a, const Point &b) {
 return real(a) != real(b)
  ? real(a) < real(b) : imag(a) < imag(b);
}
int argCmp(Point a, Point b) {
 // -1 / 0 / 1 <-> < / == / > (atan2)
 int qa = (imag(a) == 0
   ? (real(a) < 0 ? 3 : 1) : (imag(a) < 0 ? 0 : 2));
 int qb = (imag(b) == 0
   ? (real(b) < 0 ? 3 : 1) : (imag(b) < 0 ? 0 : 2));
 if (qa != qb)
  return sgn(qa - qb);
 return sgn(cross(b, a));
}
template <typename V> Real area(const V & pt) {
 coord_t ret = 0;
 for (int i = 1; i + 1 < (int)pt.size(); i++)
  ret += cross(pt[i] - pt[0], pt[i+1] - pt[0]);
 return ret / 2.0;
}
```

## 6.2 Circle Class

```
struct Circle { Point o; Real r; };

vector<Real> intersectAngle(Circle a, Circle b) {
 Real d2 = norm(a.o - b.o);
 if (norm(A.r - B.r) >= d2)
  if (A.r < B.r)
   return {-PI, PI};
  else
   return {};
 if (norm(A.r + B.r) <= d2) return {};
 Real dis = hypot(A.x - B.x, A.y - B.y);
 Real theta = atan2(B.y - A.y, B.x - A.x);
 Real phi = acos((A.r * A.r + d2 - B.r * B.r) /
   (2 * A.r * dis));
 Real L = theta - phi, R = theta + phi;
 while (L < -PI) L += PI * 2;
 while (R > PI) R -= PI * 2;
 return { L, R };
}
vector<Point> intersectPoint(Circle a, Circle b) {
 Real d=o.dis(aa.o);
 if (d >= r+aa.r || d <= fabs(r-aa.r)) return {};
 Real dt = (r*r - aa.r*aa.r)/d, d1 = (d+dt)/2;
 Point dir = (aa.o-o); dir /= d;
 Point pcrs = dir*d1 + o;
 dt=sqrt(max(0.0L, r*r - d1*d1)), dir=dir.rot90();
 return {pcrs + dir*dt, pcrs - dir*dt};
}
```

## 6.3 2D Convex Hull

```
template<typename PT>
vector<PT> buildConvexHull(vector<PT> d) {
 sort(ALL(d), [](const PT& a, const PT& b){
   return tie(a.x, a.y) < tie(b.x, b.y);});
 vector<PT> s(SZ(d)<<1);
 int o = 0;
 for(auto p: d) {
  while(o>=2 && cross(p-s[o-2],s[o-1]-s[o-2])<=0)
   o--;
  s[o++] = p;
 }
 for(int i=SZ(d)-2, t = o+1;i>=0;i--){
  while(o>=t&&cross(d[i]-s[o-2],s[o-1]-s[o-2])<=0)
   o--;
  s[o++] = d[i];
 }
 s.resize(o-1);
 return s;
}
```

## 6.4 3D Convex Hull

```
// return the faces with pt indexes
int flag[MXN][MXN];
struct Point{
 ld x,y,z;
 Point operator * (const ld &b) const {
  return (Point){x*b,y*b,z*b};}
 Point operator * (const Point &b) const {
  return(Point){y*b.z-b.y*z,z*b.x-b.z*x,x*b.y-b.x*y};
 }
};
Point ver(Point a, Point b, Point c) {
 return (b - a) * (c - a);}
vector<Face> convex_hull_3D(const vector<Point> pt) {
 int n = SZ(pt), ftop = 0;
 REP(i,n) REP(j,n) flag[i][j] = 0;
 vector<Face> now;
 now.emplace_back(0,1,2);
 now.emplace_back(2,1,0);
 for (int i=3; i<n; i++){
  ftop++; vector<Face> next;
  REP(j, SZ(now)) {
   Face& f=now[j]; int ff = 0;
   ld d=(pt[i]-pt[f.a]).dot(
     ver(pt[f.a], pt[f.b], pt[f.c]));
   if (d <= 0) next.push_back(f);
   if (d > 0) ff=ftop;
   else if (d < 0) ff=-ftop;
   flag[f.a][f.b]=flag[f.b][f.c]=flag[f.c][f.a]=ff;
  }
  REP(j, SZ(now)) {
   Face& f=now[j];
   if (flag[f.a][f.b] > 0 &&
     flag[f.a][f.b] != flag[f.b][f.a])
    next.emplace_back(f.a,f.b,i);
   if (flag[f.b][f.c] > 0 &&
     flag[f.b][f.c] != flag[f.c][f.b])
    next.emplace_back(f.b,f.c,i);
   if (flag[f.c][f.a] > 0 &&
     flag[f.c][f.a] != flag[f.a][f.c])
    next.emplace_back(f.c,f.a,i);
  }
  now=next;
 }
 return now;
}
```

## 6.5 2D Farthest Pair

```
// stk is from convex hull
n = (int)(stk.size());
```

```
int pos = 1, ans = 0; stk.push_back(stk[0]);
for(int i=0;i<n;i++) {
 while(abs(cross(stk[i+1]-stk[i],
   stk[(pos+1)%n]-stk[i])) >
   abs(cross(stk[i+1]-stk[i],
   stk[pos]-stk[i]))) pos = (pos+1)%n;
 ans = max({ans, dis(stk[i], stk[pos]),
  dis(stk[i+1], stk[pos])});
}
```

## 6.6 2D Closest Pair

```
struct cmp_y {
 bool operator()(const P& p, const P& q) const {
  return p.y < q.y;
 }
};
multiset<P, cmp_y> s;
void solve(P a[], int n) {
 sort(a, a + n, [](const P& p, const P& q) {
  return tie(p.x, p.y) < tie(q.x, q.y);
 });
 llf d = INF; int pt = 0;
 for (int i = 0; i < n; ++i) {
  while (pt < i and a[i].x - a[pt].x >= d)
   s.erase(s.find(a[pt++]));
  auto it = s.lower_bound(P(a[i].x, a[i].y - d));
  while (it != s.end() and it->y - a[i].y < d)
   d = min(d, dis(*(it++), a[i]));
  s.insert(a[i]);
 }
}
```

## 6.7 kD Closest Pair (3D ver.)

```
llf solve(vector<P> v) {
 shuffle(v.begin(), v.end(), mt19937());
 unordered_map<lld, unordered_map<lld,
  unordered_map<lld, int>>> m;
 llf d = dis(v[0], v[1]);
 auto Idx = [&d] (llf x) -> lld {
  return round(x * 2 / d) + 0.1; };
 auto rebuild_m = [&m, &v, &Idx](int k) {
  m.clear();
  for (int i = 0; i < k; ++i)
   m[Idx(v[i].x)][Idx(v[i].y)]
    [Idx(v[i].z)] = i;
 }; rebuild_m(2);
 for (size_t i = 2; i < v.size(); ++i) {
  const lld kx = Idx(v[i].x), ky = Idx(v[i].y),
     kz = Idx(v[i].z); bool found = false;
  for (int dx = -2; dx <= 2; ++dx) {
   const lld nx = dx + kx;
   if (m.find(nx) == m.end()) continue;
   auto& mm = m[nx];
   for (int dy = -2; dy <= 2; ++dy) {
    const lld ny = dy + ky;
    if (mm.find(ny) == mm.end()) continue;
    auto& mmm = mm[ny];
    for (int dz = -2; dz <= 2; ++dz) {
     const lld nz = dz + kz;
     if (mmm.find(nz) == mmm.end()) continue;
     const int p = mmm[nz];
     if (dis(v[p], v[i]) < d) {
      d = dis(v[p], v[i]);
      found = true;
     }
    }
   }
  }
  if (found) rebuild_m(i + 1);
  else m[kx][ky][kz] = i;
 }
 return d;
}
```

## 6.8 Simulated Annealing

```
llf anneal() {
 mt19937 rnd_engine( seed );
 uniform_real_distribution< llf > rnd( 0, 1 );
 const llf dT = 0.001;
 // Argument p
 llf S_cur = calc( p ), S_best = S_cur;
 for ( llf T = 2000 ; T > EPS ; T -= dT ) {
  // Modify p to p_prime
  const llf S_prime = calc( p_prime );
  const llf delta_c = S_prime - S_cur;
  llf prob = min( ( llf ) 1, exp( -delta_c / T ) );
  if ( rnd( rnd_engine ) <= prob )
   S_cur = S_prime, p = p_prime;
  if ( S_prime < S_best ) // find min
   S_best = S_prime, p_best = p_prime;
 }
 return S_best;
}
```

## 6.9 Half Plane Intersection

```
// NOTE: Point is complex<Real>
// cross(pt-line.st, line.dir)<=0 <-> pt in half plane
struct Line {
  Point st, ed;
  Point dir;
  Line (Point _s, Point _e)
   : st(_s), ed(_e), dir(_e - _s) {}
};

bool operator<(const Line &lhs, const Line &rhs) {
  if (int cmp = argCmp(lhs.dir, rhs.dir))
    return cmp == -1;
  return ori(lhs.st, lhs.ed, rhs.st) < 0;
}
Point intersect(const Line &A, const Line &B) {
  Real t = cross(B.st - A.st, B.dir) /
   cross(A.dir, B.dir);
  return A.st + t * A.dir;
}

Real HPI(vector<Line> &lines) {
  sort(lines.begin(), lines.end());
  deque<Line> que;
  deque<Point> pt;
  que.push_back(lines[0]);
  for (int i = 1; i < (int)lines.size(); i++) {
    if (argCmp(lines[i].dir, lines[i-1].dir) == 0)
     continue;
#define POP(L, R) \
    while (pt.size() > 0 \
      && ori(L.st, L.ed, pt.back()) < 0) \
      pt.pop_back(), que.pop_back(); \
    while (pt.size() > 0 \
      && ori(R.st, R.ed, pt.front()) < 0) \
      pt.pop_front(), que.pop_front();
    POP(lines[i], lines[i]);
    pt.push_back(intersect(que.back(), lines[i]));
    que.push_back(lines[i]);
  }
  POP(que.front(), que.back())
  if (que.size() <= 1 ||
    argCmp(que.front().dir, que.back().dir) == 0)
    return 0;
  pt.push_back(intersect(que.front(), que.back()));
  return area(pt);
}
```

## 6.10 Minkowski sum

```
vector<pll> Minkowski(vector<pll> A, vector<pll> B) {
 hull(A), hull(B);
 vector<pll> C(1, A[0] + B[0]), s1, s2;
 for(int i = 0; i < SZ(A); ++i)
  s1.pb(A[(i + 1) % SZ(A)] - A[i]);
 for(int i = 0; i < SZ(B); i++)
  s2.pb(B[(i + 1) % SZ(B)] - B[i]);
 for(int p1 = 0, p2 = 0; p1 < SZ(A) || p2 < SZ(B);)
  if (p2 >= SZ(B)
    || (p1 < SZ(A) && cross(s1[p1], s2[p2]) >= 0))
   C.pb(C.back() + s1[p1++]);
  else
   C.pb(C.back() + s2[p2++]);
 return hull(C), C;
}
```

## 6.11 intersection of line and circle

```
vector<pdd> line_interCircle(const pdd &p1,
    const pdd &p2,const pdd &c,const double r){
```

```
 pdd ft=foot(p1,p2,c),vec=p2-p1;
 double dis=abs(c-ft);
 if(fabs(dis-r)<eps) return vector<pdd>{ft};
 if(dis>r) return {};
 vec=vec*sqrt(r*r-dis*dis)/abs(vec);
 return vector<pdd>{ft+vec,ft-vec};
}
```

## 6.12 intersection of polygon and circle

```
// Divides into multiple triangle, and sum up
// test by HDU2892
const double PI=acos(-1);
double _area(pdd pa, pdd pb, double r){
 if(abs(pa)<abs(pb)) swap(pa, pb);
 if(abs(pb)<eps) return 0;
 double S, h, theta;
 double a=abs(pb),b=abs(pa),c=abs(pb-pa);
 double cosB = dot(pb,pb-pa) / a / c, B = acos(cosB);
 double cosC = dot(pa,pb) / a / b, C = acos(cosC);
 if(a > r){
  S = (C/2)*r*r;
  h = a*b*sin(C)/c;
  if (h < r && B < PI/2)
   S -= (acos(h/r)*r*r - h*sqrt(r*r-h*h));
 }
 else if(b > r){
  theta = PI - B - asin(sin(B)/r*a);
  S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
 }
 else S = .5*sin(C)*a*b;
 return S;
}
double area_poly_circle(const vector<pdd> poly,
   const pdd &O,const double r){
 double S=0;
 for(int i=0;i<SZ(poly);++i)
  S+=_area(poly[i]-O,poly[(i+1)%SZ(poly)]-O,r)
    *ori(O,poly[i],poly[(i+1)%SZ(poly)]);
 return fabs(S);
}
```

## 6.13 intersection of two circle

```
bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
 pdd o1 = a.O, o2 = b.O;
 double r1 = a.R, r2 = b.R, d2 = abs2(o1 - o2),
     d = sqrt(d2);
 if(d < max(r1, r2) - min(r1, r2) || d > r1 + r2)
  return 0;
 pdd u = (o1 + o2) * 0.5
  + (o1 - o2) * ((r2 * r2 - r1 * r1) / (2 * d2));
 double A = sqrt((r1 + r2 + d) * (r1 - r2 + d)
     * (r1 + r2 - d) * (-r1 + r2 + d));
 pdd v = pdd(o1.Y - o2.Y, -o1.X + o2.X) * A
  / (2 * d2);
 p1 = u + v, p2 = u - v;
 return 1;
}
```

## 6.14 tangent line of two circle

```
vector<Line> go(const Cir& c1,
   const Cir& c2, int sign1){
 // sign1 = 1 for outer tang, -1 for inter tang
 vector<Line> ret;
 double d_sq = norm2( c1.O - c2.O );
 if( d_sq < eps ) return ret;
 double d = sqrt( d_sq );
 Pt v = ( c2.O - c1.O ) / d;
 double c = ( c1.R - sign1 * c2.R ) / d;
 if( c * c > 1 ) return ret;
 double h = sqrt( max( 0.0 , 1.0 - c * c ) );
 for( int sign2 = 1 ; sign2 >= -1 ; sign2 -= 2 ){
  Pt n = { v.X * c - sign2 * h * v.Y ,
   v.Y * c + sign2 * h * v.X };
  Pt p1 = c1.O + n * c1.R;
  Pt p2 = c2.O + n * ( c2.R * sign1 );
  if( fabs( p1.X - p2.X ) < eps and
    fabs( p1.Y - p2.Y ) < eps )
   p2 = p1 + perp( c2.O - c1.O );
  ret.push_back( { p1 , p2 } );
 }
 return ret;
}
```

## 6.15 Minimum Covering Circle

```
template<typename P>
Circle getCircum(const P &a, const P &b, const P &c){
 Real a1 = a.x-b.x, b1 = a.y-b.y;
 Real c1 = (a.x+b.x)/2 * a1 + (a.y+b.y)/2 * b1;
 Real a2 = a.x-c.x, b2 = a.y-c.y;
 Real c2 = (a.x+c.x)/2 * a2 + (a.y+c.y)/2 * b2;
 Circle cc;
 cc.o.x = (c1*b2-b1*c2)/(a1*b2-b1*a2);
 cc.o.y = (a1*c2-c1*a2)/(a1*b2-b1*a2);
 cc.r = hypot(cc.o.x-a.x, cc.o.y-a.y);
 return cc;
}

template<typename P>
Circle MinCircleCover(const vector<P>& pts){
 random_shuffle(pts.begin(), pts.end());
 Circle c = { pts[0], 0 };
 for(int i=0;i<(int)pts.size();i++){
  if (dist(pts[i], c.o) <= c.r) continue;
  c = { pts[i], 0 };
  for (int j = 0; j < i; j++) {
   if(dist(pts[j], c.o) <= c.r) continue;
   c.o = (pts[i] + pts[j]) / 2;
   c.r = dist(pts[i], c.o);
   for (int k = 0; k < j; k++) {
    if (dist(pts[k], c.o) <= c.r) continue;
    c = getCircum(pts[i], pts[j], pts[k]);
   }
  }
 }
 return c;
}
```

## 6.16 KDTree (Nearest Point)

```
const int MXN = 100005;
struct KDTree {
 struct Node {
  int x,y,x1,y1,x2,y2;
  int id,f;
  Node *L, *R;
 } tree[MXN], *root;
 int n;
 LL dis2(int x1, int y1, int x2, int y2) {
  LL dx = x1-x2, dy = y1-y2;
  return dx*dx+dy*dy;
 }
 static bool cmpx(Node& a, Node& b){return a.x<b.x;}
 static bool cmpy(Node& a, Node& b){return a.y<b.y;}
 void init(vector<pair<int,int>> ip) {
  n = ip.size();
  for (int i=0; i<n; i++) {
   tree[i].id = i;
   tree[i].x = ip[i].first;
   tree[i].y = ip[i].second;
  }
  root = build_tree(0, n-1, 0);
 }
 Node* build_tree(int L, int R, int d) {
  if (L>R) return nullptr;
  int M = (L+R)/2; tree[M].f = d%2;
  nth_element(tree+L,tree+M,tree+R+1,d%2?cmpy:cmpx);
  tree[M].x1 = tree[M].x2 = tree[M].x;
  tree[M].y1 = tree[M].y2 = tree[M].y;
  tree[M].L = build_tree(L, M-1, d+1);
  if (tree[M].L) {
   tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
   tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
   tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
   tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
  }
  tree[M].R = build_tree(M+1, R, d+1);
  if (tree[M].R) {
   tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
   tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
   tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
   tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
  }
  return tree+M;
 }
 int touch(Node* r, int x, int y, LL d2){
```

```
  LL dis = sqrt(d2)+1;
  if (x<r->x1-dis || x>r->x2+dis ||
    y<r->y1-dis || y>r->y2+dis)
   return 0;
  return 1;
 }
 void nearest(Node* r,int x,int y,int &mID,LL &md2) {
  if (!r || !touch(r, x, y, md2)) return;
  LL d2 = dis2(r->x, r->y, x, y);
  if (d2 < md2 || (d2 == md2 && mID < r->id)) {
   mID = r->id;
   md2 = d2;
  }
  // search order depends on split dim
  if ((r->f == 0 && x < r->x) ||
    (r->f == 1 && y < r->y)) {
   nearest(r->L, x, y, mID, md2);
   nearest(r->R, x, y, mID, md2);
  } else {
   nearest(r->R, x, y, mID, md2);
   nearest(r->L, x, y, mID, md2);
  }
 }
 int query(int x, int y) {
  int id = 1029384756;
  LL d2 = 102938475612345678LL;
  nearest(root, x, y, id, d2);
  return id;
 }
} tree;
```

# 7 Stringology

## 7.1 Hash

```
class Hash {
 private:
  static constexpr int P = 127, Q = 1051762951;
  vector<int> h, p;
 public:
  void init(const string &s){
   h.assign(s.size()+1, 0); p.resize(s.size()+1);
   for (size_t i = 0; i < s.size(); ++i)
    h[i + 1] = add(mul(h[i], P), s[i]);
   generate(p.begin(), p.end(),[x=1,y=1,this]()
     mutable{y=x;x=mul(x,P);return y;});
  }
  int query(int l, int r){ // 1-base (l, r]
   return sub(h[r], mul(h[l], p[r-l]));}
};
```

## 7.2 Suffix Array

```
namespace sfxarray {
bool t[maxn * 2];
int hi[maxn], rev[maxn];
int _s[maxn * 2], sa[maxn * 2], c[maxn * 2];
int x[maxn], p[maxn], q[maxn * 2];
// sa[i]: sa[i]-th suffix is the \
// i-th lexigraphically smallest suffix.
// hi[i]: longest common prefix \
// of suffix sa[i] and suffix sa[i - 1].
void pre(int *sa, int *c, int n, int z) {
 memset(sa, 0, sizeof(int) * n);
 memcpy(x, c, sizeof(int) * z);
}
void induce(int *sa,int *c,int *s,bool *t,int n,int z){
 memcpy(x + 1, c, sizeof(int) * (z - 1));
 for (int i = 0; i < n; ++i)
  if (sa[i] && !t[sa[i] - 1])
   sa[x[s[sa[i] - 1]]++] = sa[i] - 1;
 memcpy(x, c, sizeof(int) * z);
 for (int i = n - 1; i >= 0; --i)
  if (sa[i] && t[sa[i] - 1])
   sa[--x[s[sa[i] - 1]]] = sa[i] - 1;
}
void sais(int *s, int *sa, int *p, int *q,
 bool *t, int *c, int n, int z) {
 bool uniq = t[n - 1] = true;
 int nn=0, nmxz=-1, *nsa = sa+n, *ns=s+n, last=-1;
 memset(c, 0, sizeof(int) * z);
 for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;
 for (int i = 0; i < z - 1; ++i) c[i + 1] += c[i];
 if (uniq) {
  for (int i = 0; i < n; ++i) sa[--c[s[i]]] = i;
  return;
 }
 for (int i = n - 2; i >= 0; --i)
  t[i] = (s[i]==s[i + 1] ? t[i + 1] : s[i]<s[i + 1]);
 pre(sa, c, n, z);
 for (int i = 1; i <= n - 1; ++i)
  if (t[i] && !t[i - 1])
   sa[--x[s[i]]] = p[q[i] = nn++] = i;
 induce(sa, c, s, t, n, z);
 for (int i = 0; i < n; ++i) {
  if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
  bool neq = last < 0 || \
   memcmp(s + sa[i], s + last,
   (p[q[sa[i]] + 1] - sa[i]) * sizeof(int));
  ns[q[last = sa[i]]] = nmxz += neq;
 }}
 sais(ns, nsa, p+nn, q+n, t+n, c+z, nn, nmxz+1);
 pre(sa, c, n, z);
 for (int i = nn - 1; i >= 0; --i)
  sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
 induce(sa, c, s, t, n, z);
}
void build(const string &s) {
 for (int i = 0; i < (int)s.size(); ++i) _s[i] = s[i];
 _s[(int)s.size()] = 0; // s shouldn't contain 0
 sais(_s, sa, p, q, t, c, (int)s.size() + 1, 256);
 for(int i = 0; i < (int)s.size(); ++i) sa[i]=sa[i+1];
 for(int i = 0; i < (int)s.size(); ++i) rev[sa[i]]=i;
 int ind = 0; hi[0] = 0;
 for (int i = 0; i < (int)s.size(); ++i) {
  if (!rev[i]) {
   ind = 0;
   continue;
  }
  while (i + ind < (int)s.size() && \
   s[i + ind] == s[sa[rev[i] - 1] + ind]) ++ind;
  hi[rev[i]] = ind ? ind-- : 0;
 }
}}
```

## 7.3 Aho-Corasick Algorithm

```
class AhoCorasick{
 private:
  static constexpr int Z = 26;
  struct node{
   node *nxt[ Z ], *fail;
   vector< int > data;
   node(): fail( nullptr ) {
    memset( nxt, 0, sizeof( nxt ) );
    data.clear();
   }
  } *rt;
  inline int Idx( char c ) { return c - 'a'; }
 public:
  void init() { rt = new node(); }
  void add( const string& s, int d ) {
   node* cur = rt;
   for ( auto c : s ) {
    if ( not cur->nxt[ Idx( c ) ] )
     cur->nxt[ Idx( c ) ] = new node();
    cur = cur->nxt[ Idx( c ) ];
   }
   cur->data.push_back( d );
  }
  void compile() {
   vector< node* > bfs;
   size_t ptr = 0;
   for ( int i = 0 ; i < Z ; ++ i ) {
    if ( not rt->nxt[ i ] ) {
     // uncomment 2 lines to make it DFA
     // rt->nxt[i] = rt;
     continue;
    }
    rt->nxt[ i ]->fail = rt;
    bfs.push_back( rt->nxt[ i ] );
   }
   while ( ptr < bfs.size() ) {
    node* u = bfs[ ptr ++ ];
    for ( int i = 0 ; i < Z ; ++ i ) {
     if ( not u->nxt[ i ] ) {
```

```
      // u->nxt[i] = u->fail->nxt[i];
      continue;
     }
     node* u_f = u->fail;
     while ( u_f ) {
      if ( not u_f->nxt[ i ] ) {
       u_f = u_f->fail; continue;
      }
      u->nxt[ i ]->fail = u_f->nxt[ i ];
      break;
     }
     if ( not u_f ) u->nxt[ i ]->fail = rt;
     bfs.push_back( u->nxt[ i ] );
    }
   }
  }
  void match( const string& s, vector< int >& ret ) {
   node* u = rt;
   for ( auto c : s ) {
    while ( u != rt and not u->nxt[ Idx( c ) ] )
     u = u->fail;
    u = u->nxt[ Idx( c ) ];
    if ( not u ) u = rt;
    node* tmp = u;
    while ( tmp != rt ) {
     for ( auto d : tmp->data )
      ret.push_back( d );
     tmp = tmp->fail;
    }
   }
  }
} ac;
```

## 7.4 Suffix Automaton

```
struct Node{
 Node *green, *edge[26];
 int max_len;
 Node(const int _max_len)
  : green(NULL), max_len(_max_len){
  memset(edge,0,sizeof(edge));
 }
} *ROOT, *LAST;
void Extend(const int c) {
 Node *cursor = LAST;
 LAST = new Node((LAST->max_len) + 1);
 for(;cursor&&!cursor->edge[c]; cursor=cursor->green)
  cursor->edge[c] = LAST;
 if (!cursor)
  LAST->green = ROOT;
 else {
  Node *potential_green = cursor->edge[c];
  if((potential_green->max_len)==(cursor->max_len+1))
   LAST->green = potential_green;
  else {
//assert(potential_green->max_len>(cursor->max_len+1));
   Node *wish = new Node((cursor->max_len) + 1);
   for(;cursor && cursor->edge[c]==potential_green;
      cursor = cursor->green)
    cursor->edge[c] = wish;
   for (int i = 0; i < 26; i++)
    wish->edge[i] = potential_green->edge[i];
   wish->green = potential_green->green;
   potential_green->green = wish;
   LAST->green = wish;
  }
 }
}
char S[10000001], A[10000001];
int N;
int main(){
 scanf("%d%s", &N, S);
 ROOT = LAST = new Node(0);
 for (int i = 0; S[i]; i++)
  Extend(S[i] - 'a');
 while (N--){
  scanf("%s", A);
  Node *cursor = ROOT;
  bool ans = true;
  for (int i = 0; A[i]; i++){
   cursor = cursor->edge[A[i] - 'a'];
   if (!cursor) {
    ans = false;
    break;
   }
  }
  puts(ans ? "Yes" : "No");
 }
 return 0;
}
```

## 7.5 KMP

```
vector<int> kmp(const string &s) {
 vector<int> f(s.size(), 0);
 /* f[i] = length of the longest prefix
   (excluding s[0:i]) such that it coincides
   with the suffix of s[0:i] of the same length */
 /* i + 1 - f[i] is the length of the
   smallest recurring period of s[0:i] */
 int k = 0;
 for (int i = 1; i < (int)s.size(); ++i) {
  while (k > 0 && s[i] != s[k]) k = f[k - 1];
  if (s[i] == s[k]) ++k;
  f[i] = k;
 }
 return f;
}
vector<int> search(const string &s, const string &t) {
 // return 0-indexed occurrence of t in s
 vector<int> f = kmp(t), r;
 for (int i = 0, k = 0; i < (int)s.size(); ++i) {
  while(k > 0 && (k==(int)t.size() || s[i]!=t[k]))
   k = f[k - 1];
  if (s[i] == t[k]) ++k;
  if (k == (int)t.size()) r.push_back(i-t.size()+1);
 }
 return res;
}
```

## 7.6 Z value

```
char s[MAXN];
int len,z[MAXN];
void Z_value() {
 int i,j,left,right;
 z[left=right=0]=len;
 for(i=1;i<len;i++) {
  j=max(min(z[i-left],right-i),0);
  for(;i+j<len&&s[i+j]==s[j];j++);
  if(i+(z[i]=j)>right)right=i+z[left=i];
 }
}
```

## 7.7 Manacher

```
int z[maxn];
int manacher(const string& s) {
 string t = ".";
 for(char c: s) t += c, t += '.';
 int l = 0, r = 0, ans = 0;
 for (int i = 1; i < t.length(); ++i) {
  z[i] = (r > i ? min(z[2 * l - i], r - i) : 1);
  while (i - z[i] >= 0 && i + z[i] < t.length()) {
   if(t[i - z[i]] == t[i + z[i]]) ++z[i];
   else break;
  }
  if (i + z[i] > r) r = i + z[i], l = i;
 }
 for(int i=1;i<t.length();++i) ans = max(ans, z[i]-1);
 return ans;
}
```

## 7.8 Lexico Smallest Rotation

```
string mcp(string s){
 int n = s.length();
 s += s;
 int i=0, j=1;
 while (i<n && j<n){
  int k = 0;
  while (k < n && s[i+k] == s[j+k]) k++;
  if (s[i+k] <= s[j+k]) j += k+1;
  else i += k+1;
  if (i == j) j++;
 }
 int ans = i < n ? i : j;
 return s.substr(ans, n);
}
```

## 7.9 BWT

```
struct BurrowsWheeler{
#define SIGMA 26
#define BASE 'a'
 vector<int> v[ SIGMA ];
 void BWT(char* ori, char* res){
  // make ori -> ori + ori
  // then build suffix array
 }
 void iBWT(char* ori, char* res){
  for( int i = 0 ; i < SIGMA ; i ++ )
   v[ i ].clear();
  int len = strlen( ori );
  for( int i = 0 ; i < len ; i ++ )
   v[ ori[i] - BASE ].push_back( i );
  vector<int> a;
  for( int i = 0 , ptr = 0 ; i < SIGMA ; i ++ )
   for( auto j : v[ i ] ){
    a.push_back( j );
    ori[ ptr ++ ] = BASE + i;
   }
  for( int i = 0 , ptr = 0 ; i < len ; i ++ ){
   res[ i ] = ori[ a[ ptr ] ];
   ptr = a[ ptr ];
  }
  res[ len ] = 0;
 }
} bwt;
```

## 7.10 Palindromic Tree

```
struct palindromic_tree{
 struct node{
  int next[26],f,len;
  int cnt,num,st,ed;
  node(int l=0):f(0),len(l),cnt(0),num(0) {
   memset(next, 0, sizeof(next)); }
 };
 vector<node> st;
 vector<char> s;
 int last,n;
 void init(){
  st.clear();s.clear();last=1; n=0;
  st.push_back(0);st.push_back(-1);
  st[0].f=1;s.push_back(-1); }
 int getFail(int x){
  while(s[n-st[x].len-1]!=s[n])x=st[x].f;
  return x;}
 void add(int c){
  s.push_back(c-='a'); ++n;
  int cur=getFail(last);
  if(!st[cur].next[c]){
   int now=st.size();
   st.push_back(st[cur].len+2);
   st[now].f=st[getFail(st[cur].f)].next[c];
   st[cur].next[c]=now;
   st[now].num=st[st[now].f].num+1;
  }
  last=st[cur].next[c];
  ++st[last].cnt;}
 int size(){ return st.size()-2;}
} pt;
int main() {
 string s; cin >> s; pt.init();
 for (int i=0; i<SZ(s); i++) {
  int prvsz = pt.size(); pt.add(s[i]);
  if (prvsz != pt.size()) {
   int r = i, l = r - pt.st[pt.last].len + 1;
   // pal @ [l,r]: s.substr(l, r-l+1)
  }
 }
 return 0;
}
```

# 8 Misc

## 8.1 Theorems

### 8.1.1 Kirchhoff's Theorem

Denote $L$ be a $n \times n$ matrix as the Laplacian matrix of graph $G$, where $L_{ii} = d(i)$, $L_{ij} = -c$ where $c$ is the number of edge $(i, j)$ in $G$.

- The number of undirected spanning in $G$ is $|\det(\tilde{L}_{11})|$.
- The number of directed spanning tree rooted at $r$ in $G$ is $|\det(\tilde{L}_{rr})|$.

### 8.1.2 Tutte's Matrix

Let $D$ be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ ($x_{ij}$ is chosen uniform randomly) if $i < j$ and $(i, j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{rank(D)}{2}$ is the maximum matching on $G$.

### 8.1.3 Cayley's Formula

- Given a degree sequence $d_1, d_2, \ldots, d_n$ for each labeled vertices, there're $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!}$ spanning trees.
- Let $T_{n,k}$ be the number of labeled forests on $n$ vertices with $k$ components, such that vertex $1, 2, \ldots, k$ belong to different components. Then $T_{n,k} = kn^{n-k-1}$.

### 8.1.4 Erdős–Gallai theorem

A sequence of non-negative integers $d_1 \geq d_2 \geq \ldots \geq d_n$ can be represented as the degree sequence of a finite simple graph on $n$ vertices if and only if $d_1 + d_2 + \ldots + d_n$ is even and

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k)$$

holds for all $1 \leq k \leq n$.

### 8.1.5 Havel–Hakimi algorithm

find the vertex who has greatest degree unused, connect it with other greatest vertex.

### 8.1.6 Hall's marriage theorem

Let $G$ be a finite bipartite graph with bipartite sets $X$ and $Y$. For a subset $W$ of $X$, let $N_G(W)$ denote the set of all vertices in $Y$ adjacent to some element of $W$. Then there is an $X$-saturating matching iff $\forall W \subseteq X, |W| \leq |N_G(W)|$

### 8.1.7 Euler's planar graph formula

$V - E + F = C + 1, E \leq 3V - 6$(?)

### 8.1.8 Pick's theorem

For simple polygon, when points are all integer, we have $A = \#\{\text{lattice points in the interior}\} + \frac{\#\{\text{lattice points on the boundary}\}}{2} - 1$

### 8.1.9 Lucas's theorem

$\binom{m}{n} \equiv \prod_{i=0}^{k} \binom{m_i}{n_i} \pmod{p}$, where $m = m_k p^k + m_{k-1} p^{k-1} + \cdots + m_1 p + m_0$, and $n = n_k p^k + n_{k-1} p^{k-1} + \cdots + n_1 p + n_0$.

## 8.2 MaximumEmptyRect

```
int max_empty_rect(int n, int m, bool blocked[N][N]) {
 static int mxu[2][N], me=0, he=1, ans=0;
 for (int i=0;i<m;i++) mxu[he][i]=0;
 for (int i=0;i<n;i++) {
  stack<PII,vector<PII>> stk;
  for (int j=0;j<m;++j) {
   if (blocked[i][j]) mxu[me][j]=0;
   else mxu[me][j]=mxu[he][j]+1;
   int la = j;
   while (!stk.empty()&&stk.top().FF>mxu[me][j]) {
    int x1 = i - stk.top().FF, x2 = i;
    int y1 = stk.top().SS, y2 = j;
    la = stk.top().SS; stk.pop();
    ans=max(ans,(x2-x1)*(y2-y1));
   }
   if (stk.empty()||stk.top().FF<mxu[me][j])
    stk.push({mxu[me][j],la});
  }
  while (!stk.empty()) {
   int x1 = i - stk.top().FF, x2 = i;
   int y1 = stk.top().SS-1, y2 = m-1;
   stk.pop(); ans=max(ans,(x2-x1)*(y2-y1));
  }
  swap(me,he);
 }
 return ans;
}
```

## 8.3 DP-opt Condition

### 8.3.1 totally monotone (concave/convex)

$\forall i < i', j < j', B[i][j] \leq B[i'][j] \implies B[i][j'] \leq B[i'][j']$
$\forall i < i', j < j', B[i][j] \geq B[i'][j] \implies B[i][j'] \geq B[i'][j']$

### 8.3.2 monge condition (concave/convex)

$\forall i < i', j < j', B[i][j] + B[i'][j'] \geq B[i][j'] + B[i'][j]$
$\forall i < i', j < j', B[i][j] + B[i'][j'] \leq B[i][j'] + B[i'][j]$

## 8.4 Convex 1D/1D DP

```
struct segment {
 int i, l, r;
 segment() {}
 segment(int a, int b, int c): i(a), l(b), r(c) {}
};
inline lld f(int l, int r){return dp[l] + w(l+1, r);}
void solve() {
 dp[0] = 0;
 deque&lt;segment&gt; dq; dq.push_back(segment(0, 1, n));
 for (int i = 1; i <= n; ++i) {
  dp[i] = f(dq.front().i, i);
  while(dq.size()&&dq.front().r<i+1) dq.pop_front();
  dq.front().l = i + 1;
  segment seg = segment(i, i + 1, n);
  while (dq.size() &&
   f(i, dq.back().l)<f(dq.back().i, dq.back().l))
    dq.pop_back();
  if (dq.size()) {
   int d = 1 << 20, c = dq.back().l;
   while (d >>= 1) if (c + d <= dq.back().r)
    if(f(i, c+d) > f(dq.back().i, c+d)) c += d;
   dq.back().r = c; seg.l = c + 1;
  }
  if (seg.l <= n) dq.push_back(seg);
 }
}
```

## 8.5 ConvexHull Optimization

```
inline lld DivCeil(lld n, lld d) { // ceil(n/d)
 return n / d + (((n < 0) != (d > 0)) && (n % d));
}
struct Line {
 static bool flag;
 lld a, b, l, r; // y=ax+b in [l, r)
 lld operator()(lld x) const { return a * x + b; }
 bool operator<(const Line& i) const {
  return flag ? tie(a, b) < tie(i.a, i.b) : l < i.l;
 }
 lld operator&(const Line& i) const {
  return DivCeil(b - i.b, i.a - a);
 }
};
bool Line::flag = true;
class ConvexHullMax {
 set<Line> L;
 public:
 ConvexHullMax() { Line::flag = true; }
 void InsertLine(lld a, lld b) { // add y = ax + b
  Line now = {a, b, -INF, INF};
  if (L.empty()) {
   L.insert(now);
   return;
  }
  Line::flag = true;
  auto it = L.lower_bound(now);
  auto prv = it == L.begin() ? it : prev(it);
  if (it != L.end() && ((it != L.begin() &&
   (*it)(it->l) >= now(it->l) &&
   (*prv)(prv->r - 1) >= now(prv->r - 1)) ||
   (it == L.begin() && it->a == now.a))) return;
  if (it != L.begin()) {
   while (prv != L.begin() &&
    (*prv)(prv->l) <= now(prv->l))
     prv = --L.erase(prv);
   if (prv == L.begin() && now.a == prv->a)
    L.erase(prv);
  }
  if (it != L.end())
   while (it != --L.end() &&
    (*it)(it->r) <= now(it->r))
     it = L.erase(it);
  if (it != L.begin()) {
   prv = prev(it);
   const_cast<Line*>(&*prv)->r=now.l=((*prv)&now);
  }
  if (it != L.end())
   const_cast<Line*>(&*it)->l=now.r=((*it)&now);
  L.insert(it, now);
 }
 lld Query(lld a) const { // query max at x=a
  if (L.empty()) return -INF;
  Line::flag = false;
  auto it = --L.upper_bound({0, 0, a, 0});
  return (*it)(a);
 }
};
```

## 8.6 Josephus Problem

```
// n people kill m for each turn
int f(int n, int m) {
 int s = 0;
 for (int i = 2; i <= n; i++)
  s = (s + m) % i;
 return s;
}
// died at kth
int kth(int n, int m, int k){
 if (m == 1) return n-1;
 for (k = k*m+m-1; k >= n; k = k-n+(k-n)/(m-1));
 return k;
}
```

## 8.7 Cactus Matching

```
vector<int> init_g[maxn],g[maxn*2];
int n,dfn[maxn],low[maxn],par[maxn],dfs_idx,bcc_id;
void tarjan(int u){
 dfn[u]=low[u]=++dfs_idx;
 for(int i=0;i<(int)init_g[u].size();i++){
  int v=init_g[u][i];
  if(v==par[u]) continue;
  if(!dfn[v]){
   par[v]=u;
   tarjan(v);
   low[u]=min(low[u],low[v]);
   if(dfn[u]<low[v]){
    g[u].push_back(v);
    g[v].push_back(u);
   }
  }else{
   low[u]=min(low[u],dfn[v]);
   if(dfn[v]<dfn[u]){
    int temp_v=u;
    bcc_id++;
    while(temp_v!=v){
     g[bcc_id+n].push_back(temp_v);
     g[temp_v].push_back(bcc_id+n);
     temp_v=par[temp_v];
    }
    g[bcc_id+n].push_back(v);
    g[v].push_back(bcc_id+n);
    reverse(g[bcc_id+n].begin(),g[bcc_id+n].end());
   }
  }
 }
}
int dp[maxn][2],min_dp[2][2],tmp[2][2],tp[2];
void dfs(int u,int fa){
 if(u<=n){
  for(int i=0;i<(int)g[u].size();i++){
   int v=g[u][i];
   if(v==fa) continue;
   dfs(v,u);
   memset(tp,0x8f,sizeof tp);
   if(v<=n){
    tp[0]=dp[u][0]+max(dp[v][0],dp[v][1]);
    tp[1]=max(
     dp[u][0]+dp[v][0]+1,
     dp[u][1]+max(dp[v][0],dp[v][1])
    );
   }else{
    tp[0]=dp[u][0]+dp[v][0];
    tp[1]=max(dp[u][0]+dp[v][1],dp[u][1]+dp[v][0]);
   }
   dp[u][0]=tp[0],dp[u][1]=tp[1];
  }
 }else{
  for(int i=0;i<(int)g[u].size();i++){
   int v=g[u][i];
   if(v==fa) continue;
```

```
   dfs(v,u);
  }
  min_dp[0][0]=0;
  min_dp[1][1]=1;
  min_dp[0][1]=min_dp[1][0]=-0x3f3f3f3f;
  for(int i=0;i<(int)g[u].size();i++){
   int v=g[u][i];
   if(v==fa) continue;
   memset(tmp,0x8f,sizeof tmp);
   tmp[0][0]=max(
    min_dp[0][0]+max(dp[v][0],dp[v][1]),
    min_dp[0][1]+dp[v][0]
   );
   tmp[0][1]=min_dp[0][0]+dp[v][0]+1;
   tmp[1][0]=max(
    min_dp[1][0]+max(dp[v][0],dp[v][1]),
    min_dp[1][1]+dp[v][0]
   );
   tmp[1][1]=min_dp[1][0]+dp[v][0]+1;
   memcpy(min_dp,tmp,sizeof tmp);
  }
  dp[u][1]=max(min_dp[0][1],min_dp[1][0]);
  dp[u][0]=min_dp[0][0];
 }
}
int main(){
 int m,a,b;
 scanf("%d%d",&n,&m);
 for(int i=0;i<m;i++){
  scanf("%d%d",&a,&b);
  init_g[a].push_back(b);
  init_g[b].push_back(a);
 }
 par[1]=-1;
 tarjan(1);
 dfs(1,-1);
 printf("%d\n",max(dp[1][0],dp[1][1]));
 return 0;
}
```

## 8.8 DLX

```
struct DLX {
 const static int maxn=210;
 const static int maxm=210;
 const static int maxnode=210*210;
 int n, m, size, row[maxnode], col[maxnode];
 int U[maxnode], D[maxnode], L[maxnode], R[maxnode];
 int H[maxn], S[maxm], ansd, ans[maxn];
 void init(int _n, int _m) {
  n = _n, m = _m;
  for(int i = 0; i <= m; ++i) {
   S[i] = 0;
   U[i] = D[i] = i;
   L[i] = i-1, R[i] = i+1;
  }
  R[L[0] = size = m] = 0;
  for(int i = 1; i <= n; ++i) H[i] = -1;
 }
 void Link(int r, int c) {
  ++S[col[++size] = c];
  row[size] = r; D[size] = D[c];
  U[D[c]] = size; U[size] = c; D[c] = size;
  if(H[r] < 0) H[r] = L[size] = R[size] = size;
  else {
   R[size] = R[H[r]];
   L[R[H[r]]] = size;
   L[size] = H[r];
   R[H[r]] = size;
  }
 }
 void remove(int c) {
  L[R[c]] = L[c]; R[L[c]] = R[c];
  for(int i = D[c]; i != c; i = D[i])
   for(int j = R[i]; j != i; j = R[j]) {
    U[D[j]] = U[j];
    D[U[j]] = D[j];
    --S[col[j]];
   }
 }
 void resume(int c) {
  L[R[c]] = c; R[L[c]] = c;
  for(int i = U[c]; i != c; i = U[i])
   for(int j = L[i]; j != i; j = L[j]) {
    U[D[j]] = j;
    D[U[j]] = j;
    ++S[col[j]];
   }
 }
 void dance(int d) {
  if(d>=ansd) return;
  if(R[0] == 0) {
   ansd = d;
   return;
  }
  int c = R[0];
  for(int i = R[0]; i; i = R[i])
   if(S[i] < S[c]) c = i;
  remove(c);
  for(int i = D[c]; i != c; i = D[i]) {
   ans[d] = row[i];
   for(int j = R[i]; j != i; j = R[j])
    remove(col[j]);
   dance(d+1);
   for(int j = L[i]; j != i; j = L[j])
    resume(col[j]);
  }
  resume(c);
 }
} sol;
```

## 8.9 Tree Knapsack

```
int dp[N][K];PII obj[N];
vector<int> G[N];
void dfs(int u, int mx){
 for(int s: G[u]) {
  if(mx < obj[s].first) continue;
  for(int i=0;i<=mx-obj[s].FF;i++)
   dp[s][i] = dp[u][i];
  dfs(s, mx - obj[s].first);
  for(int i=obj[s].FF;i<=mx;i++)
   dp[u][i] = max(dp[u][i],
    dp[s][i - obj[s].FF] + obj[s].SS);
 }
}
int main(){
 int n, k; cin >> n >> k;
 for(int i=1;i<=n;i++){
  int p; cin >> p;
  G[p].push_back(i);
  cin >> obj[i].FF >> obj[i].SS;
 }
 dfs(0, k); int ans = 0;
 for(int i=0;i<=k;i++) ans = max(ans, dp[0][i]);
 cout << ans << '\n';
 return 0;
}
```

## 8.10 N Queens Problem

```
vector< int > solve( int n ) {
 // no solution when n=2, 3
 vector< int > ret;
 if ( n % 6 == 2 ) {
  for ( int i = 2 ; i <= n ; i += 2 )
   ret.push_back( i );
  ret.push_back( 3 ); ret.push_back( 1 );
  for ( int i = 7 ; i <= n ; i += 2 )
   ret.push_back( i );
  ret.push_back( 5 );
 } else if ( n % 6 == 3 ) {
  for ( int i = 4 ; i <= n ; i += 2 )
   ret.push_back( i );
  ret.push_back( 2 );
  for ( int i = 5 ; i <= n ; i += 2 )
   ret.push_back( i );
  ret.push_back( 1 ); ret.push_back( 3 );
 } else {
  for ( int i = 2 ; i <= n ; i += 2 )
   ret.push_back( i );
  for ( int i = 1 ; i <= n ; i += 2 )
   ret.push_back( i );
 }
 return ret;
}
```

## 8.11 Aliens Optimization

```
long long Alien() {
 long long c = kInf;
 for (int d = 60; d >= 0; --d) {
  // cost can be negative, depending on the problem.
  if (c - (1LL << d) < 0) continue;
  long long ck = c - (1LL << d);
  pair<long long, int> r = check(ck);
  if (r.second == k) return r.first - ck * k;
  if (r.second < k) c = ck;
 }
 pair<long long, int> r = check(c);
 return r.first - c * k;
}
```