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1 Basic

1.1 vimrc

```

se is nu ru et tgc sc hls cin cino+=j1 sw=4 sts=4 bs=2
mouse=a encoding=utf-8 ls=2
syn on
colorscheme desert
filetype indent on
inoremap {<CR> {<CR>}<ESC>O
map <F8> <ESC>:w<CR>:!g++ "%" -o "%<" -std=c++17 -
    DKISEKI -Wall -Wextra -Wshadow -Wfatal-errors -
    Wconversion -fsanitize=address,undefined -g && echo
    success<CR>
map <F9> <ESC>:w<CR>:!g++ "%" -o "%<" -O2 -std=c++17 &&
    echo success<CR>
map <F10> <ESC>:!. / "%<" <CR>

```

1.2 Debug Macro

```

#ifdef KISEKI
#define safe cerr<<__PRETTY_FUNCTION__\
    <<" line "<<__LINE__<<" safe\n"
#define debug(a...) qwerty(#a, a)
#define orange(a...) dvorak(#a, a)
using std::cerr;
template <typename ...T>
void qwerty(const char *s, T ...a) {
    cerr << "\e[1;32m(" << s << ") = (" ;
    int cnt = sizeof...(T);
    (... , (cerr << a << (--cnt ? ", " : ") \e[0m\n")));
}
template <typename Iter>
void dvorak(const char *s, Iter L, Iter R) {
    cerr << "\e[1;32m[" << s << "] = [" ;
    for (int f = 0; L != R; ++L)
        cerr << (f++ ? ", " : "") << *L;
    cerr << "] \e[0m\n";
}
#else
#define safe ((void)0)
#define debug(...) ((void)0)
#define orange(...) ((void)0)
#endif

```

1.3 Increase Stack

```

const int size = 256 << 20;
register long rsp asm("rsp");
char *p = (char*)malloc(size)+size, *bak = (char*)rsp;
__asm__("movq %0, %%rsp\n"::"r"(p));
// main
__asm__("movq %0, %%rsp\n"::"r"(bak));

```

1.4 Pragma Optimization

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,tune=native")
```

1.5 IO Optimization

```
static inline int gc() {
    constexpr int B = 1<<20;
    static char buf[B], *p, *q;
    if(p == q &&
        (q=(p=buf)+fread(buf,1,B,stdin)) == buf)
        return EOF;
    return *p++;
}

template < typename T >
static inline bool gn( T &x ) {
    int c = gc(); T sgn = 1; x = 0;
    while(('0'>c|<'9') && c!=EOF && c!='-') c = gc();
    if(c == '-') sgn = -1, c = gc();
    if(c == EOF) return false;
    while('0'<=c&&c<='9') x = x*10 + c - '0', c = gc();
    return x *= sgn, true;
}
```

2 Data Structure

2.1 Dark Magic

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
using namespace __gnu_pbds;
// heap tags: pairing/binary/binomial/rc_binomial/thin
template<typename T>
using pbds_heap=__gnu_pbds::prioity_queue<T,less<T>, \
    pairing_heap_tag>;

// pbds_heap::point_iterator
// x = pq.push(10); pq.modify(x, 87); a.join(b);
// tree tags: rb_tree_tag/ov_tree_tag/splay_tree_tag
template<typename T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;
// find_by_order, order_of_key
// hash tables: cc_hash_table/gp_hash_table
```

2.2 Link-Cut Tree

```
template <typename Val> class LCT {
private:
    struct node {
        int pa, ch[2];
        bool rev;
        Val v, v_prod, v_rprod;
        node() : pa{0}, ch{0, 0}, rev{false}, v{}, v_prod{},
            v_rprod{} {};
    };
    vector<node> nodes;
    set<pair<int, int>> edges;
    bool is_root(int u) const {
        const int p = nodes[u].pa;
        return nodes[p].ch[0] != u and nodes[p].ch[1] != u;
    }
    bool is_rch(int u) const {
        return (not is_root(u)) and nodes[nodes[u].pa].ch[1]
            == u;
    }
    void down(int u) {
        if (auto &cnode = nodes[u]; cnode.rev) {
            if (cnode.ch[0]) set_rev(cnode.ch[0]);
            if (cnode.ch[1]) set_rev(cnode.ch[1]);
            cnode.rev = false;
        }
    }
    void up(int u) {
        auto &cnode = nodes[u];
        cnode.v_prod =
            nodes[cnode.ch[0]].v_prod * cnode.v * nodes[cnode.ch[1]].v_prod;
        cnode.v_rprod =
            nodes[cnode.ch[1]].v_rprod * cnode.v * nodes[cnode.ch[0]].v_rprod;
    }
    void set_rev(int u) {
```

```
        swap(nodes[u].ch[0], nodes[u].ch[1]);
        swap(nodes[u].v_prod, nodes[u].v_rprod);
        nodes[u].rev ^= 1;
    }
    void rotate(int u) {
        int f = nodes[u].pa, g = nodes[f].pa, l = is_rch(u);
        if (nodes[u].ch[l ^ 1])
            nodes[nodes[u].ch[l ^ 1]].pa = f;
        if (not is_root(f))
            nodes[g].ch[is_rch(f)] = u;
        nodes[f].ch[l] = nodes[u].ch[l ^ 1];
        nodes[u].ch[l ^ 1] = f;
        nodes[u].pa = g, nodes[f].pa = u;
        up(f);
    }
    void splay(int u) {
        vector<int> stk = {u};
        while (not is_root(stk.back()))
            stk.push_back(nodes[stk.back()].pa);
        for (; not stk.empty(); stk.pop_back())
            down(stk.back());
        for(int f=nodes[u].pa;!is_root(u);f=nodes[u].pa){
            if(!is_root(f))rotate(is_rch(u)==is_rch(f)?f:u);
            rotate(u);
        }
        up(u);
    }
    void access(int u) {
        int last = 0;
        for (int last = 0; u; last = u, u = nodes[u].pa) {
            splay(u);
            nodes[u].ch[1] = last;
            up(u);
        }
    }
    int find_root(int u) {
        access(u); splay(u);
        int la = 0;
        for (; u; la = u, u = nodes[u].ch[0]) down(u);
        return la;
    }
    void change_root(int u) {
        access(u); splay(u); set_rev(u);
    }
    void link(int x, int y) {
        change_root(y); nodes[y].pa = x;
    }
    void split(int x, int y) {
        change_root(x); access(y); splay(y);
    }
    void cut(int x, int y) {
        split(x, y);
        nodes[y].ch[0] = nodes[x].pa = 0;
        up(y);
    }
public:
    LCT(int n = 0) : nodes(n + 1) {}
    int add(const Val &v = {}) {
        nodes.push_back(v);
        return int(nodes.size()) - 2;
    }
    int add(Val &&v) {
        nodes.emplace_back(move(v));
        return int(nodes.size()) - 2;
    }
    void set_val(int u, const Val &v) {
        splay(++u); nodes[u].v = v; up(u);
    }
    Val query(int x, int y) {
        split(++x, ++y);
        return nodes[y].v_prod;
    }
    bool connected(int u, int v) { return find_root(++u)
        == find_root(++v); }
    void add_edge(int u, int v) {
        if (++u > ++v) swap(u, v);
        edges.emplace(u, v); link(u, v);
    }
    void del_edge(int u, int v) {
        auto k = minmax(++u, ++v);
        if (auto it = edges.find(k); it != edges.end()) {
            edges.erase(it); cut(u, v);
        }
    }
}
```

```

}
}
};

```

2.3 LiChao Segment Tree

```

struct L {
    int m, k, id;
    L() : id(-1) {}
    L(int a, int b, int c) : m(a), k(b), id(c) {}
    int at(int x) { return m * x + k; }
};

class LiChao {
private:
    int n; vector<L> nodes;
    static int lc(int x) { return 2 * x + 1; }
    static int rc(int x) { return 2 * x + 2; }
    void insert(int l, int r, int id, L ln) {
        int m = (l + r) >> 1;
        if (nodes[id].id == -1) {
            nodes[id] = ln;
            return;
        }
        bool atLeft = nodes[id].at(l) < ln.at(l);
        if (nodes[id].at(m) < ln.at(m)) {
            atLeft ^= 1;
            swap(nodes[id], ln);
        }
        if (r - l == 1) return;
        if (atLeft) insert(l, m, lc(id), ln);
        else insert(m, r, rc(id), ln);
    }
    int query(int l, int r, int id, int x) {
        int ret = 0, m = (l + r) >> 1;
        if (nodes[id].id != -1)
            ret = nodes[id].at(x);
        if (r - l == 1) return ret;
        if (x < m) return max(ret, query(l, m, lc(id), x));
        return max(ret, query(m, r, rc(id), x));
    }
public:
    LiChao(int n_) : n(n_), nodes(n * 4) {}
    void insert(L ln) { insert(0, n, 0, ln); }
    int query(int x) { return query(0, n, 0, x); }
};

```

2.4 Treap

```

namespace Treap {
#define sz(x) ((x) ? ((x)->size) : 0)
    struct node {
        int size;
        uint32_t pri;
        node *lc, *rc, *pa;
        node() : size(0), pri(rand()), lc(0), rc(0), pa(0) {}
        void pull() {
            size = 1; pa = nullptr;
            if (lc) { size += lc->size; lc->pa = this; }
            if (rc) { size += rc->size; rc->pa = this; }
        }
    };
    node* merge(node* L, node* R) {
        if (!L || !R) return L ? L : R;
        if (L->pri > R->pri) {
            L->rc = merge(L->rc, R); L->pull();
            return L;
        } else {
            R->lc = merge(L, R->lc); R->pull();
            return R;
        }
    }
    void split_by_size(node* rt, int k, node*&L, node*&R) {
        if (!rt) L = R = nullptr;
        else if (sz(rt->lc) + 1 <= k) {
            L = rt;
            split_by_size(rt->rc, k - sz(rt->lc) - 1, L->rc, R);
            L->pull();
        } else {
            R = rt;
            split_by_size(rt->lc, k, L, R->lc);
            R->pull();
        }
    }
} // sz(L) == k

```

```

int getRank(node *o) { // 1-base
    int r = sz(o->lc) + 1;
    for (; o->pa != nullptr; o = o->pa)
        if (o->pa->rc == o) r += sz(o->pa->lc) + 1;
    return r;
}
#undef sz
}

```

2.5 Linear Basis

```

template <int BITS>
struct LinearBasis {
    array<uint64_t, BITS> basis;
    Basis() { basis.fill(0); }
    void add(uint64_t x) {
        for (int i = 0; i < BITS; ++i) if ((x >> i) & 1) {
            if (basis[i] == 0) {
                basis[i] = x;
                return;
            }
            x ^= basis[i];
        }
    }
    bool ok(uint64_t x) {
        for (int i = 0; i < BITS; ++i)
            if ((x >> i) & 1) x ^= basis[i];
        return x == 0;
    }
};

```

2.6 Binary Search On Segment Tree

```

// find_first = x -> minimal x s.t. check([a, x])
// find_last = x -> maximal x s.t. check([x, b])
template <typename C>
int find_first(int l, const C &check) {
    if (l >= n) return n + 1;
    l += sz;
    for (int i = height; i > 0; i--)
        propagate(l >> i);
    Monoid sum = identity;
    do {
        while ((l & 1) == 0) l >>= 1;
        if (check(f(sum, data[l]))) {
            while (l < sz) {
                propagate(l);
                l <= 1;
                auto nxt = f(sum, data[l]);
                if (!check(nxt)) {
                    sum = nxt;
                    l++;
                }
            }
            return l + 1 - sz;
        }
        sum = f(sum, data[l++]);
    } while ((l & -l) != 1);
    return n + 1;
}

template <typename C>
int find_last(int r, const C &check) {
    if (r <= 0) return -1;
    r += sz;
    for (int i = height; i > 0; i--)
        propagate((r - 1) >> i);
    Monoid sum = identity;
    do {
        r--;
        while (r > 1 and (r & 1)) r >>= 1;
        if (check(f(data[r], sum))) {
            while (r < sz) {
                propagate(r);
                r = (r < 1) + 1;
                auto nxt = f(data[r], sum);
                if (!check(nxt)) {
                    sum = nxt;
                    r--;
                }
            }
            return r - sz;
        }
        sum = f(data[r], sum);
    } while ((r & -r) != r);
}

```

```
return -1;
}
```

3 Graph

3.1 2-SAT (SCC)

```
class TwoSat{
private:
    int n;
    vector<vector<int>> rG,G,sccs;
    vector<int> ord,idx;
    vector<bool> vis,result;
    void dfs(int u){
        vis[u]=true;
        for(int v:G[u])
            if(!vis[v]) dfs(v);
        ord.push_back(u);
    }
    void rdfs(int u){
        vis[u]=false;idx[u]=sccs.size()-1;
        sccs.back().push_back(u);
        for(int v:rG[u])
            if(vis[v])rdfs(v);
    }
public:
    void init(int n_){
        G.clear();G.resize(n=n_);
        rG.clear();rG.resize(n);
        sccs.clear();ord.clear();
        idx.resize(n);result.resize(n);
    }
    void add_edge(int u,int v){
        G[u].push_back(v);rG[v].push_back(u);
    }
    void orr(int x,int y){
        if ((x^y)==1)return;
        add_edge(x^1,y); add_edge(y^1,x);
    }
    bool solve(){
        vis.clear();vis.resize(n);
        for(int i=0;i<n;++i)
            if(not vis[i])dfs(i);
        reverse(ord.begin(),ord.end());
        for (int u:ord){
            if(!vis[u])continue;
            sccs.push_back(vector<int>());
            rdfs(u);
        }
        for(int i=0;i<n;i+=2)
            if(idx[i]==idx[i+1])
                return false;
        vector<bool> c(sccs.size());
        for(size_t i=0;i<sccs.size();++i){
            for(auto sij : sccs[i]){
                result[sij]=c[i];
                c[idx[sij^1]]=!c[i];
            }
        }
        return true;
    }
    bool get(int x){return result[x];}
    int get_id(int x){return idx[x];}
    int count(){return sccs.size();}
} sat2;
```

3.2 BCC Edge

```
class BCC_Bridge {
private:
    int n, ecnt;
    vector<vector<pair<int,int>>> G;
    vector<int> dfn, low;
    vector<bool> bridge;
    void dfs(int u, int f) {
        dfn[u] = low[u] = dfn[f] + 1;
        for (auto [v, t]: G[u]) {
            if (v == f) continue;
            if (dfn[v]) {
                low[u] = min(low[u], dfn[v]);
                continue;
            }
            dfs(v, u);
        }
    }
}
```

```
        low[u] = min(low[u], low[v]);
        if (low[v] > dfn[u]) bridge[t] = true;
    }
}
public:
    void init(int n_) {
        G.clear(); G.resize(n = n_);
        low.assign(n, ecnt = 0);
        dfn.assign(n, 0);
    }
    void add_edge(int u, int v) {
        G[u].emplace_back(v, ecnt);
        G[v].emplace_back(u, ecnt++);
    }
    void solve() {
        bridge.assign(ecnt, false);
        for (int i = 0; i < n; ++i)
            if (not dfn[i]) dfs(i, i);
    }
    bool is_bridge(int x) { return bridge[x]; }
} bcc_bridge;
```

3.3 BCC Vertex

```
class BCC_AP {
private:
    int n, ecnt;
    vector<vector<pair<int,int>>> G;
    vector<int> bcc, dfn, low, st;
    vector<bool> ap, ins;
    void dfs(int u, int f) {
        dfn[u] = low[u] = dfn[f] + 1;
        int ch = 0;
        for (auto [v, t]: G[u]) if (v != f) {
            if (not ins[t]) {
                st.push_back(t);
                ins[t] = true;
            }
            if (dfn[v]) {
                low[u] = min(low[u], dfn[v]);
                continue;
            }
            ++ch; dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (low[v] >= dfn[u]) {
                ap[u] = true;
                while (true) {
                    int eid = st.back(); st.pop_back();
                    bcc[eid] = ecnt;
                    if (eid == t) break;
                }
                ecnt++;
            }
        }
        if (ch == 1 and u == f) ap[u] = false;
    }
public:
    void init(int n_) {
        G.clear(); G.resize(n = n_);
        ecnt = 0; ap.assign(n, false);
        low.assign(n, 0); dfn.assign(n, 0);
    }
    void add_edge(int u, int v) {
        G[u].emplace_back(v, ecnt);
        G[v].emplace_back(u, ecnt++);
    }
    void solve() {
        ins.assign(ecnt, false);
        bcc.resize(ecnt); ecnt = 0;
        for (int i = 0; i < n; ++i)
            if (not dfn[i]) dfs(i, i);
    }
    int get_id(int x) { return bcc[x]; }
    int count() { return ecnt; }
    bool is_ap(int x) { return ap[x]; }
} bcc_ap;
```

3.4 Centroid Decomposition

```
struct Centroid {
    vector<vector<int64_t>> Dist;
    vector<int> Parent, Depth;
    vector<int64_t> Sub, Sub2;
    vector<int> Sz, Sz2;
    Centroid(vector<vector<pair<int, int>>> g) {
```

```

int N = g.size();
vector<bool> Vis(N);
vector<int> sz(N), mx(N);
vector<int> Path;
Dist.resize(N);
Parent.resize(N);
Depth.resize(N);
auto DfsSz = [&](auto dfs, int x) -> void {
    Vis[x] = true; sz[x] = 1; mx[x] = 0;
    for (auto [u, w] : g[x]) {
        if (Vis[u]) continue;
        dfs(dfs, u);
        sz[x] += sz[u];
        mx[x] = max(mx[x], sz[u]);
    }
    Path.push_back(x);
};
auto DfsDist = [&](auto dfs, int x, int64_t D = 0)
-> void {
    Dist[x].push_back(D); Vis[x] = true;
    for (auto [u, w] : g[x]) {
        if (Vis[u]) continue;
        dfs(dfs, u, D + w);
    }
};
auto Dfs = [&]
(auto dfs, int x, int D = 0, int p = -1) -> void {
    Path.clear(); DfsSz(DfsSz, x);
    int M = Path.size();
    int C = -1;
    for (int u : Path) {
        if (max(M - sz[u], mx[u]) * 2 <= M) C = u;
        Vis[u] = false;
    }
    DfsDist(DfsDist, C);
    for (int u : Path) Vis[u] = false;
    Parent[C] = p; Vis[C] = true;
    Depth[C] = D;
    for (auto [u, w] : g[C]) {
        if (Vis[u]) continue;
        dfs(dfs, u, D + 1, C);
    }
};
Dfs(Dfs, 0); Sub.resize(N); Sub2.resize(N);
Sz.resize(N); Sz2.resize(N);
}
void Mark(int v) {
    int x = v, z = -1;
    for (int i = Depth[v]; i >= 0; --i) {
        Sub[x] += Dist[v][i]; Sz[x]++;
        if (z != -1) {
            Sub2[z] += Dist[v][i];
            Sz2[z]++;
        }
        z = x; x = Parent[x];
    }
}
int64_t Query(int v) {
    int64_t res = 0;
    int x = v, z = -1;
    for (int i = Depth[v]; i >= 0; --i) {
        res += Sub[x] + 1LL * Sz[x] * Dist[v][i];
        if (z != -1) res -= Sub2[z] + 1LL * Sz2[z] * Dist[v][i];
        z = x; x = Parent[x];
    }
    return res;
}
};

```

3.5 Directed Minimum Spanning Tree

```

struct DirectedMST { // find maximum
    struct Edge {
        int u, v;
        int w;
        Edge(int u, int v, int w) : u(u), v(v), w(w) {}
    };
    vector<Edge> Edges;
    void clear() { Edges.clear(); }
    void addEdge(int a, int b, int w) { Edges.emplace_back(a, b, w); }
    int solve(int root, int n) {
        vector<Edge> E = Edges;

```

```

        int ans = 0;
        while (true) {
            // find best in edge
            vector<int> in(n, -inf), prv(n, -1);
            for (auto e : E)
                if (e.u != e.v && e.w > in[e.v]) {
                    in[e.v] = e.w;
                    prv[e.v] = e.u;
                }
            in[root] = 0;
            prv[root] = -1;
            for (int i = 0; i < n; i++)
                if (in[i] == -inf)
                    return -inf;
            // find cycle
            int tot = 0;
            vector<int> id(n, -1), vis(n, -1);
            for (int i = 0; i < n; i++) {
                ans += in[i];
                for (int x = i; x != -1 && id[x] == -1; x = prv[x]) {
                    if (vis[x] == i) {
                        for (int y = prv[x]; y != x; y = prv[y])
                            id[y] = tot;
                        id[x] = tot++;
                        break;
                    }
                    vis[x] = i;
                }
            }
            if (!tot)
                return ans;
            for (int i = 0; i < n; i++)
                if (id[i] == -1)
                    id[i] = tot++;
            // shrink
            for (auto &e : E) {
                if (id[e.u] != id[e.v])
                    e.w -= in[e.v];
                e.u = id[e.u], e.v = id[e.v];
            }
            n = tot;
            root = id[root];
        }
        assert(false);
    }
} DMST;

```

3.6 Dominator Tree

```

namespace dominator {
    vector<int> g[maxn], r[maxn], rdom[maxn];
    int dfn[maxn], rev[maxn], fa[maxn], sdom[maxn];
    int dom[maxn], val[maxn], rp[maxn], tk;
    void init(int n) {
        // vertices are numbered from 0 to n - 1
        fill(dfn, dfn + n, -1); fill(rev, rev + n, -1);
        fill(fa, fa + n, -1); fill(val, val + n, -1);
        fill(sdom, sdom + n, -1); fill(rp, rp + n, -1);
        fill(dom, dom + n, -1); tk = 0;
        for (int i = 0; i < n; ++i) {
            g[i].clear(); r[i].clear(); rdom[i].clear();
        }
    }
    void add_edge(int x, int y) { g[x].push_back(y); }
    void dfs(int x) {
        rev[dfn[x] = tk] = x;
        fa[tk] = sdom[tk] = val[tk] = tk; tk++;
        for (int u : g[x]) {
            if (dfn[u] == -1) dfs(u), rp[dfn[u]] = dfn[x];
            r[dfn[u]].push_back(dfn[x]);
        }
    }
    void merge(int x, int y) { fa[x] = y; }
    int find(int x, int c = 0) {
        if (fa[x] == x) return c ? -1 : x;
        int p = find(fa[x], 1);
        if (p == -1) return c ? fa[x] : val[x];
        if (sdom[val[x]] > sdom[val[fa[x]]]) val[x] = val[fa[x]];
        fa[x] = p;
        return c ? p : val[x];
    }
    vector<int> build(int s, int n) {

```



```
// return the father of each node in the dominator tree
// p[i] = -2 if i is unreachable from s
dfs(s);
for (int i = tk - 1; i >= 0; --i) {
    for (int u : r[i]) sdom[i] = min(sdom[i], sdom[find(u)]);
    if (i) rdom[sdom[i]].push_back(i);
    for (int &u : rdom[i]) {
        int p = find(u);
        if (sdom[p] == i) dom[u] = i;
        else dom[u] = p;
    }
    if (i) merge(i, rp[i]);
}
vector<int> p(n, -2); p[s] = -1;
for (int i = 1; i < tk; ++i)
    if (sdom[i] != dom[i]) dom[i] = dom[dom[i]];
for (int i = 1; i < tk; ++i) p[rev[i]] = rev[dom[i]];
return p;
}
```

3.7 Edge Coloring

```
// max(d_u) + 1 edge coloring, time: O(NM)
int C[kN][kN], G[kN][kN]; // 1-based, G: ans
void clear(int N) {
    for (int i = 0; i <= N; i++)
        for (int j = 0; j <= N; j++)
            C[i][j] = G[i][j] = 0;
}
void solve(vector<pair<int, int>> &E, int N) {
    int X[kN] = {}, a;
    auto update = [&](int u) {
        for (X[u] = 1; C[u][X[u]]; X[u]++);
    };
    auto color = [&](int u, int v, int c) {
        int p = G[u][v];
        G[u][v] = G[v][u] = c;
        C[u][c] = v, C[v][c] = u;
        C[u][p] = C[v][p] = 0;
        if (p) X[u] = X[v] = p;
        else update(u), update(v);
        return p;
    };
    auto flip = [&](int u, int c1, int c2) {
        int p = C[u][c1];
        swap(C[u][c1], C[u][c2]);
        if (p) G[u][p] = G[p][u] = c2;
        if (!C[u][c1]) X[u] = c1;
        if (!C[u][c2]) X[u] = c2;
        return p;
    };
    for (int i = 1; i <= N; i++) X[i] = 1;
    for (int t = 0; t < E.size(); t++) {
        auto [u, v] = E[t];
        int v0 = v, c = X[u], c0 = c, d;
        vector<pair<int, int>> L; int vst[kN] = {};
        while (!G[u][v0]) {
            L.emplace_back(v, d = X[v]);
            if (!C[v][c]) for(a=L.size()-1; a>=0; a--)
                c = color(u, L[a].first, c);
            else if(!C[u][d]) for(a=L.size()-1; a>=0; a--)
                color(u, L[a].first, L[a].second);
            else if (vst[d]) break;
            else vst[d] = 1, v = C[u][d];
        }
        if (!G[u][v0]) {
            for (; v; v = flip(v, c, d), swap(c, d));
            if (C[u][c0]) { a = int(L.size()) - 1;
                while (--a >= 0 && L[a].second != c);
                for(; a>=0; a--) color(u, L[a].first, L[a].second);
            } else t--;
        }
    }
}
```

3.8 Lowbit Decomposition

```
class LBD {
    int timer, chains;
    vector<vector<int>> G;
    vector<int> tl, tr, chain, head, dep, pa;
    // chains : number of chain
    // tl, tr[u] : subtree interval in the seq. of u
    // head[i] : head of the chain i
}
```

```
// chain[u] : chain id of the chain u is on
void predfs(int u, int f) {
    dep[u] = dep[pa[u] = f] + 1;
    for (int v : G[u]) if (v != f) {
        predfs(v, u);
        if (lowbit(chain[u]) < lowbit(chain[v]))
            chain[u] = chain[v];
    }
    if (chain[u] == 0) chain[u] = ++chains;
}
void dfschain(int u, int f) {
    tl[u] = timer++;
    if (head[chain[u]] == -1)
        head[chain[u]] = u;
    for (int v : G[u])
        if (v != f and chain[v] == chain[u])
            dfschain(v, u);
    for (int v : G[u])
        if (v != f and chain[v] != chain[u])
            dfschain(v, u);
    tr[u] = timer;
}
public:
    LBD(int n) : timer(0), chains(0), G(n), tl(n), tr(n),
        chain(n), head(n, -1), dep(n), pa(n) {}
    void add_edge(int u, int v) {
        G[u].push_back(v); G[v].push_back(u);
    }
    void decompose() { predfs(0, 0); dfschain(0, 0); }
    PII get_subtree(int u) { return {tl[u], tr[u]}; }
    vector<PII> get_path(int u, int v) {
        vector<PII> res;
        while (chain[u] != chain[v]) {
            if (dep[head[chain[u]]] < dep[head[chain[v]]])
                swap(u, v);
            int s = head[chain[u]];
            res.emplace_back(tl[s], tl[u] + 1);
            u = pa[s];
        }
        if (dep[u] < dep[v]) swap(u, v);
        res.emplace_back(tl[v], tl[u] + 1);
        return res;
    }
};
```

3.9 Manhattan Minimum Spanning Tree

```
typedef Point<int> P;
vector<array<int, 3>> manhattanMST(vector<P> ps) {
    vi id(sz(ps));
    iota(all(id), 0);
    vector<array<int, 3>> edges;
    rep(k, 0, 4) {
        sort(all(id), [&](int i, int j) {
            return (ps[i] - ps[j]).x < (ps[j] - ps[i]).y;
        });
        map<int, int> sweep;
        for (int i : id) {
            for (auto it = sweep.lower_bound(-ps[i].y);
                it != sweep.end(); sweep.erase(it++)) {
                int j = it->second;
                P d = ps[i] - ps[j];
                if (d.y > d.x) break;
                edges.push_back({d.y + d.x, i, j});
            }
            sweep[-ps[i].y] = i;
        }
        for (P &p : ps)
            if (k & 1) p.x = -p.x;
            else swap(p.x, p.y);
    }
    return edges; // [{w, i, j}, ...]
}
```

3.10 MaxClique

```
// contain a self loop u to u, then u won't in clique
template < size_t MAXN >
class MaxClique {
private:
    using bits = bitset< MAXN >;
    bits popped, G[ MAXN ], ans;
    size_t deg[ MAXN ], deo[ MAXN ], n;
    void sort_by_degree() {
```

```

popped.reset();
for ( size_t i = 0 ; i < n ; ++ i )
    deg[ i ] = G[ i ].count();
for ( size_t i = 0 ; i < n ; ++ i ) {
    size_t mi = MAXN, id = 0;
    for ( size_t j = 0 ; j < n ; ++ j )
        if ( not popped[ j ] and deg[ j ] < mi )
            mi = deg[ id = j ];
    popped[ deo[ i ] = id ] = 1;
    for( size_t u = G[ i ]._Find_first() ;
        u < n ; u = G[ i ]._Find_next( u ) )
        -- deg[ u ];
}
}
void BK( bits R, bits P, bits X ) {
    if (R.count()+P.count() <= ans.count()) return;
    if ( not P.count() and not X.count() ) {
        if ( R.count() > ans.count() ) ans = R;
        return;
    }
    /* greedily choose max degree as pivot
    bits cur = P | X; size_t pivot = 0, sz = 0;
    for ( size_t u = cur._Find_first() ;
        u < n ; u = cur._Find_next( u ) )
        if ( deg[ u ] > sz ) sz = deg[ pivot = u ];
    cur = P & ( ~G[ pivot ] );
    */ // or simply choose first
    bits cur = P & ( ~G[ ( P | X )._Find_first() ] );
    for ( size_t u = cur._Find_first() ;
        u < n ; u = cur._Find_next( u ) ) {
        if ( R[ u ] ) continue;
        R[ u ] = 1;
        BK( R, P & G[ u ], X & G[ u ] );
        R[ u ] = P[ u ] = 0, X[ u ] = 1;
    }
}
public:
void init( size_t n_ ) {
    n = n_;
    for ( size_t i = 0 ; i < n ; ++ i )
        G[ i ].reset();
    ans.reset();
}
void add_edges( int u, bits S ) { G[ u ] = S; }
void add_edge( int u, int v ) {
    G[ u ][ v ] = G[ v ][ u ] = 1;
}
int solve() {
    sort_by_degree(); // or simply iota( deo... )
    for ( size_t i = 0 ; i < n ; ++ i )
        deg[ i ] = G[ i ].count();
    bits pob, nob = 0; pob.set();
    for (size_t i=n; i<MAXN; ++i) pob[i] = 0;
    for ( size_t i = 0 ; i < n ; ++ i ) {
        size_t v = deo[ i ];
        bits tmp; tmp[ v ] = 1;
        BK( tmp, pob & G[ v ], nob & G[ v ] );
        pob[ v ] = 0, nob[ v ] = 1;
    }
    return static_cast< int >( ans.count() );
}
};

```

3.11 MaxCliqueDyn

```

constexpr int kN = 150;
struct MaxClique { // Maximum Clique
    bitset<kN> a[kN], cs[kN];
    int ans, sol[kN], q, cur[kN], d[kN], n;
    void init(int _n) {
        n = _n, ans = q = 0;
        for (int i = 0; i < n; i++) a[i].reset();
    }
    void addEdge(int u, int v) { a[u][v] = a[v][u] = 1; }
    void csort(vector<int> &r, vector<int> &c) {
        int mx = 1, km = max(ans - q + 1, 1), t = 0,
            m = int(r.size());
        cs[1].reset(); cs[2].reset();
        for (int i = 0; i < m; i++) {
            int p = r[i], k = 1;
            while ((cs[k] & a[p]).count()) k++;
            if (k > mx) cs[++mx + 1].reset();
            cs[k][p] = 1;

```

```

            if (k < km) r[t++] = p;
        }
        c.resize(m);
        if (t) c[t - 1] = 0;
        for (int k = km; k <= mx; k++) {
            for (int p = int(cs[k]._Find_first());
                p < kN; p = int(cs[k]._Find_next(p))) {
                r[t] = p; c[t++] = k;
            }
        }
    }
    void dfs(vector<int> &r, vector<int> &c, int l,
        bitset<kN> mask) {
        while (!r.empty()) {
            int p = r.back(); r.pop_back();
            mask[p] = 0;
            if (q + c.back() <= ans) return;
            cur[q++] = p;
            vector<int> nr, nc;
            bitset<kN> nmask = mask & a[p];
            for (int i : r)
                if (a[p][i]) nr.push_back(i);
            if (!nr.empty()) {
                if (l < 4) {
                    for (int i : nr)
                        d[i] = int((a[i] & nmask).count());
                    sort(nr.begin(), nr.end(),
                        [&](int x, int y) {
                            return d[x] > d[y];
                        });
                }
                csort(nr, nc); dfs(nr, nc, l + 1, nmask);
            } else if (q > ans) {
                ans = q; copy(cur, cur + q, sol);
            }
            c.pop_back(); q--;
        }
    }
    int solve(bitset<kN> mask) { // vertex mask
        vector<int> r, c;
        for (int i = 0; i < n; i++)
            if (mask[i]) r.push_back(i);
        for (int i = 0; i < n; i++)
            d[i] = int((a[i] & mask).count());
        sort(r.begin(), r.end(),
            [&](int i, int j) { return d[i] > d[j]; });
        csort(r, c);
        dfs(r, c, 1, mask);
        return ans; // sol[0 ~ ans-1]
    }
} graph;

```

3.12 Minimum Mean Cycle

```

/* minimum mean cycle O(VE) */
struct MMC{
#define FZ(n) memset((n),0,sizeof(n))
#define E 101010
#define V 1021
#define inf 1e9
    struct Edge { int v,u; double c; };
    int n, m, prv[V][V], prve[V][V], vst[V];
    Edge e[E];
    vector<int> edgeID, cycle, rho;
    double d[V][V];
    void init( int _n ) { n = _n; m = 0; }
    // WARNING: TYPE matters
    void add_edge( int vi , int ui , double ci )
    { e[ m ++ ] = { vi , ui , ci }; }
    void bellman_ford() {
        for(int i=0; i<n; i++) d[0][i]=0;
        for(int i=0; i<n; i++) {
            fill(d[i+1], d[i+1]+n, inf);
            for(int j=0; j<m; j++) {
                int v = e[j].v, u = e[j].u;
                if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
                    d[i+1][u] = d[i][v]+e[j].c;
                    prv[i+1][u] = v;
                    prve[i+1][u] = j;
                }
            }
        }
    }
}

```

```
double solve(){
    // returns inf if no cycle, mmc otherwise
    double mmc=inf;
    int st = -1;
    bellman_ford();
    for(int i=0; i<n; i++) {
        double avg=-inf;
        for(int k=0; k<n; k++) {
            if(d[n][i]<inf-eps)
                avg=max(avg, (d[n][i]-d[k][i])/(n-k));
            else avg=max(avg, inf);
        }
        if (avg < mmc) tie(mmc, st) = tie(avg, i);
    }
    FZ(vst);edgeID.clear();cycle.clear();rho.clear();
    for (int i=n; !vst[st]; st=prv[i--][st]) {
        vst[st]++;
        edgeID.PB(prve[i][st]);
        rho.PB(st);
    }
    while (vst[st] != 2) {
        int v = rho.back(); rho.pop_back();
        cycle.PB(v);
        vst[v]++;
    }
    reverse(ALL(edgeID));
    edgeID.resize(SZ(cycle));
    return mmc;
}
} mmc;
```

3.13 Minimum Steiner Tree

```
// Minimum Steiner Tree
// O(V 3^T + V^2 2^T)
struct SteinerTree {
#define V 33
#define T 8
#define INF 1023456789
    int n, dst[V][V], dp[1 << T][V], tdst[V];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < n; j++)
                dst[i][j] = INF * (i != j);
        }
    }
    void add_edge(int ui, int vi, int wi) {
        dst[ui][vi] = min(dst[ui][vi], wi);
        dst[vi][ui] = min(dst[vi][ui], wi);
    }
    void shortest_path() {
        for (int k = 0; k < n; k++)
            for (int i = 0; i < n; i++)
                for (int j = 0; j < n; j++)
                    dst[i][j] = min(dst[i][j], dst[i][k] + dst[k][j]);
    }
    int solve(const vector<int> &ter) {
        int t = (int)ter.size();
        for (int i = 1; i < (1 << t); i++)
            fill_n(dp[i], n, INF);
        fill_n(dp[0], n, 0);
        for (int msk = 1; msk < (1 << t); msk++) {
            if (msk == (msk & (-msk))) {
                int who = __lg(msk);
                for (int i = 0; i < n; i++)
                    dp[msk][i] = dst[ter[who]][i];
                continue;
            }
            for (int i = 0; i < n; i++)
                for (int submsk = (msk - 1) & msk; submsk; submsk = (submsk - 1) & msk)
                    dp[msk][i] = min(dp[msk][i], dp[submsk][i] + dp[msk ^ submsk][i]);
            for (int i = 0; i < n; i++) {
                tdst[i] = INF;
                for (int j = 0; j < n; j++)
                    tdst[i] = min(tdst[i], dp[msk][j] + dst[j][i]);
            }
            copy_n(tdst, n, dp[msk]);
        }
        int ans = INF;
        for (int i = 0; i < n; i++)
```

```
        ans = min(ans, dp[(1 << t) - 1][i]);
        return ans;
    }
} solver;
```

3.14 Mo's Algorithm on Tree

```
int q; vector<int> G[N];
struct Que{
    int u, v, id;
} que[ N ];
int dfn[N], dfn_, block_id[N], block_, stk[N], stk_;
void dfs( int u, int f ) {
    dfn[ u ] = dfn_++; int saved_rbp = stk_;
    for ( int v : G[ u ] ) {
        if ( v == f ) continue;
        dfs( v, u );
        if ( stk_ - saved_rbp < SQRT_N ) continue;
        for ( ++ block_ ; stk_ != saved_rbp ; )
            block_id[ stk[ -- stk_ ] ] = block_;
    }
    stk[ stk_ ++ ] = u;
}
bool inPath[ N ];
void Diff( int u ) {
    if ( inPath[ u ] ^= 1 ) { /*remove this edge*/ }
    else { /*add this edge*/ }
}
void traverse( int& origin_u, int u ) {
    for ( int g = lca( origin_u, u ) ;
        origin_u != g ; origin_u = parent_of[ origin_u ] )
        Diff( origin_u );
    for ( int v = u; v != origin_u; v = parent_of[v] )
        Diff( v );
    origin_u = u;
}
void solve() {
    dfs( 1, 1 );
    while ( stk_ ) block_id[ stk[ -- stk_ ] ] = block_;
    sort( que, que + q, [](const Que& x, const Que& y) {
        return tie( block_id[ x.u ], dfn[ x.v ] )
            < tie( block_id[ y.u ], dfn[ y.v ] );
    } );
    int U = 1, V = 1;
    for ( int i = 0 ; i < q ; ++ i ) {
        pass( U, que[ i ].u );
        pass( V, que[ i ].v );
        // we could get our answer of que[ i ].id
    }
}
/*
Method 2:
dfs u:
    push u
    iterate subtree
    push u
Let P = LCA(u, v), and St(u)<=St(v)
if (P == u) query[St(u), St(v)]
else query[Ed(u), St(v)], query[St(P), St(P)]
*/
```

3.15 Tree Hashing

```
uint64_t hsah(int u, int f) {
    uint64_t r = 127;
    for (int v : G[ u ]) if (v != f) {
        uint64_t hh = hsah(v, u);
        r=(r+(hh*hh)%1010101333)%1011820613;
    }
    return r;
}
```

3.16 Virtual Tree

```
vector<pair<int, int>> build(vector<int> vs) { // tree
    0-base
    vector<pair<int, int>> res;
    sort(vs.begin(), vs.end(), [](int i, int j) { return
        dfn[i] < dfn[j]; });
    vector<int> s = {0};
    for (int v : vs) if (v != 0) {
        int o = lca(v, s.back());
        if (o != s.back()) {
            while (s.size() >= 2 and dfn[s[s.size() - 2]] >= dfn
                [o]) {
```



```

    res.emplace_back(s[s.size() - 2], s.back());
    s.pop_back();
}
if (s.back() != o) {
    res.emplace_back(s.back(), o);
    s.back() = o;
}
}
s.push_back(v);
}
for (size_t i = 1; i < s.size(); ++i)
    res.emplace_back(s[i - 1], s[i]);
return res;
}

```

4 Matching & Flow

4.1 Bipartite Matching

```

struct BipartiteMatching {
    vector<int> X[N];
    int fX[N], fY[N], n;
    bitset<N> vis;
    bool dfs(int x) {
        for (auto i: X[x]) {
            if (vis[i]) continue;
            vis[i] = true;
            if (fY[i] == -1 || dfs(fY[i])) {
                fY[fX[x] = i] = x;
                return true;
            }
        }
        return false;
    }
    void init(int n_, int m) {
        vis.reset();
        fill(X, X + (n = n_), vector<int>());
        memset(fX, -1, sizeof(int) * n);
        memset(fY, -1, sizeof(int) * m);
    }
    void add_edge(int x, int y) {
        X[x].push_back(y);
    }
    int solve() { // return how many pair matched
        int cnt = 0;
        for (int i = 0; i < n; ++i) {
            vis.reset();
            cnt += dfs(i);
        }
        return cnt;
    }
};

```

4.2 Dijkstra Cost Flow

```

// kN = #(vertices)
// MCMF.{Init, AddEdge, MincostMaxflow}
// MincostMaxflow(source, sink, flow_limit, &cost)
// => flow
using Pii = pair<int, int>;
constexpr int kInf = 0x3f3f3f3f, kN = 500;
struct Edge {
    int to, rev, cost, flow;
};
struct MCMF { // 0-based
    int n{}, m{}, s{}, t{};
    vector<Edge> graph[kN];
    // Larger range for relabeling
    int64_t dis[kN] = {}, h[kN] = {};
    int p[kN] = {};
    void Init(int nn) {
        n = nn;
        for (int i = 0; i < n; ++i) graph[i].clear();
    }
    void AddEdge(int u, int v, int f, int c) {
        graph[u].push_back({v,
            static_cast<int>(graph[v].size()), c, f});
        graph[v].push_back(
            {u, static_cast<int>(graph[u].size()) - 1,
            -c, 0});
    }
    bool Dijkstra(int &max_flow, int64_t &cost) {
        priority_queue<Pii, vector<Pii>, greater<>> pq;
        fill_n(dis, n, kInf);

```

```

        dis[s] = 0;
        pq.emplace(0, s);
        while (!pq.empty()) {
            auto u = pq.top();
            pq.pop();
            int v = u.second;
            if (dis[v] < u.first) continue;
            for (auto &e : graph[v]) {
                auto new_dis =
                    dis[v] + e.cost + h[v] - h[e.to];
                if (e.flow > 0 && dis[e.to] > new_dis) {
                    dis[e.to] = new_dis;
                    p[e.to] = e.rev;
                    pq.emplace(dis[e.to], e.to);
                }
            }
        }
        if (dis[t] == kInf) return false;
        for (int i = 0; i < n; ++i) h[i] += dis[i];
        int d = max_flow;
        for (int u = t; u != s;
            u = graph[u][p[u]].to) {
            auto &e = graph[u][p[u]];
            d = min(d, graph[e.to][e.rev].flow);
        }
        max_flow -= d;
        cost += int64_t(d) * h[t];
        for (int u = t; u != s;
            u = graph[u][p[u]].to) {
            auto &e = graph[u][p[u]];
            e.flow += d;
            graph[e.to][e.rev].flow -= d;
        }
        return true;
    }
    int MincostMaxflow(
        int ss, int tt, int max_flow, int64_t &cost) {
        this->s = ss, this->t = tt;
        cost = 0;
        fill_n(h, n, 0);
        auto orig_max_flow = max_flow;
        while (Dijkstra(max_flow, cost) && max_flow) {}
        return orig_max_flow - max_flow;
    }
};

```

4.3 Dinic

```

template <typename Cap = int64_t>
class Dinic {
private:
    struct E {
        int to, rev;
        Cap cap;
    };
    int n, st, ed;
    vector<vector<E>> G;
    vector<int> lv, idx;
    bool BFS() {
        lv.assign(n, -1);
        queue<int> bfs;
        bfs.push(st); lv[st] = 0;
        while (not bfs.empty()) {
            int u = bfs.front(); bfs.pop();
            for (auto e: G[u]) {
                if (e.cap <= 0 or lv[e.to] != -1) continue;
                bfs.push(e.to); lv[e.to] = lv[u] + 1;
            }
        }
        return lv[ed] != -1;
    }
    Cap DFS(int u, Cap f) {
        if (u == ed) return f;
        Cap ret = 0;
        for (int &i = idx[u]; i < int(G[u].size()); ++i) {
            auto &e = G[u][i];
            if (e.cap <= 0 or lv[e.to] != lv[u] + 1) continue;
            Cap nf = DFS(e.to, min(f, e.cap));
            ret += nf; e.cap -= nf; f -= nf;
            G[e.to][e.rev].cap += nf;
            if (f == 0) return ret;
        }
        if (ret == 0) lv[u] = -1;
    }
};

```

```

    return ret;
}
public:
void init(int n_) { G.assign(n = n_, vector<E>()); }
void add_edge(int u, int v, Cap c) {
    G[u].push_back({v, int(G[v].size()), c});
    G[v].push_back({u, int(G[u].size())-1, 0});
}
Cap max_flow(int st_, int ed_) {
    st = st_, ed = ed_; Cap ret = 0;
    while (BFS()) {
        idx.assign(n, 0);
        Cap f = DFS(st, numeric_limits<Cap>::max());
        ret += f;
        if (f == 0) break;
    }
    return ret;
}
};

```

4.4 Flow Models

- Maximum/Minimum flow with lower bound / Circulation problem
 - Construct super source S and sink T .
 - For each edge (x, y, l, u) , connect $x \rightarrow y$ with capacity $u - l$.
 - For each vertex v , denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
 - If $in(v) > 0$, connect $S \rightarrow v$ with capacity $in(v)$, otherwise, connect $v \rightarrow T$ with capacity $-in(v)$.
 - To maximize, connect $t \rightarrow s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T . If $f \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
 - To minimize, let f be the maximum flow from S to T . Connect $t \rightarrow s$ with capacity ∞ and let the flow from S to T be f' . If $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, f' is the answer.
 - The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
 - Redirect every edge: $y \rightarrow x$ if $(x, y) \in M$, $x \rightarrow y$ otherwise.
 - DFS from unmatched vertices in X .
 - $x \in X$ is chosen iff x is unvisited.
 - $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
 - Construct super source S and sink T
 - For each edge (x, y, c) , connect $x \rightarrow y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \rightarrow x$ with $(cost, cap) = (-c, 1)$
 - For each edge with $c < 0$, sum these cost as K , then increase $d(y)$ by 1, decrease $d(x)$ by 1
 - For each vertex v with $d(v) > 0$, connect $S \rightarrow v$ with $(cost, cap) = (0, d(v))$
 - For each vertex v with $d(v) < 0$, connect $v \rightarrow T$ with $(cost, cap) = (0, -d(v))$
 - Flow from S to T , the answer is the cost of the flow $C + K$
- Maximum density induced subgraph
 - Binary search on answer, suppose we're checking answer T
 - Construct a max flow model, let K be the sum of all weights
 - Connect source $s \rightarrow v$, $v \in G$ with capacity K
 - For each edge (u, v, w) in G , connect $u \rightarrow v$ and $v \rightarrow u$ with capacity w
 - For $v \in G$, connect it with sink $v \rightarrow t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
 - T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
 - For each $v \in V$ create a copy v' , and connect $u' \rightarrow v'$ with weight $w(u, v)$.
 - Connect $v \rightarrow v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v .
 - Find the minimum weight perfect matching on G' .
- Project selection problem
 - If $p_v > 0$, create edge (s, v) with capacity p_v ; otherwise, create edge (v, t) with capacity $-p_v$.
 - Create edge (u, v) with capacity w with w being the cost of choosing u without choosing v .
 - The mincut is equivalent to the maximum profit of a subset of projects.
- 0/1 quadratic programming

$$\sum_x c_x x + \sum_y c_y \bar{y} + \sum_{xy} c_{xy} x \bar{y} + \sum_{xyx'y'} c_{xyx'y'} (x \bar{y} + x' \bar{y}')$$

can be minimized by the mincut of the following graph:

- Create edge (x, t) with capacity c_x and create edge (s, y) with capacity c_y .
- Create edge (x, y) with capacity c_{xy} .
- Create edge (x, y) and edge (x', y') with capacity $c_{xyx'y'}$.

4.5 General Graph Matching

```

namespace matching {
int fa[kN], pre[kN], match[kN], s[kN], v[kN];
vector<int> g[kN];
queue<int> q;
void Init(int n) {
    for (int i = 0; i <= n; ++i) match[i] = pre[i] = n;
    for (int i = 0; i < n; ++i) g[i].clear();
}
void AddEdge(int u, int v) {
    g[u].push_back(v);
    g[v].push_back(u);
}
int Find(int u) {
    return u == fa[u] ? u : fa[u] = Find(fa[u]);
}
int LCA(int x, int y, int n) {
    static int tk = 0; tk++;
    x = Find(x), y = Find(y);
    for (; ; swap(x, y)) {
        if (x != n) {
            if (v[x] == tk) return x;
            v[x] = tk;
            x = Find(pre[match[x]]);
        }
    }
}
void Blossom(int x, int y, int l) {
    while (Find(x) != l) {
        pre[x] = y, y = match[x];
        if (s[y] == 1) q.push(y), s[y] = 0;
        if (fa[x] == x) fa[x] = l;
        if (fa[y] == y) fa[y] = l;
        x = pre[y];
    }
}
bool Bfs(int r, int n) {
    for (int i = 0; i <= n; ++i) fa[i] = i, s[i] = -1;
    while (!q.empty()) q.pop();
    q.push(r);
    s[r] = 0;
    while (!q.empty()) {
        int x = q.front(); q.pop();
        for (int u : g[x]) {
            if (s[u] == -1) {
                pre[u] = x, s[u] = 1;
                if (match[u] == n) {
                    for (int a = u, b = x, last; b != n; a = last, b = pre[a])
                        last = match[b], match[b] = a, match[a] = b;
                    return true;
                }
                q.push(match[u]);
                s[match[u]] = 0;
            } else if (!s[u] && Find(u) != Find(x)) {
                int l = LCA(u, x, n);
                Blossom(x, u, l);
                Blossom(u, x, l);
            }
        }
    }
    return false;
}
int Solve(int n) {
    int res = 0;
    for (int x = 0; x < n; ++x) {
        if (match[x] == n) res += Bfs(x, n);
    }
    return res;
}
}

```

4.6 Global Min-Cut

```

const int maxn = 500 + 5;
int w[maxn][maxn], g[maxn];
bool v[maxn], del[maxn];
void add_edge(int x, int y, int c) {
    w[x][y] += c; w[y][x] += c;
}
pair<int, int> phase(int n) {
    memset(v, false, sizeof(v));
    memset(g, 0, sizeof(g));
}

```

```

int s = -1, t = -1;
while (true) {
    int c = -1;
    for (int i = 0; i < n; ++i) {
        if (del[i] || v[i]) continue;
        if (c == -1 || g[i] > g[c]) c = i;
    }
    if (c == -1) break;
    v[s = t, t = c] = true;
    for (int i = 0; i < n; ++i) {
        if (del[i] || v[i]) continue;
        g[i] += w[c][i];
    }
}
return make_pair(s, t);
}

int mincut(int n) {
    int cut = 1e9;
    memset(del, false, sizeof(del));
    for (int i = 0; i < n - 1; ++i) {
        int s, t; tie(s, t) = phase(n);
        del[t] = true; cut = min(cut, g[t]);
        for (int j = 0; j < n; ++j) {
            w[s][j] += w[t][j]; w[j][s] += w[j][t];
        }
    }
    return cut;
}

```

4.7 GomoryHu Tree

```

int g[maxn];
vector<edge> GomoryHu(int n){
    vector<edge> rt;
    for(int i=1;i<=n;++i)g[i]=1;
    for(int i=2;i<=n;++i){
        int t=g[i];
        flow.reset(); // clear flows on all edge
        rt.push_back({i,t,flow(i,t)});
        flow.walk(i); // bfs points that connected to i (use
            edges not fully flow)
        for(int j=i+1;j<=n;++j){
            if(g[j]==t && flow.connect(j))g[j]=i; // check if i
                can reach j
        }
    }
    return rt;
}

```

4.8 Kuhn Munkres

```

class KM {
private:
    static constexpr lld INF = 1LL << 60;
    vector<lld> h1,hr,slk;
    vector<int> f1,fr,pre,qu;
    vector<vector<lld>> w;
    vector<bool> vl,vr;
    int n, ql, qr;
    bool check(int x) {
        if (vl[x] = true, f1[x] != -1)
            return vr[qu[qr++]] = f1[x] = true;
        while (x != -1) swap(x, fr[f1[x] = pre[x]]);
        return false;
    }
    void bfs(int s) {
        fill(slk.begin(), slk.end(), INF);
        fill(vl.begin(), vl.end(), false);
        fill(vr.begin(), vr.end(), false);
        ql = qr = 0;
        vr[qu[qr++] = s] = true;
        while (true) {
            lld d;
            while (ql < qr) {
                for (int x = 0, y = qu[ql++]; x < n; ++x) {
                    if (!vl[x] && slk[x] >= (d = h1[x] + hr[y] - w[x][y])) {
                        if (pre[x] = y, d) slk[x] = d;
                        else if (!check(x)) return;
                    }
                }
            }
            d = INF;
            for (int x = 0; x < n; ++x)
                if (!vl[x] && d > slk[x]) d = slk[x];

```

```

        for (int x = 0; x < n; ++x) {
            if (vl[x]) h1[x] += d;
            else slk[x] -= d;
            if (vr[x]) hr[x] -= d;
        }
        for (int x = 0; x < n; ++x)
            if (!vl[x] && !slk[x] && !check(x)) return;
    }
}

public:
void init( int n_ ) {
    qu.resize(n = n_);
    f1.assign(n, -1); fr.assign(n, -1);
    hr.assign(n, 0); h1.resize(n);
    w.assign(n, vector<lld>(n));
    slk.resize(n); pre.resize(n);
    vl.resize(n); vr.resize(n);
}

void set_edge( int u, int v, lld x ) {w[u][v] = x;}

lld solve() {
    for (int i = 0; i < n; ++i)
        h1[i] = *max_element(w[i].begin(), w[i].end());
    for (int i = 0; i < n; ++i) bfs(i);
    lld res = 0;
    for (int i = 0; i < n; ++i) res += w[i][f1[i]];
    return res;
}

} km;

```

4.9 Minimum Cost Circulation

```

struct Edge { int to, cap, rev, cost; };
vector<Edge> g[kN];
int dist[kN], pv[kN], ed[kN];
bool mark[kN];
int NegativeCycle(int n) {
    memset(mark, false, sizeof(mark));
    memset(dist, 0, sizeof(dist));
    int upd = -1;
    for (int i = 0; i <= n; ++i) {
        for (int j = 0; j < n; ++j) {
            int idx = 0;
            for (auto &e : g[j]) {
                if (e.cap > 0 && dist[e.to] > dist[j] + e.cost) {
                    dist[e.to] = dist[j] + e.cost;
                    pv[e.to] = j, ed[e.to] = idx;
                    if (i == n) {
                        upd = j;
                        while (!mark[upd]) mark[upd] = 1, upd = pv[upd];
                        return upd;
                    }
                }
            }
            idx++;
        }
    }
    return -1;
}

int Solve(int n) {
    int rt = -1, ans = 0;
    while ((rt = NegativeCycle(n)) >= 0) {
        memset(mark, false, sizeof(mark));
        vector<pair<int, int>> cyc;
        while (!mark[rt]) {
            cyc.emplace_back(pv[rt], ed[rt]);
            mark[rt] = true;
            rt = pv[rt];
        }
        reverse(cyc.begin(), cyc.end());
        int cap = kInf;
        for (auto &i : cyc) {
            auto &e = g[i.first][i.second];
            cap = min(cap, e.cap);
        }
        for (auto &i : cyc) {
            auto &e = g[i.first][i.second];
            e.cap -= cap;
            g[e.to][e.rev].cap += cap;
            ans += e.cost * cap;
        }
    }
    return ans;
}

```

4.10 Minimum Cost Maximum Flow

```
class MiniCostMaxiFlow{
    using Cap = int; using Wei = int64_t;
    using PCW = pair<Cap,Wei>;
    static constexpr Cap INF_CAP = 1 << 30;
    static constexpr Wei INF_WEI = 1LL<<60;
private:
    struct Edge{
        int to, back;
        Cap cap; Wei wei;
        Edge() {}
        Edge(int a,int b, Cap c, Wei d):
            to(a),back(b),cap(c),wei(d) {}
    };
    int ori, edd;
    vector<vector<Edge>> G;
    vector<int> fa, wh;
    vector<bool> inq;
    vector<Wei> dis;
    PCW SPFA(){
        fill(inq.begin(),inq.end(),false);
        fill(dis.begin(),dis.end(),INF_WEI);
        queue<int> qq; qq.push(ori);
        dis[ori] = 0;
        while(not qq.empty()){
            int u=qq.front();qq.pop();
            inq[u] = false;
            for(int i=0;i<SZ(G[u]);++i){
                Edge e=G[u][i];
                int v=e.to; Wei d=e.wei;
                if(e.cap<=0||dis[v]<=dis[u]+d)
                    continue;
                dis[v] = dis[u] + d;
                fa[v] = u, wh[v] = i;
                if (inq[v]) continue;
                qq.push(v);
                inq[v] = true;
            }
        }
        if(dis[edd]==INF_WEI) return {-1, -1};
        Cap mw=INF_CAP;
        for(int i=edd;i!=ori;i=fa[i])
            mw=min(mw,G[fa[i]][wh[i]].cap);
        for (int i=edd;i!=ori;i=fa[i]){
            auto &eg=G[fa[i]][wh[i]];
            eg.cap -= mw;
            G[eg.to][eg.back].cap+=mw;
        }
        return {mw, dis[edd]};
    }
public:
    void init(int n){
        G.clear();G.resize(n);
        fa.resize(n);wh.resize(n);
        inq.resize(n); dis.resize(n);
    }
    void add_edge(int st, int ed, Cap c, Wei w){
        G[st].emplace_back(ed,SZ(G[ed]),c,w);
        G[ed].emplace_back(st,SZ(G[st])-1,0,-w);
    }
    PCW solve(int a, int b){
        ori = a, edd = b;
        Cap cc=0; Wei ww=0;
        while(true){
            PCW ret=SPFA();
            if(ret.first==-1) break;
            cc+=ret.first;
            ww+=ret.first * ret.second;
        }
        return {cc,ww};
    }
} mcmf;
```

4.11 Maximum Weight Graph Matching

```
struct WeightGraph {
    static const int inf = INT_MAX;
    static const int maxn = 514;
    struct edge {
        int u, v, w;
        edge(){}
        edge(int u, int v, int w): u(u), v(v), w(w) {}
    };
};
```

```
};
int n, n_x;
edge g[maxn * 2][maxn * 2];
int lab[maxn * 2];
int match[maxn * 2], slack[maxn * 2], st[maxn * 2], pa
    [maxn * 2];
int flo_from[maxn * 2][maxn + 1], S[maxn * 2], vis[
    maxn * 2];
vector<int> flo[maxn * 2];
queue<int> q;
int e_delta(const edge &e) { return lab[e.u] + lab[e.v]
    - g[e.u][e.v].w * 2; }
void update_slack(int u, int x) { if (!slack[x] ||
    e_delta(g[u][x]) < e_delta(g[slack[x]][x])) slack[x]
    = u; }
void set_slack(int x) {
    slack[x] = 0;
    for (int u = 1; u <= n; ++u)
        if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
            update_slack(u, x);
}
void q_push(int x) {
    if (x <= n) q.push(x);
    else for (size_t i = 0; i < flo[x].size(); i++)
        q.push(flo[x][i]);
}
void set_st(int x, int b) {
    st[x] = b;
    if (x > n) for (size_t i = 0; i < flo[x].size(); ++i)
        set_st(flo[x][i], b);
}
int get_pr(int b, int xr) {
    int pr = find(flo[b].begin(), flo[b].end(), xr) - flo
        [b].begin();
    if (pr % 2 == 1) {
        reverse(flo[b].begin() + 1, flo[b].end());
        return (int)flo[b].size() - pr;
    }
    return pr;
}
void set_match(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;
    edge e = g[u][v];
    int xr = flo_from[u][e.u], pr = get_pr(u, xr);
    for (int i = 0; i < pr; ++i) set_match(flo[u][i], flo
        [u][i ^ 1]);
    set_match(xr, v);
    rotate(flo[u].begin(), flo[u].begin() + pr, flo[u].
        end());
}
void augment(int u, int v) {
    for (; ; ) {
        int xnv = st[match[u]];
        set_match(u, v);
        if (!xnv) return;
        set_match(xnv, st[pa[xnv]]);
        u = st[pa[xnv]], v = xnv;
    }
}
int get_lca(int u, int v) {
    static int t = 0;
    for (++t; u || v; swap(u, v)) {
        if (u == 0) continue;
        if (vis[u] == t) return u;
        vis[u] = t;
        u = st[match[u]];
        if (u) u = st[pa[u]];
    }
    return 0;
}
void add_blossom(int u, int lca, int v) {
    int b = n + 1;
    while (b <= n_x && st[b]) ++b;
    if (b > n_x) ++n_x;
    lab[b] = 0, S[b] = 0;
    match[b] = match[lca];
    flo[b].clear();
    flo[b].push_back(lca);
    for (int x = u, y; x != lca; x = st[pa[y]])
        flo[b].push_back(x), flo[b].push_back(y = st[match[x]
            ]), q_push(y);
}
```

```

reverse(flo[b].begin() + 1, flo[b].end());
for (int x = v, y; x != lca; x = st[pa[y]])
    flo[b].push_back(x), flo[b].push_back(y = st[match[x]]), q_push(y);
set_st(b, b);
for (int x = 1; x <= n_x; ++x) g[b][x].w = g[x][b].w = 0;
for (int x = 1; x <= n; ++x) flo_from[b][x] = 0;
for (size_t i = 0; i < flo[b].size(); ++i) {
    int xs = flo[b][i];
    for (int x = 1; x <= n_x; ++x)
        if (g[b][x].w == 0 || e_delta(g[xs][x]) < e_delta(g[b][x]))
            g[b][x] = g[xs][x], g[x][b] = g[x][xs];
    for (int x = 1; x <= n; ++x)
        if (flo_from[xs][x]) flo_from[b][x] = xs;
}
set_slack(b);
}

void expand_blossom(int b) {
    for (size_t i = 0; i < flo[b].size(); ++i)
        set_st(flo[b][i], flo[b][i]);
    int xr = flo_from[b][g[b][pa[b]].u], pr = get_pr(b, xr);
    for (int i = 0; i < pr; i += 2) {
        int xs = flo[b][i], xns = flo[b][i + 1];
        pa[xs] = g[xns][xs].u;
        S[xs] = 1, S[xns] = 0;
        slack[xs] = 0, set_slack(xns);
        q_push(xns);
    }
    S[xr] = 1, pa[xr] = pa[b];
    for (size_t i = pr + 1; i < flo[b].size(); ++i) {
        int xs = flo[b][i];
        S[xs] = -1, set_slack(xs);
    }
    st[b] = 0;
}

bool on_found_edge(const edge &e) {
    int u = st[e.u], v = st[e.v];
    if (S[v] == -1) {
        pa[v] = e.u, S[v] = 1;
        int nu = st[match[v]];
        slack[v] = slack[nu] = 0;
        S[nu] = 0, q_push(nu);
    } else if (S[v] == 0) {
        int lca = get_lca(u, v);
        if (!lca) return augment(u, v), augment(v, u), true;
        else add_blossom(u, lca, v);
    }
    return false;
}

bool matching() {
    memset(S + 1, -1, sizeof(int) * n_x);
    memset(slack + 1, 0, sizeof(int) * n_x);
    q = queue<int>();
    for (int x = 1; x <= n_x; ++x)
        if (st[x] == x && !match[x]) pa[x] = 0, S[x] = 0, q_push(x);
    if (q.empty()) return false;
    for (; ; ) {
        while (q.size()) {
            int u = q.front(); q.pop();
            if (S[st[u]] == 1) continue;
            for (int v = 1; v <= n; ++v)
                if (g[u][v].w > 0 && st[u] != st[v]) {
                    if (e_delta(g[u][v]) == 0) {
                        if (on_found_edge(g[u][v])) return true;
                    } else update_slack(u, st[v]);
                }
        }
        int d = inf;
        for (int b = n + 1; b <= n_x; ++b)
            if (st[b] == b && S[b] == 1) d = min(d, lab[b] / 2);
        for (int x = 1; x <= n_x; ++x)
            if (st[x] == x && slack[x]) {
                if (S[x] == -1) d = min(d, e_delta(g[slack[x]][x]));
                else if (S[x] == 0) d = min(d, e_delta(g[slack[x]][x]) / 2);
            }
    }
}

```

```

for (int u = 1; u <= n; ++u) {
    if (S[st[u]] == 0) {
        if (lab[u] <= d) return 0;
        lab[u] -= d;
    } else if (S[st[u]] == 1) lab[u] += d;
}
for (int b = n + 1; b <= n_x; ++b)
    if (st[b] == b) {
        if (S[st[b]] == 0) lab[b] += d * 2;
        else if (S[st[b]] == 1) lab[b] -= d * 2;
    }
q = queue<int>();
for (int x = 1; x <= n_x; ++x)
    if (st[x] == x && slack[x] && st[slack[x]] != x && e_delta(g[slack[x]][x]) == 0)
        if (on_found_edge(g[slack[x]][x])) return true;
for (int b = n + 1; b <= n_x; ++b)
    if (st[b] == b && S[b] == 1 && lab[b] == 0)
        expand_blossom(b);
}
return false;
}

pair<long long, int> solve() {
    memset(match + 1, 0, sizeof(int) * n);
    n_x = n;
    int n_matches = 0;
    long long tot_weight = 0;
    for (int u = 0; u <= n; ++u) st[u] = u, flo[u].clear();
    int w_max = 0;
    for (int u = 1; u <= n; ++u)
        for (int v = 1; v <= n; ++v) {
            flo_from[u][v] = (u == v ? u : 0);
            w_max = max(w_max, g[u][v].w);
        }
    for (int u = 1; u <= n; ++u) lab[u] = w_max;
    while (matching()) ++n_matches;
    for (int u = 1; u <= n; ++u)
        if (match[u] && match[u] < u)
            tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
}

void add_edge(int ui, int vi, int wi) { g[ui][vi].w = g[vi][ui].w = wi; }
void init(int _n) {
    n = _n;
    for (int u = 1; u <= n; ++u)
        for (int v = 1; v <= n; ++v)
            g[u][v] = edge(u, v, 0);
}
}

```

5 Math

5.1 $\lfloor \frac{n}{i} \rfloor$ Enumeration

$$T_0 = 1, T_{i+1} = \lfloor \frac{n}{T_i + 1} \rfloor$$

5.2 Stirling Number

5.2.1 First Kind

$S_1(n, k)$ counts the number of permutations of n elements with k disjoint cycles.

$$S_1(n, k) = (n-1) \cdot S_1(n-1, k) + S_1(n-1, k-1)$$

$$x(x+1) \dots (x+n-1) = \sum_{k=0}^n S_1(n, k) x^k$$

$$g(x) = x(x+1) \dots (x+n-1) = \sum_{k=0}^n a_k x^k$$

$$\Rightarrow g(x+n) = \sum_{k=0}^n \frac{b_k}{(n-k)!} x^{n-k},$$

$$b_k = \sum_{i=0}^k ((n-i)! a_{n-i}) \cdot \left(\frac{n^{k-i}}{(k-i)!} \right)$$

5.2.2 Second Kind

$S_2(n, k)$ counts the number of ways to partition a set of n elements into k nonempty sets.

$$S_2(n, k) = S_2(n-1, k-1) + k \cdot S_2(n-1, k)$$

$$S_2(n, k) = \sum_{i=0}^k \binom{k}{i} i^n (-1)^{k-i} = \sum_{i=0}^k \frac{(-1)^i}{i!} \cdot \frac{(k-i)^n}{(k-i)!}$$

5.3 ax+by=gcd

```
// ax+ny = 1, ax+ny == ax == 1 (mod n)
void exgcd(lld x, lld y, lld &g, lld &a, lld &b) {
    if (y == 0) g=x, a=1, b=0;
    else exgcd(y, x%y, g, b, a), b-=(x/y)*a;
}
```

5.4 Berlekamp Massey

```
template <typename T>
vector<T> BerlekampMassey(const vector<T> &output) {
    vector<T> d(output.size() + 1), me, he;
    for (size_t f = 0, i = 1; i <= output.size(); ++i) {
        for (size_t j = 0; j < me.size(); ++j)
            d[i] += output[i - j - 2] * me[j];
        if ((d[i] -= output[i - 1]) == 0) continue;
        if (me.empty()) {
            me.resize(f = i);
            continue;
        }
        vector<T> o(i - f - 1);
        T k = -d[i] / d[f]; o.push_back(-k);
        for (T x : he) o.push_back(x * k);
        if (o.size() < me.size()) o.resize(me.size());
        for (size_t j = 0; j < me.size(); ++j) o[j] += me[j];
        if (i-f+he.size() >= me.size()) he = me, f = i;
        me = o;
    }
    return me;
}
```

5.5 Characteristic Polynomial

```
vector<vector<int>> Hessenberg(const vector<vector<int>> &A) {
    int N = A.size();
    vector<vector<int>> H = A;
    for (int i = 0; i < N - 2; ++i) {
        if (!H[i + 1][i]) {
            for (int j = i + 2; j < N; ++j) {
                if (H[j][i]) {
                    for (int k = i; k < N; ++k) swap(H[i + 1][k], H[j][k]);
                    for (int k = 0; k < N; ++k) swap(H[k][i + 1], H[k][j]);
                    break;
                }
            }
        }
        if (!H[i + 1][i]) continue;
        int val = fpow(H[i + 1][i], kP - 2);
        for (int j = i + 2; j < N; ++j) {
            int coef = 1LL * val * H[j][i] % kP;
            for (int k = i; k < N; ++k) H[j][k] = (H[j][k] + 1LL * H[i + 1][k] * (kP - coef)) % kP;
            for (int k = 0; k < N; ++k) H[k][i + 1] = (H[k][i + 1] + 1LL * H[k][j] * coef) % kP;
        }
    }
    return H;
}

vector<int> CharacteristicPoly(const vector<vector<int>> &A) {
    int N = A.size();
    auto H = Hessenberg(A);
    for (int i = 0; i < N; ++i) {
        for (int j = 0; j < N; ++j) H[i][j] = kP - H[i][j];
    }
    vector<vector<int>> P(N + 1, vector<int>(N + 1));
    P[0][0] = 1;
    for (int i = 1; i <= N; ++i) {
        P[i][0] = 0;
        for (int j = 1; j <= i; ++j) P[i][j] = P[i - 1][j - 1];
        int val = 1;
        for (int j = i - 1; j >= 0; --j) {
            int coef = 1LL * val * H[j][i] % kP;
            for (int k = 0; k <= j; ++k) P[i][k] = (P[i][k] + 1LL * P[j][k] * coef) % kP;
        }
    }
}
```

```
if (j) val = 1LL * val * (kP - H[j][j - 1]) % kP;
}
if (N & 1) {
    for (int i = 0; i <= N; ++i) P[N][i] = kP - P[N][i];
}
return P[N];
}
```

5.6 Chinese Remainder

```
lld crt(lld ans[], lld pri[], int n){
    lld M = 1, ret = 0;
    for(int i=0;i<n;i++) M *= pri[i];
    for(int i=0;i<n;i++){
        lld iv = (gcd(M/pri[i], pri[i]).FF+pri[i])%pri[i];
        ret += (ans[i]*(M/pri[i])%M * iv)%M;
        ret %= M;
    }
    return ret;
}
/*
Another:
x = a1 % m1
x = a2 % m2
g = gcd(m1, m2)
assert((a1-a2)%g==0)
[p, q] = exgcd(m2/g, m1/g)
return a2+m2*(p*(a1-a2)/g)
0 <= x < lcm(m1, m2)
*/
```

5.7 De-Bruijn

```
int res[maxn], aux[maxn], sz;
void db(int t, int p, int n, int k) {
    if (t > n) {
        if (n % p == 0)
            for (int i = 1; i <= p; ++i)
                res[sz++] = aux[i];
    } else {
        aux[t] = aux[t - p];
        db(t + 1, p, n, k);
        for (int i = aux[t - p] + 1; i < k; ++i) {
            aux[t] = i;
            db(t + 1, t, n, k);
        }
    }
}

int de_bruijn(int k, int n) {
    // return cyclic string of len k^n s.t. every string
    // of len n using k char appears as a substring.
    if (k == 1) {
        res[0] = 0;
        return 1;
    }
    for (int i = 0; i < k * n; i++) aux[i] = 0;
    sz = 0;
    db(1, 1, n, k);
    return sz;
}
```

5.8 DiscreteLog

```
template<typename Int>
Int BSGS(Int x, Int y, Int M) {
    // x^? \equiv y (mod M)
    Int t = 1, c = 0, g = 1;
    for (Int M_ = M; M_ > 0; M_ >= 1)
        g = g * x % M;
    for (g = gcd(g, M); t % g != 0; ++c) {
        if (t == y) return c;
        t = t * x % M;
    }
    if (y % g != 0) return -1;
    t /= g, y /= g, M /= g;
    Int h = 0, gs = 1;
    for (; h * h < M; ++h) gs = gs * x % M;
    unordered_map<Int, Int> bs;
    for (Int s = 0; s < h; bs[y] = ++s)
        y = y * x % M;
    for (Int s = 0; s < M; s += h) {
        t = t * gs % M;
        if (bs.count(t)) return c + s + h - bs[t];
    }
}
```

```

    }
    return -1;
}

```

5.9 Extended Euler

$$a^b \equiv \begin{cases} a^{b \bmod \varphi(m) + \varphi(m)} & \text{if } (a, m) \neq 1 \wedge b \geq \varphi(m) \\ a^{b \bmod \varphi(m)} & \text{otherwise} \end{cases} \pmod{m}$$

5.10 ExtendedFloorSum

$$g(a, b, c, n) = \sum_{i=0}^n i \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor \cdot \frac{n(n+1)}{2} + g(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1) - h(c, c-b-1, a, m-1)), & \text{otherwise} \end{cases}$$

$$h(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor^2$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor^2 \cdot (n+1) + \lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot n(n+1) + h(a \bmod c, b \bmod c, c, n) + 2\lfloor \frac{a}{c} \rfloor \cdot g(a \bmod c, b \bmod c, c, n) + 2\lfloor \frac{b}{c} \rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) - 2f(c, c-b-1, a, m-1) - f(a, b, c, n), & \text{otherwise} \end{cases}$$

5.11 Fast Fourier Transform

```

const int mod = 1000000007;
const int M1 = 985661441; // G = 3
const int M2 = 998244353;
const int M3 = 1004535809;
int superBigCRT(int64_t A, int64_t B, int64_t C) {
    static_assert(M1 <= M2 && M2 <= M3);
    constexpr int64_t r12 = modpow(M1, M2-2, M2);
    constexpr int64_t r13 = modpow(M1, M3-2, M3);
    constexpr int64_t r23 = modpow(M2, M3-2, M3);
    constexpr int64_t M1M2 = 1LL * M1 * M2 % mod;
    B = (B - A + M2) * r12 % M2;
    C = (C - A + M3) * r13 % M3;
    C = (C - B + M3) * r23 % M3;
    return (A + B * M1 + C * M1M2) % mod;
}

namespace fft {
using VI = vector<int>;
using VL = vector<long long>;
const double pi = acos(-1);
cplx omega[maxn + 1];
void prefft() {
    for (int i = 0; i <= maxn; i++)
        omega[i] = cplx(cos(2 * pi * i / maxn),
            sin(2 * pi * i / maxn));
}

void fft(vector<cplx> &v, int n) {
    int z = __builtin_ctz(n) - 1;
    for (int i = 0; i < n; ++i) {
        int x = 0, j = 0;
        for (; (1 <= j) < n; ++j) x ^= (i >= j & 1) << (z - j);
        if (x > i) swap(v[x], v[i]);
    }
    for (int s = 2; s <= n; s <= 1) {
        int z = s >> 1;
        for (int i = 0; i < n; i += s) {
            for (int k = 0; k < z; ++k) {
                cplx x = v[i + z + k] * omega[maxn / s * k];
                v[i + z + k] = v[i + k] - x;
                v[i + k] = v[i + k] + x;
            }
        }
    }
}

void ifft(vector<cplx> &v, int n) {
    fft(v, n); reverse(v.begin() + 1, v.end());
    for (int i = 0; i < n; ++i) v[i] = v[i] * cplx(1. / n, 0);
}

VL convolution(const VI &a, const VI &b) {
    // Should be able to handle N <= 10^5, C <= 10^4
}

```

```

int sz = 1;
while (sz < a.size() + b.size() - 1) sz <= 1;
vector<cplx> v(sz);
for (int i = 0; i < sz; ++i) {
    double re = i < a.size() ? a[i] : 0;
    double im = i < b.size() ? b[i] : 0;
    v[i] = cplx(re, im);
}
fft(v, sz);
for (int i = 0; i <= sz / 2; ++i) {
    int j = (sz - i) & (sz - 1);
    cplx x = (v[i] + v[j].conj()) * (v[i] - v[j].conj())
        * cplx(0, -0.25);
    if (j != i) v[j] = (v[j] + v[i].conj()) * (v[j] - v[i]
        ).conj() * cplx(0, -0.25);
    v[i] = x;
}
ifft(v, sz);
VL c(sz);
for (int i = 0; i < sz; ++i) c[i] = round(v[i].re);
return c;
}

VI convolution_mod(const VI &a, const VI &b, int p) {
    int sz = 1;
    while (sz + 1 < a.size() + b.size()) sz <= 1;
    vector<cplx> fa(sz), fb(sz);
    for (int i = 0; i < (int)a.size(); ++i)
        fa[i] = cplx(a[i] & ((1 <= 15) - 1), a[i] >> 15);
    for (int i = 0; i < (int)b.size(); ++i)
        fb[i] = cplx(b[i] & ((1 <= 15) - 1), b[i] >> 15);
    fft(fa, sz), fft(fb, sz);
    double r = 0.25 / sz;
    cplx r2(0, -1), r3(r, 0), r4(0, -r), r5(0, 1);
    for (int i = 0; i <= (sz >> 1); ++i) {
        int j = (sz - i) & (sz - 1);
        cplx a1 = (fa[i] + fa[j].conj());
        cplx a2 = (fa[i] - fa[j].conj()) * r2;
        cplx b1 = (fb[i] + fb[j].conj()) * r3;
        cplx b2 = (fb[i] - fb[j].conj()) * r4;
        if (i != j) {
            cplx c1 = (fa[j] + fa[i].conj());
            cplx c2 = (fa[j] - fa[i].conj()) * r2;
            cplx d1 = (fb[j] + fb[i].conj()) * r3;
            cplx d2 = (fb[j] - fb[i].conj()) * r4;
            fa[i] = c1 * d1 + c2 * d2 * r5;
            fb[i] = c1 * d2 + c2 * d1;
        }
        fa[j] = a1 * b1 + a2 * b2 * r5;
        fb[j] = a1 * b2 + a2 * b1;
    }
    fft(fa, sz), fft(fb, sz);
    vector<int> res(sz);
    for (int i = 0; i < sz; ++i) {
        long long a = round(fa[i].re), b = round(fb[i].re),
            c = round(fa[i].im);
        res[i] = (a + ((b % p) << 15) + ((c % p) << 30)) % p;
    }
    return res;
}

```

5.12 FloorSum

```

// @param n `n < 2^32`
// @param m `1 <= m < 2^32`
// @return sum_{i=0}^{n-1} floor((ai + b)/m) mod 2^64
llu floor_sum_unsigned(llu n, llu m, llu a, llu b) {
    llu ans = 0;
    while (true) {
        if (a >= m) {
            ans += n * (n - 1) / 2 * (a / m); a %= m;
        }
        if (b >= m) {
            ans += n * (b / m); b %= m;
        }
        llu y_max = a * n + b;
        if (y_max < m) break;
        // y_max < m * (n + 1)
        // floor(y_max / m) <= n
        n = (llu)(y_max / m), b = (llu)(y_max % m);
        swap(m, a);
    }
    return ans;
}

```

```

lld floor_sum(lld n, lld m, lld a, lld b) {
    llu ans = 0;
    if (a < 0) {
        llu a2 = (a % m + m) % m;
        ans -= 1ULL * n * (n - 1) / 2 * ((a2 - a) / m);
        a = a2;
    }
    if (b < 0) {
        llu b2 = (b % m + m) % m;
        ans -= 1ULL * n * ((b2 - b) / m);
        b = b2;
    }
    return ans + floor_sum_unsigned(n, m, a, b);
}

```

5.13 FWT

```

/* xor convolution:
 * x = (x0,x1) , y = (y0,y1)
 * z = ( x0y0 + x1y1 , x0y1 + x1y0 )
 * ==>
 * x' = ( x0+x1 , x0-x1 ) , y' = ( y0+y1 , y0-y1 )
 * z' = ( ( x0+x1 )( y0+y1 ) , ( x0-x1 )( y0-y1 ) )
 * z = (1/2) * z'
 * or convolution:
 * x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
 * and convolution:
 * x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
const LL MOD = 1e9+7;
inline void fwt( LL x[ MAXN ] , int N , bool inv=0 ) {
    for( int d = 1 ; d < N ; d <= 1 ) {
        int d2 = d<<1;
        for( int s = 0 ; s < N ; s += d2 )
            for( int i = s , j = s+d ; i < s+d ; i++, j++ ){
                LL ta = x[ i ] , tb = x[ j ];
                x[ i ] = ta+tb;
                x[ j ] = ta-tb;
                if( x[ i ] >= MOD ) x[ i ] -= MOD;
                if( x[ j ] < 0 ) x[ j ] += MOD;
            }
    }
    if( inv )
        for( int i = 0 ; i < N ; i++ ) {
            x[ i ] *= inv( N, MOD );
            x[ i ] %= MOD;
        }
}

```

5.14 Gauss Elimination

```

void gauss(vector<vector<double>> &d) {
    int n = d.size(), m = d[0].size();
    for (int i = 0; i < m; ++i) {
        int p = -1;
        for (int j = i; j < n; ++j) {
            if (fabs(d[j][i]) < eps) continue;
            if (p == -1 || fabs(d[j][i]) > fabs(d[p][i])) p = j;
        }
        if (p == -1) continue;
        for (int j = 0; j < m; ++j) swap(d[p][j], d[i][j]);
        for (int j = 0; j < n; ++j) {
            if (i == j) continue;
            double z = d[j][i] / d[i][i];
            for (int k = 0; k < m; ++k) d[j][k] -= z*d[i][k];
        }
    }
}

```

5.15 Miller Rabin

```

bool isprime(llu x){
    static llu magic[]={2,325,9375,28178,\
        450775,9780504,1795265022};
    static auto witn=[](llu a,llu u,llu n,int t)
    ->bool{
        if (!(a = mpow(a%n,u,n)))return 0;
        while(t--){
            llu a2=mul(a,a,n);
            if(a2==1 && a!=1 && a!=n-1)
                return 1;
            a = a2;
        }
        return a!=1;
    };
}

```

```

if(x<2)return 0;
if(!(x&1))return x==2;
llu x1=x-1;int t=0;
while(!(x1&1))x1>>=1,t++;
for(llu m:magic)if(witn(m,x1,x,t))return 0;
return 1;
}

```

5.16 NTT

```

template <int mod, int G, int maxn>
struct NTT {
    static_assert (maxn == (maxn & -maxn));
    int roots[maxn];
    NTT () {
        int r = modpow(G, (mod - 1) / maxn);
        for (int i = maxn >> 1; i; i >>= 1) {
            roots[i] = 1;
            for (int j = 1; j < i; j++)
                roots[i + j] = modmul(roots[i + j - 1], r);
            r = modmul(r, r);
        }
    }
    // n must be 2^k, and 0 <= F[i] < mod
    void operator()(int F[], int n, bool inv = false) {
        for (int i = 0, j = 0; i < n; i++) {
            if (i < j) swap(F[i], F[j]);
            for (int k = n>>1; (j^=k) < k; k>>=1);
        }
        for (int s = 1; s < n; s *= 2) {
            for (int i = 0; i < n; i += s * 2) {
                for (int j = 0; j < s; j++) {
                    int a = F[i+j];
                    int b = modmul(F[i+j+s], roots[s+j]);
                    F[i+j] = modadd(a, b); // a + b
                    F[i+j+s] = modsub(a, b); // a - b
                }
            }
        }
        if (inv) {
            int invn = modinv(n);
            for (int i = 0; i < n; i++)
                F[i] = modmul(F[i], invn);
            reverse(F + 1, F + n);
        }
    }
};
NTT<2013265921, 31, 1048576> ntt;

```

5.17 Range Sieve

```

const int MAX_SQRT_B = 50000;
const int MAX_L = 200000 + 5;
bool is_prime_small[MAX_SQRT_B], is_prime[MAX_L];
void sieve(lld l, lld r){ // [l, r)
    for(lld i=2;i<r;i++) is_prime_small[i] = true;
    for(lld i=l;i<r;i++) is_prime[i-l] = true;
    if(l==1) is_prime[0] = false;
    for(lld i=2;i<r;i++){
        if(!is_prime_small[i]) continue;
        for(lld j=i*i;j<r;j+=i) is_prime_small[j]=false;
        for(lld j=std::max(2LL, (1+i-1)/i)*i;j<r;j+=i)
            is_prime[j-l]=false;
    }
}

```

5.18 Partition Number

```

int b = sqrt(n);
ans[0] = tmp[0] = 1;
for (int i = 1; i <= b; i++) {
    for (int rep = 0; rep < 2; rep++)
        for (int j = i; j <= n - i * i; j++)
            modadd(tmp[j], tmp[j-i]);
    for (int j = i * i; j <= n; j++)
        modadd(ans[j], tmp[j - i * i]);
}

```

5.19 Pi Count (Linear Sieve)

```

static constexpr int N = 1000000 + 5;
lld pi[N];
vector<int> primes;
bool sieved[N];
lld cube_root(lld x){

```

```

lld s=cbrt(x-static_cast<long double>(0.1));
while(s*s*s <= x) ++s;
return s-1;
}

lld square_root(lld x){
    lld s=sqrt(x-static_cast<long double>(0.1));
    while(s*s <= x) ++s;
    return s-1;
}

void init(){
    primes.reserve(N);
    primes.push_back(1);
    for(int i=2;i<N;i++) {
        if(!sieved[i]) primes.push_back(i);
        pi[i] = !sieved[i] + pi[i-1];
        for(int p: primes) if(p > 1) {
            if(p * i >= N) break;
            sieved[p * i] = true;
            if(p % i == 0) break;
        }
    }
}

lld phi(lld m, lld n) {
    static constexpr int MM = 80000, NN = 500;
    static lld val[MM][NN];
    if(m<MM&&n<NN&&val[m][n])return val[m][n]-1;
    if(n == 0) return m;
    if(primes[n] >= m) return 1;
    lld ret = phi(m,n-1)-phi(m/primes[n],n-1);
    if(m<MM&&n<NN) val[m][n] = ret+1;
    return ret;
}

lld pi_count(lld);
lld P2(lld m, lld n) {
    lld sm = square_root(m), ret = 0;
    for(lld i = n+1;primes[i]<=sm;i++)
        ret+=pi_count(m/primes[i])-pi_count(primes[i])+1;
    return ret;
}

lld pi_count(lld m) {
    if(m < N) return pi[m];
    lld n = pi_count(cube_root(m));
    return phi(m, n) + n - 1 - P2(m, n);
}

```

5.20 Pollard Rho

```

// does not work when n is prime
// return any non-trivial factor
llu pollard_rho(llu n){
    static auto f=[](llu x,llu k,llu m){
        return add(k,mul(x,x,m),m);};
    if (!(n&1)) return 2;
    mt19937 rnd(120821011);
    while(true){
        llu y=2,yy=y,x=rnd()%n,t=1;
        for(llu sz=2;t==1;sz<=1,y=yy) {
            for(llu i=0;t==1&&i<sz;++i){
                yy=f(yy,x,n);
                t=gcd(yy>y?yy-y:y-yy,n);
            }
        }
        if(t!=1&&t!=n) return t;
    }
}

```

5.21 Polynomial Operations

```

using V = vector<int>;
#define fi(s, n) for (int i = int(s); i < int(n); ++i)
template <int mod, int G, int maxn> struct Poly : V {
    static uint32_t n2k(uint32_t n) {
        if (n <= 1) return 1;
        return 1u << (32 - __builtin_clz(n - 1));
    }
    static NTT<mod,G,maxn> ntt; // coefficients in [0, P)
    explicit Poly(int n = 1) : V(n) {}
    Poly(const V &v) : V(v) {}
    Poly(const Poly &p, size_t n) : V(n) {
        copy_n(p.data(), min(p.size(), n), data());
    }
    Poly &irev() { return reverse(data(), data() + size())
        , *this; }
    Poly &isz(int _n) { return resize(_n), *this; }

```

```

Poly &iadd(const Poly &rhs) { // n() == rhs.n()
    fi(0, size())(*this)[i] = modadd((*this)[i], rhs[i]);
    return *this;
}

Poly &imul(int k) {
    fi(0, size())(*this)[i] = modmul((*this)[i], k);
    return *this;
}

Poly Mul(const Poly &rhs) const {
    const int _n = n2k(size() + rhs.size() - 1);
    Poly X(*this, _n), Y(rhs, _n);
    ntt(X.data(), _n), ntt(Y.data(), _n);
    fi(0, _n) X[i] = modmul(X[i], Y[i]);
    ntt(X.data(), _n, true);
    return X.isz(size() + rhs.size() - 1);
}

Poly Inv() const { // coef[0] != 0
    if (size() == 1) return V{modinv(*begin())};
    const int _n = n2k(size() * 2);
    Poly Xi = Poly(*this, (size() + 1) / 2).Inv().isz(_n)
        , Y(*this, _n);
    ntt(Xi.data(), _n), ntt(Y.data(), _n);
    fi(0, _n) Xi[i] = modmul(Xi[i], modsub(2, modmul(Xi[i]
        ], Y[i])));
    ntt(Xi.data(), _n, true);
    return Xi.isz(size());
}

Poly Sqrt() const { // coef[0] \in [1, mod)^2
    if (size() == 1) return V{QuadraticResidue(*this)
        [0], mod});
    Poly X = Poly(*this, (size() + 1) / 2).Sqrt().isz(
        size());
    return X.iadd(Mul(X.Inv()).isz(size())).imul(mod / 2
        + 1);
}

pair<Poly, Poly> DivMod(const Poly &rhs) const {
    if (size() < rhs.size()) return {V{0}, *this};
    const int _n = size() - rhs.size() + 1;
    Poly X(rhs); X.irev().isz(_n);
    Poly Y(*this); Y.irev().isz(_n);
    Poly Q = Y.Mul(X.Inv()).isz(_n).irev();
    X = rhs.Mul(Q), Y = *this;
    fi(0, size()) Y[i] = modsub(Y[i], X[i]);
    return {Q, Y.isz(max<int>(1, rhs.size() - 1))};
}

Poly Dx() const {
    Poly ret(size() - 1);
    fi(0, ret.size()) ret[i] = modmul(i + 1, (*this)[i +
        1]);
    return ret.isz(max<int>(1, ret.size()));
}

Poly Sx() const {
    Poly ret(size() + 1);
    fi(0, size()) ret[i + 1] = modmul(modinv(i + 1), (*
        this)[i]);
    return ret;
}

Poly _tmul(int nn, const Poly &rhs) const {
    Poly Y = Mul(rhs).isz(size() + nn - 1);
    return V(Y.data() + size() - 1, Y.data() + Y.size());
}

V _eval(const V &x, const vector<Poly> &up) const {
    const int _n = (int)x.size();
    if (!_n) return {};
    vector<Poly> down(_n * 2);
    // down[1] = DivMod(up[1]).second;
    // fi(2, _n * 2) down[i] = down[i / 2].DivMod(up[i]).
        second;
    down[1] = Poly(up[1]).irev().isz(size()).Inv().irev()
        ._tmul(_n, *this);
    fi(2, _n * 2) down[i] = up[i ^ 1]._tmul(up[i].size()
        - 1, down[i / 2]);
    V y(_n); fi(0, _n) y[i] = down[_n + i][0];
    return y;
}

static vector<Poly> _tree1(const V &x) {
    const int _n = (int)x.size();
    vector<Poly> up(_n * 2);
    fi(0, _n) up[_n + i] = V{modsub(mod, x[i]), 1};
    for(int i=_n-1;i>0;--i) up[i] = up[i * 2].Mul(up[i *
        2 + 1]);
    return up;
}

```

```

}
V Eval(const V &x) const { return _eval(x, _tree1(x));
}
static Poly Interpolate(const V &x, const V &y) {
    const int _n = (int)x.size();
    vector<Poly> up = _tree1(x), down(_n * 2);
    V z = up[1].Dx()._eval(x, up);
    fi(0, _n) z[i] = modmul(y[i], modinv(z[i]));
    fi(0, _n) down[_n + i] = V{z[i]};
    for(int i=_n-1; i>0; --i) down[i] = down[i * 2].Mul(up[
        i * 2 + 1]).iadd(down[i * 2 + 1].Mul(up[i * 2]));
    return down[1];
}
Poly Ln() const { // coef[0] == 1
    return Dx().Mul(Inv()).Sx().isz(size());
}
Poly Exp() const { // coef[0] == 0
    if (size() == 1) return V{1};
    Poly X = Poly(*this, (size() + 1) / 2).Exp().isz(size
        ());
    Poly Y = X.Ln(); Y[0] = mod - 1;
    fi(0, size()) Y[i] = modsub((*this)[i], Y[i]);
    return X.Mul(Y).isz(size());
}
Poly Pow(const string &K) const {
    int nz = 0;
    while (nz < size() && !(*this)[nz]) ++nz;
    int nk = 0, nk2 = 0;
    for (char c : K) {
        nk = (nk * 10 + c - '0') % mod;
        nk2 = nk2 * 10 + c - '0';
        if (nk2 * nz >= size())
            return Poly(size());
        nk2 %= mod - 1;
    }
    if (!nk && !nk2) return Poly(V{1}, size());
    Poly X = V(data() + nz, data() + size() - nz * (nk2 -
        1));
    int x0 = X[0];
    return X.imul(modinv(x0)).Ln().imul(nk).Exp().imul(
        modpow(x0, nk2)).irev().isz(size()).irev();
}
Poly InvMod(int L) { // (to evaluate linear recursion)
    Poly R{1, 0}; // *this * R mod x^L = 1 (*this[0] ==
        1)
    for (int level = 0; (1 << level) < L; ++level) {
        Poly O = R.Mul(Poly(data(), min<int>(2 << level,
            size())));
        Poly Q(2 << level); Q[0] = 1;
        for (int j = (1 << level); j < (2 << level); ++j)
            Q[j] = modsub(mod, O[j]);
        R = R.Mul(Q).isz(4 << level);
    }
    return R.isz(L);
}
static int LinearRecursion(const V &a, const V &c,
    int64_t n) { // a_n = \sum c_j a_{n-j}
    const int k = (int)a.size();
    assert((int)c.size() == k + 1);
    Poly C(k + 1), W({1}, k), M = {0, 1};
    fi(1, k + 1) C[k - i] = modsub(mod, c[i]);
    C[k] = 1;
    while (n) {
        if (n % 2) W = W.Mul(M).DivMod(C).second;
        n /= 2, M = M.Mul(M).DivMod(C).second;
    }
    int ret = 0;
    fi(0, k) ret = modadd(ret, modmul(W[i], a[i]));
    return ret;
}
}
#undef fi
using Poly_t = Poly<998244353, 3, 1 << 20>;
template <> decltype(Poly_t::ntt) Poly_t::ntt = {};

```

5.22 Quadratic residue

```

struct S {
    int MOD, w;
    int64_t x, y;
    S(int m, int w=-1, int64_t x=1, int64_t y=0)
        : MOD(m), w(w), x(x), y(y) {}
    S operator*(const S &rhs) const {

```

```

    int w_ = w;
    if (w_ == -1) w_ = rhs.w;
    assert(w_ != -1 and w_ == rhs.w);
    return { MOD, w_,
        (x * rhs.x + y * rhs.y % MOD * w) % MOD,
        (x * rhs.y + y * rhs.x) % MOD };
};
int get_root(int n, int P) {
    if (P == 2 or n == 0) return n;
    if (qpow(n, (P - 1) / 2, P) != 1) return -1;
    auto check = [&](int x) {
        return qpow(x, (P - 1) / 2, P);
    };
    if (check(n) == P-1) return -1;
    int64_t a; int w; mt19937 rnd(7122);
    do { a = rnd() % P;
        w = ((a * a - n) % P + P) % P;
    } while (check(w) != P - 1);
    return qpow(S(P, w, a, 1), (P + 1) / 2).x;
}

```

5.23 Simplex

```

namespace simplex {
    // maximize c^T x under Ax <= B
    // return VD(n, -inf) if the solution doesn't exist
    // return VD(n, +inf) if the solution is unbounded
    using VD = vector<double>;
    using VVD = vector<vector<double>>;
    const double eps = 1e-9;
    const double inf = 1e+9;
    int n, m;
    VVD d;
    vector<int> p, q;
    void pivot(int r, int s) {
        double inv = 1.0 / d[r][s];
        for (int i = 0; i < m + 2; ++i)
            for (int j = 0; j < n + 2; ++j)
                if (i != r && j != s)
                    d[i][j] -= d[r][j] * d[i][s] * inv;
        for(int i=0;i<m+2;++i) if (i != r) d[i][s] *= -inv;
        for(int j=0;j<n+2;++j) if (j != s) d[r][j] *= +inv;
        d[r][s] = inv; swap(p[r], q[s]);
    }
    bool phase(int z) {
        int x = m + z;
        while (true) {
            int s = -1;
            for (int i = 0; i <= n; ++i) {
                if (!z && q[i] == -1) continue;
                if (s == -1 || d[x][i] < d[x][s]) s = i;
            }
            if (d[x][s] > -eps) return true;
            int r = -1;
            for (int i = 0; i < m; ++i) {
                if (d[i][s] < eps) continue;
                if (r == -1 || \
                    d[i][n+1]/d[i][s] < d[r][n+1]/d[r][s]) r = i;
            }
            if (r == -1) return false;
            pivot(r, s);
        }
    }
    VD solve(const VVD &a, const VD &b, const VD &c) {
        m = b.size(), n = c.size();
        d = VVD(m + 2, VD(n + 2));
        for (int i = 0; i < m; ++i)
            for (int j = 0; j < n; ++j) d[i][j] = a[i][j];
        p.resize(m), q.resize(n + 1);
        for (int i = 0; i < m; ++i)
            p[i] = n + i, d[i][n] = -1, d[i][n + 1] = b[i];
        for (int i = 0; i < n; ++i) q[i] = i, d[m][i] = -c[i];
        q[n] = -1, d[m + 1][n] = 1;
        int r = 0;
        for (int i = 1; i < m; ++i)
            if (d[i][n + 1] < d[r][n + 1]) r = i;
        if (d[r][n + 1] < -eps) {
            pivot(r, n);
            if (!phase(1) || d[m + 1][n + 1] < -eps)
                return VD(n, -inf);
            for (int i = 0; i < m; ++i) if (p[i] == -1) {
                int s = min_element(d[i].begin(), d[i].end() - 1)
                    - d[i].begin();

```



```

    pivot(i, s);
}
}
if (!phase(0)) return VD(n, inf);
VD x(n);
for (int i = 0; i < m; ++i)
    if (p[i] < n) x[p[i]] = d[i][n + 1];
return x;
}}
```

5.24 Simplex Construction

Standard form: maximize $\sum_{1 \leq i \leq n} c_i x_i$ such that for all $1 \leq j \leq m$, $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$ and $x_i \geq 0$ for all $1 \leq i \leq n$.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji} x_i \leq -b_j$
3. $\sum_{1 \leq i \leq n} A_{ji} x_i = b_j$
 - $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$
 - $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j$
4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

6 Geometry

6.1 Basic Geometry

```

#define IM imag
#define RE real
using lld = int64_t;
using llf = long double;
using Point = std::complex<lld>;
using Pointf = std::complex<llf>;
auto toPointf(Point p) { return Pointf{RE(p), IM(p)}; }
int sgn(lld x) { return (x > 0) - (x < 0); }
lld dot(Point a, Point b) { return RE(conj(a) * b); }
lld cross(Point a, Point b) { return IM(conj(a) * b); }
int ori(Point a, Point b, Point c) {
    return sgn(cross(b - a, c - a));
}
bool operator<(const Point &a, const Point &b) {
    return RE(a) != RE(b) ? RE(a) < RE(b) : IM(a) < IM(b);
}
int argCmp(Point a, Point b) {
    // -1 / 0 / 1 <-> < / == / > (atan2)
    int qa = (IM(a) == 0 ? (RE(a) < 0 ? 3 : 1) : (IM(a) < 0 ? 0 : 2));
    int qb = (IM(b) == 0 ? (RE(b) < 0 ? 3 : 1) : (IM(b) < 0 ? 0 : 2));
    if (qa != qb) return sgn(qa - qb);
    return sgn(cross(b, a));
}
template <typename V> llf area(const V &pt) {
    lld ret = 0;
    for (int i = 1; i + 1 < (int)pt.size(); i++)
        ret += cross(pt[i] - pt[0], pt[i+1] - pt[0]);
    return ret / 2.0;
}
Point rot90(Point p) { return Point{-IM(p), RE(p)}; }
Pointf project(Pointf p, Pointf q) { // p onto q
    return dot(p, q) * q / dot(q, q);
}
}
```

6.2 Segment & Line Intersection

```

struct Segment {
    Point st, dir; // represent st + t*dir for 0<=t<=1
    Segment(Point s, Point e) : st(s), dir(e - s) {}
    static bool valid(lld p, lld q) {
        // is there t s.t. 0 <= t <= 1 && qt == p ?
        if (q < 0) q = -q, p = -p;
        return 0 <= p && p <= q;
    }
};
bool isInter(Segment A, Point P) {
    if (A.dir == Point(0)) return P == A.st;
    return cross(P - A.st, A.dir) == 0 &&
        Segment::valid(dot(P - A.st, A.dir), norm(A.dir));
}
template <typename U, typename V>
```

```

bool isInter(U A, V B) {
    if (cross(A.dir, B.dir) == 0)
        return // handle parallel yourself
            isInter(A, B.st) || isInter(A, B.st+B.dir) ||
            isInter(B, A.st) || isInter(B, A.st+A.dir);
    Point D = B.st - A.st;
    lld C = cross(A.dir, B.dir);
    return U::valid(cross(D, A.dir), C) &&
        V::valid(cross(D, B.dir), C);
}
struct Line {
    Point st, ed, dir;
    Line(Point s, Point e)
        : st(s), ed(e), dir(e - s) {}
};
Pointf intersect(const Line &A, const Line &B) {
    llf t = cross(B.st - A.st, B.dir) /
        llf(cross(A.dir, B.dir));
    return toPointf(A.st) +
        Pointf(t) * toPointf(A.dir);
}
```

6.3 2D Convex Hull

```

template<typename PT>
vector<PT> buildConvexHull(vector<PT> d) {
    sort(ALL(d), [](const PT& a, const PT& b) {
        return tie(a.x, a.y) < tie(b.x, b.y); });
    vector<PT> s(SZ(d)<<1);
    int o = 0;
    for(auto p: d) {
        while(o>=2 && cross(p-s[o-2], s[o-1]-s[o-2])<=0)
            o--;
        s[o++] = p;
    }
    for(int i=SZ(d)-2, t=o+1; i>=0; i--){
        while(o>=t && cross(d[i]-s[o-2], s[o-1]-s[o-2])<=0)
            o--;
        s[o++] = d[i];
    }
    s.resize(o-1);
    return s;
}
```

6.4 3D Convex Hull

```

// return the faces with pt indexes
int flag[MXN][MXN];
struct Point {
    ld x, y, z;
    Point operator * (const ld &b) const {
        return (Point){x*b, y*b, z*b};
    }
    Point operator * (const Point &b) const {
        return (Point){y*b.z-b.y*z, z*b.x-b.z*x, x*b.y-b.x*y};
    }
};
Point ver(Point a, Point b, Point c) {
    return (b - a) * (c - a);
}
vector<Face> convex_hull_3D(const vector<Point> pt) {
    int n = SZ(pt), ftop = 0;
    REP(i, n) REP(j, n) flag[i][j] = 0;
    vector<Face> now;
    now.emplace_back(0, 1, 2);
    now.emplace_back(2, 1, 0);
    for (int i=3; i<n; i++){
        ftop++; vector<Face> next;
        REP(j, SZ(now)) {
            Face& f=now[j]; int ff = 0;
            ld d=(pt[i]-pt[f.a]).dot(
                ver(pt[f.a], pt[f.b], pt[f.c]));
            if (d <= 0) next.push_back(f);
            if (d > 0) ff=ftop;
            else if (d < 0) ff=-ftop;
            flag[f.a][f.b]=flag[f.b][f.c]=flag[f.c][f.a]=ff;
        }
        REP(j, SZ(now)) {
            Face& f=now[j];
            if (flag[f.a][f.b] > 0 &&
                flag[f.a][f.b] != flag[f.b][f.a])
                next.emplace_back(f.a, f.b, i);
            if (flag[f.b][f.c] > 0 &&
                flag[f.b][f.c] != flag[f.c][f.b])
                next.emplace_back(f.b, f.c, i);
            if (flag[f.c][f.a] > 0 &&
```

```

    flag[f.c][f.a] != flag[f.a][f.c])
    next.emplace_back(f.c, f.a, i);
}
now=next;
}
return now;
}

```

6.5 2D Farthest Pair

```

// stk is from convex hull
n = (int)(stk.size());
int pos = 1, ans = 0; stk.push_back(stk[0]);
for(int i=0; i<n; i++) {
    while(abs(cross(stk[i+1]-stk[i],
        stk[(pos+1)%n]-stk[i])) >
        abs(cross(stk[i+1]-stk[i],
            stk[pos]-stk[i]))) pos = (pos+1)%n;
    ans = max({ans, dis(stk[i], stk[pos]),
        dis(stk[i+1], stk[pos])});
}

```

6.6 2D Closest Pair

```

struct cmp_y {
    bool operator()(const P& p, const P& q) const {
        return p.y < q.y;
    }
};
multiset<P, cmp_y> s;
void solve(P a[], int n) {
    sort(a, a + n, [](const P& p, const P& q) {
        return tie(p.x, p.y) < tie(q.x, q.y);
    });
    llf d = INF; int pt = 0;
    for (int i = 0; i < n; ++i) {
        while (pt < i and a[i].x - a[pt].x >= d)
            s.erase(s.find(a[pt]));
        auto it = s.lower_bound(P(a[i].x, a[i].y - d));
        while (it != s.end() and it->y - a[i].y < d)
            d = min(d, dis(*it, a[i]));
        s.insert(a[i]);
    }
}

```

6.7 kD Closest Pair (3D ver.)

```

llf solve(vector<P> v) {
    shuffle(v.begin(), v.end(), mt19937());
    unordered_map<lld, unordered_map<lld,
        unordered_map<lld, int>>> m;
    llf d = dis(v[0], v[1]);
    auto Idx = [&d] (llf x) -> lld {
        return round(x * 2 / d) + 0.1; };
    auto rebuild_m = [&m, &v, &Idx] (int k) {
        m.clear();
        for (int i = 0; i < k; ++i)
            m[Idx(v[i].x)][Idx(v[i].y)]
                [Idx(v[i].z)] = i;
    }; rebuild_m(2);
    for (size_t i = 2; i < v.size(); ++i) {
        const lld kx = Idx(v[i].x), ky = Idx(v[i].y),
            kz = Idx(v[i].z); bool found = false;
        for (int dx = -2; dx <= 2; ++dx) {
            const lld nx = dx + kx;
            if (m.find(nx) == m.end()) continue;
            auto& mm = m[nx];
            for (int dy = -2; dy <= 2; ++dy) {
                const lld ny = dy + ky;
                if (mm.find(ny) == mm.end()) continue;
                auto& mmm = mm[ny];
                for (int dz = -2; dz <= 2; ++dz) {
                    const lld nz = dz + kz;
                    if (mmm.find(nz) == mmm.end()) continue;
                    const int p = mmm[nz];
                    if (dis(v[p], v[i]) < d) {
                        d = dis(v[p], v[i]);
                        found = true;
                    }
                }
            }
        }
        if (found) rebuild_m(i + 1);
        else m[kx][ky][kz] = i;
    }
}

```

```

}
return d;
}

```

6.8 Simulated Annealing

```

llf anneal() {
    mt19937 rnd_engine( seed );
    uniform_real_distribution< llf > rnd( 0, 1 );
    const llf dT = 0.001;
    // Argument p
    llf S_cur = calc( p ), S_best = S_cur;
    for ( llf T = 2000 ; T > EPS ; T -= dT ) {
        // Modify p to p_prime
        const llf S_prime = calc( p_prime );
        const llf delta_c = S_prime - S_cur;
        llf prob = min( ( llf ) 1, exp( -delta_c / T ) );
        if ( rnd( rnd_engine ) <= prob )
            S_cur = S_prime, p = p_prime;
        if ( S_prime < S_best ) // find min
            S_best = S_prime, p_best = p_prime;
    }
    return S_best;
}

```

6.9 Half Plane Intersection

```

// cross(pt-line.st, line.dir)<=0 <=> pt in half plane
bool operator<(const Line &lhs, const Line &rhs) {
    if (int cmp = argCmp(lhs.dir, rhs.dir))
        return cmp == -1;
    return ori(lhs.st, lhs.ed, rhs.st) < 0;
}

// intersect function is in "Segment Intersect"
llf HPI(vector<Line> &lines) {
    sort(lines.begin(), lines.end());
    deque<Line> que;
    deque<Pointf> pt;
    que.push_back(lines[0]);
    for (int i = 1; i < (int)lines.size(); i++) {
        if (argCmp(lines[i].dir, lines[i-1].dir) == 0)
            continue;
#define POP(L, R) \
        while (pt.size() > 0 \
            && ori(L.st, L.ed, pt.back()) < 0) \
            pt.pop_back(), que.pop_back(); \
        while (pt.size() > 0 \
            && ori(R.st, R.ed, pt.front()) < 0) \
            pt.pop_front(), que.pop_front();
        POP(lines[i], lines[i-1]);
        pt.push_back(intersect(que.back(), lines[i]));
        que.push_back(lines[i]);
    }
    POP(que.front(), que.back())
    if (que.size() <= 1 ||
        argCmp(que.front().dir, que.back().dir) == 0)
        return 0;
    pt.push_back(intersect(que.front(), que.back()));
    return area(pt);
}

```

6.10 Minkowski Sum

```

vector<pll> Minkowski(vector<pll> A, vector<pll> B) {
    hull(A), hull(B);
    vector<pll> C(1, A[0] + B[0]), s1, s2;
    for(int i = 0; i < SZ(A); ++i)
        s1.pb(A[(i + 1) % SZ(A)] - A[i]);
    for(int i = 0; i < SZ(B); ++i)
        s2.pb(B[(i + 1) % SZ(B)] - B[i]);
    for(int p1 = 0, p2 = 0; p1 < SZ(A) || p2 < SZ(B);)
        if (p2 >= SZ(B)
            || (p1 < SZ(A) && cross(s1[p1], s2[p2]) >= 0))
            C.pb(C.back() + s1[p1++]);
        else
            C.pb(C.back() + s2[p2++]);
    return hull(C), C;
}

```

6.11 Circle Class

```

struct Circle { Pointf o; llf r; };
vector<llf> intersectAngle(Circle A, Circle B) {

```

```

Pointf dir = B.o - A.o; llf d2 = norm(dir);
if (norm(A.r - B.r) >= d2)
    if (A.r < B.r) return {-PI, PI}; // special
    else return {};
if (norm(A.r + B.r) <= d2) return {};
llf dis = abs(dir), theta = arg(dir);
llf phi = acos((A.r * A.r + d2 - B.r * B.r) /
    (2 * A.r * dis));
llf L = theta - phi, R = theta + phi;
while (L < -PI) L += PI * 2;
while (R > PI) R -= PI * 2;
return { L, R };
}

vector<Pointf> intersectPoint(Circle a, Circle b) {
    llf d = abs(a.o - b.o);
    if (d >= b.r+a.r || d <= abs(b.r-a.r)) return {};
    llf dt = (b.r*b.r - a.r*a.r)/d, d1 = (d+dt)/2;
    Pointf dir = (a.o - b.o); dir /= d;
    Pointf pcrs = dir*d1 + b.o;
    dt=sqrt(max(0.0L, b.r*b.r-d1*d1)), dir = rot90(dir);
    return {pcrs + dir*dt, pcrs - dir*dt};
}

```

6.12 Intersection of line and Circle

```

vector<pdd> line_interCircle(const pdd &p1,
    const pdd &p2, const pdd &c, const double r) {
    pdd ft = p1 + project(c-p1, p2-p1), vec = p2-p1;
    llf dis = abs(c - ft);
    if (abs(dis - r) < eps) return {ft};
    if (dis > r) return {};
    vec = vec * sqrt(r * r - dis * dis) / abs(vec);
    return {ft + vec, ft - vec};
}

```

6.13 Intersection of Polygon and Circle

```

// Divides into multiple triangle, and sum up
// test by HDU2892
const double PI=acos(-1);
double _area(pdd pa, pdd pb, double r){
    if(abs(pa)<abs(pb)) swap(pa, pb);
    if(abs(pb)<eps) return 0;
    double S, h, theta;
    double a=abs(pb),b=abs(pa),c=abs(pb-pa);
    double cosB = dot(pb,pb-pa) / a / c, B = acos(cosB);
    double cosC = dot(pa,pb) / a / b, C = acos(cosC);
    if(a > r){
        S = (C/2)*r*r;
        h = a*b*sin(C)/c;
        if (h < r && B < PI/2)
            S -= (acos(h/r)*r*r - h*sqrt(r*r-h*h));
    }
    else if(b > r){
        theta = PI - B - asin(sin(B)/r*a);
        S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
    }
    else S = .5*sin(C)*a*b;
    return S;
}

double area_poly_circle(const vector<pdd> &poly,
    const pdd &O,const double r){
    double S=0; int N=poly.size();
    for(int i=0;i<N;++i)
        S += _area(poly[i]-O, poly[(i+1)%N]-O, r)
            * ori(O, poly[i], poly[(i+1)%N]);
    return fabs(S);
}

```

6.14 Point & Hulls Tangent

```

#define above(P, Vi, Vj) (ori(P, Vi, Vj) > 0) // true
    if Vi is above Vj
#define below(P, Vi, Vj) (ori(P, Vi, Vj) < 0) // true
    if Vi is below Vj
// Rtangent_PointPolyC(): binary search for convex
    polygon right tangent
// Input: P = a 2D point (exterior to the polygon)
//      n = number of polygon vertices
//      V = array of vertices for a 2D convex polygon
    with V[n] = V[0]
// Return: index "i" of rightmost tangent point V[i]
int Rtangent_PointPolyC(Point P, int n, Point *V) {

```

```

int a, b, c;
int upA, dnC;

if (below(P, V[1], V[0]) && !above(P, V[n - 1], V[0]))
    return 0;

for (a = 0, b = n;;) {
    c = (a + b) / 2;
    dnC = below(P, V[c + 1], V[c]);
    if (dnC && !above(P, V[c - 1], V[c]))
        return c;

    upA = above(P, V[a + 1], V[a]);
    if (upA) {
        if (dnC) {
            b = c;
        } else {
            if (above(P, V[a], V[c]))
                b = c;
            else
                a = c;
        }
    } else {
        if (!dnC) {
            a = c;
        } else {
            if (below(P, V[a], V[c]))
                b = c;
            else
                a = c;
        }
    }
}

// Ltangent_PointPolyC(): binary search for convex
    polygon left tangent
// Input: P = a 2D point (exterior to the polygon)
//      n = number of polygon vertices
//      V = array of vertices for a 2D convex polygon
    with V[n]=V[0]
// Return: index "i" of leftmost tangent point V[i]
int Ltangent_PointPolyC(Point P, int n, Point *V) {
    int a, b, c;
    int dnA, dnC;

    if (above(P, V[n - 1], V[0]) && !below(P, V[1], V[0]))
        return 0;

    for (a = 0, b = n;;) {
        c = (a + b) / 2;
        dnC = below(P, V[c + 1], V[c]);
        if (above(P, V[c - 1], V[c]) && !dnC)
            return c;
        dnA = below(P, V[a + 1], V[a]);
        if (dnA) {
            if (!dnC) {
                b = c;
            } else {
                if (below(P, V[a], V[c]))
                    b = c;
                else
                    a = c;
            }
        } else {
            if (dnC) {
                a = c;
            } else {
                if (above(P, V[a], V[c]))
                    b = c;
                else
                    a = c;
            }
        }
    }
}

```

6.15 Convex Hulls Tangent

```

// RLtangent_PolyPolyC(): get the RL tangent between
    two convex polygons
// Input: m = number of vertices in polygon 1

```

```
// V = array of vertices for convex polygon 1 with
// V[m]=V[0]
// n = number of vertices in polygon 2
// W = array of vertices for convex polygon 2 with
// W[n]=W[0]
// Output: *t1 = index of tangent point V[t1] for
// polygon 1
// *t2 = index of tangent point W[t2] for polygon
// 2
void RLtangent_PolyPolyC(int m, Point *V, int n, Point
    *W, int *t1, int *t2) {
    int ix1, ix2; // search indices for polygons 1 and 2

    // first get the initial vertex on each polygon
    ix1 = Rtangent_PointPolyC(W[0], m, V); // right
    tangent from W[0] to V
    ix2 = Ltangent_PointPolyC(V[ix1], n, W); // left
    tangent from V[ix1] to W

    // ping-pong linear search until it stabilizes
    int done = false; // flag when done
    while (done == false) {
        done = true; // assume done until...
        while (ori(W[ix2], V[ix1], V[ix1 + 1]) <= 0) {
            ++ix1; // get Rtangent from W[ix2] to V
        }
        while (ori(V[ix1], W[ix2], W[ix2 - 1]) >= 0) {
            --ix2; // get Ltangent from V[ix1] to W
            done = false; // not done if had to adjust this
        }
    }
    *t1 = ix1;
    *t2 = ix2;
    return;
}
```

6.16 Tangent line of Two Circle

```
vector<Line> go(const Cir &c1, const Cir &c2,
    int sign1) {
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    if (norm(c1.o - c2.o) < eps)
        return ret;
    llf d = abs(c1.o - c2.o);
    Pointf v = (c2.o - c1.o) / d;
    llf c = (c1.r - sign1 * c2.r) / d;
    if (c * c > 1)
        return ret;
    llf h = sqrt(max(0.0, 1.0 - c * c));
    for (int sign2: {1, -1}) {
        Pointf n = c * v + sign2 * h * rot90(v);
        Pointf p1 = c1.o + n * c1.r;
        Pointf p2 = c2.o + n * (c2.r * sign1);
        if (norm(p2 - p1) < eps)
            p2 = p1 + rot90(c2.o - c1.o);
        ret.push_back({p1, p2});
    }
    return ret;
}
```

6.17 Minimum Covering Circle

```
template<typename P>
Circle getCircum(const P &a, const P &b, const P &c){
    Real a1 = a.x-b.x, b1 = a.y-b.y;
    Real c1 = (a.x+b.x)/2 * a1 + (a.y+b.y)/2 * b1;
    Real a2 = a.x-c.x, b2 = a.y-c.y;
    Real c2 = (a.x+c.x)/2 * a2 + (a.y+c.y)/2 * b2;
    Circle cc;
    cc.o.x = (c1*b2-b1*c2)/(a1*b2-b1*a2);
    cc.o.y = (a1*c2-c1*a2)/(a1*b2-b1*a2);
    cc.r = hypot(cc.o.x-a.x, cc.o.y-a.y);
    return cc;
}
```

```
template<typename P>
Circle MinCircleCover(const vector<P>& pts){
    random_shuffle(pts.begin(), pts.end());
    Circle c = {pts[0], 0};
    for(int i=0;i<(int)pts.size();i++){
        if (dist(pts[i], c.o) <= c.r) continue;
        c = {pts[i], 0};
        for (int j = 0; j < i; j++) {
```

```
            if(dist(pts[j], c.o) <= c.r) continue;
            c.o = (pts[i] + pts[j]) / 2;
            c.r = dist(pts[i], c.o);
            for (int k = 0; k < j; k++) {
                if (dist(pts[k], c.o) <= c.r) continue;
                c = getCircum(pts[i], pts[j], pts[k]);
            }
        }
    }
    return c;
}
```

6.18 KDTree (Nearest Point)

```
const int MXN = 100005;
struct KDTree {
    struct Node {
        int x,y,x1,y1,x2,y2;
        int id,f;
        Node *L, *R;
    } tree[MXN], *root;
    int n;
    LL dis2(int x1, int y1, int x2, int y2) {
        LL dx = x1-x2, dy = y1-y2;
        return dx*dx+dy*dy;
    }
    static bool cmpx(Node& a, Node& b){return a.x<b.x;}
    static bool cmpy(Node& a, Node& b){return a.y<b.y;}
    void init(vector<pair<int,int>> ip) {
        n = ip.size();
        for (int i=0; i<n; i++) {
            tree[i].id = i;
            tree[i].x = ip[i].first;
            tree[i].y = ip[i].second;
        }
        root = build_tree(0, n-1, 0);
    }
    Node* build_tree(int L, int R, int d) {
        if (L>R) return nullptr;
        int M = (L+R)/2; tree[M].f = d%2;
        nth_element(tree+L, tree+M, tree+R+1, d%2?cmpy:cmpx);
        tree[M].x1 = tree[M].x2 = tree[M].x;
        tree[M].y1 = tree[M].y2 = tree[M].y;
        tree[M].L = build_tree(L, M-1, d+1);
        if (tree[M].L) {
            tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
            tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
            tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
            tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
        }
        tree[M].R = build_tree(M+1, R, d+1);
        if (tree[M].R) {
            tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
            tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
            tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
            tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
        }
        return tree+M;
    }
    int touch(Node* r, int x, int y, LL d2){
        LL dis = sqrt(d2)+1;
        if (x<r->x1-dis || x>r->x2+dis ||
            y<r->y1-dis || y>r->y2+dis)
            return 0;
        return 1;
    }
    void nearest(Node* r, int x, int y, int &mID, LL &md2) {
        if (!r || !touch(r, x, y, md2)) return;
        LL d2 = dis2(r->x, r->y, x, y);
        if (d2 < md2 || (d2 == md2 && mID < r->id)) {
            mID = r->id;
            md2 = d2;
        }
        // search order depends on split dim
        if ((r->f == 0 && x < r->x) ||
            (r->f == 1 && y < r->y)) {
            nearest(r->L, x, y, mID, md2);
            nearest(r->R, x, y, mID, md2);
        } else {
            nearest(r->R, x, y, mID, md2);
            nearest(r->L, x, y, mID, md2);
        }
    }
}
```

```

int query(int x, int y) {
    int id = 1029384756;
    LL d2 = 102938475612345678LL;
    nearest(root, x, y, id, d2);
    return id;
}
} tree;

```

6.19 Rotating Sweep Line

```

void rotatingSweepLine(pair<int, int> a[], int n) {
    vector<pair<int, int>> l;
    l.reserve(n * (n - 1) / 2);
    for (int i = 0; i < n; ++i)
        for (int j = i + 1; j < n; ++j)
            l.emplace_back(i, j);
    sort(l.begin(), l.end(), [&a](auto &u, auto &v){
        lld udx = a[u.first].first - a[u.second].first;
        lld udy = a[u.first].second - a[u.second].second;
        lld vdx = a[v.first].first - a[v.second].first;
        lld vdy = a[v.first].second - a[v.second].second;
        if (udx == 0 or vdx == 0) return not udx == 0;
        int s = sgn(udx * vdx);
        return udy * vdx * s < vdy * udx * s;
    });
    vector<int> idx(n), p(n);
    iota(idx.begin(), idx.end(), 0);
    sort(idx.begin(), idx.end(), [&a](int i, int j){
        return a[i] < a[j]; });
    for (int i = 0; i < n; ++i) p[idx[i]] = i;
    for (auto [i, j]: l) {
        // do here
        swap(p[i], p[j]);
        idx[p[i]] = i, idx[p[j]] = j;
    }
}

```

6.20 Circle Cover

```

const int N = 1021;
struct CircleCover {
    int C;
    Cir c[N];
    bool g[N][N], overlap[N][N];
    // Area[i] : area covered by at least i circles
    double Area[N];
    void init(int _C){ C = _C; }
    struct Teve {
        pdd p; double ang; int add;
        Teve() {}
        Teve(pdd _a, double _b, int _c):p(_a), ang(_b), add(_c){}
        bool operator<(const Teve &a)const {
            return ang < a.ang; }
    }eve[N * 2];
    // strict: x = 0, otherwise x = -1
    bool disjunct(Cir &a, Cir &b, int x) {
        return sign(abs(a.o - b.o) - a.R - b.R) > x; }
    bool contain(Cir &a, Cir &b, int x) {
        return sign(a.R - b.R - abs(a.o - b.o)) > x; }
    bool contain(int i, int j) {
        /* c[j] is non-strictly in c[i]. */
        return (sign(c[i].R - c[j].R) > 0 || (sign(c[i].R - c[j].R) == 0 && i < j)) && contain(c[i], c[j], -1); }
    void solve(){
        fill_n(Area, C + 2, 0);
        for(int i = 0; i < C; ++i)
            for(int j = 0; j < C; ++j)
                overlap[i][j] = contain(i, j);
        for(int i = 0; i < C; ++i)
            for(int j = 0; j < C; ++j)
                g[i][j] = !(overlap[i][j] || overlap[j][i] || disjunct(c[i], c[j], -1));
        for(int i = 0; i < C; ++i){
            int E = 0, cnt = 1;
            for(int j = 0; j < C; ++j)
                if(j != i && overlap[j][i])
                    ++cnt;
            for(int j = 0; j < C; ++j)
                if(i != j && g[i][j]) {
                    pdd aa, bb;
                    CCinter(c[i], c[j], aa, bb);
                    llf A = atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);

```

```

                    llf B = atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
                    eve[E++] = Teve(bb, B, 1), eve[E++] = Teve(aa, A, -1);
                    if(B > A) ++cnt;
                }
            if(E == 0) Area[cnt] += pi * c[i].R * c[i].R;
            else{
                sort(eve, eve + E);
                eve[E] = eve[0];
                for(int j = 0; j < E; ++j){
                    cnt += eve[j].add;
                    Area[cnt] += cross(eve[j].p, eve[j + 1].p) * .5;
                    double theta = eve[j + 1].ang - eve[j].ang;
                    if (theta < 0) theta += 2. * pi;
                    Area[cnt] += (theta - sin(theta)) * c[i].R * c[i].R * .5;
                }
            }
        }
    }
};

```

7 Stringology

7.1 Hash

```

class Hash {
private:
    static constexpr int P = 127, Q = 1051762951;
    vector<int> h, p;
public:
    void init(const string &s){
        h.assign(s.size() + 1, 0); p.resize(s.size() + 1);
        for (size_t i = 0; i < s.size(); ++i)
            h[i + 1] = add(mul(h[i], P), s[i]);
        generate(p.begin(), p.end(), [x=1, y=1, this]()
            mutable { y=x; x=mul(x, P); return y; });
    }
    int query(int l, int r){ // 1-base (l, r]
        return sub(h[r], mul(h[l], p[r-l])); }
};

```

7.2 Suffix Array

```

namespace sfx {
    bool _t[maxn * 2];
    int hi[maxn], rev[maxn];
    int _s[maxn * 2], sa[maxn * 2], _c[maxn * 2];
    int x[maxn], _p[maxn], _q[maxn * 2];
    // sa[i]: sa[i]-th suffix is the
    // i-th lexicographically smallest suffix.
    // hi[i]: longest common prefix
    // of suffix sa[i] and suffix sa[i - 1].
    void pre(int *a, int *c, int n, int z) {
        memset(a, 0, sizeof(int) * n);
        memcpy(x, c, sizeof(int) * z);
    }
    void induce(int *a, int *c, int *s, bool *t, int n, int z) {
        memcpy(x + 1, c, sizeof(int) * (z - 1));
        for (int i = 0; i < n; ++i)
            if (a[i] && !t[a[i] - 1])
                a[x[s[a[i] - 1]]++] = a[i] - 1;
        memcpy(x, c, sizeof(int) * z);
        for (int i = n - 1; i >= 0; --i)
            if (a[i] && t[a[i] - 1])
                a[--x[s[a[i] - 1]]] = a[i] - 1;
    }
    void sais(int *s, int *a, int *p, int *q,
        bool *t, int *c, int n, int z) {
        bool uniq = t[n - 1] = true;
        int nn=0, nmzx=-1, *nsa = a+n, *ns=s+n, last=-1;
        memset(c, 0, sizeof(int) * z);
        for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;
        for (int i = 0; i < z - 1; ++i) c[i + 1] += c[i];
        if (uniq) {
            for (int i = 0; i < n; ++i) a[--c[s[i]]] = i;
            return;
        }
        for (int i = n - 2; i >= 0; --i)
            t[i] = (s[i]==s[i + 1] ? t[i + 1] : s[i]<s[i + 1]);
        pre(a, c, n, z);
        for (int i = 1; i <= n - 1; ++i)
            if (t[i] && !t[i - 1])
                a[--x[s[i]]] = p[q[i] = nn++] = i;
        induce(a, c, s, t, n, z);
    }
}

```



```

for (int i = 0; i < n; ++i) {
    if (a[i] && t[a[i]] && !t[a[i] - 1]) {
        bool neq = last < 0 || \
            memcmp(s + a[i], s + last,
                (p[q[a[i]] + 1] - a[i]) * sizeof(int));
        ns[q[last = a[i]]] = nmzx += neq;
    }
    sais(ns, nsa, p+nn, q+n, t+n, c+z, nn, nmzx+1);
    pre(a, c, n, z);
    for (int i = nn - 1; i >= 0; --i)
        a[--x[s[p[nsa[i]]]]] = p[nsa[i]];
    induce(a, c, s, t, n, z);
}

void build(const string &s) {
    const int n = int(s.size());
    for (int i = 0; i < n; ++i) _s[i] = s[i];
    _s[n] = 0; // s shouldn't contain 0
    sais(_s, sa, _p, _q, _t, _c, n + 1, 256);
    for (int i = 0; i < n; ++i) rev[sa[i] = sa[i+1]] = i;
    int ind = hi[0] = 0;
    for (int i = 0; i < n; ++i) {
        if (!rev[i]) {
            ind = 0;
            continue;
        }
        while (i + ind < n && \
            s[i + ind] == s[sa[rev[i] - 1] + ind]) ++ind;
        hi[rev[i]] = ind ? ind-- : 0;
    }
}

```

7.3 Suffix Automaton

```

struct SuffixAutomaton {
    struct node {
        int ch[K], len, fail, cnt, indeg;
        node(int L = 0) : ch{}, len(L), fail(0), cnt(0),
            indeg(0) {}
    } st[N];
    int root, last, tot;
    void extend(int c) {
        int cur = ++tot;
        st[cur] = node(st[last].len + 1);
        while (last && !st[last].ch[c]) {
            st[last].ch[c] = cur;
            last = st[last].fail;
        }
        if (!last) {
            st[cur].fail = root;
        } else {
            int q = st[last].ch[c];
            if (st[q].len == st[last].len + 1) {
                st[cur].fail = q;
            } else {
                int clone = ++tot;
                st[clone] = st[q];
                st[clone].len = st[last].len + 1;
                st[st[cur].fail = st[q].fail = clone].cnt = 0;
                while (last && st[last].ch[c] == q) {
                    st[last].ch[c] = clone;
                    last = st[last].fail;
                }
            }
        }
        st[last = cur].cnt += 1;
    }
    void init(const char* s) {
        root = last = tot = 1;
        st[root] = node(0);
        for (char c; c = *s; ++s) extend(c - 'a');
    }
    int q[N];
    void dp() {
        for (int i = 1; i <= tot; i++) ++st[st[i].fail].indeg;
        int head = 0, tail = 0;
        for (int i = 1; i <= tot; i++)
            if (st[i].indeg == 0) q[tail++] = i;
        while (head != tail) {
            int now = q[head++];
            if (int f = st[now].fail) {
                st[f].cnt += st[now].cnt;
                if (--st[f].indeg == 0) q[tail++] = f;
            }
        }
    }
}

```

```

}
}
int run(const char* s) {
    int now = root;
    for (char c; c = *s; ++s) {
        if (!st[now].ch[c - 'a']) return 0;
        now = st[now].ch[c];
    }
    return st[now].cnt;
}
} SAM;

```

7.4 KMP

```

vector<int> kmp(const string &s) {
    vector<int> f(s.size(), 0);
    /* f[i] = length of the longest prefix
       (excluding s[0:i]) such that it coincides
       with the suffix of s[0:i] of the same length */
    /* i + 1 - f[i] is the length of the
       smallest recurring period of s[0:i] */
    int k = 0;
    for (int i = 1; i < (int)s.size(); ++i) {
        while (k > 0 && s[i] != s[k]) k = f[k - 1];
        if (s[i] == s[k]) ++k;
        f[i] = k;
    }
    return f;
}

vector<int> search(const string &s, const string &t) {
    // return 0-indexed occurrence of t in s
    vector<int> f = kmp(t), r;
    for (int i = 0, k = 0; i < (int)s.size(); ++i) {
        while (k > 0 && (k == (int)t.size() || s[i] != t[k]))
            k = f[k - 1];
        if (s[i] == t[k]) ++k;
        if (k == (int)t.size()) r.push_back(i - t.size() + 1);
    }
    return res;
}

```

7.5 Z value

```

vector<int> Zalgo(const string &s) {
    vector<int> z(s.size(), s.size());
    for (int i = 1, l = 0, r = 0; i < z[0]; ++i) {
        int j = clamp(r - i, 0, z[i - 1]);
        for (; i + j < z[0] && s[i + j] == s[j]; ++j);
        if (i + (z[i] = j) > r) r = i + z[i] = i;
    }
    return z;
}

```

7.6 Manacher

```

int z[maxn];
int manacher(const string &s) {
    string t = ". ";
    for (char c: s) t += c, t += ' ';
    int l = 0, r = 0, ans = 0;
    for (int i = 1; i < t.length(); ++i) {
        z[i] = (r > i ? min(z[2 * l - i], r - i) : 1);
        while (i - z[i] >= 0 && i + z[i] < t.length()) {
            if (t[i - z[i]] == t[i + z[i]]) ++z[i];
            else break;
        }
        if (i + z[i] > r) r = i + z[i], l = i;
    }
    for (int i = 1; i < t.length(); ++i) ans = max(ans, z[i] - 1);
    return ans;
}

```

7.7 Lexico Smallest Rotation

```

string mcp(string s) {
    int n = s.length();
    s += s; int i = 0, j = 1;
    while (i < n && j < n) {
        int k = 0;
        while (k < n && s[i + k] == s[j + k]) k++;
        ((s[i + k] <= s[j + k]) ? j : i) += k + 1;
        j += (i == j);
    }
    return s.substr(i < n ? i : j, n);
}

```

7.8 BWT

```
struct BurrowsWheeler{
#define SIGMA 26
#define BASE 'a'
vector<int> v[ SIGMA ];
void BWT(char* ori, char* res){
    // make ori -> ori + ori
    // then build suffix array
}
void iBWT(char* ori, char* res){
    for( int i = 0 ; i < SIGMA ; i ++ )
        v[ i ].clear();
    int len = strlen( ori );
    for( int i = 0 ; i < len ; i ++ )
        v[ ori[i] - BASE ].push_back( i );
    vector<int> a;
    for( int i = 0 , ptr = 0 ; i < SIGMA ; i ++ )
        for( auto j : v[ i ] ){
            a.push_back( j );
            ori[ ptr ++ ] = BASE + i;
        }
    for( int i = 0 , ptr = 0 ; i < len ; i ++ ){
        res[ i ] = ori[ a[ ptr ] ];
        ptr = a[ ptr ];
    }
    res[ len ] = 0;
}
} bwt;
```

7.9 Palindromic Tree

```
struct palindromic_tree{
    struct node{
        int next[26], f, len;
        int cnt, num, st, ed;
        node(int l=0):f(0), len(1), cnt(0), num(0) {
            memset(next, 0, sizeof(next)); }
    };
    vector<node> st;
    vector<char> s;
    int last, n;
    void init(){
        st.clear(); s.clear(); last=1; n=0;
        st.push_back(0); st.push_back(-1);
        st[0].f=1; s.push_back(-1); }
    int getFail(int x){
        while(s[n-st[x].len-1]!=s[n])x=st[x].f;
        return x; }
    void add(int c){
        s.push_back(c-'a'); ++n;
        int cur=getFail(last);
        if(!st[cur].next[c]){
            int now=st.size();
            st.push_back(st[cur].len+2);
            st[now].f=getFail(st[cur].f).next[c];
            st[cur].next[c]=now;
            st[now].num=st[st[now].f].num+1;
        }
        last=st[cur].next[c];
        ++st[last].cnt; }
    void dpent() {
        for( int i=st.size()-1; i >= 0; i-- )
            st[st[i].f].cnt += st[i].cnt;
    }
    int size(){ return st.size()-2; }
} pt;
int main() {
    string s; cin >> s; pt.init();
    for( int i=0; i<SZ(s); i++ ) {
        int prvsz = pt.size(); pt.add(s[i]);
        if( prvsz != pt.size() ) {
            int r = i, l = r - pt.st[pt.last].len + 1;
            // pal @ [l,r]: s.substr(l, r-l+1)
        }
    }
    return 0;
}
```

8 Misc

8.1 Theorems

8.1.1 Kirchhoff's Theorem

Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G , where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i, j) in G .

- The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
- The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.

8.1.2 Tutte's Matrix

Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniform randomly) if $i < j$ and $(i, j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{\text{rank}(D)}{2}$ is the maximum matching on G .

8.1.3 Cayley's Formula

- Given a degree sequence d_1, d_2, \dots, d_n for each labeled vertices, there're $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\dots(d_n-1)!}$ spanning trees.
- Let $T_{n,k}$ be the number of labeled forests on n vertices with k components, such that vertex $1, 2, \dots, k$ belong to different components. Then $T_{n,k} = k n^{n-k-1}$.

8.1.4 Erdős-Gallai theorem

A sequence of non-negative integers $d_1 \geq d_2 \geq \dots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + d_2 + \dots + d_n$ is even and

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$$

holds for all $1 \leq k \leq n$.

8.1.5 Havel-Hakimi algorithm

find the vertex who has greatest degree unused, connect it with other greatest vertex.

8.1.6 Hall's marriage theorem

Let G be a finite bipartite graph with bipartite sets X and Y . For a subset W of X , let $N_G(W)$ denote the set of all vertices in Y adjacent to some element of W . Then there is an X -saturating matching iff $\forall W \subseteq X, |W| \leq |N_G(W)|$

8.1.7 Euler's planar graph formula

$$V - E + F = C + 1, E \leq 3V - 6(?)$$

8.1.8 Pick's theorem

For simple polygon, when points are all integer, we have $A = \#\{\text{lattice points in the interior}\} + \frac{\#\{\text{lattice points on the boundary}\}}{2} - 1$

8.1.9 Lucas's theorem

$\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p}$, where $m = m_k p^k + m_{k-1} p^{k-1} + \dots + m_1 p + m_0$, and $n = n_k p^k + n_{k-1} p^{k-1} + \dots + n_1 p + n_0$.

8.1.10 Matroid Intersection

Given matroids $M_1 = (G, I_1), M_2 = (G, I_2)$, find maximum $S \in I_1 \cap I_2$. For each iteration, build the directed graph and find a shortest path from s to t .

- $s \rightarrow x : S \sqcup \{x\} \in I_1$
- $x \rightarrow t : S \sqcup \{x\} \in I_2$
- $y \rightarrow x : S \setminus \{y\} \sqcup \{x\} \in I_1$ (y is in the unique circuit of $S \sqcup \{x\}$)
- $x \rightarrow y : S \setminus \{y\} \sqcup \{x\} \in I_2$ (y is in the unique circuit of $S \sqcup \{x\}$)

Alternate the path, and $|S|$ will increase by 1. Let $R = \min(\text{rank}(I_1), \text{rank}(I_2)), N = |G|$. In each iteration, $|E| = O(RN)$. For weighted case, assign weight $-w(x)$ and $w(x)$ to $x \in S$ and $x \notin S$, resp. Use Bellman-Ford to find the weighted shortest path. The maximum iteration of Bellman-Ford is $2R + 1$.

8.2 DP-opt Condition

8.2.1 totally monotone (concave/convex)

$$\begin{aligned} \forall i < i', j < j', B[i][j] \leq B[i'][j'] &\implies B[i][j'] \leq B[i'][j'] \\ \forall i < i', j < j', B[i][j] \geq B[i'][j'] &\implies B[i][j'] \geq B[i'][j'] \end{aligned}$$

8.2.2 monge condition (concave/convex)

$$\begin{aligned} \forall i < i', j < j', B[i][j] + B[i'][j'] &\geq B[i][j'] + B[i'][j] \\ \forall i < i', j < j', B[i][j] + B[i'][j'] &\leq B[i][j'] + B[i'][j] \end{aligned}$$

8.3 Convex 1D/1D DP

```
struct segment {
    int l, r;
    segment() {}
    segment(int a, int b, int c): l(a), r(b), c(c) {}
};
inline lld f(int l, int r){return dp[l] + w(l+1, r);}
void solve() {
    dp[0] = 0;
    deque<segment> dq; dq.push_back(segment(0, 1, n));
    for (int i = 1; i <= n; ++i) {
        dp[i] = f(dq.front().l, i);
        while(dq.size() && dq.front().r < i+1) dq.pop_front();
        dq.front().l = i + 1;
        segment seg = segment(i, i + 1, n);
        while (dq.size() &&
            f(i, dq.back().l) < f(dq.back().l, dq.back().r))
            dq.pop_back();
        if (dq.size()) {
            int d = 1 << 20, c = dq.back().r;
            while (d >= 1) if (c + d <= dq.back().r)
                if (f(i, c+d) > f(dq.back().l, c+d)) c += d;
            dq.back().r = c; seg.l = c + 1;
        }
        if (seg.l <= n) dq.push_back(seg);
    }
}
```

8.4 ConvexHull Optimization

```
struct L {
    mutable int64_t a, b, p;
    bool operator<(const L &r) const { return a < r.a; }
    bool operator<(int64_t x) const { return p < x; }
};
struct DynamicHull : multiset<L, less<>> {
    static const int64_t kInf = 1e18;
    bool Isect(iterator x, iterator y) {
        auto Div = [](int64_t a, int64_t b) {
            return a / b - ((a ^ b) < 0 && a % b);
        };
        if (y == end()) { x->p = kInf; return false; }
        if (x->a == y->a) x->p = x->b > y->b ? kInf : -kInf;
        else x->p = Div(y->b - x->b, x->a - y->a);
        return x->p >= y->p;
    }
    void Insert(int64_t a, int64_t b) {
        auto z = insert({a, b, 0}); y = z++; x = y;
        while (Isect(y, z)) z = erase(z);
        if (x != begin() && Isect(--x, y)) Isect(x, y=erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            Isect(x, erase(y));
    }
    int64_t Query(int64_t x) {
        auto l = *lower_bound(x);
        return l.a * x + l.b;
    }
};
```

8.5 Josephus Problem

```
// n people kill m for each turn
int f(int n, int m) {
    int s = 0;
    for (int i = 2; i <= n; ++i)
        s = (s + m) % i;
    return s;
}
// died at kth
int kth(int n, int m, int k){
    if (m == 1) return n-1;
    for (k = k*m+m-1; k >= n; k = k-n+(k-n)/(m-1));
    return k;
}
```

8.6 Cactus Matching

```
vector<int> init_g[maxn], g[maxn*2];
int n, dfn[maxn], low[maxn], par[maxn], dfs_idx, bcc_id;
void tarjan(int u){
    dfn[u]=low[u]=++dfs_idx;
    for(int i=0; i<(int)init_g[u].size(); ++i){
        int v=init_g[u][i];
        if(v==par[u]) continue;
        if(!dfn[v]){
```

```
par[v]=u;
tarjan(v);
low[u]=min(low[u], low[v]);
if(dfn[u]<low[v]){
    g[u].push_back(v);
    g[v].push_back(u);
}
}
else{
    low[u]=min(low[u], dfn[v]);
    if(dfn[v]<dfn[u]){
        int temp_v=u;
        bcc_id++;
        while(temp_v!=v){
            g[bcc_id+n].push_back(temp_v);
            g[temp_v].push_back(bcc_id+n);
            temp_v=par[temp_v];
        }
        g[bcc_id+n].push_back(v);
        g[v].push_back(bcc_id+n);
        reverse(g[bcc_id+n].begin(), g[bcc_id+n].end());
    }
}
}
int dp[maxn][2], min_dp[2][2], tmp[2][2], tp[2];
void dfs(int u, int fa){
    if(u<=n){
        for(int i=0; i<(int)g[u].size(); ++i){
            int v=g[u][i];
            if(v==fa) continue;
            dfs(v, u);
            memset(tp, 0x8f, sizeof tp);
            if(v<=n){
                tp[0]=dp[u][0]+max(dp[v][0], dp[v][1]);
                tp[1]=max(
                    dp[u][0]+dp[v][0]+1,
                    dp[u][1]+max(dp[v][0], dp[v][1])
                );
            }
            else{
                tp[0]=dp[u][0]+dp[v][0];
                tp[1]=max(dp[u][0]+dp[v][1], dp[u][1]+dp[v][0]);
            }
            dp[u][0]=tp[0], dp[u][1]=tp[1];
        }
    }
    else{
        for(int i=0; i<(int)g[u].size(); ++i){
            int v=g[u][i];
            if(v==fa) continue;
            dfs(v, u);
        }
        min_dp[0][0]=0;
        min_dp[1][1]=1;
        min_dp[0][1]=min_dp[1][0]=-0x3f3f3f3f;
        for(int i=0; i<(int)g[u].size(); ++i){
            int v=g[u][i];
            if(v==fa) continue;
            memset(tmp, 0x8f, sizeof tmp);
            tmp[0][0]=max(
                min_dp[0][0]+max(dp[v][0], dp[v][1]),
                min_dp[0][1]+dp[v][0]
            );
            tmp[0][1]=min_dp[0][0]+dp[v][0]+1;
            tmp[1][0]=max(
                min_dp[1][0]+max(dp[v][0], dp[v][1]),
                min_dp[1][1]+dp[v][0]
            );
            tmp[1][1]=min_dp[1][0]+dp[v][0]+1;
            memcpy(min_dp, tmp, sizeof tmp);
        }
        dp[u][1]=max(min_dp[0][1], min_dp[1][0]);
        dp[u][0]=min_dp[0][0];
    }
}
int main(){
    int m, a, b;
    scanf("%d%d", &n, &m);
    for(int i=0; i<m; ++i){
        scanf("%d%d", &a, &b);
        init_g[a].push_back(b);
        init_g[b].push_back(a);
    }
    par[1]=-1;
```

```

tarjan(1);
dfs(1, -1);
printf("%d\n", max(dp[1][0], dp[1][1]));
return 0;
}

```

8.7 Tree Knapsack

```

int dp[N][K]; PII obj[N];
vector<int> G[N];
void dfs(int u, int mx){
    for(int s: G[u]) {
        if(mx < obj[s].first) continue;
        for(int i=0; i<=mx-obj[s].FF; i++)
            dp[s][i] = dp[u][i];
        dfs(s, mx - obj[s].first);
        for(int i=obj[s].FF; i<=mx; i++)
            dp[u][i] = max(dp[u][i],
                dp[s][i - obj[s].FF] + obj[s].SS);
    }
}

```

8.8 N Queens Problem

```

vector<int> solve(int n) {
    // no solution when n=2, 3
    vector<int> ret;
    if (n % 6 == 2) {
        for (int i = 2; i <= n; i += 2)
            ret.push_back(i);
        ret.push_back(3); ret.push_back(1);
        for (int i = 7; i <= n; i += 2)
            ret.push_back(i);
        ret.push_back(5);
    } else if (n % 6 == 3) {
        for (int i = 4; i <= n; i += 2)
            ret.push_back(i);
        ret.push_back(2);
        for (int i = 5; i <= n; i += 2)
            ret.push_back(i);
        ret.push_back(1); ret.push_back(3);
    } else {
        for (int i = 2; i <= n; i += 2)
            ret.push_back(i);
        for (int i = 1; i <= n; i += 2)
            ret.push_back(i);
    }
    return ret;
}

```

8.9 Aliens Optimization

```

long long Alien() {
    long long c = kInf;
    for (int d = 60; d >= 0; --d) {
        // cost can be negative, depending on the problem.
        if (c - (1LL << d) < 0) continue;
        long long ck = c - (1LL << d);
        pair<long long, int> r = check(ck);
        if (r.second == k) return r.first - ck * k;
        if (r.second < k) c = ck;
    }
    pair<long long, int> r = check(c);
    return r.first - c * k;
}

```

8.10 Hilbert Curve

```

long long hilbert(int n, int x, int y) {
    long long res = 0;
    for (int s = n / 2; s; s >>= 1) {
        int rx = (x & s) > 0, ry = (y & s) > 0;
        res += s * 111 * s * ((3 * rx) ^ ry);
        if (ry == 0) {
            if (rx == 1) x = s - 1 - x, y = s - 1 - y;
            swap(x, y);
        }
    }
    return res;
}

```

8.11 Binary Search On Fraction

```

struct Q {
    ll p, q;
    Q go(Q b, ll d) { return {p + b.p*d, q + b.q*d}; }
};
bool pred(Q);
// returns smallest p/q in [lo, hi] such that
// pred(p/q) is true, and 0 <= p, q <= N
Q frac_bs(ll N) {
    Q lo{0, 1}, hi{1, 0};
    if (pred(lo)) return lo;
    assert(pred(hi));
    bool dir = 1, L = 1, H = 1;
    for (; L || H; dir = !dir) {
        ll len = 0, step = 1;
        for (int t = 0; t < 2 && (t ? step/=2 : step*=2);)
            if (Q mid = hi.go(lo, len + step);
                mid.p > N || mid.q > N || dir ^ pred(mid))
                t++;
            else len += step;
        swap(lo, hi = hi.go(lo, len));
        (dir ? L : H) = !len;
    }
    return dir ? hi : lo;
}

```