Contents

1 Basic

1.1 vimrc

```
se is nu ru et tgc sc hls cin cino+=j1 sw=4 sts=4 bs=2 mouse=a encoding=utf-8 ls=2
syn on
colorscheme desert
filetype indent on
inoremap {<CR> {<CR>}<ESC>O
map <F8> <ESC>:w<CR>:!g++ "%" -o "%<" -std=c++17 -DKISEKI -Wall -Wextra -Wshadow -Wfatal-errors -Wconversion -fsanitize=address,undefined -g && echo success<CR>
map <F9> <ESC>:w<CR>:!g++ "%" -o "%<" -O2 -std=c++17 && echo success<CR>
map <F10> <ESC>:!./"%<"<CR>
```

1.2 Debug Macro

```
#ifdef KISEKI
#define safe cerr<<__PRETTY_FUNCTION__\
  <<" line "<<__LINE__<<" safe\n"
#define debug(a...) qwerty(#a, a)
#define orange(a...) dvorak(#a, a)
using std::cerr;
template <typename ...T>
void qwerty(const char *s, T ...a) {
  cerr << "\e[1;32m(" << s << ") = (";
  int cnt = sizeof...(T);
  (..., (cerr << a << (--cnt ? ", " : ")\e[0m\n")));
}
template <typename Iter>
void dvorak(const char *s, Iter L, Iter R) {
  cerr << "\e[1;32m[ " << s << " ] = [ ";
  for (int f = 0; L != R; ++L)
    cerr << (f++ ? ", " : "") << *L;
  cerr << " ]\e[0m\n";
}
#else
#define safe ((void)0)
#define debug(...) ((void)0)
#define orange(...) ((void)0)
#endif
```

1.3 Increase Stack

```
const int size = 256 << 20;
register long rsp asm("rsp");
char *p = (char*)malloc(size)+size, *bak = (char*)rsp;
__asm__("movq %0, %%rsp\n"::"r"(p));
// main
__asm__("movq %0, %%rsp\n"::"r"(bak));
```

1.4 Pragma Optimization

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,tune=native")
__builtin_ia32_ldmxcsr(__builtin_ia32_stmxcsr()|0x8000);
```

1.5 IO Optimization

```
static inline int gc() {
 constexpr int B = 1<<20;
 static char buf[B], *p, *q;
 if(p == q &&
  (q=(p=buf)+fread(buf,1,B,stdin)) == buf)
  return EOF;
 return *p++;
}
template < typename T >
static inline bool gn( T &x ) {
 int c = gc(); T sgn = 1; x = 0;
 while(('0'>c||c>'9') && c!=EOF && c!='-') c = gc();
 if(c == '-') sgn = -1, c = gc();
 if(c == EOF) return false;
 while('0'<=c&&c<='9') x = x*10 + c - '0', c = gc();
 return x *= sgn, true;
}
```

2 Data Structure

2.1 Dark Magic

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
using namespace __gnu_pbds;
// heap tags: paring/binary/binomial/rc_binomial/thin
template<typename T>
using pbds_heap=__gnu_pbds::prioity_queue<T,less<T>, \
                  pairing_heap_tag>;
// pbds_heap::point_iterator
// x = pq.push(10); pq.modify(x, 87); a.join(b);
// tree tags: rb_tree_tag/ov_tree_tag/splay_tree_tag
template<typename T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
// find_by_order, order_of_key
// hash tables: cc_hash_table/gp_hash_table
```

2.2 Link-Cut Tree

```
template <typename Val> class LCT {
private:
 struct node {
  int pa, ch[2];
  bool rev;
  Val v, v_prod, v_rprod;
  node() : pa{0}, ch{0, 0}, rev{false}, v{}, v_prod{},
    v_rprod{} {};
 };
 vector<node> nodes;
 set<pair<int, int>> edges;
 bool is_root(int u) const {
  const int p = nodes[u].pa;
  return nodes[p].ch[0] != u and nodes[p].ch[1] != u;
 }
 bool is_rch(int u) const {
  return (not is_root(u)) and nodes[nodes[u].pa].ch[1]
    == u;
 }
 void down(int u) {
  if (auto &cnode = nodes[u]; cnode.rev) {
   if (cnode.ch[0]) set_rev(cnode.ch[0]);
   if (cnode.ch[1]) set_rev(cnode.ch[1]);
   cnode.rev = false;
  }
 }
 void up(int u) {
  auto &cnode = nodes[u];
  cnode.v_prod =
   nodes[cnode.ch[0]].v_prod * cnode.v * nodes[cnode.ch
    [1]].v_prod;
  cnode.v_rprod =
   nodes[cnode.ch[1]].v_rprod * cnode.v * nodes[cnode.
    ch[0]].v_rprod;
 }
 void set_rev(int u) {
  swap(nodes[u].ch[0], nodes[u].ch[1]);
  swap(nodes[u].v_prod, nodes[u].v_rprod);
  nodes[u].rev ^= 1;
 }
 void rotate(int u) {
  int f = nodes[u].pa, g = nodes[f].pa, l = is_rch(u);
  if (nodes[u].ch[l ^ 1])
   nodes[nodes[u].ch[l ^ 1]].pa = f;
  if (not is_root(f))
   nodes[g].ch[is_rch(f)] = u;
  nodes[f].ch[l] = nodes[u].ch[l ^ 1];
  nodes[u].ch[l ^ 1] = f;
  nodes[u].pa = g, nodes[f].pa = u;
  up(f);
 }
 void splay(int u) {
  vector<int> stk = {u};
  while (not is_root(stk.back()))
   stk.push_back(nodes[stk.back()].pa);
  for (; not stk.empty(); stk.pop_back())
   down(stk.back());
  for(int f=nodes[u].pa;!is_root(u);f=nodes[u].pa){
   if(!is_root(f))rotate(is_rch(u)==is_rch(f)?f:u);
   rotate(u);
  }
  up(u);
 }
 void access(int u) {
  int last = 0;
  for (int last = 0; u; last = u, u = nodes[u].pa) {
   splay(u);
   nodes[u].ch[1] = last;
   up(u);
  }
 }
 int find_root(int u) {
  access(u); splay(u);
  int la = 0;
  for (; u; la = u, u = nodes[u].ch[0]) down(u);
  return la;
 }
 void change_root(int u) {
  access(u); splay(u); set_rev(u);
 }
 void link(int x, int y) {
  change_root(y); nodes[y].pa = x;
 }
 void split(int x, int y) {
  change_root(x); access(y); splay(y);
 }
 void cut(int x, int y) {
  split(x, y);
  nodes[y].ch[0] = nodes[x].pa = 0;
  up(y);
 }
public:
 LCT(int n = 0) : nodes(n + 1) {}
 int add(const Val &v = {}) {
  nodes.push_back(v);
  return int(nodes.size()) - 2;
 }
 int add(Val &&v) {
  nodes.emplace_back(move(v));
  return int(nodes.size()) - 2;
 }
 void set_val(int u, const Val &v) {
  splay(++u); nodes[u].v = v; up(u);
 }
 Val query(int x, int y) {
  split(++x, ++y);
  return nodes[y].v_prod;
 }
 bool connected(int u, int v) { return find_root(++u)
   == find_root(++v); }
 void add_edge(int u, int v) {
  if (++u > ++v) swap(u, v);
  edges.emplace(u, v); link(u, v);
 }
 void del_edge(int u, int v) {
  auto k = minmax(++u, ++v);
  if (auto it = edges.find(k); it != edges.end()) {
   edges.erase(it); cut(u, v);
  }
 }
};
```

2.3 LiChao Segment Tree

```
struct L {
```

```
 int m, k, id;
 L() : id(-1) {}
 L(int a, int b, int c) : m(a), k(b), id(c) {}
 int at(int x) { return m * x + k; }
};
class LiChao {
private:
 int n; vector<L> nodes;
 static int lc(int x) { return 2 * x + 1; }
 static int rc(int x) { return 2 * x + 2; }
 void insert(int l, int r, int id, L ln) {
  int m = (l + r) >> 1;
  if (nodes[id].id == -1) {
   nodes[id] = ln;
   return;
  }
  bool atLeft = nodes[id].at(l) < ln.at(l);
  if (nodes[id].at(m) < ln.at(m)) {
   atLeft ^= 1;
   swap(nodes[id], ln);
  }
  if (r - l == 1) return;
  if (atLeft) insert(l, m, lc(id), ln);
  else insert(m, r, rc(id), ln);
 }
 int query(int l, int r, int id, int x) {
  int ret = 0, m = (l + r) >> 1;
  if (nodes[id].id != -1)
   ret = nodes[id].at(x);
  if (r - l == 1) return ret;
  if (x < m) return max(ret, query(l, m, lc(id), x));
  return max(ret, query(m, r, rc(id), x));
 }

public:
 LiChao(int n_) : n(n_), nodes(n * 4) {}
 void insert(L ln) { insert(0, n, 0, ln); }
 int query(int x) { return query(0, n, 0, x); }
};
```

2.4 Treap

```
namespace Treap{
 #define sz( x ) ( ( x ) ? ( ( x )->size ) : 0 )
 struct node{
  int size;
  uint32_t pri;
  node *lc, *rc, *pa;
  node():size(0),pri(rand()),lc(0),rc(0),pa(0){}
  void pull() {
   size = 1; pa = nullptr;
   if ( lc ) { size += lc->size; lc->pa = this; }
   if ( rc ) { size += rc->size; rc->pa = this; }
  }
 };
 node* merge( node* L, node* R ) {
  if ( not L or not R ) return L ? L : R;
  if ( L->pri > R->pri ) {
   L->rc = merge( L->rc, R ); L->pull();
   return L;
  } else {
   R->lc = merge( L, R->lc ); R->pull();
   return R;
  }
 }
 void split_by_size( node*rt,int k,node*&L,node*&R ) {
  if ( not rt ) L = R = nullptr;
  else if( sz( rt->lc ) + 1 <= k ) {
   L = rt;
   split_by_size( rt->rc,k-sz(rt->lc)-1,L->rc,R );
   L->pull();
  } else {
   R = rt;
   split_by_size( rt->lc, k, L, R->lc );
   R->pull();
  }
 } // sz(L) == k
 int getRank(node *o) { // 1-base
  int r = sz(o->lc) + 1;
  for (;o->pa != nullptr; o = o->pa)
   if (o->pa->rc == o) r += sz(o->pa->lc) + 1;
  return r;
 }
 #undef sz
}
```

2.5 Linear Basis

```
template <int BITS>
struct LinearBasis {
 array<uint64_t, BITS> basis;
 Basis() { basis.fill(0); }
 void add(uint64_t x) {
  for (int i = 0; i < BITS; ++i) if ((x >> i) & 1) {
   if (basis[i] == 0) {
    basis[i] = x;
    return;
   }
   x ^= basis[i];
  }
 }
 bool ok(uint64_t x) {
  for (int i = 0; i < BITS; ++i)
   if ((x >> i) & 1) x ^= basis[i];
  return x == 0;
 }
};
```

2.6 Binary Search On Segment Tree

```
// find_first = x -> minimal x s.t. check( [a, x) )
// find_last = x -> maximal x s.t. check( [x, b) )
template <typename C>
int find_first(int l, const C &check) {
 if (l >= n) return n + 1;
 l += sz;
 for (int i = height; i > 0; i--)
  propagate(l >> i);
 Monoid sum = identity;
 do {
  while ((l & 1) == 0) l >>= 1;
  if (check(f(sum, data[l]))) {
   while (l < sz) {
    propagate(l);
    l <<= 1;
    auto nxt = f(sum, data[l]);
    if (not check(nxt)) {
     sum = nxt;
     l++;
    }
   }
   return l + 1 - sz;
  }
  sum = f(sum, data[l++]);
 } while ((l & -l) != l);
 return n + 1;
}
template <typename C>
int find_last(int r, const C &check) {
 if (r <= 0) return -1;
 r += sz;
 for (int i = height; i > 0; i--)
  propagate((r - 1) >> i);
 Monoid sum = identity;
 do {
  r--;
  while (r > 1 and (r & 1)) r >>= 1;
  if (check(f(data[r], sum))) {
   while (r < sz) {
    propagate(r);
    r = (r << 1) + 1;
    auto nxt = f(data[r], sum);
    if (not check(nxt)) {
     sum = nxt;
     r--;
    }
   }
   return r - sz;
  }
  sum = f(data[r], sum);
 } while ((r & -r) != r);
 return -1;
}
```

3 Graph

3.1 2-SAT (SCC)

```
class TwoSat{
private:
 int n;
 vector<vector<int>> rG,G,sccs;
 vector<int> ord,idx;
 vector<bool> vis,result;
 void dfs(int u){
  vis[u]=true;
  for(int v:G[u])
   if(!vis[v]) dfs(v);
  ord.push_back(u);
 }
 void rdfs(int u){
  vis[u]=false;idx[u]=sccs.size()-1;
  sccs.back().push_back(u);
  for(int v:rG[u])
   if(vis[v])rdfs(v);
 }
public:
 void init(int n_){
  G.clear();G.resize(n=n_);
  rG.clear();rG.resize(n);
  sccs.clear();ord.clear();
  idx.resize(n);result.resize(n);
 }
 void add_edge(int u,int v){
  G[u].push_back(v);rG[v].push_back(u);
 }
 void orr(int x,int y){
  if ((x^y)==1)return;
  add_edge(x^1,y); add_edge(y^1,x);
 }
 bool solve(){
  vis.clear();vis.resize(n);
  for(int i=0;i<n;++i)
   if(not vis[i])dfs(i);
  reverse(ord.begin(),ord.end());
  for (int u:ord){
   if(!vis[u])continue;
   sccs.push_back(vector<int>());
   rdfs(u);
  }
  for(int i=0;i<n;i+=2)
   if(idx[i]==idx[i+1])
    return false;
  vector<bool> c(sccs.size());
  for(size_t i=0;i<sccs.size();++i){
   for(auto sij : sccs[i]){
    result[sij]=c[i];
    c[idx[sij^1]]=!c[i];
   }
  }
  return true;
 }
 bool get(int x){return result[x];}
 int get_id(int x){return idx[x];}
 int count(){return sccs.size();}
} sat2;
```

3.2 BCC Edge

```
class BCC_Bridge {
 private:
  int n, ecnt;
  vector<vector<pair<int,int>>> G;
  vector<int> dfn, low;
  vector<bool> bridge;
  void dfs(int u, int f) {
   dfn[u] = low[u] = dfn[f] + 1;
   for (auto [v, t]: G[u]) {
    if (v == f) continue;
    if (dfn[v]) {
     low[u] = min(low[u], dfn[v]);
     continue;
    }
    dfs(v, u);
    low[u] = min(low[u], low[v]);
    if (low[v] > dfn[u]) bridge[t] = true;
   }
  }
 public:
  void init(int n_) {
   G.clear(); G.resize(n = n_);
   low.assign(n, ecnt = 0);
   dfn.assign(n, 0);
  }
  void add_edge(int u, int v) {
   G[u].emplace_back(v, ecnt);
   G[v].emplace_back(u, ecnt++);
  }
  void solve() {
   bridge.assign(ecnt, false);
   for (int i = 0; i < n; ++i)
    if (not dfn[i]) dfs(i, i);
  }
  bool is_bridge(int x) { return bridge[x]; }
} bcc_bridge;
```

3.3 BCC Vertex

```
class BCC_AP {
 private:
  int n, ecnt;
  vector<vector<pair<int,int>>> G;
  vector<int> bcc, dfn, low, st;
  vector<bool> ap, ins;
  void dfs(int u, int f) {
   dfn[u] = low[u] = dfn[f] + 1;
   int ch = 0;
   for (auto [v, t]: G[u]) if (v != f) {
    if (not ins[t]) {
     st.push_back(t);
     ins[t] = true;
    }
    if (dfn[v]) {
     low[u] = min(low[u], dfn[v]);
     continue;
    } ++ch; dfs(v, u);
    low[u] = min(low[u], low[v]);
    if (low[v] >= dfn[u]) {
     ap[u] = true;
     while (true) {
      int eid = st.back(); st.pop_back();
      bcc[eid] = ecnt;
      if (eid == t) break;
     }
     ecnt++;
    }
   }
   if (ch == 1 and u == f) ap[u] = false;
  }
 public:
  void init(int n_) {
   G.clear(); G.resize(n = n_);
   ecnt = 0; ap.assign(n, false);
   low.assign(n, 0); dfn.assign(n, 0);
  }
  void add_edge(int u, int v) {
   G[u].emplace_back(v, ecnt);
   G[v].emplace_back(u, ecnt++);
  }
  void solve() {
   ins.assign(ecnt, false);
   bcc.resize(ecnt); ecnt = 0;
   for (int i = 0; i < n; ++i)
    if (not dfn[i]) dfs(i, i);
  }
  int get_id(int x) { return bcc[x]; }
  int count() { return ecnt; }
  bool is_ap(int x) { return ap[x]; }
} bcc_ap;
```

3.4 Centroid Decomposition

```
struct Centroid {
 vector<vector<int64_t>> Dist;
 vector<int> Parent, Depth;
 vector<int64_t> Sub, Sub2;
 vector<int> Sz, Sz2;
 Centroid(vector<vector<pair<int, int>>> g) {
  int N = g.size();
  vector<bool> Vis(N);
  vector<int> sz(N), mx(N);
  vector<int> Path;
  Dist.resize(N);
  Parent.resize(N);
  Depth.resize(N);
```

```
  auto DfsSz = [&](auto dfs, int x) -> void {
   Vis[x] = true; sz[x] = 1; mx[x] = 0;
   for (auto [u, w] : g[x]) {
    if (Vis[u]) continue;
    dfs(dfs, u);
    sz[x] += sz[u];
    mx[x] = max(mx[x], sz[u]);
   }
   Path.push_back(x);
  };
  auto DfsDist = [&](auto dfs, int x, int64_t D = 0)
   -> void {
   Dist[x].push_back(D);Vis[x] = true;
   for (auto [u, w] : g[x]) {
    if (Vis[u]) continue;
    dfs(dfs, u, D + w);
   }
  };
  auto Dfs = [&]
   (auto dfs, int x, int D = 0, int p = -1)->void {
   Path.clear(); DfsSz(DfsSz, x);
   int M = Path.size();
   int C = -1;
   for (int u : Path) {
    if (max(M - sz[u], mx[u]) * 2 <= M) C = u;
    Vis[u] = false;
   }
   DfsDist(DfsDist, C);
   for (int u : Path) Vis[u] = false;
   Parent[C] = p; Vis[C] = true;
   Depth[C] = D;
   for (auto [u, w] : g[C]) {
    if (Vis[u]) continue;
    dfs(dfs, u, D + 1, C);
   }
  };
  Dfs(Dfs, 0); Sub.resize(N); Sub2.resize(N);
  Sz.resize(N); Sz2.resize(N);
 }
 void Mark(int v) {
  int x = v, z = -1;
  for (int i = Depth[v]; i >= 0; --i) {
   Sub[x] += Dist[v][i]; Sz[x]++;
   if (z != -1) {
    Sub2[z] += Dist[v][i];
    Sz2[z]++;
   }
   z = x; x = Parent[x];
  }
 }
 int64_t Query(int v) {
  int64_t res = 0;
  int x = v, z = -1;
  for (int i = Depth[v]; i >= 0; --i) {
   res += Sub[x] + 1LL * Sz[x] * Dist[v][i];
   if (z != -1) res-=Sub2[z]+1LL*Sz2[z]*Dist[v][i];
   z = x; x = Parent[x];
  }
  return res;
 }
};
```

3.5 Directed Minimum Spanning Tree

```
struct DirectedMST { // find maximum
 struct Edge {
  int u, v;
  int w;
  Edge(int u, int v, int w) : u(u), v(v), w(w) {}
 };
 vector<Edge> Edges;
 void clear() { Edges.clear(); }
 void addEdge(int a, int b, int w) { Edges.emplace_back
    (a, b, w); }
 int solve(int root, int n) {
  vector<Edge> E = Edges;
  int ans = 0;
  while (true) {
   // find best in edge
   vector<int> in(n, -inf), prv(n, -1);
   for (auto e : E)
    if (e.u != e.v && e.w > in[e.v]) {
     in[e.v] = e.w;
     prv[e.v] = e.u;
    }
   in[root] = 0;
   prv[root] = -1;
   for (int i = 0; i < n; i++)
    if (in[i] == -inf)
     return -inf;
   // find cycle
   int tot = 0;
   vector<int> id(n, -1), vis(n, -1);
   for (int i = 0; i < n; i++) {
    ans += in[i];
    for (int x = i; x != -1 && id[x] == -1; x = prv[x])
     {
     if (vis[x] == i) {
      for (int y = prv[x]; y != x; y = prv[y])
       id[y] = tot;
      id[x] = tot++;
      break;
     }
     vis[x] = i;
    }
   }
   if (!tot)
    return ans;
   for (int i = 0; i < n; i++)
    if (id[i] == -1)
     id[i] = tot++;
   // shrink
   for (auto &e : E) {
    if (id[e.u] != id[e.v])
     e.w -= in[e.v];
    e.u = id[e.u], e.v = id[e.v];
   }
   n = tot;
   root = id[root];
  }
  assert(false);
 }
} DMST;
```

3.6 Dominator Tree

```
namespace dominator {
vector<int> g[maxn], r[maxn], rdom[maxn];
int dfn[maxn], rev[maxn], fa[maxn], sdom[maxn];
int dom[maxn], val[maxn], rp[maxn], tk;
void init(int n) {
 // vertices are numbered from 0 to n - 1
 fill(dfn, dfn + n, -1);fill(rev, rev + n, -1);
 fill(fa, fa + n, -1);fill(val, val + n, -1);
 fill(sdom, sdom + n, -1);fill(rp, rp + n, -1);
 fill(dom, dom + n, -1); tk = 0;
 for (int i = 0; i < n; ++i) {
  g[i].clear(); r[i].clear(); rdom[i].clear();
 }
}
void add_edge(int x, int y) { g[x].push_back(y); }
void dfs(int x) {
 rev[dfn[x] = tk] = x;
 fa[tk] = sdom[tk] = val[tk] = tk; tk ++;
 for (int u : g[x]) {
  if (dfn[u] == -1) dfs(u), rp[dfn[u]] = dfn[x];
  r[dfn[u]].push_back(dfn[x]);
 }
}
void merge(int x, int y) { fa[x] = y; }
int find(int x, int c = 0) {
 if (fa[x] == x) return c ? -1 : x;
 int p = find(fa[x], 1);
 if (p == -1) return c ? fa[x] : val[x];
 if (sdom[val[x]]>sdom[val[fa[x]]]) val[x]=val[fa[x]];
 fa[x] = p;
 return c ? p : val[x];
}
vector<int> build(int s, int n) {
// return the father of each node in the dominator tree
// p[i] = -2 if i is unreachable from s
 dfs(s);
 for (int i = tk - 1; i >= 0; --i) {
  for (int u:r[i]) sdom[i]=min(sdom[i],sdom[find(u)]);
  if (i) rdom[sdom[i]].push_back(i);
  for (int &u : rdom[i]) {
```

```
   int p = find(u);
   if (sdom[p] == i) dom[u] = i;
   else dom[u] = p;
  }
  if (i) merge(i, rp[i]);
 }
 vector<int> p(n, -2); p[s] = -1;
 for (int i = 1; i < tk; ++i)
  if (sdom[i] != dom[i]) dom[i] = dom[dom[i]];
 for (int i = 1; i < tk; ++i) p[rev[i]] = rev[dom[i]];
 return p;
}}
```

3.7 Edge Coloring

```
// max(d_u) + 1 edge coloring, time: O(NM)
int C[kN][kN], G[kN][kN]; // 1-based, G: ans
void clear(int N) {
 for (int i = 0; i <= N; i++)
  for (int j = 0; j <= N; j++)
    C[i][j] = G[i][j] = 0;
}
void solve(vector<pair<int, int>> &E, int N) {
 int X[kN] = {}, a;
 auto update = [&](int u) {
  for (X[u] = 1; C[u][X[u]]; X[u]++);
 };
 auto color = [&](int u, int v, int c) {
  int p = G[u][v];
  G[u][v] = G[v][u] = c;
  C[u][c] = v, C[v][c] = u;
  C[u][p] = C[v][p] = 0;
  if (p) X[u] = X[v] = p;
  else update(u), update(v);
  return p;
 };
 auto flip = [&](int u, int c1, int c2) {
  int p = C[u][c1];
  swap(C[u][c1], C[u][c2]);
  if (p) G[u][p] = G[p][u] = c2;
  if (!C[u][c1]) X[u] = c1;
  if (!C[u][c2]) X[u] = c2;
  return p;
 };
 for (int i = 1; i <= N; i++) X[i] = 1;
 for (int t = 0; t < E.size(); t++) {
  auto [u, v] = E[t];
  int v0 = v, c = X[u], c0 = c, d;
  vector<pair<int, int>> L; int vst[kN] = {};
  while (!G[u][v0]) {
   L.emplace_back(v, d = X[v]);
   if (!C[v][c]) for(a=L.size()-1;a>=0;a--)
     c = color(u, L[a].first, c);
   else if(!C[u][d])for(a=L.size()-1;a>=0;a--)
     color(u, L[a].first, L[a].second);
   else if (vst[d]) break;
   else vst[d] = 1, v = C[u][d];
  }
  if (!G[u][v0]) {
   for (; v; v = flip(v, c, d), swap(c, d));
   if (C[u][c0]) { a = int(L.size()) - 1;
    while (--a >= 0 && L[a].second != c);
    for(;a>=0;a--)color(u,L[a].first,L[a].second);
   } else t--;
  }
 }
}
```

3.8 Lowbit Decomposition

```
class LBD {
 int timer, chains;
 vector<vector<int>> G;
 vector<int> tl, tr, chain, head, dep, pa;
 // chains : number of chain
 // tl, tr[u] : subtree interval in the seq. of u
 // head[i] : head of the chain i
 // chian[u] : chain id of the chain u is on
 void predfs(int u, int f) {
  dep[u] = dep[pa[u] = f] + 1;
  for (int v : G[u]) if (v != f) {
   predfs(v, u);
   if (lowbit(chain[u]) < lowbit(chain[v]))
    chain[u] = chain[v];
  }
  if (chain[u] == 0) chain[u] = ++chains;
 }
 void dfschain(int u, int f) {
  tl[u] = timer++;
  if (head[chain[u]] == -1)
   head[chain[u]] = u;
  for (int v : G[u])
   if (v != f and chain[v] == chain[u])
    dfschain(v, u);
  for (int v : G[u])
   if (v != f and chain[v] != chain[u])
    dfschain(v, u);
  tr[u] = timer;
 }
public:
 LBD(int n) : timer(0), chains(0), G(n), tl(n), tr(n),
    chain(n), head(n, -1), dep(n), pa(n) {}
 void add_edge(int u, int v) {
  G[u].push_back(v); G[v].push_back(u);
 }
 void decompose() { predfs(0, 0); dfschain(0, 0); }
 PII get_subtree(int u) { return {tl[u], tr[u]}; }
 vector<PII> get_path(int u, int v) {
  vector<PII> res;
  while (chain[u] != chain[v]) {
   if (dep[head[chain[u]]] < dep[head[chain[v]]])
    swap(u, v);
   int s = head[chain[u]];
   res.emplace_back(tl[s], tl[u] + 1);
   u = pa[s];
  }
  if (dep[u] < dep[v]) swap(u, v);
  res.emplace_back(tl[v], tl[u] + 1);
  return res;
 }
};
```

3.9 Manhattan Minimum Spanning Tree

```
typedef Point<int> P;
vector<array<int, 3>> manhattanMST(vector<P> ps) {
 vi id(sz(ps));
 iota(all(id), 0);
 vector<array<int, 3>> edges;
 rep(k, 0, 4) {
  sort(all(id), [&](int i, int j) {
   return (ps[i] - ps[j]).x < (ps[j] - ps[i]).y;
  });
  map<int, int> sweep;
  for (int i : id) {
   for (auto it = sweep.lower_bound(-ps[i].y);
      it != sweep.end(); sweep.erase(it++)) {
    int j = it->second;
    P d = ps[i] - ps[j];
    if (d.y > d.x) break;
    edges.push_back({d.y + d.x, i, j});
   }
   sweep[-ps[i].y] = i;
  }
  for (P &p : ps)
   if (k & 1) p.x = -p.x;
   else swap(p.x, p.y);
 }
 return edges; // [{w, i, j}, ...]
}
```

3.10 MaxClique

```
// contain a self loop u to u, than u won't in clique
template < size_t MAXN >
class MaxClique{
private:
 using bits = bitset< MAXN >;
 bits popped, G[ MAXN ], ans;
 size_t deg[ MAXN ], deo[ MAXN ], n;
 void sort_by_degree() {
  popped.reset();
  for ( size_t i = 0 ; i < n ; ++ i )
    deg[ i ] = G[ i ].count();
  for ( size_t i = 0 ; i < n ; ++ i ) {
    size_t mi = MAXN, id = 0;
    for ( size_t j = 0 ; j < n ; ++ j )
      if ( not popped[ j ] and deg[ j ] < mi )
```

```
      mi = deg[ id = j ];
    popped[ deo[ i ] = id ] = 1;
    for( size_t u = G[ i ]._Find_first() ;
     u < n ; u = G[ i ]._Find_next( u ) )
      -- deg[ u ];
  }
 }
 void BK( bits R, bits P, bits X ) {
  if (R.count()+P.count() <= ans.count()) return;
  if ( not P.count() and not X.count() ) {
   if ( R.count() > ans.count() ) ans = R;
   return;
  }
  /* greedily chosse max degree as pivot
  bits cur = P | X; size_t pivot = 0, sz = 0;
  for ( size_t u = cur._Find_first() ;
   u < n ; u = cur._Find_next( u ) )
    if ( deg[ u ] > sz ) sz = deg[ pivot = u ];
  cur = P & ( ~G[ pivot ] );
  */ // or simply choose first
  bits cur = P & (~G[ ( P | X )._Find_first() ]);
  for ( size_t u = cur._Find_first() ;
   u < n ; u = cur._Find_next( u ) ) {
   if ( R[ u ] ) continue;
   R[ u ] = 1;
   BK( R, P & G[ u ], X & G[ u ] );
   R[ u ] = P[ u ] = 0, X[ u ] = 1;
  }
 }
public:
 void init( size_t n_ ) {
  n = n_;
  for ( size_t i = 0 ; i < n ; ++ i )
   G[ i ].reset();
  ans.reset();
 }
 void add_edges( int u, bits S ) { G[ u ] = S; }
 void add_edge( int u, int v ) {
  G[ u ][ v ] = G[ v ][ u ] = 1;
 }
 int solve() {
  sort_by_degree(); // or simply iota( deo... )
  for ( size_t i = 0 ; i < n ; ++ i )
   deg[ i ] = G[ i ].count();
  bits pob, nob = 0; pob.set();
  for (size_t i=n; i<MAXN; ++i) pob[i] = 0;
  for ( size_t i = 0 ; i < n ; ++ i ) {
   size_t v = deo[ i ];
   bits tmp; tmp[ v ] = 1;
   BK( tmp, pob & G[ v ], nob & G[ v ] );
   pob[ v ] = 0, nob[ v ] = 1;
  }
  return static_cast< int >( ans.count() );
 }
};
```

3.11 MaxCliqueDyn

```
constexpr int kN = 150;
struct MaxClique { // Maximum Clique
 bitset<kN> a[kN], cs[kN];
 int ans, sol[kN], q, cur[kN], d[kN], n;
 void init(int _n) {
  n = _n, ans = q = 0;
  for (int i = 0; i < n; i++) a[i].reset();
 }
 void addEdge(int u, int v) { a[u][v] = a[v][u] = 1; }
 void csort(vector<int> &r, vector<int> &c) {
  int mx = 1, km = max(ans - q + 1, 1), t = 0,
    m = int(r.size());
  cs[1].reset(); cs[2].reset();
  for (int i = 0; i < m; i++) {
   int p = r[i], k = 1;
   while ((cs[k] & a[p]).count()) k++;
   if (k > mx) cs[++mx + 1].reset();
   cs[k][p] = 1;
   if (k < km) r[t++] = p;
  }
  c.resize(m);
  if (t) c[t - 1] = 0;
  for (int k = km; k <= mx; k++) {
   for (int p = int(cs[k]._Find_first());
      p < kN; p = int(cs[k]._Find_next(p))) {
    r[t] = p; c[t++] = k;
   }
  }
 }
 void dfs(vector<int> &r, vector<int> &c, int l,
  bitset<kN> mask) {
  while (!r.empty()) {
   int p = r.back(); r.pop_back();
   mask[p] = 0;
   if (q + c.back() <= ans) return;
   cur[q++] = p;
   vector<int> nr, nc;
   bitset<kN> nmask = mask & a[p];
   for (int i : r)
    if (a[p][i]) nr.push_back(i);
   if (!nr.empty()) {
    if (l < 4) {
     for (int i : nr)
      d[i] = int((a[i] & nmask).count());
     sort(nr.begin(), nr.end(),
      [&](int x, int y) {
       return d[x] > d[y];
      });
    }
    csort(nr, nc); dfs(nr, nc, l + 1, nmask);
   } else if (q > ans) {
    ans = q; copy(cur, cur + q, sol);
   }
   c.pop_back(); q--;
  }
 }
 int solve(bitset<kN> mask) { // vertex mask
  vector<int> r, c;
  for (int i = 0; i < n; i++)
   if (mask[i]) r.push_back(i);
  for (int i = 0; i < n; i++)
   d[i] = int((a[i] & mask).count());
  sort(r.begin(), r.end(),
   [&](int i, int j) { return d[i] > d[j]; });
  csort(r, c);
  dfs(r, c, 1, mask);
  return ans; // sol[0 ~ ans-1]
 }
} graph;
```

3.12 Minimum Mean Cycle

```
/* minimum mean cycle O(VE) */
struct MMC{
#define FZ(n) memset((n),0,sizeof(n))
#define E 101010
#define V 1021
#define inf 1e9
 struct Edge { int v,u; double c; };
 int n, m, prv[V][V], prve[V][V], vst[V];
 Edge e[E];
 vector<int> edgeID, cycle, rho;
 double d[V][V];
 void init( int _n ) { n = _n; m = 0; }
 // WARNING: TYPE matters
 void add_edge( int vi , int ui , double ci )
 { e[ m ++ ] = { vi , ui , ci }; }
 void bellman_ford() {
  for(int i=0; i<n; i++) d[0][i]=0;
  for(int i=0; i<n; i++) {
   fill(d[i+1], d[i+1]+n, inf);
   for(int j=0; j<m; j++) {
    int v = e[j].v, u = e[j].u;
    if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
     d[i+1][u] = d[i][v]+e[j].c;
     prv[i+1][u] = v;
     prve[i+1][u] = j;
    }
   }
  }
 }
 double solve(){
  // returns inf if no cycle, mmc otherwise
  double mmc=inf;
  int st = -1;
  bellman_ford();
  for(int i=0; i<n; i++) {
   double avg=-inf;
```

```
   for(int k=0; k<n; k++) {
    if(d[n][i]<inf-eps)
     avg=max(avg,(d[n][i]-d[k][i])/(n-k));
    else avg=max(avg,inf);
   }
   if (avg < mmc) tie(mmc, st) = tie(avg, i);
  }
  FZ(vst);edgeID.clear();cycle.clear();rho.clear();
  for (int i=n; !vst[st]; st=prv[i--][st]) {
   vst[st]++;
   edgeID.PB(prve[i][st]);
   rho.PB(st);
  }
  while (vst[st] != 2) {
   int v = rho.back(); rho.pop_back();
   cycle.PB(v);
   vst[v]++;
  }
  reverse(ALL(edgeID));
  edgeID.resize(SZ(cycle));
  return mmc;
 }
} mmc;
```

3.13 Minimum Steiner Tree

```
// Minimum Steiner Tree
// O(V 3^T + V^2 2^T)
struct SteinerTree {
#define V 33
#define T 8
#define INF 1023456789
 int n, dst[V][V], dp[1 << T][V], tdst[V];
 void init(int _n) {
  n = _n;
  for (int i = 0; i < n; i++) {
   for (int j = 0; j < n; j++)
    dst[i][j] = INF * (i != j);
  }
 }
 void add_edge(int ui, int vi, int wi) {
  dst[ui][vi] = min(dst[ui][vi], wi);
  dst[vi][ui] = min(dst[vi][ui], wi);
 }
 void shortest_path() {
  for (int k = 0; k < n; k++)
   for (int i = 0; i < n; i++)
    for (int j = 0; j < n; j++)
     dst[i][j] = min(dst[i][j], dst[i][k] + dst[k][j]);
 }
 int solve(const vector<int> &ter) {
  int t = (int)ter.size();
  for (int i = 1; i < (1 << t); i++)
   fill_n(dp[i], n, INF);
  fill_n(dp[0], n, 0);
  for (int msk = 1; msk < (1 << t); msk++) {
   if (msk == (msk & (-msk))) {
    int who = __lg(msk);
    for (int i = 0; i < n; i++)
     dp[msk][i] = dst[ter[who]][i];
    continue;
   }
   for (int i = 0; i < n; i++)
    for (int submsk = (msk - 1) & msk; submsk; submsk =
      (submsk - 1) & msk)
     dp[msk][i] = min(dp[msk][i], dp[submsk][i] + dp[
    msk ^ submsk][i]);
   for (int i = 0; i < n; i++) {
    tdst[i] = INF;
    for (int j = 0; j < n; j++)
     tdst[i] = min(tdst[i], dp[msk][j] + dst[j][i]);
   }
   copy_n(tdst, n, dp[msk]);
  }
  int ans = INF;
  for (int i = 0; i < n; i++)
   ans = min(ans, dp[(1 << t) - 1][i]);
  return ans;
 }
} solver;
```

3.14 Mo's Algorithm on Tree

```
int q; vector< int > G[N];
struct Que{
 int u, v, id;
} que[ N ];
int dfn[N], dfn_, block_id[N], block_, stk[N], stk_;
void dfs( int u, int f ) {
 dfn[ u ] = dfn_++; int saved_rbp = stk_;
 for ( int v : G[ u ] ) {
  if ( v == f ) continue;
  dfs( v, u );
  if ( stk_ - saved_rbp < SQRT_N ) continue;
  for ( ++ block_ ; stk_ != saved_rbp ; )
    block_id[ stk[ -- stk_ ] ] = block_;
 }
 stk[ stk_ ++ ] = u;
}
bool inPath[ N ];
void Diff( int u ) {
 if ( inPath[ u ] ^= 1 ) { /*remove this edge*/ }
 else { /*add this edge*/ }
}
void traverse( int& origin_u, int u ) {
 for ( int g = lca( origin_u, u ) ;
  origin_u != g ; origin_u = parent_of[ origin_u ] )
   Diff( origin_u );
 for (int v = u; v != origin_u; v = parent_of[v])
  Diff( v );
 origin_u = u;
}
void solve() {
 dfs( 1, 1 );
 while ( stk_ ) block_id[ stk[ -- stk_ ] ] = block_;
 sort( que, que + q, [](const Que& x, const Que& y) {
  return tie( block_id[ x.u ], dfn[ x.v ] )
      < tie( block_id[ y.u ], dfn[ y.v ] );
 } );
 int U = 1, V = 1;
 for ( int i = 0 ; i < q ; ++ i ) {
  pass( U, que[ i ].u );
  pass( V, que[ i ].v );
  // we could get our answer of que[ i ].id
 }
}
/*
Method 2:
dfs u:
 push u
 iterate subtree
 push u
Let P = LCA(u, v), and St(u)<=St(v)
if (P == u) query[St(u), St(v)]
else query[Ed(u), St(v)], query[St(P), St(P)]
*/
```

3.15 Virtural Tree

```
vector<pair<int, int>> build(vector<int> vs) { // tree
    0-base
 vector<pair<int, int>> res;
 sort(vs.begin(), vs.end(), [](int i, int j) { return
    dfn[i] < dfn[j]; });
 vector<int> s = {0};
 for (int v : vs) if (v != 0) {
  int o = lca(v, s.back());
  if (o != s.back()) {
   while (s.size() >= 2 and dfn[s[s.size() - 2]] >= dfn
     [o]) {
    res.emplace_back(s[s.size() - 2], s.back());
    s.pop_back();
   }
   if (s.back() != o) {
    res.emplace_back(s.back(), o);
    s.back() = o;
   }
  }
  s.push_back(v);
 }
 for (size_t i = 1; i < s.size(); ++i)
  res.emplace_back(s[i - 1], s[i]);
 return res;
}
```

4 Matching & Flow

4.1 Bipartite Matching

```
struct BipartiteMatching {
 vector<int> X[N];
 int fX[N], fY[N], n;
 bitset<N> vis;
 bool dfs(int x) {
  for (auto i:X[x]) {
   if (vis[i]) continue;
   vis[i] = true;
   if (fY[i]==-1 || dfs(fY[i])){
    fY[fX[x] = i] = x;
    return true;
   }
  }
  return false;
 }
 void init(int n_, int m) {
  vis.reset();
  fill(X, X + (n = n_), vector<int>());
  memset(fX, -1, sizeof(int) * n);
  memset(fY, -1, sizeof(int) * m);
 }
 void add_edge(int x, int y){
  X[x].push_back(y); }
 int solve() { // return how many pair matched
  int cnt = 0;
  for(int i=0;i<n;i++) {
   vis.reset();
   cnt += dfs(i);
  }
  return cnt;
 }
};
```

4.2 Dijkstra Cost Flow

```
// kN = #(vertices)
// MCMF.{Init, AddEdge, MincostMaxflow}
// MincostMaxflow(source, sink, flow_limit, &cost)
//  => flow
using Pii = pair<int, int>;
constexpr int kInf = 0x3f3f3f3f, kN = 500;
struct Edge {
 int to, rev, cost, flow;
};
struct MCMF { // 0-based
 int n{}, m{}, s{}, t{};
 vector<Edge> graph[kN];
 // Larger range for relabeling
 int64_t dis[kN] = {}, h[kN] = {};
 int p[kN] = {};
 void Init(int nn) {
  n = nn;
  for (int i = 0; i < n; i++) graph[i].clear();
 }
 void AddEdge(int u, int v, int f, int c) {
  graph[u].push_back({v,
   static_cast<int>(graph[v].size()), c, f});
  graph[v].push_back(
   {u, static_cast<int>(graph[u].size()) - 1,
    -c, 0});
 }
 bool Dijkstra(int &max_flow, int64_t &cost) {
  priority_queue<Pii, vector<Pii>, greater<>> pq;
  fill_n(dis, n, kInf);
  dis[s] = 0;
  pq.emplace(0, s);
  while (!pq.empty()) {
   auto u = pq.top();
   pq.pop();
   int v = u.second;
   if (dis[v] < u.first) continue;
   for (auto &e : graph[v]) {
    auto new_dis =
     dis[v] + e.cost + h[v] - h[e.to];
    if (e.flow > 0 && dis[e.to] > new_dis) {
     dis[e.to] = new_dis;
     p[e.to] = e.rev;
     pq.emplace(dis[e.to], e.to);
    }
   }
  }
  if (dis[t] == kInf) return false;
  for (int i = 0; i < n; i++) h[i] += dis[i];
  int d = max_flow;
  for (int u = t; u != s;
     u = graph[u][p[u]].to) {
   auto &e = graph[u][p[u]];
   d = min(d, graph[e.to][e.rev].flow);
  }
  max_flow -= d;
  cost += int64_t(d) * h[t];
  for (int u = t; u != s;
     u = graph[u][p[u]].to) {
   auto &e = graph[u][p[u]];
   e.flow += d;
   graph[e.to][e.rev].flow -= d;
  }
  return true;
 }
 int MincostMaxflow(
  int ss, int tt, int max_flow, int64_t &cost) {
  this->s = ss, this->t = tt;
  cost = 0;
  fill_n(h, n, 0);
  auto orig_max_flow = max_flow;
  while (Dijkstra(max_flow, cost) && max_flow) {}
  return orig_max_flow - max_flow;
 }
};
```

4.3 Dinic

```
template <typename Cap = int64_t>
class Dinic{
private:
  struct E{
    int to, rev;
    Cap cap;
  };
  int n, st, ed;
  vector<vector<E>> G;
  vector<int> lv, idx;
  bool BFS(){
    lv.assign(n, -1);
    queue<int> bfs;
    bfs.push(st); lv[st] = 0;
    while (not bfs.empty()){
      int u = bfs.front(); bfs.pop();
      for (auto e: G[u]) {
        if (e.cap <= 0 or lv[e.to]!=-1) continue;
        bfs.push(e.to); lv[e.to] = lv[u] + 1;
      }
    }
    return lv[ed] != -1;
  }
  Cap DFS(int u, Cap f){
    if (u == ed) return f;
    Cap ret = 0;
    for(int &i = idx[u]; i < int(G[u].size()); ++i) {
      auto &e = G[u][i];
      if (e.cap <= 0 or lv[e.to]!=lv[u]+1) continue;
      Cap nf = DFS(e.to, min(f, e.cap));
      ret += nf; e.cap -= nf; f -= nf;
      G[e.to][e.rev].cap += nf;
      if (f == 0) return ret;
    }
    if (ret == 0) lv[u] = -1;
    return ret;
  }
public:
  void init(int n_) { G.assign(n = n_, vector<E>()); }
  void add_edge(int u, int v, Cap c){
    G[u].push_back({v, int(G[v].size()), c});
    G[v].push_back({u, int(G[u].size())-1, 0});
  }
  Cap max_flow(int st_, int ed_){
    st = st_, ed = ed_; Cap ret = 0;
    while (BFS()) {
      idx.assign(n, 0);
      Cap f = DFS(st, numeric_limits<Cap>::max());
      ret += f;
      if (f == 0) break;
    }
```

```
    return ret;
  }
};
```

4.4 Flow Models

- Maximum/Minimum flow with lower bound / Circulation problem
  1. Construct super source S and sink T.
  2. For each edge (x, y, l, u), connect $x \to y$ with capacity $u - l$.
  3. For each vertex v, denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  4. If $in(v) > 0$, connect $S \to v$ with capacity $in(v)$, otherwise, connect $v \to T$ with capacity $-in(v)$.
     - To maximize, connect $t \to s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T. If $f \neq \sum_{v \in V, in(v)>0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
     - To minimize, let f be the maximum flow from S to T. Connect $t \to s$ with capacity ∞ and let the flow from S to T be f'. If $f + f' \neq \sum_{v \in V, in(v)>0} in(v)$, there's no solution. Otherwise, f' is the answer.
  5. The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
  1. Redirect every edge: $y \to x$ if $(x, y) \in M$, $x \to y$ otherwise.
  2. DFS from unmatched vertices in X.
  3. $x \in X$ is chosen iff x is unvisited.
  4. $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
  1. Consruct super source S and sink T
  2. For each edge (x, y, c), connect $x \to y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \to x$ with $(cost, cap) = (-c, 1)$
  3. For each edge with $c < 0$, sum these cost as K, then increase $d(y)$ by 1, decrease $d(x)$ by 1
  4. For each vertex v with $d(v) > 0$, connect $S \to v$ with $(cost, cap) = (0, d(v))$
  5. For each vertex v with $d(v) < 0$, connect $v \to T$ with $(cost, cap) = (0, -d(v))$
  6. Flow from S to T, the answer is the cost of the flow $C + K$
- Maximum density induced subgraph
  1. Binary search on answer, suppose we're checking answer T
  2. Construct a max flow model, let K be the sum of all weights
  3. Connect source $s \to v, v \in G$ with capacity K
  4. For each edge (u, v, w) in G, connect $u \to v$ and $v \to u$ with capacity w
  5. For $v \in G$, connect it with sink $v \to t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
  6. T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
  1. For each $v \in V$ create a copy v', and connect $u' \to v'$ with weight $w(u, v)$.
  2. Connect $v \to v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v.
  3. Find the minimum weight perfect matching on G'.
- Project selection problem
  1. If $p_v > 0$, create edge (s, v) with capacity p_v; otherwise, create edge (v, t) with capacity $-p_v$.
  2. Create edge (u, v) with capacity w with w being the cost of choosing u without choosing v.
  3. The mincut is equivalent to the maximum profit of a subset of projects.
- 0/1 quadratic programming

$$\sum_x c_x x + \sum_y c_y \bar{y} + \sum_{xy} c_{xy} x \bar{y} + \sum_{xyx'y'} c_{xyx'y'} (x\bar{y} + x'\bar{y'})$$

  can be minimized by the mincut of the following graph:

  1. Create edge (x, t) with capacity c_x and create edge (s, y) with capacity c_y.
  2. Create edge (x, y) with capacity c_{xy}.
  3. Create edge (x, y) and edge (x', y') with capacity $c_{xyx'y'}$.

4.5 General Graph Matching

```
namespace matching {
int fa[kN], pre[kN], match[kN], s[kN], v[kN];
vector<int> g[kN];
queue<int> q;
void Init(int n) {
 for (int i = 0; i <= n; ++i) match[i] = pre[i] = n;
 for (int i = 0; i < n; ++i) g[i].clear();
}
void AddEdge(int u, int v) {
 g[u].push_back(v);
 g[v].push_back(u);
}
int Find(int u) {
 return u == fa[u] ? u : fa[u] = Find(fa[u]);
}
int LCA(int x, int y, int n) {
 static int tk = 0; tk++;
 x = Find(x), y = Find(y);
 for (; ; swap(x, y)) {
  if (x != n) {
   if (v[x] == tk) return x;
   v[x] = tk;
   x = Find(pre[match[x]]);
  }
 }
}
void Blossom(int x, int y, int l) {
 while (Find(x) != l) {
  pre[x] = y, y = match[x];
  if (s[y] == 1) q.push(y), s[y] = 0;
  if (fa[x] == x) fa[x] = l;
  if (fa[y] == y) fa[y] = l;
  x = pre[y];
 }
}
bool Bfs(int r, int n) {
 for (int i = 0; i <= n; ++i) fa[i] = i, s[i] = -1;
 while (!q.empty()) q.pop();
 q.push(r);
 s[r] = 0;
 while (!q.empty()) {
  int x = q.front(); q.pop();
  for (int u : g[x]) {
   if (s[u] == -1) {
    pre[u] = x, s[u] = 1;
    if (match[u] == n) {
     for (int a = u, b = x, last; b != n; a = last, b =
     pre[a])
      last = match[b], match[b] = a, match[a] = b;
     return true;
    }
    q.push(match[u]);
    s[match[u]] = 0;
   } else if (!s[u] && Find(u) != Find(x)) {
    int l = LCA(u, x, n);
    Blossom(x, u, l);
    Blossom(u, x, l);
   }
  }
 }
 return false;
}
int Solve(int n) {
 int res = 0;
 for (int x = 0; x < n; ++x) {
  if (match[x] == n) res += Bfs(x, n);
 }
 return res;
}}
```

4.6 Global Min-Cut

```
const int maxn = 500 + 5;
int w[maxn][maxn], g[maxn];
bool v[maxn], del[maxn];
void add_edge(int x, int y, int c) {
 w[x][y] += c; w[y][x] += c;
}
pair<int, int> phase(int n) {
 memset(v, false, sizeof(v));
 memset(g, 0, sizeof(g));
 int s = -1, t = -1;
 while (true) {
  int c = -1;
  for (int i = 0; i < n; ++i) {
   if (del[i] || v[i]) continue;
   if (c == -1 || g[i] > g[c]) c = i;
  }
  if (c == -1) break;
  v[s = t, t = c] = true;
  for (int i = 0; i < n; ++i) {
   if (del[i] || v[i]) continue;
   g[i] += w[c][i];
  }
 }
 return make_pair(s, t);
```

```
}
int mincut(int n) {
 int cut = 1e9;
 memset(del, false, sizeof(del));
 for (int i = 0; i < n - 1; ++i) {
  int s, t; tie(s, t) = phase(n);
  del[t] = true; cut = min(cut, g[t]);
  for (int j = 0; j < n; ++j) {
   w[s][j] += w[t][j]; w[j][s] += w[j][t];
  }
 }
 return cut;
}
```

4.7 GomoryHu Tree

```
int g[maxn];
vector<edge> GomoryHu(int n){
 vector<edge> rt;
 for(int i=1;i<=n;++i)g[i]=1;
 for(int i=2;i<=n;++i){
  int t=g[i];
  flow.reset(); // clear flows on all edge
  rt.push_back({i,t,flow(i,t)});
  flow.walk(i); // bfs points that connected to i (use
    edges not fully flow)
  for(int j=i+1;j<=n;++j){
   if(g[j]==t && flow.connect(j))g[j]=i; // check if i
    can reach j
  }
 }
 return rt;
}
```

4.8 Kuhn Munkres

```
class KM {
private:
 static constexpr lld INF = 1LL << 60;
 vector<lld> hl,hr,slk;
 vector<int> fl,fr,pre,qu;
 vector<vector<lld>> w;
 vector<bool> vl,vr;
 int n, ql, qr;
 bool check(int x) {
  if (vl[x] = true, fl[x] != -1)
   return vr[qu[qr++] = fl[x]] = true;
  while (x != -1) swap(x, fr[fl[x] = pre[x]]);
  return false;
 }
 void bfs(int s) {
  fill(slk.begin(), slk.end(), INF);
  fill(vl.begin(), vl.end(), false);
  fill(vr.begin(), vr.end(), false);
  ql = qr = 0;
  vr[qu[qr++] = s] = true;
  while (true) {
   lld d;
   while (ql < qr) {
    for (int x = 0, y = qu[ql++]; x < n; ++x) {
     if(!vl[x]&&slk[x]>=(d=hl[x]+hr[y]-w[x][y])){
      if (pre[x] = y, d) slk[x] = d;
      else if (!check(x)) return;
     }
    }
   }
   d = INF;
   for (int x = 0; x < n; ++x)
    if (!vl[x] && d > slk[x]) d = slk[x];
   for (int x = 0; x < n; ++x) {
    if (vl[x]) hl[x] += d;
    else slk[x] -= d;
    if (vr[x]) hr[x] -= d;
   }
   for (int x = 0; x < n; ++x)
    if (!vl[x] && !slk[x] && !check(x)) return;
  }
 }
public:
 void init( int n_ ) {
  qu.resize(n = n_);
  fl.assign(n, -1); fr.assign(n, -1);
  hr.assign(n, 0); hl.resize(n);
  w.assign(n, vector<lld>(n));
  slk.resize(n); pre.resize(n);
  vl.resize(n); vr.resize(n);
 }
 void set_edge( int u, int v, lld x ) {w[u][v] = x;}
 lld solve() {
  for (int i = 0; i < n; ++i)
   hl[i] = *max_element(w[i].begin(), w[i].end());
  for (int i = 0; i < n; ++i) bfs(i);
  lld res = 0;
  for (int i = 0; i < n; ++i) res += w[i][fl[i]];
  return res;
 }
} km;
```

4.9 Minimum Cost Circulation

```
struct Edge { int to, cap, rev, cost; };
vector<Edge> g[kN];
int dist[kN], pv[kN], ed[kN];
bool mark[kN];
int NegativeCycle(int n) {
 memset(mark, false, sizeof(mark));
 memset(dist, 0, sizeof(dist));
 int upd = -1;
 for (int i = 0; i <= n; ++i) {
  for (int j = 0; j < n; ++j) {
   int idx = 0;
   for (auto &e : g[j]) {
    if(e.cap > 0 && dist[e.to] > dist[j] + e.cost){
     dist[e.to] = dist[j] + e.cost;
     pv[e.to] = j, ed[e.to] = idx;
     if (i == n) {
      upd = j;
      while(!mark[upd])mark[upd]=1,upd=pv[upd];
      return upd;
     }
    }
    idx++;
   }
  }
 }
 return -1;
}
int Solve(int n) {
 int rt = -1, ans = 0;
 while ((rt = NegativeCycle(n)) >= 0) {
  memset(mark, false, sizeof(mark));
  vector<pair<int, int>> cyc;
  while (!mark[rt]) {
   cyc.emplace_back(pv[rt], ed[rt]);
   mark[rt] = true;
   rt = pv[rt];
  }
  reverse(cyc.begin(), cyc.end());
  int cap = kInf;
  for (auto &i : cyc) {
   auto &e = g[i.first][i.second];
   cap = min(cap, e.cap);
  }
  for (auto &i : cyc) {
   auto &e = g[i.first][i.second];
   e.cap -= cap;
   g[e.to][e.rev].cap += cap;
   ans += e.cost * cap;
  }
 }
 return ans;
}
```

4.10 Minimum Cost Maximum Flow

```
class MiniCostMaxiFlow{
 using Cap = int; using Wei = int64_t;
 using PCW = pair<Cap,Wei>;
 static constexpr Cap INF_CAP = 1 << 30;
 static constexpr Wei INF_WEI = 1LL<<60;
private:
 struct Edge{
  int to, back;
  Cap cap; Wei wei;
  Edge() {}
  Edge(int a,int b, Cap c, Wei d):
   to(a),back(b),cap(c),wei(d) {}
 };
```

```
 int ori, edd;
 vector<vector<Edge>> G;
 vector<int> fa, wh;
 vector<bool> inq;
 vector<Wei> dis;
 PCW SPFA(){
  fill(inq.begin(),inq.end(),false);
  fill(dis.begin(),dis.end(),INF_WEI);
  queue<int> qq; qq.push(ori);
  dis[ori] = 0;
  while(not qq.empty()){
   int u=qq.front();qq.pop();
   inq[u] = false;
   for(int i=0;i<SZ(G[u]);++i){
    Edge e=G[u][i];
    int v=e.to; Wei d=e.wei;
    if(e.cap<=0||dis[v]<=dis[u]+d)
     continue;
    dis[v] = dis[u] + d;
    fa[v] = u, wh[v] = i;
    if (inq[v]) continue;
    qq.push(v);
    inq[v] = true;
   }
  }
  if(dis[edd]==INF_WEI) return {-1, -1};
  Cap mw=INF_CAP;
  for(int i=edd;i!=ori;i=fa[i])
   mw=min(mw,G[fa[i]][wh[i]].cap);
  for (int i=edd;i!=ori;i=fa[i]){
   auto &eg=G[fa[i]][wh[i]];
   eg.cap -= mw;
   G[eg.to][eg.back].cap+=mw;
  }
  return {mw, dis[edd]};
 }
public:
 void init(int n){
  G.clear();G.resize(n);
  fa.resize(n);wh.resize(n);
  inq.resize(n); dis.resize(n);
 }
 void add_edge(int st, int ed, Cap c, Wei w){
  G[st].emplace_back(ed,SZ(G[ed]),c,w);
  G[ed].emplace_back(st,SZ(G[st])-1,0,-w);
 }
 PCW solve(int a, int b){
  ori = a, edd = b;
  Cap cc=0; Wei ww=0;
  while(true){
   PCW ret=SPFA();
   if(ret.first==-1) break;
   cc+=ret.first;
   ww+=ret.first * ret.second;
  }
  return {cc,ww};
 }
} mcmf;
```

4.11 Maximum Weight Graph Matching

```
struct WeightGraph {
 static const int inf = INT_MAX;
 static const int maxn = 514;
 struct edge {
  int u, v, w;
  edge(){}
  edge(int u, int v, int w): u(u), v(v), w(w) {}
 };
 int n, n_x;
 edge g[maxn * 2][maxn * 2];
 int lab[maxn * 2];
 int match[maxn * 2], slack[maxn * 2], st[maxn * 2], pa
    [maxn * 2];
 int flo_from[maxn * 2][maxn + 1], S[maxn * 2], vis[
    maxn * 2];
 vector<int> flo[maxn * 2];
 queue<int> q;
 int e_delta(const edge &e) { return lab[e.u] + lab[e.v
    ] - g[e.u][e.v].w * 2; }
 void update_slack(int u, int x) { if (!slack[x] ||
    e_delta(g[u][x]) < e_delta(g[slack[x]][x])) slack[x
    ] = u; }
 void set_slack(int x) {
  slack[x] = 0;
  for (int u = 1; u <= n; ++u)
   if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
    update_slack(u, x);
 }
 void q_push(int x) {
  if (x <= n) q.push(x);
  else for (size_t i = 0; i < flo[x].size(); i++)
    q_push(flo[x][i]);
 }
 void set_st(int x, int b) {
  st[x] = b;
  if (x > n) for (size_t i = 0; i < flo[x].size(); ++i)
     set_st(flo[x][i], b);
 }
 int get_pr(int b, int xr) {
  int pr = find(flo[b].begin(), flo[b].end(), xr) - flo
    [b].begin();
  if (pr % 2 == 1) {
   reverse(flo[b].begin() + 1, flo[b].end());
   return (int)flo[b].size() - pr;
  }
  return pr;
 }
 void set_match(int u, int v) {
  match[u] = g[u][v].v;
  if (u <= n) return;
  edge e = g[u][v];
  int xr = flo_from[u][e.u], pr = get_pr(u, xr);
  for (int i = 0; i < pr; ++i) set_match(flo[u][i], flo
    [u][i ^ 1]);
  set_match(xr, v);
  rotate(flo[u].begin(), flo[u].begin() + pr, flo[u].
    end());
 }
 void augment(int u, int v) {
  for (; ; ) {
   int xnv = st[match[u]];
   set_match(u, v);
   if (!xnv) return;
   set_match(xnv, st[pa[xnv]]);
   u = st[pa[xnv]], v = xnv;
  }
 }
 int get_lca(int u, int v) {
  static int t = 0;
  for (++t; u || v; swap(u, v)) {
   if (u == 0) continue;
   if (vis[u] == t) return u;
   vis[u] = t;
   u = st[match[u]];
   if (u) u = st[pa[u]];
  }
  return 0;
 }
 void add_blossom(int u, int lca, int v) {
  int b = n + 1;
  while (b <= n_x && st[b]) ++b;
  if (b > n_x) ++n_x;
  lab[b] = 0, S[b] = 0;
  match[b] = match[lca];
  flo[b].clear();
  flo[b].push_back(lca);
  for (int x = u, y; x != lca; x = st[pa[y]])
   flo[b].push_back(x), flo[b].push_back(y = st[match[x
    ]]), q_push(y);
  reverse(flo[b].begin() + 1, flo[b].end());
  for (int x = v, y; x != lca; x = st[pa[y]])
   flo[b].push_back(x), flo[b].push_back(y = st[match[x
    ]]), q_push(y);
  set_st(b, b);
  for (int x = 1; x <= n_x; ++x) g[b][x].w = g[x][b].w
    = 0;
  for (int x = 1; x <= n; ++x) flo_from[b][x] = 0;
  for (size_t i = 0; i < flo[b].size(); ++i) {
   int xs = flo[b][i];
   for (int x = 1; x <= n_x; ++x)
    if (g[b][x].w == 0 || e_delta(g[xs][x]) < e_delta(g
    [b][x]))
     g[b][x] = g[xs][x], g[x][b] = g[x][xs];
   for (int x = 1; x <= n; ++x)
```

```
    if (flo_from[xs][x]) flo_from[b][x] = xs;
  }
  set_slack(b);
 }
 void expand_blossom(int b) {
  for (size_t i = 0; i < flo[b].size(); ++i)
   set_st(flo[b][i], flo[b][i]);
  int xr = flo_from[b][g[b][pa[b]].u], pr = get_pr(b,
    xr);
  for (int i = 0; i < pr; i += 2) {
   int xs = flo[b][i], xns = flo[b][i + 1];
   pa[xs] = g[xns][xs].u;
   S[xs] = 1, S[xns] = 0;
   slack[xs] = 0, set_slack(xns);
   q_push(xns);
  }
  S[xr] = 1, pa[xr] = pa[b];
  for (size_t i = pr + 1; i < flo[b].size(); ++i) {
   int xs = flo[b][i];
   S[xs] = -1, set_slack(xs);
  }
  st[b] = 0;
 }
 bool on_found_edge(const edge &e) {
  int u = st[e.u], v = st[e.v];
  if (S[v] == -1) {
   pa[v] = e.u, S[v] = 1;
   int nu = st[match[v]];
   slack[v] = slack[nu] = 0;
   S[nu] = 0, q_push(nu);
  } else if (S[v] == 0) {
   int lca = get_lca(u, v);
   if (!lca) return augment(u,v), augment(v,u), true;
   else add_blossom(u, lca, v);
  }
  return false;
 }
 bool matching() {
  memset(S + 1, -1, sizeof(int) * n_x);
  memset(slack + 1, 0, sizeof(int) * n_x);
  q = queue<int>();
  for (int x = 1; x <= n_x; ++x)
   if (st[x] == x && !match[x]) pa[x] = 0, S[x] = 0,
    q_push(x);
  if (q.empty()) return false;
  for (; ; ) {
   while (q.size()) {
    int u = q.front(); q.pop();
    if (S[st[u]] == 1) continue;
    for (int v = 1; v <= n; ++v)
     if (g[u][v].w > 0 && st[u] != st[v]) {
      if (e_delta(g[u][v]) == 0) {
       if (on_found_edge(g[u][v])) return true;
      } else update_slack(u, st[v]);
     }
   }
   int d = inf;
   for (int b = n + 1; b <= n_x; ++b)
    if (st[b] == b && S[b] == 1) d = min(d, lab[b] / 2)
    ;
   for (int x = 1; x <= n_x; ++x)
    if (st[x] == x && slack[x]) {
     if (S[x] == -1) d = min(d, e_delta(g[slack[x]][x])
    );
     else if (S[x] == 0) d = min(d, e_delta(g[slack[x
    ]][x]) / 2);
    }
   for (int u = 1; u <= n; ++u) {
    if (S[st[u]] == 0) {
     if (lab[u] <= d) return 0;
     lab[u] -= d;
    } else if (S[st[u]] == 1) lab[u] += d;
   }
   for (int b = n + 1; b <= n_x; ++b)
    if (st[b] == b) {
     if (S[st[b]] == 0) lab[b] += d * 2;
     else if (S[st[b]] == 1) lab[b] -= d * 2;
    }
   q = queue<int>();
   for (int x = 1; x <= n_x; ++x)
    if (st[x] == x && slack[x] && st[slack[x]] != x &&
    e_delta(g[slack[x]][x]) == 0)
     if (on_found_edge(g[slack[x]][x])) return true;
   for (int b = n + 1; b <= n_x; ++b)
    if (st[b] == b && S[b] == 1 && lab[b] == 0)
     expand_blossom(b);
  }
  return false;
 }
 pair<long long, int> solve() {
  memset(match + 1, 0, sizeof(int) * n);
  n_x = n;
  int n_matches = 0;
  long long tot_weight = 0;
  for (int u = 0; u <= n; ++u) st[u] = u, flo[u].clear
    ();
  int w_max = 0;
  for (int u = 1; u <= n; ++u)
   for (int v = 1; v <= n; ++v) {
    flo_from[u][v] = (u == v ? u : 0);
    w_max = max(w_max, g[u][v].w);
   }
  for (int u = 1; u <= n; ++u) lab[u] = w_max;
  while (matching()) ++n_matches;
  for (int u = 1; u <= n; ++u)
   if (match[u] && match[u] < u)
    tot_weight += g[u][match[u]].w;
  return make_pair(tot_weight, n_matches);
 }
 void add_edge(int ui, int vi, int wi) { g[ui][vi].w =
    g[vi][ui].w = wi; }
 void init(int _n) {
  n = _n;
  for (int u = 1; u <= n; ++u)
   for (int v = 1; v <= n; ++v)
    g[u][v] = edge(u, v, 0);
 }
};
```

5 Math

5.1 $\lfloor \frac{n}{i} \rfloor$ Enumeration

$T_0 = 1, T_{i+1} = \lfloor \frac{n}{\lfloor \frac{n}{T_i+1} \rfloor} \rfloor$

5.2 Strling Number

5.2.1 First Kind

$S_1(n,k)$ counts the number of permutations of n elements with k disjoint cycles.

$$S_1(n,k) = (n-1) \cdot S_1(n-1,k) + S_1(n-1,k-1)$$

$$x(x+1)\dots(x+n-1) = \sum_{k=0}^{n} S_1(n,k)x^k$$

$$g(x) = x(x+1)\dots(x+n-1) = \sum_{k=0}^{n} a_k x^k$$

$$\Rightarrow g(x+n) = \sum_{k=0}^{n} \frac{b_k}{(n-k)!} x^{n-k},$$

$$b_k = \sum_{i=0}^{k} ((n-i)! a_{n-i}) \cdot \left(\frac{n^{k-i}}{(k-i)!}\right)$$

5.2.2 Second Kind

$S_2(n,k)$ counts the number of ways to partition a set of n elements into k nonempty sets.

$$S_2(n,k) = S_2(n-1,k-1) + k \cdot S_2(n-1,k)$$

$$S_2(n,k) = \sum_{i=0}^{k} \binom{k}{i} i^n (-1)^{k-i} = \sum_{i=0}^{k} \frac{(-1)^i}{i!} \cdot \frac{(k-i)^n}{(k-i)!}$$

5.3 ax+by=gcd

```
// ax+ny = 1, ax+ny == ax == 1 (mod n)
void exgcd(lld x,lld y,lld &g,lld &a,lld &b) {
 if (y == 0) g=x,a=1,b=0;
 else exgcd(y,x%y,g,b,a),b-=(x/y)*a;
}
```

5.4 Berlekamp Massey

```
template <typename T>
vector<T> BerlekampMassey(const vector<T> &output) {
 vector<T> d(output.size() + 1), me, he;
 for (size_t f = 0, i = 1; i <= output.size(); ++i) {
  for (size_t j = 0; j < me.size(); ++j)
   d[i] += output[i - j - 2] * me[j];
  if ((d[i] -= output[i - 1]) == 0) continue;
  if (me.empty()) {
   me.resize(f = i);
   continue;
  }
  vector<T> o(i - f - 1);
  T k = -d[i] / d[f]; o.push_back(-k);
  for (T x : he) o.push_back(x * k);
  if (o.size() < me.size()) o.resize(me.size());
  for (size_t j = 0; j < me.size(); ++j) o[j] += me[j];
  if (i-f+he.size() >= me.size()) he = me, f = i;
  me = o;
 }
 return me;
}
```

5.5 Charateristic Polynomial

```
vector<vector<int>> Hessenberg(const vector<vector<int
    >> &A) {
 int N = A.size();
 vector<vector<int>> H = A;
 for (int i = 0; i < N - 2; ++i) {
  if (!H[i + 1][i]) {
   for (int j = i + 2; j < N; ++j) {
    if (H[j][i]) {
     for (int k = i; k < N; ++k) swap(H[i + 1][k], H[j
    ][k]);
     for (int k = 0; k < N; ++k) swap(H[k][i + 1], H[k
    ][j]);
     break;
    }
   }
  }
  if (!H[i + 1][i]) continue;
  int val = fpow(H[i + 1][i], kP - 2);
  for (int j = i + 2; j < N; ++j) {
   int coef = 1LL * val * H[j][i] % kP;
   for (int k = i; k < N; ++k) H[j][k] = (H[j][k] + 1LL
     * H[i + 1][k] * (kP - coef)) % kP;
   for (int k = 0; k < N; ++k) H[k][i + 1] = (H[k][i +
    1] + 1LL * H[k][j] * coef) % kP;
  }
 }
 return H;
}
vector<int> CharacteristicPoly(const vector<vector<int
    >> &A) {
 int N = A.size();
 auto H = Hessenberg(A);
 for (int i = 0; i < N; ++i) {
  for (int j = 0; j < N; ++j) H[i][j] = kP - H[i][j];
 }
 vector<vector<int>> P(N + 1, vector<int>(N + 1));
 P[0][0] = 1;
 for (int i = 1; i <= N; ++i) {
  P[i][0] = 0;
  for (int j = 1; j <= i; ++j) P[i][j] = P[i - 1][j -
    1];
  int val = 1;
  for (int j = i - 1; j >= 0; --j) {
   int coef = 1LL * val * H[j][i - 1] % kP;
   for (int k = 0; k <= j; ++k) P[i][k] = (P[i][k] + 1
    LL * P[j][k] * coef) % kP;
   if (j) val = 1LL * val * (kP - H[j][j - 1]) % kP;
  }
 }
 if (N & 1) {
  for (int i = 0; i <= N; ++i) P[N][i] = kP - P[N][i];
 }
 return P[N];
}
```

5.6 Chinese Remainder

```
lld crt(lld ans[], lld pri[], int n){
 lld M = 1, ret = 0;
 for(int i=0;i<n;i++) M *= pri[i];
 for(int i=0;i<n;i++){
  lld iv = (gcd(M/pri[i],pri[i]).FF+pri[i])%pri[i];
  ret += (ans[i]*(M/pri[i])%M * iv)%M;
  ret %= M;
 }
 return ret;
}
/*
Another:
x = a1 % m1
x = a2 % m2
g = gcd(m1, m2)
assert((a1-a2)%g==0)
[p, q] = exgcd(m2/g, m1/g)
return a2+m2*(p*(a1-a2)/g)
0 <= x < lcm(m1, m2)
*/
```

5.7 De-Bruijn

```
int res[maxn], aux[maxn], sz;
void db(int t, int p, int n, int k) {
 if (t > n) {
  if (n % p == 0)
   for (int i = 1; i <= p; ++i)
    res[sz++] = aux[i];
 } else {
  aux[t] = aux[t - p];
  db(t + 1, p, n, k);
  for (int i = aux[t - p] + 1; i < k; ++i) {
   aux[t] = i;
   db(t + 1, t, n, k);
  }
 }
}
int de_bruijn(int k, int n) {
 // return cyclic string of len k^n s.t. every string
 // of len n using k char appears as a substring.
 if (k == 1) {
  res[0] = 0;
  return 1;
 }
 for (int i = 0; i < k * n; i++) aux[i] = 0;
 sz = 0;
 db(1, 1, n, k);
 return sz;
}
```

5.8 DiscreteLog

```
template<typename Int>
Int BSGS(Int x, Int y, Int M) {
  // x^? \equiv y (mod M)
  Int t = 1, c = 0, g = 1;
  for (Int M_ = M; M_ > 0; M_ >>= 1)
    g = g * x % M;
  for (g = gcd(g, M); t % g != 0; ++c) {
    if (t == y) return c;
    t = t * x % M;
  }
  if (y % g != 0) return -1;
  t /= g, y /= g, M /= g;
  Int h = 0, gs = 1;
  for (; h * h < M; ++h) gs = gs * x % M;
  unordered_map<Int, Int> bs;
  for (Int s = 0; s < h; bs[y] = ++s)
    y = y * x % M;
  for (Int s = 0; s < M; s += h) {
    t = t * gs % M;
    if (bs.count(t)) return c + s + h - bs[t];
  }
  return -1;
}
```

5.9 Extended Euler

$$a^b \equiv \begin{cases} a^{b \bmod \varphi(m) + \varphi(m)} & \text{if } (a,m) \neq 1 \land b \geq \varphi(m) \\ a^{b \bmod \varphi(m)} & \text{otherwise} \end{cases} \pmod m$$

5.10 ExtendedFloorSum

$$g(a,b,c,n) = \sum_{i=0}^{n} i \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor \cdot \frac{n(n+1)}{2} & \\ +g(a \bmod c, b \bmod c, c, n), & a \ge c \lor b \ge c \\ 0, & n < 0 \lor a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1) & \\ -h(c, c-b-1, a, m-1)), & \text{otherwise} \end{cases}$$

$$h(a,b,c,n) = \sum_{i=0}^{n} \lfloor \frac{ai+b}{c} \rfloor^2$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor^2 \cdot (n+1) & \\ +\lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot n(n+1) & \\ +h(a \bmod c, b \bmod c, c, n) & \\ +2\lfloor \frac{a}{c} \rfloor \cdot g(a \bmod c, b \bmod c, c, n) & \\ +2\lfloor \frac{b}{c} \rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \ge c \lor b \ge c \\ 0, & n < 0 \lor a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) & \\ -2f(c, c-b-1, a, m-1) - f(a, b, c, n), & \text{otherwise} \end{cases}$$

5.11 Fast Fourier Transform

```cpp
const int mod = 1000000007;
const int M1 = 985661441; // G = 3
const int M2 = 998244353;
const int M3 = 1004535809;
int superBigCRT(int64_t A, int64_t B, int64_t C) {
  static_assert (M1 <= M2 && M2 <= M3);
  constexpr int64_t r12 = modpow(M1, M2-2, M2);
  constexpr int64_t r13 = modpow(M1, M3-2, M3);
  constexpr int64_t r23 = modpow(M2, M3-2, M3);
  constexpr int64_t M1M2 = 1LL * M1 * M2 % mod;
  B = (B - A + M2) * r12 % M2;
  C = (C - A + M3) * r13 % M3;
  C = (C - B + M3) * r23 % M3;
  return (A + B * M1 + C * M1M2) % mod;
}

namespace fft {
using VI = vector<int>;
using VL = vector<long long>;
const double pi = acos(-1);
cplx omega[maxn + 1];
void prefft() {
 for (int i = 0; i <= maxn; i++)
  omega[i] = cplx(cos(2 * pi * j / maxn),
     sin(2 * pi * j / maxn));
}
void fft(vector<cplx> &v, int n) {
 int z = __builtin_ctz(n) - 1;
 for (int i = 0; i < n; ++i) {
  int x = 0, j = 0;
  for (;(1 << j) < n;++j) x^=(i >> j & 1)<<(z - j);
  if (x > i) swap(v[x], v[i]);
 }
 for (int s = 2; s <= n; s <<= 1) {
  int z = s >> 1;
  for (int i = 0; i < n; i += s) {
   for (int k = 0; k < z; ++k) {
    cplx x = v[i + z + k] * omega[maxn / s * k];
    v[i + z + k] = v[i + k] - x;
    v[i + k] = v[i + k] + x;
   }
  }
 }
}
void ifft(vector<cplx> &v, int n) {
 fft(v, n); reverse(v.begin() + 1, v.end());
 for (int i=0;i<n;++i) v[i] = v[i] * cplx(1. / n, 0);
}
VL convolution(const VI &a, const VI &b) {
 // Should be able to handle N <= 10^5, C <= 10^4
 int sz = 1;
 while (sz < a.size() + b.size() - 1) sz <<= 1;
 vector<cplx> v(sz);
 for (int i = 0; i < sz; ++i) {
  double re = i < a.size() ? a[i] : 0;
  double im = i < b.size() ? b[i] : 0;
  v[i] = cplx(re, im);
 }
 fft(v, sz);
 for (int i = 0; i <= sz / 2; ++i) {
  int j = (sz - i) & (sz - 1);
  cplx x = (v[i] + v[j].conj()) * (v[i] - v[j].conj())
    * cplx(0, -0.25);
  if (j != i) v[j] = (v[j] + v[i].conj()) * (v[j] - v[i
    ].conj()) * cplx(0, -0.25);
  v[i] = x;
 }
 ifft(v, sz);
 VL c(sz);
 for (int i = 0; i < sz; ++i) c[i] = round(v[i].re);
 return c;
}
VI convolution_mod(const VI &a, const VI &b, int p) {
 int sz = 1;
 while (sz + 1 < a.size() + b.size()) sz <<= 1;
 vector<cplx> fa(sz), fb(sz);
 for (int i = 0; i < (int)a.size(); ++i)
  fa[i] = cplx(a[i] & ((1 << 15) - 1), a[i] >> 15);
 for (int i = 0; i < (int)b.size(); ++i)
  fb[i] = cplx(b[i] & ((1 << 15) - 1), b[i] >> 15);
 fft(fa, sz), fft(fb, sz);
 double r = 0.25 / sz;
 cplx r2(0, -1), r3(r, 0), r4(0, -r), r5(0, 1);
 for (int i = 0; i <= (sz >> 1); ++i) {
  int j = (sz - i) & (sz - 1);
  cplx a1 = (fa[i] + fa[j].conj());
  cplx a2 = (fa[i] - fa[j].conj()) * r2;
  cplx b1 = (fb[i] + fb[j].conj()) * r3;
  cplx b2 = (fb[i] - fb[j].conj()) * r4;
  if (i != j) {
   cplx c1 = (fa[j] + fa[i].conj());
   cplx c2 = (fa[j] - fa[i].conj()) * r2;
   cplx d1 = (fb[j] + fb[i].conj()) * r3;
   cplx d2 = (fb[j] - fb[i].conj()) * r4;
   fa[i] = c1 * d1 + c2 * d2 * r5;
   fb[i] = c1 * d2 + c2 * d1;
  }
  fa[j] = a1 * b1 + a2 * b2 * r5;
  fb[j] = a1 * b2 + a2 * b1;
 }
 fft(fa, sz), fft(fb, sz);
 vector<int> res(sz);
 for (int i = 0; i < sz; ++i) {
  long long a = round(fa[i].re), b = round(fb[i].re),
       c = round(fa[i].im);
  res[i] = (a+((b % p) << 15)+((c % p) << 30)) % p;
 }
 return res;
}}
```

5.12 FloorSum

```cpp
// @param n `n < 2^32`
// @param m `1 <= m < 2^32`
// @return sum_{i=0}^{n-1} floor((ai + b)/m) mod 2^64
llu floor_sum_unsigned(llu n, llu m, llu a, llu b) {
 llu ans = 0;
 while (true) {
  if (a >= m) {
   ans += n * (n - 1) / 2 * (a / m); a %= m;
  }
  if (b >= m) {
   ans += n * (b / m); b %= m;
  }
  llu y_max = a * n + b;
  if (y_max < m) break;
  // y_max < m * (n + 1)
  // floor(y_max / m) <= n
  n = (llu)(y_max / m), b = (llu)(y_max % m);
  swap(m, a);
 }
 return ans;
}
lld floor_sum(lld n, lld m, lld a, lld b) {
 llu ans = 0;
 if (a < 0) {
  llu a2 = (a % m + m) % m;
  ans -= 1ULL * n * (n - 1) / 2 * ((a2 - a) / m);
  a = a2;
 }
 if (b < 0) {
  llu b2 = (b % m + m) % m;
```

```
  ans -= 1ULL * n * ((b2 - b) / m);
  b = b2;
 }
 return ans + floor_sum_unsigned(n, m, a, b);
}
```

5.13 FWT

```
/* xor convolution:
 * x = (x0,x1) , y = (y0,y1)
 * z = ( x0y0 + x1y1 , x0y1 + x1y0 )
 * =>
 * x' = ( x0+x1 , x0-x1 ) , y' = ( y0+y1 , y0-y1 )
 * z' = ( ( x0+x1 )( y0+y1 ) , ( x0-x1 )( y0-y1 ) )
 * z = (1/2) * z''
 * or convolution:
 * x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
 * and convolution:
 * x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
const LL MOD = 1e9+7;
inline void fwt( LL x[ MAXN ] , int N , bool inv=0 ) {
 for( int d = 1 ; d < N ; d <<= 1 ) {
  int d2 = d<<1;
  for( int s = 0 ; s < N ; s += d2 )
   for( int i = s , j = s+d ; i < s+d ; i++, j++ ){
    LL ta = x[ i ] , tb = x[ j ];
    x[ i ] = ta+tb;
    x[ j ] = ta-tb;
    if( x[ i ] >= MOD ) x[ i ] -= MOD;
    if( x[ j ] < 0 ) x[ j ] += MOD;
   }
 }
 if( inv )
  for( int i = 0 ; i < N ; i++ ) {
   x[ i ] *= inv( N, MOD );
   x[ i ] %= MOD;
  }
}
```

5.14 Miller Rabin

```
bool isprime(llu x) {
 static auto witn = [](llu a, llu u, llu n, int t) {
  if (!a) return false;
  while (t--) {
   llu a2 = mmul(a, a, n);
   if (a2 == 1 && a != 1 && a != n - 1) return true;
   a = a2;
  }
  return a != 1;
 };
 if (x < 2) return false;
 if (!(x & 1)) return x == 2;
 int t = __builtin_ctzll(x - 1);
 llu odd = (x - 1) >> t;
 for (llu m:
  {2, 325, 9375, 28178, 450775, 9780504, 1795265022})
  if (witn(mpow(m % x, odd, x), odd, x, t))
   return false;
 return true;
}
```

5.15 NTT

```
template <int mod, int G, int maxn>
struct NTT {
 static_assert (maxn == (maxn & -maxn));
 int roots[maxn];
 NTT () {
  int r = modpow(G, (mod - 1) / maxn);
  for (int i = maxn >> 1; i; i >>= 1) {
   roots[i] = 1;
   for (int j = 1; j < i; j++)
    roots[i + j] = modmul(roots[i + j - 1], r);
   r = modmul(r, r);
  }
 }
 // n must be 2^k, and 0 <= F[i] < mod
 void operator()(int F[], int n, bool inv = false) {
  for (int i = 0, j = 0; i < n; i++) {
   if (i < j) swap(F[i], F[j]);
   for (int k = n>>1; (j^=k) < k; k>>=1);
  }
  for (int s = 1; s < n; s *= 2) {
   for (int i = 0; i < n; i += s * 2) {
    for (int j = 0; j < s; j++) {
     int a = F[i+j];
     int b = modmul(F[i+j+s], roots[s+j]);
     F[i+j] = modadd(a, b);  // a + b
     F[i+j+s] = modsub(a, b); // a - b
    }
   }
  }
  if (inv) {
   int invn = modinv(n);
   for (int i = 0; i < n; i++)
    F[i] = modmul(F[i], invn);
   reverse(F + 1, F + n);
  }
 }
};
NTT<2013265921, 31, 1048576> ntt;
```

5.16 Partition Number

```
int b = sqrt(n);
ans[0] = tmp[0] = 1;
for (int i = 1; i <= b; i++) {
 for (int rep = 0; rep < 2; rep++)
  for (int j = i; j <= n - i * i; j++)
   modadd(tmp[j], tmp[j-i]);
 for (int j = i * i; j <= n; j++)
  modadd(ans[j], tmp[j - i * i]);
}
```

5.17 Pi Count (Linear Sieve)

```
static constexpr int N = 1000000 + 5;
lld pi[N];
vector<int> primes;
bool sieved[N];
lld cube_root(lld x){
 lld s=cbrt(x-static_cast<long double>(0.1));
 while(s*s*s <= x) ++s;
 return s-1;
}
lld square_root(lld x){
 lld s=sqrt(x-static_cast<long double>(0.1));
 while(s*s <= x) ++s;
 return s-1;
}
void init(){
 primes.reserve(N);
 primes.push_back(1);
 for(int i=2;i<N;i++) {
  if(!sieved[i]) primes.push_back(i);
  pi[i] = !sieved[i] + pi[i-1];
  for(int p: primes) if(p > 1) {
   if(p * i >= N) break;
   sieved[p * i] = true;
   if(p % i == 0) break;
  }
 }
}
lld phi(lld m, lld n) {
 static constexpr int MM = 80000, NN = 500;
 static lld val[MM][NN];
 if(m<MM&&n<NN&&val[m][n])return val[m][n]-1;
 if(n == 0) return m;
 if(primes[n] >= m) return 1;
 lld ret = phi(m,n-1)-phi(m/primes[n],n-1);
 if(m<MM&&n<NN) val[m][n] = ret+1;
 return ret;
}
lld pi_count(lld);
lld P2(lld m, lld n) {
 lld sm = square_root(m), ret = 0;
 for(lld i = n+1;primes[i]<=sm;i++)
  ret+=pi_count(m/primes[i])-pi_count(primes[i])+1;
 return ret;
}
lld pi_count(lld m) {
 if(m < N) return pi[m];
 lld n = pi_count(cube_root(m));
 return phi(m, n) + n - 1 - P2(m, n);
}
```

5.18 Pollard Rho

```
// does not work when n is prime
// return any non-trivial factor
llu pollard_rho(llu n) {
 static auto f = [](llu x, llu k, llu m) {
    return add(k, mul(x, x, m), m); };
 if (!(n & 1)) return 2;
 mt19937 rnd(120821011);
 while (true) {
  llu y = 2, yy = y, x = rnd() % n, t = 1;
  for (llu sz = 2; t == 1; sz <<= 1, y = yy) {
   for (llu i = 0; t == 1 && i < sz; ++i) {
    yy = f(yy, x, n);
    t = gcd(yy > y ? yy - y : y - yy, n);
   }
  }
  if (t != 1 && t != n) return t;
 }
}
```

5.19 Polynomial Operations

```
using V = vector<int>;
#define fi(l, r) for (int i = int(l); i < int(r); ++i)
template <int mod, int G, int maxn> struct Poly : V {
 static uint32_t n2k(uint32_t n) {
  if (n <= 1) return 1;
  return 1u << (32 - __builtin_clz(n - 1));
 }
 static NTT<mod,G,maxn> ntt; // coefficients in [0, P)
 explicit Poly(int n = 1) : V(n) {}
 Poly(const V &v) : V(v) {}
 Poly(const Poly &p, size_t n) : V(n) {
  copy_n(p.data(), min(p.size(), n), data());
 }
 Poly &irev() { return reverse(data(), data() + size())
    , *this; }
 Poly &isz(int sz) { return resize(sz), *this; }
 Poly &iadd(const Poly &rhs) { // n() == rhs.n()
  fi(0, size())(*this)[i] = modadd((*this)[i], rhs[i]);
  return *this;
 }
 Poly &imul(int k) {
  fi(0, size())(*this)[i] = modmul((*this)[i], k);
  return *this;
 }
 Poly Mul(const Poly &rhs) const {
  const int sz = n2k(size() + rhs.size() - 1);
  Poly X(*this, sz), Y(rhs, sz);
  ntt(X.data(), sz), ntt(Y.data(), sz);
  fi(0, sz) X[i] = modmul(X[i], Y[i]);
  ntt(X.data(), sz, true);
  return X.isz(size() + rhs.size() - 1);
 }
 Poly Inv() const { // coef[0] != 0
  if (size() == 1) return V{modinv(*begin())};
  const int sz = n2k(size() * 2);
  Poly X = Poly(*this, (size() + 1) / 2).Inv().isz(sz),
     Y(*this, sz);
  ntt(X.data(), sz), ntt(Y.data(), sz);
  fi(0, sz) X[i] = modmul(X[i], modsub(2, modmul(X[i],
    Y[i])));
  ntt(X.data(), sz, true);
  return X.isz(size());
 }
 Poly Sqrt() const { // coef[0] \in [1, mod)^2
  if (size() == 1) return V{QuadraticResidue((*this)
    [0], mod)};
  Poly X = Poly(*this, (size() + 1) / 2).Sqrt().isz(
    size());
  return X.iadd(Mul(X.Inv()).isz(size())).imul(mod / 2
    + 1);
 }
 pair<Poly, Poly> DivMod(const Poly &rhs) const {
  if (size() < rhs.size()) return {V{0}, *this};
  const int sz = size() - rhs.size() + 1;
  Poly X(rhs); X.irev().isz(sz);
  Poly Y(*this); Y.irev().isz(sz);
  Poly Q = Y.Mul(X.Inv()).isz(sz).irev();
  X = rhs.Mul(Q), Y = *this;
  fi(0, size()) Y[i] = modsub(Y[i], X[i]);
  return {Q, Y.isz(max<int>(1, rhs.size() - 1))};
 }
 Poly Dx() const {
  Poly ret(size() - 1);
  fi(0, ret.size()) ret[i] = modmul(i + 1, (*this)[i +
    1]);
  return ret.isz(max<int>(1, ret.size()));
 }
 Poly Sx() const {
  Poly ret(size() + 1);
  fi(0, size()) ret[i + 1] = modmul(modinv(i + 1), (*
    this)[i]);
  return ret;
 }
 Poly Ln() const { // coef[0] == 1
  return Dx().Mul(Inv()).Sx().isz(size());
 }
 Poly Exp() const { // coef[0] == 0
  if (size() == 1) return V{1};
  Poly X = Poly(*this, (size() + 1) / 2).Exp().isz(size
    ());
  Poly Y = X.Ln(); Y[0] = mod - 1;
  fi(0, size()) Y[i] = modsub((*this)[i], Y[i]);
  return X.Mul(Y).isz(size());
 }
 Poly Pow(const string &K) const {
  int nz = 0;
  while (nz < size() && !(*this)[nz]) ++nz;
  int nk = 0, nk2 = 0;
  for (char c : K) {
   nk = (nk * 10 + c - '0') % mod;
   nk2 = nk2 * 10 + c - '0';
   if (nk2 * nz >= size())
    return Poly(size());
   nk2 %= mod - 1;
  }
  if (!nk && !nk2) return Poly(V{1}, size());
  Poly X = V(data() + nz, data() + size() - nz * (nk2 -
    1));
  int x0 = X[0];
  return X.imul(modinv(x0)).Ln().imul(nk).Exp().imul(
    modpow(x0, nk2)).irev().isz(size()).irev();
 }
 Poly InvMod(int L) { // (to evaluate linear recursion)
  Poly R{1, 0}; // *this * R mod x^L = 1 (*this[0] ==
    1)
  for (int level = 0; (1 << level) < L; ++level) {
   Poly O = R.Mul(Poly(data(), min<int>(2 << level,
    size())));
   Poly Q(2 << level); Q[0] = 1;
   for (int j = (1 << level); j < (2 << level); ++j)
    Q[j] = modsub(mod, O[j]);
   R = R.Mul(Q).isz(4 << level);
  }
  return R.isz(L);
 }
 static int LinearRecursion(const V &a, const V &c,
    int64_t n) { // a_n = \sum c_j a_(n-j)
  const int k = (int)a.size();
  assert((int)c.size() == k + 1);
  Poly C(k + 1), W({1}, k), M = {0, 1};
  fi(1, k + 1) C[k - i] = modsub(mod, c[i]);
  C[k] = 1;
  while (n) {
   if (n % 2) W = W.Mul(M).DivMod(C).second;
   n /= 2, M = M.Mul(M).DivMod(C).second;
  }
  int ret = 0;
  fi(0, k) ret = modadd(ret, modmul(W[i], a[i]));
  return ret;
 }
};
#undef fi
using Poly_t = Poly<998244353, 3, 1 << 20>;
template <> decltype(Poly_t::ntt) Poly_t::ntt = {};
```

5.20 Quadratic residue

```
struct S {
 int MOD, w;
 int64_t x, y;
 S(int m, int w_=-1, int64_t x_=1, int64_t y_=0)
  : MOD(m), w(w_), x(x_), y(y_) {}
 S operator*(const S &rhs) const {
```

```
  int w_ = w;
  if (w_ == -1) w_ = rhs.w;
  assert(w_ != -1 and w_ == rhs.w);
  return { MOD, w_,
   (x * rhs.x + y * rhs.y % MOD * w) % MOD,
   (x * rhs.y + y * rhs.x) % MOD };
 }
};
int get_root(int n, int P) {
  if (P == 2 or n == 0) return n;
  if (qpow(n, (P - 1) / 2, P) != 1) return -1;
  auto check = [&](int x) {
    return qpow(x, (P - 1) / 2, P); };
  if (check(n) == P-1) return -1;
  int64_t a; int w; mt19937 rnd(7122);
  do { a = rnd() % P;
    w = ((a * a - n) % P + P) % P;
  } while (check(w) != P - 1);
  return qpow(S(P, w, a, 1), (P + 1) / 2).x;
}
```

5.21 Simplex

```
namespace simplex {
// maximize c^Tx under Ax <= B
// return VD(n, -inf) if the solution doesn't exist
// return VD(n, +inf) if the solution is unbounded
using VD = vector<double>;
using VVD = vector<vector<double>>;
const double eps = 1e-9;
const double inf = 1e+9;
int n, m;
VVD d;
vector<int> p, q;
void pivot(int r, int s) {
 double inv = 1.0 / d[r][s];
 for (int i = 0; i < m + 2; ++i)
  for (int j = 0; j < n + 2; ++j)
   if (i != r && j != s)
    d[i][j] -= d[r][j] * d[i][s] * inv;
 for(int i=0;i<m+2;++i) if (i != r) d[i][s] *= -inv;
 for(int j=0;j<n+2;++j) if (j != s) d[r][j] *= +inv;
 d[r][s] = inv; swap(p[r], q[s]);
}
bool phase(int z) {
 int x = m + z;
 while (true) {
  int s = -1;
  for (int i = 0; i <= n; ++i) {
   if (!z && q[i] == -1) continue;
   if (s == -1 || d[x][i] < d[x][s]) s = i;
  }
  if (d[x][s] > -eps) return true;
  int r = -1;
  for (int i = 0; i < m; ++i) {
   if (d[i][s] < eps) continue;
   if (r == -1 || \
    d[i][n+1]/d[i][s] < d[r][n+1]/d[r][s]) r = i;
  }
  if (r == -1) return false;
  pivot(r, s);
 }
}
VD solve(const VVD &a, const VD &b, const VD &c) {
 m = b.size(), n = c.size();
 d = VVD(m + 2, VD(n + 2));
 for (int i = 0; i < m; ++i)
  for (int j = 0; j < n; ++j) d[i][j] = a[i][j];
 p.resize(m), q.resize(n + 1);
 for (int i = 0; i < m; ++i)
  p[i] = n + i, d[i][n] = -1, d[i][n + 1] = b[i];
 for (int i = 0; i < n; ++i) q[i] = i,d[m][i] = -c[i];
 q[n] = -1, d[m + 1][n] = 1;
 int r = 0;
 for (int i = 1; i < m; ++i)
  if (d[i][n + 1] < d[r][n + 1]) r = i;
 if (d[r][n + 1] < -eps) {
  pivot(r, n);
  if (!phase(1) || d[m + 1][n + 1] < -eps)
   return VD(n, -inf);
  for (int i = 0; i < m; ++i) if (p[i] == -1) {
   int s = min_element(d[i].begin(), d[i].end() - 1)
       - d[i].begin();
    pivot(i, s);
   }
  }
  if (!phase(0)) return VD(n, inf);
  VD x(n);
  for (int i = 0; i < m; ++i)
   if (p[i] < n) x[p[i]] = d[i][n + 1];
  return x;
}}
```

5.22 Simplex Construction

Standard form: maximize $\sum_{1\le i\le n} c_i x_i$ such that for all $1 \le j \le m$, $\sum_{1\le i\le n} A_{ji}x_i \le b_j$.and $x_i \ge 0$ for all $1 \le i \le n$.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1\le i\le n} A_{ji}x_i \ge b_j \to \sum_{1\le i\le n} -A_{ji}x_i \le -b_j$
3. $\sum_{1\le i\le n} A_{ji}x_i = b_j$
   - $\sum_{1\le i\le n} A_{ji}x_i \le b_j$
   - $\sum_{1\le i\le n} A_{ji}x_i \ge b_j$
4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

6 Geometry

6.1 Basic Geometry

```
#define IM imag
#define RE real
using lld = int64_t;
using llf = long double;
using PT = std::complex<lld>;
using PTF = std::complex<llf>;
auto toPTF(PT p) { return PTF{RE(p), IM(p)}; }
int sgn(lld x) { return (x > 0) - (x < 0); }
lld dot(PT a, PT b) { return RE(conj(a) * b); }
lld cross(PT a, PT b) { return IM(conj(a) * b); }
int ori(PT a, PT b, PT c) {
 return sgn(cross(b - a, c - a));
}
bool operator<(const PT &a, const PT &b) {
 return RE(a) != RE(b) ? RE(a) < RE(b) : IM(a) < IM(b);
}
int quad(PT p) {
 return (IM(p) == 0) // use sgn for PTF
  ? (RE(p) < 0 ? 3 : 1) : (IM(p) < 0 ? 0 : 2);
}
int argCmp(PT a, PT b) {
 // -1 / 0 / 1 <-> < / == / > (atan2)
 int qa = quad(a), qb = quad(b);
 if (qa != qb) return sgn(qa - qb);
 return sgn(cross(b, a));
}
template <typename V> llf area(const V & pt) {
 lld ret = 0;
 for (int i = 1; i + 1 < (int)pt.size(); i++)
  ret += cross(pt[i] - pt[0], pt[i+1] - pt[0]);
 return ret / 2.0;
}
PT rot90(PT p) { return PT{-IM(p), RE(p)}; }
PTF project(PTF p, PTF q) { // p onto q
 return dot(p, q) * q / dot(q, q);
}
llf FMOD(llf x) {
 if (x < -PI) x += PI * 2;
 if (x > PI) x -= PI * 2;
 return x;
}
```

6.2 Segment & Line Intersection

```
struct Segment {
 PT st, dir; // represent st + t*dir for 0<=t<=1
 Segment(PT s, PT e) : st(s), dir(e - s) {}
 static bool valid(lld p, lld q) {
  // is there t s.t. 0 <= t <= 1 && qt == p ?
  if (q < 0) q = -q, p = -p;
  return 0 <= p && p <= q;
 }
};
bool isInter(Segment A, PT P) {
```

```
 if (A.dir == PT(0)) return P == A.st;
 return cross(P - A.st, A.dir) == 0 &&
  Segment::valid(dot(P - A.st, A.dir), norm(A.dir));
}
template <typename U, typename V>
bool isInter(U A, V B) {
 if (cross(A.dir, B.dir) == 0)
  return // handle parallel yourself
   isInter(A, B.st) || isInter(A, B.st+B.dir) ||
   isInter(B, A.st) || isInter(B, A.st+A.dir);
 PT D = B.st - A.st;
 lld C = cross(A.dir, B.dir);
 return U::valid(cross(D, A.dir), C) &&
   V::valid(cross(D, B.dir), C);
}
struct Line {
 PT st, ed, dir;
 Line (PT s, PT e)
  : st(s), ed(e), dir(e - s) {}
};
PTF intersect(const Line &A, const Line &B) {
 llf t = cross(B.st - A.st, B.dir) /
  llf(cross(A.dir, B.dir));
 return toPTF(A.st) + PTF(t) * toPTF(A.dir);
}
```

6.3 2D Convex Hull

```
// returns a convex hull in counterclockwise order
// for a non-strict one, change cross >= to >
vector<PT> convex_hull(vector<PT> p) {
 sort(all(p));
 if (p[0] == p.back()) return {p[0]};
 int n = p.size(), t = 0;
 vector<PT> h(n + 1);
 for (int _ = 2, s = 0; _--; s = --t, reverse(all(p)))
  for (PT i : p) {
   while (t > s + 1 && cross(i, h[t-1], h[t-2])>=0)
    t--;
   h[t++] = i;
  }
 return h.resize(t), h;
}
```

6.4 3D Convex Hull

```
// return the faces with pt indexes
int flag[MXN][MXN];
struct Point{
 ld x,y,z;
 Point operator * (const ld &b) const {
  return (Point){x*b,y*b,z*b};}
 Point operator * (const Point &b) const {
  return(Point){y*b.z-b.y*z,z*b.x-b.z*x,x*b.y-b.x*y};
 }
};
Point ver(Point a, Point b, Point c) {
 return (b - a) * (c - a);}
vector<Face> convex_hull_3D(const vector<Point> pt) {
 int n = SZ(pt), ftop = 0;
 REP(i,n) REP(j,n) flag[i][j] = 0;
 vector<Face> now;
 now.emplace_back(0,1,2);
 now.emplace_back(2,1,0);
 for (int i=3; i<n; i++){
  ftop++; vector<Face> next;
  REP(j, SZ(now)) {
   Face& f=now[j]; int ff = 0;
   ld d=(pt[i]-pt[f.a]).dot(
     ver(pt[f.a], pt[f.b], pt[f.c]));
   if (d <= 0) next.push_back(f);
   if (d > 0) ff=ftop;
   else if (d < 0) ff=-ftop;
   flag[f.a][f.b]=flag[f.b][f.c]=flag[f.c][f.a]=ff;
  }
  REP(j, SZ(now)) {
   Face& f=now[j];
   if (flag[f.a][f.b] > 0 &&
     flag[f.a][f.b] != flag[f.b][f.a])
    next.emplace_back(f.a,f.b,i);
   if (flag[f.b][f.c] > 0 &&
     flag[f.b][f.c] != flag[f.c][f.b])
    next.emplace_back(f.b,f.c,i);
   if (flag[f.c][f.a] > 0 &&
     flag[f.c][f.a] != flag[f.a][f.c])
    next.emplace_back(f.c,f.a,i);
  }
  now=next;
 }
 return now;
}
```

6.5 2D Farthest Pair

```
// stk is from convex hull
n = (int)(stk.size());
int pos = 1, ans = 0; stk.push_back(stk[0]);
for(int i=0;i<n;i++) {
 while(abs(cross(stk[i+1]-stk[i],
   stk[(pos+1)%n]-stk[i])) >
   abs(cross(stk[i+1]-stk[i],
   stk[pos]-stk[i]))) pos = (pos+1)%n;
 ans = max({ans, dis(stk[i], stk[pos]),
  dis(stk[i+1], stk[pos])});
}
```

6.6 kD Closest Pair (3D ver.)

```
llf solve(vector<P> v) {
 shuffle(v.begin(), v.end(), mt19937());
 unordered_map<lld, unordered_map<lld,
  unordered_map<lld, int>>> m;
 llf d = dis(v[0], v[1]);
 auto Idx = [&d] (llf x) -> lld {
  return round(x * 2 / d) + 0.1; };
 auto rebuild_m = [&m, &v, &Idx](int k) {
  m.clear();
  for (int i = 0; i < k; ++i)
   m[Idx(v[i].x)][Idx(v[i].y)]
    [Idx(v[i].z)] = i;
 }; rebuild_m(2);
 for (size_t i = 2; i < v.size(); ++i) {
  const lld kx = Idx(v[i].x), ky = Idx(v[i].y),
     kz = Idx(v[i].z); bool found = false;
  for (int dx = -2; dx <= 2; ++dx) {
   const lld nx = dx + kx;
   if (m.find(nx) == m.end()) continue;
   auto& mm = m[nx];
   for (int dy = -2; dy <= 2; ++dy) {
    const lld ny = dy + ky;
    if (mm.find(ny) == mm.end()) continue;
    auto& mmm = mm[ny];
    for (int dz = -2; dz <= 2; ++dz) {
     const lld nz = dz + kz;
     if (mmm.find(nz) == mmm.end()) continue;
     const int p = mmm[nz];
     if (dis(v[p], v[i]) < d) {
      d = dis(v[p], v[i]);
      found = true;
     }
    }
   }
  }
  if (found) rebuild_m(i + 1);
  else m[kx][ky][kz] = i;
 }
 return d;
}
```

6.7 Simulated Annealing

```
llf anneal() {
 mt19937 rnd_engine( seed );
 uniform_real_distribution< llf > rnd( 0, 1 );
 const llf dT = 0.001;
 // Argument p
 llf S_cur = calc( p ), S_best = S_cur;
 for ( llf T = 2000 ; T > EPS ; T -= dT ) {
  // Modify p to p_prime
  const llf S_prime = calc( p_prime );
  const llf delta_c = S_prime - S_cur;
  llf prob = min( ( llf ) 1, exp( -delta_c / T ) );
  if ( rnd( rnd_engine ) <= prob )
   S_cur = S_prime, p = p_prime;
  if ( S_prime < S_best ) // find min
   S_best = S_prime, p_best = p_prime;
 }
 return S_best;
}
```

6.8 Half Plane Intersection

```
// cross(pt-line.st, line.dir)<=0 <-> pt in half plane
bool operator<(const Line &lhs, const Line &rhs) {
  if (int cmp = argCmp(lhs.dir, rhs.dir))
    return cmp == -1;
  return ori(lhs.st, lhs.ed, rhs.st) < 0;
}

// intersect function is in "Segment Intersect"
llf HPI(vector<Line> &lines) {
  sort(lines.begin(), lines.end());
  deque<Line> que;
  deque<PTF> pt;
  que.push_back(lines[0]);
  for (int i = 1; i < (int)lines.size(); i++) {
    if (argCmp(lines[i].dir, lines[i-1].dir) == 0)
     continue;
#define POP(L, R) \
    while (pt.size() > 0 \
      && ori(L.st, L.ed, pt.back()) < 0) \
      pt.pop_back(), que.pop_back(); \
    while (pt.size() > 0 \
      && ori(R.st, R.ed, pt.front()) < 0) \
      pt.pop_front(), que.pop_front();
    POP(lines[i], lines[i]);
    pt.push_back(intersect(que.back(), lines[i]));
    que.push_back(lines[i]);
  }
  POP(que.front(), que.back())
  if (que.size() <= 1 ||
    argCmp(que.front().dir, que.back().dir) == 0)
    return 0;
  pt.push_back(intersect(que.front(), que.back()));
  return area(pt);
}
```

6.9 Minkowski Sum

```
vector<pll> Minkowski(vector<pll> A, vector<pll> B) {
 hull(A), hull(B);
 vector<pll> C(1, A[0] + B[0]), s1, s2;
 for(int i = 0; i < SZ(A); ++i)
  s1.pb(A[(i + 1) % SZ(A)] - A[i]);
 for(int i = 0; i < SZ(B); i++)
  s2.pb(B[(i + 1) % SZ(B)] - B[i]);
 for(int p1 = 0, p2 = 0; p1 < SZ(A) || p2 < SZ(B);)
  if (p2 >= SZ(B)
    || (p1 < SZ(A) && cross(s1[p1], s2[p2]) >= 0))
   C.pb(C.back() + s1[p1++]);
  else
   C.pb(C.back() + s2[p2++]);
 return hull(C), C;
}
```

6.10 Circle Class

```
struct Circle { PTF o; llf r; };

vector<llf> intersectAngle(Circle A, Circle B) {
 PTF dir = B.o - A.o; llf d2 = norm(dir);
 if (norm(A.r - B.r) >= d2) // norm(x) := |x|^2
  if (A.r < B.r) return {-PI, PI}; // A in B
  else return {}; // B in A
 if (norm(A.r + B.r) <= d2) return {};
 llf dis = abs(dir), theta = arg(dir);
 llf phi = acos((A.r * A.r + d2 - B.r * B.r) /
   (2 * A.r * dis));
 llf L = FMOD(theta - phi), R = FMOD(theta + phi);
 return { L, R };
}

vector<PTF> intersectPoint(Circle a, Circle b) {
 llf d = abs(a.o - b.o);
 if (d >= b.r+a.r || d <= abs(b.r-a.r)) return {};
 llf dt = (b.r*b.r - a.r*a.r)/d, d1 = (d+dt)/2;
 PTF dir = (a.o - b.o) / d;
 PTF u = dir*d1 + b.o;
 PTF v = rot90(dir) * sqrt(max<llf>(0, b.r*b.r-d1*d1));
 return {u + v, u - v};
}
```

6.11 Intersection of line and Circle

```
vector<PTF> line_interCircle(const PTF &p1,
  const PTF &p2, const PTF &c, const double r) {
 PTF ft = p1 + project(c-p1, p2-p1), vec = p2-p1;
 llf dis = abs(c - ft);
 if (abs(dis - r) < eps) return {ft};
 if (dis > r) return {};
 vec = vec * sqrt(r * r - dis * dis) / abs(vec);
 return {ft + vec, ft - vec};
}
```

6.12 Intersection of Polygon and Circle

```
// Divides into multiple triangle, and sum up
// test by HDU2892
llf _area(PTF pa, PTF pb, llf r) {
 if (abs(pa) < abs(pb)) swap(pa, pb);
 if (abs(pb) < eps) return 0;
 llf S, h, theta;
 llf a = abs(pb), b = abs(pa), c = abs(pb - pa);
 llf cosB = dot(pb, pb - pa) / a / c, B = acos(cosB);
 llf cosC = dot(pa, pb) / a / b, C = acos(cosC);
 if (a > r) {
  S = (C / 2) * r * r;
  h = a * b * sin(C) / c;
  if (h < r && B < PI / 2)
   S -= (acos(h / r) * r * r - h * sqrt(r*r - h*h));
 } else if (b > r) {
  theta = PI - B - asin(sin(B) / r * a);
  S = 0.5 * a*r * sin(theta) + (C - theta) / 2 * r*r;
 } else
  S = 0.5 * sin(C) * a * b;
 return S;
}
llf area_poly_circle(const vector<PTF> &poly,
  const PTF &O, const llf r) {
 llf S = 0;
 for (int i = 0, N = poly.size(); i < N; ++i)
  S += _area(poly[i] - O, poly[(i + 1) % N] - O, r) *
     ori(O, poly[i], poly[(i + 1) % N]);
 return fabs(S);
}
```

6.13 Point & Hulls Tangent

```
#define above(P, Vi, Vj) (ori(P, Vi, Vj) > 0) // true
     if Vi is above Vj
#define below(P, Vi, Vj) (ori(P, Vi, Vj) < 0) // true
     if Vi is below Vj
// Rtangent_PointPolyC(): binary search for convex
     polygon right tangent
//  Input: P = a 2D point (exterior to the polygon)
//      n = number of polygon vertices
//      V = array of vertices for a 2D convex polygon
     with V[n] = V[0]
//  Return: index "i" of rightmost tangent point V[i]
int Rtangent_PointPolyC(PT P, int n, PT *V) {
 int a, b, c;
 int upA, dnC;

 if (below(P, V[1], V[0]) && !above(P, V[n - 1], V[0]))
  return 0;

 for (a = 0, b = n;;) {
  c = (a + b) / 2;
  dnC = below(P, V[c + 1], V[c]);
  if (dnC && !above(P, V[c - 1], V[c]))
   return c;

  upA = above(P, V[a + 1], V[a]);
  if (upA) {
   if (dnC) {
    b = c;
   } else {
    if (above(P, V[a], V[c]))
     b = c;
    else
     a = c;
   }
  } else {
   if (!dnC) {
    a = c;
   } else {
```

```
    if (below(P, V[a], V[c]))
     b = c;
    else
     a = c;
   }
  }
 }
}

// Ltangent_PointPolyC(): binary search for convex
    polygon left tangent
//  Input: P = a 2D point (exterior to the polygon)
//      n = number of polygon vertices
//      V = array of vertices for a 2D convex polygon
    with V[n]=V[0]
//  Return: index "i" of leftmost tangent point V[i]
int Ltangent_PointPolyC(PT P, int n, PT *V) {
 int a, b, c;
 int dnA, dnC;

 if (above(P, V[n - 1], V[0]) && !below(P, V[1], V[0]))
  return 0;

 for (a = 0, b = n;;) {
  c = (a + b) / 2;
  dnC = below(P, V[c + 1], V[c]);
  if (above(P, V[c - 1], V[c]) && !dnC)
   return c;
  dnA = below(P, V[a + 1], V[a]);
  if (dnA) {
   if (!dnC) {
    b = c;
   } else {
    if (below(P, V[a], V[c]))
     b = c;
    else
     a = c;
   }
  } else {
   if (dnC) {
    a = c;
   } else {
    if (above(P, V[a], V[c]))
     b = c;
    else
     a = c;
   }
  }
 }
}
```

6.14 Convex Hulls Tangent

```
// RLtangent_PolyPolyC(): get the RL tangent between
    two convex polygons
//  Input: m = number of vertices in polygon 1
//      V = array of vertices for convex polygon 1 with
     V[m]=V[0]
//      n = number of vertices in polygon 2
//      W = array of vertices for convex polygon 2 with
     W[n]=W[0]
//  Output: *t1 = index of tangent point V[t1] for
    polygon 1
//      *t2 = index of tangent point W[t2] for polygon
    2
void RLtangent_PolyPolyC(int m, PT *V, int n, PT *W,
    int *t1, int *t2) {
 int ix1, ix2; // search indices for polygons 1 and 2

 // first get the initial vertex on each polygon
 ix1 = Rtangent_PointPolyC(W[0], m, V);  // right
    tangent from W[0] to V
 ix2 = Ltangent_PointPolyC(V[ix1], n, W); // left
    tangent from V[ix1] to W

 // ping-pong linear search until it stabilizes
 int done = false; // flag when done
 while (done == false) {
  done = true; // assume done until...
  while (ori(W[ix2], V[ix1], V[ix1 + 1]) <= 0) {
   ++ix1; // get Rtangent from W[ix2] to V
  }
  while (ori(V[ix1], W[ix2], W[ix2 - 1]) >= 0) {
   --ix2;    // get Ltangent from V[ix1] to W
   done = false; // not done if had to adjust this
  }
 }
 *t1 = ix1;
 *t2 = ix2;
 return;
}
```

6.15 Tangent line of Two Circle

```
vector<Line>
tanline(const Circle &c1, const Circle &c2, int sign1){
 // sign1 = 1 for outer tang, -1 for inter tang
 vector<Line> ret;
 if (norm(c1.o - c2.o) < eps) return ret;
 llf d = abs(c1.o - c2.o);
 PTF v = (c2.o - c1.o) / d;
 llf c = (c1.r - sign1 * c2.r) / d;
 if (c * c > 1) return ret;
 llf h = sqrt(max<llf>(0, 1 - c * c));
 for (int sign2 : {1, -1}) {
  PTF n = c * v + sign2 * h * rot90(v);
  PTF p1 = c1.o + n * c1.r;
  PTF p2 = c2.o + n * (c2.r * sign1);
  if (norm(p2 - p1) < eps)
   p2 = p1 + rot90(c2.o - c1.o);
  ret.push_back({p1, p2});
 }
 return ret;
}
```

6.16 Minimum Covering Circle

```
template<typename P>
Circle getCircum(const P &a, const P &b, const P &c){
 Real a1 = a.x-b.x, b1 = a.y-b.y;
 Real c1 = (a.x+b.x)/2 * a1 + (a.y+b.y)/2 * b1;
 Real a2 = a.x-c.x, b2 = a.y-c.y;
 Real c2 = (a.x+c.x)/2 * a2 + (a.y+c.y)/2 * b2;
 Circle cc;
 cc.o.x = (c1*b2-b1*c2)/(a1*b2-b1*a2);
 cc.o.y = (a1*c2-c1*a2)/(a1*b2-b1*a2);
 cc.r = hypot(cc.o.x-a.x, cc.o.y-a.y);
 return cc;
}

template<typename P>
Circle MinCircleCover(const vector<P>& pts){
 random_shuffle(pts.begin(), pts.end());
 Circle c = { pts[0], 0 };
 for(int i=0;i<(int)pts.size();i++){
  if (dist(pts[i], c.o) <= c.r) continue;
  c = { pts[i], 0 };
  for (int j = 0; j < i; j++) {
   if(dist(pts[j], c.o) <= c.r) continue;
   c.o = (pts[i] + pts[j]) / 2;
   c.r = dist(pts[i], c.o);
   for (int k = 0; k < j; k++) {
    if (dist(pts[k], c.o) <= c.r) continue;
    c = getCircum(pts[i], pts[j], pts[k]);
   }
  }
 }
 return c;
}
```

6.17 KDTree (Nearest Point)

```
const int MXN = 100005;
struct KDTree {
 struct Node {
  int x,y,x1,y1,x2,y2;
  int id,f;
  Node *L, *R;
 } tree[MXN], *root;
 int n;
 LL dis2(int x1, int y1, int x2, int y2) {
  LL dx = x1-x2, dy = y1-y2;
  return dx*dx+dy*dy;
 }
 static bool cmpx(Node& a, Node& b){return a.x<b.x;}
 static bool cmpy(Node& a, Node& b){return a.y<b.y;}
 void init(vector<pair<int,int>> ip) {
```

```
  n = ip.size();
  for (int i=0; i<n; i++) {
   tree[i].id = i;
   tree[i].x = ip[i].first;
   tree[i].y = ip[i].second;
  }
  root = build_tree(0, n-1, 0);
 }
 Node* build_tree(int L, int R, int d) {
  if (L>R) return nullptr;
  int M = (L+R)/2; tree[M].f = d%2;
  nth_element(tree+L,tree+M,tree+R+1,d%2?cmpy:cmpx);
  tree[M].x1 = tree[M].x2 = tree[M].x;
  tree[M].y1 = tree[M].y2 = tree[M].y;
  tree[M].L = build_tree(L, M-1, d+1);
  if (tree[M].L) {
   tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
   tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
   tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
   tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
  }
  tree[M].R = build_tree(M+1, R, d+1);
  if (tree[M].R) {
   tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
   tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
   tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
   tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
  }
  return tree+M;
 }
 int touch(Node* r, int x, int y, LL d2){
  LL dis = sqrt(d2)+1;
  if (x<r->x1-dis || x>r->x2+dis ||
    y<r->y1-dis || y>r->y2+dis)
   return 0;
  return 1;
 }
 void nearest(Node* r,int x,int y,int &mID,LL &md2) {
  if (!r || !touch(r, x, y, md2)) return;
  LL d2 = dis2(r->x, r->y, x, y);
  if (d2 < md2 || (d2 == md2 && mID < r->id)) {
   mID = r->id;
   md2 = d2;
  }
  // search order depends on split dim
  if ((r->f == 0 && x < r->x) ||
    (r->f == 1 && y < r->y)) {
   nearest(r->L, x, y, mID, md2);
   nearest(r->R, x, y, mID, md2);
  } else {
   nearest(r->R, x, y, mID, md2);
   nearest(r->L, x, y, mID, md2);
  }
 }
 int query(int x, int y) {
  int id = 1029384756;
  LL d2 = 102938475612345678LL;
  nearest(root, x, y, id, d2);
  return id;
 }
} tree;
```

6.18 Rotating Sweep Line

```
void rotatingSweepLine(pair<int, int> a[], int n) {
 vector<pair<int, int>> l;
 l.reserve(n * (n - 1) / 2);
 for (int i = 0; i < n; ++i)
  for (int j = i + 1; j < n; ++j)
   l.emplace_back(i, j);
 sort(l.begin(), l.end(), [&a](auto &u, auto &v){
  lld udx = a[u.first].first - a[u.second].first;
  lld udy = a[u.first].second - a[u.second].second;
  lld vdx = a[v.first].first - a[v.second].first;
  lld vdy = a[v.first].second - a[v.second].second;
  if (udx == 0 or vdx == 0) return not udx == 0;
  int s = sgn(udx * vdx);
  return udy * vdx * s < vdy * udx * s;
 });
 vector<int> idx(n), p(n);
 iota(idx.begin(), idx.end(), 0);
 sort(idx.begin(), idx.end(), [&a](int i, int j){
  return a[i] < a[j]; });
 for (int i = 0; i < n; ++i) p[idx[i]] = i;
 for (auto [i, j]: l) {
  // do here
  swap(p[i], p[j]);
  idx[p[i]] = i, idx[p[j]] = j;
 }
}
```

6.19 Circle Cover

```
const int N = 1021;
struct CircleCover {
 int C;
 Cir c[N];
 bool g[N][N], overlap[N][N];
 // Area[i] : area covered by at least i circles
 double Area[ N ];
 void init(int _C){ C = _C;}
 struct Teve {
  PTF p; double ang; int add;
  Teve() {}
  Teve(PTF _a, double _b, int _c):p(_a), ang(_b), add(
    _c){}
  bool operator<(const Teve &a)const
  {return ang < a.ang;}
 }eve[N * 2];
 // strict: x = 0, otherwise x = -1
 bool disjuct(Cir &a, Cir &b, int x)
 {return sign(abs(a.O - b.O) - a.R - b.R) > x;}
 bool contain(Cir &a, Cir &b, int x)
 {return sign(a.R - b.R - abs(a.O - b.O)) > x;}
 bool contain(int i, int j) {
  /* c[j] is non-strictly in c[i]. */
  return (sign(c[i].R - c[j].R) > 0 || (sign(c[i].R - c
    [j].R) == 0 && i < j)) && contain(c[i], c[j], -1);
 }
 void solve(){
  fill_n(Area, C + 2, 0);
  for(int i = 0; i < C; ++i)
   for(int j = 0; j < C; ++j)
    overlap[i][j] = contain(i, j);
  for(int i = 0; i < C; ++i)
   for(int j = 0; j < C; ++j)
    g[i][j] = !(overlap[i][j] || overlap[j][i] ||
      disjuct(c[i], c[j], -1));
  for(int i = 0; i < C; ++i){
   int E = 0, cnt = 1;
   for(int j = 0; j < C; ++j)
    if(j != i && overlap[j][i])
     ++cnt;
   for(int j = 0; j < C; ++j)
    if(i != j && g[i][j]) {
     auto IP = intersectPoint(c[i], c[j]);
     PTF aa = IP[0], bb = IP[1];
     llf A = arg(aa-c[i].O), B = arg(bb-c[i].O);
     eve[E++] = Teve(bb,B,1), eve[E++]=Teve(aa,A,-1);
     if(B > A) ++cnt;
    }
   if(E == 0) Area[cnt] += pi * c[i].R * c[i].R;
   else{
    sort(eve, eve + E);
    eve[E] = eve[0];
    for(int j = 0; j < E; ++j){
     cnt += eve[j].add;
     Area[cnt] += cross(eve[j].p, eve[j + 1].p) * .5;
     double theta = eve[j + 1].ang - eve[j].ang;
     if (theta < 0) theta += 2. * pi;
     Area[cnt]+=(theta-sin(theta))*c[i].R*c[i].R*.5;
    }
   }
  }
 }
};
```

7 Stringology

7.1 Suffix Array

```
namespace sfx {
bool _t[maxn * 2];
int hi[maxn], rev[maxn];
int _s[maxn * 2], sa[maxn * 2], _c[maxn * 2];
int x[maxn], _p[maxn], _q[maxn * 2];
```

```
// sa[i]: sa[i]-th suffix is the
// i-th lexigraphically smallest suffix.
// hi[i]: longest common prefix
// of suffix sa[i] and suffix sa[i - 1].
void pre(int *a, int *c, int n, int z) {
 memset(a, 0, sizeof(int) * n);
 memcpy(x, c, sizeof(int) * z);
}
void induce(int *a,int *c,int *s,bool *t,int n,int z){
 memcpy(x + 1, c, sizeof(int) * (z - 1));
 for (int i = 0; i < n; ++i)
  if (a[i] && !t[a[i] - 1])
   a[x[s[a[i] - 1]]++] = a[i] - 1;
 memcpy(x, c, sizeof(int) * z);
 for (int i = n - 1; i >= 0; --i)
  if (a[i] && t[a[i] - 1])
   a[--x[s[a[i] - 1]]] = a[i] - 1;
}
void sais(int *s, int *a, int *p, int *q,
 bool *t, int *c, int n, int z) {
 bool uniq = t[n - 1] = true;
 int nn=0, nmxz=-1, *nsa = a+n, *ns=s+n, last=-1;
 memset(c, 0, sizeof(int) * z);
 for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;
 for (int i = 0; i < z - 1; ++i) c[i + 1] += c[i];
 if (uniq) {
  for (int i = 0; i < n; ++i) a[--c[s[i]]] = i;
  return;
 }
 for (int i = n - 2; i >= 0; --i)
  t[i] = (s[i]==s[i + 1] ? t[i + 1] : s[i]<s[i + 1]);
 pre(a, c, n, z);
 for (int i = 1; i <= n - 1; ++i)
  if (t[i] && !t[i - 1])
   a[--x[s[i]]] = p[q[i] = nn++] = i;
 induce(a, c, s, t, n, z);
 for (int i = 0; i < n; ++i) {
  if (a[i] && t[a[i]] && !t[a[i] - 1]) {
  bool neq = last < 0 || \
   memcmp(s + a[i], s + last,
   (p[q[a[i]] + 1] - a[i]) * sizeof(int));
  ns[q[last = a[i]]] = nmxz += neq;
 }}
 sais(ns, nsa, p+nn, q+n, t+n, c+z, nn, nmxz+1);
 pre(a, c, n, z);
 for (int i = nn - 1; i >= 0; --i)
  a[--x[s[p[nsa[i]]]]] = p[nsa[i]];
 induce(a, c, s, t, n, z);
}
void build(const string &s) {
 const int n = int(s.size());
 for (int i = 0; i < n; ++i) _s[i] = s[i];
 _s[n] = 0; // s shouldn't contain 0
 sais(_s, sa, _p, _q, _t, _c, n + 1, 256);
 for(int i = 0; i < n; ++i) rev[sa[i] = sa[i+1]] = i;
 int ind = hi[0] = 0;
 for (int i = 0; i < n; ++i) {
  if (!rev[i]) {
   ind = 0;
   continue;
  }
  while (i + ind < n && \
   s[i + ind] == s[sa[rev[i] - 1] + ind]) ++ind;
  hi[rev[i]] = ind ? ind-- : 0;
 }
}}
```

7.2 Suffix Automaton

```
struct SuffixAutomaton {
 struct node {
  int ch[K], len, fail, cnt, indeg;
  node(int L = 0) : ch{}, len(L), fail(0), cnt(0),
    indeg(0) {}
 } st[N];
 int root, last, tot;
 void extend(int c) {
  int cur = ++tot;
  st[cur] = node(st[last].len + 1);
  while (last && !st[last].ch[c]) {
    st[last].ch[c] = cur;
    last = st[last].fail;
  }
  if (!last) {
    st[cur].fail = root;
  } else {
    int q = st[last].ch[c];
    if (st[q].len == st[last].len + 1) {
      st[cur].fail = q;
    } else {
      int clone = ++tot;
      st[clone] = st[q];
      st[clone].len = st[last].len + 1;
      st[st[cur].fail = st[q].fail = clone].cnt = 0;
      while (last && st[last].ch[c] == q) {
        st[last].ch[c] = clone;
        last = st[last].fail;
      }
    }
  }
  st[last = cur].cnt += 1;
 }
 void init(const char* s) {
  root = last = tot = 1;
  st[root] = node(0);
  for (char c; c = *s; ++s) extend(c - 'a');
 }
 int q[N];
 void dp() {
  for (int i = 1; i <= tot; i++) ++st[st[i].fail].indeg
   ;
  int head = 0, tail = 0;
  for (int i = 1; i <= tot; i++)
    if (st[i].indeg == 0) q[tail++] = i;
  while (head != tail) {
    int now = q[head++];
    if (int f = st[now].fail) {
      st[f].cnt += st[now].cnt;
      if (--st[f].indeg == 0) q[tail++] = f;
    }
  }
 }
 int run(const char* s) {
  int now = root;
  for (char c; c = *s; ++s) {
    if (!st[now].ch[c -= 'a']) return 0;
    now = st[now].ch[c];
  }
  return st[now].cnt;
 }
} SAM;
```

7.3 Z value

```
vector<int> Zalgo(const string &s) {
 vector<int> z(s.size(), s.size());
 for (int i = 1, l = 0, r = 0; i < z[0]; ++i) {
  int j = clamp(r - i, 0, z[i - l]);
  for (; i + j < z[0] and s[i + j] == s[j]; ++j);
  if (i + (z[i] = j) > r) r = i + z[l = i];
 }
 return z;
}
```

7.4 Manacher

```
int z[maxn];
int manacher(const string& s) {
 string t = ".";
 for(char c: s) t += c, t += '.';
 int l = 0, r = 0, ans = 0;
 for (int i = 1; i < t.length(); ++i) {
  z[i] = (r > i ? min(z[2 * l - i], r - i) : 1);
  while (i - z[i] >= 0 && i + z[i] < t.length()) {
   if(t[i - z[i]] == t[i + z[i]]) ++z[i];
   else break;
  }
  if (i + z[i] > r) r = i + z[i], l = i;
 }
 for(int i=1;i<t.length();++i) ans = max(ans, z[i]-1);
 return ans;
}
```

7.5 Lexico Smallest Rotation

```
string mcp(string s) {
 int n = s.length();
```

```
 s += s; int i = 0, j = 1;
 while (i < n && j < n) {
  int k = 0;
  while (k < n && s[i + k] == s[j + k]) k++;
  ((s[i + k] <= s[j + k]) ? j : i) += k + 1;
  j += (i == j);
 }
 return s.substr(i < n ? i : j, n);
}
```

7.6 BWT

```
struct BurrowsWheeler{
#define SIGMA 26
#define BASE 'a'
 vector<int> v[ SIGMA ];
 void BWT(char* ori, char* res){
  // make ori -> ori + ori
  // then build suffix array
 }
 void iBWT(char* ori, char* res){
  for( int i = 0 ; i < SIGMA ; i ++ )
   v[ i ].clear();
  int len = strlen( ori );
  for( int i = 0 ; i < len ; i ++ )
   v[ ori[i] - BASE ].push_back( i );
  vector<int> a;
  for( int i = 0 , ptr = 0 ; i < SIGMA ; i ++ )
   for( auto j : v[ i ] ){
    a.push_back( j );
    ori[ ptr ++ ] = BASE + i;
   }
  for( int i = 0 , ptr = 0 ; i < len ; i ++ ){
   res[ i ] = ori[ a[ ptr ] ];
   ptr = a[ ptr ];
  }
  res[ len ] = 0;
 }
} bwt;
```

7.7 Palindromic Tree

```
struct palindromic_tree{
 struct node{
  int next[26],f,len;
  int cnt,num,st,ed;
  node(int l=0):f(0),len(l),cnt(0),num(0) {
   memset(next, 0, sizeof(next)); }
 };
 vector<node> st;
 vector<char> s;
 int last,n;
 void init(){
  st.clear();s.clear();last=1; n=0;
  st.push_back(0);st.push_back(-1);
  st[0].f=1;s.push_back(-1); }
 int getFail(int x){
  while(s[n-st[x].len-1]!=s[n])x=st[x].f;
  return x;}
 void add(int c){
  s.push_back(c-='a'); ++n;
  int cur=getFail(last);
  if(!st[cur].next[c]){
   int now=st.size();
   st.push_back(st[cur].len+2);
   st[now].f=st[getFail(st[cur].f)].next[c];
   st[cur].next[c]=now;
   st[now].num=st[st[now].f].num+1;
  }
  last=st[cur].next[c];
  ++st[last].cnt;}
 void dpcnt() {
  for (int i=st.size()-1; i >= 0; i--)
   st[st[i].f].cnt += st[i].cnt;
 }
 int size(){ return st.size()-2;}
} pt;
int main() {
 string s; cin >> s; pt.init();
 for (int i=0; i<SZ(s); i++) {
  int prvsz = pt.size(); pt.add(s[i]);
  if (prvsz != pt.size()) {
   int r = i, l = r - pt.st[pt.last].len + 1;
   // pal @ [l,r]: s.substr(l, r-l+1)
   }
  }
  return 0;
}
```

8 Misc

8.1 Theorems

8.1.1 Kirchhoff's Theorem

Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G, where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i, j) in G.

- The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
- The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.

8.1.2 Tutte's Matrix

Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniform randomly) if $i < j$ and $(i, j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{rank(D)}{2}$ is the maximum matching on G.

8.1.3 Cayley's Formula

- Given a degree sequence $d_1, d_2, \ldots, d_n$ for each labeled vertices, there're $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!}$ spanning trees.
- Let $T_{n,k}$ be the number of labeled forests on n vertices with k components, such that vertex $1, 2, \ldots, k$ belong to different components. Then $T_{n,k} = kn^{n-k-1}$.

8.1.4 Erdős–Gallai theorem

A sequence of non-negative integers $d_1 \geq d_2 \geq \ldots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + d_2 + \ldots + d_n$ is even and

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k)$$

holds for all $1 \leq k \leq n$.

8.1.5 Havel–Hakimi algorithm

find the vertex who has greatest degree unused, connect it with other greatest vertex.

8.1.6 Euler's planar graph formula

$V - E + F = C + 1, E \leq 3V - 6$(?)

8.1.7 Pick's theorem

For simple polygon, when points are all integer, we have $A = \#\{\text{lattice points in the interior}\} + \frac{\#\{\text{lattice points on the boundary}\}}{2} - 1$

8.1.8 Matroid Intersection

Given matroids $M_1 = (G, I_1), M_2 = (G, I_2)$, find maximum $S \in I_1 \cap I_2$. For each iteration, build the directed graph and find a shortest path from s to t.

- $s \to x : S \sqcup \{x\} \in I_1$
- $x \to t : S \sqcup \{x\} \in I_2$
- $y \to x : S \setminus \{y\} \sqcup \{x\} \in I_1$ (y is in the unique circuit of $S \sqcup \{x\}$)
- $x \to y : S \setminus \{y\} \sqcup \{x\} \in I_2$ (y is in the unique circuit of $S \sqcup \{x\}$)

Alternate the path, and $|S|$ will increase by 1. Let $R = \min(\text{rank}(I_1), \text{rank}(I_2)), N = |G|$. In each iteration, $|E| = O(RN)$. For weighted case, assign weight $-w(x)$ and $w(x)$ to $x \in S$ and $x \notin S$, resp. Use Bellman-Ford to find the weighted shortest path. The maximum iteration of Bellman-Ford is $2R + 1$.

8.2 Convex 1D/1D DP

```
struct segment {
 int i, l, r;
 segment() {}
 segment(int a, int b, int c): i(a), l(b), r(c) {}
};
inline lld f(int l, int r){return dp[l] + w(l+1, r);}
void solve() {
 dp[0] = 0;
 deque&lt;segment&gt; dq; dq.push_back(segment(0, 1, n));
 for (int i = 1; i <= n; ++i) {
  dp[i] = f(dq.front().i, i);
  while(dq.size()&&dq.front().r<i+1) dq.pop_front();
  dq.front().l = i + 1;
  segment seg = segment(i, i + 1, n);
```

```
  while (dq.size() &&
   f(i, dq.back().l)<f(dq.back().i, dq.back().l))
    dq.pop_back();
  if (dq.size()) {
   int d = 1 << 20, c = dq.back().l;
   while (d >>= 1) if (c + d <= dq.back().r)
    if(f(i, c+d) > f(dq.back().i, c+d)) c += d;
   dq.back().r = c; seg.l = c + 1;
  }
  if (seg.l <= n) dq.push_back(seg);
 }
}
```

8.3 ConvexHull Optimization

```
struct L {
 mutable int64_t a, b, p;
 bool operator<(const L &r) const { return a < r.a; }
 bool operator<(int64_t x) const { return p < x; }
};
struct DynamicHull : multiset<L, less<>> {
 static const int64_t kInf = 1e18;
 bool Isect(iterator x, iterator y) {
  auto Div = [](int64_t a, int64_t b) {
    return a / b - ((a ^ b) < 0 && a % b); }
  if (y == end()) { x->p = kInf; return false; }
  if (x->a == y->a) x->p = x->b > y->b ? kInf : -kInf;
  else x->p = Div(y->b - x->b, x->a - y->a);
  return x->p >= y->p;
 }
 void Insert(int64_t a, int64_t b) {
  auto z = insert({a, b, 0}), y = z++, x = y;
  while (Isect(y, z)) z = erase(z);
  if (x!=begin()&&Isect(--x,y)) Isect(x, y=erase(y));
  while ((y = x) != begin() && (--x)->p >= y->p)
   Isect(x, erase(y));
 }
 int64_t Query(int64_t x) {
  auto l = *lower_bound(x);
  return l.a * x + l.b;
 }
};
```

8.4 Josephus Problem

```
// n people kill m for each turn
int f(int n, int m) {
 int s = 0;
 for (int i = 2; i <= n; i++)
  s = (s + m) % i;
 return s;
}
// died at kth
int kth(int n, int m, int k){
 if (m == 1) return n-1;
 for (k = k*m+m-1; k >= n; k = k-n+(k-n)/(m-1));
 return k;
}
```

8.5 Cactus Matching

```
vector<int> init_g[maxn],g[maxn*2];
int n,dfn[maxn],low[maxn],par[maxn],dfs_idx,bcc_id;
void tarjan(int u){
 dfn[u]=low[u]=++dfs_idx;
 for(int i=0;i<(int)init_g[u].size();i++){
  int v=init_g[u][i];
  if(v==par[u]) continue;
  if(!dfn[v]){
   par[v]=u;
   tarjan(v);
   low[u]=min(low[u],low[v]);
   if(dfn[u]<low[v]){
    g[u].push_back(v);
    g[v].push_back(u);
   }
  }else{
   low[u]=min(low[u],dfn[v]);
   if(dfn[v]<dfn[u]){
    int temp_v=u;
    bcc_id++;
    while(temp_v!=v){
     g[bcc_id+n].push_back(temp_v);
     g[temp_v].push_back(bcc_id+n);
     temp_v=par[temp_v];
    }
    g[bcc_id+n].push_back(v);
    g[v].push_back(bcc_id+n);
    reverse(g[bcc_id+n].begin(),g[bcc_id+n].end());
   }
  }
 }
}
int dp[maxn][2],min_dp[2][2],tmp[2][2],tp[2];
void dfs(int u,int fa){
 if(u<=n){
  for(int i=0;i<(int)g[u].size();i++){
   int v=g[u][i];
   if(v==fa) continue;
   dfs(v,u);
   memset(tp,0x8f,sizeof tp);
   if(v<=n){
    tp[0]=dp[u][0]+max(dp[v][0],dp[v][1]);
    tp[1]=max(
     dp[u][0]+dp[v][0]+1,
     dp[u][1]+max(dp[v][0],dp[v][1])
    );
   }else{
    tp[0]=dp[u][0]+dp[v][0];
    tp[1]=max(dp[u][0]+dp[v][1],dp[u][1]+dp[v][0]);
   }
   dp[u][0]=tp[0],dp[u][1]=tp[1];
  }
 }else{
  for(int i=0;i<(int)g[u].size();i++){
   int v=g[u][i];
   if(v==fa) continue;
   dfs(v,u);
  }
  min_dp[0][0]=0;
  min_dp[1][1]=1;
  min_dp[0][1]=min_dp[1][0]=-0x3f3f3f3f;
  for(int i=0;i<(int)g[u].size();i++){
   int v=g[u][i];
   if(v==fa) continue;
   memset(tmp,0x8f,sizeof tmp);
   tmp[0][0]=max(
    min_dp[0][0]+max(dp[v][0],dp[v][1]),
    min_dp[0][1]+dp[v][0]
   );
   tmp[0][1]=min_dp[0][0]+dp[v][0]+1;
   tmp[1][0]=max(
    min_dp[1][0]+max(dp[v][0],dp[v][1]),
    min_dp[1][1]+dp[v][0]
   );
   tmp[1][1]=min_dp[1][0]+dp[v][0]+1;
   memcpy(min_dp,tmp,sizeof tmp);
  }
  dp[u][1]=max(min_dp[0][1],min_dp[1][0]);
  dp[u][0]=min_dp[0][0];
 }
}
int main(){
 int m,a,b;
 scanf("%d%d",&n,&m);
 for(int i=0;i<m;i++){
  scanf("%d%d",&a,&b);
  init_g[a].push_back(b);
  init_g[b].push_back(a);
 }
 par[1]=-1;
 tarjan(1);
 dfs(1,-1);
 printf("%d\n",max(dp[1][0],dp[1][1]));
 return 0;
}
```

8.6 Tree Knapsack

```
int dp[N][K]; PII obj[N];
vector<int> G[N];
void dfs(int u, int mx){
 for(int s: G[u]) {
  if(mx < obj[s].first) continue;
  for(int i=0;i<=mx-obj[s].FF;i++)
   dp[s][i] = dp[u][i];
  dfs(s, mx - obj[s].first);
```

```
  for(int i=obj[s].FF;i<=mx;i++)
   dp[u][i] = max(dp[u][i],
    dp[s][i - obj[s].FF] + obj[s].SS);
 }
}
```

8.7 N Queens Problem

```
vector< int > solve( int n ) {
 // no solution when n=2, 3
 vector< int > ret;
 if ( n % 6 == 2 ) {
  for ( int i = 2 ; i <= n ; i += 2 )
   ret.push_back( i );
  ret.push_back( 3 ); ret.push_back( 1 );
  for ( int i = 7 ; i <= n ; i += 2 )
   ret.push_back( i );
  ret.push_back( 5 );
 } else if ( n % 6 == 3 ) {
  for ( int i = 4 ; i <= n ; i += 2 )
   ret.push_back( i );
  ret.push_back( 2 );
  for ( int i = 5 ; i <= n ; i += 2 )
   ret.push_back( i );
  ret.push_back( 1 ); ret.push_back( 3 );
 } else {
  for ( int i = 2 ; i <= n ; i += 2 )
   ret.push_back( i );
  for ( int i = 1 ; i <= n ; i += 2 )
   ret.push_back( i );
 }
 return ret;
}
```

8.8 Binary Search On Fraction

```
struct Q {
 ll p, q;
 Q go(Q b, ll d) { return {p + b.p*d, q + b.q*d}; }
};
bool pred(Q);
// returns smallest p/q in [lo, hi] such that
// pred(p/q) is true, and 0 <= p,q <= N
Q frac_bs(ll N) {
 Q lo{0, 1}, hi{1, 0};
 if (pred(lo)) return lo;
 assert(pred(hi));
 bool dir = 1, L = 1, H = 1;
 for (; L || H; dir = !dir) {
  ll len = 0, step = 1;
  for (int t = 0; t < 2 && (t ? step/=2 : step*=2);)
   if (Q mid = hi.go(lo, len + step);
     mid.p > N || mid.q > N || dir ^ pred(mid))
    t++;
   else len += step;
  swap(lo, hi = hi.go(lo, len));
  (dir ? L : H) = !!len;
 }
 return dir ? hi : lo;
}
```