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1 Basic

1.1 vimrc

```

se is nu rnu bs=2 ru mouse=a encoding=utf-8
se cin et sw=4 sts=4 t_Co=256 tgc sc hls ls=2
syn on
colorscheme desert
filetype indent on
inoremap {<CR> {<CR>}<ESC>O
map <F8> <ESC>:w<CR>:!g++ "%> -o "%<" -O2 -std=c++17 -
DKISEKI -Wall -Wextra -Wshadow -Wfatal-errors -
Wconversion -fsanitize=address -fsanitize=undefined
-g && echo success<CR>
map <F9> <ESC>:w<CR>:!g++ "%> -o "%<" -O2 -std=c++17 -
DKISEKI && echo success<CR>
map <F10> <ESC>:!. / "%<" <CR>

```

1.2 Increase Stack

```

const int size = 256 << 20;
register long rsp asm("rsp");
char *p = (char*)malloc(size)+size, *bak = (char*)rsp;
__asm__ ("movq %0, %%rsp\n::"r"(p));
// main
__asm__ ("movq %0, %%rsp\n::"r"(bak));

```

1.3 Pragma Optimization

```

#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,tune=native")

```

1.4 IO Optimization

```

static inline int gc() {
    static char buf[ 1 << 20 ], *p = buf, *end = buf;
    if ( p == end ) {
        end = buf + fread( buf, 1, 1 << 20, stdin );
        if ( end == buf ) return EOF;
        p = buf;
    }
    return *p++;
}

template < typename T >
static inline bool gn( T &_ ) {
    register int c = gc(); register T __ = 1; _ = 0;
    while(( '0' > c || c > '9' ) && c != EOF && c != '-') c = gc();
    if(c == '-') { __ = -1; c = gc(); }
    if(c == EOF) return false;
    while( '0' <= c && c <= '9' ) _ = _ * 10 + c - '0', c = gc();
    _ *= __;
    return true;
}

```

```
template < typename T, typename ...Args >
static inline bool gn( T &x, Args &...args )
{ return gn(x) && gn(args...); }
```

2 Data Structure

2.1 BigInt

```
class BigInt{
private:
    using lld = int_fast64_t;
#define PRINTF_ARG PRIuFAST64
#define LOG_BASE_STR "g"
    static constexpr lld BASE = 1000000000;
    static constexpr int LOG_BASE = 9;
    vector<lld> dig; bool neg;
    inline int len() const { return (int) dig.size(); }
    inline int cmp_minus(const BigInt& a) const {
        if(len() == 0 && a.len() == 0) return 0;
        if(neg ^ a.neg) return a.neg ^ 1;
        if(len() != a.len())
            return neg? a.len() - len() : len() - a.len();
        for(int i=len()-1; i>=0; i--) if(dig[i] != a.dig[i])
            return neg? a.dig[i] - dig[i] : dig[i] - a.dig[i];
        return 0;
    }
    inline void trim(){
        while(!dig.empty() && !dig.back()) dig.pop_back();
        if(dig.empty()) neg = false;
    }
public:
    BigInt(): dig(vector<lld>()), neg(false){}
    BigInt(lld a): dig(vector<lld>()){
        neg = a < 0; dig.push_back(abs(a));
        trim();
    }
    BigInt(const string& a): dig(vector<lld>()){
        assert(!a.empty()); neg = (a[0] == '-');
        for(int i=((int)a.size()-1; i>=neg; i-=LOG_BASE){
            lld cur = 0;
            for(int j=min(LOG_BASE-1, i-neg); j>=0; j--){
                cur = cur*10+a[i-j]-'0';
                dig.push_back(cur);
            } trim();
        }
    }
    inline bool operator<(const BigInt& a) const {
        return cmp_minus(a) < 0;
    }
    inline bool operator<=(const BigInt& a) const {
        return cmp_minus(a) <= 0;
    }
    inline bool operator==(const BigInt& a) const {
        return cmp_minus(a) == 0;
    }
    inline bool operator!=(const BigInt& a) const {
        return cmp_minus(a) != 0;
    }
    inline bool operator>(const BigInt& a) const {
        return cmp_minus(a) > 0;
    }
    inline bool operator>=(const BigInt& a) const {
        return cmp_minus(a) >= 0;
    }
    BigInt operator-() const {
        BigInt ret = *this;
        ret.neg ^= 1; return ret;
    }
    BigInt operator+(const BigInt& a) const {
        if(neg) return -(-(*this) + (-a));
        if(a.neg) return (*this) - (-a);
        int n = max(a.len(), len());
        BigInt ret; ret.dig.resize(n);
        lld pro = 0;
        for(int i=0; i<n; i++) {
            ret.dig[i] = pro;
            if(i < a.len()) ret.dig[i] += a.dig[i];
            if(i < len()) ret.dig[i] += dig[i];
            pro = 0;
            if(ret.dig[i] >= BASE) pro = ret.dig[i]/BASE;
            ret.dig[i] -= BASE*pro;
        }
        if(pro != 0) ret.dig.push_back(pro);
        return ret;
    }
    BigInt operator-(const BigInt& a) const {
        if(neg) return -(-(*this) - (-a));
        if(a.neg) return (*this) + (-a);
        int diff = cmp_minus(a);
        if(diff < 0) return -(a - (*this));
```

```
        if(diff == 0) return 0;
        BigInt ret; ret.dig.resize(len(), 0);
        for(int i=0; i<len(); i++) {
            ret.dig[i] += dig[i];
            if(i < a.len()) ret.dig[i] -= a.dig[i];
            if(ret.dig[i] < 0){
                ret.dig[i] += BASE;
                ret.dig[i+1]--;
            }
        }
        ret.trim(); return ret;
    }
    BigInt operator*(const BigInt& a) const {
        if(!len() || !a.len()) return 0;
        BigInt ret; ret.dig.resize(len()+a.len()+1);
        ret.neg = neg ^ a.neg;
        for(int i=0; i<len(); i++)
            for(int j=0; j<a.len(); j++){
                ret.dig[i+j] += dig[i] * a.dig[j];
                if(ret.dig[i+j] >= BASE) {
                    lld x = ret.dig[i+j] / BASE;
                    ret.dig[i+j+1] += x;
                    ret.dig[i+j] -= x * BASE;
                }
            }
        ret.trim(); return ret;
    }
    BigInt operator/(const BigInt& a) const {
        assert(a.len());
        if(len() < a.len()) return 0;
        BigInt ret; ret.dig.resize(len()-a.len()+1);
        ret.neg = a.neg;
        for(int i=len()-a.len(); i>=0; i--){
            lld l = 0, r = BASE;
            while(r-l > 1){
                lld mid = (l+r)>>1;
                ret.dig[i] = mid;
                if(ret*a <= (neg? -(*this) : (*this))) l = mid;
                else r = mid;
            }
            ret.dig[i] = 1;
        }
        ret.neg ^= neg; ret.trim();
        return ret;
    }
    BigInt operator%(const BigInt& a) const {
        return (*this) - (*this) / a * a;
    }
    friend BigInt abs(BigInt a) { a.neg = 0; return a; }
    friend void swap(BigInt& a, BigInt& b){
        swap(a.dig, b.dig); swap(a.neg, b.neg);
    }
    friend istream& operator>>(istream& ss, BigInt& a){
        string s; ss >> s; a = s; return ss;
    }
    friend ostream& operator<<(ostream& o, const BigInt& a){
        if(a.len() == 0) return o << '0';
        if(a.neg) o << '-';
        o << a.dig.back();
        for(int i=a.len()-2; i>=0; i--){
            o << setw(LOG_BASE)<< setfill('0')<< a.dig[i];
        }
        return o;
    }
    inline void print() const {
        if(len() == 0){ putchar('0'); return; }
        if(neg) putchar('-');
        printf("%" PRINTF_ARG, dig.back());
        for(int i=len()-2; i>=0; i--){
            printf("%0" LOG_BASE_STR PRINTF_ARG, dig[i]);
        }
        #undef PRINTF_ARG
        #undef LOG_BASE_STR
    };
};
```

2.2 Dark Magic

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
using __gnu_pbds::pairing_heap_tag;
using __gnu_pbds::binary_heap_tag;
using __gnu_pbds::binomial_heap_tag;
using __gnu_pbds::rc_binomial_heap_tag;
using __gnu_pbds::thin_heap_tag;
```

```

template<typename T>
using pbds_heap=__gnu_pbds::priority_queue<T,less<T>,\
    pairing_heap_tag>;
// __gnu_pbds::priority_queue<T,less<T>>::
// point_iterator
// x = pq.push(10); pq.modify(x, 87); a.join(b);
using __gnu_pbds::rb_tree_tag;
using __gnu_pbds::ov_tree_tag;
using __gnu_pbds::splay_tree_tag;
template<typename T>
using ordered_set = __gnu_pbds::tree<T,\
__gnu_pbds::null_type,less<T>,rb_tree_tag,\
__gnu_pbds::tree_order_statistics_node_update>;
// find_by_order, order_of_key
template<typename A,typename B>
using hTable1=__gnu_pbds::cc_hash_table<A,B>;
template<typename A,typename B>
using hTable2=__gnu_pbds::gp_hash_table<A,B>;

```

2.3 Disjoint Set

```

class DJS {
private:
    vector< int > fa, sz, sv;
    vector< pair< int*, int > > opt;
    void assign( int *k, int v ) {
        opt.emplace_back( k, *k );
        *k = v;
    }
public:
    void init( int n ) {
        fa.resize( n ); iota( fa.begin(), fa.end(), 0 );
        sz.resize( n ); fill( sz.begin(), sz.end(), 1 );
        opt.clear();
    }
    int query( int x ) {return fa[x] == x?x:query(fa[x]);}
    void merge( int a, int b ) {
        int af = query( a ), bf = query( b );
        if( af == bf ) return;
        if( sz[ af ] < sz[ bf ] ) swap( af, bf );
        assign( &fa[ bf ], fa[ af ] );
        assign( &sz[ af ], sz[ af ] + sz[ bf ] );
    }
    void save() { sv.push_back( (int) opt.size() ); }
    void undo() {
        int ls = sv.back(); sv.pop_back();
        while ( ( int ) opt.size() > ls ) {
            pair< int*, int > cur = opt.back();
            *cur.first = cur.second;
            opt.pop_back();
        }
    }
};

```

2.4 Link-Cut Tree

```

struct Node{
    Node *par,*ch[2];
    int xor_sum,v;
    bool is_rev;
    Node(int _v){
        v=xor_sum=_v;is_rev=false;
        par=ch[0]=ch[1]=nullptr;
    }
    inline void set_rev(){is_rev^=1;swap(ch[0],ch[1]);}
    inline void down(){
        if(is_rev){
            if(ch[0]!=nullptr) ch[0]->set_rev();
            if(ch[1]!=nullptr) ch[1]->set_rev();
            is_rev=false;
        }
    }
    inline void up(){
        xor_sum=v;
        if(ch[0]!=nullptr){
            xor_sum^=ch[0]->xor_sum;
            ch[0]->par=this;
        }
        if(ch[1]!=nullptr){
            xor_sum^=ch[1]->xor_sum;
            ch[1]->par=this;
        }
    }
    inline bool is_root(){

```

```

        return par==nullptr ||\
            (par->ch[0]!=this && par->ch[1]!=this);
    }
    bool is_rch(){return !is_root() && par->ch[1]==this;}
    *node[maxn],*stk[maxn];
    int top;
    void to_child(Node* p,Node* c,bool dir){
        p->ch[dir]=c;
        p->up();
    }
    inline void rotate(Node* node){
        Node* par=node->par;
        Node* par_par=par->par;
        bool dir=node->is_rch();
        bool par_dir=par->is_rch();
        to_child(par,node->ch[!dir],dir);
        to_child(node,par,!dir);
        if(par_par!=nullptr && par_par->ch[par_dir]==par)
            to_child(par_par,node,par_dir);
        else node->par=par_par;
    }
    inline void splay(Node* node){
        Node* tmp=node;
        stk[top++]=node;
        while(!tmp->is_root()){
            tmp=tmp->par;
            stk[top++]=tmp;
        }
        while(top) stk[--top]->down();
        for(Node *fa=node->par;
            !node->is_root();
            rotate(node),fa=node->par)
            if(!fa->is_root())
                rotate(fa->is_rch()==node->is_rch()?fa:node);
    }
    inline void access(Node* node){
        Node* last=nullptr;
        while(node!=nullptr){
            splay(node);
            to_child(node,last,true);
            last=node;
            node=node->par;
        }
    }
    inline void change_root(Node* node){
        access(node);splay(node);node->set_rev();
    }
    inline void link(Node* x,Node* y){
        change_root(x);splay(x);x->par=y;
    }
    inline void split(Node* x,Node* y){
        change_root(x);access(y);splay(x);
        to_child(x,nullptr,true);y->par=nullptr;
    }
    inline void change_val(Node* node,int v){
        access(node);splay(node);node->v=v;node->up();
    }
    inline int query(Node* x,Node* y){
        change_root(x);access(y);splay(y);
        return y->xor_sum;
    }
    inline Node* find_root(Node* node){
        access(node);splay(node);
        Node* last=nullptr;
        while(node!=nullptr){
            node->down();last=node;node=node->ch[0];
        }
        return last;
    }
    set<pii> dic;
    inline void add_edge(int u,int v){
        if(u>v) swap(u,v);
        if(find_root(node[u])==find_root(node[v])) return;
        dic.insert(pii(u,v));
        link(node[u],node[v]);
    }
    inline void del_edge(int u,int v){
        if(u>v) swap(u,v);
        if(dic.find(pii(u,v))==dic.end()) return;
        dic.erase(pii(u,v));
        split(node[u],node[v]);
    }
}

```

2.5 LiChao Segment Tree

```
struct Line{
    int m, k, id;
    Line() : id( -1 ) {}
    Line( int a, int b, int c )
        : m( a ), k( b ), id( c ) {}
    int at( int x ) { return m * x + k; }
};

class LiChao {
private:
    int n; vector< Line > nodes;
    inline int lc( int x ) { return 2 * x + 1; }
    inline int rc( int x ) { return 2 * x + 2; }
    void insert( int l, int r, int id, Line ln ) {
        int m = ( l + r ) >> 1;
        if ( nodes[ id ].id == -1 ) {
            nodes[ id ] = ln;
            return;
        }
        bool atLeft = nodes[ id ].at( l ) < ln.at( l );
        if ( nodes[ id ].at( m ) < ln.at( m ) ) {
            atLeft ^= 1; swap( nodes[ id ], ln );
        }
        if ( r - l == 1 ) return;
        if ( atLeft ) insert( l, m, lc( id ), ln );
        else insert( m, r, rc( id ), ln );
    }
    int query( int l, int r, int id, int x ) {
        int ret = 0;
        if ( nodes[ id ].id != -1 )
            ret = nodes[ id ].at( x );
        int m = ( l + r ) >> 1;
        if ( r - l == 1 ) return ret;
        else if ( x < m )
            return max( ret, query( l, m, lc( id ), x ) );
        else
            return max( ret, query( m, r, rc( id ), x ) );
    }
public:
    void build( int n_ ) {
        n = n_; nodes.clear();
        nodes.resize( n << 2, Line() );
    }
    void insert( Line ln ) { insert( 0, n, 0, ln ); }
    int query( int x ) { return query( 0, n, 0, x ); }
} lichao;
```

2.6 Treap

```
namespace Treap{
#define sz( x ) ( ( x ) ? ( ( x )->size ) : 0 )
struct node{
    int size;
    uint32_t pri;
    node *lc, *rc;
    node() : size(0), pri(rand()), lc( 0 ), rc( 0 ) {}
    void pull() {
        size = 1;
        if ( lc ) size += lc->size;
        if ( rc ) size += rc->size;
    }
};

node* merge( node* L, node* R ) {
    if ( not L or not R ) return L ? L : R;
    if ( L->pri > R->pri ) {
        L->rc = merge( L->rc, R ); L->pull();
        return L;
    } else {
        R->lc = merge( L, R->lc ); R->pull();
        return R;
    }
}

void split_by_size( node*rt, int k, node*&L, node*&R ) {
    if ( not rt ) L = R = nullptr;
    else if( sz( rt->lc ) + 1 <= k ) {
        L = rt;
        split_by_size( rt->rc, k-sz(rt->lc)-1, L->rc, R );
        L->pull();
    } else {
        R = rt;
        split_by_size( rt->lc, k, L, R->lc );
        R->pull();
    }
}
```

```
}
}
#undef sz
}
```

2.7 Sparse Table

```
template < typename T, typename Cmp_ = less< T > >
class SparseTable {
private:
    vector< vector< T > > tbl;
    vector< int > lg;
    T cv( T a, T b ) {
        return Cmp_()( a, b ) ? a : b;
    }
public:
    void init( T arr[], int n ) {
        // 0-base
        lg.resize( n + 1 );
        lg[ 0 ] = -1;
        for( int i=1; i<=n; ++i ) lg[i] = lg[i>>1] + 1;
        tbl.resize( lg[n] + 1 );
        tbl[ 0 ].resize( n );
        copy( arr, arr + n, tbl[ 0 ].begin() );
        for ( int i = 1; i <= lg[ n ]; ++i ) {
            int len = 1 << ( i - 1 ), sz = 1 << i;
            tbl[ i ].resize( n - sz + 1 );
            for ( int j = 0; j <= n - sz; ++j )
                tbl[i][j] = cv(tbl[i-1][j], tbl[i-1][j+len]);
        }
    }
    T query( int l, int r ) {
        // 0-base [l, r)
        int wh = lg[ r - l ], len = 1 << wh;
        return cv( tbl[ wh ][ l ], tbl[ wh ][ r - len ] );
    }
};
```

2.8 Linear Basis

```
struct LinearBasis {
private:
    int n, sz;
    vector< llu > B;
    inline llu two( int x ){ return ( ( llu ) 1 ) << x; }
public:
    void init( int n_ ) {
        n = n_; B.clear(); B.resize( n ); sz = 0;
    }
    void insert( llu x ) {
        // add x into B
        for ( int i = n-1; i >= 0; --i ) if( two(i) & x ){
            if ( B[ i ] ) x ^= B[ i ];
            else {
                B[ i ] = x; sz++;
                for ( int j = i - 1; j >= 0; --j )
                    if( B[ j ] && ( two( j ) & B[ i ] ) )
                        B[ i ] ^= B[ j ];
                for ( int j = i + 1; j < n; ++j )
                    if ( two( i ) & B[ j ] )
                        B[ j ] ^= B[ i ];
                break;
            }
        }
    }
    inline int size() { return sz; }
    bool check( llu x ) {
        // is x in span(B) ?
        for ( int i = n-1; i >= 0; --i ) if( two(i) & x )
            if ( B[ i ] ) x ^= B[ i ];
        return true;
    }
    llu kth_small(llu k) {
        /** 1-base would always > 0 **/
        /** should check it **/
        /* if we choose at least one element
           but size(B)(vectors in B)==N(original elements)
           then we can't get 0 */
        llu ret = 0;
        for ( int i = 0; i < n; ++i ) if( B[ i ] ) {
            if ( k & 1 ) ret ^= B[ i ];
            k >>= 1;
        }
    }
}
```

```

    return ret;
}
} base;

```

3 Graph

3.1 Euler Circuit

```

bool vis[ N ]; size_t la[ K ];
void dfs( int u, vector< int >& vec ) {
    while ( la[ u ] < G[ u ].size() ) {
        if( vis[ G[ u ][ la[ u ] ].second ] ) {
            ++ la[ u ];
            continue;
        }
        int v = G[ u ][ la[ u ] ].first;
        vis[ G[ u ][ la[ u ] ].second ] = true;
        ++ la[ u ]; dfs( v, vec );
        vec.push_back( v );
    }
}

```

3.2 BCC Edge

```

class BCC_Bridge {
private:
    int n, ecnt;
    vector<vector<pair<int,int>>> G;
    vector<int> dfn, low;
    vector<bool> bridge;
    void dfs(int u, int f) {
        dfn[u] = low[u] = dfn[f] + 1;
        for (auto [v, t]: G[u]) {
            if (v == f) continue;
            if (dfn[v]) {
                low[u] = min(low[u], dfn[v]);
                continue;
            }
            dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (low[v] > dfn[u]) bridge[t] = true;
        }
    }
public:
    void init(int n_) {
        G.clear(); G.resize(n = n_);
        low.assign(n, 0); dfn.assign(n, 0);
    }
    void add_edge(int u, int v) {
        G[u].emplace_back(v, ecnt);
        G[v].emplace_back(u, ecnt++);
    }
    void solve() {
        bridge.assign(ecnt, false);
        for (int i = 0; i < n; ++i)
            if (not dfn[i]) dfs(i, i);
    }
    bool is_bridge(int x) { return bridge[x]; }
} bcc_bridge;

```

3.3 BCC Vertex

```

class BCC_AP {
private:
    int n, ecnt;
    vector<vector<pair<int,int>>> G;
    vector<int> bcc, dfn, low, st;
    vector<bool> ap, ins;
    void dfs(int u, int f) {
        dfn[u] = low[u] = dfn[f] + 1;
        int ch = 0;
        for (auto [v, t]: G[u]) if (v != f) {
            if (not ins[t]) {
                st.push_back(t);
                ins[t] = true;
            }
            if (dfn[v]) {
                low[u] = min(low[u], dfn[v]);
                continue;
            }
            ++ch; dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (low[v] >= dfn[u]) {
                ap[u] = true;
            }
        }
    }
}

```

```

    while (true) {
        int eid = st.back(); st.pop_back();
        bcc[eid] = ecnt;
        if (eid == t) break;
    }
    ecnt++;
}
if (ch == 1 and u == f) ap[u] = false;
}
public:
    void init(int n_) {
        G.clear(); G.resize(n = n_);
        ecnt = 0; ap.assign(n, false);
        low.assign(n, 0); dfn.assign(n, 0);
    }
    void add_edge(int u, int v) {
        G[u].emplace_back(v, ecnt);
        G[v].emplace_back(u, ecnt++);
    }
    void solve() {
        ins.assign(ecnt, false);
        bcc.resize(ecnt); ecnt = 0;
        for (int i = 0; i < n; ++i)
            if (not dfn[i]) dfs(i, i);
    }
    int get_id(int x) { return bcc[x]; }
    int count() { return ecnt; }
    bool is_ap(int x) { return ap[x]; }
} bcc_ap;

```

3.4 2-SAT (SCC)

```

class TwoSat{
private:
    int n;
    vector<vector<int>>> rG,G,scCs;
    vector<int> ord,idx;
    vector<bool> vis,result;
    void dfs(int u){
        vis[u]=true;
        for(int v:G[u])
            if(!vis[v]) dfs(v);
        ord.push_back(u);
    }
    void rdfs(int u){
        vis[u]=false;idx[u]=scCs.size()-1;
        scCs.back().push_back(u);
        for(int v:rG[u])
            if(vis[v])rdfs(v);
    }
public:
    void init(int n_){
        n=n_;G.clear();G.resize(n);
        rG.clear();rG.resize(n);
        scCs.clear();ord.clear();
        idx.resize(n);result.resize(n);
    }
    void add_edge(int u,int v){
        G[u].push_back(v);rG[v].push_back(u);
    }
    void orr(int x,int y){
        if ((x^y)==1)return;
        add_edge(x^1,y); add_edge(y^1,x);
    }
    bool solve(){
        vis.clear();vis.resize(n);
        for(int i=0;i<n;++i)
            if(not vis[i])dfs(i);
        reverse(ord.begin(),ord.end());
        for (int u:ord){
            if(!vis[u])continue;
            scCs.push_back(vector<int>());
            rdfs(u);
        }
        for(int i=0;i<n;i+=2)
            if(idx[i]==idx[i+1])
                return false;
        vector<bool> c(scCs.size());
        for(size_t i=0;i<scCs.size();++i){
            for(size_t j=0;j<scCs[i].size();++j){
                result[scCs[i][j]]=c[i];
                c[idx[scCs[i][j]^1]]=!c[i];
            }
        }
    }
}

```



```

    }
    }
    return true;
}
bool get(int x){return result[x];}
inline int get_id(int x){return idx[x];}
inline int count(){return sccs.size();}
} sat2;

```

3.5 Lowbit Decomposition

```

class LowbitDecomp{
private:
    int time_, chain_, LOG_N;
    vector< vector< int > > G, fa;
    vector< int > tl, tr, chain, chain_st;
    // chain_ : number of chain
    // tl, tr[ u ] : subtree interval in the seq. of u
    // chain_st[ u ] : head of the chain contains u
    // chain[ u ] : chain id of the chain u is on
    void predfs( int u, int f ) {
        chain[ u ] = 0;
        for ( int v : G[ u ] ) {
            if ( v == f ) continue;
            predfs( v, u );
            if( lowbit( chain[ u ] ) < lowbit( chain[ v ] ) )
                chain[ u ] = chain[ v ];
        }
        if ( not chain[ u ] )
            chain[ u ] = chain_ ++;
    }
    void dfschain( int u, int f ) {
        fa[ u ][ 0 ] = f;
        for ( int i = 1 ; i < LOG_N ; ++ i )
            fa[ u ][ i ] = fa[ fa[ u ][ i - 1 ] ][ i - 1 ];
        tl[ u ] = time_ ++;
        if ( not chain_st[ chain[ u ] ] )
            chain_st[ chain[ u ] ] = u;
        for ( int v : G[ u ] )
            if ( v != f and chain[ v ] == chain[ u ] )
                dfschain( v, u );
        for ( int v : G[ u ] )
            if ( v != f and chain[ v ] != chain[ u ] )
                dfschain( v, u );
        tr[ u ] = time_;
    }
    bool anc( int u, int v ) {
        return tl[ u ] <= tl[ v ] and tr[ v ] <= tr[ u ];
    }
public:
    int lca( int u, int v ) {
        if ( anc( u, v ) ) return u;
        for ( int i = LOG_N - 1 ; i >= 0 ; -- i )
            if ( not anc( fa[ u ][ i ], v ) )
                u = fa[ u ][ i ];
        return fa[ u ][ 0 ];
    }
    void init( int n ) {
        fa.assign( ++n, vector< int >( LOG_N ) );
        for ( LOG_N = 0 ; ( 1 << LOG_N ) < n ; ++ LOG_N );
        G.clear(); G.resize( n );
        tl.assign( n, 0 ); tr.assign( n, 0 );
        chain.assign( n, 0 ); chain_st.assign( n, 0 );
    }
    void add_edge( int u, int v ) {
        // 1-base
        G[ u ].push_back( v );
        G[ v ].push_back( u );
    }
    void decompose(){
        chain_ = 1;
        predfs( 1, 1 );
        time_ = 0;
        dfschain( 1, 1 );
    }
    PII get_subtree(int u) { return {tl[ u ], tr[ u ]}; }
    vector< PII > get_path( int u, int v ){
        vector< PII > res;
        int g = lca( u, v );
        while ( chain[ u ] != chain[ g ] ) {
            int s = chain_st[ chain[ u ] ];
            res.emplace_back( tl[ s ], tl[ u ] + 1 );
            u = fa[ s ][ 0 ];
        }
    }

```

```

    }
    res.emplace_back( tl[ g ], tl[ u ] + 1 );
    while ( chain[ v ] != chain[ g ] ) {
        int s = chain_st[ chain[ v ] ];
        res.emplace_back( tl[ s ], tl[ v ] + 1 );
        v = fa[ s ][ 0 ];
    }
    res.emplace_back( tl[ g ] + 1, tl[ v ] + 1 );
    return res;
    /* res : list of intervals from u to v
     * ( note only nodes work, not edge )
     * usage :
     * vector< PII > path = tree.get_path( u, v )
     * for( auto [ l, r ] : path ) {
     *     0-base [ l, r )
     * }
     */
} tree;

```

3.6 MaxClique

```

// contain a self loop u to u, than u won't in clique
template < size_t MAXN >
class MaxClique{
private:
    using bits = bitset< MAXN >;
    bits popped, G[ MAXN ], ans;
    size_t deg[ MAXN ], deo[ MAXN ], n;
    void sort_by_degree() {
        popped.reset();
        for ( size_t i = 0 ; i < n ; ++ i )
            deg[ i ] = G[ i ].count();
        for ( size_t i = 0 ; i < n ; ++ i ) {
            size_t mi = MAXN, id = 0;
            for ( size_t j = 0 ; j < n ; ++ j )
                if ( not popped[ j ] and deg[ j ] < mi )
                    mi = deg[ id = j ];
            popped[ deo[ i ] = id ] = 1;
            for( size_t u = G[ i ]._Find_first();
                u < n ; u = G[ i ]._Find_next( u ) )
                -- deg[ u ];
        }
    }
    void BK( bits R, bits P, bits X ) {
        if ( R.count()+P.count() <= ans.count() ) return;
        if ( not P.count() and not X.count() ) {
            if ( R.count() > ans.count() ) ans = R;
            return;
        }
        /* greedily choose max degree as pivot
        bits cur = P | X; size_t pivot = 0, sz = 0;
        for ( size_t u = cur._Find_first();
            u < n ; u = cur._Find_next( u ) )
            if ( deg[ u ] > sz ) sz = deg[ pivot = u ];
        cur = P & ( ~G[ pivot ] );
        */ // or simply choose first
        bits cur = P & ( ~G[ ( P | X )._Find_first() ] );
        for ( size_t u = cur._Find_first();
            u < n ; u = cur._Find_next( u ) ) {
            if ( R[ u ] ) continue;
            R[ u ] = 1;
            BK( R, P & G[ u ], X & G[ u ] );
            R[ u ] = P[ u ] = 0, X[ u ] = 1;
        }
    }
public:
    void init( size_t n ) {
        n = n_;
        for ( size_t i = 0 ; i < n ; ++ i )
            G[ i ].reset();
        ans.reset();
    }
    void add_edges( int u, bits S ) { G[ u ] = S; }
    void add_edge( int u, int v ) {
        G[ u ][ v ] = G[ v ][ u ] = 1;
    }
    int solve() {
        sort_by_degree(); // or simply iota( deo... )
        for ( size_t i = 0 ; i < n ; ++ i )
            deg[ i ] = G[ i ].count();
        bits pob, nob = 0; pob.set();
        for (size_t i=n; i<MAXN; ++i) pob[i] = 0;
    }

```

```

for ( size_t i = 0 ; i < n ; ++ i ) {
    size_t v = deo[ i ];
    bits tmp; tmp[ v ] = 1;
    BK( tmp, pob & G[ v ], nob & G[ v ] );
    pob[ v ] = 0, nob[ v ] = 1;
}
return static_cast< int >( ans.count() );
}
};

```

3.7 MaxCliqueDyn

```

constexpr int kN = 150;
struct MaxClique { // Maximum Clique
    bitset<kN> a[kN], cs[kN];
    int ans, sol[kN], q, cur[kN], d[kN], n;
    void init(int _n) {
        n = _n; for (int i = 0; i < n; i++) a[i].reset();
    }
    void addEdge(int u, int v) { a[u][v] = a[v][u] = 1; }
    void csort(vector<int> &r, vector<int> &c) {
        int mx = 1, km = max(ans - q + 1, 1), t = 0,
            m = int(r.size());
        cs[1].reset(); cs[2].reset();
        for (int i = 0; i < m; i++) {
            int p = r[i], k = 1;
            while ((cs[k] & a[p]).count()) k++;
            if (k > mx) cs[++mx].reset();
            cs[k][p] = 1;
            if (k < km) r[t++] = p;
        }
        c.resize(m);
        if (t) c[t - 1] = 0;
        for (int k = km; k <= mx; k++) {
            for (int p = int(cs[k]._Find_first());
                p < kN; p = int(cs[k]._Find_next(p))) {
                r[t] = p; c[t++] = k;
            }
        }
    }
    void dfs(vector<int> &r, vector<int> &c, int l,
        bitset<kN> mask) {
        while (!r.empty()) {
            int p = r.back(); r.pop_back();
            mask[p] = 0;
            if (q + c.back() <= ans) return;
            cur[q++] = p;
            vector<int> nr, nc;
            bitset<kN> nmask = mask & a[p];
            for (int i : r)
                if (a[p][i]) nr.push_back(i);
            if (!nr.empty()) {
                if (l < 4) {
                    for (int i : nr)
                        d[i] = int((a[i] & nmask).count());
                    sort(nr.begin(), nr.end(),
                        [&](int x, int y) {
                            return d[x] > d[y];
                        });
                }
                csort(nr, nc); dfs(nr, nc, l + 1, nmask);
            } else if (q > ans) {
                ans = q; copy(cur, cur + q, sol);
            }
            c.pop_back(); q--;
        }
    }
    int solve(bitset<kN> mask) { // vertex mask
        vector<int> r, c;
        for (int i = 0; i < n; i++)
            if (mask[i]) r.push_back(i);
        for (int i = 0; i < n; i++)
            d[i] = int((a[i] & mask).count());
        sort(r.begin(), r.end(),
            [&](int i, int j) { return d[i] > d[j]; });
        csort(r, c);
        dfs(r, c, 1, mask);
        return ans; // sol[0 ~ ans-1]
    }
} graph;

```

3.8 Virtual Tree

```

inline bool cmp(const int &i, const int &j) {
    return dfn[i] < dfn[j];
}
void build(int vectrices[], int k) {
    static int stk[MAX_N];
    sort(vectrices, vectrices + k, cmp);
    stk[sz++] = 0;
    for (int i = 0; i < k; ++i) {
        int u = vectrices[i], lca = LCA(u, stk[sz - 1]);
        if (lca == stk[sz - 1]) stk[sz++] = u;
        else {
            while (sz >= 2 && dep[stk[sz - 2]] >= dep[lca]) {
                addEdge(stk[sz - 2], stk[sz - 1]);
                sz--;
            }
            if (stk[sz - 1] != lca) {
                addEdge(lca, stk[sz - 1]);
                stk[sz++] = lca, vectrices[cnt++] = lca;
            }
            stk[sz++] = u;
        }
    }
    for (int i = 0; i < sz - 1; ++i)
        addEdge(stk[i], stk[i + 1]);
}

```

3.9 Centroid Decomposition

```

struct Centroid {
    vector<vector<int64_t>> Dist;
    vector<int> Parent, Depth;
    vector<int64_t> Sub, Sub2;
    vector<int> Sz, Sz2;
    Centroid(vector<vector<pair<int, int>>> g) {
        int N = g.size();
        vector<bool> Vis(N);
        vector<int> sz(N), mx(N);
        vector<int> Path;
        Dist.resize(N);
        Parent.resize(N);
        Depth.resize(N);
        auto DfsSz = [&](auto dfs, int x) -> void {
            Vis[x] = true; sz[x] = 1; mx[x] = 0;
            for (auto [u, w] : g[x]) {
                if (Vis[u]) continue;
                dfs(dfs, u);
                sz[x] += sz[u];
                mx[x] = max(mx[x], sz[u]);
            }
            Path.push_back(x);
        };
        auto DfsDist = [&](auto dfs, int x, int64_t D = 0)
            -> void {
            Dist[x].push_back(D); Vis[x] = true;
            for (auto [u, w] : g[x]) {
                if (Vis[u]) continue;
                dfs(dfs, u, D + w);
            }
        };
        auto Dfs = [&]
            (auto dfs, int x, int D = 0, int p = -1) -> void {
            Path.clear(); DfsSz(DfsSz, x);
            int M = Path.size();
            int C = -1;
            for (int u : Path) {
                if (max(M - sz[u], mx[u]) * 2 <= M) C = u;
                Vis[u] = false;
            }
            DfsDist(DfsDist, C);
            for (int u : Path) Vis[u] = false;
            Parent[C] = p; Vis[C] = true;
            Depth[C] = D;
            for (auto [u, w] : g[C]) {
                if (Vis[u]) continue;
                dfs(dfs, u, D + 1, C);
            }
        };
        Dfs(Dfs, 0); Sub.resize(N); Sub2.resize(N);
        Sz.resize(N); Sz2.resize(N);
    }
    void Mark(int v) {
        int x = v, z = -1;
        for (int i = Depth[v]; i >= 0; --i) {

```

```

    Sub[x] += Dist[v][i]; Sz[x]++;
    if (z != -1) {
        Sub2[z] += Dist[v][i];
        Sz2[z]++;
    }
    z = x; x = Parent[x];
}
}
int64_t Query(int v) {
    int64_t res = 0;
    int x = v, z = -1;
    for (int i = Depth[v]; i >= 0; --i) {
        res += Sub[x] + 1LL * Sz[x] * Dist[v][i];
        if (z != -1) res -= Sub2[z] + 1LL * Sz2[z] * Dist[v][i];
        z = x; x = Parent[x];
    }
    return res;
}
};

```

3.10 Tree Hashing

```

uint64_t hsah(int u, int f) {
    uint64_t r = 127;
    for (int v : G[u]) if (v != f) {
        uint64_t hh = hsah(v, u);
        r = (r + (hh * hh) % 1010101333) % 1011820613;
    }
    return r;
}

```

3.11 Minimum Mean Cycle

```

/* minimum mean cycle O(VE) */
struct MMC{
#define FZ(n) memset((n),0,sizeof(n))
#define E 101010
#define V 1021
#define inf 1e9
    struct Edge { int v,u; double c; };
    int n, m, prv[V][V], prve[V][V], vst[V];
    Edge e[E];
    vector<int> edgeID, cycle, rho;
    double d[V][V];
    void init( int _n ) { n = _n; m = 0; }
    // WARNING: TYPE matters
    void add_edge( int vi , int ui , double ci )
    { e[ m++ ] = { vi , ui , ci }; }
    void bellman_ford() {
        for(int i=0; i<n; i++) d[0][i]=0;
        for(int i=0; i<n; i++) {
            fill(d[i+1], d[i+1]+n, inf);
            for(int j=0; j<m; j++) {
                int v = e[j].v, u = e[j].u;
                if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
                    d[i+1][u] = d[i][v]+e[j].c;
                    prv[i+1][u] = v;
                    prve[i+1][u] = j;
                }
            }
        }
    }
    double solve(){
        // returns inf if no cycle, mmc otherwise
        double mmc=inf;
        int st = -1;
        bellman_ford();
        for(int i=0; i<n; i++) {
            double avg=-inf;
            for(int k=0; k<n; k++) {
                if(d[n][i]<inf-eps)
                    avg=max(avg,(d[n][i]-d[k][i])/(n-k));
                else avg=max(avg,inf);
            }
            if (avg < mmc) tie(mmc, st) = tie(avg, i);
        }
        FZ(vst);edgeID.clear();cycle.clear();rho.clear();
        for (int i=n; !vst[st]; st=prv[i--][st]) {
            vst[st]++;
            edgeID.PB(prve[i][st]);
            rho.PB(st);
        }
        while (vst[st] != 2) {
            int v = rho.back(); rho.pop_back();

```

```

            cycle.PB(v);
            vst[v]++;
        }
        reverse(ALL(edgeID));
        edgeID.resize(SZ(cycle));
        return mmc;
    }
} mmc;

```

3.12 Mo's Algorithm on Tree

```

int q; vector< int > G[N];
struct Que{
    int u, v, id;
} que[ N ];
int dfn[N], dfn_, block_id[N], block_, stk[N], stk_;
void dfs( int u, int f ) {
    dfn[ u ] = dfn_++; int saved_rbp = stk_;
    for ( int v : G[ u ] ) {
        if ( v == f ) continue;
        dfs( v, u );
        if ( stk_ - saved_rbp < SQRT_N ) continue;
        for ( ++block_ ; stk_ != saved_rbp ; )
            block_id[ stk_ -- ] = block_;
    }
    stk[ stk_ ++ ] = u;
}
bool inPath[ N ];
void Diff( int u ) {
    if ( inPath[ u ] ^ 1 ) { /*remove this edge*/ }
    else { /*add this edge*/ }
}
void traverse( int& origin_u, int u ) {
    for ( int g = lca( origin_u, u ) ;
        origin_u != g ; origin_u = parent_of[ origin_u ] )
        Diff( origin_u );
    for ( int v = u; v != origin_u; v = parent_of[v] )
        Diff( v );
    origin_u = u;
}
void solve() {
    dfs( 1, 1 );
    while ( stk_ ) block_id[ stk_ -- ] = block_;
    sort( que, que + q, [](const Que& x, const Que& y) {
        return tie( block_id[ x.u ], dfn[ x.v ] )
            < tie( block_id[ y.u ], dfn[ y.v ] );
    } );
    int U = 1, V = 1;
    for ( int i = 0 ; i < q ; ++ i ) {
        pass( U, que[ i ].u );
        pass( V, que[ i ].v );
        // we could get our answer of que[ i ].id
    }
}
/*
Method 2:
dfs u:
    push u
    iterate subtree
    push u
Let P = LCA(u, v), and St(u)<=St(v)
if (P == u) query[St(u), St(v)]
else query[Ed(u), St(v)], query[St(P), St(P)]
*/

```

3.13 Minimum Steiner Tree

```

// Minimum Steiner Tree
// O(V 3^AT + V^2 2^AT)
struct SteinerTree{
#define V 33
#define T 8
#define INF 1023456789
    int n, dst[V][V], dp[1 << T][V], tdst[V];
    void init( int _n ){
        n = _n;
        for( int i = 0 ; i < n ; i ++ ){
            for( int j = 0 ; j < n ; j ++ )
                dst[ i ][ j ] = INF;
            dst[ i ][ i ] = 0;
        }
    }
    void add_edge( int ui , int vi , int wi ){
        dst[ ui ][ vi ] = min( dst[ ui ][ vi ] , wi );
    }

```



```

    dst[ vi ][ ui ] = min( dst[ vi ][ ui ] , wi );
}
void shortest_path(){
    for( int k = 0 ; k < n ; k ++ )
        for( int i = 0 ; i < n ; i ++ )
            for( int j = 0 ; j < n ; j ++ )
                dst[ i ][ j ] = min( dst[ i ][ j ],
                    dst[ i ][ k ] + dst[ k ][ j ] );
}
int solve( const vector<int>& ter ){
    int t = (int)ter.size();
    for( int i = 0 ; i < ( 1 << t ) ; i ++ )
        for( int j = 0 ; j < n ; j ++ )
            dp[ i ][ j ] = INF;
    for( int i = 0 ; i < n ; i ++ )
        dp[ 0 ][ i ] = 0;
    for( int msk = 1 ; msk < ( 1 << t ) ; msk ++ ){
        if( msk == ( msk & (-msk) ) ){
            int who = __lg( msk );
            for( int i = 0 ; i < n ; i ++ )
                dp[ msk ][ i ] = dst[ ter[ who ] ][ i ];
            continue;
        }
        for( int i = 0 ; i < n ; i ++ )
            for( int submsk = ( msk - 1 ) & msk ; submsk ;
                submsk = ( submsk - 1 ) & msk )
                dp[ msk ][ i ] = min( dp[ msk ][ i ],
                    dp[ submsk ][ i ] +
                    dp[ msk ^ submsk ][ i ] );
        for( int i = 0 ; i < n ; i ++ ){
            tdst[ i ] = INF;
            for( int j = 0 ; j < n ; j ++ )
                tdst[ i ] = min( tdst[ i ],
                    dp[ msk ][ j ] + dst[ j ][ i ] );
        }
        for( int i = 0 ; i < n ; i ++ )
            dp[ msk ][ i ] = tdst[ i ];
    }
    int ans = INF;
    for( int i = 0 ; i < n ; i ++ )
        ans = min( ans , dp[ ( 1 << t ) - 1 ][ i ] );
    return ans;
}
} solver;

```

3.14 Directed Minimum Spanning Tree

```

template <typename T> struct DMST {
    T g[maxn][maxn], fw[maxn];
    int n, fr[maxn];
    bool vis[maxn], inc[maxn];
    void clear() {
        for(int i = 0; i < maxn; ++i) {
            for(int j = 0; j < maxn; ++j) g[i][j] = inf;
            vis[i] = inc[i] = false;
        }
    }
    void addEdge(int u, int v, T w){g[u][v]=min(g[u][v],w);}
    T operator()(int root, int _n) {
        n = _n; T ans = 0;
        if (dfs(root) != n) return -1;
        while (true) {
            for(int i = 1; i <= n; ++i) fw[i] = inf, fr[i] = i;
            for (int i = 1; i <= n; ++i) if (!inc[i]) {
                for (int j = 1; j <= n; ++j) {
                    if (!inc[j] && i != j && g[j][i] < fw[i]) {
                        fw[i] = g[j][i]; fr[i] = j;
                    }
                }
            }
            int x = -1;
            for(int i = 1; i <= n; ++i) if(i != root && !inc[i]){
                int j = i, c = 0;
                while(j != root && fr[j] != i && c <= n) ++c, j = fr[j];
                if (j == root || c > n) continue;
                else { x = i; break; }
            }
            if (!x) {
                for (int i = 1; i <= n; ++i)
                    if (i != root && !inc[i]) ans += fw[i];
                return ans;
            }
            int y = x;

```

```

            for (int i = 1; i <= n; ++i) vis[i] = false;
            do {
                ans += fw[y]; y = fr[y]; vis[y] = inc[y] = true;
            } while (y != x);
            inc[x] = false;
            for (int k = 1; k <= n; ++k) if (vis[k]) {
                for (int j = 1; j <= n; ++j) if (!vis[j]) {
                    if (g[x][j] > g[k][j]) g[x][j] = g[k][j];
                    if (g[j][k] < inf && g[j][k] - fw[k] < g[j][x])
                        g[j][x] = g[j][k] - fw[k];
                }
            }
        }
        return ans;
    }
    int dfs(int now) {
        int r = 1; vis[now] = true;
        for (int i = 1; i <= n; ++i)
            if (g[now][i] < inf && !vis[i]) r += dfs(i);
        return r;
    }
};

```

3.15 Dominator Tree

```

namespace dominator {
    vector<int> g[maxn], r[maxn], rdom[maxn];
    int dfn[maxn], rev[maxn], fa[maxn], sdom[maxn];
    int dom[maxn], val[maxn], rp[maxn], tk;
    void init(int n) {
        // vertices are numbered from 0 to n - 1
        fill(dfn, dfn + n, -1); fill(rev, rev + n, -1);
        fill(fa, fa + n, -1); fill(val, val + n, -1);
        fill(sdom, sdom + n, -1); fill(rp, rp + n, -1);
        fill(dom, dom + n, -1); tk = 0;
        for (int i = 0; i < n; ++i) {
            g[i].clear(); r[i].clear(); rdom[i].clear();
        }
    }
    void add_edge(int x, int y) { g[x].push_back(y); }
    void dfs(int x) {
        rev[dfn[x] = tk] = x;
        fa[tk] = sdom[tk] = val[tk] = tk; tk ++;
        for (int u : g[x]) {
            if (dfn[u] == -1) dfs(u), rp[dfn[u]] = dfn[x];
            r[dfn[u]].push_back(dfn[x]);
        }
    }
    void merge(int x, int y) { fa[x] = y; }
    int find(int x, int c = 0) {
        if (fa[x] == x) return c ? -1 : x;
        int p = find(fa[x], 1);
        if (p == -1) return c ? fa[x] : val[x];
        if (sdom[val[x]] > sdom[val[fa[x]]]) val[x] = val[fa[x]];
        fa[x] = p;
        return c ? p : val[x];
    }
    vector<int> build(int s, int n) {
        // return the father of each node in the dominator tree
        // p[i] = -2 if i is unreachable from s
        dfs(s);
        for (int i = tk - 1; i >= 0; --i) {
            for (int u : r[i]) sdom[i] = min(sdom[i], sdom[find(u)]);
            if (i) rdom[sdom[i]].push_back(i);
            for (int &u : rdom[i]) {
                int p = find(u);
                if (sdom[p] == i) dom[u] = i;
                else dom[u] = p;
            }
            if (i) merge(i, rp[i]);
        }
        vector<int> p(n, -2); p[s] = -1;
        for (int i = 1; i < tk; ++i)
            if (sdom[i] != dom[i]) dom[i] = dom[dom[i]];
        for (int i = 1; i < tk; ++i) p[rev[i]] = rev[dom[i]];
        return p;
    }
}

```

4 Matching & Flow

4.1 Kuhn Munkres

```

class KM {

```

```
private:
static constexpr lld INF = 1LL << 60;
vector<lld> hl, hr, slk;
vector<int> fl, fr, pre, qu;
vector<vector<lld>> w;
vector<bool> vl, vr;
int n, ql, qr;
bool check(int x) {
    if (vl[x] = true, fl[x] != -1)
        return vr[qu[qr++] = fl[x]] = true;
    while (x != -1) swap(x, fr[fl[x] = pre[x]]);
    return false;
}
void bfs(int s) {
    fill(slk.begin(), slk.end(), INF);
    fill(vl.begin(), vl.end(), false);
    fill(vr.begin(), vr.end(), false);
    ql = qr = 0;
    qu[qr++] = s;
    vr[s] = true;
    while (true) {
        lld d;
        while (ql < qr) {
            for (int x = 0, y = qu[ql++]; x < n; ++x) {
                if (!vl[x] && slk[x] >= (d = hl[x] + hr[y] - w[x][y])) {
                    if (pre[x] = y, d) slk[x] = d;
                    else if (!check(x)) return;
                }
            }
            d = INF;
            for (int x = 0; x < n; ++x)
                if (!vl[x] && d > slk[x]) d = slk[x];
            for (int x = 0; x < n; ++x) {
                if (vl[x]) hl[x] += d;
                else slk[x] -= d;
                if (vr[x]) hr[x] -= d;
            }
            for (int x = 0; x < n; ++x)
                if (!vl[x] && !slk[x] && !check(x)) return;
        }
    }
}
public:
void init(int n_) {
    n = n_; qu.resize(n);
    fl.clear(); fl.resize(n, -1);
    fr.clear(); fr.resize(n, -1);
    hr.clear(); hr.resize(n); hl.resize(n);
    w.clear(); w.resize(n, vector<lld>(n));
    slk.resize(n); pre.resize(n);
    vl.resize(n); vr.resize(n);
}
void set_edge(int u, int v, lld x) {w[u][v] = x;}
lld solve() {
    for (int i = 0; i < n; ++i)
        hl[i] = *max_element(w[i].begin(), w[i].end());
    for (int i = 0; i < n; ++i) bfs(i);
    lld res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
}
} km;
```

4.2 Bipartite Matching

```
class BipartiteMatching{
private:
vector<int> X[N], Y[N];
int fX[N], fY[N], n;
bitset<N> walked;
bool dfs(int x){
    for(auto i:X[x]){
        if(walked[i])continue;
        walked[i]=1;
        if(fY[i]==-1||dfs(fY[i])){
            fY[i]=x;fX[x]=i;
            return 1;
        }
    }
    return 0;
}
public:
void init(int _n){
```

```
n=_n; walked.reset();
for(int i=0;i<n;i++){
    X[i].clear();Y[i].clear();
    fX[i]=fY[i]=-1;
}
}
void add_edge(int x, int y){
    X[x].push_back(y); Y[y].push_back(x);
}
int solve(){
    int cnt = 0;
    for(int i=0;i<n;i++){
        walked.reset();
        if(dfs(i)) cnt++;
    }
    // return how many pair matched
    return cnt;
}
};
```

4.3 General Graph Matching

```
const int N = 514, E = (2e5) * 2;
struct Graph{
    int to[E], bro[E], head[N], e;
    int lnk[N], vis[N], stp, n;
    void init(int _n){
        stp = 0; e = 1; n = _n;
        for(int i = 0; i <= n; i++)
            head[i] = lnk[i] = vis[i] = 0;
    }
    void add_edge(int u, int v){
        // 1-base
        to[e]=v, bro[e]=head[u], head[u]=e++;
        to[e]=u, bro[e]=head[v], head[v]=e++;
    }
    bool dfs(int x){
        vis[x]=stp;
        for(int i=head[x]; i; i=bro[i]){
            int v=to[i];
            if(!lnk[v]){
                lnk[x]=v, lnk[v]=x;
                return true;
            }else if(vis[lnk[v]]<stp){
                int w=lnk[v];
                lnk[x]=v, lnk[v]=x, lnk[w]=0;
                if(dfs(w)) return true;
                lnk[w]=v, lnk[v]=w, lnk[x]=0;
            }
        }
        return false;
    }
    int solve(){
        int ans = 0;
        for(int i=1; i<=n; i++)
            if(!lnk[i]){
                stp++; ans += dfs(i);
            }
        return ans;
    }
} graph;
```

4.4 Minimum Weight Matching (Clique version)

```
struct Graph {
    // 0-base (Perfect Match)
    int n, edge[MXN][MXN];
    int match[MXN], dis[MXN], onstk[MXN];
    vector<int> stk;
    void init(int _n) {
        n = _n;
        for (int i=0; i<n; i++)
            for (int j=0; j<n; j++)
                edge[i][j] = 0;
    }
    void set_edge(int u, int v, int w) {
        edge[u][v] = edge[v][u] = w;
    }
    bool SPFA(int u){
        if (onstk[u]) return true;
        stk.PB(u);
        onstk[u] = 1;
        for (int v=0; v<n; v++){
            if (u != v && match[u] != v && !onstk[v]){
```

```

    int m = match[v];
    if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
        dis[m] = dis[u] - edge[v][m] + edge[u][v];
        onstk[v] = 1;
        stk.PB(v);
        if (SPFA(m)) return true;
        stk.pop_back();
        onstk[v] = 0;
    }
}
onstk[u] = 0;
stk.pop_back();
return false;
}

int solve() {
    // find a match
    for (int i=0; i<n; i+=2){
        match[i] = i+1;
        match[i+1] = i;
    }
    while (true){
        int found = 0;
        for (int i=0; i<n; i++){
            dis[i] = onstk[i] = 0;
        }
        for (int i=0; i<n; i++){
            stk.clear();
            if (!onstk[i] && SPFA(i)){
                found = 1;
                while (SZ(stk)>=2){
                    int u = stk.back(); stk.pop_back();
                    int v = stk.back(); stk.pop_back();
                    match[u] = v;
                    match[v] = u;
                }
            }
        }
        if (!found) break;
    }
    int ret = 0;
    for (int i=0; i<n; i++){
        ret += edge[i][match[i]];
    }
    return ret>>1;
}
} graph;

```

4.5 Minimum Cost Circulation

```

struct Edge { int to, cap, rev, cost; };
vector<Edge> g[kN];
int dist[kN], pv[kN], ed[kN];
bool mark[kN];
int NegativeCycle(int n) {
    memset(mark, false, sizeof(mark));
    memset(dist, 0, sizeof(dist));
    int upd = -1;
    for (int i = 0; i <= n; ++i) {
        for (int j = 0; j < n; ++j) {
            int idx = 0;
            for (auto &e : g[j]) {
                if (e.cap > 0 && dist[e.to] > dist[j] + e.cost){
                    dist[e.to] = dist[j] + e.cost;
                    pv[e.to] = j, ed[e.to] = idx;
                    if (i == n) {
                        upd = j;
                        while (!mark[upd]) mark[upd]=1, upd=pv[upd];
                        return upd;
                    }
                }
            }
            idx++;
        }
    }
    return -1;
}

int Solve(int n) {
    int rt = -1, ans = 0;
    while ((rt = NegativeCycle(n)) >= 0) {
        memset(mark, false, sizeof(mark));
        vector<pair<int, int>> cyc;
        while (!mark[rt]) {
            cyc.emplace_back(pv[rt], ed[rt]);

```

```

            mark[rt] = true;
            rt = pv[rt];
        }
        reverse(cyc.begin(), cyc.end());
        int cap = kInf;
        for (auto &i : cyc) {
            auto &e = g[i.first][i.second];
            cap = min(cap, e.cap);
        }
        for (auto &i : cyc) {
            auto &e = g[i.first][i.second];
            e.cap -= cap;
            g[e.to][e.rev].cap += cap;
            ans += e.cost * cap;
        }
    }
    return ans;
}

```

4.6 Flow Models

- Maximum/Minimum flow with lower bound / Circulation problem
 - Construct super source S and sink T .
 - For each edge (x, y, l, u) , connect $x \rightarrow y$ with capacity $u - l$.
 - For each vertex v , denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
 - If $in(v) > 0$, connect $S \rightarrow v$ with capacity $in(v)$, otherwise, connect $v \rightarrow T$ with capacity $-in(v)$.
 - To maximize, connect $t \rightarrow s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T . If $f \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
 - To minimize, let f be the maximum flow from S to T . Connect $t \rightarrow s$ with capacity ∞ and let the flow from S to T be f' . If $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, f' is the answer.
 - The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
 - Redirect every edge: $y \rightarrow x$ if $(x, y) \in M$, $x \rightarrow y$ otherwise.
 - DFS from unmatched vertices in X .
 - $x \in X$ is chosen iff x is unvisited.
 - $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
 - Construct super source S and sink T
 - For each edge (x, y, c) , connect $x \rightarrow y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \rightarrow x$ with $(cost, cap) = (-c, 1)$
 - For each edge with $c < 0$, sum these cost as K , then increase $d(y)$ by 1, decrease $d(x)$ by 1
 - For each vertex v with $d(v) > 0$, connect $S \rightarrow v$ with $(cost, cap) = (0, d(v))$
 - For each vertex v with $d(v) < 0$, connect $v \rightarrow T$ with $(cost, cap) = (0, -d(v))$
 - Flow from S to T , the answer is the cost of the flow $C + K$
- Maximum density induced subgraph
 - Binary search on answer, suppose we're checking answer T
 - Construct a max flow model, let K be the sum of all weights
 - Connect source $s \rightarrow v, v \in G$ with capacity K
 - For each edge (u, v, w) in G , connect $u \rightarrow v$ and $v \rightarrow u$ with capacity w
 - For $v \in G$, connect it with sink $v \rightarrow t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
 - T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
 - For each $v \in V$ create a copy v' , and connect $u' \rightarrow v'$ with weight $w(u, v)$.
 - Connect $v \rightarrow v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v .
 - Find the minimum weight perfect matching on G' .
- Project selection problem
 - If $p_v > 0$, create edge (s, v) with capacity p_v ; otherwise, create edge (v, t) with capacity $-p_v$.
 - Create edge (u, v) with capacity w with w being the cost of choosing u without choosing v .
 - The mincut is equivalent to the maximum profit of a subset of projects.
- 0/1 quadratic programming

$$\sum_x c_x x + \sum_y c_{xy} \bar{y} + \sum_{xy} c_{xy} x \bar{y} + \sum_{xyx'y'} c_{xyx'y'} (x \bar{y} + x' \bar{y}')$$

can be minimized by the mincut of the following graph:

 - Create edge (x, t) with capacity c_x and create edge (s, y) with capacity c_y .
 - Create edge (x, y) with capacity c_{xy} .
 - Create edge (x, y) and edge (x', y') with capacity $c_{xyx'y'}$.

4.7 Dinic

```
template <typename Cap = int64_t>
class Dinic{
private:
    struct Edge{
        int to, rev;
        Cap cap;
    };
    int n, st, ed;
    vector<vector<Edge>> G;
    vector<int> lv, idx;
    bool BFS(){
        fill(lv.begin(), lv.end(), -1);
        queue<int> bfs;
        bfs.push(st); lv[st] = 0;
        while(!bfs.empty()){
            int u = bfs.front(); bfs.pop();
            for(auto e: G[u]){
                if(e.cap <= 0 || lv[e.to] != -1) continue;
                bfs.push(e.to); lv[e.to] = lv[u] + 1;
            }
        }
        return (lv[ed] != -1);
    }
    Cap DFS(int u, Cap f){
        if(u == ed) return f;
        Cap ret = 0;
        for(int &i = idx[u]; i < (int)G[u].size(); ++i){
            auto &e = G[u][i];
            if(e.cap <= 0 || lv[e.to] != lv[u] + 1) continue;
            Cap nf = DFS(e.to, min(f, e.cap));
            ret += nf; e.cap -= nf; f -= nf;
            G[e.to][e.rev].cap += nf;
            if(f == 0) return ret;
        }
        if(ret == 0) lv[u] = -1;
        return ret;
    }
public:
    void init(int n_, int st_, int ed_){
        n = n_, st = st_, ed = ed_;
        G.resize(n); lv.resize(n);
        fill(G.begin(), G.end(), vector<Edge>());
    }
    void add_edge(int u, int v, Cap c){
        G[u].push_back({v, (int)G[v].size(), c});
        G[v].push_back({u, ((int)G[u].size())-1, 0});
    }
    Cap max_flow(){
        Cap ret = 0;
        while(BFS()){
            idx.assign(n, 0);
            Cap f = DFS(st, numeric_limits<Cap>::max());
            ret += f;
            if(f == 0) break;
        }
        return ret;
    }
};
```

4.8 Minimum Cost Maximum Flow

```
class MiniCostMaxiFlow{
    using Cap = int; using Wei = int64_t;
    using PCW = pair<Cap, Wei>;
    static constexpr Cap INF_CAP = 1 << 30;
    static constexpr Wei INF_WEI = 1LL << 60;
private:
    struct Edge{
        int to, back;
        Cap cap; Wei wei;
        Edge() {}
        Edge(int a, int b, Cap c, Wei d):
            to(a), back(b), cap(c), wei(d) {}
    };
    int ori, edd;
    vector<vector<Edge>> G;
    vector<int> fa, wh;
    vector<bool> inq;
    vector<Wei> dis;
    PCW SPFA(){

```

```
        fill(inq.begin(), inq.end(), false);
        fill(dis.begin(), dis.end(), INF_WEI);
        queue<int> qq; qq.push(ori);
        dis[ori] = 0;
        while(!qq.empty()){
            int u = qq.front(); qq.pop();
            inq[u] = 0;
            for(int i = 0; i < SZ(G[u]); ++i){
                Edge e = G[u][i];
                int v = e.to;
                Wei d = e.wei;
                if(e.cap <= 0 || dis[v] <= dis[u] + d)
                    continue;
                dis[v] = dis[u] + d;
                fa[v] = u, wh[v] = i;
                if(inq[v]) continue;
                qq.push(v);
                inq[v] = 1;
            }
        }
        if(dis[edd] == INF_WEI) return {-1, -1};
        Cap mw = INF_CAP;
        for(int i = edd; i != ori; i = fa[i])
            mw = min(mw, G[fa[i]][wh[i]].cap);
        for(int i = edd; i != ori; i = fa[i]){
            auto &eg = G[fa[i]][wh[i]];
            eg.cap -= mw;
            G[eg.to][eg.back].cap += mw;
        }
        return {mw, dis[edd]};
    }
public:
    void init(int a, int b, int n){
        ori = a, edd = b;
        G.clear(); G.resize(n);
        fa.resize(n); wh.resize(n);
        inq.resize(n); dis.resize(n);
    }
    void add_edge(int st, int ed, Cap c, Wei w){
        G[st].emplace_back(ed, SZ(G[ed]), c, w);
        G[ed].emplace_back(st, SZ(G[st]) - 1, 0, -w);
    }
    PCW solve(){
        /* might modify to
        cc += ret.first * ret.second
        or
        ww += ret.first * ret.second
        */
        Cap cc = 0; Wei ww = 0;
        while(true){
            PCW ret = SPFA();
            if(ret.first == -1) break;
            cc += ret.first;
            ww += ret.second;
        }
        return {cc, ww};
    }
} mcmf;
```

4.9 Global Min-Cut

```
const int maxn = 500 + 5;
int w[maxn][maxn], g[maxn];
bool v[maxn], del[maxn];
void add_edge(int x, int y, int c) {
    w[x][y] += c; w[y][x] += c;
}
pair<int, int> phase(int n) {
    memset(v, false, sizeof(v));
    memset(g, 0, sizeof(g));
    int s = -1, t = -1;
    while (true) {
        int c = -1;
        for (int i = 0; i < n; ++i) {
            if (del[i] || v[i]) continue;
            if (c == -1 || g[i] > g[c]) c = i;
        }
        if (c == -1) break;
        v[s = t, t = c] = true;
        for (int i = 0; i < n; ++i) {
            if (del[i] || v[i]) continue;
            g[i] += w[c][i];
        }
    }
}
```

```

}
return make_pair(s, t);
}
int mincut(int n) {
    int cut = 1e9;
    memset(del, false, sizeof(del));
    for (int i = 0; i < n - 1; ++i) {
        int s, t; tie(s, t) = phase(n);
        del[t] = true; cut = min(cut, g[t]);
        for (int j = 0; j < n; ++j) {
            w[s][j] += w[t][j]; w[j][s] += w[j][t];
        }
    }
    return cut;
}

```

5 Math

5.1 Prime Table

```

1002939109, 1020288887, 1028798297, 1038684299,
1041211027, 1051762951, 1058585963, 1063020809,
1147930723, 1172520109, 1183835981, 1187659051,
1241251303, 1247184097, 1255940849, 1272759031,
1287027493, 1288511629, 1294632499, 1312650799,
1868732623, 1884198443, 1884616807, 1885059541,
1909942399, 1914471137, 1923951707, 1925453197,
1979612177, 1980446837, 1989761941, 2007826547,
2008033571, 2011186739, 2039465081, 2039728567,
2093735719, 2116097521, 2123852629, 2140170259,
3148478261, 3153064147, 3176351071, 3187523093,
3196772239, 3201312913, 3203063977, 3204840059,
3210224309, 3213032591, 3217689851, 3218469083,
3219857533, 3231880427, 3235951699, 3273767923,
3276188869, 3277183181, 3282463507, 3285553889,
3319309027, 3327005333, 3327574903, 3341387953,
3373293941, 3380077549, 3380892997, 3381118801

```

5.2 $\lfloor \frac{n}{i} \rfloor$ Enumeration

$T_0 = 1, T_{i+1} = \lfloor \frac{n}{T_i + 1} \rfloor$

5.3 ax+by=gcd

```

// ax+ny = 1, ax+ny == ax == 1 (mod n)
void exgcd(lld x, lld y, lld &g, lld &a, lld &b) {
    if (y == 0) g=x, a=1, b=0;
    else exgcd(y, x%y, g, b, a), b--=(x/y)*a;
}

```

5.4 Pollard Rho

```

// does not work when n is prime
// return any non-trivial factor
llu pollard_rho(llu n){
    static auto f=[](llu x, llu k, llu m){
        return add(k, mul(x, x, m), m);
    };
    if (!(n&1)) return 2;
    mt19937 rnd(120821011);
    while(true){
        llu y=2, yy=y, x=rnd()%n, t=1;
        for(llu sz=2; t==1; sz<=1) {
            for(llu i=0; i<sz; ++i){
                if(t!=1)break;
                yy=f(yy, x, n);
                t=gcd(yy>y?yy-y:y-yy, n);
            }
            y=yy;
        }
        if(t!=1&&t!=n) return t;
    }
}

```

5.5 Pi Count (Linear Sieve)

```

static constexpr int N = 1000000 + 5;
lld pi[N];
vector<int> primes;
bool sieved[N];
lld cube_root(lld x){
    lld s=cbrt(x-static_cast<long double>(0.1));
    while(s*s*s <= x) ++s;
    return s-1;
}
lld square_root(lld x){
    lld s=sqrt(x-static_cast<long double>(0.1));
    while(s*s <= x) ++s;
    return s-1;
}

```

```

void init(){
    primes.reserve(N);
    primes.push_back(1);
    for(int i=2; i<N; i++) {
        if(!sieved[i]) primes.push_back(i);
        pi[i] = !sieved[i] + pi[i-1];
        for(int p: primes) if(p > 1) {
            if(p * i >= N) break;
            sieved[p * i] = true;
            if(p % i == 0) break;
        }
    }
}

lld phi(lld m, lld n) {
    static constexpr int MM = 80000, NN = 500;
    static lld val[MM][NN];
    if(m<MM&&n<NN&&val[m][n])return val[m][n]-1;
    if(n == 0) return m;
    if(primes[n] >= m) return 1;
    lld ret = phi(m, n-1)-phi(m/primes[n], n-1);
    if(m<MM&&n<NN) val[m][n] = ret+1;
    return ret;
}

lld pi_count(lld);
lld P2(lld m, lld n) {
    lld sm = square_root(m), ret = 0;
    for(lld i = n+1; primes[i]<=sm; i++)
        ret+=pi_count(m/primes[i])-pi_count(primes[i])+1;
    return ret;
}

lld pi_count(lld m) {
    if(m < N) return pi[m];
    lld n = pi_count(cube_root(m));
    return phi(m, n) + n - 1 - P2(m, n);
}

```

5.6 Range Sieve

```

const int MAX_SQRT_B = 50000;
const int MAX_L = 200000 + 5;

bool is_prime_small[MAX_SQRT_B];
bool is_prime[MAX_L];

void sieve(lld l, lld r){
    // [l, r)
    for(lld i=2; i<r; i++) is_prime_small[i] = true;
    for(lld i=1; i<r; i++) is_prime[i-1] = true;
    if(l==1) is_prime[0] = false;
    for(lld i=2; i<r; i++){
        if(!is_prime_small[i]) continue;
        for(lld j=i*i; j<r; j+=i) is_prime_small[j]=false;
        for(lld j=std::max(2LL, (l+i-1)/i)*i; j<r; j+=i)
            is_prime[j-1]=false;
    }
}

```

5.7 Miller Rabin

```

bool isprime(llu x){
    static llu magic[]={2, 325, 9375, 28178, \
        450775, 9780504, 1795265022};
    static auto witn=[](llu a, llu u, llu n, int t)
    ->bool{
        if (!(a = mpow(a%n, u, n)))return 0;
        while(t--){
            llu a2=mul(a, a, n);
            if(a2==1 && a!=1 && a!=n-1)
                return 1;
            a = a2;
        }
        return a!=1;
    };
    if(x<2)return 0;
    if(!(x&1))return x==2;
    llu x1=x-1; int t=0;
    while(!(x1&1))x1>>=1, t++;
    for(llu m:magic)if(witn(m, x1, x, t))return 0;
    return 1;
}

```

5.8 Inverse Element


```
// x's inverse mod k
long long GetInv(long long x, long long k){
    // k is prime: euler_(k)=k-1
    return qPow(x, euler_phi(k)-1);
}
// if you need [1, x] (most use: [1, k-1])
void solve(int x, long long k){
    inv[1] = 1;
    for(int i=2;i<x;i++){
        inv[i] = ((long long)(k - k/i) * inv[k % i]) % k;
    }
}
```

5.9 Euler Phi Function

```
/*
    extended euler:
    a^b mod p
    if gcd(a, p)==1: a^(b%phi(p))
    elif b < phi(p): a^b mod p
    else a^(b%phi(p) + phi(p))
*/
lld euler_phi(int x){
    lld r=1;
    for(int i=2;i*i<=x;++i){
        if(x%i==0){
            x/=i; r*=(i-1);
            while(x%i==0){
                x/=i; r*=i;
            }
        }
    }
    if(x>1) r*=x-1;
    return r;
}
vector<int> primes;
bool notprime[N];
lld phi[N];
void euler_sieve(int n){
    for(int i=2;i<n;i++){
        if(!notprime[i]){
            primes.push_back(i); phi[i] = i-1;
        }
        for(auto j: primes){
            if(i*j >= n) break;
            notprime[i*j] = true;
            phi[i*j] = phi[i] * phi[j];
            if(i % j == 0){
                phi[i*j] = phi[i] * j;
                break;
            }
        }
    }
}
```

5.10 Gauss Elimination

```
void gauss(vector<vector<double>> &d) {
    int n = d.size(), m = d[0].size();
    for (int i = 0; i < m; ++i) {
        int p = -1;
        for (int j = i; j < n; ++j) {
            if (fabs(d[j][i]) < eps) continue;
            if (p == -1 || fabs(d[j][i]) > fabs(d[p][i])) p=j;
        }
        if (p == -1) continue;
        for (int j = 0; j < m; ++j) swap(d[p][j], d[i][j]);
        for (int j = 0; j < n; ++j) {
            if (i == j) continue;
            double z = d[j][i] / d[i][i];
            for (int k = 0; k < m; ++k) d[j][k] -= z*d[i][k];
        }
    }
}
```

5.11 Fast Fourier Transform

```
/*
    polynomial multiply:
    DFT(a, len); DFT(b, len);
    for(int i=0;i<len;i++) c[i] = a[i]*b[i];
    iDFT(c, len);
    (len must be 2^k and >= 2*(max(a, b)))
    Hand written Cplx would be 2x faster
*/
```

```
Cplx omega[2][N];
void init_omega(int n) {
    static constexpr llf PI=acos(-1);
    const llf arg=(PI+PI)/n;
    for(int i=0;i<n;++i)
        omega[0][i]={cos(arg*i),sin(arg*i)};
    for(int i=0;i<n;++i)
        omega[1][i]=conj(omega[0][i]);
}
void tran(Cplx arr[],int n,Cplx omg[]) {
    for(int i=0,j=0;i<n;++i){
        if(i>j)swap(arr[i],arr[j]);
        for(int l=n>>1;(j^=1)<l;l>=1);
    }
    for (int l=2;l<=n;l<=1){
        int m=l>>1;
        for(auto p=arr;p!=arr+n;p+=l){
            for(int i=0;i<m;++i){
                Cplx t=omg[n/l*i]*p[m+i];
                p[m+i]=p[i]-t; p[i]+=t;
            }
        }
    }
}
void DFT(Cplx arr[],int n){tran(arr,n,omega[0]);}
void iDFT(Cplx arr[],int n){
    tran(arr,n,omega[1]);
    for(int i=0;i<n;++i) arr[i]/=n;
}
```

5.12 Chinese Remainder

```
lld crt(lld ans[], lld pri[], int n){
    lld M = 1, ret = 0;
    for(int i=0;i<n;i++) M *= pri[i];
    for(int i=0;i<n;i++){
        lld iv = (gcd(M/pri[i],pri[i]).FF+pri[i])%pri[i];
        ret += (ans[i]*(M/pri[i])%M * iv)%M;
        ret %= M;
    }
    return ret;
}
/*
    Another:
    x = a1 % m1
    x = a2 % m2
    g = gcd(m1, m2)
    assert((a1-a2)%g==0)
    [p, q] = exgcd(m2/g, m1/g)
    return a2+m2*(p*(a1-a2)/g)
    0 <= x < lcm(m1, m2)
*/
```

5.13 Berlekamp Massey

```
// x: 1-base, p[]: 0-base
template<size_t N>
vector<llf> BM(llf x[N],size_t n){
    size_t f[N]={0},t=0;llf d[N];
    vector<llf> p[N];
    for(size_t i=1,b=0;i<=n;++i) {
        for(size_t j=0;j<p[t].size();++j)
            d[i]=x[i-j-1]*p[t][j];
        if(abs(d[i]-x[i])<=EPS)continue;
        f[t]=i;if(!t){p[++t].resize(i);continue;}
        vector<llf> cur(i-f[b]-1);
        llf k=-d[i]/d[f[b]];cur.PB(-k);
        for(size_t j=0;j<p[b].size();j++){
            cur.PB(p[b][j]*k);
            if(cur.size()<p[t].size())cur.resize(p[t].size());
            for(size_t j=0;j<p[t].size();j++)cur[j]+=p[t][j];
            if(i-f[b]+p[b].size()>=p[t].size()) b=t;
            p[++t]=cur;
        }
        return p[t];
    }
}
```

5.14 NTT

```
// Remember coefficient are mod P
/* p=a*2^n+1
n  2^n      p      a      root
16  65536   65537   1      3
20  1048576 7340033 7      3 */
```

```
// (must be 2^k)
template<LL P, LL root, int MAXN>
struct NTT{
    static LL bigmod(LL a, LL b) {
        LL res = 1;
        for (LL bs = a; b; b >= 1, bs = (bs * bs) % P)
            if(b&1) res=(res*bs)%P;
        return res;
    }
    static LL inv(LL a, LL b) {
        if(a==1) return 1;
        return (((LL)(a-inv(b%a,a))*b+1)/a)%b;
    }
    LL omega[MAXN+1];
    NTT() {
        omega[0] = 1;
        LL r = bigmod(root, (P-1)/MAXN);
        for (int i=1; i<=MAXN; i++)
            omega[i] = (omega[i-1]*r)%P;
    }
    // n must be 2^k, and 0 <= a[i] < P
    void tran(int n, LL a[], bool inv_ntt=false){
        int basic = MAXN / n, theta = basic;
        for (int m = n; m >= 2; m >= 1) {
            int mh = m >> 1;
            for (int i = 0; i < mh; i++) {
                LL w = omega[i*theta*MAXN];
                for (int j = i; j < n; j += m) {
                    int k = j + mh;
                    LL x = a[j] - a[k];
                    if (x < 0) x += P;
                    a[j] += a[k];
                    if (a[j] > P) a[j] -= P;
                    a[k] = (w * x) % P;
                }
            }
            theta = (theta * 2) % MAXN;
        }
        int i = 0;
        for (int j = 1; j < n - 1; j++) {
            for (int k = n >> 1; k > (i ^ k); k >= 1);
            if (j < i) swap(a[i], a[j]);
        }
        if (inv_ntt) {
            LL ni = inv(n,P);
            reverse(a+1, a+n);
            for (i = 0; i < n; i++)
                a[i] = (a[i] * ni) % P;
        }
    }
};
const LL P=2013265921, root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;
```

5.15 Polynomial Operations

```
using VL = vector<LL>;
#define fi(s, n) for (int i=int(s); i<int(n); ++i)
#define Fi(s, n) for (int i=int(n); i>int(s); --i)
int n2k(int n) {
    int sz = 1; while (sz < n) sz <= 1;
    return sz;
}
template<int MAXN, LL P, LL RT> // MAXN = 2^k
struct Poly { // coefficients in [0, P)
    static NTT<MAXN, P, RT> ntt;
    VL coef;
    int n() const { return coef.size(); } // n()>=1
    LL *data() { return coef.data(); }
    const LL *data() const { return coef.data(); }
    LL &operator[](size_t i) { return coef[i]; }
    const LL &operator[](size_t i) const { return coef[i]; }
    Poly(initializer_list<LL> a) : coef(a) {}
    explicit Poly(int _n = 1) : coef(_n) {}
    Poly(const LL *arr, int _n) : coef(arr, arr + _n) {}
    Poly(const Poly &p, int _n) : coef(_n) {
        copy_n(p.data(), min(p.n(), _n), data());
    }
    Poly& irev(){return reverse(data(), data()+n()), *this;}
    Poly& isz(int _n) { return coef.resize(_n), *this; }
    Poly& iadd(const Poly &rhs) { // n() == rhs.n()
        fi(0, n()) if((coef[i]+=rhs[i]) >= P)coef[i]-=P;
```

```
        return *this;
    }
    Poly& imul(LL k) {
        fi(0, n()) coef[i] = coef[i] * k % P;
        return *this;
    }
    Poly Mul(const Poly &rhs) const {
        const int _n = n2k(n() + rhs.n() - 1);
        Poly X(*this, _n), Y(rhs, _n);
        ntt(X.data(), _n), ntt(Y.data(), _n);
        fi(0, _n) X[i] = X[i] * Y[i] % P;
        ntt(X.data(), _n, true);
        return X.isz(n() + rhs.n() - 1);
    }
    Poly Inv() const { // coef[0] != 0
        if (n() == 1) return {ntt.minv(coef[0])};
        const int _n = n2k(n() * 2);
        Poly Xi = Poly(*this, (n() + 1)/2).Inv().isz(_n);
        Poly Y(*this, _n);
        ntt(Xi.data(), _n), ntt(Y.data(), _n);
        fi(0, _n) {
            Xi[i] *= (2 - Xi[i] * Y[i]) % P;
            if ((Xi[i] % P) < 0) Xi[i] += P;
        }
        ntt(Xi.data(), _n, true);
        return Xi.isz(n());
    }
    Poly Sqrt() const { // Jacobi(coef[0], P) = 1
        if (n()==1) return {QuadraticResidue(coef[0], P)};
        Poly X = Poly(*this, (n()+1) / 2).Sqrt().isz(n());
        return X.iadd(Mul(X.Inv()).isz(n())).imul(P/2+1);
    }
    pair<Poly, Poly> DivMod(const Poly &rhs) const {
        // (rhs).back() != 0
        if (n() < rhs.n()) return {{0}, *this};
        const int _n = n() - rhs.n() + 1;
        Poly X(rhs); X.irev().isz(_n);
        Poly Y(*this); Y.irev().isz(_n);
        Poly Q = Y.Mul(X.Inv()).isz(_n).irev();
        X = rhs.Mul(Q), Y = *this;
        fi(0, n()) if ((Y[i] -= X[i]) < 0) Y[i] += P;
        return {Q, Y.isz(max(1, rhs.n() - 1))};
    }
    Poly Dx() const {
        Poly ret(n() - 1);
        fi(0, ret.n()) ret[i] = (i + 1) * coef[i + 1] % P;
        return ret.isz(max(1, ret.n()));
    }
    Poly Sx() const {
        Poly ret(n() + 1);
        fi(0, n()) ret[i + 1] = ntt.minv(i + 1) * coef[i] % P;
        return ret;
    }
    Poly _tmul(int nn, const Poly &rhs) const {
        Poly Y = Mul(rhs).isz(n() + nn - 1);
        return Poly(Y.data() + n() - 1, nn);
    }
    VL _eval(const VL &x, const auto up) const {
        const int _n = (int)x.size();
        if (!_n) return {};
        vector<Poly> down(_n * 2);
        down[1] = DivMod(up[1]).second;
        fi(2, _n*2) down[i]=down[i/2].DivMod(up[i]).second;
        /* down[1] = Poly(up[1]).irev().isz(n()).Inv().irev()
           ._tmul(_n, *this);
        fi(2, _n * 2) down[i] = up[i ^ 1]._tmul(up[i].n() -
            1, down[i / 2]); */
        VL y(_n);
        fi(0, _n) y[i] = down[_n + i][0];
        return y;
    }
    static vector<Poly> _tree1(const VL &x) {
        const int _n = (int)x.size();
        vector<Poly> up(_n * 2);
        fi(0, _n) up[_n + i] = {(x[i] ? P - x[i] : 0), 1};
        Fi(0, _n-1) up[i] = up[i * 2].Mul(up[i * 2 + 1]);
        return up;
    }
    VL Eval(const VL&x) const {return _eval(x, _tree1(x));}
    static Poly Interpolate(const VL &x, const VL &y) {
        const int _n = (int)x.size();
        vector<Poly> up = _tree1(x), down(_n * 2);
```

```

VL z = up[1].Dx()._eval(x, up);
fi(0, _n) z[i] = y[i] * ntt.minv(z[i]) % P;
fi(0, _n) down[_n + i] = {z[i]};
Fi(0, _n-1) down[i]=down[i * 2].Mul(up[i * 2 + 1])
    .iadd(down[i * 2 + 1].Mul(up[i * 2]));
return down[1];
}
Poly Ln() const { // coef[0] == 1
    return Dx().Mul(Inv()).Sx().isz(n());
}
Poly Exp() const { // coef[0] == 0
    if (n() == 1) return {1};
    Poly X = Poly(*this, (n() + 1)/2).Exp().isz(n());
    Poly Y = X.Ln(); Y[0] = P - 1;
    fi(0, n()) if((Y[i] = coef[i] - Y[i]) < 0)Y[i]+=P;
    return X.Mul(Y).isz(n());
}
Poly Pow(const string &K) const {
    int nz = 0;
    while (nz < n() && !coef[nz]) ++nz;
    LL nk = 0, nk2 = 0;
    for (char c : K) {
        nk = (nk * 10 + c - '0') % P;
        nk2 = nk2 * 10 + c - '0';
        if (nk2 * nz >= n()) return Poly(n());
        nk2 %= P - 1;
    }
    if (!nk && !nk2) return Poly({1}, n());
    Poly X(data() + nz, n() - nz * nk2);
    LL x0 = X[0];
    return X.imul(ntt.minv(x0)).Ln().imul(nk).Exp()
        .imul(ntt.mpow(x0, nk2)).irev().isz(n()).irev();
}
static LL LinearRecursion(const VL&a,const VL&c,LL n){
    // a_n = \sum c_j a_{n-j}
    const int k = (int)a.size();
    assert((int)c.size() == k + 1);
    Poly C(k + 1), W({1}, k), M = {0, 1};
    fi(1, k + 1) C[k - i] = c[i] ? P - c[i] : 0;
    C[k] = 1;
    while (n) {
        if (n % 2) W = W.Mul(M).DivMod(C).second;
        n /= 2, M = M.Mul(M).DivMod(C).second;
    }
    LL ret = 0;
    fi(0, k) ret = (ret + W[i] * a[i]) % P;
    return ret;
}
};
#undef fi
#undef Fi
using Poly_t = Poly<131072 * 2, 998244353, 3>;
template<> decltype(Poly_t::ntt) Poly_t::ntt = {};

```

5.16 FWT

```

/* xor convolution:
* x = (x0,x1) , y = (y0,y1)
* z = ( x0y0 + x1y1 , x0y1 + x1y0 )
* =>
* x' = ( x0+x1 , x0-x1 ) , y' = ( y0+y1 , y0-y1 )
* z' = ( ( x0+x1 )( y0+y1 ) , ( x0-x1 )( y0-y1 ) )
* z = (1/2) * z'
* or convolution:
* x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
* and convolution:
* x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
const LL MOD = 1e9+7;
inline void fwt( LL x[ MAXN ] , int N , bool inv=0 ) {
    for( int d = 1 ; d < N ; d <= 1 ) {
        int d2 = d<<1;
        for( int s = 0 ; s < N ; s += d2 )
            for( int i = s , j = s+d ; i < s+d ; i++, j++ ){
                LL ta = x[ i ] , tb = x[ j ];
                x[ i ] = ta+tb;
                x[ j ] = ta-tb;
                if( x[ i ] >= MOD ) x[ i ] -= MOD;
                if( x[ j ] < 0 ) x[ j ] += MOD;
            }
    }
    if( inv )
        for( int i = 0 ; i < N ; i++ ) {
            x[ i ] *= inv( N, MOD );

```

```

        x[ i ] %= MOD;
    }
}

```

5.17 DiscreteLog

```

lld BSGS(lld P, lld B, lld N) {
    // find B^L = N mod P
    unordered_map<lld, int> R;
    lld sq = (lld)sqrt(P);
    lld t = 1;
    for (int i = 0; i < sq; i++) {
        if (t == N) return i;
        if (!R.count(t)) R[t] = i;
        t = (t * B) % P;
    }
    lld f = inverse(t, P);
    for(int i=0;i<=sq+1;i++) {
        if (R.count(N))
            return i * sq + R[N];
        N = (N * f) % P;
    }
    return -1;
}

```

5.18 FloorSum

```

// @param n `n < 2^32`
// @param m `1 <= m < 2^32`
// @return sum_{i=0}^{n-1} floor((ai + b)/m) mod 2^64
llu floor_sum_unsigned(llu n, llu m, llu a, llu b) {
    llu ans = 0;
    while (true) {
        if (a >= m) {
            ans += n * (n - 1) / 2 * (a / m); a %= m;
        }
        if (b >= m) {
            ans += n * (b / m); b %= m;
        }
        llu y_max = a * n + b;
        if (y_max < m) break;
        // y_max < m * (n + 1)
        // floor(y_max / m) <= n
        n = (llu)(y_max / m), b = (llu)(y_max % m);
        swap(m, a);
    }
    return ans;
}
lld floor_sum(lld n, lld m, lld a, lld b) {
    assert(0 <= n && n < (1LL << 32));
    assert(1 <= m && m < (1LL << 32));
    llu ans = 0;
    if (a < 0) {
        llu a2 = (a % m + m) % m;
        ans -= 1ULL * n * (n - 1) / 2 * ((a2 - a) / m);
        a = a2;
    }
    if (b < 0) {
        llu b2 = (b % m + m) % m;
        ans -= 1ULL * n * ((b2 - b) / m);
        b = b2;
    }
    return ans + floor_sum_unsigned(n, m, a, b);
}

```

5.19 Quadratic residue

```

struct Status{
    ll x,y;
};
ll w;
Status mult(const Status& a,const Status& b,ll mod){
    Status res;
    res.x=(a.x*b.x+a.y*b.y%mod*w)%mod;
    res.y=(a.x*b.y+a.y*b.x)%mod;
    return res;
}
inline Status qpow(Status _base,ll _pow,ll _mod){
    Status res = {1, 0};
    while(_pow>0){
        if(_pow&1) res=mult(res,_base,_mod);
        _base=mult(_base,_base,_mod);
        _pow>>=1;
    }
}

```

```

    return res;
}
inline ll check(ll x, ll p) {
    return qpow_mod(x, (p-1)>>1, p);
}
inline ll get_root(ll n, ll p) {
    if(p==2) return 1;
    if(check(n,p)==p-1) return -1;
    ll a;
    while(true) {
        a=rand()%p;
        w=((a*a-n)%p+p)%p;
        if(check(w,p)==p-1) break;
    }
    Status res = {a, 1};
    res=qpow(res, (p+1)>>1, p);
    return res.x;
}

```

5.20 De-Bruijn

```

int res[maxn], aux[maxn], sz;
void db(int t, int p, int n, int k) {
    if (t > n) {
        if (n % p == 0)
            for (int i = 1; i <= p; ++i)
                res[sz++] = aux[i];
    } else {
        aux[t] = aux[t - p];
        db(t + 1, p, n, k);
        for (int i = aux[t - p] + 1; i < k; ++i) {
            aux[t] = i;
            db(t + 1, t, n, k);
        }
    }
}
int de_bruijn(int k, int n) {
    // return cyclic string of len k^n s.t. every string
    // of len n using k char appears as a substring.
    if (k == 1) {
        res[0] = 0;
        return 1;
    }
    for (int i = 0; i < k * n; i++) aux[i] = 0;
    sz = 0;
    db(1, 1, n, k);
    return sz;
}

```

5.21 Simplex Construction

Standard form: maximize $\sum_{1 \leq i \leq n} c_i x_i$ such that for all $1 \leq j \leq m$, $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$ and $x_i \geq 0$ for all $1 \leq i \leq n$.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji} x_i \leq -b_j$
3. $\sum_{1 \leq i \leq n} A_{ji} x_i = b_j$
 - $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$
 - $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j$
4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

5.22 Simplex

```

namespace simplex {
    // maximize c^T x under Ax <= B
    // return VD(n, -inf) if the solution doesn't exist
    // return VD(n, +inf) if the solution is unbounded
    using VD = vector<double>;
    using VVD = vector<vector<double>>;
    const double eps = 1e-9;
    const double inf = 1e+9;
    int n, m;
    VVD d;
    vector<int> p, q;
    void pivot(int r, int s) {
        double inv = 1.0 / d[r][s];
        for (int i = 0; i < m + 2; ++i)
            for (int j = 0; j < n + 2; ++j)
                if (i != r && j != s)
                    d[i][j] -= d[r][j] * d[i][s] * inv;
    }
}

```

```

for(int i=0;i<m+2;++i) if (i != r) d[i][s] *= -inv;
for(int j=0;j<n+2;++j) if (j != s) d[r][j] *= +inv;
d[r][s] = inv; swap(p[r], q[s]);
}
bool phase(int z) {
    int x = m + z;
    while (true) {
        int s = -1;
        for (int i = 0; i <= n; ++i) {
            if (!z && q[i] == -1) continue;
            if (s == -1 || d[x][i] < d[x][s]) s = i;
        }
        if (d[x][s] > -eps) return true;
        int r = -1;
        for (int i = 0; i < m; ++i) {
            if (d[i][s] < eps) continue;
            if (r == -1 || \
                d[i][n+1]/d[i][s] < d[r][n+1]/d[r][s]) r = i;
        }
        if (r == -1) return false;
        pivot(r, s);
    }
}
VD solve(const VVD &a, const VD &b, const VD &c) {
    m = b.size(), n = c.size();
    d = VVD(m + 2, VD(n + 2));
    for (int i = 0; i < m; ++i)
        for (int j = 0; j < n; ++j) d[i][j] = a[i][j];
    p.resize(m), q.resize(n + 1);
    for (int i = 0; i < m; ++i)
        p[i] = n + i, d[i][n] = -1, d[i][n + 1] = b[i];
    for (int i = 0; i < n; ++i) q[i] = i, d[m][i] = -c[i];
    q[n] = -1, d[m + 1][n] = 1;
    int r = 0;
    for (int i = 1; i < m; ++i)
        if (d[i][n + 1] < d[r][n + 1]) r = i;
    if (d[r][n + 1] < -eps) {
        pivot(r, n);
        if (!phase(1) || d[m + 1][n + 1] < -eps)
            return VD(n, -inf);
        for (int i = 0; i < m; ++i) if (p[i] == -1) {
            int s = min_element(d[i].begin(), d[i].end() - 1)
                - d[i].begin();
            pivot(i, s);
        }
    }
    if (!phase(0)) return VD(n, inf);
    VD x(n);
    for (int i = 0; i < m; ++i)
        if (p[i] < n) x[p[i]] = d[i][n + 1];
    return x;
}
}

```

6 Geometry

6.1 Basic Geometry

```

using coord_t = int;
using Real = double;
using Point = std::complex<coord_t>;
int sgn(coord_t x) {
    return (x > 0) - (x < 0);
}
coord_t dot(Point a, Point b) {
    return real(conj(a) * b);
}
coord_t cross(Point a, Point b) {
    return imag(conj(a) * b);
}
int ori(Point a, Point b, Point c) {
    return sgn(cross(b - a, c - a));
}
bool operator<(const Point &a, const Point &b) {
    return real(a) != real(b)
        ? real(a) < real(b) : imag(a) < imag(b);
}
int argCmp(Point a, Point b) {
    // -1 / 0 / 1 <-> < / == / > (atan2)
    int qa = (imag(a) == 0
        ? (real(a) < 0 ? 3 : 1) : (imag(a) < 0 ? 0 : 2));
    int qb = (imag(b) == 0
        ? (real(b) < 0 ? 3 : 1) : (imag(b) < 0 ? 0 : 2));
    if (qa != qb)

```

```

    return sgn(qa - qb);
    return sgn(cross(b, a));
}
template <typename V> Real area(const V & pt) {
    coord_t ret = 0;
    for (int i = 1; i + 1 < (int)pt.size(); i++)
        ret += cross(pt[i] - pt[0], pt[i+1] - pt[0]);
    return ret / 2.0;
}

```

6.2 Circle Class

```

struct Circle { Point o; Real r; };

vector<Real> intersectAngle(Circle a, Circle b) {
    Real d2 = norm(a.o - b.o);
    if (norm(A.r - B.r) >= d2)
        if (A.r < B.r)
            return {-PI, PI};
        else
            return {};
    if (norm(A.r + B.r) <= d2) return {};
    Real dis = hypot(A.x - B.x, A.y - B.y);
    Real theta = atan2(B.y - A.y, B.x - A.x);
    Real phi = acos((A.r * A.r + d2 - B.r * B.r) /
        (2 * A.r * dis));
    Real L = theta - phi, R = theta + phi;
    while (L < -PI) L += PI * 2;
    while (R > PI) R -= PI * 2;
    return {L, R};
}

vector<Point> intersectPoint(Circle a, Circle b) {
    Real d = o.dis(aa.o);
    if (d >= r+aa.r || d <= fabs(r-aa.r)) return {};
    Real dt = (r*r - aa.r*aa.r)/d, d1 = (d+dt)/2;
    Point dir = (aa.o-o); dir /= d;
    Point pcrs = dir*d1 + o;
    dt=sqrt(max(0.0L, r*r - d1*d1)), dir=dir.rot90();
    return {pcrs + dir*dt, pcrs - dir*dt};
}

```

6.3 2D Convex Hull

```

template<typename PT>
vector<PT> buildConvexHull(vector<PT> d) {
    sort(ALL(d), [](const PT& a, const PT& b){
        return tie(a.x, a.y) < tie(b.x, b.y);});
    vector<PT> s(SZ(d)<1);
    int o = 0;
    for(auto p: d) {
        while(o>=2 && cross(p-s[o-2], s[o-1]-s[o-2])<=0)
            o--;
        s[o++] = p;
    }
    for(int i=SZ(d)-2, t = o+1; i>=0; i--){
        while(o>=t && cross(d[i]-s[o-2], s[o-1]-s[o-2])<=0)
            o--;
        s[o++] = d[i];
    }
    s.resize(o-1);
    return s;
}

```

6.4 3D Convex Hull

```

// return the faces with pt indexes
int flag[MXN][MXN];
struct Point{
    ld x,y,z;
    Point operator * (const ld &b) const {
        return (Point){x*b,y*b,z*b};
    }
    Point operator * (const Point &b) const {
        return (Point){y*b.z-b.y*z, z*b.x-b.z*x, x*b.y-b.x*y};
    }
};
Point ver(Point a, Point b, Point c) {
    return (b - a) * (c - a);
}
vector<Face> convex_hull_3D(const vector<Point> pt) {
    int n = SZ(pt), ftop = 0;
    REP(i,n) REP(j,n) flag[i][j] = 0;
    vector<Face> now;
    now.emplace_back(0,1,2);
    now.emplace_back(2,1,0);
}

```

```

for (int i=3; i<n; i++){
    ftop++; vector<Face> next;
    REP(j, SZ(now)) {
        Face& f=now[j]; int ff = 0;
        ld d=(pt[i]-pt[f.a]).dot(
            ver(pt[f.a], pt[f.b], pt[f.c]));
        if (d <= 0) next.push_back(f);
        if (d > 0) ff=ftop;
        else if (d < 0) ff=-ftop;
        flag[f.a][f.b]=flag[f.b][f.c]=flag[f.c][f.a]=ff;
    }
    REP(j, SZ(now)) {
        Face& f=now[j];
        if (flag[f.a][f.b] > 0 &&
            flag[f.a][f.b] != flag[f.b][f.a])
            next.emplace_back(f.a,f.b,i);
        if (flag[f.b][f.c] > 0 &&
            flag[f.b][f.c] != flag[f.c][f.b])
            next.emplace_back(f.b,f.c,i);
        if (flag[f.c][f.a] > 0 &&
            flag[f.c][f.a] != flag[f.a][f.c])
            next.emplace_back(f.c,f.a,i);
    }
    now=next;
}
return now;
}

```

6.5 2D Farthest Pair

```

// stk is from convex hull
n = (int)(stk.size());
int pos = 1, ans = 0; stk.push_back(stk[0]);
for(int i=0; i<n; i++){
    while(abs(cross(stk[i+1]-stk[i],
        stk[(pos+1)%n]-stk[pos])) >
        abs(cross(stk[i+1]-stk[i],
        stk[pos]-stk[pos]))) pos = (pos+1)%n;
    ans = max(ans, dis(stk[i], stk[pos]));
    dis(stk[i+1], stk[pos]));
}
}

```

6.6 2D Closest Pair

```

struct cmp_y {
    bool operator()(const P& p, const P& q) const {
        return p.y < q.y;
    }
};
multiset<P, cmp_y> s;
void solve(P a[], int n) {
    sort(a, a + n, [](const P& p, const P& q) {
        return tie(p.x, p.y) < tie(q.x, q.y);
    });
    llf d = INF; int pt = 0;
    for (int i = 0; i < n; ++i) {
        while (pt < i and a[i].x - a[pt].x >= d)
            s.erase(s.find(a[pt]));
        auto it = s.lower_bound(P(a[i].x, a[i].y - d));
        while (it != s.end() and it->y - a[i].y < d)
            d = min(d, dis(*(it++), a[i]));
        s.insert(a[i]);
    }
}

```

6.7 kD Closest Pair (3D ver.)

```

llf solve(vector<P> v) {
    shuffle(v.begin(), v.end(), mt19937());
    unordered_map<lld, unordered_map<lld,
        unordered_map<lld, int>>> m;
    llf d = dis(v[0], v[1]);
    auto Idx = [&d] (llf x) -> lld {
        return round(x * 2 / d) + 0.1;
    };
    auto rebuild_m = [&m, &v, &Idx](int k) {
        m.clear();
        for (int i = 0; i < k; ++i)
            m[Idx(v[i].x)][Idx(v[i].y)]
                [Idx(v[i].z)] = i;
    }; rebuild_m(2);
    for (size_t i = 2; i < v.size(); ++i) {
        const lld kx = Idx(v[i].x), ky = Idx(v[i].y),
            kz = Idx(v[i].z); bool found = false;
        for (int dx = -2; dx <= 2; ++dx) {

```



```

const lld nx = dx + kx;
if (m.find(nx) == m.end()) continue;
auto& mm = m[nx];
for (int dy = -2; dy <= 2; ++dy) {
    const lld ny = dy + ky;
    if (mm.find(ny) == mm.end()) continue;
    auto& mmm = mm[ny];
    for (int dz = -2; dz <= 2; ++dz) {
        const lld nz = dz + kz;
        if (mmm.find(nz) == mmm.end()) continue;
        const int p = mmm[nz];
        if (dis(v[p], v[i]) < d) {
            d = dis(v[p], v[i]);
            found = true;
        }
    }
}
if (found) rebuild_m(i + 1);
else m[kx][ky][kz] = i;
}
return d;
}

```

6.8 Simulated Annealing

```

llf anneal() {
    mt19937 rnd_engine( seed );
    uniform_real_distribution< llf > rnd( 0, 1 );
    const llf dT = 0.001;
    // Argument p
    llf S_cur = calc( p ), S_best = S_cur;
    for ( llf T = 2000 ; T > EPS ; T -= dT ) {
        // Modify p to p_prime
        const llf S_prime = calc( p_prime );
        const llf delta_c = S_prime - S_cur;
        llf prob = min( ( llf ) 1, exp( -delta_c / T ) );
        if ( rnd( rnd_engine ) <= prob )
            S_cur = S_prime, p = p_prime;
        if ( S_prime < S_best ) // find min
            S_best = S_prime, p_best = p_prime;
    }
    return S_best;
}

```

6.9 Half Plane Intersection

```

// NOTE: Point is complex<Real>
// cross(pt-line.st, line.dir)<=0 <=> pt in half plane
struct Line {
    Point st, ed;
    Point dir;
    Line (Point _s, Point _e)
        : st(_s), ed(_e), dir(_e - _s) {}
};

bool operator<(const Line &lhs, const Line &rhs) {
    if (int cmp = argCmp(lhs.dir, rhs.dir))
        return cmp == -1;
    return ori(lhs.st, lhs.ed, rhs.st) < 0;
}

Point intersect(const Line &A, const Line &B) {
    Real t = cross(B.st - A.st, B.dir) /
        cross(A.dir, B.dir);
    return A.st + t * A.dir;
}

Real HPI(vector<Line> &lines) {
    sort(lines.begin(), lines.end());
    deque<Line> que;
    deque<Point> pt;
    que.push_back(lines[0]);
    for (int i = 1; i < (int)lines.size(); i++) {
        if (argCmp(lines[i].dir, lines[i-1].dir) == 0)
            continue;
#define POP(L, R) \
        while (pt.size() > 0 \
            && ori(L.st, L.ed, pt.back()) < 0) \
            pt.pop_back(), que.pop_back(); \
        while (pt.size() > 0 \
            && ori(R.st, R.ed, pt.front()) < 0) \
            pt.pop_front(), que.pop_front();
        POP(lines[i], lines[i-1]);
        pt.push_back(intersect(que.back(), lines[i]));

```

```

        que.push_back(lines[i]);
    }
    POP(que.front(), que.back())
    if (que.size() <= 1 ||
        argCmp(que.front().dir, que.back().dir) == 0)
        return 0;
    pt.push_back(intersect(que.front(), que.back()));
    return area(pt);
}

```

6.10 Minkowski sum

```

vector<p11> Minkowski(vector<p11> A, vector<p11> B) {
    hull(A), hull(B);
    vector<p11> C(1, A[0] + B[0]), s1, s2;
    for(int i = 0; i < SZ(A); ++i)
        s1.pb(A[(i + 1) % SZ(A)] - A[i]);
    for(int i = 0; i < SZ(B); i++)
        s2.pb(B[(i + 1) % SZ(B)] - B[i]);
    for(int p1 = 0, p2 = 0; p1 < SZ(A) || p2 < SZ(B);)
        if (p2 >= SZ(B)
            || (p1 < SZ(A) && cross(s1[p1], s2[p2]) >= 0))
            C.pb(C.back() + s1[p1++]);
        else
            C.pb(C.back() + s2[p2++]);
    return hull(C), C;
}

```

6.11 intersection of line and circle

```

vector<pdd> line_interCircle(const pdd &p1,
    const pdd &p2, const pdd &c, const double r){
    pdd ft=foot(p1,p2,c),vec=p2-p1;
    double dis=abs(c-ft);
    if(fabs(dis-r)<eps) return vector<pdd>{ft};
    if(dis>r) return {};
    vec=vec*sqrt(r*r-dis*dis)/abs(vec);
    return vector<pdd>{ft+vec,ft-vec};
}

```

6.12 intersection of polygon and circle

```

// Divides into multiple triangle, and sum up
// test by HDU2892
const double PI=acos(-1);
double _area(pdd pa, pdd pb, double r){
    if(abs(pa)<abs(pb)) swap(pa, pb);
    if(abs(pb)<eps) return 0;
    double S, h, theta;
    double a=abs(pb),b=abs(pa),c=abs(pb-pa);
    double cosB = dot(pb,pb-pa) / a / c, B = acos(cosB);
    double cosC = dot(pa,pb) / a / b, C = acos(cosC);
    if(a > r){
        S = (C/2)*r*r;
        h = a*b*sin(C)/c;
        if (h < r && B < PI/2)
            S -= (acos(h/r)*r*r - h*sqrt(r*r-h*h));
    }
    else if(b > r){
        theta = PI - B - asin(sin(B)/r*a);
        S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
    }
    else S = .5*sin(C)*a*b;
    return S;
}

double area_poly_circle(const vector<pdd> poly,
    const pdd &o, const double r){
    double S=0;
    for(int i=0;i<SZ(poly);++i)
        S+=_area(poly[i]-o,poly[(i+1)%SZ(poly)]-o,r)
        *ori(o,poly[i],poly[(i+1)%SZ(poly)]);
    return fabs(S);
}

```

6.13 intersection of two circle

```

bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
    pdd o1 = a.o, o2 = b.o;
    double r1 = a.R, r2 = b.R, d2 = abs2(o1 - o2),
        d = sqrt(d2);
    if(d < max(r1, r2) - min(r1, r2) || d > r1 + r2)
        return 0;
    pdd u = (o1 + o2) * 0.5
        + (o1 - o2) * ((r2 * r2 - r1 * r1) / (2 * d2));
    double A = sqrt((r1 + r2 + d) * (r1 - r2 + d)

```

```

    * (r1 + r2 - d) * (-r1 + r2 + d));
    pdd v = pdd(o1.Y - o2.Y, -o1.X + o2.X) * A
    / (2 * d2);
    p1 = u + v, p2 = u - v;
    return 1;
}

```

6.14 tangent line of two circle

```

vector<Line> go(const Cir& c1,
    const Cir& c2, int sign1){
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    double d_sq = norm2( c1.o - c2.o );
    if( d_sq < eps ) return ret;
    double d = sqrt( d_sq );
    Pt v = ( c2.o - c1.o ) / d;
    double c = ( c1.R - sign1 * c2.R ) / d;
    if( c * c > 1 ) return ret;
    double h = sqrt( max( 0.0 , 1.0 - c * c ) );
    for( int sign2 = 1 ; sign2 >= -1 ; sign2 -= 2 ){
        Pt n = { v.X * c - sign2 * h * v.Y ,
            v.Y * c + sign2 * h * v.X };
        Pt p1 = c1.o + n * c1.R;
        Pt p2 = c2.o + n * ( c2.R * sign1 );
        if( fabs( p1.X - p2.X ) < eps and
            fabs( p1.Y - p2.Y ) < eps )
            p2 = p1 + perp( c2.o - c1.o );
        ret.push_back( { p1 , p2 } );
    }
    return ret;
}

```

6.15 Minimum Covering Circle

```

template<typename P>
Circle getCircum(const P &a, const P &b, const P &c){
    Real a1 = a.x-b.x, b1 = a.y-b.y;
    Real c1 = (a.x+b.x)/2 * a1 + (a.y+b.y)/2 * b1;
    Real a2 = a.x-c.x, b2 = a.y-c.y;
    Real c2 = (a.x+c.x)/2 * a2 + (a.y+c.y)/2 * b2;
    Circle cc;
    cc.o.x = (c1*b2-b1*c2)/(a1*b2-b1*a2);
    cc.o.y = (a1*c2-c1*a2)/(a1*b2-b1*a2);
    cc.r = hypot(cc.o.x-a.x, cc.o.y-a.y);
    return cc;
}

template<typename P>
Circle MinCircleCover(const vector<P>& pts){
    random_shuffle(pts.begin(), pts.end());
    Circle c = { pts[0], 0 };
    for(int i=0;i<(int)pts.size();i++){
        if (dist(pts[i], c.o) <= c.r) continue;
        c = { pts[i], 0 };
        for (int j = 0; j < i; j++) {
            if(dist(pts[j], c.o) <= c.r) continue;
            c.o = (pts[i] + pts[j]) / 2;
            c.r = dist(pts[i], c.o);
            for (int k = 0; k < j; k++) {
                if (dist(pts[k], c.o) <= c.r) continue;
                c = getCircum(pts[i], pts[j], pts[k]);
            }
        }
    }
    return c;
}

```

6.16 KDTree (Nearest Point)

```

const int MXN = 100005;
struct KDTree {
    struct Node {
        int x,y,x1,y1,x2,y2;
        int id,f;
        Node *L, *R;
    } tree[MXN], *root;
    int n;
    LL dis2(int x1, int y1, int x2, int y2) {
        LL dx = x1-x2, dy = y1-y2;
        return dx*dx+dy*dy;
    }
    static bool cmpx(Node& a, Node& b){return a.x<b.x;}
    static bool cmpy(Node& a, Node& b){return a.y<b.y;}
}

```

```

void init(vector<pair<int,int>> ip) {
    n = ip.size();
    for (int i=0; i<n; i++) {
        tree[i].id = i;
        tree[i].x = ip[i].first;
        tree[i].y = ip[i].second;
    }
    root = build_tree(0, n-1, 0);
}

Node* build_tree(int L, int R, int d) {
    if (L>R) return nullptr;
    int M = (L+R)/2; tree[M].f = d%2;
    nth_element(tree+L, tree+M, tree+R+1, d%2?cmpy:cmpx);
    tree[M].x1 = tree[M].x2 = tree[M].x;
    tree[M].y1 = tree[M].y2 = tree[M].y;
    tree[M].L = build_tree(L, M-1, d+1);
    if (tree[M].L) {
        tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
        tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
        tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
        tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
    }
    tree[M].R = build_tree(M+1, R, d+1);
    if (tree[M].R) {
        tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
        tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
        tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
        tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
    }
    return tree+M;
}

int touch(Node* r, int x, int y, LL d2){
    LL dis = sqrt(d2)+1;
    if (x<r->x1-dis || x>r->x2+dis ||
        y<r->y1-dis || y>r->y2+dis)
        return 0;
    return 1;
}

void nearest(Node* r, int x, int y, int &mID, LL &md2) {
    if (!r || !touch(r, x, y, md2)) return;
    LL d2 = dis2(r->x, r->y, x, y);
    if (d2 < md2 || (d2 == md2 && mID < r->id)) {
        mID = r->id;
        md2 = d2;
    }
    // search order depends on split dim
    if ((r->f == 0 && x < r->x) ||
        (r->f == 1 && y < r->y)) {
        nearest(r->L, x, y, mID, md2);
        nearest(r->R, x, y, mID, md2);
    } else {
        nearest(r->R, x, y, mID, md2);
        nearest(r->L, x, y, mID, md2);
    }
}

int query(int x, int y) {
    int id = 1029384756;
    LL d2 = 102938475612345678LL;
    nearest(root, x, y, id, d2);
    return id;
}
} tree;

```

7 Stringology

7.1 Hash

```

class Hash {
private:
    static constexpr int P = 127, Q = 1051762951;
    vector<int> h, p;
public:
    void init(const string &s){
        h.assign(s.size()+1, 0); p.resize(s.size()+1);
        for (size_t i = 0; i < s.size(); ++i)
            h[i+1] = add(mul(h[i], P), s[i]);
        generate(p.begin(), p.end(), [x=1,y=1,this]()
            mutable{y=x;x=mul(x,P);return y;});
    }
    int query(int l, int r){ // 1-base [l, r]
        return sub(h[r], mul(h[l], p[r-l]));
    }
};

```

7.2 Suffix Array

```
namespace sfxarray {
bool t[maxn * 2];
int hi[maxn], rev[maxn];
int _s[maxn * 2], sa[maxn * 2], c[maxn * 2];
int x[maxn], p[maxn], q[maxn * 2];
// sa[i]: sa[i]-th suffix is the \
// i-th lexicographically smallest suffix.
// hi[i]: longest common prefix \
// of suffix sa[i] and suffix sa[i - 1].
void pre(int *sa, int *c, int n, int z) {
    memset(sa, 0, sizeof(int) * n);
    memcpy(x, c, sizeof(int) * z);
}
void induce(int *sa, int *c, int *s, bool *t, int n, int z) {
    memcpy(x + 1, c, sizeof(int) * (z - 1));
    for (int i = 0; i < n; ++i)
        if (sa[i] && !t[sa[i] - 1])
            sa[x[sa[i] - 1]]++ = sa[i] - 1;
    memcpy(x, c, sizeof(int) * z);
    for (int i = n - 1; i >= 0; --i)
        if (sa[i] && t[sa[i] - 1])
            sa[--x[sa[i] - 1]] = sa[i] - 1;
}
void sais(int *s, int *sa, int *p, int *q,
bool *t, int *c, int n, int z) {
    bool uniq = t[n - 1] = true;
    int nn=0, nmzx=-1, *nsa = sa+n, *ns=s+n, last=-1;
    memset(c, 0, sizeof(int) * z);
    for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;
    for (int i = 0; i < z - 1; ++i) c[i + 1] += c[i];
    if (uniq) {
        for (int i = 0; i < n; ++i) sa[--c[s[i]]] = i;
        return;
    }
    for (int i = n - 2; i >= 0; --i)
        t[i] = (s[i]==s[i + 1] ? t[i + 1] : s[i]<s[i + 1]);
    pre(sa, c, n, z);
    for (int i = 1; i <= n - 1; ++i)
        if (t[i] && !t[i - 1])
            sa[--x[s[i]]] = p[q[i] = nn++] = i;
    induce(sa, c, s, t, n, z);
    for (int i = 0; i < n; ++i) {
        if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
            bool neq = last < 0 || \
                memcmp(s + sa[i], s + last,
                    (p[q[sa[i]] + 1] - sa[i]) * sizeof(int));
            ns[q[last = sa[i]]] = nmzx += neq;
        }
    }
    sais(ns, nsa, p+nn, q+n, t+n, c+z, nn, nmzx+1);
    pre(sa, c, n, z);
    for (int i = nn - 1; i >= 0; --i)
        sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
    induce(sa, c, s, t, n, z);
}
void build(const string &s) {
    for (int i = 0; i < (int)s.size(); ++i) _s[i] = s[i];
    _s[(int)s.size()] = 0; // s shouldn't contain 0
    sais(_s, sa, p, q, t, c, (int)s.size() + 1, 256);
    for (int i = 0; i < (int)s.size(); ++i) sa[i]=sa[i+1];
    for (int i = 0; i < (int)s.size(); ++i) rev[sa[i]]=i;
    int ind = 0; hi[0] = 0;
    for (int i = 0; i < (int)s.size(); ++i) {
        if (!rev[i]) {
            ind = 0;
            continue;
        }
        while (i + ind < (int)s.size() && \
            s[i + ind] == s[sa[rev[i] - 1] + ind]) ++ind;
        hi[rev[i]] = ind ? ind-- : 0;
    }
}
}
```

7.3 Aho-Corasick Algorithm

```
class AhoCorasick {
private:
    static constexpr int Z = 26;
    struct node {
        node *nxt[Z], *fail;
        vector< int > data;
        node(): fail( nullptr ) {
```

```
            memset( nxt, 0, sizeof( nxt ) );
            data.clear();
        }
    } *rt;
    inline int Idx( char c ) { return c - 'a'; }
public:
    void init() { rt = new node(); }
    void add( const string& s, int d ) {
        node* cur = rt;
        for ( auto c : s ) {
            if ( not cur->nxt[ Idx( c ) ] )
                cur->nxt[ Idx( c ) ] = new node();
            cur = cur->nxt[ Idx( c ) ];
        }
        cur->data.push_back( d );
    }
    void compile() {
        vector< node* > bfs;
        size_t ptr = 0;
        for ( int i = 0 ; i < Z ; ++ i ) {
            if ( not rt->nxt[ i ] ) {
                // uncomment 2 lines to make it DFA
                // rt->nxt[i] = rt;
                continue;
            }
            rt->nxt[ i ]->fail = rt;
            bfs.push_back( rt->nxt[ i ] );
        }
        while ( ptr < bfs.size() ) {
            node* u = bfs[ ptr++ ];
            for ( int i = 0 ; i < Z ; ++ i ) {
                if ( not u->nxt[ i ] ) {
                    // u->nxt[i] = u->fail->nxt[i];
                    continue;
                }
                node* u_f = u->fail;
                while ( u_f ) {
                    if ( not u_f->nxt[ i ] ) {
                        u_f = u_f->fail; continue;
                    }
                    u->nxt[ i ]->fail = u_f->nxt[ i ];
                    break;
                }
                if ( not u_f ) u->nxt[ i ]->fail = rt;
                bfs.push_back( u->nxt[ i ] );
            }
        }
    }
    void match( const string& s, vector< int >& ret ) {
        node* u = rt;
        for ( auto c : s ) {
            while ( u != rt and not u->nxt[ Idx( c ) ] )
                u = u->fail;
            u = u->nxt[ Idx( c ) ];
            if ( not u ) u = rt;
            node* tmp = u;
            while ( tmp != rt ) {
                for ( auto d : tmp->data )
                    ret.push_back( d );
                tmp = tmp->fail;
            }
        }
    }
} ac;
```

7.4 Suffix Automaton

```
struct Node {
    Node *green, *edge[26];
    int max_len;
    Node(const int _max_len)
        : green(NULL), max_len(_max_len){
        memset(edge, 0, sizeof(edge));
    }
    *ROOT, *LAST;
    void Extend(const int c) {
        Node *cursor = LAST;
        LAST = new Node((LAST->max_len) + 1);
        for (; cursor && cursor->edge[c]; cursor=cursor->green)
            cursor->edge[c] = LAST;
        if (!cursor)
            LAST->green = ROOT;
        else {
```

```

Node *potential_green = cursor->edge[c];
if((potential_green->max_len)==(cursor->max_len+1))
    LAST->green = potential_green;
else {
    //assert(potential_green->max_len>(cursor->max_len+1));
    Node *wish = new Node((cursor->max_len) + 1);
    for(; cursor && cursor->edge[c]==potential_green;
        cursor = cursor->green)
        cursor->edge[c] = wish;
    for (int i = 0; i < 26; i++)
        wish->edge[i] = potential_green->edge[i];
    wish->green = potential_green->green;
    potential_green->green = wish;
    LAST->green = wish;
}
}
char S[10000001], A[10000001];
int N;
int main(){
    scanf("%d%s", &N, S);
    ROOT = LAST = new Node(0);
    for (int i = 0; S[i]; i++)
        Extend(S[i] - 'a');
    while (N--){
        scanf("%s", A);
        Node *cursor = ROOT;
        bool ans = true;
        for (int i = 0; A[i]; i++){
            cursor = cursor->edge[A[i] - 'a'];
            if (!cursor) {
                ans = false;
                break;
            }
        }
        puts(ans ? "Yes" : "No");
    }
    return 0;
}

```

7.5 KMP

```

vector<int> kmp(const string &s) {
    vector<int> f(s.size(), 0);
    /* f[i] = length of the longest prefix
       (excluding s[0:i]) such that it coincides
       with the suffix of s[0:i] of the same length */
    /* i + 1 - f[i] is the length of the
       smallest recurring period of s[0:i] */
    int k = 0;
    for (int i = 1; i < (int)s.size(); ++i) {
        while (k > 0 && s[i] != s[k]) k = f[k - 1];
        if (s[i] == s[k]) ++k;
        f[i] = k;
    }
    return f;
}
vector<int> search(const string &s, const string &t) {
    // return 0-indexed occurrence of t in s
    vector<int> f = kmp(t), r;
    for (int i = 0, k = 0; i < (int)s.size(); ++i) {
        while(k > 0 && (k==(int)t.size() || s[i]!=t[k]))
            k = f[k - 1];
        if (s[i] == t[k]) ++k;
        if (k == (int)t.size()) r.push_back(i-t.size()+1);
    }
    return res;
}

```

7.6 Z value

```

char s[MAXN];
int len, z[MAXN];
void Z_value() {
    int i, j, left, right;
    z[left=right=0]=len;
    for(i=1; i<len; i++) {
        j=max(min(z[i-left], right-i), 0);
        for(; i+j<len&&s[i+j]==s[j]; j++);
        if(i+(z[i]=j)>right)right=i+z[i];
    }
}

```

7.7 Manacher

```

int z[maxn];
int manacher(const string& s) {
    string t = ". ";
    for(char c:s) t += c, t += ' ';
    int l = 0, r = 0, ans = 0;
    for (int i = 1; i < t.length(); ++i) {
        z[i] = (r > i ? min(z[2 * l - i], r - i) : 1);
        while (i - z[i] >= 0 && i + z[i] < t.length()) {
            if(t[i - z[i]] == t[i + z[i]]) ++z[i];
            else break;
        }
        if (i + z[i] > r) r = i + z[i], l = i;
    }
    for(int i=1;i<t.length();++i) ans = max(ans, z[i]-1);
    return ans;
}

```

7.8 Lexico Smallest Rotation

```

string mcp(string s){
    int n = s.length();
    s += s;
    int i=0, j=1;
    while (i<n && j<n){
        int k = 0;
        while (k < n && s[i+k] == s[j+k]) k++;
        if (s[i+k] <= s[j+k]) j = j+k+1;
        else i = i+k+1;
        if (i == j) j++;
    }
    int ans = i < n ? i : j;
    return s.substr(ans, n);
}

```

7.9 BWT

```

struct BurrowsWheeler{
#define SIGMA 26
#define BASE 'a'
    vector<int> v[ SIGMA ];
    void BWT(char* ori, char* res){
        // make ori -> ori + ori
        // then build suffix array
    }
    void iBWT(char* ori, char* res){
        for( int i = 0 ; i < SIGMA ; i ++ )
            v[ i ].clear();
        int len = strlen( ori );
        for( int i = 0 ; i < len ; i ++ )
            v[ ori[i] - BASE ].push_back( i );
        vector<int> a;
        for( int i = 0 , ptr = 0 ; i < SIGMA ; i ++ )
            for( auto j : v[ i ] ){
                a.push_back( j );
                ori[ ptr ++ ] = BASE + i;
            }
        for( int i = 0 , ptr = 0 ; i < len ; i ++ ){
            res[ i ] = ori[ a[ ptr ] ];
            ptr = a[ ptr ];
        }
        res[ len ] = 0;
    }
} bwt;

```

7.10 Palindromic Tree

```

struct palindromic_tree{
    struct node{
        int next[26], f, len;
        int cnt, num, st, ed;
        node(int l=0):f(0),len(l),cnt(0),num(0) {
            memset(next, 0, sizeof(next));
        }
    };
    vector<node> st;
    vector<char> s;
    int last, n;
    void init(){
        st.clear(); s.clear(); last=1; n=0;
        st.push_back(0); st.push_back(-1);
        st[0].f=1; s.push_back(-1);
    }
    int getFail(int x){
        while(s[n-st[x].len-1]!=s[n])x=st[x].f;
        return x;
    }
}

```

```

void add(int c){
    s.push_back(c-'a'); ++n;
    int cur=getFail(last);
    if(!st[cur].next[c]){
        int now=st.size();
        st.push_back(st[cur].len+2);
        st[now].f=st[getFail(st[cur].f)].next[c];
        st[cur].next[c]=now;
        st[now].num=st[st[now].f].num+1;
    }
    last=st[cur].next[c];
    ++st[last].cnt;
}
int size(){ return st.size()-2;}
} pt;
int main() {
    string s; cin >> s; pt.init();
    for (int i=0; i<SZ(s); i++) {
        int prvsz = pt.size(); pt.add(s[i]);
        if (prvsz != pt.size()) {
            int r = i, l = r - pt.st[pt.last].len + 1;
            // pal @ [l,r]: s.substr(l, r-l+1)
        }
    }
    return 0;
}

```

8 Misc

8.1 Theorems

8.1.1 Kirchhoff's Theorem

Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G , where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i, j) in G .

- The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
- The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.

8.1.2 Tutte's Matrix

Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniform randomly) if $i < j$ and $(i, j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{\text{rank}(D)}{2}$ is the maximum matching on G .

8.1.3 Cayley's Formula

- Given a degree sequence d_1, d_2, \dots, d_n for each labeled vertices, there're $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\dots(d_n-1)!}$ spanning trees.
- Let $T_{n,k}$ be the number of labeled forests on n vertices with k components, such that vertex $1, 2, \dots, k$ belong to different components. Then $T_{n,k} = kn^{n-k-1}$.

8.1.4 Erdős-Gallai theorem

A sequence of non-negative integers $d_1 \geq d_2 \geq \dots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + d_2 + \dots + d_n$ is even and

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$$

holds for all $1 \leq k \leq n$.

8.1.5 Havel-Hakimi algorithm

find the vertex who has greatest degree unused, connect it with other greatest vertex.

8.1.6 Hall's marriage theorem

Let G be a finite bipartite graph with bipartite sets X and Y . For a subset W of X , let $N_G(W)$ denote the set of all vertices in Y adjacent to some element of W . Then there is an X -saturating matching iff $\forall W \subseteq X, |W| \leq |N_G(W)|$

8.1.7 Euler's planar graph formula

$$V - E + F = C + 1, E \leq 3V - 6(?)$$

8.1.8 Pick's theorem

For simple polygon, when points are all integer, we have $A = \#(\text{lattice points in the interior}) + \frac{\#(\text{lattice points on the boundary})}{2} - 1$

8.1.9 Lucas's theorem

$\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p}$, where $m = m_k p^k + m_{k-1} p^{k-1} + \dots + m_1 p + m_0$, and $n = n_k p^k + n_{k-1} p^{k-1} + \dots + n_1 p + n_0$.

8.2 MaximumEmptyRect

```

int max_empty_rect(int n, int m, bool blocked[N][N]) {
    static int mxu[2][N], me=0, he=1, ans=0;
    for (int i=0; i<m; i++) mxu[he][i]=0;
    for (int i=0; i<n; i++) {
        stack<PII> stk;
        for (int j=0; j<m; j++) {
            if (blocked[i][j]) mxu[me][j]=0;
            else mxu[me][j]=mxu[he][j]+1;
            int la = j;
            while (!stk.empty() && stk.top().FF > mxu[me][j]) {
                int x1 = i - stk.top().FF, x2 = i;
                int y1 = stk.top().SS, y2 = j;
                la = stk.top().SS; stk.pop();
                ans = max(ans, (x2-x1)*(y2-y1));
            }
            if (stk.empty() || stk.top().FF < mxu[me][j])
                stk.push({mxu[me][j], la});
        }
        while (!stk.empty()) {
            int x1 = i - stk.top().FF, x2 = i;
            int y1 = stk.top().SS-1, y2 = m-1;
            stk.pop(); ans = max(ans, (x2-x1)*(y2-y1));
        }
        swap(me, he);
    }
    return ans;
}

```

8.3 DP-opt Condition

8.3.1 totally monotone (concave/convex)

$$\begin{aligned} \forall i < i', j < j', B[i][j] \leq B[i'][j] &\implies B[i][j'] \leq B[i'][j'] \\ \forall i < i', j < j', B[i][j] \geq B[i'][j] &\implies B[i][j'] \geq B[i'][j'] \end{aligned}$$

8.3.2 monge condition (concave/convex)

$$\begin{aligned} \forall i < i', j < j', B[i][j] + B[i'][j'] &\geq B[i][j'] + B[i'][j] \\ \forall i < i', j < j', B[i][j] + B[i'][j'] &\leq B[i][j'] + B[i'][j] \end{aligned}$$

8.4 Convex 1D/1D DP

```

struct segment {
    int i, l, r;
    segment() {}
    segment(int a, int b, int c): i(a), l(b), r(c) {}
};
inline lld f(int l, int r){return dp[l] + w(l+1, r);}
void solve() {
    dp[0] = 0;
    deque<segment> dq; dq.push_back(segment(0, 1, n));
    for (int i = 1; i <= n; ++i) {
        dp[i] = f(dq.front().i, i);
        while(dq.size() && dq.front().r < i+1) dq.pop_front();
        dq.front().l = i + 1;
        segment seg = segment(i, i + 1, n);
        while (dq.size() && f(i, dq.back().l) < f(dq.back().i, dq.back().l))
            dq.pop_back();
        if (dq.size()) {
            int d = 1 << 20, c = dq.back().l;
            while (d >= 1) if (c + d <= dq.back().r)
                if(f(i, c+d) > f(dq.back().i, c+d)) c += d;
            dq.back().r = c; seg.l = c + 1;
        }
        if (seg.l <= n) dq.push_back(seg);
    }
}

```

8.5 ConvexHull Optimization

```

inline lld DivCeil(lld n, lld d) { // ceil(n/d)
    return n / d + ((n < 0) != (d > 0)) && (n % d);
}
struct Line {
    static bool flag;
    lld a, b, l, r; // y=ax+b in [l, r)
    lld operator()(lld x) const { return a * x + b; }
    bool operator<(const Line& i) const {
        return flag ? tie(a, b) < tie(i.a, i.b) : l < i.l;
    }
    lld operator&(const Line& i) const {
        return DivCeil(b - i.b, i.a - a);
    }
};

```



```

bool Line::flag = true;
class ConvexHullMax {
    set<Line> L;
public:
    ConvexHullMax() { Line::flag = true; }
    void InsertLine(lld a, lld b) { // add y = ax + b
        Line now = {a, b, -INF, INF};
        if (L.empty()) {
            L.insert(now);
            return;
        }
        Line::flag = true;
        auto it = L.lower_bound(now);
        auto prv = it == L.begin() ? it : prev(it);
        if (it != L.end() && ((it != L.begin() &&
            (*it)(it->l) >= now(it->l) &&
            (*prv)(prv->r - 1) >= now(prv->r - 1)) ||
            (it == L.begin() && it->a == now.a))) return;
        if (it != L.begin()) {
            while (prv != L.begin() &&
                (*prv)(prv->l) <= now(prv->l))
                prv = --L.erase(prv);
            if (prv == L.begin() && now.a == prv->a)
                L.erase(prv);
        }
        if (it != L.end())
            while (it != --L.end() &&
                (*it)(it->r) <= now(it->r))
                it = L.erase(it);
        if (it != L.begin()) {
            prv = prev(it);
            const_cast<Line*>(&*prv)->r = now.l = ((*prv)&now);
        }
        if (it != L.end())
            const_cast<Line*>(&*it)->l = now.r = ((*it)&now);
        L.insert(it, now);
    }
    lld Query(lld a) const { // query max at x=a
        if (L.empty()) return -INF;
        Line::flag = false;
        auto it = --L.upper_bound({0, 0, a, 0});
        return (*it)(a);
    }
};

```

8.6 Josephus Problem

```

// n people kill m for each turn
int f(int n, int m) {
    int s = 0;
    for (int i = 2; i <= n; i++)
        s = (s + m) % i;
    return s;
}
// died at kth
int kth(int n, int m, int k){
    if (m == 1) return n-1;
    for (k = k*m+m-1; k >= n; k = k-n+(k-n)/(m-1));
    return k;
}

```

8.7 Cactus Matching

```

vector<int> init_g[maxn], g[maxn*2];
int n, dfn[maxn], low[maxn], par[maxn], dfs_idx, bcc_id;
void tarjan(int u){
    dfn[u] = low[u] = ++dfs_idx;
    for (int i=0; i<(int)init_g[u].size(); i++){
        int v = init_g[u][i];
        if (v == par[u]) continue;
        if (!dfn[v]){
            par[v] = u;
            tarjan(v);
            low[u] = min(low[u], low[v]);
            if (dfn[u] < low[v]){
                g[u].push_back(v);
                g[v].push_back(u);
            }
        } else {
            low[u] = min(low[u], dfn[v]);
            if (dfn[v] < dfn[u]){
                int temp_v = u;
                bcc_id++;
                while (temp_v != v){

```

```

                    g[bcc_id+n].push_back(temp_v);
                    g[temp_v].push_back(bcc_id+n);
                    temp_v = par[temp_v];
                }
                g[bcc_id+n].push_back(v);
                g[v].push_back(bcc_id+n);
                reverse(g[bcc_id+n].begin(), g[bcc_id+n].end());
            }
        }
    }
    int dp[maxn][2], min_dp[2][2], tmp[2][2], tp[2];
    void dfs(int u, int fa){
        if (u <= n){
            for (int i=0; i<(int)g[u].size(); i++){
                int v = g[u][i];
                if (v == fa) continue;
                dfs(v, u);
                memset(tp, 0x8f, sizeof tp);
                if (v <= n){
                    tp[0] = dp[u][0] + max(dp[v][0], dp[v][1]);
                    tp[1] = max(
                        dp[u][0] + dp[v][0] + 1,
                        dp[u][1] + max(dp[v][0], dp[v][1])
                    );
                } else {
                    tp[0] = dp[u][0] + dp[v][0];
                    tp[1] = max(dp[u][0] + dp[v][1], dp[u][1] + dp[v][0]);
                }
                dp[u][0] = tp[0], dp[u][1] = tp[1];
            }
        } else {
            for (int i=0; i<(int)g[u].size(); i++){
                int v = g[u][i];
                if (v == fa) continue;
                dfs(v, u);
            }
            min_dp[0][0] = 0;
            min_dp[1][1] = 1;
            min_dp[0][1] = min_dp[1][0] = -0x3f3f3f3f;
            for (int i=0; i<(int)g[u].size(); i++){
                int v = g[u][i];
                if (v == fa) continue;
                memset(tmp, 0x8f, sizeof tmp);
                tmp[0][0] = max(
                    min_dp[0][0] + max(dp[v][0], dp[v][1]),
                    min_dp[0][1] + dp[v][0]
                );
                tmp[0][1] = min_dp[0][0] + dp[v][0] + 1;
                tmp[1][0] = max(
                    min_dp[1][0] + max(dp[v][0], dp[v][1]),
                    min_dp[1][1] + dp[v][0]
                );
                tmp[1][1] = min_dp[1][0] + dp[v][0] + 1;
                memcpy(min_dp, tmp, sizeof tmp);
            }
            dp[u][1] = max(min_dp[0][1], min_dp[1][0]);
            dp[u][0] = min_dp[0][0];
        }
    }
    int main(){
        int m, a, b;
        scanf("%d%d", &n, &m);
        for (int i=0; i<m; i++){
            scanf("%d%d", &a, &b);
            init_g[a].push_back(b);
            init_g[b].push_back(a);
        }
        par[1] = -1;
        tarjan(1);
        dfs(1, -1);
        printf("%d\n", max(dp[1][0], dp[1][1]));
        return 0;
    }
}

```

8.8 DLX

```

struct DLX {
    const static int maxn=210;
    const static int maxm=210;
    const static int maxnode=210*210;
    int n, m, size, row[maxnode], col[maxnode];
    int U[maxnode], D[maxnode], L[maxnode], R[maxnode];

```

```

int H[maxn], S[maxm], ansd, ans[maxn];
void init(int _n, int _m) {
    n = _n, m = _m;
    for(int i = 0; i <= m; ++i) {
        S[i] = 0;
        U[i] = D[i] = i;
        L[i] = i-1, R[i] = i+1;
    }
    R[L[0] = size = m] = 0;
    for(int i = 1; i <= n; ++i) H[i] = -1;
}
void Link(int r, int c) {
    ++S[col[++size] = c];
    row[size] = r; D[size] = D[c];
    U[D[c]] = size; U[size] = c; D[c] = size;
    if(H[r] < 0) H[r] = L[size] = R[size] = size;
    else {
        R[size] = R[H[r]];
        L[R[H[r]]] = size;
        L[size] = H[r];
        R[H[r]] = size;
    }
}
void remove(int c) {
    L[R[c]] = L[c]; R[L[c]] = R[c];
    for(int i = D[c]; i != c; i = D[i])
        for(int j = R[i]; j != i; j = R[j]) {
            U[D[j]] = U[j];
            D[U[j]] = D[j];
            --S[col[j]];
        }
}
void resume(int c) {
    L[R[c]] = c; R[L[c]] = c;
    for(int i = U[c]; i != c; i = U[i])
        for(int j = L[i]; j != i; j = L[j]) {
            U[D[j]] = j;
            D[U[j]] = j;
            ++S[col[j]];
        }
}
void dance(int d) {
    if(d>=ansd) return;
    if(R[0] == 0) {
        ansd = d;
        return;
    }
    int c = R[0];
    for(int i = R[0]; i; i = R[i])
        if(S[i] < S[c]) c = i;
    remove(c);
    for(int i = D[c]; i != c; i = D[i]) {
        ans[d] = row[i];
        for(int j = R[i]; j != i; j = R[j])
            remove(col[j]);
        dance(d+1);
        for(int j = L[i]; j != i; j = L[j])
            resume(col[j]);
    }
    resume(c);
}
} sol;

```

8.9 Tree Knapsack

```

int dp[N][K]; PII obj[N];
vector<int> G[N];
void dfs(int u, int mx){
    for(int s: G[u]) {
        if(mx < obj[s].first) continue;
        for(int i=0; i<=mx-obj[s].FF; i++)
            dp[s][i] = dp[u][i];
        dfs(s, mx - obj[s].first);
        for(int i=obj[s].FF; i<=mx; i++)
            dp[u][i] = max(dp[u][i],
                dp[s][i - obj[s].FF] + obj[s].SS);
    }
}
int main(){
    int n, k; cin >> n >> k;
    for(int i=1; i<=n; i++){
        int p; cin >> p;
        G[p].push_back(i);
    }
}

```

```

    cin >> obj[i].FF >> obj[i].SS;
}
dfs(0, k); int ans = 0;
for(int i=0; i<=k; i++) ans = max(ans, dp[0][i]);
cout << ans << '\n';
return 0;
}

```

8.10 N Queens Problem

```

vector<int> solve(int n) {
    // no solution when n=2, 3
    vector<int> ret;
    if (n % 6 == 2) {
        for (int i = 2; i <= n; i += 2)
            ret.push_back(i);
        ret.push_back(3); ret.push_back(1);
        for (int i = 7; i <= n; i += 2)
            ret.push_back(i);
        ret.push_back(5);
    } else if (n % 6 == 3) {
        for (int i = 4; i <= n; i += 2)
            ret.push_back(i);
        ret.push_back(2);
        for (int i = 5; i <= n; i += 2)
            ret.push_back(i);
        ret.push_back(1); ret.push_back(3);
    } else {
        for (int i = 2; i <= n; i += 2)
            ret.push_back(i);
        for (int i = 1; i <= n; i += 2)
            ret.push_back(i);
    }
    return ret;
}

```

8.11 Aliens Optimization

```

long long Alien() {
    long long c = kInf;
    for (int d = 60; d >= 0; --d) {
        // cost can be negative, depending on the problem.
        if (c - (1LL << d) < 0) continue;
        long long ck = c - (1LL << d);
        pair<long long, int> r = check(ck);
        if (r.second == k) return r.first - ck * k;
        if (r.second < k) c = ck;
    }
    pair<long long, int> r = check(c);
    return r.first - c * k;
}

```