

Contents

1 Basic	1	6.20 Circle Cover	23
1.1 vimrc	1	7 Stringology	23
1.2 Debug Macro	1	7.1 Hash	23
1.3 Increase Stack	1	7.2 Suffix Array	23
1.4 Pragma Optimization	2	7.3 Suffix Automaton	24
1.5 IO Optimization	2	7.4 KMP	24
2 Data Structure	2	7.5 Z value	24
2.1 Dark Magic	2	7.6 Manacher	24
2.2 Link-Cut Tree	2	7.7 Lexico Smallest Rotation	24
2.3 LiChao Segment Tree	3	7.8 BWT	25
2.4 Treap	3	7.9 Palindromic Tree	25
2.5 Linear Basis	3	8 Misc	25
2.6 Binary Search On Segment Tree	3	8.1 Theorems	25
3 Graph	4	8.1.1 Kirchhoff's Theorem	25
3.1 2-SAT (SCC)	4	8.1.2 Tutte's Matrix	25
3.2 BCC Edge	4	8.1.3 Cayley's Formula	25
3.3 BCC Vertex	4	8.1.4 Erdős-Gallai theorem	25
3.4 Centroid Decomposition	4	8.1.5 Havel-Hakimi algorithm	25
3.5 Directed Minimum Spanning Tree	5	8.1.6 Hall's marriage theorem	25
3.6 Dominator Tree	5	8.1.7 Euler's planar graph formula	25
3.7 Edge Coloring	6	8.1.8 Pick's theorem	25
3.8 Lowbit Decomposition	6	8.1.9 Lucas's theorem	25
3.9 Manhattan Minimum Spanning Tree	6	8.1.10 Matroid Intersection	25
3.10 MaxClique	6	8.2 DP-opt Condition	25
3.11 MaxCliqueDyn	7	8.2.1 totally monotone (concave/convex)	25
3.12 Minimum Mean Cycle	7	8.2.2 monge condition (concave/convex)	25
3.13 Minimum Steiner Tree	8	8.3 Convex 1D/1D DP	26
3.14 Mo's Algorithm on Tree	8	8.4 ConvexHull Optimization	26
3.15 Tree Hashing	8	8.5 Josephus Problem	26
3.16 Virtual Tree	8	8.6 Cactus Matching	26
4 Matching & Flow	9	8.7 Tree Knapsack	27
4.1 Bipartite Matching	9	8.8 N Queens Problem	27
4.2 Dijkstra Cost Flow	9	8.9 Aliens Optimization	27
4.3 Dinic	9	8.10 Hilbert Curve	27
4.4 Flow Models	10	8.11 Binary Search On Fraction	27
4.5 General Graph Matching	10		
4.6 Global Min-Cut	10		
4.7 GomoryHu Tree	11		
4.8 Kuhn Munkres	11		
4.9 Minimum Cost Circulation	11		
4.10 Minimum Cost Maximum Flow	12		
4.11 Maximum Weight Graph Matching	12		
5 Math	13		
5.1 $\lfloor \frac{n}{i} \rfloor$ Enumeration	13		
5.2 Stirling Number	13		
5.2.1 First Kind	13		
5.2.2 Second Kind	13		
5.3 $ax+by=gcd$	14		
5.4 Berlekamp Massey	14		
5.5 Characteristic Polynomial	14		
5.6 Chinese Remainder	14		
5.7 De-Brujin	14		
5.8 DiscreteLog	14		
5.9 Extended Euler	15		
5.10 ExtendedFloorSum	15		
5.11 Fast Fourier Transform	15		
5.12 FloorSum	15		
5.13 FWT	16		
5.14 Gauss Elimination	16		
5.15 Miller Rabin	16		
5.16 NTT	16		
5.17 Range Sieve	16		
5.18 Partition Number	16		
5.19 Pi Count (Linear Sieve)	16		
5.20 Pollard Rho	17		
5.21 Polynomial Operations	17		
5.22 Quadratic residue	18		
5.23 Simplex	18		
5.24 Simplex Construction	19		
6 Geometry	19		
6.1 Basic Geometry	19		
6.2 Segment & Line Intersection	19		
6.3 2D Convex Hull	19		
6.4 3D Convex Hull	19		
6.5 2D Farthest Pair	20		
6.6 2D Closest Pair	20		
6.7 kD Closest Pair (3D ver.)	20		
6.8 Simulated Annealing	20		
6.9 Half Plane Intersection	20		
6.10 Minkowski Sum	20		
6.11 Circle Class	20		
6.12 Intersection of line and Circle	21		
6.13 Intersection of Polygon and Circle	21		
6.14 Point & Hulls Tangent	21		
6.15 Convex Hulls Tangent	21		
6.16 Tangent line of Two Circle	22		
6.17 Minimum Covering Circle	22		
6.18 KDTree (Nearest Point)	22		
6.19 Rotating Sweep Line	23		

1 Basic

1.1 vimrc

```

se is nu ru et tgc sc hls cin cino+=j1 sw=4 sts=4 bs=2
mouse=a encoding=utf-8 ls=2
syn on
colorscheme desert
filetype indent on
inoremap {<CR> {<CR>}<ESC>O
map <F8> <ESC>:w<CR>:!g++ "%" -o "%<" -std=c++17 -
    DKISEKI -Wall -Wextra -Wshadow -Wfatal-errors -
    Wconversion -fsanitize=address,undefined -g && echo
    success<CR>
map <F9> <ESC>:w<CR>:!g++ "%" -o "%<" -O2 -std=c++17 &&
    echo success<CR>
map <F10> <ESC>:!. / "%<" <CR>

```

1.2 Debug Macro

```

#ifdef KISEKI
#define safe cerr<<__PRETTY_FUNCTION__\
    <<" line "<<__LINE__<<" safe\n"
#define debug(a...) qwerty(#a, a)
#define orange(a...) dvorak(#a, a)
using std::cerr;
template <typename ...T>
void qwerty(const char *s, T ...a) {
    cerr << "\e[1;32m(" << s << ") = (" ;
    int cnt = sizeof...(T);
    (... , (cerr << a << (--cnt ? ", " : ") \e[0m\n")));
}
template <typename Iter>
void dvorak(const char *s, Iter L, Iter R) {
    cerr << "\e[1;32m[" << s << " ] = [ " ;
    for (int f = 0; L != R; ++L)
        cerr << (f++ ? ", " : "") << *L;
    cerr << " ] \e[0m\n" ;
}
#else
#define safe ((void)0)
#define debug(...) ((void)0)
#define orange(...) ((void)0)
#endif

```

1.3 Increase Stack

```

const int size = 256 << 20;
register long rsp asm("rsp");
char *p = (char*)malloc(size)+size, *bak = (char*)rsp;
__asm__("movq %0, %%rsp\n"::"r"(p));
// main
__asm__("movq %0, %%rsp\n"::"r"(bak));

```

1.4 Pragma Optimization

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,tune=native")
```

1.5 IO Optimization

```
static inline int gc() {
    constexpr int B = 1<<20;
    static char buf[B], *p, *q;
    if(p == q &&
        (q=(p=buf)+fread(buf,1,B,stdin)) == buf)
        return EOF;
    return *p++;
}

template < typename T >
static inline bool gn( T &x ) {
    int c = gc(); T sgn = 1; x = 0;
    while(('0'>c|<'9') && c!=EOF && c!='-') c = gc();
    if(c == '-') sgn = -1, c = gc();
    if(c == EOF) return false;
    while('0'<=c&&c<='9') x = x*10 + c - '0', c = gc();
    return x *= sgn, true;
}
```

2 Data Structure

2.1 Dark Magic

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
using namespace __gnu_pbds;
// heap tags: pairing/binary/binomial/rc_binomial/thin
template<typename T>
using pbds_heap=__gnu_pbds::prioity_queue<T,less<T>, \
    pairing_heap_tag>;

// pbds_heap::point_iterator
// x = pq.push(10); pq.modify(x, 87); a.join(b);
// tree tags: rb_tree_tag/ov_tree_tag/splay_tree_tag
template<typename T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;
// find_by_order, order_of_key
// hash tables: cc_hash_table/gp_hash_table
```

2.2 Link-Cut Tree

```
template <typename Val> class LCT {
private:
    struct node {
        int pa, ch[2];
        bool rev;
        Val v, v_prod, v_rprod;
        node() : pa{0}, ch{0, 0}, rev{false}, v{}, v_prod{},
            v_rprod{} {};
    };
    vector<node> nodes;
    set<pair<int, int>> edges;
    bool is_root(int u) const {
        const int p = nodes[u].pa;
        return nodes[p].ch[0] != u and nodes[p].ch[1] != u;
    }
    bool is_rch(int u) const {
        return (not is_root(u)) and nodes[nodes[u].pa].ch[1]
            == u;
    }
    void down(int u) {
        if (auto &cnode = nodes[u]; cnode.rev) {
            if (cnode.ch[0]) set_rev(cnode.ch[0]);
            if (cnode.ch[1]) set_rev(cnode.ch[1]);
            cnode.rev = false;
        }
    }
    void up(int u) {
        auto &cnode = nodes[u];
        cnode.v_prod =
            nodes[cnode.ch[0]].v_prod * cnode.v * nodes[cnode.ch[1]].v_prod;
        cnode.v_rprod =
            nodes[cnode.ch[1]].v_rprod * cnode.v * nodes[cnode.ch[0]].v_rprod;
    }
    void set_rev(int u) {
```

```
        swap(nodes[u].ch[0], nodes[u].ch[1]);
        swap(nodes[u].v_prod, nodes[u].v_rprod);
        nodes[u].rev ^= 1;
    }
    void rotate(int u) {
        int f = nodes[u].pa, g = nodes[f].pa, l = is_rch(u);
        if (nodes[u].ch[l ^ 1])
            nodes[nodes[u].ch[l ^ 1]].pa = f;
        if (not is_root(f))
            nodes[g].ch[is_rch(f)] = u;
        nodes[f].ch[l] = nodes[u].ch[l ^ 1];
        nodes[u].ch[l ^ 1] = f;
        nodes[u].pa = g, nodes[f].pa = u;
        up(f);
    }
    void splay(int u) {
        vector<int> stk = {u};
        while (not is_root(stk.back()))
            stk.push_back(nodes[stk.back()].pa);
        for (; not stk.empty(); stk.pop_back())
            down(stk.back());
        for(int f=nodes[u].pa;!is_root(u);f=nodes[u].pa){
            if(!is_root(f))rotate(is_rch(u)==is_rch(f)?f:u);
            rotate(u);
        }
        up(u);
    }
    void access(int u) {
        int last = 0;
        for (int last = 0; u; last = u, u = nodes[u].pa) {
            splay(u);
            nodes[u].ch[1] = last;
            up(u);
        }
    }
    int find_root(int u) {
        access(u); splay(u);
        int la = 0;
        for (; u; la = u, u = nodes[u].ch[0]) down(u);
        return la;
    }
    void change_root(int u) {
        access(u); splay(u); set_rev(u);
    }
    void link(int x, int y) {
        change_root(y); nodes[y].pa = x;
    }
    void split(int x, int y) {
        change_root(x); access(y); splay(y);
    }
    void cut(int x, int y) {
        split(x, y);
        nodes[y].ch[0] = nodes[x].pa = 0;
        up(y);
    }
public:
    LCT(int n = 0) : nodes(n + 1) {}
    int add(const Val &v = {}) {
        nodes.push_back(v);
        return int(nodes.size()) - 2;
    }
    int add(Val &&v) {
        nodes.emplace_back(move(v));
        return int(nodes.size()) - 2;
    }
    void set_val(int u, const Val &v) {
        splay(++u); nodes[u].v = v; up(u);
    }
    Val query(int x, int y) {
        split(++x, ++y);
        return nodes[y].v_prod;
    }
    bool connected(int u, int v) { return find_root(++u)
        == find_root(++v); }
    void add_edge(int u, int v) {
        if (++u > ++v) swap(u, v);
        edges.emplace(u, v); link(u, v);
    }
    void del_edge(int u, int v) {
        auto k = minmax(++u, ++v);
        if (auto it = edges.find(k); it != edges.end()) {
            edges.erase(it); cut(u, v);
        }
    }
}
```

```

}
}
};

```

2.3 LiChao Segment Tree

```

struct L {
    int m, k, id;
    L() : id(-1) {}
    L(int a, int b, int c) : m(a), k(b), id(c) {}
    int at(int x) { return m * x + k; }
};

class LiChao {
private:
    int n; vector<L> nodes;
    static int lc(int x) { return 2 * x + 1; }
    static int rc(int x) { return 2 * x + 2; }
    void insert(int l, int r, int id, L ln) {
        int m = (l + r) >> 1;
        if (nodes[id].id == -1) {
            nodes[id] = ln;
            return;
        }
        bool atLeft = nodes[id].at(l) < ln.at(l);
        if (nodes[id].at(m) < ln.at(m)) {
            atLeft ^= 1;
            swap(nodes[id], ln);
        }
        if (r - l == 1) return;
        if (atLeft) insert(l, m, lc(id), ln);
        else insert(m, r, rc(id), ln);
    }
    int query(int l, int r, int id, int x) {
        int ret = 0, m = (l + r) >> 1;
        if (nodes[id].id != -1)
            ret = nodes[id].at(x);
        if (r - l == 1) return ret;
        if (x < m) return max(ret, query(l, m, lc(id), x));
        return max(ret, query(m, r, rc(id), x));
    }

public:
    LiChao(int n_) : n(n_), nodes(n * 4) {}
    void insert(L ln) { insert(0, n, 0, ln); }
    int query(int x) { return query(0, n, 0, x); }
};

```

2.4 Treap

```

namespace Treap {
#define sz(x) ((x) ? ((x)->size) : 0)
    struct node {
        int size;
        uint32_t pri;
        node *lc, *rc, *pa;
        node() : size(0), pri(rand()), lc(0), rc(0), pa(0) {}
        void pull() {
            size = 1; pa = nullptr;
            if (lc) { size += lc->size; lc->pa = this; }
            if (rc) { size += rc->size; rc->pa = this; }
        }
    };

    node* merge(node* L, node* R) {
        if (!L || !R) return L ? L : R;
        if (L->pri > R->pri) {
            L->rc = merge(L->rc, R); L->pull();
            return L;
        } else {
            R->lc = merge(L, R->lc); R->pull();
            return R;
        }
    }

    void split_by_size(node* rt, int k, node*&L, node*&R) {
        if (!rt) L = R = nullptr;
        else if (sz(rt->lc) + 1 <= k) {
            L = rt;
            split_by_size(rt->rc, k - sz(rt->lc) - 1, L->rc, R);
            L->pull();
        } else {
            R = rt;
            split_by_size(rt->lc, k, L, R->lc);
            R->pull();
        }
    }
}

```

```

int getRank(node *o) {
    int r = sz(o->lc);
    for (; o->pa != nullptr; o = o->pa)
        if (o->pa->rc != o) r += sz(o->pa->lc);
    return r;
}
#undef sz
}

```

2.5 Linear Basis

```

template <int BITS>
struct LinearBasis {
    array<uint64_t, BITS> basis;
    Basis() { basis.fill(0); }
    void add(uint64_t x) {
        for (int i = 0; i < BITS; ++i) if ((x >> i) & 1) {
            if (basis[i] == 0) {
                basis[i] = x;
                return;
            }
            x ^= basis[i];
        }
    }
    bool ok(uint64_t x) {
        for (int i = 0; i < BITS; ++i)
            if ((x >> i) & 1) x ^= basis[i];
        return x == 0;
    }
};

```

2.6 Binary Search On Segment Tree

```

// find_first = x -> minimal x s.t. check( [a, x) )
// find_last = x -> maximal x s.t. check( [x, b) )
template <typename C>
int find_first(int l, const C &check) {
    if (l >= n) return n + 1;
    l += sz;
    for (int i = height; i > 0; i--)
        propagate(l >> i);
    Monoid sum = identity;
    do {
        while ((l & 1) == 0) l >>= 1;
        if (check(f(sum, data[l]))) {
            while (l < sz) {
                propagate(l);
                l <= 1;
                auto nxt = f(sum, data[l]);
                if (!check(nxt)) {
                    sum = nxt;
                    l++;
                }
            }
            return l + 1 - sz;
        }
        sum = f(sum, data[l++]);
    } while ((l & -l) != 1);
    return n + 1;
}

template <typename C>
int find_last(int r, const C &check) {
    if (r <= 0) return -1;
    r += sz;
    for (int i = height; i > 0; i--)
        propagate((r - 1) >> i);
    Monoid sum = identity;
    do {
        r--;
        while (r > 1 and (r & 1)) r >>= 1;
        if (check(f(data[r], sum))) {
            while (r < sz) {
                propagate(r);
                r = (r < 1) + 1;
                auto nxt = f(data[r], sum);
                if (!check(nxt)) {
                    sum = nxt;
                    r--;
                }
            }
            return r - sz;
        }
        sum = f(data[r], sum);
    } while ((r & -r) != r);
}

```

```
return -1;
}
```

3 Graph

3.1 2-SAT (SCC)

```
class TwoSat{
private:
    int n;
    vector<vector<int>> rG,G,sccs;
    vector<int> ord,idx;
    vector<bool> vis,result;
    void dfs(int u){
        vis[u]=true;
        for(int v:G[u])
            if(!vis[v]) dfs(v);
        ord.push_back(u);
    }
    void rdfs(int u){
        vis[u]=false;idx[u]=sccs.size()-1;
        sccs.back().push_back(u);
        for(int v:rG[u])
            if(vis[v])rdfs(v);
    }
public:
    void init(int n_){
        G.clear();G.resize(n=n_);
        rG.clear();rG.resize(n);
        sccs.clear();ord.clear();
        idx.resize(n);result.resize(n);
    }
    void add_edge(int u,int v){
        G[u].push_back(v);rG[v].push_back(u);
    }
    void orr(int x,int y){
        if ((x^y)==1)return;
        add_edge(x^1,y); add_edge(y^1,x);
    }
    bool solve(){
        vis.clear();vis.resize(n);
        for(int i=0;i<n;++i)
            if(not vis[i])dfs(i);
        reverse(ord.begin(),ord.end());
        for (int u:ord){
            if(!vis[u])continue;
            sccs.push_back(vector<int>());
            rdfs(u);
        }
        for(int i=0;i<n;i+=2)
            if(idx[i]==idx[i+1])
                return false;
        vector<bool> c(sccs.size());
        for(size_t i=0;i<sccs.size();++i){
            for(auto sij : sccs[i]){
                result[sij]=c[i];
                c[idx[sij^1]]!=c[i];
            }
        }
        return true;
    }
    bool get(int x){return result[x];}
    int get_id(int x){return idx[x];}
    int count(){return sccs.size();}
} sat2;
```

3.2 BCC Edge

```
class BCC_Bridge {
private:
    int n, ecnt;
    vector<vector<pair<int,int>>> G;
    vector<int> dfn, low;
    vector<bool> bridge;
    void dfs(int u, int f) {
        dfn[u] = low[u] = dfn[f] + 1;
        for (auto [v, t]: G[u]) {
            if (v == f) continue;
            if (dfn[v]) {
                low[u] = min(low[u], dfn[v]);
                continue;
            }
            dfs(v, u);
        }
    }
}
```

```
        low[u] = min(low[u], low[v]);
        if (low[v] > dfn[u]) bridge[t] = true;
    }
}
public:
    void init(int n_) {
        G.clear(); G.resize(n = n_);
        low.assign(n, ecnt = 0);
        dfn.assign(n, 0);
    }
    void add_edge(int u, int v) {
        G[u].emplace_back(v, ecnt);
        G[v].emplace_back(u, ecnt++);
    }
    void solve() {
        bridge.assign(ecnt, false);
        for (int i = 0; i < n; ++i)
            if (not dfn[i]) dfs(i, i);
    }
    bool is_bridge(int x) { return bridge[x]; }
} bcc_bridge;
```

3.3 BCC Vertex

```
class BCC_AP {
private:
    int n, ecnt;
    vector<vector<pair<int,int>>> G;
    vector<int> bcc, dfn, low, st;
    vector<bool> ap, ins;
    void dfs(int u, int f) {
        dfn[u] = low[u] = dfn[f] + 1;
        int ch = 0;
        for (auto [v, t]: G[u]) if (v != f) {
            if (not ins[t]) {
                st.push_back(t);
                ins[t] = true;
            }
            if (dfn[v]) {
                low[u] = min(low[u], dfn[v]);
                continue;
            }
            ++ch; dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (low[v] >= dfn[u]) {
                ap[u] = true;
                while (true) {
                    int eid = st.back(); st.pop_back();
                    bcc[eid] = ecnt;
                    if (eid == t) break;
                }
                ecnt++;
            }
        }
        if (ch == 1 and u == f) ap[u] = false;
    }
public:
    void init(int n_) {
        G.clear(); G.resize(n = n_);
        ecnt = 0; ap.assign(n, false);
        low.assign(n, 0); dfn.assign(n, 0);
    }
    void add_edge(int u, int v) {
        G[u].emplace_back(v, ecnt);
        G[v].emplace_back(u, ecnt++);
    }
    void solve() {
        ins.assign(ecnt, false);
        bcc.resize(ecnt); ecnt = 0;
        for (int i = 0; i < n; ++i)
            if (not dfn[i]) dfs(i, i);
    }
    int get_id(int x) { return bcc[x]; }
    int count() { return ecnt; }
    bool is_ap(int x) { return ap[x]; }
} bcc_ap;
```

3.4 Centroid Decomposition

```
struct Centroid {
    vector<vector<int64_t>> Dist;
    vector<int> Parent, Depth;
    vector<int64_t> Sub, Sub2;
    vector<int> Sz, Sz2;
    Centroid(vector<vector<pair<int, int>>> g) {
```

```

int N = g.size();
vector<bool> Vis(N);
vector<int> sz(N), mx(N);
vector<int> Path;
Dist.resize(N);
Parent.resize(N);
Depth.resize(N);
auto DfsSz = [&](auto dfs, int x) -> void {
    Vis[x] = true; sz[x] = 1; mx[x] = 0;
    for (auto [u, w] : g[x]) {
        if (Vis[u]) continue;
        dfs(dfs, u);
        sz[x] += sz[u];
        mx[x] = max(mx[x], sz[u]);
    }
    Path.push_back(x);
};
auto DfsDist = [&](auto dfs, int x, int64_t D = 0)
-> void {
    Dist[x].push_back(D); Vis[x] = true;
    for (auto [u, w] : g[x]) {
        if (Vis[u]) continue;
        dfs(dfs, u, D + w);
    }
};
auto Dfs = [&]
(auto dfs, int x, int D = 0, int p = -1) -> void {
    Path.clear(); DfsSz(DfsSz, x);
    int M = Path.size();
    int C = -1;
    for (int u : Path) {
        if (max(M - sz[u], mx[u]) * 2 <= M) C = u;
        Vis[u] = false;
    }
    DfsDist(DfsDist, C);
    for (int u : Path) Vis[u] = false;
    Parent[C] = p; Vis[C] = true;
    Depth[C] = D;
    for (auto [u, w] : g[C]) {
        if (Vis[u]) continue;
        dfs(dfs, u, D + 1, C);
    }
};
Dfs(Dfs, 0); Sub.resize(N); Sub2.resize(N);
Sz.resize(N); Sz2.resize(N);
}

void Mark(int v) {
    int x = v, z = -1;
    for (int i = Depth[v]; i >= 0; --i) {
        Sub[x] += Dist[v][i]; Sz[x]++;
        if (z != -1) {
            Sub2[z] += Dist[v][i];
            Sz2[z]++;
        }
        z = x; x = Parent[x];
    }
}

int64_t Query(int v) {
    int64_t res = 0;
    int x = v, z = -1;
    for (int i = Depth[v]; i >= 0; --i) {
        res += Sub[x] + 1LL * Sz[x] * Dist[v][i];
        if (z != -1) res -= Sub2[z] + 1LL * Sz2[z] * Dist[v][i];
        z = x; x = Parent[x];
    }
    return res;
}
};

```

3.5 Directed Minimum Spanning Tree

```

struct DirectedMST { // find maximum
    struct Edge {
        int u, v;
        int w;
        Edge(int u, int v, int w) : u(u), v(v), w(w) {}
    };
    vector<Edge> Edges;
    void clear() { Edges.clear(); }
    void addEdge(int a, int b, int w) { Edges.emplace_back(a, b, w); }
    int solve(int root, int n) {
        vector<Edge> E = Edges;

```

```

        int ans = 0;
        while (true) {
            // find best in edge
            vector<int> in(n, -inf), prv(n, -1);
            for (auto e : E)
                if (e.u != e.v && e.w > in[e.v]) {
                    in[e.v] = e.w;
                    prv[e.v] = e.u;
                }
            in[root] = 0;
            prv[root] = -1;
            for (int i = 0; i < n; i++)
                if (in[i] == -inf)
                    return -inf;
            // find cycle
            int tot = 0;
            vector<int> id(n, -1), vis(n, -1);
            for (int i = 0; i < n; i++) {
                ans += in[i];
                for (int x = i; x != -1 && id[x] == -1; x = prv[x]) {
                    if (vis[x] == i) {
                        for (int y = prv[x]; y != x; y = prv[y])
                            id[y] = tot;
                        id[x] = tot++;
                        break;
                    }
                    vis[x] = i;
                }
            }
            if (!tot)
                return ans;
            for (int i = 0; i < n; i++)
                if (id[i] == -1)
                    id[i] = tot++;
            // shrink
            for (auto &e : E) {
                if (id[e.u] != id[e.v])
                    e.w -= in[e.v];
                e.u = id[e.u], e.v = id[e.v];
            }
            n = tot;
            root = id[root];
        }
        assert(false);
    }
} DMST;

```

3.6 Dominator Tree

```

namespace dominator {
    vector<int> g[maxn], r[maxn], rdom[maxn];
    int dfn[maxn], rev[maxn], fa[maxn], sdom[maxn];
    int dom[maxn], val[maxn], rp[maxn], tk;
    void init(int n) {
        // vertices are numbered from 0 to n - 1
        fill(dfn, dfn + n, -1); fill(rev, rev + n, -1);
        fill(fa, fa + n, -1); fill(val, val + n, -1);
        fill(sdom, sdom + n, -1); fill(rp, rp + n, -1);
        fill(dom, dom + n, -1); tk = 0;
        for (int i = 0; i < n; ++i) {
            g[i].clear(); r[i].clear(); rdom[i].clear();
        }
    }
    void add_edge(int x, int y) { g[x].push_back(y); }
    void dfs(int x) {
        rev[dfn[x] = tk] = x;
        fa[tk] = sdom[tk] = val[tk] = tk; tk++;
        for (int u : g[x]) {
            if (dfn[u] == -1) dfs(u), rp[dfn[u]] = dfn[x];
            r[dfn[u]].push_back(dfn[x]);
        }
    }
    void merge(int x, int y) { fa[x] = y; }
    int find(int x, int c = 0) {
        if (fa[x] == x) return c ? -1 : x;
        int p = find(fa[x], 1);
        if (p == -1) return c ? fa[x] : val[x];
        if (sdom[val[x]] > sdom[val[fa[x]]]) val[x] = val[fa[x]];
        fa[x] = p;
        return c ? p : val[x];
    }
    vector<int> build(int s, int n) {

```



```
// return the father of each node in the dominator tree
// p[i] = -2 if i is unreachable from s
dfs(s);
for (int i = tk - 1; i >= 0; --i) {
    for (int u : r[i]) sdom[i] = min(sdom[i], sdom[find(u)]);
    if (i) rdom[sdom[i]].push_back(i);
    for (int &u : rdom[i]) {
        int p = find(u);
        if (sdom[p] == i) dom[u] = i;
        else dom[u] = p;
    }
    if (i) merge(i, rp[i]);
}
vector<int> p(n, -2); p[s] = -1;
for (int i = 1; i < tk; ++i)
    if (sdom[i] != dom[i]) dom[i] = dom[dom[i]];
for (int i = 1; i < tk; ++i) p[rev[i]] = rev[dom[i]];
return p;
}
```

3.7 Edge Coloring

```
// max(d_u) + 1 edge coloring, time: O(NM)
int C[kN][kN], G[kN][kN]; // 1-based, G: ans
void clear(int N) {
    for (int i = 0; i <= N; i++)
        for (int j = 0; j <= N; j++)
            C[i][j] = G[i][j] = 0;
}
void solve(vector<pair<int, int>> &E, int N) {
    int X[kN] = {}, a;
    auto update = [&](int u) {
        for (X[u] = 1; C[u][X[u]]; X[u]++);
    };
    auto color = [&](int u, int v, int c) {
        int p = G[u][v];
        G[u][v] = G[v][u] = c;
        C[u][c] = v, C[v][c] = u;
        C[u][p] = C[v][p] = 0;
        if (p) X[u] = X[v] = p;
        else update(u), update(v);
        return p;
    };
    auto flip = [&](int u, int c1, int c2) {
        int p = C[u][c1];
        swap(C[u][c1], C[u][c2]);
        if (p) G[u][p] = G[p][u] = c2;
        if (!C[u][c1]) X[u] = c1;
        if (!C[u][c2]) X[u] = c2;
        return p;
    };
    for (int i = 1; i <= N; i++) X[i] = 1;
    for (int t = 0; t < E.size(); t++) {
        auto [u, v] = E[t];
        int v0 = v, c = X[u], c0 = c, d;
        vector<pair<int, int>> L; int vst[kN] = {};
        while (!G[u][v0]) {
            L.emplace_back(v, d = X[v]);
            if (!C[v][c]) for(a=L.size()-1; a>=0; a--)
                c = color(u, L[a].first, c);
            else if(!C[u][d]) for(a=L.size()-1; a>=0; a--)
                color(u, L[a].first, L[a].second);
            else if (vst[d]) break;
            else vst[d] = 1, v = C[u][d];
        }
        if (!G[u][v0]) {
            for (; v; v = flip(v, c, d), swap(c, d));
            if (C[u][c0]) { a = int(L.size()) - 1;
                while (--a >= 0 && L[a].second != c);
                for(; a>=0; a--) color(u, L[a].first, L[a].second);
            } else t--;
        }
    }
}
```

3.8 Lowbit Decomposition

```
class LBD {
    int timer, chains;
    vector<vector<int>> G;
    vector<int> tl, tr, chain, head, dep, pa;
    // chains : number of chain
    // tl, tr[u] : subtree interval in the seq. of u
    // head[i] : head of the chain i
}
```

```
// chain[u] : chain id of the chain u is on
void predfs(int u, int f) {
    dep[u] = dep[pa[u] = f] + 1;
    for (int v : G[u]) if (v != f) {
        predfs(v, u);
        if (lowbit(chain[u]) < lowbit(chain[v]))
            chain[u] = chain[v];
    }
    if (chain[u] == 0) chain[u] = ++chains;
}
void dfschain(int u, int f) {
    tl[u] = timer++;
    if (head[chain[u]] == -1)
        head[chain[u]] = u;
    for (int v : G[u])
        if (v != f and chain[v] == chain[u])
            dfschain(v, u);
    for (int v : G[u])
        if (v != f and chain[v] != chain[u])
            dfschain(v, u);
    tr[u] = timer;
}
public:
    LBD(int n) : timer(0), chains(0), G(n), tl(n), tr(n),
        chain(n), head(n, -1), dep(n), pa(n) {}
    void add_edge(int u, int v) {
        G[u].push_back(v); G[v].push_back(u);
    }
    void decompose() { predfs(0, 0); dfschain(0, 0); }
    PII get_subtree(int u) { return {tl[u], tr[u]}; }
    vector<PII> get_path(int u, int v) {
        vector<PII> res;
        while (chain[u] != chain[v]) {
            if (dep[head[chain[u]]] < dep[head[chain[v]]])
                swap(u, v);
            int s = head[chain[u]];
            res.emplace_back(tl[s], tl[u] + 1);
            u = pa[s];
        }
        if (dep[u] < dep[v]) swap(u, v);
        res.emplace_back(tl[v], tl[u] + 1);
        return res;
    }
};
```

3.9 Manhattan Minimum Spanning Tree

```
typedef Point<int> P;
vector<array<int, 3>> manhattanMST(vector<P> ps) {
    vi id(sz(ps));
    iota(all(id), 0);
    vector<array<int, 3>> edges;
    rep(k, 0, 4) {
        sort(all(id), [&](int i, int j) {
            return (ps[i] - ps[j]).x < (ps[j] - ps[i]).y;
        });
        map<int, int> sweep;
        for (int i : id) {
            for (auto it = sweep.lower_bound(-ps[i].y);
                it != sweep.end(); sweep.erase(it++)) {
                int j = it->second;
                P d = ps[i] - ps[j];
                if (d.y > d.x) break;
                edges.push_back({d.y + d.x, i, j});
            }
            sweep[-ps[i].y] = i;
        }
        for (P &p : ps)
            if (k & 1) p.x = -p.x;
            else swap(p.x, p.y);
    }
    return edges; // [{w, i, j}, ...]
}
```

3.10 MaxClique

```
// contain a self loop u to u, then u won't in clique
template < size_t MAXN >
class MaxClique {
private:
    using bits = bitset< MAXN >;
    bits popped, G[ MAXN ], ans;
    size_t deg[ MAXN ], deo[ MAXN ], n;
    void sort_by_degree() {
```

```

popped.reset();
for ( size_t i = 0 ; i < n ; ++ i )
    deg[ i ] = G[ i ].count();
for ( size_t i = 0 ; i < n ; ++ i ) {
    size_t mi = MAXN, id = 0;
    for ( size_t j = 0 ; j < n ; ++ j )
        if ( not popped[ j ] and deg[ j ] < mi )
            mi = deg[ id = j ];
    popped[ deo[ i ] = id ] = 1;
    for( size_t u = G[ i ]._Find_first() ;
        u < n ; u = G[ i ]._Find_next( u ) )
        -- deg[ u ];
}
}
void BK( bits R, bits P, bits X ) {
    if (R.count()+P.count() <= ans.count()) return;
    if ( not P.count() and not X.count() ) {
        if ( R.count() > ans.count() ) ans = R;
        return;
    }
    /* greedily choose max degree as pivot
    bits cur = P | X; size_t pivot = 0, sz = 0;
    for ( size_t u = cur._Find_first() ;
        u < n ; u = cur._Find_next( u ) )
        if ( deg[ u ] > sz ) sz = deg[ pivot = u ];
    cur = P & ( ~G[ pivot ] );
    */ // or simply choose first
    bits cur = P & ( ~G[ ( P | X )._Find_first() ] );
    for ( size_t u = cur._Find_first() ;
        u < n ; u = cur._Find_next( u ) ) {
        if ( R[ u ] ) continue;
        R[ u ] = 1;
        BK( R, P & G[ u ], X & G[ u ] );
        R[ u ] = P[ u ] = 0, X[ u ] = 1;
    }
}
public:
void init( size_t n_ ) {
    n = n_;
    for ( size_t i = 0 ; i < n ; ++ i )
        G[ i ].reset();
    ans.reset();
}
void add_edges( int u, bits S ) { G[ u ] = S; }
void add_edge( int u, int v ) {
    G[ u ][ v ] = G[ v ][ u ] = 1;
}
int solve() {
    sort_by_degree(); // or simply iota( deo... )
    for ( size_t i = 0 ; i < n ; ++ i )
        deg[ i ] = G[ i ].count();
    bits pob, nob = 0; pob.set();
    for (size_t i=n; i<MAXN; ++i) pob[i] = 0;
    for ( size_t i = 0 ; i < n ; ++ i ) {
        size_t v = deo[ i ];
        bits tmp; tmp[ v ] = 1;
        BK( tmp, pob & G[ v ], nob & G[ v ] );
        pob[ v ] = 0, nob[ v ] = 1;
    }
    return static_cast< int >( ans.count() );
}
};

```

3.11 MaxCliqueDyn

```

constexpr int kN = 150;
struct MaxClique { // Maximum Clique
    bitset<kN> a[kN], cs[kN];
    int ans, sol[kN], q, cur[kN], d[kN], n;
    void init(int _n) {
        n = _n, ans = q = 0;
        for (int i = 0; i < n; i++) a[i].reset();
    }
    void addEdge(int u, int v) { a[u][v] = a[v][u] = 1; }
    void csort(vector<int> &r, vector<int> &c) {
        int mx = 1, km = max(ans - q + 1, 1), t = 0,
            m = int(r.size());
        cs[1].reset(); cs[2].reset();
        for (int i = 0; i < m; i++) {
            int p = r[i], k = 1;
            while ((cs[k] & a[p]).count()) k++;
            if (k > mx) cs[++mx + 1].reset();
            cs[k][p] = 1;

```

```

        if (k < km) r[t++] = p;
    }
    c.resize(m);
    if (t) c[t - 1] = 0;
    for (int k = km; k <= mx; k++) {
        for (int p = int(cs[k]._Find_first());
            p < kN; p = int(cs[k]._Find_next(p))) {
            r[t] = p; c[t++] = k;
        }
    }
}
void dfs(vector<int> &r, vector<int> &c, int l,
    bitset<kN> mask) {
    while (!r.empty()) {
        int p = r.back(); r.pop_back();
        mask[p] = 0;
        if (q + c.back() <= ans) return;
        cur[q++] = p;
        vector<int> nr, nc;
        bitset<kN> nmask = mask & a[p];
        for (int i : r)
            if (a[p][i]) nr.push_back(i);
        if (!nr.empty()) {
            if (l < 4) {
                for (int i : nr)
                    d[i] = int((a[i] & nmask).count());
                sort(nr.begin(), nr.end(),
                    [&](int x, int y) {
                        return d[x] > d[y];
                    });
            }
            csort(nr, nc); dfs(nr, nc, l + 1, nmask);
        } else if (q > ans) {
            ans = q; copy(cur, cur + q, sol);
        }
        c.pop_back(); q--;
    }
}
int solve(bitset<kN> mask) { // vertex mask
    vector<int> r, c;
    for (int i = 0; i < n; i++)
        if (mask[i]) r.push_back(i);
    for (int i = 0; i < n; i++)
        d[i] = int((a[i] & mask).count());
    sort(r.begin(), r.end(),
        [&](int i, int j) { return d[i] > d[j]; });
    csort(r, c);
    dfs(r, c, 1, mask);
    return ans; // sol[0 ~ ans-1]
}
} graph;

```

3.12 Minimum Mean Cycle

```

/* minimum mean cycle O(VE) */
struct MMC{
#define FZ(n) memset((n),0,sizeof(n))
#define E 101010
#define V 1021
#define inf 1e9
    struct Edge { int v,u; double c; };
    int n, m, prv[V][V], prve[V][V], vst[V];
    Edge e[E];
    vector<int> edgeID, cycle, rho;
    double d[V][V];
    void init( int _n ) { n = _n; m = 0; }
    // WARNING: TYPE matters
    void add_edge( int vi , int ui , double ci )
    { e[ m ++ ] = { vi , ui , ci }; }
    void bellman_ford() {
        for(int i=0; i<n; i++) d[0][i]=0;
        for(int i=0; i<n; i++) {
            fill(d[i+1], d[i+1]+n, inf);
            for(int j=0; j<m; j++) {
                int v = e[j].v, u = e[j].u;
                if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
                    d[i+1][u] = d[i][v]+e[j].c;
                    prv[i+1][u] = v;
                    prve[i+1][u] = j;
                }
            }
        }
    }
}

```

```

double solve(){
    // returns inf if no cycle, mmc otherwise
    double mmc=inf;
    int st = -1;
    bellman_ford();
    for(int i=0; i<n; i++) {
        double avg=-inf;
        for(int k=0; k<n; k++) {
            if(d[n][i]<inf-eps)
                avg=max(avg, (d[n][i]-d[k][i])/(n-k));
            else avg=max(avg, inf);
        }
        if (avg < mmc) tie(mmc, st) = tie(avg, i);
    }
    FZ(vst);edgeID.clear();cycle.clear();rho.clear();
    for (int i=n; !vst[st]; st=prv[i--][st]) {
        vst[st]++;
        edgeID.PB(prve[i][st]);
        rho.PB(st);
    }
    while (vst[st] != 2) {
        int v = rho.back(); rho.pop_back();
        cycle.PB(v);
        vst[v]++;
    }
    reverse(ALL(edgeID));
    edgeID.resize(SZ(cycle));
    return mmc;
}
} mmc;

```

3.13 Minimum Steiner Tree

```

// Minimum Steiner Tree
// O(V 3^T + V^2 2^T)
struct SteinerTree {
#define V 33
#define T 8
#define INF 1023456789
    int n, dst[V][V], dp[1 << T][V], tdst[V];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < n; j++)
                dst[i][j] = INF * (i != j);
        }
    }
    void add_edge(int ui, int vi, int wi) {
        dst[ui][vi] = min(dst[ui][vi], wi);
        dst[vi][ui] = min(dst[vi][ui], wi);
    }
    void shortest_path() {
        for (int k = 0; k < n; k++)
            for (int i = 0; i < n; i++)
                for (int j = 0; j < n; j++)
                    dst[i][j] = min(dst[i][j], dst[i][k] + dst[k][j]);
    }
    int solve(const vector<int> &ter) {
        int t = (int)ter.size();
        for (int i = 1; i < (1 << t); i++)
            fill_n(dp[i], n, INF);
        fill_n(dp[0], n, 0);
        for (int msk = 1; msk < (1 << t); msk++) {
            if (msk == (msk & (-msk))) {
                int who = __lg(msk);
                for (int i = 0; i < n; i++)
                    dp[msk][i] = dst[ter[who]][i];
                continue;
            }
            for (int i = 0; i < n; i++)
                for (int submsk = (msk - 1) & msk; submsk; submsk = (submsk - 1) & msk)
                    dp[msk][i] = min(dp[msk][i], dp[submsk][i] + dp[msk ^ submsk][i]);
            for (int i = 0; i < n; i++) {
                tdst[i] = INF;
                for (int j = 0; j < n; j++)
                    tdst[i] = min(tdst[i], dp[msk][j] + dst[j][i]);
            }
            copy_n(tdst, n, dp[msk]);
        }
        int ans = INF;
        for (int i = 0; i < n; i++)

```

```

        ans = min(ans, dp[(1 << t) - 1][i]);
        return ans;
    }
} solver;

```

3.14 Mo's Algorithm on Tree

```

int q; vector<int> G[N];
struct Que{
    int u, v, id;
} que[ N ];
int dfn[N], dfn_, block_id[N], block_, stk[N], stk_;
void dfs( int u, int f ) {
    dfn[ u ] = dfn_++; int saved_rbp = stk_;
    for ( int v : G[ u ] ) {
        if ( v == f ) continue;
        dfs( v, u );
        if ( stk_ - saved_rbp < SQRT_N ) continue;
        for ( ++ block_ ; stk_ != saved_rbp ; )
            block_id[ stk[ -- stk_ ] ] = block_;
    }
    stk[ stk_ ++ ] = u;
}
bool inPath[ N ];
void Diff( int u ) {
    if ( inPath[ u ] ^= 1 ) { /*remove this edge*/ }
    else { /*add this edge*/ }
}
void traverse( int& origin_u, int u ) {
    for ( int g = lca( origin_u, u ) ;
        origin_u != g ; origin_u = parent_of[ origin_u ] )
        Diff( origin_u );
    for ( int v = u; v != origin_u; v = parent_of[v] )
        Diff( v );
    origin_u = u;
}
void solve() {
    dfs( 1, 1 );
    while ( stk_ ) block_id[ stk[ -- stk_ ] ] = block_;
    sort( que, que + q, [](const Que& x, const Que& y) {
        return tie( block_id[ x.u ], dfn[ x.v ] )
            < tie( block_id[ y.u ], dfn[ y.v ] );
    } );
    int U = 1, V = 1;
    for ( int i = 0 ; i < q ; ++ i ) {
        pass( U, que[ i ].u );
        pass( V, que[ i ].v );
        // we could get our answer of que[ i ].id
    }
}
/*
Method 2:
dfs u:
    push u
    iterate subtree
    push u
Let P = LCA(u, v), and St(u)<=St(v)
if (P == u) query[St(u), St(v)]
else query[Ed(u), St(v)], query[St(P), St(P)]
*/

```

3.15 Tree Hashing

```

uint64_t hsah(int u, int f) {
    uint64_t r = 127;
    for (int v : G[ u ]) if (v != f) {
        uint64_t hh = hsah(v, u);
        r=(r+(hh*hh)%1010101333)%1011820613;
    }
    return r;
}

```

3.16 Virtual Tree

```

vector<pair<int, int>> build(vector<int> vs) { // tree
    0-base
    vector<pair<int, int>> res;
    sort(vs.begin(), vs.end(), [](int i, int j) { return
        dfn[i] < dfn[j]; });
    vector<int> s = {0};
    for (int v : vs) if (v != 0) {
        int o = lca(v, s.back());
        if (o != s.back()) {
            while (s.size() >= 2 and dfn[s[s.size() - 2]] >= dfn
                [o]) {

```



```

    res.emplace_back(s[s.size() - 2], s.back());
    s.pop_back();
}
if (s.back() != o) {
    res.emplace_back(s.back(), o);
    s.back() = o;
}
}
s.push_back(v);
}
for (size_t i = 1; i < s.size(); ++i)
    res.emplace_back(s[i - 1], s[i]);
return res;
}

```

4 Matching & Flow

4.1 Bipartite Matching

```

struct BipartiteMatching {
    vector<int> X[N];
    int fX[N], fY[N], n;
    bitset<N> vis;
    bool dfs(int x) {
        for (auto i: X[x]) {
            if (vis[i]) continue;
            vis[i] = true;
            if (fY[i] == -1 || dfs(fY[i])) {
                fY[fX[x] = i] = x;
                return true;
            }
        }
        return false;
    }
    void init(int n_, int m) {
        vis.reset();
        fill(X, X + (n = n_), vector<int>());
        memset(fX, -1, sizeof(int) * n);
        memset(fY, -1, sizeof(int) * m);
    }
    void add_edge(int x, int y) {
        X[x].push_back(y);
    }
    int solve() { // return how many pair matched
        int cnt = 0;
        for (int i = 0; i < n; ++i) {
            vis.reset();
            cnt += dfs(i);
        }
        return cnt;
    }
};

```

4.2 Dijkstra Cost Flow

```

// kN = #(vertices)
// MCMF.{Init, AddEdge, MincostMaxflow}
// MincostMaxflow(source, sink, flow_limit, &cost)
// => flow
using Pii = pair<int, int>;
constexpr int kInf = 0x3f3f3f3f, kN = 500;
struct Edge {
    int to, rev, cost, flow;
};
struct MCMF { // 0-based
    int n{}, m{}, s{}, t{};
    vector<Edge> graph[kN];
    // Larger range for relabeling
    int64_t dis[kN] = {}, h[kN] = {};
    int p[kN] = {};
    void Init(int nn) {
        n = nn;
        for (int i = 0; i < n; ++i) graph[i].clear();
    }
    void AddEdge(int u, int v, int f, int c) {
        graph[u].push_back({v,
            static_cast<int>(graph[v].size()), c, f});
        graph[v].push_back(
            {u, static_cast<int>(graph[u].size()) - 1,
            -c, 0});
    }
    bool Dijkstra(int &max_flow, int64_t &cost) {
        priority_queue<Pii, vector<Pii>, greater<>> pq;
        fill_n(dis, n, kInf);

```

```

        dis[s] = 0;
        pq.emplace(0, s);
        while (!pq.empty()) {
            auto u = pq.top();
            pq.pop();
            int v = u.second;
            if (dis[v] < u.first) continue;
            for (auto &e : graph[v]) {
                auto new_dis =
                    dis[v] + e.cost + h[v] - h[e.to];
                if (e.flow > 0 && dis[e.to] > new_dis) {
                    dis[e.to] = new_dis;
                    p[e.to] = e.rev;
                    pq.emplace(dis[e.to], e.to);
                }
            }
        }
        if (dis[t] == kInf) return false;
        for (int i = 0; i < n; ++i) h[i] += dis[i];
        int d = max_flow;
        for (int u = t; u != s;
            u = graph[u][p[u]].to) {
            auto &e = graph[u][p[u]];
            d = min(d, graph[e.to][e.rev].flow);
        }
        max_flow -= d;
        cost += int64_t(d) * h[t];
        for (int u = t; u != s;
            u = graph[u][p[u]].to) {
            auto &e = graph[u][p[u]];
            e.flow += d;
            graph[e.to][e.rev].flow -= d;
        }
        return true;
    }
    int MincostMaxflow(
        int ss, int tt, int max_flow, int64_t &cost) {
        this->s = ss, this->t = tt;
        cost = 0;
        fill_n(h, n, 0);
        auto orig_max_flow = max_flow;
        while (Dijkstra(max_flow, cost) && max_flow) {}
        return orig_max_flow - max_flow;
    }
};

```

4.3 Dinic

```

template <typename Cap = int64_t>
class Dinic {
private:
    struct E {
        int to, rev;
        Cap cap;
    };
    int n, st, ed;
    vector<vector<E>> G;
    vector<int> lv, idx;
    bool BFS() {
        lv.assign(n, -1);
        queue<int> bfs;
        bfs.push(st); lv[st] = 0;
        while (not bfs.empty()) {
            int u = bfs.front(); bfs.pop();
            for (auto e : G[u]) {
                if (e.cap <= 0 or lv[e.to] != -1) continue;
                bfs.push(e.to); lv[e.to] = lv[u] + 1;
            }
        }
        return lv[ed] != -1;
    }
    Cap DFS(int u, Cap f) {
        if (u == ed) return f;
        Cap ret = 0;
        for (int &i = idx[u]; i < int(G[u].size()); ++i) {
            auto &e = G[u][i];
            if (e.cap <= 0 or lv[e.to] != lv[u] + 1) continue;
            Cap nf = DFS(e.to, min(f, e.cap));
            ret += nf; e.cap -= nf; f -= nf;
            G[e.to][e.rev].cap += nf;
            if (f == 0) return ret;
        }
        if (ret == 0) lv[u] = -1;
    }

```

```

    return ret;
}
public:
void init(int n_) { G.assign(n = n_, vector<E>()); }
void add_edge(int u, int v, Cap c) {
    G[u].push_back({v, int(G[v].size()), c});
    G[v].push_back({u, int(G[u].size())-1, 0});
}
Cap max_flow(int st_, int ed_) {
    st = st_, ed = ed_; Cap ret = 0;
    while (BFS()) {
        idx.assign(n, 0);
        Cap f = DFS(st, numeric_limits<Cap>::max());
        ret += f;
        if (f == 0) break;
    }
    return ret;
}
};

```

4.4 Flow Models

- Maximum/Minimum flow with lower bound / Circulation problem
 - Construct super source S and sink T .
 - For each edge (x, y, l, u) , connect $x \rightarrow y$ with capacity $u - l$.
 - For each vertex v , denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
 - If $in(v) > 0$, connect $S \rightarrow v$ with capacity $in(v)$, otherwise, connect $v \rightarrow T$ with capacity $-in(v)$.
 - To maximize, connect $t \rightarrow s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T . If $f \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
 - To minimize, let f be the maximum flow from S to T . Connect $t \rightarrow s$ with capacity ∞ and let the flow from S to T be f' . If $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, f' is the answer.
 - The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
 - Redirect every edge: $y \rightarrow x$ if $(x, y) \in M$, $x \rightarrow y$ otherwise.
 - DFS from unmatched vertices in X .
 - $x \in X$ is chosen iff x is unvisited.
 - $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
 - Construct super source S and sink T
 - For each edge (x, y, c) , connect $x \rightarrow y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \rightarrow x$ with $(cost, cap) = (-c, 1)$
 - For each edge with $c < 0$, sum these cost as K , then increase $d(y)$ by 1, decrease $d(x)$ by 1
 - For each vertex v with $d(v) > 0$, connect $S \rightarrow v$ with $(cost, cap) = (0, d(v))$
 - For each vertex v with $d(v) < 0$, connect $v \rightarrow T$ with $(cost, cap) = (0, -d(v))$
 - Flow from S to T , the answer is the cost of the flow $C + K$
- Maximum density induced subgraph
 - Binary search on answer, suppose we're checking answer T
 - Construct a max flow model, let K be the sum of all weights
 - Connect source $s \rightarrow v$, $v \in G$ with capacity K
 - For each edge (u, v, w) in G , connect $u \rightarrow v$ and $v \rightarrow u$ with capacity w
 - For $v \in G$, connect it with sink $v \rightarrow t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
 - T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
 - For each $v \in V$ create a copy v' , and connect $u' \rightarrow v'$ with weight $w(u, v)$.
 - Connect $v \rightarrow v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v .
 - Find the minimum weight perfect matching on G' .
- Project selection problem
 - If $p_v > 0$, create edge (s, v) with capacity p_v ; otherwise, create edge (v, t) with capacity $-p_v$.
 - Create edge (u, v) with capacity w with w being the cost of choosing u without choosing v .
 - The mincut is equivalent to the maximum profit of a subset of projects.
- 0/1 quadratic programming

$$\sum_x c_x x + \sum_y c_y \bar{y} + \sum_{xy} c_{xy} x \bar{y} + \sum_{xyx'y'} c_{xyx'y'} (x \bar{y} + x' \bar{y}')$$

can be minimized by the mincut of the following graph:

- Create edge (x, t) with capacity c_x and create edge (s, y) with capacity c_y .
- Create edge (x, y) with capacity c_{xy} .
- Create edge (x, y) and edge (x', y') with capacity $c_{xyx'y'}$.

4.5 General Graph Matching

```

namespace matching {
int fa[kN], pre[kN], match[kN], s[kN], v[kN];
vector<int> g[kN];
queue<int> q;
void Init(int n) {
    for (int i = 0; i <= n; ++i) match[i] = pre[i] = n;
    for (int i = 0; i < n; ++i) g[i].clear();
}
void AddEdge(int u, int v) {
    g[u].push_back(v);
    g[v].push_back(u);
}
int Find(int u) {
    return u == fa[u] ? u : fa[u] = Find(fa[u]);
}
int LCA(int x, int y, int n) {
    static int tk = 0; tk++;
    x = Find(x), y = Find(y);
    for (; ; swap(x, y)) {
        if (x != n) {
            if (v[x] == tk) return x;
            v[x] = tk;
            x = Find(pre[match[x]]);
        }
    }
}
void Blossom(int x, int y, int l) {
    while (Find(x) != l) {
        pre[x] = y, y = match[x];
        if (s[y] == 1) q.push(y), s[y] = 0;
        if (fa[x] == x) fa[x] = l;
        if (fa[y] == y) fa[y] = l;
        x = pre[y];
    }
}
bool Bfs(int r, int n) {
    for (int i = 0; i <= n; ++i) fa[i] = i, s[i] = -1;
    while (!q.empty()) q.pop();
    q.push(r);
    s[r] = 0;
    while (!q.empty()) {
        int x = q.front(); q.pop();
        for (int u : g[x]) {
            if (s[u] == -1) {
                pre[u] = x, s[u] = 1;
                if (match[u] == n) {
                    for (int a = u, b = x, last; b != n; a = last, b = pre[a])
                        last = match[b], match[b] = a, match[a] = b;
                    return true;
                }
                q.push(match[u]);
                s[match[u]] = 0;
            } else if (!s[u] && Find(u) != Find(x)) {
                int l = LCA(u, x, n);
                Blossom(x, u, l);
                Blossom(u, x, l);
            }
        }
    }
    return false;
}
int Solve(int n) {
    int res = 0;
    for (int x = 0; x < n; ++x) {
        if (match[x] == n) res += Bfs(x, n);
    }
    return res;
}
}

```

4.6 Global Min-Cut

```

const int maxn = 500 + 5;
int w[maxn][maxn], g[maxn];
bool v[maxn], del[maxn];
void add_edge(int x, int y, int c) {
    w[x][y] += c; w[y][x] += c;
}
pair<int, int> phase(int n) {
    memset(v, false, sizeof(v));
    memset(g, 0, sizeof(g));
}

```

```

int s = -1, t = -1;
while (true) {
    int c = -1;
    for (int i = 0; i < n; ++i) {
        if (del[i] || v[i]) continue;
        if (c == -1 || g[i] > g[c]) c = i;
    }
    if (c == -1) break;
    v[s = t, t = c] = true;
    for (int i = 0; i < n; ++i) {
        if (del[i] || v[i]) continue;
        g[i] += w[c][i];
    }
}
return make_pair(s, t);
}

int mincut(int n) {
    int cut = 1e9;
    memset(del, false, sizeof(del));
    for (int i = 0; i < n - 1; ++i) {
        int s, t; tie(s, t) = phase(n);
        del[t] = true; cut = min(cut, g[t]);
        for (int j = 0; j < n; ++j) {
            w[s][j] += w[t][j]; w[j][s] += w[j][t];
        }
    }
    return cut;
}

```

4.7 GomoryHu Tree

```

int g[maxn];
vector<edge> GomoryHu(int n){
    vector<edge> rt;
    for(int i=1;i<=n;++i)g[i]=1;
    for(int i=2;i<=n;++i){
        int t=g[i];
        flow.reset(); // clear flows on all edge
        rt.push_back({i,t,flow(i,t)});
        flow.walk(i); // bfs points that connected to i (use
            edges not fully flow)
        for(int j=i+1;j<=n;++j){
            if(g[j]==t && flow.connect(j))g[j]=i; // check if i
                can reach j
        }
    }
    return rt;
}

```

4.8 Kuhn Munkres

```

class KM {
private:
    static constexpr lld INF = 1LL << 60;
    vector<lld> h1,hr,slk;
    vector<int> fl,fr,pre,qu;
    vector<vector<lld>> w;
    vector<bool> vl,vr;
    int n, ql, qr;
    bool check(int x) {
        if (vl[x] = true, fl[x] != -1)
            return vr[qu[qr++] = fl[x]] = true;
        while (x != -1) swap(x, fr[fl[x] = pre[x]]);
        return false;
    }
    void bfs(int s) {
        fill(slk.begin(), slk.end(), INF);
        fill(vl.begin(), vl.end(), false);
        fill(vr.begin(), vr.end(), false);
        ql = qr = 0;
        vr[qu[qr++] = s] = true;
        while (true) {
            lld d;
            while (ql < qr) {
                for (int x = 0, y = qu[ql++]; x < n; ++x) {
                    if (!vl[x] && slk[x] >= (d = h1[x] + hr[y] - w[x][y])) {
                        if (pre[x] = y, d) slk[x] = d;
                        else if (!check(x)) return;
                    }
                }
            }
            d = INF;
            for (int x = 0; x < n; ++x)
                if (!vl[x] && d > slk[x]) d = slk[x];

```

```

        for (int x = 0; x < n; ++x) {
            if (vl[x]) h1[x] += d;
            else slk[x] -= d;
            if (vr[x]) hr[x] -= d;
        }
        for (int x = 0; x < n; ++x)
            if (!vl[x] && !slk[x] && !check(x)) return;
    }
}

public:
void init( int n_ ) {
    qu.resize(n = n_);
    fl.assign(n, -1); fr.assign(n, -1);
    hr.assign(n, 0); h1.resize(n);
    w.assign(n, vector<lld>(n));
    slk.resize(n); pre.resize(n);
    vl.resize(n); vr.resize(n);
}

void set_edge( int u, int v, lld x ) {w[u][v] = x;}

lld solve() {
    for (int i = 0; i < n; ++i)
        h1[i] = *max_element(w[i].begin(), w[i].end());
    for (int i = 0; i < n; ++i) bfs(i);
    lld res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
}

} km;

```

4.9 Minimum Cost Circulation

```

struct Edge { int to, cap, rev, cost; };
vector<Edge> g[kN];
int dist[kN], pv[kN], ed[kN];
bool mark[kN];
int NegativeCycle(int n) {
    memset(mark, false, sizeof(mark));
    memset(dist, 0, sizeof(dist));
    int upd = -1;
    for (int i = 0; i <= n; ++i) {
        for (int j = 0; j < n; ++j) {
            int idx = 0;
            for (auto &e : g[j]) {
                if (e.cap > 0 && dist[e.to] > dist[j] + e.cost) {
                    dist[e.to] = dist[j] + e.cost;
                    pv[e.to] = j, ed[e.to] = idx;
                    if (i == n) {
                        upd = j;
                        while (!mark[upd]) mark[upd] = 1, upd = pv[upd];
                        return upd;
                    }
                }
            }
            idx++;
        }
    }
    return -1;
}

int Solve(int n) {
    int rt = -1, ans = 0;
    while ((rt = NegativeCycle(n)) >= 0) {
        memset(mark, false, sizeof(mark));
        vector<pair<int, int>> cyc;
        while (!mark[rt]) {
            cyc.emplace_back(pv[rt], ed[rt]);
            mark[rt] = true;
            rt = pv[rt];
        }
        reverse(cyc.begin(), cyc.end());
        int cap = kInf;
        for (auto &i : cyc) {
            auto &e = g[i.first][i.second];
            cap = min(cap, e.cap);
        }
        for (auto &i : cyc) {
            auto &e = g[i.first][i.second];
            e.cap -= cap;
            g[e.to][e.rev].cap += cap;
            ans += e.cost * cap;
        }
    }
    return ans;
}

```

4.10 Minimum Cost Maximum Flow

```
class MiniCostMaxiFlow{
    using Cap = int; using Wei = int64_t;
    using PCW = pair<Cap,Wei>;
    static constexpr Cap INF_CAP = 1 << 30;
    static constexpr Wei INF_WEI = 1LL<<60;
private:
    struct Edge{
        int to, back;
        Cap cap; Wei wei;
        Edge() {}
        Edge(int a,int b, Cap c, Wei d):
            to(a),back(b),cap(c),wei(d) {}
    };
    int ori, edd;
    vector<vector<Edge>> G;
    vector<int> fa, wh;
    vector<bool> inq;
    vector<Wei> dis;
    PCW SPFA(){
        fill(inq.begin(),inq.end(),false);
        fill(dis.begin(),dis.end(),INF_WEI);
        queue<int> qq; qq.push(ori);
        dis[ori] = 0;
        while(not qq.empty()){
            int u=qq.front();qq.pop();
            inq[u] = false;
            for(int i=0;i<SZ(G[u]);++i){
                Edge e=G[u][i];
                int v=e.to; Wei d=e.wei;
                if(e.cap<=0||dis[v]<=dis[u]+d)
                    continue;
                dis[v] = dis[u] + d;
                fa[v] = u, wh[v] = i;
                if (inq[v]) continue;
                qq.push(v);
                inq[v] = true;
            }
        }
        if(dis[edd]==INF_WEI) return {-1, -1};
        Cap mw=INF_CAP;
        for(int i=edd;i!=ori;i=fa[i])
            mw=min(mw,G[fa[i]][wh[i]].cap);
        for (int i=edd;i!=ori;i=fa[i]){
            auto &eg=G[fa[i]][wh[i]];
            eg.cap -= mw;
            G[eg.to][eg.back].cap+=mw;
        }
        return {mw, dis[edd]};
    }
public:
    void init(int n){
        G.clear();G.resize(n);
        fa.resize(n);wh.resize(n);
        inq.resize(n); dis.resize(n);
    }
    void add_edge(int st, int ed, Cap c, Wei w){
        G[st].emplace_back(ed,SZ(G[ed]),c,w);
        G[ed].emplace_back(st,SZ(G[st])-1,0,-w);
    }
    PCW solve(int a, int b){
        ori = a, edd = b;
        Cap cc=0; Wei ww=0;
        while(true){
            PCW ret=SPFA();
            if(ret.first==-1) break;
            cc+=ret.first;
            ww+=ret.first * ret.second;
        }
        return {cc,ww};
    }
} mcmf;
```

4.11 Maximum Weight Graph Matching

```
struct WeightGraph {
    static const int inf = INT_MAX;
    static const int maxn = 514;
    struct edge {
        int u, v, w;
        edge(){}
        edge(int u, int v, int w): u(u), v(v), w(w) {}
    };
};
```

```
};
int n, n_x;
edge g[maxn * 2][maxn * 2];
int lab[maxn * 2];
int match[maxn * 2], slack[maxn * 2], st[maxn * 2], pa
    [maxn * 2];
int flo_from[maxn * 2][maxn + 1], S[maxn * 2], vis[
    maxn * 2];
vector<int> flo[maxn * 2];
queue<int> q;
int e_delta(const edge &e) { return lab[e.u] + lab[e.v]
    - g[e.u][e.v].w * 2; }
void update_slack(int u, int x) { if (!slack[x] ||
    e_delta(g[u][x]) < e_delta(g[slack[x]][x])) slack[x]
    = u; }
void set_slack(int x) {
    slack[x] = 0;
    for (int u = 1; u <= n; ++u)
        if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
            update_slack(u, x);
}
void q_push(int x) {
    if (x <= n) q.push(x);
    else for (size_t i = 0; i < flo[x].size(); i++)
        q.push(flo[x][i]);
}
void set_st(int x, int b) {
    st[x] = b;
    if (x > n) for (size_t i = 0; i < flo[x].size(); ++i)
        set_st(flo[x][i], b);
}
int get_pr(int b, int xr) {
    int pr = find(flo[b].begin(), flo[b].end(), xr) - flo
        [b].begin();
    if (pr % 2 == 1) {
        reverse(flo[b].begin() + 1, flo[b].end());
        return (int)flo[b].size() - pr;
    }
    return pr;
}
void set_match(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;
    edge e = g[u][v];
    int xr = flo_from[u][e.u], pr = get_pr(u, xr);
    for (int i = 0; i < pr; ++i) set_match(flo[u][i], flo
        [u][i ^ 1]);
    set_match(xr, v);
    rotate(flo[u].begin(), flo[u].begin() + pr, flo[u].
        end());
}
void augment(int u, int v) {
    for (; ; ) {
        int xnv = st[match[u]];
        set_match(u, v);
        if (!xnv) return;
        set_match(xnv, st[pa[xnv]]);
        u = st[pa[xnv]], v = xnv;
    }
}
int get_lca(int u, int v) {
    static int t = 0;
    for (++t; u || v; swap(u, v)) {
        if (u == 0) continue;
        if (vis[u] == t) return u;
        vis[u] = t;
        u = st[match[u]];
        if (u) u = st[pa[u]];
    }
    return 0;
}
void add_blossom(int u, int lca, int v) {
    int b = n + 1;
    while (b <= n_x && st[b]) ++b;
    if (b > n_x) ++n_x;
    lab[b] = 0, S[b] = 0;
    match[b] = match[lca];
    flo[b].clear();
    flo[b].push_back(lca);
    for (int x = u, y; x != lca; x = st[pa[y]])
        flo[b].push_back(x), flo[b].push_back(y = st[match[x]
            ]), q_push(y);
}
```

```

reverse(flo[b].begin() + 1, flo[b].end());
for (int x = v, y; x != lca; x = st[pa[y]])
    flo[b].push_back(x), flo[b].push_back(y = st[match[x]]), q_push(y);
set_st(b, b);
for (int x = 1; x <= n_x; ++x) g[b][x].w = g[x][b].w = 0;
for (int x = 1; x <= n; ++x) flo_from[b][x] = 0;
for (size_t i = 0; i < flo[b].size(); ++i) {
    int xs = flo[b][i];
    for (int x = 1; x <= n_x; ++x)
        if (g[b][x].w == 0 || e_delta(g[xs][x]) < e_delta(g[b][x]))
            g[b][x] = g[xs][x], g[x][b] = g[x][xs];
    for (int x = 1; x <= n; ++x)
        if (flo_from[xs][x]) flo_from[b][x] = xs;
}
set_slack(b);
}

void expand_blossom(int b) {
    for (size_t i = 0; i < flo[b].size(); ++i)
        set_st(flo[b][i], flo[b][i]);
    int xr = flo_from[b][g[b][pa[b]].u], pr = get_pr(b, xr);
    for (int i = 0; i < pr; i += 2) {
        int xs = flo[b][i], xns = flo[b][i + 1];
        pa[xs] = g[xns][xs].u;
        S[xs] = 1, S[xns] = 0;
        slack[xs] = 0, set_slack(xns);
        q_push(xns);
    }
    S[xr] = 1, pa[xr] = pa[b];
    for (size_t i = pr + 1; i < flo[b].size(); ++i) {
        int xs = flo[b][i];
        S[xs] = -1, set_slack(xs);
    }
    st[b] = 0;
}

bool on_found_edge(const edge &e) {
    int u = st[e.u], v = st[e.v];
    if (S[v] == -1) {
        pa[v] = e.u, S[v] = 1;
        int nu = st[match[v]];
        slack[v] = slack[nu] = 0;
        S[nu] = 0, q_push(nu);
    } else if (S[v] == 0) {
        int lca = get_lca(u, v);
        if (!lca) return augment(u, v), augment(v, u), true;
        else add_blossom(u, lca, v);
    }
    return false;
}

bool matching() {
    memset(S + 1, -1, sizeof(int) * n_x);
    memset(slack + 1, 0, sizeof(int) * n_x);
    q = queue<int>();
    for (int x = 1; x <= n_x; ++x)
        if (st[x] == x && !match[x]) pa[x] = 0, S[x] = 0, q_push(x);
    if (q.empty()) return false;
    for (; ; ) {
        while (q.size()) {
            int u = q.front(); q.pop();
            if (S[st[u]] == 1) continue;
            for (int v = 1; v <= n; ++v)
                if (g[u][v].w > 0 && st[u] != st[v]) {
                    if (e_delta(g[u][v]) == 0) {
                        if (on_found_edge(g[u][v])) return true;
                    } else update_slack(u, st[v]);
                }
        }
        int d = inf;
        for (int b = n + 1; b <= n_x; ++b)
            if (st[b] == b && S[b] == 1) d = min(d, lab[b] / 2);
        for (int x = 1; x <= n_x; ++x)
            if (st[x] == x && slack[x]) {
                if (S[x] == -1) d = min(d, e_delta(g[slack[x]][x]));
                else if (S[x] == 0) d = min(d, e_delta(g[slack[x]][x]) / 2);
            }
    }
}

```

```

for (int u = 1; u <= n; ++u) {
    if (S[st[u]] == 0) {
        if (lab[u] <= d) return 0;
        lab[u] -= d;
    } else if (S[st[u]] == 1) lab[u] += d;
}
for (int b = n + 1; b <= n_x; ++b)
    if (st[b] == b) {
        if (S[st[b]] == 0) lab[b] += d * 2;
        else if (S[st[b]] == 1) lab[b] -= d * 2;
    }
q = queue<int>();
for (int x = 1; x <= n_x; ++x)
    if (st[x] == x && slack[x] && st[slack[x]] != x && e_delta(g[slack[x]][x]) == 0)
        if (on_found_edge(g[slack[x]][x])) return true;
for (int b = n + 1; b <= n_x; ++b)
    if (st[b] == b && S[b] == 1 && lab[b] == 0)
        expand_blossom(b);
}
return false;
}

pair<long long, int> solve() {
    memset(match + 1, 0, sizeof(int) * n);
    n_x = n;
    int n_matches = 0;
    long long tot_weight = 0;
    for (int u = 0; u <= n; ++u) st[u] = u, flo[u].clear();
    int w_max = 0;
    for (int u = 1; u <= n; ++u)
        for (int v = 1; v <= n; ++v) {
            flo_from[u][v] = (u == v ? u : 0);
            w_max = max(w_max, g[u][v].w);
        }
    for (int u = 1; u <= n; ++u) lab[u] = w_max;
    while (matching()) ++n_matches;
    for (int u = 1; u <= n; ++u)
        if (match[u] && match[u] < u)
            tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
}

void add_edge(int ui, int vi, int wi) { g[ui][vi].w = g[vi][ui].w = wi; }
void init(int _n) {
    n = _n;
    for (int u = 1; u <= n; ++u)
        for (int v = 1; v <= n; ++v)
            g[u][v] = edge(u, v, 0);
}
}

```

5 Math

5.1 $\lfloor \frac{n}{i} \rfloor$ Enumeration

$$T_0 = 1, T_{i+1} = \lfloor \frac{n}{T_i + 1} \rfloor$$

5.2 Stirling Number

5.2.1 First Kind

$S_1(n, k)$ counts the number of permutations of n elements with k disjoint cycles.

$$S_1(n, k) = (n-1) \cdot S_1(n-1, k) + S_1(n-1, k-1)$$

$$x(x+1) \dots (x+n-1) = \sum_{k=0}^n S_1(n, k) x^k$$

$$g(x) = x(x+1) \dots (x+n-1) = \sum_{k=0}^n a_k x^k$$

$$\Rightarrow g(x+n) = \sum_{k=0}^n \frac{b_k}{(n-k)!} x^{n-k},$$

$$b_k = \sum_{i=0}^k ((n-i)! a_{n-i}) \cdot \left(\frac{n^{k-i}}{(k-i)!} \right)$$

5.2.2 Second Kind

$S_2(n, k)$ counts the number of ways to partition a set of n elements into k nonempty sets.

$$S_2(n, k) = S_2(n-1, k-1) + k \cdot S_2(n-1, k)$$

$$S_2(n, k) = \sum_{i=0}^k \binom{k}{i} i^n (-1)^{k-i} = \sum_{i=0}^k \frac{(-1)^i}{i!} \cdot \frac{(k-i)^n}{(k-i)!}$$

5.3 ax+by=gcd

```
// ax+ny = 1, ax+ny == ax == 1 (mod n)
void exgcd(lld x, lld y, lld &g, lld &a, lld &b) {
    if (y == 0) g=x, a=1, b=0;
    else exgcd(y, x%y, g, b, a), b-=(x/y)*a;
}
```

5.4 Berlekamp Massey

```
template <typename T>
vector<T> BerlekampMassey(const vector<T> &output) {
    vector<T> d(output.size() + 1), me, he;
    for (size_t f = 0, i = 1; i <= output.size(); ++i) {
        for (size_t j = 0; j < me.size(); ++j)
            d[i] += output[i - j - 2] * me[j];
        if ((d[i] -= output[i - 1]) == 0) continue;
        if (me.empty()) {
            me.resize(f = i);
            continue;
        }
        vector<T> o(i - f - 1);
        T k = -d[i] / d[f]; o.push_back(-k);
        for (T x : he) o.push_back(x * k);
        if (o.size() < me.size()) o.resize(me.size());
        for (size_t j = 0; j < me.size(); ++j) o[j] += me[j];
        if (i-f+he.size() >= me.size()) he = me, f = i;
        me = o;
    }
    return me;
}
```

5.5 Characteristic Polynomial

```
vector<vector<int>> Hessenberg(const vector<vector<int>> &A) {
    int N = A.size();
    vector<vector<int>> H = A;
    for (int i = 0; i < N - 2; ++i) {
        if (!H[i + 1][i]) {
            for (int j = i + 2; j < N; ++j) {
                if (H[j][i]) {
                    for (int k = i; k < N; ++k) swap(H[i + 1][k], H[j][k]);
                    for (int k = 0; k < N; ++k) swap(H[k][i + 1], H[k][j]);
                    break;
                }
            }
        }
        if (!H[i + 1][i]) continue;
        int val = fpow(H[i + 1][i], kP - 2);
        for (int j = i + 2; j < N; ++j) {
            int coef = 1LL * val * H[j][i] % kP;
            for (int k = i; k < N; ++k) H[j][k] = (H[j][k] + 1LL * H[i + 1][k] * (kP - coef)) % kP;
            for (int k = 0; k < N; ++k) H[k][i + 1] = (H[k][i + 1] + 1LL * H[k][j] * coef) % kP;
        }
    }
    return H;
}

vector<int> CharacteristicPoly(const vector<vector<int>> &A) {
    int N = A.size();
    auto H = Hessenberg(A);
    for (int i = 0; i < N; ++i) {
        for (int j = 0; j < N; ++j) H[i][j] = kP - H[i][j];
    }
    vector<vector<int>> P(N + 1, vector<int>(N + 1));
    P[0][0] = 1;
    for (int i = 1; i <= N; ++i) {
        P[i][0] = 0;
        for (int j = 1; j <= i; ++j) P[i][j] = P[i - 1][j - 1];
        int val = 1;
        for (int j = i - 1; j >= 0; --j) {
            int coef = 1LL * val * H[j][i - 1] % kP;
            for (int k = 0; k <= j; ++k) P[i][k] = (P[i][k] + 1LL * P[j][k] * coef) % kP;
        }
    }
}
```

```
if (j) val = 1LL * val * (kP - H[j][j - 1]) % kP;
}
}
if (N & 1) {
    for (int i = 0; i <= N; ++i) P[N][i] = kP - P[N][i];
}
return P[N];
}
```

5.6 Chinese Remainder

```
lld crt(lld ans[], lld pri[], int n){
    lld M = 1, ret = 0;
    for(int i=0;i<n;i++) M *= pri[i];
    for(int i=0;i<n;i++){
        lld iv = (gcd(M/pri[i], pri[i]).FF+pri[i])%pri[i];
        ret += (ans[i]*(M/pri[i])%M * iv)%M;
        ret %= M;
    }
    return ret;
}
/*
Another:
x = a1 % m1
x = a2 % m2
g = gcd(m1, m2)
assert((a1-a2)%g==0)
[p, q] = exgcd(m2/g, m1/g)
return a2+m2*(p*(a1-a2)/g)
0 <= x < lcm(m1, m2)
*/
```

5.7 De-Bruijn

```
int res[maxn], aux[maxn], sz;
void db(int t, int p, int n, int k) {
    if (t > n) {
        if (n % p == 0)
            for (int i = 1; i <= p; ++i)
                res[sz++] = aux[i];
    } else {
        aux[t] = aux[t - p];
        db(t + 1, p, n, k);
        for (int i = aux[t - p] + 1; i < k; ++i) {
            aux[t] = i;
            db(t + 1, t, n, k);
        }
    }
}

int de_bruijn(int k, int n) {
    // return cyclic string of len k^n s.t. every string
    // of len n using k char appears as a substring.
    if (k == 1) {
        res[0] = 0;
        return 1;
    }
    for (int i = 0; i < k * n; i++) aux[i] = 0;
    sz = 0;
    db(1, 1, n, k);
    return sz;
}
```

5.8 DiscreteLog

```
template<typename Int>
Int BSGS(Int x, Int y, Int M) {
    // x^? \equiv y (mod M)
    Int t = 1, c = 0, g = 1;
    for (Int M_ = M; M_ > 0; M_ >= 1)
        g = g * x % M;
    for (g = gcd(g, M); t % g != 0; ++c) {
        if (t == y) return c;
        t = t * x % M;
    }
    if (y % g != 0) return -1;
    t /= g, y /= g, M /= g;
    Int h = 0, gs = 1;
    for (; h * h < M; ++h) gs = gs * x % M;
    unordered_map<Int, Int> bs;
    for (Int s = 0; s < h; bs[y] = ++s)
        y = y * x % M;
    for (Int s = 0; s < M; s += h) {
        t = t * gs % M;
        if (bs.count(t)) return c + s + h - bs[t];
    }
}
```

```

    }
    return -1;
}

```

5.9 Extended Euler

$$a^b \equiv \begin{cases} a^{b \bmod \varphi(m) + \varphi(m)} & \text{if } (a, m) \neq 1 \wedge b \geq \varphi(m) \\ a^{b \bmod \varphi(m)} & \text{otherwise} \end{cases} \pmod{m}$$

5.10 ExtendedFloorSum

$$g(a, b, c, n) = \sum_{i=0}^n i \left\lfloor \frac{ai+b}{c} \right\rfloor$$

$$= \begin{cases} \left\lfloor \frac{a}{c} \right\rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \left\lfloor \frac{b}{c} \right\rfloor \cdot \frac{n(n+1)}{2} + g(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1) - h(c, c-b-1, a, m-1)), & \text{otherwise} \end{cases}$$

$$h(a, b, c, n) = \sum_{i=0}^n \left\lfloor \frac{ai+b}{c} \right\rfloor^2$$

$$= \begin{cases} \left\lfloor \frac{a}{c} \right\rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \left\lfloor \frac{b}{c} \right\rfloor^2 \cdot (n+1) + \left\lfloor \frac{a}{c} \right\rfloor \cdot \left\lfloor \frac{b}{c} \right\rfloor \cdot n(n+1) + h(a \bmod c, b \bmod c, c, n) + 2 \left\lfloor \frac{a}{c} \right\rfloor \cdot g(a \bmod c, b \bmod c, c, n) + 2 \left\lfloor \frac{b}{c} \right\rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) - 2f(c, c-b-1, a, m-1) - f(a, b, c, n), & \text{otherwise} \end{cases}$$

5.11 Fast Fourier Transform

```

const int mod = 1000000007;
const int M1 = 985661441; // G = 3
const int M2 = 998244353;
const int M3 = 1004535809;
int superBigCRT(int64_t A, int64_t B, int64_t C) {
    static_assert(M1 <= M2 && M2 <= M3);
    constexpr int64_t r12 = modpow(M1, M2-2, M2);
    constexpr int64_t r13 = modpow(M1, M3-2, M3);
    constexpr int64_t r23 = modpow(M2, M3-2, M3);
    constexpr int64_t M1M2 = 1LL * M1 * M2 % mod;
    B = (B - A + M2) * r12 % M2;
    C = (C - A + M3) * r13 % M3;
    C = (C - B + M3) * r23 % M3;
    return (A + B * M1 + C * M1M2) % mod;
}

namespace fft {
using VI = vector<int>;
using VL = vector<long long>;
const double pi = acos(-1);
cplx omega[maxn + 1];
void prefft() {
    for (int i = 0; i <= maxn; i++)
        omega[i] = cplx(cos(2 * pi * i / maxn),
            sin(2 * pi * i / maxn));
}

void fft(vector<cplx> &v, int n) {
    int z = __builtin_ctz(n) - 1;
    for (int i = 0; i < n; ++i) {
        int x = 0, j = 0;
        for (; (1 <= j) < n; ++j) x ^= (i >> j & 1) << (z - j);
        if (x > i) swap(v[x], v[i]);
    }
    for (int s = 2; s <= n; s <= 1) {
        int z = s >> 1;
        for (int i = 0; i < n; i += s) {
            for (int k = 0; k < z; ++k) {
                cplx x = v[i + z + k] * omega[maxn / s * k];
                v[i + z + k] = v[i + k] - x;
                v[i + k] = v[i + k] + x;
            }
        }
    }
}

void ifft(vector<cplx> &v, int n) {
    fft(v, n); reverse(v.begin() + 1, v.end());
    for (int i = 0; i < n; ++i) v[i] = v[i] * cplx(1. / n, 0);
}

VL convolution(const VI &a, const VI &b) {
    // Should be able to handle N <= 10^5, C <= 10^4
}

```

```

int sz = 1;
while (sz < a.size() + b.size() - 1) sz <= 1;
vector<cplx> v(sz);
for (int i = 0; i < sz; ++i) {
    double re = i < a.size() ? a[i] : 0;
    double im = i < b.size() ? b[i] : 0;
    v[i] = cplx(re, im);
}
fft(v, sz);
for (int i = 0; i <= sz / 2; ++i) {
    int j = (sz - i) & (sz - 1);
    cplx x = (v[i] + v[j].conj()) * (v[i] - v[j].conj())
        * cplx(0, -0.25);
    if (j != i) v[j] = (v[j] + v[i].conj()) * (v[j] - v[i]
        ).conj() * cplx(0, -0.25);
    v[i] = x;
}
ifft(v, sz);
VL c(sz);
for (int i = 0; i < sz; ++i) c[i] = round(v[i].re);
return c;
}

VI convolution_mod(const VI &a, const VI &b, int p) {
    int sz = 1;
    while (sz + 1 < a.size() + b.size()) sz <= 1;
    vector<cplx> fa(sz), fb(sz);
    for (int i = 0; i < (int)a.size(); ++i)
        fa[i] = cplx(a[i] & ((1 <= 15) - 1), a[i] >> 15);
    for (int i = 0; i < (int)b.size(); ++i)
        fb[i] = cplx(b[i] & ((1 <= 15) - 1), b[i] >> 15);
    fft(fa, sz), fft(fb, sz);
    double r = 0.25 / sz;
    cplx r2(0, -1), r3(r, 0), r4(0, -r), r5(0, 1);
    for (int i = 0; i <= (sz >> 1); ++i) {
        int j = (sz - i) & (sz - 1);
        cplx a1 = (fa[i] + fa[j].conj());
        cplx a2 = (fa[i] - fa[j].conj()) * r2;
        cplx b1 = (fb[i] + fb[j].conj()) * r3;
        cplx b2 = (fb[i] - fb[j].conj()) * r4;
        if (i != j) {
            cplx c1 = (fa[j] + fa[i].conj());
            cplx c2 = (fa[j] - fa[i].conj()) * r2;
            cplx d1 = (fb[j] + fb[i].conj()) * r3;
            cplx d2 = (fb[j] - fb[i].conj()) * r4;
            fa[i] = c1 * d1 + c2 * d2 * r5;
            fb[i] = c1 * d2 + c2 * d1;
        }
        fa[j] = a1 * b1 + a2 * b2 * r5;
        fb[j] = a1 * b2 + a2 * b1;
    }
    fft(fa, sz), fft(fb, sz);
    vector<int> res(sz);
    for (int i = 0; i < sz; ++i) {
        long long a = round(fa[i].re), b = round(fb[i].re),
            c = round(fa[i].im);
        res[i] = (a + ((b % p) << 15) + ((c % p) << 30)) % p;
    }
    return res;
}

```

5.12 FloorSum

```

// @param n `n < 2^32`
// @param m `1 <= m < 2^32`
// @return sum_{i=0}^{n-1} floor((ai + b)/m) mod 2^64
llu floor_sum_unsigned(llu n, llu m, llu a, llu b) {
    llu ans = 0;
    while (true) {
        if (a >= m) {
            ans += n * (n - 1) / 2 * (a / m); a %= m;
        }
        if (b >= m) {
            ans += n * (b / m); b %= m;
        }
        llu y_max = a * n + b;
        if (y_max < m) break;
        // y_max < m * (n + 1)
        // floor(y_max / m) <= n
        n = (llu)(y_max / m), b = (llu)(y_max % m);
        swap(m, a);
    }
    return ans;
}

```

```

lld floor_sum(lld n, lld m, lld a, lld b) {
    llu ans = 0;
    if (a < 0) {
        llu a2 = (a % m + m) % m;
        ans -= 1ULL * n * (n - 1) / 2 * ((a2 - a) / m);
        a = a2;
    }
    if (b < 0) {
        llu b2 = (b % m + m) % m;
        ans -= 1ULL * n * ((b2 - b) / m);
        b = b2;
    }
    return ans + floor_sum_unsigned(n, m, a, b);
}

```

5.13 FWT

```

/* xor convolution:
 * x = (x0,x1) , y = (y0,y1)
 * z = ( x0y0 + x1y1 , x0y1 + x1y0 )
 * ==>
 * x' = ( x0+x1 , x0-x1 ) , y' = ( y0+y1 , y0-y1 )
 * z' = ( ( x0+x1 )( y0+y1 ) , ( x0-x1 )( y0-y1 ) )
 * z = (1/2) * z'
 * or convolution:
 * x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
 * and convolution:
 * x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
const LL MOD = 1e9+7;
inline void fwt( LL x[ MAXN ] , int N , bool inv=0 ) {
    for( int d = 1 ; d < N ; d <= 1 ) {
        int d2 = d<<1;
        for( int s = 0 ; s < N ; s += d2 )
            for( int i = s , j = s+d ; i < s+d ; i++, j++ ) {
                LL ta = x[ i ] , tb = x[ j ];
                x[ i ] = ta+tb;
                x[ j ] = ta-tb;
                if( x[ i ] >= MOD ) x[ i ] -= MOD;
                if( x[ j ] < 0 ) x[ j ] += MOD;
            }
    }
    if( inv )
        for( int i = 0 ; i < N ; i++ ) {
            x[ i ] *= inv( N, MOD );
            x[ i ] %= MOD;
        }
}

```

5.14 Gauss Elimination

```

void gauss(vector<vector<double>> &d) {
    int n = d.size(), m = d[0].size();
    for (int i = 0; i < m; ++i) {
        int p = -1;
        for (int j = i; j < n; ++j) {
            if (fabs(d[j][i]) < eps) continue;
            if (p == -1 || fabs(d[j][i]) > fabs(d[p][i])) p = j;
        }
        if (p == -1) continue;
        for (int j = 0; j < m; ++j) swap(d[p][j], d[i][j]);
        for (int j = 0; j < n; ++j) {
            if (i == j) continue;
            double z = d[j][i] / d[i][i];
            for (int k = 0; k < m; ++k) d[j][k] -= z*d[i][k];
        }
    }
}

```

5.15 Miller Rabin

```

bool isprime(llu x){
    static llu magic[]={2,325,9375,28178,\
        450775,9780504,1795265022};
    static auto witn=[](llu a,llu u,llu n,int t)
    ->bool{
        if (!(a = mpow(a%n,u,n)))return 0;
        while(t--){
            llu a2=mul(a,a,n);
            if(a2==1 && a!=1 && a!=n-1)
                return 1;
            a = a2;
        }
        return a!=1;
    };
}

```

```

if(x<2)return 0;
if(!(x&1))return x==2;
llu x1=x-1;int t=0;
while(!(x1&1))x1>>=1,t++;
for(llu m:magic)if(witn(m,x1,x,t))return 0;
return 1;
}

```

5.16 NTT

```

template <int mod, int G, int maxn>
struct NTT {
    static_assert (maxn == (maxn & -maxn));
    int roots[maxn];
    NTT () {
        int r = modpow(G, (mod - 1) / maxn);
        for (int i = maxn >> 1; i; i >>= 1) {
            roots[i] = 1;
            for (int j = 1; j < i; j++)
                roots[i + j] = modmul(roots[i + j - 1], r);
            r = modmul(r, r);
        }
    }
    // n must be 2^k, and 0 <= F[i] < mod
    void operator()(int F[], int n, bool inv = false) {
        for (int i = 0, j = 0; i < n; i++) {
            if (i < j) swap(F[i], F[j]);
            for (int k = n>>1; (j^=k) < k; k>>=1);
        }
        for (int s = 1; s < n; s *= 2) {
            for (int i = 0; i < n; i += s * 2) {
                for (int j = 0; j < s; j++) {
                    int a = F[i+j];
                    int b = modmul(F[i+j+s], roots[s+j]);
                    F[i+j] = modadd(a, b); // a + b
                    F[i+j+s] = modsub(a, b); // a - b
                }
            }
        }
        if (inv) {
            int invn = modinv(n);
            for (int i = 0; i < n; i++)
                F[i] = modmul(F[i], invn);
            reverse(F + 1, F + n);
        }
    }
};
NTT<2013265921, 31, 1048576> ntt;

```

5.17 Range Sieve

```

const int MAX_SQRT_B = 50000;
const int MAX_L = 200000 + 5;
bool is_prime_small[MAX_SQRT_B], is_prime[MAX_L];
void sieve(lld l, lld r){ // [l, r)
    for(lld i=2;i<r;i++) is_prime_small[i] = true;
    for(lld i=l;i<r;i++) is_prime[i-l] = true;
    if(l==1) is_prime[0] = false;
    for(lld i=2;i<r;i++){
        if(!is_prime_small[i]) continue;
        for(lld j=i*i;j<r;j+=i) is_prime_small[j]=false;
        for(lld j=std::max(2LL, (l+i-1)/i)*i;j<r;j+=i)
            is_prime[j-l]=false;
    }
}

```

5.18 Partition Number

```

int b = sqrt(n);
ans[0] = tmp[0] = 1;
for (int i = 1; i <= b; i++) {
    for (int rep = 0; rep < 2; rep++)
        for (int j = i; j <= n - i * i; j++)
            modadd(tmp[j], tmp[j-i]);
    for (int j = i * i; j <= n; j++)
        modadd(ans[j], tmp[j - i * i]);
}

```

5.19 Pi Count (Linear Sieve)

```

static constexpr int N = 1000000 + 5;
lld pi[N];
vector<int> primes;
bool sieved[N];
lld cube_root(lld x){

```

```

lld s=cbrt(x-static_cast<long double>(0.1));
while(s*s*s <= x) ++s;
return s-1;
}
lld square_root(lld x){
lld s=sqrt(x-static_cast<long double>(0.1));
while(s*s <= x) ++s;
return s-1;
}
void init(){
primes.reserve(N);
primes.push_back(1);
for(int i=2;i<N;i++){
if(!sieved[i]) primes.push_back(i);
pi[i] = !sieved[i] + pi[i-1];
for(int p: primes) if(p > 1) {
if(p * i >= N) break;
sieved[p * i] = true;
if(p % i == 0) break;
}
}
}
lld phi(lld m, lld n) {
static constexpr int MM = 80000, NN = 500;
static lld val[MM][NN];
if(m<MM&&n<NN&&val[m][n])return val[m][n]-1;
if(n == 0) return m;
if(primes[n] >= m) return 1;
lld ret = phi(m,n-1)-phi(m/primes[n],n-1);
if(m<MM&&n<NN) val[m][n] = ret+1;
return ret;
}
lld pi_count(lld);
lld P2(lld m, lld n) {
lld sm = square_root(m), ret = 0;
for(lld i = n+1;primes[i]<=sm;i++){
ret+=pi_count(m/primes[i])-pi_count(primes[i])+1;
return ret;
}
}
lld pi_count(lld m) {
if(m < N) return pi[m];
lld n = pi_count(cube_root(m));
return phi(m, n) + n - 1 - P2(m, n);
}

```

5.20 Pollard Rho

```

// does not work when n is prime
// return any non-trivial factor
llu pollard_rho(llu n){
static auto f=[](llu x,llu k,llu m){
return add(k,mul(x,x,m),m);};
if (!(n&1)) return 2;
mt19937 rnd(120821011);
while(true){
llu y=2,yy=y,x=rnd()%n,t=1;
for(llu sz=2;t==1;sz<=1,y=yy) {
for(llu i=0;t==1&&i<sz;++i){
yy=f(yy,x,n);
t=gcd(yy>y?yy-y:y-yy,n);
}
}
if(t!=1&&t!=n) return t;
}
}

```

5.21 Polynomial Operations

```

using V = vector<int>;
#define fi(s, n) for (int i = int(s); i < int(n); ++i)
template <int mod, int G, int maxn> struct Poly : V {
static uint32_t n2k(uint32_t n) {
if (n <= 1) return 1;
return 1u << (32 - __builtin_clz(n - 1));
}
static NTT<mod,G,maxn> ntt; // coefficients in [0, P)
explicit Poly(int n = 1) : V(n) {}
Poly(const V &v) : V(v) {}
Poly(const Poly &p, size_t n) : V(n) {
copy_n(p.data(), min(p.size(), n), data());
}
Poly &irev() { return reverse(data(), data() + size()), *this; }
Poly &isz(int _n) { return resize(_n), *this; }

```

```

Poly &iadd(const Poly &rhs) { // n() == rhs.n()
fi(0, size())(*this)[i] = modadd((*this)[i], rhs[i]);
return *this;
}
Poly &imul(int k) {
fi(0, size())(*this)[i] = modmul((*this)[i], k);
return *this;
}
Poly Mul(const Poly &rhs) const {
const int _n = n2k(size() + rhs.size() - 1);
Poly X(*this, _n), Y(rhs, _n);
ntt(X.data(), _n), ntt(Y.data(), _n);
fi(0, _n) X[i] = modmul(X[i], Y[i]);
ntt(X.data(), _n, true);
return X.isz(size() + rhs.size() - 1);
}
Poly Inv() const { // coef[0] != 0
if (size() == 1) return V{modinv(*begin())};
const int _n = n2k(size() * 2);
Poly Xi = Poly(*this, (size() + 1) / 2).Inv().isz(_n),
Y(*this, _n);
ntt(Xi.data(), _n), ntt(Y.data(), _n);
fi(0, _n) Xi[i] = modmul(Xi[i], modsub(2, modmul(Xi[i], Y[i])));
ntt(Xi.data(), _n, true);
return Xi.isz(size());
}
Poly Sqrt() const { // coef[0] \in [1, mod)^2
if (size() == 1) return V{QuadraticResidue(*this)[0], mod};
Poly X = Poly(*this, (size() + 1) / 2).Sqrt().isz(size());
return X.iadd(Mul(X.Inv()).isz(size())).imul(mod / 2 + 1);
}
pair<Poly, Poly> DivMod(const Poly &rhs) const {
if (size() < rhs.size()) return {V{0}, *this};
const int _n = size() - rhs.size() + 1;
Poly X(rhs); X.irev().isz(_n);
Poly Y(*this); Y.irev().isz(_n);
Poly Q = Y.Mul(X.Inv()).isz(_n).irev();
X = rhs.Mul(Q), Y = *this;
fi(0, size()) Y[i] = modsub(Y[i], X[i]);
return {Q, Y.isz(max<int>(1, rhs.size() - 1))};
}
Poly Dx() const {
Poly ret(size() - 1);
fi(0, ret.size()) ret[i] = modmul(i + 1, (*this)[i + 1]);
return ret.isz(max<int>(1, ret.size()));
}
Poly Sx() const {
Poly ret(size() + 1);
fi(0, size()) ret[i + 1] = modmul(modinv(i + 1), (*this)[i]);
return ret;
}
Poly _tmul(int nn, const Poly &rhs) const {
Poly Y = Mul(rhs).isz(size() + nn - 1);
return V(Y.data() + size() - 1, Y.data() + Y.size());
}
V _eval(const V &x, const vector<Poly> &up) const {
const int _n = (int)x.size();
if (!_n) return {};
vector<Poly> down(_n * 2);
// down[1] = DivMod(up[1]).second;
// fi(2, _n * 2) down[i] = down[i / 2].DivMod(up[i]).second;
down[1] = Poly(up[1]).irev().isz(size()).Inv().irev()._tmul(_n, *this);
fi(2, _n * 2) down[i] = up[i ^ 1]._tmul(up[i].size() - 1, down[i / 2]);
V y(_n); fi(0, _n) y[i] = down[_n + i][0];
return y;
}
static vector<Poly> _tree1(const V &x) {
const int _n = (int)x.size();
vector<Poly> up(_n * 2);
fi(0, _n) up[_n + i] = V{modsub(mod, x[i]), 1};
for(int i=_n-1;i>0;--i) up[i] = up[i * 2].Mul(up[i * 2 + 1]);
return up;
}

```

```

}
V Eval(const V &x) const { return _eval(x, _tree1(x));
}
static Poly Interpolate(const V &x, const V &y) {
    const int _n = (int)x.size();
    vector<Poly> up = _tree1(x), down(_n * 2);
    V z = up[1].Dx()._eval(x, up);
    fi(0, _n) z[i] = modmul(y[i], modinv(z[i]));
    fi(0, _n) down[_n + i] = V{z[i]};
    for(int i=_n-1; i>0; --i) down[i] = down[i * 2].Mul(up[
        i * 2 + 1]).iadd(down[i * 2 + 1].Mul(up[i * 2]));
    return down[1];
}
Poly Ln() const { // coef[0] == 1
    return Dx().Mul(Inv()).Sx().isz(size());
}
Poly Exp() const { // coef[0] == 0
    if (size() == 1) return V{1};
    Poly X = Poly(*this, (size() + 1) / 2).Exp().isz(size
        ());
    Poly Y = X.Ln(); Y[0] = mod - 1;
    fi(0, size()) Y[i] = modsub((*this)[i], Y[i]);
    return X.Mul(Y).isz(size());
}
Poly Pow(const string &K) const {
    int nz = 0;
    while (nz < size() && !(*this)[nz]) ++nz;
    int nk = 0, nk2 = 0;
    for (char c : K) {
        nk = (nk * 10 + c - '0') % mod;
        nk2 = nk2 * 10 + c - '0';
        if (nk2 * nz >= size())
            return Poly(size());
        nk2 %= mod - 1;
    }
    if (!nk && !nk2) return Poly(V{1}, size());
    Poly X = V(data() + nz, data() + size() - nz * (nk2 -
        1));
    int x0 = X[0];
    return X.imul(modinv(x0)).Ln().imul(nk).Exp().imul(
        modpow(x0, nk2)).irev().isz(size()).irev();
}
Poly InvMod(int L) { // (to evaluate linear recursion)
    Poly R{1, 0}; // *this * R mod x^L = 1 (*this[0] ==
        1)
    for (int level = 0; (1 << level) < L; ++level) {
        Poly O = R.Mul(Poly(data(), min<int>(2 << level,
            size())));
        Poly Q(2 << level); Q[0] = 1;
        for (int j = (1 << level); j < (2 << level); ++j)
            Q[j] = modsub(mod, O[j]);
        R = R.Mul(Q).isz(4 << level);
    }
    return R.isz(L);
}
static int LinearRecursion(const V &a, const V &c,
    int64_t n) { // a_n = \sum c_j a_{n-j}
    const int k = (int)a.size();
    assert((int)c.size() == k + 1);
    Poly C(k + 1), W({1}, k), M = {0, 1};
    fi(1, k + 1) C[k - i] = modsub(mod, c[i]);
    C[k] = 1;
    while (n) {
        if (n % 2) W = W.Mul(M).DivMod(C).second;
        n /= 2, M = M.Mul(M).DivMod(C).second;
    }
    int ret = 0;
    fi(0, k) ret = modadd(ret, modmul(W[i], a[i]));
    return ret;
}
}
#undef fi
using Poly_t = Poly<998244353, 3, 1 << 20>;
template <> decltype(Poly_t::ntt) Poly_t::ntt = {};

```

5.22 Quadratic residue

```

struct S {
    int MOD, w;
    int64_t x, y;
    S(int m, int w=-1, int64_t x=1, int64_t y=0)
        : MOD(m), w(w), x(x), y(y) {}
    S operator*(const S &rhs) const {

```

```

    int w_ = w;
    if (w_ == -1) w_ = rhs.w;
    assert(w_ != -1 and w_ == rhs.w);
    return { MOD, w_,
        (x * rhs.x + y * rhs.y % MOD * w) % MOD,
        (x * rhs.y + y * rhs.x) % MOD };
};
int get_root(int n, int P) {
    if (P == 2 or n == 0) return n;
    if (qpow(n, (P - 1) / 2, P) != 1) return -1;
    auto check = [&](int x) {
        return qpow(x, (P - 1) / 2, P);
    };
    if (check(n) == P-1) return -1;
    int64_t a; int w; mt19937 rnd(7122);
    do { a = rnd() % P;
        w = ((a * a - n) % P + P) % P;
    } while (check(w) != P - 1);
    return qpow(S(P, w, a, 1), (P + 1) / 2).x;
}

```

5.23 Simplex

```

namespace simplex {
    // maximize c^T x under Ax <= B
    // return VD(n, -inf) if the solution doesn't exist
    // return VD(n, +inf) if the solution is unbounded
    using VD = vector<double>;
    using VVD = vector<vector<double>>;
    const double eps = 1e-9;
    const double inf = 1e+9;
    int n, m;
    VVD d;
    vector<int> p, q;
    void pivot(int r, int s) {
        double inv = 1.0 / d[r][s];
        for (int i = 0; i < m + 2; ++i)
            for (int j = 0; j < n + 2; ++j)
                if (i != r && j != s)
                    d[i][j] -= d[r][j] * d[i][s] * inv;
        for(int i=0;i<m+2;++i) if (i != r) d[i][s] *= -inv;
        for(int j=0;j<n+2;++j) if (j != s) d[r][j] *= +inv;
        d[r][s] = inv; swap(p[r], q[s]);
    }
    bool phase(int z) {
        int x = m + z;
        while (true) {
            int s = -1;
            for (int i = 0; i <= n; ++i) {
                if (!z && q[i] == -1) continue;
                if (s == -1 || d[x][i] < d[x][s]) s = i;
            }
            if (d[x][s] > -eps) return true;
            int r = -1;
            for (int i = 0; i < m; ++i) {
                if (d[i][s] < eps) continue;
                if (r == -1 || \
                    d[i][n+1]/d[i][s] < d[r][n+1]/d[r][s]) r = i;
            }
            if (r == -1) return false;
            pivot(r, s);
        }
    }
    VD solve(const VVD &a, const VD &b, const VD &c) {
        m = b.size(), n = c.size();
        d = VVD(m + 2, VD(n + 2));
        for (int i = 0; i < m; ++i)
            for (int j = 0; j < n; ++j) d[i][j] = a[i][j];
        p.resize(m), q.resize(n + 1);
        for (int i = 0; i < m; ++i)
            p[i] = n + i, d[i][n] = -1, d[i][n + 1] = b[i];
        for (int i = 0; i < n; ++i) q[i] = i, d[m][i] = -c[i];
        q[n] = -1, d[m + 1][n] = 1;
        int r = 0;
        for (int i = 1; i < m; ++i)
            if (d[i][n + 1] < d[r][n + 1]) r = i;
        if (d[r][n + 1] < -eps) {
            pivot(r, n);
            if (!phase(1) || d[m + 1][n + 1] < -eps)
                return VD(n, -inf);
            for (int i = 0; i < m; ++i) if (p[i] == -1) {
                int s = min_element(d[i].begin(), d[i].end() - 1)
                    - d[i].begin();

```



```

    pivot(i, s);
}
}
if (!phase(0)) return VD(n, inf);
VD x(n);
for (int i = 0; i < m; ++i)
    if (p[i] < n) x[p[i]] = d[i][n + 1];
return x;
}}
```

5.24 Simplex Construction

Standard form: maximize $\sum_{1 \leq i \leq n} c_i x_i$ such that for all $1 \leq j \leq m$, $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$ and $x_i \geq 0$ for all $1 \leq i \leq n$.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji} x_i \leq -b_j$
3. $\sum_{1 \leq i \leq n} A_{ji} x_i = b_j$
 - $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$
 - $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j$
4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

6 Geometry

6.1 Basic Geometry

```

#define IM imag
#define RE real
using lld = int64_t;
using llf = long double;
using Point = std::complex<lld>;
using Pointf = std::complex<llf>;
auto toPointf(Point p) { return Pointf{RE(p), IM(p)}; }
int sgn(lld x) { return (x > 0) - (x < 0); }
lld dot(Point a, Point b) { return RE(conj(a) * b); }
lld cross(Point a, Point b) { return IM(conj(a) * b); }
int ori(Point a, Point b, Point c) {
    return sgn(cross(b - a, c - a));
}
bool operator<(const Point &a, const Point &b) {
    return RE(a) != RE(b) ? RE(a) < RE(b) : IM(a) < IM(b);
}
int argCmp(Point a, Point b) {
    // -1 / 0 / 1 <-> < / == / > (atan2)
    int qa = (IM(a) == 0 ? (RE(a) < 0 ? 3 : 1) : (IM(a) < 0 ? 0 : 2));
    int qb = (IM(b) == 0 ? (RE(b) < 0 ? 3 : 1) : (IM(b) < 0 ? 0 : 2));
    if (qa != qb) return sgn(qa - qb);
    return sgn(cross(b, a));
}
template <typename V> llf area(const V &pt) {
    lld ret = 0;
    for (int i = 1; i + 1 < (int)pt.size(); ++i)
        ret += cross(pt[i] - pt[0], pt[i+1] - pt[0]);
    return ret / 2.0;
}
Point rot90(Point p) { return Point{-IM(p), RE(p)}; }
Pointf project(Pointf p, Pointf q) { // p onto q
    return dot(p, q) * q / dot(q, q);
}
}
```

6.2 Segment & Line Intersection

```

struct Segment {
    Point st, dir; // represent st + t*dir for 0<=t<=1
    Segment(Point s, Point e) : st(s), dir(e - s) {}
    static bool valid(lld p, lld q) {
        // is there t s.t. 0 <= t <= 1 && qt == p ?
        if (q < 0) q = -q, p = -p;
        return 0 <= p && p <= q;
    }
};
bool isInter(Segment A, Point P) {
    if (A.dir == Point(0)) return P == A.st;
    return cross(P - A.st, A.dir) == 0 &&
        Segment::valid(dot(P - A.st, A.dir), norm(A.dir));
}
template <typename U, typename V>
```

```

bool isInter(U A, V B) {
    if (cross(A.dir, B.dir) == 0)
        return // handle parallel yourself
            isInter(A, B.st) || isInter(A, B.st+B.dir) ||
            isInter(B, A.st) || isInter(B, A.st+A.dir);
    Point D = B.st - A.st;
    lld C = cross(A.dir, B.dir);
    return U::valid(cross(D, A.dir), C) &&
        V::valid(cross(D, B.dir), C);
}
struct Line {
    Point st, ed, dir;
    Line(Point s, Point e)
        : st(s), ed(e), dir(e - s) {}
};
Pointf intersect(const Line &A, const Line &B) {
    llf t = cross(B.st - A.st, B.dir) /
        llf(cross(A.dir, B.dir));
    return toPointf(A.st) +
        Pointf(t) * toPointf(A.dir);
}
```

6.3 2D Convex Hull

```

template<typename PT>
vector<PT> buildConvexHull(vector<PT> d) {
    sort(ALL(d), [](const PT& a, const PT& b) {
        return tie(a.x, a.y) < tie(b.x, b.y); });
    vector<PT> s(SZ(d)<<1);
    int o = 0;
    for(auto p: d) {
        while(o>=2 && cross(p-s[o-2], s[o-1]-s[o-2])<=0)
            o--;
        s[o++] = p;
    }
    for(int i=SZ(d)-2, t = o+1; i>=0; i--){
        while(o>=t && cross(d[i]-s[o-2], s[o-1]-s[o-2])<=0)
            o--;
        s[o++] = d[i];
    }
    s.resize(o-1);
    return s;
}
```

6.4 3D Convex Hull

```

// return the faces with pt indexes
int flag[MXN][MXN];
struct Point {
    ld x, y, z;
    Point operator * (const ld &b) const {
        return (Point){x*b, y*b, z*b};
    }
    Point operator * (const Point &b) const {
        return (Point){y*b.z-b.y*z, z*b.x-b.z*x, x*b.y-b.x*y};
    }
};
Point ver(Point a, Point b, Point c) {
    return (b - a) * (c - a);
}
vector<Face> convex_hull_3D(const vector<Point> pt) {
    int n = SZ(pt), ftop = 0;
    REP(i, n) REP(j, n) flag[i][j] = 0;
    vector<Face> now;
    now.emplace_back(0, 1, 2);
    now.emplace_back(2, 1, 0);
    for (int i=3; i<n; i++){
        ftop++; vector<Face> next;
        REP(j, SZ(now)) {
            Face& f=now[j]; int ff = 0;
            ld d=(pt[i]-pt[f.a]).dot(
                ver(pt[f.a], pt[f.b], pt[f.c]));
            if (d <= 0) next.push_back(f);
            if (d > 0) ff=ftop;
            else if (d < 0) ff=-ftop;
            flag[f.a][f.b]=flag[f.b][f.c]=flag[f.c][f.a]=ff;
        }
        REP(j, SZ(now)) {
            Face& f=now[j];
            if (flag[f.a][f.b] > 0 &&
                flag[f.a][f.b] != flag[f.b][f.a])
                next.emplace_back(f.a, f.b, i);
            if (flag[f.b][f.c] > 0 &&
                flag[f.b][f.c] != flag[f.c][f.b])
                next.emplace_back(f.b, f.c, i);
            if (flag[f.c][f.a] > 0 &&
```

```

    flag[f.c][f.a] != flag[f.a][f.c])
    next.emplace_back(f.c, f.a, i);
}
now=next;
}
return now;
}

```

6.5 2D Farthest Pair

```

// stk is from convex hull
n = (int)(stk.size());
int pos = 1, ans = 0; stk.push_back(stk[0]);
for(int i=0; i<n; i++) {
    while(abs(cross(stk[i+1]-stk[i],
        stk[(pos+1)%n]-stk[i])) >
        abs(cross(stk[i+1]-stk[i],
            stk[pos]-stk[i]))) pos = (pos+1)%n;
    ans = max({ans, dis(stk[i], stk[pos]),
        dis(stk[i+1], stk[pos])});
}

```

6.6 2D Closest Pair

```

struct cmp_y {
    bool operator()(const P& p, const P& q) const {
        return p.y < q.y;
    }
};
multiset<P, cmp_y> s;
void solve(P a[], int n) {
    sort(a, a + n, [](const P& p, const P& q) {
        return tie(p.x, p.y) < tie(q.x, q.y);
    });
    llf d = INF; int pt = 0;
    for (int i = 0; i < n; ++i) {
        while (pt < i and a[i].x - a[pt].x >= d)
            s.erase(s.find(a[pt++]));
        auto it = s.lower_bound(P(a[i].x, a[i].y - d));
        while (it != s.end() and it->y - a[i].y < d)
            d = min(d, dis(*it++, a[i]));
        s.insert(a[i]);
    }
}

```

6.7 kD Closest Pair (3D ver.)

```

llf solve(vector<P> v) {
    shuffle(v.begin(), v.end(), mt19937());
    unordered_map<lld, unordered_map<lld,
        unordered_map<lld, int>>> m;
    llf d = dis(v[0], v[1]);
    auto Idx = [&d] (llf x) -> lld {
        return round(x * 2 / d) + 0.1; };
    auto rebuild_m = [&m, &v, &Idx] (int k) {
        m.clear();
        for (int i = 0; i < k; ++i)
            m[Idx(v[i].x)][Idx(v[i].y)]
                [Idx(v[i].z)] = i;
    }; rebuild_m(2);
    for (size_t i = 2; i < v.size(); ++i) {
        const lld kx = Idx(v[i].x), ky = Idx(v[i].y),
            kz = Idx(v[i].z); bool found = false;
        for (int dx = -2; dx <= 2; ++dx) {
            const lld nx = dx + kx;
            if (m.find(nx) == m.end()) continue;
            auto& mm = m[nx];
            for (int dy = -2; dy <= 2; ++dy) {
                const lld ny = dy + ky;
                if (mm.find(ny) == mm.end()) continue;
                auto& mmm = mm[ny];
                for (int dz = -2; dz <= 2; ++dz) {
                    const lld nz = dz + kz;
                    if (mmm.find(nz) == mmm.end()) continue;
                    const int p = mmm[nz];
                    if (dis(v[p], v[i]) < d) {
                        d = dis(v[p], v[i]);
                        found = true;
                    }
                }
            }
        }
        if (found) rebuild_m(i + 1);
        else m[kx][ky][kz] = i;
    }
}

```

```

}
return d;
}

```

6.8 Simulated Annealing

```

llf anneal() {
    mt19937 rnd_engine( seed );
    uniform_real_distribution< llf > rnd( 0, 1 );
    const llf dT = 0.001;
    // Argument p
    llf S_cur = calc( p ), S_best = S_cur;
    for ( llf T = 2000 ; T > EPS ; T -= dT ) {
        // Modify p to p_prime
        const llf S_prime = calc( p_prime );
        const llf delta_c = S_prime - S_cur;
        llf prob = min( ( llf ) 1, exp( -delta_c / T ) );
        if ( rnd( rnd_engine ) <= prob )
            S_cur = S_prime, p = p_prime;
        if ( S_prime < S_best ) // find min
            S_best = S_prime, p_best = p_prime;
    }
    return S_best;
}

```

6.9 Half Plane Intersection

```

// cross(pt-line.st, line.dir)<=0 <=> pt in half plane
bool operator<(const Line &lhs, const Line &rhs) {
    if (int cmp = argCmp(lhs.dir, rhs.dir))
        return cmp == -1;
    return ori(lhs.st, lhs.ed, rhs.st) < 0;
}

// intersect function is in "Segment Intersect"
llf HPI(vector<Line> &lines) {
    sort(lines.begin(), lines.end());
    deque<Line> que;
    deque<Pointf> pt;
    que.push_back(lines[0]);
    for (int i = 1; i < (int)lines.size(); i++) {
        if (argCmp(lines[i].dir, lines[i-1].dir) == 0)
            continue;
#define POP(L, R) \
        while (pt.size() > 0 \
            && ori(L.st, L.ed, pt.back()) < 0) \
            pt.pop_back(), que.pop_back(); \
        while (pt.size() > 0 \
            && ori(R.st, R.ed, pt.front()) < 0) \
            pt.pop_front(), que.pop_front();
        POP(lines[i], lines[i-1]);
        pt.push_back(intersect(que.back(), lines[i]));
        que.push_back(lines[i]);
    }
    POP(que.front(), que.back())
    if (que.size() <= 1 ||
        argCmp(que.front().dir, que.back().dir) == 0)
        return 0;
    pt.push_back(intersect(que.front(), que.back()));
    return area(pt);
}

```

6.10 Minkowski Sum

```

vector<pll> Minkowski(vector<pll> A, vector<pll> B) {
    hull(A), hull(B);
    vector<pll> C(1, A[0] + B[0]), s1, s2;
    for(int i = 0; i < SZ(A); ++i)
        s1.pb(A[(i + 1) % SZ(A)] - A[i]);
    for(int i = 0; i < SZ(B); ++i)
        s2.pb(B[(i + 1) % SZ(B)] - B[i]);
    for(int p1 = 0, p2 = 0; p1 < SZ(A) || p2 < SZ(B);)
        if (p2 >= SZ(B)
            || (p1 < SZ(A) && cross(s1[p1], s2[p2]) >= 0))
            C.pb(C.back() + s1[p1++]);
        else
            C.pb(C.back() + s2[p2++]);
    return hull(C), C;
}

```

6.11 Circle Class

```

struct Circle { Pointf o; llf r; };
vector<llf> intersectAngle(Circle A, Circle B) {

```

```

Pointf dir = B.o - A.o; llf d2 = norm(dir);
if (norm(A.r - B.r) >= d2)
    if (A.r < B.r) return {-PI, PI}; // special
    else return {};
if (norm(A.r + B.r) <= d2) return {};
llf dis = abs(dir), theta = arg(dir);
llf phi = acos((A.r * A.r + d2 - B.r * B.r) /
    (2 * A.r * dis));
llf L = theta - phi, R = theta + phi;
while (L < -PI) L += PI * 2;
while (R > PI) R -= PI * 2;
return { L, R };
}

vector<Pointf> intersectPoint(Circle a, Circle b) {
    llf d = abs(a.o - b.o);
    if (d >= b.r+a.r || d <= abs(b.r-a.r)) return {};
    llf dt = (b.r*b.r - a.r*a.r)/d, d1 = (d+dt)/2;
    Pointf dir = (a.o - b.o); dir /= d;
    Pointf pcrs = dir*d1 + b.o;
    dt=sqrt(max(0.0L, b.r*b.r-d1*d1)), dir = rot90(dir);
    return {pcrs + dir*dt, pcrs - dir*dt};
}

```

6.12 Intersection of line and Circle

```

vector<pdd> line_interCircle(const pdd &p1,
    const pdd &p2, const pdd &c, const double r) {
    pdd ft = p1 + project(c-p1, p2-p1), vec = p2-p1;
    llf dis = abs(c - ft);
    if (abs(dis - r) < eps) return {ft};
    if (dis > r) return {};
    vec = vec * sqrt(r * r - dis * dis) / abs(vec);
    return {ft + vec, ft - vec};
}

```

6.13 Intersection of Polygon and Circle

```

// Divides into multiple triangle, and sum up
// test by HDU2892
const double PI=acos(-1);
double _area(pdd pa, pdd pb, double r){
    if(abs(pa)<abs(pb)) swap(pa, pb);
    if(abs(pb)<eps) return 0;
    double S, h, theta;
    double a=abs(pb),b=abs(pa),c=abs(pb-pa);
    double cosB = dot(pb,pb-pa) / a / c, B = acos(cosB);
    double cosC = dot(pa,pb) / a / b, C = acos(cosC);
    if(a > r){
        S = (C/2)*r*r;
        h = a*b*sin(C)/c;
        if (h < r && B < PI/2)
            S -= (acos(h/r)*r*r - h*sqrt(r*r-h*h));
    }
    else if(b > r){
        theta = PI - B - asin(sin(B)/r*a);
        S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
    }
    else S = .5*sin(C)*a*b;
    return S;
}

double area_poly_circle(const vector<pdd> &poly,
    const pdd &O,const double r){
    double S=0; int N=poly.size();
    for(int i=0;i<N;++i)
        S += _area(poly[i]-O, poly[(i+1)%N]-O, r)
            * ori(0, poly[i], poly[(i+1)%N]);
    return fabs(S);
}

```

6.14 Point & Hulls Tangent

```

#define above(P, Vi, Vj) (ori(P, Vi, Vj) > 0) // true
    if Vi is above Vj
#define below(P, Vi, Vj) (ori(P, Vi, Vj) < 0) // true
    if Vi is below Vj
// Rtangent_PointPolyC(): binary search for convex
    polygon right tangent
// Input: P = a 2D point (exterior to the polygon)
//      n = number of polygon vertices
//      V = array of vertices for a 2D convex polygon
    with V[n] = V[0]
// Return: index "i" of rightmost tangent point V[i]
int Rtangent_PointPolyC(Point P, int n, Point *V) {

```

```

int a, b, c;
int upA, dnC;

if (below(P, V[1], V[0]) && !above(P, V[n - 1], V[0]))
    return 0;

for (a = 0, b = n;;) {
    c = (a + b) / 2;
    dnC = below(P, V[c + 1], V[c]);
    if (dnC && !above(P, V[c - 1], V[c]))
        return c;

    upA = above(P, V[a + 1], V[a]);
    if (upA) {
        if (dnC) {
            b = c;
        } else {
            if (above(P, V[a], V[c]))
                b = c;
            else
                a = c;
        }
    } else {
        if (!dnC) {
            a = c;
        } else {
            if (below(P, V[a], V[c]))
                b = c;
            else
                a = c;
        }
    }
}

// Ltangent_PointPolyC(): binary search for convex
    polygon left tangent
// Input: P = a 2D point (exterior to the polygon)
//      n = number of polygon vertices
//      V = array of vertices for a 2D convex polygon
    with V[n]=V[0]
// Return: index "i" of leftmost tangent point V[i]
int Ltangent_PointPolyC(Point P, int n, Point *V) {
    int a, b, c;
    int dnA, dnC;

    if (above(P, V[n - 1], V[0]) && !below(P, V[1], V[0]))
        return 0;

    for (a = 0, b = n;;) {
        c = (a + b) / 2;
        dnC = below(P, V[c + 1], V[c]);
        if (above(P, V[c - 1], V[c]) && !dnC)
            return c;
        dnA = below(P, V[a + 1], V[a]);
        if (dnA) {
            if (!dnC) {
                b = c;
            } else {
                if (below(P, V[a], V[c]))
                    b = c;
                else
                    a = c;
            }
        } else {
            if (dnC) {
                a = c;
            } else {
                if (above(P, V[a], V[c]))
                    b = c;
                else
                    a = c;
            }
        }
    }
}

```

6.15 Convex Hulls Tangent

```

// RLtangent_PolyPolyC(): get the RL tangent between
    two convex polygons
// Input: m = number of vertices in polygon 1

```

```
// V = array of vertices for convex polygon 1 with
// V[m]=V[0]
// n = number of vertices in polygon 2
// W = array of vertices for convex polygon 2 with
// W[n]=W[0]
// Output: *t1 = index of tangent point V[t1] for
// polygon 1
// *t2 = index of tangent point W[t2] for polygon
// 2
void RLtangent_PolyPolyC(int m, Point *V, int n, Point
    *W, int *t1, int *t2) {
    int ix1, ix2; // search indices for polygons 1 and 2

    // first get the initial vertex on each polygon
    ix1 = Rtangent_PointPolyC(W[0], m, V); // right
    tangent from W[0] to V
    ix2 = Ltangent_PointPolyC(V[ix1], n, W); // left
    tangent from V[ix1] to W

    // ping-pong linear search until it stabilizes
    int done = false; // flag when done
    while (done == false) {
        done = true; // assume done until...
        while (ori(W[ix2], V[ix1], V[ix1 + 1]) <= 0) {
            ++ix1; // get Rtangent from W[ix2] to V
        }
        while (ori(V[ix1], W[ix2], W[ix2 - 1]) >= 0) {
            --ix2; // get Ltangent from V[ix1] to W
            done = false; // not done if had to adjust this
        }
    }
    *t1 = ix1;
    *t2 = ix2;
    return;
}
```

6.16 Tangent line of Two Circle

```
vector<Line> go(const Cir &c1, const Cir &c2,
    int sign1) {
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    if (norm(c1.o - c2.o) < eps)
        return ret;
    llf d = abs(c1.o - c2.o);
    Pointf v = (c2.o - c1.o) / d;
    llf c = (c1.r - sign1 * c2.r) / d;
    if (c * c > 1)
        return ret;
    llf h = sqrt(max(0.0, 1.0 - c * c));
    for (int sign2: {1, -1}) {
        Pointf n = c * v + sign2 * h * rot90(v);
        Pointf p1 = c1.o + n * c1.r;
        Pointf p2 = c2.o + n * (c2.r * sign1);
        if (norm(p2 - p1) < eps)
            p2 = p1 + rot90(c2.o - c1.o);
        ret.push_back({p1, p2});
    }
    return ret;
}
```

6.17 Minimum Covering Circle

```
template<typename P>
Circle getCircum(const P &a, const P &b, const P &c){
    Real a1 = a.x-b.x, b1 = a.y-b.y;
    Real c1 = (a.x+b.x)/2 * a1 + (a.y+b.y)/2 * b1;
    Real a2 = a.x-c.x, b2 = a.y-c.y;
    Real c2 = (a.x+c.x)/2 * a2 + (a.y+c.y)/2 * b2;
    Circle cc;
    cc.o.x = (c1*b2-b1*c2)/(a1*b2-b1*a2);
    cc.o.y = (a1*c2-c1*a2)/(a1*b2-b1*a2);
    cc.r = hypot(cc.o.x-a.x, cc.o.y-a.y);
    return cc;
}
```

```
template<typename P>
Circle MinCircleCover(const vector<P>& pts){
    random_shuffle(pts.begin(), pts.end());
    Circle c = { pts[0], 0 };
    for(int i=0;i<(int)pts.size();i++){
        if (dist(pts[i], c.o) <= c.r) continue;
        c = { pts[i], 0 };
        for (int j = 0; j < i; j++) {
```

```
            if(dist(pts[j], c.o) <= c.r) continue;
            c.o = (pts[i] + pts[j]) / 2;
            c.r = dist(pts[i], c.o);
            for (int k = 0; k < j; k++) {
                if (dist(pts[k], c.o) <= c.r) continue;
                c = getCircum(pts[i], pts[j], pts[k]);
            }
        }
    }
    return c;
}
```

6.18 KDTree (Nearest Point)

```
const int MXN = 100005;
struct KDTree {
    struct Node {
        int x,y,x1,y1,x2,y2;
        int id,f;
        Node *L, *R;
    } tree[MXN], *root;
    int n;
    LL dis2(int x1, int y1, int x2, int y2) {
        LL dx = x1-x2, dy = y1-y2;
        return dx*dx+dy*dy;
    }
    static bool cmpx(Node& a, Node& b){return a.x<b.x;}
    static bool cmpy(Node& a, Node& b){return a.y<b.y;}
    void init(vector<pair<int,int>> ip) {
        n = ip.size();
        for (int i=0; i<n; i++) {
            tree[i].id = i;
            tree[i].x = ip[i].first;
            tree[i].y = ip[i].second;
        }
        root = build_tree(0, n-1, 0);
    }
    Node* build_tree(int L, int R, int d) {
        if (L>R) return nullptr;
        int M = (L+R)/2; tree[M].f = d%2;
        nth_element(tree+L, tree+M, tree+R+1, d%2?cmpy:cmpx);
        tree[M].x1 = tree[M].x2 = tree[M].x;
        tree[M].y1 = tree[M].y2 = tree[M].y;
        tree[M].L = build_tree(L, M-1, d+1);
        if (tree[M].L) {
            tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
            tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
            tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
            tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
        }
        tree[M].R = build_tree(M+1, R, d+1);
        if (tree[M].R) {
            tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
            tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
            tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
            tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
        }
        return tree+M;
    }
    int touch(Node* r, int x, int y, LL d2){
        LL dis = sqrt(d2)+1;
        if (x<r->x1-dis || x>r->x2+dis ||
            y<r->y1-dis || y>r->y2+dis)
            return 0;
        return 1;
    }
    void nearest(Node* r, int x, int y, int &mID, LL &md2) {
        if (!r || !touch(r, x, y, md2)) return;
        LL d2 = dis2(r->x, r->y, x, y);
        if (d2 < md2 || (d2 == md2 && mID < r->id)) {
            mID = r->id;
            md2 = d2;
        }
        // search order depends on split dim
        if ((r->f == 0 && x < r->x) ||
            (r->f == 1 && y < r->y)) {
            nearest(r->L, x, y, mID, md2);
            nearest(r->R, x, y, mID, md2);
        } else {
            nearest(r->R, x, y, mID, md2);
            nearest(r->L, x, y, mID, md2);
        }
    }
}
```

```

int query(int x, int y) {
    int id = 1029384756;
    LL d2 = 102938475612345678LL;
    nearest(root, x, y, id, d2);
    return id;
}
} tree;

```

6.19 Rotating Sweep Line

```

void rotatingSweepLine(pair<int, int> a[], int n) {
    vector<pair<int, int>> l;
    l.reserve(n * (n - 1) / 2);
    for (int i = 0; i < n; ++i)
        for (int j = i + 1; j < n; ++j)
            l.emplace_back(i, j);
    sort(l.begin(), l.end(), [&a](auto &u, auto &v){
        lld udx = a[u.first].first - a[u.second].first;
        lld udy = a[u.first].second - a[u.second].second;
        lld vdx = a[v.first].first - a[v.second].first;
        lld vdy = a[v.first].second - a[v.second].second;
        if (udx == 0 or vdx == 0) return not udx == 0;
        int s = sgn(udx * vdx);
        return udy * vdx * s < vdy * udx * s;
    });
    vector<int> idx(n), p(n);
    iota(idx.begin(), idx.end(), 0);
    sort(idx.begin(), idx.end(), [&a](int i, int j){
        return a[i] < a[j]; });
    for (int i = 0; i < n; ++i) p[idx[i]] = i;
    for (auto [i, j]: l) {
        // do here
        swap(p[i], p[j]);
        idx[p[i]] = i, idx[p[j]] = j;
    }
}

```

6.20 Circle Cover

```

const int N = 1021;
struct CircleCover {
    int C;
    Cir c[N];
    bool g[N][N], overlap[N][N];
    // Area[i] : area covered by at least i circles
    double Area[N];
    void init(int _C){ C = _C; }
    struct Teve {
        pdd p; double ang; int add;
        Teve() {}
        Teve(pdd _a, double _b, int _c):p(_a), ang(_b), add(_c){}
        bool operator<(const Teve &a)const {
            return ang < a.ang; }
    }eve[N * 2];
    // strict: x = 0, otherwise x = -1
    bool disjunct(Cir &a, Cir &b, int x) {
        return sign(abs(a.o - b.o) - a.R - b.R) > x; }
    bool contain(Cir &a, Cir &b, int x) {
        return sign(a.R - b.R - abs(a.o - b.o)) > x; }
    bool contain(int i, int j) {
        /* c[j] is non-strictly in c[i]. */
        return (sign(c[i].R - c[j].R) > 0 || (sign(c[i].R - c[j].R) == 0 && i < j)) && contain(c[i], c[j], -1); }
    void solve(){
        fill_n(Area, C + 2, 0);
        for(int i = 0; i < C; ++i)
            for(int j = 0; j < C; ++j)
                overlap[i][j] = contain(i, j);
        for(int i = 0; i < C; ++i)
            for(int j = 0; j < C; ++j)
                g[i][j] = !(overlap[i][j] || overlap[j][i] || disjunct(c[i], c[j], -1));
        for(int i = 0; i < C; ++i){
            int E = 0, cnt = 1;
            for(int j = 0; j < C; ++j)
                if(j != i && overlap[j][i])
                    ++cnt;
            for(int j = 0; j < C; ++j)
                if(i != j && g[i][j]) {
                    pdd aa, bb;
                    CCinter(c[i], c[j], aa, bb);
                    llf A = atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);

```

```

                    llf B = atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
                    eve[E++] = Teve(bb, B, 1), eve[E++] = Teve(aa, A, -1);
                    if(B > A) ++cnt;
                }
            if(E == 0) Area[cnt] += pi * c[i].R * c[i].R;
            else{
                sort(eve, eve + E);
                eve[E] = eve[0];
                for(int j = 0; j < E; ++j){
                    cnt += eve[j].add;
                    Area[cnt] += cross(eve[j].p, eve[j + 1].p) * .5;
                    double theta = eve[j + 1].ang - eve[j].ang;
                    if (theta < 0) theta += 2. * pi;
                    Area[cnt] += (theta - sin(theta)) * c[i].R * c[i].R * .5;
                }
            }
        }
    }
};

```

7 Stringology

7.1 Hash

```

class Hash {
private:
    static constexpr int P = 127, Q = 1051762951;
    vector<int> h, p;
public:
    void init(const string &s){
        h.assign(s.size() + 1, 0); p.resize(s.size() + 1);
        for (size_t i = 0; i < s.size(); ++i)
            h[i + 1] = add(mul(h[i], P), s[i]);
        generate(p.begin(), p.end(), [x=1, y=1, this]()
            mutable { y=x; x=mul(x, P); return y; });
    }
    int query(int l, int r){ // 1-base (l, r]
        return sub(h[r], mul(h[l], p[r-l])); }
};

```

7.2 Suffix Array

```

namespace sfx {
    bool _t[maxn * 2];
    int hi[maxn], rev[maxn];
    int _s[maxn * 2], sa[maxn * 2], _c[maxn * 2];
    int x[maxn], _p[maxn], _q[maxn * 2];
    // sa[i]: sa[i]-th suffix is the
    // i-th lexicographically smallest suffix.
    // hi[i]: longest common prefix
    // of suffix sa[i] and suffix sa[i - 1].
    void pre(int *a, int *c, int n, int z) {
        memset(a, 0, sizeof(int) * n);
        memcpy(x, c, sizeof(int) * z);
    }
    void induce(int *a, int *c, int *s, bool *t, int n, int z) {
        memcpy(x + 1, c, sizeof(int) * (z - 1));
        for (int i = 0; i < n; ++i)
            if (a[i] && !t[a[i] - 1])
                a[x[s[a[i] - 1]]++] = a[i] - 1;
        memcpy(x, c, sizeof(int) * z);
        for (int i = n - 1; i >= 0; --i)
            if (a[i] && t[a[i] - 1])
                a[--x[s[a[i] - 1]]] = a[i] - 1;
    }
    void sais(int *s, int *a, int *p, int *q,
        bool *t, int *c, int n, int z) {
        bool uniq = t[n - 1] = true;
        int nn=0, nmxz=-1, *nsa = a+n, *ns=s+n, last=-1;
        memset(c, 0, sizeof(int) * z);
        for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;
        for (int i = 0; i < z - 1; ++i) c[i + 1] += c[i];
        if (uniq) {
            for (int i = 0; i < n; ++i) a[--c[s[i]]] = i;
            return;
        }
        for (int i = n - 2; i >= 0; --i)
            t[i] = (s[i]==s[i + 1] ? t[i + 1] : s[i]<s[i + 1]);
        pre(a, c, n, z);
        for (int i = 1; i <= n - 1; ++i)
            if (t[i] && !t[i - 1])
                a[--x[s[i]]] = p[q[i] = nn++] = i;
        induce(a, c, s, t, n, z);
    }
}

```



```

for (int i = 0; i < n; ++i) {
    if (a[i] && t[a[i]] && !t[a[i] - 1]) {
        bool neq = last < 0 || \
            memcmp(s + a[i], s + last,
                (p[q[a[i]] + 1] - a[i]) * sizeof(int));
        ns[q[last = a[i]]] = nmzx += neq;
    }
    sais(ns, nsa, p+nn, q+n, t+n, c+z, nn, nmzx+1);
    pre(a, c, n, z);
    for (int i = nn - 1; i >= 0; --i)
        a[--x[s[p[nsa[i]]]]] = p[nsa[i]];
    induce(a, c, s, t, n, z);
}

void build(const string &s) {
    const int n = int(s.size());
    for (int i = 0; i < n; ++i) _s[i] = s[i];
    _s[n] = 0; // s shouldn't contain 0
    sais(_s, sa, _p, _q, _t, _c, n + 1, 256);
    for (int i = 0; i < n; ++i) rev[sa[i] = sa[i+1]] = i;
    int ind = hi[0] = 0;
    for (int i = 0; i < n; ++i) {
        if (!rev[i]) {
            ind = 0;
            continue;
        }
        while (i + ind < n && \
            s[i + ind] == s[sa[rev[i] - 1] + ind]) ++ind;
        hi[rev[i]] = ind ? ind-- : 0;
    }
}

```

7.3 Suffix Automaton

```

struct SuffixAutomaton {
    struct node {
        int ch[K], len, fail, cnt, indeg;
        node(int L = 0) : ch{}, len(L), fail(0), cnt(0),
            indeg(0) {}
    } st[N];
    int root, last, tot;
    void extend(int c) {
        int cur = ++tot;
        st[cur] = node(st[last].len + 1);
        while (last && !st[last].ch[c]) {
            st[last].ch[c] = cur;
            last = st[last].fail;
        }
        if (!last) {
            st[cur].fail = root;
        } else {
            int q = st[last].ch[c];
            if (st[q].len == st[last].len + 1) {
                st[cur].fail = q;
            } else {
                int clone = ++tot;
                st[clone] = st[q];
                st[clone].len = st[last].len + 1;
                st[st[cur].fail = st[q].fail = clone].cnt = 0;
                while (last && st[last].ch[c] == q) {
                    st[last].ch[c] = clone;
                    last = st[last].fail;
                }
            }
        }
        st[last = cur].cnt += 1;
    }
    void init(const char* s) {
        root = last = tot = 1;
        st[root] = node(0);
        for (char c; c = *s; ++s) extend(c - 'a');
    }
    int q[N];
    void dp() {
        for (int i = 1; i <= tot; i++) ++st[st[i].fail].indeg;
        int head = 0, tail = 0;
        for (int i = 1; i <= tot; i++)
            if (st[i].indeg == 0) q[tail++] = i;
        while (head != tail) {
            int now = q[head++];
            if (int f = st[now].fail) {
                st[f].cnt += st[now].cnt;
                if (--st[f].indeg == 0) q[tail++] = f;
            }
        }
    }
}

```

```

}
}
int run(const char* s) {
    int now = root;
    for (char c; c = *s; ++s) {
        if (!st[now].ch[c - 'a']) return 0;
        now = st[now].ch[c];
    }
    return st[now].cnt;
}
} SAM;

```

7.4 KMP

```

vector<int> kmp(const string &s) {
    vector<int> f(s.size(), 0);
    /* f[i] = length of the longest prefix
       (excluding s[0:i]) such that it coincides
       with the suffix of s[0:i] of the same length */
    /* i + 1 - f[i] is the length of the
       smallest recurring period of s[0:i] */
    int k = 0;
    for (int i = 1; i < (int)s.size(); ++i) {
        while (k > 0 && s[i] != s[k]) k = f[k - 1];
        if (s[i] == s[k]) ++k;
        f[i] = k;
    }
    return f;
}

vector<int> search(const string &s, const string &t) {
    // return 0-indexed occurrence of t in s
    vector<int> f = kmp(t), r;
    for (int i = 0, k = 0; i < (int)s.size(); ++i) {
        while (k > 0 && (k == (int)t.size() || s[i] != t[k]))
            k = f[k - 1];
        if (s[i] == t[k]) ++k;
        if (k == (int)t.size()) r.push_back(i - t.size() + 1);
    }
    return res;
}

```

7.5 Z value

```

vector<int> Zalgo(const string &s) {
    vector<int> z(s.size(), s.size());
    for (int i = 1, l = 0, r = 0; i < z[0]; ++i) {
        int j = clamp(r - i, 0, z[i - 1]);
        for (; i + j < z[0] && s[i + j] == s[j]; ++j);
        if (i + (z[i] = j) > r) r = i + z[i];
    }
    return z;
}

```

7.6 Manacher

```

int z[maxn];
int manacher(const string &s) {
    string t = ". ";
    for (char c: s) t += c, t += ' ';
    int l = 0, r = 0, ans = 0;
    for (int i = 1; i < t.length(); ++i) {
        z[i] = (r > i ? min(z[2 * l - i], r - i) : 1);
        while (i - z[i] >= 0 && i + z[i] < t.length()) {
            if (t[i - z[i]] == t[i + z[i]]) ++z[i];
            else break;
        }
        if (i + z[i] > r) r = i + z[i], l = i;
    }
    for (int i = 1; i < t.length(); ++i) ans = max(ans, z[i] - 1);
    return ans;
}

```

7.7 Lexico Smallest Rotation

```

string mcp(string s) {
    int n = s.length();
    s += s; int i = 0, j = 1;
    while (i < n && j < n) {
        int k = 0;
        while (k < n && s[i + k] == s[j + k]) k++;
        ((s[i + k] <= s[j + k]) ? j : i) += k + 1;
        j += (i == j);
    }
    return s.substr(i < n ? i : j, n);
}

```

7.8 BWT

```
struct BurrowsWheeler{
#define SIGMA 26
#define BASE 'a'
    vector<int> v[ SIGMA ];
    void BWT(char* ori, char* res){
        // make ori -> ori + ori
        // then build suffix array
    }
    void iBWT(char* ori, char* res){
        for( int i = 0 ; i < SIGMA ; i ++ )
            v[ i ].clear();
        int len = strlen( ori );
        for( int i = 0 ; i < len ; i ++ )
            v[ ori[i] - BASE ].push_back( i );
        vector<int> a;
        for( int i = 0 , ptr = 0 ; i < SIGMA ; i ++ )
            for( auto j : v[ i ] ){
                a.push_back( j );
                ori[ ptr ++ ] = BASE + i;
            }
        for( int i = 0 , ptr = 0 ; i < len ; i ++ ){
            res[ i ] = ori[ a[ ptr ] ];
            ptr = a[ ptr ];
        }
        res[ len ] = 0;
    }
} bwt;
```

7.9 Palindromic Tree

```
struct palindromic_tree{
    struct node{
        int next[26], f, len;
        int cnt, num, st, ed;
        node(int l=0):f(0), len(1), cnt(0), num(0) {
            memset(next, 0, sizeof(next)); }
    };
    vector<node> st;
    vector<char> s;
    int last, n;
    void init(){
        st.clear(); s.clear(); last=1; n=0;
        st.push_back(0); st.push_back(-1);
        st[0].f=1; s.push_back(-1); }
    int getFail(int x){
        while(s[n-st[x].len-1]!=s[n])x=st[x].f;
        return x; }
    void add(int c){
        s.push_back(c-'a'); ++n;
        int cur=getFail(last);
        if(!st[cur].next[c]){
            int now=st.size();
            st.push_back(st[cur].len+2);
            st[now].f=getFail(st[cur].f).next[c];
            st[cur].next[c]=now;
            st[now].num=st[st[now].f].num+1;
        }
        last=st[cur].next[c];
        ++st[last].cnt;
    }
    void dpent() {
        for( int i=st.size()-1; i >= 0; i-- )
            st[st[i].f].cnt += st[i].cnt;
    }
    int size(){ return st.size()-2; }
} pt;

int main() {
    string s; cin >> s; pt.init();
    for( int i=0; i<SZ(s); i++ ) {
        int prvsz = pt.size(); pt.add(s[i]);
        if( prvsz != pt.size() ) {
            int r = i, l = r - pt.st[pt.last].len + 1;
            // pal @ [l,r]: s.substr(l, r-l+1)
        }
    }
    return 0;
}
```

8 Misc

8.1 Theorems

8.1.1 Kirchhoff's Theorem

Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G , where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i, j) in G .

- The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
- The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.

8.1.2 Tutte's Matrix

Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniform randomly) if $i < j$ and $(i, j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{\text{rank}(D)}{2}$ is the maximum matching on G .

8.1.3 Cayley's Formula

- Given a degree sequence d_1, d_2, \dots, d_n for each labeled vertices, there're $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\dots(d_n-1)!}$ spanning trees.
- Let $T_{n,k}$ be the number of labeled forests on n vertices with k components, such that vertex $1, 2, \dots, k$ belong to different components. Then $T_{n,k} = k n^{n-k-1}$.

8.1.4 Erdős-Gallai theorem

A sequence of non-negative integers $d_1 \geq d_2 \geq \dots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + d_2 + \dots + d_n$ is even and

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$$

holds for all $1 \leq k \leq n$.

8.1.5 Havel-Hakimi algorithm

find the vertex who has greatest degree unused, connect it with other greatest vertex.

8.1.6 Hall's marriage theorem

Let G be a finite bipartite graph with bipartite sets X and Y . For a subset W of X , let $N_G(W)$ denote the set of all vertices in Y adjacent to some element of W . Then there is an X -saturating matching iff $\forall W \subseteq X, |W| \leq |N_G(W)|$

8.1.7 Euler's planar graph formula

$$V - E + F = C + 1, E \leq 3V - 6(?)$$

8.1.8 Pick's theorem

For simple polygon, when points are all integer, we have $A = \#\{\text{lattice points in the interior}\} + \frac{\#\{\text{lattice points on the boundary}\}}{2} - 1$

8.1.9 Lucas's theorem

$\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p}$, where $m = m_k p^k + m_{k-1} p^{k-1} + \dots + m_1 p + m_0$, and $n = n_k p^k + n_{k-1} p^{k-1} + \dots + n_1 p + n_0$.

8.1.10 Matroid Intersection

Given matroids $M_1 = (G, I_1), M_2 = (G, I_2)$, find maximum $S \in I_1 \cap I_2$. For each iteration, build the directed graph and find a shortest path from s to t .

- $s \rightarrow x : S \sqcup \{x\} \in I_1$
- $x \rightarrow t : S \sqcup \{x\} \in I_2$
- $y \rightarrow x : S \setminus \{y\} \sqcup \{x\} \in I_1$ (y is in the unique circuit of $S \sqcup \{x\}$)
- $x \rightarrow y : S \setminus \{y\} \sqcup \{x\} \in I_2$ (y is in the unique circuit of $S \sqcup \{x\}$)

Alternate the path, and $|S|$ will increase by 1. Let $R = \min(\text{rank}(I_1), \text{rank}(I_2)), N = |G|$. In each iteration, $|E| = O(RN)$. For weighted case, assign weight $-w(x)$ and $w(x)$ to $x \in S$ and $x \notin S$, resp. Use Bellman-Ford to find the weighted shortest path. The maximum iteration of Bellman-Ford is $2R + 1$.

8.2 DP-opt Condition

8.2.1 totally monotone (concave/convex)

$$\begin{aligned} \forall i < i', j < j', B[i][j] \leq B[i'][j'] &\implies B[i][j'] \leq B[i'][j'] \\ \forall i < i', j < j', B[i][j] \geq B[i'][j'] &\implies B[i][j'] \geq B[i'][j'] \end{aligned}$$

8.2.2 monge condition (concave/convex)

$$\begin{aligned} \forall i < i', j < j', B[i][j] + B[i'][j'] &\geq B[i][j'] + B[i'][j] \\ \forall i < i', j < j', B[i][j] + B[i'][j'] &\leq B[i][j'] + B[i'][j] \end{aligned}$$

8.3 Convex 1D/1D DP

```
struct segment {
    int l, r;
    segment() {}
    segment(int a, int b, int c): l(a), r(b), c(c) {}
};
inline lld f(int l, int r){return dp[l] + w(l+1, r);}
void solve() {
    dp[0] = 0;
    deque<segment> dq; dq.push_back(segment(0, 1, n));
    for (int i = 1; i <= n; ++i) {
        dp[i] = f(dq.front().l, i);
        while(dq.size() && dq.front().r < i+1) dq.pop_front();
        dq.front().l = i + 1;
        segment seg = segment(i, i + 1, n);
        while (dq.size() &&
            f(i, dq.back().l) < f(dq.back().l, dq.back().r))
            dq.pop_back();
        if (dq.size()) {
            int d = 1 << 20, c = dq.back().r;
            while (d >= 1) if (c + d <= dq.back().r)
                if (f(i, c+d) > f(dq.back().l, c+d)) c += d;
            dq.back().r = c; seg.l = c + 1;
        }
        if (seg.l <= n) dq.push_back(seg);
    }
}
```

8.4 ConvexHull Optimization

```
struct L {
    mutable int64_t a, b, p;
    bool operator<(const L &r) const { return a < r.a; }
    bool operator<(int64_t x) const { return p < x; }
};
struct DynamicHull : multiset<L, less<>> {
    static const int64_t kInf = 1e18;
    bool Isect(iterator x, iterator y) {
        auto Div = [](int64_t a, int64_t b) {
            return a / b - ((a ^ b) < 0 && a % b);
        };
        if (y == end()) { x->p = kInf; return false; }
        if (x->a == y->a) x->p = x->b > y->b ? kInf : -kInf;
        else x->p = Div(y->b - x->b, x->a - y->a);
        return x->p >= y->p;
    }
    void Insert(int64_t a, int64_t b) {
        auto z = insert({a, b, 0}); y = z++; x = y;
        while (Isect(y, z)) z = erase(z);
        if (x != begin() && Isect(--x, y)) Isect(x, y=erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            Isect(x, erase(y));
    }
    int64_t Query(int64_t x) {
        auto l = *lower_bound(x);
        return l.a * x + l.b;
    }
};
```

8.5 Josephus Problem

```
// n people kill m for each turn
int f(int n, int m) {
    int s = 0;
    for (int i = 2; i <= n; ++i)
        s = (s + m) % i;
    return s;
}
// died at kth
int kth(int n, int m, int k){
    if (m == 1) return n-1;
    for (k = k*m+m-1; k >= n; k = k-n+(k-n)/(m-1));
    return k;
}
```

8.6 Cactus Matching

```
vector<int> init_g[maxn], g[maxn*2];
int n, dfn[maxn], low[maxn], par[maxn], dfs_idx, bcc_id;
void tarjan(int u){
    dfn[u]=low[u]=++dfs_idx;
    for(int i=0; i<(int)init_g[u].size(); ++i){
        int v=init_g[u][i];
        if(v==par[u]) continue;
        if(!dfn[v]){
```

```
par[v]=u;
tarjan(v);
low[u]=min(low[u], low[v]);
if(dfn[u]<low[v]){
    g[u].push_back(v);
    g[v].push_back(u);
}
}
else{
    low[u]=min(low[u], dfn[v]);
    if(dfn[v]<dfn[u]){
        int temp_v=u;
        bcc_id++;
        while(temp_v!=v){
            g[bcc_id+n].push_back(temp_v);
            g[temp_v].push_back(bcc_id+n);
            temp_v=par[temp_v];
        }
        g[bcc_id+n].push_back(v);
        g[v].push_back(bcc_id+n);
        reverse(g[bcc_id+n].begin(), g[bcc_id+n].end());
    }
}
}
}
int dp[maxn][2], min_dp[2][2], tmp[2][2], tp[2];
void dfs(int u, int fa){
    if(u<=n){
        for(int i=0; i<(int)g[u].size(); ++i){
            int v=g[u][i];
            if(v==fa) continue;
            dfs(v, u);
            memset(tp, 0x8f, sizeof tp);
            if(v<=n){
                tp[0]=dp[u][0]+max(dp[v][0], dp[v][1]);
                tp[1]=max(
                    dp[u][0]+dp[v][0]+1,
                    dp[u][1]+max(dp[v][0], dp[v][1])
                );
            }
            else{
                tp[0]=dp[u][0]+dp[v][0];
                tp[1]=max(dp[u][0]+dp[v][1], dp[u][1]+dp[v][0]);
            }
            dp[u][0]=tp[0], dp[u][1]=tp[1];
        }
    }
    else{
        for(int i=0; i<(int)g[u].size(); ++i){
            int v=g[u][i];
            if(v==fa) continue;
            dfs(v, u);
        }
        min_dp[0][0]=0;
        min_dp[1][1]=1;
        min_dp[0][1]=min_dp[1][0]=-0x3f3f3f3f;
        for(int i=0; i<(int)g[u].size(); ++i){
            int v=g[u][i];
            if(v==fa) continue;
            memset(tmp, 0x8f, sizeof tmp);
            tmp[0][0]=max(
                min_dp[0][0]+max(dp[v][0], dp[v][1]),
                min_dp[0][1]+dp[v][0]
            );
            tmp[0][1]=min_dp[0][0]+dp[v][0]+1;
            tmp[1][0]=max(
                min_dp[1][0]+max(dp[v][0], dp[v][1]),
                min_dp[1][1]+dp[v][0]
            );
            tmp[1][1]=min_dp[1][0]+dp[v][0]+1;
            memcpy(min_dp, tmp, sizeof tmp);
        }
        dp[u][1]=max(min_dp[0][1], min_dp[1][0]);
        dp[u][0]=min_dp[0][0];
    }
}
}
int main(){
    int m, a, b;
    scanf("%d%d", &n, &m);
    for(int i=0; i<m; ++i){
        scanf("%d%d", &a, &b);
        init_g[a].push_back(b);
        init_g[b].push_back(a);
    }
    par[1]=-1;
```

```

tarjan(1);
dfs(1, -1);
printf("%d\n", max(dp[1][0], dp[1][1]));
return 0;
}

```

8.7 Tree Knapsack

```

int dp[N][K]; PII obj[N];
vector<int> G[N];
void dfs(int u, int mx){
    for(int s: G[u]) {
        if(mx < obj[s].first) continue;
        for(int i=0; i<=mx-obj[s].FF; i++)
            dp[s][i] = dp[u][i];
        dfs(s, mx - obj[s].first);
        for(int i=obj[s].FF; i<=mx; i++)
            dp[u][i] = max(dp[u][i],
                dp[s][i - obj[s].FF] + obj[s].SS);
    }
}

```

8.8 N Queens Problem

```

vector<int> solve(int n) {
    // no solution when n=2, 3
    vector<int> ret;
    if (n % 6 == 2) {
        for (int i = 2; i <= n; i += 2)
            ret.push_back(i);
        ret.push_back(3); ret.push_back(1);
        for (int i = 7; i <= n; i += 2)
            ret.push_back(i);
        ret.push_back(5);
    } else if (n % 6 == 3) {
        for (int i = 4; i <= n; i += 2)
            ret.push_back(i);
        ret.push_back(2);
        for (int i = 5; i <= n; i += 2)
            ret.push_back(i);
        ret.push_back(1); ret.push_back(3);
    } else {
        for (int i = 2; i <= n; i += 2)
            ret.push_back(i);
        for (int i = 1; i <= n; i += 2)
            ret.push_back(i);
    }
    return ret;
}

```

8.9 Aliens Optimization

```

long long Alien() {
    long long c = kInf;
    for (int d = 60; d >= 0; --d) {
        // cost can be negative, depending on the problem.
        if (c - (1LL << d) < 0) continue;
        long long ck = c - (1LL << d);
        pair<long long, int> r = check(ck);
        if (r.second == k) return r.first - ck * k;
        if (r.second < k) c = ck;
    }
    pair<long long, int> r = check(c);
    return r.first - c * k;
}

```

8.10 Hilbert Curve

```

long long hilbert(int n, int x, int y) {
    long long res = 0;
    for (int s = n / 2; s; s >>= 1) {
        int rx = (x & s) > 0, ry = (y & s) > 0;
        res += s * 111 * s * ((3 * rx) ^ ry);
        if (ry == 0) {
            if (rx == 1) x = s - 1 - x, y = s - 1 - y;
            swap(x, y);
        }
    }
    return res;
}

```

8.11 Binary Search On Fraction

```

struct Q {
    ll p, q;
    Q go(Q b, ll d) { return {p + b.p*d, q + b.q*d}; }
};
bool pred(Q);
// returns smallest p/q in [lo, hi] such that
// pred(p/q) is true, and 0 <= p, q <= N
Q frac_bs(ll N) {
    Q lo{0, 1}, hi{1, 0};
    if (pred(lo)) return lo;
    assert(pred(hi));
    bool dir = 1, L = 1, H = 1;
    for (; L || H; dir = !dir) {
        ll len = 0, step = 1;
        for (int t = 0; t < 2 && (t ? step/=2 : step*=2);)
            if (Q mid = hi.go(lo, len + step);
                mid.p > N || mid.q > N || dir ^ pred(mid))
                t++;
            else len += step;
        swap(lo, hi = hi.go(lo, len));
        (dir ? L : H) = !len;
    }
    return dir ? hi : lo;
}

```