Contents

1 Basic

1.1 vimrc

```
set nu rnu cin ts=4 sw=4 autoread hls
sy on
map<leader>b :w<bar>!g++ -std=c++17 '%' -DKEV -fsanitize=undefined -o /tmp/.run<CR>
map<leader>r :w<bar>!cat 01.in && echo "---" && /tmp/.run < 01.in<CR>
map<leader>i :!/tmp/.run<CR>
map<leader>c I//<Esc>
map<leader>y :%y+<CR>
map<leader>l :%d<bar>0r ~/t.cpp<CR>
```

1.2 Default code

```
#include <bits/stdc++.h>
using namespace std;
using i64 = long long;
using ll = long long;
#define SZ(v) (ll)((v).size())
#define pb emplace_back
#define AI(i) begin(i), end(i)
#define X first
#define Y second
template<class T> bool chmin(T &a, T b) {
  return b < a && (a = b, true); }
template<class T> bool chmax(T &a, T b) {
  return a < b && (a = b, true); }
#ifdef KEV
#define DE(args...) kout("[ " + string(#args) + " ] = ", args)
void kout() { cerr << endl; }
template<class T, class ...U> void kout(T a, U ...b) { cerr << a << ' ', kout(b...); }
template<class T> void debug(T l, T r) {
  while (l != r) cerr << *l << " \n"[next(l)==r], ++l; }
#else
#define DE(...) 0
#define debug(...) 0
#endif
int main() {
  cin.tie(nullptr)->sync_with_stdio(false);
  return 0;
}
```

1.3 Fast Integer Input

```
char buf[1 << 16], *p1 = buf, *p2 = buf;
char get() {
  if (p1 == p2) {
    p1 = buf;
    p2 = p1 + fread(buf, 1, sizeof(buf), stdin);
  }
  if (p1 == p2)
    return -1;
  return *p1++;
}
char readChar() {
  char c = get();
  while (isspace(c))
    c = get();
  return c;
}
int readInt() {
  int x = 0;
  char c = get();
  while (!isdigit(c))
    c = get();
  while (isdigit(c)) {
    x = 10 * x + c - '0';
    c = get();
  }
  return x;
}
```

1.4 Pragma optimization

```
#pragma GCC optimize("Ofast", "no-stack-protector", "no-math-errno", "unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4,sse4.2,popcnt,abm,mmx,avx,tune=native,arch=core-avx2,tune=core-avx2")
#pragma GCC ivdep
```

2 Flows, Matching

2.1 Flow

```
template <typename F>
struct Flow {
  static constexpr F INF = numeric_limits<F>::max() / 2;
  struct Edge {
    int to;
    F cap;
    Edge(int to, F cap) : to(to), cap(cap) {}
  };
  int n;
  vector<Edge> e;
  vector<vector<int>> adj;
  vector<int> cur, h;
  Flow(int n) : n(n), adj(n) {}
  bool bfs(int s, int t) {
    h.assign(n, -1);
    queue<int> q;
    h[s] = 0;
    q.push(s);
    while (!q.empty()) {
      int u = q.front();
      q.pop();
      for (int i : adj[u]) {
        auto [v, c] = e[i];
        if (c > 0 && h[v] == -1) {
          h[v] = h[u] + 1;
          if (v == t) { return true; }
          q.push(v);
        }
      }
    }
    return false;
  }
  F dfs(int u, int t, F f) {
    if (u == t) { return f; }
    F r = f;
    for (int &i = cur[u]; i < int(adj[u].size()); i++) {
      int j = adj[u][i];
      auto [v, c] = e[j];
      if (c > 0 && h[v] == h[u] + 1) {
        F a = dfs(v, t, min(r, c));
        e[j].cap -= a;
        e[j ^ 1].cap += a;
        r -= a;
        if (r == 0) { return f; }
      }
    }
    return f - r;
  }
  // can be bidirectional
  void addEdge(int u, int v, F cf = INF, F cb = 0) {
    adj[u].push_back(e.size()), e.emplace_back(v, cf);
```

```
    adj[v].push_back(e.size()), e.
        emplace_back(u, cb);
  }
  F maxFlow(int s, int t) {
    F ans = 0;
    while (bfs(s, t)) {
      cur.assign(n, 0);
      ans += dfs(s, t, INF);
    }
    return ans;
  }
  // do max flow first
  vector<int> minCut() {
    vector<int> res(n);
    for (int i = 0; i < n; i++) { res[i]
        = h[i] != -1; }
    return res;
  }
};
```

2.2 MCMF

```
template <class Flow, class Cost>
struct MinCostMaxFlow {
public:
  static constexpr Flow flowINF =
      numeric_limits<Flow>::max();
  static constexpr Cost costINF =
      numeric_limits<Cost>::max();
  MinCostMaxFlow() {}
  MinCostMaxFlow(int n) : n(n), g(n) {}
  int addEdge(int u, int v, Flow cap,
      Cost cost) {
    int m = int(pos.size());
    pos.push_back({u, int(g[u].size())});
    g[u].push_back({v, int(g[v].size()),
        cap, cost});
    g[v].push_back({u, int(g[u].size()) -
         1, 0, -cost});
    return m;
  }
  struct edge {
    int u, v;
    Flow cap, flow;
    Cost cost;
  };
  edge getEdge(int i) {
    auto _e = g[pos[i].first][pos[i].
        second];
    auto _re = g[_e.v][_e.rev];
    return {pos[i].first, _e.v, _e.cap +
        _re.cap, _re.cap, _e.cost};
  }
  vector<edge> edges() {
    int m = int(pos.size());
    vector<edge> result(m);
    for (int i = 0; i < m; i++) { result[
        i] = getEdge(i); }
    return result;
  }
  pair<Flow, Cost> maxFlow(int s, int t,
      Flow flow_limit = flowINF) {
      return slope(s, t, flow_limit).
      back(); }
  vector<pair<Flow, Cost>> slope(int s,
      int t, Flow flow_limit = flowINF)
      {
    vector<Cost> dual(n, 0), dis(n);
    vector<int> pv(n), pe(n), vis(n);
    auto dualRef = [&]() {
      fill(dis.begin(), dis.end(),
          costINF);
      fill(pv.begin(), pv.end(), -1);
      fill(pe.begin(), pe.end(), -1);
      fill(vis.begin(), vis.end(), false)
          ;
      struct Q {
        Cost key;
        int u;
        bool operator<(Q o) const {
            return key > o.key; }
      };
      priority_queue<Q> h;
      dis[s] = 0;
      h.push({0, s});
      while (!h.empty()) {
        int u = h.top().u;
        h.pop();
        if (vis[u]) { continue; }
        vis[u] = true;
        if (u == t) { break; }
        for (int i = 0; i < int(g[u].size
            ()); i++) {
          auto e = g[u][i];
          if (vis[e.v] || e.cap == 0)
              continue;
          Cost cost = e.cost - dual[e.v]
              + dual[u];
          if (dis[e.v] - dis[u] > cost) {
            dis[e.v] = dis[u] + cost;
            pv[e.v] = u;
            pe[e.v] = i;
            h.push({dis[e.v], e.v});
          }
        }
      }
      if (!vis[t]) { return false; }
      for (int v = 0; v < n; v++) {
        if (!vis[v]) continue;
        dual[v] -= dis[t] - dis[v];
      }
      return true;
    };
    Flow flow = 0;
    Cost cost = 0, prevCost = -1;
    vector<pair<Flow, Cost>> result;
    result.push_back({flow, cost});
    while (flow < flow_limit) {
      if (!dualRef()) break;
      Flow c = flow_limit - flow;
      for (int v = t; v != s; v = pv[v])
          {
        c = min(c, g[pv[v]][pe[v]].cap);
      }
      for (int v = t; v != s; v = pv[v])
          {
        auto& e = g[pv[v]][pe[v]];
        e.cap -= c;
        g[v][e.rev].cap += c;
      }
      Cost d = -dual[s];
      flow += c;
      cost += c * d;
      if (prevCost == d) { result.
          pop_back(); }
      result.push_back({flow, cost});
      prevCost = cost;
    }
    return result;
  }
private:
  int n;
  struct _edge {
    int v, rev;
    Flow cap;
    Cost cost;
  };
  vector<pair<int, int>> pos;
  vector<vector<_edge>> g;
};
```

2.3 GomoryHu Tree

```
auto gomory(int n, vector<array<int, 3>>
    e) {
  Flow<int, int> mf(n);
  for (auto [u, v, c] : e) { mf.addEdge(u
      , v, c, c); }
  vector<array<int, 3>> res;
  vector<int> p(n);
  for (int i = 1; i < n; i++) {
    for (int j = 0; j < int(e.size()); j
        ++) { mf.e[j << 1].cap = mf.e[j
        << 1 | 1].cap = e[j][2]; }
    int f = mf.maxFlow(i, p[i]);
    auto cut = mf.minCut();
    for (int j = i + 1; j < n; j++) { if
        (cut[i] == cut[j] && p[i] == p[j
        ]) { p[j] = i; }}
    res.push_back({f, i, p[i]});
  }
  return res;
}
```

2.4 Global Minimum Cut

```
// O(V ^ 3)
template <typename F>
struct GlobalMinCut {
  static constexpr int INF =
      numeric_limits<F>::max() / 2;
  int n;
  vector<int> vis, wei;
  vector<vector<int>> adj;
  GlobalMinCut(int n) : n(n), vis(n), wei
      (n), adj(n, vector<int>(n)) {}
  void addEdge(int u, int v, int w){
    adj[u][v] += w;
    adj[v][u] += w;
  }
  int solve() {
    int sz = n;
    int res = INF, x = -1, y = -1;
    auto search = [&]() {
      fill(vis.begin(), vis.begin() + sz,
          0);
      fill(wei.begin(), wei.begin() + sz,
          0);
      x = y = -1;
      int mx, cur;
      for (int i = 0; i < sz; i++) {
        mx = -1, cur = 0;
        for (int j = 0; j < sz; j++) {
          if (wei[j] > mx) {
            mx = wei[j], cur = j;
          }
        }
        vis[cur] = 1, wei[cur] = -1;
        x = y;
        y = cur;
        for (int j = 0; j < sz; j++) {
          if (!vis[j]) {
            wei[j] += adj[cur][j];
          }
        }
      }
      return mx;
    };
    while (sz > 1) {
      res = min(res, search());
      for (int i = 0; i < sz; i++) {
        adj[x][i] += adj[y][i];
        adj[i][x] = adj[x][i];
      }
      for (int i = 0; i < sz; i++) {
        adj[y][i] = adj[sz - 1][i];
        adj[i][y] = adj[i][sz - 1];
      }
      sz--;
    }
    return res;
  }
};
```

2.5 Bipartite Matching

```
struct BipartiteMatching {
  int n, m;
  vector<vector<int>> adj;
  vector<int> l, r, dis, cur;
  BipartiteMatching(int n, int m) : n(n),
       m(m), adj(n), l(n, -1), r(m, -1),
       dis(n), cur(n) {}
  void addEdge(int u, int v) { adj[u].
      push_back(v); }
  void bfs() {
    vector<int> q;
    for (int u = 0; u < n; u++) {
      if (l[u] == -1) {
        q.push_back(u), dis[u] = 0;
      } else {
        dis[u] = -1;
      }
    }
    for (int i = 0; i < int(q.size()); i
        ++) {
      int u = q[i];
      for (auto v : adj[u]) {
        if (r[v] != -1 && dis[r[v]] ==
            -1) {
          dis[r[v]] = dis[u] + 1;
```

```
        q.push_back(r[v]);
      }
    }
  }
}
bool dfs(int u) {
  for (int &i = cur[u]; i < int(adj[u].size()); i++) {
    int v = adj[u][i];
    if (r[v] == -1 || dis[r[v]] == dis[u] + 1 && dfs(r[v])) {
      l[u] = v, r[v] = u;
      return true;
    }
  }
  return false;
}
int maxMatching() {
  int match = 0;
  while (true) {
    bfs();
    fill(cur.begin(), cur.end(), 0);
    int cnt = 0;
    for (int u = 0; u < n; u++) {
      if (l[u] == -1) {
        cnt += dfs(u);
      }
    }
    if (cnt == 0) {
      break;
    }
    match += cnt;
  }
  return match;
}
auto minVertexCover() {
  vector<int> L, R;
  for (int u = 0; u < n; u++) {
    if (dis[u] == -1) {
      L.push_back(u);
    } else if (l[u] != -1) {
      R.push_back(l[u]);
    }
  }
  return pair(L, R);
}
};
```

2.6 GeneralMatching

```
struct GeneralMatching {
  int n;
  vector<vector<int>> adj;
  vector<int> match;
  GeneralMatching(int n) : n(n), adj(n), match(n, -1) {}
  void addEdge(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
  }
  int maxMatching() {
    vector<int> vis(n), link(n), f(n), dep(n);
    auto find = [&](int u) {
      while (f[u] != u) { u = f[u] = f[f[u]]; }
      return u;
    };
    auto lca = [&](int u, int v) {
      u = find(u);
      v = find(v);
      while (u != v) {
        if (dep[u] < dep[v]) { swap(u, v); }
        u = find(link[match[u]]);
      }
      return u;
    };
    queue<int> q;
    auto blossom = [&](int u, int v, int p) {
      while (find(u) != p) {
        link[u] = v;
        v = match[u];
        if (vis[v] == 0) {
          vis[v] = 1;
          q.push(v);
        }
        f[u] = f[v] = p;
        u = link[v];
      }
    };
    auto augment = [&](int u) {
      while (!q.empty()) { q.pop(); }
      iota(f.begin(), f.end(), 0);
      fill(vis.begin(), vis.end(), -1);
      q.push(u), vis[u] = 1, dep[u] = 0;
      while (!q.empty()){
        int u = q.front();
        q.pop();
        for (auto v : adj[u]) {
          if (vis[v] == -1) {
            vis[v] = 0;
            link[v] = u;
            dep[v] = dep[u] + 1;
            if (match[v] == -1) {
              for (int x = v, y = u, tmp; y != -1; x = tmp, y = x == -1 ? -1 : link[x]) {
                tmp = match[y], match[x] = y, match[y] = x;
              }
              return true;
            }
            q.push(match[v]), vis[match[v]] = 1, dep[match[v]] = dep[u] + 2;
          } else if (vis[v] == 1 && find(v) != find(u)) {
            int p = lca(u, v);
            blossom(u, v, p), blossom(v, u, p);
          }
        }
      }
      return false;
    };
    int res = 0;
    for (int u = 0; u < n; ++u) { if (match[u] == -1) { res += augment(u); } }
    return res;
  }
};
```

2.7 Kuhn Munkres

```
// need perfect matching or not : w intialize with -INF / 0
template <typename Cost>
struct KM {
  static constexpr Cost INF = numeric_limits<Cost>::max() / 2;
  int n;
  vector<Cost> hl, hr, slk;
  vector<int> l, r, pre, vl, vr;
  queue<int> q;
  vector<vector<Cost>> w;
  KM(int n) : n(n), hl(n), hr(n), slk(n), l(n, -1), r(n, -1), pre(n), vl(n), vr(n),
    w(n, vector<Cost>(n, -INF)) {}
  bool check(int x) {
    vl[x] = true;
    if (l[x] != -1) {
      q.push(l[x]);
      return vr[l[x]] = true;
    }
    while (x != -1) { swap(x, r[l[x] = pre[x]]); }
    return false;
  }
  void bfs(int s) {
    fill(slk.begin(), slk.end(), INF);
    fill(vl.begin(), vl.end(), false);
    fill(vr.begin(), vr.end(), false);
    q = {};
    q.push(s);
    vr[s] = true;
    while (true) {
      Cost d;
      while (!q.empty()) {
        int y = q.front();
        q.pop();
        for (int x = 0; x < n; ++x) {
          if (!vl[x] && slk[x] >= (d = hl[x] + hr[y] - w[x][y])) {
            pre[x] = y;
            if (d != 0) {
              slk[x] = d;
            } else if (!check(x)) {
              return;
            }
          }
        }
      }
      d = INF;
      for (int x = 0; x < n; ++x) { if (!vl[x] && d > slk[x]) { d = slk[x]; }}
      for (int x = 0; x < n; ++x) {
        if (vl[x]) {
          hl[x] += d;
        } else {
          slk[x] -= d;
        }
        if (vr[x]) { hr[x] -= d; }
      }
      for (int x = 0; x < n; ++x) { if (!vl[x] && !slk[x] && !check(x)) { return; }}
    }
  }
  void addEdge(int u, int v, Cost x) { w[u][v] = max(w[u][v], x); }
  Cost solve() {
    for (int i = 0; i < n; ++i) { hl[i] = *max_element(w[i].begin(), w[i].end()); }
    for (int i = 0; i < n; ++i) { bfs(i); }
    Cost res = 0;
    for (int i = 0; i < n; ++i) { res += w[i][l[i]]; }
    return res;
  }
};
```

2.8 Flow Models

- Maximum/Minimum flow with lower bound / Circulation problem
  1. Construct super source S and sink T.
  2. For each edge (x, y, l, u), connect $x \to y$ with capacity $u - l$.
  3. For each vertex v, denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  4. If $in(v) > 0$, connect $S \to v$ with capacity $in(v)$, otherwise, connect $v \to T$ with capacity $-in(v)$.
     - To maximize, connect $t \to s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T. If $f \neq \sum_{v \in V, in(v)>0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
     - To minimize, let f be the maximum flow from S to T. Connect $t \to s$ with capacity ∞ and let the flow from S to T be f'. If $f + f' \neq \sum_{v \in V, in(v)>0} in(v)$, there's no solution. Otherwise, f' is the answer.
  5. The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
  1. Redirect every edge: $y \to x$ if $(x, y) \in M$, $x \to y$ otherwise.
  2. DFS from unmatched vertices in X.
  3. $x \in X$ is chosen iff x is unvisited.
  4. $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
  1. Consruct super source S and sink T
  2. For each edge (x, y, c), connect $x \to y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \to x$ with $(cost, cap) = (-c, 1)$

3. For each edge with $c < 0$, sum these cost as K, then increase $d(y)$ by 1, decrease $d(x)$ by 1
4. For each vertex v with $d(v) > 0$, connect $S \to v$ with $(cost, cap) = (0, d(v))$
5. For each vertex v with $d(v) < 0$, connect $v \to T$ with $(cost, cap) = (0, -d(v))$
6. Flow from S to T, the answer is the cost of the flow $C + K$

- Maximum density induced subgraph
  1. Binary search on answer, suppose we're checking answer T
  2. Construct a max flow model, let K be the sum of all weights
  3. Connect source $s \to v$, $v \in G$ with capacity K
  4. For each edge (u, v, w) in G, connect $u \to v$ and $v \to u$ with capacity w
  5. For $v \in G$, connect it with sink $v \to t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
  6. T is a valid answer if the maximum flow $f < K|V|$

- Minimum weight edge cover
  1. For each $v \in V$ create a copy v', and connect $u' \to v'$ with weight $w(u, v)$.
  2. Connect $v \to v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v.
  3. Find the minimum weight perfect matching on G'.

- Project selection problem
  1. If $p_v > 0$, create edge (s, v) with capacity p_v; otherwise, create edge (v, t) with capacity $-p_v$.
  2. Create edge (u, v) with capacity w with w being the cost of choosing u without choosing v.
  3. The mincut is equivalent to the maximum profit of a subset of projects.

- 0/1 quadratic programming

  $$\sum_x c_x x + \sum_y c_y \bar{y} + \sum_{xy} c_{xy} x \bar{y} + \sum_{xyx'y'} c_{xyx'y'} (x$$

  can be minimized by the mincut of the following graph:

  1. Create edge (x, t) with capacity c_x and create edge (s, y) with capacity c_y.
  2. Create edge (x, y) with capacity c_{xy}.
  3. Create edge (x, y) and edge (x', y') with capacity $c_{xyx'y'}$.

3 Data Structure

3.1 <ext/pbds>

```cpp
#include <bits/extc++.h>
#include <ext/rope>
using namespace __gnu_pbds;
using namespace __gnu_cxx;
#include <ext/pb_ds/assoc_container.hpp>
typedef tree<int, null_type, std::less<
    int>, rb_tree_tag,
    tree_order_statistics_node_update>
    tree_set;
typedef cc_hash_table<int, int> umap;
typedef priority_queue<int> heap;

int main() {
  // rb tree
  tree_set s;
  s.insert(71); s.insert(22);
  assert(*s.find_by_order(0) == 22);
      assert(*s.find_by_order(1) == 71);
  assert(s.order_of_key(22) == 0); assert
      (s.order_of_key(71) == 1);
  s.erase(22);
  assert(*s.find_by_order(0) == 71);
      assert(s.order_of_key(71) == 0);
  // mergable heap
  heap a, b; a.join(b);
  // persistant
  rope<char> r[2];
  r[1] = r[0];
  std::string st = "abc";
  r[1].insert(0, st.c_str());
  r[1].erase(1, 1);
  std::cout << r[1].substr(0, 2) << std::
      endl;
  return 0;
}
```

3.2 Li Chao Tree

```cpp
constexpr i64 INF = 4e18;
struct Line {
  i64 a, b;
  Line() : a(0), b(INF) {}
  Line(i64 a, i64 b) : a(a), b(b) {}
  i64 operator()(i64 x) { return a * x +
      b; }
};
// [, ) !!!!!!!!!!!
struct Lichao {
  int n;
  vector<int> vals;
  vector<Line> lines;
  Lichao() {}
  void init(const vector<int> &v) {
    n = v.size();
    vals = v;
    sort(vals.begin(), vals.end());
    vals.erase(unique(vals.begin(), vals.
        end()), vals.end());
    lines.assign(4 * n, {});
  }
  int get(int x) { return lower_bound(
      vals.begin(), vals.end(), x) -
      vals.begin(); }
  void apply(Line p, int id, int l, int r
      ) {
    Line &q = lines[id];
    if (p(vals[l]) < q(vals[l])) { swap(p
        , q); }
    if (l + 1 == r) { return; }
    int m = l + r >> 1;
    if (p(vals[m]) < q(vals[m])) {
      swap(p, q);
      apply(p, id << 1, l, m);
    } else {
      apply(p, id << 1 | 1, m, r);
    }
  }
  void add(int ql, int qr, Line p) {
    ql = get(ql), qr = get(qr);
    auto go = [&](auto go, int id, int l,
        int r) -> void {
      if (qr <= l || r <= ql) { return; }
      if (ql <= l && r <= qr) {
        apply(p, id, l, r);
        return;
      }
      int m = l + r >> 1;
      go(go, id << 1, l, m);
      go(go, id << 1 | 1, m, r);
    };
    go(go, 1, 0, n);
  }
  i64 query(int p) {
    p = get(p);
    auto go = [&](auto go, int id, int l,
        int r) -> i64 {
      if (l + 1 == r) { return lines[id](
          vals[p]); }
      int m = l + r >> 1;
      return min(lines[id](vals[p]), p <
          m ? go(go, id << 1, l, m) : go
          (go, id << 1 | 1, m, r));
    };
    return go(go, 1, 0, n);
  }
};
```

3.3 Treap

```cpp
struct Treap {
    Treap *lc = nullptr, *rc = nullptr;
    int sz = 1;
    unsigned w = rng();
    i64 m = 0, b = 0, val = 0;
};
int size(Treap *t) {
    return t == nullptr ? 0 : t->sz;
}
void apply(Treap *t, i64 m, i64 b) {
    t->m += m;
    t->b += b;
    t->val += m * size(t->lc) + b;
}
void pull(Treap *t) {
    t->sz = size(t->lc) + size(t->rc) +
        1;
}
void push(Treap *t) {
    if (t->lc != nullptr) {
        apply(t->lc, t->m, t->b);
    }
    if (t->rc != nullptr) {
        apply(t->rc, t->m, t->b + t->m *
            (size(t->lc) + 1));
    }
    t->m = t->b = 0;
}
pair<Treap*, Treap*> split(Treap *t, int
    s) {
    if (t == nullptr) { return {t, t}; }
    push(t);
    Treap *a, *b;
    if (s <= size(t->lc)) {
        b = t;
        tie(a, b->lc) = split(t->lc, s);
    } else {
        a = t;
        tie(a->rc, b) = split(t->rc, s -
            size(t->lc) - 1);
    }
    pull(t);
    return {a, b};
}
Treap* merge(Treap *t1, Treap *t2) {
    if (t1 == nullptr) { return t2; }
    if (t2 == nullptr) { return t1; }
    push(t1), push(t2);
    if (t1->w > t2->w) {
        t1->rc = merge(t1->rc, t2);
        pull(t1);
        return t1;
    } else {
        t2->lc = merge(t1, t2->lc);
        pull(t2);
        return t2;
    }
}
int rnk(Treap *t, i64 val) {
    int res = 0;
    while (t != nullptr) {
        push(t);
        if (val <= t->val) {
            res += size(t->lc) + 1;
            t = t->rc;
        } else {
            t = t->lc;
        }
    }
    return res;
}
Treap* join(Treap *t1, Treap *t2) {
    if (size(t1) > size(t2)) {
        swap(t1, t2);
    }
    Treap *t = nullptr;
    while (t1 != nullptr) {
        auto [u1, v1] = split(t1, 1);
        t1 = v1;
        int r = rnk(t2, u1->val);
        if (r > 0) {
            auto [u2, v2] = split(t2, r);
            t = merge(t, u2);
            t2 = v2;
        }
        t = merge(t, u1);
    }
    t = merge(t, t2);
    return t;
}
```

3.4 Link-Cut Tree

```
struct Splay {
  array<Splay*, 2> ch = {nullptr, nullptr
      };
  Splay* fa = nullptr;
  int sz = 1;
  bool rev = false;
  Splay() {}
  void applyRev(bool x) {
    if (x) {
      swap(ch[0], ch[1]);
      rev ^= 1;
    }
  }
  void push() {
    for (auto k : ch) {
      if (k) {
        k->applyRev(rev);
      }
    }
    rev = false;
  }
  void pull() {
    sz = 1;
    for (auto k : ch) {
      if (k) {
      }
    }
  }
  int relation() { return this == fa->ch
      [1]; }
  bool isRoot() { return !fa || fa->ch[0]
       != this && fa->ch[1] != this; }
  void rotate() {
    Splay *p = fa;
    bool x = !relation();
    p->ch[!x] = ch[x];
    if (ch[x]) { ch[x]->fa = p; }
    fa = p->fa;
    if (!p->isRoot()) { p->fa->ch[p->
        relation()] = this; }
    ch[x] = p;
    p->fa = this;
    p->pull();
  }
  void splay() {
    vector<Splay*> s;
    for (Splay *p = this; !p->isRoot(); p
         = p->fa) { s.push_back(p->fa);
        }
    while (!s.empty()) {
      s.back()->push();
      s.pop_back();
    }
    push();
    while (!isRoot()) {
      if (!fa->isRoot()) {
        if (relation() == fa->relation())
            {
          fa->rotate();
        } else {
          rotate();
        }
      }
      rotate();
    }
    pull();
  }
  void access() {
    for (Splay *p = this, *q = nullptr; p
        ; q = p, p = p->fa) {
      p->splay();
      p->ch[1] = q;
      p->pull();
    }
    splay();
  }
  void makeRoot() {
    access();
    applyRev(true);
  }
  Splay* findRoot() {
    access();
    Splay *p = this;
    while (p->ch[0]) { p = p->ch[0]; }
    p->splay();
    return p;
  }
  friend void split(Splay *x, Splay *y) {
    x->makeRoot();
    y->access();
  }
  // link if not connected
  friend void link(Splay *x, Splay *y) {
    x->makeRoot();
    if (y->findRoot() != x) {
      x->fa = y;
    }
  }
  // delete edge if doesn't exist
  friend void cut(Splay *x, Splay *y) {
    split(x, y);
    if (x->fa == y && !x->ch[1]) {
      x->fa = y->ch[0] = nullptr;
      x->pull();
    }
  }
  bool connected(Splay *x, Splay *y) {
    return x->findRoot() == y->findRoot()
        ;
  }
};
```

4 Graph

4.1 2-Edge-Connected Components

```
struct EBCC {
  int n, cnt = 0, T = 0;
  vector<vector<int>> adj, comps;
  vector<int> stk, dfn, low, id;
  EBCC(int n) : n(n), adj(n), dfn(n, -1),
       low(n), id(n, -1) {}
  void addEdge(int u, int v) { adj[u].
      push_back(v), adj[v].push_back(u);
      }
  void build() { for (int i = 0; i < n; i
      ++) { if (dfn[i] == -1) { dfs(i,
      -1); }}}
  void dfs(int u, int p) {
    dfn[u] = low[u] = T++;
    stk.push_back(u);
    for (auto v : adj[u]) {
      if (v == p) { continue; }
      if (dfn[v] == -1) {
        dfs(v, u);
        low[u] = min(low[u], low[v]);
      } else if (id[v] == -1) {
        low[u] = min(low[u], dfn[v]);
      }
    }
    if (dfn[u] == low[u]) {
      int x;
      comps.emplace_back();
      do {
        x = stk.back();
        comps.back().push_back(x);
        id[x] = cnt;
        stk.pop_back();
      } while (x != u);
      cnt++;
    }
  }
};
```

4.2 2-Vertex-Connected Components

```
// is articulation point if appear in >=
    2 comps
auto dfs = [&](auto dfs, int u, int p) ->
     void {
  dfn[u] = low[u] = T++;
  for (auto v : adj[u]) {
    if (v == p) { continue; }
    if (dfn[v] == -1) {
      stk.push_back(v);
      dfs(dfs, v, u);
      low[u] = min(low[u], low[v]);
      if (low[v] >= dfn[u]) {
        comps.emplace_back();
        int x;
        do {
          x = stk.back();
          cnt[x]++;
          stk.pop_back();
        } while (x != v);
        comps.back().push_back(u);
        cnt[u]++;
      }
    } else {
      low[u] = min(low[u], dfn[v]);
    }
  }
};
for (int i = 0; i < n; i++) {
  if (!adj[i].empty()) {
    dfs(dfs, i, -1);
  } else {
    comps.push_back({i});
  }
}
```

4.3 3-Edge-Connected Components

```
// DSU
struct ETCC {
  int n, cnt = 0;
  vector<vector<int>> adj, comps;
  vector<int> in, out, low, up, nx, id;
  ETCC(int n) : n(n), adj(n), in(n, -1),
      out(in), low(n), up(n), nx(in), id
      (in) {}
  void addEdge(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
  }
  void build() {
    int T = 0;
    DSU d(n);
    auto merge = [&](int u, int v) {
      d.join(u, v);
      up[u] += up[v];
    };
    auto dfs = [&](auto dfs, int u, int p
        ) -> void {
      in[u] = low[u] = T++;
      for (auto v : adj[u]) {
        if (v == u) { continue; }
        if (v == p) {
          p = -1;
          continue;
        }
        if (in[v] == -1) {
          dfs(dfs, v, u);
          if (nx[v] == -1 && up[v] <= 1)
              {
            up[u] += up[v];
            low[u] = min(low[u], low[v]);
            continue;
          }
          if (up[v] == 0) { v = nx[v]; }
          if (low[u] > low[v]) { low[u] =
               low[v], swap(nx[u], v); }
          while (v != -1) { merge(u, v);
               v = nx[v]; }
        } else if (in[v] < in[u]) {
          low[u] = min(low[u], in[v]);
          up[u]++;
        } else {
          for (int &x = nx[u]; x != -1 &&
               in[x] <= in[v] && in[v] <
               out[x]; x = nx[x]) {
            merge(u, x);
          }
          up[u]--;
        }
      }
      out[u] = T;
    };
    for (int i = 0; i < n; i++) { if (in[
        i] == -1) { dfs(dfs, i, -1); }}
    for (int i = 0; i < n; i++) { if (d.
        find(i) == i) { id[i] = cnt++;
```

```
    }}
    comps.resize(cnt);
    for (int i = 0; i < n; i++) { comps[id[d.find(i)]].push_back(i); }
  }
};
```

4.4 Heavy-Light Decomposition

```
struct HLD {
  int n, cur = 0;
  vector<int> sz, top, dep, par, tin, tout, seq;
  vector<vector<int>> adj;
  HLD(int n) : n(n), sz(n, 1), top(n), dep(n), par(n), tin(n), tout(n), seq(n), adj(n) {}
  void addEdge(int u, int v) { adj[u].push_back(v), adj[v].push_back(u); }
  void build(int root = 0) {
    top[root] = root, dep[root] = 0, par[root] = -1;
    dfs1(root), dfs2(root);
  }
  void dfs1(int u) {
    if (auto it = find(adj[u].begin(), adj[u].end(), par[u]); it != adj[u].end()) {
      adj[u].erase(it);
    }
    for (auto &v : adj[u]) {
      par[v] = u;
      dep[v] = dep[u] + 1;
      dfs1(v);
      sz[u] += sz[v];
      if (sz[v] > sz[adj[u][0]]) { swap(v, adj[u][0]); }
    }
  }
  void dfs2(int u) {
    tin[u] = cur++;
    seq[tin[u]] = u;
    for (auto v : adj[u]) {
      top[v] = v == adj[u][0] ? top[u] : v;
      dfs2(v);
    }
    tout[u] = cur - 1;
  }
  int lca(int u, int v) {
    while (top[u] != top[v]) {
      if (dep[top[u]] > dep[top[v]]) {
        u = par[top[u]];
      } else {
        v = par[top[v]];
      }
    }
    return dep[u] < dep[v] ? u : v;
  }
  int dist(int u, int v) { return dep[u] + dep[v] - 2 * dep[lca(u, v)]; }
  int jump(int u, int k) {
    if (dep[u] < k) { return -1; }
    int d = dep[u] - k;
    while (dep[top[u]] > d) { u = par[top[u]]; }
    return seq[tin[u] - dep[u] + d];
  }
  // u is v's ancestor
  bool isAncestor(int u, int v) { return tin[u] <= tin[v] && tin[v] <= tout[u]; }
  // root's parent is itself
  int rootedParent(int r, int u) {
    if (r == u) { return u; }
    if (isAncestor(r, u)) { return par[u]; }
    auto it = upper_bound(adj[u].begin(), adj[u].end(), r, [&](int x, int y) {
      return tin[x] < tin[y];
    }) - 1;
    return *it;
  }
  // rooted at u, v's subtree size
  int rootedSize(int r, int u) {
    if (r == u) { return n; }
    if (isAncestor(u, r)) { return sz[u]; }
    return n - sz[rootedParent(r, u)];
  }
  int rootedLca(int r, int a, int b) {
    return lca(a, b) ^ lca(a, r) ^ lca(b, r); }
};
```

4.5 Centroid Decomposition

```
vector<int> sz(n), vis(n);
auto build = [&](auto build, int u, int p) -> void {
  sz[u] = 1;
  for (auto v : g[u]) {
    if (v != p && !vis[v]) {
      build(build, v, u);
      sz[u] += sz[v];
    }
  }
};
auto find = [&](auto find, int u, int p, int tot) -> int {
  for (auto v : g[u]) {
    if (v != p && !vis[v] && 2 * sz[v] > tot) {
      return find(find, v, u, tot);
    }
  }
  return u;
};

auto dfs = [&](auto dfs, int cen) -> void {
  build(build, cen, -1);
  cen = find(find, cen, -1, sz[cen]);
  vis[cen] = 1;
  build(build, cen, -1);

  for (auto v : g[cen]) {
    if (!vis[v]) {
      dfs(dfs, v);
    }
  }
};
dfs(dfs, 0);
```

4.6 Strongly Connected Components

```
struct SCC {
  int n, cnt = 0, cur = 0;
  vector<int> id, dfn, low, stk;
  vector<vector<int>> adj, comps;
  void addEdge(int u, int v) { adj[u].push_back(v); }
  SCC(int n) : n(n), id(n, -1), dfn(n, -1), low(n, -1), adj(n) {}
  void build() {
    auto dfs = [&](auto dfs, int u) -> void {
      dfn[u] = low[u] = cur++;
      stk.push_back(u);
      for (auto v : adj[u]) {
        if (dfn[v] == -1) {
          dfs(dfs, v);
          low[u] = min(low[u], low[v]);
        } else if (id[v] == -1) {
          low[u] = min(low[u], dfn[v]);
        }
      }
      if (dfn[u] == low[u]) {
        int v;
        comps.emplace_back();
        do {
          v = stk.back();
          comps.back().push_back(v);
          id[v] = cnt;
          stk.pop_back();
        } while (u != v);
        cnt++;
      }
    };
    for (int i = 0; i < n; i++) { if (dfn[i] == -1) { dfs(dfs, i); }}
    for (int i = 0; i < n; i++) { id[i] = cnt - 1 - id[i]; }
    reverse(comps.begin(), comps.end());
  }
  // the comps are in topological sorted order
};
```

4.7 2-SAT

```
struct TwoSat {
  int n, N;
  vector<vector<int>> adj;
  vector<int> ans;
  TwoSat(int n) : n(n), N(n), adj(2 * n) {}
  // u == x
  void addClause(int u, bool x) { adj[2 * u + !x].push_back(2 * u + x); }
  // u == x || v == y
  void addClause(int u, bool x, int v, bool y) {
    adj[2 * u + !x].push_back(2 * v + y);
    adj[2 * v + !y].push_back(2 * u + x);
  }
  // u == x -> v == y
  void addImply(int u, bool x, int v, bool y) { addClause(u, !x, v, y); }
  void addVar() {
    adj.emplace_back(), adj.emplace_back();
    N++;
  }
  // at most one in var is true
  // adds prefix or as supplementary variables
  void atMostOne(const vector<pair<int, bool>> &vars) {
    int sz = vars.size();
    for (int i = 0; i < sz; i++) {
      addVar();
      auto [u, x] = vars[i];
      addImply(u, x, N - 1, true);
      if (i > 0) {
        addImply(N - 2, true, N - 1, true);
        addClause(u, !x, N - 2, false);
      }
    }
  }
  // does not return supplementary variables from atMostOne()
  bool satisfiable() {
    // run tarjan scc on 2 * N
    for (int i = 0; i < 2 * N; i++) { if (dfn[i] == -1) { dfs(dfs, i); }}
    for (int i = 0; i < N; i++) { if (id[2 * i] == id[2 * i + 1]) { return false; }}
    ans.resize(n);
    for (int i = 0; i < n; i++) { ans[i] = id[2 * i] > id[2 * i + 1]; }
    return true;
  }
};
```

4.8 count 3-cycles and 4-cycles

```
sort(ord.begin(), ord.end(), [&](auto i, auto j) { return pair(deg[i], i) > pair(deg[j], j); });
for (int i = 0; i < n; i++) { rnk[ord[i]] = i; }
if (rnk[u] < rnk[v]) { dag[u].push_back(v); }
// c3
for (int x = 0; x < n; x++) {
  for (auto y : dag[x]) { vis[y] = 1; }
  for (auto y : dag[x]) { for (auto z : dag[y]) { ans += vis[z]; }}
  for (auto y : dag[x]) { vis[y] = 0; }
```

```
}
// c4
for (int x = 0; x < n; x++) {
  for (auto y : dag[x]) { for (auto z :
      adj[y]) { if (rnk[z] > rnk[x]) {
      ans += vis[z]++; }}}
  for (auto y : dag[x]) { for (auto z :
      adj[y]) { if (rnk[z] > rnk[x]) {
      vis[z]--; }}}
}
```

4.9 Minimum Mean Cycle

create a new vertex S, connect S to all vertices with arbitrary weight (0). Let $f_i(u)$ be the shortest path from S to u with exactly i edges.

$$ans = \min_{f_{n+1}(i)!=\infty} \max_{j=1}^{n} \frac{f_{n+1}(i) - f_j(i)}{n+1-j}$$

4.10 Directed Minimum Spanning Tree

```
// DSU with rollback
template <typename Cost>
struct DMST {
  int n;
  vector<int> s, t, lc, rc, h;
  vector<Cost> c, tag;
  DMST(int n) : n(n), h(n, -1) {}
  void addEdge(int u, int v, Cost w) {
    int id = s.size();
    s.push_back(u), t.push_back(v), c.
        push_back(w);
    lc.push_back(-1), rc.push_back(-1);
    tag.emplace_back();
    h[v] = merge(h[v], id);
  }
  pair<Cost, vector<int>> build(int root
      = 0) {
    DSU d(n);
    Cost res{};
    vector<int> vis(n, -1), path(n), q(n)
        , in(n, -1);
    vis[root] = root;
    vector<pair<int, vector<int>>> cycles
        ;
    for (auto r = 0; r < n; ++r) {
      auto u = r, b = 0, w = -1;
      while (!~vis[u]) {
        if (!~h[u]) { return {-1, {}}; }
        push(h[u]);
        int e = h[u];
        res += c[e], tag[h[u]] -= c[e];
        h[u] = pop(h[u]);
        q[b] = e, path[b++] = u, vis[u] =
            r;
        u = d.find(s[e]);
        if (vis[u] == r) {
          int cycle = -1, e = b;
          do {
            w = path[--b];
            cycle = merge(cycle, h[w]);
          } while (d.join(u, w));
          u = d.find(u);
          h[u] = cycle, vis[u] = -1;
          cycles.emplace_back(u, vector<
              int>(q.begin() + b, q.
              begin() + e));
        }
      }
      for (auto i = 0; i < b; ++i) { in[d
          .find(t[q[i]])] = q[i]; }
    }
    reverse(cycles.begin(), cycles.end())
        ;
    for (const auto &[u, comp] : cycles)
        {
      int count = int(comp.size()) - 1;
      d.back(count);
      int ine = in[u];
      for (auto e : comp) { in[d.find(t[e
          ])] = e; }
      in[d.find(t[ine])] = ine;
    }
    vector<int> par;
    par.reserve(n);
    for (auto i : in) { par.push_back(i
        != -1 ? s[i] : -1); }
    return {res, par};
  }
  void push(int u) {
    c[u] += tag[u];
    if (int l = lc[u]; l != -1) { tag[l]
        += tag[u]; }
    if (int r = rc[u]; r != -1) { tag[r]
        += tag[u]; }
    tag[u] = 0;
  }
  int merge(int u, int v) {
    if (u == -1 || v == -1) { return u !=
        -1 ? u : v; }
    push(u);
    push(v);
    if (c[u] > c[v]) { swap(u, v); }
    rc[u] = merge(v, rc[u]);
    swap(lc[u], rc[u]);
    return u;
  }
  int pop(int u) {
    push(u);
    return merge(lc[u], rc[u]);
  }
};
```

4.11 Maximum Clique

```
pair<int, vector<int>> maxClique(int n,
    const vector<bitset<N>> adj) {
  int mx = 0;
  vector<int> ans, cur;
  auto rec = [&](auto rec, bitset<N> s)
      -> void {
    int sz = s.count();
    if (int(cur.size()) > mx) { mx = cur.
        size(), ans = cur; }
    if (int(cur.size()) + sz <= mx) {
        return; }
    int e1 = -1, e2 = -1;
    vector<int> d(n);
    for (int i = 0; i < n; i++) {
      if (s[i]) {
        d[i] = (adj[i] & s).count();
        if (e1 == -1 || d[i] > d[e1]) {
            e1 = i; }
        if (e2 == -1 || d[i] < d[e2]) {
            e2 = i; }
      }
    }
    if (d[e1] >= sz - 2) {
      cur.push_back(e1);
      auto s1 = adj[e1] & s;
      rec(rec, s1);
      cur.pop_back();
      return;
    }
    cur.push_back(e2);
    auto s2 = adj[e2] & s;
    rec(rec, s2);
    cur.pop_back();
    s.reset(e2);
    rec(rec, s);
  };
  bitset<N> all;
  for (int i = 0; i < n; i++) {
    all.set(i);
  }
  rec(rec, all);
  return pair(mx, ans);
}
```

4.12 Dominator Tree

```
// res : parent of each vertex in
    dominator tree, -1 is root, -2 if
    not in tree
struct DominatorTree {
  int n, cur = 0;
  vector<int> dfn, rev, fa, sdom, dom,
      val, rp, res;
  vector<vector<int>> adj, rdom, r;
  DominatorTree(int n) : n(n), dfn(n, -1)
      , res(n, -2), adj(n), rdom(n), r(n
      ) {
    rev = fa = sdom = dom = val = rp =
        dfn;
  }
  void addEdge(int u, int v) {
    adj[u].push_back(v);
  }
  void dfs(int u) {
    dfn[u] = cur;
    rev[cur] = u;
    fa[cur] = sdom[cur] = val[cur] = cur;
    cur++;
    for (int v : adj[u]) {
      if (dfn[v] == -1) {
        dfs(v);
        rp[dfn[v]] = dfn[u];
      }
      r[dfn[v]].push_back(dfn[u]);
    }
  }
  int find(int u, int c) {
    if (fa[u] == u) { return c != 0 ? -1
        : u; }
    int p = find(fa[u], 1);
    if (p == -1) { return c != 0 ? fa[u]
        : val[u]; }
    if (sdom[val[u]] > sdom[val[fa[u]]])
        { val[u] = val[fa[u]]; }
    fa[u] = p;
    return c != 0 ? p : val[u];
  }
  void build(int s = 0) {
    dfs(s);
    for (int i = cur - 1; i >= 0; i--) {
      for (int u : r[i]) { sdom[i] = min(
          sdom[i], sdom[find(u, 0)]); }
      if (i > 0) { rdom[sdom[i]].
          push_back(i); }
      for (int u : rdom[i]) {
        int p = find(u, 0);
        if (sdom[p] == i) {
          dom[u] = i;
        } else {
          dom[u] = p;
        }
      }
      if (i > 0) { fa[i] = rp[i]; }
    }
    res[s] = -1;
    for (int i = 1; i < cur; i++) { if (
        sdom[i] != dom[i]) { dom[i] =
        dom[dom[i]]; }}
    for (int i = 1; i < cur; i++) { res[
        rev[i]] = rev[dom[i]]; }
  }
};
```

4.13 Edge Coloring

```
// bipartite
e[i] = pair(u, v + a), deg[u]++, deg[v +
    a]++;
int col = *max_element(deg.begin(), deg.
    end());
vector<int> ans(m, -1);
vector has(a + b, vector<pair<int, int>>(
    col, {-1, -1}));
for (int i = 0; i < m; i++) {
  auto [u, v] = e[i];
  vector<int> c;
  for (auto x : {u, v}) {
    c.push_back(0);
    while (has[x][c.back()].first != -1)
        { c.back()++; }
  }
  if (c[0] != c[1]) {
    auto dfs = [&](auto dfs, int u, int x
        ) -> void {
      auto [v, i] = has[u][c[x]];
      if (v != -1) {
        if (has[v][c[x ^ 1]].first != -1)
            {
          dfs(dfs, v, x ^ 1);
        } else {
```

```
        has[v][c[x]] = {-1, -1};
      }
      has[u][c[x ^ 1]] = {v, i}, has[v][c[x ^ 1]] = {u, i};
      ans[i] = c[x ^ 1];
    }
  };
  dfs(dfs, v, 0);
}
has[u][c[0]] = {v, i};
has[v][c[0]] = {u, i};
ans[i] = c[0];
}
// general
auto vizing(int n, const vector<pair<int, int>> &e) {
  vector<int> deg(n);
  for (auto [u, v] : e) {
    deg[u]++, deg[v]++;
  }
  int col = *max_element(deg.begin(), deg.end()) + 1;
  vector<int> free(n);
  vector ans(n, vector<int>(n, -1));
  vector at(n, vector<int>(col, -1));
  auto update = [&](int u) {
    free[u] = 0;
    while (at[u][free[u]] != -1) {
      free[u]++;
    }
  };
  auto color = [&](int u, int v, int c1) {
    int c2 = ans[u][v];
    ans[u][v] = ans[v][u] = c1;
    at[u][c1] = v, at[v][c1] = u;
    if (c2 != -1) {
      at[u][c2] = at[v][c2] = -1;
      free[u] = free[v] = c2;
    } else {
      update(u), update(v);
    }
    return c2;
  };
  auto flip = [&](int u, int c1, int c2) {
    int v = at[u][c1];
    swap(at[u][c1], at[u][c2]);
    if (v != -1) {
      ans[u][v] = ans[v][u] = c2;
    }
    if (at[u][c1] == -1) {
      free[u] = c1;
    }
    if (at[u][c2] == -1) {
      free[u] = c2;
    }
    return v;
  };
  for (int i = 0; i < int(e.size()); i++) {
    auto [u, v1] = e[i];
    int v2 = v1, c1 = free[u], c2 = c1, d;
    vector<pair<int, int>> fan;
    vector<int> vis(col);
    while (ans[u][v1] == -1) {
      fan.emplace_back(v2, d = free[v2]);
      if (at[v2][c2] == -1) {
        for (int j = int(fan.size()) - 1; j >= 0; j--) {
          c2 = color(u, fan[j].first, c2);
        }
      } else if (at[u][d] == -1) {
        for (int j = int(fan.size()) - 1; j >= 0; j--) {
          color(u, fan[j].first, fan[j].second);
        }
      } else if (vis[d] == 1) {
        break;
      } else {
        vis[d] = 1, v2 = at[u][d];
      }
    }
    if (ans[u][v1] == -1) {
      while (v2 != -1) {
        v2= flip(v2, c2, d);
        swap(c2, d);
      }
      if (at[u][c1] != -1) {
        int j = int(fan.size()) - 2;
        while (j >= 0 && fan[j].second != c2) {
          j--;
        }
        while (j >= 0) {
          color(u, fan[j].first, fan[j].second);
          j--;
        }
      } else {
        i--;
      }
    }
  }
  return pair(col, ans);
}
```

5 String

5.1 Prefix Function

```
template <typename T>
vector<int> prefixFunction(const T &s) {
  int n = int(s.size());
  vector<int> p(n);
  for (int i = 1; i < n; i++) {
    int j = p[i - 1];
    while (j > 0 && s[i] != s[j]) { j = p[j - 1]; }
    if (s[i] == s[j]) { j++; }
    p[i] = j;
  }
  return p;
}
```

5.2 Z Function

```
template <typename T>
vector<int> zFunction(const T &s) {
  int n = int(s.size());
  if (n == 0) return {};
  vector<int> z(n);
  for (int i = 1, j = 0; i < n; i++) {
    int &k = z[i];
    k = j + z[j] <= i ? 0 : min(j + z[j] - i, z[i - j]);
    while (i + k < n && s[k] == s[i + k]) { k++; }
    if (j + z[j] < i + z[i]) { j = i; }
  }
  z[0] = n;
  return z;
}
```

5.3 Suffix Array

```
// need to discretize
struct SuffixArray {
  int n;
  vector<int> sa, as, ha;
  template <typename T>
  vector<int> sais(const T &s) {
    int n = s.size(), m = *max_element(s.begin(), s.end()) + 1;
    vector<int> pos(m + 1), f(n);
    for (auto ch : s) { pos[ch + 1]++; }
    for (int i = 0; i < m; i++) { pos[i + 1] += pos[i]; }
    for (int i = n - 2; i >= 0; i--) { f[i] = s[i] != s[i + 1] ? s[i] < s[i + 1] : f[i + 1]; }
    vector<int> x(m), sa(n);
    auto induce = [&](const vector<int> &ls) {
      fill(sa.begin(), sa.end(), -1);
      auto L = [&](int i) { if (i >= 0 && !f[i]) { sa[x[s[i]]++] = i; }};
      auto S = [&](int i) { if (i >= 0 && f[i]) { sa[--x[s[i]]] = i; }};
      for (int i = 0; i < m; i++) { x[i] = pos[i + 1]; }
      for (int i = int(ls.size()) - 1; i >= 0; i--) { S(ls[i]); }
      for (int i = 0; i < m; i++) { x[i] = pos[i]; }
      L(n - 1);
      for (int i = 0; i < n; i++) { L(sa[i] - 1); }
      for (int i = 0; i < m; i++) { x[i] = pos[i + 1]; }
      for (int i = n - 1; i >= 0; i--) { S(sa[i] - 1); }
    };
    auto ok = [&](int i) { return i == n || !f[i - 1] && f[i]; };
    auto same = [&](int i, int j) {
      do { if (s[i++] != s[j++]) { return false; }} while (!ok(i) && !ok(j));
      return ok(i) && ok(j);
    };
    vector<int> val(n), lms;
    for (int i = 1; i < n; i++) { if (ok(i)) { lms.push_back(i); }}
    induce(lms);
    if (!lms.empty()) {
      int p = -1, w = 0;
      for (auto v : sa) {
        if (v != 0 && ok(v)) {
          if (p != -1 && same(p, v)) { w--; }
          val[p = v] = w++;
        }
      }
      auto b = lms;
      for (auto &v : b) { v = val[v]; }
      b = sais(b);
      for (auto &v : b) { v = lms[v]; }
      induce(b);
    }
    return sa;
  }
template <typename T>
  SuffixArray(const T &s) : n(s.size()), sa(sais(s)), as(n), ha(n - 1) {
    for (int i = 0; i < n; i++) { as[sa[i]] = i; }
    for (int i = 0, j = 0; i < n; ++i) {
      if (as[i] == 0) {
        j = 0;
      } else {
        for (j -= j > 0; i + j < n && sa[as[i] - 1] + j < n && s[i + j] == s[sa[as[i] - 1] + j]; ) { ++j; }
        ha[as[i] - 1] = j;
      }
    }
  }
};
```

5.4 Manacher's Algorithm

```
// returns radius of t, length of s : rad(t) - 1, radius of s : rad(t) / 2
vector<int> manacher(string s) {
  string t = "#";
  for (auto c : s) { t += c, t += '#'; }
  int n = t.size();
  vector<int> r(n);
  for (int i = 0, j = 0; i < n; i++) {
    if (2 * j - i >= 0 && j + r[j] > i) {
      r[i] = min(r[2 * j - i], j + r[j] - i); }
    while (i - r[i] >= 0 && i + r[i] < n && t[i - r[i]] == t[i + r[i]]) {
      r[i]++; }
    if (i + r[i] > j + r[j]) { j = i; }
  }
  return r;
}
```

5.5 Aho-Corasick Automaton

```
constexpr int K = 26;
struct Node {
  array<int, K> nxt;
  int fail = -1;
  // other vars
  Node() { nxt.fill(-1); }
};
vector<Node> aho(1);
for (int i = 0; i < n; i++) {
  string s;
  cin >> s;
  int u = 0;
  for (auto ch : s) {
    int c = ch - 'a';
    if (aho[u].nxt[c] == -1) {
      aho[u].nxt[c] = aho.size();
      aho.emplace_back();
    }
    u = aho[u].nxt[c];
  }
}
vector<int> q;
for (auto &i : aho[0].nxt) {
  if (i == -1) {
    i = 0;
  } else {
    q.push_back(i);
    aho[i].fail = 0;
  }
}
for (int i = 0; i < int(q.size()); i++) {
  int u = q[i];
  if (u > 0) {
    // maintain
  }
  for (int c = 0; c < K; c++) {
    if (int v = aho[u].nxt[c]; v != -1) {
      aho[v].fail = aho[aho[u].fail].nxt[c];
      q.push_back(v);
    } else {
      aho[u].nxt[c] = aho[aho[u].fail].nxt[c];
    }
  }
}
```

5.6 Suffix Automaton

```
struct SAM {
  static constexpr int A = 26;
  struct Node {
    int len = 0, link = -1, cnt = 0;
    array<int, A> nxt;
    Node() { nxt.fill(-1); }
  };
  vector<Node> t;
  SAM() : t(1) {}
  int size() { return t.size(); }
  Node& operator[](int i) { return t[i]; }
  int newNode() {
    t.emplace_back();
    return t.size() - 1;
  }
  int extend(int p, int c) {
    int cur = newNode();
    t[cur].len = t[p].len + 1;
    t[cur].cnt = 1;
    while (p != -1 && t[p].nxt[c] == -1) {
      t[p].nxt[c] = cur;
      p = t[p].link;
    }
    if (p == -1) {
      t[cur].link = 0;
    } else {
      int q = t[p].nxt[c];
      if (t[p].len + 1 == t[q].len) {
        t[cur].link = q;
      } else {
        int clone = newNode();
        t[clone].len = t[p].len + 1;
        t[clone].link = t[q].link;
        t[clone].nxt = t[q].nxt;
        while (p != -1 && t[p].nxt[c] == q) {
          t[p].nxt[c] = clone;
          p = t[p].link;
        }
        t[q].link = t[cur].link = clone;
      }
    }
    return cur;
  }
};
```

5.7 Lexicographically Smallest Rotation

```
template <typename T>
T minRotation(T s) {
  int n = s.size();
  int i = 0, j = 1;
  s.insert(s.end(), s.begin(), s.end());
  while (i < n && j < n) {
    int k = 0;
    while (k < n && s[i + k] == s[j + k]) {
      k++;
    }
    if (s[i + k] <= s[j + k]) {
      j += k + 1;
    } else {
      i += k + 1;
    }
    if (i == j) {
      j++;
    }
  }
  int ans = i < n ? i : j;
  return T(s.begin() + ans, s.begin() + ans + n);
}
```

5.8 EER Tree

```
// cnt : occurrences, (dfs fail tree)
// num : number of pal ending here
struct PAM {
  static constexpr int A = 26;
  struct Node {
    int len = 0, link = 0, cnt = 0, num = 0;
    array<int, A> nxt{};
    Node() {}
  };
  vector<Node> t;
  int suf = 1;
  string s;
  PAM() : t(2) { t[0].len = -1; }
  int size() { return t.size(); }
  Node& operator[](int i) { return t[i]; }
  int newNode() {
    t.emplace_back();
    return t.size() - 1;
  }
  bool add(int c, char offset = 'a') {
    int pos = s.size();
    s += c + offset;
    int cur = suf, curlen = 0;
    while (true) {
      curlen = t[cur].len;
      if (pos - 1 - curlen >= 0 && s[pos - 1 - curlen] == s[pos]) {
        break; }
      cur = t[cur].link;
    }
    if (t[cur].nxt[c]) {
      suf = t[cur].nxt[c];
      t[suf].cnt++;
      return false;
    }
    suf = newNode();
    t[suf].len = t[cur].len + 2;
    t[suf].cnt = t[suf].num = 1;
    t[cur].nxt[c] = suf;
    if (t[suf].len == 1) {
      t[suf].link = 1;
      return true;
    }
    while (true) {
      cur = t[cur].link;
      curlen = t[cur].len;
      if (pos - 1 - curlen >= 0 && s[pos - 1 - curlen] == s[pos]) {
        t[suf].link = t[cur].nxt[c];
        break;
      }
    }
    t[suf].num += t[t[suf].link].num;
    return true;
  }
};
```

6 Math

6.1 Extended GCD

```
array<i64, 3> extgcd(i64 a, i64 b) {
  if (b == 0) { return {a, 1, 0}; }
  auto [g, x, y] = extgcd(b, a % b);
  return {g, y, x - a / b * y};
}
```

6.2 Chinese Remainder Theorem

```
// returns (rem, mod), n = 0 return (0, 1), no solution return (0, 0)
pair<i64, i64> crt(vector<i64> r, vector<i64> m) {
  int n = r.size();
  for (int i = 0; i < n; i++) {
    r[i] %= m[i];
    if (r[i] < 0) { r[i] += m[i]; }
  }
  i64 r0 = 0, m0 = 1;
  for (int i = 0; i < n; i++) {
    i64 r1 = r[i], m1 = m[i];
    if (m0 < m1) { swap(r0, r1), swap(m0, m1); }
    if (m0 % m1 == 0) {
      if (r0 % m1 != r1) { return {0, 0}; }
      continue;
    }
    auto [g, a, b] = extgcd(m0, m1);
    i64 u1 = m1 / g;
    if ((r1 - r0) % g != 0) { return {0, 0}; }
    i64 x = (r1 - r0) / g % u1 * a % u1;
    r0 += x * m0;
    m0 *= u1;
    if (r0 < 0) { r0 += m0; }
  }
  return {r0, m0};
}
```

6.3 NTT and polynomials

```
template <int P>
struct Modint {
  int v;
  // need constexpr, constructor, +-*, qpow, inv()
};
template<int P>
constexpr Modint<P> findPrimitiveRoot() {
  Modint<P> i = 2;
  int k = __builtin_ctz(P - 1);
  while (true) {
    if (i.qpow((P - 1) / 2).v != 1) { break; }
    i = i + 1;
  }
  return i.qpow(P - 1 >> k);
}
template <int P>
constexpr Modint<P> primitiveRoot = findPrimitiveRoot<P>();
vector<int> rev;
template <int P>
```

```cpp
vector<Modint<P>> roots{0, 1};
template <int P>
void dft(vector<Modint<P>> &a) {
  int n = a.size();
  if (n == 1) { return; }
  if (int(rev.size()) != n) {
    int k = __builtin_ctz(n) - 1;
    rev.resize(n);
    for (int i = 0; i < n; i++) { rev[i]
        = rev[i >> 1] >> 1 | (i & 1) <<
        k; }
  }
  for (int i = 0; i < n; i++) { if (rev[i
      ] < i) { swap(a[i], a[rev[i]]); }}
  if (roots<P>.size() < n) {
    int k = __builtin_ctz(roots<P>.size()
        );
    roots<P>.resize(n);
    while ((1 << k) < n) {
      auto e = Modint<P>(primitiveRoot<P
          >).qpow(P - 1 >> k + 1);
      for (int i = 1 << k - 1; i < 1 << k
          ; i++) {
        roots<P>[2 * i] = roots<P>[i];
        roots<P>[2 * i + 1] = roots<P>[i]
            * e;
      }
      k++;
    }
  }
  // fft : just do roots[i] = exp(2 * PI
      / n * i * complex<double>(0, 1))
  for (int k = 1; k < n; k *= 2) {
    for (int i = 0; i < n; i += 2 * k) {
      for (int j = 0; j < k; j++) {
        Modint<P> u = a[i + j];
        Modint<P> v = a[i + j + k] *
            roots<P>[k + j];
        // fft : v = a[i + j + k] * roots
            [n / (2 * k) * j]
        a[i + j] = u + v;
        a[i + j + k] = u - v;
      }
    }
  }
}
template <int P>
void idft(vector<Modint<P>> &a) {
  int n = a.size();
  reverse(a.begin() + 1, a.end());
  dft(a);
  Modint<P> x = (1 - P) / n;
  for (int i = 0; i < n; i++) { a[i] = a[
      i] * x; }
}
template <int P>
struct Poly : vector<Modint<P>> {
  using Mint = Modint<P>;
  Poly() {}
  explicit Poly(int n) : vector<Mint>(n)
      {}
  explicit Poly(const vector<Mint> &a) :
      vector<Mint>(a) {}
  explicit Poly(const initializer_list<
      Mint> &a) : vector<Mint>(a) {}
template<class F>
  explicit Poly(int n, F f) : vector<Mint
      >(n) { for (int i = 0; i < n; i++)
       { (*this)[i] = f(i); }}
template<class InputIt>
  explicit constexpr Poly(InputIt first,
      InputIt last) : vector<Mint>(first
      , last) {}
  Poly mulxk(int k) {
    auto b = *this;
    b.insert(b.begin(), k, 0);
    return b;
  }
  Poly modxk(int k) {
    k = min(k, int(this->size()));
    return Poly(this->begin(), this->
        begin() + k);
  }
  Poly divxk(int k) {
    if (this->size() <= k) { return Poly
        (); }
    return Poly(this->begin() + k, this->
        end());
  }
  friend Poly operator+(const Poly &a,
      const Poly &b) {
    Poly res(max(a.size(), b.size()));
    for (int i = 0; i < int(a.size()); i
        ++) { res[i] = res[i] + a[i]; }
    for (int i = 0; i < int(b.size()); i
        ++) { res[i] = res[i] + b[i]; }
    return res;
  }
  friend Poly operator-(const Poly &a,
      const Poly &b) {
    Poly res(max(a.size(), b.size()));
    for (int i = 0; i < int(a.size()); i
        ++) { res[i] = res[i] + a[i]; }
    for (int i = 0; i < int(b.size()); i
        ++) { res[i] = res[i] - b[i]; }
    return res;
  }
  friend Poly operator*(Poly a, Poly b) {
    if (a.empty() || b.empty()) { return
        Poly(); }
    int sz = 1, tot = a.size() + b.size()
        - 1;
    while (sz < tot) { sz *= 2; }
    a.resize(sz);
    b.resize(sz);
    dft(a);
    dft(b);
    for (int i = 0; i < sz; i++) { a[i] =
        a[i] * b[i]; }
    idft(a);
    a.resize(tot);
    return a;
  }
  friend Poly operator*(Poly a, Mint b) {
    for (int i = 0; i < int(a.size()); i
        ++) { a[i] = a[i] * b; }
    return a;
  }
  Poly derivative()  {
    if (this->empty()) { return Poly(); }
    Poly res(this->size() - 1);
    for (int i = 0; i < this->size() - 1;
        ++i) { res[i] = (i + 1) * (*
        this)[i + 1]; }
    return res;
  }
  Poly integral() {
    Poly res(this->size() + 1);
    for (int i = 0; i < this->size(); ++i
        ) { res[i + 1] = (*this)[i] *
        Mint(i + 1).inv(); }
    return res;
  }
  Poly inv(int m) {
    // a[0] != 0
    Poly x({(*this)[0].inv()});
    int k = 1;
    while (k < m) {
      k *= 2;
      x = (x * (Poly({2}) - modxk(k) * x)
          ).modxk(k);
    }
    return x.modxk(m);
  }
  Poly log(int m)  {
    return (derivative() * inv(m)).
        integral().modxk(m);
  }
  Poly exp(int m) {
    Poly x({1});
    int k = 1;
    while (k < m) {
      k *= 2;
      x = (x * (Poly({1}) - x.log(k) +
          modxk(k))).modxk(k);
    }
    return x.modxk(m);
  }
  Poly pow(i64 k, int m) {
    if (k == 0) { return Poly(m, [&](int
        i) { return i == 0; }); }
    int i = 0;
    while (i < this->size() && (*this)[i
        ].v == 0) { i++; }
    if (i == this->size() || __int128(i)
        * k >= m) { return Poly(m); }
    Mint v = (*this)[i];
    auto f = divxk(i) * v.inv();
    return (f.log(m - i * k) * k).exp(m -
         i * k).mulxk(i * k) * v.qpow(k)
        ;
  }
  Poly sqrt(int m) {
    // a[0] == 1, otherwise quadratic
        residue?
    Poly x({1});
    int k = 1;
    while (k < m) {
      k *= 2;
      x = (x + (modxk(k) * x.inv(k)).
          modxk(k)) * ((P + 1) / 2);
    }
    return x.modxk(m);
  }
  Poly mulT(Poly b) const {
    if (b.empty()) { return Poly(); }
    int n = b.size();
    reverse(b.begin(), b.end());
    return (*this * b).divxk(n - 1);
  }
  vector<Mint> evaluate(vector<Mint> x) {
    if (this->empty()) { return vector<
        Mint>(x.size()); }
    int n = max(x.size(), this->size());
    vector<Poly> q(4 * n);
    vector<Mint> ans(x.size());
    x.resize(n);
    auto build = [&](auto build, int id,
        int l, int r) -> void {
      if (r - l == 1) {
        q[id] = Poly({1, -x[l].v});
      } else {
        int m = (l + r) / 2;
        build(build, 2 * id, l, m);
        build(build, 2 * id + 1, m, r);
        q[id] = q[2 * id] * q[2 * id +
            1];
      }
    };
    build(build, 1, 0, n);
    auto work = [&](auto work, int id,
        int l, int r, const Poly &num)
        -> void {
      if (r - l == 1) {
        if (l < int(ans.size())) { ans[l]
            = num[0]; }
      } else {
        int m = (l + r) / 2;
        work(work, 2 * id, l, m, num.mulT
            (q[2 * id + 1]).modxk(m - l)
            );
        work(work, 2 * id + 1, m, r, num.
            mulT(q[2 * id]).modxk(r - m)
            );
      }
    };
    work(work, 1, 0, n, mulT(q[1].inv(n))
        );
    return ans;
  }
};
template <int P>
Poly<P> interpolate(vector<Modint<P>> x,
    vector<Modint<P>> y) {
  // f(xi) = yi
  int n = x.size();
  vector<Poly<P>> p(4 * n), q(4 * n);
  auto dfs1 = [&](auto dfs1, int id, int
      l, int r) -> void {
    if (l == r) {
      p[id] = Poly<P>({-x[l].v, 1});
      return;
    }
    int m = l + r >> 1;
    dfs1(dfs1, id << 1, l, m);
    dfs1(dfs1, id << 1 | 1, m + 1, r);
    p[id] = p[id << 1] * p[id << 1 | 1];
  };
```

```
  dfs1(dfs1, 1, 0, n - 1);
  Poly<P> f = Poly<P>(p[1].derivative().
      evaluate(x));
  auto dfs2 = [&](auto dfs2, int id, int
      l, int r) -> void {
    if (l == r) {
      q[id] = Poly<P>({y[l] * f[l].inv()
          });
      return;
    }
    int m = l + r >> 1;
    dfs2(dfs2, id << 1, l, m);
    dfs2(dfs2, id << 1 | 1, m + 1, r);
    q[id] = q[id << 1] * p[id << 1 | 1] +
        q[id << 1 | 1] * p[id << 1];
  };
  dfs2(dfs2, 1, 0, n - 1);
  return q[1];
}
auto shift = [&](FPS f, int k) {
  FPS a(n + 1), b(n + 1);
  Mint powk = 1;
  for (int i = 0; i <= n; i++) {
    a[i] = ifact[i] * powk;
    b[i] = fact[i] * f[i];
    powk = powk * k;
  }
  reverse(b.begin(), b.end());
  auto g = a * b;
  g.resize(n + 1);
  reverse(g.begin(), g.end());
  for (int i = 0; i <= n; i++) {
    g[i] = g[i] * ifact[i];
  }
  return g;
};
```

6.4 Any Mod NTT

```
constexpr int P0 = 998244353, P1 =
    1004535809, P2 = 469762049;
constexpr i64 P01 = 1LL * P0 * P1;
constexpr int inv0 = Modint<P1>(P0).inv()
    .v;
constexpr int inv01 = Modint<P2>(P01).inv
    ().v;
for (int i = 0; i < int(c.size()); i++) {
  i64 x = 1LL * (c1[i] - c0[i] + P1) % P1
      * inv0 % P1 * P0 + c0[i];
  c[i] = ((c2[i] - x % P2 + P2) % P2 *
      inv01 % P2 * (P01 % P) % P + x) %
      P;
}
```

6.5 Newton's Method

$$Q_{k+1} = Q_k - \frac{F(Q_k)}{F'(Q_k)} \pmod{x^{2^{k+1}}}$$

6.6 Fast Walsh-Hadamard Transform

1. XOR Convolution
   - $f(A) = (f(A_0) + f(A_1), f(A_0) - f(A_1))$
   - $f^{-1}(A) = (f^{-1}(\frac{A_0+A_1}{2}), f^{-1}(\frac{A_0-A_1}{2}))$
2. OR Convolution
   - $f(A) = (f(A_0), f(A_0) + f(A_1))$
   - $f^{-1}(A) = (f^{-1}(A_0), f^{-1}(A_1) - f^{-1}(A_0))$
3. AND Convolution
   - $f(A) = (f(A_0) + f(A_1), f(A_1))$
   - $f^{-1}(A) = (f^{-1}(A_0) - f^{-1}(A_1), f^{-1}(A_1))$

6.7 Simplex Algorithm

Description: maximize $\mathbf{c}^T\mathbf{x}$ subject to $A\mathbf{x} \le \mathbf{b}$ and $\mathbf{x} \ge 0$. Returns $-\infty$ if infeasible and ∞ if unbounded.

```
const double eps = 1e-9;
const double inf = 1e+9;
int n, m;
vector<vector<double>> d;
vector<int> p, q;
void pivot(int r, int s) {
  double inv = 1.0 / d[r][s];
  for (int i = 0; i < m + 2; ++i) {
    for (int j = 0; j < n + 2; ++j) {
      if (i != r && j != s) d[i][j] -= d[
          r][j] * d[i][s] * inv;
    }
  }
  for (int i = 0; i < m + 2; ++i) if (i
      != r) d[i][s] *= -inv;
  for (int j = 0; j < n + 2; ++j) if (j
      != s) d[r][j] *= +inv;
  d[r][s] = inv;
  swap(p[r], q[s]);
}
bool phase(int z) {
  int x = m + z;
  while (true) {
    int s = -1;
    for (int i = 0; i <= n; ++i) {
      if (!z && q[i] == -1) continue;
      if (s == -1 || d[x][i] < d[x][s]) s
          = i;
    }
    if (d[x][s] > -eps) return true;
    int r = -1;
    for (int i = 0; i < m; ++i) {
      if (d[i][s] < eps) continue;
      if (r == -1 || d[i][n + 1] / d[i][s
          ] < d[r][n + 1] / d[r][s]) r =
          i;
    }
    if (r == -1) return false;
    pivot(r, s);
  }
}
vector<double> solve(const vector<vector<
    double>> &a, const vector<double> &b
    , const vector<double> &c) {
  m = b.size(), n = c.size();
  d = vector<vector<double>>(m + 2,
      vector<double>(n + 2));
  for (int i = 0; i < m; ++i) {
    for (int j = 0; j < n; ++j) d[i][j] =
        a[i][j];
  }
  p.resize(m), q.resize(n + 1);
  for (int i = 0; i < m; ++i) p[i] = n +
      i, d[i][n] = -1, d[i][n + 1] = b[i
      ];
  for (int i = 0; i < n; ++i) q[i] = i, d
      [m][i] = -c[i];
  q[n] = -1, d[m + 1][n] = 1;
  int r = 0;
  for (int i = 1; i < m; ++i) if (d[i][n
      + 1] < d[r][n + 1]) r = i;
  if (d[r][n + 1] < -eps) {
    pivot(r, n);
    if (!phase(1) || d[m + 1][n + 1] < -
        eps) return vector<double>(n, -
        inf);
    for (int i = 0; i < m; ++i) if (p[i]
        == -1) {
      int s = min_element(d[i].begin(), d
          [i].end() - 1) - d[i].begin();
      pivot(i, s);
    }
  }
  if (!phase(0)) return vector<double>(n,
      inf);
  vector<double> x(n);
  for (int i = 0; i < m; ++i) if (p[i] <
      n) x[p[i]] = d[i][n + 1];
  return x;
}
```

6.7.1 Construction

Standard form: maximize $\mathbf{c}^T\mathbf{x}$ subject to $A\mathbf{x} \le \mathbf{b}$ and $\mathbf{x} \ge 0$.
Dual LP: minimize $\mathbf{b}^T\mathbf{y}$ subject to $A^T\mathbf{y} \ge \mathbf{c}$ and $\mathbf{y} \ge 0$.
$\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$ are optimal if and only if for all $i \in [1, n]$, either $\bar{x}_i = 0$ or $\sum_{j=1}^{m} A_{ji}\bar{y}_j = c_i$ holds and for all $i \in [1, m]$ either $\bar{y}_i = 0$ or $\sum_{j=1}^{n} A_{ij}\bar{x}_j = b_j$ holds.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1\le i\le n} A_{ji}x_i \ge b_j \to \sum_{1\le i\le n} -A_{ji}x_i \le -b_j$
3. $\sum_{1\le i\le n} A_{ji}x_i = b_j$
   - $\sum_{1\le i\le n} A_{ji}x_i \le b_j$
   - $\sum_{1\le i\le n} A_{ji}x_i \ge b_j$
4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

6.8 Subset Convolution

Description: $h(s) = \sum_{s' \subseteq s} f(s')g(s \setminus s')$

```
vector<int> SubsetConv(int n, const
    vector<int> &f, const vector<int> &g
    ) {
  const int m = 1 << n;
  vector<vector<int>> a(n + 1, vector<int
      >(m)), b(n + 1, vector<int>(m));
  for (int i = 0; i < m; ++i) {
    a[__builtin_popcount(i)][i] = f[i];
    b[__builtin_popcount(i)][i] = g[i];
  }
  for (int i = 0; i <= n; ++i) {
    for (int j = 0; j < n; ++j) {
      for (int s = 0; s < m; ++s) {
        if (s >> j & 1) {
          a[i][s] += a[i][s ^ (1 << j)];
          b[i][s] += b[i][s ^ (1 << j)];
        }
      }
    }
  }
  vector<vector<int>> c(n + 1, vector<int
      >(m));
  for (int s = 0; s < m; ++s) {
    for (int i = 0; i <= n; ++i) {
      for (int j = 0; j <= i; ++j) c[i][s
          ] += a[j][s] * b[i - j][s];
    }
  }
  for (int i = 0; i <= n; ++i) {
    for (int j = 0; j < n; ++j) {
      for (int s = 0; s < m; ++s) {
        if (s >> j & 1) c[i][s] -= c[i][s
            ^ (1 << j)];
      }
    }
  }
  vector<int> res(m);
  for (int i = 0; i < m; ++i) res[i] = c[
      __builtin_popcount(i)][i];
  return res;
}
```

6.9 Berlekamp Massey Algorithm

```
// find \sum a_(i- j)c_j = 0 for d <= i
template <typename T>
vector<T> berlekampMassey(const vector<T>
    &a) {
  vector<T> c(1, 1), oldC(1);
  int oldI = -1;
  T oldD = 1;
  for (int i = 0; i < int(a.size()); i++)
      {
    T d = 0;
    for (int j = 0; j < int(c.size()); j
        ++) { d += c[j] * a[i - j]; }
    if (d == 0) { continue; }
    T mul = d / oldD;
    vector<T> nc = c;
    nc.resize(max(int(c.size()), i - oldI
        + int(oldC.size())));
    for (int j = 0; j < int(oldC.size());
        j++) { nc[j + i - oldI] -= oldC
        [j] * mul; }
    if (i - int(c.size()) > oldI - int(
        oldC.size())) {
      oldI = i;
      oldD = d;
```

```
      swap(oldC, c);
    }
    swap(c, nc);
  }
  return c;
}
```

6.10 Fast Linear Recurrence

```
// p : a[0] ~ a[d - 1]
// q : a[i] = \sum a[i - j]q[j]
template <typename T>
T linearRecurrence(vector<T> p, vector<T>
      q, i64 n) {
  int d = q.size() - 1;
  assert(int(p.size()) == d);
  p = p * q;
  p.resize(d);
  while (n > 0) {
    auto nq = q;
    for (int i = 1; i <= d; i += 2) {
      nq[i] *= -1;
    }
    auto np = p * nq;
    nq = q * nq;
    for (int i = 0; i < d; i++) {
      p[i] = np[i * 2 + n % 2];
    }
    for (int i = 0; i <= d; i++) {
      q[i] = nq[i * 2];
    }
    n /= 2;
  }
  return p[0] / q[0];
}
```

6.11 Prime check and factorize

```
i64 mul(i64 a, i64 b, i64 mod) {}
i64 qpow(i64 x, i64 p, i64 mod) {}
bool isPrime(i64 n) {
  if (n == 1) { return false; }
  int r = __builtin_ctzll(n - 1);
  i64 d = n - 1 >> r;
  auto checkComposite = [&](i64 p) {
    i64 x = qpow(p, d, n);
    if (x == 1 || x == n - 1) { return
        false; }
    for (int i = 1; i < r; i++) {
      x = mul(x, x, n);
      if (x == n - 1) { return false; }
    }
    return true;
  };
  for (auto p : {2, 3, 5, 7, 11, 13, 17,
        19, 23, 29, 31, 37}) {
    if (n == p) {
      return true;
    } else if (checkComposite(p)) {
      return false;
    }
  }
  return true;
}
vector<i64> pollardRho(i64 n) {
  vector<i64> res;
  auto work = [&](auto work, i64 n) {
    if (n <= 10000) {
      for (int i = 2; i * i <= n; i++) {
        while (n % i == 0) {
          res.push_back(i);
          n /= i;
        }
      }
      if (n > 1) { res.push_back(n); }
      return;
    } else if (isPrime(n)) {
      res.push_back(n);
      return;
    }
    i64 x0 = 2;
    auto f = [&](i64 x) { return (mul(x,
        x, n) + 1) % n; };
    while (true) {
      i64 x = x0, y = x0, d = 1, power =
          1, lam = 0, v = 1;
      while (d == 1) {
        y = f(y);
        ++lam;
        v = mul(v, abs(x - y), n);
        if (lam % 127 == 0) {
          d = gcd(v, n);
          v = 1;
        }
        if (power == lam) {
          x = y;
          power *= 2;
          lam = 0;
          d = gcd(v, n);
          v = 1;
        }
      }
      if (d != n) {
        work(work, d);
        work(work, n / d);
        return;
      }
      ++x0;
    }
  };
  work(work, n);
  sort(res.begin(), res.end());
  return res;
}
```

6.12 Count Primes leq n

```
// __attribute__((target("avx2"),
     optimize("O3", "unroll-loops")))
i64 primeCount(const i64 n) {
  if (n <= 1) { return 0; }
  if (n == 2) { return 1; }
  const int v = sqrtl(n);
  int s = (v + 1) / 2;
  vector<int> smalls(s), roughs(s), skip(
      v + 1);
  vector<i64> larges(s);
  iota(smalls.begin(), smalls.end(), 0);
  for (int i = 0; i < s; i++) {
    roughs[i] = 2 * i + 1;
    larges[i] = (n / roughs[i] - 1) / 2;
  }
  const auto half = [](int n) -> int {
      return (n - 1) >> 1; };
  int pc = 0;
  for (int p = 3; p <= v; p += 2) {
    if (skip[p]) { continue; }
    int q = p * p;
    if (1LL * q * q > n) { break; }
    skip[p] = true;
    for (int i = q; i <= v; i += 2 * p)
        skip[i] = true;
    int ns = 0;
    for (int k = 0; k < s; k++) {
      int i = roughs[k];
      if (skip[i]) { continue; }
      i64 d = 1LL * i * p;
      larges[ns] = larges[k] - (d <= v ?
          larges[smalls[d / 2] - pc] :
          smalls[half(n / d)]) + pc;
      roughs[ns++] = i;
    }
    s = ns;
    for (int i = half(v), j = v / p - 1 |
        1; j >= p; j -= 2) {
      int c = smalls[j / 2] - pc;
      for (int e = j * p / 2; i >= e; i
          --) { smalls[i] -= c; }
    }
    pc++;
  }
  larges[0] += 1LL * (s + 2 * (pc - 1)) *
      (s - 1) / 2;
  for (int k = 1; k < s; k++) { larges[0]
      -= larges[k]; }
  for (int l = 1; l < s; l++) {
    i64 q = roughs[l];
    i64 M = n / q;
    int e = smalls[half(M / q)] - pc;
    if (e <= l) { break; }
    i64 t = 0;
    for (int k = l + 1; k <= e; k++) { t
        += smalls[half(M / roughs[k])]; }
    larges[0] += t - 1LL * (e - l) * (pc
        + l - 1);
  }
  return larges[0] + 1;
}
```

6.13 Discrete Logarithm

```
// return min x >= 0 s.t. a ^ x = b mod m
    , 0 ^ 0 = 1, -1 if no solution
// (I think) if you want x > 0 (m != 1),
    remove if (b == k) return add;
int discreteLog(int a, int b, int m) {
  if (m == 1) {
    return 0;
  }
  a %= m, b %= m;
  int k = 1, add = 0, g;
  while ((g = gcd(a, m)) > 1) {
    if (b == k) {
      return add;
    } else if (b % g) {
      return -1;
    }
    b /= g, m /= g, ++add;
    k = 1LL * k * a / g % m;
  }
  if (b == k) {
    return add;
  }
  int n = sqrt(m) + 1;
  int an = 1;
  for (int i = 0; i < n; ++i) {
    an = 1LL * an * a % m;
  }
  unordered_map<int, int> vals;
  for (int q = 0, cur = b; q < n; ++q) {
    vals[cur] = q;
    cur = 1LL * a * cur % m;
  }
  for (int p = 1, cur = k; p <= n; ++p) {
    cur = 1LL * cur * an % m;
    if (vals.count(cur)) {
      int ans = n * p - vals[cur] + add;
      return ans;
    }
  }
  return -1;
}
```

6.14 Quadratic Residue

```
// rng
int jacobi(int a, int m) {
  int s = 1;
  while (m > 1) {
    a %= m;
    if (a == 0) { return 0; }
    int r = __builtin_ctz(a);
    if (r % 2 == 1 && (m + 2 & 4) != 0) {
        s = -s; }
    a >>= r;
    if ((a & m & 2) != 0) { s = -s; }
    swap(a, m);
  }
  return s;
}
int quadraticResidue(int a, int p) {
  if (p == 2) { return a % 2; }
  int j = jacobi(a, p);
  if (j == 0 || j == -1) { return j; }
  int b, d;
  while (true) {
    b = rng() % p;
    d = (1LL * b * b + p - a) % p;
    if (jacobi(d, p) == -1) { break; }
  }
  int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp
      ;
  for (int e = p + 1 >> 1; e > 0; e >>=
      1) {
    if (e % 2 == 1) {
      tmp = (1LL * g0 * f0 + 1LL * d * g1
          % p * f1 % p) % p;
      g1 = (1LL * g0 * f1 + 1LL * g1 * f0
          ) % p;
      g0 = tmp;
    }
```

```cpp
    tmp = (1LL * f0 * f0 + 1LL * d * f1 %
        p * f1 % p) % p;
    f1 = 2LL * f0 * f1 % p;
    f0 = tmp;
  }
  return g0;
}
```

6.15 Characteristic Polynomial

```cpp
vector<vector<int>> Hessenberg(const
    vector<vector<int>> &A) {
  int N = A.size();
  vector<vector<int>> H = A;
  for (int i = 0; i < N - 2; ++i) {
    if (!H[i + 1][i]) {
      for (int j = i + 2; j < N; ++j) {
        if (H[j][i]) {
          for (int k = i; k < N; ++k)
              swap(H[i + 1][k], H[j][k])
              ;
          for (int k = 0; k < N; ++k)
              swap(H[k][i + 1], H[k][j])
              ;
          break;
        }
      }
    }
    if (!H[i + 1][i]) continue;
    int val = fpow(H[i + 1][i], kP - 2);
    for (int j = i + 2; j < N; ++j) {
      int coef = 1LL * val * H[j][i] % kP
          ;
      for (int k = i; k < N; ++k) H[j][k]
           = (H[j][k] + 1LL * H[i + 1][k
          ] * (kP - coef)) % kP;
      for (int k = 0; k < N; ++k) H[k][i
          + 1] = (H[k][i + 1] + 1LL * H[
          k][j] * coef) % kP;
    }
  }
  return H;
}
vector<int> CharacteristicPoly(const
    vector<vector<int>> &A) {
  int N = A.size();
  auto H = Hessenberg(A);
  for (int i = 0; i < N; ++i) {
    for (int j = 0; j < N; ++j) H[i][j] =
         kP - H[i][j];
  }
  vector<vector<int>> P(N + 1, vector<int
      >(N + 1));
  P[0][0] = 1;
  for (int i = 1; i <= N; ++i) {
    P[i][0] = 0;
    for (int j = 1; j <= i; ++j) P[i][j]
        = P[i - 1][j - 1];
    int val = 1;
    for (int j = i - 1; j >= 0; --j) {
      int coef = 1LL * val * H[j][i - 1]
          % kP;
      for (int k = 0; k <= j; ++k) P[i][k
          ] = (P[i][k] + 1LL * P[j][k] *
           coef) % kP;
      if (j) val = 1LL * val * (kP - H[j
          ][j - 1]) % kP;
    }
  }
  if (N & 1) {
    for (int i = 0; i <= N; ++i) P[N][i]
        = kP - P[N][i];
  }
  return P[N];
}
```

6.16 Linear Sieve Related

```cpp
vector<int> minp(N + 1), primes, mobius(N
     + 1);
mobius[1] = 1;
for (int i = 2; i <= N; i++) {
  if (!minp[i]) {
    primes.push_back(i);
    minp[i] = i;
    mobius[i] = -1;
  }
  for (int p : primes) {
    if (p > N / i) {
      break;
    }
    minp[p * i] = p;
    mobius[p * i] = -mobius[i];
    if (i % p == 0) {
      mobius[p * i] = 0;
      break;
    }
  }
}
```

6.17 De Bruijn Sequence

```cpp
int res[kN], aux[kN], a[kN], sz;
void Rec(int t, int p, int n, int k) {
  if (t > n) {
    if (n % p == 0)
      for (int i = 1; i <= p; ++i) res[sz
          ++] = aux[i];
  } else {
    aux[t] = aux[t - p];
    Rec(t + 1, p, n, k);
    for (aux[t] = aux[t - p] + 1; aux[t]
         < k; ++aux[t]) Rec(t + 1, t, n,
         k);
  }
}
int DeBruijn(int k, int n) {
  // return cyclic string of length k^n
      such that every string of length n
       using k character appears as a
      substring.
  if (k == 1) return res[0] = 0, 1;
  fill(aux, aux + k * n, 0);
  return sz = 0, Rec(1, 1, n, k), sz;
}
```

6.18 Floor Sum

```cpp
// \sum {i = 0} {n} floor((a * i + b) / c
    )
i64 floorSum(i64 a, i64 b, i64 c, i64 n)
    {
  if (n < 0) { return 0; }
  if (n == 0) { return b / c; }
  if (a == 0) { return b / c * (n + 1); }
  i64 res = 0;
  if (a >= c) { res += a / c * n * (n +
      1) / 2, a %= c; }
  if (b >= c) { res += b / c * (n + 1), b
       %= c; }
  i64 m = (a * n + b) / c;
  return res + n * m - (m == 0 ? 0 :
      floorSum(c, c - b - 1, a, m - 1));
}
```

6.19 More Floor Sum

- $m = \lfloor \frac{an+b}{c} \rfloor$

$$g(a,b,c,n) = \sum_{i=0}^{n} i \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor \cdot \frac{n(n+1)}{2} \\ +g(a \bmod c, b \bmod c, c, n), & a \ge c \vee b \ge c \\ 0, & n < 0 \vee a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1) \\ -h(c, c-b-1, a, m-1)), & \text{otherwise} \end{cases}$$

$$h(a,b,c,n) = \sum_{i=0}^{n} \lfloor \frac{ai+b}{c} \rfloor^2$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor^2 \cdot (n+1) \\ +\lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot n(n+1) \\ +h(a \bmod c, b \bmod c, c, n) \\ +2\lfloor \frac{a}{c} \rfloor \cdot g(a \bmod c, b \bmod c, c, n) \\ +2\lfloor \frac{b}{c} \rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \ge c \vee b \ge c \\ 0, & n < 0 \vee a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) \\ -2f(c, c-b-1, a, m-1) - f(a,b,c,n), & \text{otherwise} \end{cases}$$

6.20 Min Mod Linear

```cpp
// \min i : [0, n) (a * i + b) % m
// ok in 1e9
int minModLinear(int n, int m, int a, int
     b, int cnt = 1, int p = 1, int q =
    1) {
  if (a == 0) { return b; }
  if (cnt % 2 == 1) {
    if (b >= a) {
      int t = (m - b + a - 1) / a;
      int c = (t - 1) * p + q;
      if (n <= c) { return b; }
      n -= c;
      b += a * t - m;
    }
    b = a - 1 - b;
  } else {
    if (b < m - a) {
      int t = (m - b - 1) / a;
      int c = t * p;
      if (n <= c) { return (n - 1) / p *
          a + b; }
      n -= c;
      b += a * t;
    }
    b = m - 1 - b;
  }
  cnt++;
  int d = m / a;
  int c = minModLinear(n, a, m % a, b,
      cnt, (d - 1) * p + q, d * p + q);
  return cnt % 2 == 1 ? m - 1 - c : a - 1
       - c;
}
```

6.21 Count of subsets with sum (mod P) leq T

```cpp
int n, T;
cin >> n >> T;
vector<int> cnt(T + 1);
for (int i = 0; i < n; i++) {
  int a;
  cin >> a;
  cnt[a]++;
}
vector<Mint> inv(T + 1);
for (int i = 1; i <= T; i++) {
  inv[i] = i == 1 ? 1 : -P / i * inv[P %
      i];
}
FPS f(T + 1);
for (int i = 1; i <= T; i++) {
  for (int j = 1; j * i <= T; j++) {
    f[i * j] = f[i * j] + (j % 2 == 1 ? 1
         : -1) * cnt[i] * inv[j];
  }
}
f = f.exp(T + 1);
```

6.22 Theorem

- Kirchhoff's Theorem

  Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G, where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i,j) in G.

  - The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
  - The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.

- Tutte's Matrix

  Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniformly at random) if $i < j$ and $(i,j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{rank(D)}{2}$ is the maximum matching on G.

- Cayley's Formula

  - Given a degree sequence $d_1, d_2, \ldots, d_n$ for each *labeled* vertices, there are

  $$\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!}$$

  spanning trees.

  - Let $T_{n,k}$ be the number of *labeled* forests on n vertices with k components, such that vertex $1, 2, \ldots, k$ belong to different components. Then $T_{n,k} = kn^{n-k-1}$.

- Erdős–Gallai Theorem

  A sequence of non-negative integers $d_1 \geq d_2 \geq \ldots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + d_2 + \ldots + d_n$ is even and

  $$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k)$$

  holds for all $1 \leq k \leq n$.

- Burnside's Lemma

  Let X be a set and G be a group that acts on X. For $g \in G$, denote by X^g the elements fixed by g:

  $$X^g = \{x \in X \mid gx \in X\}$$

  Then
  $$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

- Gale–Ryser theorem

  A pair of sequences of nonnegative integers $a_1 \geq \cdots \geq a_n$ and $b_1, \ldots, b_n$ is bigraphic if and only if $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i$ and $\sum_{i=1}^{k} a_i \leq \sum_{i=1}^{n} \min(b_i, k)$ holds for every $1 \leq k \leq n$.
  Sequences a and b called bigraphic if there is a labeled simple bipartite graph such that a and b is the degree sequence of this bipartite graph.

- Fulkerson–Chen–Anstee theorem

  A sequence $(a_1, b_1), \ldots, (a_n, b_n)$ of nonnegative integer pairs with $a_1 \geq \cdots \geq a_n$ is digraphic if and only if $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i$ and $\sum_{i=1}^{k} a_i \leq \sum_{i=1}^{k} \min(b_i, k-1) + \sum_{i=k+1}^{n} \min(b_i, k)$ holds for every $1 \leq k \leq n$.
  Sequences a and b called digraphic if there is a labeled simple directed graph such that each vertex v_i has indegree a_i and outdegree b_i.

- Pick's theorem

  For simple polygon, when points are all integer, we have $A = \#\{\text{lattice points in the interior}\} + \frac{\#\{\text{lattice points on the boundary}\}}{2} - 1$

- Möbius inversion formula

  - $f(n) = \sum_{d \mid n} g(d) \Leftrightarrow g(n) = \sum_{d \mid n} \mu(d) f(\frac{n}{d})$
  - $f(n) = \sum_{n \mid d} g(d) \Leftrightarrow g(n) = \sum_{n \mid d} \mu(\frac{d}{n}) f(d)$

- Spherical cap

  - A portion of a sphere cut off by a plane.
  - r: sphere radius, a: radius of the base of the cap, h: height of the cap, θ: $\arcsin(a/r)$.
  - Volume $= \pi h^2(3r - h)/3 = \pi h(3a^2 + h^2)/6 = \pi r^3(2 + \cos\theta)(1 - \cos\theta)^2/3$.
  - Area $= 2\pi rh = \pi(a^2 + h^2) = 2\pi r^2(1 - \cos\theta)$.

- The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200000 for $n < 1e19$.
- The number of ways of writing n as a sum of positive integers, disregarding the order of the summands. $1, 1, 2, 3, 5, 7, 11, 15, 22, 30$ for $n = 0 \sim 9$, 627 for $n = 20$, $\sim 2e5$ for $n = 50$, $\sim 2e8$ for $n = 100$.
- Total number of partitions of n distinct elements: $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975,$ $27644437, 190899322, \ldots$.

- Ordinary Generating Function $A(x) = \sum_{i \geq 0} a_i x^i$

  - $A(rx) \Rightarrow r^n a_n$
  - $A(x) + B(x) \Rightarrow a_n + b_n$
  - $A(x)B(x) \Rightarrow \sum_{i=0}^{n} a_i b_{n-i}$
  - $A(x)^k \Rightarrow \sum_{i_1+i_2+\cdots+i_k=n} a_{i_1} a_{i_2} \ldots a_{i_k}$
  - $xA(x)' \Rightarrow na_n$
  - $\frac{A(x)}{1-x} \Rightarrow \sum_{i=0}^{n} a_i$

- Exponential Generating Function $A(x) = \sum_{i \geq 0} \frac{a_i}{i!} x_i$

  - $A(x) + B(x) \Rightarrow a_n + b_n$
  - $A^{(k)}(x) \Rightarrow a_{n+k}$
  - $A(x)B(x) \Rightarrow \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$
  - $A(x)^k \Rightarrow \sum_{i_1+i_2+\cdots+i_k=n} \binom{n}{i_1, i_2, \ldots, i_k} a_{i_1} a_{i_2} \ldots a_{i_k}$
  - $xA(x) \Rightarrow na_n$

- Special Generating Function

  - $(1+x)^n = \sum_{i \geq 0} \binom{n}{i} x^i$
  - $\frac{1}{(1-x)^n} = \sum_{i \geq 0} \binom{i}{n-1} x^i$

- Bernoulli numbers

  $B_0 = 1, B_1^{\pm} = \pm\frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$

  $\sum_{j=0}^{m} \binom{m+1}{j} B_j = 0$, EGF is $B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}$.

  $S_m(n) = \sum_{k=1}^{n} k^m = \frac{1}{m+1} \sum_{k=0}^{m} \binom{m+1}{k} B_k^+ n^{m+1-k}$

- Stirling numbers of the second kind Partitions of n distinct elements into exactly k groups.

  $S(n,k) = S(n-1, k-1) + kS(n-1, k)$, $S(n,1) = S(n,n) = 1$

  $S(n,k) = \frac{1}{k!} \sum_{i=0}^{k} (-1)^{k-i} \binom{k}{i} i^n$

  $x^n = \sum_{i=0}^{n} S(n,i)(x)_i$

- Pentagonal number theorem

  $\prod_{n=1}^{\infty}(1 - x^n) = 1 + \sum_{k=1}^{\infty}(-1)^k \left(x^{k(3k+1)/2} + x^{k(3k-1)/2}\right)$

- Eulerian numbers

  Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

  $E(n,k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$

  $E(n,0) = E(n, n-1) = 1$

  $E(n,k) = \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$

7 Dynamic Programming

7.1 Dynamic Convex Hull

```
struct Line {
  // kx + b
  mutable i64 k, b, p;
  bool operator<(const Line& o) const {
      return k < o.k; }
  bool operator<(i64 x) const { return p
      < x; }
};
struct DynamicConvexHullMax : multiset<
    Line, less<>> {
  // (for doubles, use INF = 1/.0, div(a,
      b) = a/b)
  static constexpr i64 INF =
      numeric_limits<i64>::max();
  i64 div(i64 a, i64 b) {
    // floor
    return a / b - ((a ^ b) < 0 && a % b)
        ;
  }
  bool isect(iterator x, iterator y) {
    if (y == end()) return x->p = INF, 0;
    if (x->k == y->k) x->p = x->b > y->b
        ? INF : -INF;
    else x->p = div(y->b - x->b, x->k - y
        ->k);
    return x->p >= y->p;
  }
  void add(i64 k, i64 b) {
    auto z = insert({k, b, 0}), y = z++,
        x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y))
        isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p
         >= y->p)
      isect(x, erase(y));
  }
  i64 query(i64 x) {
    if (empty()) {
      return -INF;
    }
    auto l = *lower_bound(x);
    return l.k * x + l.b;
  }
};
```

7.2 $1D/1D$ Convex Optimization

```
struct segment {
  int i, l, r;
  segment(int a, int b, int c): i(a), l(b
      ), r(c) {}
};
inline long long f(int l, int r) { return
     dp[l] + w(l + 1, r); }
void solve() {
  dp[0] = 0ll;
  deque&lt;segment&gt; deq; deq.push_back(
      segment(0, 1, n));
  for (int i = 1; i <= n; ++i) {
  dp[i] = f(deq.front().i, i);
  while (deq.size() && deq.front().r < i
      + 1) deq.pop_front();
  deq.front().l = i + 1;
  segment seg = segment(i, i + 1, n);
  while (deq.size() && f(i, deq.back().l)
       < f(deq.back().i, deq.back().l))
      deq.pop_back();
  if (deq.size()) {
    int d = 1048576, c = deq.back().l;
    while (d >>= 1) if (c + d <= deq.back
        ().r) {
    if (f(i, c + d) > f(deq.back().i, c +
        d)) c += d;
    }
    deq.back().r = c; seg.l = c + 1;
  }
  if (seg.l <= n) deq.push_back(seg);
  }
}
```

7.3 Conditon

7.3.1 Totally Monotone (Concave/Convex)

$\forall i < i', j < j', B[i][j] \leq B[i'][j] \implies B[i][j'] \leq B[i'][j']$
$\forall i < i', j < j', B[i][j] \geq B[i'][j] \implies B[i][j'] \geq B[i'][j']$

7.3.2 Monge Condition (Concave/Convex)

$\forall i < i', j < j', B[i][j] + B[i'][j'] \geq B[i][j'] + B[i'][j]$
$\forall i < i', j < j', B[i][j] + B[i'][j'] \leq B[i][j'] + B[i'][j]$

7.3.3 Optimal Split Point

If

$$B[i][j] + B[i+1][j+1] \geq B[i][j+1] + B[i+1][j]$$

then

$$H_{i,j-1} \le H_{i,j} \le H_{i+1,j}$$

8 Geometry

8.1 Basic

```cpp
using Real = double; // modify these if needed
constexpr Real eps = 1e-9;
int sign(T x) { return (x > 0) - (x < 0); }
int sign(Real x) { return (x > eps) - (x < -eps); }
int cmp(T a, T b) { return sign(a - b); }
struct P {
  T x = 0, y = 0;
  P(T x = 0, T y = 0) : x(x), y(y) {}
  -, +*/, ==!=<, - (unary)
};
struct L {
  P<T> a, b;
  L(P<T> a = {}, P<T> b = {}) : a(a), b(b) {}
};
T dot(P<T> a, P<T> b) { return a.x * b.x + a.y * b.y; }
T square(P<T> a) { return dot(a, a); }
Real length(P<T> a) { return sqrtl(square(a)); }
Real dist(P<T> a, P<T> b) { return length(a - b); }
T cross(P<T> a, P<T> b) { return a.x * b.y - a.y * b.x; }
T cross(P<T> p, P<T> a, P<T> b) { return cross(a - p, b - p); }
P<Real> normal(P<T> a) {
  Real len = length(a);
  return P<Real>(a.x / len, a.y / len);
}
bool up(P<T> a) { return sign(a.y) > 0 || sign(a.y) == 0 && sign(a.x) > 0; }
// 3 colinear? please remember to remove (0, 0)
bool polar(P<T> a, P<T> b) {
  bool ua = up(a), ub = up(b);
  return ua != ub ? ua : sign(cross(a, b)) == 1;
}
bool sameDirection(P<T> a, P<T> b) { return sign(cross(a, b)) == 0 && sign(dot(a, b)) == 1; }
// 1/0/1 if on a->b's left/ /right
int side(P<T> p, P<T> a, P<T> b) { return sign(cross(p, a, b)); }
int side(P<T> p, L<T> l) { return side(p, l.a, l.b); }
P<T> rotate90(P<T> p) { return {-p.y, p.x}; }
P<Real> rotate(P<Real> p, Real ang) { return {p.x * cos(ang) - p.y * sin(ang), p.x * sin(ang) + p.y * cos(ang)}; }
Real angle(P<T> p) { return atan2(p.y, p.x); }
P<T> direction(L<T> l) { return l.b - l.a; }
bool sameDirection(L<T> l1, L<T> l2) { return sameDirection(direction(l1), direction(l2)); }
P<Real> projection(P<Real> p, L<Real> l) {
  auto d = direction(l);
  return l.a + d * (dot(p - l.a, d) / square(d));
}
P<Real> reflection(P<Real> p, L<Real> l) { return projection(p, l) * 2 - p; }
Real pointToLineDist(P<Real> p, L<Real> l) { return dist(p, projection(p, l)); }
// better use integers if you don't need exact coordinate
// l <= r is not explicitly required
P<Real> lineIntersection(L<T> l1, L<T> l2) { return l1.a - direction(l1) * (Real(cross(direction(l2), l1.a - l2.a)) / cross(direction(l2), direction(l1))); }
bool between(T m, T l, T r) { return cmp(l, m) == 0 || cmp(m, r) == 0 || l < m != r < m; }
bool pointOnSeg(P<T> p, L<T> l) { return side(p, l) == 0 && between(p.x, l.a.x, l.b.x) && between(p.y, l.a.y, l.b.y); }
bool pointStrictlyOnSeg(P<T> p, L<T> l) { return side(p, l) == 0 && sign(dot(p - l.a, direction(l))) * sign(dot(p - l.b, direction(l))) < 0; }
bool overlap(T l1, T r1, T l2, T r2) {
  if (l1 > r1) { swap(l1, r1); }
  if (l2 > r2) { swap(l2, r2); }
  return cmp(r1, l2) != -1 && cmp(r2, l1) != -1;
}
bool segIntersect(L<T> l1, L<T> l2) {
  auto [p1, p2] = l1;
  auto [q1, q2] = l2;
  return overlap(p1.x, p2.x, q1.x, q2.x) && overlap(p1.y, p2.y, q1.y, q2.y) &&
         side(p1, l2) * side(p2, l2) <= 0 &&
         side(q1, l1) * side(q2, l1) <= 0;
}
// parallel intersecting is false
bool segStrictlyIntersect(L<T> l1, L<T> l2) {
  auto [p1, p2] = l1;
  auto [q1, q2] = l2;
  return side(p1, l2) * side(p2, l2) < 0 &&
         side(q1, l1) * side(q2, l1) < 0;
}
// parallel or intersect at source doesn't count
bool rayIntersect(L<T> l1, L<T> l2) {
  int x = sign(cross(l1.b - l1.a, l2.b - l2.a));
  return x == 0 ? false : side(l1.a, l2) == x && side(l2.a, l1) == -x;
}
Real pointToSegDist(P<T> p, L<T> l) {
  auto d = direction(l);
  if (sign(dot(p - l.a, d)) >= 0 && sign(dot(p - l.b, d)) <= 0) {
    return 1.0L * cross(p, l.a, l.b) / dist(l.a, l.b);
  } else {
    return min(dist(p, l.a), dist(p, l.b));
  }
}
Real segDist(L<T> l1, L<T> l2) {
  if (segIntersect(l1, l2)) { return 0; }
  return min({pointToSegDist(l1.a, l2), pointToSegDist(l1.b, l2),
             pointToSegDist(l2.a, l1), pointToSegDist(l2.b, l1)});
}
// 2 times area
T area(vector<P<T>> a) {
  T res = 0;
  int n = a.size();
  for (int i = 0; i < n; i++) { res += cross(a[i], a[(i + 1) % n]); }
  return res;
}
bool pointInPoly(P<T> p, vector<P<T>> a) {
  int n = a.size(), res = 0;
  for (int i = 0; i < n; i++) {
    P<T> u = a[i], v = a[(i + 1) % n];
    if (pointOnSeg(p, {u, v})) { return 1; }
    if (cmp(u.y, v.y) <= 0) { swap(u, v); }
    if (cmp(p.y, u.y) > 0 || cmp(p.y, v.y) <= 0) { continue; }
    res ^= cross(p, u, v) > 0;
  }
  return res;
}
```

8.2 Convex Hull and related

```cpp
vector<P<T>> convexHull(vector<P<T>> a) {
  int n = a.size();
  if (n <= 1) { return a; }
  sort(a.begin(), a.end());
  a.resize(unique(a.begin(), a.end()), a.end());
  vector<P<T>> b(2 * n);
  int j = 0;
  for (int i = 0; i < n; b[j++] = a[i++]) {
    while (j >= 2 && side(b[j - 2], b[j - 1], a[i]) <= 0) { j--; }
  }
  for (int i = n - 2, k = j; i >= 0; b[j++] = a[i--]) {
    while (j > k && side(b[j - 2], b[j - 1], a[i]) <= 0) { j--; }
  }
  b.resize(j - 1);
  return b;
}
// nonstrict : change <= 0 to < 0
// warning : if all point on same line will return {1, 2, 3, 2}
```

8.3 Half Plane Intersection

```cpp
vector<P<Real>> halfPlaneIntersection(vector<L<Real>> a) {
  sort(a.begin(), a.end(), [&](auto l1, auto l2) {
    if (sameDirection(l1, l2)) {
      return side(l1.a, l2) > 0;
    } else {
      return polar(direction(l1), direction(l2));
    }
  });
  deque<L<Real>> dq;
  auto check = [&](L<Real> l, L<Real> l1, L<Real> l2) { return side(lineIntersection(l1, l2), l) > 0; };
  for (int i = 0; i < int(a.size()); i++) {
    if (i > 0 && sameDirection(a[i], a[i - 1])) { continue; }
    while (int(dq.size()) > 1 && !check(a[i], dq.end()[-2], dq.back())) { dq.pop_back(); }
    while (int(dq.size()) > 1 && !check(a[i], dq[1], dq[0])) { dq.pop_front(); }
    dq.push_back(a[i]);
  }
  while (int(dq.size()) > 2 && !check(dq[0], dq.end()[-2], dq.back())) { dq.pop_back(); }
  while (int(dq.size()) > 2 && !check(dq.back(), dq[1], dq[0])) { dq.pop_front(); }
  vector<P<Real>> res;
  dq.push_back(dq[0]);
  for (int i = 0; i + 1 < int(dq.size()); i++) { res.push_back(lineIntersection(dq[i], dq[i + 1])); }
  return res;
}
```

8.4 Triangle Centers

```cpp
// radius: (a + b + c) * r / 2 = A or pointToLineDist
P<Real> inCenter(P<Real> a, P<Real> b, P<Real> c) {
  Real la = length(b - c), lb = length(c - a), lc = length(a - b);
  return (a * la + b * lb + c * lc) / (la + lb + lc);
}
// used in min enclosing circle
```

```
P<Real> circumCenter(P<Real> a, P<Real> b
    , P<Real> c) {
  P<Real> ba = b - a, ca = c - a;
  Real db = square(ba), dc = square(ca),
      d = 2 * cross(ba, ca);
  return a - P<Real>(ba.y * dc - ca.y *
      db, ca.x * db - ba.x * dc) / d;
}
P<Real> orthoCenter(P<Real> a, P<Real> b,
     P<Real> c) {
  L<Real> u(c, P<Real>(c.x - a.y + b.y, c
      .y + a.x - b.x));
  L<Real> v(b, P<Real>(b.x - a.y + c.y, b
      .y + a.x - c.x));
  return lineIntersection(u, v);
}
```

8.5 Circle

```
const Real PI = acos(-1);
struct Circle {
  P<Real> o;
  Real r;
  Circle(P<Real> o = {}, Real r = 0) : o(
      o), r(r) {}
};
// actually counts number of tangent
    lines
int typeOfCircles(Circle c1, Circle c2) {
  auto [o1, r1] = c1;
  auto [o2, r2] = c2;
  Real d = dist(o1, o2);
  if (cmp(d, r1 + r2) == 1) { return 4; }
  if (cmp(d, r1 + r2) == 0) { return 3; }
  if (cmp(d, abs(r1 - r2)) == 1) { return
       2; }
  if (cmp(d, abs(r1 - r2)) == 0) { return
       1; }
  return 0;
}
// aligned l.a -> l.b;
vector<P<Real>> circleLineIntersection(
    Circle c, L<Real> l) {
  P<Real> p = projection(c.o, l);
  Real h = c.r * c.r - square(p - c.o);
  if (sign(h) < 0) { return {}; }
  P<Real> q = normal(direction(l)) *
      sqrtl(c.r * c.r - square(p - c.o))
      ;
  return {p - q, p + q};
}
// circles shouldn't be identical
// duplicated if only one intersection,
    aligned c1 counterclockwise
vector<P<Real>> circleIntersection(Circle
     c1, Circle c2) {
  int type = typeOfCircles(c1, c2);
  if (type == 0 || type == 4) { return
      {}; }
  auto [o1, r1] = c1;
  auto [o2, r2] = c2;
  Real d = clamp(dist(o1, o2), abs(r1 -
      r2), r1 + r2);
  Real y = (r1 * r1 + d * d - r2 * r2) /
      (2 * d), x = sqrtl(r1 * r1 - y * y
      );
  P<Real> dir = normal(o2 - o1), q1 = o1
      + dir * y, q2 = rotate90(dir) * x;
  return {q1 - q2, q1 + q2};
}
// counterclockwise, on circle -> no
    tangent
vector<P<Real>> pointCircleTangent(P<Real
    > p, Circle c) {
  Real x = square(p - c.o), d = x - c.r *
       c.r;
  if (sign(d) <= 0) { return {}; }
  P<Real> q1 = c.o + (p - c.o) * (c.r * c
      .r / x), q2 = rotate90(p - c.o) *
      (c.r * sqrt(d) / x);
  return {q1 - q2, q1 + q2};
}
// one-point tangent lines are not
    returned
vector<L<Real>> externalTangent(Circle c1
    , Circle c2) {
  auto [o1, r1] = c1;
  auto [o2, r2] = c2;
  vector<L<Real>> res;
  if (cmp(r1, r2) == 0) {
    P dr = rotate90(normal(o2 - o1)) * r1
        ;
    res.emplace_back(o1 + dr, o2 + dr);
    res.emplace_back(o1 - dr, o2 - dr);
  } else {
    P p = (o2 * r1 - o1 * r2) / (r1 - r2)
        ;
    auto ps = pointCircleTangent(p, c1),
         qs = pointCircleTangent(p, c2);
    for (int i = 0; i < int(min(ps.size()
        , qs.size())); i++) { res.
        emplace_back(ps[i], qs[i]); }
  }
  return res;
}
vector<L<Real>> internalTangent(Circle c1
    , Circle c2) {
  auto [o1, r1] = c1;
  auto [o2, r2] = c2;
  vector<L<Real>> res;
  P<Real> p = (o1 * r2 + o2 * r1) / (r1 +
      r2);
  auto ps = pointCircleTangent(p, c1), qs
       = pointCircleTangent(p, c2);
  for (int i = 0; i < int(min(ps.size(),
      qs.size())); i++) { res.
      emplace_back(ps[i], qs[i]); }
  return res;
}
// OAB and circle directed area
Real triangleCircleIntersectionArea(P<
    Real> p1, P<Real> p2, Real r) {
  auto angle = [&](P<Real> p1, P<Real> p2
      ) { return atan2l(cross(p1, p2),
      dot(p1, p2)); };
  vector<P<Real>> v =
      circleLineIntersection(Circle(P<
      Real>(), r), L<Real>(p1, p2));
  if (v.empty()) { return r * r * angle(
      p1, p2) / 2; }
  bool b1 = cmp(square(p1), r * r) == 1,
      b2 = cmp(square(p2), r * r) == 1;
  if (b1 && b2) {
    if (sign(dot(p1 - v[0], p2 - v[0]))
        <= 0 && sign(dot(p1 - v[0], p2 -
        v[0])) <= 0) {
      return r * r * (angle(p1, v[0]) +
          angle(v[1], p2)) / 2 + cross(v
          [0], v[1]) / 2;
    } else {
      return r * r * angle(p1, p2) / 2;
    }
  } else if (b1) {
    return (r * r * angle(p1, v[0]) +
        cross(v[0], p2)) / 2;
  } else if (b2) {
    return (cross(p1, v[1]) + r * r *
        angle(v[1], p2)) / 2;
  } else {
    return cross(p1, p2) / 2;
  }
}
Real polyCircleIntersectionArea(const
    vector<P<Real>> &a, Circle c) {
  int n = a.size();
  Real ans = 0;
  for (int i = 0; i < n; i++) {
    ans += triangleCircleIntersectionArea
        (a[i], a[(i + 1) % n], c.r);
  }
  return ans;
}
Real circleIntersectionArea(Circle a,
    Circle b) {
  int t = typeOfCircles(a, b);
  if (t >= 3) {
    return 0;
  } else if (t <= 1) {
    Real r = min(a.r, b.r);
    return r * r * PI;
  }
  Real res = 0, d = dist(a.o, b.o);
  for (int i = 0; i < 2; ++i) {
    Real alpha = acos((b.r * b.r + d * d
        - a.r * a.r) / (2 * b.r * d));
    Real s = alpha * b.r * b.r;
    Real t = b.r * b.r * sin(alpha) * cos
        (alpha);
    res += s - t;
    swap(a, b);
  }
  return res;
}
```

8.6 Delaunay Triangulation

```
const P<i64> pINF = P<i64>(1e18, 1e18);
using i128 = __int128_t;
struct Quad {
  P<i64> origin;
  Quad *rot = nullptr, *onext = nullptr;
  bool used = false;
  Quad* rev() const { return rot->rot; }
  Quad* lnext() const { return rot->rev()
      ->onext->rot; }
  Quad* oprev() const { return rot->onext
      ->rot; }
  P<i64> dest() const { return rev()->
      origin; }
};
Quad* makeEdge(P<i64> from, P<i64> to) {
  Quad *e1 = new Quad, *e2 = new Quad, *
      e3 = new Quad, *e4 = new Quad;
  e1->origin = from;
  e2->origin = to;
  e3->origin = e4->origin = pINF;
  e1->rot = e3;
  e2->rot = e4;
  e3->rot = e2;
  e4->rot = e1;
  e1->onext = e1;
  e2->onext = e2;
  e3->onext = e4;
  e4->onext = e3;
  return e1;
}
void splice(Quad *a, Quad *b) {
  swap(a->onext->rot->onext, b->onext->
      rot->onext);
  swap(a->onext, b->onext);
}
void delEdge(Quad *e) {
  splice(e, e->oprev());
  splice(e->rev(), e->rev()->oprev());
  delete e->rev()->rot;
  delete e->rev();
  delete e->rot;
  delete e;
}
Quad *connect(Quad *a, Quad *b) {
  Quad *e = makeEdge(a->dest(), b->origin
      );
  splice(e, a->lnext());
  splice(e->rev(), b);
  return e;
}
bool onLeft(P<i64> p, Quad *e) { return
    side(p, e->origin, e->dest()) > 0; }
bool onRight(P<i64> p, Quad *e) { return
    side(p, e->origin, e->dest()) < 0; }
template <class T>
T det3(T a1, T a2, T a3, T b1, T b2, T b3
    , T c1, T c2, T c3) {
  return a1 * (b2 * c3 - c2 * b3) - a2 *
      (b1 * c3 - c1 * b3) + a3 * (b1 *
      c2 - c1 * b2);
}
bool inCircle(P<i64> a, P<i64> b, P<i64>
    c, P<i64> d) {
  auto f = [&](P<i64> a, P<i64> b, P<i64>
       c) {
    return det3<i128>(a.x, a.y, square(a)
        , b.x, b.y, square(b), c.x, c.y,
        square(c));
  };
  i128 det = f(a, c, d) + f(a, b, c) - f(
      b, c, d) - f(a, b, d);
  return det > 0;
}
pair<Quad*, Quad*> build(int l, int r,
    vector<P<i64>> &p) {
```

```
  if (r - l == 2) {
    Quad *res = makeEdge(p[l], p[l + 1]);
    return pair(res, res->rev());
  } else if (r - l == 3) {
    Quad *a = makeEdge(p[l], p[l + 1]), *
        b = makeEdge(p[l + 1], p[l + 2])
        ;
    splice(a->rev(), b);
    int sg = sign(cross(p[l], p[l + 1], p
        [l + 2]));
    if (sg == 0) { return pair(a, b->rev
        ()); }
    Quad *c = connect(b, a);
    if (sg == 1) {
      return pair(a, b->rev());
    } else {
      return pair(c->rev(), c);
    }
  }
  int m = l + r >> 1;
  auto [ldo, ldi] = build(l, m, p);
  auto [rdi, rdo] = build(m, r, p);
  while (true) {
    if (onLeft(rdi->origin, ldi)) {
      ldi = ldi->lnext();
      continue;
    }
    if (onRight(ldi->origin, rdi)) {
      rdi = rdi->rev()->onext;
      continue;
    }
    break;
  }
  Quad *basel = connect(rdi->rev(), ldi);
  auto valid = [&](Quad *e) { return
      onRight(e->dest(), basel); };
  if (ldi->origin == ldo->origin) { ldo =
      basel->rev(); }
  if (rdi->origin == rdo->origin) { rdo =
      basel; }
  while (true) {
    Quad *lcand = basel->rev()->onext;
    if (valid(lcand)) {
      while (inCircle(basel->dest(),
          basel->origin, lcand->dest(),
          lcand->onext->dest())) {
        Quad *t = lcand->onext;
        delEdge(lcand);
        lcand = t;
      }
    }
    Quad *rcand = basel->oprev();
    if (valid(rcand)) {
      while (inCircle(basel->dest(),
          basel->origin, rcand->dest(),
          rcand->oprev()->dest())) {
        Quad *t = rcand->oprev();
        delEdge(rcand);
        rcand = t;
      }
    }
    if (!valid(lcand) && !valid(rcand)) {
        break; }
    if (!valid(lcand) || valid(rcand) &&
        inCircle(lcand->dest(), lcand->
        origin, rcand->origin, rcand->
        dest())) {
      basel = connect(rcand, basel->rev()
          );
    } else {
      basel = connect(basel->rev(), lcand
          ->rev());
    }
  }
  return pair(ldo, rdo);
}
vector<array<P<i64>, 3>> delaunay(vector<
    P<i64>> p) {
  sort(p.begin(), p.end());
  auto res = build(0, p.size(), p);
  Quad *e = res.first;
  vector<Quad*> edges = {e};
  while (sign(cross(e->onext->dest(), e->
      dest(), e->origin)) == -1) { e = e
      ->onext; }
  auto add = [&]() {
    Quad *cur = e;
    do {
      cur->used = true;
      p.push_back(cur->origin);
      edges.push_back(cur->rev());
      cur = cur->lnext();
    } while (cur != e);
  };
  add();
  p.clear();
  int i = 0;
  while (i < int(edges.size())) { if (!(e
      = edges[i++])->used) { add(); }}
  vector<array<P<i64>, 3>> ans(p.size() /
      3);
  for (int i = 0; i < int(p.size()); i++)
      { ans[i / 3][i % 3] = p[i]; }
  return ans;
}
```

8.7 ConvexHull Operations (yhchang3)

```
给定凸包，log n 内完成各种询问，具体操作
    有：
1. 判定一个点是否在凸包内
2. 询问凸包外的点到凸包的两个切点
3. 询问一个向量关于凸包的切点
4. 询问一条直线和凸包的交点
 INF 为坐标范围，需要定义点类大于号
 改成实数只需修改 sign 函数，以及把 long
     long 改为 double 即可
 构造函数时传入凸包要求无重点，面积非空，
     以及 pair(x,y) 的最小点放在第一个

const int INF = 1e9;
struct Convex {
  int n;
  vector<Point> a, upper, lower;
  Convex(vector<Point> a_) : a(a_) {
    n = a_.size();
    int ptr = 0;
    for (int i = 1; i < n; i++) {
      if (a[ptr] < a[i]) ptr = i;
    }
    for (int i = 0; i <= ptr; i++) {
      lower.push_back(a[i]);
    }
    for (int i = ptr; i < n; i++) {
      upper.push_back(a[i]);
    }
    upper.push_back(a[0]);
  }
  int sign(long long x) { return x < 0 ?
      -1 : x > 0; }
  pair<long long, int> get_tangent(vector
      <Point> &convex, Point vec) {
    int l = 0, r = int(convex.size()) -
        2;
    for ( ; l + 1 < r; ) {
      int mid = (l + r) / 2;
      if (sign((convex[mid + 1] - convex[
          mid]).det(vec)) > 0) r = mid;
      else l = mid;
    }
    return max(make_pair(vec.det(convex[r
        ]), r), make_pair(vec.det(convex
        [0]), 0));
  }
  void update_tangent(const Point &p, int
       id, int &i0, int &i1) {
    if ((a[i0] - p).det(a[id] - p) > 0)
        i0 = id;
    if ((a[i1] - p).det(a[id] - p) < 0)
        i1 = id;
  }
  void binary_search(int l, int r, Point
      p, int &i0, int &i1) {
    if (l == r) { return; }
    update_tangent(p, l % n, i0, i1);
    int sl = sign((a[l % n] - p).det(a[(l
         + 1) % n] - p));
    for ( ; l + 1 < r; ) {
      int mid = (l + r) / 2;
      int smid = sign((a[mid % n] - p).
          det(a[(mid + 1) % n] - p));
      if (smid == sl) l = mid;
      else r = mid;
    }
    update_tangent(p, r % n, i0, i1);
  }
  int binary_search(Point u, Point v, int
       l, int r) {
    int sl = sign((v - u).det(a[l % n] -
        u));
    for( ; l + 1 < r; ) {
      int mid=(l+r)/2;
      int smid = sign((v - u).det(a[mid %
           n] - u));
      if(smid==sl)l=mid;
      else r = mid;
    }
    return l % n;
  }
  // 判定点是否在凸包内，在边界返回 true
  bool contain(Point p) {
    if (p.x < lower[0].x || p.x > lower.
        back().x) return false;
    int id = lower_bound(lower.begin(),
        lower.end(), Point(p.x, -INF)) -
         lower.begin();
    if (lower[id].x == p.x) {
      if (lower[id].y > p.y) return false
          ;
    } else if ((lower[id - 1] - p).det(
        lower[id] - p) < 0) return false
        ;
    id = lower_bound(upper.begin(), upper
        .end(), Point(p.x, INF), greater
        <Point>()) - upper.begin();
    if (upper[id].x == p.x) {
      if (upper[id].y < p.y) return false
          ;
    } else if ((upper[id - 1] - p).det(
        upper[id] - p) < 0) return false
        ;
    return true;
  }
  // 求点 p 关于凸包的两个切点，如果在凸
      包外则有序返回编号，共线的多个切点
      返回任意一个，否则返回 false
  bool get_tangent(Point p, int &i0, int
      &i1) {
    if (contain(p)) return false;
    i0 = i1 = 0;
    int id = lower_bound(lower.begin(),
        lower.end(),p) - lower.begin();
    binary_search(0, id, p, i0, i1);
    binary_search(id, (int)lower.size(),
        p, i0, i1);
    id = lower_bound(upper.begin(), upper
        .end(), p, greater<Point>()) -
        upper.begin();
    binary_search((int)lower.size() - 1,
        (int)lower.size() - 1 + id, p,
        i0, i1);
    binary_search((int)lower.size() - 1 +
         id, (int)lower.size() - 1 + (
        int)upper.size(), p, i0, i1);
    return true;
  }
  // 求凸包上和向量 vec 叉积最大的点，返
      回编号，共线的多个切点返回任意一个
  int get_tangent(Point vec) {
    pair<long long, int> ret =
        get_tangent(upper, vec);
    ret.second = (ret.second + int(lower.
        size()) - 1) % n;
    ret = max(ret, get_tangent(lower, vec
        ));
    return ret.second;
  }
  // 求凸包和直线 u,v 的交点，如果无严格
      相交返回 false. 如果有则是和(i,
      next(i)) 的交点，两个点无序，交在
      点上不确定返回前后两条线段其中之一
  bool get_intersection(Point p, Point v,
       int &i0, int &i1) {
    int p0 = get_tangent(u - v), p1 =
        get_tangent(v - u);
    if (sign((v - u).det(a[p0] - u)) *
        sign((v - u).det(a[p1] - u)) <
        0) {
      if (p0 > p1) swap(p0, p1);
      i0 = binary_search(u, v, p0, p1);
```

```
      i1 = binary_search(u, v, p1, p0 + n);
      return true;
    } else { return false; }
  }
```

9 Miscellaneous

9.1 Cactus 1

```
auto work = [&](const vector<int> cycle) {
  // merge cycle info to u?
  int len = cycle.size(), u = cycle[0];
};
auto dfs = [&](auto dfs, int u, int p) {
  par[u] = p;
  vis[u] = 1;
  for (auto v : adj[u]) {
    if (v == p) { continue; }
    if (vis[v] == 0) {
      dfs(dfs, v, u);
      if (!cyc[v]) { // merge dp }
    } else if (vis[v] == 1) {
      for (int w = u; w != v; w = par[w]) {
        cyc[w] = 1;
      }
    } else {
      vector<int> cycle = {u};
      for (int w = v; w != u; w = par[w]) { cycle.push_back(w); }
      work(cycle);
    }
  }

  vis[u] = 2;
};
```

9.2 Cactus 2

```
// a component contains no articulation point, so P2 is a component
// but not a vertex biconnected component by definition
// resulting bct is rooted
struct BlockCutTree {
  int n, square = 0, cur = 0;
  vector<int> low, dfn, stk;
  vector<vector<int>> adj, bct;
  BlockCutTree(int n) : n(n), low(n), dfn(n, -1), adj(n), bct(n) {}
  void build() { dfs(0); }
  void addEdge(int u, int v) { adj[u].push_back(v), adj[v].push_back(u); }
  void dfs(int u) {
    low[u] = dfn[u] = cur++;
    stk.push_back(u);
    for (auto v : adj[u]) {
      if (dfn[v] == -1) {
        dfs(v);
        low[u] = min(low[u], low[v]);
        if (low[v] == dfn[u]) {
          bct.emplace_back();
          int x;
          do {
            x = stk.back();
            stk.pop_back();
            bct.back().push_back(x);
          } while (x != v);
          bct[u].push_back(n + square);
          square++;
        }
      } else {
        low[u] = min(low[u], dfn[v]);
      }
    }
  }
};
```

9.3 Dancing Links

```
#include <bits/stdc++.h>
using namespace std;
// tioj 1333
#define TRAV(i, link, start) for (int i = link[start]; i != start; i = link[i])
const int NN = 40000, RR = 200;
template<bool E> // E: Exact, NN: num of 1s, RR: num of rows
struct DLX {
  int lt[NN], rg[NN], up[NN], dn[NN], rw[NN], cl[NN], bt[NN], s[NN], head, sz, ans;
  int rows, columns;
  bool vis[NN];
  bitset<RR> sol, cur; // not sure
  void remove(int c) {
    if (E) lt[rg[c]] = lt[c], rg[lt[c]] = rg[c];
    TRAV(i, dn, c) {
      if (E) {
        TRAV(j, rg, i)
          up[dn[j]] = up[j], dn[up[j]] = dn[j], --s[cl[j]];
      } else {
        lt[rg[i]] = lt[i], rg[lt[i]] = rg[i];
      }
    }
  }
  void restore(int c) {
    TRAV(i, up, c) {
      if (E) {
        TRAV(j, lt, i)
          ++s[cl[j]], up[dn[j]] = j, dn[up[j]] = j;
      } else {
        lt[rg[i]] = rg[lt[i]] = i;
      }
    }
    if (E) lt[rg[c]] = c, rg[lt[c]] = c;
  }
  void init(int c) {
    rows = 0, columns = c;
    for (int i = 0; i < c; ++i) {
      up[i] = dn[i] = bt[i] = i;
      lt[i] = i == 0 ? c : i - 1;
      rg[i] = i == c - 1 ? c : i + 1;
      s[i] = 0;
    }
    rg[c] = 0, lt[c] = c - 1;
    up[c] = dn[c] = -1;
    head = c, sz = c + 1;
  }
  void insert(const vector<int> &col) {
    if (col.empty()) return;
    int f = sz;
    for (int i = 0; i < (int)col.size(); ++i) {
      int c = col[i], v = sz++;
      dn[bt[c]] = v;
      up[v] = bt[c], bt[c] = v;
      rg[v] = (i + 1 == (int)col.size() ? f : v + 1);
      rw[v] = rows, cl[v] = c;
      ++s[c];
      if (i > 0) lt[v] = v - 1;
    }
    ++rows, lt[f] = sz - 1;
  }
  int h() {
    int ret = 0;
    fill_n(vis, sz, false);
    TRAV(x, rg, head) {
      if (vis[x]) continue;
      vis[x] = true, ++ret;
      TRAV(i, dn, x) TRAV(j, rg, i) vis[cl[j]] = true;
    }
    return ret;
  }
  void dfs(int dep) {
    if (dep + (E ? 0 : h()) >= ans) return;
    if (rg[head] == head) return sol = cur, ans = dep, void();
    if (dn[rg[head]] == rg[head]) return;
    int w = rg[head];
    TRAV(x, rg, head) if (s[x] < s[w]) w = x;
    if (E) remove(w);
    TRAV(i, dn, w) {
      if (!E) remove(i);
      TRAV(j, rg, i) remove(E ? cl[j] : j);
      cur.set(rw[i]), dfs(dep + 1), cur.reset(rw[i]);
      TRAV(j, lt, i) restore(E ? cl[j] : j);
      if (!E) restore(i);
    }
    if (E) restore(w);
  }
  int solve() {
    for (int i = 0; i < columns; ++i)
      dn[bt[i]] = i, up[i] = bt[i];
    ans = 1e9, sol.reset(), dfs(0);
    return ans;
  }
};
int main() {
    int n, m; cin >> n >> m;
    DLX<true> solver;
    solver.init(m);
    for (int i = 0; i < n; i++){
        vector<int> add;
        for (int j = 0; j < m; j++){
            int x; cin >> x;
            if (x == 1) {
                add.push_back(j);
            }
        }
        solver.insert(add);
    }
    cout << solver.solve() << '\n';
    return 0;
}
```

9.4 Offline Dynamic MST

```
int cnt[maxn], cost[maxn], st[maxn], ed[maxn];
pair<int, int> qr[maxn];
// qr[i].first = id of edge to be changed, qr[i].second = weight after operation
// cnt[i] = number of operation on edge i
// call solve(0, q - 1, v, 0), where v contains edges i such that cnt[i] == 0

void contract(int l, int r, vector<int> v, vector<int> &x, vector<int> &y) {
  sort(v.begin(), v.end(), [&](int i, int j) {
    if (cost[i] == cost[j]) return i < j;
    return cost[i] < cost[j];
    });
  djs.save();
  for (int i = l; i <= r; ++i) djs.merge(st[qr[i].first], ed[qr[i].first]);
  for (int i = 0; i < (int)v.size(); ++i) {
    if (djs.find(st[v[i]]) != djs.find(ed[v[i]])) {
      x.push_back(v[i]);
      djs.merge(st[v[i]], ed[v[i]]);
    }
  }
  djs.undo();
  djs.save();
  for (int i = 0; i < (int)x.size(); ++i) djs.merge(st[x[i]], ed[x[i]]);
  for (int i = 0; i < (int)v.size(); ++i) {
    if (djs.find(st[v[i]]) != djs.find(ed[v[i]])) {
      y.push_back(v[i]);
      djs.merge(st[v[i]], ed[v[i]]);
    }
  }
  djs.undo();
```

```
}

void solve(int l, int r, vector<int> v,
    long long c) {
  if (l == r) {
    cost[qr[l].first] = qr[l].second;
    if (st[qr[l].first] == ed[qr[l].first
        ]) {
      printf("%lld\n", c);
      return;
    }
    int minv = qr[l].second;
    for (int i = 0; i < (int)v.size(); ++
        i) minv = min(minv, cost[v[i]]);
    printf("%lld\n", c + minv);
    return;
  }
  int m = (l + r) >> 1;
  vector<int> lv = v, rv = v;
  vector<int> x, y;
  for (int i = m + 1; i <= r; ++i) {
    cnt[qr[i].first]--;
    if (cnt[qr[i].first] == 0) lv.
        push_back(qr[i].first);
  }
  contract(l, m, lv, x, y);
  long long lc = c, rc = c;
  djs.save();
  for (int i = 0; i < (int)x.size(); ++i)
      {
    lc += cost[x[i]];
    djs.merge(st[x[i]], ed[x[i]]);
  }
  solve(l, m, y, lc);
  djs.undo();
  x.clear(), y.clear();
  for (int i = m + 1; i <= r; ++i) cnt[qr
      [i].first]++;
  for (int i = l; i <= m; ++i) {
    cnt[qr[i].first]--;
    if (cnt[qr[i].first] == 0) rv.
        push_back(qr[i].first);
  }
  contract(m + 1, r, rv, x, y);
  djs.save();
  for (int i = 0; i < (int)x.size(); ++i)
      {
    rc += cost[x[i]];
    djs.merge(st[x[i]], ed[x[i]]);
  }
  solve(m + 1, r, y, rc);
  djs.undo();
  for (int i = l; i <= m; ++i) cnt[qr[i].
      first]++;
}
```

9.5 Matroid Intersection

- $x \to y$ if $S - \{x\} \cup \{y\} \in I_1$ with $cost(\{y\})$.
- $source \to y$ if $S \cup \{y\} \in I_1$ with $cost(\{y\})$.
- $y \to x$ if $S - \{x\} \cup \{y\} \in I_2$ with $-cost(\{y\})$.
- $y \to sink$ if $S \cup \{y\} \in I_2$ with $-cost(\{y\})$.

Augmenting path is shortest path from source to sink.

9.6 Euler Tour

```
vector<int> euler, vis(V);
auto dfs = [&](auto dfs, int u) -> void {
  while (!adj[u].empty()) {
    while (!adj[u].empty() && del[adj[u].
        back()[1]]) {
      adj[u].pop_back();
    }
    if (!adj[u].empty()) {
      auto [v, i] = adj[u].back();
      del[i] = true;
      dfs(dfs, v);
    }
  }
  euler.push_back(u);
};
dfs(dfs, 0);
reverse(euler.begin(), euler.end());
```

9.7 SegTree Beats

```
struct SegmentTree {
  int n;
  struct node {
    i64 mx1, mx2, mxc;
    i64 mn1, mn2, mnc;
    i64 add;
    i64 sum;
    node(i64 v = 0) {
      mx1 = mn1 = sum = v;
      mxc = mnc = 1;
      add = 0;
      mx2 = -9e18, mn2 = 9e18;
    }
  };
  vector<node> t;
  // build
  void push(int id, int l, int r) {
    auto& c = t[id];
    int m = l + r >> 1;
    if (c.add != 0) {
      apply_add(id << 1, l, m, c.add);
      apply_add(id << 1 | 1, m + 1, r, c.
          add);
      c.add = 0;
    }
    apply_min(id << 1, l, m, c.mn1);
    apply_min(id << 1 | 1, m + 1, r, c.
        mn1);
    apply_max(id << 1, l, m, c.mx1);
    apply_max(id << 1 | 1, m + 1, r, c.
        mx1);
  }
  void apply_add(int id, int l, int r,
      i64 v) {
    if (v == 0) {
      return;
    }
    auto& c = t[id];
    c.add += v;
    c.sum += v * (r - l + 1);
    c.mx1 += v;
    c.mn1 += v;
    if (c.mx2 != -9e18) {
      c.mx2 += v;
    }
    if (c.mn2 != 9e18) {
      c.mn2 += v;
    }
  }
  void apply_min(int id, int l, int r,
      i64 v) {
    auto& c = t[id];
    if (v <= c.mn1) {
      return;
    }
    c.sum -= c.mn1 * c.mnc;
    c.mn1 = v;
    c.sum += c.mn1 * c.mnc;
    if (l == r || v >= c.mx1) {
      c.mx1 = v;
    } else if (v > c.mx2) {
      c.mx2 = v;
    }
  }
  void apply_max(int id, int l, int r,
      i64 v) {
    auto& c = t[id];
    if (v >= c.mx1) {
      return;
    }
    c.sum -= c.mx1 * c.mxc;
    c.mx1 = v;
    c.sum += c.mx1 * c.mxc;
    if (l == r || v <= c.mn1) {
      c.mn1 = v;
    } else if (v < c.mn2) {
      c.mn2 = v;
    }
  }
  void pull(int id) {
    auto &c = t[id], &lc = t[id << 1], &
        rc = t[id << 1 | 1];
    c.sum = lc.sum + rc.sum;
    if (lc.mn1 == rc.mn1) {
      c.mn1 = lc.mn1;
      c.mn2 = min(lc.mn2, rc.mn2);
      c.mnc = lc.mnc + rc.mnc;
    } else if (lc.mn1 < rc.mn1) {
      c.mn1 = lc.mn1;
      c.mn2 = min(lc.mn2, rc.mn1);
      c.mnc = lc.mnc;
    } else {
      c.mn1 = rc.mn1;
      c.mn2 = min(lc.mn1, rc.mn2);
      c.mnc = rc.mnc;
    }
    if (lc.mx1 == rc.mx1) {
      c.mx1 = lc.mx1;
      c.mx2 = max(lc.mx2, rc.mx2);
      c.mxc = lc.mxc + rc.mxc;
    } else if (lc.mx1 > rc.mx1) {
      c.mx1 = lc.mx1;
      c.mx2 = max(lc.mx2, rc.mx1);
      c.mxc = lc.mxc;
    } else {
      c.mx1 = rc.mx1;
      c.mx2 = max(lc.mx1, rc.mx2);
      c.mxc = rc.mxc;
    }
  }
  void range_chmin(int id, int l, int r,
      int ql, int qr, i64 v) {
    if (r < ql || l > qr || v >= t[id].
        mx1) {
      return;
    }
    if (ql <= l && r <= qr && v > t[id].
        mx2) {
      apply_max(id, l, r, v);
      return;
    }
    push(id, l, r);
    int m = l + r >> 1;
    range_chmin(id << 1, l, m, ql, qr, v)
        ;
    range_chmin(id << 1 | 1, m + 1, r, ql
        , qr, v);
    pull(id);
  }
  void range_chmin(int ql, int qr, i64 v)
      {
    range_chmin(1, 0, n - 1, ql, qr, v);
  }
  void range_chmax(int id, int l, int r,
      int ql, int qr, i64 v) {
    if (r < ql || l > qr || v <= t[id].
        mn1) {
      return;
    }
    if (ql <= l && r <= qr && v < t[id].
        mn2) {
      apply_min(id, l, r, v);
      return;
    }
    push(id, l, r);
    int m = l + r >> 1;
    range_chmax(id << 1, l, m, ql, qr, v)
        ;
    range_chmax(id << 1 | 1, m + 1, r, ql
        , qr, v);
    pull(id);
  }
  void range_chmax(int ql, int qr, i64 v)
      {
    range_chmax(1, 0, n - 1, ql, qr, v);
  }
  void range_add(int id, int l, int r,
      int ql, int qr, i64 v) {
    if (r < ql || l > qr) {
      return;
    }
    if (ql <= l && r <= qr) {
      apply_add(id, l, r, v);
      return;
    }
    push(id, l, r);
    int m = l + r >> 1;
    range_add(id << 1, l, m, ql, qr, v);
    range_add(id << 1 | 1, m + 1, r, ql,
        qr, v);
    pull(id);
  }
  void range_add(int ql, int qr, i64 v) {
    range_add(1, 0, n - 1, ql, qr, v);
  }
```

```
  i64 range_sum(int id, int l, int r, int
      ql, int qr) {
    if (r < ql || l > qr) {
      return 0;
    }
    if (ql <= l && r <= qr) {
      return t[id].sum;
    }
    push(id, l, r);
    int m = l + r >> 1;
    return range_sum(id << 1, l, m, ql,
        qr) + range_sum(id << 1 | 1, m +
         1, r, ql, qr);
  }
  i64 range_sum(int ql, int qr) {
    return range_sum(1, 0, n - 1, ql, qr)
        ;
  }
};
```

9.8 unorganized

```
const int N = 1021;
struct CircleCover {
  int C;
  Cir c[N];
  bool g[N][N], overlap[N][N];
  // Area[i] : area covered by at least i
        circles
  double Area[ N ];
  void init(int _C){ C = _C;}
  struct Teve {
    pdd p; double ang; int add;
    Teve() {}
    Teve(pdd _a, double _b, int _c):p(_a)
        , ang(_b), add(_c){}
    bool operator<(const Teve &a)const
    {return ang < a.ang;}
  }eve[N * 2];
  // strict: x = 0, otherwise x = -1
  bool disjuct(Cir &a, Cir &b, int x)
  {return sign(abs(a.O - b.O) - a.R - b.R
        ) > x;}
  bool contain(Cir &a, Cir &b, int x)
  {return sign(a.R - b.R - abs(a.O - b.O)
        ) > x;}
  bool contain(int i, int j) {
    /* c[j] is non-strictly in c[i]. */
    return (sign(c[i].R - c[j].R) > 0 ||
        (sign(c[i].R - c[j].R) == 0 && i
         < j)) && contain(c[i], c[j],
         -1);
  }
  void solve(){
    fill_n(Area, C + 2, 0);
    for(int i = 0; i < C; ++i)
      for(int j = 0; j < C; ++j)
        overlap[i][j] = contain(i, j);
    for(int i = 0; i < C; ++i)
      for(int j = 0; j < C; ++j)
        g[i][j] = !(overlap[i][j] ||
            overlap[j][i] ||
           disjuct(c[i], c[j], -1));
    for(int i = 0; i < C; ++i){
      int E = 0, cnt = 1;
      for(int j = 0; j < C; ++j)
        if(j != i && overlap[j][i])
          ++cnt;
      for(int j = 0; j < C; ++j)
        if(i != j && g[i][j]) {
          pdd aa, bb;
          CCinter(c[i], c[j], aa, bb);
          double A = atan2(aa.Y - c[i].O.
              Y, aa.X - c[i].O.X);
          double B = atan2(bb.Y - c[i].O.
              Y, bb.X - c[i].O.X);
          eve[E++] = Teve(bb, B, 1), eve[
              E++] = Teve(aa, A, -1);
          if(B > A) ++cnt;
        }
      if(E == 0) Area[cnt] += pi * c[i].R
           * c[i].R;
      else{
        sort(eve, eve + E);
        eve[E] = eve[0];
        for(int j = 0; j < E; ++j){
          cnt += eve[j].add;
          Area[cnt] += cross(eve[j].p,
              eve[j + 1].p) * .5;
          double theta = eve[j + 1].ang -
               eve[j].ang;
          if (theta < 0) theta += 2. * pi
              ;
          Area[cnt] += (theta - sin(theta
              )) * c[i].R * c[i].R * .5;
        }
      }
    }
  }
};

struct Point {
  double x, y, z;
  Point(double _x = 0, double _y = 0,
      double _z = 0): x(_x), y(_y), z(_z
      ){}
  Point(pdd p) { x = p.X, y = p.Y, z =
      abs2(p); }
};
Point operator-(Point p1, Point p2)
{ return Point(p1.x - p2.x, p1.y - p2.y,
     p1.z - p2.z); }
Point operator+(Point p1, Point p2)
{ return Point(p1.x + p2.x, p1.y + p2.y,
     p1.z + p2.z); }
Point operator*(Point p1, double v)
{ return Point(p1.x * v, p1.y * v, p1.z *
      v); }
Point operator/(Point p1, double v)
{ return Point(p1.x / v, p1.y / v, p1.z /
      v); }
Point cross(Point p1, Point p2)
{ return Point(p1.y * p2.z - p1.z * p2.y,
     p1.z * p2.x - p1.x * p2.z, p1.x *
     p2.y - p1.y * p2.x); }
double dot(Point p1, Point p2)
{ return p1.x * p2.x + p1.y * p2.y + p1.z
      * p2.z; }
double abs(Point a)
{ return sqrt(dot(a, a)); }
Point cross3(Point a, Point b, Point c)
{ return cross(b - a, c - a); }
double area(Point a, Point b, Point c)
{ return abs(cross3(a, b, c)); }
double volume(Point a, Point b, Point c,
    Point d)
{ return dot(cross3(a, b, c), d - a); }
//Azimuthal angle (longitude) to x-axis
     in interval [-pi, pi]
double phi(Point p) { return atan2(p.y, p
     .x); }
//Zenith angle (latitude) to the z-axis
     in interval [0, pi]
double theta(Point p) { return atan2(sqrt
     (p.x * p.x + p.y * p.y), p.z); }
Point masscenter(Point a, Point b, Point
     c, Point d)
{ return (a + b + c + d) / 4; }
pdd proj(Point a, Point b, Point c, Point
     u) {
// proj. u to the plane of a, b, and c
  Point e1 = b - a;
  Point e2 = c - a;
  e1 = e1 / abs(e1);
  e2 = e2 - e1 * dot(e2, e1);
  e2 = e2 / abs(e2);
  Point p = u - a;
  return pdd(dot(p, e1), dot(p, e2));
}
Point rotate_around(Point p, double angle
    , Point axis) {
  double s = sin(angle), c = cos(angle);
  Point u = axis / abs(axis);
  return u * dot(u, p) * (1 - c) + p * c
      + cross(u, p) * s;
}

struct convex_hull_3D {
struct Face {
  int a, b, c;
  Face(int ta, int tb, int tc): a(ta), b(
      tb), c(tc) {}
}; // return the faces with pt indexes
vector<Face> res;
vector<Point> P;
convex_hull_3D(const vector<Point> &_P):
    res(), P(_P) {
// all points coplanar case will WA, O(n
     ^2)
  int n = SZ(P);
  if (n <= 2) return; // be careful about
       edge case
  // ensure first 4 points are not
       coplanar
  swap(P[1], *find_if(ALL(P), [&](auto p)
        { return sign(abs2(P[0] - p)) !=
       0; }));
  swap(P[2], *find_if(ALL(P), [&](auto p)
        { return sign(abs2(cross3(p, P
       [0], P[1]))) != 0; }));
  swap(P[3], *find_if(ALL(P), [&](auto p)
        { return sign(volume(P[0], P[1],
       P[2], p)) != 0; }));
  vector<vector<int>> flag(n, vector<int
       >(n));
  res.emplace_back(0, 1, 2); res.
       emplace_back(2, 1, 0);
  for (int i = 3; i < n; ++i) {
    vector<Face> next;
    for (auto f : res) {
      int d = sign(volume(P[f.a], P[f.b],
           P[f.c], P[i]));
      if (d <= 0) next.pb(f);
      int ff = (d > 0) - (d < 0);
      flag[f.a][f.b] = flag[f.b][f.c] =
           flag[f.c][f.a] = ff;
    }
    for (auto f : res) {
      auto F = [&](int x, int y) {
        if (flag[x][y] > 0 && flag[y][x]
             <= 0)
          next.emplace_back(x, y, i);
      };
      F(f.a, f.b); F(f.b, f.c); F(f.c, f.
           a);
    }
    res = next;
  }
}
bool same(Face s, Face t) {
  if (sign(volume(P[s.a], P[s.b], P[s.c],
       P[t.a])) != 0) return 0;
  if (sign(volume(P[s.a], P[s.b], P[s.c],
       P[t.b])) != 0) return 0;
  if (sign(volume(P[s.a], P[s.b], P[s.c],
       P[t.c])) != 0) return 0;
  return 1;
}
int polygon_face_num() {
  int ans = 0;
  for (int i = 0; i < SZ(res); ++i)
    ans += none_of(res.begin(), res.begin
         () + i, [&](Face g) { return
         same(res[i], g); });
  return ans;
}
double get_volume() {
  double ans = 0;
  for (auto f : res)
    ans += volume(Point(0, 0, 0), P[f.a],
         P[f.b], P[f.c]);
  return fabs(ans / 6);
}
double get_dis(Point p, Face f) {
  Point p1 = P[f.a], p2 = P[f.b], p3 = P[
       f.c];
  double a = (p2.y - p1.y) * (p3.z - p1.z
       ) - (p2.z - p1.z) * (p3.y - p1.y);
  double b = (p2.z - p1.z) * (p3.x - p1.x
       ) - (p2.x - p1.x) * (p3.z - p1.z);
  double c = (p2.x - p1.x) * (p3.y - p1.y
       ) - (p2.y - p1.y) * (p3.x - p1.x);
  double d = 0 - (a * p1.x + b * p1.y + c
        * p1.z);
  return fabs(a * p.x + b * p.y + c * p.z
        + d) / sqrt(a * a + b * b + c * c
       );
}
};
// n^2 delaunay: facets with negative z
    normal of
```

```
// convexhull of (x, y, x^2 + y^2), use a pseudo-point
// (0, 0, inf) to avoid degenerate case

vector<pdd> cut(vector<pdd> poly, pdd s, pdd e) {
  vector<pdd> res;
  for (int i = 0; i < SZ(poly); ++i) {
    pdd cur = poly[i], prv = i ? poly[i - 1] : poly.back();
    bool side = ori(s, e, cur) < 0;
    if (side != (ori(s, e, prv) < 0))
      res.pb(intersect(s, e, cur, prv));
    if (side)
      res.pb(cur);
  }
  return res;
}

// p, q is convex
double TwoConvexHullMinDist(Point P[], Point Q[], int n, int m) {
  int YMinP = 0, YMaxQ = 0;
  double tmp, ans = 999999999;
  for (i = 0; i < n; ++i) if(P[i].y < P[YMinP].y) YMinP = i;
  for (i = 0; i < m; ++i) if(Q[i].y > Q[YMaxQ].y) YMaxQ = i;
  P[n] = P[0], Q[m] = Q[0];
  for (int i = 0; i < n; ++i) {
    while (tmp = Cross(Q[YMaxQ + 1] - P[YMinP + 1], P[YMinP] - P[YMinP + 1]) > Cross(Q[YMaxQ] - P[YMinP + 1], P[YMinP] - P[YMinP + 1]))
      YMaxQ = (YMaxQ + 1) % m;
    if (tmp < 0) ans = min(ans, PointToSegDist(P[YMinP], P[YMinP + 1], Q[YMaxQ]));
    else ans = min(ans, TwoSegMinDist(P[YMinP], P[YMinP + 1], Q[YMaxQ], Q[YMaxQ + 1]));
    YMinP = (YMinP + 1) % n;
  }
  return ans;
}

template <typename F, typename C> class MCMF {
  static constexpr F INF_F = numeric_limits<F>::max();
  static constexpr C INF_C = numeric_limits<C>::max();
  vector<tuple<int, int, F, C>> es;
  vector<vector<int>> g;
  vector<F> f;
  vector<C> d;
  vector<int> pre, inq;
  void spfa(int s) {
    fill(inq.begin(), inq.end(), 0);
    fill(d.begin(), d.end(), INF_C);
    fill(pre.begin(), pre.end(), -1);

    queue<int> q;
    d[s] = 0;
    q.push(s);
    while (!q.empty()) {
      int u = q.front();
      inq[u] = false;
      q.pop();
      for (int j : g[u]) {
        int to = get<1>(es[j]);
        C w = get<3>(es[j]);
        if (f[j] == 0 || d[to] <= d[u] + w)
          continue;
        d[to] = d[u] + w;
        pre[to] = j;
        if (!inq[to]) {
          inq[to] = true;
          q.push(to);
        }
      }
    }
  }
public:
  MCMF(int n) : g(n), pre(n), inq(n) {}
  void add_edge(int s, int t, F c, C w) {
    g[s].push_back(es.size());
    es.emplace_back(s, t, c, w);
    g[t].push_back(es.size());
    es.emplace_back(t, s, 0, -w);
  }
  pair<F, C> solve(int s, int t, C mx = INF_C / INF_F) {
    add_edge(t, s, INF_F, -mx);
    f.resize(es.size()), d.resize(es.size());
    for (F I = INF_F ^ (INF_F / 2); I; I >>= 1) {
      for (auto &fi : f)
        fi *= 2;
      for (size_t i = 0; i < f.size(); i += 2) {
        auto [u, v, c, w] = es[i];
        if ((c & I) == 0)
          continue;
        if (f[i]) {
          f[i] += 1;
          continue;
        }
        spfa(v);
        if (d[u] == INF_C || d[u] + w >= 0) {
          f[i] += 1;
          continue;
        }
        f[i + 1] += 1;
        while (u != v) {
          int x = pre[u];
          f[x] -= 1;
          f[x ^ 1] += 1;
          u = get<0>(es[x]);
        }
      }
    }
    C w = 0;
    for (size_t i = 1; i + 2 < f.size(); i += 2)
      w -= f[i] * get<3>(es[i]);
    return {f.back(), w};
  }
};
  auto [f, c] = mcmf.solve(s, t, 1e12);
  cout << f << ' ' << c << '\n';

void MoAlgoOnTree() {
  Dfs(0, -1);
  vector<int> euler(tk);
  for (int i = 0; i < n; ++i) {
    euler[tin[i]] = i;
    euler[tout[i]] = i;
  }
  vector<int> l(q), r(q), qr(q), sp(q, -1);
  for (int i = 0; i < q; ++i) {
    if (tin[u[i]] > tin[v[i]]) swap(u[i], v[i]);
    int z = GetLCA(u[i], v[i]);
    sp[i] = z[i];
    if (z == u) l[i] = tin[u[i]], r[i] = tin[v[i]];
    else l[i] = tout[u[i]], r[i] = tin[v[i]];
    qr[i] = i;
  }
  sort(qr.begin(), qr.end(), [&](int i, int j) {
      if (l[i] / kB == l[j] / kB) return r[i] < r[j];
      return l[i] / kB < l[j] / kB;
    });
  vector<bool> used(n);
  // Add(v): add/remove v to/from the path based on used[v]
  for (int i = 0, tl = 0, tr = -1; i < q; ++i) {
    while (tl < l[qr[i]]) Add(euler[tl++]);
    while (tl > l[qr[i]]) Add(euler[--tl]);
    while (tr > r[qr[i]]) Add(euler[tr--]);
    while (tr < r[qr[i]]) Add(euler[++tr]);
    // add/remove LCA(u, v) if necessary
  }
}

for (int l = 0, r = -1; auto [ql, qr, i] : qs) {
  if (ql / B == qr / B) {
      for (int j = ql; j <= qr; j++) {
          cntSmall[a[j]]++;
          ans[i] = max(ans[i], 1LL * b[a[j]] * cntSmall[a[j]]);
      }
      for (int j = ql; j <= qr; j++) {
          cntSmall[a[j]]--;
      }
      continue;
  }
  if (int block = ql / B; block != lst) {
      int x = min((block + 1) * B, n);
      while (r + 1 < x) { add(++r); }
      while (r >= x) { del(r--); }
      while (l < x) { del(l++); }
      mx = 0;
      lst = block;
  }
  while (r < qr) { add(++r); }
  i64 tmpMx = mx;
  int tmpL = l;
  while (l > ql) { add(--l); }
  ans[i] = mx;
  mx = tmpMx;
  while (l < tmpL) { del(l++); }
}

typedef pair<ll,int> T;
typedef struct heap* ph;
struct heap { // min heap
  ph l = NULL, r = NULL;
  int s = 0; T v; // s: path to leaf
  heap(T _v):v(_v) {}
};
ph meld(ph p, ph q) {
  if (!p || !q) return p?:q;
  if (p->v > q->v) swap(p,q);
  ph P = new heap(*p); P->r = meld(P->r,q);
  if (!P->l || P->l->s < P->r->s) swap(P->l,P->r);
  P->s = (P->r?P->r->s:0)+1; return P;
}
ph ins(ph p, T v) { return meld(p, new heap(v)); }
ph pop(ph p) { return meld(p->l,p->r); }
int N,M,src,des,K;
ph cand[MX];
vector<array<int,3>> adj[MX], radj[MX];
pi pre[MX];
ll dist[MX];
struct state {
  int vert; ph p; ll cost;
  bool operator<(const state& s) const {
      return cost > s.cost; }
};
int main() {
  setIO(); re(N,M,src,des,K);
  F0R(i,M) {
    int u,v,w; re(u,v,w);
    adj[u].pb({v,w,i}); radj[v].pb({u,w,i}); // vert, weight, label
  }
  priority_queue<state> ans;
  {
    F0R(i,N) dist[i] = INF, pre[i] = {-1,-1};
    priority_queue<T,vector<T>,greater<T>> pq;
    auto ad = [&](int a, ll b, pi ind) {
      if (dist[a] <= b) return;
      pre[a] = ind; pq.push({dist[a] = b, a});
    };
    ad(des,0,{-1,-1});
    vi seq;
    while (sz(pq)) {
```

```
      auto a = pq.top(); pq.pop();
      if (a.f > dist[a.s]) continue;
      seq.pb(a.s); trav(t,radj[a.s]) ad(t[0],a.f+t[1],{t[2],a.s}); // edge index, vert
    }
    trav(t,seq) {
      trav(u,adj[t]) if (u[2] != pre[t].f && dist[u[0]] != INF) {
        ll cost = dist[u[0]]+u[1]-dist[t];
        cand[t] = ins(cand[t],{cost,u[0]});
      }
      if (pre[t].f != -1) cand[t] = meld(cand[t],cand[pre[t].s]);
      if (t == src) {
        ps(dist[t]); K --;
        if (cand[t]) ans.push(state{t,cand[t],dist[t]+cand[t]->v.f});
      }
    }
  }
  F0R(i,K) {
    if (!sz(ans)) {
      ps(-1);
      continue;
    }
    auto a = ans.top(); ans.pop();
    int vert = a.vert;
    ps(a.cost);
    if (a.p->l) {
      ans.push(state{vert,a.p->l,a.cost+a.p->l->v.f-a.p->v.f});
    }
    if (a.p->r) {
      ans.push(state{vert,a.p->r,a.cost+a.p->r->v.f-a.p->v.f});
    }
    int V = a.p->v.s;
    if (cand[V]) ans.push(state{V,cand[V],a.cost+cand[V]->v.f});
  }
}
```

```
// Minimum Steiner Tree, O(V 3^T + V^2 2^T)
struct SteinerTree { // 0-base
  static const int T = 10, N = 105, INF = 1e9;
  int n, dst[N][N], dp[1 << T][N], tdst[N];
  int vcost[N]; // the cost of vertexs
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) {
      for (int j = 0; j < n; ++j) dst[i][j] = INF;
      dst[i][i] = vcost[i] = 0;
    }
  }
  void add_edge(int ui, int vi, int wi) {
    dst[ui][vi] = min(dst[ui][vi], wi);
  }
  void shortest_path() {
    for (int k = 0; k < n; ++k)
      for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)
          dst[i][j] =
            min(dst[i][j], dst[i][k] + dst[k][j]);
  }
  int solve(const vector<int> &ter) {
    shortest_path();
    int t = SZ(ter);
    for (int i = 0; i < (1 << t); ++i)
      for (int j = 0; j < n; ++j) dp[i][j] = INF;
    for (int i = 0; i < n; ++i) dp[0][i] = vcost[i];
    for (int msk = 1; msk < (1 << t); ++msk) {
      if (!(msk & (msk - 1))) {
        int who = __lg(msk);
        for (int i = 0; i < n; ++i)
          dp[msk][i] =
            vcost[ter[who]] + dst[ter[who]][i];
      }
      for (int i = 0; i < n; ++i)
        for (int submsk = (msk - 1) & msk; submsk;
             submsk = (submsk - 1) & msk)
          dp[msk][i] = min(dp[msk][i],
            dp[submsk][i] + dp[msk ^ submsk][i] -
              vcost[i]);
      for (int i = 0; i < n; ++i) {
        tdst[i] = INF;
        for (int j = 0; j < n; ++j)
          tdst[i] =
            min(tdst[i], dp[msk][j] + dst[j][i]);
      }
      for (int i = 0; i < n; ++i) dp[msk][i] = tdst[i];
    }
    int ans = INF;
    for (int i = 0; i < n; ++i)
      ans = min(ans, dp[(1 << t) - 1][i]);
    return ans;
  }
};
```

```
llf simp(llf l, llf r) {
llf m = (l + r) / 2;
return (f(l) + f(r) + 4.0 * f(m)) * (r - l) / 6.0;
}
llf F(llf L, llf R, llf v, llf eps) {
llf M = (L + R) / 2, vl = simp(L, M), vr = simp(M, R);
if (abs(vl + vr - v) <= 15 * eps)
return vl + vr + (vl + vr - v) / 15.0;
return F(L, M, vl, eps / 2.0) +
F(M, R, vr, eps / 2.0);
} // call F(l, r, simp(l, r), 1e-6)
```

```
pair<int, int> get_tangent(const vector<P> &v, P p) {
const auto gao = [&, N = int(v.size())](int s) {
  const auto lt = [&](int x, int y) {
return ori(p, v[x % N], v[y % N]) == s; };
int l = 0, r = N; bool up = lt(0, 1);
while (r - l > 1) {
int m = (l + r) / 2;
if (lt(m, 0) ? up : !lt(m, m+1)) r = m;
else l = m;
}
return (lt(l, r) ? r : l) % N;
}; // test @ codeforces.com/gym/101201/problem/E
return {gao(-1), gao(1)}; // (a,b):ori(p,v[a],v[b])<0
} // plz ensure that point strictly out of hull
```

```
1: Initialize m □ M and w □ W to free
2: while □ free man m who has a woman w to propose to do
3: w □ first woman on m' s list to whom m has not yet proposed
4: if □ some pair (m', w) then
5: if w prefers m to m' then
6: m' □ free
7: (m, w) □ engaged
8: end if
9: else
10: (m, w) □ engaged
11: end if
12: end while
```

```
// virtual tree
vector<pair<int, int>> build(vector<int> vs, int r) {
  vector<pair<int, int>> res;
  sort(vs.begin(), vs.end(), [](int i, int j) {
    return dfn[i] < dfn[j]; });
  vector<int> s = {r};
  for (int v : vs) if (v != r) {
    if (int o = lca(v, s.back()); o != s.back()) {
      while (s.size() >= 2) {
        if (dfn[s[s.size() - 2]] < dfn[o]) break;
        res.emplace_back(s[s.size() - 2], s.back());
        s.pop_back();
      }
      if (s.back() != o) {
        res.emplace_back(o, s.back());
        s.back() = o;
      }
    }
    s.push_back(v);
  }
  for (size_t i = 1; i < s.size(); ++i)
    res.emplace_back(s[i - 1], s[i]);
  return res; // (x, y): x->y
}
```

```
#define pb emplace_back
#define rep(i, l, r) for (int i=(l); i<=(r); ++i)
struct WeightGraph { // 1-based
  static const int inf = INT_MAX;
  struct edge { int u, v, w; }; int n, nx;
  vector<int> lab; vector<vector<edge>> g;
  vector<int> slack, match, st, pa, S, vis;
  vector<vector<int>> flo, flo_from;
      queue<int> q;
  WeightGraph(int n_) : n(n_), nx(n * 2), lab(nx + 1),
    g(nx + 1, vector<edge>(nx + 1)),slack(nx + 1),
    flo(nx + 1), flo_from(nx + 1, vector(n + 1, 0)) {
    match = st = pa = S = vis = slack;
    rep(u, 1, n) rep(v, 1, n) g[u][v] = {u, v, 0};
  }
  int ED(edge e) {
    return lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2; }
  void update_slack(int u, int x, int &s) {
    if (!s || ED(g[u][x]) < ED(g[s][x])) s = u; }
  void set_slack(int x) {
    slack[x] = 0;
    for (int u = 1; u <= n; ++u)
      if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
        update_slack(u, x, slack[x]);
  }
  void q_push(int x) {
    if (x <= n) q.push(x);
    else for (int y : flo[x]) q_push(y);
  }
  void set_st(int x, int b) {
    st[x] = b;
    if (x > n) for (int y : flo[x]) set_st(y, b);
  }
  vector<int> split_flo(auto &f, int xr) {
    auto it = find(all(f), xr);
    if (auto pr = it - f.begin(); pr % 2 == 1)
      reverse(1 + all(f)), it = f.end() - pr;
    auto res = vector(f.begin(), it);
    return f.erase(f.begin(), it), res;
  }
  void set_match(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;
    int xr = flo_from[u][g[u][v].u];
```

```
    auto &f = flo[u], z = split_flo(f, xr
        );
    rep(i, 0, int(z.size())-1) set_match(
        z[i], z[i ^ 1]);
    set_match(xr, v); f.insert(f.end(),
        all(z));
  }
  void augment(int u, int v) {
    for (;;) {
      int xnv = st[match[u]]; set_match(u
        , v);
      if (!xnv) return;
      set_match(xnv, st[pa[xnv]]);
      u = st[pa[xnv]], v = xnv;
    }
  }
  int lca(int u, int v) {
    static int t = 0; ++t;
    for (++t; u || v; swap(u, v)) if (u)
        {
      if (vis[u] == t) return u;
      vis[u] = t; u = st[match[u]];
      if (u) u = st[pa[u]];
    }
    return 0;
  }
  void add_blossom(int u, int o, int v) {
    int b = int(find(n + 1 + all(st), 0)
        - begin(st));
    lab[b] = 0, S[b] = 0; match[b] =
        match[o];
    vector<int> f = {o};
    for (int x = u, y; x != o; x = st[pa[
        y]])
      f.pb(x), f.pb(y = st[match[x]]),
          q_push(y);
    reverse(1 + all(f));
    for (int x = v, y; x != o; x = st[pa[
        y]])
      f.pb(x), f.pb(y = st[match[x]]),
          q_push(y);
    flo[b] = f; set_st(b, b);
    for (int x = 1; x <= nx; ++x)
      g[b][x].w = g[x][b].w = 0;
    for (int x = 1; x <= n; ++x) flo_from
        [b][x] = 0;
    for (int xs : flo[b]) {
      for (int x = 1; x <= nx; ++x)
        if (g[b][x].w == 0 || ED(g[xs][x
            ]) < ED(g[b][x]))
          g[b][x] = g[xs][x], g[x][b] = g
              [x][xs];
      for (int x = 1; x <= n; ++x)
        if (flo_from[xs][x]) flo_from[b][
            x] = xs;
    }
    set_slack(b);
  }
  void expand_blossom(int b) {
    for (int x : flo[b]) set_st(x, x);
    int xr = flo_from[b][g[b][pa[b]].u],
        xs = -1;
    for (int x : split_flo(flo[b], xr)) {
      if (xs == -1) { xs = x; continue; }
      pa[xs] = g[x][xs].u; S[xs] = 1, S[x
          ] = 0;
      slack[xs] = 0; set_slack(x); q_push
          (x); xs = -1;
    }
    for (int x : flo[b])
      if (x == xr) S[x] = 1, pa[x] = pa[b
          ];
      else S[x] = -1, set_slack(x);
    st[b] = 0;
  }
  bool on_found_edge(const edge &e) {
    if (int u = st[e.u], v = st[e.v]; S[v
        ] == -1) {
      int nu = st[match[v]]; pa[v] = e.u;
           S[v] = 1;
      slack[v] = slack[nu] = 0; S[nu] =
          0; q_push(nu);
    } else if (S[v] == 0) {
      if (int o = lca(u, v)) add_blossom(
          u, o, v);
      else return augment(u, v), augment(
          v, u), true;
    }
    return false;
  }
  bool matching() {
    ranges::fill(S, -1); ranges::fill(
        slack, 0);
    q = queue<int>();
    for (int x = 1; x <= nx; ++x)
      if (st[x] == x && !match[x])
        pa[x] = 0, S[x] = 0, q_push(x);
    if (q.empty()) return false;
    for (;;) {
      while (q.size()) {
        int u = q.front(); q.pop();
        if (S[st[u]] == 1) continue;
        for (int v = 1; v <= n; ++v)
          if (g[u][v].w > 0 && st[u] !=
              st[v]) {
            if (ED(g[u][v]) != 0)
              update_slack(u, st[v],
                  slack[st[v]]);
            else if (on_found_edge(g[u][v
                ])) return true;
          }
      }
      int d = inf;
      for (int b = n + 1; b <= nx; ++b)
        if (st[b] == b && S[b] == 1)
          d = min(d, lab[b] / 2);
      for (int x = 1; x <= nx; ++x)
        if (int s = slack[x]; st[x] == x
            && s && S[x] <= 0)
          d = min(d, ED(g[s][x]) / (S[x]
              + 2));
      for (int u = 1; u <= n; ++u)
        if (S[st[u]] == 1) lab[u] += d;
        else if (S[st[u]] == 0) {
          if (lab[u] <= d) return false;
          lab[u] -= d;
        }
      rep(b, n + 1, nx) if (st[b] == b &&
          S[b] >= 0)
        lab[b] += d * (2 - 4 * S[b]);
      for (int x = 1; x <= nx; ++x)
        if (int s = slack[x]; st[x] == x
            &&
            s && st[s] != x && ED(g[s][x
                ]) == 0)
          if (on_found_edge(g[s][x]))
            return true;
      for (int b = n + 1; b <= nx; ++b)
        if (st[b] == b && S[b] == 1 &&
            lab[b] == 0)
          expand_blossom(b);
    }
    return false;
  }
  pair<lld, int> solve() {
    ranges::fill(match, 0);
    rep(u, 0, n) st[u] = u, flo[u].clear
        ();
    int w_max = 0;
    rep(u, 1, n) rep(v, 1, n) {
      flo_from[u][v] = (u == v ? u : 0);
      w_max = max(w_max, g[u][v].w);
    }
    for (int u = 1; u <= n; ++u) lab[u] =
         w_max;
    int n_matches = 0; lld tot_weight =
        0;
    while (matching()) ++n_matches;
    rep(u, 1, n) if (match[u] && match[u]
         < u)
      tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight,
        n_matches);
  }
  void set_edge(int u, int v, int w) {
    g[u][v].w = g[v][u].w = w; }
};

// 2D range add, range sum in log^2
struct seg {
  int l, r;
  ll sum, lz;
  seg *ch[2]{};
  seg(int _l, int _r) : l(_l), r(_r), sum
      (0), lz(0) {}
  void push() {
    if (lz) ch[0]->add(l, r, lz), ch[1]->
        add(l, r, lz), lz = 0;
  }
  void pull() { sum = ch[0]->sum + ch
      [1]->sum; }
  void add(int _l, int _r, ll d) {
    if (_l <= l && r <= _r) {
      sum += d * (r - l), lz += d;
      return;
    }
    if (!ch[0]) ch[0] = new seg(l, l + r
        >> 1), ch[1] = new seg(l + r >>
        1, r);
    push();
    if (_l < l + r >> 1) ch[0]->add(_l,
        _r, d);
    if (l + r >> 1 < _r) ch[1]->add(_l,
        _r, d);
    pull();
  }
  ll qsum(int _l, int _r) {
    if (_l <= l && r <= _r) return sum;
    if (!ch[0]) return lz * (min(r, _r) -
         max(l, _l));
    push();
    ll res = 0;
    if (_l < l + r >> 1) res += ch[0]->
        qsum(_l, _r);
    if (l + r >> 1 < _r) res += ch[1]->
        qsum(_l, _r);
    return res;
  }
};
struct seg2 {
  int l, r;
  seg v, lz;
  seg2 *ch[2]{};
  seg2(int _l, int _r) : l(_l), r(_r), v
      (0, N), lz(0, N) {
    if (l < r - 1) ch[0] = new seg2(l, l
        + r >> 1), ch[1] = new seg2(l +
        r >> 1, r);
  }
  void add(int _l, int _r, int _l2, int
      _r2, ll d) {
    v.add(_l2, _r2, d * (min(r, _r) - max
        (l, _l)));
    if (_l <= l && r <= _r)
      return lz.add(_l2, _r2, d), void(0)
          ;
    if (_l < l + r >> 1)
        ch[0]->add(_l, _r, _l2, _r2, d);
    if (l + r >> 1 < _r)
        ch[1]->add(_l, _r, _l2, _r2, d);
  }
  ll qsum(int _l, int _r, int _l2, int
      _r2) {
    if (_l <= l && r <= _r) return v.qsum
        (_l2, _r2);
    ll d = min(r, _r) - max(l, _l);
    ll res = lz.qsum(_l2, _r2) * d;
    if (_l < l + r >> 1)
        res += ch[0]->qsum(_l, _r, _l2,
            _r2);
    if (l + r >> 1 < _r)
        res += ch[1]->qsum(_l, _r, _l2,
            _r2);
    return res;
  }
};
```

PPPPPPartition number

```
ans[0] = tmp[0] = 1;
for (int i = 1; i * i <= n; i++) {
  for (int rep = 0; rep < 2; rep++)
    for (int j = i; j <= n - i * i; j++)
      modadd(tmp[j], tmp[j-i]);
  for (int j = i * i; j <= n; j++)
    modadd(ans[j], tmp[j - i * i]);
}
```

```cpp
vector<string> Duval(const string& s){//b
    b abb a
  vector<string> fact;int n=s.size();
  for(int i=0;i<n;){
    int j=i+1,k=i;
    for(;j<n&&s[k]<=s[j];j++)  if(s[k]<s
        [j])  k=i;else  k++;
    for(;i<=k;i+=j-k)  fact.emplace_back
        (s.substr(i,j-k));
}
  return fact;
 }

struct AC {
    static constexpr int A = 26;
    struct Node {
        array<int, A> nxt;
        int fail = -1;
        Node() { nxt.fill(-1); }
    };
    vector<Node> t;
    AC() : t(1) {}
    int size() { return t.size(); }
    Node& operator[](int i) { return t[i
        ]; }
    int add(const string &s, char offset
        = 'a') {
        int u = 0;
        for (auto ch : s) {
            int c = ch - offset;
            if (t[u].nxt[c] == -1) {
                t[u].nxt[c] = t.size();
                t.emplace_back();
            }
            u = t[u].nxt[c];
        }
        return u;
    }
    void build() {
        vector<int> q;
        for (auto &i : t[0].nxt) {
            if (i == -1) {
                i = 0;
            } else {
                q.push_back(i);
                t[i].fail = 0;
            }
        }

        for (int i = 0; i < int(q.size())
            ; i++) {
            int u = q[i];
            if (u > 0) {
                // maintain here?
            }
            for (int c = 0; c < A; c++) {
                if (int v = t[u].nxt[c];
                    v != -1) {
                    t[v].fail = t[t[u].
                        fail].nxt[c];
                    q.push_back(v);
                } else {
                    t[u].nxt[c] = t[t[u].
                        fail].nxt[c];
                }
            }
        }
    }
};

/* bool pred(int a, int b);
f(0) ~ f(n - 1) is a cyclic-shift U-
    function
return idx s.t. pred(x, idx) is false
    forall x*/
int cyc_tsearch(int n, auto pred) {
  if (n == 1) return 0;
  int l = 0, r = n; bool rv = pred(1, 0);
  while (r - l > 1) {
    int m = (l + r) / 2;
    if (pred(0, m) ? rv: pred(m, (m + 1)
        % n)) r = m;
    else l = m;
  }
  return pred(l, r % n) ? l : r % n;
}

//
    ---------------------------------------

// intersection of line and hull
int TangentDir(vector<pll> &C, pll dir) {
  return cyc_tsearch(SZ(C), [&](int a,
      int b) {
    return cross(dir, C[a]) > cross(dir,
        C[b]);
  });
}
#define cmpL(i) sign(cross(C[i] - a, b -
    a))
pii lineHull(pll a, pll b, vector<pll> &C
    ) {
  int A = TangentDir(C, a - b);
  int B = TangentDir(C, b - a);
  int n = SZ(C);
  if (cmpL(A) < 0 || cmpL(B) > 0)
    return pii(-1, -1); // no collision
  auto gao = [&](int l, int r) {
    for (int t = l; (l + 1) % n != r; ) {
      int m = ((l + r + (l < r ? 0 : n))
          / 2) % n;
      (cmpL(m) == cmpL(t) ? l : r) = m;
    }
    return (l + !cmpL(r)) % n;
  };
  pii res = pii(gao(B, A), gao(A, B)); //
      (i, j)
  if (res.X == res.Y) // touching the
      corner i
    return pii(res.X, -1);
  if (!cmpL(res.X) && !cmpL(res.Y)) //
      along side i, i+1
    switch ((res.X - res.Y + n + 1) % n)
        {
      case 0: return pii(res.X, res.X);
      case 2: return pii(res.Y, res.Y);
    }
  /* crossing sides (i, i+1) and (j, j+1)
  crossing corner i is treated as side (i
      , i+1)
  returned in the same order as the line
      hits the convex */
  return res;
} // convex cut: (r, l]

//-------------------------------

vector<pll> Minkowski(vector<pll> A,
    vector<pll> B) {
  hull(A), hull(B);
  vector<pll> C(1, A[0] + B[0]), s1, s2;
  for (int i = 0; i < SZ(A); ++i)
    s1.pb(A[(i + 1) % SZ(A)] - A[i]);
  for (int i = 0; i < SZ(B); i++)
    s2.pb(B[(i + 1) % SZ(B)] - B[i]);
  for (int i = 0, j = 0; i < SZ(A) || j <
       SZ(B);)
    if (j >= SZ(B) || (i < SZ(A) && cross
        (s1[i], s2[j]) >= 0))
      C.pb(B[j % SZ(B)] + A[i++]);
    else
      C.pb(A[i % SZ(A)] + B[j++]);
  return hull(C), C;
}

bool PointInConvex(const vector<pll> &C,
    pll p, bool strict = true) {
  int a = 1, b = SZ(C) - 1, r = !strict;
  if (SZ(C) == 0) return false;
  if (SZ(C) < 3) return r && btw(C[0], C.
      back(), p);
  if (ori(C[0], C[a], C[b]) > 0) swap(a,
      b);
  if (ori(C[0], C[a], p) >= r || ori(C
      [0], C[b], p) <= -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (ori(C[0], C[c], p) > 0 ? b : a) = c;
  }
  return ori(C[a], C[b], p) < r;
}

double rat(pll a, pll b) {
  return sign(b.X) ? (double)a.X / b.X :
      (double)a.Y / b.Y;
} // all poly. should be ccw
double polyUnion(vector<vector<pll>> &
    poly) {
  double res = 0;
  for (auto &p : poly)
    for (int a = 0; a < SZ(p); ++a) {
      pll A = p[a], B = p[(a + 1) % SZ(p)
          ];
      vector<pair<double, int>> segs =
          {{0, 0}, {1, 0}};
      for (auto &q : poly) {
        if (&p == &q) continue;
        for (int b = 0; b < SZ(q); ++b) {
          pll C = q[b], D = q[(b + 1) %
              SZ(q)];
          int sc = ori(A, B, C), sd = ori
              (A, B, D);
          if (sc != sd && min(sc, sd) <
              0) {
            double sa = cross(D - C, A -
                C), sb = cross(D - C, B
                - C);
            segs.emplace_back(sa / (sa -
                sb), sign(sc - sd));
          }
          if (!sc && !sd && &q < &p &&
              sign(dot(B - A, D - C)) >
              0) {
            segs.emplace_back(rat(C - A,
                B - A), 1);
            segs.emplace_back(rat(D - A,
                B - A), -1);
          }
        }
      }
      sort(ALL(segs));
      for (auto &s : segs) s.X = clamp(s.
          X, 0.0, 1.0);
      double sum = 0;
      int cnt = segs[0].second;
      for (int j = 1; j < SZ(segs); ++j)
          {
        if (!cnt) sum += segs[j].X - segs
            [j - 1].X;
        cnt += segs[j].Y;
      }
      res += cross(A, B) * sum;
    }
  return res / 2;
}

/* The point should be strictly out of
    hull
  return arbitrary point on the tangent
    line */
pii get_tangent(vector<pll> &C, pll p) {
  auto gao = [&](int s) {
    return cyc_tsearch(SZ(C), [&](int x,
        int y)
    { return ori(p, C[x], C[y]) == s; });
  };
  return pii(gao(1), gao(-1));
} // return (a, b), ori(p, C[a], C[b]) >=
    0

double ConvexHullDist(vector<pdd> A,
    vector<pdd> B) {
  for (auto &p : B) p = {-p.X, -p.Y};
  auto C = Minkowski(A, B); // assert
      SZ(C) > 0
  if (PointInConvex(C, pdd(0, 0)))
      return 0;
  double ans = PointSegDist(C.back(), C
      [0], pdd(0, 0));
  for (int i = 0; i + 1 < SZ(C); ++i) {
      ans = min(ans, PointSegDist(C[i],
          C[i + 1], pdd(0, 0)));
  }
  return ans;
}

// return q's relation with circumcircle
    of tri(p[0],p[1],p[2])
bool in_cc(const array<pll, 3> &p, pll q)
    {
```

```cpp
    __int128 det = 0;
    for (int i = 0; i < 3; ++i)
      det += __int128(abs2(p[i]) - abs2(q))
           * cross(p[(i + 1) % 3] - q, p[(
          i + 2) % 3] - q);
    return det > 0; // in: >0, on: =0, out:
        <0
}

// 0 : not intersect
// 1 : strictly intersect
// 2 : overlap
// 3 : intersect at endpoint
template<class T>
std::tuple<int, Point<T>, Point<T>>
     segmentIntersection(Line<T> l1, Line
     <T> l2) {
    if (std::max(l1.a.x, l1.b.x) < std::
         min(l2.a.x, l2.b.x)) {
        return {0, Point<T>(), Point<T>()
            };
    }
    if (std::min(l1.a.x, l1.b.x) > std::
         max(l2.a.x, l2.b.x)) {
        return {0, Point<T>(), Point<T>()
            };
    }
    if (std::max(l1.a.y, l1.b.y) < std::
         min(l2.a.y, l2.b.y)) {
        return {0, Point<T>(), Point<T>()
            };
    }
    if (std::min(l1.a.y, l1.b.y) > std::
         max(l2.a.y, l2.b.y)) {
        return {0, Point<T>(), Point<T>()
            };
    }
    if (cross(l1.b - l1.a, l2.b - l2.a)
         == 0) {
        if (cross(l1.b - l1.a, l2.a - l1.
            a) != 0) {
            return {0, Point<T>(), Point<
                T>()};
        } else {
            auto maxx1 = std::max(l1.a.x,
                 l1.b.x);
            auto minx1 = std::min(l1.a.x,
                 l1.b.x);
            auto maxy1 = std::max(l1.a.y,
                 l1.b.y);
            auto miny1 = std::min(l1.a.y,
                 l1.b.y);
            auto maxx2 = std::max(l2.a.x,
                 l2.b.x);
            auto minx2 = std::min(l2.a.x,
                 l2.b.x);
            auto maxy2 = std::max(l2.a.y,
                 l2.b.y);
            auto miny2 = std::min(l2.a.y,
                 l2.b.y);
            Point<T> p1(std::max(minx1,
                 minx2), std::max(miny1,
                 miny2));
            Point<T> p2(std::min(maxx1,
                 maxx2), std::min(maxy1,
                 maxy2));
            if (!pointOnSegment(p1, l1))
                 {
                std::swap(p1.y, p2.y);
            }
            if (p1 == p2) {
                return {3, p1, p2};
            } else {
                return {2, p1, p2};
            }
        }
    }
    auto cp1 = cross(l2.a - l1.a, l2.b -
         l1.a);
    auto cp2 = cross(l2.a - l1.b, l2.b -
         l1.b);
    auto cp3 = cross(l1.a - l2.a, l1.b -
         l2.a);
    auto cp4 = cross(l1.a - l2.b, l1.b -
         l2.b);
    if ((cp1 > 0 && cp2 > 0) || (cp1 < 0
         && cp2 < 0) || (cp3 > 0 && cp4 >
          0) || (cp3 < 0 && cp4 < 0)) {
        return {0, Point<T>(), Point<T>()
            };
    }

    Point p = lineIntersection(l1, l2);
    if (cp1 != 0 && cp2 != 0 && cp3 != 0
         && cp4 != 0) {
        return {1, p, p};
    } else {
        return {3, p, p};
    }
}

template<class T>
bool segmentInPolygon(Line<T> l, std::
     vector<Point<T>> p) {
    int n = p.size();
    if (!pointInPolygon(l.a, p)) {
        return false;
    }
    if (!pointInPolygon(l.b, p)) {
        return false;
    }
    for (int i = 0; i < n; i++) {
        auto u = p[i];
        auto v = p[(i + 1) % n];
        auto w = p[(i + 2) % n];
        auto [t, p1, p2] =
            segmentIntersection(l, Line(
            u, v));

        if (t == 1) {
            return false;
        }
        if (t == 0) {
            continue;
        }
        if (t == 2) {
            if (pointOnSegment(v, l) && v
                 != l.a && v != l.b) {
                if (cross(v - u, w - v) >
                     0) {
                    return false;
                }
            }
        } else {
            if (p1 != u && p1 != v) {
                if (pointOnLineLeft(l.a,
                     Line(v, u))
                    || pointOnLineLeft(l.
                        b, Line(v, u)))
                        {
                    return false;
                }
            } else if (p1 == v) {
                if (l.a == v) {
                    if (pointOnLineLeft(u
                        , l)) {
                        if (
                            pointOnLineLeft
                            (w, l)
                           &&
                                pointOnLineLeft
                                (w, Line
                                (u, v)))
                             {
                            return false;
                        }
                    } else {
                        if (
                            pointOnLineLeft
                            (w, l)
                           ||
                                pointOnLineLeft
                                (w, Line
                                (u, v)))
                             {
                            return false;
                        }
                    }
                } else if (l.b == v) {
                    if (pointOnLineLeft(u
                        , Line(l.b, l.a)
                        )) {
                        if (
                            pointOnLineLeft
                            (w, Line(l.b
                            , l.a))
                           &&
                                pointOnLineLef
                                (w, Line
                                (u, v)))
                             {
                            return false;
                        }
                    } else {
                        if (
                            pointOnLineLeft
                            (w, Line(l.b
                            , l.a))
                           ||
                                pointOnLineLef
                                (w, Line
                                (u, v)))
                             {
                            return false;
                        }
                    }
                } else {
                    if (pointOnLineLeft(u
                        , l)) {
                        if (
                            pointOnLineLeft
                            (w, Line(l.b
                            , l.a))
                           ||
                                pointOnLineLef
                                (w, Line
                                (u, v)))
                             {
                            return false;
                        }
                    } else {
                        if (
                            pointOnLineLeft
                            (w, l)
                           ||
                                pointOnLineLef
                                (w, Line
                                (u, v)))
                             {
                            return false;
                        }
                    }
                }
            }
        }
    }
    return true;
}
```