Contents

1 Basic

1.1 vimrc

```
se is nu ru et tgc sc hls cin cino+=j1 sw=2 sts=2 bs=2
    mouse=a "encoding=utf-8 ls=2
syn on | colo desert | filetype indent on
inoremap {<CR> {<CR>}<ESC>O
map <F8> <ESC>:w<CR>:!g++ "%" -o "%<" -g -std=gnu++20 -
    DCKISEKI -Wall -Wextra -Wshadow -Wfatal-errors -
    Wconversion -fsanitize=address,undefined && echo
    success<CR>
map <F9> <ESC>:w<CR>:!g++ "%" -o "%<" -O2 -g -std=gnu
    ++20 && echo success<CR>
map <F10> <ESC>:!./"%<"<CR>
ca Hash w !cpp -dD -P -fpreprocessed \| tr -d '[:space
    :]' \| md5sum \| cut -c-6
" setxkbmap -option caps:ctrl_modifier
```

1.2 Debug Macro [b78d75]

```
#ifdef CKISEKI
#define safe cerr<<__PRETTY_FUNCTION__<<" line "<<
    __LINE__<<" safe\n"
#define debug(a...) debug_(#a, a)
#define orange(a...) orange_(#a, a)
template <typename ...T>
void debug_(const char *s, T ...a) {
  cerr << "\e[1;32m(" << s << ") = (";
  int cnt = sizeof...(T);
  (..., (cerr << a << (--cnt ? ", " : ")\e[0m\n")));
}
template <typename I>
void orange_(const char *s, I L, I R) {
  cerr << "\e[1;32m[ " << s << " ] = [ ";
  for (int f = 0; L != R; ++L)
    cerr << (f++ ? ", " : "") << *L;
  cerr << " ]\e[0m\n";
}
#else
#define safe ((void)0)
#define debug(...) safe
#define orange(...) safe
#endif
```

1.3 Increase Stack

```
const int size = 256 << 20;
register long rsp asm("rsp");
char *p = (char*)malloc(size)+size, *bak = (char*)rsp;
__asm__("movq %0, %%rsp\n"::"r"(p));
// main
__asm__("movq %0, %%rsp\n"::"r"(bak));
```

1.4 Pragma Optimization [f63b0a]

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,tune=native")
__builtin_ia32_ldmxcsr(__builtin_ia32_stmxcsr()|0x8000)
```

1.5 IO Optimization [c9494b]

```
static inline int gc() {
 constexpr int B = 1<<20; static char buf[B], *p, *q;
 if (p == q) q = (p = buf) + fread(buf, 1, B, stdin);
 return q == buf ? EOF : *p++;
}
```

2 Data Structure

2.1 Dark Magic [095f25]

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
using namespace __gnu_pbds;
// heap tags: paring/binary/binomial/rc_binomial/thin
template<typename T>
using pbds_heap=__gnu_pbds::prioity_queue<T,less<T>, \
                  pairing_heap_tag>;
// pbds_heap::point_iterator
// x = pq.push(10); pq.modify(x, 87); a.join(b);
// tree tags: rb_tree_tag/ov_tree_tag/splay_tree_tag
template<typename T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
// find_by_order, order_of_key
// hash tables: cc_hash_table/gp_hash_table
```

2.2 Link-Cut Tree [7ce2b4]

```
template <typename Val, typename SVal> class LCT {
 struct node {
  int pa, ch[2];
  bool rev;
  Val v, prod, rprod;
  SVal sv, sub, vir;
  node() : pa{0}, ch{0, 0}, rev{false}, v{}, prod{},
    rprod{}, sv{}, sub{}, vir{} {};
 };
#define cur o[u]
#define lc cur.ch[0]
#define rc cur.ch[1]
 vector<node> o;
 bool is_root(int u) const {
  return o[cur.pa].ch[0]!=u && o[cur.pa].ch[1]!=u;
 }
 bool is_rch(int u) const {
  return o[cur.pa].ch[1] == u && !is_root(u);
 }
 void down(int u) {
  if (not cur.rev) return;
  if (lc) set_rev(lc);
  if (rc) set_rev(rc);
  cur.rev = false;
 }
 void up(int u) {
  cur.prod = o[lc].prod * cur.v * o[rc].prod;
  cur.rprod = o[rc].rprod * cur.v * o[lc].rprod;
  cur.sub = cur.vir + o[lc].sub + o[rc].sub + cur.sv;
 }
 void set_rev(int u) {
  swap(lc, rc);
  swap(cur.prod, cur.rprod);
  cur.rev ^= 1;
 }
 void rotate(int u) {
  int f=cur.pa, g=o[f].pa, l=is_rch(u);
  if (cur.ch[l ^ 1]) o[cur.ch[l ^ 1]].pa = f;
  if (not is_root(f)) o[g].ch[is_rch(f)] = u;
  o[f].ch[l] = cur.ch[l ^ 1];
  cur.ch[l ^ 1] = f;
  cur.pa = g, o[f].pa = u;
  up(f);
 }
 void splay(int u) {
  vector<int> stk = {u};
  while (not is_root(stk.back()))
   stk.push_back(o[stk.back()].pa);
  while (not stk.empty()) {
   down(stk.back());
   stk.pop_back();
  }
  for (int f = cur.pa; not is_root(u); f = cur.pa) {
   if(!is_root(f))rotate(is_rch(u)==is_rch(f)?f:u);
   rotate(u);
  }
  up(u);
 }
 void access(int x) {
  for (int u = x, last = 0; u; u = cur.pa) {
   splay(u);
   cur.vir = cur.vir + o[rc].sub - o[last].sub;
   rc = last; up(last = u);
  }
  splay(x);
 }
 int find_root(int u) {
  int la = 0;
  for (access(u); u; u = lc) down(la = u);
  return la;
 }
 void split(int x, int y) {change_root(x);access(y);}
 void change_root(int u) { access(u); set_rev(u); }
public:
 LCT(int n = 0) : o(n + 1) {}
 int add(const Val &v = {}) {
  o.push_back(v);
  return int(o.size()) - 2;
 }
 int add(Val &&v) {
  o.emplace_back(move(v));
  return int(o.size()) - 2;
 }
 void set_val(int u, const Val &v) {
  splay(++u); cur.v = v; up(u);
 }
 void set_sval(int u, const SVal &v) {
  splay(++u); cur.sv = v; up(u);
 }
 Val query(int x, int y) {
  split(++x, ++y); return o[y].prod;
 }
 SVal subtree(int p, int u) {
  change_root(++p); access(++u);
  return cur.vir + cur.sv;
 }
 bool connected(int u, int v) {
  return find_root(++u) == find_root(++v); }
 void link(int x, int y) {
  change_root(++x); access(++y);
  o[y].vir = o[y].vir + o[x].sub;
  up(o[x].pa = y);
 }
 void cut(int x, int y) {
  split(++x, ++y);
  o[y].ch[0] = o[x].pa = 0; up(y);
 }
#undef cur
#undef lc
#undef rc
};
```

2.3 LiChao Segment Tree [b9c827]

```
struct L {
 int m, k, id;
 L() : id(-1) {}
 L(int a, int b, int c) : m(a), k(b), id(c) {}
 int at(int x) { return m * x + k; }
};
class LiChao {
private:
 int n; vector<L> nodes;
 static int lc(int x) { return 2 * x + 1; }
 static int rc(int x) { return 2 * x + 2; }
 void insert(int l, int r, int id, L ln) {
  int m = (l + r) >> 1;
  if (nodes[id].id == -1)
   return nodes[id] = ln, void();
  bool atLeft = nodes[id].at(l) < ln.at(l);
  if (nodes[id].at(m) < ln.at(m))
   atLeft ^= 1, swap(nodes[id], ln);
  if (r - l == 1) return;
  if (atLeft) insert(l, m, lc(id), ln);
  else insert(m, r, rc(id), ln);
 }
 int query(int l, int r, int id, int x) {
  int m = (l + r) >> 1, ret = 0;
  if (nodes[id].id != -1) ret = nodes[id].at(x);
  if (r - l == 1) return ret;
  if (x < m) return max(ret, query(l, m, lc(id), x));
  return max(ret, query(m, r, rc(id), x));
 }
public:
 LiChao(int n_) : n(n_), nodes(n * 4) {}
 void insert(L ln) { insert(0, n, 0, ln); }
 int query(int x) { return query(0, n, 0, x); }
};
```

2.4 Treap [ae576c]

```
__gnu_cxx::sfmt19937 rnd(7122);
namespace Treap {
struct node {
 int size, pri; node *lc, *rc, *pa;
 node() : size(1), pri(rnd()), lc(0), rc(0), pa(0) {}
 void pull() {
  size = 1; pa = 0;
  if (lc) { size += lc->size; lc->pa = this; }
  if (rc) { size += rc->size; rc->pa = this; }
 }
};
int SZ(node *x) { return x ? x->size : 0; }
node *merge(node *L, node *R) {
```

```cpp
 if (not L or not R) return L ? L : R;
 if (L->pri > R->pri)
  return L->rc = merge(L->rc, R), L->pull(), L;
 else
  return R->lc = merge(L, R->lc), R->pull(), R;
}
void splitBySize(node *o, int k, node *&L, node *&R) {
 if (not o) L = R = 0;
 else if (int s = SZ(o->lc) + 1; s <= k)
  L=o, splitBySize(o->rc, k-s, L->rc, R), L->pull();
 else
  R=o, splitBySize(o->lc, k, L, R->lc), R->pull();
} // SZ(L) == k
int getRank(node *o) { // 1-base
 int r = SZ(o->lc) + 1;
 for (; o->pa; o = o->pa)
  if (o->pa->rc == o) r += SZ(o->pa->lc) + 1;
 return r;
}
} // namespace Treap
```

2.5 Linear Basis [138d5d]

```cpp
template <int BITS, typename S = int> struct Basis {
 static constexpr S MIN = numeric_limits<S>::min();
 array<pair<llu, S>, BITS> b;
 Basis() { b.fill({0, MIN}); }
 void add(llu x, S p) {
  for (int i = BITS-1; i>=0; i--) if (x >> i & 1) {
   if (b[i].first == 0) return b[i]={x, p}, void();
   if (b[i].second < p)
    swap(b[i].first, x), swap(b[i].second, p);
   x ^= b[i].first;
  }
 }
 optional<llu> query_kth(llu v, llu k) {
  vector<pair<llu, int>> o;
  for (int i = 0; i < BITS; i++)
   if (b[i].first) o.emplace_back(b[i].first, i);
  if (k >= (1ULL << o.size())) return {};
  for (int i = int(o.size()) - 1; i >= 0; i--)
   if ((k >> i & 1) ^ (v >> o[i].second & 1))
    v ^= o[i].first;
  return v;
 }
 Basis filter(S l) {
  Basis res = *this;
  for (int i = 0; i < BITS; i++)
   if (res.b[i].second < l) res.b[i] = {0, MIN};
  return res;
 }
};
```

2.6 Binary Search On Segtree [29b3cb]

```cpp
// find_first = x -> minimal x s.t. check( [a, x) )
// find_last = x -> maximal x s.t. check( [x, b) )
template <typename C>
int find_first(int l, const C &check) {
 if (l >= n) return n + 1;
 l += sz;
 for (int i = hei; i > 0; i--) propagate(l >> i);
 Monoid sum = identity;
 do {
  while ((l & 1) == 0) l >>= 1;
  if (check(f(sum, data[l]))) {
   while (l < sz) {
    propagate(l); l <<= 1;
    if (auto nxt = f(sum,data[l]); not check(nxt))
     sum = nxt, l++;
   }
   return l + 1 - sz;
  }
  sum = f(sum, data[l++]);
 } while ((l & -l) != l);
 return n + 1;
}
template <typename C>
int find_last(int r, const C &check) {
 if (r <= 0) return -1;
 r += sz;
 for (int i = hei; i > 0; i--) propagate((r-1) >> i);
 Monoid sum = identity;
 do {
  r--;
  while (r > 1 and (r & 1)) r >>= 1;
  if (check(f(data[r], sum))) {
   while (r < sz) {
    propagate(r); r = (r << 1) + 1;
    if (auto nxt = f(data[r],sum); not check(nxt))
     sum = nxt, r--;
   }
   return r - sz;
  }
  sum = f(data[r], sum);
 } while ((r & -r) != r);
 return -1;
}
```

3 Graph

3.1 2-SAT (SCC) [76434f]

```cpp
class TwoSat { // test @ CSES Giant Pizza
private:
 int n; vector<vector<int>> G, rG, sccs;
 vector<int> ord, idx, vis, res;
 void dfs(int u) {
  vis[u] = true;
  for (int v : G[u]) if (!vis[v]) dfs(v);
  ord.push_back(u);
 }
 void rdfs(int u) {
  vis[u] = false; idx[u] = sccs.size() - 1;
  sccs.back().push_back(u);
  for (int v : rG[u]) if (vis[v]) rdfs(v);
 }
public:
 TwoSat(int n_) : n(n_), G(n), rG(n), idx(n), vis(n),
    res(n) {}
 void add_edge(int u, int v) {
  G[u].push_back(v); rG[v].push_back(u);
 }
 void orr(int x, int y) {
  if ((x ^ y) == 1) return;
  add_edge(x ^ 1, y); add_edge(y ^ 1, x);
 }
 bool solve() {
  for (int i = 0; i < n; ++i) if (not vis[i]) dfs(i);
  reverse(ord.begin(), ord.end());
  for (int u : ord)
   if (vis[u]) sccs.emplace_back(), rdfs(u);
  for (int i = 0; i < n; i += 2)
   if (idx[i] == idx[i + 1]) return false;
  vector<bool> c(sccs.size());
  for (size_t i = 0; i < sccs.size(); ++i)
   for (int z : sccs[i])
    res[z] = c[i], c[idx[z ^ 1]] = !c[i];
  return true;
 }
 bool get(int x) { return res[x]; }
 int get_id(int x) { return idx[x]; }
 int count() { return sccs.size(); }
};
```

3.2 BCC [6ac6db]

```cpp
class BCC {
 int n, ecnt, bcnt;
 vector<vector<pair<int, int>>> g;
 vector<int> dfn, low, bcc, stk;
 vector<bool> ap, bridge;
 void dfs(int u, int f) {
  dfn[u] = low[u] = dfn[f] + 1;
  int ch = 0;
  for (auto [v, t] : g[u]) if (bcc[t] == -1) {
   bcc[t] = 0; stk.push_back(t);
   if (dfn[v]) {
    low[u] = min(low[u], dfn[v]);
    continue;
   }
   ++ch, dfs(v, u);
   low[u] = min(low[u], low[v]);
   if (low[v] > dfn[u]) bridge[t] = true;
   if (low[v] < dfn[u]) continue;
   ap[u] = true;
   while (not stk.empty()) {
```

```
    int o = stk.back(); stk.pop_back();
    bcc[o] = bcnt;
    if (o == t) break;
   }
   bcnt += 1;
  }
  ap[u] = ap[u] and (ch != 1 or u != f);
 }
public:
 BCC(int n_) : n(n_), ecnt(0), bcnt(0), g(n), dfn(n),
    low(n), stk(), ap(n) {}
 void add_edge(int u, int v) {
  g[u].emplace_back(v, ecnt);
  g[v].emplace_back(u, ecnt++);
 }
 void solve() {
  bridge.assign(ecnt, false); bcc.assign(ecnt, -1);
  for (int i = 0; i < n; ++i) if (!dfn[i]) dfs(i, i);
 }
 int bcc_id(int x) const { return bcc[x]; }
 bool is_ap(int x) const { return ap[x]; }
 bool is_bridge(int x) const { return bridge[x]; }
};
```

3.3 Round Square Tree [528440]

```
struct RST {
 int n; vector<vector<int>> T;
 RST(auto &G) : n(G.size()), T(n) {
  vector<int> stk, vis(n), low(n);
  auto dfs = [&](auto self, int u, int d) -> void {
   low[u] = vis[u] = d; stk.push_back(u);
   for (int v : G[u]) if (!vis[v]) {
    self(self, v, d + 1);
    if (low[v] == vis[u]) {
     int cnt = T.size(); T.emplace_back();
     for (int x = -1; x != v; stk.pop_back())
      T[cnt].push_back(x = stk.back());
     T[u].push_back(cnt); // T is rooted
    } else low[u] = min(low[u], low[v]);
   } else low[u] = min(low[u], vis[v]);
  };
  for (int u = 0; u < N; u++)
   if (!vis[u]) dfs(dfs, u, 1);
 } // T may be forest; after dfs, stk are the roots
}; // test @ 2020 Shanghai K
```

3.4 Edge TCC [5a2668]

```
vector<vector<int>> ETCC(auto &adj) {
 const int n = static_cast<int>(adj.size());
 vector<int> up(n), low(n), in, out, nx, id;
 in = out = nx = id = vector<int>(n, -1);
 int dfc = 0, cnt = 0; Dsu dsu(n);
 auto merge = [&](int u, int v) {
  dsu.join(u, v); up[u] += up[v]; };
 auto dfs = [&](auto self, int u, int p) -> void {
  in[u] = low[u] = dfc++;
  for (int v : adj[u]) if (v != u) {
   if (v == p) { p = -1; continue; }
   if (in[v] == -1) {
    self(self, v, u);
    if (nx[v] == -1 && up[v] <= 1) {
     up[u] += up[v]; low[u] = min(low[u], low[v]);
     continue;
    }
    if (up[v] == 0) v = nx[v];
    if (low[u] > low[v])
     low[u] = low[v], swap(nx[u], v);
    for (; v != -1; v = nx[v]) merge(u, v);
   } else if (in[v] < in[u]) {
    low[u] = min(low[u], in[v]); up[u]++;
   } else {
    for (int &x = nx[u]; x != -1 &&
      in[x] <= in[v] && in[v] < out[x]; x = nx[x])
     merge(u, x);
    up[u]--;
   }
  }
  out[u] = dfc;
 };
 for (int i = 0; i < n; i++)
  if (in[i] == -1) dfs(dfs, i, -1);
 for (int i = 0; i < n; i++)
  if (dsu.anc(i) == i) id[i] = cnt++;
 vector<vector<int>> comps(cnt);
 for (int i = 0; i < n; i++)
  comps[id[dsu.anc(i)]].push_back(i);
 return comps;
} // test @ yosupo judge
```

3.5 Centroid Decomposition [63b2fb]

```
struct Centroid {
 using G = vector<vector<pair<int, int>>>;
 vector<vector<int64_t>> Dist;
 vector<int> Pa, Dep;
 vector<int64_t> Sub, Sub2;
 vector<int> Cnt, Cnt2;
 vector<int> vis, sz, mx, tmp;
 void DfsSz(const G &g, int x) {
  vis[x] = true, sz[x] = 1, mx[x] = 0;
  for (auto [u, w] : g[x]) if (not vis[u]) {
   DfsSz(g, u); sz[x] += sz[u];
   mx[x] = max(mx[x], sz[u]);
  }
  tmp.push_back(x);
 }
 void DfsDist(const G &g, int x, int64_t D = 0) {
  Dist[x].push_back(D); vis[x] = true;
  for (auto [u, w] : g[x])
   if (not vis[u]) DfsDist(g, u, D + w);
 }
 void DfsCen(const G &g, int x, int D = 0, int p = -1)
    {
  tmp.clear(); DfsSz(g, x);
  int M = tmp.size(), C = -1;
  for (int u : tmp) {
   if (max(M - sz[u], mx[u]) * 2 <= M) C = u;
   vis[u] = false;
  }
  DfsDist(g, C);
  for (int u : tmp) vis[u] = false;
  Pa[C] = p, vis[C] = true, Dep[C] = D;
  for (auto [u, w] : g[C])
   if (not vis[u]) DfsCen(g, u, D + 1, C);
 }
 Centroid(int N, G g)
   : Sub(N), Sub2(N), Cnt(N), Cnt2(N), Dist(N), Pa(N),
    Dep(N), vis(N), sz(N), mx(N) { DfsCen(g, 0); }
 void Mark(int v) {
  int x = v, z = -1;
  for (int i = Dep[v]; i >= 0; --i) {
   Sub[x] += Dist[v][i], Cnt[x]++;
   if (z != -1)
    Sub2[z] += Dist[v][i], Cnt2[z]++;
   x = Pa[z = x];
  }
 }
 int64_t Query(int v) {
  int64_t res = 0;
  int x = v, z = -1;
  for (int i = Dep[v]; i >= 0; --i) {
   res += Sub[x] + 1LL * Cnt[x] * Dist[v][i];
   if (z != -1)
    res -= Sub2[z] + 1LL * Cnt2[z] * Dist[v][i];
   x = Pa[z = x];
  }
  return res;
 }
};
```

3.6 DMST [0ae901]

```
using D = int64_t;
struct E { int s, t; D w; }; // 0-base
vector<int> dmst(const vector<E> &e, int n, int root) {
 using PQ = pair<min_heap<pair<D, int>>, D>;
 auto push = [](PQ &pq, pair<D, int> v) {
  pq.first.emplace(v.first - pq.second, v.second);
 };
 auto top = [](const PQ &pq) -> pair<D, int> {
  auto r = pq.first.top();
  return {r.first + pq.second, r.second};
 };
 auto join = [&push, &top](PQ &a, PQ &b) {
  if (a.first.size() < b.first.size()) swap(a, b);
  while (!b.first.empty()) {
```

```
   push(a, top(b));
   b.first.pop();
  }
 };
 vector<PQ> h(n * 2);
 for (size_t i = 0; i < e.size(); ++i)
  push(h[e[i].t], {e[i].w, i});
 vector<int> a(n*2), v(n*2, -1), pa(n*2, -1), r(n*2);
 iota(a.begin(), a.end(), 0);
 auto o = [&](int x) { int y;
  for (y = x; a[y] != y; y = a[y]);
  for (int ox = x; x != y; ox = x)
   x = a[x], a[ox] = y;
  return y;
 };
 v[root] = n + 1;
 int pc = n;
 for (int i = 0; i < n; ++i) if (v[i] == -1) {
  for (int p = i; v[p] == -1 || v[p] == i; p = o(e[r[p]].s)) {
   if (v[p] == i) {
    int q = p; p = pc++;
    do {
     h[q].second = -h[q].first.top().first;
     join(h[pa[q] = a[q] = p], h[q]);
    } while ((q = o(e[r[q]].s)) != p);
   }
   v[p] = i;
   while (!h[p].first.empty() && o(e[top(h[p]).second].s) == p)
    h[p].first.pop();
   r[p] = top(h[p]).second;
  }
 }
 vector<int> ans;
 for (int i = pc - 1; i >= 0; i--) if (i != root && v[i] != n) {
  for (int f = e[r[i]].t; f != -1 && v[f] != n; f = pa[f])
   v[f] = n;
  ans.push_back(r[i]);
 }
 return ans; // default minimize, returns edgeid array
}
```

3.7 Dominator Tree [ea5b7c]

```
struct Dominator {
 vector<vector<int>> g, r, rdom; int tk;
 vector<int> dfn, rev, fa, sdom, dom, val, rp;
 Dominator(int n) : g(n), r(n), rdom(n), tk(0) {
  dfn = rev = fa = sdom = dom =
   val = rp = vector<int>(n, -1); }
 void add_edge(int x, int y) { g[x].push_back(y); }
 void dfs(int x) {
  rev[dfn[x] = tk] = x;
  fa[tk] = sdom[tk] = val[tk] = tk; tk++;
  for (int u : g[x]) {
   if (dfn[u] == -1) dfs(u), rp[dfn[u]] = dfn[x];
   r[dfn[u]].push_back(dfn[x]);
  }
 }
 void merge(int x, int y) { fa[x] = y; }
 int find(int x, int c = 0) {
  if (fa[x] == x) return c ? -1 : x;
  if (int p = find(fa[x], 1); p != -1) {
   if (sdom[val[x]] > sdom[val[fa[x]]])
    val[x] = val[fa[x]];
   fa[x] = p;
   return c ? p : val[x];
  } else return c ? fa[x] : val[x];
 }
 vector<int> build(int s, int n) {
  // return the father of each node in dominator tree
  dfs(s); // p[i] = -2 if i is unreachable from s
  for (int i = tk - 1; i >= 0; --i) {
   for (int u : r[i])
    sdom[i] = min(sdom[i], sdom[find(u)]);
   if (i) rdom[sdom[i]].push_back(i);
   for (int u : rdom[i]) {
    int p = find(u);
    dom[u] = (sdom[p] == i ? i : p);
   }
   if (i) merge(i, rp[i]);
  }
  vector<int> p(n, -2); p[s] = -1;
  for (int i = 1; i < tk; ++i)
   if (sdom[i] != dom[i]) dom[i] = dom[dom[i]];
  for (int i = 1; i < tk; ++i)
   p[rev[i]] = rev[dom[i]];
  return p;
 } // test @ yosupo judge
};
```

3.8 Edge Coloring [029763]

```
// max(d_u) + 1 edge coloring, time: O(NM)
int C[kN][kN], G[kN][kN]; // 1-based, G: ans
void clear(int N) {
 for (int i = 0; i <= N; i++)
  for (int j = 0; j <= N; j++)
   C[i][j] = G[i][j] = 0;
}
void solve(vector<pair<int, int>> &E, int N) {
 int X[kN] = {}, a;
 auto update = [&](int u) {
  for (X[u] = 1; C[u][X[u]]; X[u]++);
 };
 auto color = [&](int u, int v, int c) {
  int p = G[u][v];
  G[u][v] = G[v][u] = c;
  C[u][c] = v, C[v][c] = u;
  C[u][p] = C[v][p] = 0;
  if (p) X[u] = X[v] = p;
  else update(u), update(v);
  return p;
 };
 auto flip = [&](int u, int c1, int c2) {
  int p = C[u][c1];
  swap(C[u][c1], C[u][c2]);
  if (p) G[u][p] = G[p][u] = c2;
  if (!C[u][c1]) X[u] = c1;
  if (!C[u][c2]) X[u] = c2;
  return p;
 };
 for (int i = 1; i <= N; i++) X[i] = 1;
 for (int t = 0; t < E.size(); t++) {
  auto [u, v] = E[t];
  int v0 = v, c = X[u], c0 = c, d;
  vector<pair<int, int>> L; int vst[kN] = {};
  while (!G[u][v0]) {
   L.emplace_back(v, d = X[v]);
   if (!C[v][c]) for(a=L.size()-1;a>=0;a--)
     c = color(u, L[a].first, c);
   else if(!C[u][d])for(a=L.size()-1;a>=0;a--)
     color(u, L[a].first, L[a].second);
   else if (vst[d]) break;
   else vst[d] = 1, v = C[u][d];
  }
  if (!G[u][v0]) {
   for (; v; v = flip(v, c, d), swap(c, d));
   if (C[u][c0]) { a = int(L.size()) - 1;
    while (--a >= 0 && L[a].second != c);
    for(;a>=0;a--)color(u,L[a].first,L[a].second);
   } else t--;
  }
 }
}
```

3.9 Lowbit Decomposition [aa3f57]

```
class LBD {
 int timer, chains;
 vector<vector<int>> G;
 vector<int> tl, tr, chain, head, dep, pa;
 // chains : number of chain
 // tl, tr[u] : subtree interval in the seq. of u
 // head[i] : head of the chain i
 // chian[u] : chain id of the chain u is on
 void predfs(int u, int f) {
  dep[u] = dep[pa[u] = f] + 1;
  for (int v : G[u]) if (v != f) {
   predfs(v, u);
   if (lowbit(chain[u]) < lowbit(chain[v]))
    chain[u] = chain[v];
  }
  if (chain[u] == 0) chain[u] = ++chains;
```

```
 }
 void dfschain(int u, int f) {
  tl[u] = timer++;
  if (head[chain[u]] == -1)
   head[chain[u]] = u;
  for (int v : G[u])
   if (v != f and chain[v] == chain[u])
    dfschain(v, u);
  for (int v : G[u])
   if (v != f and chain[v] != chain[u])
    dfschain(v, u);
  tr[u] = timer;
 }
public:
 LBD(int n) : timer(0), chains(0), G(n), tl(n), tr(n),
     chain(n), head(n, -1), dep(n), pa(n) {}
 void add_edge(int u, int v) {
  G[u].push_back(v); G[v].push_back(u);
 }
 void decompose() { predfs(0, 0); dfschain(0, 0); }
 PII get_subtree(int u) { return {tl[u], tr[u]}; }
 vector<PII> get_path(int u, int v) {
  vector<PII> res;
  while (chain[u] != chain[v]) {
   if (dep[head[chain[u]]] < dep[head[chain[v]]])
    swap(u, v);
   int s = head[chain[u]];
   res.emplace_back(tl[s], tl[u] + 1);
   u = pa[s];
  }
  if (dep[u] < dep[v]) swap(u, v);
  res.emplace_back(tl[v], tl[u] + 1);
  return res;
 }
};
```

3.10 Manhattan MST [df6f59]

```
typedef Point<int> P;
vector<array<int, 3>> manhattanMST(vector<P> ps) {
 vi id(sz(ps));
 iota(all(id), 0);
 vector<array<int, 3>> edges;
 rep(k, 0, 4) {
  sort(all(id), [&](int i, int j) {
   return (ps[i] - ps[j]).x < (ps[j] - ps[i]).y;
  });
  map<int, int> sweep;
  for (int i : id) {
   for (auto it = sweep.lower_bound(-ps[i].y);
      it != sweep.end(); sweep.erase(it++)) {
    int j = it->second;
    P d = ps[i] - ps[j];
    if (d.y > d.x) break;
    edges.push_back({d.y + d.x, i, j});
   }
   sweep[-ps[i].y] = i;
  }
  for (P &p : ps)
   if (k & 1) p.x = -p.x;
   else swap(p.x, p.y);
 }
 return edges; // [{w, i, j}, ...]
}
```

3.11 MaxClique [293730]

```
// contain a self loop u to u, than u won't in clique
template <size_t maxn> class MaxClique {
private:
 using bits = bitset<maxn>;
 bits popped, G[maxn], ans;
 size_t deg[maxn], deo[maxn], n;
 void sort_by_degree() {
  popped.reset();
  for (size_t i = 0; i < n; ++i)
   deg[i] = G[i].count();
  for (size_t i = 0; i < n; ++i) {
   size_t mi = maxn, id = 0;
   for (size_t j = 0; j < n; ++j)
    if (not popped[j] and deg[j] < mi)
     mi = deg[id = j];
   popped[deo[i] = id] = 1;
   for (size_t u = G[i]._Find_first(); u < n;
     u = G[i]._Find_next(u))
    --deg[u];
  }
 }
 void BK(bits R, bits P, bits X) {
  if (R.count() + P.count() <= ans.count()) return;
  if (not P.count() and not X.count()) {
   if (R.count() > ans.count()) ans = R;
   return;
  }
  /* greedily chosse max degree as pivot
  bits cur = P | X; size_t pivot = 0, sz = 0;
  for ( size_t u = cur._Find_first() ;
   u < n ; u = cur._Find_next( u ) )
    if ( deg[ u ] > sz ) sz = deg[ pivot = u ];
  cur = P & ( ~G[ pivot ] );
  */ // or simply choose first
  bits cur = P & (~G[(P | X)._Find_first()]);
  for (size_t u = cur._Find_first(); u < n;
    u = cur._Find_next(u)) {
   if (R[u]) continue;
   R[u] = 1;
   BK(R, P & G[u], X & G[u]);
   R[u] = P[u] = 0, X[u] = 1;
  }
 }
public:
 void init(size_t n_) {
  n = n_;
  for (size_t i = 0; i < n; ++i) G[i].reset();
  ans.reset();
 }
 void add_edges(int u, bits S) { G[u] = S; }
 void add_edge(int u, int v) { G[u][v] = G[v][u] = 1; }
 int solve() {
  sort_by_degree(); // or simply iota( deo... )
  for (size_t i = 0; i < n; ++i)
   deg[i] = G[i].count();
  bits pob, nob = 0; pob.set();
  for (size_t i = n; i < maxn; ++i) pob[i] = 0;
  for (size_t i = 0; i < n; ++i) {
   size_t v = deo[i];
   bits tmp;
   tmp[v] = 1;
   BK(tmp, pob & G[v], nob & G[v]);
   pob[v] = 0, nob[v] = 1;
  }
  return static_cast<int>(ans.count());
 }
};
```

3.12 Minimum Mean Cycle [e23bc0]

```
// WARNING: TYPE matters
struct Edge { int s, t; llf c; };
llf solve(vector<Edge> &e, int n) {
 // O(VE), returns inf if no cycle, mmc otherwise
 vector<VI> prv(n + 1, VI(n)), prve = prv;
 vector<vector<llf>> d(n + 1, vector<llf>(n, inf));
 d[0] = vector<llf>(n, 0);
 for (int i = 0; i < n; i++) {
  for (int j = 0; j < (int)e.size(); j++) {
   auto [s, t, c] = e[j];
   if (d[i][s] < inf && d[i + 1][t] > d[i][s] + c) {
    d[i + 1][t] = d[i][s] + c;
    prv[i + 1][t] = s; prve[i + 1][t] = j;
   }
  }
 }
 llf mmc = inf; int st = -1;
 for (int i = 0; i < n; i++) {
  llf avg = -inf;
  for (int k = 0; k < n; k++) {
   if (d[n][i] < inf - eps)
    avg = max(avg, (d[n][i] - d[k][i]) / (n - k));
   else avg = inf;
  }
  if (avg < mmc) tie(mmc, st) = tie(avg, i);
 }
 if (st == -1) return inf;
 vector<int> vst(n), eid, cycle, rho;
 for (int i = n; !vst[st]; st = prv[i--][st]) {
  vst[st]++; eid.emplace_back(prve[i][st]);
```

```
  rho.emplace_back(st);
 }
 while (vst[st] != 2) {
  int v = rho.back(); rho.pop_back();
  cycle.emplace_back(v); vst[v]++;
 }
 reverse(all(eid)); eid.resize(cycle.size());
 return mmc;
}
```

3.13 Mo's Algorithm on Tree

```
dfs u:
 push u
 iterate subtree
 push u
Let P = LCA(u, v) with St(u)<=St(v)
if (P == u) query[St(u), St(v)]
else query[Ed(u), St(v)], query[St(P), St(P)]
```

3.14 Tree Hashing [707efa]

```
llu F(llu z) { // xorshift64star from iwiwi
 z ^= z >> 12; z ^= z << 25; z ^= z >> 27;
 return z * 2685821657736338717LL;
}
llu hsah(int u, int f) {
 llu r = 127; // bigger?
 for (int v : G[u]) if (v != f) r += F( hsah(v, u) );
 return F(r);
} // test @ UOJ 763
```

3.15 Virtual Tree [ad5cf5]

```
vector<pair<int, int>> build(vector<int> vs, int r) {
 vector<pair<int, int>> res;
 sort(vs.begin(), vs.end(), [](int i, int j) {
  return dfn[i] < dfn[j]; });
 vector<int> s = {r};
 for (int v : vs) if (v != r) {
  if (int o = lca(v, s.back()); o != s.back()) {
   while (s.size() >= 2) {
    if (dfn[s[s.size() - 2]] < dfn[o]) break;
    res.emplace_back(s[s.size() - 2], s.back());
    s.pop_back();
   }
   if (s.back() != o) {
    res.emplace_back(o, s.back());
    s.back() = o;
   }
  }
  s.push_back(v);
 }
 for (size_t i = 1; i < s.size(); ++i)
  res.emplace_back(s[i - 1], s[i]);
 return res; // (x, y): x->y
}
```

4 Matching & Flow

4.1 HopcroftKarp [4e7e69]

```
struct HK {
 vector<int> l, r, a, p; int ans;
 HK(int n, int m, auto &g) : l(n,-1),r(m,-1),ans(0) {
  for (bool match = true; match; ) {
   match = false; a.assign(n, -1); p = a;
   queue<int> q; int z;
   for (int i = 0; i < n; i++)
    if (l[i] == -1) q.push(a[i] = p[i] = i);
   // bitset<maxn> nvis, t; nvis.set();
   while (!q.empty()) {
    int x = q.front(); q.pop();
    if (l[a[x]] != -1) continue;
    // or use _Find_first and _Find_next here
    for (int y: g[x]) {
     // nvis.reset(y);
     if (r[y] == -1) {
      while (y != -1)
       r[y] = x, swap(l[x], y), x = p[x];
      match = true; ans++; break;
     } else if (p[r[y]] == -1)
      q.push(z = r[y]), p[z] = x, a[z] = a[x];
    }
```

```
    }
   }
  }
};
```

4.2 Dijkstra Cost Flow [06a723]

```
template <typename F, typename C> class MCMF {
 static constexpr F INF_F = numeric_limits<F>::max();
 static constexpr C INF_C = numeric_limits<C>::max();

 struct E {
  int to, r;
  F f; C c;
  E() {}
  E(int a, int b, F x, C y)
   : to(a), r(b), f(x), c(y) {}
 };
 vector<vector<E>> g;
 vector<pair<int, int>> f;
 vector<F> up;
 vector<C> d, h;
 optional<pair<F, C>> step(int S, int T) {
  priority_queue<pair<C, int>> q;
  q.emplace(d[S] = 0, S), up[S] = INF_F;
  while (not q.empty()) {
   auto [l, u] = q.top(); q.pop();
   if (up[u] == 0 or l != -d[u]) continue;
   for (int i = 0; i < int(g[u].size()); ++i) {
    auto e = g[u][i]; int v = e.to;
    auto nd = d[u] + e.c + h[u] - h[v];
    if (e.f <= 0 or d[v] <= nd)
     continue;
    f[v] = {u, i};
    up[v] = min(up[u], e.f);
    q.emplace(-(d[v] = nd), v);
   }
  }
  if (d[T] == INF_C) return nullopt;
  for (size_t i = 0; i < d.size(); i++) h[i]+=d[i];
  for (int i = T; i != S; i = f[i].first) {
   auto &eg = g[f[i].first][f[i].second];
   eg.f -= up[T];
   g[eg.to][eg.r].f += up[T];
  }
  return pair{up[T], h[T]};
 }

public:
 MCMF(int n) : g(n),f(n),up(n),d(n, INF_C),h(n) {}
 void add_edge(int s, int t, F c, C w) {
  g[s].emplace_back(t, int(g[t].size()), c, w);
  g[t].emplace_back(s, int(g[s].size()) - 1, 0, -w);
 }
 pair<F, C> solve(int a, int b) {
  F c = 0; C w = 0;
  while (auto r = step(a, b)) {
   c += r->first, w += r->first * r->second;
   fill(d.begin(), d.end(), INF_C);
  }
  return {c, w};
 }
};
```

4.3 Dinic [ebd802]

```
template <typename Cap = int64_t>
class Dinic{
private:
  struct E{
    int to, rev;
    Cap cap;
  };
  int n, st, ed;
  vector<vector<E>> G;
  vector<int> lv, idx;
  bool BFS(){
    lv.assign(n, -1);
    queue<int> bfs;
    bfs.push(st); lv[st] = 0;
    while (not bfs.empty()){
      int u = bfs.front(); bfs.pop();
      for (auto e: G[u]) {
        if (e.cap <= 0 or lv[e.to]!=-1) continue;
```

```
        bfs.push(e.to); lv[e.to] = lv[u] + 1;
      }
    }
    return lv[ed] != -1;
  }
  Cap DFS(int u, Cap f){
    if (u == ed) return f;
    Cap ret = 0;
    for(int &i = idx[u]; i < int(G[u].size()); ++i) {
      auto &e = G[u][i];
      if (e.cap <= 0 or lv[e.to]!=lv[u]+1) continue;
      Cap nf = DFS(e.to, min(f, e.cap));
      ret += nf; e.cap -= nf; f -= nf;
      G[e.to][e.rev].cap += nf;
      if (f == 0) return ret;
    }
    if (ret == 0) lv[u] = -1;
    return ret;
  }
public:
  void init(int n_) { G.assign(n = n_, vector<E>()); }
  void add_edge(int u, int v, Cap c){
    G[u].push_back({v, int(G[v].size()), c});
    G[v].push_back({u, int(G[u].size())-1, 0});
  }
  Cap max_flow(int st_, int ed_){
    st = st_, ed = ed_; Cap ret = 0;
    while (BFS()) {
      idx.assign(n, 0);
      Cap f = DFS(st, numeric_limits<Cap>::max());
      ret += f;
      if (f == 0) break;
    }
    return ret;
  }
};
```

4.4 Flow Models

- Maximum/Minimum flow with lower bound / Circulation problem
  1. Construct super source S and sink T.
  2. For each edge (x, y, l, u), connect $x \to y$ with capacity $u - l$.
  3. For each vertex v, denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  4. If $in(v) > 0$, connect $S \to v$ with capacity $in(v)$, otherwise, connect $v \to T$ with capacity $-in(v)$.
     - To maximize, connect $t \to s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T. If $f \neq \sum_{v \in V, in(v)>0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
     - To minimize, let f be the maximum flow from S to T. Connect $t \to s$ with capacity ∞ and let the flow from S to T be f'. If $f + f' \neq \sum_{v \in V, in(v)>0} in(v)$, there's no solution. Otherwise, f' is the answer.
  5. The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.

- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
  1. Redirect every edge: $y \to x$ if $(x, y) \in M$, $x \to y$ otherwise.
  2. DFS from unmatched vertices in X.
  3. $x \in X$ is chosen iff x is unvisited.
  4. $y \in Y$ is chosen iff y is visited.

- Minimum cost cyclic flow
  1. Consruct super source S and sink T
  2. For each edge (x, y, c), connect $x \to y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \to x$ with $(cost, cap) = (-c, 1)$
  3. For each edge with $c < 0$, sum these cost as K, then increase $d(y)$ by 1, decrease $d(x)$ by 1
  4. For each vertex v with $d(v) > 0$, connect $S \to v$ with $(cost, cap) = (0, d(v))$
  5. For each vertex v with $d(v) < 0$, connect $v \to T$ with $(cost, cap) = (0, -d(v))$
  6. Flow from S to T, the answer is the cost of the flow $C + K$

- Maximum density induced subgraph
  1. Binary search on answer, suppose we're checking answer T
  2. Construct a max flow model, let K be the sum of all weights
  3. Connect source $s \to v, v \in G$ with capacity K
  4. For each edge (u, v, w) in G, connect $u \to v$ and $v \to u$ with capacity w
  5. For $v \in G$, connect it with sink $v \to t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
  6. T is a valid answer if the maximum flow $f < K|V|$

- Minimum weight edge cover
  1. For each $v \in V$ create a copy v', and connect $u' \to v'$ with weight $w(u, v)$.
  2. Connect $v \to v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v.
  3. Find the minimum weight perfect matching on G'.

- Project selection problem
  1. If $p_v > 0$, create edge (s, v) with capacity p_v; otherwise, create edge (v, t) with capacity $-p_v$.
  2. Create edge (u, v) with capacity w with w being the cost of choosing u without choosing v.
  3. The mincut is equivalent to the maximum profit of a subset of projects.

4.5 General Graph Matching [00732c]

```
namespace matching {
int fa[kN], pre[kN], match[kN], s[kN], v[kN];
vector<int> g[kN];
queue<int> q;
void Init(int n) {
 for (int i = 0; i <= n; ++i) match[i] = pre[i] = n;
 for (int i = 0; i < n; ++i) g[i].clear();
}
void AddEdge(int u, int v) {
 g[u].push_back(v);
 g[v].push_back(u);
}
int Find(int u) {
 return u == fa[u] ? u : fa[u] = Find(fa[u]);
}
int LCA(int x, int y, int n) {
 static int tk = 0; tk++;
 x = Find(x), y = Find(y);
 for (; ; swap(x, y)) {
  if (x != n) {
   if (v[x] == tk) return x;
   v[x] = tk;
   x = Find(pre[match[x]]);
  }
 }
}
void Blossom(int x, int y, int l) {
 while (Find(x) != l) {
  pre[x] = y, y = match[x];
  if (s[y] == 1) q.push(y), s[y] = 0;
  if (fa[x] == x) fa[x] = l;
  if (fa[y] == y) fa[y] = l;
  x = pre[y];
 }
}
bool Bfs(int r, int n) {
 for (int i = 0; i <= n; ++i) fa[i] = i, s[i] = -1;
 while (!q.empty()) q.pop();
 q.push(r);
 s[r] = 0;
 while (!q.empty()) {
  int x = q.front(); q.pop();
  for (int u : g[x]) {
   if (s[u] == -1) {
    pre[u] = x, s[u] = 1;
    if (match[u] == n) {
     for (int a = u, b = x, last; b != n; a = last, b =
     pre[a])
      last = match[b], match[b] = a, match[a] = b;
     return true;
    }
    q.push(match[u]);
    s[match[u]] = 0;
   } else if (!s[u] && Find(u) != Find(x)) {
    int l = LCA(u, x, n);
    Blossom(x, u, l);
    Blossom(u, x, l);
   }
  }
 }
 return false;
}
int Solve(int n) {
 int res = 0;
 for (int x = 0; x < n; ++x) {
  if (match[x] == n) res += Bfs(x, n);
 }
 return res;
}}
```

4.6 Global Min-Cut [1f0306]

```
const int maxn = 500 + 5;
int w[maxn][maxn], g[maxn];
bool v[maxn], del[maxn];
void add_edge(int x, int y, int c) {
 w[x][y] += c; w[y][x] += c;
}
pair<int, int> phase(int n) {
 memset(v, false, sizeof(v));
 memset(g, 0, sizeof(g));
 int s = -1, t = -1;
 while (true) {
  int c = -1;
  for (int i = 0; i < n; ++i) {
   if (del[i] || v[i]) continue;
   if (c == -1 || g[i] > g[c]) c = i;
  }
  if (c == -1) break;
  v[s = t, t = c] = true;
  for (int i = 0; i < n; ++i) {
   if (del[i] || v[i]) continue;
   g[i] += w[c][i];
  }
 }
 return make_pair(s, t);
}
int mincut(int n) {
 int cut = 1e9;
 memset(del, false, sizeof(del));
 for (int i = 0; i < n - 1; ++i) {
  int s, t; tie(s, t) = phase(n);
  del[t] = true; cut = min(cut, g[t]);
  for (int j = 0; j < n; ++j) {
   w[s][j] += w[t][j]; w[j][s] += w[j][t];
  }
 }
 return cut;
}
```

4.7 GomoryHu Tree [f8938f]

```
int g[maxn];
vector<edge> GomoryHu(int n){
 vector<edge> rt;
 for(int i=1;i<=n;++i)g[i]=1;
 for(int i=2;i<=n;++i){
  int t=g[i];
  flow.reset(); // clear flows on all edge
  rt.push_back({i,t,flow(i,t)});
  flow.walk(i); // bfs points that connected to i (use
    edges not fully flow)
  for(int j=i+1;j<=n;++j){
   if(g[j]==t && flow.connect(j))g[j]=i; // check if i
    can reach j
  }
 }
 return rt;
}
```

4.8 Kuhn Munkres [1d3c40]

```
class KM {
private:
 static constexpr lld INF = 1LL << 60;
 vector<lld> hl,hr,slk;
 vector<int> fl,fr,pre,qu;
 vector<vector<lld>> w;
 vector<bool> vl,vr;
 int n, ql, qr;
 bool check(int x) {
  if (vl[x] = true, fl[x] != -1)
   return vr[qu[qr++] = fl[x]] = true;
  while (x != -1) swap(x, fr[fl[x] = pre[x]]);
  return false;
 }
 void bfs(int s) {
  fill(slk.begin(), slk.end(), INF);
  fill(vl.begin(), vl.end(), false);
  fill(vr.begin(), vr.end(), false);
  ql = qr = 0;
  vr[qu[qr++] = s] = true;
  while (true) {
   lld d;
   while (ql < qr) {
    for (int x = 0, y = qu[ql++]; x < n; ++x) {
     if(!vl[x]&&slk[x]>=(d=hl[x]+hr[y]-w[x][y])){
      if (pre[x] = y, d) slk[x] = d;
      else if (!check(x)) return;
     }
    }
   }
   d = INF;
   for (int x = 0; x < n; ++x)
    if (!vl[x] && d > slk[x]) d = slk[x];
   for (int x = 0; x < n; ++x) {
    if (vl[x]) hl[x] += d;
    else slk[x] -= d;
    if (vr[x]) hr[x] -= d;
   }
   for (int x = 0; x < n; ++x)
    if (!vl[x] && !slk[x] && !check(x)) return;
  }
 }
public:
 void init( int n_ ) {
  qu.resize(n = n_);
  fl.assign(n, -1); fr.assign(n, -1);
  hr.assign(n, 0); hl.resize(n);
  w.assign(n, vector<lld>(n));
  slk.resize(n); pre.resize(n);
  vl.resize(n); vr.resize(n);
 }
 void set_edge( int u, int v, lld x ) {w[u][v] = x;}
 lld solve() {
  for (int i = 0; i < n; ++i)
   hl[i] = *max_element(w[i].begin(), w[i].end());
  for (int i = 0; i < n; ++i) bfs(i);
  lld res = 0;
  for (int i = 0; i < n; ++i) res += w[i][fl[i]];
  return res;
 }
} km;
```

4.9 Minimum Cost Circulation [d99194]

```
struct Edge { int to, cap, rev, cost; };
vector<Edge> g[kN];
int dist[kN], pv[kN], ed[kN];
bool mark[kN];
int NegativeCycle(int n) {
 memset(mark, false, sizeof(mark));
 memset(dist, 0, sizeof(dist));
 int upd = -1;
 for (int i = 0; i <= n; ++i) {
  for (int j = 0; j < n; ++j) {
   int idx = 0;
   for (auto &e : g[j]) {
    if(e.cap > 0 && dist[e.to] > dist[j] + e.cost){
     dist[e.to] = dist[j] + e.cost;
     pv[e.to] = j, ed[e.to] = idx;
     if (i == n) {
      upd = j;
      while(!mark[upd])mark[upd]=1,upd=pv[upd];
      return upd;
     }
    }
    idx++;
   }
  }
 }
 return -1;
}
int Solve(int n) {
 int rt = -1, ans = 0;
 while ((rt = NegativeCycle(n)) >= 0) {
  memset(mark, false, sizeof(mark));
  vector<pair<int, int>> cyc;
  while (!mark[rt]) {
   cyc.emplace_back(pv[rt], ed[rt]);
   mark[rt] = true;
   rt = pv[rt];
  }
  reverse(cyc.begin(), cyc.end());
  int cap = kInf;
  for (auto &i : cyc) {
   auto &e = g[i.first][i.second];
```

```
   cap = min(cap, e.cap);
  }
  for (auto &i : cyc) {
   auto &e = g[i.first][i.second];
   e.cap -= cap;
   g[e.to][e.rev].cap += cap;
   ans += e.cost * cap;
  }
 }
 return ans;
}
```

4.10 Minimum Cost Max Flow [6d1b01]

```
template <typename F, typename C> class MCMF {
 static constexpr F INF_F = numeric_limits<F>::max();
 static constexpr C INF_C = numeric_limits<C>::max();
 struct E {
  int to, r;
  F f; C c;
  E() {}
  E(int a, int b, F x, C y)
   : to(a), r(b), f(x), c(y) {}
 };
 vector<vector<E>> g;
 vector<pair<int, int>> f;
 vector<bool> inq;
 vector<F> up; vector<C> d;
 optional<pair<F, C>> step(int S, int T) {
  queue<int> q;
  for (q.push(S), d[S] = 0, up[S] = INF_F;
    not q.empty(); q.pop()) {
   int u = q.front(); inq[u] = false;
   if (up[u] == 0) continue;
   for (int i = 0; i < int(g[u].size()); ++i) {
    auto e = g[u][i]; int v = e.to;
    if (e.f <= 0 or d[v] <= d[u] + e.c)
     continue;
    d[v] = d[u] + e.c; f[v] = {u, i};
    up[v] = min(up[u], e.f);
    if (not inq[v]) q.push(v);
    inq[v] = true;
   }
  }
  if (d[T] == INF_C) return nullopt;
  for (int i = T; i != S; i = f[i].first) {
   auto &eg = g[f[i].first][f[i].second];
   eg.f -= up[T];
   g[eg.to][eg.r].f += up[T];
  }
  return pair{up[T], d[T]};
 }
public:
 MCMF(int n) : g(n),f(n),inq(n),up(n),d(n,INF_C) {}
 void add_edge(int s, int t, F c, C w) {
  g[s].emplace_back(t, int(g[t].size()), c, w);
  g[t].emplace_back(s, int(g[s].size()) - 1, 0, -w);
 }
 pair<F, C> solve(int a, int b) {
  F c = 0; C w = 0;
  while (auto r = step(a, b)) {
   c += r->first, w += r->first * r->second;
   fill(inq.begin(), inq.end(), false);
   fill(d.begin(), d.end(), INF_C);
  }
  return {c, w};
 }
};
```

4.11 Weighted Matching [60ca53]

```
struct WeightGraph {
 static const int inf = INT_MAX;
 static const int maxn = 514;
 struct edge {
  int u, v, w;
  edge(){}
  edge(int u, int v, int w): u(u), v(v), w(w) {}
 };
 int n, n_x;
 edge g[maxn * 2][maxn * 2];
 int lab[maxn * 2];
 int match[maxn * 2], slack[maxn * 2], st[maxn * 2], pa
    [maxn * 2];
 int flo_from[maxn * 2][maxn + 1], S[maxn * 2], vis[
    maxn * 2];
 vector<int> flo[maxn * 2];
 queue<int> q;
 int e_delta(const edge &e) { return lab[e.u] + lab[e.v
    ] - g[e.u][e.v].w * 2; }
 void update_slack(int u, int x) { if (!slack[x] ||
    e_delta(g[u][x]) < e_delta(g[slack[x]][x])) slack[x
    ] = u; }
 void set_slack(int x) {
  slack[x] = 0;
  for (int u = 1; u <= n; ++u)
   if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
    update_slack(u, x);
 }
 void q_push(int x) {
  if (x <= n) q.push(x);
  else for (size_t i = 0; i < flo[x].size(); i++)
    q_push(flo[x][i]);
 }
 void set_st(int x, int b) {
  st[x] = b;
  if (x > n) for (size_t i = 0; i < flo[x].size(); ++i)
     set_st(flo[x][i], b);
 }
 int get_pr(int b, int xr) {
  int pr = find(flo[b].begin(), flo[b].end(), xr) - flo
    [b].begin();
  if (pr % 2 == 1) {
   reverse(flo[b].begin() + 1, flo[b].end());
   return (int)flo[b].size() - pr;
  }
  return pr;
 }
 void set_match(int u, int v) {
  match[u] = g[u][v].v;
  if (u <= n) return;
  edge e = g[u][v];
  int xr = flo_from[u][e.u], pr = get_pr(u, xr);
  for (int i = 0; i < pr; ++i) set_match(flo[u][i], flo
    [u][i ^ 1]);
  set_match(xr, v);
  rotate(flo[u].begin(), flo[u].begin() + pr, flo[u].
    end());
 }
 void augment(int u, int v) {
  for (; ; ) {
   int xnv = st[match[u]];
   set_match(u, v);
   if (!xnv) return;
   set_match(xnv, st[pa[xnv]]);
   u = st[pa[xnv]], v = xnv;
  }
 }
 int get_lca(int u, int v) {
  static int t = 0;
  for (++t; u || v; swap(u, v)) {
   if (u == 0) continue;
   if (vis[u] == t) return u;
   vis[u] = t;
   u = st[match[u]];
   if (u) u = st[pa[u]];
  }
  return 0;
 }
 void add_blossom(int u, int lca, int v) {
  int b = n + 1;
  while (b <= n_x && st[b]) ++b;
  if (b > n_x) ++n_x;
  lab[b] = 0, S[b] = 0;
  match[b] = match[lca];
  flo[b].clear();
  flo[b].push_back(lca);
  for (int x = u, y; x != lca; x = st[pa[y]])
   flo[b].push_back(x), flo[b].push_back(y = st[match[x
    ]]), q_push(y);
  reverse(flo[b].begin() + 1, flo[b].end());
  for (int x = v, y; x != lca; x = st[pa[y]])
   flo[b].push_back(x), flo[b].push_back(y = st[match[x
    ]]), q_push(y);
  set_st(b, b);
```

```cpp
  for (int x = 1; x <= n_x; ++x) g[b][x].w = g[x][b].w = 0;
  for (int x = 1; x <= n; ++x) flo_from[b][x] = 0;
  for (size_t i = 0; i < flo[b].size(); ++i) {
   int xs = flo[b][i];
   for (int x = 1; x <= n_x; ++x)
    if (g[b][x].w == 0 || e_delta(g[xs][x]) < e_delta(g[b][x]))
     g[b][x] = g[xs][x], g[x][b] = g[x][xs];
   for (int x = 1; x <= n; ++x)
    if (flo_from[xs][x]) flo_from[b][x] = xs;
  }
  set_slack(b);
 }
 void expand_blossom(int b) {
  for (size_t i = 0; i < flo[b].size(); ++i)
   set_st(flo[b][i], flo[b][i]);
  int xr = flo_from[b][g[b][pa[b]].u], pr = get_pr(b, xr);
  for (int i = 0; i < pr; i += 2) {
   int xs = flo[b][i], xns = flo[b][i + 1];
   pa[xs] = g[xns][xs].u;
   S[xs] = 1, S[xns] = 0;
   slack[xs] = 0, set_slack(xns);
   q_push(xns);
  }
  S[xr] = 1, pa[xr] = pa[b];
  for (size_t i = pr + 1; i < flo[b].size(); ++i) {
   int xs = flo[b][i];
   S[xs] = -1, set_slack(xs);
  }
  st[b] = 0;
 }
 bool on_found_edge(const edge &e) {
  int u = st[e.u], v = st[e.v];
  if (S[v] == -1) {
   pa[v] = e.u, S[v] = 1;
   int nu = st[match[v]];
   slack[v] = slack[nu] = 0;
   S[nu] = 0, q_push(nu);
  } else if (S[v] == 0) {
   int lca = get_lca(u, v);
   if (!lca) return augment(u,v), augment(v,u), true;
   else add_blossom(u, lca, v);
  }
  return false;
 }
 bool matching() {
  memset(S + 1, -1, sizeof(int) * n_x);
  memset(slack + 1, 0, sizeof(int) * n_x);
  q = queue<int>();
  for (int x = 1; x <= n_x; ++x)
   if (st[x] == x && !match[x]) pa[x] = 0, S[x] = 0, q_push(x);
  if (q.empty()) return false;
  for (; ; ) {
   while (q.size()) {
    int u = q.front(); q.pop();
    if (S[st[u]] == 1) continue;
    for (int v = 1; v <= n; ++v)
     if (g[u][v].w > 0 && st[u] != st[v]) {
      if (e_delta(g[u][v]) == 0) {
       if (on_found_edge(g[u][v])) return true;
      } else update_slack(u, st[v]);
     }
   }
   int d = inf;
   for (int b = n + 1; b <= n_x; ++b)
    if (st[b] == b && S[b] == 1) d = min(d, lab[b] / 2);
   for (int x = 1; x <= n_x; ++x)
    if (st[x] == x && slack[x]) {
     if (S[x] == -1) d = min(d, e_delta(g[slack[x]][x]));
     else if (S[x] == 0) d = min(d, e_delta(g[slack[x]][x]) / 2);
    }
   for (int u = 1; u <= n; ++u) {
    if (S[st[u]] == 0) {
     if (lab[u] <= d) return 0;
     lab[u] -= d;
    } else if (S[st[u]] == 1) lab[u] += d;
   }
   for (int b = n + 1; b <= n_x; ++b)
    if (st[b] == b) {
     if (S[st[b]] == 0) lab[b] += d * 2;
     else if (S[st[b]] == 1) lab[b] -= d * 2;
    }
   q = queue<int>();
   for (int x = 1; x <= n_x; ++x)
    if (st[x] == x && slack[x] && st[slack[x]] != x && e_delta(g[slack[x]][x]) == 0)
     if (on_found_edge(g[slack[x]][x])) return true;
   for (int b = n + 1; b <= n_x; ++b)
    if (st[b] == b && S[b] == 1 && lab[b] == 0)
     expand_blossom(b);
  }
  return false;
 }
 pair<long long, int> solve() {
  memset(match + 1, 0, sizeof(int) * n);
  n_x = n;
  int n_matches = 0;
  long long tot_weight = 0;
  for (int u = 0; u <= n; ++u) st[u] = u, flo[u].clear();
  int w_max = 0;
  for (int u = 1; u <= n; ++u)
   for (int v = 1; v <= n; ++v) {
    flo_from[u][v] = (u == v ? u : 0);
    w_max = max(w_max, g[u][v].w);
   }
  for (int u = 1; u <= n; ++u) lab[u] = w_max;
  while (matching()) ++n_matches;
  for (int u = 1; u <= n; ++u)
   if (match[u] && match[u] < u)
    tot_weight += g[u][match[u]].w;
  return make_pair(tot_weight, n_matches);
 }
 void add_edge(int ui, int vi, int wi) { g[ui][vi].w = g[vi][ui].w = wi; }
 void init(int _n) {
  n = _n;
  for (int u = 1; u <= n; ++u)
   for (int v = 1; v <= n; ++v)
    g[u][v] = edge(u, v, 0);
 }
};
```

5 Math

5.1 Common Bounds

$$p(0) = 1,\ p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \approx 0.145/n \cdot \mathrm{exp}(2.56\sqrt{n})$$

n	100	1e3	1e6	1e9	1e12	1e15	1e18
$\max_{i \le n}(d(i))$	12	32	240	1344	6720	26880	103680

n	1	2	3	4	5	6	7	8	9	10
$\binom{2n}{n}$	2	6	20	70	252	924	3432	12870	48620	184756

5.2 Strling Number

First Kind

$S_1(n, k)$ counts the number of permutations of n elements with k disjoint cycles.

$$S_1(n,k) = (n-1) \cdot S_1(n-1,k) + S_1(n-1,k-1)$$

$$x(x+1)\dots(x+n-1) = \sum_{k=0}^{n} S_1(n,k) x^k$$

$$g(x) = x(x+1)\dots(x+n-1) = \sum_{k=0}^{n} a_k x^k$$

$$\Rightarrow g(x+n) = \sum_{k=0}^{n} \frac{b_k}{(n-k)!} x^{n-k},$$

$$b_k = \sum_{i=0}^{k} ((n-i)! a_{n-i}) \cdot (\frac{n^{k-i}}{(k-i)!})$$

Second Kind

$S_2(n,k)$ counts the number of ways to partition a set of n elements into k nonempty sets.

$$S_2(n,k) = S_2(n-1,k-1) + k \cdot S_2(n-1,k)$$

$$S_2(n,k) = \sum_{i=0}^{k} \binom{k}{i} i^n (-1)^{k-i} = \sum_{i=0}^{k} \frac{(-1)^i}{i!} \cdot \frac{(k-i)^n}{(k-i)!}$$

5.3 ax+by=gcd [d0cbdd]

```
// ax+ny = 1, ax+ny == ax == 1 (mod n)
void exgcd(lld x,lld y,lld &g,lld &a,lld &b) {
 if (y == 0) g=x,a=1,b=0;
 else exgcd(y,x%y,g,b,a),b-=(x/y)*a;
}
```

5.4 Chinese Remainder [d69e74]

```
// please ensure r_i\in[0,m_i)
bool crt(lld &m1, lld &r1, lld m2, lld r2) {
  if (m2 > m1) swap(m1, m2), swap(r1, r2);
  lld g, a, b; exgcd(m1, m2, g, a, b);
  if ((r2 - r1) % g != 0) return false;
  m2 /= g; lld D = (r2 - r1) / g % m2 * a % m2;
  r1 += (D < 0 ? D + m2 : D) * m1; m1 *= m2;
  assert (r1 >= 0 && r1 < m1);
  return true;
}
```

5.5 De-Bruijn [7f536e]

```
int res[maxn], aux[maxn], sz;
void db(int t, int p, int n, int k) {
 if (t > n) {
  if (n % p == 0)
   for (int i = 1; i <= p; ++i)
    res[sz++] = aux[i];
 } else {
  aux[t] = aux[t - p];
  db(t + 1, p, n, k);
  for (int i = aux[t - p] + 1; i < k; ++i) {
   aux[t] = i;
   db(t + 1, t, n, k);
  }
 }
}
int de_bruijn(int k, int n) {
 // return cyclic string of len k^n s.t. every string
 // of len n using k char appears as a substring.
 if (k == 1) {
  res[0] = 0;
  return 1;
 }
 for (int i = 0; i < k * n; i++) aux[i] = 0;
 sz = 0;
 db(1, 1, n, k);
 return sz;
}
```

5.6 DiscreteLog [86e463]

```
template<typename Int>
Int BSGS(Int x, Int y, Int M) {
 // x^? \equiv y (mod M)
 Int t = 1, c = 0, g = 1;
 for (Int M_ = M; M_ > 0; M_ >>= 1)
  g = g * x % M;
 for (g = gcd(g, M); t % g != 0; ++c) {
  if (t == y) return c;
  t = t * x % M;
 }
 if (y % g != 0) return -1;
 t /= g, y /= g, M /= g;
 Int h = 0, gs = 1;
 for (; h * h < M; ++h) gs = gs * x % M;
 unordered_map<Int, Int> bs;
 for (Int s = 0; s < h; bs[y] = ++s)
  y = y * x % M;
 for (Int s = 0; s < M; s += h) {
  t = t * gs % M;
  if (bs.count(t)) return c + s + h - bs[t];
 }
 return -1;
}
```

5.7 Quadratic residue [1eabad]

```
int get_root(int n, int P) { // ensure 0 <= n < p
 if (P == 2 or n == 0) return n;
 auto check = [&](int x) {
  return modpow(x, (P - 1) / 2, P); };
 if (check(n) != 1) return -1;
 mt19937 rnd(7122); lld z = 1, w;
 while (check(w = (z * z - n + P) % P) != P - 1)
  z = rnd() % P;
 const auto M = [P, w](auto &u, auto &v) {
  auto [a, b] = u; auto [c, d] = v;
  return make_pair((a * c + b * d % P * w) % P,
    (a * d + b * c) % P);
 };
 pair<lld, lld> r(1, 0), e(z, 1);
 for (int w = (P + 1) / 2; w; w >>= 1, e = M(e, e))
  if (w & 1) r = M(r, e);
 return r.first; // sqrt(n) mod P where P is prime
}
```

5.8 Extended Euler

$$a^b \equiv \begin{cases} a^{(b \mod \varphi(m)) + \varphi(m)} & \text{if } (a,m) \neq 1 \land b \geq \varphi(m) \\ a^{b \mod \varphi(m)} & \text{otherwise} \end{cases} \pmod m$$

5.9 Extended FloorSum

$$g(a,b,c,n) = \sum_{i=0}^{n} i \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor \cdot \frac{n(n+1)}{2} & \\ +g(a \bmod c, b \bmod c, c, n), & a \geq c \lor b \geq c \\ 0, & n < 0 \lor a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1) & \\ -h(c, c-b-1, a, m-1)), & \text{otherwise} \end{cases}$$

$$h(a,b,c,n) = \sum_{i=0}^{n} \lfloor \frac{ai+b}{c} \rfloor^2$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor^2 \cdot (n+1) & \\ +\lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot n(n+1) & \\ +h(a \bmod c, b \bmod c, c, n) & \\ +2\lfloor \frac{a}{c} \rfloor \cdot g(a \bmod c, b \bmod c, c, n) & \\ +2\lfloor \frac{b}{c} \rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \geq c \lor b \geq c \\ 0, & n < 0 \lor a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) & \\ -2f(c, c-b-1, a, m-1) - f(a,b,c,n), & \text{otherwise} \end{cases}$$

5.10 FloorSum [bda6b2]

```
// @param n `n < 2^32`
// @param m `1 <= m < 2^32`
// @return sum_{i=0}^{n-1} floor((ai + b)/m) mod 2^64
llu floor_sum_unsigned(llu n, llu m, llu a, llu b) {
 llu ans = 0;
 while (true) {
  if (a >= m) {
   ans += n * (n - 1) / 2 * (a / m); a %= m;
  }
  if (b >= m) {
   ans += n * (b / m); b %= m;
  }
  llu y_max = a * n + b;
  if (y_max < m) break;
  // y_max < m * (n + 1)
  // floor(y_max / m) <= n
  n = (llu)(y_max / m), b = (llu)(y_max % m);
  swap(m, a);
 }
 return ans;
}
lld floor_sum(lld n, lld m, lld a, lld b) {
 llu ans = 0;
 if (a < 0) {
  llu a2 = (a % m + m) % m;
  ans -= 1ULL * n * (n - 1) / 2 * ((a2 - a) / m);
  a = a2;
 }
 if (b < 0) {
  llu b2 = (b % m + m) % m;
  ans -= 1ULL * n * ((b2 - b) / m);
  b = b2;
 }
 return ans + floor_sum_unsigned(n, m, a, b);
}
```

5.11 ModMin [07d5e1]

```
// min{k | l <= ((ak) mod m) <= r}
optional<uint64_t> mod_min(uint32_t a, uint32_t m,
    uint32_t l, uint32_t r) {
  if (a == 0) return l ? nullopt : 0;
  if (auto k = uint64_t(l + a - 1) / a; k * a <= r)
    return k;
  auto b = m / a, c = m % a;
  if (auto y = mod_min(c, a, a - r % a, a - l % a))
    return (l + *y * c + a - 1) / a + *y * b;
 return nullopt;
}
```

5.12 Fast Fourier Transform [993ee3]

```
namespace fft {
using VI = vector<int>;
using VL = vector<long long>;
const double pi = acos(-1);
cplx omega[maxn + 1];
void prefft() {
 for (int i = 0; i <= maxn; i++)
  omega[i] = cplx(cos(2 * pi * j / maxn),
     sin(2 * pi * j / maxn));
}
void fft(vector<cplx> &v, int n) {
 int z = __builtin_ctz(n) - 1;
 for (int i = 0; i < n; ++i) {
  int x = 0, j = 0;
  for (;(1 << j) < n;++j) x^=(i >> j & 1)<<(z - j);
  if (x > i) swap(v[x], v[i]);
 }
 for (int s = 2; s <= n; s <<= 1) {
  int z = s >> 1;
  for (int i = 0; i < n; i += s) {
   for (int k = 0; k < z; ++k) {
    cplx x = v[i + z + k] * omega[maxn / s * k];
    v[i + z + k] = v[i + k] - x;
    v[i + k] = v[i + k] + x;
   }
  }
 }
}
void ifft(vector<cplx> &v, int n) {
 fft(v, n); reverse(v.begin() + 1, v.end());
 for (int i=0;i<n;++i) v[i] = v[i] * cplx(1. / n, 0);
}
VL convolution(const VI &a, const VI &b) {
 // Should be able to handle N <= 10^5, C <= 10^4
 int sz = 1;
 while (sz < a.size() + b.size() - 1) sz <<= 1;
 vector<cplx> v(sz);
 for (int i = 0; i < sz; ++i) {
  double re = i < a.size() ? a[i] : 0;
  double im = i < b.size() ? b[i] : 0;
  v[i] = cplx(re, im);
 }
 fft(v, sz);
 for (int i = 0; i <= sz / 2; ++i) {
  int j = (sz - i) & (sz - 1);
  cplx x = (v[i] + v[j].conj()) * (v[i] - v[j].conj())
    * cplx(0, -0.25);
  if (j != i) v[j] = (v[j] + v[i].conj()) * (v[j] - v[i
    ].conj()) * cplx(0, -0.25);
  v[i] = x;
 }
 ifft(v, sz);
 VL c(sz);
 for (int i = 0; i < sz; ++i) c[i] = round(v[i].re);
 return c;
}
VI convolution_mod(const VI &a, const VI &b, int p) {
 int sz = 1;
 while (sz + 1 < a.size() + b.size()) sz <<= 1;
 vector<cplx> fa(sz), fb(sz);
 for (int i = 0; i < (int)a.size(); ++i)
  fa[i] = cplx(a[i] & ((1 << 15) - 1), a[i] >> 15);
 for (int i = 0; i < (int)b.size(); ++i)
  fb[i] = cplx(b[i] & ((1 << 15) - 1), b[i] >> 15);
 fft(fa, sz), fft(fb, sz);
 double r = 0.25 / sz;
 cplx r2(0, -1), r3(r, 0), r4(0, -r), r5(0, 1);
 for (int i = 0; i <= (sz >> 1); ++i) {
  int j = (sz - i) & (sz - 1);
  cplx a1 = (fa[i] + fa[j].conj());
  cplx a2 = (fa[i] - fa[j].conj()) * r2;
  cplx b1 = (fb[i] + fb[j].conj()) * r3;
  cplx b2 = (fb[i] - fb[j].conj()) * r4;
  if (i != j) {
   cplx c1 = (fa[j] + fa[i].conj());
   cplx c2 = (fa[j] - fa[i].conj()) * r2;
   cplx d1 = (fb[j] + fb[i].conj()) * r3;
   cplx d2 = (fb[j] - fb[i].conj()) * r4;
   fa[i] = c1 * d1 + c2 * d2 * r5;
   fb[i] = c1 * d2 + c2 * d1;
  }
  fa[j] = a1 * b1 + a2 * b2 * r5;
  fb[j] = a1 * b2 + a2 * b1;
 }
 fft(fa, sz), fft(fb, sz);
 vector<int> res(sz);
 for (int i = 0; i < sz; ++i) {
  long long a = round(fa[i].re), b = round(fb[i].re),
       c = round(fa[i].im);
  res[i] = (a+((b % p) << 15)+((c % p) << 30)) % p;
 }
 return res;
}}
```

5.13 FWT [c5167a]

```
/* or convolution:
 * x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
 * and convolution:
 * x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
void fwt(int x[], int N, bool inv = false) {
 for (int d = 1; d < N; d <<= 1)
  for (int s = 0; s < N; s += d * 2)
   for (int i = s; i < s + d; i++) {
    int j = i + d, ta = x[i], tb = x[j];
    x[i] = modadd(ta, tb);
    x[j] = modsub(ta, tb);
   }
 if (inv) {
  const int invn = modinv(N);
  for (int i = 0; i < N; i++)
   x[i] = modmul(x[i], invn);
 }
}
```

5.14 CRT for arbitrary mod [7272c4]

```
const int mod = 1000000007;
const int M1 = 985661441; // G = 3 for M1, M2, M3
const int M2 = 998244353;
const int M3 = 1004535809;
int superBigCRT(int64_t A, int64_t B, int64_t C) {
  static_assert (M1 <= M2 && M2 <= M3);
  constexpr int64_t r12 = modpow(M1, M2-2, M2);
  constexpr int64_t r13 = modpow(M1, M3-2, M3);
  constexpr int64_t r23 = modpow(M2, M3-2, M3);
  constexpr int64_t M1M2 = 1LL * M1 * M2 % mod;
  B = (B - A + M2) * r12 % M2;
  C = (C - A + M3) * r13 % M3;
  C = (C - B + M3) * r23 % M3;
  return (A + B * M1 + C * M1M2) % mod;
}
```

5.15 NTT [946e8e]

```
template <int mod, int G, int maxn> struct NTT {
 static_assert (maxn == (maxn & -maxn));
 int roots[maxn];
 NTT () {
  int r = modpow(G, (mod - 1) / maxn);
  for (int i = maxn >> 1; i; i >>= 1) {
   roots[i] = 1;
   for (int j = 1; j < i; j++)
    roots[i + j] = modmul(roots[i + j - 1], r);
   r = modmul(r, r);
  }
 }
 // n must be 2^k, and 0 <= F[i] < mod
 void operator()(int F[], int n, bool inv = false) {
  for (int i = 0, j = 0; i < n; i++) {
   if (i < j) swap(F[i], F[j]);
```

```
    for (int k = n>>1; (j^=k) < k; k>>=1);
   }
   for (int s = 1; s < n; s *= 2) {
    for (int i = 0; i < n; i += s * 2) {
     for (int j = 0; j < s; j++) {
      int a = F[i+j];
      int b = modmul(F[i+j+s], roots[s+j]);
      F[i+j] = modadd(a, b);  // a + b
      F[i+j+s] = modsub(a, b); // a - b
     }
    }
   }
   if (inv) {
    int invn = modinv(n);
    for (int i = 0; i < n; i++)
     F[i] = modmul(F[i], invn);
    reverse(F + 1, F + n);
   }
  }
};
```

5.16 Partition Number [9bb845]

```
ans[0] = tmp[0] = 1;
for (int i = 1; i * i <= n; i++) {
 for (int rep = 0; rep < 2; rep++)
  for (int j = i; j <= n - i * i; j++)
   modadd(tmp[j], tmp[j-i]);
 for (int j = i * i; j <= n; j++)
  modadd(ans[j], tmp[j - i * i]);
}
```

5.17 Pi Count (+Linear Sieve) [47e0de]

```
static constexpr int N = 1000000 + 5;
lld pi[N];
vector<int> primes;
bool sieved[N];
lld cube_root(lld x){
 lld s=cbrt(x-static_cast<long double>(0.1));
 while(s*s*s <= x) ++s;
 return s-1;
}
lld square_root(lld x){
 lld s=sqrt(x-static_cast<long double>(0.1));
 while(s*s <= x) ++s;
 return s-1;
}
void init(){
 primes.reserve(N);
 primes.push_back(1);
 for(int i=2;i<N;i++) {
  if(!sieved[i]) primes.push_back(i);
  pi[i] = !sieved[i] + pi[i-1];
  for(int p: primes) if(p > 1) {
   if(p * i >= N) break;
   sieved[p * i] = true;
   if(p % i == 0) break;
  }
 }
}
lld phi(lld m, lld n) {
 static constexpr int MM = 80000, NN = 500;
 static lld val[MM][NN];
 if(m<MM&&n<NN&&val[m][n])return val[m][n]-1;
 if(n == 0) return m;
 if(primes[n] >= m) return 1;
 lld ret = phi(m,n-1)-phi(m/primes[n],n-1);
 if(m<MM&&n<NN) val[m][n] = ret+1;
 return ret;
}
lld pi_count(lld);
lld P2(lld m, lld n) {
 lld sm = square_root(m), ret = 0;
 for(lld i = n+1;primes[i]<=sm;i++)
  ret+=pi_count(m/primes[i])-pi_count(primes[i])+1;
 return ret;
}
lld pi_count(lld m) {
 if(m < N) return pi[m];
 lld n = pi_count(cube_root(m));
 return phi(m, n) + n - 1 - P2(m, n);
}
```

5.18 Miller Rabin [0edab2]

```
bool isprime(llu x) {
 static auto witn = [](llu a, llu n, int t) {
  if (!a) return false;
  while (t--) {
   llu a2 = mmul(a, a, n);
   if (a2 == 1 && a != 1 && a != n - 1) return true;
   a = a2;
  }
  return a != 1;
 };
 if (x < 2) return false;
 if (!(x & 1)) return x == 2;
 int t = __builtin_ctzll(x - 1);
 llu odd = (x - 1) >> t;
 for (llu m:
  {2, 325, 9375, 28178, 450775, 9780504, 1795265022})
  if (witn(mpow(m % x, odd, x), x, t))
   return false;
 return true;
}
```

5.19 Pollard Rho [2ac1ad]

```
// does not work when n is prime
// return any non-trivial factor
llu pollard_rho(llu n) {
 static auto f = [](llu x, llu k, llu m) {
    return add(k, mul(x, x, m), m); };
 if (!(n & 1)) return 2;
 mt19937 rnd(120821011);
 while (true) {
  llu y = 2, yy = y, x = rnd() % n, t = 1;
  for (llu sz = 2; t == 1; sz <<= 1, y = yy) {
   for (llu i = 0; t == 1 && i < sz; ++i) {
    yy = f(yy, x, n);
    t = gcd(yy > y ? yy - y : y - yy, n);
   }
  }
  if (t != 1 && t != n) return t;
 }
}
```

5.20 Berlekamp Massey [a94d00]

```
template <typename T>
vector<T> BerlekampMassey(const vector<T> &output) {
 vector<T> d(output.size() + 1), me, he;
 for (size_t f = 0, i = 1; i <= output.size(); ++i) {
  for (size_t j = 0; j < me.size(); ++j)
   d[i] += output[i - j - 2] * me[j];
  if ((d[i] -= output[i - 1]) == 0) continue;
  if (me.empty()) {
   me.resize(f = i);
   continue;
  }
  vector<T> o(i - f - 1);
  T k = -d[i] / d[f]; o.push_back(-k);
  for (T x : he) o.push_back(x * k);
  if (o.size() < me.size()) o.resize(me.size());
  for (size_t j = 0; j < me.size(); ++j) o[j] += me[j];
  if (i-f+he.size() >= me.size()) he = me, f = i;
  me = o;
 }
 return me;
}
```

5.21 Charateristic Polynomial [e006eb]

```
#define rep(x, y, z) for (int x=y; x<z; x++)
using VI = vector<int>; using VVI = vector<VI>;
void Hessenberg(VVI &H, int N) {
 for (int i = 0; i < N - 2; ++i) {
  for (int j = i + 1; j < N; ++j) if (H[j][i]) {
   rep(k, i, N) swap(H[i+1][k], H[j][k]);
   rep(k, 0, N) swap(H[k][i+1], H[k][j]);
   break;
  }
  if (!H[i + 1][i]) continue;
  for (int j = i + 2; j < N; ++j) {
   int co = mul(modinv(H[i + 1][i]), H[j][i]);
   rep(k, i, N) subeq(H[j][k], mul(H[i+1][k], co));
   rep(k, 0, N) addeq(H[k][i+1], mul(H[k][j], co));
  }
```

```cpp
 }
}
VI CharacteristicPoly(VVI &A) {
 int N = A.size(); Hessenberg(A, N);
 VVI P(N + 1, VI(N + 1)); P[0][0] = 1;
 for (int i = 1; i <= N; ++i) {
  rep(j, 0, i+1) P[i][j] = j ? P[i-1][j-1] : 0;
  for (int j = i - 1, val = 1; j >= 0; --j) {
   int co = mul(val, A[j][i - 1]);
   rep(k, 0, j+1) subeq(P[i][k], mul(P[j][k], co));
   if (j) val = mul(val, A[j][j - 1]);
  }
 }
 if (N & 1) for (int &pi: P[N]) pi = sub(0, pi);
 return P[N]; // test: 2021 PTZ Korea K
}
```

5.22 Polynomial Operations [d40491]

```cpp
using V = vector<int>;
#define fi(l, r) for (int i = int(l); i < int(r); ++i)
template <int mod, int G, int maxn> struct Poly : V {
 static uint32_t n2k(uint32_t n) {
  if (n <= 1) return 1;
  return 1u << (32 - __builtin_clz(n - 1));
 }
 static NTT<mod,G,maxn> ntt; // coefficients in [0, P)
 explicit Poly(int n = 1) : V(n) {}
 Poly(const V &v) : V(v) {}
 Poly(const Poly &p, size_t n) : V(n) {
  copy_n(p.data(), min(p.size(), n), data());
 }
 Poly &irev() { return reverse(data(), data() + size())
    , *this; }
 Poly &isz(int sz) { return resize(sz), *this; }
 Poly &iadd(const Poly &rhs) { // n() == rhs.n()
  fi(0, size())(*this)[i] = modadd((*this)[i], rhs[i]);
  return *this;
 }
 Poly &imul(int k) {
  fi(0, size())(*this)[i] = modmul((*this)[i], k);
  return *this;
 }
 Poly Mul(const Poly &rhs) const {
  const int sz = n2k(size() + rhs.size() - 1);
  Poly X(*this, sz), Y(rhs, sz);
  ntt(X.data(), sz), ntt(Y.data(), sz);
  fi(0, sz) X[i] = modmul(X[i], Y[i]);
  ntt(X.data(), sz, true);
  return X.isz(size() + rhs.size() - 1);
 }
 Poly Inv() const { // coef[0] != 0
  if (size() == 1) return V{modinv(*begin())};
  const int sz = n2k(size() * 2);
  Poly X = Poly(*this, (size() + 1) / 2).Inv().isz(sz),
     Y(*this, sz);
  ntt(X.data(), sz), ntt(Y.data(), sz);
  fi(0, sz) X[i] = modmul(X[i], modsub(2, modmul(X[i],
    Y[i])));
  ntt(X.data(), sz, true);
  return X.isz(size());
 }
 Poly Sqrt() const { // coef[0] \in [1, mod)^2
  if (size() == 1) return V{QuadraticResidue((*this)
    [0], mod)};
  Poly X = Poly(*this, (size() + 1) / 2).Sqrt().isz(
    size());
  return X.iadd(Mul(X.Inv()).isz(size())).imul(mod / 2
    + 1);
 }
 pair<Poly, Poly> DivMod(const Poly &rhs) const {
  if (size() < rhs.size()) return {V{0}, *this};
  const int sz = size() - rhs.size() + 1;
  Poly X(rhs); X.irev().isz(sz);
  Poly Y(*this); Y.irev().isz(sz);
  Poly Q = Y.Mul(X.Inv()).isz(sz).irev();
  X = rhs.Mul(Q), Y = *this;
  fi(0, size()) Y[i] = modsub(Y[i], X[i]);
  return {Q, Y.isz(max<int>(1, rhs.size() - 1))};
 }
 Poly Dx() const {
  Poly ret(size() - 1);
  fi(0, ret.size()) ret[i] = modmul(i + 1, (*this)[i +
    1]);
  return ret.isz(max<int>(1, ret.size()));
 }
 Poly Sx() const {
  Poly ret(size() + 1);
  fi(0, size()) ret[i + 1] = modmul(modinv(i + 1), (*
    this)[i]);
  return ret;
 }
 Poly Ln() const { // coef[0] == 1; res[0] == 0
  return Dx().Mul(Inv()).Sx().isz(size());
 }
 Poly Exp() const { // coef[0] == 0; res[0] == 1
  if (size() == 1) return V{1};
  Poly X = Poly(*this, (size() + 1) / 2).Exp().isz(size
    ());
  Poly Y = X.Ln(); Y[0] = mod - 1;
  fi(0, size()) Y[i] = modsub((*this)[i], Y[i]);
  return X.Mul(Y).isz(size());
 }
 Poly Pow(const string &K) const {
  int nz = 0;
  while (nz < size() && !(*this)[nz]) ++nz;
  int nk = 0, nk2 = 0;
  for (char c : K) {
   nk = (nk * 10 + c - '0') % mod;
   nk2 = nk2 * 10 + c - '0';
   if (nk2 * nz >= size())
    return Poly(size());
   nk2 %= mod - 1;
  }
  if (!nk && !nk2) return Poly(V{1}, size());
  Poly X = V(data() + nz, data() + size() - nz * (nk2 -
     1));
  int x0 = X[0];
  return X.imul(modinv(x0)).Ln().imul(nk).Exp().imul(
    modpow(x0, nk2)).irev().isz(size()).irev();
 }
 V Eval(V x) const {
  if (x.empty()) return {};
  const size_t n = max(x.size(), size());
  vector<Poly> t(n * 2, V{1, 0}), f(n * 2);
  for (size_t i = 0; i < x.size(); ++i)
   t[n + i] = V{1, mod-x[i]};
  for (size_t i = n - 1; i > 0; --i)
   t[i] = t[i * 2].Mul(t[i * 2 + 1]);
  f[1] = Poly(*this, n).irev().Mul(t[1].Inv()).isz(n).
    irev();
  for (size_t i = 1; i < n; ++i) {
   auto o = f[i]; auto sz = o.size();
   f[i*2] = o.irev().Mul(t[i*2+1]).isz(sz).irev().isz(t
    [i*2].size());
   f[i*2+1] = o.Mul(t[i*2]).isz(sz).irev().isz(t[i
    *2+1].size());
  }
  for (size_t i=0;i<x.size();++i) x[i] = f[n+i][0];
  return x;
 }

 static int LinearRecursion(const V &a, const V &c,
    int64_t n) { // a_n = \sum c_j a_(n-j)
  const int k = (int)a.size();
  assert((int)c.size() == k + 1);
  Poly C(k + 1), W({1}, k), M = {0, 1};
  fi(1, k + 1) C[k - i] = modsub(mod, c[i]);
  C[k] = 1;
  while (n) {
   if (n % 2) W = W.Mul(M).DivMod(C).second;
   n /= 2, M = M.Mul(M).DivMod(C).second;
  }
  int ret = 0;
  fi(0, k) ret = modadd(ret, modmul(W[i], a[i]));
  return ret;
 }
};
#undef fi
using Poly_t = Poly<998244353, 3, 1 << 20>;
template <> decltype(Poly_t::ntt) Poly_t::ntt = {};
```

5.23 Simplex [e975d5]

```cpp
namespace simplex {
```

```
// maximize c^Tx under Ax <= B
// return VD(n, -inf) if the solution doesn't exist
// return VD(n, +inf) if the solution is unbounded
using VD = vector<double>;
using VVD = vector<vector<double>>;
const double eps = 1e-9;
const double inf = 1e+9;
int n, m;
VVD d;
vector<int> p, q;
void pivot(int r, int s) {
 double inv = 1.0 / d[r][s];
 for (int i = 0; i < m + 2; ++i)
  for (int j = 0; j < n + 2; ++j)
   if (i != r && j != s)
    d[i][j] -= d[r][j] * d[i][s] * inv;
 for(int i=0;i<m+2;++i) if (i != r) d[i][s] *= -inv;
 for(int j=0;j<n+2;++j) if (j != s) d[r][j] *= +inv;
 d[r][s] = inv; swap(p[r], q[s]);
}
bool phase(int z) {
 int x = m + z;
 while (true) {
  int s = -1;
  for (int i = 0; i <= n; ++i) {
   if (!z && q[i] == -1) continue;
   if (s == -1 || d[x][i] < d[x][s]) s = i;
  }
  if (d[x][s] > -eps) return true;
  int r = -1;
  for (int i = 0; i < m; ++i) {
   if (d[i][s] < eps) continue;
   if (r == -1 || \
    d[i][n+1]/d[i][s] < d[r][n+1]/d[r][s]) r = i;
  }
  if (r == -1) return false;
  pivot(r, s);
 }
}
VD solve(const VVD &a, const VD &b, const VD &c) {
 m = b.size(), n = c.size();
 d = VVD(m + 2, VD(n + 2));
 for (int i = 0; i < m; ++i)
  for (int j = 0; j < n; ++j) d[i][j] = a[i][j];
 p.resize(m), q.resize(n + 1);
 for (int i = 0; i < m; ++i)
  p[i] = n + i, d[i][n] = -1, d[i][n + 1] = b[i];
 for (int i = 0; i < n; ++i) q[i] = i,d[m][i] = -c[i];
 q[n] = -1, d[m + 1][n] = 1;
 int r = 0;
 for (int i = 1; i < m; ++i)
  if (d[i][n + 1] < d[r][n + 1]) r = i;
 if (d[r][n + 1] < -eps) {
  pivot(r, n);
  if (!phase(1) || d[m + 1][n + 1] < -eps)
   return VD(n, -inf);
  for (int i = 0; i < m; ++i) if (p[i] == -1) {
   int s = min_element(d[i].begin(), d[i].end() - 1)
       - d[i].begin();
   pivot(i, s);
  }
 }
 if (!phase(0)) return VD(n, inf);
 VD x(n);
 for (int i = 0; i < m; ++i)
  if (p[i] < n) x[p[i]] = d[i][n + 1];
 return x;
}}
```

5.24 Simplex Construction

Standard form: maximize $\sum_{1\le i\le n} c_i x_i$ such that for all $1 \le j \le m$, $\sum_{1\le i\le n} A_{ji}x_i \le b_j$ and $x_i \ge 0$ for all $1 \le i \le n$.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1\le i\le n} A_{ji}x_i \ge b_j \to \sum_{1\le i\le n} -A_{ji}x_i \le -b_j$
3. $\sum_{1\le i\le n} A_{ji}x_i = b_j$
   - $\sum_{1\le i\le n} A_{ji}x_i \le b_j$
   - $\sum_{1\le i\le n} A_{ji}x_i \ge b_j$
4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

6 Geometry

6.1 Basic Geometry [17fa9b]

```
#define IM imag
#define RE real
using lld = int64_t;
using llf = long double;
using PT = std::complex<lld>;
using PTF = std::complex<llf>;
using P = PT;
llf abs(P p) { return sqrtl(norm(p)); }
PTF toPTF(PT p) { return PTF{RE(p), IM(p)}; }
int sgn(lld x) { return (x > 0) - (x < 0); }
lld dot(P a, P b) { return RE(conj(a) * b); }
lld cross(P a, P b) { return IM(conj(a) * b); }
int ori(P a, P b, P c) {
 return sgn(cross(b - a, c - a));
}
int quad(P p) {
 return (IM(p) == 0) // use sgn for PTF
  ? (RE(p) < 0 ? 3 : 1) : (IM(p) < 0 ? 0 : 2);
}
int argCmp(P a, P b) {
 // returns 0/+-1, starts from theta = -PI
 int qa = quad(a), qb = quad(b);
 if (qa != qb) return sgn(qa - qb);
 return sgn(cross(b, a));
}
template <typename V> llf area(const V & pt) {
 lld ret = 0;
 for (int i = 1; i + 1 < (int)pt.size(); i++)
  ret += cross(pt[i] - pt[0], pt[i+1] - pt[0]);
 return ret / 2.0;
}
P rot90(P p) { return P{-IM(p), RE(p)}; }
PTF project(PTF p, PTF q) { // p onto q
 return dot(p, q) * q / dot(q, q); // dot<llf>
}
```

6.2 2D Convex Hull [ecba37]

```
// from NaCl, counterclockwise, be careful of n<=2
vector<P> convex_hull(vector<P> v) {
 sort(all(v)); // by X then Y
 if (v[0] == v.back()) return {v[0]};
 int t = 0, s = 1; vector<P> h(v.size() + 1);
 for (int _ = 2; _--; s = t--, reverse(all(v)))
  for (P p : v) {
   while (t>s && ori(p, h[t-1], h[t-2]) >= 0) t--;
   h[t++] = p;
  }
 return h.resize(t), h;
}
```

6.3 2D Farthest Pair [ceb2ae]

```
// p is CCW convex hull w/o colinear points
int n = p.size(), pos = 1; lld ans = 0;
for (int i = 0; i < n; i++) {
 P e = p[(i + 1) % n] - p[i];
 while (cross(e, p[(pos + 1) % n] - p[i]) >
     cross(e, p[pos] - p[i]))
  pos = (pos + 1) % n;
 for (int j: {i, (i + 1) % n})
  ans = max(ans, norm(p[pos] - p[j]));
} // tested @ AOJ CGL_4_B
```

6.4 MinMax Enclosing Rect [c66dbf]

```
// from 8BQube, plz ensure p is strict convex hull
const llf INF = 1e18, qi = acos(-1) / 2 * 3;
pair<llf, llf> solve(vector<P> &p) {
#define Z(v) (p[v] - p[i])
 llf mx = 0, mn = INF;
 int n = (int)p.size(); p.emplace_back(p[0]);
 for (int i = 0, u = 1, r = 1, l = 1; i < n; ++i) {
  P e = Z(i + 1);
  while (cross(e, Z(u + 1)) > cross(e, Z(u)))
   u = (u + 1) % n;
  while (dot(e, Z(r + 1)) > dot(e, Z(r)))
   r = (r + 1) % n;
  if (!i) l = (r + 1) % n;
  while (dot(e, Z(l + 1)) < dot(e, Z(l)))
   l = (l + 1) % n;
  P D = p[r] - p[l];
  mn = min(mn, dot(e, D) / llf(norm(e)) * cross(e, Z(u)
    ));
```

```
  llf B = sqrt(norm(D)) * sqrt(norm(Z(u)));
  llf deg = (qi - acos(dot(D, Z(u)) / B)) / 2;
  mx = max(mx, B * sin(deg) * sin(deg));
 }
 return {mn, mx};
}
```

6.5 Minkowski Sum [c71bec]

```
// A, B are convex hull rotate to min by (X, Y)
vector<P> Minkowski(vector<P> A, vector<P> B) {
 vector<P> C(1, A[0] + B[0]), s1, s2;
 const int N = (int)A.size(), M = (int)B.size();
 for(int i = 0; i < N; ++i)
  s1.pb(A[(i + 1) % N] - A[i]);
 for(int i = 0; i < M; i++)
  s2.pb(B[(i + 1) % M] - B[i]);
 for(int i = 0, j = 0; i < N || j < M;)
  if (j >= N || (i < M && cross(s1[i], s2[j]) >= 0))
   C.pb(C.back() + s1[i++]);
  else
   C.pb(C.back() + s2[j++]);
 return hull(C), C;
}
```

6.6 Segment Intersection [60d016]

```
struct Seg { // closed segment
 P st, dir; // represent st + t*dir for 0<=t<=1
 Seg(P s, P e) : st(s), dir(e - s) {}
 static bool valid(lld p, lld q) {
  // is there t s.t. 0 <= t <= 1 && qt == p ?
  if (q < 0) q = -q, p = -p;
  return 0 <= p && p <= q;
 }
 vector<P> ends() const { return { st, st + dir }; }
};
template <typename T> bool isInter(T A, P p) {
 if (A.dir == P(0)) return p == A.st; // BE CAREFUL
 return cross(p - A.st, A.dir) == 0 &&
  T::valid(dot(p - A.st, A.dir), norm(A.dir));
}
template <typename U, typename V>
bool isInter(U A, V B) {
 if (cross(A.dir, B.dir) == 0) { // BE CAREFUL
  bool res = false;
  for (P p: A.ends()) res |= isInter(B, p);
  for (P p: B.ends()) res |= isInter(A, p);
  return res;
 }
 P D = B.st - A.st; lld C = cross(A.dir, B.dir);
 return U::valid(cross(D, B.dir), C) &&
  V::valid(cross(D, A.dir), C);
}
```

6.7 Half Plane Intersection [e98068]

```
struct Line {
 P st, ed, dir;
 Line (P s, P e) : st(s), ed(e), dir(e - s) {}
}; using L = const Line &;
PTF intersect(L A, L B) {
 llf t = cross(B.st - A.st, B.dir) /
  llf(cross(A.dir, B.dir));
 return toPTF(A.st) + toPTF(A.dir) * t; // C^3 / C^2
}
bool cov(L l, L A, L B) {
 i128 u = cross(B.st-A.st, B.dir);
 i128 v = cross(A.dir, B.dir);
 // ori(l.st, l.ed, A.st + A.dir*(u/v)) <= 0?
 i128 x = RE(A.dir) * u + RE(A.st - l.st) * v;
 i128 y = IM(A.dir) * u + IM(A.st - l.st) * v;
 return sgn(x*IM(l.dir) - y*RE(l.dir)) * sgn(v) >= 0;
} // x, y are C^3, also sgn<i128> is needed
bool operator<(L a, L b) {
 if (int c = argCmp(a.dir, b.dir)) return c == -1;
 return ori(a.st, a.ed, b.st) < 0;
}
// cross(pt-line.st, line.dir)<=0 <-> pt in half plane
// the half plane is the LHS when going from st to ed
llf HPI(vector<Line> &q) {
 sort(q.begin(), q.end());
 int n = (int)q.size(), l = 0, r = -1;
 for (int i = 0; i < n; i++) {
  if (i && !argCmp(q[i].dir, q[i-1].dir)) continue;
  while (l < r && cov(q[i], q[r-1], q[r])) --r;
  while (l < r && cov(q[i], q[l], q[l+1])) ++l;
  q[++r] = q[i];
 }
 while (l < r && cov(q[l], q[r-1], q[r])) --r;
 while (l < r && cov(q[r], q[l], q[l+1])) ++l;
 n = r - l + 1; // q[l .. r] are the lines
 if (n <= 1 || !argCmp(q[l].dir, q[r].dir)) return 0;
 vector<PTF> pt(n);
 for (int i = 0; i < n; i++)
  pt[i] = intersect(q[i+l], q[(i+1)%n+l]);
 return area(pt);
} // test @ 2020 Nordic NCPC : BigBrother
```

6.8 SegmentDist (Sausage) [9d8603]

```
// be careful of abs<complex<int>> (replace _abs below)
llf PointSegDist(P A, Seg B) {
 if (B.dir == P(0)) return _abs(A - B.st);
 if (sgn(dot(A - B.st, B.dir)) *
   sgn(dot(A - B.ed, B.dir)) <= 0)
  return abs(cross(A - B.st, B.dir)) / _abs(B.dir);
 return min(_abs(A - B.st), _abs(A - B.ed));
}
llf SegSegDist(const Seg &s1, const Seg &s2) {
 if (isInter(s1, s2)) return 0;
 return min({
   PointSegDist(s1.st, s2),
   PointSegDist(s1.ed, s2),
   PointSegDist(s2.st, s1),
   PointSegDist(s2.ed, s1) });
} // test @ QOJ2444 / PTZ19 Summer.D3
```

6.9 Rotating Sweep Line [1d9b4d]

```
void rotatingSweepLine(P a[], int n) {
 vector<pair<int,int>> ls; ls.reserve(n*(n-1)/2);
 for (int i = 0; i < n; ++i)
  for (int j = i + 1; j < n; ++j)
   ls.emplace_back(i, j);
 sort(all(ls), [&a](auto &u, auto &v){
  P zu = a[u.first] - a[u.second];
  P zv = a[v.first] - a[v.second];
  int s = sgn(RE(zu)) * sgn(RE(zv));
  if (s == 0) return RE(zu) != 0;
  return sgn(cross(zu, zv)) * s > 0;
 });
 vector<int> idx(n), p(n);
 iota(all(idx), 0);
 sort(all(idx), [&a](int i, int j) {
  return cmpxy(a[i], a[j]); });
 for (int i = 0; i < n; ++i) p[idx[i]] = i;
 for (auto [i, j]: ls) {
  // do here
  assert (abs(p[i] - p[j]) == 1);
  swap(p[i], p[j]); idx[p[i]] = i; idx[p[j]] = j;
 } // consider swap same slope together?
}
```

6.10 Point In Simple Polygon [22ef0b]

```
bool PIP(vector<P> &p, P z, bool strict = true) {
 int cnt = 0, n = p.size();
 for (int i = 0; i < n; i++) {
  P A = p[i], B = p[(i + 1) % n];
  if (isInter(Seg(A, B), z)) return !strict;
  auto zy = IM(z), Ay = IM(A), By = IM(B);
  cnt ^= ((zy<Ay) - (zy<By)) * ori(z, A, B) > 0;
 }
 return cnt;
}
```

6.11 Point In Hull (Fast) [906873]

```
bool PIH(vector<P> &h, P z, bool strict = true) {
 int n = (int)h.size(), a = 1, b = n - 1, r = !strict;
 if (n < 3) return r && isInter(Seg(h[0], h[n-1]), z);
 if (ori(h[0],h[a],h[b]) > 0) swap(a, b);
 if (ori(h[0],h[a],z) >= r || ori(h[0],h[b],z) <= -r)
  return false;
 while (abs(a - b) > 1) {
  int c = (a + b) / 2;
  (ori(h[0], h[c], z) > 0 ? b : a) = c;
 }
```

```
 return ori(h[a], h[b], z) < r;
}
```

6.12 Tangent of Points To Hull [6d7cd7]

```
pair<int, int> get_tangent(const vector<P> &v, P p) {
 const auto gao = [&, N = int(v.size())](int s) {
  const auto lt = [&](int x, int y) {
   return ori(p, v[x % N], v[y % N]) == s; };
  int l = 0, r = N; bool up = lt(0, 1);
  while (r - l > 1) {
   int m = (l + r) / 2;
   if (lt(m, 0) ? up : !lt(m, m+1)) r = m;
   else l = m;
  }
  return (lt(l, r) ? r : l) % N;
 }; // test @ codeforces.com/gym/101201/problem/E
 return {gao(-1), gao(1)}; // (a,b):ori(p,v[a],v[b])<0
} // plz ensure that point strictly out of hull
```

6.13 Circle Class & Intersection [5111af]

```
llf FMOD(llf x) {
 if (x < -PI) x += PI * 2;
 if (x > PI) x -= PI * 2;
 return x;
}
struct Cir { PTF o; llf r; };
// be carefule when tangent
vector<llf> intersectAngle(Cir a, Cir b) {
 PTF dir = b.o - a.o; llf d2 = norm(dir);
 if (norm(a.r - b.r) >= d2) { // norm(x) := |x|^2
  if (a.r < b.r) return {-PI, PI}; // a in b
  else return {}; // b in a
 } else if (norm(a.r + b.r) <= d2) return {};
 llf dis = abs(dir), theta = arg(dir);
 llf phi = acos((a.r * a.r + d2 - b.r * b.r) /
   (2 * a.r * dis)); // is acos_safe needed ?
 llf L = FMOD(theta - phi), R = FMOD(theta + phi);
 return { L, R };
}
vector<PTF> intersectPoint(Cir a, Cir b) {
 llf d = abs(a.o - b.o);
 if (d > b.r+a.r || d < abs(b.r-a.r)) return {};
 llf dt = (b.r*b.r - a.r*a.r)/d, d1 = (d+dt)/2;
 PTF dir = (a.o - b.o) / d;
 PTF u = dir * d1 + b.o;
 PTF v = rot90(dir) * sqrt(max(0.0L, b.r*b.r-d1*d1));
 return {u + v, u - v};
} // test @ AOJ CGL probs
```

6.14 Circle Common Tangent [5ff02c]

```
// be careful of tangent / exact same circle
// sign1 = 1 for outer tang, -1 for inter tang
vector<Line> common_tan(const Cir &a, const Cir &b, int
     sign1) {
 if (norm(a.o - b.o) < eps) return {};
 llf d = abs(a.o - b.o), c = (a.r - sign1 * b.r) / d;
 PTF v = (b.o - a.o) / d;
 if (c * c > 1) return {};
 if (abs(c * c - 1) < eps) {
  PTF p = a.o + c * v * a.r;
  return {Line(p, p + rot90(b.o - a.o))};
 }
 vector<Line> ret; llf h = sqrt(max(0.0L, 1-c*c));
 for (int sign2 : {1, -1}) {
  PTF n = c * v + sign2 * h * rot90(v);
  PTF p1 = a.o + n * a.r;
  PTF p2 = b.o + n * (b.r * sign1);
  ret.emplace_back(p1, p2);
 }
 return ret;
}
```

6.15 Line-Circle Intersection [12b42a]

```
vector<PTF> LineCircleInter(PTF p1, PTF p2, PTF o, llf
    r) {
 PTF ft = p1 + project(o-p1, p2-p1), vec = p2-p1;
 llf dis = abs(o - ft);
 if (abs(dis - r) < eps) return {ft};
 if (dis > r) return {};
 vec = vec * sqrt(r * r - dis * dis) / abs(vec);
 return {ft + vec, ft - vec}; // sqrt_safe?
}
```

6.16 Poly-Circle Intersection [242a4e]

```
// Divides into multiple triangle, and sum up
// from 8BQube, test by HDU2892 & AOJ CGL_7_H
llf _area(PTF pa, PTF pb, llf r) {
 if (abs(pa) < abs(pb)) swap(pa, pb);
 if (abs(pb) < eps) return 0;
 llf S, h, theta;
 llf a = abs(pb), b = abs(pa), c = abs(pb - pa);
 llf cB = dot(pb, pb-pa) / a / c, B = acos_safe(cB);
 llf cC = dot(pa, pb) / a / b, C = acos_safe(cC);
 if (a > r) {
  S = (C / 2) * r * r; h = a * b * sin(C) / c;
  if (h < r && B < PI / 2)
   S -= (acos_safe(h/r)*r*r - h*sqrt_safe(r*r-h*h));
 } else if (b > r) {
  theta = PI - B - asin_safe(sin(B) / r * a);
  S = 0.5 * a*r*sin(theta) + (C-theta)/2 * r * r;
 } else
  S = 0.5 * sin(C) * a * b;
 return S;
}
llf area_poly_circle(const vector<PTF> &poly, PTF O,
    llf r) {
 llf S = 0;
 for (int i = 0, N = poly.size(); i < N; ++i)
  S += _area(poly[i] - O, poly[(i + 1) % N] - O, r) *
     ori(O, poly[i], poly[(i + 1) % N]);
 return abs(S);
}
```

6.17 Minimum Covering Circle [3a9017]

```
// be careful of type
Cir getCircum(P a, P b, P c){
 llf a1 = a.x-b.x, b1 = a.y-b.y;
 llf c1 = (a.x+b.x)/2 * a1 + (a.y+b.y)/2 * b1;
 llf a2 = a.x-c.x, b2 = a.y-c.y;
 llf c2 = (a.x+c.x)/2 * a2 + (a.y+c.y)/2 * b2;
 Cir cc;
 cc.o.x = (c1*b2-b1*c2)/(a1*b2-b1*a2);
 cc.o.y = (a1*c2-c1*a2)/(a1*b2-b1*a2);
 cc.r = hypot(cc.o.x-a.x, cc.o.y-a.y);
 return cc;
}
Cir minCircleCover(vector<P> &pts) {
 shuffle(pts.begin(), pts.end(), mt19937(114514));
 Cir c = { pts[0], 0 };
 for(int i = 0; i < (int)pts.size(); i++) {
  if (dist(pts[i], c.o) <= c.r) continue;
  c = { pts[i], 0 };
  for (int j = 0; j < i; j++) {
   if (dist(pts[j], c.o) <= c.r) continue;
   c.o = (pts[i] + pts[j]) / llf(2);
   c.r = dist(pts[i], c.o);
   for (int k = 0; k < j; k++) {
    if (dist(pts[k], c.o) <= c.r) continue;
    c = getCircum(pts[i], pts[j], pts[k]);
   }
  }
 }
 return c;
}
```

6.18 Circle Union [1a5265]

```
#define eb emplace_back
struct Teve { // test@SPOJ N=1000, 0.3~0.5s
 PTF p; llf a; int add; // point, ang, add
 Teve(PTF x, llf y, int z) : p(x), a(y), add(z) {}
 bool operator<(Teve &b) const { return a < b.a; }
};
// strict: x = 0, otherwise x = -1
bool disjunct(Cir &a, Cir &b, int x)
{ return sgn(abs(a.o - b.o) - a.r - b.r) > x; }
bool contain(Cir &a, Cir &b, int x)
{ return sgn(a.r - b.r - abs(a.o - b.o)) > x; }
vector<llf> CircleUnion(vector<Cir> &c) {
 // area[i] : area covered by at least i circles
 int N = (int)c.size(); vector<llf> area(N + 1);
 vector<vector<int>> overlap(N, vector<int>(N));
 auto g = overlap; // use simple 2darray to speedup
 for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j) {
```

```
   /* c[j] is non-strictly in c[i]. */
   overlap[i][j] = i != j &&
    (sgn(c[i].r - c[j].r) > 0 ||
     (sgn(c[i].r - c[j].r) == 0 && i < j)) &&
    contain(c[i], c[j], -1);
  }
 for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
   g[i][j] = i != j && !(overlap[i][j] ||
     overlap[j][i] || disjunct(c[i], c[j], -1));
 for (int i = 0; i < N; ++i) {
  vector<Teve> eve; int cnt = 1;
  for (int j = 0; j < N; ++j) cnt += overlap[j][i];
  // if (cnt > 1) continue; (if only need area[1])
  for (int j = 0; j < N; ++j) if (g[i][j]) {
   auto IP = intersectPoint(c[i], c[j]);
   PTF aa = IP[1], bb = IP[0];
   llf A = arg(aa - c[i].o), B = arg(bb - c[i].o);
   eve.eb(bb, B, 1); eve.eb(aa, A, -1);
   if (B > A) ++cnt;
  }
  if (eve.empty()) area[cnt] += PI*c[i].r*c[i].r;
  else {
   sort(eve.begin(), eve.end());
   eve.eb(eve[0]); eve.back().a += PI * 2;
   for (size_t j = 0; j + 1 < eve.size(); j++) {
    cnt += eve[j].add;
    area[cnt] += cross(eve[j].p, eve[j+1].p) *.5;
    llf t = eve[j + 1].a - eve[j].a;
    area[cnt] += (t-sin(t)) * c[i].r * c[i].r *.5;
   }
  }
 }
 return area;
}
```

6.19 Polygon Union [2bff43]

```
llf rat(P a, P b) { return sgn(RE(b)) ? llf(RE(a))/RE(b
    ) : llf(IM(a))/IM(b); }
llf polyUnion(vector<vector<P>>& poly) {
 llf ret = 0; // area of poly[i] must be non-negative
 rep(i,0,sz(poly)) rep(v,0,sz(poly[i])) {
  P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])];
  vector<pair<llf, int>> segs{{0, 0}, {1, 0}};
  rep(j,0,sz(poly)) if (i != j) {
   rep(u,0,sz(poly[j])) {
    P C = poly[j][u], D = poly[j][(u + 1) % sz(poly[j])
    ];
    if (int sc = ori(A, B, C), sd = ori(A, B, D); sc !=
      sd) {
     llf sa = cross(D-C, A-C), sb = cross(D-C, B-C);
     if (min(sc, sd) < 0)
      segs.emplace_back(sa / (sa - sb), sgn(sc - sd));
    } else if (!sc && !sd && j<i && sgn(dot(B-A,D-C))
    >0){
     segs.emplace_back(rat(C - A, B - A), 1);
     segs.emplace_back(rat(D - A, B - A), -1);
    }
   }
  }
  sort(segs.begin(), segs.end());
  for (auto &s : segs) s.first = clamp<llf>(s.first, 0,
     1);
  llf sum = 0;
  int cnt = segs[0].second;
  rep(j,1,sz(segs)) {
   if (!cnt) sum += segs[j].first - segs[j - 1].first;
   cnt += segs[j].second;
  }
  ret += cross(A,B) * sum;
 }
 return ret / 2;
}
```

6.20 3D Convex Hull [93b153]

```
// return the faces with pt indexes
struct P3 { lld x,y,z;
 P3 operator * (const P3 &b) const {
  return(P3){y*b.z-b.y*z,z*b.x-b.z*x,x*b.y-b.x*y};
 } };
struct Face { int a, b, c;
 Face(int ta,int tb,int tc):a(ta),b(tb),c(tc){} };
P3 ver(P3 a, P3 b, P3 c) { return (b - a) * (c - a); }
// plz ensure that first 4 points are not coplanar
// all points coplanar case will WA
vector<Face> convex_hull_3D(const vector<P3> &pt) {
 int n = int(pt.size()); vector<Face> now;
 if (n <= 2) return {}; // be careful about edge case
 vector<vector<int>> flag(n, vector<int>(n));
 now.emplace_back(0,1,2); now.emplace_back(2,1,0);
 for (int i = 3; i < n; i++) {
  vector<Face> next;
  for (const auto &f : now) {
   lld d = (pt[i] - pt[f.a]).dot(
     ver(pt[f.a], pt[f.b], pt[f.c]));
   if (d <= 0) next.push_back(f);
   int ff = (d > 0) - (d < 0);
   flag[f.a][f.b]=flag[f.b][f.c]=flag[f.c][f.a]=ff;
  }
  for (const auto &f : now) {
   const auto F = [&](int x, int y) {
    if (flag[x][y] > 0 && flag[y][x] <= 0)
     next.emplace_back(x, y, i);
   };
   F(f.a, f.b); F(f.b, f.c); F(f.c, f.a);
  }
  now = next;
 }
 return now;
}
// delaunay: facets with negative z normal of
// convexhull of (x, y, x^2 + y^2)
```

6.21 Delaunay [7f0d57]

```
/* A triangulation such that no points will strictly
inside circumcircle of any triangle.
find(root, p) : return a triangle contain given point
add_point : add a point into triangulation
Region of triangle u: iterate each u.e[i].tri,
each points are u.p[(i+1)%3], u.p[(i+2)%3]
Voronoi diagram: for each triangle in `res`,
the bisector of all its edges will split the region. */
#define L(i) ((i)==0 ? 2 : (i)-1)
#define R(i) ((i)==2 ? 0 : (i)+1)
#define F3 for (int i = 0; i < 3; i++)
bool in_cc(const array<P,3> &p, P q) {
  i128 det = 0;
  F3 det += i128(norm(p[i]) - norm(q)) *
   cross(p[R(i)] - q, p[L(i)] - q);
  return det > 0;
}
struct Tri;
struct E {
 Tri *t; int side; E() : t(0), side(0) {}
 E(Tri *t_, int side_) : t(t_), side(side_){}
};
struct Tri {
 array<P,3> p; array<Tri*,3> ch; array<E,3> e;
 Tri(P a = 0, P b = 0, P c = 0) : p{a, b, c}, ch{} {}
 bool has_chd() const { return ch[0] != nullptr; }
 bool contains(P q) const {
  F3 if (ori(p[i], p[R(i)], q) < 0) return false;
  return true;
 }
} pool[maxn * 10], *it;
void link(E a, E b) {
 if (a.t) a.t->e[a.side] = b;
 if (b.t) b.t->e[b.side] = a;
}
struct Trigs {
 Tri *root;
 Trigs() { // should at least contain all points
  root = // C = 100*MAXC^2 or just MAXC?
   new(it++) Tri(P(-C, -C), P(C*2, -C), P(-C, C*2));
 }
 void add_point(P p) { add_point(find(p, root), p); }
 static Tri* find(P p, Tri *r) {
  while (r->has_chd()) for (Tri *c: r->ch)
    if (c && c->contains(p)) { r = c; break; }
  return r;
 }
 void add_point(Tri *r, P p) {
  array<Tri*, 3> t; /* split into 3 triangles */
  F3 t[i] = new (it++) Tri(r->p[i], r->p[R(i)], p);
```

```
  F3 link(E(t[i], 0), E(t[R(i)], 1));
  F3 link(E(t[i], 2), r->e[L(i)]);
  r->ch = t;
  F3 flip(t[i], 2);
 }
 void flip(Tri* A, int a) {
  auto [B, b] = A->e[a]; /* flip edge between A,B */
  if (!B || !in_cc(A->p, B->p[b])) return;
  Tri *X = new(it++)Tri(A->p[R(a)],B->p[b],A->p[a]);
  Tri *Y = new(it++)Tri(B->p[R(b)],A->p[a],B->p[b]);
  link(E(X,0), E(Y,0));
  link(E(X,1), A->e[L(a)]); link(E(X,2), B->e[R(b)]);
  link(E(Y,1), B->e[L(b)]); link(E(Y,2), A->e[R(a)]);
  A->ch = B->ch = {X, Y, nullptr};
  flip(X, 1); flip(X, 2); flip(Y, 1); flip(Y, 2);
 }
};
vector<Tri*> res; set<Tri*> vis;
void go(Tri *now) { // store all tri into res
 if (!vis.insert(now).second) return;
 if (!now->has_chd()) return res.push_back(now);
 for (Tri *c: now->ch) if (c) go(c);
}
void build(vector<P> ps) {
 it = pool; res.clear(); vis.clear();
 shuffle(ps.begin(), ps.end(), mt19937(114514));
 Trigs tr; for (P p: ps) tr.add_point(p);
 go(tr.root); // use `res` afterwards
}
```

6.22 kd Tree (Nearest Point) [f87996]

```
struct KDTree {
 struct Node {
  int x, y, x1, y1, x2, y2, id, f;
  Node *L, *R;
 } tree[maxn], *root;
 lld dis2(int x1, int y1, int x2, int y2) {
  lld dx = x1 - x2, dy = y1 - y2;
  return dx * dx + dy * dy;
 }
 static bool cmpx(Node& a, Node& b){return a.x<b.x;}
 static bool cmpy(Node& a, Node& b){return a.y<b.y;}
 void init(vector<pair<int,int>> &ip) {
  const int n = ip.size();
  for (int i = 0; i < n; i++) {
   tree[i].id = i;
   tree[i].x = ip[i].first;
   tree[i].y = ip[i].second;
  }
  root = build_tree(0, n-1, 0);
 }
 Node* build_tree(int L, int R, int d) {
  if (L>R) return nullptr;
  int M = (L+R)/2; tree[M].f = d%2;
  nth_element(tree+L,tree+M,tree+R+1,d%2?cmpy:cmpx);
  tree[M].x1 = tree[M].x2 = tree[M].x;
  tree[M].y1 = tree[M].y2 = tree[M].y;
  tree[M].L = build_tree(L, M-1, d+1);
  tree[M].R = build_tree(M+1, R, d+1);
  for (Node *s: {tree[M].L, tree[M].R}) if (s) {
   tree[M].x1 = min(tree[M].x1, s->x1);
   tree[M].x2 = max(tree[M].x2, s->x2);
   tree[M].y1 = min(tree[M].y1, s->y1);
   tree[M].y2 = max(tree[M].y2, s->y2);
  }
  return tree+M;
 }
 bool touch(int x, int y, lld d2, Node *r){
  lld d = sqrt(d2)+1;
  return x >= r->x1 - d && x <= r->x2 + d &&
         y >= r->y1 - d && y <= r->y2 + d;
 }
 using P = pair<lld, int>;
 void dfs(int x, int y, P &mn, Node *r) {
  if (!r || !touch(x, y, mn.first, r)) return;
  mn = min(mn, P(dis2(r->x, r->y, x, y), r->id));
  // search order depends on split dim
  if (r->f == 1 ? y < r->y : x < r->x) {
   dfs(x, y, mn, r->L);
   dfs(x, y, mn, r->R);
  } else {
   dfs(x, y, mn, r->R);
   dfs(x, y, mn, r->L);
  }
 }
 int query(int x, int y) {
  P mn(INF, -1);
  dfs(x, y, mn, root);
  return mn.second;
 }
} tree;
```

6.23 kd Closest Pair (3D ver.) [84d9eb]

```
llf solve(vector<P> v) {
 shuffle(v.begin(), v.end(), mt19937());
 unordered_map<lld, unordered_map<lld,
  unordered_map<lld, int>>> m;
 llf d = dis(v[0], v[1]);
 auto Idx = [&d] (llf x) -> lld {
  return round(x * 2 / d) + 0.1; };
 auto rebuild_m = [&m, &v, &Idx](int k) {
  m.clear();
  for (int i = 0; i < k; ++i)
   m[Idx(v[i].x)][Idx(v[i].y)]
    [Idx(v[i].z)] = i;
 }; rebuild_m(2);
 for (size_t i = 2; i < v.size(); ++i) {
  const lld kx = Idx(v[i].x), ky = Idx(v[i].y),
     kz = Idx(v[i].z); bool found = false;
  for (int dx = -2; dx <= 2; ++dx) {
   const lld nx = dx + kx;
   if (m.find(nx) == m.end()) continue;
   auto& mm = m[nx];
   for (int dy = -2; dy <= 2; ++dy) {
    const lld ny = dy + ky;
    if (mm.find(ny) == mm.end()) continue;
    auto& mmm = mm[ny];
    for (int dz = -2; dz <= 2; ++dz) {
     const lld nz = dz + kz;
     if (mmm.find(nz) == mmm.end()) continue;
     const int p = mmm[nz];
     if (dis(v[p], v[i]) < d) {
      d = dis(v[p], v[i]);
      found = true;
     }
    }
   }
  }
  if (found) rebuild_m(i + 1);
  else m[kx][ky][kz] = i;
 }
 return d;
}
```

6.24 Simulated Annealing [4e0fe5]

```
llf anneal() {
 mt19937 rnd_engine(seed);
 uniform_real_distribution<llf> rnd(0, 1);
 const llf dT = 0.001;
 // Argument p
 llf S_cur = calc(p), S_best = S_cur;
 for (llf T = 2000; T > EPS; T -= dT) {
  // Modify p to p_prime
  const llf S_prime = calc(p_prime);
  const llf delta_c = S_prime - S_cur;
  llf prob = min((llf)1, exp(-delta_c / T));
  if (rnd(rnd_engine) <= prob)
   S_cur = S_prime, p = p_prime;
  if (S_prime < S_best) // find min
   S_best = S_prime, p_best = p_prime;
 }
 return S_best;
}
```

7 Stringology

7.1 Hash [7afe3e]

```
class Hash {
 private:
  static constexpr int P = 127, Q = 1051762951;
  vector<int> h, p;
 public:
  void init(const string &s){
```

```
  h.assign(s.size()+1, 0); p.resize(s.size()+1);
  for (size_t i = 0; i < s.size(); ++i)
   h[i + 1] = add(mul(h[i], P), s[i]);
  generate(p.begin(), p.end(),[x=1,y=1,this]()
    mutable{y=x;x=mul(x,P);return y;});
 }
 int query(int l, int r){ // 1-base (l, r]
  return sub(h[r], mul(h[l], p[r-l]));}
};
```

7.2 Suffix Array [2846f0]

```
namespace sfx {
bool _t[maxn * 2];
int hi[maxn], rev[maxn];
int _s[maxn * 2], sa[maxn * 2], _c[maxn * 2];
int x[maxn], _p[maxn], _q[maxn * 2];
// sa[i]: sa[i]-th suffix is the
// i-th lexigraphically smallest suffix.
// hi[i]: longest common prefix
// of suffix sa[i] and suffix sa[i - 1].
void pre(int *a, int *c, int n, int z) {
 memset(a, 0, sizeof(int) * n);
 memcpy(x, c, sizeof(int) * z);
}
void induce(int *a,int *c,int *s,bool *t,int n,int z){
 memcpy(x + 1, c, sizeof(int) * (z - 1));
 for (int i = 0; i < n; ++i)
  if (a[i] && !t[a[i] - 1])
   a[x[s[a[i] - 1]]++] = a[i] - 1;
 memcpy(x, c, sizeof(int) * z);
 for (int i = n - 1; i >= 0; --i)
  if (a[i] && t[a[i] - 1])
   a[--x[s[a[i] - 1]]] = a[i] - 1;
}
void sais(int *s, int *a, int *p, int *q,
 bool *t, int *c, int n, int z) {
 bool uniq = t[n - 1] = true;
 int nn=0, nmxz=-1, *nsa = a+n, *ns=s+n, last=-1;
 memset(c, 0, sizeof(int) * z);
 for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;
 for (int i = 0; i < z - 1; ++i) c[i + 1] += c[i];
 if (uniq) {
  for (int i = 0; i < n; ++i) a[--c[s[i]]] = i;
  return;
 }
 for (int i = n - 2; i >= 0; --i)
  t[i] = (s[i]==s[i + 1] ? t[i + 1] : s[i]<s[i + 1]);
 pre(a, c, n, z);
 for (int i = 1; i <= n - 1; ++i)
  if (t[i] && !t[i - 1])
   a[--x[s[i]]] = p[q[i] = nn++] = i;
 induce(a, c, s, t, n, z);
 for (int i = 0; i < n; ++i) {
  if (a[i] && t[a[i]] && !t[a[i] - 1]) {
  bool neq = last < 0 ||
   memcmp(s + a[i], s + last,
   (p[q[a[i]] + 1] - a[i]) * sizeof(int));
  ns[q[last = a[i]]] = nmxz += neq;
 }}
 sais(ns, nsa, p+nn, q+n, t+n, c+z, nn, nmxz+1);
 pre(a, c, n, z);
 for (int i = nn - 1; i >= 0; --i)
  a[--x[s[p[nsa[i]]]]] = p[nsa[i]];
 induce(a, c, s, t, n, z);
}
void build(const string &s) {
 const int n = int(s.size());
 for (int i = 0; i < n; ++i) _s[i] = s[i];
 _s[n] = 0; // s shouldn't contain 0
 sais(_s, sa, _p, _q, _t, _c, n + 1, 256);
 for(int i = 0; i < n; ++i) rev[sa[i] = sa[i+1]] = i;
 int ind = hi[0] = 0;
 for (int i = 0; i < n; ++i) {
  if (!rev[i]) {
   ind = 0;
   continue;
  }
  while (i + ind < n &&
   s[i + ind] == s[sa[rev[i] - 1] + ind]) ++ind;
  hi[rev[i]] = ind ? ind-- : 0;
 }
}}
```

7.3 Ex SAM [a56a7c]

```
struct exSAM {
 int len[maxn * 2], link[maxn * 2]; // maxlen, suflink
 int next[maxn * 2][maxc], tot; // [0, tot), root = 0
 int ord[maxn * 2]; // topo. order
 int cnt[maxn * 2]; // occurence
 int newnode() {
  fill_n(next[tot], maxc, 0);
  return len[tot] = cnt[tot] = link[tot] = 0, tot++;
 }
 void init() { tot = 0, newnode(), link[0] = -1; }
 int insertSAM(int last, int c) {
  int cur = next[last][c];
  len[cur] = len[last] + 1;
  int p = link[last];
  while (p != -1 && !next[p][c])
   next[p][c] = cur, p = link[p];
  if (p == -1) return link[cur] = 0, cur;
  int q = next[p][c];
  if (len[p] + 1 == len[q]) return link[cur] = q, cur;
  int clone = newnode();
  for (int i = 0; i < maxc; ++i)
   next[clone][i] = len[next[q][i]] ? next[q][i] : 0;
  len[clone] = len[p] + 1;
  while (p != -1 && next[p][c] == q)
   next[p][c] = clone, p = link[p];
  link[link[cur] = clone] = link[q];
  link[q] = clone;
  return cur;
 }
 void insert(const string &s) {
  int cur = 0;
  for (auto ch : s) {
   int &nxt = next[cur][int(ch - 'a')];
   if (!nxt) nxt = newnode();
   cnt[cur = nxt] += 1;
  }
 }
 void build() {
  queue<int> q; q.push(0);
  while (!q.empty()) {
   int cur = q.front(); q.pop();
   for (int i = 0; i < maxc; ++i)
    if (next[cur][i]) q.push(insertSAM(cur, i));
  }
  vector<int> lc(tot);
  for (int i = 1; i < tot; ++i) ++lc[len[i]];
  partial_sum(all(lc), lc.begin());
  for (int i = 1; i < tot; ++i) lenSorted[--lc[len[i]]]
     = i;
 }
 void solve() {
  for (int i = tot - 2; i >= 0; --i)
   cnt[link[lenSorted[i]]] += cnt[lenSorted[i]];
 }
};
```

7.4 Z value [6a7fd0]

```
vector<int> Zalgo(const string &s) {
 vector<int> z(s.size(), s.size());
 for (int i = 1, l = 0, r = 0; i < z[0]; ++i) {
  int j = clamp(r - i, 0, z[i - l]);
  for (; i + j < z[0] and s[i + j] == s[j]; ++j);
  if (i + (z[i] = j) > r) r = i + z[l = i];
 }
 return z;
}
```

7.5 Manacher [365720]

```
int z[maxn];
int manacher(const string& s) {
 string t = ".";
 for(char c: s) t += c, t += '.';
 int l = 0, r = 0, ans = 0;
 for (int i = 1; i < t.length(); ++i) {
  z[i] = (r > i ? min(z[2 * l - i], r - i) : 1);
  while (i - z[i] >= 0 && i + z[i] < t.length()) {
   if(t[i - z[i]] == t[i + z[i]]) ++z[i];
   else break;
  }
  if (i + z[i] > r) r = i + z[i], l = i;
```

```
 }
 for(int i=1;i<t.length();++i) ans = max(ans, z[i]-1);
 return ans;
}
```

7.6 Lexico Smallest Rotation [0e9fb8]

```
string mcp(string s) {
 int n = s.length();
 s += s; int i = 0, j = 1;
 while (i < n && j < n) {
  int k = 0;
  while (k < n && s[i + k] == s[j + k]) k++;
  ((s[i + k] <= s[j + k]) ? j : i) += k + 1;
  j += (i == j);
 }
 return s.substr(i < n ? i : j, n);
}
```

7.7 Main Lorentz [615b8f]

```
vector<pair<int, int>> rep[kN]; // 0-base [l, r]
void main_lorentz(const string &s, int sft = 0) {
 const int n = s.size();
 if (n == 1) return;
 const int nu = n / 2, nv = n - nu;
 const string u = s.substr(0, nu), v = s.substr(nu),
   ru(u.rbegin(), u.rend()), rv(v.rbegin(), v.rend());
 main_lorentz(u, sft), main_lorentz(v, sft + nu);
 const auto z1 = Zalgo(ru), z2 = Zalgo(v + '#' + u),
       z3 = Zalgo(ru + '#' + rv), z4 = Zalgo(v);
 auto get_z = [](const vector<int> &z, int i) {
  return (0 <= i and i < (int)z.size()) ? z[i] : 0; };
 auto add_rep = [&](bool left, int c, int l, int k1,
   int k2) {
  const int L = max(1, l - k2), R = min(l - left, k1);
  if (L > R) return;
  if (left) rep[l].emplace_back(sft + c - R, sft + c -
    L);
  else rep[l].emplace_back(sft + c - R - l + 1, sft + c
     - L - l + 1);
 };
 for (int cntr = 0; cntr < n; cntr++) {
  int l, k1, k2;
  if (cntr < nu) {
   l = nu - cntr;
   k1 = get_z(z1, nu - cntr);
   k2 = get_z(z2, nv + 1 + cntr);
  } else {
   l = cntr - nu + 1;
   k1 = get_z(z3, nu + 1 + nv - 1 - (cntr - nu));
   k2 = get_z(z4, (cntr - nu) + 1);
  }
  if (k1 + k2 >= l)
   add_rep(cntr < nu, cntr, l, k1, k2);
 }
}
```

7.8 BWT [5a9b3a]

```
vector<int> v[SIGMA];
void BWT(char *ori, char *res) {
 // make ori -> ori + ori
 // then build suffix array
}
void iBWT(char *ori, char *res) {
 for (int i = 0; i < SIGMA; i++) v[i].clear();
 const int len = strlen(ori);
 for (int i = 0; i < len; i++)
  v[ori[i] - 'a'].push_back(i);
 vector<int> a;
 for (int i = 0, ptr = 0; i < SIGMA; i++)
  for (int j : v[i]) {
   a.push_back(j);
   ori[ptr++] = 'a' + i;
  }
 for (int i = 0, ptr = 0; i < len; i++) {
  res[i] = ori[a[ptr]];
  ptr = a[ptr];
 }
 res[len] = 0;
}
```

7.9 Palindromic Tree [0673ee]

```
struct PalindromicTree {
 struct node {
  int nxt[26], f, len; // num = depth of fail link
  int cnt, num;     // = #pal_suffix of this node
  node(int l = 0) : nxt{}, f(0), len(l), cnt(0), num(0)
     {}
 };
 vector<node> st; vector<char> s; int last, n;
 void init() {
  st.clear(); s.clear();
  last = 1; n = 0;
  st.push_back(0); st.push_back(-1);
  st[0].f = 1; s.push_back(-1);
 }
 int getFail(int x) {
  while (s[n - st[x].len - 1] != s[n]) x = st[x].f;
  return x;
 }
 void add(int c) {
  s.push_back(c -= 'a'); ++n;
  int cur = getFail(last);
  if (!st[cur].nxt[c]) {
   int now = st.size();
   st.push_back(st[cur].len + 2);
   st[now].f = st[getFail(st[cur].f)].nxt[c];
   st[cur].nxt[c] = now;
   st[now].num = st[st[now].f].num + 1;
  }
  last = st[cur].nxt[c]; ++st[last].cnt;
 }
 void dpcnt() { // cnt = #occurence in whole str
  for (int i = st.size() - 1; i >= 0; i--)
   st[st[i].f].cnt += st[i].cnt;
 }
 int size() { return st.size() - 2; }
} pt;
/* usage
string s; cin >> s; pt.init();
for (int i = 0; i < SZ(s); i++) {
 int prvsz = pt.size(); pt.add(s[i]);
 if (prvsz != pt.size()) {
  int r = i, l = r - pt.st[pt.last].len + 1;
  // pal @ [l,r]: s.substr(l, r-l+1)
 }
} */
```

8 Misc

8.1 Theorems

Sherman-Morrison formula

$\left(A+uv^{\intercal}\right)^{-1} = A^{-1} - \frac{A^{-1}uv^{\intercal}A^{-1}}{1+v^{\intercal}A^{-1}u}$

Kirchhoff's Theorem

Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G, where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i,j) in G.

- The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
- The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.

Tutte's Matrix

Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniform randomly) if $i < j$ and $(i,j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{rank(D)}{2}$ is the maximum matching on G.

Cayley's Formula

- Given a degree sequence $d_1, d_2, \ldots, d_n$ for each labeled vertices, there're $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!}$ spanning trees.
- Let $T_{n,k}$ be the number of labeled forests on n vertices with k components, such that vertex $1, 2, \ldots, k$ belong to different components. Then $T_{n,k} = kn^{n-k-1}$.

Erdős–Gallai theorem

A sequence of non-negative integers $d_1 \geq d_2 \geq \ldots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + d_2 + \ldots + d_n$ is even and

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k)$$

holds for all $1 \leq k \leq n$.

Havel–Hakimi algorithm

find the vertex who has greatest degree unused, connect it with other greatest vertex.

Euler's planar graph formula

$V - E + F = C + 1$. $E \le 3V - 6$ (when $V \ge 3$)

Pick's theorem

For simple polygon, when points are all integer, we have A = #{lattice points in the interior} + $\frac{\text{\#\{lattice points on the boundary\}}}{2} - 1$

Matroid Intersection

Given matroids $M_1 = (G, I_1), M_2 = (G, I_2)$, find maximum $S \in I_1 \cap I_2$. For each iteration, build the directed graph and find a shortest path from s to t.

- $s \to x : S \sqcup \{x\} \in I_1$
- $x \to t : S \sqcup \{x\} \in I_2$
- $y \to x : S \setminus \{y\} \sqcup \{x\} \in I_1$ (y is in the unique circuit of $S \sqcup \{x\}$)
- $x \to y : S \setminus \{y\} \sqcup \{x\} \in I_2$ (y is in the unique circuit of $S \sqcup \{x\}$)

Alternate the path, and $|S|$ will increase by 1. Let R = $\min(\text{rank}(I_1), \text{rank}(I_2)), N = |G|$. In each iteration, $|E| = O(RN)$. For weighted case, assign weight $-w(x)$ and $w(x)$ to $x \in S$ and $x \notin S$, resp. Use Bellman-Ford to find the weighted shortest path. The maximum iteration of Bellman-Ford is $2R + 1$.

8.2 Weight Matroid Intersection [c376a9]

```
struct Matroid {
 Matroid(bitset<N>); // init from an independent set
 bool can_add(int);  // check if break independence
 Matroid remove(int); // removing from the set
};
auto matroid_intersection(const vector<int> &w) {
 const int n = w.size(); bitset<N> S;
 for (int sz = 1; sz <= n; sz++) {
  Matroid M1(S), M2(S); vector<vector<pii>> e(n + 2);
  for (int j = 0; j < n; j++) if (!S[j]) {
   if (M1.can_add(j)) e[n].eb(j, -w[j]);
   if (M2.can_add(j)) e[j].eb(n + 1, 0);
  }
  for (int i = 0; i < n; i++) if (S[i]) {
   Matroid T1 = M1.remove(i), T2 = M2.remove(i);
   for (int j = 0; j < n; j++) if (!S[j]) {
    if (T1.can_add(j)) e[i].eb(j, -w[j]);
    if (T2.can_add(j)) e[j].eb(i, w[i]);
   }
  } // maybe implicit build graph for more speed
  vector<pii> d(n + 2, {INF, 0}); d[n] = {0, 0};
  vector<int> prv(n + 2, -1);
  // change to SPFA for more speed, if necessary
  bool upd = 1;
  while (upd) {
   upd = 0;
   for (int u = 0; u < n + 2; u++)
    for (auto [v, c] : e[u]) {
     pii x(d[u].first + c, d[u].second + 1);
     if (x < d[v]) d[v] = x, prv[v] = u, upd = 1;
    }
  }
  if (d[n + 1].first >= INF) break;
  for (int x = prv[n+1]; x!=n; x = prv[x]) S.flip(x);
  // S is the max-weighted independent set w/ size sz
 }
 return S;
} // from Nacl
```

8.3 Bitset LCS [5e6c56]

```
scanf("%d%d", &n, &m), u = n / 64 + 1;
for (int i = 1, c; i <= n; i++)
 scanf("%d", &c), p[c].set(i);
for (int i = 1, c; i <= m; i++) {
 scanf("%d", &c), (g = f) |= p[c];
 f.shiftLeftByOne(), f.set(0);
 ((f = g - f) ^= g) &= g;
}
printf("%d\n", f.count());
```

8.4 Prefix Substring LCS [78a378]

```
void all_lcs(string s, string t) { // 0-base
 vector<int> h(SZ(t));
 iota(ALL(h), 0);
 for (int a = 0; a < SZ(s); ++a) {
  int v = -1;
  for (int c = 0; c < SZ(t); ++c)
   if (s[a] == t[c] || h[c] < v)
    swap(h[c], v);
  // LCS(s[0, a], t[b, c]) =
  // c - b + 1 - sum([h[i] >= b] | i <= c)
  // h[i] might become -1 !!
 }
}
```

8.5 Convex 1D/1D DP [6e0124]

```
struct segment {
 int i, l, r;
 segment() {}
 segment(int a, int b, int c): i(a), l(b), r(c) {}
};
void solve() {
 auto f = [](int l, int r){return dp[l] + w(l+1, r);};
 dp[0] = 0;
 deque&lt;segment&gt; dq; dq.push_back(segment(0, 1, n));
 for (int i = 1; i <= n; ++i) {
  dp[i] = f(dq.front().i, i);
  while(dq.size()&&dq.front().r<i+1) dq.pop_front();
  dq.front().l = i + 1;
  segment seg = segment(i, i + 1, n);
  while (dq.size() &&
   f(i, dq.back().l)<f(dq.back().i, dq.back().l))
    dq.pop_back();
  if (dq.size()) {
   int d = 1 << 20, c = dq.back().l;
   while (d >>= 1) if (c + d <= dq.back().r)
    if(f(i, c+d) > f(dq.back().i, c+d)) c += d;
   dq.back().r = c; seg.l = c + 1;
  }
  if (seg.l <= n) dq.push_back(seg);
 }
}
```

8.6 ConvexHull Optimization [25eb56]

```
struct L {
 mutable lld a, b, p;
 bool operator<(const L &r) const {
  return a < r.a; /* here */ }
 bool operator<(lld x) const { return p < x; }
};
lld Div(lld a, lld b) {
 return a / b - ((a ^ b) < 0 && a % b); };
struct DynamicHull : multiset<L, less<>> {
 static const lld kInf = 1e18;
 bool Isect(iterator x, iterator y) {
  if (y == end()) { x->p = kInf; return false; }
  if (x->a == y->a)
   x->p = x->b > y->b ? kInf : -kInf; /* here */
  else x->p = Div(y->b - x->b, x->a - y->a);
  return x->p >= y->p;
 }
 void Insert(lld a, lld b) {
  auto z = insert({a, b, 0}), y = z++, x = y;
  while (Isect(y, z)) z = erase(z);
  if (x!=begin()&&Isect(--x,y)) Isect(x, y=erase(y));
  while ((y = x) != begin() && (--x)->p >= y->p)
   Isect(x, erase(y));
 }
 lld Query(lld x) {  // default chmax
  auto l = *lower_bound(x); // to chmin:
  return l.a * x + l.b;    // modify the 2 "<>"
 }
};
```

8.7 Josephus Problem [f4494f]

```
// n people kill m for each turn
int f(int n, int m) {
 int s = 0;
 for (int i = 2; i <= n; i++)
  s = (s + m) % i;
 return s;
}
// died at kth
int kth(int n, int m, int k){
 if (m == 1) return n-1;
 for (k = k*m+m-1; k >= n; k = k-n+(k-n)/(m-1));
 return k;
}
```

8.8 Tree Knapsack [87db92]

```
int dp[N][K]; PII obj[N];
vector<int> G[N];
void dfs(int u, int mx){
 for(int s: G[u]) {
  if(mx < obj[s].first) continue;
  for(int i=0;i<=mx-obj[s].FF;i++)
   dp[s][i] = dp[u][i];
  dfs(s, mx - obj[s].first);
  for(int i=obj[s].FF;i<=mx;i++)
   dp[u][i] = max(dp[u][i],
    dp[s][i - obj[s].FF] + obj[s].SS);
 }
}
```

8.9 N Queens Problem [31f83e]

```
void solve(VI &ret, int n) { // no sol when n=2,3
 if (n % 6 == 2) {
  for (int i = 2; i <= n; i += 2) ret.push_back(i);
  ret.push_back(3); ret.push_back(1);
  for (int i = 7; i <= n; i += 2) ret.push_back(i);
  ret.push_back(5);
 } else if (n % 6 == 3) {
  for (int i = 4; i <= n; i += 2) ret.push_back(i);
  ret.push_back(2);
  for (int i = 5; i <= n; i += 2) ret.push_back(i);
  ret.push_back(1); ret.push_back(3);
 } else {
  for (int i = 2; i <= n; i += 2) ret.push_back(i);
  for (int i = 1; i <= n; i += 2) ret.push_back(i);
 }
}
```

8.10 Stable Marriage

Initialize $m \in M$ and $w \in W$ to *free*
while $\exists$ *free* man m who has a woman w to propose to **do**
  $w \leftarrow$ first woman on m's list to whom m has not yet proposed
  if $\exists$ some pair (m', w) **then**
    if w prefers m to m' **then**
      $m' \leftarrow$ *free*
      $(m, w) \leftarrow$ *engaged*
    end if
  else
    $(m, w) \leftarrow$ *engaged*
  end if
end while

8.11 Binary Search On Fraction [765c5a]

```
struct Q {
 ll p, q;
 Q go(Q b, ll d) { return {p + b.p*d, q + b.q*d}; }
};
bool pred(Q);
// returns smallest p/q in [lo, hi] such that
// pred(p/q) is true, and 0 <= p,q <= N
Q frac_bs(ll N) {
 Q lo{0, 1}, hi{1, 0};
 if (pred(lo)) return lo;
 assert(pred(hi));
 bool dir = 1, L = 1, H = 1;
 for (; L || H; dir = !dir) {
  ll len = 0, step = 1;
  for (int t = 0; t < 2 && (t ? step/=2 : step*=2);)
   if (Q mid = hi.go(lo, len + step);
     mid.p > N || mid.q > N || dir ^ pred(mid))
    t++;
   else len += step;
  swap(lo, hi = hi.go(lo, len));
  (dir ? L : H) = !!len;
 }
 return dir ? hi : lo;
}
```