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1 Basic

1.1 Default Code

```

1 #include <bits/stdc++.h>
2 using namespace std;
2 using lld = int64_t;
2 using llu = uint64_t;
2 using llf = long double;
2 using PII = pair<int,int>;
3 using PIL = pair<int,lld>;
4 using PLI = pair<lld,int>;
4 using PLL = pair<lld,lld>;
4 template<typename T>
4 using maxHeap = priority_queue<T,vector<T>,less<T>>;
4 template<typename T>
5 using minHeap = priority_queue<T,vector<T>,greater<T>>;
5 #define FF first
5 #define SS second
6 #define SZ(x) ((int)(x).size())
6 #define ALL(x) begin(x), end(x)
6 #define PB push_back
7 #define WC(x) while((x)-->0)
7 template<typename Iter>
8 ostream& out(ostream& s, Iter b, Iter e) {
8     s<<"[";
8     for ( auto it=b; it!=e; it++ ) s<<(it==b?" ":"")<<*
9         it;
9     s<<"]";
9     return s;
10 }
10 template<typename A, typename B>
10 ostream& operator <<( ostream& s, const pair<A,B> &p )
10 { return s<<"("<<p.FF<<" "<<p.SS<<" )"; }
10 template<typename T>
11 ostream& operator <<( ostream& s, const vector<T> &c )
11 { return out(s,ALL(c)); }
11 bool debug = 0;
11 #define DUMP(x) if(debug) cerr<<"_PRETTY_FUNCTION_<<"
12 "<<"_LINE_<<" - "<<(#x)<<"="<<(x)<<"'\n'
12 template<typename T>
12 void DEBUG(const T& x){if(debug) cerr<<x;}
12 template<typename T, typename... Args>
12 void DEBUG(const T& head,const Args& ...tail){
13     if(debug){cerr<<head; DEBUG(tail...);}
13 }
13 int main(int argc, char* argv[]){
14     if(argc>1 and string(argv[1])=="-D") debug=1;
15     if(!debug){ios_base::sync_with_stdio(0);cin.tie(0)}
15     ;}
15     return 0;
16 }

```

1.2 IncreaseStackSize

```

//stack resize
asm( "mov %0,%esp\n" ::"g"(mem+10000000) );
//change esp to rsp if 64-bit system

//stack resize (linux)
#include <sys/resource.h>
void increase_stack_size() {
    const rlim_t ks = 64*1024*1024;
    struct rlimit rl;
    int res=getrlimit(RLIMIT_STACK, &rl);
    if(res==0){
        if(rl.rlim_cur<ks){
            rl.rlim_cur=ks;
            res=setrlimit(RLIMIT_STACK, &rl);
        }
    }
}

// craziest way
static void run_with_stack_size(void (*func)(), size_t
    stsize) {
    char *stack, *send;
    stack=(char *)malloc(stsize);
    send=stack+stsize-16;
    send=(char *) ((uintptr_t)send/16*16);
    asm volatile(
        "mov %%rsp, (%0)\n"

```

```

    "mov %0, %%rsp\n"
    :
    : "r" (send));
func();
asm volatile(
    "mov (%0), %%rsp\n"
    :
    : "r" (send));
free(stack);
}

```

1.3 Pragma optimization

```

#pragma GCC optimize("Ofast,no-stack-protector,no-math-
    errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4,popcnt,abm
    ,mmx,avx,tune=native")

```

1.4 Debugger

```

#!/usr/bin/env python3
import subprocess, platform

os_name = platform.system()
cmd = []
prefix = ""

if os_name == 'Windows':
    cmd = ["cmd", "/C"]
else:
    cmd = ["bash", "-c"]
    prefix = "./"

def GetTestData(exe):
    myout = subprocess.check_output(cmd + ["%s%s"%(
        prefix, exe)])
    return myout.decode("utf8")

def Judge(a,b,testdata):
    f = open("test.in", "w+")
    f.write(testdata)
    f.close()
    myout = subprocess.check_output(cmd + ["%s%s < test.
        in"%(prefix, a)])
    ansout = subprocess.check_output(cmd + ["%s%s < test
        .in"%(prefix, b)])
    if not myout == ansout:
        print("answer: %s"%ansout.decode("utf8"),end="")
        print("output: %s"%myout.decode("utf8"),end="")
        print("WA!")
        return False
    return True

if __name__ == '__main__':
    cnt = 0
    isOK = True
    while isOK:
        cnt += 1
        print(cnt)
        isOK = Judge("1397.exe", "test.exe", GetTestData(
            "gen.exe"))

```

1.5 Quick Random

```

// PRNG {{{
template<class T, T x1, T x2, T x3, int y1, int y2, int
    y3>
struct PRNG {
    using S = typename std::make_signed<T>::type;
    T s;
    PRNG(T _s = 0) : s(_s) {}
    T next() {
        T z = (s += x1);
        z = (z ^ (z >> y1)) * x2;
        z = (z ^ (z >> y2)) * x3;
        return z ^ (z >> y3);
    }
    T next(T n) { return next() % n; }
    S next(S l, S r) { return l + next(r - l + 1); }
    T operator()() { return next(); }
    T operator()(T n) { return next(n); }
}

```

```

S operator()(S l, S r) { return next(l, r); }
static T gen(T s) { return PRNG(s)(); }
template<class U>
void shuffle(U first, U last) {
    size_t n = last - first;
    for (size_t i = 0; i < n; i++) swap(first[i],
        first[next(i + 1)]);
}

};

using R32 = PRNG<uint32_t, 0x9E3779B1, 0x85EBCA6B, 0
    xC2B2AE35, 16, 13, 16>;
R32 r32;

using R64 = PRNG<uint64_t, 0x9E3779B97F4A7C15, 0
    xBF58476D1CE4E5B9, 0x94D049BB133111EB, 30, 27, 31>;
R64 r64;
// }}}

```

1.6 IO Optimization

```

static inline int gc() {
    static char buf[1 << 20], *p = buf, *end = buf;
    if (p == end) {
        if ((end = buf + fread(buf, 1, 1 << 20, stdin))
            == buf) return EOF;
        p = buf;
    }
    return *p++;
}

template<typename T>
static inline bool gn(T &_){
    register int c = gc(); register T __ = 1; _ = 0;
    while(!isdigit(c) and c!=EOF and c!='-') c = gc();
    if(c == '-') { __ = -1; c = gc(); }
    if(c == EOF) return false;
    while(isdigit(c)) _ = _ * 10 + c - '0', c = gc();
    _ *= __;
    return true;
}

template <typename T, typename ...Args>
static inline bool gn(T &x, Args& ...args){return gn(x)
    and gn(args...);}

```

2 Data Structure

2.1 Bigint

```

class BigInt{
private:
    using lld = int_fast64_t;
#define PRINTF_ARG PRIuFAST64
#define LOG_BASE_STR "9"
    static constexpr lld BASE = 1000000000;
    static constexpr int LOG_BASE = 9;
    vector<lld> dig;
    bool neg;
    inline int len(){const{return (int)dig.size();}}
    inline int cmp_minus(const BigInt& a) const {
        if(len() == 0 and a.len() == 0) return 0;
        if(neg ^ a.neg) return (int)a.neg*2 - 1;
        if(len() != a.len()) return neg?a.len()-len():len
            ()-a.len();
        for(int i=len()-1;i>=0;i--) if(dig[i] != a.dig[i]
            ) {
            return neg?a.dig[i]-dig[i]:dig[i]-a.dig[i];
        }
        return 0;
    }
    inline void trim(){
        while(!dig.empty() and dig.back()==0) dig.
            pop_back();
        if(dig.empty()) neg = false;
    }
public:
    BigInt(): dig(vector<lld>()), neg(false){}
    BigInt(lld a): dig(vector<lld>()){
        neg = a<0; dig.push_back(abs(a));
        trim();
    }
}

```

```

}
BigInt(const string& a): dig(vector<lld>()) {
    assert(!a.empty()); neg = (a[0]=='-');
    for(int i=((int)(a.size()))-1; i>=neg; i-=LOG_BASE)
    {
        lld cur = 0;
        for(int j=min(LOG_BASE-1, i-neg); j>=0; j--) cur
            = cur*10+a[i-j]-'0';
        dig.push_back(cur);
    } trim();
}

inline bool operator<(const BigInt& a) const {return
    cmp_minus(a)<0;}
inline bool operator<=(const BigInt& a) const {return
    cmp_minus(a)<=0;}
inline bool operator==(const BigInt& a) const {return
    cmp_minus(a)==0;}
inline bool operator!=(const BigInt& a) const {return
    cmp_minus(a)!=0;}
inline bool operator>(const BigInt& a) const {return
    cmp_minus(a)>0;}
inline bool operator>=(const BigInt& a) const {return
    cmp_minus(a)>=0;}

BigInt operator-() const {
    BigInt ret = *this;
    ret.neg ^= 1;
    return ret;
}

BigInt operator+(const BigInt& a) const {
    if(neg) return -(*this)+(-a);
    if(a.neg) return (*this)-(-a);
    int n = max(a.len(), len());
    BigInt ret; ret.dig.resize(n);
    lld pro = 0;
    for(int i=0; i<n; i++) {
        ret.dig[i] = pro;
        if(i < a.len()) ret.dig[i] += a.dig[i];
        if(i < len()) ret.dig[i] += dig[i];
        pro = 0;
        if(ret.dig[i] >= BASE) pro = ret.dig[i]/BASE;
        ret.dig[i] -= BASE*pro;
    }
    if(pro != 0) ret.dig.push_back(pro);
    return ret;
}

BigInt operator-(const BigInt& a) const {
    if(neg) return -(*this) - (-a);
    if(a.neg) return (*this) + (-a);
    int diff = cmp_minus(a);
    if(diff < 0) return -(a - (*this));
    if(diff == 0) return 0;
    BigInt ret; ret.dig.resize(len(), 0);
    for(int i=0; i<len(); i++) {
        ret.dig[i] += dig[i];
        if(i < a.len()) ret.dig[i] -= a.dig[i];
        if(ret.dig[i] < 0) {
            ret.dig[i] += BASE;
            ret.dig[i+1]--;
        }
    }
    ret.trim();
    return ret;
}

BigInt operator*(const BigInt& a) const {
    if(len()==0 or a.len()==0) return 0;
    BigInt ret; ret.dig.resize(len()+a.len()+1);
    ret.neg = neg ^ a.neg;
    for(int i=0; i<len(); i++) for(int j=0; j<a.len(); j
        ++){
        ret.dig[i+j] += dig[i] * a.dig[j];
        if(ret.dig[i+j] >= BASE) {
            lld x = ret.dig[i+j] / BASE;
            ret.dig[i+j+1] += x;
            ret.dig[i+j] -= x * BASE;
        }
    }
    ret.trim();
    return ret;
}

BigInt operator/(const BigInt& a) const {
    assert(a.len());
    if(len() < a.len()) return 0;
    BigInt ret; ret.dig.resize(len()-a.len()+1);
    ret.neg = a.neg;
    for(int i=len()-a.len(); i>=0; i--) {
        lld l = 0, r = BASE;

```

```

        while(r-l > 1) {
            lld mid = (l+r)>>1;
            ret.dig[i] = mid;
            if(ret*a <= (neg?-(*this):(*this))) l = mid;
            else r = mid;
        }
        ret.dig[i] = l;
    }
    ret.neg ^= neg; ret.trim();
    return ret;
}

BigInt operator%(const BigInt& a) const {
    return (*this) - (*this) / a * a;
}

friend BigInt abs(BigInt a) {
    a.neg = 1; return a;
}

friend void swap(BigInt& a, BigInt& b) {
    swap(a.dig, b.dig); swap(a.neg, b.neg);
}

friend istream& operator>>(istream& ss, BigInt& a) {
    string s; ss >> s;
    a = s;
    return ss;
}

friend ostream& operator<<(ostream& ss, const
    BigInt& a) {
    if(a.len() == 0) return ss << '0';
    if(a.neg) ss << '-';
    ss << a.dig.back();
    for(int i=a.len()-2; i>=0; i--) ss << setw(LOG_BASE
        ) << setfill('0') << a.dig[i];
    return ss;
}

inline void print() const {
    if(len() == 0) {putchar('0'); return;}
    if(neg) putchar('-');
    printf("%" PRINTF_ARG, dig.back());
    for(int i=len()-2; i>=0; i--) printf("%0"
        LOG_BASE_STR PRINTF_ARG, dig[i]);
}

#undef PRINTF_ARG
#undef LOG_BASE_STR
};

```

2.2 Linear Basis

```

struct LinearBasis {
private:
    int n, sz;
    vector<llu> B;
    inline llu two(int x) {return ((llu)1)<<x;}
public:
    void init(int n_) {
        n = n_; B.clear();
        B.resize(n); sz = 0;
    }

    void insert(llu x) {
        // add x into B
        for(int i=n-1; i>=0; i--) if(two(i) & x) {
            if(B[i] x ^= B[i];
            else {
                B[i] = x; sz++;
                for(int j=i-1; j>=0; j--)
                    if(B[j] and two(j) & B[i])
                        B[i] ^= B[j];
                for(int j=i+1; j<n; j++)
                    if(two(i) & B[j])
                        B[j] ^= B[i];
                break;
            }
        }
    }

    inline int size() {return sz;}
    bool check(llu x) {
        // is x in span(B) ?
        for(int i=n-1; i>=0; i--) if(two(i) & x) {
            if(B[i] x ^= B[i];
            else return false;
        }
        return true;
    }

    llu kth_small(llu k) {
        /** 1-base would always > 0 **/

```

```

    /** should check it */
    // if we should choose at most one element
    // but size(B)(vectors in B) == N(original
    // elements)
    // then we can't get 0
    ll ret = 0;
    for(int i=0;i<n;i++) if(B[i]) {
        if(k & 1) ret ^= B[i];
        k >>= 1;
    }
    return ret;
}
} base;

```

2.3 unordered_map

```

#include <ext/pb_ds/assoc_container.hpp>
using __gnu_pbds::cc_hash_table;
using __gnu_pbds::gp_hash_table;
template<typename A, typename B> using hTable1 =
    cc_hash_table<A,B>;
template<typename A, typename B> using hTable2 =
    gp_hash_table<A,B>;

```

2.4 extc_balance_tree

```

#include <functional>
#include <ext/pb_ds/assoc_container.hpp>
using std::less;
using std::greater;
using __gnu_pbds::tree;
using __gnu_pbds::rb_tree_tag;
using __gnu_pbds::ov_tree_tag;
using __gnu_pbds::splay_tree_tag;
using __gnu_pbds::null_type;
using __gnu_pbds::tree_order_statistics_node_update;

template<typename T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;

template<typename A, B>
using ordered_map = tree<A, B, less<A>, rb_tree_tag,
    tree_order_statistics_node_update>;

int main() {
    ordered_set<int> ss;
    ordered_map<int,int> mm;
    ss.insert(1);
    ss.insert(5);
    assert(*ss.find_by_order(0)==1);
    assert(ss.order_of_key(-1)==0);
    assert(ss.order_of_key(87)==2);
    return 0;
}

```

2.5 extc_heap

```

#include <functional>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
using std::less;
using std::greater;
using __gnu_pbds::priority_queue;
using __gnu_pbds::pairing_heap_tag;
using __gnu_pbds::binary_heap_tag;
using __gnu_pbds::binomial_heap_tag;
using __gnu_pbds::rc_binomial_heap_tag;
using __gnu_pbds::thin_heap_tag;

int main() {
    priority_queue<int,less<int>,pairing_heap_tag> pq1,
        pq2;
    pq1.push(1);
    pq2.push(2);
    pq1.join(pq2);
    assert(pq2.size()==0);
    auto it = pq1.push(87);
    pq1.modify(it, 19);
}

```

```

while(!pq1.empty()){
    pq1.top();
    pq1.pop();
}
return 0;
}

```

2.6 SkewHeap

```

#include <functional>
using std::less;

template<typename T, typename cmp=less<T> >
class SkewHeap{
private:
    struct SkewNode{
        T x;
        SkewNode *lc, *rc;
        SkewNode(T a=0):x(a), lc(nullptr), rc(nullptr) {}
    } *root;
    cmp CMP_;
    size_t count;
    SkewNode* Merge(SkewNode* a, SkewNode* b){
        if(!a or !b) return a?a:b;
        if(CMP_(a->x, b->x)) swap(a, b);
        a->rc = Merge(a->rc, b);
        swap(a->lc, a->rc);
        return a;
    }
    void clear(SkewNode*& a){
        if(!a) return;
        clear(a->lc); clear(a->rc);
        delete a; a = nullptr;
    }
public:
    SkewHeap(): root(nullptr), count(0){}
    bool empty(){return count==0;}
    size_t size(){return count;}
    T top(){return root->x;}
    void clear(){clear(root);count = 0;}
    void push(const T& x){
        SkewNode* a = new SkewNode(x);
        count += 1;
        root = Merge(root, a);
    }
    void join(SkewHeap& a){
        count += a.count; a.count = 0;
        root = Merge(root, a.root);
    }
    void pop(){
        count -= 1;
        SkewNode* rt = Merge(root->lc, root->rc);
        delete root; root = rt;
    }
    friend void swap(SkewHeap& a, SkewHeap& b){
        swap(a.root, b.root);
    }
};

```

2.7 Disjoint Set

```

class DJS{
private:
    vector<int> fa, sz, sv;
    vector<pair<int*, int>> opt;
    inline void assign(int *k, int v){
        opt.emplace_back(k, *k);
        *k = v;
    }
public:
    inline void init(int n){
        fa.resize(n); iota(fa.begin(), fa.end(), 0);
        sz.resize(n); fill(sz.begin(), sz.end(), 1);
        opt.clear();
    }
    int query(int x){
        if(fa[x] == x) return x;
        return query(fa[x]);
    }
    inline void merge(int a, int b){
        int af = query(a), bf = query(b);
    }
};

```

```

    if (af == bf) return;
    if (sz[af] < sz[bf]) swap(af, bf);
    assign(&fa[bf], fa[af]);
    assign(&sz[af], sz[af]+sz[bf]);
}
inline void save() {sv.push_back((int)opt.size());}
inline void undo() {
    int ls = sv.back(); sv.pop_back();
    while((int)opt.size() > ls){
        pair<int*, int> cur=opt.back();
        *cur.first = cur.second;
        opt.pop_back();
    }
}
};

```

2.8 Treap

```

class Treap{
private:
    const int MEM = 500000 + 5;
    unsigned seed;
    inline unsigned myrand(){
        static unsigned seed = time(NULL);
        seed = seed*seed*127 + seed*227 + 2147483587;
        seed ^= seed*97;
        seed /= 7123;
        return seed;
    }
    struct node{
        node *lc, *rc;
        int pri, size, val;
        node(){}
        node(int x):
            lc(nullptr),
            rc(nullptr),
            pri(myrand()),
            size(1),
            val(x)
        {}
        inline void pull(){
            size = 1;
            if(lc) size += lc->size;
            if(rc) size += rc->size;
        }
    } *root, pool[MEM];
    int mem_;
    inline node* new_node(int x){
        static int mem_ = 0;
        assert(mem_ < MEM);
        pool[mem_] = node(x);
        return &pool[mem_++];
    }
    inline int sz(node* x){return x?x->size:0;}
    node* merge(node *a, node *b){
        if(!a or !b) return a?a:b;
        if(a->pri > b->pri){
            a->rc = merge(a->rc, b);
            a->pull();
            return a;
        }else{
            b->lc = merge(a, b->lc);
            b->pull();
            return b;
        }
    }
    void split(Treap* t, int k, Treap*& a, Treap*& b){
        if(!t) a=b=nullptr;
        else if (sz(t->lc) < k){
            a = t;
            split(t->rc, k - sz(t->lc) - 1, a->rc, b);
            a->pull();
        }else{
            b = t;
            split(t->lc, k, a, b->lc);
            b->pull();
        }
    }
    int oOk(node* rr, int x){
        if(rr==NULL) return 0;
        if((rr->val) < x) return gSize(rr->l)+oOk(rr->r,
            x)+1;
        else return oOk(rr->l, x);
    }
}

```

```

public:
    Treap(){root=nullptr;seed=time(NULL);mem_=0;}
    void do_something_at(int l, int r){
        // 1-base [l, r]
        split(root, l-1, tl, root);
        split(root, r-l+1, root, tr);
        root = merge(tl, merge(root, tr));
    }
    void insert(int x){
        node *a, *b;
        split(root, x, a, b);
        root = merge(merge(a, new node(x)), b);
        root->size = gSize(root->l)+gSize(root->r)+1;
    }
    void remove(int x){
        //need debug may contain bugs
        node *a, *b, *c, *d;
        split(root, x, a, b);
        a->size = gSize(a->l)+gSize(a->r);
        split(a, x-1, c, d);
        root = merge(b, c);
        root->size = gSize(root->l)+gSize(root->r);
        delete d;
    }
    int order_of_key(int x){return oOk(root,x);}
};

```

2.9 SparseTable

```

template<typename T, typename Cmp_=std::less<T>>
class SparseTable{
private:
    vector<vector<T>> table;
    vector<int> lg;
    T cmp_(T a, T b){
        return Cmp_()(a, b)?a:b;
    }
public:
    void init(T arr[], int n){
        // 0-base
        lg.resize(n+1);
        lg[0] = -1, lg[1] = 0;
        for(int i=2;i<=n;i++) lg[i] = lg[i>>1]+1;
        table.resize(lg[n]+1);
        table[0].resize(n);
        for(int i=0;i<n;i++) table[0][i] = arr[i];
        for(int i=1;i<=lg[n];i++){
            int len = 1<<(i-1), sz = 1<<i;
            table[i].resize(n-sz+1);
            for(int j=0;j<=n-sz;j++){
                table[i][j] = cmp_(table[i-1][j], table
                    [i-1][j+len]);
            }
        }
    }
    T query(int l, int r){
        // 0-base [l, r]
        int wh = lg[r-l], len=1<<wh;
        return cmp_(table[wh][l], table[wh][r-len]);
    }
};

```

2.10 FenwickTree

```

#include <vector>
using std::vector;

template<typename T>
class BIT{
#define ALL(x) begin(x), end(x)
private:
    vector<T> arr;
    int n;
    inline int lowbit(int x){return x & (-x);}
    T query(int x){
        T ret = 0;
        while(x > 0){
            ret += arr[x];
            x -= lowbit(x);
        }
        return ret;
    }
}

```

```

public:
    void init(int n_){
        n = n_;
        arr.resize(n);
        fill(arr.begin(), arr.end(), 0);
    }
    void modify(int pos, T v){
        while(pos < n){
            arr[pos] += v;
            pos += lowbit(pos);
        }
    }
    T query(int l, int r){
        // 1-base (l, r]
        return query(r) - query(l);
    }
}

#undef ALL
};

template<typename T>
class BIT{
#define ALL(x) begin(x), end(x)
private:
    vector<T> arr;
    int n;
    inline int lowbit(int x){return x & (-x);}
    void add(int s, int v){
        while(s){
            arr[s] += v;
            s -= lowbit(s);
        }
    }
public:
    void init(int n_){
        n = n_;
        arr.resize(n);
        fill(ALL(arr), 0);
    }
    void add(int l, int r, T v){
        // 1-base (l, r]
        add(l, -v);
        add(r, v);
    }
    T query(int x){
        T r = 0;
        while(x < size()){
            r += arr[x];
            x += lowbit(x);
        }
        return r;
    }
}

#undef ALL
};

```

3 Graph

3.1 BCC Edge

```

class BCC{
private:
    vector<int> low, dfn;
    int cnt;
    vector<bool> bcc;
    vector<vector<PII>> G;
    void dfs(int w, int f){
        dfn[w] = cnt++;
        low[w] = dfn[w];
        for(auto i: G[w]){
            int u = i.FF, t = i.SS;
            if(u == f) continue;
            if(dfn[u] != 0){
                low[w] = min(low[w], dfn[u]);
            } else{
                dfs(u, w);
                low[w] = min(low[w], low[u]);
                if(low[u] > dfn[w]) bcc[t] = true;
            }
        }
    }
public:
    void init(int n, int m){
        G.resize(n);

```

```

        fill(G.begin(), G.end(), vector<PII>());
        bcc.clear(); bcc.resize(m);
        low.clear(); low.resize(n);
        dfn.clear(); dfn.resize(n);
        cnt = 0;
    }
    void add_edge(int u, int v){
        // should check for multiple edge
        G[u].PB({v, cnt});
        G[v].PB({u, cnt});
        cnt++;
    }
    void solve(){cnt = 1; dfs(0, 0);}
    // the id will be same as insert order, 0-base
    bool is_bcc(int x){return bcc[x];}
} bcc;

```

3.2 BCC Vertex

```

class BCC{
private:
    vector<vector<pair<int,int>>> G;
    vector<int> dfn, low, id, sz;
    vector<bool> vis, ap;
    int n, ecnt, bcnt;
    void tarjan(int u, int f, int d){
        vis[u] = true;
        dfn[u] = low[u] = d;
        int child = 0;
        for(auto e: G[u]) if(e.first != f){
            int v = e.first;
            if(vis[v]){
                low[u] = min(low[u], dfn[v]);
            } else{
                tarjan(v, u, d+1);
                if(low[v] >= dfn[u]) ap[u] = true;
                low[u] = min(low[u], low[v]);
                child += 1;
            }
        }
        if(dfn[u] == 0 and child <= 1) ap[u] = false;
    }
    void bfs_bcc(int x){
        // not sure
        queue<int> bfs;
        bfs.push(x); vis[x] = true;
        while(!bfs.empty()){
            int u = bfs.front(); bfs.pop();
            for(auto e: G[u]){
                id[e.second] = bcnt;
                if(ap[e.first] or vis[e.first]) continue;
                bfs.push(e.first); vis[e.first] = true;
                sz[bcnt] += 1;
            }
        }
    }
public:
    void init(int n_){
        n = n_; G.clear(); G.resize(n);
        dfn.resize(n); low.resize(n);
        vis.clear(); vis.resize(n);
        ap.clear(); ap.resize(n);
        ecnt = 0, bcnt = 0;
    }
    void add_edge(int u, int v){
        assert(0 <= u and u < n);
        assert(0 <= v and v < n);
        G[u].emplace_back(v, ecnt);
        G[v].emplace_back(u, ecnt);
        ecnt += 1;
    }
    void solve(){
        for(int i=0; i<n; i++) if(!vis[i]) {
            tarjan(i, i, 0);
        }
        id.resize(ecnt);
        vis.clear(); vis.resize(n);
        sz.clear(); sz.resize(n);
        for(int i=0; i<n; i++) if(ap[i]){
            bfs_bcc(i); bcnt += 1;
        }
    }
    bool isAP(int x){return ap[x];}
}

```

```

int count(){return bcnt;}
// bcc_id of edges by insert order (0-base)
int get_id(int x){return id[x];}
// bcc size by bcc_id
int get_size(int x){return sz[x];}
} bcc;

```

3.3 Strongly Connected Components

```

class SCC{
private:
    int n, num_;
    vector<vector<int>> G, rG;
    vector<int> ord, num;
    bool vis[N];
    void dfs(int u){
        if(vis[u]) return;
        vis[u]=1;
        for(auto v: G[u]) dfs(v);
        ord.PB(u);
    }
    void rdfs(int u){
        if(vis[u]) return;
        num[u] = num_;
        vis[u] = 1;
        for(auto v: rG[u]) rdfs(v);
    }
public:
    inline void init(int n_){
        G.resize(n); rG.resize(n);
        n=n_, num_=0;
        num.resize(n);
        for(int i=0;i<n;i++) G[i].clear();
        for(int i=0;i<n;i++) rG[i].clear();
    }
    inline void add_edge(int st, int ed){
        G[st].PB(ed);
        rG[ed].PB(st);
    }
    void solve(){
        memset(vis, 0, sizeof(vis));
        for(int i=0;i<n;i++){
            if(!vis[i]) dfs(i);
        }
        reverse(ALL(ord));
        memset(vis, 0, sizeof(vis));
        for(auto i: ord){
            if(!vis[i]){
                rdfs(i);
                num_++;
            }
        }
        inline int get_id(int x){return num[x];}
        inline int count(){return num_;}
    }
} scc;

```

3.4 Bipartite Matching

```

#include <bits/stdc++.h>
using namespace std;
#define N 500

class BipartiteMatching{
private:
    vector<int> X[N], Y[N];
    int fX[N], fY[N], n;
    bitset<N> walked;
    bool dfs(int x){
        for(auto i:X[x]){
            if(walked[i]) continue;
            walked[i]=1;
            if(fY[i]==-1||dfs(fY[i])){
                fY[i]=x; fX[x]=i;
                return 1;
            }
        }
        return 0;
    }
public:
    void init(int _n){
        n=_n;
    }

```

```

for(int i=0;i<n;i++){
    X[i].clear();
    Y[i].clear();
    fX[i]=fY[i]=-1;
}
walked.reset();
}
void AddEdge(int x, int y){
    X[x].push_back(y);
    Y[y].push_back(x);
}
int solve(){
    int cnt = 0;
    for(int i=0;i<n;i++){
        walked.reset();
        if(dfs(i)) cnt++;
    }
    // return how many pair matched
    return cnt;
}
};

```

3.5 MinimumCostMaximumFlow

```

class MiniCostMaxiFlow{
    typedef int CapT;
    typedef lld WeiT;
    typedef pair<CapT, WeiT> PCW;
    const CapT INF_CAP = 1<<30;
    const WeiT INF_WEI = 1LL<<60;
    const int MAXV = N;
private:
    struct Edge{
        int to, back;
        WeiT wei;
        CapT cap;
        Edge(){}
        Edge(int a, int b, WeiT c, CapT d): to(a), back
            (b), wei(c), cap(d) {}
    };
    int ori, edd, V;
    vector<Edge> G[MAXV];
    int fa[MAXV], wh[MAXV];
    bool inq[MAXV];
    WeiT dis[MAXV];
    PCW SPFA(){
        for(int i=0;i<V;i++) inq[i]=0;
        for(int i=0;i<V;i++) dis[i]=INF_WEI;
        queue<int> qq;
        qq.push(ori);
        dis[ori]=0;
        while(!qq.empty()){
            int u = qq.front(); qq.pop();
            inq[u]=0;
            for(int i=0;i<SZ(G[u]);i++){
                Edge e = G[u][i];
                int v = e.to;
                WeiT d = e.wei;
                if(e.cap > 0 and dis[v] > dis[u]+d){
                    dis[v]=dis[u]+d;
                    fa[v]=u;
                    wh[v] = i;
                    if(inq[v]) continue;
                    qq.push(v);
                    inq[v]=1;
                }
            }
        }
        if(dis[edd]==INF_WEI) return {-1, -1};
        CapT mw=INF_CAP;
        for(int i=edd;i!=ori;i=fa[i]){
            mw = min(mw, G[fa[i]][wh[i]].cap);
        }
        for(int i=edd;i!=ori;i=fa[i]){
            auto &eg = G[fa[i]][wh[i]];
            eg.cap -= mw;
            G[eg.to][eg.back].cap += mw;
        }
        return {mw, dis[edd]};
    }
public:
    void init(int a, int b, int n=MAXV){
        V=n;
        ori = a;
    }

```



```

    edd = b;
    for(int i=0;i<n;i++) G[i].clear();
}
void addEdge(int st, int ed, WeiT w, CapT c){
    G[st].PB(Edge(ed, SZ(G[ed]), w, c));
    G[ed].PB(Edge(st, SZ(G[st])-1, -w, 0));
}
PCW solve(){
    CapT cc=0; WeiT ww=0;
    while(true){
        PCW ret = SPFA();
        if(ret.FF===-1) break;
        cc += ret.FF;
        ww += ret.SS;
    }
    return {cc, ww};
}
} mcmf;

```

3.6 MaximumFlow

```

class Dinic{
private:
    using CapT = int64_t;
    struct Edge{
        int to, rev;
        CapT cap;
    };
    int n, st, ed;
    vector<vector<Edge>> G;
    vector<int> lv;
    bool BFS(){
        fill(lv.begin(), lv.end(), -1);
        queue<int> bfs;
        bfs.push(st);
        lv[st] = 0;
        while(!bfs.empty()){
            int u = bfs.front(); bfs.pop();
            for(auto e: G[u]){
                if(e.cap <= 0 or lv[e.to]!==-1) continue;
                lv[e.to] = lv[u] + 1;
                bfs.push(e.to);
            }
        }
        return (lv[ed]!==-1);
    }
    CapT DFS(int u, CapT f){
        if(u == ed) return f;
        CapT ret = 0;
        for(auto& e: G[u]){
            if(e.cap <= 0 or lv[e.to]!==(lv[u]+1)) continue;
            CapT nf = DFS(e.to, min(f, e.cap));
            ret += nf; e.cap -= nf; f -= nf;
            G[e.to][e.rev].cap += nf;
            if(f == 0) return ret;
        }
        if(ret == 0) lv[u] = -1;
        return ret;
    }
public:
    void init(int n_, int st_, int ed_){
        n = n_, st = st_, ed = ed_;
        G.resize(n); lv.resize(n);
        fill(G.begin(), G.end(), vector<Edge>());
    }
    void add_edge(int u, int v, CapT c){
        G[u].push_back({v, (int) G[v].size(), c});
        G[v].push_back({u, (int) G[u].size()-1, 0});
    }
    CapT max_flow(){
        CapT ret = 0;
        while(BFS()){
            CapT f = DFS(st, numeric_limits<CapT>::max());
            ret += f;
            if(f == 0) break;
        }
        return ret;
    }
} flow;

```

3.7 Kuhn Munkres

```

struct KM{
    // Maximum Bipartite Weighted Matching (Perfect Match)
    static const int MXN = 650;
    static const int INF = 2147483647; // LL
    int n, match[MXN], vx[MXN], vy[MXN];
    int edge[MXN][MXN], lx[MXN], ly[MXN], slack[MXN];
    // ^^^^ LL
    void init(int _n){
        n = _n;
        for(int i=0; i<n; i++) for(int j=0; j<n; j++)
            edge[i][j] = 0;
    }
    void addEdge(int x, int y, int w) // LL
    { edge[x][y] = w; }
    bool DFS(int x){
        vx[x] = 1;
        for (int y=0; y<n; y++){
            if (vy[y]) continue;
            if (lx[x]+ly[y] > edge[x][y]){
                slack[y]=min(slack[y], lx[x]+ly[y]-edge[x][y]);
            } else {
                vy[y] = 1;
                if (match[y] == -1 || DFS(match[y]))
                    { match[y] = x; return true; }
            }
        }
        return false;
    }
    int solve(){
        fill(match, match+n, -1);
        fill(lx, lx+n, -INF); fill(ly, ly+n, 0);
        for (int i=0; i<n; i++){
            for (int j=0; j<n; j++)
                lx[i] = max(lx[i], edge[i][j]);
            for (int i=0; i<n; i++){
                fill(slack, slack+n, INF);
                while (true){
                    fill(vx, vx+n, 0); fill(vy, vy+n, 0);
                    if ( DFS(i) ) break;
                    int d = INF; // long long
                    for (int j=0; j<n; j++)
                        if (!vy[j]) d = min(d, slack[j]);
                    for (int j=0; j<n; j++){
                        if (vx[j]) lx[j] -= d;
                        if (vy[j]) ly[j] += d;
                        else slack[j] -= d;
                    }
                }
            }
        }
        int res=0;
        for (int i=0; i<n; i++)
            res += edge[match[i]][i];
        return res;
    }
} graph;

```

3.8 2-SAT

```

// 2-SAT solver based on Kosaraju's algorithm.
// Variables are 0-based. Positive variables are stored
// in vertices 2n, corresponding negative variables
// in 2n+1
// TODO: This is quite slow (3x-4x slower than Gabow's
// algorithm)
struct TwoSat {
    int n;
    vector<vector<int>> adj, radj, scc;
    vector<int> sid, vis, val;
    stack<int> stk;
    int scnt;

    // n: number of variables, including negations
    TwoSat(int n): n(n), adj(n), radj(n), sid(n), vis(n),
        val(n, -1) {}

    // adds an implication
    void impl(int x, int y) { adj[x].push_back(y); radj[y]
        .push_back(x); }
    // adds a disjunction
    void vee(int x, int y) { impl(x^1, y); impl(y^1, x);
    }
}

```



```
// forces variables to be equal
void eq(int x, int y) { impl(x, y); impl(y, x); impl(
    x^1, y^1); impl(y^1, x^1); }
// forces variable to be true
void tru(int x) { impl(x^1, x); }

void dfs1(int x) {
    if (vis[x]++) return;
    for (int i = 0; i < adj[x].size(); i++) {
        dfs1(adj[x][i]);
    }
    stk.push(x);
}

void dfs2(int x) {
    if (!vis[x]) return; vis[x] = 0;
    sid[x] = scnt; scc.back().push_back(x);
    for (int i = 0; i < radj[x].size(); i++) {
        dfs2(radj[x][i]);
    }
}

// returns true if satisfiable, false otherwise
// on completion, val[x] is the assigned value of
// variable x
// note, val[x] = 0 implies val[x^1] = 1
bool two_sat() {
    scnt = 0;
    for (int i = 0; i < n; i++) {
        dfs1(i);
    }
    while (!stk.empty()) {
        int v = stk.top(); stk.pop();
        if (vis[v]) {
            scc.push_back(vector<int>());
            dfs2(v);
            scnt++;
        }
    }
    for (int i = 0; i < n; i += 2) {
        if (sid[i] == sid[i+1]) return false;
    }
    vector<int> must(scnt);
    for (int i = 0; i < scnt; i++) {
        for (int j = 0; j < scc[i].size(); j++) {
            val[scc[i][j]] = must[i];
            must[sid[scc[i][j]^1]] = !must[i];
        }
    }
    return true;
}
};
```

3.9 HeavyLightDecomp

```
#define REP(i, s, e) for(int i = (s); i <= (e); i++)
#define REPD(i, s, e) for(int i = (s); i >= (e); i--)
const int MAXN = 100010;
const int LOG = 19;
struct HLD{
    int n;
    vector<int> g[MAXN];
    int sz[MAXN], dep[MAXN];
    int ts, tid[MAXN], tdi[MAXN], tl[MAXN], tr[MAXN];
    // ts : timestamp , useless after yutruli
    // tid[ u ] : pos. of node u in the seq.
    // tdi[ i ] : node at pos i of the seq.
    // tl , tr[ u ] : subtree interval in the seq. of
    // node u
    int prt[MAXN][LOG], head[MAXN];
    // head[ u ] : head of the chain contains u
    void dfssz(int u, int p){
        dep[u] = dep[p] + 1;
        prt[u][0] = p; sz[u] = 1; head[u] = u;
        for(int& v:g[u]) if(v != p){
            dep[v] = dep[u] + 1;
            dfssz(v, u);
            sz[u] += sz[v];
        }
    }
    void dfs1(int u){
        ts++;
        tid[u] = tl[u] = tr[u] = ts;
        tdi[tid[u]] = u;
```

```
sort(ALL(g[u]),
    [&](int a, int b){return sz[a] > sz[b];});
bool flag = 1;
for(int& v:g[u]) if(v != prt[u][0]){
    if(flag) head[v] = head[u], flag = 0;
    dfs1(v);
    tr[u] = tr[v];
}
}
inline int lca(int a, int b){
    if(dep[a] > dep[b]) swap(a, b);
    int diff = dep[b] - dep[a];
    REPD(k, LOG-1, 0) if(diff & (1<<k)){
        b = prt[b][k];
    }
    if(a == b) return a;
    REPD(k, LOG-1, 0) if(prt[a][k] != prt[b][k]){
        a = prt[a][k]; b = prt[b][k];
    }
    return prt[a][0];
}
void init( int _n ){
    n = _n; REP( i , 1 , n ) g[ i ].clear();
}
void addEdge( int u , int v ){
    g[ u ].push_back( v );
    g[ v ].push_back( u );
}
void yutruli(){
    dfssz(1, 0);
    ts = 0;
    dfs1(1);
    REP(k, 1, LOG-1) REP(i, 1, n)
        prt[i][k] = prt[prt[i][k-1]][k-1];
}
vector< PII > getPath( int u , int v ){
    vector< PII > res;
    while( tid[ u ] < tid[ head[ v ] ] ){
        res.push_back( PII(tid[ head[ v ] ] , tid[ v ] ) );
        v = prt[ head[ v ] ][ 0 ];
    }
    res.push_back( PII( tid[ u ] , tid[ v ] ) );
    reverse( ALL( res ) );
    return res;
}
/* res : list of intervals from u to v
 * u must be ancestor of v
 * usage :
 * vector< PII >& path = tree.getPath( u , v )
 * for( PII tp : path ) {
 *     int l , r; tie( l , r ) = tp;
 *     upd( l , r );
 *     uu = tree.tdi[ l ] , vv = tree.tdi[ r ];
 *     uu ~> vv is a heavy path on tree
 * }
 */
}
} tree;
```

4 Math

4.1 Prime Table

```
// 1000000000 < primes < 2147483647
1002939109, 1020288887, 1028798297, 1038684299,
1041211027, 1051762951, 1058585963, 1063020809,
1094763083, 1106384353, 1120154459, 1140593173,
1147930723, 1172520109, 1183835981, 1187659051,
1241251303, 1247184097, 1255940849, 1272759031,
1287027493, 1288511629, 1294632499, 1312650799,
1314753281, 1320080669, 1321970357, 1333133947,
1337684419, 1353508067, 1358715989, 1364961029,
1366046831, 1376536367, 1381705499, 1410637769,
1411311571, 1422795043, 1437499801, 1495803851,
1511764363, 1526710979, 1538018089, 1542373769,
1545326953, 1549429633, 1556212739, 1575971759,
1586465261, 1608336427, 1609783001, 1620728569,
1643267081, 1652401603, 1656717203, 1660920671,
1666858577, 1669260361, 1670240317, 1678791131,
1685583143, 1725964619, 1734856421, 1743134179,
1761537223, 1774260193, 1778872889, 1781930609,
1803000149, 1814256623, 1834876331, 1839154463,
```

```
1840044389, 1843241713, 1856039431, 1868564531,
1868732623, 1884198443, 1884616807, 1885059541,
1909942399, 1914471137, 1923951707, 1925453197,
1937719153, 1954649041, 1958915237, 1970709803,
1979612177, 1980446837, 1989761941, 2007826547,
2008033571, 2011186739, 2039465081, 2039728567,
2093735719, 2116097521, 2123852629, 2140170259
```

```
// 2147483647 < primes < 4000000000
```

```
3148478261, 3153064147, 3176351071, 3187523093,
3196772239, 3201312913, 3203063977, 3204840059,
3210224309, 3213032591, 3217689851, 3218469083,
3219857533, 3231880427, 3235951699, 3273767923,
3276188869, 3277183181, 3282463507, 3285553889,
3319309027, 3327005333, 3327574903, 3341387953,
3373293941, 3380077549, 3380892997, 3381118801,
3384716479, 3386991323
```

倒數枚舉

```
T_0 = 1, T_{i+1} = n // (n // (T_i + 1))
```

4.2 ax+by=gcd

```
// By Adrien1018 (not knowing how to use.
// ax+ny = 1, ax+ny == ax == 1 (mod n)
tuple<int, int, int> extended_gcd(int a, int b) {
    if (!b) return make_tuple(a, 1, 0);
    int d, x, y;
    tie(d, x, y) = extended_gcd(b, a % b);
    return make_tuple(d, y, x - (a / b) * y);
}
// ax+by = gcd (by Eddy1021
PII gcd(int a, int b){
    if(b == 0) return {1, 0};
    PII q = gcd(b, a % b);
    return {q.second, q.first - q.second * (a / b)};
}
```

4.3 Pollard Rho

```
// coded by hanhanW
// does not work when n is prime
long long modit(long long x, long long mod) {
    if(x >= mod) x -= mod;
    //if(x < 0) x += mod;
    return x;
}
long long mult(long long x, long long y, long long mod) {
    long long s = 0, m = x % mod;
    while(y) {
        if(y & 1) s = modit(s + m, mod);
        y >>= 1;
        m = modit(m + m, mod);
    }
    return s;
}
long long f(long long x, long long mod) {
    return modit(mult(x, x, mod) + 1, mod);
}
long long pollard_rho(long long n) {
    if(!(n & 1)) return 2;
    while(true) {
        long long y = 2, x = rand() % (n - 1) + 1, res = 1;
        for(int sz = 2; res == 1; sz *= 2) {
            for(int i = 0; i < sz && res <= 1; i++) {
                x = f(x, n);
                res = __gcd(abs(x - y), n);
            }
            y = x;
        }
        if(res != 0 && res != n) return res;
    }
}
```

4.4 Pi Count (Linear Sieve)

```
static constexpr int N = 1000000 + 5;
lld pi[N];
vector<int> primes;
```

```
bool sieved[N];
lld cube_root(lld x) {
    lld s = static_cast<lld>(cbrt(x - static_cast<long
        double>(0.1)));
    while(s * s * s <= x) ++s;
    return s - 1;
}
lld square_root(lld x) {
    lld s = static_cast<lld>(sqrt(x - static_cast<long
        double>(0.1)));
    while(s * s <= x) ++s;
    return s - 1;
}
void init() {
    primes.reserve(N);
    primes.push_back(1);
    for(int i = 2; i < N; i++) {
        if(!sieved[i]) primes.push_back(i);
        pi[i] = !sieved[i] + pi[i - 1];
        for(int p: primes) if(p > 1) {
            if(p * i >= N) break;
            sieved[p * i] = true;
            if(p % i == 0) break;
        }
    }
}
lld phi(lld m, lld n) {
    static constexpr int MM = 80000, NN = 500;
    static lld val[MM][NN];
    if(m < MM and n < NN and val[m][n]) return val[m][n]
        - 1;
    if(n == 0) return m;
    if(primes[n] >= m) return 1;
    lld ret = phi(m, n - 1) - phi(m / primes[n], n - 1);
    if(m < MM and n < NN) val[m][n] = ret + 1;
    return ret;
}
lld pi_count(lld);
lld P2(lld m, lld n) {
    lld sm = square_root(m), ret = 0;
    for(lld i = n + 1; primes[i] <= sm; i++)
        ret += pi_count(m / primes[i]) - pi_count(primes[i]
            ) + 1;
    return ret;
}
lld pi_count(lld m) {
    if(m < N) return pi[m];
    lld n = pi_count(cube_root(m));
    return phi(m, n) + n - 1 - P2(m, n);
}
```

4.5 NloglogN Sieve

```
void Sieve(int n) {
    for(int i = 2; i <= n; i++) {
        if(notprime[i]) continue;
        primes.push_back(i);
        for(int j = i * i; j <= n; j += i) notprime[j] = true;
    }
}
```

4.6 Range Sieve

```
#include <algorithm>
typedef long long lld;
const int MAX_SQRT_B = 50000;
const int MAX_L = 200000 + 5;

bool is_prime_small[MAX_SQRT_B];
bool is_prime[MAX_L];
void sieve(lld, lld);

void sieve(lld l, lld r) {
    // [l, r)
    for(lld i = 2; i * i < r; i++) is_prime_small[i] = true;
    for(lld i = l; i < r; i++) is_prime[i - 1] = true;
    if(l == 1) is_prime[0] = false;
    for(lld i = 2; i * i < r; i++) {
        if(!is_prime_small[i]) continue;
        for(lld j = i * i; j * j < r; j += i) is_prime_small[j] =
            false;
    }
}
```

```

        for(lld j=std::max(2LL, (1+i-1)/i)*i;j<r;j+=i)
            is_prime[j-1]=false;
    }
}

```

4.7 Miller Rabin

```

lld modu(lld a, lld m){
    while(a >= m) a -= m;
    return a;
}

lld mul(lld a, lld b, lld m){
    if(a < b) swap(a, b);
    lld ret = 0;
    while(b){
        if(b & 1) ret = modu(ret+a, m);
        a = modu(a+a, m);
        b >>= 1;
    }
    return ret;
}

lld qPow(lld a, lld k, lld m){
    lld ret = 1;
    a %= m;
    while(k){
        if(k & 1) ret = mul(ret, a, m);
        a = mul(a, a, m);
        k >>= 1;
    }
    return modu(ret, m);
}

bool witness(lld a, lld s, int t, lld n){
    lld b = qPow(a, s, n);
    if(b == 0) return false;
    while(t--){
        lld bb = mul(b, b, n);
        if(bb == 1 and b != 1 and b != n-1) return true;
        b = bb;
    }
    return b != 1;
}

bool miller_rabin(lld n){
    if(n < 2) return false;
    if(!(n & 1)) return (n==2);
    lld x = n-1; int t = 0;
    while(!(x&1)) x >>= 1, t++;
    lld sprp[] = {2,325,9375,28178,450775,9780504,1795265022};
    for(int i=0;i<7;i++){
        if(witness(sprp[i]%n, x, t, n)) return false;
    }
    return true;
}

```

4.8 Inverse Element

```

// x's inverse mod k
// if k is prime
long long GetInv(long long x, long long k){
    return qPow(x, k-2);
}

// x's inverse mod k
// if k is not prime
long long GetInv(long long x, long long k){
    return qPow(x, Euler(k)-1);
}

// or extended_gcd(x, k).second
// if you need [1, x] (most use: [1, k-1])
void solve(int x, long long k){
    inv[1] = 1;
    for(int i=2;i<=k;i++){
        inv[i] = ((long long)(k - k/i) * inv[k % i]) % k;
    }
}

```

4.9 Euler Phi Function

```

/*
    extended euler:
    a^b mod p
    if gcd(a, p)==1: a^(b%phi(p))
    elif b < phi(p): a^b mod p
    else a^(b%phi(p) + phi(p))
*/
inline int64_t Euler(int x){
    int64_t r=1;
    for(int i=2;i<=x;++i){
        if(x%i==0){
            x/=i;
            r*=(i-1);
            while(x%i==0){
                x/=i;
                r*=i;
            }
        }
    }
    if(x>1) r*=x-1;
    return r;
}

vector<int> primes;
bool notprime[N];
int64_t phi[N];
inline void euler_sieve(int n){
    for(int i=2;i<=n;i++){
        if(!notprime[i]){
            primes.push_back(i);
            phi[i] = i-1;
        }
        for(auto j: primes){
            if(i*j >= n) break;
            notprime[i*j] = true;
            phi[i*j] = phi[i] * phi[j];
            if(i % j == 0){
                phi[i*j] = phi[i] * j;
                break;
            }
        }
    }
}

```

4.10 Gauss Elimination

```

typedef long double llf;
const int N = 300;
const llf EPS = 1e-8;

// make m[i][i] = x, m[i][j] = 0
// v is for solving equation:
// for(int i=0;i<n;i++) ans[pos[i]] = val[i]/mtx[i][pos[i]];
// for(int i=0;i<n;i++) cout << ans[i] << '\n';
bool Gauss(llf m[N][N], llf v[N], int n, int pos[N]){
    for(int i=0;i<n;i++){
        int x=-1, y=-1; llf m = 0;
        for(int j=i;j<n;j++){
            if(fabs(m[j][pos[k]])>m){
                m = fabs(m[j][pos[k]]);
                x = j, y = k;
            }
        }
        if(x==-1 or y==-1) return false;
        swap(m[x], m[i]);
        swap(v[x], v[i]);
        swap(pos[y], pos[i]);
        for(int j=i+1;j<n;j++){
            llf xi = m[j][pos[i]]/m[i][pos[i]];
            for(int k=0;k<n;k++){
                m[j][pos[k]] -= xi*m[i][pos[k]];
                v[j] -= xi*v[i];
            }
        }
    }
    for(int i=n-1;i>=0;i--){
        for(int j=i-1;j>=0;j--){
            llf xi = m[j][pos[i]]/m[i][pos[i]];
            for(int k=0;k<n;k++){
                m[j][pos[k]] -= xi*m[i][pos[k]];
                v[j] -= xi*v[i];
            }
        }
    }
    return true;
}

```

```
}
/*
```

4.11 Fast Fourier Transform

```
/*
    polynomial multiply:
    FFT(a, N, true);
    FFT(b, N, true);
    for(int i=0;i<MAXN;i++) c[i] = a[i]*b[i];
    FFT(c, N, false);
    yeah~ go result in c
    (N must be 2^k and >= len(a)+len(b))
*/
typedef long double llf;
typedef complex<llf> cplx;
const int MAXN = 262144;
const llf PI = acos((-1));

cplx A[MAXN], B[MAXN], C[MAXN], omega[MAXN+1];

void init_omega(){
    const cplx I = {0, 1};
    for(int i=0;i<MAXN;i++) omega[i] = exp(i*2*PI/MAXN
        *I);
}

void FFT(cplx arr[], int n, bool ori){
    // n must be 2^k
    int theta = MAXN / n;
    for(int len=n;len>=2;len>=1){
        int tot = len>>1;
        for(int i=0;i<tot;i++){
            cplx omg = omega[ori?i*theta%MAXN:MAXN-(i*
                theta%MAXN)];
            for(int j=i;j<n;j+=len){
                int k = j+tot;
                cplx x = arr[j] - arr[k];
                arr[j] += arr[k];
                arr[k] = omg * x;
            }
            theta = (theta * 2) % MAXN;
        }
        int i = 0;
        for(int j=1;j<n-1;j++){
            for(int k=n>>1;k>(i^=k);k>=1);
            if(j < i) swap(arr[j], arr[i]);
        }
        if(ori) return;
        for(int i=0;i<n;i++) arr[i] /= n;
    }
}
```

4.12 Chinese Remainder

```
// ax+ny = 1, ax+ny == ax == 1 (mod n)
pair<lld, lld> exgcd(lld a, lld b){
    if(b == 0) return {1, 0};
    pair<lld, lld> q = gcd(b, a % b);
    return {q.second, q.first - q.second * (a / b)};
}

lld crt(lld ans[], lld pri[], int n){
    lld M = 1;
    for(int i=0;i<n;i++) M *= pri[i];
    lld ret = 0;
    for(int i=0;i<n;i++){
        lld inv = (gcd(M/pri[i], pri[i]).first + pri[i]
            )%pri[i];
        ret += (ans[i]*(M/pri[i])%M * inv)%M;
        ret %= M;
    }
    return ret;
}

/*
Another:
x = a1 % m1
x = a2 % m2
g = gcd(m1, m2)
assert((a1-a2)%g==0)
[p, q] = exgcd(m2/g, m1/g)
return a2+m2*(p*(a1-a2)/g)
0 <= x < lcm(m1, m2)
*/
```

4.13 NTT

```
typedef long long LL;
// Remember coefficient are mod P
/* p=a*2^n+1
    n    2^n    p    a    root
    5    32    97    3    5
    6    64    193    3    5
    7    128    257    2    3
    8    256    257    1    3
    9    512    7681    15    17
    10   1024    12289    12    11
    11   2048    12289    6    11
    12   4096    12289    3    11
    13   8192    40961    5    3
    14   16384    65537    4    3
    15   32768    65537    2    3
    16   65536    65537    1    3
    17   131072    786433    6    10
    18   262144    786433    3    10 (605028353,
        2308, 3)
    19   524288    5767169    11    3
    20   1048576    7340033    7    3
    21   2097152    23068673    11    3
    22   4194304    104857601    25    3
    23   8388608    167772161    20    3
    24   16777216    167772161    10    3
    25   33554432    167772161    5    3 (1107296257, 33,
        10)
    26   67108864    469762049    7    3
    27   134217728    2013265921    15    31 */
// (must be 2^k)
// To implement poly. multiply:
// NTT<P, root, MAXN> ntt;
// ntt( n , a ); // or ntt.tran( n , a );
// ntt( n , b );
// for( int i = 0 ; i < n ; i++ )
//     c[ i ] = a[ i ] * b[ i ];
// ntt( n , c , 1 );
//
// then you have the result in c :: [LL]

template<LL P, LL root, int MAXN>
struct NTT{
    static LL bigmod(LL a, LL b) {
        LL res = 1;
        for (LL bs = a; b; b >>= 1, bs = (bs * bs) % P)
            if(b&1) res=(res*bs)%P;
        return res;
    }
    static LL inv(LL a, LL b) {
        if(a==1) return 1;
        return ((LL) (a-inv(b%a,a))*b+1)/a%b;
    }
    LL omega[MAXN+1];
    NTT() {
        omega[0] = 1;
        LL r = bigmod(root, (P-1)/MAXN);
        for (int i=1; i<=MAXN; i++)
            omega[i] = (omega[i-1]*r)%P;
    }
    // n must be 2^k
    void tran(int n, LL a[], bool inv_ntt=false){
        int basic = MAXN / n;
        int theta = basic;
        for (int m = n; m >= 2; m >>= 1) {
            int mh = m >> 1;
            for (int i = 0; i < mh; i++) {
                LL w = omega[i*theta%MAXN];
                for (int j = i; j < n; j += m) {
                    int k = j + mh;
                    LL x = a[j] - a[k];
                    if (x < 0) x += P;
                    a[j] += a[k];
                    if (a[j] > P) a[j] -= P;
                    a[k] = (w * x) % P;
                }
            }
            theta = (theta * 2) % MAXN;
        }
    }
}
```

```

int i = 0;
for (int j = 1; j < n - 1; j++) {
    for (int k = n >> 1; k > (i ^ k); k >>= 1)
        ;
    if (j < i) swap(a[i], a[j]);
}
if (inv_ntt) {
    LL ni = inv(n,P);
    reverse(a+1, a+n);
    for (i = 0; i < n; i++)
        a[i] = (a[i] * ni) % P;
}
}
void operator()(int n, LL a[], bool inv_ntt=false)
{
    tran(n, a, inv_ntt);
}
};

const LL P=2013265921,root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;

```

4.14 DiscreteLog

```

// Baby-step Giant-step Algorithm
// a x + by = g
void exgcd(long long x, long long y, long long &g,
           long long &a, long long &b) {
    if (y == 0)
        g = x, a = 1, b = 0;
    else
        exgcd(y, x%y, g, b, a), b -= (x/y) * a;
}
long long inverse(long long x, long long p) {
    long long g, b, r;
    exgcd(x, p, g, r, b);
    if (g < 0) r = -r;
    return (r%p + p)%p;
}
long long BSGS(long long P, long long B, long long N) {
    // find B^L = N mod P
    unordered_map<long long, int> R;
    long long sq = (long long) sqrt(P);
    long long t = 1, f;
    for (int i = 0; i < sq; i++) {
        if (t == N)
            return i;
        if (!R.count(t))
            R[t] = i;
        t = (t * B) % P;
    }
    f = inverse(t, P);
    for (int i = 0; i <= sq+1; i++) {
        if (R.count(N))
            return i * sq + R[N];
        N = (N * f) % P;
    }
    return -1;
}

```

5 Geometry

5.1 Point Class

```

template<typename T>
struct Point{
    typedef long double llf;
    static constexpr llf EPS = 1e-8;
    T x, y;
    Point(T __=0, T __=0): x(__), y(__){}
    template<typename T2>
    Point(const Point<T2>& a): x(a.x), y(a.y){}
    inline llf theta() const {
        return atan2((llf)y, (llf)x);
    }
    inline llf dis() const {
        return hypot((llf)x, (llf)y);
    }
};

```

```

inline llf dis(const Point& o) const {
    return hypot((llf)(x-o.x), (llf)(y-o.y));
}
Point operator-(const Point& o) const {
    return Point(x-o.x, y-o.y);
}
Point operator+=(const Point& o) {
    x+=o.x, y+=o.y;
    return *this;
}
Point operator+(const Point& o) const {
    return Point(x+o.x, y+o.y);
}
Point operator+=(const Point& o) {
    x+=o.x, y+=o.y;
    return *this;
}
Point operator*(const T& k) const {
    return Point(x*k, y*k);
}
Point operator*=(const T& k) {
    x*=k, y*=k;
    return *this;
}
Point operator/(const T& k) const {
    return Point(x/k, y/k);
}
Point operator/=(const T& k) {
    x/=k, y/=k;
    return *this;
}
Point operator-() const {
    return Point(-x, -y);
}
Point rot90() const {
    return Point(-y, x);
}
template<typename T2>
bool in(const Circle<T2>& a) const {
    /* Add struct Circle at top */
    return a.o.dis(*this)+EPS <= a.r;
}
bool equal(const Point& o, true_type) const {
    return fabs(x-o.x) < EPS and fabs(y-o.y) < EPS;
}
bool equal(const Point& o, false_type) const {
    return tie(x, y) == tie(o.x, o.y);
}
bool operator==(const Point& o) const {
    return equal(o, is_floating_point<T>());
}
bool operator!=(const Point& o) const {
    return !(*this == o);
}
bool operator<(const Point& o) const {
    return theta() < o.theta();
    // sort like what pairs did
    // if(is_floating_point<T>()) return fabs(x-o.x)
    // <EPS?y<o.y:x<o.x;
    // else return tie(x, y) < tie(o.x, o.y);
}
friend inline T cross(const Point& a, const Point&
                      b){
    return a.x*b.y - b.x*a.y;
}
friend inline T dot(const Point& a, const Point& b)
{
    return a.x*b.x + a.y*b.y;
}
friend ostream& operator<<(ostream& ss, const Point
& o){
    ss<<"("<<o.x<<"", "<<o.y<<"")";
    return ss;
}
};

```

5.2 Circle Class

```

template<typename T>
struct Circle{
    static constexpr llf EPS = 1e-8;
    Point<T> o;
    T r;
};

```

```
vector<Point<llf>> operator&(const Circle& aa)
{
    const {
        // https://www.cnblogs.com/wangzming/p/8338142.html
        llf d=o.dis(aa.o);
        if(d > r+aa.r+EPS or d < fabs(r-aa.r)-EPS)
            return {};
        llf dt = (r*r - aa.r*aa.r)/d, d1 = (d+dt)/2;
        Point<llf> dir = (aa.o-o); dir /= d;
        Point<llf> pcrs = dir*d1 + o;
        dt=sqrt(max(0.0L, r*r - d1*d1)), dir=dir.rot90
            ();
        return {pcrs + dir*dt, pcrs - dir*dt};
    }
};
```

5.3 Line Class

```
const Point<long double> INF_P(-1e20, 1e20);
const Point<long double> NOT_EXIST(1e20, 1e-20);
template<typename T>
struct Line{
    static constexpr long double EPS = 1e-8;
    // ax+by+c = 0
    T a, b, c;
    Line(): a(0), b(1), c(0){}
    Line(T __, T ___, T ___): a(__), b(___), c(___){
        assert(fabs(a)>EPS or fabs(b)>EPS);
    }
    template<typename T2>
    Line(const Line<T2>& x): a(x.a), b(x.b), c(x.c){}
    typedef Point<long double> Pt;
    bool equal(const Line& o, true_type) const {
        return fabs(a-o.a) < EPS and fabs(b-o.b) < EPS
            and fabs(c-o.c) < EPS;
    }
    bool euqal(const Line& o, false_type) const {
        return a==o.a and b==o.b and c==o.c;
    }
    bool operator==(const Line& o) const {
        return euqal(o, is_floating_point<T>());
    }
    bool operator!=(const Line& o) const {
        return !(*this == o);
    }
    friend inline bool on_line__(const Point<T>& p,
        const Line& l, true_type){
        return fabs(l.a*p.x + l.b*p.y + l.c) < EPS;
    }
    friend inline bool on_line__(const Point<T>& p,
        const Line& l, false_type){
        return l.a*p.x + l.b*p.y + l.c == 0;
    }
    friend inline bool on_line(const Point<T>&p const
        Line& l){
        return on_line__(p, l, is_floating_point<T>());
    }
    friend inline bool is_parallel__(const Line& x,
        const Line& y, true_type){
        return fabs(x.a*y.b - x.b*y.a) < EPS;
    }
    friend inline bool is_parallel__(const Line& x,
        const Line& y, false_type){
        return x.a*y.b == x.b*y.a;
    }
    friend inline bool is_parallel(const Line& x, const
        Line& y){
        return is_parallel__(x, y, is_floating_point<T>
            >());
    }
    friend inline Pt get_inter(const Line& x, const
        Line& y){
        typedef long double llf;
        if(x==y) return INF_P;
        if(is_parallel(x, y)) return NOT_EXIST;
        llf delta = x.a*y.b - x.b*y.a;
        llf delta_x = x.b*y.c - x.c*y.b;
        llf delta_y = x.c*y.a - x.a*y.c;
        return Pt(delta_x / delta, delta_y / delta);
    }
    friend ostream& operator<<(ostream& ss, const Line&
        o){
        ss<<o.a<<"x+"<<o.b<<"y+"<<o.c<<"=0";
        return ss;
    }
};
```

```
};
template<typename T>
inline Line<T> get_line(const Point<T>& a, const Point<
    T>& b){
    return Line<T>(a.y-b.y, b.x-a.x, (b.y-a.y)*a.x-(b.x
        -a.x)*a.y);
}
```

5.4 Segment Class

```
const long double EPS = 1e-8;
template<typename T>
struct Segment{
    // p1.x < p2.x
    Line<T> base;
    Point<T> p1, p2;
    Segment(): base(Line<T>()), p1(Point<T>()), p2(
        Point<T>()){
        assert(on_line(p1, base) and on_line(p2, base))
            ;
    }
    Segment(Line<T> __, Point<T> ___, Point<T> ___): base
        (__), p1(___), p2(___){
        assert(on_line(p1, base) and on_line(p2, base))
            ;
    }
    template<typename T2>
    Segment(const Segment<T2>& __): base(__.base), p1(
        __.p1), p2(__.p2) {}
    typedef Point<long double> Pt;
    friend bool on_segment(const Point<T>& p, const
        Segment& l){
        if(on_line(p, l.base))
            return (l.p1.x-p.x)*(p.x-l.p2.x)>=0 and (l.
                p1.y-p.y)*(p.y-l.p2.y)>=0;
        return false;
    }
    friend bool have_inter(const Segment& a, const
        Segment& b){
        if(is_parallel(a.base, b.base)){
            return on_segment(a.p1, b) or on_segment(a.
                p2, b) or on_segment(b.p1, a) or
                on_segment(b.p2, a);
        }
        Pt inter = get_inter(a.base, b.base);
        return on_segment(inter, a) and on_segment(
            inter, b);
    }
    friend inline Pt get_inter(const Segment& a, const
        Segment& b){
        if(!have_inter(a, b)){
            return NOT_EXIST;
        }else if(is_parallel(a.base, b.base)){
            if(a.p1 == b.p1){
                if(on_segment(a.p2, b) or on_segment(b.
                    p2, a)) return INF_P;
                else return a.p1;
            }else if(a.p1 == b.p2){
                if(on_segment(a.p2, b) or on_segment(b.
                    p1, a)) return INF_P;
                else return a.p1;
            }else if(a.p2 == b.p1){
                if(on_segment(a.p1, b) or on_segment(b.
                    p2, a)) return INF_P;
                else return a.p2;
            }else if(a.p2 == b.p2){
                if(on_segment(a.p1, b) or on_segment(b.
                    p1, a)) return INF_P;
                else return a.p2;
            }
        }
        return INF_P;
    }
    return get_inter(a.base, b.base);
}
friend ostream& operator<<(ostream& ss, const
    Segment& o){
    ss<<o.base<<"", "<<o.p1<<" ~ "<<o.p2;
    return ss;
}
};
template<typename T>
inline Segment<T> get_segment(const Point<T>& a, const
    Point<T>& b){
```

```

    return Segment<T>(get_line(a, b), a, b);
}

```

5.5 Triangle Circumcentre

```

template<typename T>
Circle<llf> get_circum(const Point<T>& a, const Point<T>
>& b, const Point<T>& c) {
    llf a1 = a.x-b.x;
    llf b1 = a.y-b.y;
    llf c1 = (a.x+b.x)/2 * a1 + (a.y+b.y)/2 * b1;

    llf a2 = a.x-c.x;
    llf b2 = a.y-c.y;
    llf c2 = (a.x+c.x)/2 * a2 + (a.y+c.y)/2 * b2;

    Circle<llf> cc;
    cc.o.x = (c1*b2-b1*c2)/(a1*b2-b1*a2);
    cc.o.y = (a1*c2-c1*a2)/(a1*b2-b1*a2);
    cc.r = hypot(cc.o.x-a.x, cc.o.y-a.y);
    return cc;
}

```

5.6 2D Convex Hull

```

template<typename T>
class ConvexHull_2D {
private:
    typedef Point<T> PT;
    vector<PT> dots;
    struct myhash {
        uint64_t operator()(const PT& a) const {
            uint64_t xx=0, yy=0;
            memcpy(&xx, &a.x, sizeof(a.x));
            memcpy(&yy, &a.y, sizeof(a.y));
            uint64_t ret = xx*17+yy*31;
            ret = (ret ^ (ret >> 16))*0x9E3779B1;
            ret = (ret ^ (ret >> 13))*0xC2B2AE35;
            ret = ret ^ xx;
            return (ret ^ (ret << 3)) * yy;
        }
    };
    unordered_set<PT, myhash> in_hull;
public:
    inline void init() {in_hull.clear();dots.clear();}
    void insert(const PT& x) {dots.PB(x);}
    void solve() {
        sort(ALL(dots), [](const PT& a, const PT& b) {
            return tie(a.x, a.y) < tie(b.x, b.y);
        });
        vector<PT> stk(SZ(dots)<<1);
        int top = 0;
        for(auto p: dots) {
            while(top >= 2 and cross(p-stk[top-2], stk[
top-1]-stk[top-2]) <= 0)
                top--;
            stk[top++] = p;
        }
        for(int i=SZ(dots)-2, t = top+1; i>=0; i--){
            while(top >= t and cross(dots[i]-stk[top
-2], stk[top-1]-stk[top-2]) <= 0)
                top--;
            stk[top++] = dots[i];
        }
        stk.resize(top-1);
        swap(stk, dots);
        for(auto i: stk) in_hull.insert(i);
    }
    vector<PT> get() {return dots;}
    inline bool in_it(const PT& x) {
        return in_hull.find(x)!=in_hull.end();
    }
};

```

5.7 2D Farthest Pair

```

// stk is from convex hull
n = (int)(stk.size());
int pos = 1, ans = 0; stk.push_back(arr[0]);

```

```

for(int i=0; i<n; i++){
    while(abs(cross(stk[i+1]-stk[i], stk[(pos+1)%n]-stk
[i]))\
        > abs(cross(stk[i+1]-stk[i], stk[pos]-stk[i
]))) pos = (pos+1)%n;
    ans = max((ans, dis(stk[i], stk[pos])), dis(stk[i
+1], stk[pos]));
}

```

5.8 SimulateAnnealing

```

#include <random>
#include <functional>
#include <utility>
#include <algorithm>
using namespace std;

double getY(double);

int main() {
    int rr, ll;
    default_random_engine rEng(time(NULL));
    uniform_real_distribution<double> Range(-1,1);
    uniform_real_distribution<double> expR(0,1);
    auto Random=bind(Range,rEng), expRand=bind(expR,
rEng);
    int step=0;
    double pace=rr-ll, mini=0.95; // need to search for
it
    double x=max(min(Random()*pace+ll, rr), ll), y=getY
(x);
    while(pace>=1e-7) {
        double newX = max(min(x + Random()*pace, rr),
ll);
        double newY = getY(newX);
        if(newY < y || expRand() < exp(-step))
            x=newX, y=newY;
        step++;
        pace*=mini;
    }
}

```

5.9 Ternary Search on Interger

```

int TernarySearch(int l, int r) {
    // (l, r]
    while (r - l > 1) {
        int mid = (l + r) >> 1;
        if (f(mid) > f(mid + 1)) r = mid;
        else l = mid;
    }
    return l+1;
}

```

5.10 Minimum Covering Circle

```

template<typename T>
Circle<llf> MinCircleCover(const vector<Point<T>>& pts)
{
    random_shuffle(ALL(pts));
    Circle<llf> c = {pts[0], 0};
    int n = SZ(pts);
    for(int i=0; i<n; i++){
        if(pts[i].in(c)) continue;
        c = {pts[i], 0};
        for(int j=0; j<i; j++){
            if(pts[j].in(c)) continue;
            c.o = (pts[i] + pts[j]) / 2;
            c.r = pts[i].dis(c.o);
            for(int k=0; k<j; k++){
                if(pts[k].in(c)) continue;
                c = get_circum(pts[i], pts[j], pts[k]);
            }
        }
    }
    return c;
}

```


5.11 KDTree (Nearest Point)

```
const int MXN = 100005;
struct KDTree {
    struct Node {
        int x,y,x1,y1,x2,y2;
        int id,f;
        Node *L, *R;
    }tree[MXN];
    int n;
    Node *root;
    LL dis2(int x1, int y1, int x2, int y2) {
        LL dx = x1-x2;
        LL dy = y1-y2;
        return dx*dx+dy*dy;
    }
    static bool cmpx(Node& a, Node& b){ return a.x<b.x; }
    static bool cmpy(Node& a, Node& b){ return a.y<b.y; }
    void init(vector<pair<int,int>> ip) {
        n = ip.size();
        for (int i=0; i<n; i++) {
            tree[i].id = i;
            tree[i].x = ip[i].first;
            tree[i].y = ip[i].second;
        }
        root = build_tree(0, n-1, 0);
    }
    Node* build_tree(int L, int R, int dep) {
        if (L>R) return nullptr;
        int M = (L+R)/2;
        tree[M].f = dep%2;
        nth_element(tree+L, tree+M, tree+R+1, tree[M].f ?
            cmpy : cmpx);
        tree[M].x1 = tree[M].x2 = tree[M].x;
        tree[M].y1 = tree[M].y2 = tree[M].y;

        tree[M].L = build_tree(L, M-1, dep+1);
        if (tree[M].L) {
            tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
            tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
            tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
            tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
        }
        tree[M].R = build_tree(M+1, R, dep+1);
        if (tree[M].R) {
            tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
            tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
            tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
            tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
        }
        return tree+M;
    }
    int touch(Node* r, int x, int y, LL d2){
        LL dis = sqrt(d2)+1;
        if (x<r->x1-dis || x>r->x2+dis ||
            y<r->y1-dis || y>r->y2+dis)
            return 0;
        return 1;
    }
    void nearest(Node* r, int x, int y,
        int &mID, LL &md2){
        if (!r || !touch(r, x, y, md2)) return;
        LL d2 = dis2(r->x, r->y, x, y);
        if (d2 < md2 || (d2 == md2 && mID < r->id)) {
            mID = r->id;
            md2 = d2;
        }
        // search order depends on split dim
        if ((r->f == 0 && x < r->x) ||
            (r->f == 1 && y < r->y)) {
            nearest(r->L, x, y, mID, md2);
            nearest(r->R, x, y, mID, md2);
        } else {
            nearest(r->R, x, y, mID, md2);
            nearest(r->L, x, y, mID, md2);
        }
    }
    int query(int x, int y) {
        int id = 1029384756;
        LL d2 = 102938475612345678LL;
        nearest(root, x, y, id, d2);
        return id;
    }
}tree;
```

6 Stringology

6.1 Hash

```
const int N = 1000000;
class Hash{
private:
    const int p = 127, q = 1208220623;
    int sz, prefix[N], power[N];
    inline int add(int x, int y){return x+y>=q?x+y-q:x+y;}
    inline int sub(int x, int y){return x-y<0?x-y+q:x-y;}
    inline int mul(int x, int y){return 1LL*x*y%q;}
public:
    void init(const std::string &x){
        sz = x.size();
        prefix[0]=0;
        for(int i=1;i<=sz;i++) prefix[i]=add(mul(prefix[i-1], p), x[i-1]);
        power[0]=1;
        for(int i=1;i<=sz;i++) power[i]=mul(power[i-1], p);
    }
    int query(int l, int r){
        // 1-base (l, r]
        return sub(prefix[r], mul(prefix[l], power[r-l]));
    }
};
```

6.2 Suffix Array

```
//help by http://www.geeksforgeeks.org/suffix-array-set-2-a-nlognlogn-algorithm/
#include <bits/stdc++.h>
using namespace std;
#define PB push_back

struct sfx{
    int index;
    int r,nr;
};

char str[N + 10];
int len;

vector<sfx> srs[N + 10];
int mapping[N + 10];
sfx sa[N + 10];

bool cmp(sfx a,sfx b){
    if(a.r==b.r){
        return a.nr<b.nr;
    }else{
        return a.r<b.r;
    }
}

void SA();
void radixSort();

int main(){
    gets(str);
    len = strlen(str);
    SA();
    for(int i=0;i<len;i++){
        printf("%d\n",sa[i].index);
    }
    return 0;
}

void SA(){
    for(int i=0;i<len;i++){
        sa[i].index = i;
        sa[i].r=str[i];
        sa[i].nr=(i+1>len)?0:str[i+1];
    }
    //sort(sa,sa+len,cmp);
    radixSort();
    for(int j=2;j<=len;j*=2){
        int cnt=1;
```

```

    int rr = sa[0].r;
    sa[0].r=cnt;
    mapping[sa[0].index]=0;
    for(int i=1;i<len;i++){
        if(sa[i].r == rr && sa[i].nr == sa[i-1].nr)
        {
            rr=sa[i].r;
            sa[i].r=cnt;
        }else{
            rr=sa[i].r;
            sa[i].r=++cnt;
        }
        mapping[sa[i].index]=i;
    }
    for(int i=0;i<len;i++){
        int nn = sa[i].index+j;
        sa[i].nr = (nn>=len)?0:sa[mapping[nn]].r;
    }
    //sort(sa, sa+len, cmp);
    radixSort();
}

void radixSort(){
    int m = 0;
    for(int i=0;i<len;i++){
        srs[sa[i].nr].PB(sa[i]);
        m=max(m,sa[i].nr);
    }
    int cnt=0;
    for(int i=0;i<=m;i++){
        if(srs[i].empty())continue;
        for(auto j:srs[i]){
            sa[cnt++] = j;
        }
        srs[i].clear();
    }
    m = 0;
    for(int i=0;i<len;i++){
        srs[sa[i].r].PB(sa[i]);
        m=max(m,sa[i].r);
    }
    cnt=0;
    for(int i=0;i<=m;i++){
        if(srs[i].empty())continue;
        for(auto j:srs[i]){
            sa[cnt++] = j;
        }
        srs[i].clear();
    }
}

```

6.3 KMP

```

int F[N<<1];
void KMP(char s1[], char s2[], int n, int m){
    // make F[] for s1+'\0'+s2;
    char ss[N<<1];
    int len = n+m+1;
    for(int i=0;i<n;i++) ss[i] = s1[i];
    ss[n] = '\0';
    for(int i=0;i<m;i++) ss[i+1+n] = s2[i];
    F[0] = F[1] = 0;
    for(int i=1;i<len;i++){
        int j = F[i];
        while(j > 0 and ss[i]!=ss[j]) j = F[j];
        F[i+1] = (ss[i]==ss[j]?j+1:0);
    }
    // just find (F[len2+i] == len2), i from 1 to len+1
    // for matching
}

// [0, i]是個循環字串，且循環節為i-f[i]:
// if(f[i]>0 and i%(i-f[i])==0) cout << i << " " << i/(
    i-f[i]) << '\n';

```

6.4 Z value

```

char s[MAXN];
int len,z[MAXN];
void Z_value() {
    int i,j,left,right;

```

```

    left=right=0; z[0]=len;
    for(i=1;i<len;i++) {
        j=max(min(z[i-left],right-i),0);
        for(;i+j<len&&s[i+j]==s[j];j++);
        z[i]=j;
        if(i+z[i]>right) {
            right=i+z[i];
            left=i;
        }
    }
}

```

6.5 Lexicographically Smallest Rotation

```

string mcp(string s){
    int n = s.length();
    s += s;
    int i=0, j=1;
    while (i<n && j<n){
        int k = 0;
        while (k < n && s[i+k] == s[j+k]) k++;
        if (s[i+k] <= s[j+k]) j += k+1;
        else i += k+1;
        if (i == j) j++;
    }
    int ans = i < n ? i : j;
    return s.substr(ans, n);
}

```

7 Problems

7.1 MaximumEmptyRect

```

int largest_empty_rectangle(){
    int max_area = 0;
    for (int i=1; i<=n; ++i) {
        for (int j=1; j<=n; ++j)
            if (array[i][j]) wl[j] = wl[j-1] + 1;
            else wl[j] = 0;
        for (int j=n; j>=1; --j)
            if (array[i][j]) wr[j] = wr[j+1] + 1;
            else wr[j] = 0;
        for (int j=1; j<=n; ++j)
            if (array[i][j]) h[j] = h[j] + 1;
            else h[j] = 0;
        for (int j=1; j<=n; ++j)
            if (l[j] == 0) l[j] = wl[j];
            else l[j] = min(wl[j], l[j]);
        for (int j=1; j<=n; ++j)
            if (r[j] == 0) r[j] = wr[j];
            else r[j] = min(wr[j], r[j]);
        for (int j=1; j<=n; ++j)
            max_area = max(max_area, (l[j] + r[j] - 1)
                * h[j]);
    }
    return max_area;
}

```