

# Contents

## 1 Basic

1.1	vimrc	1
1.2	Debug Macro	1
1.3	SVG Writer	1
1.4	Pragma Optimization	1
1.5	IO Optimization	1
1.6	Increase Stack	2

## 2 Data Structure

2.1	Dark Magic	2
2.2	Link-Cut Tree	2
2.3	LiChao Segment Tree	2
2.4	Treap	3
2.5	Linear Basis	3
2.6	Binary Search on Segtree	3

## 3 Graph

3.1	SCC	3
3.2	2-SAT	3
3.3	BCC	3
3.4	Round Square Tree	4
3.5	Edge TCC	4
3.6	Bipolar Orientation	4
3.7	DMST	4
3.8	Dominator Tree	5
3.9	Edge Coloring	5
3.10	Centroid Decomp.	5
3.11	Lowbit Decomp.	6
3.12	Virtual Tree	6
3.13	Tree Hashing	6
3.14	Mo's Algo on Tree	6
3.15	Count Cycles	6
3.16	Maximal Clique	6
3.17	Maximum Clique	7
3.18	Min Mean Cycle	7

## 4 Flow & Matching

4.1	HopcroftKarp	7
4.2	Kuhn Munkres	7
4.3	Flow Models	8
4.4	Dinic	8
4.5	HLPP	8
4.6	Global Min-Cut	9
4.7	GomoryHu Tree	9
4.8	MCMF	9
4.9	Dijkstra Cost Flow	9
4.10	Min Cost Circulation	9
4.11	General Matching	10
4.12	Weighted Matching	10

## 5 Math

5.1	Common Bounds	11
5.2	Equations	11
5.3	Extended FloorSum	11
5.4	Integer Division	12
5.5	FloorSum	12
5.6	ModMin	12
5.7	Floor Monoid Product	12
5.8	$ax+by=gcd$	12
5.9	Chinese Remainder	12
5.10	DiscreteLog	12
5.11	Quadratic Residue	12
5.12	FWT	12
5.13	Packed FFT	12
5.14	CRT for arbitrary mod	13
5.15	NTT / FFT	13
5.16	Formal Power Series	13
5.17	Partition Number	14
5.18	Pi Count	14
5.19	Miller Rabin	14
5.20	Pollard Rho	14
5.21	Barrett Reduction	14
5.22	Montgomery	14
5.23	Berlekamp Massey	14
5.24	Gauss Elimination	15
5.25	CharPoly	15
5.26	Simplex	15
5.27	Simplex Construction	15
5.28	Adaptive Simpson	16

5.29	Golden Ratio Search	16
------	---------------------	----

## 6 Geometry

6.1	Basic Geometry	16
6.2	2D Convex Hull	16
6.3	2D Farthest Pair	16
6.4	MinMax Enclosing Rect	16
6.5	Minkowski Sum	16
6.6	Segment Intersection	16
6.7	Halfplane Intersection	17
6.8	SegmentDist (Sausage)	17
6.9	Rotating Sweep Line	17
6.10	Hull Cut	17
6.11	Point In Hull	17
6.12	Point In Polygon	17
6.13	Point In Polygon (Fast)	17
6.14	Cyclic Ternary Search	18
6.15	Tangent of Points to Hull	18
6.16	Circle Class & Intersection	18
6.17	Circle Common Tangent	18
6.18	Line-Circle Intersection	18
6.19	Poly-Circle Intersection	18
6.20	Min Covering Circle	18
6.21	Circle Union	19
6.22	Polygon Union	19
6.23	3D Point	19
6.24	3D Convex Hull	19
6.25	3D Projection	20
6.26	3D Skew Line Nearest Point	20
6.27	Delaunay	20
6.28	Build Voronoi	20
6.29	kd Tree (Nearest Point)	20
6.30	kd Closest Pair (3D ver.)	21
6.31	Simulated Annealing	21
6.32	Triangle Centers	21

## 7 Stringology

7.1	Hash	21
7.2	Suffix Array	21
7.3	Suffix Array Tools	21
7.4	Ex SAM	22
7.5	KMP	22
7.6	Z value	22
7.7	Manacher	22
7.8	Lyndon Factorization	22
7.9	Main Lorentz	22
7.10	BWT	23
7.11	Palindromic Tree	23

## 8 Misc

8.1	Theorems	23
8.2	Stable Marriage	24
8.3	Weight Matroid Intersection	24
8.4	Bitset LCS	24
8.5	Prefix Substring LCS	24
8.6	Convex ID/ID DP	24
8.7	ConvexHull Optimization	24
8.8	Min Plus Convolution	25
8.9	SMAWK	25
8.10	De-Bruijn	25
8.11	Josephus Problem	25
8.12	N Queens Problem	25
8.13	Tree Knapsack	25
8.14	Manhattan MST	25
8.15	Binary Search On Fraction	25
8.16	Nim Product	26

## 1 Basic

### 1.1 vimrc

```
se is nu ru et tgc sc hls cin cino+=j1 sw=2 sts=2 bs=2
mouse=a "encoding=utf-8 ls=2
syn on | colo desert | filetype indent on
inoremap {<CR> {<CR>}<ESC>O
map <F8> <ESC>:w<CR>:!g++ "%<" -o "%<" -g -std=gnu++20 -
    DCKISEKI -Wall -Wextra -Wshadow -Wfatal-errors -
    Wconversion -fsanitize=address,undefined,float-
    divide-by-zero,float-cast-overflow && echo success<
    CR>
map <F9> <ESC>:w<CR>:!g++ "%<" -o "%<" -O2 -g -std=gnu
    ++20 && echo success<CR>
map <F10> <ESC>:!/ "%<"<CR>
ca Hash w !cpp -dD -P -fpreprocessed \ | tr -d '[:space
    :]' \ | md5sum \ | cut -c-6
let c_no_curly_error=1
" setxkbmap -option caps:ctrl_modifier
```

### 1.2 Debug Macro [a45c59]

```
#define all(x) begin(x), end(x)
#ifndef CKISEKI
#include <experimental/iterator>
#define safe cerr<<__PRETTY_FUNCTION__<<" line "<<
    __LINE__<<" safe\n"
#define debug(a...) debug(#a, a)
#define orange(a...) orange(#a, a)
void debug_(auto s, auto ...a) {
    cerr << "[1;32m(" << s << ") = (" ;
    int f = 0;
    (... , (cerr << (f++ ? ", " : "") << a));
    cerr << ")]e[0m\n";
}
void orange_(auto s, auto L, auto R) {
    cerr << "[1;33m[ " << s << " ] = [ " ;
    using namespace experimental;
    copy(L, R, make_ostream_joiner(cerr, ", "));
    cerr << " ]e[0m\n";
}
#else
#define safe ((void)0)
#define debug(...) safe
#define orange(...) safe
#endif
```

### 1.3 SVG Writer [ac13c8]

```
#ifndef CKISEKI
class SVG {
    void p(string_view s) { o << s; }
    void p(string_view s, auto v, auto... vs) {
        auto i = s.find('$');
        o << s.substr(0, i) << v, p(s.substr(i + 1), vs...);
    }
    ostream o; string c = "red";
public:
    SVG(auto f, auto x1, auto y1, auto x2, auto y2) : o(f) {
        p("<svg xmlns='http://www.w3.org/2000/svg' "
            "viewBox='$ $ $ $'\n"
            "<style>{*stroke-width:0.5%;}</style>\n",
            x1, -y2, x2 - x1, y2 - y1);
        ~SVG() { p("</svg>\n"); }
    void color(string nc) { c = nc; }
    void line(auto x1, auto y1, auto x2, auto y2) {
        p("<line x1='$' y1='$' x2='$' y2='$' stroke='$'>\n",
            x1, -y1, x2, -y2, c);
    }
    void circle(auto x, auto y, auto r) {
        p("<circle cx='$' cy='$' r='$' stroke='$' "
            "fill='none'/>\n", x, -y, r, c);
    }
    void text(auto x, auto y, string s, int w = 12) {
        p("<text x='$' y='$' font-size='$px'>${s}</text>\n",
            x, -y, w, s);
    }; // write wrapper for complex if use complex
#else
class SVG { SVG(auto ...) {} }; // you know how to
#endif
```

### 1.4 Pragma Optimization [6006f6]

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,arch=skylake")
__builtin_ia32_ldmxcsr(__builtin_ia32_stmxcsr()|0x8040)
```

### 1.5 IO Optimization [c9494b]

```
static inline int gc() {
```

```
constexpr int B = 1<<20; static char buf[B], *p, *q;
if (p == q) q = (p = buf) + fread(buf, 1, B, stdin);
return q == buf ? EOF : *p++;
}
```

## 1.6 Increase Stack [b6856c]

```
const int size = 256 << 20;
register long rsp asm("rsp");
char *p = (char*)malloc(size)+size, *bak = (char*)rsp;
__asm__ ("movq %0, %%rsp\n"::"r"(p));
// main
__asm__ ("movq %0, %%rsp\n"::"r"(bak));
```

## 2 Data Structure

### 2.1 Dark Magic [095f25]

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
using namespace __gnu_pbds;
// heap tags: pairing/binary/binomial/rc_binomial/thin
template<typename T>
using pbds_heap=__gnu_pbds::prioity_queue<T,less<T>, \
    pairing_heap_tag>;
// pbds_heap::point_iterator
// x = pq.push(10); pq.modify(x, 87); a.join(b);
// tree tags: rb_tree_tag/ov_tree_tag/splay_tree_tag
template<typename T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;
// find_by_order, order_of_key
// hash tables: cc_hash_table/gp_hash_table
```

### 2.2 Link-Cut Tree [60627f]

```
template <typename Val, typename SVal> class LCT {
    struct node {
        int pa, ch[2];
        bool rev;
        Val v, prod, rprod;
        SVal sv, sub, vir;
        node() : pa{0}, ch{0, 0}, rev{false}, v{}, prod{},
            rprod{}, sv{}, sub{}, vir{} {};
    };
    #define cur o[u]
    #define lc cur.ch[0]
    #define rc cur.ch[1]
    vector<node> o;
    bool is_root(int u) const {
        return o[cur.pa].ch[0] != u && o[cur.pa].ch[1] != u;
    }
    bool is_rch(int u) const {
        return o[cur.pa].ch[1] == u && !is_root(u);
    }
    void down(int u) {
        if (not cur.rev) return;
        if (lc) set_rev(lc);
        if (rc) set_rev(rc);
        cur.rev = false;
    }
    void up(int u) {
        cur.prod = o[lc].prod * cur.v * o[rc].prod;
        cur.rprod = o[rc].rprod * cur.v * o[lc].rprod;
        cur.sub = cur.vir + o[lc].sub + o[rc].sub + cur.sv;
    }
    void set_rev(int u) {
        swap(lc, rc); swap(cur.prod, cur.rprod);
        cur.rev ^= 1;
    }
    void rotate(int u) {
        int f = cur.pa, g = o[f].pa, l = is_rch(u);
        if (cur.ch[l ^ 1]) o[cur.ch[l ^ 1]].pa = f;
        if (not is_root(f)) o[g].ch[is_rch(f)] = u;
        o[f].ch[l] = cur.ch[l ^ 1];
        cur.ch[l ^ 1] = f;
        cur.pa = g, o[f].pa = u;
        up(f);
    }
    void splay(int u) {
        vector<int> stk = {u};
        while (not is_root(stk.back()))
            stk.push_back(o[stk.back()].pa);
        while (not stk.empty()) {
            down(stk.back());
            stk.pop_back();
        }
        for (int f = cur.pa; not is_root(u); f = cur.pa) {
            if (!is_root(f))
```

```
        rotate(is_rch(u) == is_rch(f) ? f : u);
        rotate(u);
    }
    up(u);
}
void access(int x) {
    for (int u = x, last = 0; u; u = cur.pa) {
        splay(u);
        cur.vir = cur.vir + o[rc].sub - o[last].sub;
        rc = last; up(last = u);
    }
    splay(x);
}
int find_root(int u) {
    int la = 0;
    for (access(u); u; u = lc) down(la = u);
    return la;
}
void split(int x, int y) { chroot(x); access(y); }
void chroot(int u) { access(u); set_rev(u); }
public:
    LCT(int n = 0) : o(n + 1) {}
    int add(const Val &v = {}) {
        o.push_back(v);
        return int(o.size()) - 2;
    }
    void set_val(int u, const Val &v) {
        splay(++u); cur.v = v; up(u);
    }
    void set_sval(int u, const SVal &v) {
        access(++u); cur.sv = v; up(u);
    }
    Val query(int x, int y) {
        split(++x, ++y); return o[y].prod;
    }
    SVal subtree(int p, int u) {
        chroot(++p); access(++u);
        return cur.vir + cur.sv;
    }
    bool connected(int u, int v) {
        return find_root(++u) == find_root(++v);
    }
    void link(int x, int y) {
        chroot(++x); access(++y);
        o[y].vir = o[y].vir + o[x].sub;
        up(o[x].pa = y);
    }
    void cut(int x, int y) {
        split(++x, ++y);
        o[y].ch[0] = o[x].pa = 0; up(y);
    }
    #undef cur
    #undef lc
    #undef rc
};
```

### 2.3 LiChao Segment Tree [b9c827]

```
struct L {
    int m, k, id;
    L() : id(-1) {}
    L(int a, int b, int c) : m(a), k(b), id(c) {}
    int at(int x) { return m * x + k; }
};
class LiChao {
private:
    int n; vector<L> nodes;
    static int lc(int x) { return 2 * x + 1; }
    static int rc(int x) { return 2 * x + 2; }
    void insert(int l, int r, int id, L ln) {
        int m = (l + r) >> 1;
        if (nodes[id].id == -1)
            return nodes[id] = ln, void();
        bool atLeft = nodes[id].at(l) < ln.at(l);
        if (nodes[id].at(m) < ln.at(m))
            atLeft ^= 1, swap(nodes[id], ln);
        if (r - l == 1) return;
        if (atLeft) insert(l, m, lc(id), ln);
        else insert(m, r, rc(id), ln);
    }
    int query(int l, int r, int id, int x) {
        int m = (l + r) >> 1, ret = 0;
        if (nodes[id].id != -1) ret = nodes[id].at(x);
        if (r - l == 1) return ret;
```

```

    if (x < m) return max(ret, query(l, m, lc(id), x));
    return max(ret, query(m, r, rc(id), x));
}
public:
    LiChao(int n_) : n(n_), nodes(n * 4) {}
    void insert(L ln) { insert(0, n, 0, ln); }
    int query(int x) { return query(0, n, 0, x); }
};

```

## 2.4 Treap [ae576c]

```

__gnu_cxx::sfmt19937 rnd(7122); // <ext/random>
namespace Treap {
    struct node {
        int size, pri; node *lc, *rc, *pa;
        node() : size(1), pri(rnd()), lc(0), rc(0), pa(0) {}
        void pull() {
            size = 1; pa = 0;
            if (lc) { size += lc->size; lc->pa = this; }
            if (rc) { size += rc->size; rc->pa = this; }
        }
    };
    int SZ(node *x) { return x ? x->size : 0; }
    node *merge(node *L, node *R) {
        if (not L or not R) return L ? L : R;
        if (L->pri > R->pri)
            return L->rc = merge(L->rc, R), L->pull(), L;
        else
            return R->lc = merge(L, R->lc), R->pull(), R;
    }
    void splitBySize(node *o, int k, node *&L, node *&R) {
        if (not o) L = R = 0;
        else if (int s = SZ(o->lc) + 1; s <= k)
            L = o, splitBySize(o->rc, k-s, L->rc, R), L->pull();
        else
            R = o, splitBySize(o->lc, k, L, R->lc), R->pull();
    } // SZ(L) == k
    int getRank(node *o) { // 1-base
        int r = SZ(o->lc) + 1;
        for (; o->pa; o = o->pa)
            if (o->pa->rc == o) r += SZ(o->pa->lc) + 1;
        return r;
    }
} // namespace Treap

```

## 2.5 Linear Basis [138d5d]

```

template <int BITS, typename S = int> struct Basis {
    static constexpr S MIN = numeric_limits<S>::min();
    array<pair<llu, S>, BITS> b;
    Basis() { b.fill({0, MIN}); }
    void add(llu x, S p) {
        for (int i = BITS-1; i>=0; i--) if (x >> i & 1) {
            if (b[i].first == 0) return b[i]={x, p}, void();
            if (b[i].second < p)
                swap(b[i].first, x), swap(b[i].second, p);
            x ^= b[i].first;
        }
    }
    optional<llu> query_kth(llu v, llu k) {
        vector<pair<llu, int>> o;
        for (int i = 0; i < BITS; i++)
            if (b[i].first) o.emplace_back(b[i].first, i);
        if (k >= (1ULL << o.size())) return {};
        for (int i = int(o.size()) - 1; i >= 0; i--)
            if ((k >> i & 1) ^ (v >> o[i].second & 1))
                v ^= o[i].first;
        return v;
    }
    Basis filter(S l) {
        Basis res = *this;
        for (int i = 0; i < BITS; i++)
            if (res.b[i].second < l) res.b[i] = {0, MIN};
        return res;
    }
};

```

## 2.6 Binary Search on Segtree [6c61c0]

```

// find_first = l -> minimal x s.t. check([l, x])
// find_last = r -> maximal x s.t. check([x, r])
int find_first(int l, auto &&check) {
    if (l >= n) return n + 1;
    l += sz; push(l); Monoid sum; // identity
    do {
        while ((l & 1) == 0) l >>= 1;
        if (auto s = sum + nd[l]; check(s)) {

```

```

            while (l < sz) {
                prop(l); l = (l << 1);
                if (auto nxt = sum + nd[l]; not check(nxt))
                    sum = nxt, l++;
            }
            return l + 1 - sz;
        } else sum = s, l++;
    } while (lowbit(l) != l);
    return n + 1;
}
int find_last(int r, auto &&check) {
    if (r <= 0) return -1;
    r += sz; push(r - 1); Monoid sum; // identity
    do {
        r--;
        while (r > 1 and (r & 1)) r >>= 1;
        if (auto s = nd[r] + sum; check(s)) {
            while (r < sz) {
                prop(r); r = (r << 1) | 1;
                if (auto nxt = nd[r] + sum; not check(nxt))
                    sum = nxt, r--;
            }
            return r - sz;
        } else sum = s;
    } while (lowbit(r) != r);
    return -1;
}

```

## 3 Graph

### 3.1 SCC [16c7d6]

```

class SCC { // test @ library checker
protected:
    int n, dfc, nscc; vector<vector<int>> G;
    vector<int> vis, low, idx, stk;
    void dfs(int i) {
        vis[i] = low[i] = ++dfc; stk.push_back(i);
        for (int j : G[i])
            if (!vis[j])
                dfs(j), low[i] = min(low[i], low[j]);
            else if (vis[j] != -1)
                low[i] = min(low[i], vis[j]);
        if (low[i] == vis[i])
            for (idx[i] = nscc++; vis[i] != -1;) {
                int x = stk.back(); stk.pop_back();
                idx[x] = idx[i]; vis[x] = -1;
            }
    }
public:
    SCC(int n_) : n(n_), dfc(0), nscc(0), G(n),
        vis(n), low(n), idx(n) {}
    void add_edge(int u, int v) { G[u].push_back(v); }
    void solve() {
        for (int i = 0; i < n; i++) if (!vis[i]) dfs(i);
    }
    int get_id(int x) { return idx[x]; }
    int count() { return nscc; }
}; // dag edges point from idx large to idx small

```

### 3.2 2-SAT [ca961f]

```

struct TwoSat : SCC {
    void orr(int x, int y) {
        if ((x ^ y) == 1) return;
        add_edge(x ^ 1, y); add_edge(y ^ 1, x);
    }
    vector<int> solve2sat() {
        solve(); vector<int> res(n);
        for (int i = 0; i < n; i += 2)
            if (idx[i] == idx[i + 1]) return {};
        for (int i = 0; i < n; i++)
            res[i] = idx[i] < idx[i ^ 1];
        return res;
    }
};

```

### 3.3 BCC [6ac6db]

```

class BCC {
    int n, ecnt, bcnt;
    vector<vector<pair<int, int>>> g;
    vector<int> dfn, low, bcc, stk;
    vector<bool> ap, bridge;
    void dfs(int u, int f) {
        dfn[u] = low[u] = dfn[f] + 1;
        int ch = 0;
        for (auto [v, t] : g[u]) if (bcc[t] == -1) {
            bcc[t] = 0; stk.push_back(t);

```

```

if (dfn[v]) {
    low[u] = min(low[u], dfn[v]);
    continue;
}
++ch, dfs(v, u);
low[u] = min(low[u], low[v]);
if (low[v] > dfn[u]) bridge[t] = true;
if (low[v] < dfn[u]) continue;
ap[u] = true;
while (not stk.empty()) {
    int o = stk.back(); stk.pop_back();
    bcc[o] = bcnt;
    if (o == t) break;
}
bcnt += 1;
}
ap[u] = ap[u] and (ch != 1 or u != f);
}
public:
BCC(int n_) : n(n_), ecnt(0), bcnt(0), g(n), dfn(n),
    low(n), stk(), ap(n) {}
void add_edge(int u, int v) {
    g[u].emplace_back(v, ecnt);
    g[v].emplace_back(u, ecnt++);
}
void solve() {
    bridge.assign(ecnt, false); bcc.assign(ecnt, -1);
    for (int i = 0; i < n; ++i) if (!dfn[i]) dfs(i, i);
}
int bcc_id(int x) const { return bcc[x]; }
bool is_ap(int x) const { return ap[x]; }
bool is_bridge(int x) const { return bridge[x]; }
};

```

### 3.4 Round Square Tree [cf6d74]

```

struct RST { // be careful about isolate point
    int n; vector<vector<int>> T;
    RST(auto &G) : n(G.size()), T(n) {
        vector<int> stk, vis(n), low(n);
        auto dfs = [&](auto self, int u, int d) -> void {
            low[u] = vis[u] = d; stk.push_back(u);
            for (int v : G[u]) if (!vis[v]) {
                self(self, v, d + 1);
                if (low[v] == vis[u]) {
                    int cnt = T.size(); T.emplace_back();
                    for (int x = -1; x != v; stk.pop_back())
                        T[cnt].push_back(x = stk.back());
                    T[u].push_back(cnt); // T is rooted
                } else low[u] = min(low[u], low[v]);
            } else low[u] = min(low[u], vis[v]);
        };
        for (int u = 0; u < n; u++)
            if (!vis[u]) dfs(dfs, u, 1);
    } // T may be forest; after dfs, stk are the roots
}; // test @ 2020 Shanghai K

```

### 3.5 Edge TCC [5a2668]

```

vector<vector<int>> ETCC(auto &adj) {
    const int n = static_cast<int>(adj.size());
    vector<int> up(n), low(n), in, out, nx, id;
    in = out = nx = id = vector<int>(n, -1);
    int dfc = 0, cnt = 0; Dsu dsu(n);
    auto merge = [&](int u, int v) {
        dsu.join(u, v); up[u] += up[v]; };
    auto dfs = [&](auto self, int u, int p) -> void {
        in[u] = low[u] = dfc++;
        for (int v : adj[u]) if (v != u) {
            if (v == p) { p = -1; continue; }
            if (in[v] == -1) {
                self(self, v, u);
                if (nx[v] == -1 && up[v] <= 1) {
                    up[u] += up[v]; low[u] = min(low[u], low[v]);
                    continue;
                }
            }
            if (up[v] == 0) v = nx[v];
            if (low[u] > low[v])
                low[u] = low[v], swap(nx[u], v);
            for (; v != -1; v = nx[v]) merge(u, v);
        } else if (in[v] < in[u]) {
            low[u] = min(low[u], in[v]); up[u]++;
        } else {
            for (int &x = nx[u]; x != -1 &&
                in[x] <= in[v] && in[v] < out[x]; x = nx[x])

```

```

                merge(u, x);
                up[u]--;
            }
        }
        out[u] = dfc;
    };
    for (int i = 0; i < n; i++)
        if (in[i] == -1) dfs(dfs, i, -1);
    for (int i = 0; i < n; i++)
        if (dsu.anc(i) == i) id[i] = cnt++;
    vector<vector<int>> comps(cnt);
    for (int i = 0; i < n; i++)
        comps[id[dsu.anc(i)]].push_back(i);
    return comps;
} // test @ yosupo judge

```

### 3.6 Bipolar Orientation [9d5557]

```

struct BipolarOrientation {
    int n; vector<vector<int>> g;
    vector<int> vis, low, pa, sgn, ord;
    BipolarOrientation(int n_) : n(n_),
        g(n), vis(n), low(n), pa(n, -1), sgn(n) {}
    void dfs(int i) {
        ord.push_back(i); low[i] = vis[i] = int(ord.size());
        for (int j : g[i])
            if (!vis[j])
                pa[j] = i, dfs(j), low[i] = min(low[i], low[j]);
            else
                low[i] = min(low[i], vis[j]);
    }
    vector<int> solve(int S, int T) {
        g[S].insert(g[S].begin(), T); dfs(S);
        vector<int> nxt(n, -1), prv(n, -1);
        nxt[S] = T; prv[T] = S; sgn[S] = -1;
        for (int i : ord) if (i != S && i != T) {
            int p = pa[i], l = ord[low[i] - 1];
            if (sgn[l] > 0) { // insert after
                nxt[i] = nxt[p]; prv[i] = p;
                if (nxt[p] != -1) prv[nxt[p]] = i;
                nxt[p] = i;
            } else {
                prv[i] = prv[p]; nxt[i] = p;
                if (prv[p] != -1) nxt[prv[p]] = i;
                prv[p] = i;
            }
            sgn[p] = -sgn[l];
        }
        vector<int> v;
        for (int x = S; x != -1; x = nxt[x]) v.push_back(x);
        return v;
    } // S, T are unique source / unique sink
    void add_edge(int a, int b) {
        g[a].emplace_back(b); g[b].emplace_back(a);
    }; // 存在 ST 雙極定向 iff 連接 (S,T) 後整張圖點雙連通
};

```

### 3.7 DMST [f4317e]

```

using lld = int64_t;
struct E { int s, t; lld w; }; // 0-base
struct PQ {
    struct P {
        lld v; int i;
        bool operator>(const P &b) const { return v > b.v; }
    };
    min_heap<P> pq; lld tag;
    void push(P p) { p.v -= tag; pq.emplace(p); }
    P top() { P p = pq.top(); p.v += tag; return p; }
    void join(PQ &b) {
        if (pq.size() < b.pq.size())
            swap(pq, b.pq), swap(tag, b.tag);
        while (!b.pq.empty()) push(b.top()), b.pq.pop();
    };
};
vector<int> dmst(const vector<E> &e, int n, int root) {
    vector<PQ> h(n * 2);
    for (int i = 0; i < int(e.size()); ++i)
        h[e[i].t].push({e[i].w, i});
    vector<int> a(n * 2); iota(all(a), 0);
    vector<int> v(n * 2, -1), pa(n * 2, -1), r(n * 2);
    auto o = [&](auto Y, int x) -> int {
        return x == a[x] ? x : a[x] = Y(Y, a[x]); };
    auto S = [&](int i) { return o(o, e[i].s); };
    int pc = v[root] = n;
    for (int i = 0; i < n; ++i) if (v[i] == -1)

```



```

for (int p = i; v[p]<0 || v[p]==i; p = S(r[p])) {
    if (v[p] == i)
        for (int q = pc++; p != q; p = S(r[p])) {
            h[p].tag -= h[p].top().v; h[q].join(h[p]);
            pa[p] = a[p] = q;
        }
    while (S(h[p].top().i) == p) h[p].pq.pop();
    v[p] = i; r[p] = h[p].top().i;
}
vector<int> ans;
for (int i = pc - 1; i >= 0; i--) if (v[i] != n) {
    for (int f = e[r[i]].t; f!=-1 && v[f]!=n; f = pa[f])
        v[f] = n;
    ans.push_back(r[i]);
}
return ans; // default minimize, returns edgeid array
}

```

### 3.8 Dominator Tree [ea5b7c]

```

struct Dominator {
    vector<vector<int>> g, r, rdom; int tk;
    vector<int> dfn, rev, fa, sdom, dom, val, rp;
    Dominator(int n) : g(n), r(n), rdom(n), tk(0) {
        dfn = rev = fa = sdom = dom =
            val = rp = vector<int>(n, -1);
    }
    void add_edge(int x, int y) { g[x].push_back(y); }
    void dfs(int x) {
        rev[dfn[x] = tk] = x;
        fa[tk] = sdom[tk] = val[tk] = tk; tk++;
        for (int u : g[x]) {
            if (dfn[u] == -1) dfs(u), rp[dfn[u]] = dfn[x];
            r[dfn[u]].push_back(dfn[x]);
        }
    }
    void merge(int x, int y) { fa[x] = y; }
    int find(int x, int c = 0) {
        if (fa[x] == x) return c ? -1 : x;
        if (int p = find(fa[x], 1); p != -1) {
            if (sdom[val[x]] > sdom[val[fa[x]]])
                val[x] = val[fa[x]];
            fa[x] = p;
            return c ? p : val[x];
        } else return c ? fa[x] : val[x];
    }
    vector<int> build(int s, int n) {
        // return the father of each node in dominator tree
        dfs(s); // p[i] = -2 if i is unreachable from s
        for (int i = tk - 1; i >= 0; --i) {
            for (int u : r[i])
                sdom[i] = min(sdom[i], sdom[find(u)]);
            if (i) rdom[sdom[i]].push_back(i);
            for (int u : rdom[i]) {
                int p = find(u);
                dom[u] = (sdom[p] == i ? i : p);
            }
            if (i) merge(i, rp[i]);
        }
        vector<int> p(n, -2); p[s] = -1;
        for (int i = 1; i < tk; ++i)
            if (sdom[i] != dom[i]) dom[i] = dom[dom[i]];
        for (int i = 1; i < tk; ++i)
            p[rev[i]] = rev[dom[i]];
        return p;
    } // test @ yosupo judge
};

```

### 3.9 Edge Coloring [029763]

```

// max(d_u) + 1 edge coloring, time: O(NM)
int C[kN][kN], G[kN][kN]; // 1-based, G: ans
void clear(int N) {
    for (int i = 0; i <= N; i++)
        for (int j = 0; j <= N; j++)
            C[i][j] = G[i][j] = 0;
}
void solve(vector<pair<int, int>> &E, int N) {
    int X[kN] = {}, a;
    auto update = [&](int u) {
        for (X[u] = 1; C[u][X[u]]; X[u]++);
    };
    auto color = [&](int u, int v, int c) {
        int p = G[u][v];
        G[u][v] = G[v][u] = c;
        C[u][c] = v, C[v][c] = u;
    };
}

```

```

C[u][p] = C[v][p] = 0;
if (p) X[u] = X[v] = p;
else update(u), update(v);
return p;
};
auto flip = [&](int u, int c1, int c2) {
    int p = C[u][c1];
    swap(C[u][c1], C[u][c2]);
    if (p) G[u][p] = G[p][u] = c2;
    if (!C[u][c1]) X[u] = c1;
    if (!C[u][c2]) X[u] = c2;
    return p;
};
for (int i = 1; i <= N; i++) X[i] = 1;
for (int t = 0; t < E.size(); t++) {
    auto [u, v] = E[t];
    int v0 = v, c = X[u], c0 = c, d;
    vector<pair<int, int>> L; int vst[kN] = {};
    while (!G[u][v0]) {
        L.emplace_back(v, d = X[v]);
        if (!C[v][c]) for (a=L.size()-1;a>=0;a--)
            c = color(u, L[a].first, c);
        else if (!C[u][d]) for (a=L.size()-1;a>=0;a--)
            color(u, L[a].first, L[a].second);
        else if (vst[d]) break;
        else vst[d] = 1, v = C[u][d];
    }
    if (!G[u][v0]) {
        for (; v; v = flip(v, c, d), swap(c, d));
        if (C[u][c0]) { a = int(L.size()) - 1;
            while (--a >= 0 && L[a].second != c);
            for (; a >= 0; a--) color(u, L[a].first, L[a].second);
        } else t--;
    }
}
}

```

### 3.10 Centroid Decomp. [670cdd]

```

class Centroid {
    vector<vector<pair<int, int>>> g; // g[u] = {(v, w)}
    vector<int> pa, dep, vis, sz, mx;
    vector<vector<int64_t>> Dist;
    vector<int64_t> Sub, Sub2;
    vector<int> Cnt, Cnt2;
    void DfsSz(vector<int> &tmp, int x) {
        vis[x] = true, sz[x] = 1, mx[x] = 0;
        for (auto [u, w] : g[x]) if (not vis[u]) {
            DfsSz(tmp, u); sz[x] += sz[u];
            mx[x] = max(mx[x], sz[u]);
        }
        tmp.push_back(x);
    }
    void DfsDist(int x, int64_t D = 0) {
        Dist[x].push_back(D); vis[x] = true;
        for (auto [u, w] : g[x])
            if (not vis[u]) DfsDist(u, D + w);
    }
    void DfsCen(int x, int D, int p) {
        vector<int> tmp; DfsSz(tmp, x);
        int M = int(tmp.size()), C = -1;
        for (int u : tmp)
            if (max(M - sz[u], mx[u]) * 2 <= M) C = u;
        for (int u : tmp) vis[u] = false;
        DfsDist(C);
        for (int u : tmp) vis[u] = false;
        pa[C] = p, vis[C] = true, dep[C] = D;
        for (auto [u, w] : g[C])
            if (not vis[u]) DfsCen(u, D + 1, C);
    }
public:
    Centroid(int N) : g(N), pa(N), dep(N),
        vis(N), sz(N), mx(N), Dist(N),
        Sub(N), Sub2(N), Cnt(N), Cnt2(N) {}
    void AddEdge(int u, int v, int w) {
        g[u].emplace_back(v, w);
        g[v].emplace_back(u, w);
    }
    void Build() { DfsCen(0, 0, -1); }
    void Mark(int v) {
        int x = v, z = -1;
        for (int i = dep[v]; i >= 0; --i) {
            Sub[x] += Dist[v][i], Cnt[x]++;
        }
    }
}

```

```

    if (z != -1)
        Sub2[z] += Dist[v][i], Cnt2[z]++;
    x = pa[z = x];
}
}
int64_t Query(int v) {
    int64_t res = 0;
    int x = v, z = -1;
    for (int i = dep[v]; i >= 0; --i) {
        res += Sub[x] + 1LL * Cnt[x] * Dist[v][i];
        if (z != -1)
            res -= Sub2[z] + 1LL * Cnt2[z] * Dist[v][i];
        x = pa[z = x];
    }
    return res;
}; // pa, dep are centroid tree attributes

```

### 3.11 Lowbit Decomp. [d1d724]

```

class LBD {
    int timer, chains;
    vector<vector<int>> G;
    vector<int> tl, tr, chain, head, dep, pa;
    // chains : number of chain
    // tl, tr[u] : subtree interval in the seq. of u
    // head[i] : head of the chain i
    // chain[u] : chain id of the chain u is on
    void predfs(int u, int f) {
        dep[u] = dep[pa[u] = f] + 1;
        for (int v : G[u]) if (v != f) {
            predfs(v, u);
            if (lowbit(chain[u]) < lowbit(chain[v]))
                chain[u] = chain[v];
        }
        if (chain[u] == 0) chain[u] = ++chains;
    }
    void dfschain(int u, int f) {
        tl[u] = timer++;
        if (head[chain[u]] == -1)
            head[chain[u]] = u;
        for (int v : G[u])
            if (v != f and chain[v] == chain[u])
                dfschain(v, u);
        for (int v : G[u])
            if (v != f and chain[v] != chain[u])
                dfschain(v, u);
        tr[u] = timer;
    }
public:
    LBD(auto &&G_) : n((int)size(G_)),
        timer(0), chains(0), G(G_), tl(n), tr(n),
        chain(n), head(n + 1, -1), dep(n), pa(n)
    { predfs(0, 0); dfschain(0, 0); }
    PII get_subtree(int u) { return {tl[u], tr[u]}; }
    vector<PII> get_path(int u, int v) {
        vector<PII> res;
        while (chain[u] != chain[v]) {
            if (dep[head[chain[u]]] < dep[head[chain[v]]])
                swap(u, v);
            int s = head[chain[u]];
            res.emplace_back(tl[s], tl[u] + 1);
            u = pa[s];
        }
        if (dep[u] < dep[v]) swap(u, v);
        res.emplace_back(tl[v], tl[u] + 1);
        return res;
    }
}; // 記得在資結上對點的修改要改成對其 dfs 序的修改

```

### 3.12 Virtual Tree [ad5cf5]

```

vector<pair<int, int>> build(vector<int> vs, int r) {
    vector<pair<int, int>> res;
    sort(vs.begin(), vs.end(), [](int i, int j) {
        return dfn[i] < dfn[j]; });
    vector<int> s = {r};
    for (int v : vs) if (v != r) {
        if (int o = lca(v, s.back()); o != s.back()) {
            while (s.size() >= 2) {
                if (dfn[s[s.size() - 2]] < dfn[o]) break;
                res.emplace_back(s[s.size() - 2], s.back());
                s.pop_back();
            }
            if (s.back() != o) {

```

```

                res.emplace_back(o, s.back());
                s.back() = o;
            }
        }
        s.push_back(v);
    }
    for (size_t i = 1; i < s.size(); ++i)
        res.emplace_back(s[i - 1], s[i]);
    return res; // (x, y): x->y
}

```

### 3.13 Tree Hashing [d6a9f9]

```

vector<int> g[maxn]; ll u h[maxn];
ll F(ll u) { // xorshift64star from iwiwi
    u ^= u >> 12; u ^= u << 25; u ^= u >> 27;
    return u * 2685821657736338717LL;
}
ll hsah(int u, int f) {
    ll r = 127; // bigger?
    for (int v : g[u]) if (v != f) r += hsah(v, u);
    return h[u] = F(r);
} // test @ UOJ 763 & yosupo library checker

```

### 3.14 Mo's Algo on Tree

```

dfs u:
    push u
    iterate subtree
    push u
Let P = LCA(u, v) with St(u) <= St(v)
if (P == u) query[St(u), St(v)]
else query[Ed(u), St(v)], query[St(P), St(P)]
3.15 Count Cycles [c7e8f2]
// ord = sort by deg decreasing, rk[ord[i]] = i
// D[i] = edge point from rk small to rk big
for (int x : ord) { // c3
    for (int y : D[x]) vis[y] = 1;
    for (int y : D[x]) for (int z : D[y]) c3 += vis[z];
    for (int y : D[x]) vis[y] = 0;
}
for (int x : ord) { // c4
    for (int y : D[x]) for (int z : adj[y])
        if (rk[z] > rk[x]) c4 += vis[z]++;
    for (int y : D[x]) for (int z : adj[y])
        if (rk[z] > rk[x]) --vis[z];
} // both are O(M*sqrt(M)), test @ 2022 CCPC guangzhou

```

### 3.16 Maximal Clique [293730]

```

// contain a self loop u to u, then u won't in clique
template <size_t maxn> class MaxClique {
private:
    using bits = bitset<maxn>;
    bits popped, G[maxn], ans;
    size_t deg[maxn], deo[maxn], n;
    void sort_by_degree() {
        popped.reset();
        for (size_t i = 0; i < n; ++i)
            deg[i] = G[i].count();
        for (size_t i = 0; i < n; ++i) {
            size_t mi = maxn, id = 0;
            for (size_t j = 0; j < n; ++j)
                if (not popped[j] and deg[j] < mi)
                    mi = deg[id = j];
            popped[deo[i] = id] = 1;
            for (size_t u = G[id]._Find_first(); u < n;
                u = G[id]._Find_next(u))
                --deg[u];
        }
    }
    void BK(bits R, bits P, bits X) {
        if (R.count() + P.count() <= ans.count()) return;
        if (not P.count() and not X.count()) {
            if (R.count() > ans.count()) ans = R;
            return;
        }
        /* greedily choose max degree as pivot
        bits cur = P | X; size_t pivot = 0, sz = 0;
        for (size_t u = cur._Find_first();
            u < n; u = cur._Find_next(u))
            if (deg[u] > sz) sz = deg[pivot = u];
        cur = P & (~G[pivot]);
        */ // or simply choose first
        bits cur = P & (~G[pivot]);
        for (size_t u = cur._Find_first(); u < n;
            u = cur._Find_next(u)) {

```

```

    if (R[u]) continue;
    R[u] = 1;
    BK(R, P & G[u], X & G[u]);
    R[u] = P[u] = 0, X[u] = 1;
}
}
public:
void init(size_t n_) {
    n = n_;
    for (size_t i = 0; i < n; ++i) G[i].reset();
    ans.reset();
}
void add_edges(int u, bits S) { G[u] = S; }
void add_edge(int u, int v) { G[u][v] = G[v][u] = 1; }
int solve() {
    sort_by_degree(); // or simply iota(deo...)
    for (size_t i = 0; i < n; ++i)
        deg[i] = G[i].count();
    bits pob, nob = 0; pob.set();
    for (size_t i = n; i < maxn; ++i) pob[i] = 0;
    for (size_t i = 0; i < n; ++i) {
        size_t v = deo[i];
        bits tmp;
        tmp[v] = 1;
        BK(tmp, pob & G[v], nob & G[v]);
        pob[v] = 0, nob[v] = 1;
    }
    return static_cast<int>(ans.count());
}
};

```

### 3.17 Maximum Clique [aee5d8]

```

constexpr size_t KN = 150; using bits = bitset<KN>;
struct MaxClique {
    bits G[KN], cs[KN];
    int ans, sol[KN], q, cur[KN], d[KN], n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].reset();
    }
    void add_edge(int u, int v) { G[u][v] = G[v][u] = 1; }
    void pre_dfs(vector<int> &v, int i, bits mask) {
        if (i < 4) {
            for (int x : v) d[x] = (int)(G[x] & mask).count();
            sort(all(v), [&](int x, int y) {
                return d[x] > d[y]; });
        }
        vector<int> c(v.size());
        cs[1].reset(), cs[2].reset();
        int l = max(ans - q + 1, 1), r = 2, tp = 0, k;
        for (int p : v) {
            for (k = 1; (cs[k] & G[p]).any(); ++k);
            if (k >= r) cs[++r].reset();
            cs[k][p] = 1;
            if (k < l) v[tp++] = p;
        }
        for (k = l; k < r; ++k)
            for (auto p = cs[k]._Find_first();
                 p < KN; p = cs[k]._Find_next(p))
                v[tp] = (int)p, c[tp] = k, ++tp;
        dfs(v, c, i + 1, mask);
    }
    void dfs(vector<int> &v, vector<int> &c,
            int i, bits mask) {
        while (!v.empty()) {
            int p = v.back(); v.pop_back(); mask[p] = 0;
            if (q + c.back() <= ans) return;
            cur[q++] = p;
            vector<int> nr;
            for (int x : v) if (G[p][x]) nr.push_back(x);
            if (!nr.empty()) pre_dfs(nr, i, mask & G[p]);
            else if (q > ans) ans = q, copy_n(cur, q, sol);
            c.pop_back(); --q;
        }
    }
    int solve() {
        vector<int> v(n); iota(all(v), 0);
        ans = q = 0; pre_dfs(v, 0, bits(string(n, '1')));
        return ans; // sol[0 ~ ans-1]
    }
} cliq; // test @ yosupo judge

```

### 3.18 Min Mean Cycle [e23bc0]

```

// WARNING: TYPE matters
struct Edge { int s, t; llf c; };
llf solve(vector<Edge> &e, int n) {
    // O(VE), returns inf if no cycle, mmc otherwise
    vector<VI> prv(n + 1, VI(n)), prve = prv;
    vector<vector<llf>> d(n + 1, vector<llf>(n, inf));
    d[0] = vector<llf>(n, 0);
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < (int)e.size(); j++) {
            auto [s, t, c] = e[j];
            if (d[i][s] < inf && d[i + 1][t] > d[i][s] + c) {
                d[i + 1][t] = d[i][s] + c;
                prv[i + 1][t] = s; prve[i + 1][t] = j;
            }
        }
    }
    llf mmc = inf; int st = -1;
    for (int i = 0; i < n; i++) {
        llf avg = -inf;
        for (int k = 0; k < n; k++) {
            if (d[n][i] < inf - eps)
                avg = max(avg, (d[n][i] - d[k][i]) / (n - k));
            else avg = inf;
        }
        if (avg < mmc) tie(mmc, st) = tie(avg, i);
    }
    if (st == -1) return inf;
    vector<int> vst(n), eid, cycle, rho;
    for (int i = n; !vst[st]; st = prv[i--][st]) {
        vst[st]++; eid.emplace_back(prve[i][st]);
        rho.emplace_back(st);
    }
    while (vst[st] != 2) {
        int v = rho.back(); rho.pop_back();
        cycle.emplace_back(v); vst[v]++;
    }
    reverse(all(eid)); eid.resize(cycle.size());
    return mmc;
}

```

## 4 Flow & Matching

### 4.1 HopcroftKarp [930040]

```

struct HK {
    vector<int> l, r, a, p; int ans;
    HK(int n, int m, auto &g) : l(n, -1), r(m, -1), ans(0) {
        for (bool match = true; match;) {
            match = false; a.assign(n, -1); p.assign(m, -1);
            queue<int> q;
            for (int i = 0; i < n; i++)
                if (l[i] == -1) q.push(a[i] = p[i] = i);
            // bitset<maxn> nvis, t; nvis.set();
            while (!q.empty()) {
                int z, x = q.front(); q.pop();
                if (l[a[x]] != -1) continue;
                for (int y : g[x]) { // or iterate t = g[x]&nvis
                    // nvis.reset(y);
                    if (r[y] == -1) {
                        for (z = y; z != -1; )
                            r[z] = x, swap(l[x], z), x = p[x];
                        match = true; ++ans; break;
                    } else if (p[r[y]] == -1)
                        q.push(z = r[y]), p[z] = x, a[z] = a[x];
                }
            }
        }
    }
};

```

### 4.2 Kuhn Munkres [2c09ed]

```

struct KM { // maximize, test @ UOJ 80
    int n, l, r; lld ans; // fl and fr are the match
    vector<lld> hl, hr; vector<int> fl, fr, pre, q;
    void bfs(const auto &w, int s) {
        vector<int> vl(n), vr(n); vector<lld> slk(n, INF);
        l = r = 0; vr[q[r++]] = s;
        const auto check = [&](int x) -> bool {
            if (vl[x] || slk[x] > 0) return true;
            vl[x] = true; slk[x] = INF;
            if (fl[x] != -1) return vr[q[r++]] = fl[x] = true;
            while (x != -1) swap(x, fr[fl[x] = pre[x]]);
            return false;
        };
        while (true) {

```

```

while (l < r)
    for (int x = 0, y = q[l++]; x < n; ++x) if (!vl[x])
        if (chmin(slk[x], hl[x] + hr[y] - w[x][y]))
            if (pre[x] = y, !check(x)) return;
    lld d = ranges::min(slk);
    for (int x = 0; x < n; ++x)
        vl[x] ? hl[x] += d : slk[x] -= d;
    for (int x = 0; x < n; ++x) if (vr[x]) hr[x] -= d;
    for (int x = 0; x < n; ++x) if (!check(x)) return;
}
}
KM(int n_, const auto &w) : n(n_), ans(0),
    hl(n), hr(n), fl(n, -1), fr(fl), pre(n), q(n) {
    for (int i = 0; i < n; ++i) hl[i] = ranges::max(w[i]);
    for (int i = 0; i < n; ++i) bfs(w, i);
    for (int i = 0; i < n; ++i) ans += w[i][fl[i]];
}
};

```

### 4.3 Flow Models

- Maximum/Minimum flow with lower bound / Circulation problem
  - Construct super source  $S$  and sink  $T$ .
  - For each edge  $(x, y, l, u)$ , connect  $x \rightarrow y$  with capacity  $u - l$ .
  - For each vertex  $v$ , denote by  $in(v)$  the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  - If  $in(v) > 0$ , connect  $S \rightarrow v$  with capacity  $in(v)$ , otherwise, connect  $v \rightarrow T$  with capacity  $-in(v)$ .
    - To maximize, connect  $t \rightarrow s$  with capacity  $\infty$  (skip this in circulation problem), and let  $f$  be the maximum flow from  $S$  to  $T$ . If  $f \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise, the maximum flow from  $s$  to  $t$  is the answer. Also,  $f$  is a mincost valid flow.
    - To minimize, let  $f$  be the maximum flow from  $S$  to  $T$ . Connect  $t \rightarrow s$  with capacity  $\infty$  and let the flow from  $S$  to  $T$  be  $f'$ . If  $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise,  $f'$  is the answer.
  - The solution of each edge  $e$  is  $l_e + f_e$ , where  $f_e$  corresponds to the flow of edge  $e$  on the graph.
- Construct minimum vertex cover from maximum matching  $M$  on bipartite graph  $(X, Y)$ 
  - Redirect every edge:  $y \rightarrow x$  if  $(x, y) \in M$ ,  $x \rightarrow y$  otherwise.
  - DFS from unmatched vertices in  $X$ .
  - $x \in X$  is chosen iff  $x$  is unvisited;  $y \in Y$  is chosen iff  $y$  is visited.
- Minimum cost cyclic flow
  - Construct super source  $S$  and sink  $T$
  - For each edge  $(x, y, c)$ , connect  $x \rightarrow y$  with  $(cost, cap) = (c, 1)$  if  $c > 0$ , otherwise connect  $y \rightarrow x$  with  $(cost, cap) = (-c, 1)$
  - For each edge with  $c < 0$ , sum these cost as  $K$ , then increase  $d(y)$  by 1, decrease  $d(x)$  by 1
  - For each vertex  $v$  with  $d(v) > 0$ , connect  $S \rightarrow v$  with  $(cost, cap) = (0, d(v))$
  - For each vertex  $v$  with  $d(v) < 0$ , connect  $v \rightarrow T$  with  $(cost, cap) = (0, -d(v))$
  - Flow from  $S$  to  $T$ , the answer is the cost of the flow  $C + K$
- Maximum density induced subgraph
  - Binary search on answer, suppose we're checking answer  $T$
  - Construct a max flow model, let  $K$  be the sum of all weights
  - Connect source  $s \rightarrow v, v \in G$  with capacity  $K$
  - For each edge  $(u, v, w)$  in  $G$ , connect  $u \rightarrow v$  and  $v \rightarrow u$  with capacity  $w$
  - For  $v \in G$ , connect it with sink  $v \rightarrow t$  with capacity  $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
  - $T$  is a valid answer if the maximum flow  $f < K|V|$
- Minimum weight edge cover
  - For each  $v \in V$  create a copy  $v'$ , and connect  $u' \rightarrow v'$  with weight  $w(u, v)$ .
  - Connect  $v \rightarrow v'$  with weight  $2\mu(v)$ , where  $\mu(v)$  is the cost of the cheapest edge incident to  $v$ .
  - Find the minimum weight perfect matching on  $G'$ .
- Project selection cheat sheet:  $S, T$  分別代表 0, 1 側, 最小化總花費。
 

$i$ 為 0 時花費 $c$	$(i, T, c)$
$i$ 為 1 時花費 $c$	$(S, i, c)$
$i \in I$ 有任何一個為 0 時花費 $c$	$(i, w, \infty), (w, T, c)$
$i \in I$ 有任何一個為 1 時花費 $c$	$(S, w, c), (w, i, \infty)$
$i$ 為 0 時得到 $c$	直接得到 $c; (S, i, c)$
$i$ 為 1 時得到 $c$	直接得到 $c; (i, T, c)$
$i$ 為 $0, j$ 為 1 時花費 $c$	$(i, j, c)$
$i, j$ 不同時花費 $c$	$(i, j, c), (j, i, c)$
$i, j$ 同時是 0 時得到 $c$	直接得到 $c; (S, w, c), (w, i, \infty), (w, j, \infty)$
$i, j$ 同時是 1 時得到 $c$	直接得到 $c; (i, w, \infty), (j, w, \infty), (w, T, c)$
- Submodular functions minimization
  - For a function  $f : 2^V \rightarrow \mathbb{R}$ ,  $f$  is a submodular function iff
    - $\forall S, T \subseteq V, f(S) + f(T) \geq f(S \cup T) + f(S \cap T)$ , or
    - $\forall X \subseteq Y \subseteq V, x \notin Y, f(X \cup \{x\}) - f(X) \geq f(Y \cup \{x\}) - f(Y)$ .
  - To minimize  $\sum_i \theta_i(x_i) + \sum_{i < j} \phi_{ij}(x_i, x_j) + \sum_{i < j < k} \psi_{ijk}(x_i, x_j, x_k)$
  - If  $\theta_i(1) \geq \theta_i(0)$ , add edge  $(S, i, \theta_i(1) - \theta_i(0))$  and  $\theta_i(0)$  to answer; otherwise,  $(i, T, \theta_i(0) - \theta_i(1))$  and  $\theta_i(1)$ .
  - Add edges  $(i, j, \phi_{ij}(0, 1) + \phi_{ij}(1, 0) - \phi_{ij}(0, 0) - \phi_{ij}(1, 1))$ .
  - Denote  $x_{ijk}$  as helper nodes. Let  $P = \psi_{ijk}(0, 0, 0) + \psi_{ijk}(0, 1, 1) + \psi_{ijk}(1, 0, 1) + \psi_{ijk}(1, 1, 0) - \psi_{ijk}(0, 0, 1) - \psi_{ijk}(0, 1, 0) - \psi_{ijk}(1, 1, 1)$ . Add  $-P$  to answer. If  $P \geq 0$ , add edges  $(i, x_{ijk}, P), (j, x_{ijk}, P), (k, x_{ijk}, P), (x_{ijk}, T, P)$ ; otherwise  $(x_{ijk}, i, -P), (x_{ijk}, j, -P), (x_{ijk}, k, -P), (S, x_{ijk}, -P)$ .
  - The minimum cut of this graph will be the the minimum value of the function above.

### 4.4 Dinic [32c53e]

```

template <typename Cap = int64_t> class Dinic {
private:
    struct E { int to, rev; Cap cap; }; int n, st, ed;
    vector<vector<E>> G; vector<size_t> lv, idx;
    bool BFS(int k) {
        lv.assign(n, 0); idx.assign(n, 0);
        queue<int> bfs; bfs.push(st); lv[st] = 1;
        while (!bfs.empty() and not lv[ed]) {
            int u = bfs.front(); bfs.pop();
            for (auto e: G[u]) if (e.cap >> k and !lv[e.to])
                bfs.push(e.to), lv[e.to] = lv[u] + 1;
        }
        return lv[ed];
    }
    Cap DFS(int u, Cap f = numeric_limits<Cap>::max()) {
        if (u == ed) return f;
        Cap ret = 0;
        for (auto &i = idx[u]; i < G[u].size(); ++i) {
            auto &[to, rev, cap] = G[u][i];
            if (cap <= 0 or lv[to] != lv[u] + 1) continue;
            Cap nf = DFS(to, min(f, cap));
            ret += nf; cap -= nf; f -= nf;
            G[to][rev].cap += nf;
            if (f == 0) return ret;
        }
        if (ret == 0) lv[u] = 0;
        return ret;
    }
public:
    void init(int n_) { G.assign(n = n_, vector<E>()); }
    void add_edge(int u, int v, Cap c) {
        G[u].push_back({v, G[v].size(), c});
        G[v].push_back({u, G[u].size() - 1, 0});
    }
    Cap max_flow(int st_, int ed_) {
        st = st_, ed = ed_; Cap ret = 0;
        for (int i = 63; i >= 0; --i)
            while (BFS(i)) ret += DFS(st);
        return ret;
    }
}; // test @ luogu P3376

```

### 4.5 HLPP [198e4e]

```

template <typename T> struct HLPP {
    struct Edge { int to, rev; T flow, cap; };
    int n, mx; vector<vector<Edge>> adj; vector<T> excess;
    vector<int> d, cnt, active; vector<vector<int>> B;
    void add_edge(int u, int v, int f) {
        Edge a{v, (int)size(adj[v]), 0, f};
        Edge b{u, (int)size(adj[u]), 0, 0};
        adj[u].push_back(a), adj[v].push_back(b);
    }
    void enqueue(int v) {
        if (!active[v] && excess[v] > 0 && d[v] < n) {
            mx = max(mx, d[v]);
            B[d[v]].push_back(v); active[v] = 1;
        }
    }
    void push(int v, Edge &e) {
        T df = min(excess[v], e.cap - e.flow);
        if (df <= 0 || d[v] != d[e.to] + 1) return;
        e.flow += df, adj[e.to][e.rev].flow -= df;
        excess[e.to] += df, excess[v] -= df;
        enqueue(e.to);
    }
    void gap(int k) {
        for (int v = 0; v < n; v++) if (d[v] >= k)
            cnt[d[v]]--, d[v] = n, cnt[d[v]]++;
    }
    void relabel(int v) {
        cnt[d[v]]--; d[v] = n;
        for (auto e : adj[v])
            if (e.cap > e.flow) d[v] = min(d[v], d[e.to] + 1);
        cnt[d[v]]++; enqueue(v);
    }
    void discharge(int v) {
        for (auto &e : adj[v])
            if (excess[v] > 0) push(v, e);
        else break;
        if (excess[v] <= 0) return;
        if (cnt[d[v]] == 1) gap(d[v]);
        else relabel(v);
    }
};

```



```

}
T max_flow(int s, int t) {
    for (auto &e : adj[s]) excess[s] += e.cap;
    cnt[0] = n; enqueue(s); active[t] = 1;
    for (mx = 0; mx >= 0; )
        if (!B[mx].empty()) {
            int v = B[mx].back(); B[mx].pop_back();
            active[v] = 0; discharge(v);
        } else --mx;
    return excess[t];
}
HLPP(int _n) : n(_n), adj(n), excess(n),
    d(n), cnt(n + 1), active(n), B(n) {}
};

```

#### 4.6 Global Min-Cut [ae7013]

```

void add_edge(auto &w, int u, int v, int c) {
    w[u][v] += c; w[v][u] += c; }
auto phase(const auto &w, int n, vector<int> id) {
    vector<int> g(n); int s = -1, t = -1;
    while (!id.empty()) {
        int c = -1;
        for (int i : id) if (c == -1 || g[i] > g[c]) c = i;
        s = t; t = c;
        id.erase(ranges::find(id, c));
        for (int i : id) g[i] += w[c][i];
    }
    return tuple{s, t, g[t]};
}
lld mincut(auto w, int n) {
    lld cut = numeric_limits<lld>::max();
    vector<int> id(n); iota(all(id), 0);
    for (int i = 0; i < n - 1; ++i) {
        auto [s, t, gt] = phase(w, n, id);
        id.erase(ranges::find(id, t));
        cut = min(cut, gt);
        for (int j = 0; j < n; ++j)
            w[s][j] += w[t][j], w[j][s] += w[j][t];
    }
    return cut;
} // O(V^3), can be O(VE + V^2 log V)?

```

#### 4.7 GomoryHu Tree [5eddb29]

```

vector<tuple<int, int, int>> GomoryHu(int n) {
    vector<tuple<int, int, int>> rt;
    vector<int> g(n);
    for (int i = 1; i < n; ++i) {
        int t = g[i];
        auto f = flow;
        rt.emplace_back(f.max_flow(i, t), i, t);
        f.walk(i); // bfs points that connected to i (use
            // edges with .cap > 0)
        for (int j = i + 1; j < n; ++j)
            if (g[j] == t && f.connect(j)) // check if i can reach j
                g[j] = i;
    }
    return rt;
}
/* for our dinic:
* void walk(int) { BFS(0); }
* bool connect(int i) { return lv[i]; } */

```

#### 4.8 MCMF [0df510]

```

template <typename F, typename C> class MCMF {
    static constexpr F INF_F = numeric_limits<F>::max();
    static constexpr C INF_C = numeric_limits<C>::max();
    struct E { int to, r; F f; C c; };
    vector<vector<E>> g; vector<pair<int, int>> f;
    vector<int> inq; vector<F> up; vector<C> d;
    optional<pair<F, C>> step(int S, int T) {
        queue<int> q;
        for (q.push(S), d[S] = 0, up[S] = INF_F;
            not q.empty(); q.pop()) {
            int u = q.front(); inq[u] = false;
            if (up[u] == 0) continue;
            for (int i = 0; i < int(g[u].size()); ++i) {
                auto e = g[u][i]; int v = e.to;
                if (e.f <= 0 || d[v] <= d[u] + e.c) continue;
                d[v] = d[u] + e.c; f[v] = {u, i};
                up[v] = min(up[u], e.f);
                if (not inq[v]) q.push(v);
                inq[v] = true;
            }
        }
    }
};

```

```

if (d[T] == INF_C) return nullopt;
for (int i = T; i != S; i = f[i].first) {
    auto &eg = g[f[i].first][f[i].second];
    eg.f -= up[T]; g[eg.to][eg.r].f += up[T];
}
return pair{up[T], d[T]};
}
public:
MCMF(int n) : g(n), f(n), inq(n), up(n), d(n, INF_C) {}
void add_edge(int s, int t, F c, C w) {
    g[s].emplace_back(t, int(g[t].size()), c, w);
    g[t].emplace_back(s, int(g[s].size()) - 1, 0, -w);
}
pair<F, C> solve(int a, int b) {
    F c = 0; C w = 0;
    while (auto r = step(a, b)) {
        c += r->first, w += r->first * r->second;
        ranges::fill(inq, false); ranges::fill(d, INF_C);
    }
    return {c, w};
}
};

```

#### 4.9 Dijkstra Cost Flow [d0cfd9]

```

template <typename F, typename C> class MCMF {
    static constexpr F INF_F = numeric_limits<F>::max();
    static constexpr C INF_C = numeric_limits<C>::max();
    struct E { int to, r; F f; C c; };
    vector<vector<E>> g; vector<pair<int, int>> f;
    vector<F> up; vector<C> d, h;
    optional<pair<F, C>> step(int S, int T) {
        priority_queue<pair<C, int>> q;
        q.emplace(d[S] = 0, S), up[S] = INF_F;
        while (not q.empty()) {
            auto [l, u] = q.top(); q.pop();
            if (up[u] == 0 || l != -d[u]) continue;
            for (int i = 0; i < int(g[u].size()); ++i) {
                auto e = g[u][i]; int v = e.to;
                auto nd = d[u] + e.c + h[u] - h[v];
                if (e.f <= 0 || d[v] <= nd) continue;
                f[v] = {u, i}; up[v] = min(up[u], e.f);
                q.emplace(-(d[v] = nd), v);
            }
        }
        if (d[T] == INF_C) return nullopt;
        for (size_t i = 0; i < d.size(); ++i) h[i] += d[i];
        for (int i = T; i != S; i = f[i].first) {
            auto &eg = g[f[i].first][f[i].second];
            eg.f -= up[T]; g[eg.to][eg.r].f += up[T];
        }
        return pair{up[T], h[T]};
    }
public:
MCMF(int n) : g(n), f(n), up(n), d(n, INF_C) {}
void add_edge(int s, int t, F c, C w) {
    g[s].emplace_back(t, int(g[t].size()), c, w);
    g[t].emplace_back(s, int(g[s].size()) - 1, 0, -w);
}
pair<F, C> solve(int a, int b) {
    h.assign(g.size(), 0);
    F c = 0; C w = 0;
    while (auto r = step(a, b)) {
        c += r->first, w += r->first * r->second;
        fill(d.begin(), d.end(), INF_C);
    }
    return {c, w};
}
};

```

#### 4.10 Min Cost Circulation [3f7d84]

```

template <typename F, typename C>
struct MinCostCirculation {
    struct ep { int to; F flow; C cost; };
    int n; vector<int> vis; int visc;
    vector<int> fa, fae; vector<vector<int>> g;
    vector<ep> e; vector<C> pi;
    MinCostCirculation(int n_) : n(n_), vis(n), visc(0), g
        (n), pi(n) {}
    void add_edge(int u, int v, F fl, C cs) {
        g[u].emplace_back((int)e.size());
        e.emplace_back(v, fl, cs);
        g[v].emplace_back((int)e.size());
        e.emplace_back(u, 0, -cs);
    }
};

```

```

}
C phi(int x) {
    if (fa[x] == -1) return 0;
    if (vis[x] == visc) return pi[x];
    vis[x] = visc;
    return pi[x] = phi(fa[x]) - e[fae[x]].cost;
}
int lca(int u, int v) {
    for (; u != -1 || v != -1; swap(u, v)) if (u != -1) {
        if (vis[u] == visc) return u;
        vis[u] = visc;
        u = fa[u];
    }
    return -1;
}
void pushflow(int x, C &cost) {
    int v = e[x ^ 1].to, u = e[x].to;
    ++visc;
    if (int w = lca(u, v); w == -1) {
        while (v != -1)
            swap(x ^ 1, fae[v]), swap(u, fa[v]), swap(u, v);
    } else {
        int z = u, dir = 0; F f = e[x].flow;
        vector<int> cyc = {x};
        for (int d : {0, 1})
            for (int i = (d ? u : v); i != w; i = fa[i]) {
                cyc.push_back(fae[i] ^ d);
                if (chmin(f, e[fae[i] ^ d].flow)) z = i, dir = d;
            }
        for (int i : cyc) {
            e[i].flow -= f; e[i ^ 1].flow += f;
            cost += f * e[i].cost;
        }
        if (dir) x ^ 1, swap(u, v);
        while (u != z)
            swap(x ^ 1, fae[v]), swap(u, fa[v]), swap(u, v);
    }
}
void dfs(int u) {
    vis[u] = visc;
    for (int i : g[u])
        if (int v = e[i].to; vis[v] != visc and e[i].flow)
            fa[v] = u, fae[v] = i, dfs(v);
}
C simplex() {
    C cost = 0;
    fa.assign(g.size(), -1); fae.assign(e.size(), -1);
    ++visc; dfs(0);
    for (int fail = 0; fail < ssize(e); )
        for (int i = 0; i < ssize(e); i++)
            if (e[i].flow and e[i].cost < phi(e[i ^ 1].to) -
                phi(e[i].to))
                fail = 0, pushflow(i, cost), ++visc;
            else ++fail;
    return cost;
}
};

```

## 4.11 General Matching [5f2293]

```

struct Matching {
    queue<int> q; int ans, n;
    vector<int> fa, s, v, pre, match;
    int Find(int u) {
        return u == fa[u] ? u : fa[u] = Find(fa[u]);
    }
    int LCA(int x, int y) {
        static int tk = 0; tk++; x = Find(x); y = Find(y);
        for (; swap(x, y)) if (x != n) {
            if (v[x] == tk) return x;
            v[x] = tk;
            x = Find(pre[match[x]]);
        }
    }
    void Blossom(int x, int y, int l) {
        for (; Find(x) != l; x = pre[y]) {
            pre[x] = y, y = match[x];
            if (s[y] == 1) q.push(y), s[y] = 0;
            for (int z : {x, y}) if (fa[z] == z) fa[z] = l;
        }
    }
    bool Bfs(auto &&g, int r) {
        iota(all(fa), 0); ranges::fill(s, -1);
        q = queue<int>(); q.push(r); s[r] = 0;

```

```

    for (; !q.empty(); q.pop()) {
        for (int x = q.front(); int u : g[x])
            if (s[u] == -1) {
                if (pre[u] = x, s[u] = 1, match[u] == n) {
                    for (int a = u, b = x, last;
                        b != n; a = last, b = pre[a])
                        last = match[b], match[b] = a, match[a] = b;
                    return true;
                }
                q.push(match[u]); s[match[u]] = 0;
            } else if (!s[u] && Find(u) != Find(x)) {
                int l = LCA(u, x);
                Blossom(x, u, l); Blossom(u, x, l);
            }
        }
        return false;
    }
    Matching(auto &&g) : ans(0), n(int(g.size())),
        fa(n+1), s(n+1), v(n+1), pre(n+1, n), match(n+1, n) {
        for (int x = 0; x < n; ++x)
            if (match[x] == n) ans += Bfs(g, x);
    } // match[x] == n means not matched
}; // test @ yosupo judge

```

## 4.12 Weighted Matching [94ca35]

```

#define pb emplace_back
#define rep(i, l, r) for (int i=(l); i<=(r); ++i)
struct WeightGraph { // 1-based
    static const int inf = INT_MAX;
    struct edge { int u, v, w; }; int n, nx;
    vector<int> lab; vector<vector<edge>> g;
    vector<int> slack, match, st, pa, S, vis;
    vector<vector<int>> flo, flo_from; queue<int> q;
    WeightGraph(int n_) : n(n_), nx(n * 2), lab(nx + 1),
        g(nx + 1, vector<edge>(nx + 1)), slack(nx + 1),
        flo(nx + 1), flo_from(nx + 1, vector(n + 1, 0)) {
        match = st = pa = S = vis = slack;
        rep(u, 1, n) rep(v, 1, n) g[u][v] = {u, v, 0};
    }
    int ED(edge e) {
        return lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2;
    }
    void update_slack(int u, int x, int &s) {
        if (!s || ED(g[u][x]) < ED(g[s][x])) s = u;
    }
    void set_slack(int x) {
        slack[x] = 0;
        for (int u = 1; u <= n; ++u)
            if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
                update_slack(u, x, slack[x]);
    }
    void q_push(int x) {
        if (x <= n) q.push(x);
        else for (int y : flo[x]) q_push(y);
    }
    void set_st(int x, int b) {
        st[x] = b;
        if (x > n) for (int y : flo[x]) set_st(y, b);
    }
    vector<int> split_flo(auto &f, int xr) {
        auto it = find(all(f), xr);
        if (auto pr = it - f.begin(); pr % 2 == 1)
            reverse(1 + all(f), it = f.end() - pr);
        auto res = vector(f.begin(), it);
        return f.erase(f.begin(), it), res;
    }
    void set_match(int u, int v) {
        match[u] = g[u][v].v;
        if (u <= n) return;
        int xr = flo_from[u][g[u][v].u];
        auto &f = flo[u], z = split_flo(f, xr);
        rep(i, 0, int(z.size())-1) set_match(z[i], z[i ^ 1]);
        set_match(xr, v); f.insert(f.end(), all(z));
    }
    void augment(int u, int v) {
        for (;) {
            int xnv = st[match[u]]; set_match(u, v);
            if (!xnv) return;
            set_match(xnv, st[pa[xnv]]);
            u = st[pa[xnv]], v = xnv;
        }
    }
    int lca(int u, int v) {
        static int t = 0; ++t;

```

```

for (++t; u || v; swap(u, v)) if (u) {
    if (vis[u] == t) return u;
    vis[u] = t; u = st[match[u]];
    if (u) u = st[pa[u]];
}
return 0;
}

void add_blossom(int u, int o, int v) {
    int b = int(find(n + 1 + all(st), 0) - begin(st));
    lab[b] = 0, S[b] = 0; match[b] = match[o];
    vector<int> f = {o};
    for (int x = u, y; x != o; x = st[pa[y]])
        f.pb(x), f.pb(y = st[match[x]]), q_push(y);
    reverse(1 + all(f));
    for (int x = v, y; x != o; x = st[pa[y]])
        f.pb(x), f.pb(y = st[match[x]]), q_push(y);
    flo[b] = f; set_st(b, b);
    for (int x = 1; x <= nx; ++x)
        g[b][x].w = g[x][b].w = 0;
    for (int x = 1; x <= n; ++x) flo_from[b][x] = 0;
    for (int xs : flo[b]) {
        for (int x = 1; x <= nx; ++x)
            if (g[b][x].w == 0 || ED(g[xs][x]) < ED(g[b][x]))
                g[b][x] = g[xs][x], g[x][b] = g[x][xs];
        for (int x = 1; x <= n; ++x)
            if (flo_from[xs][x]) flo_from[b][x] = xs;
    }
    set_slack(b);
}

void expand_blossom(int b) {
    for (int x : flo[b]) set_st(x, x);
    int xr = flo_from[b][g[b][pa[b]].u], xs = -1;
    for (int x : split_flo(flo[b], xr)) {
        if (xs == -1) { xs = x; continue; }
        pa[xs] = g[x][xs].u; S[xs] = 1, S[x] = 0;
        slack[xs] = 0; set_slack(x); q_push(x); xs = -1;
    }
    for (int x : flo[b])
        if (x == xr) S[x] = 1, pa[x] = pa[b];
        else S[x] = -1, set_slack(x);
    st[b] = 0;
}

bool on_found_edge(const edge &e) {
    if (int u = st[e.u], v = st[e.v]; S[v] == -1) {
        int nu = st[match[v]]; pa[v] = e.u; S[v] = 1;
        slack[v] = slack[nu] = 0; S[nu] = 0; q_push(nu);
    } else if (S[v] == 0) {
        if (int o = lca(u, v)) add_blossom(u, o, v);
        else return augment(u, v), augment(v, u), true;
    }
    return false;
}

bool matching() {
    ranges::fill(S, -1); ranges::fill(slack, 0);
    q = queue<int>();
    for (int x = 1; x <= nx; ++x)
        if (st[x] == x && !match[x])
            pa[x] = 0, S[x] = 0, q_push(x);
    if (q.empty()) return false;
    for (;;) {
        while (q.size()) {
            int u = q.front(); q.pop();
            if (S[st[u]] == 1) continue;
            for (int v = 1; v <= n; ++v)
                if (g[u][v].w > 0 && st[u] != st[v]) {
                    if (ED(g[u][v]) != 0)
                        update_slack(u, st[v], slack[st[v]]);
                    else if (on_found_edge(g[u][v])) return true;
                }
        }
        int d = inf;
        for (int b = n + 1; b <= nx; ++b)
            if (st[b] == b && S[b] == 1)
                d = min(d, lab[b] / 2);
        for (int x = 1; x <= nx; ++x)
            if (int s = slack[x]; st[x] == x && s && S[x] <= 0)
                d = min(d, ED(g[s][x]) / (S[x] + 2));
        for (int u = 1; u <= n; ++u)
            if (S[st[u]] == 1) lab[u] += d;
            else if (S[st[u]] == 0) {
                if (lab[u] <= d) return false;
                lab[u] -= d;
            }
    }
}

```

```

}
rep(b, n + 1, nx) if (st[b] == b && S[b] >= 0)
    lab[b] += d * (2 - 4 * S[b]);
for (int x = 1; x <= nx; ++x)
    if (int s = slack[x]; st[x] == x &&
        s && st[s] != x && ED(g[s][x]) == 0)
        if (on_found_edge(g[s][x])) return true;
for (int b = n + 1; b <= nx; ++b)
    if (st[b] == b && S[b] == 1 && lab[b] == 0)
        expand_blossom(b);
}
return false;
}

pair<lld, int> solve() {
    ranges::fill(match, 0);
    rep(u, 0, n) st[u] = u, flo[u].clear();
    int w_max = 0;
    rep(u, 1, n) rep(v, 1, n) {
        flo_from[u][v] = (u == v ? u : 0);
        w_max = max(w_max, g[u][v].w);
    }
    for (int u = 1; u <= n; ++u) lab[u] = w_max;
    int n_matches = 0; lld tot_weight = 0;
    while (matching()) ++n_matches;
    rep(u, 1, n) if (match[u] && match[u] < u)
        tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
}

void set_edge(int u, int v, int w) {
    g[u][v].w = g[v][u].w = w;
}
};

```

## 5 Math

### 5.1 Common Bounds

$n$	2	3	4	5	6	7	8	9	20	50	100	$n$	100	1e3	1e6	1e9	1e12	1e15	1e18
$p(n)$	2	3	5	7	11	15	22	30	627	2e5	2e8	$d(i)$	12	32	240	1344	6720	26880	103680
$n$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15				
$\binom{2n}{n}$	2	6	20	70	252	924	3432	12870	48620	184756	7e5	2e6	1e7	4e7	1.5e8				
$n$	2	3	4	5	6	7	8	9	10	11	12	13							
$B_n$	2	5	15	52	203	877	4140	21147	115975	7e5	4e6	3e7							

### 5.2 Equations

#### Stirling Number of the First Kind

$S_1(n, k)$  counts the number of permutations of  $n$  elements with  $k$  disjoint cycles.

- $S_1(n, k) = (n-1) \cdot S_1(n-1, k) + S_1(n-1, k-1)$
- $S_1(n, i) = [x^i] \left( \prod_{i=0}^{n-1} (x+i) \right)$ , use D&Q and Taylor shift.
- $S_1(i, k) = \frac{i!}{k!} [x^i] \left( \sum_{j \geq 1} \frac{x^j}{j} \right)^k$

#### Stirling Number of the Second Kind

$S_2(n, k)$  counts the number of ways to partition a set of  $n$  elements into  $k$  nonempty sets.

- $S_2(n, k) = S_2(n-1, k-1) + k \cdot S_2(n-1, k)$
- $S_2(n, k) = \sum_{i=0}^k \binom{k}{i} i^n (-1)^{k-i} = \sum_{i=0}^k \frac{(-1)^i}{i!} \cdot \frac{(k-i)^n}{(k-i)!}$
- $S_2(i, k) = \frac{i!}{k!} [x^i] (e^x - 1)^k$

#### Derivatives/Integrals

Integration by parts:  $\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$

$$\left| \begin{array}{l} \frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2} \\ \frac{d}{dx} \tan x = 1 + \tan^2 x \quad \int \tan ax = -\frac{\ln |\cos ax|}{a} \\ \int e^{-x^2} = \frac{\sqrt{\pi}}{2} \operatorname{erf}(x) \quad \int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1) \end{array} \right|$$

$$\int \sqrt{a^2 + x^2} = \frac{1}{2} \left( x \sqrt{a^2 + x^2} + a^2 \operatorname{asinh}(x/a) \right)$$

#### Extended Euler

$$a^b \equiv \begin{cases} a^{(b \bmod \varphi(m)) + \varphi(m)} & \text{if } (a, m) \neq 1 \wedge b \geq \varphi(m) \\ a^{b \bmod \varphi(m)} & \text{otherwise} \end{cases} \pmod{m}$$

#### Pentagonal Number Theorem

$$\prod_{n=1}^{\infty} (1 - x^n) = \sum_{k=-\infty}^{\infty} (-1)^k x^{k(3k-1)/2} = \left( \sum p(n) x^n \right)^{-1}$$

### 5.3 Extended FloorSum

$$g(a, b, c, n) = \sum_{i=0}^n i \left\lfloor \frac{ai+b}{c} \right\rfloor$$

$$= \begin{cases} \left\lfloor \frac{a}{c} \right\rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \left\lfloor \frac{b}{c} \right\rfloor \cdot \frac{n(n+1)}{2} + g(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1) - h(c, c-b-1, a, m-1)), & \text{otherwise} \end{cases}$$

$$h(a, b, c, n) = \sum_{i=0}^n \left\lfloor \frac{ai+b}{c} \right\rfloor^2$$

$$= \begin{cases} \left\lfloor \frac{a}{c} \right\rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \left\lfloor \frac{b}{c} \right\rfloor^2 \cdot (n+1) \\ + \left\lfloor \frac{a}{c} \right\rfloor \cdot \left\lfloor \frac{b}{c} \right\rfloor \cdot n(n+1) \\ + h(a \bmod c, b \bmod c, c, n) \\ + 2 \left\lfloor \frac{a}{c} \right\rfloor \cdot g(a \bmod c, b \bmod c, c, n) \\ + 2 \left\lfloor \frac{b}{c} \right\rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) \\ - 2f(c, c-b-1, a, m-1) - f(a, b, c, n), & \text{otherwise} \end{cases}$$

## 5.4 Integer Division [cd017d]

```
lld fdiv(lld a, lld b)
{ return a / b - (a % b && (a < 0) ^ (b < 0)); }
lld cdiv(lld a, lld b)
{ return a / b + (a % b && (a < 0) ^ (b > 0)); }
```

## 5.5 FloorSum [fb5917]

```
// @param n 'n < 2^32'
// @param m '1 <= m < 2^32'
// @return sum_{i=0}^{n-1} floor((ai + b)/m) mod 2^64
llu floor_sum_unsigned(llu n, llu m, llu a, llu b) {
    llu ans = 0;
    while (true) {
        if (a >= m) ans += n*(n-1)/2 * (a/m), a %= m;
        if (b >= m) ans += n * (b/m), b %= m;
        if ((llu) y_max = a * n + b; y_max >= m) {
            n = (llu)(y_max / m), b = (llu)(y_max % m);
            swap(m, a);
        } else break;
    }
    return ans;
}
lld floor_sum(lld n, lld m, lld a, lld b) {
    llu ans = 0;
    if (a < 0) {
        llu a2 = (a % m + m), d = (a2 - a) / m;
        ans -= 1ULL * n * (n - 1) / 2 * d; a = a2;
    }
    if (b < 0) {
        llu b2 = (b % m + m), d = (b2 - b) / m;
        ans -= 1ULL * n * d; b = b2;
    }
    return ans + floor_sum_unsigned(n, m, a, b);
}
```

## 5.6 ModMin [253e4d]

```
// min{k | l <= ((ak) mod m) <= r}
optional<llu> mod_min(u32 a, u32 m, u32 l, u32 r) {
    if (a == 0) return l ? nullopt : 0;
    if (auto k = llu(l + a - 1) / a; k * a <= r)
        return k;
    auto b = m / a, c = m % a;
    if (auto y = mod_min(c, a, a - r % a, a - l % a))
        return (l + *y * c + a - 1) / a + *y * b;
    return nullopt;
}
```

## 5.7 Floor Monoid Product [416e89]

```
/* template <typename T>
T brute(llu a, llu b, llu c, llu n, T U, T R) {
    T res;
    for (llu i = 1, l = 0; i <= n; i++, res = res * R)
        for (llu r = (a*i+b)/c; l < r; ++l) res = res * U;
    return res;
} */
template <typename T>
T euclid(llu a, llu b, llu c, llu n, T U, T R) {
    if (!n) return T{};
    if (b >= c)
        return mpow(U, b / c) * euclid(a, b % c, c, n, U, R);
    if (a >= c)
        return euclid(a % c, b, c, n, U, mpow(U, a / c) * R);
    llu m = (u128(a) * n + b) / c;
    if (!m) return mpow(R, n);
    return mpow(R, (c - b - 1) / a) * U
        * euclid(c, (c - b - 1) % a, a, m - 1, R, U)
        * mpow(R, n - (u128(c) * m - b - 1) / a);
}
// time complexity is O(log max(a, b, c))
// UUUU R UUUUU R ... UUU R 共 N 個 R, 最後一個必是 R
// 一直到第 k 個 R 前總共有 (ak+b)/c 個 U
5.8 ax+by=gcd [d0cbdd]
```

```
// ax+ny = 1, ax+ny == ax == 1 (mod n)
void exgcd(lld x, lld y, lld &g, lld &a, lld &b) {
    if (y == 0) g = x, a = 1, b = 0;
    else exgcd(y, x % y, g, b, a), b -= (x / y) * a;
}
```

## 5.9 Chinese Remainder [d69e74]

```
// please ensure r_i \in [0, m_i)
bool crt(lld &m1, lld &r1, lld m2, lld r2) {
    if (m2 > m1) swap(m1, m2), swap(r1, r2);
    lld g, a, b; exgcd(m1, m2, g, a, b);
    if ((r2 - r1) % g != 0) return false;
    m2 /= g; lld D = (r2 - r1) / g % m2 * a % m2;
    r1 += (D < 0 ? D + m2 : D) * m1; m1 *= m2;
    assert(r1 >= 0 && r1 < m1);
    return true;
}
```

## 5.10 DiscreteLog [86e463]

```
template<typename Int>
Int BSGS(Int x, Int y, Int M) {
    // x^? \equiv y (mod M)
    Int t = 1, c = 0, g = 1;
    for (Int M_ = M; M_ > 0; M_ >>= 1) g = g * x % M;
    for (g = gcd(g, M); t % g != 0; ++c) {
        if (t == y) return c;
        t = t * x % M;
    }
    if (y % g != 0) return -1;
    t /= g, y /= g, M /= g;
    Int h = 0, gs = 1;
    for (; h * h < M; ++h) gs = gs * x % M;
    unordered_map<Int, Int> bs;
    for (Int s = 0; s < h; bs[y] = ++s) y = y * x % M;
    for (Int s = 0; s < M; s += h) {
        t = t * gs % M;
        if (bs.count(t)) return c + s + h - bs[t];
    }
    return -1;
}
```

## 5.11 Quadratic Residue [f0baec]

```
int get_root(int n, int P) { // ensure 0 <= n < p
    if (P == 2 or n == 0) return n;
    auto check = [&](lld x) {
        return modpow(int(x), (P - 1) / 2, P);
    };
    if (check(n) != 1) return -1;
    mt19937 rnd(7122); lld z = 1, w;
    while (check(w = (z * z - n + P) % P) != P - 1)
        z = rnd() % P;
    const auto M = [P, w](auto &u, auto &v) {
        auto [a, b] = u; auto [c, d] = v;
        return make_pair((a * c + b * d % P * w) % P,
            (a * d + b * c) % P);
    };
    pair<lld, lld> r(1, 0), e(z, 1);
    for (int q = (P + 1) / 2; q; q >>= 1, e = M(e, e))
        if (q & 1) r = M(r, e);
    return int(r.first); // sqrt(n) mod P where P is prime
}
```

## 5.12 FWT [f82550]

```
/* or convolution:
* x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
* and convolution:
* x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
void fwt(int x[], int N, bool inv = false) {
    for (int d = 1; d < N; d <<= 1)
        for (int s = 0; s < N; s += d * 2)
            for (int i = s; i < s + d; i++) {
                int j = i + d, ta = x[i], tb = x[j];
                x[i] = add(ta, tb);
                x[j] = sub(ta, tb);
            }
    if (inv) {
        const int invn = modinv(N);
        for (int i = 0; i < N; i++)
            x[i] = mul(x[i], invn);
    }
}
```

## 5.13 Packed FFT [0a6af5]

```
VL convolution(const VI &a, const VI &b) {
    if (a.empty() || b.empty()) return {};
    const int sz = bit_ceil(a.size() + b.size() - 1);
    // Should be able to handle N <= 10^5, C <= 10^4
```



```

vector<P> v(sz);
for (size_t i = 0; i < a.size(); ++i) v[i].RE(a[i]);
for (size_t i = 0; i < b.size(); ++i) v[i].IM(b[i]);
fft(v.data(), sz, /*inv=*/false);
auto rev = v; reverse(1 + all(rev));
for (int i = 0; i < sz; ++i) {
    P A = (v[i] + conj(rev[i])) / P(2, 0);
    P B = (v[i] - conj(rev[i])) / P(0, 2);
    v[i] = A * B;
}
VL c(sz); fft(v.data(), sz, /*inv=*/true);
for (int i = 0; i < sz; ++i) c[i] = roundl(RE(v[i]));
return c;
}
VI convolution_mod(const VI &a, const VI &b) {
    if (a.empty() || b.empty()) return {};
    const int sz = bit_ceil(a.size() + b.size() - 1);
    vector<P> fa(sz), fb(sz);
    for (size_t i = 0; i < a.size(); ++i)
        fa[i] = P(a[i] & ((1 << 15) - 1), a[i] >> 15);
    for (size_t i = 0; i < b.size(); ++i)
        fb[i] = P(b[i] & ((1 << 15) - 1), b[i] >> 15);
    fft(fa.data(), sz); fft(fb.data(), sz);
    auto rfa = fa; reverse(1 + all(rfa));
    for (int i = 0; i < sz; ++i) fa[i] *= fb[i];
    for (int i = 0; i < sz; ++i) fb[i] *= conj(rfa[i]);
    fft(fa.data(), sz, true); fft(fb.data(), sz, true);
    vector<int> res(sz);
    for (int i = 0; i < sz; ++i) {
        lld A = (lld)roundl(RE((fa[i] + fb[i]) / P(2, 0)));
        lld C = (lld)roundl(IM((fa[i] - fb[i]) / P(0, 2)));
        lld B = (lld)roundl(IM(fa[i])); B %= p; C %= p;
        res[i] = (A + (B << 15) + (C << 30)) % p;
    }
    return res;
} // test @ yosupo judge with long double

```

## 5.14 CRT for arbitrary mod [e4dde7]

```

const int mod = 1000000007;
const int M1 = 985661441; // G = 3 for M1, M2, M3
const int M2 = 998244353;
const int M3 = 1004535809;
int superBigCRT(lld A, lld B, lld C) {
    static_assert(M1 < M2 && M2 < M3);
    constexpr lld r12 = modpow(M1, M2-2, M2);
    constexpr lld r13 = modpow(M1, M3-2, M3);
    constexpr lld r23 = modpow(M2, M3-2, M3);
    constexpr lld M1M2 = 1LL * M1 * M2 % mod;
    B = (B - A + M2) * r12 % M2;
    C = (C - A + M3) * r13 % M3;
    C = (C - B + M3) * r23 % M3;
    return (A + B * M1 + C * M1M2) % mod;
}

```

## 5.15 NTT / FFT [41cf12]

```

template <int mod, int G, int maxn> struct NTT {
    static_assert(maxn == (maxn & -maxn));
    int roots[maxn];
    NTT () {
        int r = modpow(G, (mod - 1) / maxn);
        for (int i = maxn >> 1; i; i >>= 1) {
            roots[i] = 1;
            for (int j = 1; j < i; j++)
                roots[i + j] = mul(roots[i + j - 1], r);
            r = mul(r, r);
            // for (int j = 0; j < i; j++) // FFT (tested)
            // roots[i+j] = polar<llf>(1, PI * j / i);
        }
    }
    // n must be 2^k, and 0 <= F[i] < mod
    template <typename T>
    void operator()(int F[], T n, bool inv = false) {
        for (T i = 0, j = 0; i < n; i++) {
            if (i < j) swap(F[i], F[j]);
            for (T k = n >> 1; (j ^= k) < k; k >>= 1);
        }
        for (T s = 1; s < n; s *= 2) {
            for (T i = 0; i < n; i += s * 2) {
                for (T j = 0; j < s; j++) {
                    int a = F[i+j], b = mul(F[i+j+s], roots[s+j]);
                    F[i+j] = add(a, b); // a + b
                    F[i+j+s] = sub(a, b); // a - b
                }
            }
        }
    }
}

```

```

}
}
if (inv) {
    int iv = modinv(int(n));
    for (T i = 0; i < n; i++) F[i] = mul(F[i], iv);
    reverse(F + 1, F + n);
}
}
};

```

## 5.16 Formal Power Series [c6b99a]

```

#define fi(l, r) for (size_t i = (l); i < (r); i++)
using S = vector<int>;
auto Mul(auto a, auto b, size_t sz) {
    a.resize(sz), b.resize(sz);
    ntt(a.data(), sz); ntt(b.data(), sz);
    fi(0, sz) a[i] = mul(a[i], b[i]);
    return ntt(a.data(), sz, true), a;
}
S Newton(const S &v, int init, auto &&iter) {
    S Q = { init };
    for (int sz = 2; Q.size() < v.size(); sz *= 2) {
        S A{begin(v), begin(v) + min(sz, int(v.size()))};
        A.resize(sz * 2), Q.resize(sz * 2);
        iter(Q, A, sz * 2); Q.resize(sz);
    }
    return Q.resize(v.size()), Q;
}
S Inv(const S &v) { // v[0] != 0
    return Newton(v, modinv(v[0]),
        [](S &X, S &A, int sz) {
            ntt(X.data(), sz), ntt(A.data(), sz);
            for (int i = 0; i < sz; i++)
                X[i] = mul(X[i], sub(2, mul(X[i], A[i])));
            ntt(X.data(), sz, true);
        });
}
S Dx(S A) {
    fi(1, A.size()) A[i - 1] = mul(i, A[i]);
    return A.empty() ? A : (A.pop_back(), A);
}
S Sx(S A) {
    A.insert(A.begin(), 0);
    fi(1, A.size()) A[i] = mul(modinv(int(i)), A[i]);
    return A;
}
S Ln(const S &A) { // coef[0] == 1; res[0] == 0
    auto B = Sx(Mul(Dx(A), Inv(A), bit_ceil(A.size()*2)));
    return B.resize(A.size()), B;
}
S Exp(const S &v) { // coef[0] == 0; res[0] == 1
    return Newton(v, 1,
        [](S &X, S &A, int sz) {
            auto Y = X; Y.resize(sz / 2); Y = Ln(Y);
            fi(0, Y.size()) Y[i] = sub(A[i], Y[i]);
            Y[0] = add(Y[0], 1); X = Mul(X, Y, sz);
        });
}
S Pow(S a, lld M) { // period mod*(mod-1)
    assert(!a.empty() && a[0] != 0);
    const auto imul = [&a](int s) {
        for (int &x: a) x = mul(x, s);
    };
    auto c = a[0];
    imul(modinv(c)); a = Ln(a); imul(int(M % mod));
    a = Exp(a); imul(modpow(c, int(M % (mod - 1))));
    return a; // mod x^N where N=a.size()
}
S Sqrt(const S &v) { // need: QuadraticResidue
    assert(!v.empty() && v[0] != 0);
    const int r = get_root(v[0]); assert(r != -1);
    return Newton(v, r,
        [](S &X, S &A, int sz) {
            auto Y = X; Y.resize(sz / 2);
            auto B = Mul(A, Inv(Y), sz);
            for (int i = 0, inv2 = mod / 2 + 1; i < sz; i++)
                X[i] = mul(inv2, add(X[i], B[i]));
        });
}
S Mul(auto &&a, auto &&b) {
    const auto n = a.size() + b.size() - 1;
    auto R = Mul(a, b, bit_ceil(n));
    return R.resize(n), R;
}
S MulT(S a, S b, size_t k) {
    assert(b.size()); reverse(all(b)); auto R = Mul(a, b);
    R = vector(R.begin() + b.size() - 1, R.end());
}

```

```

return R.resize(k), R;
}
S Eval(const S &f, const S &x) {
    if (f.empty()) return vector(x.size(), 0);
    const int n = int(max(x.size(), f.size()));
    auto q = vector(n * 2, S(2, 1)); S ans(n);
    fi(0, x.size()) q[i + n][1] = sub(0, x[i]);
    for (int i = n - 1; i > 0; i--)
        q[i] = Mul(q[i << 1], q[i << 1 | 1]);
    q[1] = Mult(f, Inv(q[1]), n);
    for (int i = 1; i < n; i++) {
        auto L = q[i << 1], R = q[i << 1 | 1];
        q[i << 1 | 0] = Mult(q[i], R, L.size());
        q[i << 1 | 1] = Mult(q[i], L, R.size());
    }
    for (int i = 0; i < n; i++) ans[i] = q[i + n][0];
    return ans.resize(x.size()), ans;
}
pair<S, S> DivMod(const S &A, const S &B) {
    assert(!B.empty() && B.back() != 0);
    if (A.size() < B.size()) return {{}, A};
    const auto sz = A.size() - B.size() + 1;
    S X = B; reverse(all(X)); X.resize(sz);
    S Y = A; reverse(all(Y)); Y.resize(sz);
    S Q = Mul(Inv(X), Y);
    Q.resize(sz); reverse(all(Q)); X = Mul(Q, B); Y = A;
    fi(0, Y.size()) Y[i] = sub(Y[i], X[i]);
    while (Y.size() && Y.back() == 0) Y.pop_back();
    while (Q.size() && Q.back() == 0) Q.pop_back();
    return {Q, Y};
} // empty means zero polynomial
int LinearRecursionKth(S a, S c, int64_t k) {
    const auto d = a.size(); assert(c.size() == d + 1);
    const auto sz = bit_ceil(2 * d + 1), o = sz / 2;
    S q = c; for (int &x: q) x = sub(0, x); q[0]=1;
    S p = Mul(a, q); p.resize(sz); q.resize(sz);
    for (int r; r = (k & 1), k >= 1) {
        fill(d + all(p), 0); fill(d + 1 + all(q), 0);
        ntt(p.data(), sz); ntt(q.data(), sz);
        for (size_t i = 0; i < sz; i++)
            p[i] = mul(p[i], q[(i + o) & (sz - 1)]);
        for (size_t i = 0, j = o; j < sz; i++, j++)
            q[i] = q[j] = mul(q[i], q[j]);
        ntt(p.data(), sz, true); ntt(q.data(), sz, true);
        for (size_t i = 0; i < d; i++) p[i] = p[i << 1 | r];
        for (size_t i = 0; i <= d; i++) q[i] = q[i << 1];
    } // Bostan-Mori
    return mul(p[0], modinv(q[0]));
} // a_n = \sum c_j a_{n-j}, c_0 is not used

```

### 5.17 Partition Number [9bb845]

```

ans[0] = tmp[0] = 1;
for (int i = 1; i * i <= n; i++) {
    for (int rep = 0; rep < 2; rep++)
        for (int j = i; j <= n - i * i; j++)
            modadd(tmp[j], tmp[j - i]);
    for (int j = i * i; j <= n; j++)
        modadd(ans[j], tmp[j - i * i]);
}

```

### 5.18 Pi Count [715863]

```

struct S { int rough; lld large; int id; };
lld PrimeCount(lld n) { // n ~ 10^13 => < 1s
    if (n <= 1) return 0;
    const int v = static_cast<int>(sqrtl(n)); int pc = 0;
    vector<int> smalls(v + 1), skip(v + 1); vector<S> z;
    for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
    for (int i : views::iota(0, (v + 1) / 2))
        z.emplace_back(2*i+1, (n / (2*i+1) + 1) / 2, i);
    for (int p = 3; p <= v; ++p)
        if (smalls[p] > smalls[p - 1]) {
            const int q = p * p; ++pc;
            if (1LL * q * q > n) break;
            skip[p] = 1;
            for (int i = q; i <= v; i += 2 * p) skip[i] = 1;
            int ns = 0;
            for (auto e : z) if (!skip[e.rough]) {
                lld d = 1LL * e.rough * p;
                e.large += pc - (d <= v ? z[smalls[d] - pc].large :
                    smalls[n / d]);
                e.id = ns; z[ns++] = e;
            }
            z.resize(ns);
        }
}

```

```

for (int j = v / p; j >= p; --j) {
    int c = smalls[j] - pc, e = min(j * p + p, v + 1);
    for (int i = j * p; i < e; ++i) smalls[i] -= c;
}
}
lld ans = z[0].large; z.erase(z.begin());
for (auto &[rough, large, k] : z) {
    const lld m = n / rough; --k;
    ans -= large - (pc + k);
    for (auto [p, _, l] : z)
        if (l >= k || p * p > m) break;
    else ans += smalls[m / p] - (pc + l);
}
return ans;
} // test @ yosupo library checker w/ n=1e11, 68ms

```

### 5.19 Miller Rabin [fbd812]

```

bool isprime(llu x) {
    auto witn = [&](llu a, int t) {
        for (llu a2; t--; a = a2) {
            a2 = mmul(a, a, x);
            if (a2 == 1 && a != 1 && a != x - 1) return true;
        }
        return a != 1;
    };
    if (x <= 2 || ~x & 1) return x == 2;
    int t = countr_zero(x - 1); llu odd = (x - 1) >> t;
    for (llu m : {2, 325, 9375, 28178, 450775, 9780504, 1795265022})
        if (m % x != 0 && witn(mpow(m % x, odd, x), t))
            return false;
    return true;
} // test @ luogu 143 & yosupo judge, ~1700ms for Q=1e5
// if use montgomery, ~250ms for Q=1e5

```

### 5.20 Pollard Rho [57ad88]

```

// does not work when n is prime or n == 1
// return any non-trivial factor
llu pollard_rho(llu n) {
    static mt19937_64 rnd(120821011);
    if (!(n & 1)) return 2;
    llu y = 2, z = y, c = rnd() % n, p = 1, i = 0, t;
    auto f = [&](llu x) {
        return madd(mmul(x, x, n), c, n);
    };
    do {
        p = mmul(msub(z = f(f(z)), y = f(y), n), p, n);
        if (++i &= 63) if (i == (i & -i)) t = gcd(p, n);
    } while (t == 1);
    return t == n ? pollard_rho(n) : t;
} // test @ yosupo judge, ~270ms for Q=100
// if use montgomery, ~70ms for Q=100

```

### 5.21 Barrett Reduction [d44617]

```

struct FastMod {
    using Big = __uint128_t; llu b, m;
    FastMod(llu b) : b(b), m(-1ULL / b) {}
    llu reduce(llu a) { // a % b
        llu r = a - (llu)((Big(m) * a) >> 64) * b;
        return r >= b ? r - b : r;
    }
};

```

### 5.22 Montgomery [648fb3]

```

struct Mont { // Montgomery multiplication
    constexpr static int W = 64, L = 6;
    llu mod, R1, R2, xinv;
    void set_mod(llu _mod) {
        mod = _mod; assert(mod & 1); xinv = 1;
        for (int j = 0; j < L; j++) xinv *= 2 - xinv * mod;
        assert(xinv * mod == 1);
        const u128 R = (u128(1) << W) % mod;
        R1 = llu(R); R2 = llu(R * R % mod);
    }
    llu redc(llu a, llu b) const {
        u128 T = u128(a) * b, m = -llu(T) * xinv;
        T += m * mod; T >>= W;
        return llu(T >= mod ? T - mod : T);
    }
    llu from(llu x) const {
        assert(x < mod); return redc(x, R2);
    }
    llu get(llu a) const { return redc(a, 1); }
    llu one() const { return R1; }
} mont;
// a * b % mod == get(redc(from(a), from(b)))

```

### 5.23 Berlekamp Massey [a94d00]

```

template <typename T>
vector<T> BerlekampMassey(const vector<T> &output) {
    vector<T> d(output.size() + 1, me, he);
    for (size_t f = 0, i = 1; i <= output.size(); ++i) {
        for (size_t j = 0; j < me.size(); ++j)
            d[i] += output[i - j - 2] * me[j];
        if ((d[i] - output[i - 1]) == 0) continue;
        if (me.empty()) {
            me.resize(f = i);
            continue;
        }
        vector<T> o(i - f - 1);
        T k = -d[i] / d[f]; o.push_back(-k);
        for (T x : he) o.push_back(x * k);
        if (o.size() < me.size()) o.resize(me.size());
        for (size_t j = 0; j < me.size(); ++j) o[j] += me[j];
        if (i-f+he.size() >= me.size()) he = me, f = i;
        me = o;
    }
    return me;
}

```

## 5.24 Gauss Elimination [1f5f8c]

```

using VI = vector<int>;
using VVI = vector<VI>;
pair<VI, VVI> gauss(VVI A, VI b) { // solve Ax=b
    const int N = (int)A.size(), M = (int)A[0].size();
    vector<int> depv, free(M, true); int rk = 0;
    for (int i = 0; i < M; ++i) {
        int p = -1;
        for (int j = rk; j < N; ++j)
            if (p == -1 || abs(A[j][i]) > abs(A[p][i]))
                p = j;
        if (p == -1 || A[p][i] == 0) continue;
        swap(A[p], A[rk]); swap(b[p], b[rk]);
        const int inv = modinv(A[rk][i]);
        for (int &x : A[rk]) x = mul(x, inv);
        b[rk] = mul(b[rk], inv);
        for (int j = 0; j < N; ++j) if (j != rk) {
            int z = A[j][i];
            for (int k = 0; k < M; ++k)
                A[j][k] = sub(A[j][k], mul(z, A[rk][k]));
            b[j] = sub(b[j], mul(z, b[rk]));
        }
        depv.push_back(i); free[i] = false; ++rk;
    }
    for (int i = rk; i < N; ++i)
        if (b[i] != 0) return {{}, {}}; // not consistent
    VI x(M); VVI h;
    for (int i = 0; i < rk; ++i) x[depv[i]] = b[i];
    for (int i = 0; i < M; ++i) if (free[i]) {
        h.emplace_back(M); h.back()[i] = 1;
        for (int j = 0; j < rk; ++j)
            h.back()[depv[j]] = sub(0, A[j][i]);
    }
    return {x, h}; // solution = x + span(h[i])
}

```

## 5.25 CharPoly [cd559d]

```

#define rep(x, y, z) for (int x=y; x<z; x++)
using VI = vector<int>; using VVI = vector<VI>;
void Hessenberg(VVI &H, int N) {
    for (int i = 0; i < N - 2; ++i) {
        for (int j = i + 1; j < N; ++j) if (H[j][i]) {
            rep(k, i, N) swap(H[i+1][k], H[j][k]);
            rep(k, 0, N) swap(H[k][i+1], H[k][j]);
            break;
        }
        if (!H[i + 1][i]) continue;
        for (int j = i + 2; j < N; ++j) {
            int co = mul(modinv(H[i + 1][i]), H[j][i]);
            rep(k, i, N) subeq(H[j][k], mul(H[i+1][k], co));
            rep(k, 0, N) addeq(H[k][i+1], mul(H[k][j], co));
        }
    }
}
VI CharacteristicPoly(VVI A) {
    int N = (int)A.size(); Hessenberg(A, N);
    VVI P(N + 1, VI(N + 1)); P[0][0] = 1;
    for (int i = 1; i <= N; ++i) {
        rep(j, 0, i+1) P[i][j] = j * P[i-1][j-1] : 0;
        for (int j = i - 1, val = 1; j >= 0; --j) {
            int co = mul(val, A[j][i - 1]);

```

```

            rep(k, 0, j+1) subeq(P[i][k], mul(P[j][k], co));
            if (j) val = mul(val, A[j][j - 1]);
        }
    }
    if (N & 1) for (int &x: P[N]) x = sub(0, x);
    return P[N]; // test: 2021 PTZ Korea K
}

```

## 5.26 Simplex [c9c93b]

```

namespace simplex {
    // maximize c^T x under Ax <= B and x >= 0
    // return VD(n, -inf) if the solution doesn't exist
    // return VD(n, +inf) if the solution is unbounded
    using VD = vector<llf>;
    using VVD = vector<vector<llf>>;
    const llf eps = 1e-9, inf = 1e+9;
    int n, m; VVD d; vector<int> p, q;
    void pivot(int r, int s) {
        llf inv = 1.0 / d[r][s];
        for (int i = 0; i < m + 2; ++i)
            for (int j = 0; j < n + 2; ++j)
                if (i != r && j != s)
                    d[i][j] -= d[r][j] * d[i][s] * inv;
        for (int i=0; i<m+2; ++i) if (i != r) d[i][s] *= -inv;
        for (int j=0; j<n+2; ++j) if (j != s) d[r][j] *= +inv;
        d[r][s] = inv; swap(p[r], q[s]);
    }
    bool phase(int z) {
        int x = m + 2;
        while (true) {
            int s = -1;
            for (int i = 0; i <= n; ++i) {
                if (!z && q[i] == -1) continue;
                if (s == -1 || d[x][i] < d[x][s]) s = i;
            }
            if (s == -1 || d[x][s] > -eps) return true;
            int r = -1;
            for (int i = 0; i < m; ++i) {
                if (d[i][s] < eps) continue;
                if (r == -1 ||
                    d[i][n+1]/d[i][s] < d[r][n+1]/d[r][s]) r = i;
            }
            if (r == -1) return false;
            pivot(r, s);
        }
    }
    VD solve(const VVD &a, const VD &b, const VD &c) {
        m = (int)b.size(), n = (int)c.size();
        d = VVD(m + 2, VD(n + 2));
        for (int i = 0; i < m; ++i)
            for (int j = 0; j < n; ++j) d[i][j] = a[i][j];
        p.resize(m), q.resize(n + 1);
        for (int i = 0; i < m; ++i)
            p[i] = n + i, d[i][n] = -1, d[i][n + 1] = b[i];
        for (int i = 0; i < n; ++i) q[i] = i, d[m][i] = -c[i];
        q[n] = -1, d[m + 1][n] = 1;
        int r = 0;
        for (int i = 1; i < m; ++i)
            if (d[i][n + 1] < d[r][n + 1]) r = i;
        if (d[r][n + 1] < -eps) {
            pivot(r, n);
            if (!phase(1) || d[m + 1][n + 1] < -eps)
                return VD(n, -inf);
            for (int i = 0; i < m; ++i) if (p[i] == -1) {
                int s = min_element(d[i].begin(), d[i].end() - 1)
                    - d[i].begin();
                pivot(i, s);
            }
        }
        if (!phase(0)) return VD(n, inf);
        VD x(n);
        for (int i = 0; i < m; ++i)
            if (p[i] < n) x[p[i]] = d[i][n + 1];
        return x;
    }
} // use double instead of long double if possible

```

## 5.27 Simplex Construction

Standard form: maximize  $\sum_{1 \leq i \leq n} c_i x_i$  such that  $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$  for all  $1 \leq j \leq m$  and  $x_i \geq 0$  for all  $1 \leq i \leq n$ .

1. In case of minimization, let  $c'_i = -c_i$
2.  $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji} x_i \leq -b_j$
3.  $\sum_{1 \leq i \leq n} A_{ji} x_i = b_j \rightarrow \text{add } \leq \text{ and } \geq$ .
4. If  $x_i$  has no lower bound, replace  $x_i$  with  $x_i - x'_i$

## 5.28 Adaptive Simpson [b8cef9]

```
llf integrate(auto &&f, llf L, llf R) {
    auto simp = [&](llf l, llf r) {
        llf m = (l + r) / 2;
        return (f(l) + f(r) + 4.0 * f(m)) * (r - l) / 6.0;
    };
    auto F = [&](auto Y, llf l, llf r, llf v, llf eps) {
        llf m = (l+r)/2, vl = simp(l, m), vr = simp(m, r);
        if (abs(vl + vr - v) <= 15 * eps)
            return vl + vr + (vl + vr - v) / 15.0;
        return Y(Y, l, m, vl, eps / 2.0) +
            Y(Y, m, r, vr, eps / 2.0);
    };
    return F(F, L, R, simp(L, R), 1e-6);
}
```

## 5.29 Golden Ratio Search [376bcb]

```
llf gss(llf a, llf b, auto &&f) {
    llf r = (sqrt(5)-1)/2, eps = 1e-7;
    llf x1 = b - r*(b-a), x2 = a + r*(b-a);
    llf f1 = f(x1), f2 = f(x2);
    while (b-a > eps)
        if (f1 < f2) { //change to > to find maximum
            b = x2; x2 = x1; f2 = f1;
            x1 = b - r*(b-a); f1 = f(x1);
        } else {
            a = x1; x1 = x2; f1 = f2;
            x2 = a + r*(b-a); f2 = f(x2);
        }
    return a;
}
```

# 6 Geometry

## 6.1 Basic Geometry [1d2d70]

```
#define IM imag
#define RE real
using lld = int64_t;
using llf = long double;
using PT = complex<lld>;
using PF = complex<llf>;
using P = PT;

llf abs(P p) { return sqrtl(norm(p)); }
PF toPF(PT p) { return PF{RE(p), IM(p)}; }
int sgn(lld x) { return (x > 0) - (x < 0); }
lld dot(P a, P b) { return RE(conj(a) * b); }
lld cross(P a, P b) { return IM(conj(a) * b); }
int ori(P a, P b, P c) {
    return sgn(cross(b - a, c - a));
}

int quad(P p) {
    return (IM(p) == 0) // use sgn for PF
        ? (RE(p) < 0 ? 3 : 1) : (IM(p) < 0 ? 0 : 2);
}

int argCmp(P a, P b) {
    // returns 0/+1, starts from theta = -PI
    int qa = quad(a), qb = quad(b);
    if (qa != qb) return sgn(qa - qb);
    return sgn(cross(b, a));
}

P rot90(P p) { return P{-IM(p), RE(p)}; }
template <typename V> llf area(const V & pt) {
    lld ret = 0;
    for (int i = 1; i + 1 < (int)pt.size(); i++)
        ret += cross(pt[i] - pt[0], pt[i+1] - pt[0]);
    return ret / 2.0;
}

template <typename V> PF center(const V & pt) {
    P ret = 0; lld A = 0;
    for (int i = 1; i + 1 < (int)pt.size(); i++) {
        lld cur = cross(pt[i] - pt[0], pt[i+1] - pt[0]);
        ret += (pt[i] + pt[i + 1] + pt[0]) * cur; A += cur;
    }
    return toPF(ret) / llf(A * 3);
}

PF project(PF p, PF q) { // p onto q
    return dot(p, q) * q / dot(q, q); // dot<llf>
}
```

## 6.2 2D Convex Hull [ecba37]

```
// from NaCl, counterclockwise, be careful of n<=2
vector<P> convex_hull(vector<P> v) {
    sort(all(v)); // by X then Y
    if (v[0] == v.back()) return {v[0]};
    int t = 0, s = 1; vector<P> h(v.size() + 1);
```

```
for (int _ = 2; _--; s = t--, reverse(all(v)))
    for (P p : v) {
        while (t > s && ori(p, h[t-1], h[t-2]) >= 0) t--;
        h[t++] = p;
    }
    return h.resize(t), h;
}
```

## 6.3 2D Farthest Pair [8b5844]

```
// p is CCW convex hull w/o colinear points
int n = (int)p.size(), pos = 1; lld ans = 0;
for (int i = 0; i < n; i++) {
    P e = p[(i + 1) % n] - p[i];
    while (cross(e, p[(pos + 1) % n] - p[i]) >
        cross(e, p[pos] - p[i]))
        pos = (pos + 1) % n;
    for (int j: {i, (i + 1) % n})
        ans = max(ans, norm(p[pos] - p[j]));
} // tested @ AOJ CGL_4_B
```

## 6.4 MinMax Enclosing Rect [e4470c]

```
// from 8BQube, plz ensure p is strict convex hull
const llf INF = 1e18, qi = acos(-1) / 2 * 3;
pair<llf, llf> solve(const vector<P> &p) {
    llf mx = 0, mn = INF; int n = (int)p.size();
    for (int i = 0, u = 1, r = 1, l = 1; i < n; ++i) {
        #define Z(v) (p[(v) % n] - p[i])
        P e = Z(i + 1);
        while (cross(e, Z(u + 1)) > cross(e, Z(u))) ++u;
        while (dot(e, Z(r + 1)) > dot(e, Z(r))) ++r;
        if (!i) l = r + 1;
        while (dot(e, Z(l + 1)) < dot(e, Z(l))) ++l;
        P D = p[r % n] - p[l % n];
        llf H = cross(e, Z(u)) / llf(norm(e));
        mn = min(mn, dot(e, D) * H);
        llf B = sqrt(norm(D)) * sqrt(norm(Z(u)));
        llf deg = (qi - acos(dot(D, Z(u)) / B)) / 2;
        mx = max(mx, B * sin(deg) * sin(deg));
    }
    return {mn, mx};
} // test @ UVA 819
```

## 6.5 Minkowski Sum [602806]

```
// A, B are strict convex hull rotate to min by (X, Y)
vector<P> Minkowski(vector<P> A, vector<P> B) {
    const int N = (int)A.size(), M = (int)B.size();
    vector<P> sa(N), sb(M), C(N + M + 1);
    for (int i = 0; i < N; i++) sa[i] = A[(i+1)%N] - A[i];
    for (int i = 0; i < M; i++) sb[i] = B[(i+1)%M] - B[i];
    C[0] = A[0] + B[0];
    for (int i = 0, j = 0; i < N || j < M; ) {
        P e = (j >= M || (i < N && cross(sa[i], sb[j]) >= 0))
            ? sa[i++] : sb[j++];
        C[i + j] = e;
    }
    partial_sum(all(C), C.begin()); C.pop_back();
    return convex_hull(C); // just to remove colinear
}
```

## 6.6 Segment Intersection [60d016]

```
struct Seg { // closed segment
    P st, dir; // represent st + t*dir for 0<=t<=1
    Seg(P s, P e) : st(s), dir(e - s) {}
    static bool valid(lld p, lld q) {
        // is there t s.t. 0 <= t <= 1 && qt == p ?
        if (q < 0) q = -q, p = -p;
        return 0 <= p && p <= q;
    }
    vector<P> ends() const { return { st, st + dir }; }
};

template <typename T> bool isInter(T A, P p) {
    if (A.dir == P(0)) return p == A.st; // BE CAREFUL
    return cross(p - A.st, A.dir) == 0 &&
        T::valid(dot(p - A.st, A.dir), norm(A.dir));
}

template <typename U, typename V>
bool isInter(U A, V B) {
    if (cross(A.dir, B.dir) == 0) { // BE CAREFUL
        bool res = false;
        for (P p: A.ends()) res |= isInter(B, p);
        for (P p: B.ends()) res |= isInter(A, p);
        return res;
    }
    P D = B.st - A.st; lld C = cross(A.dir, B.dir);
    return U::valid(cross(D, B.dir), C) &&
```



```
V::valid(cross(D, A.dir), C);
}
```

## 6.7 Halfplane Intersection [31e216]

```
struct Line {
    P st, ed, dir;
    Line(P s, P e) : st(s), ed(e), dir(e - s) {}
}; using LN = const Line &;
PF intersect(LN A, LN B) {
    llf t = cross(B.st - A.st, B.dir) /
        llf(cross(A.dir, B.dir));
    return toPF(A.st) + toPF(A.dir) * t; // C^3 / C^2
}
bool cov(LN l, LN A, LN B) {
    i128 u = cross(B.st - A.st, B.dir);
    i128 v = cross(A.dir, B.dir);
    // ori(l.st, l.ed, A.st + A.dir*(u/v)) <= 0?
    i128 x = RE(A.dir) * u + RE(A.st - l.st) * v;
    i128 y = IM(A.dir) * u + IM(A.st - l.st) * v;
    return sgn(x*IM(l.dir) - y*RE(l.dir)) * sgn(v) >= 0;
} // x, y are C^3, also sgn<i128> is needed
bool operator<(LN a, LN b) {
    if (int c = argCmp(a.dir, b.dir)) return c == -1;
    return ori(a.st, a.ed, b.st) < 0;
}
// cross(pt-line.st, line.dir)<=0 <-> pt in half plane
// the half plane is the LHS when going from st to ed
llf HPI(vector<Line> &q) {
    sort(q.begin(), q.end());
    int n = (int)q.size(), l = 0, r = -1;
    for (int i = 0; i < n; i++) {
        if (i && !argCmp(q[i].dir, q[i-1].dir)) continue;
        while (l < r && cov(q[i], q[r-1], q[r])) --r;
        while (l < r && cov(q[i], q[l], q[l+1])) ++l;
        q[++r] = q[i];
    }
    while (l < r && cov(q[l], q[r-1], q[r])) --r;
    while (l < r && cov(q[r], q[l], q[l+1])) ++l;
    n = r - l + 1; // q[l .. r] are the lines
    if (n <= 1 || !argCmp(q[l].dir, q[r].dir)) return 0;
    vector<P> pt(n);
    for (int i = 0; i < n; i++)
        pt[i] = intersect(q[i+l], q[(i+1)%n+l]);
    return area(pt);
} // test @ 2020 Nordic NCP C : BigBrother
```

## 6.8 SegmentDist (Sausage) [9d8603]

```
// be careful of abs<complex<int>> (replace _abs below)
llf PointSegDist(P A, Seg B) {
    if (B.dir == P(0)) return _abs(A - B.st);
    if (sgn(dot(A - B.st, B.dir)) *
        sgn(dot(A - B.ed, B.dir)) <= 0)
        return abs(cross(A - B.st, B.dir)) / _abs(B.dir);
    return min(_abs(A - B.st), _abs(A - B.ed));
}
llf SegSegDist(const Seg &s1, const Seg &s2) {
    if (isInter(s1, s2)) return 0;
    return min({
        PointSegDist(s1.st, s2),
        PointSegDist(s1.ed, s2),
        PointSegDist(s2.st, s1),
        PointSegDist(s2.ed, s1) });
} // test @ QOJ2444 / PTZ19 Summer.D3
```

## 6.9 Rotating Sweep Line [8aff27]

```
struct Event {
    P d; int u, v;
    bool operator<(const Event &b) const {
        return sgn(cross(d, b.d)) > 0;
    }
};
P makePositive(P z) { return cmpxy(z, 0) ? -z : z; }
void rotatingSweepLine(const vector<P> &p) {
    const int n = int(p.size());
    vector<Event> e; e.reserve(n * (n - 1) / 2);
    for (int i = 0; i < n; i++)
        for (int j = i + 1; j < n; j++)
            e.emplace_back(makePositive(p[i] - p[j]), i, j);
    sort(all(e));
    vector<int> ord(n), pos(n);
    iota(all(ord), 0);
    sort(all(ord), [&p](int i, int j) {
        return cmpxy(p[i], p[j]);
    });
    for (int i = 0; i < n; i++) pos[ord[i]] = i;
    const auto makeReverse = [](auto &v) {
```

```
sort(all(v)); v.erase(unique(all(v)), v.end());
vector<pair<int, int>> segs;
for (size_t i = 0, j = 0; i < v.size(); i = j) {
    for (; j < v.size() && v[j] - v[i] <= j - i; j++);
    segs.emplace_back(v[i], v[j - 1] + 1 + 1);
}
return segs;
};
for (size_t i = 0, j = 0; i < e.size(); i = j) {
    /* do here */
    vector<size_t> tmp;
    for (; j < e.size() && !(e[i] < e[j]); j++)
        tmp.push_back(min(pos[e[j].u], pos[e[j].v]));
    for (auto [l, r] : makeReverse(tmp)) {
        reverse(ord.begin() + l, ord.begin() + r);
        for (int t = l; t < r; t++) pos[ord[t]] = t;
    }
}
}
```

## 6.10 Hull Cut [277def]

```
vector<P> cut(const vector<P> &p, P s, P e) {
    vector<P> res;
    for (size_t i = 0; i < p.size(); i++) {
        P cur = p[i], prv = i ? p[i-1] : p.back();
        bool side = ori(s, e, cur) < 0;
        if (side != (ori(s, e, prv) < 0))
            res.push_back(intersect({s, e}, {cur, prv}));
        if (side) res.push_back(cur);
    } // P is complex<llf>
    return res; // hull intersection with halfplane
} // left of the line s -> e
```

## 6.11 Point In Hull [13edeb]

```
bool isAnti(P a, P b) {
    return cross(a, b) == 0 && dot(a, b) <= 0;
}
bool PIH(const vector<P> &h, P z, bool strict = true) {
    int n = (int)h.size(), a = 1, b = n - 1, r = !strict;
    if (n < 3) return r && isAnti(h[0] - z, h[n-1] - z);
    if (ori(h[0], h[a], h[b]) > 0) swap(a, b);
    if (ori(h[0], h[a], z) >= r || ori(h[0], h[b], z) <= -r)
        return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        (ori(h[0], h[c], z) > 0 ? b : a) = c;
    }
    return ori(h[a], h[b], z) < r;
}
```

## 6.12 Point In Polygon [037c52]

```
bool PIP(const vector<P> &p, P z, bool strict = true) {
    int cnt = 0, n = (int)p.size();
    for (int i = 0; i < n; i++) {
        P A = p[i], B = p[(i + 1) % n];
        if (isInter(Seg(A, B), z)) return !strict;
        auto zy = IM(z), Ay = IM(A), By = IM(B);
        cnt ^= ((zy < Ay) - (zy < By)) * ori(z, A, B) > 0;
    }
    return cnt;
}
```

## 6.13 Point In Polygon (Fast) [2cd3d6]

```
vector<int> PIPfast(vector<P> p, vector<P> q) {
    const int N = int(p.size()), Q = int(q.size());
    vector<pair<P, int>> evt; vector<Seg> edge;
    for (int i = 0; i < N; i++) {
        int a = i, b = (i + 1) % N;
        P A = p[a], B = p[b];
        assert(A < B || B < A); // std::operator<
        if (B < A) swap(A, B);
        evt.emplace_back(A, i); evt.emplace_back(B, ~i);
        edge.emplace_back(A, B);
    }
    for (int i = 0; i < Q; i++)
        evt.emplace_back(q[i], i + N);
    sort(all(evt));
    auto vtx = p; sort(all(vtx));
    auto eval = [](const Seg &a, lld x) -> llf {
        if (RE(a.dir) == 0) {
            assert(x == RE(a.st));
            return IM(a.st) + llf(IM(a.dir)) / 2;
        }
        llf t = (x - RE(a.st)) / llf(RE(a.dir));
        return IM(a.st) + IM(a.dir) * t;
    };
```

```

};
lld cur_x = 0;
auto cmp = [&](const Seg &a, const Seg &b) -> bool {
    if (int s = sgn(eval(a, cur_x) - eval(b, cur_x)))
        return s == -1; // be careful: sgn<llf>, sgn<lld>
    int s = sgn(cross(b.dir, a.dir));
    if (cur_x != RE(a.st) && cur_x != RE(b.st)) s *= -1;
    return s == -1;
};
namespace pbds {
    pbds::tree<Seg, int, decltype(cmp),
        pbds::rb_tree_tag,
        pbds::tree_order_statistics_node_update> st(cmp);
    auto answer = [&](P ep) {
        if (binary_search(all(vtx), ep))
            return 1; // on vertex
        Seg H(ep, ep); // ??
        auto it = st.lower_bound(H);
        if (it != st.end() && isInter(it->first, ep))
            return 1; // on edge
        if (it != st.begin() && isInter(prev(it)->first, ep))
            return 1; // on edge
        auto rk = st.order_of_key(H);
        return rk % 2 == 0 ? 0 : 2; // 0: outside, 2: inside
    };
    vector<int> ans(Q);
    for (auto [ep, i] : evt) {
        cur_x = RE(ep);
        if (i < 0) { // remove
            st.erase(edge[-i]);
        } else if (i < N) { // insert
            auto [it, succ] = st.insert({edge[i], i});
            assert(succ);
        } else ans[i - N] = answer(ep);
    }
    return ans;
} // test @ AOJ CGL_3_C

```

#### 6.14 Cyclic Ternary Search [162adf]

```

int cyclic_ternary_search(int N, auto &&lt;_>) {
    auto lt = [&](int x, int y) {
        return lt_(x % N, y % N);
    };
    int l = 0, r = N; bool up = lt(0, 1);
    while (r - l > 1) {
        int m = (l + r) / 2;
        if (lt(m, 0) ? up : !lt(m, m+1)) r = m;
        else l = m;
    }
    return (lt(l, r) ? r : l) % N;
}

```

#### 6.15 Tangent of Points to Hull [8e1343]

```

pair<int, int> get_tangent(const vector<P> &v, P p) {
    auto gao = [&](int s) {
        return cyclic_ternary_search(v.size(),
            [&](int x, int y) {
                return ori(p, v[x], v[y]) == s;
            });
    }; // test @ codeforces.com/gym/101201/problem/E
    return {gao(1), gao(-1)}; // (a,b):ori(p,v[a],v[b])<0
} // plz ensure that point strictly out of hull
    // if colinear, returns arbitrary point on line

```

#### 6.16 Circle Class & Intersection [d5df51]

```

llf FMOD(llf x) {
    if (x < -PI) x += PI * 2;
    if (x > PI) x -= PI * 2;
    return x;
}
struct Cir { PF o; llf r; };
// be carefule when tangent
vector<llf> intersectAngle(Cir a, Cir b) {
    PF dir = b.o - a.o; llf d2 = norm(dir);
    if (norm(a.r - b.r) >= d2) { // norm(x) := |x|^2
        if (a.r < b.r) return {-PI, PI}; // a in b
        else return {}; // b in a
    } else if (norm(a.r + b.r) <= d2) return {};
    llf dis = abs(dir), theta = arg(dir);
    llf phi = acos((a.r * a.r + d2 - b.r * b.r) /
        (2 * a.r * dis)); // is acos_safe needed?
    llf L = FMOD(theta - phi), R = FMOD(theta + phi);
    return {L, R};
}
vector<PF> intersectPoint(Cir a, Cir b) {
    llf d = abs(a.o - b.o);

```

```

    if (d > b.r + a.r || d < abs(b.r - a.r)) return {};
    llf dt = (b.r * b.r - a.r * a.r) / d, d1 = (d + dt) / 2;
    PF dir = (a.o - b.o) / d;
    PF u = dir * d1 + b.o;
    PF v = rot90(dir) * sqrt(max(0.0L, b.r * b.r - d1 * d1));
    return {u + v, u - v};
} // test @ AOJ CGL probs

```

#### 6.17 Circle Common Tangent [d97f1c]

```

// be careful of tangent / exact same circle
// sign1 = 1 for outer tang, -1 for inter tang
vector<Line> common_tan(const Cir &a, const Cir &b, int
    sign1) {
    if (norm(a.o - b.o) < eps) return {};
    llf d = abs(a.o - b.o), c = (a.r - sign1 * b.r) / d;
    PF v = (b.o - a.o) / d;
    if (c * c > 1) return {};
    if (abs(c * c - 1) < eps) {
        PF p = a.o + c * v * a.r;
        return {Line(p, p + rot90(b.o - a.o))};
    }
    vector<Line> ret; llf h = sqrt(max(0.0L, 1 - c * c));
    for (int sign2 : {1, -1}) {
        PF n = c * v + sign2 * h * rot90(v);
        PF p1 = a.o + n * a.r;
        PF p2 = b.o + n * (b.r * sign1);
        ret.emplace_back(p1, p2);
    }
    return ret;
}

```

#### 6.18 Line-Circle Intersection [10786a]

```

vector<PF> LineCircleInter(PF p1, PF p2, PF o, llf r) {
    PF ft = p1 + project(o - p1, p2 - p1), vec = p2 - p1;
    llf dis = abs(o - ft);
    if (abs(dis - r) < eps) return {ft};
    if (dis > r) return {};
    vec = vec * sqrt(r * r - dis * dis) / abs(vec);
    return {ft + vec, ft - vec}; // sqrt_safe?
}

```

#### 6.19 Poly-Circle Intersection [8e5133]

```

// Divides into multiple triangle, and sum up
// from 8BQube, test by HDU2892 & AOJ CGL_7_H
llf _area(PF pa, PF pb, llf r) {
    if (abs(pa) < abs(pb)) swap(pa, pb);
    if (abs(pb) < eps) return 0;
    llf S, h, theta;
    llf a = abs(pb), b = abs(pa), c = abs(pb - pa);
    llf cB = dot(pb, pb - pa) / a / c, B = acos_safe(cB);
    llf cC = dot(pa, pb) / a / b, C = acos_safe(cC);
    if (a > r) {
        S = (C / 2) * r * r; h = a * b * sin(C) / c;
        if (h < r && B < PI / 2)
            S -= (acos_safe(h/r) * r * r - h * sqrt_safe(r * r - h * h));
    } else if (b > r) {
        theta = PI - B - asin_safe(sin(B) / r * a);
        S = 0.5 * a * r * sin(theta) + (C - theta) / 2 * r * r;
    } else
        S = 0.5 * sin(C) * a * b;
    return S;
}
llf area_poly_circle(const vector<PF> &v, PF o, llf r) {
    llf S = 0;
    for (size_t i = 0, N = v.size(); i < N; ++i)
        S += _area(v[i] - o, v[(i + 1) % N] - o, r) *
            ori(o, v[i], v[(i + 1) % N]);
    return abs(S);
}

```

#### 6.20 Min Covering Circle [92bb15]

```

Cir getCircum(P a, P b, P c) { // P = complex<llf>
    P z1 = a - b, z2 = a - c; llf D = cross(z1, z2) * 2;
    auto c1 = dot(a + b, z1), c2 = dot(a + c, z2);
    P o = rot90(c2 * z1 - c1 * z2) / D;
    return {o, abs(o - a)};
}
Cir minCircleCover(vector<P> p) {
    assert(!p.empty());
    ranges::shuffle(p, mt19937(114514));
    Cir c = {0, 0};
    for (size_t i = 0; i < p.size(); i++) {
        if (abs(p[i] - c.o) <= c.r) continue;
        c = {p[i], 0};
    }

```

```

for (size_t j = 0; j < i; j++) {
    if (abs(p[j] - c.o) <= c.r) continue;
    c.o = (p[i] + p[j]) / llf(2);
    c.r = abs(p[i] - c.o);
    for (size_t k = 0; k < j; k++) {
        if (abs(p[k] - c.o) <= c.r) continue;
        c = getCircum(p[i], p[j], p[k]);
    }
}
return c;
} // test @ TIOJ 1093 & luogu P1742

```

## 6.21 Circle Union [073c1c]

```

#define eb emplace_back
struct Teve { // test@SPOJ N=1000, 0.3~0.5s
    PF p; llf a; int add; // point, ang, add
    Teve(PF x, llf y, int z) : p(x), a(y), add(z) {}
    bool operator<(Teve &b) const { return a < b.a; }
};
// strict: x = 0, otherwise x = -1
bool disjunct(Cir &a, Cir &b, int x)
{ return sgn(abs(a.o - b.o) - a.r - b.r) > x; }
bool contain(Cir &a, Cir &b, int x)
{ return sgn(a.r - b.r - abs(a.o - b.o)) > x; }
vector<llf> CircleUnion(vector<Cir> &c) {
    // area[i] : area covered by at least i circles
    int N = (int)c.size(); vector<llf> area(N + 1);
    vector<vector<int>> overlap(N, vector<int>(N));
    auto g = overlap; // use simple 2darray to speedup
    for (int i = 0; i < N; ++i)
        for (int j = 0; j < N; ++j) {
            /* c[j] is non-strictly in c[i]. */
            overlap[i][j] = i != j &&
                (sgn(c[i].r - c[j].r) > 0 ||
                 (sgn(c[i].r - c[j].r) == 0 && i < j)) &&
                contain(c[i], c[j], -1);
        }
    for (int i = 0; i < N; ++i)
        for (int j = 0; j < N; ++j)
            g[i][j] = i != j && !(overlap[i][j] ||
                overlap[j][i] || disjunct(c[i], c[j], -1));
    for (int i = 0; i < N; ++i) {
        vector<Teve> eve; int cnt = 1;
        for (int j = 0; j < N; ++j) cnt += overlap[j][i];
        // if (cnt > 1) continue; (if only need area[1])
        for (int j = 0; j < N; ++j) if (g[i][j]) {
            auto IP = intersectPoint(c[i], c[j]);
            PF aa = IP[1], bb = IP[0];
            llf A = arg(aa - c[i].o), B = arg(bb - c[i].o);
            eve.eb(bb, B, 1); eve.eb(aa, A, -1);
            if (B > A) ++cnt;
        }
        if (eve.empty()) area[cnt] += PI*c[i].r*c[i].r;
        else {
            sort(eve.begin(), eve.end());
            eve.eb(eve[0]); eve.back().a += PI * 2;
            for (size_t j = 0; j + 1 < eve.size(); j++) {
                cnt += eve[j].add;
                area[cnt] += cross(eve[j].p, eve[j+1].p) *.5;
                llf t = eve[j + 1].a - eve[j].a;
                area[cnt] += (t-sin(t)) * c[i].r * c[i].r *.5;
            }
        }
    }
    return area;
}

```

## 6.22 Polygon Union [42e75b]

```

llf polyUnion(const vector<vector<P>> &p) {
    vector<tuple<P, P, int>> seg;
    for (int i = 0; i < ssize(p); i++)
        for (int j = 0, m = int(p[i].size()); j < m; j++)
            seg.emplace_back(p[i][j], p[i][(j + 1) % m], i);
    llf ret = 0; // area of p[i] must be non-negative
    for (auto [A, B, i] : seg) {
        vector<pair<llf, int>> evt{{0, 0}, {1, 0}};
        for (auto [C, D, j] : seg) {
            int sc = ori(A, B, C), sd = ori(A, B, D);
            if (sc != sd && i != j && min(sc, sd) < 0) {
                llf sa = cross(D-C, A-C), sb = cross(D-C, B-C);
                evt.emplace_back(sa / (sa - sb), sgn(sc - sd));
            } else if (!sc && !sd && j < i

```

```

                && sgn(dot(B - A, D - C)) > 0) {
                    evt.emplace_back(real((C - A) / (B - A)), 1);
                    evt.emplace_back(real((D - A) / (B - A)), -1);
                }
            }
            for (auto &[q, _] : evt) q = clamp<llf>(q, 0, 1);
            sort(evt.begin(), evt.end());
            llf sum = 0, last = 0; int cnt = 0;
            for (auto [q, c] : evt) {
                if (!cnt) sum += q - last;
                cnt += c; last = q;
            }
            ret += cross(A, B) * sum;
        }
    }
    return ret / 2;
}

```

## 6.23 3D Point [46b73b]

```

struct P3 {
    lld x, y, z;
    P3 operator^(const P3 &b) const {
        return {y*b.z-b.y*z, z*b.x-b.z*x, x*b.y-b.x*y};
    }
    //Azimuthal angle (longitude) to x-axis. \in [-pi, pi]
    llf phi() const { return atan2(y, x); }
    //Zenith angle (latitude) to the z-axis. \in [0, pi]
    llf theta() const { return atan2(sqrt(x*x+y*y), z); }
};
P3 ver(P3 a, P3 b, P3 c) { return (b - a) ^ (c - a); }
lld volume(P3 a, P3 b, P3 c, P3 d) {
    return dot(ver(a, b, c), d - a);
}
P3 rotate_around(P3 p, llf angle, P3 axis) {
    llf s = sin(angle), c = cos(angle);
    P3 u = normalize(axis);
    return u*dot(u, p)*(1-c) + p * c + cross(u, p)*s;
}

```

## 6.24 3D Convex Hull [01652a]

```

struct Face {
    int a, b, c;
    Face(int ta, int tb, int tc) : a(ta), b(tb), c(tc) {}
};
auto preprocess(const vector<P3> &pt) {
    auto G = pt.begin();
    auto a = find_if(all(pt), [&](P3 z) {
        return z != *G; }) - G;
    auto b = find_if(all(pt), [&](P3 z) {
        return ver(*G, pt[a], z) != P3(0, 0, 0); }) - G;
    auto c = find_if(all(pt), [&](P3 z) {
        return volume(*G, pt[a], pt[b], z) != 0; }) - G;
    vector<size_t> id;
    for (size_t i = 0; i < pt.size(); i++)
        if (i != a && i != b && i != c) id.push_back(i);
    return tuple{a, b, c, id};
}
// return the faces with pt indexes
// all points coplanar case will WA
vector<Face> convex_hull_3D(const vector<P3> &pt) {
    const int n = int(pt.size());
    if (n <= 3) return {}; // be careful about edge case
    vector<vector<int>> z(n, vector<int>(n));
    auto [a, b, c, ord] = preprocess(pt);
    now.emplace_back(a, b, c); now.emplace_back(c, b, a);
    for (auto i : ord) {
        vector<Face> next;
        for (const auto &f : now) {
            lld v = volume(pt[f.a], pt[f.b], pt[f.c], pt[i]);
            if (v <= 0) next.push_back(f);
            z[f.a][f.b] = z[f.b][f.c] = z[f.c][f.a] = sgn(v);
        }
        const auto F = [&](int x, int y) {
            if (z[x][y] > 0 && z[y][x] <= 0)
                next.emplace_back(x, y, i);
        };
        for (const auto &f : now)
            F(f.a, f.b), F(f.b, f.c), F(f.c, f.a);
        now = next;
    }
    return now;
}
// n^2 delaunay: facets with negative z normal of

```

```
// convexhull of (x, y, x^2 + y^2), use a pseudo-point
// (0, 0, inf) to avoid degenerate case
// test @ SPOJ CH3D
// llf area = 0, vol = 0; // surface area / volume
// for (auto [a, b, c]: faces)
// area += abs(ver(p[a], p[b], p[c]))/2.0,
// vol += volume(P3(0, 0, 0), p[a], p[b], p[c])/6.0;
```

## 6.25 3D Projection [68f350]

```
using P3F = valarray<llf>;
P3F toP3F(P3 p) { return {p.x, p.y, p.z}; }
llf dot(P3F a, P3F b) {
    return a[0]*b[0]+a[1]*b[1]+a[2]*b[2];
}
P3F housev(P3 A, P3 B, int s) {
    const llf a = abs(A), b = abs(B);
    return toP3F(A) / a + s * toP3F(B) / b;
}
P project(P3 p, P3 q) {
    P3 o(0, 0, 1);
    P3F u = housev(q, o, q.z > 0 ? 1 : -1);
    auto pf = toP3F(p);
    auto np = pf - 2 * u * dot(u, pf) / dot(u, u);
    return P(np[0], np[1]);
} // project p onto the plane q^Tx = 0
```

## 6.26 3D Skew Line Nearest Point

- $L_1: \mathbf{v}_1 = \mathbf{p}_1 + t_1 \mathbf{d}_1, L_2: \mathbf{v}_2 = \mathbf{p}_2 + t_2 \mathbf{d}_2$
- $\mathbf{n} = \mathbf{d}_1 \times \mathbf{d}_2$
- $\mathbf{n}_1 = \mathbf{d}_1 \times \mathbf{n}, \mathbf{n}_2 = \mathbf{d}_2 \times \mathbf{n}$
- $\mathbf{c}_1 = \mathbf{p}_1 + \frac{(\mathbf{p}_2 - \mathbf{p}_1) \cdot \mathbf{n}_2}{\mathbf{d}_1 \cdot \mathbf{n}_2} \mathbf{d}_1, \mathbf{c}_2 = \mathbf{p}_2 + \frac{(\mathbf{p}_1 - \mathbf{p}_2) \cdot \mathbf{n}_1}{\mathbf{d}_2 \cdot \mathbf{n}_1} \mathbf{d}_2$

## 6.27 Delaunay [3a4ff1]

```
/* please ensure input points are unique */
/* A triangulation such that no points will strictly
inside circumcircle of any triangle. C should be big
enough s.t. the initial triangle contains all points */
#define L(i) ((i)==0 ? 2 : (i)-1)
#define R(i) ((i)==2 ? 0 : (i)+1)
#define F3 for (int i = 0; i < 3; i++)
bool is_inf(P z) { return RE(z) <= -C || RE(z) >= C; }
bool in_cc(const array<P,3> &p, P q) {
    i128 inf_det = 0, det = 0, inf_N, N;
    F3 {
        if (is_inf(p[i]) && is_inf(q)) continue;
        else if (is_inf(p[i])) inf_N = 1, N = -norm(q);
        else if (is_inf(q)) inf_N = -1, N = norm(p[i]);
        else inf_N = 0, N = norm(p[i]) - norm(q);
        lld D = cross(p[R(i)] - q, p[L(i)] - q);
        inf_det += inf_N * D; det += N * D;
    }
    return inf_det != 0 ? inf_det > 0 : det > 0;
}
P v[maxn];
struct Tri;
struct E {
    Tri *t; int side;
    E(Tri *t_=0, int side_=0) : t(t_), side(side_) {}
};
struct Tri {
    array<int,3> p; array<Tri*,3> ch; array<E,3> e;
    Tri(int a=0, int b=0, int c=0) : p{a, b, c}, ch{} {}
    bool has_chd() const { return ch[0] != nullptr; }
    bool contains(int q) const {
        F3 if (ori(v[p[i]], v[p[R(i)]], v[q]) < 0)
            return false;
        return true;
    }
    bool check(int q) const {
        return in_cc({v[p[0]], v[p[1]], v[p[2]]}, v[q]);
    }
} pool[maxn * 10], *it, *root;
void link(const E &a, const E &b) {
    if (a.t) a.t->e[a.side] = b;
    if (b.t) b.t->e[b.side] = a;
}
void flip(Tri *A, int a) {
    auto [B, b] = A->e[a]; /* flip edge between A,B */
    if (!B || !A->check(B->p[b])) return;
    Tri *X = new (it++) Tri(A->p[R(a)], B->p[b], A->p[a]);
    Tri *Y = new (it++) Tri(B->p[R(b)], A->p[a], B->p[b]);
    link(E(X, 0), E(Y, 0));
    link(E(X, 1), A->e[L(a)]); link(E(X, 2), B->e[R(b)]);
    link(E(Y, 1), B->e[L(b)]); link(E(Y, 2), A->e[R(a)]);
    A->ch = B->ch = {X, Y, nullptr};
}
```

```
flip(X, 1); flip(X, 2); flip(Y, 1); flip(Y, 2);
}
void add_point(int p) {
    Tri *r = root;
    while (r->has_chd()) for (Tri *c: r->ch)
        if (c && c->contains(p)) { r = c; break; }
    array<Tri*, 3> t; /* split into 3 triangles */
    F3 t[i] = new (it++) Tri(r->p[i], r->p[R(i)], p);
    F3 link(E(t[i], 0), E(t[R(i)], 1));
    F3 link(E(t[i], 2), r->e[L(i)]);
    r->ch = t;
    F3 flip(t[i], 2);
}
auto build(const vector<P> &p) {
    it = pool; int n = (int)p.size();
    vector<int> ord(n); iota(all(ord), 0);
    shuffle(all(ord), mt19937(114514));
    root = new (it++) Tri(n, n + 1, n + 2);
    copy_n(p.data(), n, v); v[n++] = P(-C, -C);
    v[n++] = P(C * 2, -C); v[n++] = P(-C, C * 2);
    for (int i : ord) add_point(i);
    vector<array<int, 3>> res;
    for (Tri *now = pool; now != it; now++)
        if (!now->has_chd()) res.push_back(now->p);
    return res;
}
```

## 6.28 Build Voronoi [94f000]

```
void build_voronoi_cells(auto &&p, auto &&res) {
    vector<vector<int>> adj(p.size());
    for (auto f: res) F3 {
        int a = f[i], b = f[R(i)];
        if (a >= p.size() || b >= p.size()) continue;
        adj[a].emplace_back(b);
    }
    // use `adj` and `p` and HPI to build cells
    for (size_t i = 0; i < p.size(); i++) {
        vector<Line> ls = frame; // the frame
        for (int j : adj[i]) {
            P m = p[i] + p[j], d = rot90(p[j] - p[i]);
            assert(norm(d) != 0);
            ls.emplace_back(m, m + d); // doubled coordinate
        } // HPI(ls)
    }
}
```

## 6.29 kd Tree (Nearest Point) [f733e5]

```
struct KDTree {
    struct Node {
        int x, y, x1, y1, x2, y2, id, f; Node *L, *R;
    } tree[maxn], *root;
    lld dis2(int x1, int y1, int x2, int y2) {
        lld dx = x1 - x2, dy = y1 - y2;
        return dx * dx + dy * dy;
    }
    static bool cmpx(Node& a, Node& b) { return a.x < b.x; }
    static bool cmpy(Node& a, Node& b) { return a.y < b.y; }
    void init(vector<pair<int,int>> &ip) {
        for (int i = 0; i < ssize(ip); i++)
            tie(tree[i].x, tree[i].y) = ip[i], tree[i].id = i;
        root = build(0, (int)ip.size()-1, 0);
    }
    Node* build(int L, int R, int d) {
        if (L > R) return nullptr;
        int M = (L+R)/2;
        nth_element(tree+L, tree+M, tree+R+1, d%2?cmpx:cmpy);
        Node &o = tree[M]; o.f = d % 2;
        o.x1 = o.x2 = o.x; o.y1 = o.y2 = o.y;
        o.L = build(L, M-1, d+1); o.R = build(M+1, R, d+1);
        for (Node *s: {o.L, o.R}) if (s) {
            o.x1 = min(o.x1, s->x1); o.x2 = max(o.x2, s->x2);
            o.y1 = min(o.y1, s->y1); o.y2 = max(o.y2, s->y2);
        }
        return tree+M;
    }
    bool touch(int x, int y, lld d2, Node *r){
        lld d = (lld)sqrt(d2)+1;
        return x >= r->x1 - d && x <= r->x2 + d &&
            y >= r->y1 - d && y <= r->y2 + d;
    }
}
using P = pair<lld, int>;
void dfs(int x, int y, P &mn, Node *r) {
    if (!r || !touch(x, y, mn.first, r)) return;
}
```



```

mn = min(mn, P(dis2(r->x, r->y, x, y), r->id));
if (r->f == 1 ? y < r->y : x < r->x)
    dfs(x, y, mn, r->L), dfs(x, y, mn, r->R);
else
    dfs(x, y, mn, r->R), dfs(x, y, mn, r->L);
}
int query(int x, int y) {
    P mn(INF, -1); dfs(x, y, mn, root);
    return mn.second;
}
} tree;

```

### 6.30 kd Closest Pair (3D ver.) [84d9eb]

```

llf solve(vector<P> v) {
    shuffle(v.begin(), v.end(), mt19937());
    unordered_map<lld, unordered_map<lld,
        unordered_map<lld, int>>> m;
    llf d = dis(v[0], v[1]);
    auto Idx = [&d] (llf x) -> lld {
        return round(x * 2 / d) + 0.1; };
    auto rebuild_m = [&m, &v, &Idx](int k) {
        m.clear();
        for (int i = 0; i < k; ++i)
            m[Idx(v[i].x)][Idx(v[i].y)]
                [Idx(v[i].z)] = i;
        ; rebuild_m(2);
        for (size_t i = 2; i < v.size(); ++i) {
            const lld kx = Idx(v[i].x), ky = Idx(v[i].y),
                kz = Idx(v[i].z); bool found = false;
            for (int dx = -2; dx <= 2; ++dx) {
                const lld nx = dx + kx;
                if (m.find(nx) == m.end()) continue;
                auto& mm = m[nx];
                for (int dy = -2; dy <= 2; ++dy) {
                    const lld ny = dy + ky;
                    if (mm.find(ny) == mm.end()) continue;
                    auto& mmm = mm[ny];
                    for (int dz = -2; dz <= 2; ++dz) {
                        const lld nz = dz + kz;
                        if (mmm.find(nz) == mmm.end()) continue;
                        const int p = mmm[nz];
                        if (dis(v[p], v[i]) < d) {
                            d = dis(v[p], v[i]);
                            found = true;
                        }
                    }
                }
            }
            if (found) rebuild_m(i + 1);
            else m[kx][ky][kz] = i;
        }
        return d;
    }
}

```

### 6.31 Simulated Annealing [4e0fe5]

```

llf anneal() {
    mt19937 rnd_engine(seed);
    uniform_real_distribution<llf> rnd(0, 1);
    const llf dT = 0.001;
    // Argument p
    llf S_cur = calc(p), S_best = S_cur;
    for (llf T = 2000; T > EPS; T -= dT) {
        // Modify p to p_prime
        const llf S_prime = calc(p_prime);
        const llf delta_c = S_prime - S_cur;
        llf prob = min((llf)1, exp(-delta_c / T));
        if (rnd(rnd_engine) <= prob)
            S_cur = S_prime, p = p_prime;
        if (S_prime < S_best) // find min
            S_best = S_prime, p_best = p_prime;
    }
    return S_best;
}

```

### 6.32 Triangle Centers [adb146]

```

O = ... // see min circle cover
G = (A + B + C) / 3;
H = G * 3 - O * 2; // orthogonal center
llf a = abs(B - C), b = abs(A - C), c = abs(A - B);
I = (a * A + b * B + c * C) / (a + b + c);
// FermatPoint: minimizes sum of distance
// if max. angle >= 120 deg then vertex
// otherwise, make eq. triangle AB'C, CA'B, BC'A
// line AA', BB', CC' intersects at P

```

## 7 Stringology

### 7.1 Hash [ce7fad]

```

template <int P = 127, int Q = 1051762951>
class Hash {
    vector<int> h, p;
public:
    Hash(const auto &s) : h(s.size()+1), p(s.size()+1) {
        for (size_t i = 0; i < s.size(); ++i)
            h[i + 1] = add(mul(h[i], P), s[i]);
        generate(all(p), [x = 1, y = 1, this]() mutable {
            return y = x, x = mul(x, P), y; });
    }
    int query(int l, int r) const { // 0-base [l, r)
        return sub(h[r], mul(h[l], p[r - l]));
    }
};

```

### 7.2 Suffix Array [da27c7]

```

auto sais(const auto &s) {
    const int n = (int)s.size(), z = ranges::max(s) + 1;
    vector<int> c(z); for (int x : s) ++c[x];
    partial_sum(all(c), begin(c));
    vector<int> sa(n); auto I = views::iota(0, n);
    if (ranges::max(c) <= 1) {
        for (int i : I) sa[--c[s[i]]] = i;
        return sa;
    }
    vector<bool> t(n); t[n - 1] = true;
    for (int i = n - 2; i >= 0; --i)
        t[i] = (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
    auto is_lms = views::filter([&t](int x) {
        return x && t[x] && !t[x - 1]; });
    auto induce = [&] {
        for (auto x = c; int y : sa)
            if (y-- if (!t[y]) sa[x[s[y]] - 1]++ = y;
            for (auto x = c; int y : sa | views::reverse)
                if (y-- if (t[y]) sa[--x[s[y]]] = y;
    };
    vector<int> lms, q(n); lms.reserve(n);
    for (auto x = c; int i : I | is_lms) {
        q[i] = int(lms.size());
        lms.push_back(sa[--x[s[i]]] = i);
    }
    induce(); vector<int> ns(lms.size());
    for (int j = -1, nz = 0; int i : sa | is_lms) {
        if (j >= 0) {
            int len = min({n - i, n - j, lms[q[i] + 1] - i});
            ns[q[i]] = nz += lexicographical_compare(
                begin(s) + j, begin(s) + j + len,
                begin(s) + i, begin(s) + i + len);
        }
        j = i;
    }
    ranges::fill(sa, 0); auto nsa = sais(ns);
    for (auto x = c; int y : nsa | views::reverse)
        y = lms[y], sa[--x[s[y]]] = y;
    return induce(), sa;
}
// sa[i]: sa[i]-th suffix is the
// i-th lexicographically smallest suffix.
// hi[i]: LCP of suffix sa[i] and suffix sa[i - 1].
struct Suffix {
    int n; vector<int> sa, hi, rev;
    Suffix(const auto &s) : n((int)s.size()),
        hi(n), rev(n) {
        vector<int> _s(n + 1); // _s[n] = 0;
        copy(all(s), begin(_s)); // s shouldn't contain 0
        sa = sais(_s); sa.erase(sa.begin());
        for (int i = 0; i < n; ++i) rev[sa[i]] = i;
        for (int i = 0, h = 0; i < n; ++i) {
            if (!rev[i]) { h = 0; continue; }
            for (int j = sa[rev[i] - 1]; i + h < n && j + h < n
                && s[i + h] == s[j + h];) ++h;
            hi[rev[i]] = h ? h-- : 0;
        }
    }
};

```

### 7.3 Suffix Array Tools [be0061]

```

struct OfflineGetRange : Suffix {
    vector<vector<pair<int, int>>> qs; int qid;
    OfflineGetRange(const auto &s)
        : Suffix(s), qs(n), qid(0) {}
};

```

```

int offline_get_range(int x, int len) {
    return qs[len].eb(rev[x], qid), qid++;
}
vector<pair<int,int>> solve_get_range() {
    vector<pair<int,int>> ans(qid); Dsu dsu(n);
    for (int i = 1; i < n; i++) qs[hi[i]].eb(i, -1);
    for (int i = n - 1; i >= 0; i--)
        for (auto [pos, id] : qs[i] | views::reverse)
            if (id == -1) dsu.join(pos - 1, pos);
            else ans[id] =
                {dsu.get_min(pos), dsu.get_max(pos) + 1};
    return qs.assign(n), qid = 0, ans;
}
};
template <int LG = 20> struct SparseTableSA : Suffix {
    array<vector<int>, LG> mn;
    SparseTableSA(const auto &s) : Suffix(s), mn{hi} {
        for (int l = 0; l + 1 < LG; l++) { mn[l+1].resize(n);
            for (int i = 0, len = 1 << l; i + len < n; i++)
                mn[l + 1][i] = min(mn[l][i], mn[l][i + len]);
        }
    }
    int lcp(int a, int b) {
        if (a == b) return n - a;
        a = rev[a] + 1, b = rev[b] + 1;
        if (a > b) swap(a, b);
        const int lg = __lg(b - a);
        return min(mn[lg][a], mn[lg][b - (1 << lg)]);
    }
    pair<int,int> get_range(int x, int len) { // WIP
        int a = rev[x] + 1, b = rev[x] + 1;
        for (int l = LG - 1; l >= 0; l--) {
            const int s = 1 << l;
            if (a + s <= n && mn[l][a] >= len) a += s;
            if (b - s >= 0 && mn[l][b - s] >= len) b -= s;
        }
        return {b - 1, a};
    }
};

```

## 7.4 Ex SAM [58374b]

```

struct exSAM {
    int len[maxn * 2], link[maxn * 2]; // maxlen, suflink
    int next[maxn * 2][maxc], tot; // [0, tot), root = 0
    int ord[maxn * 2]; // topo. order (sort by length)
    int cnt[maxn * 2]; // occurrence
    int newnode() {
        fill_n(next[tot], maxc, 0);
        return len[tot] = cnt[tot] = link[tot] = 0, tot++;
    }
    void init() { tot = 0, newnode(), link[0] = -1; }
    int insertSAM(int last, int c) {
        int cur = next[last][c];
        len[cur] = len[last] + 1;
        int p = link[last];
        while (p != -1 && !next[p][c])
            next[p][c] = cur, p = link[p];
        if (p == -1) return link[cur] = 0, cur;
        int q = next[p][c];
        if (len[p] + 1 == len[q]) return link[cur] = q, cur;
        int clone = newnode();
        for (int i = 0; i < maxc; ++i)
            next[clone][i] = len[next[q][i]] ? next[q][i] : 0;
        len[clone] = len[p] + 1;
        while (p != -1 && next[p][c] == q)
            next[p][c] = clone, p = link[p];
        link[link[cur] = clone] = link[q];
        link[q] = clone;
        return cur;
    }
    void insert(const string &s) {
        int cur = 0;
        for (char ch : s) {
            int &nxt = next[cur][int(ch - 'a')];
            if (!nxt) nxt = newnode();
            cnt[cur = nxt] += 1;
        }
    }
    void build() {
        queue<int> q; q.push(0);
        while (!q.empty()) {
            int cur = q.front(); q.pop();

```

```

            for (int i = 0; i < maxc; ++i)
                if (next[cur][i]) q.push(insertSAM(cur, i));
        }
        vector<int> lc(tot);
        for (int i = 1; i < tot; ++i) ++lc[len[i]];
        partial_sum(all(lc), lc.begin());
        for (int i = 1; i < tot; ++i) ord[--lc[len[i]]] = i;
    }
    void solve() {
        for (int i = tot - 2; i >= 0; --i)
            cnt[link[ord[i]]] += cnt[ord[i]];
    }
};

```

## 7.5 KMP [281185]

```

vector<int> kmp(const auto &s) {
    vector<int> f(s.size());
    for (int i = 1, k = 0; i < (int)s.size(); ++i) {
        while (k > 0 && s[i] != s[k]) k = f[k - 1];
        f[i] = (k += (s[i] == s[k]));
    }
    return f;
}
vector<int> search(const auto &s, const auto &t) {
    // return 0-indexed occurrence of t in s
    vector<int> f = kmp(t), r;
    for (int i = 0, k = 0; i < (int)s.size(); ++i) {
        while (k > 0 && s[i] != t[k]) k = f[k - 1];
        k += (s[i] == t[k]);
        if (k == (int)t.size()) {
            r.push_back(i - t.size() + 1);
            k = f[k - 1];
        }
    }
    return r;
}

```

## 7.6 Z value [6a7fd0]

```

vector<int> Zalgo(const string &s) {
    vector<int> z(s.size(), s.size());
    for (int i = 1, l = 0, r = 0; i < z[0]; ++i) {
        int j = clamp(r - i, 0, z[i - l]);
        for (; i + j < z[0] && s[i + j] == s[j]; ++j);
        if (i + (z[i] = j) > r) r = i + z[i] = i;
    }
    return z;
}

```

## 7.7 Manacher [c938a9]

```

vector<int> manacher(const string &s) {
    const int n = (int)S.size(), m = n * 2 + 1;
    vector<int> z(m);
    string t = "."; for (char c : S) t += c, t += '.';
    for (int i = 1, l = 0, r = 0; i < m; ++i) {
        z[i] = (r > i ? min(z[2 * l - i], r - i) : 1);
        while (i - z[i] >= 0 && i + z[i] < m) {
            if (t[i - z[i]] == t[i + z[i]]) ++z[i];
            else break;
        }
        if (i + z[i] > r) r = i + z[i], l = i;
    }
    return z; // the palindrome lengths are z[i] - 1
}

```

```

/* for (int i = 1; i + 1 < m; ++i) {
    int l = (i - z[i] + 2) / 2, r = (i + z[i]) / 2;
    if (l != r) // [l, r) is maximal palindrome
} */

```

## 7.8 Lyndon Factorization [d22cc9]

```

// partition s = w[0] + w[1] + ... + w[k-1],
// w[0] >= w[1] >= ... >= w[k-1]
// each w[i] strictly smaller than all its suffix
void duval(const auto &s, auto &&report) {
    for (int n = (int)s.size(), i = 0, j, k; i < n; ) {
        for (j = i + 1, k = i; j < n && s[k] <= s[j]; j++)
            k = (s[k] < s[j] ? i : k + 1);
        // if (i < n / 2 && j >= n / 2) {
        // for min cyclic shift, call duval(s + s)
        // then here s.substr(i, n / 2) is min cyclic shift
        // }
        for (; i <= k; i += j - k)
            report(i, j - k); // s.substr(l, len)
    }
}

```

```

} // tested @ luogu 6114, 1368 & UVA 719

```

## 7.9 Main Lorentz [615b8f]

```
vector<pair<int, int>> rep[kN]; // 0-base [l, r]
void main_lorentz(const string &s, int sft = 0) {
    const int n = s.size();
    if (n == 1) return;
    const int nu = n / 2, nv = n - nu;
    const string u = s.substr(0, nu), v = s.substr(nu,
        ru(u.rbegin(), u.rend()), rv(v.rbegin(), v.rend()));
    main_lorentz(u, sft), main_lorentz(v, sft + nu);
    const auto z1 = Zalgo(ru), z2 = Zalgo(v + '#' + u),
        z3 = Zalgo(ru + '#' + rv), z4 = Zalgo(v);
    auto get_z = [](const vector<int> &z, int i) {
        return (0 <= i and i < (int)z.size()) ? z[i] : 0; };
    auto add_rep = [&](bool left, int c, int l, int k1,
        int k2) {
        const int L = max(1, l - k2), R = min(l - left, k1);
        if (L > R) return;
        if (left) rep[l].emplace_back(sft + c - R, sft + c -
            L);
        else rep[l].emplace_back(sft + c - R - l + 1, sft + c
            - L - l + 1);
    };
    for (int cnt = 0; cnt < n; cnt++) {
        int l, k1, k2;
        if (cnt < nu) {
            l = nu - cnt;
            k1 = get_z(z1, nu - cnt);
            k2 = get_z(z2, nv + 1 + cnt);
        } else {
            l = cnt - nu + 1;
            k1 = get_z(z3, nu + 1 + nv - 1 - (cnt - nu));
            k2 = get_z(z4, (cnt - nu) + 1);
        }
        if (k1 + k2 >= l)
            add_rep(cnt < nu, cnt, l, k1, k2);
    }
}
```

## 7.10 BWT [5a9b3a]

```
vector<int> v[SIGMA];
void BWT(char *ori, char *res) {
    // make ori -> ori + ori
    // then build suffix array
}
void iBWT(char *ori, char *res) {
    for (int i = 0; i < SIGMA; i++) v[i].clear();
    const int len = strlen(ori);
    for (int i = 0; i < len; i++)
        v[ori[i] - 'a'].push_back(i);
    vector<int> a;
    for (int i = 0, ptr = 0; i < SIGMA; i++)
        for (int j : v[i]) {
            a.push_back(j);
            ori[ptr++] = 'a' + i;
        }
    for (int i = 0, ptr = 0; i < len; i++) {
        res[i] = ori[a[ptr]];
        ptr = a[ptr];
    }
    res[len] = 0;
}
```

## 7.11 Palindromic Tree [0673ee]

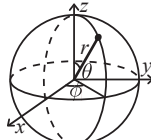
```
struct PalindromicTree {
    struct node {
        int nxt[26], f, len; // num = depth of fail link
        int cnt, num; // = #pal_suffix of this node
        node(int l = 0) : nxt{}, f(0), len(l), cnt(0), num(0) {}
    };
    vector<node> st; vector<char> s; int last, n;
    void init() {
        st.clear(); s.clear();
        last = 1; n = 0;
        st.push_back(0); st.push_back(-1);
        st[0].f = 1; s.push_back(-1);
    }
    int getFail(int x) {
        while (s[n - st[x].len - 1] != s[n]) x = st[x].f;
        return x;
    }
    void add(int c) {
        s.push_back(c == 'a'); ++n;
        int cur = getFail(last);
```

```
if (!st[cur].nxt[c]) {
    int now = st.size();
    st.push_back(st[cur].len + 2);
    st[now].f = st[getFail(st[cur].f)].nxt[c];
    st[cur].nxt[c] = now;
    st[now].num = st[st[now].f].num + 1;
}
last = st[cur].nxt[c]; ++st[last].cnt;
}
void dpcnt() { // cnt = #occurrence in whole str
    for (int i = st.size() - 1; i >= 0; i--)
        st[st[i].f].cnt += st[i].cnt;
}
int size() { return st.size() - 2; }
} pt;
/* usage
string s; cin >> s; pt.init();
for (int i = 0; i < SZ(s); i++) {
    int prvsz = pt.size(); pt.add(s[i]);
    if (prvsz != pt.size()) {
        int r = i, l = r - pt.st[pt.last].len + 1;
        // pal @ [l,r]: s.substr(l, r-l+1)
    }
} */
```

## 8 Misc

### 8.1 Theorems

#### Spherical Coordinate

$$\begin{aligned} x &= r \sin \theta \cos \phi \\ y &= r \sin \theta \sin \phi \\ z &= r \cos \theta \end{aligned}$$


$$\begin{aligned} r &= \sqrt{x^2 + y^2 + z^2} \\ \theta &= \arccos(z / \sqrt{x^2 + y^2 + z^2}) \\ \phi &= \operatorname{atan2}(y, x) \end{aligned}$$

#### Spherical Cap

- A portion of a sphere cut off by a plane.
- $r$ : sphere radius,  $a$ : radius of the base of the cap,  $h$ : height of the cap,  $\theta$ :  $\arcsin(a/r)$ .
- Volume  $= \pi h^2(3r - h)/3 = \pi h(3a^2 + h^2)/6 = \pi r^3(2 + \cos \theta)(1 - \cos \theta)^2/3$ .
- Area  $= 2\pi r h = \pi(a^2 + h^2) = 2\pi r^2(1 - \cos \theta)$ .

#### Sherman-Morrison formula

$$(A + uv^T)^{-1} = A^{-1} - \frac{A^{-1}uv^T A^{-1}}{1 + v^T A^{-1}u}$$

#### Kirchhoff's Theorem

Denote  $L$  be a  $n \times n$  matrix as the Laplacian matrix of graph  $G$ , where  $L_{ii} = d(i)$ ,  $L_{ij} = -c$  where  $c$  is the number of edge  $(i, j)$  in  $G$ .

- The number of undirected spanning in  $G$  is  $\det(\tilde{L}_{11})$ .
- The number of directed spanning tree rooted at  $r$  in  $G$  is  $\det(\tilde{L}_{rr})$ .

#### BEST Theorem

$$\#[\text{Eulerian circuits}] = \#[\text{arborescences rooted at } 1] \cdot \prod_{v \in V} (\deg(v) - 1)!$$

#### Random Walk on Graph

Let  $P$  be the transition matrix of a strongly connected directed graph,  $\sum_j P_{i,j} = 1$ . Let  $F_{i,j}$  be the expected time to reach  $j$  from  $i$ . Let  $g_i$  be the expected time from  $i$  to  $i$ ,  $G = \text{diag}(g)$  and  $J$  be a matrix all of 1, i.e.  $J_{i,j} = 1$ . Then,  $F = J - G + PF$ .

First solve  $G$ : let  $\pi P = \pi$  be a stationary distribution. Then  $\pi_i g_i = 1$ . The rank of  $I - P$  is  $n - 1$ , so we first solve a special solution  $X$  such that  $(I - P)X = J - G$  and adjust  $X$  to  $F$  by  $F_{i,j} = X_{i,j} - X_{j,j}$ .

#### Tutte Matrix

For  $i < j$ ,  $d_{ij} = x_{ij}$  (in practice, a random number) if  $(i, j) \in E$ , otherwise  $d_{ij} = 0$ . For  $i \geq j$ ,  $d_{ij} = -d_{ji}$ .  $\frac{\text{rank}(D)}{2}$  is the maximum matching.

#### Cayley's Formula

- Given a degree sequence  $d_1, d_2, \dots, d_n$  for each labeled vertices, there're  $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\dots(d_n-1)!}$  spanning trees.
- Let  $T_{n,k}$  be the number of labeled forests on  $n$  vertices with  $k$  components, such that vertex  $1, 2, \dots, k$  belong to different components. Then  $T_{n,k} = kn^{n-k-1}$ .

#### Erdős-Gallai theorem

A sequence of non-negative integers  $d_1 \geq d_2 \geq \dots \geq d_n$  can be represented as the degree sequence of a finite simple graph on  $n$  vertices if and only if  $d_1 + d_2 + \dots + d_n$  is even and  $\sum_{i=1}^n d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$  holds for all  $1 \leq k \leq n$ .

#### Havel-Hakimi algorithm

Find the vertex who has greatest degree unused, connect it with other greatest vertex.

#### Gale-Ryser theorem

A pair of sequences of nonnegative integers  $a_1 \geq \dots \geq a_n$  and  $b_1, \dots, b_n$  is bigraphic if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and  $\sum_{i=1}^k a_i \leq \sum_{i=1}^n \min(b_i, k)$  holds for every  $1 \leq k \leq n$ .

#### Fulkerson-Chen-Anstee theorem

A sequence  $(a_1, b_1), \dots, (a_n, b_n)$  of nonnegative integer pairs with  $a_1 \geq \dots \geq a_n$  is digraphic if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and  $\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k-1) + \sum_{i=k+1}^n \min(b_i, k)$  holds for every  $1 \leq k \leq n$ .

#### Euler's planar graph formula

$$V - E + F = C + 1, E \leq 3V - 6 \text{ (when } V \geq 3)$$

**Pick's theorem**

For simple polygon, when points are all integer, we have  $A = \#\{\text{lattice points in the interior}\} + \frac{1}{2} \#\{\text{lattice points on the boundary}\} - 1$

**Matroid**

- $B \subseteq A \wedge A \in \mathcal{I} \Rightarrow B \in \mathcal{I}$ .
  - If  $A, B \in \mathcal{I}$  and  $|A| > |B|$ , then  $\exists x \in A \setminus B, B \cup \{x\} \in \mathcal{I}$ .
- |                       |  |
|-----------------------|--|
| Linear matroid        | $A \in \mathcal{I}$ iff linear indep.                                    |
| Graphic matroid       | $\mathcal{I}$ = forests of undirected graph                              |
| Colorful matroid (EX) | Each color $c$ has an upper bound $R_c$ .                                |
| Transversal matroid   | $A \in \mathcal{I}$ iff $\exists$ matching $M$ whose right part is $A$ . |
| Bond matroid          | $A \in \mathcal{I}$ iff $G$ is connected after removing edges $A$ .      |
| Dual matroid          | $A \in \mathcal{I}'$ iff there is a basis $\subseteq E \setminus A$      |
| Truncated matroid     | $A \in \mathcal{I}'$ iff $A \in \mathcal{I} \wedge  A  \leq k$           |

**Matroid Intersection**

Given matroids  $M_1 = (G, \mathcal{I}_1), M_2 = (G, \mathcal{I}_2)$ , find maximum  $S \in \mathcal{I}_1 \cap \mathcal{I}_2$ . For each iteration, build the directed graph and find a shortest path from  $s$  to  $t$ .

- $s \rightarrow x : S \cup \{x\} \in \mathcal{I}_1$
- $x \rightarrow t : S \cup \{x\} \in \mathcal{I}_2$
- $y \rightarrow x : S \setminus \{y\} \cup \{x\} \in \mathcal{I}_1$  ( $y$  is in the unique circuit of  $S \cup \{x\}$ )
- $x \rightarrow y : S \setminus \{y\} \cup \{x\} \in \mathcal{I}_2$  ( $y$  is in the unique circuit of  $S \cup \{x\}$ )

Alternate the path, and  $|S|$  will increase by 1. In each iteration,  $|E| = O(RN)$ , where  $R = \min(\text{rank}(\mathcal{I}_1), \text{rank}(\mathcal{I}_2))$ ,  $N = |G|$ . For weighted case, assign weight  $-w(x)$  and  $w(x)$  to  $x \in S$  and  $x \notin S$ , resp. Find the shortest path by Bellman-Ford. The maximum iteration of Bellman-Ford is  $2R + 1$ .

**Dual of LP**

Primal	Dual
Maximize $c^T x$ s.t. $Ax \leq b, x \geq 0$	Minimize $b^T y$ s.t. $A^T y \geq c, y \geq 0$
Maximize $c^T x$ s.t. $Ax \leq b$	Minimize $b^T y$ s.t. $A^T y = c, y \geq 0$
Maximize $c^T x$ s.t. $Ax = b, x \geq 0$	Minimize $b^T y$ s.t. $A^T y \geq c$

**Dual of Min Cost b-Flow**

- Capacity  $c_{uv}$ , Flow  $f_{uv}$ , Cost  $w_{uv}$ , Required Flow difference for vertex  $b_u$ .
- If all  $w_{uv}$  are integers, then optimal solution can happen when all  $p_u$  are integers.

$$\min \sum_{uv} w_{uv} f_{uv} \text{ s.t. } -f_{uv} \geq -c_{uv}, \sum_v f_{vu} - \sum_v f_{uv} = -b_u$$

$$\Leftrightarrow \min \sum_u b_u p_u + \sum_{uv} c_{uv} \max(0, p_v - p_u - w_{uv}) \text{ s.t. } p_u \geq 0$$

**Minimax Theorem**

Let  $f : X \times Y \rightarrow \mathbb{R}$  be continuous where  $X \subseteq \mathbb{R}^n, Y \subseteq \mathbb{R}^m$  are compact and convex. If  $f(\cdot, y) : X \rightarrow \mathbb{R}$  is concave for fixed  $y$ , and  $f(x, \cdot) : Y \rightarrow \mathbb{R}$  is convex for fixed  $x$ , then  $\max_{x \in X} \min_{y \in Y} f(x, y) = \min_{y \in Y} \max_{x \in X} f(x, y)$ , e.g.  $f(x, y) = x^T A y$  for

zero-sum matrix game.

**Parallel Axis Theorem**

The second moment of area is  $I_z = \iint x^2 + y^2 dA$ .  $I_{z'} = I_z + A d^2$  where  $d$  is the distance between two parallel axis  $z, z'$ .

**8.2 Stable Marriage**

- 1: Initialize  $m \in M$  and  $w \in W$  to free
- 2: while  $\exists$  free man  $m$  who has a woman  $w$  to propose to do
- 3:    $w \leftarrow$  first woman on  $m$ 's list to whom  $m$  has not yet proposed
- 4:   if  $\exists$  some pair  $(m', w)$  then
- 5:     if  $w$  prefers  $m$  to  $m'$  then
- 6:        $m' \leftarrow$  free
- 7:        $(m, w) \leftarrow$  engaged
- 8:     end if
- 9:   else
- 10:     $(m, w) \leftarrow$  engaged
- 11:   end if
- 12: end while

**8.3 Weight Matroid Intersection [d00ee8]**

```

struct Matroid {
    Matroid(bitset<N>); // init from an independent set
    bool can_add(int); // check if break independence
    Matroid remove(int); // removing from the set
};

auto matroid_intersection(const vector<int> &w) {
    const int n = (int)w.size(); bitset<N> S;
    for (int sz = 1; sz <= n; sz++) {
        Matroid M1(S), M2(S); vector<vector<pii>> e(n + 2);
        for (int j = 0; j < n; j++) if (!S[j]) {
            if (M1.can_add(j)) e[n].eb(j, -w[j]);
            if (M2.can_add(j)) e[j].eb(n + 1, 0);
        }
        for (int i = 0; i < n; i++) if (S[i]) {
            Matroid T1 = M1.remove(i), T2 = M2.remove(i);
            for (int j = 0; j < n; j++) if (!S[j]) {
                if (T1.can_add(j)) e[i].eb(j, -w[j]);
                if (T2.can_add(j)) e[j].eb(i, w[i]);
            }
        }
    } // maybe implicit build graph for more speed
    vector<pii> d(n + 2, {INF, 0}); d[n] = {0, 0};
    vector<int> prv(n + 2, -1);
    // change to SPFA for more speed, if necessary
    for (int upd = 1; upd--;) {
        for (int u = 0; u < n; u++) {
            for (auto [v, c] : e[u]) {
                pii x(d[u].first + c, d[u].second + 1);

```

```

        if (x < d[v]) d[v] = x, prv[v] = u, upd = 1;
    }
    if (d[n + 1].first >= INF) break;
    for (int x = prv[n + 1]; x != n; x = prv[x]) S.flip(x);
    // S is the max-weighted independent set w/ size sz
}
return S;
} // from Nacl

```

**8.4 Bitset LCS [4155ab]**

```

cin >> n >> m;
for (int i = 1, x; i <= n; ++i)
    cin >> x, p[x].set(i);
for (int i = 1, x; i <= m; ++i) {
    cin >> x, (g = f) |= p[x];
    f.shiftLeftByOne(), f.set(0);
    ((f = g - f) ^= g) &= g;
}
cout << f.count() << '\n';

```

**8.5 Prefix Substring LCS [7d8faf]**

```

void all_lcs(string S, string T) { // 0-base
    vector<size_t> h(T.size()); iota(all(h), 1);
    for (size_t a = 0; a < S.size(); ++a) {
        for (size_t c = 0, v = 0; c < T.size(); ++c)
            if (S[a] == T[c] || h[c] < v) swap(h[c], v);
        // here, LCS(s[0, a], t[b, c]) =
        // c - b + 1 - sum([h[i] > b] | i <= c)
    }
} // test @ yosupo judge

```

**8.6 Convex 1D/1D DP [e5ab4b]**

```

struct S { int l, r; };
auto solve(int n, int k, auto &w) {
    vector<int64_t> dp(n + 1);
    auto f = [&](int l, int r) -> int64_t {
        if (r - l > k) return -INF;
        return dp[l] + w(l + 1, r);
    };
    dp[0] = 0;
    deque<S> dq; dq.emplace_back(0, 1, n);
    for (int i = 1; i <= n; ++i) {
        dp[i] = f(dq.front().i, i);
        while (!dq.empty() && dq.front().r <= i)
            dq.pop_front();
        dq.front().l = i + 1;
        while (!dq.empty() &&
            f(i, dq.back().l) >= f(dq.back().i, dq.back().l))
            dq.pop_back();
        int p = i + 1;
        if (!dq.empty()) {
            auto [j, l, r] = dq.back();
            for (int s = 1 << 20; s; s >= 1)
                if (l + s <= n && f(i, l + s) < f(j, l + s))
                    l += s;
            dq.back().r = l; p = l + 1;
        }
        if (p <= n) dq.emplace_back(i, p, n);
    }
    return dp;
} // test @ tioj 烏龜疊疊樂

```

**8.7 ConvexHull Optimization [b4318e]**

```

struct L {
    mutable lld a, b, p;
    bool operator<(const L &r) const {
        return a < r.a; /* here */
    }
    bool operator<(lld x) const { return p < x; }
};

lld Div(lld a, lld b) {
    return a / b - ((a ^ b) < 0 && a % b);
}
struct DynamicHull : multiset<L, less<>> {
    static const lld kInf = 1e18;
    bool Isect(iterator x, iterator y) {
        if (y == end()) { x->p = kInf; return false; }
        if (x->a == y->a)
            x->p = x->b > y->b ? kInf : -kInf; /* here */
        else x->p = Div(y->b - x->b, x->a - y->a);
        return x->p >= y->p;
    }

    void Insert(lld a, lld b) {
        auto z = insert({a, b, 0}), y = z++, x = y;
        while (Isect(y, z)) z = erase(z);
        if (x != begin() && Isect(--x, y)) Isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)

```



```

    Isect(x, erase(y));
}
lld Query(lld x) { // default chmax
    auto l = *lower_bound(x); // to chmin:
    return l.a * x + l.b; // modify the 2 "<>"
}
};

```

## 8.8 Min Plus Convolution [464dcd]

```

// a is convex a[i+1]-a[i] <= a[i+2]-a[i+1]
vector<int> min_plus_convolution(auto &a, auto &b) {
    const int n = (int)a.size(), m = (int)b.size();
    vector<int> c(n + m - 1, numeric_limits<int>::max());
    auto dc = [&](auto Y, int l, int r, int jl, int jr) {
        if (l > r) return;
        int mid = (l + r) / 2, from = -1, &best = c[mid];
        for (int j = jl; j <= jr; j++)
            if (int i = mid - j; i >= 0 && i < n)
                if (best > a[i] + b[j]) best = a[i] + b[j], from = j;
        Y(Y, l, mid - 1, jl, from); Y(Y, mid + 1, r, from, jr);
    };
    return dc(dc, 0, n - 1 + m - 1, 0, m - 1), c;
}

```

## 8.9 SMAWK [75314c]

```

// For all 2x2 submatrix:
// If M[i][0] < M[i][1], M[0][0] < M[0][1]
// If M[i][0] == M[i][1], M[0][0] <= M[0][1]
// M[i][ans_i] is the best value in the i-th row
VI smawk(int N, int M, auto &&select) {
    auto dc = [&](auto self, const VI &r, const VI &c) ->
        VI {
        if (r.empty()) return {};
        const int n = (int)r.size(); VI ans(n), nr, nc;
        for (int i : c) {
            while (!nc.empty() && select(r[nc.size() - 1], nc.
                back(), i))
                nc.pop_back();
            if ((int)nc.size() < n) nc.push_back(i);
        }
        for (int i = 1; i < n; i += 2) nr.push_back(r[i]);
        const auto na = self(self, nr, nc);
        for (int i = 1; i < n; i += 2) ans[i] = na[i >> 1];
        for (int i = 0, j = 0; i < n; i += 2) {
            ans[i] = nc[j];
            const int end = i + 1 == n ? nc.back() : ans[i + 1];
            while (nc[j] != end)
                if (select(r[i], ans[i], nc[++j])) ans[i] = nc[j];
        }
        return ans;
    };
    VI R(N), C(M); iota(all(R), 0), iota(all(C), 0);
    return dc(dc, R, C);
}
// if f(r, v) is better than f(r, v), return true
bool min_plus_conv_select(int r, int u, int v) {
    auto f = [](int i, int j) {
        if (0 <= i - j && i - j < n) return b[j] + a[i - j];
        return 2100000000 + (i - j);
    };
    return f(r, u) > f(r, v);
}

```

## 8.10 De-Bruijn [aa7700]

```

vector<int> de_bruijn(int k, int n) {
    // return cyclic string of len k^n s.t. every string
    // of len n using k char appears as a substring.
    vector<int> aux(n + 1), res;
    auto db = [&](auto self, int t, int p) -> void {
        if (t <= n)
            for (int i = aux[t - p]; i < k; ++i, p = t)
                aux[t] = i, self(self, t + 1, p);
        else if (n % p == 0) for (int i = 1; i <= p; ++i)
            res.push_back(aux[i]);
    };
    return db(db, 1, 1), res;
}

```

## 8.11 Josephus Problem [7f9ceb]

```

lld f(lld n, lld m, lld k) { // n people kill m for
    each turn
    lld s = (m - 1) % (n - k); // O(k)
    for (lld i = n - k + 1; i <= n; i++) s = (s + m) % i;
    return s;
}

```

```

lld kth(lld n, lld m, lld k) { // died at kth
    if (m == 1) return k; // O(m log(n))
    for (k = k + m - 1; k >= n; k = k - n + (k - n) / (m - 1));
    return k;
} // k and result are 0-based, test @ CF 101955

```

## 8.12 N Queens Problem [31f83e]

```

void solve(VI &ret, int n) { // no sol when n=2,3
    if (n % 6 == 2) {
        for (int i = 2; i <= n; i += 2) ret.push_back(i);
        ret.push_back(3); ret.push_back(1);
        for (int i = 7; i <= n; i += 2) ret.push_back(i);
        ret.push_back(5);
    } else if (n % 6 == 3) {
        for (int i = 4; i <= n; i += 2) ret.push_back(i);
        ret.push_back(2);
        for (int i = 5; i <= n; i += 2) ret.push_back(i);
        ret.push_back(1); ret.push_back(3);
    } else {
        for (int i = 2; i <= n; i += 2) ret.push_back(i);
        for (int i = 1; i <= n; i += 2) ret.push_back(i);
    }
}

```

## 8.13 Tree Knapsack [f42766]

```

vector<int> G[N]; int dp[N][K]; pair<int,int> obj[N];
void dfs(int u, int mx) {
    for (int s : G[u]) {
        auto [w, v] = obj[s];
        if (mx < w) continue;
        for (int i = 0; i <= mx - w; i++)
            dp[s][i] = dp[u][i];
        dfs(s, mx - w);
        for (int i = w; i <= mx; i++)
            dp[u][i] = max(dp[u][i], dp[s][i - w] + v);
    }
}

```

## 8.14 Manhattan MST [1008bc]

```

vector<array<int, 3>> manhattanMST(vector<P> ps) {
    vector<int> id(ps.size()); iota(all(id), 0);
    vector<array<int, 3>> edges;
    for (int k = 0; k < 4; k++) {
        sort(all(id), [&](int i, int j) {
            return (ps[i] - ps[j]).x < (ps[j] - ps[i]).y; });
        map<int, int> sweep;
        for (int i : id) {
            for (auto it = sweep.lower_bound(-ps[i].y);
                it != sweep.end(); sweep.erase(it++)) {
                if (P d = ps[i] - ps[it->second]; d.y > d.x) break;
                else edges.push_back({d.y + d.x, i, it->second});
            }
            sweep[-ps[i].y] = i;
        }
        for (P &p : ps)
            if (k & 1) p.x = -p.x;
            else swap(p.x, p.y);
    }
    return edges; // [{w, i, j}, ...]
} // test @ yosupo judge

```

## 8.15 Binary Search On Fraction [765c5a]

```

struct Q {
    ll p, q;
    Q go(Q b, ll d) { return {p + b.p*d, q + b.q*d}; }
};
bool pred(Q);
// returns smallest p/q in [lo, hi] such that
// pred(p/q) is true, and 0 <= p, q <= N
Q frac_bs(ll N) {
    Q lo{0, 1}, hi{1, 0};
    if (pred(lo)) return lo;
    assert(pred(hi));
    bool dir = 1, L = 1, H = 1;
    for (; L || H; dir = !dir) {
        ll len = 0, step = 1;
        for (int t = 0; t < 2 && (t ? step/=2 : step*=2);)
            if (Q mid = hi.go(lo, len + step);
                mid.p > N || mid.q > N || dir ^ pred(mid))
                t++;
            else len += step;
        swap(lo, hi = hi.go(lo, len));
        (dir ? L : H) = !len;
    }
    return dir ? hi : lo;
}

```

```
}

```

## 8.16 Nim Product [4ac1ce]

```
#define rep(i, r) for (int i = 0; i < r; i++)
struct NimProd {
    llu bit_prod[64][64] {}, prod[8][8][256][256] {};
    NimProd() {
        rep(i, 64) rep(j, 64) if (i & j) {
            int a = lowbit(i & j);
            bit_prod[i][j] = bit_prod[i ^ a][j] ^
                bit_prod[(i ^ a) | (a-1)][(j ^ a) | (i & (a-1))];
        } else bit_prod[i][j] = 1ULL << (i | j);
        rep(e, 8) rep(f, 8) rep(x, 256) rep(y, 256)
            rep(i, 8) if (x >> i & 1) rep(j, 8) if (y >> j & 1)
                prod[e][f][x][y] ^= bit_prod[e * 8 + i][f * 8 + j];
    }
    llu operator()(llu a, llu b) const {
        llu r = 0;
        rep(e, 8) rep(f, 8)
            r ^= prod[e][f][a >> (e*8) & 255][b >> (f*8) & 255];
        return r;
    }
};
```