Contents

1 Basic

1.1 vimrc

```
se is nu bs=2 ru mouse=a encoding=utf-8 ls=2
se cin cino+=j1 et sw=4 sts=4 tgc sc hls
syn on
colorscheme desert
filetype indent on
inoremap {<CR> {<CR>}<ESC>O
map <F8> <ESC>:w<CR>:!g++ "%" -o "%<" -std=c++17 -
    DKISEKI -Wall -Wextra -Wshadow -Wfatal-errors -
    Wconversion -fsanitize=address,undefined -g && echo
     success<CR>
map <F9> <ESC>:w<CR>:!g++ "%" -o "%<" -O2 -std=c++17 &&
     echo success<CR>
map <F10> <ESC>:!./"%<"<CR>
```

1.2 Debug Macro

```
#ifdef KISEKI
#define safe cerr<<__PRETTY_FUNCTION__\
  <<" line "<<__LINE__<<" safe\n"
#define debug(a...) qwerty(#a, a)
#define orange(a...) dvorak(#a, a)
using std::cerr;
template <typename ...T>
void qwerty(const char *s, T ...a) {
  cerr << "\e[1;32m(" << s << ") = (";
  int cnt = sizeof...(T);
  (..., (cerr << a << (--cnt ? ", " : ")\e[0m\n")));
}
template <typename Iter>
void dvorak(const char *s, Iter L, Iter R) {
  cerr << "\e[1;32m[ " << s << " ] = [ ";
  for (int f = 0; L != R; ++L)
    cerr << (f++ ? ", " : "") << *L;
  cerr << " ]\e[0m\n";
}
#else
#define safe ((void)0)
#define debug(...) ((void)0)
#define orange(...) ((void)0)
#endif
```

1.3 Increase Stack

```
const int size = 256 << 20;
register long rsp asm("rsp");
char *p = (char*)malloc(size)+size, *bak = (char*)rsp;
__asm__("movq %0, %%rsp\n"::"r"(p));
// main
__asm__("movq %0, %%rsp\n"::"r"(bak));
```

1.4 Pragma Optimization

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,tune=native")
```

1.5 IO Optimization

```
static inline int gc() {
  constexpr int B = 1<<20;
  static char buf[B], *p, *q;
  if(p == q &&
    (q=(p=buf)+fread(buf,1,B,stdin)) == buf)
   return EOF;
  return *p++;
}
template < typename T >
static inline bool gn( T &x ) {
 int c = gc(); T sgn = 1; x = 0;
 while(('0'>c||c>'9') && c!=EOF && c!='-') c = gc();
 if(c == '-') sgn = -1, c = gc();
 if(c == EOF) return false;
 while('0'<=c&&c<='9') x = x*10 + c - '0', c = gc();
 return x *= sgn, true;
}
```

2 Data Structure

2.1 Dark Magic

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
using namespace __gnu_pbds;
// heap tags: paring/binary/binomial/rc_binomial/thin
template<typename T>
using pbds_heap=__gnu_pbds::prioity_queue<T,less<T>, \
                  pairing_heap_tag>;
// pbds_heap::point_iterator
// x = pq.push(10); pq.modify(x, 87); a.join(b);
// tree tags: rb_tree_tag/ov_tree_tag/splay_tree_tag
template<typename T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
// find_by_order, order_of_key
// hash tables: cc_hash_table/gp_hash_table
```

2.2 Link-Cut Tree

```
struct Node{
 Node *par,*ch[2];
 int xor_sum,v;
 bool is_rev;
 Node(int _v){
  v=xor_sum=_v;is_rev=false;
  par=ch[0]=ch[1]=nullptr;
 }
 inline void set_rev(){is_rev^=1;swap(ch[0],ch[1]);}
 inline void down(){
  if(is_rev){
   if(ch[0]!=nullptr) ch[0]->set_rev();
   if(ch[1]!=nullptr) ch[1]->set_rev();
   is_rev=false;
  }
 }
 inline void up(){
  xor_sum=v;
  if(ch[0]!=nullptr){
   xor_sum^=ch[0]->xor_sum;
   ch[0]->par=this;
  }
  if(ch[1]!=nullptr){
   xor_sum^=ch[1]->xor_sum;
   ch[1]->par=this;
  }
 }
 inline bool is_root(){
  return par==nullptr ||\
   (par->ch[0]!=this && par->ch[1]!=this);
 }
 bool is_rch(){return !is_root() && par->ch[1]==this;}
} *node[maxn],*stk[maxn];
int top;
void to_child(Node* p,Node* c,bool dir){
 p->ch[dir]=c;
 p->up();
}
inline void rotate(Node* node){
 Node* par=node->par;
 Node* par_par=par->par;
 bool dir=node->is_rch();
 bool par_dir=par->is_rch();
 to_child(par,node->ch[!dir],dir);
 to_child(node,par,!dir);
 if(par_par!=nullptr && par_par->ch[par_dir]==par)
  to_child(par_par,node,par_dir);
 else node->par=par_par;
}
inline void splay(Node* node){
 Node* tmp=node;
 stk[top++]=node;
 while(!tmp->is_root()){
  tmp=tmp->par;
  stk[top++]=tmp;
 }
 while(top) stk[--top]->down();
 for(Node *fa=node->par;
  !node->is_root();
  rotate(node),fa=node->par)
  if(!fa->is_root())
   rotate(fa->is_rch()==node->is_rch()?fa:node);
}
inline void access(Node* node){
 Node* last=nullptr;
 while(node!=nullptr){
  splay(node);
  to_child(node,last,true);
  last=node;
  node=node->par;
 }
}
inline void change_root(Node* node){
 access(node);splay(node);node->set_rev();
}
inline void link(Node* x,Node* y){
 change_root(x);splay(x);x->par=y;
}
inline void split(Node* x,Node* y){
 change_root(x);access(y);splay(x);
 to_child(x,nullptr,true);y->par=nullptr;
}
inline void change_val(Node* node,int v){
 access(node);splay(node);node->v=v;node->up();
}
inline int query(Node* x,Node* y){
 change_root(x);access(y);splay(y);
 return y->xor_sum;
}
inline Node* find_root(Node* node){
 access(node);splay(node);
 Node* last=nullptr;
 while(node!=nullptr){
  node->down();last=node;node=node->ch[0];
 }
 return last;
}
set<pii> dic;
inline void add_edge(int u,int v){
 if(u>v) swap(u,v);
 if(find_root(node[u])==find_root(node[v])) return;
 dic.insert(pii(u,v));
 link(node[u],node[v]);
}
inline void del_edge(int u,int v){
 if(u>v) swap(u,v);
 if(dic.find(pii(u,v))==dic.end()) return;
 dic.erase(pii(u,v));
 split(node[u],node[v]);
}
```

2.3 LiChao Segment Tree

```
struct Line{
 int m, k, id;
 Line() : id( -1 ) {}
 Line( int a, int b, int c )
  : m( a ), k( b ), id( c ) {}
 int at( int x ) { return m * x + k; }
```

```
};
class LiChao {
 private:
  int n; vector< Line > nodes;
  inline int lc( int x ) { return 2 * x + 1; }
  inline int rc( int x ) { return 2 * x + 2; }
  void insert( int l, int r, int id, Line ln ) {
   int m = ( l + r ) >> 1;
   if ( nodes[ id ].id == -1 ) {
    nodes[ id ] = ln;
    return;
   }
   bool atLeft = nodes[ id ].at( l ) < ln.at( l );
   if ( nodes[ id ].at( m ) < ln.at( m ) ) {
    atLeft ^= 1; swap( nodes[ id ], ln );
   }
   if ( r - l == 1 ) return;
   if ( atLeft ) insert( l, m, lc( id ), ln );
   else insert( m, r, rc( id ), ln );
  }
  int query( int l, int r, int id, int x ) {
   int ret = 0;
   if ( nodes[ id ].id != -1 )
    ret = nodes[ id ].at( x );
   int m = ( l + r ) >> 1;
   if ( r - l == 1 ) return ret;
   else if ( x < m )
    return max( ret, query( l, m, lc( id ), x ) );
   else
    return max( ret, query( m, r, rc( id ), x ) );
  }
 public:
  void build( int n_ ) {
   n = n_; nodes.clear();
   nodes.resize( n << 2, Line() );
  }
  void insert( Line ln ) { insert( 0, n, 0, ln ); }
  int query( int x ) { return query( 0, n, 0, x ); }
} lichao;
```

2.4 Treap

```
namespace Treap{
 #define sz( x ) ( ( x ) ? ( ( x )->size ) : 0 )
 struct node{
  int size;
  uint32_t pri;
  node *lc, *rc;
  node() : size(0), pri(rand()), lc( 0 ), rc( 0 ) {}
  void pull() {
   size = 1;
   if ( lc ) size += lc->size;
   if ( rc ) size += rc->size;
  }
 };
 node* merge( node* L, node* R ) {
  if ( not L or not R ) return L ? L : R;
  if ( L->pri > R->pri ) {
   L->rc = merge( L->rc, R ); L->pull();
   return L;
  } else {
   R->lc = merge( L, R->lc ); R->pull();
   return R;
  }
 }
 void split_by_size( node*rt,int k,node*&L,node*&R ) {
  if ( not rt ) L = R = nullptr;
  else if( sz( rt->lc ) + 1 <= k ) {
   L = rt;
   split_by_size( rt->rc,k-sz(rt->lc)-1,L->rc,R );
   L->pull();
  } else {
   R = rt;
   split_by_size( rt->lc, k, L, R->lc );
   R->pull();
  }
 }
 #undef sz
}
```

2.5 Sparse Table

```
template < typename T, typename Cmp_ = less< T > >
class SparseTable {
private:
 vector< vector< T > > tbl;
 vector< int > lg;
 T cv( T a, T b ) {
  return Cmp_()( a, b ) ? a : b;
 }
public:
 void init( T arr[], int n ) {
  // 0-base
  lg.resize( n + 1 );
  lg[ 0 ] = -1;
  for( int i=1 ; i<=n ; ++i ) lg[i] = lg[i>>1] + 1;
  tbl.resize( lg[n] + 1 );
  tbl[ 0 ].resize( n );
  copy( arr, arr + n, tbl[ 0 ].begin() );
  for ( int i = 1 ; i <= lg[ n ] ; ++ i ) {
   int len = 1 << ( i - 1 ), sz = 1 << i;
   tbl[ i ].resize( n - sz + 1 );
   for ( int j = 0 ; j <= n - sz ; ++ j )
    tbl[i][j] = cv(tbl[i-1][j], tbl[i-1][j+len]);
  }
 }
 T query( int l, int r ) {
  // 0-base [l, r)
  int wh = lg[ r - l ], len = 1 << wh;
  return cv( tbl[ wh ][ l ], tbl[ wh ][ r - len ] );
 }
};
```

2.6 Linear Basis

```
template <int BITS>
struct LinearBasis {
 array<uint64_t, BITS> basis;
 Basis() { basis.fill(0); }
 void add(uint64_t x) {
  for (int i = 0; i < BITS; ++i) if ((x >> i) & 1) {
   if (basis[i] == 0) {
    basis[i] = x;
    return;
   }
   x ^= basis[i];
  }
 }
 bool ok(uint64_t x) {
  for (int i = 0; i < BITS; ++i)
   if ((x >> i) & 1) x ^= basis[i];
  return x == 0;
 }
};
```

2.7 Binary Search On Segment Tree

```
// find_first = x -> minimal x s.t. check( [a, x) )
// find_last = x -> maximal x s.t. check( [x, b) )
template <typename C>
int find_first(int l, const C &check) {
 if (l >= n)
  return n;
 l += sz;
 for (int i = height; i > 0; i--)
  propagate(l >> i);
 Monoid sum = identity;
 do {
  while ((l & 1) == 0)
   l >>= 1;
  if (check(f(sum, data[l]))) {
   while (l < sz) {
    propagate(l);
    l <<= 1;
    auto nxt = f(sum, data[l]);
    if (not check(nxt)) {
     sum = nxt;
     l++;
    }
   }
   return l + 1 - sz;
  }
  sum = f(sum, data[l++]);
 } while ((l & -l) != l);
 return n;
}

template <typename C>
```

```
int find_last(int r, const C &check) {
 if (r <= 0)
  return -1;
 r += sz;
 for (int i = height; i > 0; i--)
  propagate((r - 1) >> i);
 Monoid sum = identity;
 do {
  r--;
  while (r > 1 and (r & 1))
   r >>= 1;
  if (check(f(data[r], sum))) {
   while (r < sz) {
    propagate(r);
    r = (r << 1) + 1;
    auto nxt = f(data[r], sum);
    if (not check(nxt)) {
     sum = nxt;
     r--;
    }
   }
   return r - sz;
  }
  sum = f(data[r], sum);
 } while ((r & -r) != r);
 return -1;
}
```

3 Graph

3.1 BCC Edge

```
class BCC_Bridge {
 private:
  int n, ecnt;
  vector<vector<pair<int,int>>> G;
  vector<int> dfn, low;
  vector<bool> bridge;
  void dfs(int u, int f) {
   dfn[u] = low[u] = dfn[f] + 1;
   for (auto [v, t]: G[u]) {
    if (v == f) continue;
    if (dfn[v]) {
     low[u] = min(low[u], dfn[v]);
     continue;
    }
    dfs(v, u);
    low[u] = min(low[u], low[v]);
    if (low[v] > dfn[u]) bridge[t] = true;
   }
  }
 public:
  void init(int n_) {
   G.clear(); G.resize(n = n_);
   low.assign(n, ecnt = 0);
   dfn.assign(n, 0);
  }
  void add_edge(int u, int v) {
   G[u].emplace_back(v, ecnt);
   G[v].emplace_back(u, ecnt++);
  }
  void solve() {
   bridge.assign(ecnt, false);
   for (int i = 0; i < n; ++i)
    if (not dfn[i]) dfs(i, i);
  }
  bool is_bridge(int x) { return bridge[x]; }
} bcc_bridge;
```

3.2 BCC Vertex

```
class BCC_AP {
 private:
  int n, ecnt;
  vector<vector<pair<int,int>>> G;
  vector<int> bcc, dfn, low, st;
  vector<bool> ap, ins;
  void dfs(int u, int f) {
   dfn[u] = low[u] = dfn[f] + 1;
   int ch = 0;
   for (auto [v, t]: G[u]) if (v != f) {
    if (not ins[t]) {
     st.push_back(t);
     ins[t] = true;
    }
    if (dfn[v]) {
     low[u] = min(low[u], dfn[v]);
     continue;
    } ++ch; dfs(v, u);
    low[u] = min(low[u], low[v]);
    if (low[v] >= dfn[u]) {
     ap[u] = true;
     while (true) {
      int eid = st.back(); st.pop_back();
      bcc[eid] = ecnt;
      if (eid == t) break;
     }
     ecnt++;
    }
   }
   if (ch == 1 and u == f) ap[u] = false;
  }
 public:
  void init(int n_) {
   G.clear(); G.resize(n = n_);
   ecnt = 0; ap.assign(n, false);
   low.assign(n, 0); dfn.assign(n, 0);
  }
  void add_edge(int u, int v) {
   G[u].emplace_back(v, ecnt);
   G[v].emplace_back(u, ecnt++);
  }
  void solve() {
   ins.assign(ecnt, false);
   bcc.resize(ecnt); ecnt = 0;
   for (int i = 0; i < n; ++i)
    if (not dfn[i]) dfs(i, i);
  }
  int get_id(int x) { return bcc[x]; }
  int count() { return ecnt; }
  bool is_ap(int x) { return ap[x]; }
} bcc_ap;
```

3.3 2-SAT (SCC)

```
class TwoSat{
 private:
  int n;
  vector<vector<int>> rG,G,sccs;
  vector<int> ord,idx;
  vector<bool> vis,result;
  void dfs(int u){
   vis[u]=true;
   for(int v:G[u])
    if(!vis[v]) dfs(v);
   ord.push_back(u);
  }
  void rdfs(int u){
   vis[u]=false;idx[u]=sccs.size()-1;
   sccs.back().push_back(u);
   for(int v:rG[u])
    if(vis[v])rdfs(v);
  }
 public:
  void init(int n_){
   n=n_;G.clear();G.resize(n);
   rG.clear();rG.resize(n);
   sccs.clear();ord.clear();
   idx.resize(n);result.resize(n);
  }
  void add_edge(int u,int v){
   G[u].push_back(v);rG[v].push_back(u);
  }
  void orr(int x,int y){
   if ((x^y)==1)return;
   add_edge(x^1,y); add_edge(y^1,x);
  }
  bool solve(){
   vis.clear();vis.resize(n);
   for(int i=0;i<n;++i)
    if(not vis[i])dfs(i);
   reverse(ord.begin(),ord.end());
   for (int u:ord){
    if(!vis[u])continue;
    sccs.push_back(vector<int>());
    rdfs(u);
```

```
  }
  for(int i=0;i<n;i+=2)
   if(idx[i]==idx[i+1])
    return false;
  vector<bool> c(sccs.size());
  for(size_t i=0;i<sccs.size();++i){
   for(size_t j=0;j<sccs[i].size();++j){
    result[sccs[i][j]]=c[i];
    c[idx[sccs[i][j]^1]]=!c[i];
   }
  }
  return true;
 }
 bool get(int x){return result[x];}
 inline int get_id(int x){return idx[x];}
 inline int count(){return sccs.size();}
} sat2;
```

3.4 Lowbit Decomposition

```
class LowbitDecomp{
private:
 int time_, chain_, LOG_N;
 vector< vector< int > > G, fa;
 vector< int > tl, tr, chain, chain_st;
 // chain_ : number of chain
 // tl, tr[ u ] : subtree interval in the seq. of u
 // chain_st[ u ] : head of the chain contains u
 // chian[ u ] : chain id of the chain u is on
 void predfs( int u, int f ) {
  chain[ u ] = 0;
  for ( int v : G[ u ] ) {
   if ( v == f ) continue;
   predfs( v, u );
   if( lowbit( chain[ u ] ) < lowbit( chain[ v ] ) )
    chain[ u ] = chain[ v ];
  }
  if ( not chain[ u ] )
   chain[ u ] = chain_ ++;
 }
 void dfschain( int u, int f ) {
  fa[ u ][ 0 ] = f;
  for ( int i = 1 ; i < LOG_N ; ++ i )
   fa[ u ][ i ] = fa[ fa[ u ][ i - 1 ] ][ i - 1 ];
  tl[ u ] = time_++;
  if ( not chain_st[ chain[ u ] ] )
   chain_st[ chain[ u ] ] = u;
  for ( int v : G[ u ] )
   if ( v != f and chain[ v ] == chain[ u ] )
    dfschain( v, u );
  for ( int v : G[ u ] )
   if ( v != f and chain[ v ] != chain[ u ] )
    dfschain( v, u );
  tr[ u ] = time_;
 }
 bool anc( int u, int v ) {
  return tl[ u ] <= tl[ v ] and tr[ v ] <= tr[ u ];
 }
public:
 int lca( int u, int v ) {
  if ( anc( u, v ) ) return u;
  for ( int i = LOG_N - 1 ; i >= 0 ; -- i )
   if ( not anc( fa[ u ][ i ], v ) )
    u = fa[ u ][ i ];
  return fa[ u ][ 0 ];
 }
 void init( int n ) {
  fa.assign( ++n, vector< int >( LOG_N ) );
  for ( LOG_N = 0 ; ( 1 << LOG_N ) < n ; ++ LOG_N );
  G.clear(); G.resize( n );
  tl.assign( n, 0 ); tr.assign( n, 0 );
  chain.assig( n, 0 ); chain_st.assign( n, 0 );
 }
 void add_edge( int u , int v ) {
  // 1-base
  G[ u ].push_back( v );
  G[ v ].push_back( u );
 }
 void decompose(){
  chain_ = 1;
  predfs( 1, 1 );
  time_ = 0;
  dfschain( 1, 1 );
 }
 PII get_subtree(int u) { return {tl[ u ],tr[ u ] }; }
 vector< PII > get_path( int u , int v ){
  vector< PII > res;
  int g = lca( u, v );
  while ( chain[ u ] != chain[ g ] ) {
   int s = chain_st[ chain[ u ] ];
   res.emplace_back( tl[ s ], tl[ u ] + 1 );
   u = fa[ s ][ 0 ];
  }
  res.emplace_back( tl[ g ], tl[ u ] + 1 );
  while ( chain[ v ] != chain[ g ] ) {
   int s = chain_st[ chain[ v ] ];
   res.emplace_back( tl[ s ], tl[ v ] + 1 );
   v = fa[ s ][ 0 ];
  }
  res.emplace_back( tl[ g ] + 1, tl[ v ] + 1 );
  return res;
  /* res : list of intervals from u to v
   * ( note only nodes work, not edge )
   * usage :
   * vector< PII >& path = tree.get_path( u , v )
   * for( auto [ l, r ] : path ) {
   *  0-base [ l, r )
   * }
   */
 }
} tree;
```

3.5 MaxClique

```
// contain a self loop u to u, than u won't in clique
template < size_t MAXN >
class MaxClique{
private:
 using bits = bitset< MAXN >;
 bits popped, G[ MAXN ], ans;
 size_t deg[ MAXN ], deo[ MAXN ], n;
 void sort_by_degree() {
  popped.reset();
  for ( size_t i = 0 ; i < n ; ++ i )
    deg[ i ] = G[ i ].count();
  for ( size_t i = 0 ; i < n ; ++ i ) {
    size_t mi = MAXN, id = 0;
    for ( size_t j = 0 ; j < n ; ++ j )
      if ( not popped[ j ] and deg[ j ] < mi )
        mi = deg[ id = j ];
    popped[ deo[ i ] = id ] = 1;
    for( size_t u = G[ i ]._Find_first() ;
     u < n ; u = G[ i ]._Find_next( u ) )
       -- deg[ u ];
  }
 }
 void BK( bits R, bits P, bits X ) {
  if (R.count()+P.count() <= ans.count()) return;
  if ( not P.count() and not X.count() ) {
   if ( R.count() > ans.count() ) ans = R;
   return;
  }
  /* greedily chosse max degree as pivot
  bits cur = P | X; size_t pivot = 0, sz = 0;
  for ( size_t u = cur._Find_first() ;
   u < n ; u = cur._Find_next( u ) )
    if ( deg[ u ] > sz ) sz = deg[ pivot = u ];
  cur = P & ( ~G[ pivot ] );
  */ // or simply choose first
  bits cur = P & (~G[ ( P | X )._Find_first() ]);
  for ( size_t u = cur._Find_first() ;
   u < n ; u = cur._Find_next( u ) ) {
   if ( R[ u ] ) continue;
   R[ u ] = 1;
   BK( R, P & G[ u ], X & G[ u ] );
   R[ u ] = P[ u ] = 0, X[ u ] = 1;
  }
 }
public:
 void init( size_t n_ ) {
  n = n_;
  for ( size_t i = 0 ; i < n ; ++ i )
   G[ i ].reset();
  ans.reset();
 }
 void add_edges( int u, bits S ) { G[ u ] = S; }
```

```
 void add_edge( int u, int v ) {
  G[ u ][ v ] = G[ v ][ u ] = 1;
 }
 int solve() {
  sort_by_degree(); // or simply iota( deo... )
  for ( size_t i = 0 ; i < n ; ++ i )
   deg[ i ] = G[ i ].count();
  bits pob, nob = 0; pob.set();
  for (size_t i=n; i<MAXN; ++i) pob[i] = 0;
  for ( size_t i = 0 ; i < n ; ++ i ) {
   size_t v = deo[ i ];
   bits tmp; tmp[ v ] = 1;
   BK( tmp, pob & G[ v ], nob & G[ v ] );
   pob[ v ] = 0, nob[ v ] = 1;
  }
  return static_cast< int >( ans.count() );
 }
};
```

3.6 MaxCliqueDyn

```
constexpr int kN = 150;
struct MaxClique { // Maximum Clique
 bitset<kN> a[kN], cs[kN];
 int ans, sol[kN], q, cur[kN], d[kN], n;
 void init(int _n) {
  n = _n, ans = q = 0;
  for (int i = 0; i < n; i++) a[i].reset();
 }
 void addEdge(int u, int v) { a[u][v] = a[v][u] = 1; }
 void csort(vector<int> &r, vector<int> &c) {
  int mx = 1, km = max(ans - q + 1, 1), t = 0,
    m = int(r.size());
  cs[1].reset(); cs[2].reset();
  for (int i = 0; i < m; i++) {
   int p = r[i], k = 1;
   while ((cs[k] & a[p]).count()) k++;
   if (k > mx) cs[++mx + 1].reset();
   cs[k][p] = 1;
   if (k < km) r[t++] = p;
  }
  c.resize(m);
  if (t) c[t - 1] = 0;
  for (int k = km; k <= mx; k++) {
   for (int p = int(cs[k]._Find_first());
      p < kN; p = int(cs[k]._Find_next(p))) {
    r[t] = p; c[t++] = k;
   }
  }
 }
 void dfs(vector<int> &r, vector<int> &c, int l,
  bitset<kN> mask) {
  while (!r.empty()) {
   int p = r.back(); r.pop_back();
   mask[p] = 0;
   if (q + c.back() <= ans) return;
   cur[q++] = p;
   vector<int> nr, nc;
   bitset<kN> nmask = mask & a[p];
   for (int i : r)
    if (a[p][i]) nr.push_back(i);
   if (!nr.empty()) {
    if (l < 4) {
     for (int i : nr)
      d[i] = int((a[i] & nmask).count());
     sort(nr.begin(), nr.end(),
      [&](int x, int y) {
       return d[x] > d[y];
      });
    }
    csort(nr, nc); dfs(nr, nc, l + 1, nmask);
   } else if (q > ans) {
    ans = q; copy(cur, cur + q, sol);
   }
   c.pop_back(); q--;
  }
 }
 int solve(bitset<kN> mask) { // vertex mask
  vector<int> r, c;
  for (int i = 0; i < n; i++)
   if (mask[i]) r.push_back(i);
  for (int i = 0; i < n; i++)
   d[i] = int((a[i] & mask).count());
  sort(r.begin(), r.end(),
   [&](int i, int j) { return d[i] > d[j]; });
  csort(r, c);
  dfs(r, c, 1, mask);
  return ans; // sol[0 ~ ans-1]
 }
} graph;
```

3.7 Virtual Tree

```
inline bool cmp(const int &i, const int &j) {
 return dfn[i] < dfn[j];
}
void build(int vectrices[], int k) {
 static int stk[MAX_N];
 sort(vectrices, vectrices + k, cmp);
 stk[sz++] = 0;
 for (int i = 0; i < k; ++i) {
  int u = vectrices[i], lca = LCA(u, stk[sz - 1]);
  if (lca == stk[sz - 1]) stk[sz++] = u;
  else {
   while (sz >= 2 && dep[stk[sz - 2]] >= dep[lca]) {
    addEdge(stk[sz - 2], stk[sz - 1]);
    sz--;
   }
   if (stk[sz - 1] != lca) {
    addEdge(lca, stk[--sz]);
    stk[sz++] = lca, vectrices[cnt++] = lca;
   }
   stk[sz++] = u;
  }
 }
 for (int i = 0; i < sz - 1; ++i)
  addEdge(stk[i], stk[i + 1]);
}
```

3.8 Centroid Decomposition

```
struct Centroid {
 vector<vector<int64_t>> Dist;
 vector<int> Parent, Depth;
 vector<int64_t> Sub, Sub2;
 vector<int> Sz, Sz2;
 Centroid(vector<vector<pair<int, int>>> g) {
  int N = g.size();
  vector<bool> Vis(N);
  vector<int> sz(N), mx(N);
  vector<int> Path;
  Dist.resize(N);
  Parent.resize(N);
  Depth.resize(N);
  auto DfsSz = [&](auto dfs, int x) -> void {
   Vis[x] = true; sz[x] = 1; mx[x] = 0;
   for (auto [u, w] : g[x]) {
    if (Vis[u]) continue;
    dfs(dfs, u);
    sz[x] += sz[u];
    mx[x] = max(mx[x], sz[u]);
   }
   Path.push_back(x);
  };
  auto DfsDist = [&](auto dfs, int x, int64_t D = 0)
   -> void {
   Dist[x].push_back(D);Vis[x] = true;
   for (auto [u, w] : g[x]) {
    if (Vis[u]) continue;
    dfs(dfs, u, D + w);
   }
  };
  auto Dfs = [&]
   (auto dfs, int x, int D = 0, int p = -1)->void {
   Path.clear(); DfsSz(DfsSz, x);
   int M = Path.size();
   int C = -1;
   for (int u : Path) {
    if (max(M - sz[u], mx[u]) * 2 <= M) C = u;
    Vis[u] = false;
   }
   DfsDist(DfsDist, C);
   for (int u : Path) Vis[u] = false;
   Parent[C] = p; Vis[C] = true;
   Depth[C] = D;
   for (auto [u, w] : g[C]) {
    if (Vis[u]) continue;
```

```
    dfs(dfs, u, D + 1, C);
   }
  };
  Dfs(Dfs, 0); Sub.resize(N); Sub2.resize(N);
  Sz.resize(N); Sz2.resize(N);
 }
 void Mark(int v) {
  int x = v, z = -1;
  for (int i = Depth[v]; i >= 0; --i) {
   Sub[x] += Dist[v][i]; Sz[x]++;
   if (z != -1) {
    Sub2[z] += Dist[v][i];
    Sz2[z]++;
   }
   z = x; x = Parent[x];
  }
 }
 int64_t Query(int v) {
  int64_t res = 0;
  int x = v, z = -1;
  for (int i = Depth[v]; i >= 0; --i) {
   res += Sub[x] + 1LL * Sz[x] * Dist[v][i];
   if (z != -1) res-=Sub2[z]+1LL*Sz2[z]*Dist[v][i];
   z = x; x = Parent[x];
  }
  return res;
 }
};
```

3.9 Tree Hashing

```
uint64_t hsah(int u, int f) {
 uint64_t r = 127;
 for (int v : G[ u ]) if (v != f) {
  uint64_t hh = hsah(v, u);
  r=(r+(hh*hh)%1010101333)%1011820613;
 }
 return r;
}
```

3.10 Minimum Mean Cycle

```
/* minimum mean cycle O(VE) */
struct MMC{
#define FZ(n) memset((n),0,sizeof(n))
#define E 101010
#define V 1021
#define inf 1e9
 struct Edge { int v,u; double c; };
 int n, m, prv[V][V], prve[V][V], vst[V];
 Edge e[E];
 vector<int> edgeID, cycle, rho;
 double d[V][V];
 void init( int _n ) { n = _n; m = 0; }
 // WARNING: TYPE matters
 void add_edge( int vi , int ui , double ci )
 { e[ m ++ ] = { vi , ui , ci }; }
 void bellman_ford() {
  for(int i=0; i<n; i++) d[0][i]=0;
  for(int i=0; i<n; i++) {
   fill(d[i+1], d[i+1]+n, inf);
   for(int j=0; j<m; j++) {
    int v = e[j].v, u = e[j].u;
    if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
     d[i+1][u] = d[i][v]+e[j].c;
     prv[i+1][u] = v;
     prve[i+1][u] = j;
    }
   }
  }
 }
 double solve(){
  // returns inf if no cycle, mmc otherwise
  double mmc=inf;
  int st = -1;
  bellman_ford();
  for(int i=0; i<n; i++) {
   double avg=-inf;
   for(int k=0; k<n; k++) {
    if(d[n][i]<inf-eps)
     avg=max(avg,(d[n][i]-d[k][i])/(n-k));
    else avg=max(avg,inf);
   }
   if (avg < mmc) tie(mmc, st) = tie(avg, i);
  }
  FZ(vst);edgeID.clear();cycle.clear();rho.clear();
  for (int i=n; !vst[st]; st=prv[i--][st]) {
   vst[st]++;
   edgeID.PB(prve[i][st]);
   rho.PB(st);
  }
  while (vst[st] != 2) {
   int v = rho.back(); rho.pop_back();
   cycle.PB(v);
   vst[v]++;
  }
  reverse(ALL(edgeID));
  edgeID.resize(SZ(cycle));
  return mmc;
 }
} mmc;
```

3.11 Mo's Algorithm on Tree

```
int q; vector< int > G[N];
struct Que{
 int u, v, id;
} que[ N ];
int dfn[N], dfn_, block_id[N], block_, stk[N], stk_;
void dfs( int u, int f ) {
 dfn[ u ] = dfn_++; int saved_rbp = stk_;
 for ( int v : G[ u ] ) {
  if ( v == f ) continue;
  dfs( v, u );
  if ( stk_ - saved_rbp < SQRT_N ) continue;
  for ( ++ block_ ; stk_ != saved_rbp ; )
    block_id[ stk[ -- stk_ ] ] = block_;
 }
 stk[ stk_ ++ ] = u;
}
bool inPath[ N ];
void Diff( int u ) {
 if ( inPath[ u ] ^= 1 ) { /*remove this edge*/ }
 else { /*add this edge*/ }
}
void traverse( int& origin_u, int u ) {
 for ( int g = lca( origin_u, u ) ;
  origin_u != g ; origin_u = parent_of[ origin_u ] )
   Diff( origin_u );
 for (int v = u; v != origin_u; v = parent_of[v])
  Diff( v );
 origin_u = u;
}
void solve() {
 dfs( 1, 1 );
 while ( stk_ ) block_id[ stk[ -- stk_ ] ] = block_;
 sort( que, que + q, [](const Que& x, const Que& y) {
  return tie( block_id[ x.u ], dfn[ x.v ] )
      < tie( block_id[ y.u ], dfn[ y.v ] );
 } );
 int U = 1, V = 1;
 for ( int i = 0 ; i < q ; ++ i ) {
  pass( U, que[ i ].u );
  pass( V, que[ i ].v );
  // we could get our answer of que[ i ].id
 }
}
/*
Method 2:
dfs u:
 push u
 iterate subtree
 push u
Let P = LCA(u, v), and St(u)<=St(v)
if (P == u) query[St(u), St(v)]
else query[Ed(u), St(v)], query[St(P), St(P)]
*/
```

3.12 Minimum Steiner Tree

```
// Minimum Steiner Tree
// O(V 3^T + V^2 2^T)
struct SteinerTree{
#define V 33
#define T 8
#define INF 1023456789
 int n , dst[V][V] , dp[1 << T][V] , tdst[V];
 void init( int _n ){
```

```
  n = _n;
  for( int i = 0 ; i < n ; i ++ ){
   for( int j = 0 ; j < n ; j ++ )
    dst[ i ][ j ] = INF;
   dst[ i ][ i ] = 0;
  }
 }
 void add_edge( int ui , int vi , int wi ){
  dst[ ui ][ vi ] = min( dst[ ui ][ vi ] , wi );
  dst[ vi ][ ui ] = min( dst[ vi ][ ui ] , wi );
 }
 void shortest_path(){
  for( int k = 0 ; k < n ; k ++ )
   for( int i = 0 ; i < n ; i ++ )
    for( int j = 0 ; j < n ; j ++ )
     dst[ i ][ j ] = min( dst[ i ][ j ],
        dst[ i ][ k ] + dst[ k ][ j ] );
 }
 int solve( const vector<int>& ter ){
  int t = (int)ter.size();
  for( int i = 0 ; i < ( 1 << t ) ; i ++ )
   for( int j = 0 ; j < n ; j ++ )
    dp[ i ][ j ] = INF;
  for( int i = 0 ; i < n ; i ++ )
   dp[ 0 ][ i ] = 0;
  for( int msk = 1 ; msk < ( 1 << t ) ; msk ++ ){
   if( msk == ( msk & (-msk) ) ){
    int who = __lg( msk );
    for( int i = 0 ; i < n ; i ++ )
     dp[ msk ][ i ] = dst[ ter[ who ] ][ i ];
    continue;
   }
   for( int i = 0 ; i < n ; i ++ )
    for( int submsk = ( msk - 1 ) & msk ; submsk ;
         submsk = ( submsk - 1 ) & msk )
      dp[ msk ][ i ] = min( dp[ msk ][ i ],
              dp[ submsk ][ i ] +
              dp[ msk ^ submsk ][ i ] );
   for( int i = 0 ; i < n ; i ++ ){
    tdst[ i ] = INF;
    for( int j = 0 ; j < n ; j ++ )
     tdst[ i ] = min( tdst[ i ],
           dp[ msk ][ j ] + dst[ j ][ i ] );
   }
   for( int i = 0 ; i < n ; i ++ )
    dp[ msk ][ i ] = tdst[ i ];
  }
  int ans = INF;
  for( int i = 0 ; i < n ; i ++ )
   ans = min( ans , dp[ ( 1 << t ) - 1 ][ i ] );
  return ans;
 }
} solver;
```

3.13 Directed Minimum Spanning Tree

```
struct DirectedMST { // find maximum
 struct Edge {
  int u, v;
  int w;
  Edge(int u, int v, int w) : u(u), v(v), w(w) {}
 };
 vector<Edge> Edges;
 void clear() { Edges.clear(); }
 void addEdge(int a, int b, int w) { Edges.emplace_back
    (a, b, w); }
 int solve(int root, int n) {
  vector<Edge> E = Edges;
  int ans = 0;
  while (true) {
   // find best in edge
   vector<int> in(n, -inf), prv(n, -1);
   for (auto e : E)
    if (e.u != e.v && e.w > in[e.v]) {
     in[e.v] = e.w;
     prv[e.v] = e.u;
    }
   in[root] = 0;
   prv[root] = -1;
   for (int i = 0; i < n; i++)
    if (in[i] == -inf)
     return -inf;
   // find cycle
   int tot = 0;
   vector<int> id(n, -1), vis(n, -1);
   for (int i = 0; i < n; i++) {
    ans += in[i];
    for (int x = i; x != -1 && id[x] == -1; x = prv[x])
     {
     if (vis[x] == i) {
      for (int y = prv[x]; y != x; y = prv[y])
       id[y] = tot;
      id[x] = tot++;
      break;
     }
     vis[x] = i;
    }
   }
   if (!tot)
    return ans;
   for (int i = 0; i < n; i++)
    if (id[i] == -1)
     id[i] = tot++;
   // shrink
   for (auto &e : E) {
    if (id[e.u] != id[e.v])
     e.w -= in[e.v];
    e.u = id[e.u], e.v = id[e.v];
   }
   n = tot;
   root = id[root];
  }
  assert(false);
 }
} DMST;
```

3.14 Dominator Tree

```
namespace dominator {
vector<int> g[maxn], r[maxn], rdom[maxn];
int dfn[maxn], rev[maxn], fa[maxn], sdom[maxn];
int dom[maxn], val[maxn], rp[maxn], tk;
void init(int n) {
 // vertices are numbered from 0 to n - 1
 fill(dfn, dfn + n, -1);fill(rev, rev + n, -1);
 fill(fa, fa + n, -1);fill(val, val + n, -1);
 fill(sdom, sdom + n, -1);fill(rp, rp + n, -1);
 fill(dom, dom + n, -1); tk = 0;
 for (int i = 0; i < n; ++i) {
  g[i].clear(); r[i].clear(); rdom[i].clear();
 }
}
void add_edge(int x, int y) { g[x].push_back(y); }
void dfs(int x) {
 rev[dfn[x] = tk] = x;
 fa[tk] = sdom[tk] = val[tk] = tk; tk ++;
 for (int u : g[x]) {
  if (dfn[u] == -1) dfs(u), rp[dfn[u]] = dfn[x];
  r[dfn[u]].push_back(dfn[x]);
 }
}
void merge(int x, int y) { fa[x] = y; }
int find(int x, int c = 0) {
 if (fa[x] == x) return c ? -1 : x;
 int p = find(fa[x], 1);
 if (p == -1) return c ? fa[x] : val[x];
 if (sdom[val[x]]>sdom[val[fa[x]]]) val[x]=val[fa[x]];
 fa[x] = p;
 return c ? p : val[x];
}
vector<int> build(int s, int n) {
// return the father of each node in the dominator tree
// p[i] = -2 if i is unreachable from s
 dfs(s);
 for (int i = tk - 1; i >= 0; --i) {
  for (int u:r[i]) sdom[i]=min(sdom[i],sdom[find(u)]);
  if (i) rdom[sdom[i]].push_back(i);
  for (int &u : rdom[i]) {
   int p = find(u);
   if (sdom[p] == i) dom[u] = i;
   else dom[u] = p;
  }
  if (i) merge(i, rp[i]);
 }
 vector<int> p(n, -2); p[s] = -1;
 for (int i = 1; i < tk; ++i)
```

```
  if (sdom[i] != dom[i]) dom[i] = dom[dom[i]];
 for (int i = 1; i < tk; ++i) p[rev[i]] = rev[dom[i]];
 return p;
}}
```

3.15 Edge Coloring

```
// max(d_u) + 1 edge coloring, time: O(NM)
int C[kN][kN], G[kN][kN]; // 1-based, G: ans
void clear(int N) {
 for (int i = 0; i <= N; i++)
  for (int j = 0; j <= N; j++)
    C[i][j] = G[i][j] = 0;
}
void solve(vector<pair<int, int>> &E, int N) {
 int X[kN] = {}, a;
 auto update = [&](int u) {
  for (X[u] = 1; C[u][X[u]]; X[u]++);
 };
 auto color = [&](int u, int v, int c) {
  int p = G[u][v];
  G[u][v] = G[v][u] = c;
  C[u][c] = v, C[v][c] = u;
  C[u][p] = C[v][p] = 0;
  if (p) X[u] = X[v] = p;
  else update(u), update(v);
  return p;
 };
 auto flip = [&](int u, int c1, int c2) {
  int p = C[u][c1];
  swap(C[u][c1], C[u][c2]);
  if (p) G[u][p] = G[p][u] = c2;
  if (!C[u][c1]) X[u] = c1;
  if (!C[u][c2]) X[u] = c2;
  return p;
 };
 for (int i = 1; i <= N; i++) X[i] = 1;
 for (int t = 0; t < E.size(); t++) {
  auto [u, v] = E[t];
  int v0 = v, c = X[u], c0 = c, d;
  vector<pair<int, int>> L; int vst[kN] = {};
  while (!G[u][v0]) {
   L.emplace_back(v, d = X[v]);
   if (!C[v][c]) for(a=L.size()-1;a>=0;a--)
     c = color(u, L[a].first, c);
   else if(!C[u][d])for(a=L.size()-1;a>=0;a--)
     color(u, L[a].first, L[a].second);
   else if (vst[d]) break;
   else vst[d] = 1, v = C[u][d];
  }
  if (!G[u][v0]) {
   for (; v; v = flip(v, c, d), swap(c, d));
   if (C[u][c0]) { a = int(L.size()) - 1;
    while (--a >= 0 && L[a].second != c);
    for(;a>=0;a--)color(u,L[a].first,L[a].second);
   } else t--;
  }
 }
}
```

4 Matching & Flow

4.1 Kuhn Munkres

```
class KM {
private:
 static constexpr lld INF = 1LL << 60;
 vector<lld> hl,hr,slk;
 vector<int> fl,fr,pre,qu;
 vector<vector<lld>> w;
 vector<bool> vl,vr;
 int n, ql, qr;
 bool check(int x) {
  if (vl[x] = true, fl[x] != -1)
   return vr[qu[qr++] = fl[x]] = true;
  while (x != -1) swap(x, fr[fl[x] = pre[x]]);
  return false;
 }
 void bfs(int s) {
  fill(slk.begin(), slk.end(), INF);
  fill(vl.begin(), vl.end(), false);
  fill(vr.begin(), vr.end(), false);
  ql = qr = 0;
  qu[qr++] = s;
  vr[s] = true;
  while (true) {
   lld d;
   while (ql < qr) {
    for (int x = 0, y = qu[ql++]; x < n; ++x) {
     if(!vl[x]&&slk[x]>=(d=hl[x]+hr[y]-w[x][y])){
      if (pre[x] = y, d) slk[x] = d;
      else if (!check(x)) return;
     }
    }
   }
   d = INF;
   for (int x = 0; x < n; ++x)
    if (!vl[x] && d > slk[x]) d = slk[x];
   for (int x = 0; x < n; ++x) {
    if (vl[x]) hl[x] += d;
    else slk[x] -= d;
    if (vr[x]) hr[x] -= d;
   }
   for (int x = 0; x < n; ++x)
    if (!vl[x] && !slk[x] && !check(x)) return;
  }
 }
public:
 void init( int n_ ) {
  n = n_; qu.resize(n);
  fl.clear(); fl.resize(n, -1);
  fr.clear(); fr.resize(n, -1);
  hr.clear(); hr.resize(n); hl.resize(n);
  w.clear(); w.resize(n, vector<lld>(n));
  slk.resize(n); pre.resize(n);
  vl.resize(n); vr.resize(n);
 }
 void set_edge( int u, int v, lld x ) {w[u][v] = x;}
 lld solve() {
  for (int i = 0; i < n; ++i)
   hl[i] = *max_element(w[i].begin(), w[i].end());
  for (int i = 0; i < n; ++i) bfs(i);
  lld res = 0;
  for (int i = 0; i < n; ++i) res += w[i][fl[i]];
  return res;
 }
} km;
```

4.2 Bipartite Matching

```
class BipartiteMatching{
private:
 vector<int> X[N], Y[N];
 int fX[N], fY[N], n;
 bitset<N> walked;
 bool dfs(int x){
  for(auto i:X[x]){
   if(walked[i])continue;
   walked[i]=1;
   if(fY[i]==-1||dfs(fY[i])){
    fY[i]=x;fX[x]=i;
    return 1;
   }
  }
  return 0;
 }
public:
 void init(int _n){
  n=_n; walked.reset();
  for(int i=0;i<n;i++){
   X[i].clear();Y[i].clear();
   fX[i]=fY[i]=-1;
  }
 }
 void add_edge(int x, int y){
  X[x].push_back(y); Y[y].push_back(y);
 }
 int solve(){
  int cnt = 0;
  for(int i=0;i<n;i++){
   walked.reset();
   if(dfs(i)) cnt++;
  }
  // return how many pair matched
  return cnt;
 }
```

```
};
```

4.3 General Graph Matching

```
namespace matching {
int fa[kN], pre[kN], match[kN], s[kN], v[kN];
vector<int> g[kN];
queue<int> q;
void Init(int n) {
 for (int i = 0; i <= n; ++i) match[i] = pre[i] = n;
 for (int i = 0; i < n; ++i) g[i].clear();
}
void AddEdge(int u, int v) {
 g[u].push_back(v);
 g[v].push_back(u);
}
int Find(int u) {
 return u == fa[u] ? u : fa[u] = Find(fa[u]);
}
int LCA(int x, int y, int n) {
 static int tk = 0; tk++;
 x = Find(x), y = Find(y);
 for (; ; swap(x, y)) {
  if (x != n) {
   if (v[x] == tk) return x;
   v[x] = tk;
   x = Find(pre[match[x]]);
  }
 }
}
void Blossom(int x, int y, int l) {
 while (Find(x) != l) {
  pre[x] = y, y = match[x];
  if (s[y] == 1) q.push(y), s[y] = 0;
  if (fa[x] == x) fa[x] = l;
  if (fa[y] == y) fa[y] = l;
  x = pre[y];
 }
}
bool Bfs(int r, int n) {
 for (int i = 0; i <= n; ++i) fa[i] = i, s[i] = -1;
 while (!q.empty()) q.pop();
 q.push(r);
 s[r] = 0;
 while (!q.empty()) {
  int x = q.front(); q.pop();
  for (int u : g[x]) {
   if (s[u] == -1) {
    pre[u] = x, s[u] = 1;
    if (match[u] == n) {
     for (int a = u, b = x, last; b != n; a = last, b =
     pre[a])
      last = match[b], match[b] = a, match[a] = b;
     return true;
    }
    q.push(match[u]);
    s[match[u]] = 0;
   } else if (!s[u] && Find(u) != Find(x)) {
    int l = LCA(u, x, n);
    Blossom(x, u, l);
    Blossom(u, x, l);
   }
  }
 }
 return false;
}
int Solve(int n) {
 int res = 0;
 for (int x = 0; x < n; ++x) {
  if (match[x] == n) res += Bfs(x, n);
 }
 return res;
}}
```

4.4 Minimum Weight Matching (Clique version)

```
struct Graph {
 // 0-base (Perfect Match)
 int n, edge[MXN][MXN];
 int match[MXN],dis[MXN],onstk[MXN];
 vector<int> stk;
 void init(int _n) {
  n = _n;
  for (int i=0; i<n; i++)
   for (int j=0; j<n; j++)
    edge[i][j] = 0;
 }
 void set_edge(int u, int v, int w) {
  edge[u][v] = edge[v][u] = w;
 }
 bool SPFA(int u){
  if (onstk[u]) return true;
  stk.PB(u);
  onstk[u] = 1;
  for (int v=0; v<n; v++){
   if (u != v && match[u] != v && !onstk[v]){
    int m = match[v];
    if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
     dis[m] = dis[u] - edge[v][m] + edge[u][v];
     onstk[v] = 1;
     stk.PB(v);
     if (SPFA(m)) return true;
     stk.pop_back();
     onstk[v] = 0;
    }
   }
  }
  onstk[u] = 0;
  stk.pop_back();
  return false;
 }

 int solve() {
  // find a match
  for (int i=0; i<n; i+=2){
   match[i] = i+1;
   match[i+1] = i;
  }
  while (true){
   int found = 0;
   for (int i=0; i<n; i++)
    dis[i] = onstk[i] = 0;
   for (int i=0; i<n; i++){
    stk.clear();
    if (!onstk[i] && SPFA(i)){
     found = 1;
     while (SZ(stk)>=2){
      int u = stk.back(); stk.pop_back();
      int v = stk.back(); stk.pop_back();
      match[u] = v;
      match[v] = u;
     }
    }
   }
   if (!found) break;
  }
  int ret = 0;
  for (int i=0; i<n; i++)
   ret += edge[i][match[i]];
  return ret>>1;
 }
} graph;
```

4.5 Minimum Cost Circulation

```
struct Edge { int to, cap, rev, cost; };
vector<Edge> g[kN];
int dist[kN], pv[kN], ed[kN];
bool mark[kN];
int NegativeCycle(int n) {
 memset(mark, false, sizeof(mark));
 memset(dist, 0, sizeof(dist));
 int upd = -1;
 for (int i = 0; i <= n; ++i) {
  for (int j = 0; j < n; ++j) {
   int idx = 0;
   for (auto &e : g[j]) {
    if(e.cap > 0 && dist[e.to] > dist[j] + e.cost){
     dist[e.to] = dist[j] + e.cost;
     pv[e.to] = j, ed[e.to] = idx;
     if (i == n) {
      upd = j;
      while(!mark[upd])mark[upd]=1,upd=pv[upd];
      return upd;
     }
    }
    idx++;
```

```
    }
   }
  }
  return -1;
}
int Solve(int n) {
  int rt = -1, ans = 0;
  while ((rt = NegativeCycle(n)) >= 0) {
   memset(mark, false, sizeof(mark));
   vector<pair<int, int>> cyc;
   while (!mark[rt]) {
    cyc.emplace_back(pv[rt], ed[rt]);
    mark[rt] = true;
    rt = pv[rt];
   }
   reverse(cyc.begin(), cyc.end());
   int cap = kInf;
   for (auto &i : cyc) {
    auto &e = g[i.first][i.second];
    cap = min(cap, e.cap);
   }
   for (auto &i : cyc) {
    auto &e = g[i.first][i.second];
    e.cap -= cap;
    g[e.to][e.rev].cap += cap;
    ans += e.cost * cap;
   }
  }
  return ans;
}
```

4.6 Flow Models

- Maximum/Minimum flow with lower bound / Circulation problem
  1. Construct super source S and sink T.
  2. For each edge (x, y, l, u), connect $x \to y$ with capacity $u - l$.
  3. For each vertex v, denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  4. If $in(v) > 0$, connect $S \to v$ with capacity $in(v)$, otherwise, connect $v \to T$ with capacity $-in(v)$.
     - To maximize, connect $t \to s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T. If $f \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
     - To minimize, let f be the maximum flow from S to T. Connect $t \to s$ with capacity ∞ and let the flow from S to T be f'. If $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, f' is the answer.
  5. The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
  1. Redirect every edge: $y \to x$ if $(x, y) \in M$, $x \to y$ otherwise.
  2. DFS from unmatched vertices in X.
  3. $x \in X$ is chosen iff x is unvisited.
  4. $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
  1. Consruct super source S and sink T
  2. For each edge (x, y, c), connect $x \to y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \to x$ with $(cost, cap) = (-c, 1)$
  3. For each edge with $c < 0$, sum these cost as K, then increase $d(y)$ by 1, decrease $d(x)$ by 1
  4. For each vertex v with $d(v) > 0$, connect $S \to v$ with $(cost, cap) = (0, d(v))$
  5. For each vertex v with $d(v) < 0$, connect $v \to T$ with $(cost, cap) = (0, -d(v))$
  6. Flow from S to T, the answer is the cost of the flow $C + K$
- Maximum density induced subgraph
  1. Binary search on answer, suppose we're checking answer T
  2. Construct a max flow model, let K be the sum of all weights
  3. Connect source $s \to v$, $v \in G$ with capacity K
  4. For each edge (u, v, w) in G, connect $u \to v$ and $v \to u$ with capacity w
  5. For $v \in G$, connect it with sink $v \to t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
  6. T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
  1. For each $v \in V$ create a copy v', and connect $u' \to v'$ with weight $w(u, v)$.
  2. Connect $v \to v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v.
  3. Find the minimum weight perfect matching on G'.
- Project selection problem
  1. If $p_v > 0$, create edge (s, v) with capacity p_v; otherwise, create edge (v, t) with capacity $-p_v$.
  2. Create edge (u, v) with capacity w with w being the cost of choosing u without choosing v.
  3. The mincut is equivalent to the maximum profit of a subset of projects.
- 0/1 quadratic programming

  $$\sum_x c_x x + \sum_y c_y \bar{y} + \sum_{xy} c_{xy} x\bar{y} + \sum_{xyx'y'} c_{xyx'y'}(x\bar{y} + x'\bar{y'})$$

  can be minimized by the mincut of the following graph:
  1. Create edge (x, t) with capacity c_x and create edge (s, y) with capacity c_y.
  2. Create edge (x, y) with capacity c_{xy}.
  3. Create edge (x, y) and edge (x', y') with capacity $c_{xyx'y'}$.

4.7 Dinic

```
template <typename Cap = int64_t>
class Dinic{
private:
  struct E{
    int to, rev;
    Cap cap;
  };
  int n, st, ed;
  vector<vector<E>> G;
  vector<int> lv, idx;
  bool BFS(){
    lv.assign(n, -1);
    queue<int> bfs;
    bfs.push(st); lv[st] = 0;
    while (not bfs.empty()){
      int u = bfs.front(); bfs.pop();
      for (auto e: G[u]) {
        if (e.cap <= 0 or lv[e.to]!=-1) continue;
        bfs.push(e.to); lv[e.to] = lv[u] + 1;
      }
    }
    return lv[ed] != -1;
  }
  Cap DFS(int u, Cap f){
    if (u == ed) return f;
    Cap ret = 0;
    for(int &i = idx[u]; i < int(G[u].size()); ++i) {
      auto &e = G[u][i];
      if (e.cap <= 0 or lv[e.to]!=lv[u]+1) continue;
      Cap nf = DFS(e.to, min(f, e.cap));
      ret += nf; e.cap -= nf; f -= nf;
      G[e.to][e.rev].cap += nf;
      if (f == 0) return ret;
    }
    if (ret == 0) lv[u] = -1;
    return ret;
  }
public:
  void init(int n_) { G.assign(n = n_, vector<E>()); }
  void add_edge(int u, int v, Cap c){
    G[u].push_back({v, int(G[v].size()), c});
    G[v].push_back({u, int(G[u].size())-1, 0});
  }
  Cap max_flow(int st_, int ed_){
    st = st_, ed = ed_; Cap ret = 0;
    while (BFS()) {
      idx.assign(n, 0);
      Cap f = DFS(st, numeric_limits<Cap>::max());
      ret += f;
      if (f == 0) break;
    }
    return ret;
  }
};
```

4.8 Minimum Cost Maximum Flow

```
class MiniCostMaxiFlow{
 using Cap = int; using Wei = int64_t;
 using PCW = pair<Cap,Wei>;
 static constexpr Cap INF_CAP = 1 << 30;
 static constexpr Wei INF_WEI = 1LL<<60;
private:
 struct Edge{
  int to, back;
  Cap cap; Wei wei;
  Edge() {}
  Edge(int a,int b, Cap c, Wei d):
   to(a),back(b),cap(c),wei(d)
```

```
  {}
 };
 int ori, edd;
 vector<vector<Edge>> G;
 vector<int> fa, wh;
 vector<bool> inq;
 vector<Wei> dis;
 PCW SPFA(){
  fill(inq.begin(),inq.end(),false);
  fill(dis.begin(),dis.end(),INF_WEI);
  queue<int> qq; qq.push(ori);
  dis[ori] = 0;
  while(not qq.empty()){
   int u=qq.front();qq.pop();
   inq[u] = false;
   for(int i=0;i<SZ(G[u]);++i){
    Edge e=G[u][i];
    int v=e.to; Wei d=e.wei;
    if(e.cap<=0||dis[v]<=dis[u]+d)
     continue;
    dis[v] = dis[u] + d;
    fa[v] = u, wh[v] = i;
    if (inq[v]) continue;
    qq.push(v);
    inq[v] = true;
   }
  }
  if(dis[edd]==INF_WEI) return {-1, -1};
  Cap mw=INF_CAP;
  for(int i=edd;i!=ori;i=fa[i])
   mw=min(mw,G[fa[i]][wh[i]].cap);
  for (int i=edd;i!=ori;i=fa[i]){
   auto &eg=G[fa[i]][wh[i]];
   eg.cap -= mw;
   G[eg.to][eg.back].cap+=mw;
  }
  return {mw, dis[edd]};
 }
public:
 void init(int a,int b,int n){
  ori=a,edd=b;
  G.clear();G.resize(n);
  fa.resize(n);wh.resize(n);
  inq.resize(n); dis.resize(n);
 }
 void add_edge(int st, int ed, Cap c, Wei w){
  G[st].emplace_back(ed,SZ(G[ed]),c,w);
  G[ed].emplace_back(st,SZ(G[st])-1,0,-w);
 }
 PCW solve(){
  Cap cc=0; Wei ww=0;
  while(true){
   PCW ret=SPFA();
   if(ret.first==-1) break;
   cc+=ret.first;
   ww+=ret.first * ret.second;
  }
  return {cc,ww};
 }
} mcmf;
```

4.9 GomoryHu Tree

```
int g[maxn];
vector<edge> GomoryHu(int n){
 vector<edge> rt;
 for(int i=1;i<=n;++i)g[i]=1;
 for(int i=2;i<=n;++i){
  int t=g[i];
  flow.reset(); // clear flows on all edge
  rt.push_back({i,t,flow(i,t)});
  flow.walk(i); // bfs points that connected to i (use
    edges not fully flow)
  for(int j=i+1;j<=n;++j){
   if(g[j]==t && flow.connect(j))g[j]=i; // check if i
    can reach j
  }
 }
 return rt;
}
```

4.10 Global Min-Cut

```
const int maxn = 500 + 5;
int w[maxn][maxn], g[maxn];
bool v[maxn], del[maxn];
void add_edge(int x, int y, int c) {
 w[x][y] += c; w[y][x] += c;
}
pair<int, int> phase(int n) {
 memset(v, false, sizeof(v));
 memset(g, 0, sizeof(g));
 int s = -1, t = -1;
 while (true) {
  int c = -1;
  for (int i = 0; i < n; ++i) {
   if (del[i] || v[i]) continue;
   if (c == -1 || g[i] > g[c]) c = i;
  }
  if (c == -1) break;
  v[s = t, t = c] = true;
  for (int i = 0; i < n; ++i) {
   if (del[i] || v[i]) continue;
   g[i] += w[c][i];
  }
 }
 return make_pair(s, t);
}
int mincut(int n) {
 int cut = 1e9;
 memset(del, false, sizeof(del));
 for (int i = 0; i < n - 1; ++i) {
  int s, t; tie(s, t) = phase(n);
  del[t] = true; cut = min(cut, g[t]);
  for (int j = 0; j < n; ++j) {
   w[s][j] += w[t][j]; w[j][s] += w[j][t];
  }
 }
 return cut;
}
```

5 Math

5.1 Prime Table

1002939109, 1020288887, 1028798297, 1038684299, 1041211027, 1051762951, 1058585963, 1063020809, 1147930723, 1172520109, 1183835981, 1187659051, 1241251303, 1247184097, 1255940849, 1272759031, 1287027493, 1288511629, 1294632499, 1312650799, 1868732623, 1884198443, 1884616807, 1885059541, 1909942399, 1914471137, 1923951707, 1925453197, 1979612177, 1980446837, 1989761941, 2007826547, 2008033571, 2011186739, 2039465081, 2039728567, 2093735719, 2116097521, 2123852629, 2140170259, 3148478261, 3153064147, 3176351071, 3187523093, 3196772239, 3201312913, 3203063977, 3204840059, 3210224309, 3213032591, 3217689851, 3218469083, 3219857533, 3231880427, 3235951699, 3273767923, 3276188869, 3277183181, 3282463507, 3285553889, 3319309027, 3327005333, 3327574903, 3341387953, 3373293941, 3380077549, 3380892997, 3381118801

5.2 $\lfloor \frac{n}{i} \rfloor$ Enumeration

$T_0 = 1, T_{i+1} = \lfloor \frac{n}{\lfloor \frac{n}{T_i+1} \rfloor} \rfloor$

5.3 ax+by=gcd

```
// ax+ny = 1, ax+ny == ax == 1 (mod n)
void exgcd(lld x,lld y,lld &g,lld &a,lld &b) {
 if (y == 0) g=x,a=1,b=0;
 else exgcd(y,x%y,g,b,a),b-=(x/y)*a;
}
```

5.4 Pollard Rho

```
// does not work when n is prime
// return any non-trivial factor
llu pollard_rho(llu n){
 static auto f=[](llu x,llu k,llu m){
  return add(k,mul(x,x,m),m);
 };
 if (!(n&1)) return 2;
 mt19937 rnd(120821011);
 while(true){
  llu y=2,yy=y,x=rnd()%n,t=1;
  for(llu sz=2;t==1;sz<<=1) {
   for(llu i=0;i<sz;++i){
    if(t!=1)break;
    yy=f(yy,x,n);
    t=gcd(yy>y?yy-y:y-yy,n);
```

```
    }
    y=yy;
   }
   if(t!=1&&t!=n) return t;
  }
}
```

5.5 Pi Count (Linear Sieve)

```
static constexpr int N = 1000000 + 5;
lld pi[N];
vector<int> primes;
bool sieved[N];
lld cube_root(lld x){
 lld s=cbrt(x-static_cast<long double>(0.1));
 while(s*s*s <= x) ++s;
 return s-1;
}
lld square_root(lld x){
 lld s=sqrt(x-static_cast<long double>(0.1));
 while(s*s <= x) ++s;
 return s-1;
}
void init(){
 primes.reserve(N);
 primes.push_back(1);
 for(int i=2;i<N;i++) {
  if(!sieved[i]) primes.push_back(i);
  pi[i] = !sieved[i] + pi[i-1];
  for(int p: primes) if(p > 1) {
   if(p * i >= N) break;
   sieved[p * i] = true;
   if(p % i == 0) break;
  }
 }
}
lld phi(lld m, lld n) {
 static constexpr int MM = 80000, NN = 500;
 static lld val[MM][NN];
 if(m<MM&&n<NN&&val[m][n])return val[m][n]-1;
 if(n == 0) return m;
 if(primes[n] >= m) return 1;
 lld ret = phi(m,n-1)-phi(m/primes[n],n-1);
 if(m<MM&&n<NN) val[m][n] = ret+1;
 return ret;
}
lld pi_count(lld);
lld P2(lld m, lld n) {
 lld sm = square_root(m), ret = 0;
 for(lld i = n+1;primes[i]<=sm;i++)
  ret+=pi_count(m/primes[i])-pi_count(primes[i])+1;
 return ret;
}
lld pi_count(lld m) {
 if(m < N) return pi[m];
 lld n = pi_count(cube_root(m));
 return phi(m, n) + n - 1 - P2(m, n);
}
```

5.6 Strling Number

5.6.1 First Kind

$S_1(n,k)$ counts the number of permutations of n elements with k disjoint cycles.

$$S_1(n,k) = (n-1)\cdot S_1(n-1,k) + S_1(n-1,k-1)$$

$$x(x+1)\dots(x+n-1) = \sum_{k=0}^{n} S_1(n,k)x^k$$

$$g(x) = x(x+1)\dots(x+n-1) = \sum_{k=0}^{n} a_k x^k$$

$$\Rightarrow g(x+n) = \sum_{k=0}^{n} \frac{b_k}{(n-k)!}x^{n-k},$$

$$b_k = \sum_{i=0}^{k}((n-i)!a_{n-i})\cdot(\frac{n^{k-i}}{(k-i)!})$$

5.6.2 Second Kind

$S_2(n,k)$ counts the number of ways to partition a set of n elements into k nonempty sets.

$$S_2(n,k) = S_2(n-1,k-1) + k\cdot S_2(n-1,k)$$

$$S_2(n,k) = \sum_{i=0}^{k}\binom{k}{i}i^n(-1)^{k-i} = \sum_{i=0}^{k}\frac{(-1)^i}{i!}\cdot\frac{(k-i)^n}{(k-i)!}$$

5.7 Range Sieve

```
const int MAX_SQRT_B = 50000;
const int MAX_L = 200000 + 5;

bool is_prime_small[MAX_SQRT_B];
bool is_prime[MAX_L];

void sieve(lld l, lld r){
 // [l, r)
 for(lld i=2;i*i<r;i++) is_prime_small[i] = true;
 for(lld i=l;i<r;i++) is_prime[i-l] = true;
 if(l==1) is_prime[0] = false;
 for(lld i=2;i*i<r;i++){
  if(!is_prime_small[i]) continue;
  for(lld j=i*i;j*j<r;j+=i) is_prime_small[j]=false;
  for(lld j=std::max(2LL, (l+i-1)/i)*i;j<r;j+=i)
    is_prime[j-l]=false;
 }
}
```

5.8 Miller Rabin

```
bool isprime(llu x){
 static llu magic[]={2,325,9375,28178,\
          450775,9780504,1795265022};
 static auto witn=[](llu a,llu u,llu n,int t)
 ->bool{
  if (!(a = mpow(a%n,u,n)))return 0;
  while(t--){
   llu a2=mul(a,a,n);
   if(a2==1 && a!=1 && a!=n-1)
    return 1;
   a = a2;
  }
  return a!=1;
 };
 if(x<2)return 0;
 if(!(x&1))return x==2;
 llu x1=x-1;int t=0;
 while(!(x1&1))x1>>=1,t++;
 for(llu m:magic)if(witn(m,x1,x,t))return 0;
 return 1;
}
```

5.9 Extended Euler

$$a^b \equiv \begin{cases} a^{b \bmod \varphi(m)+\varphi(m)} & \text{if } (a,m)\neq 1 \land b\geq\varphi(m) \\ a^{b \bmod \varphi(m)} & \text{otherwise} \end{cases} \pmod m$$

5.10 Gauss Elimination

```
void gauss(vector<vector<double>> &d) {
 int n = d.size(), m = d[0].size();
 for (int i = 0; i < m; ++i) {
  int p = -1;
  for (int j = i; j < n; ++j) {
   if (fabs(d[j][i]) < eps) continue;
   if (p == -1 || fabs(d[j][i])>fabs(d[p][i])) p=j;
  }
  if (p == -1) continue;
  for (int j = 0; j < m; ++j) swap(d[p][j], d[i][j]);
  for (int j = 0; j < n; ++j) {
   if (i == j) continue;
   double z = d[j][i] / d[i][i];
   for (int k = 0; k < m; ++k) d[j][k] -= z*d[i][k];
  }
 }
}
```

5.11 Fast Fourier Transform

```
const int mod = 1000000007;

const int M1 = 985661441; // G = 3
const int M2 = 998244353;
const int M3 = 1004535809;

int superBigCRT(int64_t A, int64_t B, int64_t C) {
  static_assert (M1 <= M2 && M2 <= M3);
  constexpr int64_t r12 = modpow(M1, M2-2, M2);
```

```
  constexpr int64_t r13 = modpow(M1, M3-2, M3);
  constexpr int64_t r23 = modpow(M2, M3-2, M3);
  constexpr int64_t M1M2 = 1LL * M1 * M2 % mod;
  B = (B - A + M2) * r12 % M2;
  C = (C - A + M3) * r13 % M3;
  C = (C - B + M3) * r23 % M3;
  return (A + B * M1 + C * M1M2) % mod;
}

namespace fft {
using VI = vector<int>;
using VL = vector<long long>;
const double pi = acos(-1);
cplx omega[maxn + 1];
void prefft() {
 for (int i = 0; i <= maxn; i++)
  omega[i] = cplx(cos(2 * pi * j / maxn),
     sin(2 * pi * j / maxn));
}
void fft(vector<cplx> &v, int n) {
 int z = __builtin_ctz(n) - 1;
 for (int i = 0; i < n; ++i) {
  int x = 0, j = 0;
  for (;(1 << j) < n;++j) x^=(i >> j & 1)<<(z - j);
  if (x > i) swap(v[x], v[i]);
 }
 for (int s = 2; s <= n; s <<= 1) {
  int z = s >> 1;
  for (int i = 0; i < n; i += s) {
   for (int k = 0; k < z; ++k) {
    cplx x = v[i + z + k] * omega[maxn / s * k];
    v[i + z + k] = v[i + k] - x;
    v[i + k] = v[i + k] + x;
   }
  }
 }
}
void ifft(vector<cplx> &v, int n) {
 fft(v, n);
 reverse(v.begin() + 1, v.end());
 for (int i=0;i<n;++i) v[i] = v[i] * cplx(1. / n, 0);
}
VL convolution(const VI &a, const VI &b) {
 // Should be able to handle N <= 10^5, C <= 10^4
 int sz = 1;
 while (sz < a.size() + b.size() - 1) sz <<= 1;
 vector<cplx> v(sz);
 for (int i = 0; i < sz; ++i) {
  double re = i < a.size() ? a[i] : 0;
  double im = i < b.size() ? b[i] : 0;
  v[i] = cplx(re, im);
 }
 fft(v, sz);
 for (int i = 0; i <= sz / 2; ++i) {
  int j = (sz - i) & (sz - 1);
  cplx x = (v[i] + v[j].conj()) * (v[i] - v[j].conj())
    * cplx(0, -0.25);
  if (j != i) v[j] = (v[j] + v[i].conj()) * (v[j] - v[i
    ].conj()) * cplx(0, -0.25);
  v[i] = x;
 }
 ifft(v, sz);
 VL c(sz);
 for (int i = 0; i < sz; ++i) c[i] = round(v[i].re);
 return c;
}
VI convolution_mod(const VI &a, const VI &b, int p) {
 int sz = 1;
 while (sz + 1 < a.size() + b.size()) sz <<= 1;
 vector<cplx> fa(sz), fb(sz);
 for (int i = 0; i < (int)a.size(); ++i)
  fa[i] = cplx(a[i] & ((1 << 15) - 1), a[i] >> 15);
 for (int i = 0; i < (int)b.size(); ++i)
  fb[i] = cplx(b[i] & ((1 << 15) - 1), b[i] >> 15);
 fft(fa, sz), fft(fb, sz);
 double r = 0.25 / sz;
 cplx r2(0, -1), r3(r, 0), r4(0, -r), r5(0, 1);
 for (int i = 0; i <= (sz >> 1); ++i) {
  int j = (sz - i) & (sz - 1);
  cplx a1 = (fa[i] + fa[j].conj());
  cplx a2 = (fa[i] - fa[j].conj()) * r2;
  cplx b1 = (fb[i] + fb[j].conj()) * r3;
  cplx b2 = (fb[i] - fb[j].conj()) * r4;
  if (i != j) {
   cplx c1 = (fa[j] + fa[i].conj());
   cplx c2 = (fa[j] - fa[i].conj()) * r2;
   cplx d1 = (fb[j] + fb[i].conj()) * r3;
   cplx d2 = (fb[j] - fb[i].conj()) * r4;
   fa[i] = c1 * d1 + c2 * d2 * r5;
   fb[i] = c1 * d2 + c2 * d1;
  }
  fa[j] = a1 * b1 + a2 * b2 * r5;
  fb[j] = a1 * b2 + a2 * b1;
 }
 fft(fa, sz), fft(fb, sz);
 vector<int> res(sz);
 for (int i = 0; i < sz; ++i) {
  long long a = round(fa[i].re), b = round(fb[i].re),
       c = round(fa[i].im);
  res[i] = (a+((b % p) << 15)+((c % p) << 30)) % p;
 }
 return res;
}}
```

5.12 Chinese Remainder

```
lld crt(lld ans[], lld pri[], int n){
 lld M = 1, ret = 0;
 for(int i=0;i<n;i++) M *= pri[i];
 for(int i=0;i<n;i++){
  lld iv = (gcd(M/pri[i],pri[i]).FF+pri[i])%pri[i];
  ret += (ans[i]*(M/pri[i])%M * iv)%M;
  ret %= M;
 }
 return ret;
}
/*
Another:
x = a1 % m1
x = a2 % m2
g = gcd(m1, m2)
assert((a1-a2)%g==0)
[p, q] = exgcd(m2/g, m1/g)
return a2+m2*(p*(a1-a2)/g)
0 <= x < lcm(m1, m2)
*/
```

5.13 Berlekamp Massey

```
// x: 1-base, p[]: 0-base
template<size_t N>
vector<llf> BM(llf x[N],size_t n){
 size_t f[N]={0},t=0;llf d[N];
 vector<llf> p[N];
 for(size_t i=1,b=0;i<=n;++i) {
  for(size_t j=0;j<p[t].size();++j)
   d[i]+=x[i-j-1]*p[t][j];
  if(abs(d[i]-=x[i])<=EPS)continue;
  f[t]=i;if(!t){p[++t].resize(i);continue;}
  vector<llf> cur(i-f[b]-1);
  llf k=-d[i]/d[f[b]];cur.PB(-k);
  for(size_t j=0;j<p[b].size();j++)
   cur.PB(p[b][j]*k);
  if(cur.size()<p[t].size())cur.resize(p[t].size());
  for(size_t j=0;j<p[t].size();j++)cur[j]+=p[t][j];
  if(i-f[b]+p[b].size()>=p[t].size()) b=t;
  p[++t]=cur;
 }
 return p[t];
}
```

5.14 NTT

```
template <int mod, int G, int maxn>
struct NTT {
 static_assert (maxn == (maxn & -maxn));
 int roots[maxn];
 NTT () {
  int r = modpow(G, (mod - 1) / maxn);
  for (int i = maxn >> 1; i; i >>= 1) {
   roots[i] = 1;
   for (int j = 1; j < i; j++)
    roots[i + j] = modmul(roots[i + j - 1], r);
   r = modmul(r, r);
  }
 }
```

```
 // n must be 2^k, and 0 <= F[i] < mod
 void inplace_ntt(int n, int F[], bool inv = false) {
  for (int i = 0, j = 0; i < n; i++) {
   if (i < j) swap(F[i], F[j]);
   for (int k = n>>1; (j^=k) < k; k>>=1);
  }
  for (int s = 1; s < n; s *= 2) {
   for (int i = 0; i < n; i += s * 2) {
    for (int j = 0; j < s; j++) {
     int a = F[i+j];
     int b = modmul(F[i+j+s], roots[s+j]);
     F[i+j] = modadd(a, b);  // a + b
     F[i+j+s] = modsub(a, b); // a - b
    }
   }
  }
  if (inv) {
   int invn = modinv(n);
   for (int i = 0; i < n; i++)
    F[i] = modmul(F[i], invn);
   reverse(F + 1, F + n);
  }
 }
};
const int P=2013265921,root=31;
const int MAXN=1<<20;
NTT<P, root, MAXN> ntt;
```

5.15 Polynomial Operations

```
using VL = vector<LL>;
#define fi(s, n) for (int i=int(s); i<int(n); ++i)
#define Fi(s, n) for (int i=int(n); i>int(s); --i)
int n2k(int n) {
 int sz = 1; while (sz < n) sz <<= 1;
 return sz;
}
template<int MAXN, LL P, LL RT> // MAXN = 2^k
struct Poly { // coefficients in [0, P)
 static NTT<MAXN, P, RT> ntt;
 VL coef;
 int n() const { return coef.size(); } // n()>=1
 LL *data() { return coef.data(); }
 const LL *data() const { return coef.data(); }
 LL &operator[](size_t i) { return coef[i]; }
 const LL &operator[](size_t i)const{return coef[i];}
 Poly(initializer_list<LL> a) : coef(a) { }
 explicit Poly(int _n = 1) : coef(_n) { }
 Poly(const LL *arr, int _n) : coef(arr, arr + _n) {}
 Poly(const Poly &p, int _n) : coef(_n) {
  copy_n(p.data(), min(p.n(), _n), data());
 }
 Poly& irev(){return reverse(data(),data()+n()),*this;}
 Poly& isz(int _n) { return coef.resize(_n), *this; }
 Poly& iadd(const Poly &rhs) { // n() == rhs.n()
  fi(0, n()) if ((coef[i]+=rhs[i]) >= P)coef[i]-=P;
  return *this;
 }
 Poly& imul(LL k) {
  fi(0, n()) coef[i] = coef[i] * k % P;
  return *this;
 }
 Poly Mul(const Poly &rhs) const {
  const int _n = n2k(n() + rhs.n() - 1);
  Poly X(*this, _n), Y(rhs, _n);
  ntt(X.data(), _n), ntt(Y.data(), _n);
  fi(0, _n) X[i] = X[i] * Y[i] % P;
  ntt(X.data(), _n, true);
  return X.isz(n() + rhs.n() - 1);
 }
 Poly Inv() const { // coef[0] != 0
  if (n() == 1) return {ntt.minv(coef[0])};
  const int _n = n2k(n() * 2);
  Poly Xi = Poly(*this, (n() + 1)/2).Inv().isz(_n);
  Poly Y(*this, _n);
  ntt(Xi.data(), _n), ntt(Y.data(), _n);
  fi(0, _n) {
   Xi[i] *= (2 - Xi[i] * Y[i]) % P;
   if ((Xi[i] %= P) < 0) Xi[i] += P;
  }
  ntt(Xi.data(), _n, true);
  return Xi.isz(n());
 }
Poly Sqrt() const { // Jacobi(coef[0], P) = 1
 if (n()==1) return {QuadraticResidue(coef[0], P)};
 Poly X = Poly(*this, (n()+1) / 2).Sqrt().isz(n());
 return X.iadd(Mul(X.Inv()).isz(n())).imul(P/2+1);
}
pair<Poly, Poly> DivMod(const Poly &rhs) const {
 // (rhs.)back() != 0
 if (n() < rhs.n()) return {{0}, *this};
 const int _n = n() - rhs.n() + 1;
 Poly X(rhs); X.irev().isz(_n);
 Poly Y(*this); Y.irev().isz(_n);
 Poly Q = Y.Mul(X.Inv()).isz(_n).irev();
 X = rhs.Mul(Q), Y = *this;
 fi(0, n()) if ((Y[i] -= X[i]) < 0) Y[i] += P;
 return {Q, Y.isz(max(1, rhs.n() - 1))};
}
Poly Dx() const {
 Poly ret(n() - 1);
 fi(0, ret.n()) ret[i] = (i + 1) * coef[i + 1] % P;
 return ret.isz(max(1, ret.n()));
}
Poly Sx() const {
 Poly ret(n() + 1);
 fi(0, n()) ret[i + 1]=ntt.minv(i + 1)*coef[i] % P;
 return ret;
}
Poly _tmul(int nn, const Poly &rhs) const {
 Poly Y = Mul(rhs).isz(n() + nn - 1);
 return Poly(Y.data() + n() - 1, nn);
}
VL _eval(const VL &x, const auto up)const{
 const int _n = (int)x.size();
 if (!_n) return {};
 vector<Poly> down(_n * 2);
 down[1] = DivMod(up[1]).second;
 fi(2,_n*2) down[i]=down[i/2].DivMod(up[i]).second;
 /* down[1] = Poly(up[1]).irev().isz(n()).Inv().irev()
    ._tmul(_n, *this);
 fi(2, _n * 2) down[i] = up[i ^ 1]._tmul(up[i].n() -
    1, down[i / 2]); */
 VL y(_n);
 fi(0, _n) y[i] = down[_n + i][0];
 return y;
}
static vector<Poly> _tree1(const VL &x) {
 const int _n = (int)x.size();
 vector<Poly> up(_n * 2);
 fi(0, _n) up[_n + i] = {(x[i] ? P - x[i] : 0), 1};
 Fi(0, _n-1) up[i] = up[i * 2].Mul(up[i * 2 + 1]);
 return up;
}
VL Eval(const VL&x)const{return _eval(x,_tree1(x));}
static Poly Interpolate(const VL &x, const VL &y) {
 const int _n = (int)x.size();
 vector<Poly> up = _tree1(x), down(_n * 2);
 VL z = up[1].Dx()._eval(x, up);
 fi(0, _n) z[i] = y[i] * ntt.minv(z[i]) % P;
 fi(0, _n) down[_n + i] = {z[i]};
 Fi(0, _n-1) down[i]=down[i * 2].Mul(up[i * 2 + 1])
  .iadd(down[i * 2 + 1].Mul(up[i * 2]));
 return down[1];
}
Poly Ln() const { // coef[0] == 1
 return Dx().Mul(Inv()).Sx().isz(n());
}
Poly Exp() const { // coef[0] == 0
 if (n() == 1) return {1};
 Poly X = Poly(*this, (n() + 1)/2).Exp().isz(n());
 Poly Y = X.Ln(); Y[0] = P - 1;
 fi(0, n()) if((Y[i] = coef[i] - Y[i]) < 0)Y[i]+=P;
 return X.Mul(Y).isz(n());
}
Poly Pow(const string &K) const {
 int nz = 0;
 while (nz < n() && !coef[nz]) ++nz;
 LL nk = 0, nk2 = 0;
 for (char c : K) {
  nk = (nk * 10 + c - '0') % P;
  nk2 = nk2 * 10 + c - '0';
  if (nk2 * nz >= n()) return Poly(n());
  nk2 %= P - 1;
 }
```

```cpp
  if (!nk && !nk2) return Poly({1}, n());
  Poly X(data() + nz, n() - nz * nk2);
  LL x0 = X[0];
  return X.imul(ntt.minv(x0)).Ln().imul(nk).Exp()
   .imul(ntt.mpow(x0, nk2)).irev().isz(n()).irev();
 }
 Poly InvMod(int L) { // (to evaluate linear recursion)
  Poly R{1, 0}; // *this * R mod x^L = 1 (*this[0] ==
    1)
  for (int level = 0; (1 << level) < L; ++level) {
   Poly O = R.Mul(Poly(data(), min(2 << level, n())));
   Poly Q(2 << level); Q[0] = 1;
   for (int j = (1 << level); j < (2 << level); ++j)
    Q[j] = (P - O[j]) % P;
   R = R.Mul(Q).isz(4 << level);
  }
  return R.isz(L);
 }
 static LL LinearRecursion(const VL&a,const VL&c,LL n){
  // a_n = \sum c_j a_(n-j)
  const int k = (int)a.size();
  assert((int)c.size() == k + 1);
  Poly C(k + 1), W({1}, k), M = {0, 1};
  fi(1, k + 1) C[k - i] = c[i] ? P - c[i] : 0;
  C[k] = 1;
  while (n) {
   if (n % 2) W = W.Mul(M).DivMod(C).second;
   n /= 2, M = M.Mul(M).DivMod(C).second;
  }
  LL ret = 0;
  fi(0, k) ret = (ret + W[i] * a[i]) % P;
  return ret;
 }
};
#undef fi
#undef Fi
using Poly_t = Poly<131072 * 2, 998244353, 3>;
template<> decltype(Poly_t::ntt) Poly_t::ntt = {};
```

5.16 FWT

```cpp
/* xor convolution:
 * x = (x0,x1) , y = (y0,y1)
 * z = ( x0y0 + x1y1 , x0y1 + x1y0 )
 * =>
 * x' = ( x0+x1 , x0-x1 ) , y' = ( y0+y1 , y0-y1 )
 * z' = ( ( x0+x1 )( y0+y1 ) , ( x0-x1 )( y0-y1 ) )
 * z = (1/2) * z''
 * or convolution:
 * x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
 * and convolution:
 * x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
const LL MOD = 1e9+7;
inline void fwt( LL x[ MAXN ] , int N , bool inv=0 ) {
 for( int d = 1 ; d < N ; d <<= 1 ) {
  int d2 = d<<1;
  for( int s = 0 ; s < N ; s += d2 )
   for( int i = s , j = s+d ; i < s+d ; i++, j++ ){
    LL ta = x[ i ] , tb = x[ j ];
    x[ i ] = ta+tb;
    x[ j ] = ta-tb;
    if( x[ i ] >= MOD ) x[ i ] -= MOD;
    if( x[ j ] < 0 ) x[ j ] += MOD;
   }
 }
 if( inv )
  for( int i = 0 ; i < N ; i++ ) {
   x[ i ] *= inv( N, MOD );
   x[ i ] %= MOD;
  }
}
```

5.17 DiscreteLog

```cpp
template<typename Int>
Int BSGS(Int x, Int y, Int M) {
  // x^? \equiv y (mod M)
  Int t = 1, c = 0, g = 1;
  for (Int M_ = M; M_ > 0; M_ >>= 1)
    g = g * x % M;
  for (g = gcd(g, M); t % g != 0; ++c) {
    if (t == y) return c;
    t = t * x % M;
  }
  if (y % g != 0) return -1;
  t /= g, y /= g, M /= g;
  Int h = 0, gs = 1;
  for (; h * h < M; ++h) gs = gs * x % M;
  unordered_map<Int, Int> bs;
  for (Int s = 0; s < h; bs[y] = ++s)
    y = y * x % M;
  for (Int s = 0; s < M; s += h) {
    t = t * gs % M;
    if (bs.count(t)) return c + s + h - bs[t];
  }
  return -1;
}
```

5.18 FloorSum

```cpp
// @param n `n < 2^32`
// @param m `1 <= m < 2^32`
// @return sum_{i=0}^{n-1} floor((ai + b)/m) mod 2^64
llu floor_sum_unsigned(llu n, llu m, llu a, llu b) {
 llu ans = 0;
 while (true) {
  if (a >= m) {
   ans += n * (n - 1) / 2 * (a / m); a %= m;
  }
  if (b >= m) {
   ans += n * (b / m); b %= m;
  }
  llu y_max = a * n + b;
  if (y_max < m) break;
  // y_max < m * (n + 1)
  // floor(y_max / m) <= n
  n = (llu)(y_max / m), b = (llu)(y_max % m);
  swap(m, a);
 }
 return ans;
}
lld floor_sum(lld n, lld m, lld a, lld b) {
 llu ans = 0;
 if (a < 0) {
  llu a2 = (a % m + m) % m;
  ans -= 1ULL * n * (n - 1) / 2 * ((a2 - a) / m);
  a = a2;
 }
 if (b < 0) {
  llu b2 = (b % m + m) % m;
  ans -= 1ULL * n * ((b2 - b) / m);
  b = b2;
 }
 return ans + floor_sum_unsigned(n, m, a, b);
}
```

5.19 Quadratic residue

```cpp
struct S {
 int MOD, w;
 int64_t x, y;
 S(int m, int w_=-1, int64_t x_=1, int64_t y_=0)
  : MOD(m), w(w_), x(x_), y(y_) {}
 S operator*(const S &rhs) const {
  int w_ = w;
  if (w_ == -1) w_ = rhs.w;
  assert(w_ != -1 and w_ == rhs.w);
  return { MOD, w_,
   (x * rhs.x + y * rhs.y % MOD * w) % MOD,
   (x * rhs.y + y * rhs.x) % MOD };
 }
};
int get_root(int n, int P) {
  if (P == 2 or n == 0) return n;
  if (qpow(n, (P - 1) / 2, P) != 1) return -1;
  auto check = [&](int x) {
    return qpow(x, (P - 1) / 2, P); };
  if (check(n) == P-1) return -1;
  int64_t a; int w; mt19937 rnd(7122);
  do { a = rnd() % P;
    w = ((a * a - n) % P + P) % P;
  } while (check(w) != P - 1);
  return qpow(S(P, w, a, 1), (P + 1) / 2).x;
}
```

5.20 De-Bruijn

```
int res[maxn], aux[maxn], sz;
void db(int t, int p, int n, int k) {
 if (t > n) {
  if (n % p == 0)
   for (int i = 1; i <= p; ++i)
    res[sz++] = aux[i];
 } else {
  aux[t] = aux[t - p];
  db(t + 1, p, n, k);
  for (int i = aux[t - p] + 1; i < k; ++i) {
   aux[t] = i;
   db(t + 1, t, n, k);
  }
 }
}
int de_bruijn(int k, int n) {
 // return cyclic string of len k^n s.t. every string
 // of len n using k char appears as a substring.
 if (k == 1) {
  res[0] = 0;
  return 1;
 }
 for (int i = 0; i < k * n; i++) aux[i] = 0;
 sz = 0;
 db(1, 1, n, k);
 return sz;
}
```

5.21 Simplex Construction

Standard form: maximize $\sum_{1\le i\le n} c_i x_i$ such that for all $1 \le j \le m$, $\sum_{1\le i\le n} A_{ji}x_i \le b_j$.and $x_i \ge 0$ for all $1 \le i \le n$.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1\le i\le n} A_{ji}x_i \ge b_j \to \sum_{1\le i\le n} -A_{ji}x_i \le -b_j$
3. $\sum_{1\le i\le n} A_{ji}x_i = b_j$
   - $\sum_{1\le i\le n} A_{ji}x_i \le b_j$
   - $\sum_{1\le i\le n} A_{ji}x_i \ge b_j$
4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

5.22 Simplex

```
namespace simplex {
// maximize c^Tx under Ax <= B
// return VD(n, -inf) if the solution doesn't exist
// return VD(n, +inf) if the solution is unbounded
using VD = vector<double>;
using VVD = vector<vector<double>>;
const double eps = 1e-9;
const double inf = 1e+9;
int n, m;
VVD d;
vector<int> p, q;
void pivot(int r, int s) {
 double inv = 1.0 / d[r][s];
 for (int i = 0; i < m + 2; ++i)
  for (int j = 0; j < n + 2; ++j)
   if (i != r && j != s)
    d[i][j] -= d[r][j] * d[i][s] * inv;
 for(int i=0;i<m+2;++i) if (i != r) d[i][s] *= -inv;
 for(int j=0;j<n+2;++j) if (j != s) d[r][j] *= +inv;
 d[r][s] = inv; swap(p[r], q[s]);
}
bool phase(int z) {
 int x = m + z;
 while (true) {
  int s = -1;
  for (int i = 0; i <= n; ++i) {
   if (!z && q[i] == -1) continue;
   if (s == -1 || d[x][i] < d[x][s]) s = i;
  }
  if (d[x][s] > -eps) return true;
  int r = -1;
  for (int i = 0; i < m; ++i) {
   if (d[i][s] < eps) continue;
   if (r == -1 || \
    d[i][n+1]/d[i][s] < d[r][n+1]/d[r][s]) r = i;
  }
  if (r == -1) return false;
  pivot(r, s);
 }
}
VD solve(const VVD &a, const VD &b, const VD &c) {
 m = b.size(), n = c.size();
 d = VVD(m + 2, VD(n + 2));
 for (int i = 0; i < m; ++i)
  for (int j = 0; j < n; ++j) d[i][j] = a[i][j];
 p.resize(m), q.resize(n + 1);
 for (int i = 0; i < m; ++i)
  p[i] = n + i, d[i][n] = -1, d[i][n + 1] = b[i];
 for (int i = 0; i < n; ++i) q[i] = i,d[m][i] = -c[i];
 q[n] = -1, d[m + 1][n] = 1;
 int r = 0;
 for (int i = 1; i < m; ++i)
  if (d[i][n + 1] < d[r][n + 1]) r = i;
 if (d[r][n + 1] < -eps) {
  pivot(r, n);
  if (!phase(1) || d[m + 1][n + 1] < -eps)
   return VD(n, -inf);
  for (int i = 0; i < m; ++i) if (p[i] == -1) {
   int s = min_element(d[i].begin(), d[i].end() - 1)
       - d[i].begin();
   pivot(i, s);
  }
 }
 if (!phase(0)) return VD(n, inf);
 VD x(n);
 for (int i = 0; i < m; ++i)
  if (p[i] < n) x[p[i]] = d[i][n + 1];
 return x;
}}
```

5.23 Charateristic Polynomial

```
vector<vector<int>> Hessenberg(const vector<vector<int
    >> &A) {
 int N = A.size();
 vector<vector<int>> H = A;
 for (int i = 0; i < N - 2; ++i) {
  if (!H[i + 1][i]) {
   for (int j = i + 2; j < N; ++j) {
    if (H[j][i]) {
     for (int k = i; k < N; ++k) swap(H[i + 1][k], H[j
    ][k]);
     for (int k = 0; k < N; ++k) swap(H[k][i + 1], H[k
    ][j]);
     break;
    }
   }
  }
  if (!H[i + 1][i]) continue;
  int val = fpow(H[i + 1][i], kP - 2);
  for (int j = i + 2; j < N; ++j) {
   int coef = 1LL * val * H[j][i] % kP;
   for (int k = i; k < N; ++k) H[j][k] = (H[j][k] + 1LL
     * H[i + 1][k] * (kP - coef)) % kP;
   for (int k = 0; k < N; ++k) H[k][i + 1] = (H[k][i +
    1] + 1LL * H[k][j] * coef) % kP;
  }
 }
 return H;
}
vector<int> CharacteristicPoly(const vector<vector<int
    >> &A) {
 int N = A.size();
 auto H = Hessenberg(A);
 for (int i = 0; i < N; ++i) {
  for (int j = 0; j < N; ++j) H[i][j] = kP - H[i][j];
 }
 vector<vector<int>> P(N + 1, vector<int>(N + 1));
 P[0][0] = 1;
 for (int i = 1; i <= N; ++i) {
  P[i][0] = 0;
  for (int j = 1; j <= i; ++j) P[i][j] = P[i - 1][j -
    1];
  int val = 1;
  for (int j = i - 1; j >= 0; --j) {
   int coef = 1LL * val * H[j][i - 1] % kP;
   for (int k = 0; k <= j; ++k) P[i][k] = (P[i][k] + 1
    LL * P[j][k] * coef) % kP;
   if (j) val = 1LL * val * (kP - H[j][j - 1]) % kP;
  }
 }
```

```
 if (N & 1) {
  for (int i = 0; i <= N; ++i) P[N][i] = kP - P[N][i];
 }
 return P[N];
}
```

6 Geometry

6.1 Basic Geometry

```
using coord_t = int;
using Real = double;
using Point = std::complex<coord_t>;
int sgn(coord_t x) {
 return (x > 0) - (x < 0);
}
coord_t dot(Point a, Point b) {
 return real(conj(a) * b);
}
coord_t cross(Point a, Point b) {
 return imag(conj(a) * b);
}
int ori(Point a, Point b, Point c) {
 return sgn(cross(b - a, c - a));
}
bool operator<(const Point &a, const Point &b) {
 return real(a) != real(b)
  ? real(a) < real(b) : imag(a) < imag(b);
}
int argCmp(Point a, Point b) {
 // -1 / 0 / 1 <-> < / == / > (atan2)
 int qa = (imag(a) == 0
   ? (real(a) < 0 ? 3 : 1) : (imag(a) < 0 ? 0 : 2));
 int qb = (imag(b) == 0
   ? (real(b) < 0 ? 3 : 1) : (imag(b) < 0 ? 0 : 2));
 if (qa != qb)
  return sgn(qa - qb);
 return sgn(cross(b, a));
}
template <typename V> Real area(const V & pt) {
 coord_t ret = 0;
 for (int i = 1; i + 1 < (int)pt.size(); i++)
  ret += cross(pt[i] - pt[0], pt[i+1] - pt[0]);
 return ret / 2.0;
}
```

6.2 Circle Class

```
struct Circle { Point o; Real r; };

vector<Real> intersectAngle(Circle a, Circle b) {
 Real d2 = norm(a.o - b.o);
 if (norm(A.r - B.r) >= d2)
  if (A.r < B.r)
   return {-PI, PI};
  else
   return {};
 if (norm(A.r + B.r) <= d2) return {};
 Real dis = hypot(A.x - B.x, A.y - B.y);
 Real theta = atan2(B.y - A.y, B.x - A.x);
 Real phi = acos((A.r * A.r + d2 - B.r * B.r) /
   (2 * A.r * dis));
 Real L = theta - phi, R = theta + phi;
 while (L < -PI) L += PI * 2;
 while (R > PI) R -= PI * 2;
 return { L, R };
}

vector<Point> intersectPoint(Circle a, Circle b) {
 Real d=o.dis(aa.o);
 if (d >= r+aa.r || d <= fabs(r-aa.r)) return {};
 Real dt = (r*r - aa.r*aa.r)/d, d1 = (d+dt)/2;
 Point dir = (aa.o-o); dir /= d;
 Point pcrs = dir*d1 + o;
 dt=sqrt(max(0.0L, r*r - d1*d1)), dir=dir.rot90();
 return {pcrs + dir*dt, pcrs - dir*dt};
}
```

6.3 2D Convex Hull

```
template<typename PT>
vector<PT> buildConvexHull(vector<PT> d) {
 sort(ALL(d), [](const PT& a, const PT& b){
   return tie(a.x, a.y) < tie(b.x, b.y);});
 vector<PT> s(SZ(d)<<1);
 int o = 0;
 for(auto p: d) {
  while(o>=2 && cross(p-s[o-2],s[o-1]-s[o-2])<=0)
   o--;
  s[o++] = p;
 }
 for(int i=SZ(d)-2, t = o+1;i>=0;i--){
  while(o>=t&&cross(d[i]-s[o-2],s[o-1]-s[o-2])<=0)
   o--;
  s[o++] = d[i];
 }
 s.resize(o-1);
 return s;
}
```

6.4 3D Convex Hull

```
// return the faces with pt indexes
int flag[MXN][MXN];
struct Point{
 ld x,y,z;
 Point operator * (const ld &b) const {
  return (Point){x*b,y*b,z*b};}
 Point operator * (const Point &b) const {
  return(Point){y*b.z-b.y*z,z*b.x-b.z*x,x*b.y-b.x*y};
 }
};
Point ver(Point a, Point b, Point c) {
 return (b - a) * (c - a);}
vector<Face> convex_hull_3D(const vector<Point> pt) {
 int n = SZ(pt), ftop = 0;
 REP(i,n) REP(j,n) flag[i][j] = 0;
 vector<Face> now;
 now.emplace_back(0,1,2);
 now.emplace_back(2,1,0);
 for (int i=3; i<n; i++){
  ftop++; vector<Face> next;
  REP(j, SZ(now)) {
   Face& f=now[j]; int ff = 0;
   ld d=(pt[i]-pt[f.a]).dot(
     ver(pt[f.a], pt[f.b], pt[f.c]));
   if (d <= 0) next.push_back(f);
   if (d > 0) ff=ftop;
   else if (d < 0) ff=-ftop;
   flag[f.a][f.b]=flag[f.b][f.c]=flag[f.c][f.a]=ff;
  }
  REP(j, SZ(now)) {
   Face& f=now[j];
   if (flag[f.a][f.b] > 0 &&
     flag[f.a][f.b] != flag[f.b][f.a])
    next.emplace_back(f.a,f.b,i);
   if (flag[f.b][f.c] > 0 &&
     flag[f.b][f.c] != flag[f.c][f.b])
    next.emplace_back(f.b,f.c,i);
   if (flag[f.c][f.a] > 0 &&
     flag[f.c][f.a] != flag[f.a][f.c])
    next.emplace_back(f.c,f.a,i);
  }
  now=next;
 }
 return now;
}
```

6.5 2D Farthest Pair

```
// stk is from convex hull
n = (int)(stk.size());
int pos = 1, ans = 0; stk.push_back(stk[0]);
for(int i=0;i<n;i++) {
 while(abs(cross(stk[i+1]-stk[i],
   stk[(pos+1)%n]-stk[i])) >
   abs(cross(stk[i+1]-stk[i],
   stk[pos]-stk[i]))) pos = (pos+1)%n;
 ans = max({ans, dis(stk[i], stk[pos]),
  dis(stk[i+1], stk[pos])});
}
```

6.6 2D Closest Pair

```
struct cmp_y {
 bool operator()(const P& p, const P& q) const {
  return p.y < q.y;
 }
```

```
};
multiset<P, cmp_y> s;
void solve(P a[], int n) {
 sort(a, a + n, [](const P& p, const P& q) {
  return tie(p.x, p.y) < tie(q.x, q.y);
 });
 llf d = INF; int pt = 0;
 for (int i = 0; i < n; ++i) {
  while (pt < i and a[i].x - a[pt].x >= d)
   s.erase(s.find(a[pt++]));
  auto it = s.lower_bound(P(a[i].x, a[i].y - d));
  while (it != s.end() and it->y - a[i].y < d)
   d = min(d, dis(*(it++), a[i]));
  s.insert(a[i]);
 }
}
```

6.7 kD Closest Pair (3D ver.)

```
llf solve(vector<P> v) {
 shuffle(v.begin(), v.end(), mt19937());
 unordered_map<lld, unordered_map<lld,
  unordered_map<lld, int>>> m;
 llf d = dis(v[0], v[1]);
 auto Idx = [&d] (llf x) -> lld {
  return round(x * 2 / d) + 0.1; };
 auto rebuild_m = [&m, &v, &Idx](int k) {
  m.clear();
  for (int i = 0; i < k; ++i)
   m[Idx(v[i].x)][Idx(v[i].y)]
    [Idx(v[i].z)] = i;
 }; rebuild_m(2);
 for (size_t i = 2; i < v.size(); ++i) {
  const lld kx = Idx(v[i].x), ky = Idx(v[i].y),
     kz = Idx(v[i].z); bool found = false;
  for (int dx = -2; dx <= 2; ++dx) {
   const lld nx = dx + kx;
   if (m.find(nx) == m.end()) continue;
   auto& mm = m[nx];
   for (int dy = -2; dy <= 2; ++dy) {
    const lld ny = dy + ky;
    if (mm.find(ny) == mm.end()) continue;
    auto& mmm = mm[ny];
    for (int dz = -2; dz <= 2; ++dz) {
     const lld nz = dz + kz;
     if (mmm.find(nz) == mmm.end()) continue;
     const int p = mmm[nz];
     if (dis(v[p], v[i]) < d) {
      d = dis(v[p], v[i]);
      found = true;
     }
    }
   }
  }
  if (found) rebuild_m(i + 1);
  else m[kx][ky][kz] = i;
 }
 return d;
}
```

6.8 Simulated Annealing

```
llf anneal() {
 mt19937 rnd_engine( seed );
 uniform_real_distribution< llf > rnd( 0, 1 );
 const llf dT = 0.001;
 // Argument p
 llf S_cur = calc( p ), S_best = S_cur;
 for ( llf T = 2000 ; T > EPS ; T -= dT ) {
  // Modify p to p_prime
  const llf S_prime = calc( p_prime );
  const llf delta_c = S_prime - S_cur;
  llf prob = min( ( llf ) 1, exp( -delta_c / T ) );
  if ( rnd( rnd_engine ) <= prob )
   S_cur = S_prime, p = p_prime;
  if ( S_prime < S_best ) // find min
   S_best = S_prime, p_best = p_prime;
 }
 return S_best;
}
```

6.9 Half Plane Intersection

```
// NOTE: Point is complex<Real>
// cross(pt-line.st, line.dir)<=0 <-> pt in half plane
struct Line {
  Point st, ed;
  Point dir;
  Line (Point _s, Point _e)
   : st(_s), ed(_e), dir(_e - _s) {}
};

bool operator<(const Line &lhs, const Line &rhs) {
  if (int cmp = argCmp(lhs.dir, rhs.dir))
    return cmp == -1;
  return ori(lhs.st, lhs.ed, rhs.st) < 0;
}
Point intersect(const Line &A, const Line &B) {
  Real t = cross(B.st - A.st, B.dir) /
   cross(A.dir, B.dir);
  return A.st + t * A.dir;
}

Real HPI(vector<Line> &lines) {
  sort(lines.begin(), lines.end());
  deque<Line> que;
  deque<Point> pt;
  que.push_back(lines[0]);
  for (int i = 1; i < (int)lines.size(); i++) {
    if (argCmp(lines[i].dir, lines[i-1].dir) == 0)
     continue;
#define POP(L, R) \
    while (pt.size() > 0 \
      && ori(L.st, L.ed, pt.back()) < 0) \
      pt.pop_back(), que.pop_back(); \
    while (pt.size() > 0 \
      && ori(R.st, R.ed, pt.front()) < 0) \
      pt.pop_front(), que.pop_front();
    POP(lines[i], lines[i]);
    pt.push_back(intersect(que.back(), lines[i]));
    que.push_back(lines[i]);
  }
  POP(que.front(), que.back())
  if (que.size() <= 1 ||
    argCmp(que.front().dir, que.back().dir) == 0)
    return 0;
  pt.push_back(intersect(que.front(), que.back()));
  return area(pt);
}
```

6.10 Minkowski sum

```
vector<pll> Minkowski(vector<pll> A, vector<pll> B) {
 hull(A), hull(B);
 vector<pll> C(1, A[0] + B[0]), s1, s2;
 for(int i = 0; i < SZ(A); ++i)
  s1.pb(A[(i + 1) % SZ(A)] - A[i]);
 for(int i = 0; i < SZ(B); i++)
  s2.pb(B[(i + 1) % SZ(B)] - B[i]);
 for(int p1 = 0, p2 = 0; p1 < SZ(A) || p2 < SZ(B);)
  if (p2 >= SZ(B)
    || (p1 < SZ(A) && cross(s1[p1], s2[p2]) >= 0))
   C.pb(C.back() + s1[p1++]);
  else
   C.pb(C.back() + s2[p2++]);
 return hull(C), C;
}
```

6.11 intersection of line and circle

```
vector<pdd> line_interCircle(const pdd &p1,
    const pdd &p2,const pdd &c,const double r){
 pdd ft=foot(p1,p2,c),vec=p2-p1;
 double dis=abs(c-ft);
 if(fabs(dis-r)<eps) return vector<pdd>{ft};
 if(dis>r) return {};
 vec=vec*sqrt(r*r-dis*dis)/abs(vec);
 return vector<pdd>{ft+vec,ft-vec};
}
```

6.12 intersection of polygon and circle

```
// Divides into multiple triangle, and sum up
// test by HDU2892
const double PI=acos(-1);
double _area(pdd pa, pdd pb, double r){
 if(abs(pa)<abs(pb)) swap(pa, pb);
```

```
 if(abs(pb)<eps) return 0;
 double S, h, theta;
 double a=abs(pb),b=abs(pa),c=abs(pb-pa);
 double cosB = dot(pb,pb-pa) / a / c, B = acos(cosB);
 double cosC = dot(pa,pb) / a / b, C = acos(cosC);
 if(a > r){
  S = (C/2)*r*r;
  h = a*b*sin(C)/c;
  if (h < r && B < PI/2)
   S -= (acos(h/r)*r*r - h*sqrt(r*r-h*h));
 }
 else if(b > r){
  theta = PI - B - asin(sin(B)/r*a);
  S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
 }
 else S = .5*sin(C)*a*b;
 return S;
}
double area_poly_circle(const vector<pdd> poly,
  const pdd &O,const double r){
 double S=0;
 for(int i=0;i<SZ(poly);++i)
  S+=_area(poly[i]-O,poly[(i+1)%SZ(poly)]-O,r)
    *ori(O,poly[i],poly[(i+1)%SZ(poly)]);
 return fabs(S);
}
```

6.13 intersection of two circle

```
bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
 pdd o1 = a.O, o2 = b.O;
 double r1 = a.R, r2 = b.R, d2 = abs2(o1 - o2),
     d = sqrt(d2);
 if(d < max(r1, r2) - min(r1, r2) || d > r1 + r2)
  return 0;
 pdd u = (o1 + o2) * 0.5
  + (o1 - o2) * ((r2 * r2 - r1 * r1) / (2 * d2));
 double A = sqrt((r1 + r2 + d) * (r1 - r2 + d)
     * (r1 + r2 - d) * (-r1 + r2 + d));
 pdd v = pdd(o1.Y - o2.Y, -o1.X + o2.X) * A
  / (2 * d2);
 p1 = u + v, p2 = u - v;
 return 1;
}
```

6.14 tangent line of two circle

```
vector<Line> go(const Cir& c1,
  const Cir& c2, int sign1){
 // sign1 = 1 for outer tang, -1 for inter tang
 vector<Line> ret;
 double d_sq = norm2( c1.O - c2.O );
 if( d_sq < eps ) return ret;
 double d = sqrt( d_sq );
 Pt v = ( c2.O - c1.O ) / d;
 double c = ( c1.R - sign1 * c2.R ) / d;
 if( c * c > 1 ) return ret;
 double h = sqrt( max( 0.0 , 1.0 - c * c ) );
 for( int sign2 = 1 ; sign2 >= -1 ; sign2 -= 2 ){
  Pt n = { v.X * c - sign2 * h * v.Y ,
   v.Y * c + sign2 * h * v.X };
  Pt p1 = c1.O + n * c1.R;
  Pt p2 = c2.O + n * ( c2.R * sign1 );
  if( fabs( p1.X - p2.X ) < eps and
    fabs( p1.Y - p2.Y ) < eps )
   p2 = p1 + perp( c2.O - c1.O );
  ret.push_back( { p1 , p2 } );
 }
 return ret;
}
```

6.15 Minimum Covering Circle

```
template<typename P>
Circle getCircum(const P &a, const P &b, const P &c){
 Real a1 = a.x-b.x, b1 = a.y-b.y;
 Real c1 = (a.x+b.x)/2 * a1 + (a.y+b.y)/2 * b1;
 Real a2 = a.x-c.x, b2 = a.y-c.y;
 Real c2 = (a.x+c.x)/2 * a2 + (a.y+c.y)/2 * b2;
 Circle cc;
 cc.o.x = (c1*b2-b1*c2)/(a1*b2-b1*a2);
 cc.o.y = (a1*c2-c1*a2)/(a1*b2-b1*a2);
 cc.r = hypot(cc.o.x-a.x, cc.o.y-a.y);
 return cc;
}

template<typename P>
Circle MinCircleCover(const vector<P>& pts){
 random_shuffle(pts.begin(), pts.end());
 Circle c = { pts[0], 0 };
 for(int i=0;i<(int)pts.size();i++){
  if (dist(pts[i], c.o) <= c.r) continue;
  c = { pts[i], 0 };
  for (int j = 0; j < i; j++) {
   if(dist(pts[j], c.o) <= c.r) continue;
   c.o = (pts[i] + pts[j]) / 2;
   c.r = dist(pts[i], c.o);
   for (int k = 0; k < j; k++) {
    if (dist(pts[k], c.o) <= c.r) continue;
    c = getCircum(pts[i], pts[j], pts[k]);
   }
  }
 }
 return c;
}
```

6.16 KDTree (Nearest Point)

```
const int MXN = 100005;
struct KDTree {
 struct Node {
  int x,y,x1,y1,x2,y2;
  int id,f;
  Node *L, *R;
 } tree[MXN], *root;
 int n;
 LL dis2(int x1, int y1, int x2, int y2) {
  LL dx = x1-x2, dy = y1-y2;
  return dx*dx+dy*dy;
 }
 static bool cmpx(Node& a, Node& b){return a.x<b.x;}
 static bool cmpy(Node& a, Node& b){return a.y<b.y;}
 void init(vector<pair<int,int>> ip) {
  n = ip.size();
  for (int i=0; i<n; i++) {
   tree[i].id = i;
   tree[i].x = ip[i].first;
   tree[i].y = ip[i].second;
  }
  root = build_tree(0, n-1, 0);
 }
 Node* build_tree(int L, int R, int d) {
  if (L>R) return nullptr;
  int M = (L+R)/2; tree[M].f = d%2;
  nth_element(tree+L,tree+M,tree+R+1,d%2?cmpy:cmpx);
  tree[M].x1 = tree[M].x2 = tree[M].x;
  tree[M].y1 = tree[M].y2 = tree[M].y;
  tree[M].L = build_tree(L, M-1, d+1);
  if (tree[M].L) {
   tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
   tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
   tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
   tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
  }
  tree[M].R = build_tree(M+1, R, d+1);
  if (tree[M].R) {
   tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
   tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
   tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
   tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
  }
  return tree+M;
 }
 int touch(Node* r, int x, int y, LL d2){
  LL dis = sqrt(d2)+1;
  if (x<r->x1-dis || x>r->x2+dis ||
    y<r->y1-dis || y>r->y2+dis)
   return 0;
  return 1;
 }
 void nearest(Node* r,int x,int y,int &mID,LL &md2) {
  if (!r || !touch(r, x, y, md2)) return;
  LL d2 = dis2(r->x, r->y, x, y);
  if (d2 < md2 || (d2 == md2 && mID < r->id)) {
   mID = r->id;
   md2 = d2;
  }
```

```
  // search order depends on split dim
  if ((r->f == 0 && x < r->x) ||
    (r->f == 1 && y < r->y)) {
   nearest(r->L, x, y, mID, md2);
   nearest(r->R, x, y, mID, md2);
  } else {
   nearest(r->R, x, y, mID, md2);
   nearest(r->L, x, y, mID, md2);
  }
 }
 int query(int x, int y) {
  int id = 1029384756;
  LL d2 = 102938475612345678LL;
  nearest(root, x, y, id, d2);
  return id;
 }
} tree;
```

7 Stringology

7.1 Hash

```
class Hash {
 private:
  static constexpr int P = 127, Q = 1051762951;
  vector<int> h, p;
 public:
  void init(const string &s){
   h.assign(s.size()+1, 0); p.resize(s.size()+1);
   for (size_t i = 0; i < s.size(); ++i)
    h[i + 1] = add(mul(h[i], P), s[i]);
   generate(p.begin(), p.end(),[x=1,y=1,this]()
     mutable{y=x;x=mul(x,P);return y;});
  }
  int query(int l, int r){ // 1-base (l, r]
   return sub(h[r], mul(h[l], p[r-l]));}
};
```

7.2 Suffix Array

```
namespace sfxarray {
bool t[maxn * 2];
int hi[maxn], rev[maxn];
int _s[maxn * 2], sa[maxn * 2], c[maxn * 2];
int x[maxn], p[maxn], q[maxn * 2];
// sa[i]: sa[i]-th suffix is the \
// i-th lexigraphically smallest suffix.
// hi[i]: longest common prefix \
// of suffix sa[i] and suffix sa[i - 1].
void pre(int *sa, int *c, int n, int z) {
 memset(sa, 0, sizeof(int) * n);
 memcpy(x, c, sizeof(int) * z);
}
void induce(int *sa,int *c,int *s,bool *t,int n,int z){
 memcpy(x + 1, c, sizeof(int) * (z - 1));
 for (int i = 0; i < n; ++i)
  if (sa[i] && !t[sa[i] - 1])
   sa[x[s[sa[i] - 1]]++] = sa[i] - 1;
 memcpy(x, c, sizeof(int) * z);
 for (int i = n - 1; i >= 0; --i)
  if (sa[i] && t[sa[i] - 1])
   sa[--x[s[sa[i] - 1]]] = sa[i] - 1;
}
void sais(int *s, int *sa, int *p, int *q,
 bool *t, int *c, int n, int z) {
 bool uniq = t[n - 1] = true;
 int nn=0, nmxz=-1, *nsa = sa+n, *ns=s+n, last=-1;
 memset(c, 0, sizeof(int) * z);
 for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;
 for (int i = 0; i < z - 1; ++i) c[i + 1] += c[i];
 if (uniq) {
  for (int i = 0; i < n; ++i) sa[--c[s[i]]] = i;
  return;
 }
 for (int i = n - 2; i >= 0; --i)
  t[i] = (s[i]==s[i + 1] ? t[i + 1] : s[i]<s[i + 1]);
 pre(sa, c, n, z);
 for (int i = 1; i <= n - 1; ++i)
  if (t[i] && !t[i - 1])
   sa[--x[s[i]]] = p[q[i] = nn++] = i;
 induce(sa, c, s, t, n, z);
 for (int i = 0; i < n; ++i) {
  if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
  bool neq = last < 0 || \
   memcmp(s + sa[i], s + last,
   (p[q[sa[i]] + 1] - sa[i]) * sizeof(int));
  ns[q[last = sa[i]]] = nmxz += neq;
 }}
 sais(ns, nsa, p+nn, q+n, t+n, c+z, nn, nmxz+1);
 pre(sa, c, n, z);
 for (int i = nn - 1; i >= 0; --i)
  sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
 induce(sa, c, s, t, n, z);
}
void build(const string &s) {
 for (int i = 0; i < (int)s.size(); ++i) _s[i] = s[i];
 _s[(int)s.size()] = 0; // s shouldn't contain 0
 sais(_s, sa, p, q, t, c, (int)s.size() + 1, 256);
 for(int i = 0; i < (int)s.size(); ++i) sa[i]=sa[i+1];
 for(int i = 0; i < (int)s.size(); ++i) rev[sa[i]]=i;
 int ind = 0; hi[0] = 0;
 for (int i = 0; i < (int)s.size(); ++i) {
  if (!rev[i]) {
   ind = 0;
   continue;
  }
  while (i + ind < (int)s.size() && \
   s[i + ind] == s[sa[rev[i] - 1] + ind]) ++ind;
  hi[rev[i]] = ind ? ind-- : 0;
 }
}}
```

7.3 Suffix Automaton

```
struct Node{
 Node *green, *edge[26];
 int max_len;
 Node(const int _max_len)
  : green(NULL), max_len(_max_len){
  memset(edge,0,sizeof(edge));
 }
} *ROOT, *LAST;
void Extend(const int c) {
 Node *cursor = LAST;
 LAST = new Node((LAST->max_len) + 1);
 for(;cursor&&!cursor->edge[c]; cursor=cursor->green)
  cursor->edge[c] = LAST;
 if (!cursor)
  LAST->green = ROOT;
 else {
  Node *potential_green = cursor->edge[c];
  if((potential_green->max_len)==(cursor->max_len+1))
   LAST->green = potential_green;
  else {
//assert(potential_green->max_len>(cursor->max_len+1));
   Node *wish = new Node((cursor->max_len) + 1);
   for(;cursor && cursor->edge[c]==potential_green;
      cursor = cursor->green)
    cursor->edge[c] = wish;
   for (int i = 0; i < 26; i++)
    wish->edge[i] = potential_green->edge[i];
   wish->green = potential_green->green;
   potential_green->green = wish;
   LAST->green = wish;
  }
 }
}
char S[10000001], A[10000001];
int N;
int main(){
 scanf("%d%s", &N, S);
 ROOT = LAST = new Node(0);
 for (int i = 0; S[i]; i++)
  Extend(S[i] - 'a');
 while (N--){
  scanf("%s", A);
  Node *cursor = ROOT;
  bool ans = true;
  for (int i = 0; A[i]; i++){
   cursor = cursor->edge[A[i] - 'a'];
   if (!cursor) {
    ans = false;
    break;
   }
  }
  puts(ans ? "Yes" : "No");
```

```
 }
 return 0;
}
```

7.4 KMP

```
vector<int> kmp(const string &s) {
 vector<int> f(s.size(), 0);
 /* f[i] = length of the longest prefix
   (excluding s[0:i]) such that it coincides
   with the suffix of s[0:i] of the same length */
 /* i + 1 - f[i] is the length of the
   smallest recurring period of s[0:i] */
 int k = 0;
 for (int i = 1; i < (int)s.size(); ++i) {
  while (k > 0 && s[i] != s[k]) k = f[k - 1];
  if (s[i] == s[k]) ++k;
  f[i] = k;
 }
 return f;
}
vector<int> search(const string &s, const string &t) {
 // return 0-indexed occurrence of t in s
 vector<int> f = kmp(t), r;
 for (int i = 0, k = 0; i < (int)s.size(); ++i) {
  while(k > 0 && (k==(int)t.size() || s[i]!=t[k]))
   k = f[k - 1];
  if (s[i] == t[k]) ++k;
  if (k == (int)t.size()) r.push_back(i-t.size()+1);
 }
 return res;
}
```

7.5 Z value

```
char s[MAXN];
int len,z[MAXN];
void Z_value() {
 int i,j,left,right;
 z[left=right=0]=len;
 for(i=1;i<len;i++) {
  j=max(min(z[i-left],right-i),0);
  for(;i+j<len&&s[i+j]==s[j];j++);
  if(i+(z[i]=j)>right)right=i+z[left=i];
 }
}
```

7.6 Manacher

```
int z[maxn];
int manacher(const string& s) {
 string t = ".";
 for(char c: s) t += c, t += '.';
 int l = 0, r = 0, ans = 0;
 for (int i = 1; i < t.length(); ++i) {
  z[i] = (r > i ? min(z[2 * l - i], r - i) : 1);
  while (i - z[i] >= 0 && i + z[i] < t.length()) {
   if(t[i - z[i]] == t[i + z[i]]) ++z[i];
   else break;
  }
  if (i + z[i] > r) r = i + z[i], l = i;
 }
 for(int i=1;i<t.length();++i) ans = max(ans, z[i]-1);
 return ans;
}
```

7.7 Lexico Smallest Rotation

```
string mcp(string s){
 int n = s.length();
 s += s;
 int i=0, j=1;
 while (i<n && j<n){
  int k = 0;
  while (k < n && s[i+k] == s[j+k]) k++;
  if (s[i+k] <= s[j+k]) j += k+1;
  else i += k+1;
  if (i == j) j++;
 }
 int ans = i < n ? i : j;
 return s.substr(ans, n);
}
```

7.8 BWT

```
struct BurrowsWheeler{
#define SIGMA 26
#define BASE 'a'
 vector<int> v[ SIGMA ];
 void BWT(char* ori, char* res){
  // make ori -> ori + ori
  // then build suffix array
 }
 void iBWT(char* ori, char* res){
  for( int i = 0 ; i < SIGMA ; i ++ )
   v[ i ].clear();
  int len = strlen( ori );
  for( int i = 0 ; i < len ; i ++ )
   v[ ori[i] - BASE ].push_back( i );
  vector<int> a;
  for( int i = 0 , ptr = 0 ; i < SIGMA ; i ++ )
   for( auto j : v[ i ] ){
    a.push_back( j );
    ori[ ptr ++ ] = BASE + i;
   }
  for( int i = 0 , ptr = 0 ; i < len ; i ++ ){
   res[ i ] = ori[ a[ ptr ] ];
   ptr = a[ ptr ];
  }
  res[ len ] = 0;
 }
} bwt;
```

7.9 Palindromic Tree

```
struct palindromic_tree{
 struct node{
  int next[26],f,len;
  int cnt,num,st,ed;
  node(int l=0):f(0),len(l),cnt(0),num(0) {
   memset(next, 0, sizeof(next)); }
 };
 vector<node> st;
 vector<char> s;
 int last,n;
 void init(){
  st.clear();s.clear();last=1; n=0;
  st.push_back(0);st.push_back(-1);
  st[0].f=1;s.push_back(-1); }
 int getFail(int x){
  while(s[n-st[x].len-1]!=s[n])x=st[x].f;
  return x;}
 void add(int c){
  s.push_back(c-='a'); ++n;
  int cur=getFail(last);
  if(!st[cur].next[c]){
   int now=st.size();
   st.push_back(st[cur].len+2);
   st[now].f=st[getFail(st[cur].f)].next[c];
   st[cur].next[c]=now;
   st[now].num=st[st[now].f].num+1;
  }
  last=st[cur].next[c];
  ++st[last].cnt;}
 void dpcnt() {
  for (int i=st.size()-1; i >= 0; i--)
   st[st[i].f].cnt += st[i].cnt;
 }
 int size(){ return st.size()-2;}
} pt;
int main() {
 string s; cin >> s; pt.init();
 for (int i=0; i<SZ(s); i++) {
  int prvsz = pt.size(); pt.add(s[i]);
  if (prvsz != pt.size()) {
   int r = i, l = r - pt.st[pt.last].len + 1;
   // pal @ [l,r]: s.substr(l, r-l+1)
  }
 }
 return 0;
}
```

8 Misc

8.1 Theorems

8.1.1 Kirchhoff's Theorem

Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G, where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i, j) in G.

- The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
- The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.

8.1.2 Tutte's Matrix

Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniform randomly) if $i < j$ and $(i, j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{rank(D)}{2}$ is the maximum matching on G.

8.1.3 Cayley's Formula

- Given a degree sequence $d_1, d_2, \ldots, d_n$ for each labeled vertices, there're $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!}$ spanning trees.
- Let $T_{n,k}$ be the number of labeled forests on n vertices with k components, such that vertex $1, 2, \ldots, k$ belong to different components. Then $T_{n,k} = kn^{n-k-1}$.

8.1.4 Erdős–Gallai theorem

A sequence of non-negative integers $d_1 \geq d_2 \geq \ldots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + d_2 + \ldots + d_n$ is even and

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k)$$

holds for all $1 \leq k \leq n$.

8.1.5 Havel–Hakimi algorithm

find the vertex who has greatest degree unused, connect it with other greatest vertex.

8.1.6 Hall's marriage theorem

Let G be a finite bipartite graph with bipartite sets X and Y. For a subset W of X, let $N_G(W)$ denote the set of all vertices in Y adjacent to some element of W. Then there is an X-saturating matching iff $\forall W \subseteq X, |W| \leq |N_G(W)|$

8.1.7 Euler's planar graph formula

$V - E + F = C + 1, E \leq 3V - 6$(?)

8.1.8 Pick's theorem

For simple polygon, when points are all integer, we have $A = \#\{\text{lattice points in the interior}\} + \frac{\#\{\text{lattice points on the boundary}\}}{2} - 1$

8.1.9 Lucas's theorem

$\binom{m}{n} \equiv \prod_{i=0}^{k} \binom{m_i}{n_i} \pmod{p}$, where $m = m_k p^k + m_{k-1} p^{k-1} + \cdots + m_1 p + m_0$, and $n = n_k p^k + n_{k-1} p^{k-1} + \cdots + n_1 p + n_0$.

8.1.10 Matroid Intersection

Given matroids $M_1 = (G, I_1), M_2 = (G, I_2)$, find maximum $S \in I_1 \cap I_2$. For each iteration, build the directed graph and find a shortest path from s to t.

- $s \to x : S \sqcup \{x\} \in I_1$
- $x \to t : S \sqcup \{x\} \in I_2$
- $y \to x : S \setminus \{y\} \sqcup \{x\} \in I_1$ (y is in the unique circuit of $S \sqcup \{x\}$)
- $x \to y : S \setminus \{y\} \sqcup \{x\} \in I_2$ (y is in the unique circuit of $S \sqcup \{x\}$)

Alternate the path, and $|S|$ will increase by 1. Let $R = \min(\text{rank}(I_1), \text{rank}(I_2)), N = |G|$. In each iteration, $|E| = O(RN)$. For weighted case, assign weight $-w(x)$ and $w(x)$ to $x \in S$ and $x \notin S$, resp. Use Bellman-Ford to find the weighted shortest path. The maximum iteration of Bellman-Ford is $2R + 1$.

8.2 DP-opt Condition

8.2.1 totally monotone (concave/convex)

$\forall i < i', j < j', B[i][j] \leq B[i'][j] \implies B[i][j'] \leq B[i'][j']$
$\forall i < i', j < j', B[i][j] \geq B[i'][j] \implies B[i][j'] \geq B[i'][j']$

8.2.2 monge condition (concave/convex)

$\forall i < i', j < j', B[i][j] + B[i'][j'] \geq B[i][j'] + B[i'][j]$
$\forall i < i', j < j', B[i][j] + B[i'][j'] \leq B[i][j'] + B[i'][j]$

8.3 Convex 1D/1D DP

```
struct segment {
 int i, l, r;
 segment() {}
 segment(int a, int b, int c): i(a), l(b), r(c) {}
};
inline lld f(int l, int r){return dp[l] + w(l+1, r);}
void solve() {
 dp[0] = 0;
 deque&lt;segment&gt; dq; dq.push_back(segment(0, 1, n));
 for (int i = 1; i <= n; ++i) {
  dp[i] = f(dq.front().i, i);
  while(dq.size()&&dq.front().r<i+1) dq.pop_front();
  dq.front().l = i + 1;
  segment seg = segment(i, i + 1, n);
  while (dq.size() &&
   f(i, dq.back().l)<f(dq.back().i, dq.back().l))
    dq.pop_back();
  if (dq.size()) {
   int d = 1 << 20, c = dq.back().l;
   while (d >>= 1) if (c + d <= dq.back().r)
    if(f(i, c+d) > f(dq.back().i, c+d)) c += d;
   dq.back().r = c; seg.l = c + 1;
  }
  if (seg.l <= n) dq.push_back(seg);
 }
}
```

8.4 ConvexHull Optimization

```
struct Line {
 mutable int64_t a, b, p;
 bool operator<(const Line &rhs) const { return a < rhs
    .a; }
 bool operator<(int64_t x) const { return p < x; }
};
struct DynamicHull : multiset<Line, less<>> {
 static const int64_t kInf = 1e18;
 bool Isect(iterator x, iterator y) {
  auto Div = [](int64_t a, int64_t b) {
    return a / b - ((a ^ b) < 0 && a % b); }
  if (y == end()) { x->p = kInf; return false; }
  if (x->a == y->a) x->p = x->b > y->b ? kInf : -kInf;
  else x->p = Div(y->b - x->b, x->a - y->a);
  return x->p >= y->p;
 }
 void Insert(int64_t a, int64_t b) {
  auto z = insert({a, b, 0}), y = z++, x = y;
  while (Isect(y, z)) z = erase(z);
  if (x != begin() && Isect(--x, y)) Isect(x, y = erase
    (y));
  while ((y = x) != begin() && (--x)->p >= y->p) Isect(
    x, erase(y));
 }
 int64_t Query(int64_t x) {
  auto l = *lower_bound(x);
  return l.a * x + l.b;
 }
};
```

8.5 Josephus Problem

```
// n people kill m for each turn
int f(int n, int m) {
 int s = 0;
 for (int i = 2; i <= n; i++)
  s = (s + m) % i;
 return s;
}
// died at kth
int kth(int n, int m, int k){
 if (m == 1) return n-1;
 for (k = k*m+m-1; k >= n; k = k-n+(k-n)/(m-1));
 return k;
}
```

8.6 Cactus Matching

```
vector<int> init_g[maxn],g[maxn*2];
int n,dfn[maxn],low[maxn],par[maxn],dfs_idx,bcc_id;
void tarjan(int u){
 dfn[u]=low[u]=++dfs_idx;
 for(int i=0;i<(int)init_g[u].size();i++){
  int v=init_g[u][i];
```

```
  if(v==par[u]) continue;
  if(!dfn[v]){
   par[v]=u;
   tarjan(v);
   low[u]=min(low[u],low[v]);
   if(dfn[u]<low[v]){
    g[u].push_back(v);
    g[v].push_back(u);
   }
  }else{
   low[u]=min(low[u],dfn[v]);
   if(dfn[v]<dfn[u]){
    int temp_v=u;
    bcc_id++;
    while(temp_v!=v){
     g[bcc_id+n].push_back(temp_v);
     g[temp_v].push_back(bcc_id+n);
     temp_v=par[temp_v];
    }
    g[bcc_id+n].push_back(v);
    g[v].push_back(bcc_id+n);
    reverse(g[bcc_id+n].begin(),g[bcc_id+n].end());
   }
  }
 }
}
int dp[maxn][2],min_dp[2][2],tmp[2][2],tp[2];
void dfs(int u,int fa){
 if(u<=n){
  for(int i=0;i<(int)g[u].size();i++){
   int v=g[u][i];
   if(v==fa) continue;
   dfs(v,u);
   memset(tp,0x8f,sizeof tp);
   if(v<=n){
    tp[0]=dp[u][0]+max(dp[v][0],dp[v][1]);
    tp[1]=max(
     dp[u][0]+dp[v][0]+1,
     dp[u][1]+max(dp[v][0],dp[v][1])
    );
   }else{
    tp[0]=dp[u][0]+dp[v][0];
    tp[1]=max(dp[u][0]+dp[v][1],dp[u][1]+dp[v][0]);
   }
   dp[u][0]=tp[0],dp[u][1]=tp[1];
  }
 }else{
  for(int i=0;i<(int)g[u].size();i++){
   int v=g[u][i];
   if(v==fa) continue;
   dfs(v,u);
  }
  min_dp[0][0]=0;
  min_dp[1][1]=1;
  min_dp[0][1]=min_dp[1][0]=-0x3f3f3f3f;
  for(int i=0;i<(int)g[u].size();i++){
   int v=g[u][i];
   if(v==fa) continue;
   memset(tmp,0x8f,sizeof tmp);
   tmp[0][0]=max(
    min_dp[0][0]+max(dp[v][0],dp[v][1]),
    min_dp[0][1]+dp[v][0]
   );
   tmp[0][1]=min_dp[0][0]+dp[v][0]+1;
   tmp[1][0]=max(
    min_dp[1][0]+max(dp[v][0],dp[v][1]),
    min_dp[1][1]+dp[v][0]
   );
   tmp[1][1]=min_dp[1][0]+dp[v][0]+1;
   memcpy(min_dp,tmp,sizeof tmp);
  }
  dp[u][1]=max(min_dp[0][1],min_dp[1][0]);
  dp[u][0]=min_dp[0][0];
 }
}
int main(){
 int m,a,b;
 scanf("%d%d",&n,&m);
 for(int i=0;i<m;i++){
  scanf("%d%d",&a,&b);
  init_g[a].push_back(b);
  init_g[b].push_back(a);
 }
 par[1]=-1;
 tarjan(1);
 dfs(1,-1);
 printf("%d\n",max(dp[1][0],dp[1][1]));
 return 0;
}
```

8.7 DLX

```
struct DLX {
 const static int maxn=210;
 const static int maxm=210;
 const static int maxnode=210*210;
 int n, m, size, row[maxnode], col[maxnode];
 int U[maxnode], D[maxnode], L[maxnode], R[maxnode];
 int H[maxn], S[maxm], ansd, ans[maxn];
 void init(int _n, int _m) {
  n = _n, m = _m;
  for(int i = 0; i <= m; ++i) {
   S[i] = 0;
   U[i] = D[i] = i;
   L[i] = i-1, R[i] = i+1;
  }
  R[L[0] = size = m] = 0;
  for(int i = 1; i <= n; ++i) H[i] = -1;
 }
 void Link(int r, int c) {
  ++S[col[++size] = c];
  row[size] = r; D[size] = D[c];
  U[D[c]] = size; U[size] = c; D[c] = size;
  if(H[r] < 0) H[r] = L[size] = R[size] = size;
  else {
   R[size] = R[H[r]];
   L[R[H[r]]] = size;
   L[size] = H[r];
   R[H[r]] = size;
  }
 }
 void remove(int c) {
  L[R[c]] = L[c]; R[L[c]] = R[c];
  for(int i = D[c]; i != c; i = D[i])
   for(int j = R[i]; j != i; j = R[j]) {
    U[D[j]] = U[j];
    D[U[j]] = D[j];
    --S[col[j]];
   }
 }
 void resume(int c) {
  L[R[c]] = c; R[L[c]] = c;
  for(int i = U[c]; i != c; i = U[i])
   for(int j = L[i]; j != i; j = L[j]) {
    U[D[j]] = j;
    D[U[j]] = j;
    ++S[col[j]];
   }
 }
 void dance(int d) {
  if(d>=ansd) return;
  if(R[0] == 0) {
   ansd = d;
   return;
  }
  int c = R[0];
  for(int i = R[0]; i; i = R[i])
   if(S[i] < S[c]) c = i;
  remove(c);
  for(int i = D[c]; i != c; i = D[i]) {
   ans[d] = row[i];
   for(int j = R[i]; j != i; j = R[j])
    remove(col[j]);
   dance(d+1);
   for(int j = L[i]; j != i; j = L[j])
    resume(col[j]);
  }
  resume(c);
 }
} sol;
```

8.8 Tree Knapsack

```
int dp[N][K];PII obj[N];
vector<int> G[N];
void dfs(int u, int mx){
```

```
 for(int s: G[u]) {
  if(mx < obj[s].first) continue;
  for(int i=0;i<=mx-obj[s].FF;i++)
   dp[s][i] = dp[u][i];
  dfs(s, mx - obj[s].first);
  for(int i=obj[s].FF;i<=mx;i++)
   dp[u][i] = max(dp[u][i],
    dp[s][i - obj[s].FF] + obj[s].SS);
 }
}
int main(){
 int n, k; cin >> n >> k;
 for(int i=1;i<=n;i++){
  int p; cin >> p;
  G[p].push_back(i);
  cin >> obj[i].FF >> obj[i].SS;
 }
 dfs(0, k); int ans = 0;
 for(int i=0;i<=k;i++) ans = max(ans, dp[0][i]);
 cout << ans << '\n';
 return 0;
}
```

8.9 N Queens Problem

```
vector< int > solve( int n ) {
 // no solution when n=2, 3
 vector< int > ret;
 if ( n % 6 == 2 ) {
  for ( int i = 2 ; i <= n ; i += 2 )
   ret.push_back( i );
  ret.push_back( 3 ); ret.push_back( 1 );
  for ( int i = 7 ; i <= n ; i += 2 )
   ret.push_back( i );
  ret.push_back( 5 );
 } else if ( n % 6 == 3 ) {
  for ( int i = 4 ; i <= n ; i += 2 )
   ret.push_back( i );
  ret.push_back( 2 );
  for ( int i = 5 ; i <= n ; i += 2 )
   ret.push_back( i );
  ret.push_back( 1 ); ret.push_back( 3 );
 } else {
  for ( int i = 2 ; i <= n ; i += 2 )
   ret.push_back( i );
  for ( int i = 1 ; i <= n ; i += 2 )
   ret.push_back( i );
 }
 return ret;
}
```

8.10 Aliens Optimization

```
long long Alien() {
 long long c = kInf;
 for (int d = 60; d >= 0; --d) {
  // cost can be negative, depending on the problem.
  if (c - (1LL << d) < 0) continue;
  long long ck = c - (1LL << d);
  pair<long long, int> r = check(ck);
  if (r.second == k) return r.first - ck * k;
  if (r.second < k) c = ck;
 }
 pair<long long, int> r = check(c);
 return r.first - c * k;
}
```

8.11 To Check When Submit

- Array out of bound.
- long long / double cast.
- Initialization.
- Delete all debug code.
- Check sample testcase after simple modification.