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1 Basic

1.1 Default Code

```
#include <bits/stdc++.h>
using namespace std;
using lld = int64_t;
using llu = uint64_t;
using llf = long double;
using PII = pair<int,int>;
using PIL = pair<int, lld>;
using PLI = pair<lld,int>;
using PLL = pair<lld, lld>;
template<typename T>
using maxHeap = priority_queue<T, vector<T>, less<T>>;
template<typename T>
using minHeap = priority_queue<T, vector<T>, greater<T>>;
#define FF first
 #define SS second
#define SZ(x) (int)((x).size())
#define ALL(x) begin(x), end(x)
#define PB push_back
#define WC(x) while((x)--)
template<typename Iter>
ostream& _out(ostream &s, Iter b, Iter e) {
  s<<"[";
  for ( auto it=b; it!=e; it++ ) s<<(it==b?"":" ")<<*it</pre>
  s<<"]";
  return s;
template<typename A, typename B>
ostream& operator <<( ostream &s, const pair<A,B> &p )
    { return s<<"("<<p.FF<<","<<p.SS<<")"; }
template<typename T>
ostream& operator << ( ostream &s, const vector<T> &c )
    { return _out(s,ALL(c)); }
bool debug = \overline{0};
template<typename T>
void DEBUG(const T& x) {if(debug) cerr<<x;}</pre>
template<typename T, typename... Args>
 void DEBUG(const T& head,const Args& ...tail) {
  if(debug) {cerr<<head; DEBUG(tail...);}</pre>
int main(int argc, char* argv[]){
  if(argc>1 and string(argv[1]) == "-D") debug=1;
   if(!debug) {ios_base::sync_with_stdio(0);cin.tie(0);}
  return 0;
```

1.2 IncreaseStackSize

```
//stack resize
asm("mov %0, %%esp\n" :: "g"(mem+10000000));
//change esp to rsp if 64-bit system
//stack resize (linux)
#include <sys/resource.h>
void increase_stack_size() {
  const rlim_t ks = 64*1024*1024;
  struct rlimit rl;
  int res=getrlimit(RLIMIT STACK, &rl);
  if(res==0){
    if(rl.rlim_cur<ks){</pre>
      rl.rlim cur=ks;
      res=setrlimit(RLIMIT STACK, &rl);
    }
  }
}
// craziest way
static void run with stack size(void (*func)(), size t
   stsize) {
  char *stack, *send;
stack=(char *)malloc(stsize);
  send=stack+stsize-16;
  send=(char *)((uintptr_t)send/16*16);
  asm volatile(
    "mov %%rsp, (%0)\n"
    "mov %0, %%rsp\n"
```

```
:
    : "r" (send));
func();
asm volatile(
    "mov (%0), %%rsp\n"
:
    : "r" (send));
free(stack);
}
```

1.3 Pragma optimization

```
#pragma GCC optimize("Ofast,no-stack-protector,no-math-
errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,sse4,popcnt,abm
,mmx,avx,tune=native")
```

1.4 Debugger

```
#! /usr/bin/env python3
import subprocess, platform
os_name = platform.system()
cmd = []
prefix = ""
if os name == 'Windows':
 cmd=["cmd", "/C"]
 cmd = ["bash", "-c"]
 prefix = "./"
def GetTestData(exe):
 myout=subprocess.check output(cmd+["%s%s"%(prefix,
     exe)])
 return myout.decode("utf8")
def Judge(a,b,testdata):
 f = open("test.in", "w+")
 f.write(testdata)
 f.close()
 c=subprocess.check_output(cmd+["%s%s < test.in"%(
      prefix, a)])
 d=subprocess.check_output(cmd+["%s%s < test.in"%(
      prefix, b)])
 if not c == d:
   print("answer: %s"%c.decode("utf8"),end="")
   print("output: %s"%d.decode("utf8"), end="")
    print("WA!")
    return False
 return True
if __name__ == '__main__
 cnt = 0
 isOK = True
 while isOK:
   cnt += 1
   print(cnt)
    isOK = Judge("1234.exe", "test.exe", GetTestData("
        gen.exe"))
```

1.5 Quick Random

```
template<class T,T x1,T x2,T x3,int y1,int y2,int y3>
struct PRNG {
  using S = typename std::make signed<T>::type;
  PRNG (T
          s = 0) : s(s) {}
 T next() {
   T z = (s += x1);

z = (z ^ (z >> y1)) * x2;
    z = (z ^ (z >> y2)) * x3;
    return z ^ (z >> y3);
 T next(T n) { return next() % n; }
 S \text{ next}(S 1, S r) \{ \text{ return } 1 + \text{ next}(r - 1 + 1); \}
 T operator()() { return next(); }
  T operator()(T n) { return next(n); }
 S operator()(S 1, S r) { return next(1, r); }
  static T gen(T s) { return PRNG(s)(); }
  template<class U>
```

1.6 IO Optimization

```
static inline int gc() {
  static char buf[1 << 20], *p = buf, *end = buf;</pre>
  if (p == end) {
    if ((end = buf + fread(buf, 1, 1 << 20, stdin)) ==</pre>
         buf) return EOF;
    p = buf;
  return *p++;
template<typename T>
static inline bool gn(T &_){
  register int c = gc(); register T
                                              = 1;
  while(!isdigit(c) and c!=EOF and c!='-') c = gc();
if(c == '-') { __ = -1; c = gc(); }
  if(c == '-') { __ = -1; c
if(c == EOF) return false;
  while(isdigit(c)) _ = _ * 10 + c - '0', c = gc();
  *= __;
return true;
template <typename T, typename ...Args>
static inline bool gn(T &x, Args& ...args)(return gn(x)
      and gn(args...);}
```

2 Data Structure

2.1 Bigint

```
class BigInt{
    using lld = int fast64 t;
    #define PRINTF ARG PRIdFAST64
    #define LOG_BASE_STR "9"
    static constexpr lld BASE = 1000000000;
    static constexpr int LOG BASE = 9;
    vector<lld> dig;
    bool neg;
    inline int len()const{return (int)dig.size();}
    inline int cmp_minus(const BigInt& a) const {
      if(len() == 0 and a.len() == 0) return 0;
      if (neg ^ a.neg) return (int)a.neg*2 - 1;
      if(len() != a.len()) return neg?a.len()-len():len
          ()-a.len();
      for (int i=len()-1;i>=0;i--) if (dig[i] != a.dig[i
          ]) {
        return neg?a.dig[i]-dig[i]:dig[i]-a.dig[i];
      return 0;
    inline void trim(){
      while(!dig.empty() and dig.back()==0) dig.
          pop back();
      if(dig.empty()) neg = false;
  public:
    BigInt(): dig(vector<lld>()), neg(false){}
    BigInt(lld a): dig(vector<lld>()) {
      neg = a<0; dig.push back(abs(a));</pre>
      trim();
    BigInt(const string& a): dig(vector<lld>()){
      assert(!a.empty()); neg = (a[0] == '-');
      for (int i=((int)(a.size()))-1;i>=neq;i-=LOG BASE)
        lld cur = 0:
```

```
for(int j=min(LOG BASE-1, i-neg);j>=0;j--) cur
        = cur*10+a[i-j]-'0';
    dig.push back(cur);
  } trim();
inline bool operator<(const BigInt& a)const{return</pre>
    cmp minus(a)<0;}
inline bool operator<=(const BigInt& a)const{return</pre>
     cmp_minus(a) <=0;}</pre>
inline bool operator==(const BigInt& a)const{return
     cmp minus(a) == 0; }
inline bool operator!=(const BigInt& a)const{return
     cmp minus(a)!=0;}
inline bool operator>(const BigInt& a)const{return
    cmp minus(a)>0;}
inline bool operator>=(const BigInt& a)const{return
    cmp minus(a)>=0;}
BigInt operator-() const {
  BigInt ret = *this;
  ret.neg ^= 1;
  return ret;
BigInt operator+(const BigInt& a) const {
  if (neg) return -(-(*this)+(-a));
  if(a.neg) return (*this)-(-a);
  int n = max(a.len(), len());
  BigInt ret; ret.dig.resize(n);
  11d pro = 0;
  for (int i=0;i<n;i++) {</pre>
    ret.dig[i] = pro;
    if(i < a.len()) ret.dig[i] += a.dig[i];</pre>
    if(i < len()) ret.dig[i] += dig[i];</pre>
    pro = 0;
    if(ret.dig[i] >= BASE) pro = ret.dig[i]/BASE;
    ret.dig[i] -= BASE*pro;
  if(pro != 0) ret.dig.push_back(pro);
  return ret;
BigInt operator-(const BigInt& a) const {
  if (neg) return -(-(*this) - (-a));
  if(a.neg) return (*this) + (-a);
  int diff = cmp_minus(a);
  if(diff < 0) return -(a - (*this));</pre>
  if(diff == 0) return 0;
  BigInt ret; ret.dig.resize(len(), 0);
  for(int i=0;i<len();i++) {</pre>
    ret.dig[i] += dig[i];
    if(i < a.len()) ret.dig[i] -= a.dig[i];</pre>
    if(ret.dig[i] < 0){
      ret.dig[i] += BASE;
      ret.dig[i+1]--;
    }
  ret.trim();
  return ret;
BigInt operator*(const BigInt& a) const {
  if(len()==0 or a.len()==0) return 0;
  BigInt ret; ret.dig.resize(len()+a.len()+1);
  ret.neg = neg ^ a.neg;
  for (int i=0; i < len(); i++) for (int j=0; j < a.len(); j</pre>
      ++) {
    ret.dig[i+j] += dig[i] * a.dig[j];
    if (ret.dig[i+j] >= BASE) {
      lld x = ret.dig[i+j] / BASE;
      ret.dig[i+j+1] += x;
      ret.dig[i+j] -= x * BASE;
    }
  ret.trim();
  return ret;
BigInt operator/(const BigInt& a) const {
  assert(a.len());
  if(len() < a.len()) return 0;</pre>
  BigInt ret; ret.dig.resize(len()-a.len()+1);
  ret.neg = a.neg;
  for(int i=len()-a.len();i>=0;i--){
    lld l = 0, r = BASE;
    while (r-1 > 1) {
      11d \ mid = (1+r) >> 1;
      ret.dig[i] = mid;
      if(ret*a <= (neg?-(*this):(*this))) 1 = mid;</pre>
      else r = mid;
```

```
ret.dig[i] = 1;
      ret.neg ^= neg; ret.trim();
    BigInt operator%(const BigInt& a) const {
      return (*this) - (*this) / a * a;
    friend BigInt abs(BigInt a) {
      a.neg = 1; return a;
    friend void swap(BigInt& a, BigInt& b) {
      swap(a.dig, b.dig); swap(a.neg, b.neg);
    friend istream& operator>>(istream& ss, BigInt& a){
      string s; ss >> s;
      a = s;
      return ss;
    friend ostream& operator<<(ostream& ss. const
        BigInt& a) {
      if(a.len() == 0) return ss << '0';</pre>
      if(a.neg) ss << '-';
      ss << a.dig.back();
      for(int i=a.len()-2;i>=0;i--) ss << setw(LOG BASE</pre>
          ) << setfill('0') << a.dig[i];
      return ss;
    inline void print() const {
      if(len() == 0) {putchar('0'); return;}
      if(neg) putchar('-');
      printf("%" PRINTF ARG, dig.back());
      for(int i=len()-2;i>=0;i--) printf("%0"
          LOG BASE STR PRINTF ARG, dig[i]);
    #undef PRINTF ARG
    #undef LOG_BASE_STR
}:
```

2.2 unordered_map

2.3 extc balance tree

```
#include <ext/pb ds/assoc_container.hpp>
using __gnu_pbds::tree;
using __gnu_pbds::rb_tree_tag;
using __gnu_pbds::ov_tree_tag;
using __gnu_pbds::splay_tree_tag;
using __gnu_pbds::null_type;
using
       __gnu_pbds::tree_order_statistics_node_update;
template<typename T>
using ordered set = tree<T, null type, less<T>,
    rb tree tag, tree order statistics node update>;
template<typename A, B>
using ordered_map = tree<A, B, less<A>, rb_tree_tag,
    tree order statistics node update>;
int main(){
  ordered set<int> ss;
  ordered map<int, int> mm;
  ss.insert(1);
  ss.insert(5);
  assert(*ss.find_by_order(0)==1);
assert(ss.order_of_key(-1)==0);
  assert(ss.order of key(87) == 2);
  return 0;
```

2.4 extc_heap

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
```

```
using __gnu_pbds::priority queue;
using __gnu_pbds::pairing_heap_tag;
using __gnu_pbds::binary_heap_tag;
using _
        gnu pbds::binomial heap tag;
using __gnu_pbds::rc_binomial_heap_tag;
using __gnu_pbds::thin_heap_tag;
int main(){
 priority queue<int,less<int>,pairing heap tag> pq1,
      pq2;
 pq1.push(1);
 pq2.push(2);
 pq1.join(pq2);
  assert (pq2.size() == 0);
 auto it = pq1.push(87);
 pq1.modify(it, 19);
 while(!pq1.empty()){
   pq1.top();
   pq1.pop();
 return 0;
```

2.5 SkewHeap

```
template<typename T, typename cmp=less<T> >
class SkewHeap{
private:
 struct SkewNode {
   T x;
   SkewNode *lc, *rc;
   SkewNode(T a=0):x(a), lc(nullptr), rc(nullptr){}
 } *root;
 cmp CMP ;
 size t count;
 SkewNode* Merge (SkewNode* a, SkewNode* b) {
   if(!a or !b) return a?a:b;
    if (CMP (a->x, b->x)) swap(a, b);
   a->rc = Merge(a->rc, b);
   swap(a->lc, a->rc);
   return a:
  void clear(SkewNode*& a) {
   if(!a) return;
    clear(a->lc); clear(a->rc);
   delete a; a = nullptr;
public:
 SkewHeap(): root(nullptr), count(0){}
 bool empty() {return count==0;}
 size t size() {return count;}
 T top() {return root->x;}
 void clear() {clear(root);count = 0;}
 void push(const T& x) {
   SkewNode* a = new SkewNode(x);
   count += 1;
   root = Merge(root, a);
 void join(SkewHeap& a){
   count += a.count; a.count = 0;
   root = Merge(root, a.root);
 void pop(){
    count -= 1;
    SkewNode* rt = Merge(root->lc, root->rc);
   delete root; root = rt;
 friend void swap(SkewHeap& a, SkewHeap& b){
    swap(a.root, b.root);
```

2.6 Disjoint Set

```
class DJS{
private:
   vector<int> fa, sz, sv;
   vector<pair<int*, int>> opt;
   inline void assign(int *k, int v) {
      opt.emplace_back(k, *k);
      *k = v;
   }
```

```
public:
  inline void init(int n) {
    fa.resize(n); iota(fa.begin(), fa.end(), 0);
    sz.resize(n); fill(sz.begin(), sz.end(), 1);
    opt.clear();
  int query(int x) {
    if(fa[x] == x) return x;
    return query(fa[x]);
  inline void merge(int a, int b) {
    int af = query(a), bf = query(b);
    if(af == bf) return;
    if(sz[af] < sz[bf]) swap(af, bf);</pre>
    assign(&fa[bf], fa[af]);
    assign(&sz[af], sz[af]+sz[bf]);
  inline void save() {sv.push_back((int)opt.size());}
  inline void undo() {
    int ls = sv.back(); sv.pop_back();
    while((int)opt.size() > ls) {
      pair<int*, int> cur=opt.back();
       *cur.first = cur.second;
      opt.pop back();
};
```

2.7 Treap

```
namespace Treap{
  \#define sz(x) ((x)? ((x)->size):0)
  \#define sm(x) ((x)? ((x)->sum):0)
  struct node{
    int size, cnt, sum;
    uint32_t pri;
    node *\overline{l}c, *rc;
    node(): size( 0 ), cnt( 0 ), sum( 0 ), pri( rand()
        ),
    lc( nullptr ), rc( nullptr ) {}
node( int x ): size( 1 ), cnt( x ), sum( x ), pri(
        rand()),
             lc( nullptr ), rc( nullptr ) {}
    void pull() {
      sum = cnt;
       if ( lc ) sum += lc->sum;
      if ( rc ) sum += rc->sum;
      size = 1;
      if ( lc ) size += lc->size;
       if ( rc ) size += rc->size;
  }:
  node* merge( node* L, node* R ) {
    if ( not L or not R ) return L ? L : R;
    if ( L->pri > R->pri ) {
      L->rc = merge( L->rc, R );
      L->pull();
      return L:
    } else {
      R->lc = merge(L, R->lc);
      R->pull();
      return R;
  void split by size( node* rt, int k, node*& L, node*&
       R ) {
    if ( not rt ) L = R = nullptr;
     else if( sz( rt->lc ) + 1 <= k ) {
       L = rt;
       split by size( rt->rc, k - sz( rt->lc ) - 1, L->
          rc, R);
      L->pull();
     } else {
       R = rt;
       split by size( rt->lc, k, L, R->lc );
       R->pull();
    }
  void split_by_sum( node* rt, int k, node*& L, node*&
       R ) {
     if ( not rt ) L = R = nullptr;
    else if ( sm( rt->lc ) + rt->cnt <= k ) {
      L = rt;
```

2.8 SparseTable

```
template<typename T, typename Cmp_=less<T>>
class SparseTable{
  vector<vector<T>> table;
  vector<int> lg;
  T cmp_(T a, T b) {
    return Cmp_()(a, b)?a:b;
public:
  void init(T arr[], int n){
    // 0-base
    lq.resize(n+1);
    lg[0] = -1, lg[1] = 0;
    for (int i=2;i<=n;i++) lg[i] = lg[i>>1]+1;
    table.resize(lg[n]+1);
    table[0].resize(n);
    for (int i=0;i<n;i++) table[0][i] = arr[i];</pre>
    for (int i=1;i<=lg[n];i++) {</pre>
      int len = 1 << (i-1), sz = 1 << i;
      table[i].resize(n-sz+1);
      for (int j=0; j<=n-sz; j++) {</pre>
        table[i][j] = cmp (table[i-1][j], table[i-1][j+
             len]);
    }
  T query(int 1, int r) {
     // 0-base [1, r)
    int wh = lg[r-1], len=1 << wh;
    return cmp_(table[wh][1], table[wh][r-len]);
};
```

2.9 Linear Basis

```
struct LinearBasis{
private:
  int n, sz;
  vector<llu> B;
  inline llu two(int x) {return ((llu)1) <<x;}</pre>
public:
  void init(int n ) {
    n = n ; B.clear();
    B.resize(n); sz = 0;
  void insert(llu x){
    // add x into B
    for (int i=n-1;i>=0;i--) if (two(i) & x) {
      if(B[i]) x ^= B[i];
      else{
        B[i] = x; sz++;
        for (int j=i-1; j>=0; j--)
          if(B[j] and two(j) & B[i])
            B[i] ^= B[j];
        for (int j=i+1; j<n; j++)</pre>
          if(two(i) & B[j])
            B[j] ^= B[i];
        break;
    }
  inline int size() {return sz;}
  bool check(llu x) {
     // is x in span(B) ?
    for (int i=n-1;i>=0;i--) if (two(i) & x) {
      if(B[i]) x ^= B[i];
      else return false;
```

```
return true;
}
llu kth_small(llu k) {
    /** 1-base would always > 0 **/
    /** should check it **/
    /* if we choose at least one element
        but size(B) (vectors in B) ==N(original elements)
        then we can't get 0 */
    llu ret = 0;
    for(int i=0;i<n;i++) if(B[i]) {
        if(k & 1) ret ^= B[i];
        k >>= 1;
    }
    return ret;
}
base;
```

3 Graph

3.1 BCC Edge

```
class BCC{
private:
  vector<int> low, dfn;
  int cnt;
  vector<bool> bcc;
  vector<vector<PII>> G:
  void dfs(int w, int f) {
    dfn[w] = cnt++;
    low[w] = dfn[w];
    for(auto i: G[w]){
      int u = i.FF, t = i.SS;
      if(u == f) continue;
      if(dfn[u]!=0){
       low[w] = min(low[w], dfn[u]);
      }else{
        dfs(u, w);
        low[w] = min(low[w], low[u]);
        if(low[u] > dfn[w]) bcc[t] = true;
    }
public:
  void init(int n, int m) {
    G.resize(n);
    fill(G.begin(), G.end(), vector<PII>());
    bcc.clear(); bcc.resize(m);
    low.clear(); low.resize(n);
    dfn.clear(); dfn.resize(n);
   cnt = 0;
  void add edge(int u, int v) {
    // should check for multiple edge
    G[u].PB({v, cnt});
    G[v].PB({u, cnt});
    cnt++;
  void solve() {cnt = 1;dfs(0, 0);}
  // the id will be same as insert order, 0-base
  bool is bcc(int x) {return bcc[x];}
```

3.2 BCC Vertex

```
class BCC{
private:
   vector<vector<pair<int,int>>> G;
   vector<int> dfn, low, id, sz;
   vector<bool> vis, ap;
   int n, ecnt, bcnt;
   void tarjan(int u, int f, int d) {
      vis[u] = true;
      dfn[u] = low[u] = d;
      int child = 0;
      for(auto e: G[u]) if(e.first != f) {
       int v = e.first;
      if(vis[v]) {
            low[u] = min(low[u], dfn[v]);
            }else{
```

```
tarjan(v, u, d+1);
        if(low[v] >= dfn[u]) ap[u] = true;
        low[u] = min(low[u], low[v]);
        child += 1;
    if(dfn[u] == 0 and child <= 1) ap[u] = false;</pre>
  void bfs bcc(int x) {
    // not sure
    queue<int> bfs;
    bfs.push(x); vis[x] = true;
    while(!bfs.empty()){
      int u = bfs.front(); bfs.pop();
      for(auto e: G[u]){
        id[e.second] = bcnt;
        if(ap[e.first] or vis[e.first]) continue;
        bfs.push(e.first); vis[e.first] = true;
        sz[bcnt] += 1;
   }
public:
 void init(int n ) {
   n = n ; G.clear(); G.resize(n);
    dfn.resize(n); low.resize(n);
    vis.clear(); vis.resize(n);
    ap.clear(); ap.resize(n);
    ecnt = 0, bcnt = 0;
 void add edge(int u, int v) {
   assert(0 \le u \text{ and } u \le n);
    assert(0 \le v and v \le n);
    G[u].emplace back(v, ecnt);
    G[v].emplace back(u, ecnt);
   ecnt += 1;
 void solve() {
    for(int i=0;i<n;i++) if(!vis[i]) {</pre>
      tarjan(i, i, 0);
    id.resize(ecnt);
   vis.clear(); vis.resize(n);
    sz.clear(); sz.resize(n);
    for (int i=0;i<n;i++) if (ap[i]) {</pre>
      bfs bcc(i); bcnt += 1;
 bool isAP(int x) {return ap[x];}
 int count() {return bcnt;}
  // bcc id of edges by insert order (0-base)
 int get id(int x) {return id[x];}
  // bcc size by bcc id
 int get size(int x) {return sz[x];}
} bcc;
```

3.3 Strongly Connected Components

```
class SCC{
private:
 int n, num ;
  vector<vector<int>> G, rG;
  vector<int> ord, num;
 bool vis[N];
  void dfs(int u) {
    if(vis[u]) return;
    vis[u]=1;
    for (auto v: G[u]) dfs(v);
    ord.PB(u);
  void rdfs(int u) {
   if(vis[u]) return;
    num[u] = num_;
    vis[u] = 1;
    for(auto v: rG[u]) rdfs(v);
public:
 inline void init(int n_){
    n=n_, num_=0;
    G.resize(n); rG.resize(n);
    num.resize(n);
    for (int i=0;i<n;i++) G[i].clear();</pre>
    for (int i=0;i<n;i++) rG[i].clear();</pre>
```

```
inline void add_edge(int st, int ed) {
   G[st].PB(ed);
   rG[ed].PB(st);
}
void solve() {
   memset(vis, 0, sizeof(vis));
   for(int i=0;i<n;i++) {
      if(!vis[i]) dfs(i);
   }
   reverse(ALL(ord));
   memset(vis, 0, sizeof(vis));
   for(auto i: ord) {
      if(!vis[i]) {
       rdfs(i);
       num_++;
      }
   }
   inline int get_id(int x) {return num[x];}
   inline int count() {return num_;}
}</pre>
```

3.4 Bipartite Matching

```
class BipartiteMatching{
  private:
     vector<int> X[N], Y[N];
     int fX[N], fY[N], n;
     bitset<N> walked;
    bool dfs(int x) {
      for(auto i:X[x]){
         if (walked[i]) continue;
         walked[i]=1;
         if(fY[i] ==-1||dfs(fY[i])){
           fY[i]=x; fX[x]=i;
           return 1;
         }
       return 0;
  public:
     void init(int n) {
      n=n;
       for (int i=0;i<n;i++) {</pre>
         X[i].clear();
         Y[i].clear();
         fX[i]=fY[i]=-1;
       walked.reset();
     void add edge(int x, int v) {
       X[x].push_back(y);
       Y[y].push back(y);
     int solve(){
      int cnt = 0;
       for (int i=0;i<n;i++) {</pre>
        walked.reset();
         if(dfs(i)) cnt++;
       // return how many pair matched
       return cnt;
};
```

3.5 MinimumCostMaximumFlow

```
{}
  }:
  int ori, edd;
  vector<vector<Edge>> G;
  vector<int> fa, wh;
  vector<bool> inq;
  vector<WeiT> dis;
  PCW SPFA() {
    fill(inq.begin(),inq.end(),false);
    fill(dis.begin(), dis.end(), INF WEI);
    queue<int> qq; qq.push(ori);
    dis[ori]=0;
    while(!qq.empty()){
      int u=qq.front();qq.pop();
      inq[u] = 0;
      for (int i=0;i<SZ(G[u]);++i) {</pre>
        Edge e=G[u][i];
        int v=e.to;
        WeiT d=e.wei;
        if(e.cap<=0||dis[v]<=dis[u]+d)
          continue;
        dis[v]=dis[u]+d;
        fa[v]=u,wh[v]=i;
        if(inq[v]) continue;
        qq.push(v);
        inq[v]=1;
    if (dis[edd] == INF WEI)
      return {-1,-1};
    CapT mw=INF CAP;
    for (int i=edd; i!=ori; i=fa[i])
      mw=min(mw,G[fa[i]][wh[i]].cap);
    for (int i=edd;i!=ori;i=fa[i]) {
      auto &eg=G[fa[i]][wh[i]];
      eg.cap-=mw;
      G[eg.to][eg.back].cap+=mw;
    return {mw,dis[edd]};
public:
  void init(int a, int b, int n) {
    ori=a,edd=b;
    G.clear();G.resize(n);
    fa.resize(n); wh.resize(n);
    inq.resize(n); dis.resize(n);
  void add edge(int st,int ed,WeiT w,CapT c) {
    G[st].emplace_back(ed,SZ(G[ed]),w,c);
    G[ed].emplace\_back(st,SZ(G[st])-1,-w,0);
  PCW solve(){
    CapT cc=0; WeiT ww=0;
    while(true){
      PCW ret=SPFA();
      if(ret.first==-1) break;
      cc+=ret.first;
      ww+=ret.second;
    return {cc,ww};
} mcmf;
```

3.6 MaximumFlow

```
class Dinic{
private:
 using CapT = int64_t;
  struct Edge{
    int to, rev;
   CapT cap;
 };
 int n, st, ed;
 vector<vector<Edge>> G;
  vector<int> lv;
 bool BFS() {
   fill(lv.begin(), lv.end(), -1);
    queue<int> bfs;
   bfs.push(st);
   lv[st] = 0;
    while(!bfs.empty()){
      int u = bfs.front(); bfs.pop();
      for(auto e: G[u]){
        if(e.cap <= 0 or lv[e.to]!=-1) continue;</pre>
```

```
lv[e.to] = lv[u] + 1;
        bfs.push(e.to);
      }
    return (lv[ed]!=-1);
  CapT DFS(int u, CapT f) {
    if(u == ed) return f;
    CapT ret = 0;
    for(auto& e: G[u]){
      if(e.cap <= 0 or lv[e.to]!=lv[u]+1) continue;</pre>
      CapT nf = DFS(e.to, min(f, e.cap));
      ret += nf; e.cap -= nf; f -= nf;
      G[e.to][e.rev].cap += nf;
      if(f == 0) return ret;
    if (ret == 0) lv[u] = -1;
    return ret;
public:
  void init(int n_, int st_, int ed_) {
    n = n_, st = st_, ed = ed_;
    G.resize(n); lv.resize(n);
    fill(G.begin(), G.end(), vector<Edge>());
  void add edge(int u, int v, CapT c) {
    G[u].push_back({v, (int)(G[v].size()), c});
    G[v].push\_back({u, (int)(G[u].size())-1, 0});
  CapT max_flow() {
    CapT ret = 0;
    while (BFS()) {
      CapT f = DFS(st, numeric_limits<CapT>::max());
      ret += f;
      if(f == 0) break;
    return ret;
} flow;
```

3.7 Kuhn Munkres

```
struct KM{
  static constexpr lld INF = 1LL<<60;</pre>
  lld w[N][N], lx[N], ly[N], slack[N];
  int match[N], n, vx[N], vy[N], step_;
void init(int n_) {
    n=n , step=0;
    memset(w,0,sizeof(w));
    memset(lx,0,sizeof(lx));
    memset(ly,0,sizeof(ly));
    memset(slack, 0, sizeof(slack));
    memset(match, 0, sizeof(match));
    memset(vx,0,sizeof(vx));
    memset(vy,0,sizeof(vy));
  void add edge(int u,int v,lld w) {w[u][v]=w;}
  bool dfs(int x) {
    vx[x] = step_;
for (int i = 0; i < n; ++i) {</pre>
      if (vy[i]==step ) continue;
      if (lx[x] + ly[i] > w[x][i]) {
        slack[i] = min(slack[i], lx[x] + ly[i] - w[x][i]
             ]);
        continue;
      vy[i] = step_;
      if (match[i] == -1 || dfs(match[i])) {
        match[i] = x;
        return true;
      }
    }
    return false;
  lld solve() {
    fill n(match, n, -1);
    fill n(lx, n, -INF);
    fill_n(ly, n, 0);
    for (int i = 0; i < n; ++i)</pre>
      for (int j = 0; j < n; ++j)</pre>
        lx[i] = max(lx[i], w[i][j]);
    for (int i = 0; i < n; ++i) {
      fill_n(slack, n, INF);
      while (true) {
```

```
step_++;
    if (dfs(i)) break;
    lld dlt = INF;
    for (int j = 0; j < n; ++j) if (vy[j] != step_)
        dlt = min(dlt, slack[j]);
    for (int j = 0; j < n; ++j) {
        if (vx[j]==step_) lx[j] -= dlt;
        if (vy[j]==step_) ly[j] += dlt;
        else slack[j] -= dlt;
    }
}
lld res = 0;
for (int i = 0; i < n; ++i) res += w[match[i]][i];
    return res;
}
km;</pre>
```

3.8 2-SAT

```
// 2-SAT solver based on Kosaraju's algorithm.
// Variables are 0-based. Positive variables are stored
     in vertices 2n, corresponding negative variables
// TODO: This is quite slow (3x-4x slower than Gabow's
    algorithm)
struct TwoSat {
 int n;
  vector<vector<int> > adj, radj, scc;
 vector<int> sid, vis, val;
 stack<int> stk;
 int scnt;
  // n: number of variables, including negations
 \label{eq:twoSat} TwoSat\left( \begin{array}{c} \textbf{int} \\ \end{array} n\right): \ n\left( n\right) \text{, adj}\left( n\right) \text{, radj}\left( n\right) \text{, sid}\left( n\right) \text{, vis}\left( n\right) \text{,} \\
       val(n, -1) {}
    adds an implication
 void impl(int x, int y) { adj[x].push back(y); radj[y
       ].push back(x); }
  // adds a disjunction
 void vee (int x, int y) { impl(x^1, y); impl(y^1, x);
      }
  // forces variables to be equal
 void eq(int x, int y) { impl(x, y); impl(y, x); impl(
       x^1, y^1; impl(y^1, x^1);
  // forces variable to be true
  void tru(int x) { impl(x^1, x); }
  void dfs1(int x) {
    if (vis[x]++) return;
    for (int i = 0; i < adj[x].size(); i++) {</pre>
      dfs1(adj[x][i]);
    stk.push(x);
  void dfs2(int x) {
    if (!vis[x]) return; vis[x] = 0;
    sid[x] = scnt; scc.back().push back(x);
    for (int i = 0; i < radj[x].size(); i++) {</pre>
      dfs2(radj[x][i]);
  // returns true if satisfiable, false otherwise
  // on completion, val[x] is the assigned value of
       variable x
  // note, val[x] = 0 implies val[x^1] = 1
 bool two sat() {
    scnt = 0;
    for (int i = 0; i < n; i++) {</pre>
      dfs1(i);
    while (!stk.empty()) {
      int v = stk.top(); stk.pop();
      if (vis[v]) {
        scc.push_back(vector<int>());
         dfs2(v);
        scnt++;
      }
    for (int i = 0; i < n; i += 2) {</pre>
      if (sid[i] == sid[i+1]) return false;
    vector<int> must(scnt);
    for (int i = 0; i < scnt; i++) {</pre>
      for (int j = 0; j < scc[i].size(); j++) {</pre>
        val[scc[i][j]] = must[i];
```

3.9 HeavyLightDecomp

```
#define REP(i, s, e) for(int i = (s); i <= (e); i++)
#define REPD(i, s, e) for(int i = (s); i \ge (e); i = -(e)
const int MAXN = 100010;
const int LOG = 19;
struct HLD{
  int n;
  vector<int> g[MAXN];
  int sz[MAXN], dep[MAXN];
  int ts, tid[MAXN], tdi[MAXN], tl[MAXN], tr[MAXN];
     ts : timestamp , useless after yutruli
  // tid[u]: pos. of node u in the seq.
     tdi[i]: node at pos i of the seq.
     tl , tr[ u ] : subtree interval in the seq. of
      node u
  int prt[MAXN][LOG], head[MAXN];
  // head[ u ] : head of the chain contains u
  void dfssz(int u, int p) {
    dep[u] = dep[p] + 1;
    prt[u][0] = p; sz[u] = 1; head[u] = u;
    for(int& v:g[u]) if(v != p){
      dep[v] = dep[u] + 1;
      dfssz(v, u);
      sz[u] += sz[v];
    }
  void dfshl(int u) {
    ts++;
    tid[u] = tl[u] = tr[u] = ts;
    tdi[tid[u]] = u;
    sort (ALL (q[u]),
         [&] (int a, int b) {return sz[a] > sz[b];});
    bool flag = 1;
    for(int& v:g[u]) if(v != prt[u][0]){
      if(flag) head[v] = head[u], flag = 0;
      dfshl(v);
      tr[u] = tr[v];
    }
  inline int lca(int a, int b) {
    if(dep[a] > dep[b]) swap(a, b);
    int diff = dep[b] - dep[a];
    REPD(k, LOG-1, 0) if(diff & (1 << k)){}
     b = prt[b][k];
    if(a == b) return a;
    REPD(k, LOG-1, 0) if(prt[a][k] != prt[b][k]){
     a = prt[a][k]; b = prt[b][k];
    return prt[a][0];
  void init( int _n ){
   n = _n; REP(i, 1, n) g[i].clear();
  void addEdge( int u , int v ){
    g[ u ].push back( v );
    g[ v ].push back( u );
  void yutruli(){
    dfssz(1, 0);
    t.s = 0:
    dfshl(1);
    REP(k, 1, LOG-1) REP(i, 1, n)
     prt[i][k] = prt[prt[i][k-1]][k-1];
  vector< PII > getPath( int u , int v ){
    vector< PII > res;
    while( tid[ u ] < tid[ head[ v ] ] ) {</pre>
     res.push back( PII(tid[ head[ v ] ] , tid[ v ]) )
      v = prt[ head[ v ] ][ 0 ];
    res.push back( PII( tid[ u ] , tid[ v ] ) );
    reverse( ALL( res ) );
    return res;
    /* res : list of intervals from u to v
```

```
* u must be ancestor of v
* usage :
    * vector< PII >& path = tree.getPath( u , v )
    * for( PII tp : path ) {
    * int 1 , r;tie( 1 , r ) = tp;
    * upd( 1 , r );
    * uu = tree.tdi[ 1 ] , vv = tree.tdi[ r ];
    * uu ~> vv is a heavy path on tree
    * }
    */
}
tree;
```

3.10 MaxClique

```
struct MaxClique {
    int n, deg[maxn], ans;
    bitset<maxn> adj[maxn];
    vector<pair<int, int>> edge;
    void init(int _n) {
         n = _n;
         for (int i = 0; i < n; ++i) adj[i].reset();</pre>
         for (int i = 0; i < n; ++i) deg[i] = 0;</pre>
         edge.clear();
    void add edge(int a, int b) {
         edge.emplace_back(a, b);
         ++deg[a]; ++deg[b];
    int solve() {
         vector<int> ord;
         for (int i = 0; i < n; ++i) ord.push back(i);</pre>
         sort(ord.begin(), ord.end(), [&](const int &a,
             const int &b) { return deg[a] < deg[b]; });</pre>
         vector<int> id(n);
         for (int i = 0; i < n; ++i) id[ord[i]] = i;</pre>
         for (auto e : edge) {
             int u = id[e.first], v = id[e.second];
             adj[u][v] = adj[v][u] = true;
         bitset<maxn> r, p;
for (int i = 0; i < n; ++i) p[i] = true;</pre>
         ans = 0;
         dfs(r, p);
         return ans;
    void dfs(bitset<maxn> r, bitset<maxn> p) {
         if (p.count() == 0) return ans = max(ans, (int)
             r.count()), void();
         if ((r | p).count() <= ans) return;
int now = p._Find_first();</pre>
         bitset<maxn> cur = p & ~adj[now];
         for (now = cur. Find first(); now < n; now =</pre>
             cur. Find next(now)) {
             r[now] = \overline{true};
             dfs(r, p & adj[now]);
             r[now] = false;
             p[now] = false;
    }
};
```

4 Math

4.1 Prime Table

```
\begin{array}{c} 1002939109, 1020288887, 1028798297, 1038684299, \\ 1041211027, 1051762951, 1058585963, 1063020809, \\ 1147930723, 1172520109, 1183835981, 1187659051, \\ 1241251303, 1247184097, 1255940849, 1272759031, \\ 1287027493, 1288511629, 1294632499, 1312650799, \\ 1868732623, 1884198443, 1884616807, 1885059541, \\ 1909942399, 1914471137, 1923951707, 1925453197, \\ 1979612177, 1980446837, 1989761941, 2007826547, \\ 2008033571, 2011186739, 2039465081, 2039728567, \\ 2093735719, 2116097521, 2123852629, 2140170259, \\ 3148478261, 3153064147, 3176351071, 3187523093, \\ 3196772239, 3201312913, 3203063977, 3204840059, \\ 3210224309, 3213032591, 3217689851, 3218469083, \\ 3219857533, 3231880427, 3235951699, 3273767923, \\ 3276188869, 3277183181, 3282463507, 3285553889, \\ 3319309027, 3327005333, 3327574903, 3341387953, \\ 3373293941, 3380077549, 3380892997, 3381118801 \\ \end{array}
```

4.2 $\left| \frac{n}{i} \right|$ Enumeration

```
T_0 = 1, T_{i+1} = \lfloor \frac{n}{\lfloor \frac{n}{T_i + 1} \rfloor} \rfloor
```

4.3 ax+by=gcd

```
// By Adrien1018 (not knowing how to use.
// ax+ny = 1, ax+ny == ax == 1 (mod n)
tuple<int, int, int> extended_gcd(int a, int b) {
   if (!b) return make_tuple(a, 1, 0);
   int d, x, y;
   tie(d, x, y) = extended_gcd(b, a % b);
   return make_tuple(d, y, x - (a / b) * y);
}
```

4.4 Pollard Rho

```
// does not work when n is prime
11d modit(lld x,lld mod) {
  if(x \ge mod) x = mod;
  //if(x<0) x+=mod;
  return x;
1ld mult(lld x,lld y,lld mod) {
  11d s=0, m=x mod;
  while(y) {
    if(y&1) s=modit(s+m, mod);
    \forall >>=1;
    m=modit(m+m, mod);
  return s;
11d f(11d x,11d mod) {
  return modit (mult (x, x, mod) +1, mod);
lld pollard rho(lld n) {
  if(!(n&1)) return 2;
  while (true) {
    11d y=2, x=rand()%(n-1)+1, res=1;
    for (int sz=2; res==1; sz*=2) {
      for (int i=0; i<sz && res<=1; i++) {</pre>
        x = f(x, n);
        res = \underline{gcd}(abs(x-y), n);
      y = x;
    if (res!=0 && res!=n) return res;
```

4.5 Pi Count (Linear Sieve)

```
static constexpr int N = 1000000 + 5;
lld pi[N];
vector<int> primes;
bool sieved[N];
lld cube_root(lld x) {
  11d s = static_cast<11d>(cbrt(x - static_cast<long))</pre>
      double>(0.1)));
  while(s*s*s <= x) ++s;</pre>
  return s-1;
lld square_root(lld x){
  11d s = static_cast<11d>(sqrt(x - static_cast<long</pre>
      double>(0.1)));
  while (s*s <= x) ++s;
  return s-1;
void init(){
  primes.reserve(N);
  primes.push back(1);
  for (int i=2;i<N;i++) {</pre>
    if(!sieved[i]) primes.push_back(i);
    pi[i] = !sieved[i] + pi[i-1];
    for(int p: primes) if(p > 1) {
      if(p * i >= N) break;
      sieved[p * i] = true;
      if(p % i == 0) break;
```

```
}
lld phi(lld m, lld n) {
  static constexpr int MM = 80000, NN = 500;
  static lld val[MM][NN];
  if(m < MM and n < NN and val[m][n]) return val[m][n]</pre>
      - 1;
 if(n == 0) return m;
  if(primes[n] >= m) return 1;
  lld ret = phi(m, n - 1) - phi(m / primes[n], n - 1);
  if (m < MM and n < NN) val[m][n] = ret + 1;</pre>
  return ret;
lld pi count(lld);
11d P2(11d m, 11d n) {
  11d sm = square root(m), ret = 0;
  for(lld i = n+1; primes[i] <= sm; i++)</pre>
    ret += pi_count(m / primes[i]) - pi_count(primes[i
        ]) + 1;
  return ret;
lld pi_count(lld m) {
  if(m < N) return pi[m];</pre>
  11d n = pi_count(cube_root(m));
  return phi(m, n) + n - 1 - P2(m, n);
```

4.6 NloglogN Sieve

```
void Sieve(int n) {
  for(int i=2;i<=n;i++) {
    if(notprime[i]) continue;
    primes.push_back(i);
    for(int j=i*i;j<=n;j+=i) notprime[i]=true;
  }
}</pre>
```

4.7 Range Sieve

4.8 Miller Rabin

```
lld modu(lld a, lld m) {
 while(a >= m) a -= m;
 return a;
lld mul(lld a, lld b, lld m) {
 if(a < b) swap(a, b);
 11d ret = 0;
 while(b){
   if(b & 1) ret = modu(ret+a, m);
    a = modu(a+a, m);
   b >>= 1:
 return ret;
lld qPow(lld a, lld k, lld m) {
 11d ret = 1;
 a %= m;
 while (k) {
   if(k & 1) ret = mul(ret, a, m);
```

```
a = mul(a, a, m);
    k >>= 1:
  return modu(ret, m);
bool witness(lld a, lld s, int t, lld n) {
  lld b = qPow(a, s, n);
  if(b == 0) return false;
  while(t--){
    lld bb = mul(b, b, n);
    if(bb == 1 and b != 1 and b != n-1) return true;
    b = bb:
  return b != 1;
bool miller rabin(lld n) {
  if(n < 2) return false;</pre>
  if(!(n & 1)) return (n==2);
  lld x = n-1; int t = 0;
  while(!(x&1)) x >>= 1, t++;
  lld sprp[]
      {2,325,9375,28178,450775,9780504,1795265022};
  for (int i=0;i<7;i++) {</pre>
    if(witness(sprp[i]%n, x, t, n)) return false;
  return true;
```

4.9 Inverse Element

```
// x's inverse mod k
long long GetInv(long long x, long long k) {
    // k is prime: euler_(k)=k-1
    return qPow(x, euler_phi(k)-1);
}
// if you need [1, x] (most use: [1, k-1]
void solve(int x, long long k) {
    inv[1] = 1;
    for(int i=2;i<x;i++)
        inv[i] = ((long long)(k - k/i) * inv[k % i]) % k;
}</pre>
```

4.10 Euler Phi Function

```
extended euler:
   a^b mod p
   if gcd(a, p) ==1: a^(b%phi(p))
   elif b < phi(p): a^b mod p
   else a^(b%phi(p) + phi(p))
lld euler phi(int x){
  11d r=1;
  for (int i=2;i*i<=x;++i) {</pre>
    if(x%i==0){
      x/=i:
       r*=(i-1);
       while (x%i==0) {
        x/=i;
         r*=i;
      }
  if (x>1) r*=x-1;
  return r;
vector<int> primes;
bool notprime[N];
lld phi[N];
void euler_sieve(int n) {
  for (int i=2;i<n;i++) {</pre>
    if(!notprime[i]){
      primes.push back(i);
      phi[i] = i - \overline{1};
    for(auto j: primes) {
  if(i*j >= n) break;
       notprime[i*j] = true;
      phi[i*j] = phi[i] * phi[j];
       if(i % j == 0) {
         phi[i*j] = phi[i] * j;
```

```
| break;
| }
| }
| }
```

4.11 Gauss Elimination

```
typedef long double llf;
const int N = 300;
const llf EPS = 1e-8;
// make m[i][i] = x, m[i][j] = 0
// v is for solving equation:
// for(int i=0;i<n;i++) ans[pos[i]] = val[i]/mtx[i][pos
    [i]];
// for(int i=0;i<n;i++) cout << ans[i] << '\n';
bool Gauss(llf m[N][N], llf v[N], int n, int pos[N]){
 for (int i=0; i<n; i++) {</pre>
    int x=-1, y=-1; llf e = 0;
for(int j=i;j<n;j++) for(int k=i;k<n;k++) {</pre>
      if (fabs(m[j][pos[k]])>e) {
        e = fabs(m[j][pos[k]]);
        x = j, y = k;
      }
    if(x==-1 or y==-1) return false;
    swap(m[x], m[i]);
    swap(v[x], v[i]);
    swap(pos[y], pos[i]);
    for (int j=i+1; j<n; j++) {</pre>
      llf xi = m[j][pos[i]]/m[i][pos[i]];
      for(int k=0; k<n; k++) m[j][pos[k]] -= xi*m[i][pos[</pre>
          k]];
      v[j] = xi*v[i];
    }
  for (int i=n-1;i>=0;i--) {
    for (int j=i-1; j>=0; j--) {
      llf xi = m[j][pos[i]]/m[i][pos[i]];
      for (int k=0; k<n; k++) m[j][pos[k]] -= xi*m[i][pos[</pre>
          k]];
      v[j] = xi*v[i];
   }
 return true;
```

4.12 Fast Fourier Transform

```
polynomial multiply:
   FFT(a, N, true);
   FFT(b, N, true);
   for (int i=0; i < MAXN; i++) c[i] = a[i] * b[i];
   FFT(c, N, false);
   yeah~ go result in c
   (N must be 2^k and >= len(a) + len(b))
typedef long double llf;
typedef complex<llf> cplx;
const int MAXN = 262144;
const llf PI = acos((llf)-1);
cplx A[MAXN], B[MAXN], C[MAXN], omega[MAXN+1];
void init omega() {
  const cplx I = \{0, 1\};
  for (int i=0;i<=MAXN;i++) omega[i] = exp(i*2*PI/MAXN*I</pre>
      );
void FFT(cplx arr[], int n, bool ori){
  // n must be 2^k
  int theta = MAXN / n;
  for (int len=n;len>=2;len>>=1) {
    int tot = len>>1;
    for (int i=0;i<tot;i++) {</pre>
      cplx omg = omega[ori?i*theta%MAXN:MAXN-(i*theta%
         MAXN)];
      for(int j=i;j<n;j+=len) {</pre>
        int k = j+tot;
```

```
cplx x = arr[j] - arr[k];
    arr[j] += arr[k];
    arr[k] = omg * x;
}
theta = (theta * 2) % MAXN;
}
int i = 0;
for(int j=1;j<n-1;j++) {
    for(int k=n>>1;k>(i^=k);k>>=1);
    if(j < i) swap(arr[j], arr[i]);
}
if(ori) return;
for(int i=0;i<n;i++) arr[i] /= n;
}</pre>
```

4.13 Chinese Remainder

```
lld crt(lld ans[], lld pri[], int n) {
  11d M = 1;
  for (int i=0; i < n; i++) M *= pri[i];</pre>
  11d ret = 0;
  for (int i=0;i<n;i++) {</pre>
    lld inv = (gcd(M/pri[i], pri[i]).first + pri[i])%
        pri[i];
    ret += (ans[i] * (M/pri[i]) %M * inv) %M;
   ret %= M;
  return ret;
}
Another:
x = a1 % m1
x = a2 \% m2
g = gcd(m1, m2)
assert((a1-a2)%g==0)
[p, q] = exgcd(m2/g, m1/g)
return a2+m2*(p*(a1-a2)/g)
0 <= x < 1cm(m1, m2)
```

4.14 NTT

```
' Remember coefficient are mod P
/* p=a*2^n+1
      2^n
65536
  n
                                      root
                                 а
                     65537
   16
                                      3 */
   20 1048576
                     7340033
// (must be 2^k)
template<LL P, LL root, int MAXN>
struct NTT{
  static LL bigmod(LL a, LL b) {
    LL res = 1;
    for (LL bs = a; b; b >>= 1, bs = (bs * bs) % P)
     if(b&1) res=(res*bs)%P;
    return res;
  static LL inv(LL a, LL b) {
    if(a==1)return 1;
    return (((LL)(a-inv(b%a,a))*b+1)/a)%b;
  LL omega[MAXN+1];
  NTT() {
    omega[0] = 1;
    LL r = bigmod(root, (P-1)/MAXN);
    for (int i=1; i<=MAXN; i++)</pre>
      omega[i] = (omega[i-1]*r)%P;
  // n must be 2^k
  void tran(int n, LL a[], bool inv_ntt=false) {
    int basic = MAXN / n , theta = basic;
    for (int m = n; m >= 2; m >>= 1) {
      int mh = m >> 1;
      for (int i = 0; i < mh; i++) {</pre>
        LL w = omega[i*theta%MAXN];
        for (int j = i; j < n; j += m) {
  int k = j + mh;</pre>
          LL x = a[j] - a[k];
          if (x < 0) x += P;
          a[j] += a[k];
          if (a[j] > P) a[j] -= P;
          a[k] = (w * x) % P;
```

```
}
theta = (theta * 2) % MAXN;

int i = 0;
for (int j = 1; j < n - 1; j++) {
    for (int k = n >> 1; k > (i ^= k); k >>= 1);
    if (j < i) swap(a[i], a[j]);
}
if (inv_ntt) {
    LL ni = inv(n,P);
    reverse( a+1 , a+n );
    for (i = 0; i < n; i++)
        a[i] = (a[i] * ni) % P;
}
};
const LL P=2013265921, root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;
```

4.15 DiscreteLog

```
// Baby-step Giant-step Algorithm
// a x + bv = a
void exgcd(long long x, long long y, long long &g,
   long long &a, long long &b) {
  if (y == 0)
   g = x, a = 1, b = 0;
  else
   exgcd(y, x%y, g, b, a), b = (x/y) * a;
long long inverse(long long x, long long p) {
 long long g, b, r;
 exgcd(x, p, g, r, b);
if (g < 0) r = -r;
 return (r%p + p)%p;
long long BSGS (long long P, long long B, long long N) {
 // find B^L = N \mod P
  unordered map<long long, int> R;
  long long sq = (long long) sqrt(P);
 long long t = 1, f;
 for (int i = 0; i < sq; i++) {</pre>
   if (t == N)
     return i;
    if (!R.count(t))
   R[t] = i;

t = (t * B) % P;
 f = inverse(t, P);
 for (int i = 0; i <= sq+1; i++) {</pre>
   if (R.count(N))
      return i * sq + R[N];
   N = (N * f) % P;
 return -1;
```

5 Geometry

5.1 Point Class

```
template<typename T>
struct Point{
  typedef long double llf;
  static constexpr llf EPS = 1e-8;
  T x, y;
  Point(T _=0, T _=0): x(_), y(__){}
  template<typename T2>
    Point(const Point<T2>& a): x(a.x), y(a.y){}
  inline llf theta() const {
    return atan2((llf)y, (llf)x);
  }
  inline llf dis() const {
    return hypot((llf)x, (llf)y);
  }
  inline llf dis(const Point& o) const {
    return hypot((llf)(x-o.x), (llf)(y-o.y));
  }
```

```
Point operator-(const Point& o) const {
     return Point(x-o.x, y-o.y);
   Point operator -= (const Point& o) {
    x-=o.x, y-=o.y;
return *this;
   Point operator+(const Point& o) const {
    return Point(x+o.x, y+o.y);
   Point operator+=(const Point& o) {
     x+=o.x, y+=o.y;
return *this;
   Point operator*(const T& k) const {
    return Point(x*k, y*k);
   Point operator*=(const T& k) {
    x*=k, y*=k;
    return *this;
  Point operator/(const T& k) const {
     return Point(x/k, y/k);
   Point operator/=(const T& k) {
    x/=k, y/=k;
     return *this;
   Point operator-() const {
    return Point(-x, -y);
   Point rot90() const {
     return Point(-y, x);
   template<typename T2>
  bool in(const Circle<T2>& a) const {
     /* Add struct Circle at top */
     return a.o.dis(*this) +EPS <= a.r;</pre>
   bool equal(const Point& o, true type) const {
    return fabs(x-o.x) < EPS and fabs(y-o.y) < EPS;
  bool equal(const Point& o, false_type) const {
     return tie(x, y) == tie(o.x, o.y);
  bool operator==(const Point& o) const {
     return equal(o, is floating point<T>());
  bool operator!=(const Point& o) const {
    return ! (*this == 0);
  bool operator<(const Point& o) const {</pre>
     return theta() < o.theta();</pre>
     // sort like what pairs did
     // if(is floating point<T>()) return fabs(x-o.x)<
         EPS?y<o.y:x<o.x;
     // else return tie(x, y) < tie(o.x, o.y);
   friend inline T cross(const Point& a, const Point& b)
     return a.x*b.y - b.x*a.y;
   friend inline T dot(const Point& a, const Point &b) {
     return a.x*b.x + a.y*b.y;
   friend ostream& operator<<(ostream& ss, const Point&</pre>
     ss<<"("<<o.x<<", "<<o.y<<")";
     return ss;
};
```

5.2 Circle Class

```
template<typename T>
struct Circle{
    static constexpr llf EPS = 1e-8;
    Point<T> o;
    T r;
    vector<Point<1lf>> operator&(const Circle& aa)const{
        // https://www.cnblogs.com/wangzming/p/8338142.html
        llf d=o.dis(aa.o);
```

```
if(d > r+aa.r+EPS or d < fabs(r-aa.r)-EPS) return
{};

llf dt = (r*r - aa.r*aa.r)/d, d1 = (d+dt)/2;

Point<llf> dir = (aa.o-o); dir /= d;

Point<llf> pcrs = dir*d1 + o;

dt=sqrt(max(0.0L, r*r - d1*d1)), dir=dir.rot90();

return {pcrs + dir*dt, pcrs - dir*dt};
};
```

5.3 Line Class

```
const Point<long double> INF_P(-1e20, 1e20);
const Point<long double> NOT EXIST(1e20, 1e-20);
template<tvpename T>
struct Line{
 static constexpr long double EPS = 1e-8;
  // ax+by+c = 0
  T a, b, c;
 Line(): a(0), b(1), c(0) {}
Line(T , T , T ): a(
   ine(T _, T _, T __): a(_), b(_),
  assert(fabs(a)>EPS or fabs(b)>EPS);
 Line(T , T , T
                                     _), c(___){
  template<typename T2>
   Line(const Line<T2>& x): a(x.a), b(x.b), c(x.c) {}
  typedef Point<long double> Pt;
 bool equal(const Line& o, true type) const {
    return fabs(a-o.a) < EPS and fabs(b-o.b) < EPS and
        fabs(c-o.b) < EPS;
 bool euqal(const Line& o, false type) const {
   return a==o.a and b==o.b and c==o.c;
 bool operator==(const Line& o) const {
   return euqal(o, is floating point<T>());
 bool operator!=(const Line& o) const {
    return ! (*this == 0);
 friend inline bool on_line__(const Point<T>& p, const
       Line& 1, true_type) {
    return fabs(1.a*p.x + 1.b*p.y + 1.c) < EPS;
  friend inline bool on_line__(const Point<T>& p, const
      Line& l, false type) {
    return 1.a*p.x + 1.b*p.y + 1.c == 0;
  friend inline bool on line(const Point<T>&p const
    return on line (p, l, is floating point<T>());
  friend inline bool is_parallel__(const Line& x, const
       Line& y, true_type) {
    return fabs(x.a*y.b - x.b*y.a) < EPS;</pre>
 friend inline bool is_parallel__(const Line& x, const
       Line& y, false_type) {
    return x.a*y.b == x.b*y.a;
 friend inline bool is parallel(const Line& x, const
      Line& y) {
    return is_parallel__(x, y, is_floating_point<T>());
  friend inline Pt get inter(const Line& x, const Line&
      у) {
    typedef long double llf;
    if(x==y) return INF P;
    if(is_parallel(x, y)) return NOT_EXIST;
    llf delta = x.a*y.b - x.b*y.a;
    llf delta x = x.b*y.c - x.c*y.b;
   llf delta_y = x.c*y.a - x.a*y.c;
return Pt(delta_x / delta, delta_y / delta);
  friend ostream& operator<<(ostream& ss, const Line& o</pre>
    ss<<o.a<<"x+"<<o.b<<"y+"<<o.c<<"=0";
    return ss;
 }
template<typename T>
inline Line<T> get line(const Point<T>& a, const Point<</pre>
  return Line<T>(a.y-b.y, b.x-a.x, (b.y-a.y) *a.x-(b.x-a
      .x) *a.y);
```

5.4 Triangle Circumcentre

}

5.5 2D Convex Hull

```
template<tvpename T>
class ConvexHull 2D{
private:
  typedef Point<T> PT;
  vector<PT> dots;
  struct myhash{
    uint64 t operator()(const PT& a) const {
      uint\overline{6}4 t xx=0, yy=0;
      memcpy(&xx, &a.x, sizeof(a.x));
      memcpy(&yy, &a.y, sizeof(a.y));
      uint64 t ret = xx*17+yy*31;
      ret = (\text{ret } ^ (\text{ret } >> 16))*0x9E3779B1;
      ret = (ret ^ (ret >> 13)) *0xC2B2AE35;
      ret = ret ^ xx;
      return (ret ^ (ret << 3)) * yy;</pre>
  } ;
  unordered set<PT, myhash> in hull;
public:
  inline void init() {in hull.clear(); dots.clear();}
  void insert(const PT& x) {dots.PB(x);}
  void solve(){
    sort(ALL(dots), [](const PT& a, const PT& b){
      return tie(a.x, a.y) < tie(b.x, b.v);</pre>
    });
    vector<PT> stk(SZ(dots)<<1);
    int top = 0;
    for (auto p: dots) {
      while(top >= 2 and cross(p-stk[top-2], stk[top
          -1]-stk[top-2]) <= 0)
        top --;
      stk[top++] = p;
    for (int i=SZ(dots)-2, t = top+1;i>=0;i--) {
      while(top >= t and cross(dots[i]-stk[top-2], stk[
         top-1]-stk[top-2]) <= 0)
      stk[top++] = dots[i];
    stk.resize(top-1);
    swap(stk, dots);
    for(auto i: stk) in hull.insert(i);
  vector<PT> get() {return dots;}
  inline bool in it(const PT& x) {
    return in hull.find(x)!=in hull.end();
```

5.6 2D Farthest Pair

```
// stk is from convex hull
n = (int) (stk.size());
int pos = 1, ans = 0; stk.push_back(arr[0]);
for(int i=0;i<n;i++) {</pre>
```

5.7 2D Cosest Pair

```
struct Point{
  llf x, y;
  llf dis;
} arr[N];
inline llf get dis(Point a, Point b){
 return hypot(a.x-b.x, a.y-b.y);
llf solve(){
 int cur = rand()%n;
  for(int i=0;i<n;i++) arr[i].dis = get dis(arr[cur],</pre>
     arr[i]);
 sort(arr, arr+n, [](Point a, Point b){return a.dis <
      b.dis; });
 llf ans = 1e50;
 for (int i=0; i<n; i++) {</pre>
   for (int j=i+1; j<n; j++) {</pre>
      if(arr[j].dis - arr[i].dis > ans) break;
      ans = min(ans, get_dis(arr[i], arr[j]));
 return ans;
```

5.8 SimulateAnnealing

```
double getY(double);
int main() {
   int rr, ll;
   default_random_engine rEng(time(NULL));
   uniform_real_distribution<double> Range(-1,1);
   uniform_real_distribution<double> expR(0,1);
   auto Random=bind(Range,rEng), expRand=bind(expR,rEng)
   ;
   int step=0;
   double pace=rr-ll, mini=0.95; // need to search for
        it
   double x=max(min(Random()*pace+ll, rr), ll), y=getY(x
        );
   while(pace>=1e-7) {
        double newX = max(min(x + Random()*pace, rr), ll);
        double newY = getY(newX);
        if(newY < y || expRand() < exp(-step))
            x=newX, y=newY;
        step++;
        pace*=mini;
    }
}</pre>
```

5.9 Ternary Search on Integer

```
int TernarySearch(int 1, int r) {
    // (1, r]
    while (r - 1 > 1) {
        int mid = (1 + r) >> 1;
        if (f(mid) > f(mid + 1)) r = mid;
        else 1 = mid;
    }
    return 1+1;
}
```

5.10 Minimum Covering Circle

```
template<typename T>
Circle<llf> MinCircleCover(const vector<Point<T>>& pts)
  random shuffle (ALL (pts));
  Circle < llf > c = {pts[0], 0};
  int n = SZ(pts);
  for (int i=0; i < n; i++) {</pre>
    if(pts[i].in(c)) continue;
    c = \{pts[i], 0\};
    for (int j=0; j<i; j++) {</pre>
      if(pts[j].in(c)) continue;
       c.o = (pts[i] + pts[j]) / 2;
       c.r = pts[i].dis(c.o);
       for (int k=0; k<j; k++) {</pre>
        if(pts[k].in(c)) continue;
         c = get_circum(pts[i], pts[j], pts[k]);
    }
  return c;
```

5.11 KDTree (Nearest Point)

```
const int MXN = 100005;
struct KDTree {
  struct Node {
    int x,y,x1,y1,x2,y2;
    int id,f;
   Node *L, *R;
  }tree[MXN];
  int n;
  Node *root;
  LL dis2(int x1, int y1, int x2, int y2) {
    LL dx = x1-x2;
    LL dy = y1-y2;
    return dx*dx+dy*dy;
  static bool cmpx(Node& a, Node& b) { return a.x<b.x; }</pre>
  static bool cmpy(Node& a, Node& b) { return a.y<b.y; }</pre>
  void init(vector<pair<int,int>> ip) {
    n = ip.size();
    for (int i=0; i<n; i++) {</pre>
      tree[i].id = i;
      tree[i].x = ip[i].first;
      tree[i].y = ip[i].second;
    root = build tree(0, n-1, 0);
  Node* build tree(int L, int R, int dep) {
    if (L>R) return nullptr;
    int M = (L+R)/2;
    tree[M].f = dep%2;
    nth element(tree+L, tree+M, tree+R+1, tree[M].f?
        cmpy : cmpx);
    tree[M].x1 = tree[M].x2 = tree[M].x;
    tree[M].y1 = tree[M].y2 = tree[M].y;
    tree[M].L = build tree(L, M-1, dep+1);
    if (tree[M].L) {
      tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
      tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
      tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
      tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
    tree[M].R = build tree(M+1, R, dep+1);
    if (tree[M].R) {
      tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
      tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
      tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
      tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
    return tree+M;
  int touch(Node* r, int x, int y, LL d2){
    LL dis = sqrt(d2)+1;
    if (x<r->x1-dis || x>r->x2+dis ||
        y < r > y1-dis \mid \mid y>r > y2+dis)
      return 0;
    return 1;
  void nearest(Node* r, int x, int y,
               int &mID, LL &md2) {
    if (!r || !touch(r, x, y, md2)) return;
```

```
LL d2 = dis2(r->x, r->y, x, y);
    if (d2 < md2 | | (d2 == md2 && mID < r->id)) {
      mTD = r - > id:
      md2 = d2;
    // search order depends on split dim
    if ((r->f == 0 \&\& x < r->x) | |
        (r->f == 1 && y < r->y))
      nearest(r->L, x, y, mID, md2);
      nearest(r->R, x, y, mID, md2);
    } else {
      nearest(r->R, x, y, mID, md2);
      nearest(r->L, x, y, mID, md2);
  int query(int x, int y) {
    int id = 1029384756;
    LL d2 = 102938475612345678LL;
    nearest(root, x, y, id, d2);
    return id;
}tree;
```

6 Stringology

6.1 Hash

```
class Hash{
private:
  static const int N = 1000000;
  const int p = 127, q = 1208220623;
  int sz, prefix[N], power[N];
inline int add(int x, int y){return x+y>=q?x+y-q:x+y
      ; }
  inline int sub(int x, int y) {return x-y<0?x-y+q:x-y;}</pre>
  inline int mul(int x, int y) {return 1LL*x*y%q;}
public:
  void init(const std::string &x) {
    sz = x.size();
    prefix[0]=0;
    for (int i=1;i<=sz;i++) prefix[i]=add(mul(prefix[i</pre>
         -1], p), x[i-1]);
    power[0]=1;
    for (int i=1;i<=sz;i++) power[i]=mul(power[i-1], p);</pre>
  int query(int 1, int r) {
       ' 1-base (1, r]
    return sub(prefix[r], mul(prefix[l], power[r-l]));
};
```

6.2 Suffix Array

```
//help by http://www.geeksforgeeks.org/suffix-array-set
    -2-a-nlognlogn-algorithm/
struct sfx{
  int index;
 int r,nr;
char str[N + 10];
int len;
vector<sfx> srs[N + 10];
int mapping[N + 10];
sfx sa[N + 10];
bool cmp(sfx a,sfx b) {
  if(a.r==b.r){
    return a.nr<b.nr;</pre>
  }else{
   return a.r<b.r;</pre>
void SA() {
 len = strlen(str);
  for (int i=0;i<len;i++) {</pre>
    sa[i].index = i;
    sa[i].r=str[i];
    sa[i].nr=(i+1>=len)?0:str[i+1];
  //sort(sa,sa+len,cmp);
  radixSort();
```

```
for (int i=2;i<=len;i*=2) {</pre>
    int cnt=1:
    int rr = sa[0].r;
    sa[0].r=cnt;
    mapping[sa[0].index]=0;
    for(int i=1;i<len;i++) {</pre>
      if(sa[i].r == rr && sa[i].nr == sa[i-1].nr){
        rr=sa[i].r;
         sa[i].r=cnt;
      }else{
        rr=sa[i].r;
        sa[i].r=++cnt;
      mapping[sa[i].index]=i;
    for(int i=0;i<len;i++) {</pre>
      int nn = sa[i].index+j;
      sa[i].nr = (nn>=len)?0:sa[mapping[nn]].r;
    //sort(sa, sa+len, cmp);
    radixSort();
void radixSort() {
  int m = 0;
  for (int i=0;i<len;i++) {</pre>
    srs[sa[i].nr].PB(sa[i]);
    m=max(m,sa[i].nr);
  int cnt=0;
  for (int i=0;i<=m;i++) {</pre>
    if(srs[i].empty())continue;
    for(auto j:srs[i]){
      sa[cnt++] = j;
    srs[i].clear();
  m = 0;
  for (int i=0;i<len;i++) {</pre>
    srs[sa[i].r].PB(sa[i]);
    m=max(m,sa[i].r);
  cnt=0;
  for (int i=0;i<=m;i++) {</pre>
    if(srs[i].empty())continue;
    for(auto j:srs[i]){
      sa[cnt++] = j;
    srs[i].clear();
```

6.3 KMP

```
int F[N<<1];</pre>
void KMP(char s1[], char s2[], int n, int m) {
  // make F[] for s1+'\0'+s2;
  char ss[N<<1];</pre>
  int len = n+m+1;
  for (int i=0;i<n;i++) ss[i] = s1[i];</pre>
  ss[n] = ' \setminus 0';
  for (int i=0;i<m;i++) ss[i+1+n] = s2[i];</pre>
  F[0] = F[1] = 0;
  for (int i=1;i<len;i++) {</pre>
    int j = F[i];
    while (j > 0 \text{ and } ss[i]!=ss[j]) j = F[j];
    F[i+1] = (ss[i]==ss[j]?j+1:0);
  // just find (F[len2+i] == len2), i from 1 to len+1
       for matching
  [0, i]是個循環字串,且循環節為i-f[i]:
  if(f[i]>0 \text{ and } i\%(i-f[i])==0) \text{ cout } << i << " " << i/(i
       -f[i]) << ' \n';
```

6.4 Z value

```
char s[MAXN];
int len,z[MAXN];
```

```
void Z_value() {
   int i,j,left,right;
   left=right=0; z[0]=len;
   for(i=1;i<len;i++) {
      j=max(min(z[i-left],right-i),0);
      for(;i+j<len&&s[i+j]==s[j];j++);
      z[i]=j;
      if(i+z[i]>right) {
        right=i+z[i];
        left=i;
      }
   }
}
```

6.5 Lexicographically Smallest Rotation

```
string mcp(string s) {
  int n = s.length();
  s += s;
  int i=0, j=1;
  while (i<n && j<n) {
    int k = 0;
    while (k < n && s[i+k] == s[j+k]) k++;
    if (s[i+k] <= s[j+k]) j += k+1;
    else i += k+1;
    if (i == j) j++;
  }
  int ans = i < n ? i : j;
  return s.substr(ans, n);
}</pre>
```

7 Misc

7.1 MaximumEmptyRect

```
int largest_empty_rectangle() {
  int max_area = 0;
  for (int i=1; i<=n; ++i) {</pre>
    for (int j=1; j<=n; ++j)</pre>
      if (array[i][j]) wl[j] = wl[j-1] + 1;
      else wl[j] = 0;
    for (int j=n; j>=1; --j)
      if (array[i][j]) wr[j] = wr[j+1] + 1;
      else wr[j] = 0;
    for (int j=1; j<=n; ++j)</pre>
      if (array[i][j]) h[j] = h[j] + 1;
      else h[j] = 0;
     for (int j=1; j<=n; ++j)</pre>
      if (1[j] == 0) 1[j] = w1[j];
      else l[j] = min(wl[j], l[j]);
    for (int j=1; j<=n; ++j)</pre>
      if (r[j] == 0) r[j] = wr[j];
      else r[j] = min(wr[j], r[j]);
    for (int j=1; j<=n; ++j)</pre>
      \max \text{ area} = \max(\max \text{ area}, (l[j] + r[j] - 1) * h[j]
  return max_area;
```

7.2 DP-opt Condition

7.2.1 totally monotone (concave/convex)

```
\begin{array}{l} \forall i < i', j < j', B[i][j] \leq B[i'][j] \implies B[i][j'] \leq B[i'][j'] \\ \forall i < i', j < j', B[i][j] \geq B[i'][j] \implies B[i][j'] \geq B[i'][j'] \end{array}
```

7.2.2 monge condition (concave/convex)

```
\begin{array}{l} \forall i < i', j < j', B[i][j] + B[i'][j'] \geq B[i][j'] + B[i'][j] \\ \forall i < i', j < j', B[i][j] + B[i'][j'] \leq B[i][j'] + B[i'][j] \end{array}
```

7.3 Convex 1D/1D DP

```
struct segment {
  int i, 1, r;
  segment() {}
  segment(int a, int b, int c): i(a), l(b), r(c) {}
inline long long f(int 1, int r) {
 return dp[l] + w(l + 1, r);
void solve() {
 dp[0] = 011;
  deque<segment> deq; deq.push_back(segment(0, 1, n));
  for (int i = 1; i <= n; ++i)
    dp[i] = f(deq.front().i, i);
    while (deq.size() && deq.front().r < i + 1) deq.</pre>
       pop_front();
    deq.front().l = i + 1;
    segment seg = segment(i, i + 1, n);
    while (deq.size() && f(i, deq.back().l) < f(deq.
        back().i, deq.back().l)) deq.pop_back();
    if (deq.size()) {
      int d = 1048576, c = deq.back().1;
      while (d >>= 1) if (c + d <= deq.back().r) {</pre>
        if (f(i, c + d) > f(deq.back().i, c + d)) c +=
            d;
      deq.back().r = c; seg.l = c + 1;
    if (seg.l <= n) deq.push back(seg);</pre>
```