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1 Basic

1.1 vimrc

```
se nu rnu bs=2 ru mouse=a cin et ts=4 sw=4 sts=4 syn on filetype indent on inoremap {<CR> {<CR>}<Esc>0
```

1.2 IncreaseStackSize

```
//stack resize(change esp to rsp if 64-bit system)
asm( "mov %0,%%esp\n" ::"g"(mem+10000000) );
// craziest way
static void run_stack_sz(void(*func)(),size_t stsize){
    char *stack, *send;
    stack=(char *)malloc(stsize);
    send=stack+stsize-16;
    send=(char *)((uintptr_t)send/16*16);
asm volatile(
        "mov %%rsp, (%0)\n"
        "mov %0, %%rsp\n"
        :
        : "r" (send));
    func();
    asm volatile(
        "mov (%0), %%rsp\n"
        :
        : "r" (send));
    free(stack);
}
```

1.3 Pragma optimization

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,tune=native")
```

1.4 Debugger

```
#! /usr/bin/env python3
 import subprocess as sp
 os_name = __import__('platform').system()
cmd,prefix = [],""
if os_name == 'Windows':
  cmd=["cmd", "/C"]
 else:
   cmd = ["bash", "-c"]
prefix = "./"
 def GetTestData(exe):
   myout=sp.check_output(cmd+["%s%s"%(prefix, exe)])
    return myout.decode("utf8")
 def Judge(a,b,testdata):
    f = open("test.in", "w+
                                .
"w+")
   f.write(testdata)
   f.close()
   c=sp.check_output(cmd+["%s%s<test.in"%(prefix, a)])
d=sp.check_output(cmd+["%s%s<test.in"%(prefix, b)])</pre>
    if not c == d:
      print("answer: %s"%c.decode("utf8"),end="")
print("output: %s"%d.decode("utf8"),end="")
print("WA!")
      return False
   return True
 if __name__ == '__main__':
   cnt = 0
    isOK = True
    while isOK:
      cnt += 1
      print(cnt)
      isOK=Judge("sol", "mysol", GetTestData("gen"))
```

1.5 Quick Random

```
template < class T,T x1,T x2,T x3,int y1,int y2,int y3>
  using S = typename std::make_signed<T>::type;
   T s:
  PRNG(T
             _s = 0) : s(_s) \{ \}
   T next() {
     T z = (s += x1);
     z = (z ^ (z >> y1)) * x2;
z = (z ^ (z >> y2)) * x3;
return z ^ (z >> y3);
  T next(T n) { return next() % n; }
S next(S 1, S r){return l+next(r-l+1);}
T operator()() { return next(); }
T operator()(T n) { return next(n); }
S operator()(S 1, S r) { return next(l, r); }
  static T gen(T s) { return PRNG(s)(); }
  template < class U>
  void shuffle(U first,U last){
     size_t n=last-first;
     for(size_t i=0;i<n;i++)</pre>
        swap(first[i],first[next(i+1)]);
  }
using R32=PRNG<uint32_t,0x9E3779B1,0x85EBCA6B,
0xC2B2AE35,16,13,16>;
using R64=PRNG<uint64_t,0x9E3779B97F4A7C15,</pre>
0xBF58476D1CE4E5B9,0x94D049BB133111EB,30,27,31>;
R64 r64:
```

1.6 IO Optimization

```
static inline int gc() {
  static char buf[ 1 << 20 ], *p = buf, *end = buf;</pre>
  if ( p == end ) {
    end = buf + fread( buf, 1, 1 << 20, stdin );
    if ( end == buf ) return EOF;
    p = buf;
  return *p++;
template < typename T >
static inline bool gn( T &_ ) {
  register int c = gc(); register T __ = 1; _ = 0;
  while(('0'>c||c>'9') && c!=EOF && c!='-') c = gc();
  if(c == '-') { __ = -1; c = if(c == EOF) return false;
                     = -1; c = gc(); }
  while('0'<=c&c<='9') _{-} = _{-} * 10 + c - '0', c = gc();
  return true;
template < typename T, typename ...Args >
static inline bool gn( T &x, Args &...args )
{ return gn(x) && gn(args...); }
```

2 Data Structure

2.1 Bigint

```
class BigInt{
  private:
    using lld = int_fast64_t;
    #define PRINTF_ARG PRIdFAST64
  #define LOG_BASE_STR "9"
  static constexpr lld BASE = 10000000000;
  static constexpr int LOG_BASE = 9;
  vector<lld> dig;
  bool neg;
  inline int len() const { return (int) dig.size(); }
  inline int cmp_minus(const BigInt& a) const {
    if(len() == 0 && a.len() == 0) return 0;
    if(neg ^ a.neg)return (int)a.neg*2 - 1;
    if(len()!=a.len())
      return neg?a.len()-len():len()-a.len();
  for(int i=len()-1;i>=0;i--) if(dig[i]!=a.dig[i])
    return neg?a.dig[i]-dig[i]:dig[i]-a.dig[i];
```

```
return 0:
  inline void trim(){
    while(!dig.empty()&&!dig.back())dig.pop_back();
    if(dig.empty()) neg = false;
public:
  BigInt(): dig(vector<lld>()), neg(false){}
  BigInt(lld a): dig(vector<lld>()){
    neg = a<0; dig.push_back(abs(a));</pre>
    trim();
  BigInt(const string& a): dig(vector<lld>()){
    11d cur = 0;
      for(int j=min(LOG_BASE-1,i-neg);j>=0;j--)
        cur = cur*10+a[i-j]-'0';
      dig.push_back(cur);
    } trim();
  inline bool operator<(const BigInt& a)const</pre>
    {return cmp_minus(a)<0;}
  inline bool operator<=(const BigInt& a)const</pre>
    {return cmp_minus(a)<=0;}
  inline bool operator==(const BigInt& a)const
    {return cmp_minus(a)==0;}
  inline bool operator!=(const BigInt& a)const
    {return cmp_minus(a)!=0;}
  inline bool operator>(const BigInt& a)const
    {return cmp_minus(a)>0;}
  inline bool operator>=(const BigInt& a)const
    {return cmp_minus(a)>=0;}
  BigInt operator-() const {
  BigInt ret = *this;
    ret.neg ^= 1;
    return ret;
  BigInt operator+(const BigInt& a) const {
    if(neg) return -(-(*this)+(-a));
if(a.neg) return (*this)-(-a);
    int n = max(a.len(), len());
    BigInt ret; ret.dig.resize(n);
    11d pro = 0;
    for(int i=0;i<n;i++) {</pre>
      ret.dig[i] = pro;
      if(i < a.len()) ret.dig[i] += a.dig[i];</pre>
      if(i < len()) ret.dig[i] += dig[i];</pre>
      if(ret.dig[i] >= BASE) pro = ret.dig[i]/BASE;
      ret.dig[i] -= BASE*pro;
    if(pro != 0) ret.dig.push_back(pro);
    return ret;
  BigInt operator-(const BigInt& a) const {
    if(neg) return -(-(*this) - (-a));
if(a.neg) return (*this) + (-a);
int diff = cmp_minus(a);
    if(diff < 0) return -(a - (*this));</pre>
    if(diff == 0) return 0;
    BigInt ret; ret.dig.resize(len(), 0);
    for(int i=0;i<len();i++) {</pre>
      ret.dig[i] += dig[i];
      if(i < a.len()) ret.dig[i] -= a.dig[i];</pre>
      if(ret.dig[i] < 0){</pre>
        ret.dig[i] += BASE;
        ret.dig[i+1]--;
    ret.trim();
    return ret;
  BigInt operator*(const BigInt& a) const {
    if(!len()||!a.len()) return 0;
    BigInt ret; ret.dig.resize(len()+a.len()+1);
    ret.neg = neg ^ a.neg;
    for(int i=0;i<len();i++)</pre>
      for(int j=0;j<a.len();j++){</pre>
        ret.dig[i+j] += dig[i] * a.dig[j];
        if(ret.dig[i+j] >= BASE) {
          1ld x = ret.dig[i+j] / BASE;
           ret.dig[i+j+1] += x;
ret.dig[i+j] -= x * BASE;
      }
```

```
ret.trim():
       return ret;
     BigInt operator/(const BigInt& a) const {
       assert(a.len());
       if(len() < a.len()) return 0;</pre>
       BigInt ret; ret.dig.resize(len()-a.len()+1);
       ret.neg = a.neg;
       for(int i=len()-a.len();i>=0;i--){
         11d 1 = 0, r = BASE;
         \label{eq:while} \mbox{while} (\mbox{r-l} > 1) \{
           lld mid = (l+r)>>1;
            ret.dig[i] = mid;
           if(ret*a<=(neg?-(*this):(*this))) 1 = mid;</pre>
           else r = mid;
         ret.dig[i] = 1;
       ret.neg ^= neg; ret.trim();
       return ret:
     BigInt operator%(const BigInt& a) const {
       return (*this) - (*this) / a * a;
     friend BigInt abs(BigInt a){
       a.neg = 1; return a;
     friend void swap(BigInt& a, BigInt& b){
       swap(a.dig, b.dig); swap(a.neg, b.neg);
     friend istream& operator>>(istream& ss, BigInt& a){
       string s; ss >> s; a = s;
       return ss;
     friend ostream& operator<<(ostream& ss, BigInt& a){</pre>
       if(a.len() == 0) return ss << '0';
if(a.neg) ss << '-';
       ss << a.dig.back();</pre>
       for(int i=a.len()-2;i>=0;i--)
         ss<<setw(LOG_BASE)<<setfill('0')<<a.dig[i];</pre>
     inline void print() const {
       if(len() == 0){putchar('0');return;}
       if(neg) putchar('-');
printf("%" PRINTF_ARG, dig.back());
       for(int i=len()-2;i>=0;i--)
         printf("%0" LOG_BASE_STR PRINTF_ARG, dig[i]);
     #undef PRINTF_ARG
     #undef LOG_BASE_STR
};
```

2.2 Dark Magic

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
using __gnu_pbds::pairing_heap_tag;
using __gnu_pbds::binary_heap_tag;
using __gnu_pbds::binomial_heap_tag;
using __gnu_pbds::rc_binomial_heap_tag;
        _gnu_pbds::thin_heap_tag;
using
template<typename T>
using pbds_heap=__gnu_pbds::prioity_queue<T,less<T>,\
                                         pairing_heap_tag>;
using __gnu_pbds::rb_tree_tag;
using __gnu_pbds::ov_tree_tag;
      __gnu_pbds::splay_tree_tag;
template<typename T>
using ordered_set = __gnu_pbds::tree<T,\
    __gnu_pbds::null_type,less<T>,rb_tree_tag,\
  _gnu_pbds::tree_order_statistics_node_update>;
template<typename A, typename B>
using hTable1=__gnu_pbds::cc_hash_table<A,B>;
template<typename A, typename B>
using hTable2=__gnu_pbds::gp_hash_table<A,B>;
int main(){
  ordered_set<int> ss;
  ss.insert(1); ss.insert(5);
  assert(*ss.find_by_order(0)==1);
  assert(ss.order_of_key(-1)==0);
  pbds_heap pq1, pq2;
  pq1.push(1); pq2.push(2);
  pq1.join(pq2);
```

```
assert(pq2.size()==0);
auto it = pq1.push(87);
pq1.modify(it, 19);
return 0;
}
```

2.3 SkewHeap

```
template < typename T, typename cmp = less< T > >
class SkewHeap{
private:
  struct SkewNode{
    SkewNode *lc, *rc;
    SkewNode( T a = 0) : x( a ), lc( 0 ), rc( 0 ) {}
  cmp CMP_;
  size_t count;
  SkewNode* Merge( SkewNode* a, SkewNode* b ) {
    if ( !a or !b ) return a ? a : b;
if ( CMP_( a->x, b->x ) ) swap( a, b );
    a -> rc = Merge( a->rc, b );
    swap( a -> lc, a->rc );
    return a;
public:
  SkewHeap(): root( 0 ), count( 0 ) {}
  size_t size() { return count;
  bool empty() { return count == 0; }
  T top() { return root->x; }
  void clear(){ root = 0; count = 0; }
  void push ( const T& x ) {
   SkewNode* a = new SkewNode( x );
    count += 1; root = Merge( root, a );
  void join( SkewHeap& a ) {
    count += a.count; a.count = 0;
    root = Merge( root, a.root );
  void pop() {
    count--; root = Merge( root->lc, root->rc );
  friend void swap( SkewHeap& a, SkewHeap& b ) {
    swap( a.root, b.root ); swap( a.count, b.count );
```

2.4 Disjoint Set

```
class DJS{
private:
  vector< int > fa, sz, sv;
  vector< pair< int*, int > > opt;
inline void assign( int *k, int v ) {
     opt.emplace_back( k, *k );
     *k = v;
public:
  inline void init( int n ) {
     fa.resize( n ); iota( fa.begin(), fa.end(), 0 );
     sz.resize( n ); fill( sz.begin(), sz.end(), 1 );
     opt.clear();
  int query( int x ) {
  return ( fa[ x ] == x ) ? x : query( fa[ x ] );
  inline void merge( int a, int b ) {
  int af = query( a ), bf = query( b );
     if( af == bf ) return;
     if( sz[ af ] < sz[ bf ] ) swap( af, bf );</pre>
     assign( &fa[ bf ], fa[ af ] );
assign( &sz[ af ], sz[ af ] + sz[ bf ] );
   inline void save() {sv.push_back( (int)opt.size() );}
  inline void undo() {
     int ls = sv.back(); sv.pop_back();
     while ( ( int ) opt.size() > ls ) {
  pair< int*, int > cur = opt.back();
        *cur.first = cur.second;
        opt.pop back();
  }
```

2.5 Link-Cut Tree

|};

```
struct Node{
  Node *par, *ch[2];
  int xor_sum,v;
  bool is_rev;
  Node(int _v){
    v=xor_sum=_v;
    par=nullptr;
    ch[0]=ch[1]=nullptr;
    is_rev=false;
  inline void set_rev(){
    is_rev^=1;
    swap(ch[0],ch[1]);
  inline void down(){
    if(is_rev){
      if(ch[0]!=nullptr) ch[0]->set_rev();
      if(ch[1]!=nullptr) ch[1]->set_rev();
      is_rev=false;
    }
  inline void up(){
    xor_sum=v;
    if(ch[0]!=nullptr){
      xor_sum^=ch[0]->xor_sum;
      ch[0]->par=this;
    if(ch[1]!=nullptr){
      xor_sum^=ch[1]->xor_sum;
      ch[1]->par=this;
    }
  inline bool is_root(){
    return par==nullptr ||\
      (par->ch[0]!=this && par->ch[1]!=this);
  inline bool is_rch(){
    return !is_root() && par->ch[1]==this;
} *node[maxn],*stk[maxn];
int top;
void to_child(Node* p,Node* c,bool dir){
  p->ch[dir]=c;
  p->up();
inline void rotate(Node* node){
  Node* par=node->par;
Node* par_par=par->par;
  bool dir=node->is_rch()
  bool par_dir=par->is_rch();
  to_child(par, node -> ch[!dir], dir);
  to_child(node,par,!dir);
  if(par_par!=nullptr && par_par->ch[par_dir]==par)
    to_child(par_par,node,par_dir);
  else node->par=par_par;
inline void splay(Node* node){
  Node* tmp=node;
  stk[top++]=node;
  while(!tmp->is_root()){
    tmp=tmp->par;
    stk[top++]=tmp;
  while(top) stk[--top]->down();
for(Node *fa=node->par;
   !node->is_root();
   rotate(node),fa=node->par)
    if(!fa->is_root())
      rotate(fa->is_rch()==node->is_rch()?fa:node);
inline void access(Node* node){
  Node* last=nullptr;
  while(node!=nullptr){
    splay(node);
    to_child(node,last,true);
    last=node;
    node=node->par;
  }
inline void change_root(Node* node){
```

```
access(node);
  splay(node);
  node->set_rev();
inline void link(Node* x,Node* y){
  change_root(x);
  splay(x);
  x->par=y;
inline void split(Node* x,Node* y){
  change_root(x);
  access(y);
  splay(x)
  to_child(x,nullptr,true);
  y->par=nullptr;
inline void change_val(Node* node,int v){
  access(node);
  splay(node);
  node->v=v:
  node->up();
inline int query(Node* x,Node* y){
  change_root(x);
  access(y);
  splay(y);
  return y->xor_sum;
inline Node* find_root(Node* node){
  access(node);
  splay(node);
  Node* last=nullptr;
  while(node!=nullptr){
    node->down();
    last=node;
    node=node->ch[0];
  return last;
set<pii> dic;
inline void add_edge(int u,int v){
  if(u>v) swap(u,v);
  if(find_root(node[u])==find_root(node[v])) return;
  dic.insert(pii(u,v));
  link(node[u],node[v]);
inline void del_edge(int u,int v){
  if(u>v) swap(u,v);
  if(dic.find(pii(u,v))==dic.end()) return;
  dic.erase(pii(u,v));
  split(node[u],node[v]);
```

2.6 LiChao Segment Tree

```
struct Line{
  int m, k, id;
Line() : id( -1 ) {}
Line( int a, int b, int c )
    : m( a ), k( b ), id( c ) {}
  int at( int x ) { return m * x + k; }
};
class LiChao {
  private:
     int n; vector< Line > nodes;
     inline int lc( int x ) { return 2 * x + 1;
inline int rc( int x ) { return 2 * x + 2;
     void insert( int l, int r, int id, Line ln ) {
  int m = ( l + r ) >> 1;
        if ( nodes[ id ].id == -1 ) {
          nodes[ id ] = ln;
          return;
        bool atLeft = nodes[ id ].at( l ) < ln.at( l );</pre>
        if ( nodes[ id ].at( m ) < ln.at( m ) ) {</pre>
          atLeft ^= 1;
          swap( nodes[ id ], ln );
       if ( r - l == 1 ) return;
       if ( atLeft ) insert( l, m, lc( id ), ln );
       else insert( m, r, rc( id ), ln );
     int query( int 1, int r, int id, int x ) {
```

```
int ret = 0;
if ( nodes[ id ].id != -1 )
        ret = nodes[ id ].at( x );
      int m = ( 1 + r ) >> 1;
      if ( r - l == 1 ) return ret;
      else if (x < m)
        return max( ret, query( 1, m, lc( id ), x ) );
      else
        return max( ret, query( m, r, rc( id ), x ) );
  public:
    void build( int n_ ) {
      n = n_; nodes.clear();
      nodes.resize( n << 2, Line() );</pre>
    void insert( Line ln ) {
      insert( 0, n, 0, ln );
    int query( int x ) {
      return query( 0, n, 0, x );
} lichao;
```

2.7 Treap

```
namespace Treap{
 #define sz( x ) ( ( x ) ? ( ( x )->size ) : 0 )
  struct node{
    int size;
    uint32_t pri;
    node *\overline{1}c, *rc;
    node() : size(0), pri(rand()), lc( 0 ), rc( 0 ) {}
    void pull() {
      size = 1;
      if ( lc ) size += lc->size;
      if ( rc ) size += rc->size;
    }
  node* merge( node* L, node* R ) {
    if ( not L or not R ) return L ? L : R;
    if ( L->pri > R->pri ) {
      L->rc = merge( L->rc, R );
      L->pull();
      return L;
    } else {
  R->lc = merge( L, R->lc );
      R->pull();
      return R;
  void split_by_size( node*rt,int k,node*&L,node*&R ) {
   if ( not rt ) L = R = nullptr;
    else if( sz( rt->lc ) + 1 <= k ) {
      split_by_size( rt->rc,k-sz(rt->lc)-1,L->rc,R );
      L->pull();
    } else {
      R = rt:
      split_by_size( rt->lc, k, L, R->lc );
      R->pull();
    }
  #undef sz
```

2.8 SparseTable

```
template < typename T, typename Cmp_ = less< T > >
class SparseTable {
private:
    vector< vector< T > > tbl;
    vector< int > lg;
    T cv( T a, T b ) {
        return Cmp_()( a, b ) ? a : b;
    }
public:
    void init( T arr[], int n ) {
        // 0-base
        lg.resize( n + 1 );
        lg[ 0 ] = -1;
        for( int i=1 ; i<=n ; ++i ) lg[i] = lg[i>>1] + 1;
        tbl.resize( lg[n] + 1 );
```

```
tbl[ 0 ].resize( n );
copy( arr, arr + n, tbl[ 0 ].begin() );
for ( int i = 1 ; i <= lg[ n ] ; ++ i ) {
   int len = 1 << (i - 1 ), sz = 1 << i;
   tbl[ i ].resize( n - sz + 1 );
   for ( int j = 0 ; j <= n - sz ; ++ j )
        tbl[i][j] = cv(tbl[i-1][j], tbl[i-1][j+len]);
}

T query( int l, int r ) {
   // 0-base [l, r)
   int wh = lg[ r - l ], len = 1 << wh;
   return cv( tbl[ wh ][ l ], tbl[ wh ][ r - len ] );
}
};</pre>
```

2.9 Linear Basis

```
struct LinearBasis {
private:
  int n, sz;
   vector< llu > B;
   inline llu two( int x ){ return ( ( llu ) 1 ) << x; }</pre>
public:
   void init( int n_ ) {
     n = n_; B.clear();
     B.resize( n ); sz = 0;
   void insert( llu x ) {
     // add x into B
     for ( int i = n-1; i >= 0 ; --i ) if( two(i) & x ){
  if ( B[ i ] ) x ^= B[ i ];
        else
          Ise {
B[ i ] = x; sz++;
for ( int j = i - 1; j >= 0; -- j )
   if( B[ j ] && ( two( j ) & B[ i ] ) )
      B[ i ] ^= B[ j ];
for (int j = i + 1; j < n; ++ j )
   if ( two( i ) & B[ j ] )
      B[ j ] ^= B[ i ];
break:</pre>
           break;
     }
   inline int size() { return sz; }
   bool check( llu x )
     // is x in span(B) ?
for ( int i = n-1 ; i >= 0 ; --i ) if( two(i) & x )
        if( B[ i ] ) x ^= B[ i ];
        else return false;
     return true;
   llu kth_small(llu k) {
     /** 1-base would always > 0 **/
     /** should check it **/
      /* if we choose at least one element
         but size(B)(vectors in B)==N(original elements)
         then we can't get 0 */
     11u ret = 0;
      for ( int i = 0 ; i < n ; ++ i ) if( B[ i ] ) {
   if( k & 1 ) ret ^= B[ i ];</pre>
        k \gg 1;
     return ret;
} base;
```

3 Graph

3.1 BCC Edge

```
class BCC{
private:
  vector< int > low, dfn;
  int cnt;
  vector< bool > bridge;
  vector< vector< PII > > G;
  void dfs( int w, int f ) {
    dfn[ w ] = cnt++;
    low[ w ] = dfn[ w ];
```

```
for ( auto [ u, t ] : G[ w ] ) {
  if ( u == f ) continue;
  if ( dfn[ u ] != 0 ) {
            low[ w ] = min( low[ w ], dfn[ u ] );
            dfs( u, w );
low[ w ] = min( low[ w ], low[ u ] );
dfn[ w ] ) bridge[ t
            if ( low[ u ] > dfn[ w ] ) bridge[ t ] = true;
     }
public:
   void init( int n, int m ) {
  G.resize( n );
      fill( G.begin(), G.end(), vector< PII >() );
      bridge.clear(); bridge.resize( m );
low.clear(); low.resize( n );
      dfn.clear(); dfn.resize( n );
      cnt = 0;
   void add_edge( int u, int v ) {
   // should check for multiple edge
     G[ u ].emplace_back( v, cnt );
G[ v ].emplace_back( u, cnt ++ );
   void solve(){ cnt = 1; dfs( 0, 0 ); }
// the id will be same as insert order, 0-base
   bool is_bridge( int x ) { return bridge[ x ]; }
} bcc;
```

3.2 BCC Vertex

```
class BCC{
  private:
    int n, ecnt;
    vector< vector< pair< int, int > > > G;
    vector< int > low, dfn, id;
    vector< bool > vis, ap;
    void dfs( int u, int f, int d ) {
       int child = 0;
      dfn[ u ] = low[ u ] = d; vis[ u ] = true;
for ( auto e : G[ u ] ) if ( e.first != f ) {
   if ( vis[ e.first ] ) {
            low[ u ] = min( low[ u ], dfn[ e.first ] );
            dfs( e.first, u, d + 1 ); child ++;
low[ u ] = min( low[ u ], low[ e.first ] );
            if ( low[ e.first ] >= d ) ap[ u ] = true;
       if ( u == f and child <= 1 ) ap[ u ] = false;</pre>
    void mark( int u, int idd ) {
    // really?????????
       if ( ap[ u ] ) return;
            ( auto e : G[ u ] )
         if( id[ e.second ] != -1 ) {
            id[ e.second ] = idd;
            mark( e.first, idd );
  public:
    void init( int n_ ) {
       ecnt = 0, n = n_{j};
       G.clear(); G.resize( n );
       low.resize( n ); dfn.resize( n );
       ap.clear(); ap.resize( n );
       vis.clear(); vis.resize( n );
     void add_edge( int u, int v ) {
       G[ u ].emplace_back( v, ecnt );
       G[ v ].emplace_back( u, ecnt ++ );
     void solve() {
       for ( int i = 0 ; i < n ; ++ i )
  if ( not vis[ i ] ) dfs( i, i, 0 );</pre>
       id.resize( ecnt );
       fill( id.begin(), id.end(), -1 );
       ecnt = 0;
       for ( int i = 0 ; i < n ; ++ i )
  if ( ap[ i ] ) for ( auto e : G[ i ] )</pre>
            if( id[ e.second ] != -1 ) {
              id[ e.second ] = ecnt;
              mark( e.first, ecnt ++ );
```

```
}
  int get_id( int x ) { return id[ x ]; }
  int count() { return ecnt; }
  bool is_ap( int u ) { return ap[ u ]; }
} bcc;
```

3.3 Bipartite Matching

```
class BipartiteMatching{
   private:
     vector<int> X[N], Y[N];
     int fX[N], fY[N], n;
     bitset<N> walked;
     bool dfs(int x)
       for(auto i:X[x]){
         if(walked[i])continue;
         walked[i]=1;
         if(fY[i]==-1||dfs(fY[i])){
           fY[i]=x;fX[x]=i;
           return 1:
         }
       }
       return 0;
   public:
     void init(int _n){
       n=_n;
       for(int i=0;i<n;i++){</pre>
         X[i].clear();
         Y[i].clear();
         fX[i]=fY[i]=-1;
       walked.reset();
     void add_edge(int x, int y){
       X[x].push_back(y);
       Y[y].push_back(y);
     int solve(){
       int cnt = 0;
       for(int i=0;i<n;i++){</pre>
         walked.reset();
         if(dfs(i)) cnt++;
       // return how many pair matched
       return cnt;
|};
```

3.4 Minimum Cost Maximum Flow

```
class MiniCostMaxiFlow{
  using CapT = int;
  using WeiT = int64_t;
  using PCW = pair<CapT,WeiT>;
  static constexpr CapT INF_CAP = 1 << 30;</pre>
  static constexpr WeiT INF_WEI = 1LL<<60;</pre>
private:
  struct Edge{
    int to, back;
    WeiT wei;
    CapT cap;
    Edge() {}
    Edge(int a,int b,WeiT c,CapT d):
      to(a),back(b),wei(c),cap(d)
    {}
  int ori, edd;
  vector<vector<Edge>> G;
  vector<int> fa, wh;
  vector<bool> inq;
  vector<WeiT> dis;
  PCW SPFA(){
    fill(inq.begin(),inq.end(),false);
fill(dis.begin(),dis.end(),INF_WEI);
    queue<int> qq; qq.push(ori);
    dis[ori]=0;
    while(!qq.empty()){
      int u=qq.front();qq.pop();
      inq[u] = 0;
```

```
for(int i=0;i<SZ(G[u]);++i){</pre>
        Edge e=G[u][i];
        int v=e.to;
        WeiT d=e.wei;
        if(e.cap<=0||dis[v]<=dis[u]+d)</pre>
          continue
        dis[v]=dis[u]+d;
        fa[v]=u,wh[v]=i;
        if(inq[v]) continue;
        qq.push(v);
        inq[v]=1;
      }
    if(dis[edd]==INF_WEI)
      return {-1,-1};
    CapT mw=INF_CAP;
    for(int i=edd;i!=ori;i=fa[i])
      mw=min(mw,G[fa[i]][wh[i]].cap);
    for (int i=edd;i!=ori;i=fa[i]){
      auto &eg=G[fa[i]][wh[i]];
      eg.cap-=mw;
      G[eg.to][eg.back].cap+=mw;
    return {mw,dis[edd]};
public:
  void init(int a,int b,int n){
    ori=a,edd=b;
    G.clear();G.resize(n);
    fa.resize(n); wh.resize(n);
    inq.resize(n); dis.resize(n);
  void add_edge(int st,int ed,WeiT w,CapT c){
    G[st].emplace_back(ed,SZ(G[ed]),w,c);
    G[ed].emplace_back(st,SZ(G[st])-1,-w,0);
  PCW solve(){
    /* might modify to
    cc += ret.first * ret.second
    ww += ret.first * ret.second
    CapT cc=0; WeiT ww=0;
    while(true){
      PCW ret=SPFA();
      if(ret.first==-1) break;
      cc+=ret.first;
      ww+=ret.second;
    return {cc,ww};
  }
} mcmf;
```

3.5 General Graph Matching

```
const int N = 514, E = (2e5) * 2;
struct Graph{
  int to[E],bro[E],head[N],e;
  int lnk[N],vis[N],stp,n;
  void init( int _n ){
    stp = 0; e = 1; n = _n;
for( int i = 1 ; i <= n ; i ++ )
      lnk[i] = vis[i] = 0;
  void add_edge(int u,int v){
    to[e]=v,bro[e]=head[u],head[u]=e++;
    to[e]=u,bro[e]=head[v],head[v]=e++;
  bool dfs(int x){
    vis[x]=stp;
    for(int i=head[x];i;i=bro[i]){
      int v=to[i];
      if(!lnk[v]){
        lnk[x]=v, lnk[v]=x;
        return true;
      }else if(vis[lnk[v]]<stp){</pre>
        int w=lnk[v];
        lnk[x]=v, lnk[v]=x, lnk[w]=0;
        if(dfs(w)){
          return true;
         lnk[w]=v, lnk[v]=w, lnk[x]=0;
      }
```

```
return false:
  int solve(){
    int ans = 0;
     for(int i=1;i<=n;i++)</pre>
       if(!lnk[i]){
         stp++; ans += dfs(i);
    return ans;
} graph;
3.6 Dinic
class Dinic{
private:
  using CapT = int64_t;
  struct Edge{
    int to, rev;
    CapT cap;
  int n, st, ed;
  vector<vector<Edge>> G;
  vector<int> lv;
  bool BFS(){
    fill(lv.begin(), lv.end(), -1);
    queue<int> bfs;
    bfs.push(st);
    lv[st] = 0;
    while(!bfs.empty()){
       int u = bfs.front(); bfs.pop();
       for(auto e: G[u]){
         if(e.cap <= 0 or lv[e.to]!=-1) continue;</pre>
         lv[e.to] = lv[u] + 1;
         bfs.push(e.to);
       }
    return (lv[ed]!=-1);
  CapT DFS(int u, CapT f){
    if(u == ed) return f;
    CapT ret = 0;
     for(auto& e: G[u]){
       if(e.cap <= 0 or lv[e.to]!=lv[u]+1) continue;</pre>
       CapT nf = DFS(e.to, min(f, e.cap));
ret += nf; e.cap -= nf; f -= nf;
       G[e.to][e.rev].cap += nf;
       if(f == 0) return ret;
    if(ret == 0) lv[u] = -1;
    return ret:
public:
  void init(int n_, int st_, int ed_){
    n = n_, st = st_, ed = ed_;
G.resize(n); lv.resize(n);
    fill(G.begin(), G.end(), vector<Edge>());
  void add_edge(int u, int v, CapT c){
    G[u].push_back({v, (int)G[v].size(), c});
G[v].push_back({u, ((int)G[u].size())-1, 0});
  CapT max_flow(){
```

3.7 Kuhn Munkres

CapT ret = 0;
while(BFS()){

ret += f:

return ret;

} flow;

if(f == 0) break;

```
struct KM{
    static constexpr lld INF = 1LL<<60;
    lld w[N][N], lx[N], ly[N], slack[N];
    int match[N], n, vx[N], vy[N], step_;
    void init(int n_){
        n=n_, step_=0;
        memset(w,0,sizeof(w));</pre>
```

CapT f = DFS(st, numeric_limits<CapT>::max());

```
memset(lx,0,sizeof(lx));
     memset(ly,0,sizeof(ly));
     memset(slack,0,sizeof(slack));
     memset(match,0,sizeof(match));
     memset(vx,0,sizeof(vx));
     memset(vy,0,sizeof(vy));
  void add_edge(int u,int v,lld c){w[u][v]=c;}
  bool dfs(int x) {
     vx[x] = step_;
     for (int i = 0; i < n; ++i) {</pre>
       if (vy[i]==step_) continue;
       if (lx[x] + ly[i] > w[x][i])
         slack[i] = min(slack[i], lx[x] + ly[i] - w[x][i]
              ]);
         continue;
       vy[i] = step_;
       if (match[i] == -1 || dfs(match[i])) {
         match[i] = x;
         return true;
     return false;
  11d solve() {
     fill_n(match, n, -1);
     fill_n(lx, n, -INF);
    fill_n(ly, n, 0);

for (int i = 0; i < n; ++i)

for (int j = 0; j < n; ++j)
         lx[i] = max(lx[i], w[i][j]);
     for (int i = 0; i < n; ++i) {</pre>
       fill_n(slack, n, INF);
       while (true) {
         step_++
         if (dfs(i)) break;
         11d dlt = INF;
         for (int j = 0; j < n; ++j) if (vy[j] != step_)</pre>
         dlt = min(dlt, slack[j]);
for (int j = 0; j < n; ++j) {
   if (vx[j]==step_) lx[j] -= dlt;</pre>
            if (vy[j]==step_) ly[j] += dlt;
            else slack[j] -= dlt;
         }
       }
     ild res = 0;
     for (int i = 0; i < n; ++i) res += w[match[i]][i];</pre>
     return res;
} km;
```

3.8 2-SAT

```
class TwoSat{
 private:
   int n;
    vector<vector<int>> rG,G,sccs;
    vector<int> ord,idx;
    vector<bool> vis,result;
    void dfs(int u){
      vis[u]=true;
      for(int v:G[u])
        if(!vis[v])
          dfs(v);
      ord.push_back(u);
    void rdfs(int u){
      vis[u]=false;
      idx[u]=sccs.size()-1;
      sccs.back().push_back(u);
      for(int v:rG[u])
        if(vis[v])
          rdfs(v);
  public:
    void init(int n_){
     n=n_;
      G.clear();
      G.resize(n);
      rG.clear();
      rG.resize(n);
      sccs.clear();
```

```
ord.clear():
      idx.resize(n);
      result.resize(n);
    void add_edge(int u,int v){
      G[u].push_back(v);
      rG[v].push_back(u);
    void orr(int x,int y){
      if ((x^y)==1)return;
      add_edge(x^1,y);
      add_edge(y^1,x);
    bool solve(){
      vis.clear();
      vis.resize(n);
      for(int i=0;i<n;++i)</pre>
         if(not vis[i])
          dfs(i);
      reverse(ord.begin(),ord.end());
      for (int u:ord){
        if(!vis[u])
           continue
         sccs.push_back(vector<int>());
        rdfs(u);
      for(int i=0;i<n;i+=2)</pre>
        if(idx[i]==idx[i+1])
           return false;
      vector<bool> c(sccs.size());
      for(size_t i=0;i<sccs.size();++i){</pre>
        for(size_t j=0;j<sccs[i].size();++j){</pre>
           result[sccs[i][j]]=c[i];
           c[idx[sccs[i][j]^1]]=!c[i];
      }
      return true;
    bool get(int x){return result[x];}
    inline int get_id(int x){return idx[x];}
    inline int count(){return sccs.size();}
} sat2:
```

3.9 Lowbit Decomposition

```
class LowbitDecomp{
private:
  int time_, chain_, LOG_N;
  vector< vector< int > > G, fa;
  vector< int > tl, tr, chain, chain_st;
  // chain_ : number of chain
// tl, tr[ u ] : subtree interval in the seq. of u
  // chain_st[ u ] : head of the chain contains u
// chian[ u ] : chain id of the chain u is on
  inline int lowbit( int x ) {
    return x & ( -x );
  void predfs( int u, int f ) {
    chain[ u ] = 0;
     for ( int v : G[ u ] ) {
       if ( v == f ) continue;
       predfs( v, u );
       if( lowbit( chain[ u ] ) < lowbit( chain[ v ] ) )</pre>
         chain[ u ] = chain[ v ];
    if ( not chain[ u ] )
       chain[ u ] = chain_ ++;
  void dfschain( int u, int f ) {
    fa[ u ][ 0 ] = f;
for ( int i = 1 ; i < LOG_N ; ++ i )
       fa[ u ][ i ] = fa[ fa[ u ][ i - 1 ] ][ i - 1 ];
    tl[ u ] = time_++;
    if ( not chain_st[ chain[ u ] ] )
       chain_st[ chain[ u ] ] = u;
    for ( int v : G[ u ] )
  if ( v != f and chain[ v ] == chain[ u ] )
    dfschain( v, u );
for ( int v : G[ u ] )
  if ( v != f and chain[ v ] != chain[ u ] )
         dfschain( v, u );
    tr[ u ] = time_;
  inline bool anc( int u, int v ) {
```

```
return tl[ u ] <= tl[ v ] \</pre>
       and tr[v] <= tr[u];
public:
  inline int lca( int u, int v ) {
     if ( anc( u, v ) ) return u;
for ( int i = LOG_N - 1; i >= 0; -- i )
       if ( not anc( fa[ u ][ i ], v ) )
  u = fa[ u ][ i ];
     return fa[ u ][ 0 ];
  void init( int n ) {
     n ++; for ( LOG_N = 0 ; ( 1 << LOG_N ) < n ; ++ LOG_N );
     fa.clear();
     fa.resize( n, vector< int >( LOG_N ) );
     G.clear(); G.resize( n );
    tl.clear(); tl.resize( n );
tr.clear(); tr.resize( n );
     chain.clear(); chain.resize( n );
     chain_st.clear(); chain_st.resize( n );
  void add_edge( int u , int v ) {
     // 1-base
     G[ u ].push_back( v );
    G[ v ].push_back( u );
  void decompose(){
     chain_ = 1;
     predfs(1, 1);
     time_ = 0;
     dfschain( 1, 1 );
  PII get_inter( int u ) { return {tl[ u ], tr[ u ]}; }
  vector< PII > get_path( int u , int v ){
  vector< PII > res;
     int g = lca( u, v );
     while ( chain[ u ] != chain[ g ] ) {
       int s = chain_st[ chain[ u ] ];
res.emplace_back( tl[ s ], tl[ u ] + 1 );
       u = fa[ s ][ 0 ];
    res.emplace_back( tl[ g ], tl[ u ] + 1 );
while ( chain[ v ] != chain[ g ] ) {
  int s = chain_st[ chain[ v ] ];
       res.emplace_back( tl[ s ], tl[ v ] + 1 );
       v = fa[s][0];
     res.emplace_back( tl[ g ] + 1, tl[ v ] + 1 );
     return res;
     /* res : list of intervals from u to v
* ( note only nodes work, not edge )
      * usage :
      * vector< PII >& path = tree.get_path( u , v )
      * for( auto [ l, r ] : path ) {
          0-base [ l, r )
} tree;
```

3.10 MaxClique

```
#define N 111
struct MaxClique{ // 0-base
  typedef bitset< N > Int;
  Int linkto[ N ] , v[ N ];
  int n;
  void init( int _n ){
    n = _n;
for( int i = 0 ; i < n ; i ++ ){</pre>
      linkto[ i ].reset();
      v[ i ].reset();
  void add_edge( int a , int b ){
  v[ a ][ b ] = v[ b ][ a ] = 1;
  int popcount(const Int& val)
  { return val.count(); }
  int lowbit(const Int& val)
  { return val._Find_first(); }
  int ans , stk[ N ];
  int id[ N ] , di[ N ] , deg[ N ];
```

```
Int cans:
  void maxclique(int elem_num, Int candi){
     if(elem_num > ans){
       ans = elem_num;
       cans.reset();
       for( int i = 0 ; i < elem_num ; i ++ )
  cans[ id[ stk[ i ] ] ] = 1;</pre>
     int potential = elem_num + popcount(candi);
     if(potential <= ans) return;</pre>
     int pivot = lowbit(candi);
     Int smaller_candi = candi & (~linkto[pivot]);
     while(smaller_candi.count() && potential>ans){
  int next = lowbit(smaller_candi);
       candi[ next ] = !candi[ next ];
       smaller_candi[next] = !smaller_candi[next];
       potential --
       if(next!=pivot
          &&!(smaller_candi&linkto[next]).count())
            continue:
       stk[elem num] = next;
       maxclique(elem_num+1, candi&linkto[next]);
  int solve(){
     for( int i = 0 ; i < n ; i ++ ){</pre>
       id[ i ] = i;
       deg[ i ] = v[ i ].count();
     sort( id , id + n , [&](int id1, int id2){
    return deg[id1] > deg[id2]; } );
     for( int i = 0 ; i < n ; i ++ )</pre>
       di[ id[ i ] ] = i;
     for( int i = 0 ; i < n ; i ++ )</pre>
       for( int j = 0; j < n; j ++ )
  if( v[ i ][ j ] )
    linkto[ di[ i ] ][ di[ j ] ] = 1;</pre>
     Int cand; cand.reset();
     for( int i = 0 ; i < n ; i ++ )</pre>
       cand[ i ] = 1;
     ans = 1;
     cans.reset(); cans[ 0 ] = 1;
     maxclique(0, cand);
     return ans;
} solver;
3.11 Min-Cut
```

```
const int maxn = 500 + 5;
int w[maxn][maxn], g[maxn];
bool v[maxn], del[maxn];
void add_edge(int x, int y, int c) {
    w[x][y] += c;
    w[y][x] += c;
pair<int, int> phase(int n) {
    memset(v, false, sizeof(v));
    memset(g, 0, sizeof(g));
    int s = -1, t = -1;
    while (true) {
        int c = -1;
        for (int i = 0; i < n; ++i) {</pre>
            if (del[i] || v[i]) continue;
            if (c == -1 || g[i] > g[c]) c = i;
        if (c == -1) break;
        v[c] = true;
        for (int i = 0; i < n; ++i) {</pre>
            if (del[i] || v[i]) continue;
            g[i] += w[c][i];
    return make_pair(s, t);
}
int mincut(int n) {
    int cut = 1e9;
    memset(del, false, sizeof(del));
    for (int i = 0; i < n - 1; ++i)</pre>
        int s, t; tie(s, t) = phase(n);
```

```
del[t] = true;
    cut = min(cut, g[t]);
    for (int j = 0; j < n; ++j) {
        w[s][j] += w[t][j];
        w[j][s] += w[j][t];
    }
}
return cut;
}</pre>
```

3.12 Vitural Tree

```
inline bool cmp(const int &i, const int &j) {
  return dfn[i] < dfn[j];</pre>
void build(int vectrices[], int k) {
  static int stk[MAX_N];
  sort(vectrices, vectrices + k, cmp);
  stk[sz++] = 0;
  for (int i = 0; i < k; ++i) {
  int u = vectrices[i], lca = LCA(u, stk[sz - 1]);</pre>
    if (lca == stk[sz - 1]) stk[sz++] = u;
    else {
       while (sz \ge 2 \&\& dep[stk[sz - 2]] \ge dep[lca]) {
         addEdge(stk[sz - 2], stk[sz - 1]);
      if (stk[sz - 1] != lca) {
        addEdge(lca, stk[--sz]);
        stk[sz++] = lca, vectrices[cnt++] = lca;
      stk[sz++] = u;
    }
  for (int i = 0; i < sz - 1; ++i)
    addEdge(stk[i], stk[i + 1]);
}
```

4 Math

4.1 Prime Table

```
1002939109, 1020288887, 1028798297, 1038684299, \\ 1041211027, 1051762951, 1058585963, 1063020809, \\ 1147930723, 1172520109, 1183835981, 1187659051, \\ 1241251303, 1247184097, 1255940849, 1272759031, \\ 1287027493, 1288511629, 1294632499, 1312650799, \\ 1868732623, 1884198443, 1884616807, 1885059541, \\ 1909942399, 1914471137, 1923951707, 1925453197, \\ 1979612177, 1980446837, 1989761941, 2007826547, \\ 2008033571, 2011186739, 2039465081, 2039728567, \\ 2093735719, 2116097521, 2123852629, 2140170259, \\ 3148478261, 3153064147, 3176351071, 3187523093, \\ 3196772239, 3201312913, 3203063977, 3204840059, \\ 3210224309, 3213032591, 3217689851, 3218469083, \\ 3219857533, 3231880427, 3235951699, 3273767923, \\ 3276188869, 3277183181, 3282463507, 3285553889, \\ 3319309027, 3327005333, 3327574903, 3341387953, \\ 3373293941, 3380077549, 3380892997, 3381118801
```

4.2 $\left| \frac{n}{i} \right|$ Enumeration

```
T_0 = 1, T_{i+1} = \lfloor \frac{n}{\lfloor \frac{n}{T_i + 1} \rfloor} \rfloor
```

4.3 ax+by=gcd

```
// ax+ny = 1, ax+ny == ax == 1 (mod n)
void exgcd(lld x,lld y,lld &g,lld &a,lld &b) {
  if (y == 0) g=x,a=1,b=0;
  else
    exgcd(y,x%y,g,b,a),b-=(x/y)*a;
}
```

4.4 Pollard Rho

```
// does not work when n is prime
// return any non-trivial factor
1lu pollard_rho(llu n){
  static auto f=[](llu x,llu k,llu m){
    return add(k,mul(x,x,m),m);
  if (!(n&1)) return 2;
  mt19937 rnd(120821011);
  while(true){
    llu y=2,yy=y,x=rnd()%n,t=1;
    for(llu sz=2;t==1;sz<<=1) {</pre>
      for(llu i=0;i<sz;++i){</pre>
        if(t!=1)break;
        yy=f(yy,x,n);
        t=gcd(yy>y?yy-y:y-yy,n);
      }
      y=yy;
    if(t!=1&&t!=n) return t;
}
4.5 Pi Count (Linear Sieve)
static constexpr int N = 1000000 + 5;
11d pi[N];
```

```
vector<int> primes;
bool sieved[N];
lld cube_root(lld x){
  lld s=cbrt(x-static cast<long double>(0.1));
  while(s*s*s <= x) ++s;
  return s-1;
11d square_root(11d x){
  lld s=sqrt(x-static_cast<long double>(0.1));
  while(s*s <= x) ++s;</pre>
  return s-1;
void init(){
  primes.reserve(N);
  primes.push_back(1);
  for(int i=2;i<N;i++) {</pre>
     if(!sieved[i]) primes.push_back(i);
     pi[i] = !sieved[i] + pi[i-1];
     for(int p: primes) if(p > 1) {
   if(p * i >= N) break;
   sieved[p * i] = true;
   if(p * i >= N)
       if(p % i == 0) break;
  }
ild phi(lld m, lld n) {
   static constexpr int MM = 80000, NN = 500;
  static lld val[MM][NN];
  if(m<MM&&n<NN&&val[m][n])return val[m][n]-1;</pre>
  if(n == 0) return m;
  if(primes[n] >= m) return 1;
lld ret = phi(m,n-1)-phi(m/primes[n],n-1);
  if(m<MM&&n<NN) val[m][n] = ret+1;</pre>
  return ret;
11d pi_count(11d);
11d P2(11d m, 11d n) {
  11d sm = square_root(m), ret = 0;
  for(lld i = n+1;primes[i]<=sm;i++)</pre>
     ret+=pi_count(m/primes[i])-pi_count(primes[i])+1;
  return ret;
1ld pi_count(lld m) {
  if(m < N) return pi[m];
lld n = pi_count(cube_root(m));</pre>
  return phi(m, n) + n - 1 - P2(m, n);
```

4.6 Range Sieve

```
const int MAX_SQRT_B = 50000;
const int MAX_L = 200000 + 5;
bool is_prime_small[MAX_SQRT_B];
bool is_prime[MAX_L];
```

4.7 Miller Rabin

```
bool isprime(llu x){
  static llu magic[]={2,325,9375,28178,\
                     450775,9780504,1795265022};
  static auto witn=[](llu a,llu u,llu n,int t){
    a = mpow(a,u,n);
    if (!a)return 0;
    while(t--){
      1lu a2=mul(a,a,n);
      if(a2==1 && a!=1 && a!=n-1)
        return 1:
      a = a2:
    }
    return a!=1;
  if(x<2)return 0;
  if(!(x&1))return x==2;
  llu x1=x-1; int t=0;
  while(!(x1&1))x1>>=1,t++;
  for(llu m:magic)
    if(witn(m,x1,x,t))
      return 0;
  return 1;
}
```

4.8 Inverse Element

```
// x's inverse mod k
long long GetInv(long long x, long long k){
    // k is prime: euler_(k)=k-1
    return qPow(x, euler_phi(k)-1);
}
// if you need [1, x] (most use: [1, k-1]
void solve(int x, long long k){
    inv[1] = 1;
    for(int i=2;i<x;i++)
        inv[i] = ((long long)(k - k/i) * inv[k % i]) % k;
}</pre>
```

4.9 Euler Phi Function

```
extended euler:
   a^b mod p
   if gcd(a, p)==1: a^{(b\%phi(p))}
   elif b < phi(p): a^b mod p
   else a^(b\%phi(p) + phi(p))
lld euler_phi(int x){
  11d r=1;
  for(int i=2;i*i<=x;++i){</pre>
    if(x%i==0){
      x/=i;
      r*=(i-1);
      while(x%i==0){
        x/=i;
        r*=i;
      }
    }
  if(x>1) r*=x-1;
  return r;
vector<int> primes;
```

```
bool notprime[N];
lld phi[N];
void euler_sieve(int n){
   for(int i=2;i<n;i++){
        if(!notprime[i]){
        primes.push_back(i);
        phi[i] = i-1;
    }
   for(auto j: primes){
        if(i*j >= n) break;
        notprime[i*j] = true;
        phi[i*j] = phi[i] * phi[j];
        if(i % j == 0){
            phi[i*j] = phi[i] * j;
            break;
        }
    }
   }
}
```

4.10 Gauss Elimination

```
typedef long double llf;
const int N = 300;
const llf EPS = 1e-8;
// make m[i][i] = x, m[i][j] = 0
  v is for solving equation:
// for(int i=0;i<n;i++) ans[pos[i]] = val[i]/mtx[i][pos
    [i]];
bool Gauss(llf m[N][N], llf v[N], int n, int pos[N]){
  for(int i=0;i<n;i++)</pre>
    int x=-1, y=-1; llf e = 0;
    for(int j=i;j<n;j++) for(int k=i;k<n;k++){</pre>
      if(fabs(m[j][pos[k]])>e){
        e = fabs(m[j][pos[k]]);
        x = j, y = k;
      }
    if(x==-1 or y==-1) return false;
    swap(m[x], m[i]);
swap(v[x], v[i]);
    swap(pos[y], pos[i]);
    for(int j=i+1;j<n;j++){
    llf xi = m[j][pos[i]]/m[i][pos[i]];</pre>
      for(int k=0;k<n;k++) m[j][pos[k]] -= xi*m[i][pos[</pre>
          k]];
      v[j] -= xi*v[i];
  for(int i=n-1;i>=0;i--){
    for(int j=i-1;j>=0;j--){
      11f xi = m[j][pos[i]]/m[i][pos[i]];
      for(int k=0;k<n;k++) m[j][pos[k]] -= xi*m[i][pos[</pre>
          k]];
      v[j] -= xi*v[i];
    }
  return true;
```

4.11 Fast Fourier Transform

```
polynomial multiply:
    pFT(a, len); DFT(b, len);
    for(int i=0;i<len;i++) c[i] = a[i]*b[i];
    iDFT(c, len);
    (len must be 2^k and >= 2*(max(a, b)))
    Hand written Cplx would be 2x faster
    */
Cplx omega[2][N];
void init_omega(int n) {
    static constexpr llf PI=acos(-1);
    const llf arg=(PI+PI)/n;
    for(int i=0;i<n;++i)
        omega[0][i]={cos(arg*i),sin(arg*i)};
    for(int i=0;i<n;++i)
        omega[1][i]=conj(omega[0][i]);
}
void tran(Cplx arr[],int n,Cplx omg[]) {</pre>
```

```
for(int i=0,j=0;i<n;++i){</pre>
    if(i>j)swap(arr[i],arr[j]);
    for(int l=n>>1;(j^=l)<l;l>>=1);
  for (int l=2;l<=n;l<<=1){</pre>
    int m=1>>1;
    for(auto p=arr;p!=arr+n;p+=1){
      for(int i=0;i<m;++i){</pre>
        Cplx t=omg[n/1*i]*p[m+i];
        p[m+i]=p[i]-t;
        p[i]+=t;
      }
   }
 }
void DFT(Cplx arr[],int n){
 tran(arr,n,omega[0]);
void iDFT(Cplx arr[],int n){
  tran(arr,n,omega[1]);
  for(int i=0;i<n;++i)arr[i]/=n;</pre>
```

4.12 Chinese Remainder

```
1ld crt(lld ans[], lld pri[], int n){
  11d M = 1:
  for(int i=0;i<n;i++) M *= pri[i];</pre>
  11d ret = 0;
  for(int i=0;i<n;i++){</pre>
    lld inv = (gcd(M/pri[i], pri[i]).first + pri[i])%
        pri[i];
    ret += (ans[i]*(M/pri[i])%M * inv)%M;
    ret %= M;
  return ret;
Another:
x = a1 \% m1
x = a2 \% m2
g = gcd(m1, m2)
assert((a1-a2)\%g==0)
[p, q] = exgcd(m2/g, m1/g)
return a2+m2*(p*(a1-a2)/g)
\theta \leftarrow x \leftarrow lcm(m1, m2)
```

4.13 Berlekamp Massey

```
// x: 1-base, p[]: 0-base
template < size t N>
vector<llf> BM(llf x[N], size_t n){
  size_t f[N]={0},t=0;llf d[N];
  vector<llf> p[N];
  for(size_t i=1,b=0;i<=n;++i) {</pre>
    for(size_t j=0;j<p[t].size();++j)
d[i]+=x[i-j-1]*p[t][j];
if(abs(d[i]-=x[i])<=EPS)continue;</pre>
    f[t]=i;if(!t){p[++t].resize(i);continue;}
    vector<llf> cur(i-f[b]-1);
    11f k=-d[i]/d[f[b]];cur.PB(-k);
    for(size_t j=0;j<p[b].size();j++)
  cur.PB(p[b][j]*k);</pre>
    if(cur.size()<p[t].size())cur.resize(p[t].size());</pre>
     for(size_t j=0;j<p[t].size();j++)cur[j]+=p[t][j];</pre>
    if(i-f[b]+p[b].size()>=p[t].size()) b=t;
    p[++t]=cur;
  return p[t];
```

4.14 NTT

```
// (must be 2^k)
template < LL P, LL root, int MAXN>
struct NTT{
  static LL bigmod(LL a, LL b) {
     LL res = 1;
     for (LL bs = a; b; b >>= 1, bs = (bs * bs) % P)
       if(b&1) res=(res*bs)%P;
     return res;
  static LL inv(LL a, LL b) {
     if(a==1)return 1;
     return (((LL)(a-inv(b%a,a))*b+1)/a)%b;
  LL omega[MAXN+1];
  NTT() {
    omega[0] = 1;
     LL r = bigmod(root, (P-1)/MAXN);
     for (int i=1; i<=MAXN; i++)</pre>
       omega[i] = (omega[i-1]*r)%P;
  // n must be 2^k
  void tran(int n, LL a[], bool inv_ntt=false){
  int basic = MAXN / n , theta = basic;
     for (int m = n; m >= 2; m >>= 1) {
       int mh = m >> 1;
       for (int i = 0; i < mh; i++) {
   LL w = omega[i*theta%MAXN];</pre>
         for (int j = i; j < n; j += m) {
  int k = j + mh;</pre>
            LL x = a[j] - a[k];
            if (x < 0) x += P;
            a[j] += a[k];
            if (a[j] > P) a[j] -= P;
            a[k] = (w * x) \% P;
       theta = (theta * 2) % MAXN;
     int i = 0;
     for (int j = 1; j < n - 1; j++) {
  for (int k = n >> 1; k > (i ^= k); k >>= 1);
       if (j < i) swap(a[i], a[j]);</pre>
     if (inv_ntt) {
       LL ni = inv(n,P);
       reverse( a+1 , a+n );
for (i = 0; i < n; i++)
         a[i] = (a[i] * ni) % P;
  }
};
const LL P=2013265921, root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;
```

4.15 Sqrt of Polynomial

```
const int mod = (119 << 23) + 1;
int inv_temp[400010];
void poly_inv(int *f, int *inv, int len) {
  int *inv_t = inv_temp, *g = inv;
  g[0] = get_inv(f[0]);
  for (int 1 = 2; 1 <= len; 1 <<= 1, swap(g, inv_t)) {
  for (int i = 0; i < 1; i++) {</pre>
       inv_t[i] = f[i];
       g[i+1] = inv_t[i+1] = 0;
     exec_ntt(inv_t, 1 << 1, 1);
     exec_ntt(g, 1 << 1, 1);
for (int i = 0; i < 2 * 1; i++)
  inv_t[i] = (ll)inv_t[i] * g[i] % mod;
for (int i = 0; i < 2 * 1; i++) {</pre>
       if (inv_t[i])
         inv_t[i] = mod - inv_t[i];
       inv_t[i] += 2, inv_t[i] %= mod;
     for (int i = 0; i < 2 * 1; i++)
       inv_t[i] = (11)inv_t[i] * g[i] % mod;
     exec_ntt(inv_t, l << 1, -1);
     for (int i = 0; i < 1; i++)</pre>
       inv_t[i + 1] = 0;
  for (int i = 0; i < len; i++)</pre>
     inv[i] = g[i];
```

```
int sqrt_temp[400010], inv_t[400010];
void poly_sqrt(int *f, int *sqrt_pol, int len) {
   int *g = sqrt_pol, *t = sqrt_temp, inv2 = get_inv(2);
   for (int 1 = 2; 1 <= len; 1 <<= 1, swap(g, t)) {
  for (int i = 0; i < 1; i++)</pre>
        t[i] = f[i], t[i + 1] = g[i + 1] = inv_t[i] = 0;
      poly_inv(g, inv_t, 1);
     for (int i = 1; i < 2 * 1; i++)
        inv_t[i] = 0;
     exec_ntt(g, 1 << 1, 1);
     exec_ntt(inv_t, 1 << 1, 1);
exec_ntt(t, 1 << 1, 1);
     for (int i = 0; i < (1 << 1); i++)
        t[i]=(ll)inv2*(g[i]+(ll)t[i]*inv_t[i] % mod)%mod;
     exec_ntt(t, 1 << 1, -1);
for (int i = 0; i < 1; i++)
        t[i + 1] = 0;
   for (int i = 0; i < len; i++)</pre>
     sqrt_pol[i] = g[i];
int c[400010], inv[400010], sqrt_pol[400010];
int main(){
  int n, m, x;
scanf("%d%d", &n, &m);
   for (int i = 0; i < n; i++)</pre>
     scanf("%d", &x);
     if (x <= m)
        c[x] = mod - 4;
   c[0]++, c[0] \% = mod;
   int len = 1;
  while (len <= m)len <<= 1;</pre>
  poly_sqrt(c, sqrt_pol, len);
  sqrt_pol[0]++, sqrt_pol[0] %= mod;
  poly_inv(sqrt_pol, inv, len);
for (int i = 1; i <= m; i++)
    printf("%d\n", (inv[i] + inv[i]) % mod);</pre>
  puts("");
   return 0;
```

4.16 FWT

```
/* xor convolution:
 * x = (x0, x1) , y = (y0, y1)
* z = (x0y0 + x1y1 , x0y1 + x1y0 )
 * x' = (x0+x1, x0-x1), y' = (y0+y1, y0-y1)
* z' = ((x0+x1)(y0+y1), (x0-x1)(y0-y1))
* z = (1/2) * z''
 * or convolution:
 * x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
 * and convolution:
 * x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
const LL MOD = 1e9+7;
inline void fwt( LL x[ MAXN ] , int N , bool inv=0 ) {
  for( int d = 1 ; d < N ; d <<= 1 ) {</pre>
     int d2 = d<<1;
     for( int s = 0 ; s < N ; s += d2 )</pre>
        for( int i = s , j = s+d ; i < s+d ; i++, j++ ){
  LL ta = x[ i ] , tb = x[ j ];</pre>
          x[i] = ta+tb;
          x[j] = ta-tb;
if(x[i] >= MOD ) x[i] -= MOD;
           if( x[ j ] < 0 ) x[ j ] += MOD;</pre>
   if( inv )
     for( int i = 0 ; i < N ; i++ ) {
    x[ i ] *= inv( N, MOD );
    x[ i ] %= MOD;</pre>
}
```

4.17 DiscreteLog

```
// Baby-step Giant-step Algorithm
lld BSGS(lld P, lld B, lld N) {
```

```
// find B^L = N mod P
unordered_map<lld, int> R;
lld sq = (lld)sqrt(P);
lld t = 1;
for (int i = 0; i < sq; i++) {
   if (t == N)
        return i;
   if (!R.count(t))
        R[t] = i;
   t = (t * B) % P;
}
lld f = inverse(t, P);
for(int i=0;i<=sq+1;i++) {
   if (R.count(N))
        return i * sq + R[N];
   N = (N * f) % P;
}
return -1;
}</pre>
```

4.18 Quadratic residue

```
struct Status{
 11 x,y;
11 w;
Status mult(const Status& a,const Status& b,ll mod){
  res.x=(a.x*b.x+a.y*b.y%mod*w)%mod;
  res.y=(a.x*b.y+a.y*b.x)%mod;
  return res;
inline Status qpow(Status _base,11 _pow,11 _mod){
  Status res;
  res.x=1, res.y=0;
  while(_pow>0){
    if(_pow&1) res=mult(res,_base,_mod);
    _base=mult(_base,_base,_mod);
    _pow>>=1;
  return res;
inline 11 check(11 x,11 p){
  return qpow_mod(x,(p-1)>>1,p);
inline ll get_root(ll n,ll p){
  if(p==2) return 1;
  if(check(n,p)==p-1) return -1;
  while(true){
   a=rand()%p;
    w=((a*a-n)%p+p)%p;
    if(check(w,p)==p-1) break;
  Status res;
  res.x=a:
  res.y=1;
  res=qpow(res,(p+1)>>1,p);
  return res.x;
```

4.19 De-Bruijn

```
int res[maxn], aux[maxn], sz;
void db(int t, int p, int n, int k) {
  if (t > n) {
    if (n % p == 0)
       for (int i = 1; i <= p; ++i)</pre>
        res[sz++] = aux[i];
  } else {
    aux[t] = aux[t - p];
    db(t + 1, p, n, k);
     for (int i = aux[t - p] + 1; i < k; ++i) {</pre>
       aux[t] = i;
       db(\bar{t} + 1, t, n, k);
    }
  }
int de_bruijn(int k, int n) {
 // return cyclic string of len k^n s.t. every string
  // of len n using k char appears as a substring.
  if (k == 1) {
```

```
res[0] = 0;
  return 1;
}
for (int i = 0; i < k * n; i++) aux[i] = 0;
sz = 0;
db(1, 1, n, k);
return sz;
}</pre>
```

5 Geometry

5.1 Point Class

```
template<typename T>
struct Point{
  typedef long double llf;
  static constexpr llf EPS = 1e-8;
  Тх, у;
 Point(T _=0, T __=0): x(_), y(__){}
template<typename T2>
    Point(const Point<T2>& a): x(a.x), y(a.y){}
  inline llf theta() const
    return atan2((llf)y, (llf)x);
  inline llf dis() const {
    return hypot((llf)x, (llf)y);
  inline llf dis(const Point& o) const {
   return hypot((llf)(x-o.x), (llf)(y-o.y));
 Point operator-(const Point& o) const {
    return Point(x-o.x, y-o.y);
 Point operator -= (const Point& o){
   x-=o.x, y-=o.y;
return *this;
  Point operator+(const Point& o) const {
   return Point(x+o.x, y+o.y);
 Point operator+=(const Point& o){
    x+=0.x, y+=0.y;
    return *this;
  Point operator*(const T& k) const {
    return Point(x*k, y*k);
  Point operator*=(const T& k){
   x^*=k, y^*=k;
    return *this;
  Point operator/(const T& k) const {
    return Point(x/k, y/k);
  Point operator/=(const T& k){
   x/=k, y/=k;
    return *this;
  Point operator-() const {
   return Point(-x, -y);
  Point rot90() const {
    return Point(-y, x);
  template<typename T2>
  bool in(const Circle<T2>& a) const {
    /* Add struct Circle at top */
    return a.o.dis(*this)+EPS <= a.r;</pre>
  bool equal(const Point& o, true_type) const {
    return fabs(x-o.x) < EPS and fabs(y-o.y) < EPS;
  bool equal(const Point& o, false_type) const {
   return tie(x, y) == tie(o.x, o.y);
  bool operator==(const Point& o) const +
    return equal(o, is_floating_point<T>());
  bool operator!=(const Point& o) const {
   return !(*this == 0);
  bool operator<(const Point& o) const {</pre>
```

5.2 Circle Class

```
template < typename T>
struct Circle{
    static constexpr llf EPS = 1e-8;
    Point < T > 0;
    T r;
    vector < Point < llf >> operator & (const Circle & aa) const {
        // https://www.cnblogs.com/wangzming/p/8338142.html
        llf d=0.dis(aa.0);
        if(d > r+aa.r+EPS or d < fabs(r-aa.r)-EPS) return
            {};
        llf dt = (r*r - aa.r*aa.r)/d, d1 = (d+dt)/2;
        Point < llf > dir = (aa.o-o); dir /= d;
        Point < llf > pcrs = dir*d1 + o;
        dt = sqrt(max(0.0L, r*r - d1*d1)), dir=dir.rot90();
        return {pcrs + dir*dt, pcrs - dir*dt};
    }
};
```

5.3 Line Class

```
const Point<long double> INF_P(-1e20, 1e20);
const Point<long double> NOT_EXIST(1e20, 1e-20);
template<typename T>
struct Line{
  static constexpr long double EPS = 1e-8;
  // ax+by+c = 0
  T a, b, c;
  Line(): a(0), b(1), c(0){}
                       _): a(_), b(_
  Line(T
            Т
    assert(fabs(a)>EPS or fabs(b)>EPS);
  template<typename T2>
    Line(const Line<T2>& x): a(x.a), b(x.b), c(x.c){}
  typedef Point<long double> Pt;
  bool equal(const Line& o, true_type) const {
    return fabs(a-o.a) < EPS and fabs(b-o.b) < EPS and
        fabs(c-o.b) < EPS;</pre>
  bool euqal(const Line& o, false_type) const {
    return a==o.a and b==o.b and c==o.c;
  bool operator==(const Line& o) const {
    return euqal(o, is_floating_point<T>());
  bool operator!=(const Line& o) const {
    return !(*this == 0);
  friend inline bool on_line__(const Point<T>& p, const
       Line& 1, true_type){
    return fabs(l.a*p.x + l.b*p.y + l.c) < EPS;</pre>
  friend inline bool on_line__(const Point<T>& p, const
    Line& 1, false_type){
return 1.a*p.x + 1.b*p.y + 1.c == 0;
  friend inline bool on_line(const Point<T>&p const
      Line& 1){
    return on_line__(p, 1, is_floating_point<T>());
```

```
friend inline bool is_parallel__(const Line& x, const
       Line& y, true_type){
    return fabs(x.a*y.b - x.b*y.a) < EPS;</pre>
  friend inline bool is_parallel__(const Line& x, const
       Line& y, false_type){
    return x.a*y.b == x.b*y.a;
  friend inline bool is_parallel(const Line& x, const
      Line& y){
    return is_parallel__(x, y, is_floating_point<T>());
  friend inline Pt get_inter(const Line& x, const Line&
    typedef long double llf;
    if(x==y) return INF_P;
    if(is_parallel(x, y)) return NOT_EXIST;
    llf delta = x.a*y.b - x.b*y.a;
    llf delta_x = x.b*y.c - x.c*y.b;
    llf delta_y = x.c*y.a - x.a*y.c;
    return Pt(delta_x / delta, delta_y / delta);
  friend ostream& operator<<(ostream& ss, const Line& o</pre>
    ss<<o.a<<"x+"<<o.b<<"y+"<<o.c<<"=0";
    return ss;
 }
};
template<typename T>
inline Line<T> get_line(const Point<T>& a, const Point<</pre>
    T>& b){}
  return Line<T>(a.y-b.y, b.x-a.x, (b.y-a.y)*a.x-(b.x-a
      .x)*a.y);
```

5.4 Triangle Circumcentre

5.5 2D Convex Hull

```
template<typename T>
class ConvexHull 2D{
private:
  typedef Point<T> PT;
  vector<PT> dots;
  struct myhash{
    uint64_t operator()(const PT& a) const {
      uint64_t xx=0, yy=0;
      memcpy(&xx, &a.x, sizeof(a.x));
      memcpy(&yy, &a.y, sizeof(a.y));
uint64_t ret = xx*17+yy*31;
      ret = (ret ^ (ret >> 16))*0x9E3779B1;
      ret = (ret ^ (ret >> 13))*0xC2B2AE35;
      ret = ret ^ xx;
      return (ret ^ (ret << 3)) * yy;</pre>
  unordered_set<PT, myhash> in_hull;
public:
  inline void init(){in_hull.clear();dots.clear();}
  void insert(const PT& x){dots.PB(x);}
  void solve(){
    sort(ALL(dots), [](const PT& a, const PT& b){
      return tie(a.x, a.y) < tie(b.x, b.y);</pre>
    });
    vector<PT> stk(SZ(dots)<<1);</pre>
```

```
int top = 0;
  for(auto p: dots){
    while(top >= 2 and cross(p-stk[top-2], stk[top
        -1]-stk[top-2]) <= 0)
    stk[top++] = p;
  for(int i=SZ(dots)-2, t = top+1;i>=0;i--){
    while(top >= t and cross(dots[i]-stk[top-2], stk[
       top-1]-stk[top-2]) <= 0)
      top --;
    stk[top++] = dots[i];
  stk.resize(top-1);
  swap(stk, dots);
  for(auto i: stk) in_hull.insert(i);
vector<PT> get(){return dots;}
inline bool in_it(const PT& x){
  return in_hull.find(x)!=in_hull.end();
```

5.6 2D Farthest Pair

5.7 2D Cosest Pair

```
struct Point{
  11f x, y;
  llf dis;
} arr[N];
inline llf get_dis(Point a, Point b){
  return hypot(a.x-b.x, a.y-b.y);
11f solve(){
  int cur = rand()%n;
  for(int i=0;i<n;i++) arr[i].dis = get_dis(arr[cur],</pre>
      arr[i]);
  sort(arr, arr+n, [](Point a, Point b){return a.dis <</pre>
      b.dis;});
  11f ans = 1e50;
  for(int i=0;i<n;i++){</pre>
    for(int j=i+1;j<n;j++){</pre>
      if(arr[j].dis - arr[i].dis > ans) break;
      ans = min(ans, get_dis(arr[i], arr[j]));
    }
  return ans;
}
```

5.8 SimulateAnnealing

```
double getY(double);
int main(){
  int rr, 11;
  default_random_engine rEng(time(NULL));
  uniform_real_distribution<double> Range(-1,1);
  uniform_real_distribution<double> expR(0,1);
  auto Random=bind(Range,rEng), expRand=bind(expR,rEng)
  ;
  int step=0;
  double pace=rr-ll, mini=0.95; // need to search for
    it
  double x=max(min(Random()*pace+ll, rr), 11), y=getY(x
    );
```

```
while(pace>=1e-7){
   double newX = max(min(x + Random()*pace, rr), ll);
   double newY = getY(newX);
   if(newY < y || expRand() < exp(-step))
        x=newX, y=newY;
   step++;
   pace*=mini;
}</pre>
```

5.9 Ternary Search on Integer

```
int TernarySearch(int 1, int r) {
    // (L, r]
    while (r - 1 > 1){
        int mid = (l + r)>>1;
        if (f(mid) > f(mid + 1)) r = mid;
        else l = mid;
    }
    return l+1;
}
```

5.10 Minimum Covering Circle

```
template<typename T>
Circle<llf> MinCircleCover(const vector<Point<T>>& pts)
  random_shuffle(ALL(pts));
  Circle<llf> c = \{pts[0], 0\};
  int n = SZ(pts);
  for(int i=0;i<n;i++){</pre>
   if(pts[i].in(c)) continue;
    c = {pts[i], 0};
    for(int j=0;j<i;j++){</pre>
      if(pts[j].in(c)) continue;
      c.o = (pts[i] + pts[j]) / 2;
      c.r = pts[i].dis(c.o);
      for(int k=0;k<j;k++){</pre>
        if(pts[k].in(c)) continue;
        c = get_circum(pts[i], pts[j], pts[k]);
      }
   }
  return c;
```

5.11 KDTree (Nearest Point)

```
const int MXN = 100005;
struct KDTree {
  struct Node {
    int x,y,x1,y1,x2,y2;
    int id,f;
Node *L, *R;
  }tree[MXN];
  int n;
Node *root;
  LL dis2(int x1, int y1, int x2, int y2) {
    LL dx = x1-x2;
    LL dy = y1-y2;
    return dx*dx+dy*dy;
  static bool cmpx(Node& a, Node& b){ return a.x<b.x; }
static bool cmpy(Node& a, Node& b){ return a.y<b.y; }</pre>
  void init(vector<pair<int,int>> ip) {
    n = ip.size();
    for (int i=0; i<n; i++) {</pre>
       tree[i].id = i;
       tree[i].x = ip[i].first;
      tree[i].y = ip[i].second;
    root = build_tree(0, n-1, 0);
  Node* build_tree(int L, int R, int dep) {
    if (L>R) return nullptr;
    int M = (L+R)/2;
    tree[M].f = dep%2;
    nth_element(tree+L, tree+M, tree+R+1, tree[M].f ?
         cmpy : cmpx);
```

```
tree[M].x1 = tree[M].x2 = tree[M].x;
     tree[M].y1 = tree[M].y2 = tree[M].y;
     tree[M].L = build_tree(L, M-1, dep+1);
     if (tree[M].L) {
       tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
       tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
       tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
     tree[M].R = build_tree(M+1, R, dep+1);
     if (tree[M].R) {
       tree[M].x1 = min(tree[M].x1, tree[M].R->x1);

tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
       tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
       tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
     return tree+M;
  int touch(Node* r, int x, int y, LL d2){
     LL dis = sqrt(d2)+1;
     if (x<r->x1-dis || x>r->x2+dis ||
         y<r->y1-dis || y>r->y2+dis)
       return 0;
     return 1;
  void nearest(Node* r, int x, int y,
                  int &mID, LL &md2){
     if (!r || !touch(r, x, y, md2)) return;
LL d2 = dis2(r->x, r->y, x, y);
     if (d2 < md2 || (d2 == md2 && mID < r->id)) {}
       mID = r \rightarrow id;
       md2 = d2;
     // search order depends on split dim
     if ((r->f == 0 && x < r->x) ||
          (r->f == 1 && y < r->y)) {
       nearest(r->L, x, y, mID, md2);
nearest(r->R, x, y, mID, md2);
     } else {
       nearest(r->R, x, y, mID, md2);
       nearest(r->L, x, y, mID, md2);
    }
  int query(int x, int y) {
     int id = 1029384756;
     LL d2 = 102938475612345678LL;
     nearest(root, x, y, id, d2);
     return id:
}tree;
```

6 Stringology

6.1 Hash

```
class Hash{
 private:
   static const int N = 1000000;
   const int p = 127, q = 1208220623;
   int sz, prefix[N], power[N];
inline int add(int x, int y){return x+y>=q?x+y-q:x+y
   inline int sub(int x, int y){return x-y<0?x-y+q:x-y;}</pre>
   inline int mul(int x, int y){return 1LL*x*y%q;}
 public:
   void init(const std::string &x){
     sz = x.size();
     prefix[0]=0;
     for(int i=1;i<=sz;i++) prefix[i]=add(mul(prefix[i</pre>
         -1], p), x[i-1]);
     power[0]=1;
     for(int i=1;i<=sz;i++) power[i]=mul(power[i-1], p);</pre>
   int query(int 1, int r){
     // 1-base (l, r]
     return sub(prefix[r], mul(prefix[l], power[r-l]));
};
```

6.2 Suffix Array

```
//help by http://www.geeksforgeeks.org/suffix-array-set
     -2-a-nlognlogn-algorithm/
struct sfx{
  int index;
  int r,nr;
char str[N + 10];
int len;
vector<sfx> srs[N + 10];
int mapping[N + 10];
sfx sa[N + 10];
bool cmp(sfx a,sfx b){
  if(a.r==b.r){
    return a.nr<b.nr;</pre>
  }else{
    return a.r<b.r;</pre>
  }
void SA(){
  len = strlen(str);
  for(int i=0;i<len;i++){</pre>
    sa[i].index = i;
    sa[i].r=str[i];
    sa[i].nr=(i+1>=len)?0:str[i+1];
  //sort(sa,sa+len,cmp);
  radixSort();
  for(int j=2;j<=len;j*=2){</pre>
    int cnt=1;
    int rr = sa[0].r;
    sa[0].r=cnt;
    mapping[sa[0].index]=0;
    for(int i=1;i<len;i++){</pre>
      if(sa[i].r == rr && sa[i].nr == sa[i-1].nr){
         rr=sa[i].r;
         sa[i].r=cnt;
      }else{
        rr=sa[i].r;
         sa[i].r=++cnt;
      mapping[sa[i].index]=i;
    for(int i=0;i<len;i++){</pre>
      int nn = sa[i].index+j;
      sa[i].nr = (nn>=len)?0:sa[mapping[nn]].r;
    //sort(sa, sa+len, cmp);
    radixSort();
void radixSort(){
  int m = 0;
  for(int i=0;i<len;i++){</pre>
    srs[sa[i].nr].PB(sa[i]);
    m=max(m,sa[i].nr);
  int cnt=0;
  for(int i=0;i<=m;i++){</pre>
    if(srs[i].empty())continue;
    for(auto j:srs[i]){
    sa[cnt++] = j;
    }
    srs[i].clear();
  for(int i=0;i<len;i++){</pre>
    srs[sa[i].r].PB(sa[i]);
    m=max(m,sa[i].r);
  cnt=0;
  for(int i=0;i<=m;i++){</pre>
    if(srs[i].empty())continue;
    for(auto j:srs[i]){
      sa[cnt++] = j;
    srs[i].clear();
  }
}
```

```
inline int Idx( char c ) { return c - 'a'; }
   public:
     void init() { rt = new node(); }
     void add( const string& s, int d ) {
       node* cur = rt;
       for ( auto c : s ) {
         if ( not cur->nxt[ Idx( c ) ] )
           cur->nxt[ Idx( c ) ] = new node();
          cur = cur - > nxt[Idx(c)];
       }
       cur->data.push back( d );
     void compile() {
  vector< node* > bfs;
       size_t ptr = 0;
for ( int i = 0 ; i < Z ; ++ i ) {</pre>
         if ( not rt->nxt[ i ] )
            continue;
          rt->nxt[ i ]->fail = rt;
         bfs.push_back( rt->nxt[ i ] );
       while ( ptr < bfs.size() ) {
  node* u = bfs[ ptr ++ ];
  for ( int i = 0 ; i < Z ; ++ i ) {</pre>
            if ( not u->nxt[ i ] )
              continue;
            node* u_f = u->fail;
            while ( u_f )
              if ( not u_f->nxt[ i ] ) {
                u_f = u_f->fail;
                continue;
              }
              u->nxt[ i ]->fail = u_f->nxt[ i ];
            if ( not u f ) u->nxt[ i ]->fail = rt;
            bfs.push_back( u->nxt[ i ] );
       }
     void match( const string& s, vector< int >& ret ) {
       node* u = rt;
       for ( auto c : s ) {
          while ( u != rt and not u->nxt[ Idx( c ) ] )
           u = u->fail;
          u = u \rightarrow nxt[Idx(c)];
         if ( not u ) u = rt;
node* tmp = u;
          while ( tmp != rt ) {
            for ( auto d : tmp->data )
              ret.push_back( d );
            tmp = tmp->fail;
       }
} ac;
```

6.4 Suffix Automaton

class AhoCorasick{
 private:

struct node{

} *rt;

static constexpr int Z = 26;

memset(nxt, 0, sizeof(nxt));

node *nxt[Z], *fail;

vector< int > data; node(): fail(nullptr) {

data.clear();

```
struct Node{
  Node *green, *edge[26];
  int max_len;
  Node(const int _max_len)
    : green(NULL), max_len(_max_len){
      memset(edge,0,sizeof(edge));
  }
} *ROOT, *LAST;

void Extend(const int c) {
  Node *cursor = LAST;
  LAST = new Node((LAST->max_len) + 1);
  for(;cursor&&!cursor->edge[c]; cursor=cursor->green)
      cursor->edge[c] = LAST;
  if (!cursor)
```

6.3 Aho-Corasick Algorithm

```
LAST->green = ROOT;
  else {
    Node *potential_green = cursor->edge[c];
    if((potential_green->max_len)==(cursor->max_len+1))
      LAST->green = potential_green;
    else {
//assert(potential_green->max_len>(cursor->max_len+1));
      Node *wish = new Node((cursor->max_len) + 1);
      for(;cursor && cursor->edge[c]==potential_green;
           cursor = cursor->green)
        cursor->edge[c] = wish;
      for (int i = 0; i < 26; i++)
        wish->edge[i] = potential_green->edge[i];
      wish->green = potential_green->green;
      potential_green->green = wish;
      LAST->green = wish;
 }
char S[10000001], A[10000001];
int N;
int main(){
 scanf("%d%s", &N, S);
ROOT = LAST = new Node(0);
  for (int i = 0; S[i]; i++)
    Extend(S[i] - 'a');
 while (N--){
   scanf("%s", A);
   Node *cursor = ROOT;
    bool ans = true;
    for (int i = 0; A[i]; i++){
      cursor = cursor->edge[A[i] - 'a'];
      if (!cursor) {
        ans = false;
        break;
      }
    puts(ans ? "Yes" : "No");
  return 0;
```

6.5 KMP

```
int F[N<<1];</pre>
void KMP(char s1[], char s2[], int n, int m){
  // make F[] for s1+'\0'+s2;
  char ss[N<<1];</pre>
  int len = n+m+1;
  for(int i=0;i<n;i++) ss[i] = s1[i];</pre>
  ss[n] = ' \setminus 0'
  for(int i=0;i<m;i++) ss[i+1+n] = s2[i];</pre>
  F[0] = F[1] = 0;
  for(int i=1;i<len;i++){</pre>
    int j = F[i];
    while(j > 0 and ss[i]!=ss[j]) j = F[j];
    F[i+1] = (ss[i]==ss[j]?j+1:0);
  // just find (F[len2+i] == len2)
  // i from 1 to len+1 for matching
  [0, i]是個循環字串,且循環節為i-f[i]:
  if(f[i]>0 and i%(i-f[i])==0)
cout << i << " " << i/(i-f[i]) << '\n';
```

6.6 Z value

```
char s[MAXN];
int len,z[MAXN];
void Z_value() {
   int i,j,left,right;
   left=right=0; z[0]=len;
   for(i=1;i<len;i++) {
      j=max(min(z[i-left],right-i),0);
      for(;i+j<len&&s[i+j]==s[j];j++);
      z[i]=j;
      if(i+z[i]>right) {
        right=i+z[i];
      left=i;
   }
```

```
6.7 Lexicographically Smallest Rotation
```

```
string mcp(string s){
  int n = s.length();
  s += s;
  int i=0, j=1;
  while (i<n && j<n){
    int k = 0;
    while (k < n && s[i+k] == s[j+k]) k++;
    if (s[i+k] <= s[j+k]) j += k+1;
    if (i == j) j++;
  }
  int ans = i < n ? i : j;
  return s.substr(ans, n);
}</pre>
```

6.8 BWT

}

}

```
struct BurrowsWheeler{
#define SIGMA 26
#define BASE 'a'
  vector<int> v[ SIGMA ];
void BWT(char* ori, char* res){
   // make ori -> ori + ori
     // then build suffix array
   void iBWT(char* ori, char* res){
     for( int i = 0 ; i < SIGMA ; i ++ )</pre>
       v[ i ].clear();
     int len = strlen( ori );
     for( int i = 0 ; i < len ; i ++ )
  v[ ori[i] - BASE ].push_back( i );</pre>
     vector<int> a;
     for( int i = 0 , ptr = 0 ; i < SIGMA ; i ++ )</pre>
        for( auto j : v[ i ] ){
          a.push_back( j );
ori[ ptr ++ ] = BASE + i;
     for( int i = 0 , ptr = 0 ; i < len ; i ++ ){</pre>
        res[ i ] = ori[ a[ ptr ] ];
       ptr = a[ ptr ];
     res[ len ] = 0;
} bwt;
```

6.9 Palindromic Tree

```
struct palindromic_tree{
  struct node{
    int next[26],fail,len;
    int cnt,num,st,ed;
    node(int l=0):fail(0),len(1),cnt(0),num(0){
      for(int i=0;i<26;++i)next[i]=0;</pre>
    }
  };
  vector<node> state;
  vector<char> s;
  int last,n;
  void init(){
    state.clear();
    s.clear();
    last=1;
    n=0;
    state.push_back(0);
    state.push_back(-1);
    state[0].fail=1;
    s.push_back(-1);
  int get_fail(int x){
    while(s[n-state[x].len-1]!=s[n])x=state[x].fail;
    return x;
```

```
void add(int c){
   s.push_back(c-='a');
   ++n;
   int cur=get_fail(last);
   if(!state[cur].next[c]){
     int now=state.size();
     state.push_back(state[cur].len+2);
     state[now].fail=state[get_fail(state[cur].fail)].
         next[c];
     state[cur].next[c]=now;
     state[now].num=state[state[now].fail].num+1;
   last=state[cur].next[c];
   ++state[last].cnt;
  int size(){
   return state.size()-2;
}pt;
int main() {
 string s;
 cin >> s;
  pt.init();
  for (int i=0; i<SZ(s); i++) {</pre>
   int prvsz = pt.size();
   pt.add(s[i]);
   if (prvsz != pt.size()) {
     int r = i;
     }
 return 0;
```

7 Misc

7.1 Degree Sequence Validity

7.1.1 Erdős-Gallai Theorem

```
d_1 \geq \cdots \geq d_n is a valid degree sequence iff \sum_{k=1}^n d_k \equiv 0 \mod 2 \forall 1 \leq k \leq n, \sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i,k)
```

7.1.2 Havel-Hakimi algorithm

find the vertex who has greatest degree unused, connect it with other greatest vertex.

7.2 MaximumEmptyRect

```
int max_empty_rect(int n, int m, bool blocked[N][N]){
  static int mxu[2][N], me=0,he=1,ans=0;
  for(int i=0;i<m;i++) mxu[he][i]=0;</pre>
  for(int i=0;i<n;i++){</pre>
    stack<PII, vector<PII>> stk;
    for(int j=0;j<m;++j){</pre>
      if(blocked[i][j]) mxu[me][j]=0;
      else mxu[me][j]=mxu[he][j]+1;
      int la = j;
      while(!stk.empty()&&stk.top().FF>mxu[me][j]){
        int x1 = i - stk.top().FF, x2 = i;
        int y1 = stk.top().SS, y2 = j;
        la = stk.top().SS; stk.pop();
        ans=max(ans,(x2-x1)*(y2-y1));
      if(stk.empty()||stk.top().FF<mxu[me][j])</pre>
        stk.push({mxu[me][j],la});
    while(!stk.empty()){
      int x1 = i - stk.top().FF, x2 = i;
      int y1 = stk.top().SS-1, y2 = m-1;
      stk.pop();
      ans=max(ans,(x2-x1)*(y2-y1));
    swap(me,he);
```

```
return ans;
}
```

7.3 DP-opt Condition

7.3.1 totally monotone (concave/convex)

```
\begin{array}{l} \forall i < i', j < j' \text{, } B[i][j] \leq B[i'][j] \implies B[i][j'] \leq B[i'][j'] \\ \forall i < i', j < j' \text{, } B[i][j] \geq B[i'][j] \implies B[i][j'] \geq B[i'][j'] \end{array}
```

7.3.2 monge condition (concave/convex)

```
\begin{array}{l} \forall i < i', j < j' \text{, } B[i][j] + B[i'][j'] \geq B[i][j'] + B[i'][j] \\ \forall i < i', j < j' \text{, } B[i][j] + B[i'][j'] \leq B[i][j'] + B[i'][j] \end{array}
```

7.4 Convex 1D/1D DP

```
struct segment {
  int i, 1, r;
  segment() {}
  segment(int a, int b, int c): i(a), l(b), r(c) {}
inline long long f(int 1, int r) {
  return dp[1] + w(1 + 1, r);
void solve() {
  dp[0] = 011;
  deque<segment> deq; deq.push_back(segment(0, 1, n));
  for (int i = 1; i <= n; ++i) {</pre>
    dp[i] = f(deq.front().i, i);
    while (deq.size() && deq.front().r < i + 1) deq.</pre>
         pop_front();
    deq.front().l = i + 1;
segment seg = segment(i, i + 1, n);
     while (deq.size() && f(i, deq.back().1) < f(deq.</pre>
         back().i, deq.back().1)) deq.pop_back();
     if (deq.size()) {
       int d = 1048576, c = deq.back().1;
while (d >>= 1) if (c + d <= deq.back().r) {
         if (f(i, c + d) > f(deq.back().i, c + d)) c +=
       deq.back().r = c; seg.l = c + 1;
     if (seg.1 <= n) deq.push_back(seg);</pre>
```

7.5 Josephus Problem

```
// n people kill m for each turn
int f(int n, int m) {
  int s = 0;
  for (int i = 2; i <= n; i++)
    s = (s + m) % i;
  return s;
}
// died at kth
int kth(int n, int m, int k){
  if (m == 1) return n-1;
  for (k = k*m+m-1; k >= n; k = k-n+(k-n)/(m-1));
  return k;
}
```

7.6 Cactus Matching

```
const int maxn=200010;
vector<int> init_g[maxn],g[maxn*2];
int dfn[maxn],low[maxn],par[maxn],dfs_idx,bcc_id;
int n;
void tarjan(int u){
   dfn[u]=low[u]=++dfs_idx;
   for(int i=0;i<(int)init_g[u].size();i++){
     int v=init_g[u][i];</pre>
```

```
if(v==par[u]) continue;
                                                               printf("%d\n",max(dp[1][0],dp[1][1]));
    if(!dfn[v]){
                                                               return 0;
      par[v]=u;
      tarjan(v);
      low[u]=min(low[u],low[v]);
      if(dfn[u]<low[v]){</pre>
        g[u].push_back(v);
        g[v].push_back(u);
    }else{
      low[u]=min(low[u],dfn[v]);
      if(dfn[v]<dfn[u]){</pre>
        int temp_v=u;
        bcc_id++;
        while(temp_v!=v){
          g[bcc_id+n].push_back(temp_v);
          g[temp_v].push_back(bcc_id+n);
          temp_v=par[temp_v];
        g[bcc_id+n].push_back(v);
        g[v].push_back(bcc_id+n);
        reverse(g[bcc_id+n].begin(),g[bcc_id+n].end());
      }
   }
 }
int dp[maxn][2],min_dp[2][2],tmp[2][2],tp[2];
void dfs(int u,int fa){
  if(u<=n){</pre>
    for(int i=0;i<(int)g[u].size();i++){</pre>
      int v=g[u][i];
      if(v==fa) continue;
      dfs(v,u);
      memset(tp,0x8f,sizeof tp);
      if(v<=n){</pre>
        tp[0]=dp[u][0]+max(dp[v][0],dp[v][1]);
        tp[1]=max(
          dp[u][0]+dp[v][0]+1;
          dp[u][1]+max(dp[v][0],dp[v][1])
      }else{
        tp[0]=dp[u][0]+dp[v][0];
        tp[1]=max(dp[u][0]+dp[v][1],dp[u][1]+dp[v][0]);
      dp[u][0]=tp[0],dp[u][1]=tp[1];
  }else{
    for(int i=0;i<(int)g[u].size();i++){</pre>
      int v=g[u][i];
      if(v==fa) continue;
      dfs(v,u);
    min_dp[0][0]=0;
    min_dp[1][1]=1;
    for(int i=0;i<(int)g[u].size();i++){</pre>
      int v=g[u][i];
      if(v==fa) continue;
      memset(tmp,0x8f,sizeof tmp);
      tmp[0][0]=max(
        min_dp[0][0]+max(dp[v][0],dp[v][1]),
        min_dp[0][1]+dp[v][0]
      tmp[0][1]=min\_dp[0][0]+dp[v][0]+1;
      tmp[1][0]=max(
        min_dp[1][0]+max(dp[v][0],dp[v][1]),
        min_dp[1][1]+dp[v][0]
      tmp[1][1]=min_dp[1][0]+dp[v][0]+1;
      memcpy(min_dp,tmp,sizeof tmp);
    dp[u][1]=max(min_dp[0][1],min_dp[1][0]);
    dp[u][0]=min_dp[0][0];
  }
int main(){
 int m,a,b;
 scanf("%d%d",&n,&m);
for(int i=0;i<m;i++){</pre>
    scanf("%d%d",&a,&b);
    init_g[a].push_back(b);
    init_g[b].push_back(a);
  par[1]=-1;
  tarjan(1);
  dfs(1,-1);
```