

Contents

1 Basic	1	7 Stringology	21
1.1 vimrc	1	7.1 Hash	21
1.2 Debug Macro	1	7.2 Suffix Array	21
1.3 Increase Stack	1	7.3 Suffix Automaton	21
1.4 Pragma Optimization	2	7.4 Z value	22
1.5 IO Optimization	2	7.5 Manacher	22
2 Data Structure	2	7.6 Lexico Smallest Rotation	22
2.1 Dark Magic	2	7.7 Main Lorentz	22
2.2 Link-Cut Tree	2	7.8 BWT	22
2.3 LiChao Segment Tree	2	7.9 Palindromic Tree	23
2.4 Treap	3	8 Misc	23
2.5 Linear Basis	3	8.1 Theorems	23
2.6 Binary Search On Segment Tree	3	8.1.1 Sherman-Morrison formula	23
3 Graph	3	8.1.2 Kirchhoff's Theorem	23
3.1 2-SAT (SCC)	3	8.1.3 Tutte's Matrix	23
3.2 BCC	4	8.1.4 Cayley's Formula	23
3.3 Round Square Tree	4	8.1.5 Erdős-Gallai theorem	23
3.4 Centroid Decomposition	4	8.1.6 Havel-Hakimi algorithm	23
3.5 Directed Minimum Spanning Tree	5	8.1.7 Euler's planar graph formula	23
3.6 Dominator Tree	5	8.1.8 Pick's theorem	23
3.7 Edge Coloring	5	8.1.9 Matroid Intersection	23
3.8 Lowbit Decomposition	6	8.2 Bitset LCS	23
3.9 Manhattan Minimum Spanning Tree	6	8.3 Prefix Substring LCS	23
3.10 MaxClique	6	8.4 Convex 1D/1D DP	23
3.11 Minimum Mean Cycle	7	8.5 ConvexHull Optimization	24
3.12 Mo's Algorithm on Tree	7	8.6 Josephus Problem	24
3.13 Virtual Tree	7	8.7 Tree Knapsack	24
4 Matching & Flow	7	8.8 N Queens Problem	24
4.1 Bipartite Matching	7	8.9 Stable Marriage	24
4.2 Dijkstra Cost Flow	8	8.10 Binary Search On Fraction	24
4.3 Dinic	8	1 Basic	
4.4 Flow Models	8	1.1 vimrc	
4.5 General Graph Matching	9	<code>se is nu ru et tgc sc hls cin cino+=j1 sw=4 sts=4 bs=2</code>	
4.6 Global Min-Cut	9	<code>mouse=a "encoding=utf-8 ls=2</code>	
4.7 GomoryHu Tree	9	<code>syn on</code>	
4.8 Kuhn Munkres	9	<code>colo desert</code>	
4.9 Minimum Cost Circulation	10	<code>filetype indent on</code>	
4.10 Minimum Cost Maximum Flow	10	<code>inoremap {<CR> {<CR>}<ESC>O</code>	
4.11 Maximum Weight Graph Matching	10	<code>map <F8> <ESC>:w<CR>:!g++ "%" -o "%<" -std=c++17 -</code>	
5 Math	12	<code>DCKISEKI -Wall -Wextra -Wshadow -Wfatal-errors -</code>	
5.1 Common Bounds	12	<code>Wconversion -fsanitize=address,undefined -g && echo</code>	
5.2 Stirling Number	12	<code>success<CR></code>	
5.2.1 First Kind	12	<code>map <F9> <ESC>:w<CR>:!g++ "%" -o "%<" -O2 -std=c++17 &&</code>	
5.2.2 Second Kind	12	<code>echo success<CR></code>	
5.3 $ax+by=gcd$	12	<code>map <F10> <ESC>:!. / "%<"<CR></code>	
5.4 Chinese Remainder	12	<code>ca Hash w !cpp -dD -P -fpreprocessed \ tr -d '[:space</code>	
5.5 De-Bruijn	12	<code>:]' \ md5sum \ cut -c-6</code>	
5.6 DiscreteLog	12	<code>" setxkbmap -option caps:ctrl_modifier</code>	
5.7 Quadratic residue	13	1.2 Debug Macro [b78d75]	
5.8 Extended Euler	13	<code>#ifdef CKISEKI</code>	
5.9 ExtendedFloorSum	13	<code>#define safe cerr<<__PRETTY_FUNCTION__<<" line "<<</code>	
5.10 FloorSum	13	<code>__LINE__<<" safe\n"</code>	
5.11 FloorSum	13	<code>#define debug(a...) debug_(#a, a)</code>	
5.12 Fast Fourier Transform	13	<code>#define orange(a...) orange_(#a, a)</code>	
5.13 FWT	14	<code>template <typename ...T></code>	
5.14 CRT for arbitrary mod	14	<code>void debug_(const char *s, T ...a) {</code>	
5.15 NTT	14	<code>cerr << "\e[1;32m(" << s << ") = (";</code>	
5.16 Partition Number	14	<code>int cnt = sizeof...(T);</code>	
5.17 Pi Count (+Linear Sieve)	14	<code>(..., (cerr << a << (--cnt ? ", " : ") \e[0m\n")));</code>	
5.18 Miller Rabin	14	<code>} template <typename I></code>	
5.19 Pollard Rho	15	<code>void orange_(const char *s, I L, I R) {</code>	
5.20 Berlekamp Massey	15	<code>cerr << "\e[1;32m[" << s << "] = [";</code>	
5.21 Characteristic Polynomial	15	<code>for (int f = 0; L != R; ++L)</code>	
5.22 Polynomial Operations	15	<code>cerr << (f++ ? ", " : "") << *L;</code>	
5.23 Simplex	16	<code>cerr << "] \e[0m\n";</code>	
5.24 Simplex Construction	17	<code>} #else</code>	
6 Geometry	17	<code>#define safe ((void)0)</code>	
6.1 Basic Geometry	17	<code>#define debug(...) safe</code>	
6.2 Segment Intersection	17	<code>#define orange(...) safe</code>	
6.3 Half Plane Intersection	17	<code>#endif</code>	
6.4 Rotating Sweep Line	17	1.3 Increase Stack	
6.5 2D Convex Hull	18	<code>const int size = 256 << 20;</code>	
6.6 Minkowski Sum	18	<code>register long rsp asm("rsp");</code>	
6.7 2D Farthest Pair	18	<code>char *p = (char*)malloc(size)+size, *bak = (char*)rsp;</code>	
6.8 MinMaxEnclosingRectangle	18	<code>__asm__("movq %, %%rsp\n": "r"(p));</code>	
6.9 Convex Hulls Tangent	18	<code>// main</code>	
6.10 Point & Hulls Tangent	18	<code>__asm__("movq %0, %%rsp\n": "r"(bak));</code>	
6.11 Circle Class	18		
6.12 Tangent Line of Two Circle	19		
6.13 Intersection of Line and Circle	19		
6.14 Intersection of Polygon and Circle	19		
6.15 3D Convex Hull	19		
6.16 Minimum Covering Circle	19		
6.17 Polygon Union	19		
6.18 Circle Union	20		
6.19 KDTree (Nearest Point)	20		
6.20 kD Closest Pair (3D ver.)	21		
6.21 Simulated Annealing	21		

1.4 Pragma Optimization

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,tune=native")
__builtin_ia32_ldmxcsr(__builtin_ia32_stmxcsr()|0x8000)
```

1.5 IO Optimization

```
static inline int gc() {
    constexpr int B = 1<<20;
    static char buf[B], *p, *q;
    if(p == q &&
        (q=(p=buf)+fread(buf,1,B,stdin)) == buf)
        return EOF;
    return *p++;
}
```

2 Data Structure

2.1 Dark Magic

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
using namespace __gnu_pbds;
// heap tags: paring/binary/binomial/rc_binomial/thin
template<typename T>
using pbds_heap=__gnu_pbds::priority_queue<T,less<T>, \
    pairing_heap_tag>;
// pbds_heap::point_iterator
// x = pq.push(10); pq.modify(x, 87); a.join(b);
// tree tags: rb_tree_tag/ov_tree_tag/splay_tree_tag
template<typename T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;
// find_by_order, order_of_key
// hash tables: cc_hash_table/gp_hash_table
```

2.2 Link-Cut Tree [7ce2b4]

```
template <typename Val, typename SVal> class LCT {
    struct node {
        int pa, ch[2];
        bool rev;
        Val v, prod, rprod;
        SVal sv, sub, vir;
        node() : pa{0}, ch{0, 0}, rev{false}, v{}, prod{},
            rprod{}, sv{}, sub{}, vir{} {};
    };
    #define cur o[u]
    #define lc cur.ch[0]
    #define rc cur.ch[1]
    vector<node> o;
    bool is_root(int u) const {
        return o[cur.pa].ch[0]!=u && o[cur.pa].ch[1]!=u;
    }
    bool is_rch(int u) const {
        return o[cur.pa].ch[1] == u && !is_root(u);
    }
    void down(int u) {
        if (not cur.rev) return;
        if (lc) set_rev(lc);
        if (rc) set_rev(rc);
        cur.rev = false;
    }
    void up(int u) {
        cur.prod = o[lc].prod * cur.v * o[rc].prod;
        cur.rprod = o[rc].rprod * cur.v * o[lc].rprod;
        cur.sub = cur.vir + o[lc].sub + o[rc].sub + cur.sv;
    }
    void set_rev(int u) {
        swap(lc, rc);
        swap(cur.prod, cur.rprod);
        cur.rev ^= 1;
    }
    void rotate(int u) {
        int f=cur.pa, g=o[f].pa, l=is_rch(u);
        if (cur.ch[l ^ 1]) o[cur.ch[l ^ 1]].pa = f;
        if (not is_root(f)) o[g].ch[is_rch(f)] = u;
        o[f].ch[l] = cur.ch[l ^ 1];
        cur.ch[l ^ 1] = f;
        cur.pa = g, o[f].pa = u;
        up(f);
    }
}
```

```
void splay(int u) {
    vector<int> stk = {u};
    while (not is_root(stk.back()))
        stk.push_back(o[stk.back()].pa);
    while (not stk.empty()) {
        down(stk.back());
        stk.pop_back();
    }
    for (int f = cur.pa; not is_root(u); f = cur.pa) {
        if (!is_root(f)) rotate(is_rch(u)==is_rch(f)?f:u);
        rotate(u);
    }
    up(u);
}
void access(int x) {
    for (int u = x, last = 0; u; u = cur.pa) {
        splay(u);
        cur.vir = cur.vir + o[rc].sub - o[last].sub;
        rc = last; up(last = u);
    }
    splay(x);
}
int find_root(int u) {
    int la = 0;
    for (access(u); u; u = lc) down(la = u);
    return la;
}
void split(int x, int y) {change_root(x);access(y);}
void change_root(int u) { access(u); set_rev(u); }
public:
    LCT(int n = 0) : o(n + 1) {}
    int add(const Val &v = {}) {
        o.push_back(v);
        return int(o.size()) - 2;
    }
    int add(Val &&v) {
        o.emplace_back(move(v));
        return int(o.size()) - 2;
    }
    void set_val(int u, const Val &v) {
        splay(++u); cur.v = v; up(u);
    }
    void set_sval(int u, const SVal &v) {
        splay(++u); cur.sv = v; up(u);
    }
    Val query(int x, int y) {
        split(++x, ++y); return o[y].prod;
    }
    SVal subtree(int p, int u) {
        change_root(++p); access(++u);
        return cur.vir + cur.sv;
    }
    bool connected(int u, int v) {
        return find_root(++u) == find_root(++v);
    }
    void link(int x, int y) {
        change_root(++x); access(++y);
        o[y].vir = o[y].vir + o[x].sub;
        up(o[x].pa = y);
    }
    void cut(int x, int y) {
        split(++x, ++y);
        o[y].ch[0] = o[x].pa = 0; up(y);
    }
    #undef cur
    #undef lc
    #undef rc
};
```

2.3 LiChao Segment Tree [c45c03]

```
struct L {
    int m, k, id;
    L() : id(-1) {}
    L(int a, int b, int c) : m(a), k(b), id(c) {}
    int at(int x) { return m * x + k; }
};
class LiChao {
private:
    int n; vector<L> nodes;
    static int lc(int x) { return 2 * x + 1; }
    static int rc(int x) { return 2 * x + 2; }
    void insert(int l, int r, int id, L ln) {
        int m = (l + r) >> 1;
```

```

if (nodes[id].id == -1) {
    nodes[id] = ln;
    return;
}
bool atLeft = nodes[id].at(l) < ln.at(l);
if (nodes[id].at(m) < ln.at(m)) {
    atLeft ^= 1;
    swap(nodes[id], ln);
}
if (r - l == 1) return;
if (atLeft) insert(l, m, lc(id), ln);
else insert(m, r, rc(id), ln);
}
int query(int l, int r, int id, int x) {
    int ret = 0, m = (l + r) >> 1;
    if (nodes[id].id != -1)
        ret = nodes[id].at(x);
    if (r - l == 1) return ret;
    if (x < m) return max(ret, query(l, m, lc(id), x));
    return max(ret, query(m, r, rc(id), x));
}

public:
LiChao(int n_) : n(n_), nodes(n * 4) {}
void insert(L ln) { insert(0, n, 0, ln); }
int query(int x) { return query(0, n, 0, x); }
};

```

2.4 Treap [82fac7]

```

__gnu_cxx::sfmt19937 rnd(7122);
namespace Treap{
#define sz(x) ( (x) ? ( (x)->size ) : 0 )
struct node{
    int size; uint32_t pri;
    node *lc, *rc, *pa;
    node():size(0),pri(rnd()),lc(0),rc(0),pa(0){}
    void pull() {
        size = 1; pa = nullptr;
        if ( lc ) { size += lc->size; lc->pa = this; }
        if ( rc ) { size += rc->size; rc->pa = this; }
    }
};
node* merge( node* L, node* R ) {
    if ( not L or not R ) return L ? L : R;
    if ( L->pri > R->pri ) {
        L->rc = merge( L->rc, R ); L->pull();
        return L;
    } else {
        R->lc = merge( L, R->lc ); R->pull();
        return R;
    }
}
void split_by_size( node*rt, int k, node*&L, node*&R ) {
    if ( not rt ) L = R = nullptr;
    else if( sz( rt->lc ) + 1 <= k ) {
        L = rt;
        split_by_size( rt->rc, k-sz(rt->lc)-1, L->rc, R );
        L->pull();
    } else {
        R = rt;
        split_by_size( rt->lc, k, L, R->lc );
        R->pull();
    }
} // sz(L) == k
int getRank(node *o) { // 1-base
    int r = sz(o->lc) + 1;
    for (;o->pa != nullptr; o = o->pa)
        if (o->pa->rc == o) r += sz(o->pa->lc) + 1;
    return r;
}
#undef sz
}

```

2.5 Linear Basis

```

template <int BITS, typename S = int> struct Basis {
    static constexpr auto MIN = numeric_limits<S>::min();
    array<pair<uint64_t, S>, BITS> b;
    Basis() { b.fill({0, MIN}); }
    void add(uint64_t x, S p) {
        for (int i = BITS-1; i>=0; i--) if ((x >> i) & 1) {
            if (b[i].first == 0) return b[i]={x, p}, void();
            if (b[i].second < p)

```

```

                swap(b[i].first, x), swap(b[i].second, p);
                x ^= b[i].first;
            }
        }
        optional<uint64_t> query_kth(uint64_t v, uint64_t k){
            vector<pair<uint64_t, int>> o;
            for (int i = 0; i < BITS; i++)
                if (b[i].first < o.empty()) o.emplace_back(b[i].first, i);
            if (k >= (1ULL << o.size())) return {};
            for (int i = int(o.size()) - 1; i >= 0; i--)
                if ((k >> i & 1) ^ (v >> o[i].second & 1))
                    v ^= o[i].first;
            return v;
        }
        Basis filter(S l) {
            Basis res = *this;
            for (int i = 0; i < BITS; i++)
                if (res.b[i].second < l) res.b[i] = {0, MIN};
            return res;
        }
};

```

2.6 Binary Search On Segment Tree

```

// find_first = x -> minimal x s.t. check( [a, x) )
// find_last = x -> maximal x s.t. check( [x, b) )
template <typename C>
int find_first(int l, const C &check) {
    if (l >= n) return n + 1;
    l += sz;
    for (int i = height; i > 0; i--)
        propagate(l >> i);
    Monoid sum = identity;
    do {
        while ((l & 1) == 0) l >>= 1;
        if (check(f(sum, data[l]))) {
            while (l < sz) {
                propagate(l);
                l <<= 1;
                auto nxt = f(sum, data[l]);
                if (not check(nxt)) {
                    sum = nxt;
                    l++;
                }
            }
            return l + 1 - sz;
        }
        sum = f(sum, data[l++]);
    } while ((l & -l) != l);
    return n + 1;
}

template <typename C>
int find_last(int r, const C &check) {
    if (r <= 0) return -1;
    r += sz;
    for (int i = height; i > 0; i--)
        propagate((r - 1) >> i);
    Monoid sum = identity;
    do {
        r--;
        while (r > 1 and (r & 1)) r >>= 1;
        if (check(f(data[r], sum))) {
            while (r < sz) {
                propagate(r);
                r = (r << 1) + 1;
                auto nxt = f(data[r], sum);
                if (not check(nxt)) {
                    sum = nxt;
                    r--;
                }
            }
            return r - sz;
        }
        sum = f(data[r], sum);
    } while ((r & -r) != r);
    return -1;
}

```

3 Graph

3.1 2-SAT (SCC) [3cc0b4]

```

class TwoSat{

```

```

private:
    int n;
    vector<vector<int>> rG,G,sccs;
    vector<int> ord,idx;
    vector<bool> vis,result;
    void dfs(int u){
        vis[u]=true;
        for(int v:G[u])
            if(!vis[v]) dfs(v);
        ord.push_back(u);
    }
    void rdfs(int u){
        vis[u]=false;idx[u]=sccs.size()-1;
        sccs.back().push_back(u);
        for(int v:rG[u])
            if(vis[v])rdfs(v);
    }
public:
    void init(int n_){
        G.clear();G.resize(n=n_);
        rG.clear();rG.resize(n);
        sccs.clear();ord.clear();
        idx.resize(n);result.resize(n);
    }
    void add_edge(int u,int v){
        G[u].push_back(v);rG[v].push_back(u);
    }
    void orr(int x,int y){
        if ((x^y)==1)return;
        add_edge(x^1,y); add_edge(y^1,x);
    }
    bool solve(){
        vis.clear();vis.resize(n);
        for(int i=0;i<n;++i)
            if(not vis[i])dfs(i);
        reverse(ord.begin(),ord.end());
        for (int u:ord){
            if(!vis[u])continue;
            sccs.push_back(vector<int>());
            rdfs(u);
        }
        for(int i=0;i<n;i+=2)
            if(idx[i]==idx[i+1])
                return false;
        vector<bool> c(sccs.size());
        for(size_t i=0;i<sccs.size();++i){
            for(auto sij : sccs[i]){
                result[sij]=c[i];
                c[idx[sij^1]]=!c[i];
            }
        }
        return true;
    }
    bool get(int x){return result[x];}
    int get_id(int x){return idx[x];}
    int count(){return sccs.size();}
} sat2;

```

3.2 BCC [4ef534]

```

class BCC {
    int n, ecnt, bcnt;
    vector<vector<pair<int, int>>> g;
    vector<int> dfn, low, bcc, stk;
    vector<bool> ap, bridge;
    void dfs(int u, int f) {
        dfn[u] = low[u] = dfn[f] + 1;
        int ch = 0;
        for (auto [v, t] : g[u]) if (bcc[t] == -1) {
            bcc[t] = 0;stk.push_back(t);
            if (dfn[v]) {
                low[u] = min(low[u], dfn[v]);
                continue;
            }
            ++ch, dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (low[v] > dfn[u]) bridge[t] = true;
            if (low[v] < dfn[u]) continue;
            ap[u] = true;
            while (not stk.empty()) {
                int o = stk.back(); stk.pop_back();
                bcc[o] = bcnt;
                if (o == t)break;
            }
        }
    }
}

```

```

    }
    bcnt += 1;
}
ap[u] = ap[u] and (ch != 1 or u != f);
}

public:
    BCC(int n_) : n(n_), ecnt(0), bcnt(0), g(n), dfn(n),
        low(n), stk(), ap(n) {}
    void add_edge(int u, int v) {
        g[u].emplace_back(v, ecnt);
        g[v].emplace_back(u, ecnt++);
    }
    void solve() {
        bridge.assign(ecnt, false);
        bcc.assign(ecnt, -1);
        for (int i = 0; i < n; ++i)
            if (not dfn[i]) dfs(i, i);
    }
    int bcc_id(int x) const { return bcc[x]; }
    bool is_ap(int x) const { return ap[x]; }
    bool is_bridge(int x) const { return bridge[x]; }
};

```

3.3 Round Square Tree [df7723]

```

int N, M, cnt;
std::vector<int> G[maxn], T[maxn * 2];

int dfn[maxn], low[maxn], dfc;
int stk[maxn], tp;

void Tarjan(int u) {
    low[u] = dfn[u] = ++dfc;
    stk[++tp] = u;
    for (int v : G[u]) {
        if (!dfn[v]) {
            Tarjan(v);
            low[u] = std::min(low[u], low[v]);
            if (low[v] == dfn[u]) {
                ++cnt;
                for (int x = 0; x != v; --tp) {
                    x = stk[tp];
                    T[cnt].push_back(x);
                    T[x].push_back(cnt);
                }
                T[cnt].push_back(u);
                T[u].push_back(cnt);
            }
        } else
            low[u] = std::min(low[u], dfn[v]);
    }
}

int main() { // ...
    cnt = N;
    for (int u = 1; u <= N; ++u)
        if (!dfn[u]) Tarjan(u), --tp;
}

```

3.4 Centroid Decomposition [63b2fb]

```

struct Centroid {
    using G = vector<vector<pair<int, int>>>
    vector<vector<int64_t>> Dist;
    vector<int> Pa, Dep;
    vector<int64_t> Sub, Sub2;
    vector<int> Cnt, Cnt2;
    vector<int> vis, sz, mx, tmp;
    void DfsSz(const G &g, int x) {
        vis[x] = true, sz[x] = 1, mx[x] = 0;
        for (auto [u, w] : g[x]) if (not vis[u]) {
            DfsSz(g, u); sz[x] += sz[u];
            mx[x] = max(mx[x], sz[u]);
        }
        tmp.push_back(x);
    }
    void DfsDist(const G &g, int x, int64_t D = 0) {
        Dist[x].push_back(D); vis[x] = true;
        for (auto [u, w] : g[x])
            if (not vis[u]) DfsDist(g, u, D + w);
    }
    void DfsCen(const G &g, int x, int D = 0, int p = -1)
    {

```

```

tmp.clear(); DfsSz(g, x);
int M = tmp.size(), C = -1;
for (int u : tmp) {
    if (max(M - sz[u], mx[u]) * 2 <= M) C = u;
    vis[u] = false;
}
DfsDist(g, C);
for (int u : tmp) vis[u] = false;
Pa[C] = p, vis[C] = true, Dep[C] = D;
for (auto [u, w] : g[C])
    if (not vis[u]) DfsCen(g, u, D + 1, C);
}
Centroid(int N, G g) {
    Sub(N), Sub2(N), Cnt(N), Cnt2(N), Dist(N), Pa(N),
    Dep(N), vis(N), sz(N), mx(N) { DfsCen(g, 0); }
void Mark(int v) {
    int x = v, z = -1;
    for (int i = Dep[v]; i >= 0; --i) {
        Sub[x] += Dist[v][i], Cnt[x]++;
        if (z != -1)
            Sub2[z] += Dist[v][i], Cnt2[z]++;
        x = Pa[z = x];
    }
}
int64_t Query(int v) {
    int64_t res = 0;
    int x = v, z = -1;
    for (int i = Dep[v]; i >= 0; --i) {
        res += Sub[x] + 1LL * Cnt[x] * Dist[v][i];
        if (z != -1)
            res -= Sub2[z] + 1LL * Cnt2[z] * Dist[v][i];
        x = Pa[z = x];
    }
    return res;
}
};

```

3.5 Directed Minimum Spanning Tree

```

using D = int64_t;
struct E {
    int s, t; // 0-base
    D w;
};
vector<int> dmst(const vector<E> &e, int n, int root) {
    using PQ = pair<min_heap<pair<D, int>>, D>;
    auto push = [](PQ &pq, pair<D, int> v) {
        pq.first.emplace(v.first - pq.second, v.second);
    };
    auto top = [](const PQ &pq) -> pair<D, int> {
        auto r = pq.first.top();
        return {r.first + pq.second, r.second};
    };
    auto join = [&push, &top](PQ &a, PQ &b) {
        if (a.first.size() < b.first.size()) swap(a, b);
        while (!b.first.empty()) {
            push(a, top(b));
            b.first.pop();
        }
    };
    vector<PQ> h(n * 2);
    for (size_t i = 0; i < e.size(); ++i)
        push(h[e[i].t], {e[i].w, i});
    vector<int> a(n * 2), v(n * 2, -1), pa(n * 2, -1), r(n
        * 2);
    iota(a.begin(), a.end(), 0);
    auto o = [&](int x) { int y;
        for (y = x; a[y] != y; y = a[y]);
        for (int ox = x; x != y; ox = x)
            x = a[x], a[ox] = y;
        return y;
    };
    v[root] = n + 1;
    int pc = n;
    for (int i = 0; i < n; ++i) if (v[i] == -1) {
        for (int p = i; v[p] == -1 || v[p] == i; p = o(e[r[p]
            ].s)) {
            if (v[p] == i) {
                int q = p; p = pc++;
                do {
                    h[q].second = -h[q].first.top().first;
                    join(h[pa[q] = a[q] = p], h[q]);
                } while ((q = o(e[r[q]].s)) != p);
            }
        }
    }
}

```

```

}
v[p] = i;
while (!h[p].first.empty() && o(e[top(h[p]).second].
    s) == p)
    h[p].first.pop();
r[p] = top(h[p]).second;
}
}
vector<int> ans;
for (int i = pc - 1; i >= 0; i--) if (i != root && v[i]
    != n) {
    for (int f = e[r[i]].t; f != -1 && v[f] != n; f = pa[
        f])
        v[f] = n;
    ans.push_back(r[i]);
}
return ans;
}
}

```

3.6 Dominator Tree [0631b2]

```

namespace dominator {
vector<int> g[maxn], r[maxn], rdom[maxn];
int dfn[maxn], rev[maxn], fa[maxn], sdom[maxn];
int dom[maxn], val[maxn], rp[maxn], tk;
void init(int n) {
    // vertices are numbered from 0 to n - 1
    fill(dfn, dfn + n, -1); fill(rev, rev + n, -1);
    fill(fa, fa + n, -1); fill(val, val + n, -1);
    fill(sdom, sdom + n, -1); fill(rp, rp + n, -1);
    fill(dom, dom + n, -1); tk = 0;
    for (int i = 0; i < n; ++i) {
        g[i].clear(); r[i].clear(); rdom[i].clear();
    }
}
void add_edge(int x, int y) { g[x].push_back(y); }
void dfs(int x) {
    rev[dfn[x] = tk] = x;
    fa[tk] = sdom[tk] = val[tk] = tk; tk++;
    for (int u : g[x]) {
        if (dfn[u] == -1) dfs(u), rp[dfn[u]] = dfn[x];
        r[dfn[u]].push_back(dfn[x]);
    }
}
void merge(int x, int y) { fa[x] = y; }
int find(int x, int c = 0) {
    if (fa[x] == x) return c ? -1 : x;
    int p = find(fa[x], 1);
    if (p == -1) return c ? fa[x] : val[x];
    if (sdom[val[x]] > sdom[val[fa[x]]]) val[x] = val[fa[x]];
    fa[x] = p;
    return c ? p : val[x];
}
vector<int> build(int s, int n) {
    // return the father of each node in the dominator tree
    // p[i] = -2 if i is unreachable from s
    dfs(s);
    for (int i = tk - 1; i >= 0; --i) {
        for (int u : r[i]) sdom[i] = min(sdom[i], sdom[find(u)]);
        if (i) rdom[sdom[i]].push_back(i);
        for (int &u : rdom[i]) {
            int p = find(u);
            if (sdom[p] == i) dom[u] = i;
            else dom[u] = p;
        }
        if (i) merge(i, rp[i]);
    }
    vector<int> p(n, -2); p[s] = -1;
    for (int i = 1; i < tk; ++i)
        if (sdom[i] != dom[i]) dom[i] = dom[dom[i]];
    for (int i = 1; i < tk; ++i) p[rev[i]] = rev[dom[i]];
    return p;
}
}

```

3.7 Edge Coloring

```

// max(d_u) + 1 edge coloring, time: O(NM)
int C[kN][kN], G[kN][kN]; // 1-based, G: ans
void clear(int N) {
    for (int i = 0; i <= N; i++)
        for (int j = 0; j <= N; j++)
            C[i][j] = G[i][j] = 0;
}
void solve(vector<pair<int, int>> &E, int N) {

```



```

int X[kN] = {}, a;
auto update = [&](int u) {
    for (X[u] = 1; C[u][X[u]]; X[u]++);
};
auto color = [&](int u, int v, int c) {
    int p = G[u][v];
    G[u][v] = G[v][u] = c;
    C[u][c] = v, C[v][c] = u;
    C[u][p] = C[v][p] = 0;
    if (p) X[u] = X[v] = p;
    else update(u), update(v);
    return p;
};
auto flip = [&](int u, int c1, int c2) {
    int p = C[u][c1];
    swap(C[u][c1], C[u][c2]);
    if (p) G[u][p] = G[p][u] = c2;
    if (!C[u][c1]) X[u] = c1;
    if (!C[u][c2]) X[u] = c2;
    return p;
};
for (int i = 1; i <= N; i++) X[i] = 1;
for (int t = 0; t < E.size(); t++) {
    auto [u, v] = E[t];
    int v0 = v, c = X[u], c0 = c, d;
    vector<pair<int, int>> L; int vst[kN] = {};
    while (!G[u][v0]) {
        L.emplace_back(v, d = X[v]);
        if (!C[v][c]) for(a=L.size()-1; a>=0; a--)
            c = color(u, L[a].first, c);
        else if(!C[u][d]) for(a=L.size()-1; a>=0; a--)
            color(u, L[a].first, L[a].second);
        else if (vst[d]) break;
        else vst[d] = 1, v = C[u][d];
    }
    if (!G[u][v0]) {
        for (; v; v = flip(v, c, d), swap(c, d));
        if (C[u][c0]) { a = int(L.size()) - 1;
            while (--a >= 0 && L[a].second != c);
            for(; a>=0; a--) color(u, L[a].first, L[a].second);
        } else t--;
    }
}
}
}

```

3.8 Lowbit Decomposition [aa3f57]

```

class LBD {
    int timer, chains;
    vector<vector<int>> G;
    vector<int> tl, tr, chain, head, dep, pa;
    // chains : number of chain
    // tl, tr[u] : subtree interval in the seq. of u
    // head[i] : head of the chain i
    // chain[u] : chain id of the chain u is on
    void predfs(int u, int f) {
        dep[u] = dep[pa[u]] = f + 1;
        for (int v : G[u]) if (v != f) {
            predfs(v, u);
            if (lowbit(chain[u]) < lowbit(chain[v]))
                chain[u] = chain[v];
        }
        if (chain[u] == 0) chain[u] = ++chains;
    }
    void dfschain(int u, int f) {
        tl[u] = timer++;
        if (head[chain[u]] == -1)
            head[chain[u]] = u;
        for (int v : G[u])
            if (v != f and chain[v] == chain[u])
                dfschain(v, u);
        for (int v : G[u])
            if (v != f and chain[v] != chain[u])
                dfschain(v, u);
        tr[u] = timer;
    }
public:
    LBD(int n) : timer(0), chains(0), G(n), tl(n), tr(n),
        chain(n), head(n, -1), dep(n), pa(n) {}
    void add_edge(int u, int v) {
        G[u].push_back(v); G[v].push_back(u);
    }
    void decompose() { predfs(0, 0); dfschain(0, 0); }
}

```

```

PII get_subtree(int u) { return {tl[u], tr[u]}; }
vector<PII> get_path(int u, int v) {
    vector<PII> res;
    while (chain[u] != chain[v]) {
        if (dep[head[chain[u]]] < dep[head[chain[v]]])
            swap(u, v);
        int s = head[chain[u]];
        res.emplace_back(tl[s], tl[u] + 1);
        u = pa[s];
    }
    if (dep[u] < dep[v]) swap(u, v);
    res.emplace_back(tl[v], tl[u] + 1);
    return res;
}
}
}

```

3.9 Manhattan Minimum Spanning Tree

```

typedef Point<int> P;
vector<array<int, 3>> manhattanMST(vector<P> ps) {
    vi id(sz(ps));
    iota(all(id), 0);
    vector<array<int, 3>> edges;
    rep(k, 0, 4) {
        sort(all(id), [&](int i, int j) {
            return (ps[i] - ps[j]).x < (ps[j] - ps[i]).y;
        });
        map<int, int> sweep;
        for (int i : id) {
            for (auto it = sweep.lower_bound(-ps[i].y);
                it != sweep.end(); sweep.erase(it++)) {
                int j = it->second;
                P d = ps[i] - ps[j];
                if (d.y > d.x) break;
                edges.push_back({d.y + d.x, i, j});
            }
            sweep[-ps[i].y] = i;
        }
        for (P &p : ps)
            if (k & 1) p.x = -p.x;
            else swap(p.x, p.y);
    }
    return edges; // [{w, i, j}, ...]
}

```

3.10 MaxClique [293730]

```

// contain a self loop u to u, than u won't in clique
template <size_t maxn> class MaxClique {
private:
    using bits = bitset<maxn>;
    bits popped, G[maxn], ans;
    size_t deg[maxn], deo[maxn], n;
    void sort_by_degree() {
        popped.reset();
        for (size_t i = 0; i < n; ++i)
            deg[i] = G[i].count();
        for (size_t i = 0; i < n; ++i) {
            size_t mi = maxn, id = 0;
            for (size_t j = 0; j < n; ++j)
                if (not popped[j] and deg[j] < mi)
                    mi = deg[id = j];
            popped[deo[i] = id] = 1;
            for (size_t u = G[i]._Find_first(); u < n;
                u = G[i]._Find_next(u))
                --deg[u];
        }
    }
    void BK(bits R, bits P, bits X) {
        if (R.count() + P.count() <= ans.count()) return;
        if (not P.count() and not X.count()) {
            if (R.count() > ans.count()) ans = R;
            return;
        }
        /* greedily choose max degree as pivot
        bits cur = P | X; size_t pivot = 0, sz = 0;
        for (size_t u = cur._Find_first();
            u < n; u = cur._Find_next(u))
            if (deg[u] > sz) sz = deg[pivot = u];
        cur = P & (~G[pivot]);
        */ // or simply choose first
        bits cur = P & (~G[P | X]._Find_first());
        for (size_t u = cur._Find_first(); u < n;
            u = cur._Find_next(u)) {

```

```

    if (R[u]) continue;
    R[u] = 1;
    BK(R, P & G[u], X & G[u]);
    R[u] = P[u] = 0, X[u] = 1;
}
}
public:
void init(size_t n_) {
    n = n_;
    for (size_t i = 0; i < n; ++i) G[i].reset();
    ans.reset();
}
void add_edges(int u, bits S) { G[u] = S; }
void add_edge(int u, int v) { G[u][v] = G[v][u] = 1; }
int solve() {
    sort_by_degree(); // or simply iota( deo... )
    for (size_t i = 0; i < n; ++i)
        deg[i] = G[i].count();
    bits pob, nob = 0; pob.set();
    for (size_t i = n; i < maxn; ++i) pob[i] = 0;
    for (size_t i = 0; i < n; ++i) {
        size_t v = deo[i];
        bits tmp;
        tmp[v] = 1;
        BK(tmp, pob & G[v], nob & G[v]);
        pob[v] = 0, nob[v] = 1;
    }
    return static_cast<int>(ans.count());
}
};

```

3.11 Minimum Mean Cycle

```

const int inf = 1e9, V = 1021, E = 101010;
struct MMC { // O(VE)
#define FZ(n) memset((n), 0, sizeof(n))
    struct Edge { int v, u; double c; };
    int n, m, prv[V][V], prve[V][V], vst[V];
    Edge e[E];
    vector<int> edgeID, cycle, rho;
    double d[V][V];
    void init(int _n) {
        n = _n;
        m = 0;
    }
    // WARNING: TYPE matters
    void add_edge(int v, int u, double c) { e[m++] = {v, u, c}; }
    void bellman_ford() {
        for (int i = 0; i < n; i++)
            d[0][i] = 0;
        for (int i = 0; i < n; i++) {
            fill(d[i + 1], d[i + 1] + n, inf);
            for (int j = 0; j < m; j++) {
                int v = e[j].v, u = e[j].u;
                if (d[i][v] < inf && d[i + 1][u] > d[i][v] + e[j].c) {
                    d[i + 1][u] = d[i][v] + e[j].c;
                    prv[i + 1][u] = v;
                    prve[i + 1][u] = j;
                }
            }
        }
    }
    double solve() {
        // returns inf if no cycle, mmc otherwise
        double mmc = inf;
        int st = -1;
        bellman_ford();
        for (int i = 0; i < n; i++) {
            double avg = -inf;
            for (int k = 0; k < n; k++) {
                if (d[n][i] < inf - eps)
                    avg = max(avg, (d[n][i] - d[k][i]) / (n - k));
                else
                    avg = max(avg, inf);
            }
            if (avg < mmc)
                tie(mmc, st) = tie(avg, i);
        }
        FZ(vst);
        edgeID.clear();
        cycle.clear();
    }
};

```

```

rho.clear();
for (int i = n; !vst[st]; st = prv[i--][st]) {
    vst[st]++;
    edgeID.emplace_back(prve[i][st]);
    rho.emplace_back(st);
}
while (vst[st] != 2) {
    int v = rho.back(); rho.pop_back();
    cycle.emplace_back(v);
    vst[v]++;
}
reverse(all(edgeID));
edgeID.resize(cycle.size());
return mmc;
}
} mmc;

```

3.12 Mo's Algorithm on Tree

```

dfs u:
    push u
    iterate subtree
    push u
Let P = LCA(u, v) with St(u) <= St(v)
if (P == u) query[St(u), St(v)]
else query[Ed(u), St(v)], query[St(P), St(P)]

```

3.13 Virtual Tree [ad5cf5]

```

vector<pair<int, int>> build(vector<int> vs, int r) {
    vector<pair<int, int>> res;
    sort(vs.begin(), vs.end(), [](int i, int j) {
        return dfn[i] < dfn[j]; });
    vector<int> s = {r};
    for (int v : vs) if (v != r) {
        if (int o = lca(v, s.back()); o != s.back()) {
            while (s.size() >= 2) {
                if (dfn[s[s.size() - 2]] < dfn[o]) break;
                res.emplace_back(s[s.size() - 2], s.back());
                s.pop_back();
            }
            if (s.back() != o) {
                res.emplace_back(o, s.back());
                s.back() = o;
            }
        }
        s.push_back(v);
    }
    for (size_t i = 1; i < s.size(); ++i)
        res.emplace_back(s[i - 1], s[i]);
    return res; // (x, y): x->y
}

```

4 Matching & Flow

4.1 Bipartite Matching [2fc9c7]

```

struct BipartiteMatching {
    vector<int> X[N];
    int fX[N], fY[N], n;
    bitset<N> vis;
    bool dfs(int x) {
        for (auto i : X[x]) if (not vis[i]) {
            vis[i] = true;
            if (fY[i] == -1 || dfs(fY[i])) {
                fY[fX[x] = i] = x;
                return true;
            }
        }
        return false;
    }
    void init(int n_, int m) {
        fill_n(X, n = n_, vector<int>());
        memset(fX, -1, sizeof(int) * n);
        memset(fY, -1, sizeof(int) * m);
    }
    void add_edge(int x, int y) { X[x].push_back(y); }
    int solve() { // return how many pair matched
        int cnt = 0;
        for (int i = 0; i < n; i++) {
            vis.reset();
            cnt += dfs(i);
        }
        return cnt;
    }
};

```

```
}
};
```

4.2 Dijkstra Cost Flow [06a723]

```
template <typename F, typename C> class MCMF {
    static constexpr F INF_F = numeric_limits<F>::max();
    static constexpr C INF_C = numeric_limits<C>::max();

    struct E {
        int to, r;
        F f; C c;
        E() {}
        E(int a, int b, F x, C y)
            : to(a), r(b), f(x), c(y) {}
    };
    vector<vector<E>> g;
    vector<pair<int, int>> f;
    vector<F> up;
    vector<C> d, h;
    optional<pair<F, C>> step(int S, int T) {
        priority_queue<pair<C, int>> q;
        q.emplace(d[S] = 0, S), up[S] = INF_F;
        while (not q.empty()) {
            auto [l, u] = q.top(); q.pop();
            if (up[u] == 0 or l != -d[u]) continue;
            for (int i = 0; i < int(g[u].size()); ++i) {
                auto e = g[u][i]; int v = e.to;
                auto nd = d[u] + e.c + h[u] - h[v];
                if (e.f <= 0 or d[v] <= nd)
                    continue;
                f[v] = {u, i};
                up[v] = min(up[u], e.f);
                q.emplace(-(d[v] = nd), v);
            }
        }
        if (d[T] == INF_C) return nullopt;
        for (size_t i = 0; i < d.size(); i++) h[i] += d[i];
        for (int i = T; i != S; i = f[i].first) {
            auto &eg = g[f[i].first][f[i].second];
            eg.f -= up[T];
            g[eg.to][eg.r].f += up[T];
        }
        return pair{up[T], h[T]};
    }

public:
    MCMF(int n) : g(n), f(n), up(n), d(n, INF_C), h(n) {}
    void add_edge(int s, int t, F c, C w) {
        g[s].emplace_back(t, int(g[t].size()), c, w);
        g[t].emplace_back(s, int(g[s].size()) - 1, 0, -w);
    }
    pair<F, C> solve(int a, int b) {
        F c = 0; C w = 0;
        while (auto r = step(a, b)) {
            c += r->first, w += r->first * r->second;
            fill(d.begin(), d.end(), INF_C);
        }
        return {c, w};
    }
};
```

4.3 Dinic [ebd802]

```
template <typename Cap = int64_t>
class Dinic {
private:
    struct E {
        int to, rev;
        Cap cap;
    };
    int n, st, ed;
    vector<vector<E>> G;
    vector<int> lv, idx;
    bool BFS() {
        lv.assign(n, -1);
        queue<int> bfs;
        bfs.push(st); lv[st] = 0;
        while (not bfs.empty()) {
            int u = bfs.front(); bfs.pop();
            for (auto e : G[u]) {
                if (e.cap <= 0 or lv[e.to] != -1) continue;
                bfs.push(e.to); lv[e.to] = lv[u] + 1;
            }
        }
    }
};
```

```
}
return lv[ed] != -1;
}
Cap DFS(int u, Cap f) {
    if (u == ed) return f;
    Cap ret = 0;
    for (int &i = idx[u]; i < int(G[u].size()); ++i) {
        auto &e = G[u][i];
        if (e.cap <= 0 or lv[e.to] != lv[u] + 1) continue;
        Cap nf = DFS(e.to, min(f, e.cap));
        ret += nf; e.cap -= nf; f -= nf;
        G[e.to][e.rev].cap += nf;
        if (f == 0) return ret;
    }
    if (ret == 0) lv[u] = -1;
    return ret;
}
public:
    void init(int n_) { G.assign(n = n_, vector<E>()); }
    void add_edge(int u, int v, Cap c) {
        G[u].push_back({v, int(G[v].size()), c});
        G[v].push_back({u, int(G[u].size()) - 1, 0});
    }
    Cap max_flow(int st_, int ed_) {
        st = st_, ed = ed_; Cap ret = 0;
        while (BFS()) {
            idx.assign(n, 0);
            Cap f = DFS(st, numeric_limits<Cap>::max());
            ret += f;
            if (f == 0) break;
        }
        return ret;
    }
};
```

4.4 Flow Models

- Maximum/Minimum flow with lower bound / Circulation problem
 - Construct super source S and sink T .
 - For each edge (x, y, l, u) , connect $x \rightarrow y$ with capacity $u - l$.
 - For each vertex v , denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
 - If $in(v) > 0$, connect $S \rightarrow v$ with capacity $in(v)$, otherwise, connect $v \rightarrow T$ with capacity $-in(v)$.
 - To maximize, connect $t \rightarrow s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T . If $f \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
 - To minimize, let f be the maximum flow from S to T . Connect $t \rightarrow s$ with capacity ∞ and let the flow from S to T be f' . If $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, f' is the answer.
 - The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
 - Redirect every edge: $y \rightarrow x$ if $(x, y) \in M$, $x \rightarrow y$ otherwise.
 - DFS from unmatched vertices in X .
 - $x \in X$ is chosen iff x is unvisited.
 - $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
 - Construct super source S and sink T
 - For each edge (x, y, c) , connect $x \rightarrow y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \rightarrow x$ with $(cost, cap) = (-c, 1)$
 - For each edge with $c < 0$, sum these cost as K , then increase $d(y)$ by 1, decrease $d(x)$ by 1
 - For each vertex v with $d(v) > 0$, connect $S \rightarrow v$ with $(cost, cap) = (0, d(v))$
 - For each vertex v with $d(v) < 0$, connect $v \rightarrow T$ with $(cost, cap) = (0, -d(v))$
 - Flow from S to T , the answer is the cost of the flow $C + K$
- Maximum density induced subgraph
 - Binary search on answer, suppose we're checking answer T
 - Construct a max flow model, let K be the sum of all weights
 - Connect source $s \rightarrow v, v \in G$ with capacity K
 - For each edge (u, v, w) in G , connect $u \rightarrow v$ and $v \rightarrow u$ with capacity w
 - For $v \in G$, connect it with sink $v \rightarrow t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
 - T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
 - For each $v \in V$ create a copy v' , and connect $u' \rightarrow v'$ with weight $w(u, v)$.
 - Connect $v \rightarrow v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v .
 - Find the minimum weight perfect matching on G' .

- Project selection problem
 1. If $p_v > 0$, create edge (s, v) with capacity p_v ; otherwise, create edge (v, t) with capacity $-p_v$.
 2. Create edge (u, v) with capacity w with w being the cost of choosing u without choosing v .
 3. The mincut is equivalent to the maximum profit of a subset of projects.
- 0/1 quadratic programming

$$\sum_x c_x x + \sum_y c_y y + \sum_{xy} c_{xy} x y + \sum_{xyx'y'} c_{xyx'y'} (x y + x' y')$$

can be minimized by the mincut of the following graph:

1. Create edge (x, t) with capacity c_x and create edge (s, y) with capacity c_y .
2. Create edge (x, y) with capacity c_{xy} .
3. Create edge (x, y) and edge (x', y') with capacity $c_{xyx'y'}$.

4.5 General Graph Matching [00732c]

```
namespace matching {
int fa[kN], pre[kN], match[kN], s[kN], v[kN];
vector<int> g[kN];
queue<int> q;
void Init(int n) {
for (int i = 0; i <= n; ++i) match[i] = pre[i] = n;
for (int i = 0; i < n; ++i) g[i].clear();
}
void AddEdge(int u, int v) {
g[u].push_back(v);
g[v].push_back(u);
}
int Find(int u) {
return u == fa[u] ? u : fa[u] = Find(fa[u]);
}
int LCA(int x, int y, int n) {
static int tk = 0; tk++;
x = Find(x), y = Find(y);
for (; ; swap(x, y)) {
if (x != n) {
if (v[x] == tk) return x;
v[x] = tk;
x = Find(pre[match[x]]);
}
}
}
void Blossom(int x, int y, int l) {
while (Find(x) != l) {
pre[x] = y, y = match[x];
if (s[y] == 1) q.push(y), s[y] = 0;
if (fa[x] == x) fa[x] = l;
if (fa[y] == y) fa[y] = l;
x = pre[y];
}
}
bool Bfs(int r, int n) {
for (int i = 0; i <= n; ++i) fa[i] = i, s[i] = -1;
while (!q.empty()) q.pop();
q.push(r);
s[r] = 0;
while (!q.empty()) {
int x = q.front(); q.pop();
for (int u : g[x]) {
if (s[u] == -1) {
pre[u] = x, s[u] = 1;
if (match[u] == n) {
for (int a = u, b = x, last; b != n; a = last, b = pre[a])
last = match[b], match[b] = a, match[a] = b;
return true;
}
q.push(match[u]);
s[match[u]] = 0;
} else if (!s[u] && Find(u) != Find(x)) {
int l = LCA(u, x, n);
Blossom(x, u, l);
Blossom(u, x, l);
}
}
}
return false;
}
int Solve(int n) {
int res = 0;
for (int x = 0; x < n; ++x) {
if (match[x] == n) res += Bfs(x, n);
}
return res;
}
```

```
for (int x = 0; x < n; ++x) {
if (match[x] == n) res += Bfs(x, n);
}
return res;
}
```

4.6 Global Min-Cut

```
const int maxn = 500 + 5;
int w[maxn][maxn], g[maxn];
bool v[maxn], del[maxn];
void add_edge(int x, int y, int c) {
w[x][y] += c; w[y][x] += c;
}
pair<int, int> phase(int n) {
memset(v, false, sizeof(v));
memset(g, 0, sizeof(g));
int s = -1, t = -1;
while (true) {
int c = -1;
for (int i = 0; i < n; ++i) {
if (del[i] || v[i]) continue;
if (c == -1 || g[i] > g[c]) c = i;
}
if (c == -1) break;
v[s = t, t = c] = true;
for (int i = 0; i < n; ++i) {
if (del[i] || v[i]) continue;
g[i] += w[c][i];
}
}
return make_pair(s, t);
}
int mincut(int n) {
int cut = 1e9;
memset(del, false, sizeof(del));
for (int i = 0; i < n - 1; ++i) {
int s, t; tie(s, t) = phase(n);
del[t] = true; cut = min(cut, g[t]);
for (int j = 0; j < n; ++j) {
w[s][j] += w[t][j]; w[j][s] += w[j][t];
}
}
return cut;
}
```

4.7 GomoryHu Tree

```
int g[maxn];
vector<edge> GomoryHu(int n){
vector<edge> rt;
for(int i=1;i<=n;++i)g[i]=1;
for(int i=2;i<=n;++i){
int t=g[i];
flow.reset(); // clear flows on all edge
rt.push_back({i,t,flow(i,t)});
flow.walk(i); // bfs points that connected to i (use
edges not fully flow)
for(int j=i+1;j<=n;++j){
if(g[j]==t && flow.connect(j))g[j]=i; // check if i
can reach j
}
}
return rt;
}
```

4.8 Kuhn Munkres [1d3c40]

```
class KM {
private:
static constexpr lld INF = 1LL << 60;
vector<lld> hl, hr, slk;
vector<int> fl, fr, pre, qu;
vector<vector<lld>> w;
vector<bool> vl, vr;
int n, ql, qr;
bool check(int x) {
if (vl[x] == true, fl[x] != -1)
return vr[qu[qr++]] = fl[x] == true;
while (x != -1) swap(x, fr[fl[x] = pre[x]]);
return false;
}
void bfs(int s) {
fill(slk.begin(), slk.end(), INF);
for (int x = 0; x < n; ++x) {
if (match[x] == n) res += Bfs(x, n);
}
return res;
}
```

```

fill(vl.begin(), vl.end(), false);
fill(vr.begin(), vr.end(), false);
ql = qr = 0;
vr[qu[qr++]] = s;
while (true) {
    lld d;
    while (ql < qr) {
        for (int x = 0, y = qu[ql++]; x < n; ++x) {
            if (!vl[x] && slk[x] >= (d = hl[x] + hr[y] - w[x][y])) {
                if (pre[x] == y, d) slk[x] = d;
                else if (!check(x)) return;
            }
        }
    }
    d = INF;
    for (int x = 0; x < n; ++x)
        if (!vl[x] && d > slk[x]) d = slk[x];
    for (int x = 0; x < n; ++x) {
        if (vl[x]) hl[x] += d;
        else slk[x] -= d;
        if (vr[x]) hr[x] -= d;
    }
    for (int x = 0; x < n; ++x)
        if (!vl[x] && !slk[x] && !check(x)) return;
}
}
public:
void init( int n_ ) {
    qu.resize(n = n_);
    fl.assign(n, -1); fr.assign(n, -1);
    hr.assign(n, 0); hl.resize(n);
    w.assign(n, vector<lld>(n));
    slk.resize(n); pre.resize(n);
    vl.resize(n); vr.resize(n);
}
void set_edge( int u, int v, lld x ) {w[u][v] = x;}
lld solve() {
    for (int i = 0; i < n; ++i)
        hl[i] = *max_element(w[i].begin(), w[i].end());
    for (int i = 0; i < n; ++i) bfs(i);
    lld res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
}
} km;

```

4.9 Minimum Cost Circulation [d99194]

```

struct Edge { int to, cap, rev, cost; };
vector<Edge> g[kN];
int dist[kN], pv[kN], ed[kN];
bool mark[kN];
int NegativeCycle(int n) {
    memset(mark, false, sizeof(mark));
    memset(dist, 0, sizeof(dist));
    int upd = -1;
    for (int i = 0; i <= n; ++i) {
        for (int j = 0; j < n; ++j) {
            int idx = 0;
            for (auto &e : g[j]) {
                if (e.cap > 0 && dist[e.to] > dist[j] + e.cost) {
                    dist[e.to] = dist[j] + e.cost;
                    pv[e.to] = j, ed[e.to] = idx;
                    if (i == n) {
                        upd = j;
                        while (!mark[upd]) mark[upd] = 1, upd = pv[upd];
                        return upd;
                    }
                }
            }
            idx++;
        }
    }
    return -1;
}
int Solve(int n) {
    int rt = -1, ans = 0;
    while ((rt = NegativeCycle(n)) >= 0) {
        memset(mark, false, sizeof(mark));
        vector<pair<int, int>> cyc;
        while (!mark[rt]) {
            cyc.emplace_back(pv[rt], ed[rt]);
            mark[rt] = true;

```

```

            rt = pv[rt];
        }
        reverse(cyc.begin(), cyc.end());
        int cap = kInf;
        for (auto &i : cyc) {
            auto &e = g[i.first][i.second];
            cap = min(cap, e.cap);
        }
        for (auto &i : cyc) {
            auto &e = g[i.first][i.second];
            e.cap -= cap;
            g[e.to][e.rev].cap += cap;
            ans += e.cost * cap;
        }
    }
    return ans;
}

```

4.10 Minimum Cost Maximum Flow [6d1b01]

```

template <typename F, typename C> class MCMF {
    static constexpr F INF_F = numeric_limits<F>::max();
    static constexpr C INF_C = numeric_limits<C>::max();

    struct E {
        int to, r;
        F f; C c;
        E() {}
        E(int a, int b, F x, C y)
            : to(a), r(b), f(x), c(y) {}
    };
    vector<vector<E>> g;
    vector<pair<int, int>> f;
    vector<bool> inq;
    vector<F> up; vector<C> d;
    optional<pair<F, C>> step(int S, int T) {
        queue<int> q;
        for (q.push(S), d[S] = 0, up[S] = INF_F;
             !q.empty(); q.pop()) {
            int u = q.front(); inq[u] = false;
            if (up[u] == 0) continue;
            for (int i = 0; i < int(g[u].size()); ++i) {
                auto e = g[u][i]; int v = e.to;
                if (e.f <= 0 or d[v] <= d[u] + e.c)
                    continue;
                d[v] = d[u] + e.c; f[v] = {u, i};
                up[v] = min(up[u], e.f);
                if (not inq[v]) q.push(v);
                inq[v] = true;
            }
        }
        if (d[T] == INF_C) return nullopt;
        for (int i = T; i != S; i = f[i].first) {
            auto &eg = g[f[i].first][f[i].second];
            eg.f -= up[T];
            g[eg.to][eg.r].f += up[T];
        }
        return pair{up[T], d[T]};
    }

public:
    MCMF(int n) : g(n), f(n), inq(n), up(n), d(n, INF_C) {}
    void add_edge(int s, int t, F c, C w) {
        g[s].emplace_back(t, int(g[t].size()), c, w);
        g[t].emplace_back(s, int(g[s].size()) - 1, 0, -w);
    }
    pair<F, C> solve(int a, int b) {
        F c = 0; C w = 0;
        while (auto r = step(a, b)) {
            c += r->first, w += r->first * r->second;
            fill(inq.begin(), inq.end(), false);
            fill(d.begin(), d.end(), INF_C);
        }
        return {c, w};
    }
};

```

4.11 Maximum Weight Graph Matching [60ca53]

```

struct WeightGraph {
    static const int inf = INT_MAX;

```

```

static const int maxn = 514;
struct edge {
    int u, v, w;
    edge(){}
    edge(int u, int v, int w): u(u), v(v), w(w) {}
};
int n, n_x;
edge g[maxn * 2][maxn * 2];
int lab[maxn * 2];
int match[maxn * 2], slack[maxn * 2], st[maxn * 2], pa
    [maxn * 2];
int flo_from[maxn * 2][maxn + 1], S[maxn * 2], vis[
    maxn * 2];
vector<int> flo[maxn * 2];
queue<int> q;
int e_delta(const edge &e) { return lab[e.u] + lab[e.v]
    - g[e.u][e.v].w * 2; }
void update_slack(int u, int x) { if (!slack[x] ||
    e_delta(g[u][x]) < e_delta(g[slack[x]][x])) slack[x]
    = u; }
void set_slack(int x) {
    slack[x] = 0;
    for (int u = 1; u <= n; ++u)
        if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
            update_slack(u, x);
}
void q_push(int x) {
    if (x <= n) q.push(x);
    else for (size_t i = 0; i < flo[x].size(); ++i)
        q.push(flo[x][i]);
}
void set_st(int x, int b) {
    st[x] = b;
    if (x > n) for (size_t i = 0; i < flo[x].size(); ++i)
        set_st(flo[x][i], b);
}
int get_pr(int b, int xr) {
    int pr = find(flo[b].begin(), flo[b].end(), xr) - flo
        [b].begin();
    if (pr % 2 == 1) {
        reverse(flo[b].begin() + 1, flo[b].end());
        return (int)flo[b].size() - pr;
    }
    return pr;
}
void set_match(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;
    edge e = g[u][v];
    int xr = flo_from[u][e.u], pr = get_pr(u, xr);
    for (int i = 0; i < pr; ++i) set_match(flo[u][i], flo
        [u][i ^ 1]);
    set_match(xr, v);
    rotate(flo[u].begin(), flo[u].begin() + pr, flo[u].
        end());
}
void augment(int u, int v) {
    for (; ; ) {
        int xnv = st[match[u]];
        set_match(u, v);
        if (!xnv) return;
        set_match(xnv, st[pa[xnv]]);
        u = st[pa[xnv]], v = xnv;
    }
}
int get_lca(int u, int v) {
    static int t = 0;
    for (++t; u || v; swap(u, v)) {
        if (u == 0) continue;
        if (vis[u] == t) return u;
        vis[u] = t;
        u = st[match[u]];
        if (u) u = st[pa[u]];
    }
    return 0;
}
void add_blossom(int u, int lca, int v) {
    int b = n + 1;
    while (b <= n_x && st[b]) ++b;
    if (b > n_x) ++n_x;
    lab[b] = 0, S[b] = 0;
    match[b] = match[lca];
    flo[b].clear();
    flo[b].push_back(lca);
    for (int x = u, y; x != lca; x = st[pa[y]])
        flo[b].push_back(x), flo[b].push_back(y = st[match[x]
            ]), q.push(y);
    reverse(flo[b].begin() + 1, flo[b].end());
    for (int x = v, y; x != lca; x = st[pa[y]])
        flo[b].push_back(x), flo[b].push_back(y = st[match[x]
            ]), q.push(y);
    set_st(b, b);
    for (int x = 1; x <= n_x; ++x) g[b][x].w = g[x][b].w
        = 0;
    for (int x = 1; x <= n; ++x) flo_from[b][x] = 0;
    for (size_t i = 0; i < flo[b].size(); ++i) {
        int xs = flo[b][i];
        for (int x = 1; x <= n_x; ++x)
            if (g[b][x].w == 0 || e_delta(g[xs][x]) < e_delta(g
                [b][x]))
                g[b][x] = g[xs][x], g[x][b] = g[x][xs];
        for (int x = 1; x <= n; ++x)
            if (flo_from[xs][x]) flo_from[b][x] = xs;
    }
    set_slack(b);
}
void expand_blossom(int b) {
    for (size_t i = 0; i < flo[b].size(); ++i)
        set_st(flo[b][i], flo[b][i]);
    int xr = flo_from[b][g[b][pa[b]].u], pr = get_pr(b,
        xr);
    for (int i = 0; i < pr; i += 2) {
        int xs = flo[b][i], xns = flo[b][i + 1];
        pa[xs] = g[xns][xs].u;
        S[xs] = 1, S[xns] = 0;
        slack[xs] = 0, set_slack(xns);
        q.push(xns);
    }
    S[xr] = 1, pa[xr] = pa[b];
    for (size_t i = pr + 1; i < flo[b].size(); ++i) {
        int xs = flo[b][i];
        S[xs] = -1, set_slack(xs);
    }
    st[b] = 0;
}
bool on_found_edge(const edge &e) {
    int u = st[e.u], v = st[e.v];
    if (S[v] == -1) {
        pa[v] = e.u, S[v] = 1;
        int nu = st[match[v]];
        slack[v] = slack[nu] = 0;
        S[nu] = 0, q.push(nu);
    } else if (S[v] == 0) {
        int lca = get_lca(u, v);
        if (!lca) return augment(u, v), augment(v, u), true;
        else add_blossom(u, lca, v);
    }
    return false;
}
bool matching() {
    memset(S + 1, -1, sizeof(int) * n_x);
    memset(slack + 1, 0, sizeof(int) * n_x);
    q = queue<int>();
    for (int x = 1; x <= n_x; ++x)
        if (st[x] == x && !match[x]) pa[x] = 0, S[x] = 0,
            q.push(x);
    if (q.empty()) return false;
    for (; ; ) {
        while (q.size()) {
            int u = q.front(); q.pop();
            if (S[st[u]] == 1) continue;
            for (int v = 1; v <= n; ++v)
                if (g[u][v].w > 0 && st[u] != st[v]) {
                    if (e_delta(g[u][v]) == 0) {
                        if (on_found_edge(g[u][v])) return true;
                    } else update_slack(u, st[v]);
                }
        }
        int d = inf;
        for (int b = n + 1; b <= n_x; ++b)
            if (st[b] == b && S[b] == 1) d = min(d, lab[b] / 2)
                ;
        for (int x = 1; x <= n_x; ++x)
            if (st[x] == x && slack[x]) {

```

```

    if (S[x] == -1) d = min(d, e_delta(g[slack[x]])[x])
    );
    else if (S[x] == 0) d = min(d, e_delta(g[slack[x]]
    ][x]) / 2);
    }
    for (int u = 1; u <= n; ++u) {
        if (S[st[u]] == 0) {
            if (lab[u] <= d) return 0;
            lab[u] -= d;
        } else if (S[st[u]] == 1) lab[u] += d;
    }
    for (int b = n + 1; b <= n_x; ++b)
        if (st[b] == b) {
            if (S[st[b]] == 0) lab[b] += d * 2;
            else if (S[st[b]] == 1) lab[b] -= d * 2;
        }
    q = queue<int>();
    for (int x = 1; x <= n_x; ++x)
        if (st[x] == x && slack[x] && st[slack[x]] != x &&
            e_delta(g[slack[x]])[x] == 0)
            if (on_found_edge(g[slack[x]])[x]) return true;
    for (int b = n + 1; b <= n_x; ++b)
        if (st[b] == b && S[b] == 1 && lab[b] == 0)
            expand_blossom(b);
    }
    return false;
}
pair<long long, int> solve() {
    memset(match + 1, 0, sizeof(int) * n);
    n_x = n;
    int n_matches = 0;
    long long tot_weight = 0;
    for (int u = 0; u <= n; ++u) st[u] = u, flo[u].clear
    ();
    int w_max = 0;
    for (int u = 1; u <= n; ++u)
        for (int v = 1; v <= n; ++v) {
            flo_from[u][v] = (u == v ? u : 0);
            w_max = max(w_max, g[u][v].w);
        }
    for (int u = 1; u <= n; ++u) lab[u] = w_max;
    while (matching()) ++n_matches;
    for (int u = 1; u <= n; ++u)
        if (match[u] && match[u] < u)
            tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
}
void add_edge(int ui, int vi, int wi) { g[ui][vi].w =
    g[vi][ui].w = wi; }
void init(int _n) {
    n = _n;
    for (int u = 1; u <= n; ++u)
        for (int v = 1; v <= n; ++v)
            g[u][v] = edge(u, v, 0);
}
};

```

5 Math

5.1 Common Bounds

$$p(0) = 1, p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k - 1)/2)$$

$$p(n) \approx 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	100	1e3	1e6	1e9	1e12	1e15	1e18
$\max_{i \leq n}(d(i))$	12	32	240	1344	6720	26880	103680

n	1	2	3	4	5	6	7	8	9	10
$\frac{n}{\binom{2n}{n}}$	2	6	20	70	252	924	3432	12870	48620	184756

5.2 Stirling Number

5.2.1 First Kind

$S_1(n, k)$ counts the number of permutations of n elements with k disjoint cycles.

$$S_1(n, k) = (n - 1) \cdot S_1(n - 1, k) + S_1(n - 1, k - 1)$$

$$x(x + 1) \dots (x + n - 1) = \sum_{k=0}^n S_1(n, k) x^k$$

$$g(x) = x(x + 1) \dots (x + n - 1) = \sum_{k=0}^n a_k x^k$$

$$\Rightarrow g(x + n) = \sum_{k=0}^n \frac{b_k}{(n - k)!} x^{n - k},$$

$$b_k = \sum_{i=0}^k ((n - i)! a_{n-i}) \cdot \left(\frac{n^{k-i}}{(k-i)!} \right)$$

5.2.2 Second Kind

$S_2(n, k)$ counts the number of ways to partition a set of n elements into k nonempty sets.

$$S_2(n, k) = S_2(n - 1, k - 1) + k \cdot S_2(n - 1, k)$$

$$S_2(n, k) = \sum_{i=0}^k \binom{k}{i} i^n (-1)^{k-i} = \sum_{i=0}^k \frac{(-1)^i}{i!} \cdot \frac{(k-i)^n}{(k-i)!}$$

5.3 ax+by=gcd

```

// ax+ny = 1, ax+ny == ax == 1 (mod n)
void exgcd(lld x, lld y, lld &g, lld &a, lld &b) {
    if (y == 0) g=x, a=1, b=0;
    else exgcd(y, x%y, g, b, a), b-=(x/y)*a;
}

```

5.4 Chinese Remainder

```

// please ensure r_i \in [0, m_i)
bool crt(lld &m1, lld &r1, lld m2, lld r2) {
    if (m2 > m1) swap(m1, m2), swap(r1, r2);
    lld g, a, b; exgcd(m1, m2, g, a, b);
    if ((r2 - r1) % g != 0) return false;
    m2 /= g; lld D = (r2 - r1) / g % m2 * a % m2;
    r1 += (D < 0 ? D + m2 : D) * m1; m1 *= m2;
    assert(r1 >= 0 && r1 < m1);
    return true;
}

```

5.5 De-Bruijn

```

int res[maxn], aux[maxn], sz;
void db(int t, int p, int n, int k) {
    if (t > n) {
        if (n % p == 0)
            for (int i = 1; i <= p; ++i)
                res[sz++] = aux[i];
    } else {
        aux[t] = aux[t - p];
        db(t + 1, p, n, k);
        for (int i = aux[t - p] + 1; i < k; ++i) {
            aux[t] = i;
            db(t + 1, t, n, k);
        }
    }
}
int de_bruijn(int k, int n) {
    // return cyclic string of len k^n s.t. every string
    // of len n using k char appears as a substring.
    if (k == 1) {
        res[0] = 0;
        return 1;
    }
    for (int i = 0; i < k * n; ++i) aux[i] = 0;
    sz = 0;
    db(1, 1, n, k);
    return sz;
}

```

5.6 DiscreteLog [86e463]

```

template<typename> Int>
Int BSGS(Int x, Int y, Int M) {
    // x^? \equiv y (mod M)
    Int t = 1, c = 0, g = 1;
    for (Int M_ = M; M_ > 0; M_ >>= 1)
        g = g * x % M;
    for (g = gcd(g, M); t % g != 0; ++c) {
        if (t == y) return c;
        t = t * x % M;
    }
    if (y % g != 0) return -1;
    t /= g, y /= g, M /= g;
    Int h = 0, gs = 1;
}

```

```

for (; h * h < M; ++h) gs = gs * x % M;
unordered_map<Int, Int> bs;
for (Int s = 0; s < h; bs[y] = ++s)
    y = y * x % M;
for (Int s = 0; s < M; s += h) {
    t = t * gs % M;
    if (bs.count(t)) return c + s + h - bs[t];
}
return -1;
}

```

5.7 Quadratic residue [14d6e4]

```

struct S {
    int MOD, w;
    int64_t x, y;
    S(int m, int w_=-1, int64_t x_=1, int64_t y_=0)
        : MOD(m), w(w_), x(x_), y(y_) {}
    S operator*(const S &rhs) const {
        int w_ = w;
        if (w_ == -1) w_ = rhs.w;
        assert(w_ != -1 and w_ == rhs.w);
        return { MOD, w_,
            (x * rhs.x + y * rhs.y % MOD * w) % MOD,
            (x * rhs.y + y * rhs.x) % MOD };
    }
};

int get_root(int n, int P) {
    if (P == 2 or n == 0) return n;
    auto check = [&](int x) {
        return qpow(x, (P - 1) / 2, P);
    };
    if (check(n) != 1) return -1;
    int64_t a; int w; mt19937 rnd(7122);
    do { a = rnd() % P;
        w = ((a * a - n) % P + P) % P;
    } while (check(w) != P - 1);
    return qpow(S(P, w, a, 1), (P + 1) / 2).x;
}

```

5.8 Extended Euler

$$a^b \equiv \begin{cases} a^{(b \bmod \varphi(m)) + \varphi(m)} & \text{if } (a, m) \neq 1 \wedge b \geq \varphi(m) \\ a^{b \bmod \varphi(m)} & \text{otherwise} \end{cases} \pmod{m}$$

5.9 ExtendedFloorSum

$$g(a, b, c, n) = \sum_{i=0}^n i \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor \cdot \frac{n(n+1)}{2} \\ + g(a \bmod c, b \bmod c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1) \\ - h(c, c-b-1, a, m-1)), & \text{otherwise} \end{cases}$$

$$h(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor^2$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor^2 \cdot (n+1) \\ + \lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot n(n+1) \\ + h(a \bmod c, b \bmod c, n) \\ + 2 \lfloor \frac{a}{c} \rfloor \cdot g(a \bmod c, b \bmod c, n) \\ + 2 \lfloor \frac{b}{c} \rfloor \cdot f(a \bmod c, b \bmod c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) \\ - 2f(c, c-b-1, a, m-1) - f(a, b, c, n), & \text{otherwise} \end{cases}$$

5.10 FloorSum [bda6b2]

```

// @param n `n < 2^32`
// @param m `1 <= m < 2^32`
// @return sum_{i=0}^{n-1} floor((ai+b)/m) mod 2^64
llu floor_sum_unsigned(llu n, llu m, llu a, llu b) {
    llu ans = 0;
    while (true) {
        if (a >= m) {
            ans += n * (n - 1) / 2 * (a / m); a %= m;
        }
        if (b >= m) {
            ans += n * (b / m); b %= m;
        }
        llu y_max = a * n + b;
        if (y_max < m) break;
        // y_max < m * (n + 1)
        // floor(y_max / m) <= n

```

```

    n = (llu)(y_max / m), b = (llu)(y_max % m);
    swap(m, a);
}
return ans;
}

llu floor_sum(llu n, llu m, llu a, llu b) {
    llu ans = 0;
    if (a < 0) {
        llu a2 = (a % m + m) % m;
        ans -= 1ULL * n * (n - 1) / 2 * ((a2 - a) / m);
        a = a2;
    }
    if (b < 0) {
        llu b2 = (b % m + m) % m;
        ans -= 1ULL * n * ((b2 - b) / m);
        b = b2;
    }
    return ans + floor_sum_unsigned(n, m, a, b);
}

```

5.11 FloorSum

```

// min{k | l <= ((ak) mod m) <= r}
optional<uint64_t> mod_min(uint32_t a, uint32_t m,
    uint32_t l, uint32_t r) {
    if (a == 0) return l ? nullopt : 0;
    if (auto k = uint64_t(l + a - 1) / a; k * a <= r)
        return k;
    auto b = m / a, c = m % a;
    if (auto y = mod_min(c, a, a - r % a, a - l % a))
        return (l + *y * c + a - 1) / a + *y * b;
    return nullopt;
}

```

5.12 Fast Fourier Transform

```

namespace fft {
using VI = vector<int>;
using VL = vector<long long>;
const double pi = acos(-1);
cplx omega[maxn + 1];
void prefft() {
    for (int i = 0; i <= maxn; i++)
        omega[i] = cplx(cos(2 * pi * i / maxn),
            sin(2 * pi * i / maxn));
}

void fft(vector<cplx> &v, int n) {
    int z = __builtin_ctz(n) - 1;
    for (int i = 0; i < n; ++i) {
        int x = 0, j = 0;
        for (; (1 << j) < n; ++j) x ^= (i >> j & 1) << (z - j);
        if (x > i) swap(v[x], v[i]);
    }
    for (int s = 2; s <= n; s <= 1) {
        int z = s >> 1;
        for (int i = 0; i < n; i += s) {
            for (int k = 0; k < z; ++k) {
                cplx x = v[i + z + k] * omega[maxn / s * k];
                v[i + z + k] = v[i + k] - x;
                v[i + k] = v[i + k] + x;
            }
        }
    }
}

void ifft(vector<cplx> &v, int n) {
    fft(v, n); reverse(v.begin() + 1, v.end());
    for (int i = 0; i < n; ++i) v[i] = v[i] * cplx(1. / n, 0);
}

VL convolution(const VI &a, const VI &b) {
    // Should be able to handle N <= 10^5, C <= 10^4
    int sz = 1;
    while (sz < a.size() + b.size() - 1) sz <= 1;
    vector<cplx> v(sz);
    for (int i = 0; i < sz; ++i) {
        double re = i < a.size() ? a[i] : 0;
        double im = i < b.size() ? b[i] : 0;
        v[i] = cplx(re, im);
    }
    fft(v, sz);
    for (int i = 0; i <= sz / 2; ++i) {
        int j = (sz - i) & (sz - 1);
        cplx x = (v[i] + v[j].conj()) * (v[i] - v[j].conj())
            * cplx(0, -0.25);

```



```

    if (j != i) v[j] = (v[j] + v[i].conj()) * (v[j] - v[i]
        ].conj()) * cplx(0, -0.25);
    v[i] = x;
}
ifft(v, sz);
VL c(sz);
for (int i = 0; i < sz; ++i) c[i] = round(v[i].re);
return c;
}
VI convolution_mod(const VI &a, const VI &b, int p) {
    int sz = 1;
    while (sz + 1 < a.size() + b.size()) sz <<= 1;
    vector<cplx> fa(sz), fb(sz);
    for (int i = 0; i < (int)a.size(); ++i)
        fa[i] = cplx(a[i] & ((1 << 15) - 1), a[i] >> 15);
    for (int i = 0; i < (int)b.size(); ++i)
        fb[i] = cplx(b[i] & ((1 << 15) - 1), b[i] >> 15);
    fft(fa, sz), fft(fb, sz);
    double r = 0.25 / sz;
    cplx r2(0, -1), r3(r, 0), r4(0, -r), r5(0, 1);
    for (int i = 0; i <= (sz >> 1); ++i) {
        int j = (sz - i) & (sz - 1);
        cplx a1 = (fa[i] + fa[j].conj());
        cplx a2 = (fa[i] - fa[j].conj()) * r2;
        cplx b1 = (fb[i] + fb[j].conj()) * r3;
        cplx b2 = (fb[i] - fb[j].conj()) * r4;
        if (i != j) {
            cplx c1 = (fa[j] + fa[i].conj());
            cplx c2 = (fa[j] - fa[i].conj()) * r2;
            cplx d1 = (fb[j] + fb[i].conj()) * r3;
            cplx d2 = (fb[j] - fb[i].conj()) * r4;
            fa[i] = c1 * d1 + c2 * d2 * r5;
            fb[i] = c1 * d2 + c2 * d1;
        }
        fa[j] = a1 * b1 + a2 * b2 * r5;
        fb[j] = a1 * b2 + a2 * b1;
    }
    fft(fa, sz), fft(fb, sz);
    vector<int> res(sz);
    for (int i = 0; i < sz; ++i) {
        long long a = round(fa[i].re), b = round(fb[i].re),
            c = round(fa[i].im);
        res[i] = (a + ((b % p) << 15) + ((c % p) << 30)) % p;
    }
    return res;
}
}

```

5.13 FWT

```

/* or convolution:
 * x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
 * and convolution:
 * x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
void fwt(int x[], int N, bool inv = false) {
    for (int d = 1; d < N; d <<= 1)
        for (int s = 0; s < N; s += d * 2)
            for (int i = s; i < s + d; i++) {
                int j = i + d, ta = x[i], tb = x[j];
                x[i] = modadd(ta, tb);
                x[j] = modsub(ta, tb);
            }
    if (inv) {
        const int invn = modinv(N);
        for (int i = 0; i < N; i++)
            x[i] = modmul(x[i], invn);
    }
}

```

5.14 CRT for arbitrary mod

```

const int mod = 1000000007;
const int M1 = 985661441; // G = 3 for M1, M2, M3
const int M2 = 998244353;
const int M3 = 1004535809;
int superBigCRT(int64_t A, int64_t B, int64_t C) {
    static_assert(M1 <= M2 && M2 <= M3);
    constexpr int64_t r12 = modpow(M1, M2-2, M2);
    constexpr int64_t r13 = modpow(M1, M3-2, M3);
    constexpr int64_t r23 = modpow(M2, M3-2, M3);
    constexpr int64_t M1M2 = 1LL * M1 * M2 % mod;
    B = (B - A + M2) * r12 % M2;
    C = (C - A + M3) * r13 % M3;
    C = (C - B + M3) * r23 % M3;
    return (A + B * M1 + C * M1M2) % mod;
}

```

5.15 NTT [946e8e]

```

template <int mod, int G, int maxn> struct NTT {
    static_assert(maxn == (maxn & -maxn));
    int roots[maxn];
    NTT() {
        int r = modpow(G, (mod - 1) / maxn);
        for (int i = maxn >> 1; i; i >>= 1) {
            roots[i] = 1;
            for (int j = 1; j < i; j++)
                roots[i + j] = modmul(roots[i + j - 1], r);
            r = modmul(r, r);
        }
    }
    // n must be 2^k, and 0 <= F[i] < mod
    void operator()(int F[], int n, bool inv = false) {
        for (int i = 0, j = 0; i < n; i++) {
            if (i < j) swap(F[i], F[j]);
            for (int k = n >> 1; (j ^= k) < k; k >>= 1);
        }
        for (int s = 1; s < n; s *= 2) {
            for (int i = 0; i < n; i += s * 2) {
                for (int j = 0; j < s; j++) {
                    int a = F[i+j];
                    int b = modmul(F[i+j+s], roots[s+j]);
                    F[i+j] = modadd(a, b); // a + b
                    F[i+j+s] = modsub(a, b); // a - b
                }
            }
        }
        if (inv) {
            int invn = modinv(n);
            for (int i = 0; i < n; i++)
                F[i] = modmul(F[i], invn);
            reverse(F + 1, F + n);
        }
    }
};

```

5.16 Partition Number

```

ans[0] = tmp[0] = 1;
for (int i = 1; i * i <= n; i++) {
    for (int rep = 0; rep < 2; rep++)
        for (int j = i; j <= n - i * i; j++)
            modadd(tmp[j], tmp[j-i]);
    for (int j = i * i; j <= n; j++)
        modadd(ans[j], tmp[j - i * i]);
}

```

5.17 Pi Count (+Linear Sieve)

```

static constexpr int N = 1000000 + 5;
lld pi[N];
vector<int> primes;
bool sieved[N];
lld cube_root(lld x){
    lld s=cbrt(x-static_cast<long double>(0.1));
    while(s*s*s <= x) ++s;
    return s-1;
}
lld square_root(lld x){
    lld s=sqrt(x-static_cast<long double>(0.1));
    while(s*s <= x) ++s;
    return s-1;
}
void init(){
    primes.reserve(N);
    primes.push_back(1);
    for(int i=2;i<N;i++) {
        if(!sieved[i]) primes.push_back(i);
        pi[i] = !sieved[i] + pi[i-1];
        for(int p: primes) if(p > 1) {
            if(p * i >= N) break;
            sieved[p * i] = true;
            if(p % i == 0) break;
        }
    }
}
lld phi(lld m, lld n) {
    static constexpr int MM = 80000, NN = 500;
    static lld val[MM][NN];
}

```

```

if(m<MM&&n<NN&&val[m][n])return val[m][n]-1;
if(n == 0) return m;
if(primes[n] >= m) return 1;
lld ret = phi(m,n-1)-phi(m/primes[n],n-1);
if(m<MM&&n<NN) val[m][n] = ret+1;
return ret;
}
lld pi_count(lld);
lld P2(lld m, lld n) {
    lld sm = square_root(m), ret = 0;
    for(lld i = n+1;primes[i]<=sm;i++)
        ret+=pi_count(m/primes[i])-pi_count(primes[i])+1;
    return ret;
}
lld pi_count(lld m) {
    if(m < N) return pi[m];
    lld n = pi_count(cube_root(m));
    return phi(m, n) + n - 1 - P2(m, n);
}

```

5.18 Miller Rabin [0edab2]

```

bool isprime(llu x) {
    static auto witn = [](llu a, llu n, int t) {
        if (!a) return false;
        while (t--) {
            llu a2 = mmul(a, a, n);
            if (a2 == 1 && a != 1 && a != n - 1) return true;
            a = a2;
        }
        return a != 1;
    };
    if (x < 2) return false;
    if (!(x & 1)) return x == 2;
    int t = __builtin_ctzll(x - 1);
    llu odd = (x - 1) >> t;
    for (llu m:
        {2, 325, 9375, 28178, 450775, 9780504, 1795265022})
        if (witn(mpow(m % x, odd, x), x, t))
            return false;
    return true;
}

```

5.19 Pollard Rho [2ac1ad]

```

// does not work when n is prime
// return any non-trivial factor
llu pollard_rho(llu n) {
    static auto f = [](llu x, llu k, llu m) {
        return add(k, mul(x, x, m), m);
    };
    if (!(n & 1)) return 2;
    mt19937 rnd(120821011);
    while (true) {
        llu y = 2, yy = y, x = rnd() % n, t = 1;
        for (llu sz = 2; t == 1; sz <= 1, y = yy) {
            for (llu i = 0; t == 1 && i < sz; ++i) {
                yy = f(yy, x, n);
                t = gcd(yy > y ? yy - y : y - yy, n);
            }
            if (t != 1 && t != n) return t;
        }
    }
}

```

5.20 Berlekamp Massey

```

template <typename T>
vector<T> BerlekampMassey(const vector<T> &output) {
    vector<T> d(output.size() + 1), me, he;
    for (size_t f = 0, i = 1; i <= output.size(); ++i) {
        for (size_t j = 0; j < me.size(); ++j)
            d[i] += output[i - j - 2] * me[j];
        if ((d[i] -= output[i - 1]) == 0) continue;
        if (me.empty()) {
            me.resize(f = i);
            continue;
        }
        vector<T> o(i - f - 1);
        T k = -d[i] / d[f]; o.push_back(-k);
        for (T x : he) o.push_back(x * k);
        if (o.size() < me.size()) o.resize(me.size());
        for (size_t j = 0; j < me.size(); ++j) o[j] += me[j];
        if (i-f+he.size() >= me.size()) he = me, f = i;
        me = o;
    }
}

```

```

}
return me;
}

```

5.21 Characteristic Polynomial

```

#define rep(x, y, z) for (int x=y; x<z; x++)
using VI = vector<int>; using VVI = vector<VI>;
VVI Hessenberg(VVI H) { // test: 2021 PTZ korea K
    int N = H.size();
    for (int i = 0; i < N - 2; ++i) {
        if (!H[i + 1][i]) {
            for (int j = i + 2; j < N; ++j) {
                if (H[j][i]) {
                    rep(k, i, N) swap(H[i+1][k], H[j][k]);
                    rep(k, 0, N) swap(H[k][i+1], H[k][j]);
                    break;
                }
            }
        }
        if (!H[i + 1][i]) continue;
        int val = modinv(H[i + 1][i]);
        for (int j = i + 2; j < N; ++j) {
            int co = mul(val, H[j][i]);
            rep(k, i, N) subeq(H[j][k], mul(H[i+1][k], co));
            rep(k, 0, N) addeq(H[k][i+1], mul(H[k][j], co));
        }
    }
    return H;
}
VI CharacteristicPoly(const VVI &A) {
    int N = A.size(); auto H = Hessenberg(A);
    VVI P(N + 1, VI(N + 1)); P[0][0] = 1;
    for (int i = 1; i <= N; ++i) {
        P[i][0] = 0;
        for (int j = 1; j <= i; ++j) P[i][j] = P[i-1][j-1];
        for (int j = i - 1, val = 1; j >= 0; --j) {
            int co = mul(val, H[j][i - 1]);
            rep(k, 0, j+1) subeq(P[i][k], mul(P[j][k], co));
            if (j) val = mul(val, H[j][j - 1]);
        }
    }
    if (N & 1) for (int &pi: P[N]) pi = sub(0, pi);
    return P[N];
}

```

5.22 Polynomial Operations

```

using V = vector<int>;
#define fi(l, r) for (int i = int(l); i < int(r); ++i)
template <int mod, int G, int maxn> struct Poly : V {
    static uint32_t n2k(uint32_t n) {
        if (n <= 1) return 1;
        return 1u << (32 - __builtin_clz(n - 1));
    }
    static NTT<mod,G,maxn> ntt; // coefficients in [0, P)
    explicit Poly(int n = 1) : V(n) {}
    Poly(const V &v) : V(v) {}
    Poly(const Poly &p, size_t n) : V(n) {
        copy_n(p.data(), min(p.size(), n), data());
    }
    Poly &rev() { return reverse(data(), data() + size()), *this; }
    Poly &isz(int sz) { return resize(sz), *this; }
    Poly &iadd(const Poly &rhs) { // n() == rhs.n()
        fi(0, size())(*this)[i] = modadd((*this)[i], rhs[i]);
        return *this;
    }
    Poly &imul(int k) {
        fi(0, size())(*this)[i] = modmul((*this)[i], k);
        return *this;
    }
    Poly Mul(const Poly &rhs) const {
        const int sz = n2k(size() + rhs.size() - 1);
        Poly X(*this, sz), Y(rhs, sz);
        ntt(X.data(), sz), ntt(Y.data(), sz);
        fi(0, sz) X[i] = modmul(X[i], Y[i]);
        ntt(X.data(), sz, true);
        return X.isz(size() + rhs.size() - 1);
    }
    Poly Inv() const { // coef[0] != 0
        if (size() == 1) return V{modinv(*begin())};
        const int sz = n2k(size() * 2);
    }
}

```

```

Poly X = Poly(*this, (size() + 1) / 2).Inv().isz(sz),
    Y(*this, sz);
ntt(X.data(), sz), ntt(Y.data(), sz);
fi(0, sz) X[i] = modmul(X[i], modsub(2, modmul(X[i],
    Y[i])));
ntt(X.data(), sz, true);
return X.isz(size());
}
Poly Sqrt() const { // coef[0] \in [1, mod)^2
    if (size() == 1) return V{QuadraticResidue(*this)
        [0], mod};
    Poly X = Poly(*this, (size() + 1) / 2).Sqrt().isz(
        size());
    return X.iadd(Mul(X.Inv()).isz(size())).imul(mod / 2
        + 1);
}
pair<Poly, Poly> DivMod(const Poly &rhs) const {
    if (size() < rhs.size()) return {V{0}, *this};
    const int sz = size() - rhs.size() + 1;
    Poly X(rhs); X.irev().isz(sz);
    Poly Y(*this); Y.irev().isz(sz);
    Poly Q = Y.Mul(X.Inv()).isz(sz).irev();
    X = rhs.Mul(Q), Y = *this;
    fi(0, size()) Y[i] = modsub(Y[i], X[i]);
    return {Q, Y.isz(max<int>(1, rhs.size() - 1))};
}
Poly Dx() const {
    Poly ret(size() - 1);
    fi(0, ret.size()) ret[i] = modmul(i + 1, (*this)[i +
        1]);
    return ret.isz(max<int>(1, ret.size()));
}
Poly Sx() const {
    Poly ret(size() + 1);
    fi(0, size()) ret[i + 1] = modmul(modinv(i + 1), (*
        this)[i]);
    return ret;
}
Poly Ln() const { // coef[0] == 1
    return Dx().Mul(Inv()).Sx().isz(size());
}
Poly Exp() const { // coef[0] == 0
    if (size() == 1) return V{1};
    Poly X = Poly(*this, (size() + 1) / 2).Exp().isz(size
        ());
    Poly Y = X.Ln(); Y[0] = mod - 1;
    fi(0, size()) Y[i] = modsub((*this)[i], Y[i]);
    return X.Mul(Y).isz(size());
}
Poly Pow(const string &K) const {
    int nz = 0;
    while (nz < size() && !(*this)[nz]) ++nz;
    int nk = 0, nk2 = 0;
    for (char c : K) {
        nk = (nk * 10 + c - '0') % mod;
        nk2 = nk2 * 10 + c - '0';
        if (nk2 * nz >= size())
            return Poly(size());
        nk2 %= mod - 1;
    }
    if (!nk && !nk2) return Poly(V{1}, size());
    Poly X = V(data() + nz, data() + size() - nz * (nk2 -
        1));
    int x0 = X[0];
    return X.imul(modinv(x0)).Ln().imul(nk).Exp().imul(
        modpow(x0, nk2)).irev().isz(size()).irev();
}
V Eval(V x) const {
    if (x.empty()) return {};
    const size_t n = max(x.size(), size());
    vector<Poly> t(n * 2, V{1, 0}), f(n * 2);
    for (size_t i = 0; i < x.size(); ++i)
        t[n + i] = V{1, mod-x[i]};
    for (size_t i = n - 1; i > 0; --i)
        t[i] = t[i * 2].Mul(t[i * 2 + 1]);
    f[1] = Poly(*this, n).irev().Mul(t[1].Inv()).isz(n).
        irev();
    for (size_t i = 1; i < n; ++i) {
        auto o = f[i]; auto sz = o.size();
        f[i*2] = o.irev().Mul(t[i*2+1]).isz(sz).irev().isz(t
            [i*2].size());
    }
}

```

```

    f[i*2+1] = o.Mul(t[i*2]).isz(sz).irev().isz(t[i
        *2+1].size());
}
for (size_t i=0; i<x.size(); ++i) x[i] = f[n+i][0];
return x;
}

static int LinearRecursion(const V &a, const V &c,
    int64_t n) { // a_n = \sum c_j a_{n-j}
    const int k = (int)a.size();
    assert((int)c.size() == k + 1);
    Poly C(k + 1), W({1}, k), M = {0, 1};
    fi(1, k + 1) C[k - i] = modsub(mod, c[i]);
    C[k] = 1;
    while (n) {
        if (n % 2) W = W.Mul(M).DivMod(C).second;
        n /= 2, M = M.Mul(M).DivMod(C).second;
    }
    int ret = 0;
    fi(0, k) ret = modadd(ret, modmul(W[i], a[i]));
    return ret;
}
};
#undef fi
using Poly_t = Poly<998244353, 3, 1 << 20>;
template <> decltype(Poly_t::ntt) Poly_t::ntt = {};

```

5.23 Simplex [e975d5]

```

namespace simplex {
    // maximize c^T x under Ax <= B
    // return VD(n, -inf) if the solution doesn't exist
    // return VD(n, +inf) if the solution is unbounded
    using VD = vector<double>;
    using VVD = vector<vector<double>>;
    const double eps = 1e-9;
    const double inf = 1e+9;
    int n, m;
    VVD d;
    vector<int> p, q;
    void pivot(int r, int s) {
        double inv = 1.0 / d[r][s];
        for (int i = 0; i < m + 2; ++i)
            for (int j = 0; j < n + 2; ++j)
                if (i != r && j != s)
                    d[i][j] -= d[r][j] * d[i][s] * inv;
        for(int i=0;i<m+2;++i) if (i != r) d[i][s] *= -inv;
        for(int j=0;j<n+2;++j) if (j != s) d[r][j] *= +inv;
        d[r][s] = inv; swap(p[r], q[s]);
    }
    bool phase(int z) {
        int x = m + z;
        while (true) {
            int s = -1;
            for (int i = 0; i <= n; ++i) {
                if (!z && q[i] == -1) continue;
                if (s == -1 || d[x][i] < d[x][s]) s = i;
            }
            if (d[x][s] > -eps) return true;
            int r = -1;
            for (int i = 0; i < m; ++i) {
                if (d[i][s] < eps) continue;
                if (r == -1 || \
                    d[i][n+1]/d[i][s] < d[r][n+1]/d[r][s]) r = i;
            }
            if (r == -1) return false;
            pivot(r, s);
        }
    }
    VD solve(const VVD &a, const VD &b, const VD &c) {
        m = b.size(), n = c.size();
        d = VVD(m + 2, VD(n + 2));
        for (int i = 0; i < m; ++i)
            for (int j = 0; j < n; ++j) d[i][j] = a[i][j];
        p.resize(m), q.resize(n + 1);
        for (int i = 0; i < m; ++i)
            p[i] = n + i, d[i][n] = -1, d[i][n + 1] = b[i];
        for (int i = 0; i < n; ++i) q[i] = i, d[m][i] = -c[i];
        q[n] = -1, d[m + 1][n] = 1;
        int r = 0;
        for (int i = 1; i < m; ++i)
            if (d[i][n + 1] < d[r][n + 1]) r = i;
        if (d[r][n + 1] < -eps) {

```

```

pivot(r, n);
if (!phase(1) || d[m + 1][n + 1] < -eps)
    return VD(n, -inf);
for (int i = 0; i < m; ++i) if (p[i] == -1) {
    int s = min_element(d[i].begin(), d[i].end() - 1)
        - d[i].begin();
    pivot(i, s);
}
}
if (!phase(0)) return VD(n, inf);
VD x(n);
for (int i = 0; i < m; ++i)
    if (p[i] < n) x[p[i]] = d[i][n + 1];
return x;
}
}

```

5.24 Simplex Construction

Standard form: maximize $\sum_{1 \leq i \leq n} c_i x_i$ such that for all $1 \leq j \leq m$, $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$ and $x_i \geq 0$ for all $1 \leq i \leq n$.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji} x_i \leq -b_j$
3. $\sum_{1 \leq i \leq n} A_{ji} x_i = b_j$
 - $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$
 - $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j$
4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

6 Geometry

6.1 Basic Geometry [3c676b]

```

#define IM imag
#define RE real
using lld = int64_t;
using llf = long double;
using PT = std::complex<lld>;
using PTF = std::complex<llf>;
using P = PT;
PTF toPTF(PT p) { return PTF(RE(p), IM(p)); }
int sgn(lld x) { return (x > 0) - (x < 0); }
lld dot(P a, P b) { return RE(conj(a) * b); }
lld cross(P a, P b) { return IM(conj(a) * b); }
int ori(P a, P b, P c) {
    return sgn(cross(b - a, c - a));
}
int quad(P p) {
    return (IM(p) == 0) // use sgn for PTF
        ? (RE(p) < 0 ? 3 : 1) : (IM(p) < 0 ? 0 : 2);
}
int argCmp(P a, P b) {
    // returns 0/+1, starts from theta = -PI
    int qa = quad(a), qb = quad(b);
    if (qa != qb) return sgn(qa - qb);
    return sgn(cross(b, a));
}
template <typename V> llf area(const V & pt) {
    lld ret = 0;
    for (int i = 1; i + 1 < (int)pt.size(); ++i)
        ret += cross(pt[i] - pt[0], pt[i+1] - pt[0]);
    return ret / 2.0;
}
P rot90(P p) { return P{-IM(p), RE(p)}; }
PTF project(PTF p, PTF q) { // p onto q
    return dot(p, q) * q / dot(q, q); // dot<llf>
}

```

6.2 Segment Intersection [999844]

```

struct Segment { // closed segment
    PT st, dir; // represent st + t*dir for 0<=t<=1
    Segment(PT s, PT e) : st(s), dir(e - s) {}
    static bool valid(lld p, lld q) {
        // is there t s.t. 0 <= t <= 1 && qt == p ?
        if (q < 0) q = -q, p = -p;
        return 0 <= p && p <= q;
    }
    vector<PT> ends() const { return { st, st + dir }; }
};
template <typename T> bool isInter(T A, PT p) {

```

```

    if (A.dir == PT(0)) return p == A.st; // BE CAREFUL
    return cross(p - A.st, A.dir) == 0 &&
        T::valid(dot(p - A.st, A.dir), norm(A.dir));
}
template <typename U, typename V>
bool isInter(U A, V B) {
    if (cross(A.dir, B.dir) == 0) { // BE CAREFUL
        bool res = false;
        for (PT p: A.ends()) res |= isInter(B, p);
        for (PT p: B.ends()) res |= isInter(A, p);
        return res;
    }
    PT D = B.st - A.st;
    lld C = cross(A.dir, B.dir);
    return U::valid(cross(D, B.dir), C) &&
        V::valid(cross(D, A.dir), C);
}

```

6.3 Half Plane Intersection [a2b53a]

```

struct Line {
    PT st, ed, dir;
    Line(PT s, PT e)
        : st(s), ed(e), dir(e - s) {}
};
PTF intersect(const Line &A, const Line &B) {
    llf t = cross(B.st - A.st, B.dir) /
        llf(cross(A.dir, B.dir));
    return toPTF(A.st) + PTF(t) * toPTF(A.dir);
}
// cross(pt-line.st, line.dir)<=0 <=> pt in half plane
// the LHS when going from st to ed
bool operator<(const Line &lhs, const Line &rhs) {
    if (int cmp = argCmp(lhs.dir, rhs.dir))
        return cmp == -1;
    return ori(lhs.st, lhs.ed, rhs.st) < 0;
}
// be careful about the type of `ori` and `area`
llf HPI(vector<Line> &lines) {
    sort(lines.begin(), lines.end());
    deque<Line> que;
    deque<PTF> pt;
    que.push_back(lines[0]);
    for (int i = 1; i < (int)lines.size(); ++i) {
        if (argCmp(lines[i].dir, lines[i-1].dir) == 0)
            continue;
#define POP(L, R) \
        while (pt.size() > 0 \
            && ori(L.st, L.ed, pt.back()) < 0) \
            pt.pop_back(), que.pop_back(); \
        while (pt.size() > 0 \
            && ori(R.st, R.ed, pt.front()) < 0) \
            pt.pop_front(), que.pop_front();
        POP(lines[i], lines[i-1]);
        pt.push_back(intersect(que.back(), lines[i]));
        que.push_back(lines[i]);
    }
    POP(que.front(), que.back())
    if (que.size() <= 1 ||
        argCmp(que.front().dir, que.back().dir) == 0)
        return 0;
    pt.push_back(intersect(que.front(), que.back()));
    return area(pt);
}

```

6.4 Rotating Sweep Line [f8c127]

```

void rotatingSweepLine(pair<int, int> a[], int n) {
    vector<pair<int, int>> l;
    l.reserve(n * (n - 1) / 2);
    for (int i = 0; i < n; ++i)
        for (int j = i + 1; j < n; ++j)
            l.emplace_back(i, j);
    sort(l.begin(), l.end(), [&a](auto &u, auto &v){
        lld udx = a[u.first].first - a[u.second].first;
        lld udy = a[u.first].second - a[u.second].second;
        lld vdx = a[v.first].first - a[v.second].first;
        lld vdy = a[v.first].second - a[v.second].second;
        if (udx == 0 || vdx == 0) return not udx == 0;
        int s = sgn(udx * vdx);
        return udy * vdx * s < vdy * udx * s;
    });
    vector<int> idx(n), p(n);
    iota(idx.begin(), idx.end(), 0);
}

```

```

sort(idx.begin(), idx.end(), [&a](int i, int j){
    return a[i] < a[j]; });
for (int i = 0; i < n; ++i) p[idx[i]] = i;
for (auto [i, j]: l) {
    // do here
    swap(p[i], p[j]);
    idx[p[i]] = i, idx[p[j]] = j;
}
}

```

6.5 2D Convex Hull [ecba37]

```

// from NaCl, counterclockwise, be careful of n<=2
vector<P> convex_hull(vector<P> v) {
    sort(all(v)); // by X then Y
    if (v[0] == v.back()) return {v[0]};
    int t = 0, s = 1; vector<P> h(v.size() + 1);
    for (int _ = 2; _--; s = t--, reverse(all(v)))
        for (P p : v) {
            while (t > s && ori(p, h[t-1], h[t-2]) >= 0) t--;
            h[t++] = p;
        }
    return h.resize(t), h;
}

```

6.6 Minkowski Sum [c71bec]

```

// A, B are convex hull sort by (X, Y)
vector<P> Minkowski(vector<P> A, vector<P> B) {
    vector<P> C(1, A[0] + B[0]), s1, s2;
    const int N = (int)A.size(), M = (int)B.size();
    for (int i = 0; i < N; ++i)
        s1.pb(A[(i + 1) % N] - A[i]);
    for (int i = 0; i < M; ++i)
        s2.pb(B[(i + 1) % M] - B[i]);
    for (int i = 0, j = 0; i < N || j < M; )
        if (j >= N || (i < M && cross(s1[i], s2[j]) >= 0))
            C.pb(C.back() + s1[i++]);
        else
            C.pb(C.back() + s2[j++]);
    return hull(C), C;
}

```

6.7 2D Farthest Pair [83d60a]

```

// stk is from convex hull
n = (int)(stk.size());
int pos = 1, ans = 0; stk.push_back(stk[0]);
for (int i=0; i<n; i++) {
    while (abs(cross(stk[i+1]-stk[i],
        stk[(pos+1)%n]-stk[i])) >
        abs(cross(stk[i+1]-stk[i],
            stk[pos]-stk[i]))) pos = (pos+1)%n;
    ans = max({ans, dis(stk[i], stk[pos]),
        dis(stk[i+1], stk[pos])});
}

```

6.8 MinMaxEnclosingRectangle [c66dbf]

```

// from 8BQube, please ensure p is convex hull
const llf INF = 1e18, qi = acos(-1) / 2 * 3;
pair<llf, llf> solve(vector<P> &p) {
#define Z(v) (p[v] - p[i])
    llf mx = 0, mn = INF;
    int n = (int)p.size(); p.emplace_back(p[0]);
    for (int i = 0, u = 1, r = 1, l = 1; i < n; ++i) {
        P e = Z(i + 1);
        while (cross(e, Z(u + 1)) > cross(e, Z(u)))
            u = (u + 1) % n;
        while (dot(e, Z(r + 1)) > dot(e, Z(r)))
            r = (r + 1) % n;
        if (!i) l = (r + 1) % n;
        while (dot(e, Z(l + 1)) < dot(e, Z(l)))
            l = (l + 1) % n;
        P D = p[r] - p[l];
        mn = min(mn, dot(e, D) / llf(norm(e)) * cross(e, Z(u)));
        llf B = sqrt(norm(D)) * sqrt(norm(Z(u)));
        llf deg = (qi - acos(dot(D, Z(u)) / B)) / 2;
        mx = max(mx, B * sin(deg) * sin(deg));
    }
    return {mn, mx};
}

```

6.9 Convex Hulls Tangent [e9c73c]

```

// RLtangent_PolyPolyC(): get the RL tangent between
// two convex polygons
// Input:
// m = number of vertices in polygon 1
// V = array of vertices for convex polygon 1 with V[m]
// =V[0]
// n = number of vertices in polygon 2
// W = array of vertices for convex polygon 2 with W[n]
// =W[0]
// Output:
// *t1 = index of tangent point V[t1] for polygon 1
// *t2 = index of tangent point W[t2] for polygon 2
void RLtangent_PolyPolyC(int m, PT *V, int n, PT *W,
    int *t1, int *t2) {
    // * first get the initial vertex on each polygon
    // right tangent from W[0] to V
    int ix1 = Rtangent_PointPolyC(W[0], m, V);
    // left tangent from V[ix1] to W
    int ix2 = Ltangent_PointPolyC(V[ix1], n, W);
    // * ping-pong linear search until it stabilizes
    for (bool done = false; not done; ) {
        done = true; // assume done until...
        while (ori(W[ix2], V[ix1], V[ix1 + 1]) <= 0)
            ++ix1; // get Rtangent from W[ix2] to V
        while (ori(V[ix1], W[ix2], W[ix2 + 1]) >= 0) {
            --ix2; // get Ltangent from V[ix1] to W
            done = false; // not done if had to adjust this
        }
    }
    *t1 = ix1, *t2 = ix2;
}

```

6.10 Point & Hulls Tangent [c1f735]

```

// ref: codeforces.com/gym/101201/submission/36665988
// return nullopt means point is IN or ON hull
optional<pair<int, int>> point_to_hull_tangent(const
    vector<P> &v, P p) {
    const int N = v.size();
    if (N <= 2) return {}; // BE CAREFUL
    if (p == v[0]) return {};
    const auto cmp = [w = conj(v[0] - p)](P a, P b) {
        int qa = quad(a * w), qb = quad(b * w);
        if (qa != qb) return sgn(qa - qb);
        return sgn(cross(b, a));
    };
    const auto gao = [&](int s) {
        const auto lt = [&](int x, int y) {
            return cmp(v[x%N]-p, v[y%N]-p) == s; };
        bool up = lt(0, 1);
        int l = 0, r = N;
        while (r - l > 1) {
            int m = (l + r) / 2;
            if (lt(m, 0)) {
                if (up) r = m;
                else l = m;
            } else {
                if (lt(m, m + 1)) l = m;
                else r = m;
            }
        }
        return lt(l, r) ? r : l;
    };
    int a = gao(-1) % N, b = gao(1) % N;
    assert(cmp(v[b] - p, v[a] - p) <= 0);
    if (cmp(v[a] - p, p - v[b]) >= 0) return {};
    return make_pair(a, b);
}

```

6.11 Circle Class [8bf42c]

```

llf FMOD(llf x) {
    if (x < -PI) x += PI * 2;
    if (x > PI) x -= PI * 2;
    return x;
}
struct Circle { PTF o; llf r; };
vector<llf> intersectAngle(Circle A, Circle B) {
    PTF dir = B.o - A.o; llf d2 = norm(dir);
    if (norm(A.r - B.r) >= d2) // norm(x) := |x|^2
        if (A.r < B.r) return {-PI, PI}; // A in B
        else return {}; // B in A
}

```



```

if (norm(A.r + B.r) <= d2) return {};
llf dis = abs(dir), theta = arg(dir);
llf phi = acos((A.r * A.r + d2 - B.r * B.r) /
    (2 * A.r * dis));
llf L = FMOD(theta - phi), R = FMOD(theta + phi);
return { L, R };
}

vector<PTF> intersectPoint(Circle a, Circle b) {
    llf d = abs(a.o - b.o);
    if (d >= b.r+a.r || d <= abs(b.r-a.r)) return {};
    llf dt = (b.r*b.r - a.r*a.r)/d, d1 = (d+dt)/2;
    PTF dir = (a.o - b.o) / d;
    PTF u = dir*d1 + b.o;
    PTF v = rot90(dir) * sqrt(max<llf>(0, b.r*b.r-d1*d1));
    return {u + v, u - v};
}

```

6.12 Tangent Line of Two Circle [40c677]

```

vector<Line>
tanline(const Circle &c1, const Circle &c2, int sign1){
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    if (norm(c1.o - c2.o) < eps) return ret;
    llf d = abs(c1.o - c2.o);
    PTF v = (c2.o - c1.o) / d;
    llf c = (c1.r - sign1 * c2.r) / d;
    if (c * c > 1) return ret;
    llf h = sqrt(max<llf>(0, 1 - c * c));
    for (int sign2 : {1, -1}) {
        PTF n = c * v + sign2 * h * rot90(v);
        PTF p1 = c1.o + n * c1.r;
        PTF p2 = c2.o + n * (c2.r * sign1);
        if (norm(p2 - p1) < eps)
            p2 = p1 + rot90(c2.o - c1.o);
        ret.push_back({p1, p2});
    }
    return ret;
}

```

6.13 Intersection of Line and Circle [5c86ef]

```

vector<PTF> line_interCircle(const PTF &p1,
    const PTF &p2, const PTF &c, const double r) {
    PTF ft = p1 + project(c-p1, p2-p1), vec = p2-p1;
    llf dis = abs(c - ft);
    if (abs(dis - r) < eps) return {ft};
    if (dis > r) return {};
    vec = vec * sqrt(r * r - dis * dis) / abs(vec);
    return {ft + vec, ft - vec};
}

```

6.14 Intersection of Polygon and Circle [1504f9]

```

// Divides into multiple triangle, and sum up
// test by HDU2892
llf _area(PTF pa, PTF pb, llf r) {
    if (abs(pa) < abs(pb)) swap(pa, pb);
    if (abs(pb) < eps) return 0;
    llf S, h, theta;
    llf a = abs(pb), b = abs(pa), c = abs(pb - pa);
    llf cosB = dot(pb, pb - pa) / a / c, B = acos(cosB);
    llf cosC = dot(pa, pb) / a / b, C = acos(cosC);
    if (a > r) {
        S = (C / 2) * r * r;
        h = a * b * sin(C) / c;
        if (h < r && B < PI / 2)
            S -= (acos(h / r) * r * r - h * sqrt(r*r - h*h));
    } else if (b > r) {
        theta = PI - B - asin(sin(B) / r * a);
        S = 0.5 * a*r * sin(theta) + (C - theta) / 2 * r*r;
    } else
        S = 0.5 * sin(C) * a * b;
    return S;
}

llf area_poly_circle(const vector<PTF> &poly,
    const PTF &O, const llf r) {
    llf S = 0;
    for (int i = 0, N = poly.size(); i < N; ++i)
        S += _area(poly[i] - O, poly[(i + 1) % N] - O, r) *
            ori(O, poly[i], poly[(i + 1) % N]);
    return fabs(S);
}

```

6.15 3D Convex Hull [93b153]

```

// return the faces with pt indexes
struct P3 { llf x,y,z;
    P3 operator * (const P3 &b) const {
        return (P3){y*b.z-b.y*z,z*b.x-b.z*x,x*b.y-b.x*y};
    }
};

struct Face { int a, b, c;
    Face(int ta,int tb,int tc):a(ta),b(tb),c(tc){} };
P3 ver(P3 a, P3 b, P3 c) { return (b - a) * (c - a); }
// plz ensure that first 4 points are not coplanar
// all points coplanar case will WA
vector<Face> convex_hull_3D(const vector<P3> &pt) {
    int n = int(pt.size()); vector<Face> now;
    if (n <= 2) return {}; // be careful about edge case
    vector<vector<int>> flag(n, vector<int>(n));
    now.emplace_back(0,1,2); now.emplace_back(2,1,0);
    for (int i = 3; i < n; i++) {
        vector<Face> next;
        for (const auto &f : now) {
            llf d = (pt[i] - pt[f.a]).dot(
                ver(pt[f.a], pt[f.b], pt[f.c]));
            if (d <= 0) next.push_back(f);
            int ff = (d > 0) - (d < 0);
            flag[f.a][f.b]=flag[f.b][f.c]=flag[f.c][f.a]=ff;
        }
        for (const auto &f : now) {
            const auto F = [&](int x, int y) {
                if (flag[x][y] > 0 && flag[y][x] <= 0)
                    next.emplace_back(x, y, i);
            };
            F(f.a, f.b); F(f.b, f.c); F(f.c, f.a);
        }
        now = next;
    }
    return now;
}

// delaunay: facets with negative z normal of
// convexhull of (x, y, x^2 + y^2)

```

6.16 Minimum Covering Circle [a26cde]

```

template<typename P>
Circle getCircum(const P &a, const P &b, const P &c){
    Real a1 = a.x-b.x, b1 = a.y-b.y;
    Real c1 = (a.x+b.x)/2 * a1 + (a.y+b.y)/2 * b1;
    Real a2 = a.x-c.x, b2 = a.y-c.y;
    Real c2 = (a.x+c.x)/2 * a2 + (a.y+c.y)/2 * b2;
    Circle cc;
    cc.o.x = (c1*b2-b1*c2)/(a1*b2-b1*a2);
    cc.o.y = (a1*c2-c1*a2)/(a1*b2-b1*a2);
    cc.r = hypot(cc.o.x-a.x, cc.o.y-a.y);
    return cc;
}

```

```

template<typename P>
Circle MinCircleCover(const vector<P>& pts){
    random_shuffle(pts.begin(), pts.end());
    Circle c = { pts[0], 0 };
    for(int i=0;i<(int)pts.size();i++){
        if (dist(pts[i], c.o) <= c.r) continue;
        c = { pts[i], 0 };
        for (int j = 0; j < i; j++) {
            if(dist(pts[j], c.o) <= c.r) continue;
            c.o = (pts[i] + pts[j]) / 2;
            c.r = dist(pts[i], c.o);
            for (int k = 0; k < j; k++) {
                if (dist(pts[k], c.o) <= c.r) continue;
                c = getCircum(pts[i], pts[j], pts[k]);
            }
        }
    }
    return c;
}

```

6.17 Polygon Union [5c0b42]

```

llf rat(P a, P b) { return sgn(RE(b)) ? llf(RE(a))/RE(b)
    : llf(IM(a))/IM(b); }
llf polyUnion(vector<vector<P>>& poly) {
    llf ret = 0; // area of poly[i] must be non-negative
    rep(i,0,sz(poly)) rep(v,0,sz(poly[i])) {
        P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])];
        vector<pair<llf, int>> segs{{0, 0}, {1, 0}};
    }
}

```

```

rep(j,0,sz(poly)) if (i != j) {
    rep(u,0,sz(poly[j])) {
        P C = poly[j][u], D = poly[j][(u + 1) % sz(poly[j])]
    };
    if (int sc = ori(A, B, C), sd = ori(A, B, D); sc !=
        sd) {
        llf sa = cross(D-C, A-C), sb = cross(D-C, B-C);
        if (min(sc, sd) < 0)
            segs.emplace_back(sa / (sa - sb), sgn(sc - sd));
    } else if (!sc && !sd && j < i && sgn(dot(B-A,D-C))
        > 0){
        segs.emplace_back(rat(C - A, B - A), 1);
        segs.emplace_back(rat(D - A, B - A), -1);
    }
}
sort(segs.begin(), segs.end());
for (auto &s : segs) s.first = clamp<llf>(s.first, 0,
    1);;
llf sum = 0;
int cnt = segs[0].second;
rep(j,1,sz(segs)) {
    if (!cnt) sum += segs[j].first - segs[j - 1].first;
    cnt += segs[j].second;
}
ret += cross(A,B) * sum;
}
return ret / 2;
}

```

6.18 Circle Union [a783a1]

```

#define eb emplace_back
struct CircleCover { // test@SPOJ N=1000, 0.3~0.5s
    struct Teve {
        PTF p; llf ang; int add;
        Teve(PTF a, llf b, int c) : p(a), ang(b), add(c) {}
        bool operator<(const Teve &a)
            const { return ang < a.ang; }
    };
    // strict: x = 0, otherwise x = -1
    bool disjunct(Cir &a, Cir &b, int x)
    { return sgn(abs(a.o - b.o) - a.r - b.r) > x; }
    bool contain(Cir &a, Cir &b, int x)
    { return sgn(a.r - b.r - abs(a.o - b.o)) > x; }
    vector<llf> solve(vector<Cir> c) {
        // area[i] : area covered by at least i circles
        int N = c.size(); vector<llf> area(N + 1);
        vector<vector<int>> overlap(N, vector<int>(N));
        auto g = overlap; // use simple 2darray to speedup
        for (int i = 0; i < N; ++i)
            for (int j = 0; j < N; ++j) {
                /* c[j] is non-strictly in c[i]. */
                overlap[i][j] = i != j &&
                    (sgn(c[i].r - c[j].r) > 0 ||
                     (sgn(c[i].r - c[j].r) == 0 && i < j)) &&
                    contain(c[i], c[j], -1);
            }
        for (int i = 0; i < N; ++i)
            for (int j = 0; j < N; ++j)
                g[i][j] = i != j && !(overlap[i][j] ||
                    overlap[j][i] || disjunct(c[i], c[j], -1));
        for (int i = 0; i < N; ++i) {
            vector<Teve> eve; int cnt = 1;
            for (int j = 0; j < N; ++j)
                if (overlap[j][i]) ++cnt;
            // if (cnt > 1) continue; (if only need area[1])
            for (int j = 0; j < N; ++j) if (g[i][j]) {
                auto IP = intersectPoint(c[i], c[j]);
                PTF aa = IP[1], bb = IP[0];
                llf A = arg(aa - c[i].o), B = arg(bb - c[i].o);
                eve.eb(bb, B, 1); eve.eb(aa, A, -1);
                if (B > A) ++cnt;
            }
            if (eve.empty())
                area[cnt] += pi * c[i].r * c[i].r;
            else {
                sort(eve.begin(), eve.end());
                eve.eb(eve[0]); eve.back().ang += 2. * pi;
                for (size_t j = 0; j + 1 < eve.size(); j++) {
                    cnt += eve[j].add;
                    area[cnt] += cross(eve[j].p, eve[j+1].p) * .5;
                    llf t = eve[j + 1].ang - eve[j].ang;

```

```

                area[cnt] += (t-sin(t)) * c[i].r * c[i].r * .5;
            }
        }
        return area;
    }
} CC0;

```

6.19 KDTree (Nearest Point)

```

const int MXN = 100005;
struct KDTree {
    struct Node {
        int x,y,x1,y1,x2,y2;
        int id,f;
        Node *L, *R;
    } tree[MXN], *root;
    int n;
    LL dis2(int x1, int y1, int x2, int y2) {
        LL dx = x1-x2, dy = y1-y2;
        return dx*dx+dy*dy;
    }
    static bool cmpx(Node& a, Node& b){return a.x<b.x;}
    static bool cmpy(Node& a, Node& b){return a.y<b.y;}
    void init(vector<pair<int,int>> ip) {
        n = ip.size();
        for (int i=0; i<n; i++) {
            tree[i].id = i;
            tree[i].x = ip[i].first;
            tree[i].y = ip[i].second;
        }
        root = build_tree(0, n-1, 0);
    }
    Node* build_tree(int L, int R, int d) {
        if (L>R) return nullptr;
        int M = (L+R)/2; tree[M].f = d%2;
        nth_element(tree+L, tree+M, tree+R+1, d%2?cmpy:cmpx);
        tree[M].x1 = tree[M].x2 = tree[M].x;
        tree[M].y1 = tree[M].y2 = tree[M].y;
        tree[M].L = build_tree(L, M-1, d+1);
        if (tree[M].L) {
            tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
            tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
            tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
            tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
        }
        tree[M].R = build_tree(M+1, R, d+1);
        if (tree[M].R) {
            tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
            tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
            tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
            tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
        }
        return tree+M;
    }
    int touch(Node* r, int x, int y, LL d2){
        LL dis = sqrt(d2)+1;
        if (x<r->x1-dis || x>r->x2+dis ||
            y<r->y1-dis || y>r->y2+dis)
            return 0;
        return 1;
    }
    void nearest(Node* r, int x, int y, int &mID, LL &md2) {
        if (!r || !touch(r, x, y, md2)) return;
        LL d2 = dis2(r->x, r->y, x, y);
        if (d2 < md2 || (d2 == md2 && mID < r->id)) {
            mID = r->id;
            md2 = d2;
        }
        // search order depends on split dim
        if ((r->f == 0 && x < r->x) ||
            (r->f == 1 && y < r->y)) {
            nearest(r->L, x, y, mID, md2);
            nearest(r->R, x, y, mID, md2);
        } else {
            nearest(r->R, x, y, mID, md2);
            nearest(r->L, x, y, mID, md2);
        }
    }
    int query(int x, int y) {
        int id = 1029384756;
        LL d2 = 102938475612345678LL;
        nearest(root, x, y, id, d2);
    }
}

```

```

    return id;
}
} tree;

```

6.20 kD Closest Pair (3D ver.)

```

llf solve(vector<P> v) {
    shuffle(v.begin(), v.end(), mt19937());
    unordered_map<lld, unordered_map<lld,
        unordered_map<lld, int>>> m;
    llf d = dis(v[0], v[1]);
    auto Idx = [&d] (llf x) -> lld {
        return round(x * 2 / d) + 0.1; };
    auto rebuild_m = [&m, &v, &Idx] (int k) {
        m.clear();
        for (int i = 0; i < k; ++i)
            m[Idx(v[i].x)][Idx(v[i].y)]
                [Idx(v[i].z)] = i;
    }; rebuild_m(2);
    for (size_t i = 2; i < v.size(); ++i) {
        const lld kx = Idx(v[i].x), ky = Idx(v[i].y),
            kz = Idx(v[i].z); bool found = false;
        for (int dx = -2; dx <= 2; ++dx) {
            const lld nx = dx + kx;
            if (m.find(nx) == m.end()) continue;
            auto& mm = m[nx];
            for (int dy = -2; dy <= 2; ++dy) {
                const lld ny = dy + ky;
                if (mm.find(ny) == mm.end()) continue;
                auto& mmm = mm[ny];
                for (int dz = -2; dz <= 2; ++dz) {
                    const lld nz = dz + kz;
                    if (mmm.find(nz) == mmm.end()) continue;
                    const int p = mmm[nz];
                    if (dis(v[p], v[i]) < d) {
                        d = dis(v[p], v[i]);
                        found = true;
                    }
                }
            }
        }
        if (found) rebuild_m(i + 1);
        else m[kx][ky][kz] = i;
    }
    return d;
}

```

6.21 Simulated Annealing

```

llf anneal() {
    mt19937 rnd_engine( seed );
    uniform_real_distribution< llf > rnd( 0, 1 );
    const llf dT = 0.001;
    // Argument p
    llf S_cur = calc( p ), S_best = S_cur;
    for ( llf T = 2000 ; T > EPS ; T -= dT ) {
        // Modify p to p_prime
        const llf S_prime = calc( p_prime );
        const llf delta_c = S_prime - S_cur;
        llf prob = min( ( llf ) 1, exp( -delta_c / T ) );
        if ( rnd( rnd_engine ) <= prob )
            S_cur = S_prime, p = p_prime;
        if ( S_prime < S_best ) // find min
            S_best = S_prime, p_best = p_prime;
    }
    return S_best;
}

```

7 Stringology

7.1 Hash [7afe3e]

```

class Hash {
private:
    static constexpr int P = 127, Q = 1051762951;
    vector<int> h, p;
public:
    void init(const string &s){
        h.assign(s.size()+1, 0); p.resize(s.size()+1);
        for (size_t i = 0; i < s.size(); ++i)
            h[i + 1] = add(mul(h[i], P), s[i]);
        generate(p.begin(), p.end(), [x=1,y=1,this]()
            mutable{x=y;x=mul(x,P);return y;});
    }
}

```

```

int query(int l, int r){ // 1-base (l, r]
    return sub(h[r], mul(h[l], p[r-l]));
};

```

7.2 Suffix Array [4e7e80]

```

namespace sfx {
    bool _t[maxn * 2];
    int hi[maxn], rev[maxn];
    int _s[maxn * 2], sa[maxn * 2], _c[maxn * 2];
    int x[maxn], _p[maxn], _q[maxn * 2];
    // sa[i]: sa[i]-th suffix is the
    // i-th lexicographically smallest suffix.
    // hi[i]: longest common prefix
    // of suffix sa[i] and suffix sa[i - 1].
    void pre(int *a, int *c, int n, int z) {
        memset(a, 0, sizeof(int) * n);
        memcpy(x, c, sizeof(int) * z);
    }
    void induce(int *a, int *c, int *s, bool *t, int n, int z){
        memcpy(x + 1, c, sizeof(int) * (z - 1));
        for (int i = 0; i < n; ++i)
            if (a[i] && !t[a[i] - 1])
                a[x[s[a[i] - 1]]++] = a[i] - 1;
        memcpy(x, c, sizeof(int) * z);
        for (int i = n - 1; i >= 0; --i)
            if (a[i] && t[a[i] - 1])
                a[--x[s[a[i] - 1]]] = a[i] - 1;
    }
    void sais(int *s, int *a, int *p, int *q,
        bool *t, int *c, int n, int z) {
        bool uniq = t[n - 1] = true;
        int nn=0, nmxz=-1, *nsa = a+n, *ns=s+n, last=-1;
        memset(c, 0, sizeof(int) * z);
        for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;
        for (int i = 0; i < z - 1; ++i) c[i + 1] += c[i];
        if (uniq) {
            for (int i = 0; i < n; ++i) a[--c[s[i]]] = i;
            return;
        }
        for (int i = n - 2; i >= 0; --i)
            t[i] = (s[i]==s[i + 1] ? t[i + 1] : s[i]<s[i + 1]);
        pre(a, c, n, z);
        for (int i = 1; i <= n - 1; ++i)
            if (t[i] && !t[i - 1])
                a[--x[s[i]]] = p[q[i] = nn++] = i;
        induce(a, c, s, t, n, z);
        for (int i = 0; i < n; ++i) {
            if (a[i] && t[a[i]] && !t[a[i] - 1]) {
                bool neq = last < 0 || \
                    memcmp(s + a[i], s + last,
                        (p[q[a[i]] + 1] - a[i]) * sizeof(int));
                ns[q[last = a[i]]] = nmxz += neq;
            }
        }
        sais(ns, nsa, p+nn, q+n, t+n, c+z, nn, nmxz+1);
        pre(a, c, n, z);
        for (int i = nn - 1; i >= 0; --i)
            a[--x[s[p[nsa[i]]]]] = p[nsa[i]];
        induce(a, c, s, t, n, z);
    }
    void build(const string &s) {
        const int n = int(s.size());
        for (int i = 0; i < n; ++i) _s[i] = s[i];
        _s[n] = 0; // s shouldn't contain 0
        sais(_s, sa, _p, _q, _t, _c, n + 1, 256);
        for (int i = 0; i < n; ++i) rev[sa[i] = sa[i+1]] = i;
        int ind = hi[0] = 0;
        for (int i = 0; i < n; ++i) {
            if (!rev[i]) {
                ind = 0;
                continue;
            }
            while (i + ind < n && \
                s[i + ind] == s[sa[rev[i] - 1] + ind]) ++ind;
            hi[rev[i]] = ind ? ind-- : 0;
        }
    }
}

```

7.3 Suffix Automaton [bf53b9]

```

struct SuffixAutomaton {
    struct node {
        int ch[K], len, fail, cnt, indeg;
    };
};

```

```

node(int L = 0) : ch{}, len(L), fail(0), cnt(0),
    indeg(0) {}
} st[N];
int root, last, tot;
void extend(int c) {
    int cur = ++tot;
    st[cur] = node(st[last].len + 1);
    while (last && !st[last].ch[c]) {
        st[last].ch[c] = cur;
        last = st[last].fail;
    }
    if (!last) {
        st[cur].fail = root;
    } else {
        int q = st[last].ch[c];
        if (st[q].len == st[last].len + 1) {
            st[cur].fail = q;
        } else {
            int clone = ++tot;
            st[clone] = st[q];
            st[clone].len = st[last].len + 1;
            st[st[cur].fail = st[q].fail = clone].cnt = 0;
            while (last && st[last].ch[c] == q) {
                st[last].ch[c] = clone;
                last = st[last].fail;
            }
        }
    }
    st[last = cur].cnt += 1;
}
void init(const char* s) {
    root = last = tot = 1;
    st[root] = node(0);
    for (char c; c = *s; ++s) extend(c - 'a');
}
int q[N];
void dp() {
    for (int i = 1; i <= tot; i++) ++st[st[i].fail].indeg;
    int head = 0, tail = 0;
    for (int i = 1; i <= tot; i++)
        if (st[i].indeg == 0) q[tail++] = i;
    while (head != tail) {
        int now = q[head++];
        if (int f = st[now].fail) {
            st[f].cnt += st[now].cnt;
            if (--st[f].indeg == 0) q[tail++] = f;
        }
    }
}
int run(const char* s) {
    int now = root;
    for (char c; c = *s; ++s) {
        if (!st[now].ch[c - 'a']) return 0;
        now = st[now].ch[c];
    }
    return st[now].cnt;
}
} SAM;

```

7.4 Z value [6a7fd0]

```

vector<int> Zalgo(const string &s) {
    vector<int> z(s.size(), s.size());
    for (int i = 1, l = 0, r = 0; i < z[0]; ++i) {
        int j = clamp(r - i, 0, z[i - l]);
        for (; i + j < z[0] && s[i + j] == s[j]; ++j);
        if (i + (z[i] = j) > r) r = i + z[l = i];
    }
    return z;
}

```

7.5 Manacher [365720]

```

int z[maxn];
int manacher(const string& s) {
    string t = ". ";
    for(char c: s) t += c, t += ' ';
    int l = 0, r = 0, ans = 0;
    for (int i = 1; i < t.length(); ++i) {
        z[i] = (r > i ? min(z[2 * l - i], r - i) : 1);
        while (i - z[i] >= 0 && i + z[i] < t.length()) {
            if (t[i - z[i]] == t[i + z[i]]) ++z[i];
            else break;
        }
    }
}

```

```

}
if (i + z[i] > r) r = i + z[i], l = i;
}
for(int i=1;i<t.length();++i) ans = max(ans, z[i]-1);
return ans;
}

```

7.6 Lexico Smallest Rotation [0e9fb8]

```

string mcp(string s) {
    int n = s.length();
    s += s; int i = 0, j = 1;
    while (i < n && j < n) {
        int k = 0;
        while (k < n && s[i + k] == s[j + k]) k++;
        ((s[i + k] <= s[j + k]) ? j : i) += k + 1;
        j += (i == j);
    }
    return s.substr(i < n ? i : j, n);
}

```

7.7 Main Lorentz

```

vector<pair<int, int>> rep[kN]; // 0-base [l, r]
void main_lorentz(const string &s, int sft = 0) {
    const int n = s.size();
    if (n == 1) return;
    const int nu = n / 2, nv = n - nu;
    const string u = s.substr(0, nu), v = s.substr(nu);
    ru(u.rbegin(), u.rend(), rv(v.rbegin(), v.rend()));
    main_lorentz(u, sft), main_lorentz(v, sft + nu);
    const auto z1 = Zalgo(ru), z2 = Zalgo(v + '#' + u),
        z3 = Zalgo(ru + '#' + rv), z4 = Zalgo(v);
    auto get_z = [](const std::vector<int> &z, int i) {
        return (0 <= i && i < (int)z.size()) ? z[i] : 0;
    };
    auto add_rep = [&](bool left, int c, int l, int k1,
        int k2) {
        const int L = max(1, l - k2), R = std::min(l - left,
            k1);
        if (L > R) return;
        if (left) rep[l].emplace_back(sft + c - R, sft + c -
            L);
        else rep[l].emplace_back(sft + c - R - l + 1, sft + c -
            L - l + 1);
    };
    for (int cntr = 0; cntr < n; cntr++) {
        int l, k1, k2;
        if (cntr < nu) {
            l = nu - cntr;
            k1 = get_z(z1, nu - cntr);
            k2 = get_z(z2, nv + 1 + cntr);
        } else {
            l = cntr - nu + 1;
            k1 = get_z(z3, nu + 1 + nv - 1 - (cntr - nu));
            k2 = get_z(z4, (cntr - nu) + 1);
        }
        if (k1 + k2 >= l)
            add_rep(cntr < nu, cntr, l, k1, k2);
    }
}

```

7.8 BWT

```

struct BurrowsWheeler{
#define SIGMA 26
#define BASE 'a'
    vector<int> v[ SIGMA ];
    void BWT(char* ori, char* res){
        // make ori -> ori + ori
        // then build suffix array
    }
    void iBWT(char* ori, char* res){
        for( int i = 0 ; i < SIGMA ; i ++ )
            v[ i ].clear();
        int len = strlen( ori );
        for( int i = 0 ; i < len ; i ++ )
            v[ ori[i] - BASE ].push_back( i );
        vector<int> a;
        for( int i = 0 , ptr = 0 ; i < SIGMA ; i ++ )
            for( auto j : v[ i ] ){
                a.push_back( j );
                ori[ ptr ++ ] = BASE + i;
            }
        for( int i = 0 , ptr = 0 ; i < len ; i ++ ){

```

```

    res[ i ] = ori[ a[ ptr ] ];
    ptr = a[ ptr ];
}
res[ len ] = 0;
}
} bwt;

```

7.9 Palindromic Tree

```

struct palindromic_tree{
    struct node{
        int next[26], f, len;
        int cnt, num, st, ed; // num = depth of fail link
        node(int l=0): f(0), len(l), cnt(0), num(0) {
            memset(next, 0, sizeof(next));
        }
    };
    vector<node> st;
    vector<char> s;
    int last, n;
    void init(){
        st.clear(); s.clear(); last=1; n=0;
        st.push_back(0); st.push_back(-1);
        st[0].f=1; s.push_back(-1);
    }
    int getFail(int x){
        while(s[n-st[x].len-1] != s[n]) x=st[x].f;
        return x;
    }
    void add(int c){
        s.push_back(c-'a'); ++n;
        int cur=getFail(last);
        if(!st[cur].next[c]){
            int now=st.size();
            st.push_back(st[cur].len+2);
            st[now].f=st[getFail(st[cur].f)].next[c];
            st[cur].next[c]=now;
            st[now].num=st[st[now].f].num+1;
        }
        last=st[cur].next[c];
        ++st[last].cnt;
    }
    void dpcnt() { // cnt = #occurrence in whole str
        for (int i=st.size()-1; i >= 0; i--)
            st[st[i].f].cnt += st[i].cnt;
    }
    int size(){ return st.size()-2; }
} pt;
int main() {
    string s; cin >> s; pt.init();
    for (int i=0; i<SZ(s); i++) {
        int prvsz = pt.size(); pt.add(s[i]);
        if (prvsz != pt.size()) {
            int r = i, l = r - pt.st[pt.last].len + 1;
            // pal @ [l,r]: s.substr(l, r-l+1)
        }
    }
    return 0;
}

```

8 Misc

8.1 Theorems

8.1.1 Sherman-Morrison formula

$$(A + uv^T)^{-1} = A^{-1} - \frac{A^{-1}uv^TA^{-1}}{1 + v^TA^{-1}u}$$

8.1.2 Kirchhoff's Theorem

Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G , where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i, j) in G .

- The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
- The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.

8.1.3 Tutte's Matrix

Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniform randomly) if $i < j$ and $(i, j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{\text{rank}(D)}{2}$ is the maximum matching on G .

8.1.4 Cayley's Formula

- Given a degree sequence d_1, d_2, \dots, d_n for each labeled vertices, there're $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\dots(d_n-1)!}$ spanning trees.
- Let $T_{n,k}$ be the number of labeled forests on n vertices with k components, such that vertex $1, 2, \dots, k$ belong to different components. Then $T_{n,k} = kn^{n-k-1}$.

8.1.5 Erdős–Gallai theorem

A sequence of non-negative integers $d_1 \geq d_2 \geq \dots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + d_2 + \dots + d_n$ is even and

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$$

holds for all $1 \leq k \leq n$.

8.1.6 Havel–Hakimi algorithm

find the vertex who has greatest degree unused, connect it with other greatest vertex.

8.1.7 Euler's planar graph formula

$$V - E + F = C + 1. \quad E \leq 3V - 6 \quad (\text{when } V \geq 3)$$

8.1.8 Pick's theorem

For simple polygon, when points are all integer, we have $A = \# \{\text{lattice points in the interior}\} + \frac{\# \{\text{lattice points on the boundary}\}}{2} - 1$

8.1.9 Matroid Intersection

Given matroids $M_1 = (G, I_1)$, $M_2 = (G, I_2)$, find maximum $S \in I_1 \cap I_2$. For each iteration, build the directed graph and find a shortest path from s to t .

- $s \rightarrow x : S \sqcup \{x\} \in I_1$
- $x \rightarrow t : S \sqcup \{x\} \in I_2$
- $y \rightarrow x : S \setminus \{y\} \sqcup \{x\} \in I_1$ (y is in the unique circuit of $S \sqcup \{x\}$)
- $x \rightarrow y : S \setminus \{y\} \sqcup \{x\} \in I_2$ (y is in the unique circuit of $S \sqcup \{x\}$)

Alternate the path, and $|S|$ will increase by 1. Let $R = \min(\text{rank}(I_1), \text{rank}(I_2))$, $N = |G|$. In each iteration, $|E| = O(RN)$. For weighted case, assign weight $-w(x)$ and $w(x)$ to $x \in S$ and $x \notin S$, resp. Use Bellman-Ford to find the weighted shortest path. The maximum iteration of Bellman-Ford is $2R + 1$.

8.2 Bitset LCS

```

scanf("%d%d", &n, &m), u = n / 64 + 1;
for (int i = 1, c; i <= n; i++)
    scanf("%d", &c), p[c].set(i);
for (int i = 1, c; i <= m; i++) {
    scanf("%d", &c), (g = f) |= p[c];
    f.shiftLeftByOne(), f.set(0);
    ((f = g - f) ^= g) &= g;
}
printf("%d\n", f.count());

```

8.3 Prefix Substring LCS

```

void all_lcs(string s, string t) { // 0-base
    vector<int> h(SZ(t));
    iota(ALL(h), 0);
    for (int a = 0; a < SZ(s); ++a) {
        int v = -1;
        for (int c = 0; c < SZ(t); ++c)
            if (s[a] == t[c] || h[c] < v)
                swap(h[c], v);
        // LCS(s[0, a], t[b, c]) =
        // c - b + 1 - sum([h[i] >= b] | i <= c)
        // h[i] might become -1 !!
    }
}

```

8.4 Convex 1D/1D DP

```

struct segment {
    int i, l, r;
    segment() {}
    segment(int a, int b, int c): i(a), l(b), r(c) {}
};
inline lld f(int l, int r){return dp[l] + w(l+1, r);}
void solve() {
    dp[0] = 0;
    deque<segment> dq; dq.push_back(segment(0, 1, n));
    for (int i = 1; i <= n; ++i) {
        dp[i] = f(dq.front().i, i);
        while(dq.size() && dq.front().r < i+1) dq.pop_front();
        dq.front().l = i + 1;
        segment seg = segment(i, i + 1, n);
        while (dq.size() &&
            f(i, dq.back().l) < f(dq.back().i, dq.back().l))
            dq.pop_back();
    }
}

```



```

if (dq.size()) {
    int d = 1 << 20, c = dq.back().l;
    while (d >= 1) if (c + d <= dq.back().r)
        if (f(i, c+d) > f(dq.back().i, c+d)) c += d;
        dq.back().r = c; seg.l = c + 1;
    }
    if (seg.l <= n) dq.push_back(seg);
}
}

```

8.5 ConvexHull Optimization [bcdclā]

```

struct L {
    mutable int64_t a, b, p;
    bool operator<(const L &r) const {
        return a < r.a; /* here */
    }
    bool operator<(int64_t x) const { return p < x; }
};

struct DynamicHull : multiset<L, less<>> {
    static const int64_t kInf = 1e18;
    bool Isect(iterator x, iterator y) {
        auto Div = [] (int64_t a, int64_t b) {
            return a / b - ((a ^ b) < 0 && a % b); };
        if (y == end()) { x->p = kInf; return false; }
        if (x->a == y->a)
            x->p = x->b > y->b ? kInf : -kInf; /* here */
        else x->p = Div(y->b - x->b, x->a - y->a);
        return x->p >= y->p;
    }
    void Insert(int64_t a, int64_t b) {
        auto z = insert({a, b, 0}); y = z++, x = y;
        while (Isect(y, z)) z = erase(z);
        if (x != begin() && Isect(--x, y)) Isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            Isect(x, erase(y));
    }
    int64_t Query(int64_t x) { // default chmax
        auto l = *lower_bound(x); // to chmin:
        return l.a * x + l.b; // modify the 2 "<>"
    }
};

```

8.6 Josephus Problem

```

// n people kill m for each turn
int f(int n, int m) {
    int s = 0;
    for (int i = 2; i <= n; i++)
        s = (s + m) % i;
    return s;
}

// died at kth
int kth(int n, int m, int k) {
    if (m == 1) return n-1;
    for (k = k*m+m-1; k >= n; k = k-n+(k-n)/(m-1));
    return k;
}

```

8.7 Tree Knapsack

```

int dp[N][K]; PII obj[N];
vector<int> G[N];
void dfs(int u, int mx) {
    for (int s: G[u]) {
        if (mx < obj[s].first) continue;
        for (int i=0; i<=mx-obj[s].FF; i++)
            dp[s][i] = dp[u][i];
        dfs(s, mx - obj[s].first);
        for (int i=obj[s].FF; i<=mx; i++)
            dp[u][i] = max(dp[u][i],
                dp[s][i - obj[s].FF] + obj[s].SS);
    }
}

```

8.8 N Queens Problem

```

vector<int> solve(int n) {
    // no solution when n=2, 3
    vector<int> ret;
    if (n % 6 == 2) {
        for (int i = 2; i <= n; i += 2)
            ret.push_back(i);
        ret.push_back(3); ret.push_back(1);
        for (int i = 7; i <= n; i += 2)
            ret.push_back(i);
    }
}

```

```

ret.push_back(5);
} else if (n % 6 == 3) {
    for (int i = 4; i <= n; i += 2)
        ret.push_back(i);
    ret.push_back(2);
    for (int i = 5; i <= n; i += 2)
        ret.push_back(i);
    ret.push_back(1); ret.push_back(3);
} else {
    for (int i = 2; i <= n; i += 2)
        ret.push_back(i);
    for (int i = 1; i <= n; i += 2)
        ret.push_back(i);
}
return ret;
}

```

8.9 Stable Marriage

```

1: Initialize  $m \in M$  and  $w \in W$  to free
2: while  $\exists$  free man  $m$  who has a woman  $w$  to propose to do
3:    $w \leftarrow$  first woman on  $m$ 's list to whom  $m$  has not yet proposed
4:   if  $\exists$  some pair  $(m', w')$  then
5:     if  $w$  prefers  $m$  to  $m'$  then
6:        $m' \leftarrow$  free
7:        $(m, w) \leftarrow$  engaged
8:     end if
9:   else
10:     $(m, w) \leftarrow$  engaged
11:   end if
12: end while

```

8.10 Binary Search On Fraction [765c5a]

```

struct Q {
    ll p, q;
    Q go(Q b, ll d) { return {p + b.p*d, q + b.q*d}; }
};

bool pred(Q);
// returns smallest p/q in [lo, hi] such that
// pred(p/q) is true, and  $0 \leq p, q \leq N$ 
Q frac_bs(ll N) {
    Q lo{0, 1}, hi{1, 0};
    if (pred(lo)) return lo;
    assert(pred(hi));
    bool dir = 1, L = 1, H = 1;
    for (; L || H; dir = !dir) {
        ll len = 0, step = 1;
        for (int t = 0; t < 2 && (t ? step/=2 : step*=2);)
            if (Q mid = hi.go(lo, len + step);
                mid.p > N || mid.q > N || dir ^ pred(mid))
                t++;
            else len += step;
        swap(lo, hi = hi.go(lo, len));
        (dir ? L : H) = !len;
    }
    return dir ? hi : lo;
}

```