

## 1 Basic

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## 1.1 Shell script

```
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp> //rb_tree
using namespace __gnu_pbds;
typedef __gnu_pbds::priority_queue<int> heap;
int main() {
    heap h1, h2;
    h1.push(1), h1.push(3);
    h2.push(2), h2.push(4);
    h1.join(h2);
    cout << h1.size() << h2.size() << h1.top() << endl;
    //404
    tree<ll, null_type, less<ll>, rb_tree_tag,
        tree_order_statistics_node_update> st;
    tree<ll, ll, less<ll>, rb_tree_tag,
        tree_order_statistics_node_update> mp;
    for (int x : {0, 2, 3, 4}) st.insert(x);
    cout << *st.find_by_order(2) << st.order_of_key(1)
        << endl; //31
}
// int128 t, float128 t
```

## 1.6 Texas hold'em

```
char suit[4]={'C','D','H','Y'}, ranks[13]={'2','3','4','5','6','7','8','9','T','J','Q','K','A'};
int rk[256];
/*
for(int i=0;i<13;++i)
    rk[ranks[i]]=i;
for(int i=0;i<4;++i)
    rk[suit[i]]=i;
*/
struct cards{
    vector<pii> v;
    int suit_count[4], hands;
    void reset(){v.clear(), FILL(suit_count, 0), hands=-1;}
    void insert(char a, char b){//suit, rank
        ++suit_count[rk[a]];
        int flag=0;
        for(auto &i:v)
            if(i.Y==rk[b])
            {
                ++i.X, flag=1;
                break;
            }
        if(!flag) v.pb(pii(1, rk[b]));
    }
    void insert(string s){insert(s[0], s[1]);}
    void ready(){
        int Straight=0, Flush=(max_element(suit_count, suit_count+4)==5);
        sort(ALL(v), [](ii a, ii b){return a>b;});
        if(SZ(v)==5&&v[0].Y==v[1].Y+1&&v[1].Y==v[2].Y+1&&v[2].Y==v[3].Y+1&&v[3].Y==v[4].Y+1)
            Straight=1;
        else if(SZ(v)==5&&v[0].Y==12&&v[1].Y==3&&v[2].Y==2&&v[3].Y==1&&v[4].Y==0)
            Straight=1;
        if(Straight&&Flush) hands=1;
        else if(v[0].X==4) hands=2;
        else if(v[0].X==3&&v[1].X==2) hands=3;
        else if(Flush) hands=4;
        else if(Straight) hands=5;
        else if(v[0].X==3) hands=6;
        else if(v[0].X==2&&v[1].X==2) hands=7;
        else if(v[0].X==2) hands=8;
        else hands=9;
    }
    bool operator>(const cards &a) const{
        if(hands==a.hands) return v>a.v;
        return hands<a.hands;
    }
};
```

## 2 Graph

### 2.1 BCC Vertex

```
vector<int> G[N]; //1-base
vector<int> nG[N], bcc[N];
int low[N], dfn[N], Time;
int bcc_id[N], bcc_cnt; //1-base
bool is_cut[N]; //whether is av
bool cir[N];
int st[N], top;

void dfs(int u, int pa = -1) {
    int child = 0;
    low[u] = dfn[u] = ++Time;
    st[top++] = u;
    for(int v : G[u])
        if(!dfn[v]) {
            dfs(v, u), ++child;
            low[u] = min(low[u], low[v]);
            if(dfn[u] <= low[v]) {
                is_cut[u]=1;
                bcc[++bcc_cnt].clear();
                int t;
```

```
                do {
                    bcc_id[t = st[--top]] = bcc_cnt;
                    bcc[bcc_cnt].push_back(t);
                } while(t != v);
                bcc_id[u]=bcc_cnt;
                bcc[bcc_cnt].pb(u);
            }
        }
        else if(dfn[v] < dfn[u] && v!=pa)
            low[u] = min(low[u], dfn[v]);
    if(pa == -1 && child < 2)
        is_cut[u] = 0;
}

void bcc_init(int n) {
    Time = bcc_cnt = top = 0;
    for(int i = 1; i <= n; ++i)
        G[i].clear(), dfn[i] = bcc_id[i] = is_cut[i] = 0;
}

void bcc_solve(int n) {
    for(int i = 1; i <= n; ++i)
        if(!dfn[i])
            dfs(i);
    // circle-square tree
    for(int i = 1; i <= n; ++i)
        if(is_cut[i])
            bcc_id[i] = ++bcc_cnt, cir[bcc_cnt] = 1;
    for(int i = 1; i <= bcc_cnt && !cir[i]; ++i)
        for(int j : bcc[i])
            if(is_cut[j])
                nG[i].pb(bcc_id[j]), nG[bcc_id[j]].pb(i);
}
```

### 2.2 Bridge

```
int low[N], dfn[N], Time; // 1-base
vector<pii> G[N], edge;
vector<bool> is_bridge;

void init(int n) {
    Time = 0;
    for(int i = 1; i <= n; ++i)
        G[i].clear(), low[i] = dfn[i] = 0;
}

void add_edge(int a, int b) {
    G[a].pb(pii(b, SZ(edge))), G[b].pb(pii(a, SZ(edge)));
    edge.pb(pii(a, b));
}

void dfs(int u, int f) {
    dfn[u] = low[u] = ++Time;
    for(auto i : G[u])
        if(!dfn[i.X])
            dfs(i.X, i.Y), low[u] = min(low[u], low[i.X]);
        else if(i.Y != f)
            low[u] = min(low[u], dfn[i.X]);
    if(low[u] == dfn[u] && f != -1)
        is_bridge[f] = 1;
}

void solve(int n) {
    is_bridge.resize(SZ(edge));
    for(int i = 1; i <= n; ++i)
        if(!dfn[i])
            dfs(i, -1);
}
```

### 2.3 Strongly Connected Components

```
struct Strongly_CC{//1-base
    int low[N], dfn[N], bln[N], sz[N], n, Time, nScc;
    bitset<N> instack;
```

```

stack<int> st;
vector<int> G[N], SCC[N];
void init(int _n) {
    n = _n;
    for(int i = 1; i <= n; ++i)
        G[i].clear();
}
void add_edge(int a, int b) {
    G[a].pb(b);
}
void dfs(int u) {
    dfn[u] = low[u] = ++Time;
    instack[u] = 1, st.push(u);
    for(int i : G[u])
        if(!dfn[i]) dfs(i), low[u] = min(low[i], low[u]);
        else if(instack[i] && dfn[i] < dfn[u])
            low[u] = min(low[u], dfn[i]);
    if(low[u] == dfn[u]) {
        int tmp;
        do {
            tmp = st.top(), st.pop();
            instack[tmp] = 0, bln[tmp] = nScc;
        } while(tmp != u);
        ++nScc;
    }
}
void solve() {
    Time = nScc = 0;
    for(int i = 1; i <= n; ++i)
        SCC[i].clear(), low[i] = dfn[i] = bln[i] =
            sz[i] = 0;
    for(int i = 1; i <= n; ++i)
        if(!dfn[i])
            dfs(i);
    for(int i = 1; i <= n; ++i)
        ++sz[bln[i]], SCC[bln[i]].pb(i);
}
};

```

## 2.4 MinimumMeanCycle

```

ll road[N][N]; //input here
struct MinimumMeanCycle{
    ll dp[N + 5][N], n;
    pll solve() {
        ll a = -1, b = -1, L = n+1;
        for(int i = 2; i <= L; ++i)
            for(int k = 0; k < n; ++k)
                for(int j = 0; j < n; ++j)
                    dp[i][j] = min(dp[i - 1][k] + road[k][j], dp[i][j]);
        for(int i = 0; i < n; ++i) {
            if(dp[L][i] >= INF) continue;
            ll ta = 0, tb = 1;
            for(int j = 1; j < n; ++j)
                if(dp[j][i] < INF && ta * (L - j) < (dp[L][i] -
                    dp[j][i]) * tb)
                    ta = dp[L][i] - dp[j][i], tb = L - j;
            if(ta == 0) continue;
            if(a == -1 || a * tb > ta * b)
                a = ta, b = tb;
        }
        if(a != -1) {
            ll g = __gcd(a, b);
            return pll(a / g, b / g);
        }
        return pll(-1LL, -1LL);
    }
}
void init(int _n){
    n = _n;
    for(int i = 0; i < n; ++i)
        for(int j = 0; j < n; ++j)
            dp[i + 2][j] = INF;
}
};

```

## 2.5 Virtual Tree

```

void insert(int x){
    if(top == -1)
        return st[++top] = x, void();
    int p = LCA(st[top], x);
    if(p == st[top])
        return st[++top] = x, void();
    while(dep[st[top - 1]] > dep[p])
        vG[st[top - 1]].pb(st[top]), --top;
    vG[p].pb(st[top]), --top;
    if(st[top] != p) st[++top] = p;
    st[++top] = x;
}
void ending(){
    while(top > 0)
        vG[st[top - 1]].pb(st[top]), --top;
}
void reset(int x){
    for(int i : vG[x])
        reset(i);
    vG[x].clear();
}

```

## 2.6 Maximum Clique

```

struct Maximum_Clique{
    typedef bitset<MAXN> bst;
    bst N[MAXN], empty;
    int p[MAXN], n, ans;
    void BronKerbosch2(bst R, bst P, bst X){
        if(P == empty && X == empty)
            return ans = max(ans, (int)R.count()), void();
        bst tmp = P | X;
        int u;
        if((R | P | X).count() <= ans) return;
        for(int uu = 0; uu < n; ++uu){
            u = p[uu];
            if(tmp[u] == 1) break;
        }
        //if (double(clock()) / CLOCKS_PER_SEC > .999) return;
        bst now2 = P & ~N[u];
        for(int vv = 0; vv < n; ++vv){
            int v = p[vv];
            if(now2[v] == 1){
                R[v] = 1;
                BronKerbosch2(R, P & N[v], X & N[v]);
                R[v] = 0, P[v] = 0, X[v] = 1;
            }
        }
    }
    void init(int _n){n = _n;
        for(int i = 0; i < n; ++i) N[i].reset();
    }
    void add_edge(int u, int v){N[u][v] = N[v][u] = 1;}
    int solve(){//remember srand
        bst R, P, X;
        ans = 0, P.flip();
        for(int i = 0; i < n; ++i) p[i] = i;
        random_shuffle(p, p + n, BronKerbosch2(R, P, X));
        return ans;
    }
};

```

## 2.7 MinimumSteinerTree

```

// Minimum Steiner Tree
// O(V^3 T + V^2 2^T)
struct SteinerTree{//0-base
    static const int T = 9, MAXN = 70, INF = 1e9;
    int n, dst[MAXN][MAXN], dp[1 < T][MAXN], tdst[MAXN];
    int vcost[MAXN]; //the cost of vertexs
    void init(int _n){n = _n;
        for(int i = 0; i < n; ++i){
            for(int j = 0; j < n; ++j)
                dst[i][j] = INF;
            dst[i][i] = vcost[i] = 0;
        }
    }
}

```

```

}
void add_edge(int ui, int vi, int wi){
    dst[ui][vi]=min(dst[ui][vi], wi);
    //dst[vi][ui]=min(dst[vi][ui], wi);
}
void shortest_path(){
    for(int k=0; k<n; ++k)
        for(int i=0; i<n; ++i)
            for(int j=0; j<n; ++j)
                dst[i][j]=min(dst[i][j], dst[i][k]+
                    dst[k][j]);
}
int solve(const vector<int>& ter){
    shortest_path();
    int t=SZ(ter);
    for(int i=0; i<(1<t); ++i)
        for(int j=0; j<n; ++j)
            dp[i][j]=INF;
    for(int i=0; i<n; ++i)
        dp[0][i] = vcost[i];
    for(int msk=1; msk<(1<t); ++msk){
        if(msk==(msk&(-msk))){
            int who=__lg( msk );
            for(int i=0; i<n; ++i)
                dp[msk][i] = vcost[ter[who]] + dst[
                    ter[who]][i];
            continue;
        }
        for(int i=0; i<n; ++i)
            for(int submsk=(msk-1)&msk; submsk; submsk=(
                submsk-1)&msk)
                dp[msk][i] = min(dp[msk][i], dp[submsk
                    ][i] + dp[msk^submsk][i] - vcost[i
                    ]);
        for(int i=0; i<n; ++i){
            tdst[i]=INF;
            for(int j=0; j<n; ++j)
                tdst[i]=min(tdst[i], dp[msk][j]+dst[
                    j][i]);
        }
        for(int i=0; i<n; ++i)
            dp[msk][i]=tdst[i];
    }
    int ans=INF;
    for(int i=0; i<n; ++i)
        ans=min(ans, dp[(1<t)-1][i]);
    return ans;
}
};

```

## 2.8 Dominator Tree

```

struct dominator_tree{//1-base
    int n;
    vector<int> G[MAXN], rG[MAXN];
    int pa[MAXN], dfn[MAXN], id[MAXN], dfnCnt;
    int semi[MAXN], idom[MAXN], best[MAXN];
    vector<int> tree[MAXN]; //dominator_tree
    void init(int _n){
        n=_n;
        for(int i=1; i<=n; ++i) G[i].clear(), rG[i].clear();
    }
    void add_edge(int u, int v){
        G[u].push_back(v);
        rG[v].push_back(u);
    }
    void dfs(int u){
        id[dfn[u]]=++dfnCnt;
        for(auto v:G[u]) if(!dfn[v]){
            dfs(v); pa[dfn[v]]=dfn[u];
        }
    }
    int find(int y, int x){
        if(y<=x) return y;
        int tmp=find(pa[y], x);
        if(semi[best[y]]>semi[best[pa[y]]])
            best[y]=best[pa[y]];
        return pa[y]=tmp;
    }
    void tarjan(int root){

```

```

        dfnCnt=0;
        for(int i=1; i<=n; ++i){
            dfn[i]=idom[i]=0;
            tree[i].clear();
            best[i]=semi[i]=i;
        }
        dfs(root);
        for(int i=dfnCnt; i>1; --i){
            int u=id[i];
            for(auto v:rG[u]) if(v=dfn[v]){
                find(v, i);
                semi[i]=min(semi[i], semi[best[v]]);
            }
            tree[semi[i]].push_back(i);
            for(auto v:tree[pa[i]]){
                find(v, pa[i]);
                idom[v] = semi[best[v]]==pa[i] ? pa[i] : best[v
                    ];
            }
            tree[pa[i]].clear();
        }
        for(int i=2; i<=dfnCnt; ++i){
            if(idom[i]!=semi[i]) idom[i]=idom[idom[i]];
            tree[id[idom[i]]].push_back(id[i]);
        }
    }
};

```

## 2.9 Minimum Arborescence

```

#define INF 0x3f3f3f3f
template<typename T>
struct zhu_liu{//O(VE)
    static const int MAXN=110;
    struct edge{
        int u, v;
        T w;
        edge(int u=0, int v=0, T w=0):u(u), v(v), w(w){}
    };
    vector<edge> E; // 0-base
    int pe[MAXN], id[MAXN], vis[MAXN];
    T in[MAXN];
    void init(){E.clear();}
    void add_edge(int u, int v, T w){
        if(u!=v) E.push_back(edge(u, v, w));
    }
    T build(int root, int n){
        T ans=0;
        for(;;){
            for(int u=0; u<n; ++u) in[u]=INF;
            for(size_t i=0; i<E.size(); ++i)
                if(E[i].u!=E[i].v && E[i].w<in[E[i].v])
                    pe[E[i].v]=i, in[E[i].v]=E[i].w;
            for(int u=0; u<n; ++u) //no solution
                if(u!=root && in[u]==INF) return -INF;
            int cntnode=0;
            memset(id, -1, sizeof(int)*n);
            memset(vis, -1, sizeof(int)*n);
            for(int u=0; u<n; ++u){
                if(u!=root) ans+=in[u];
                int v=u;
                for(;; vis[v]!=u && id[v]==-1 && v!=root; v=E[pe[v]].u)
                    vis[v]=u;
                if(v!=root && id[v]==-1){
                    for(int x=E[pe[v]].u; x!=v; x=E[pe[x]].u)
                        id[x]=cntnode;
                    id[v]=cntnode++;
                }
            }
            if(!cntnode) break; //no cycle
            for(int u=0; u<n; ++u) if(id[u]==-1) id[u]=cntnode++;
            for(size_t i=0; i<E.size(); ++i){
                int v=E[i].v;
                E[i].u=id[E[i].u];
                E[i].v=id[E[i].v];
                if(E[i].u!=E[i].v) E[i].w-=in[v];
            }
            n=cntnode;
            root=id[root];

```

```

    }
    return ans;
}
};

```

## 2.10 Vizing's theorem

```

namespace vizing { // returns edge coloring in adjacent
    matrix G, 1 - based
int C[kN][kN], G[kN][kN];
void clear(int N) {
    for (int i = 0; i <= N; i++) {
        for (int j = 0; j <= N; j++) C[i][j] = G[i][j]
            = 0;
    }
}
void solve(vector<pair<int, int>> &E, int N, int M) {
    int X[kN] = {}, a;
    auto update = [&](int u) {
        for (X[u] = 1; C[u][X[u]]; X[u]++);
    };
    auto color = [&](int u, int v, int c) {
        int p = G[u][v];
        G[u][v] = G[v][u] = c;
        C[u][c] = v, C[v][c] = u;
        C[u][p] = C[v][p] = 0;
        if (p) X[u] = X[v] = p;
        else update(u), update(v);
        return p;
    };
    auto flip = [&](int u, int c1, int c2) {
        int p = C[u][c1];
        swap(C[u][c1], C[u][c2]);
        if (p) G[u][p] = G[p][u] = c2;
        if (!C[u][c1]) X[u] = c1;
        if (!C[u][c2]) X[u] = c2;
        return p;
    };
    for (int i = 1; i <= N; i++) X[i] = 1;
    for (int t = 0; t < E.size(); t++) {
        int u = E[t].first, v0 = E[t].second, v = v0,
            c0 = X[u], c = c0, d;
        vector<pair<int, int>> L;
        int vst[kN] = {};
        while (!G[u][v0]) {
            L.emplace_back(v, d = X[v]);
            if (!C[v][c]) for (a = (int)L.size() - 1; a
                >= 0; a--) c = color(u, L[a].first, c)
                ;
            else if (!C[u][d]) for (a = (int)L.size() -
                1; a >= 0; a--) color(u, L[a].first, L
                [a].second);
            else if (vst[d]) break;
            else vst[d] = 1, v = C[u][d];
        }
        if (!G[u][v0]) {
            for (; v; v = flip(v, c, d), swap(c, d));
            if (C[u][c0]) {
                for (a = (int)L.size() - 2; a >= 0 && L
                    [a].second != c; a--);
                for (; a >= 0; a--) color(u, L[a].first
                    , L[a].second);
            } else t--;
        }
    }
}
}
}

```

## 2.11 Theory

$|\text{Maximum independent edge set}| = |V| - |\text{Minimum edge cover}|$   
 $|\text{Maximum independent set}| = |V| - |\text{Minimum vertex cover}|$   
 A sequence of non-negative integers  $d_1 \geq \dots \geq d_n$  can be represented as the degree sequence of a finite simple graph on  $n$  vertices if and only if  $d_1 + \dots + d_n$  is even and
 
$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k) \text{ holds for every } k \text{ in } 1 \leq k \leq n.$$

## 3 Data Structure

### 3.1 Leftist Tree

```

struct node{
    ll v,data,sz,sum;
    node *l,*r;
    node(ll k):v(0),data(k),sz(1),l(0),r(0),sum(k){}
};
ll sz(node *p){return p ? p->sz : 0;}
ll V(node *p){return p ? p->v : -1;}
ll sum(node *p){return p ? p->sum : 0;}
node* merge(node *a,node *b){
    if(!a || !b) return a ? a : b;
    if(a->data<b->data) swap(a,b);
    a->r=merge(a->r,b);
    if(V(a->r)>V(a->l)) swap(a->r,a->l);
    a->v=V(a->r)+1,a->sz=sz(a->l)+sz(a->r)+1;
    a->sum=sum(a->l)+sum(a->r)+a->data;
    return a;
}
void pop(node *&o){
    node *tmp=o;
    o=merge(o->l,o->r);
    delete tmp;
}

```

### 3.2 Heavy light Decomposition

```

struct Heavy_light_Decomposition{//1-base
    int n,ulink[10005],deep[10005],mxson[10005],w[10005],
        pa[10005];
    int t,pl[10005],data[10005],dt[10005],bln[10005],edge
        [10005],et;
    vector<pii> G[10005];
    void init(int _n){n=_n,t=0,et=1;
        for(int i=1;i<=n;++i) G[i].clear(),mxson[i]=0;
    }
    void add_edge(int a,int b,int w){
        G[a].pb(pii(b,et)),G[b].pb(pii(a,et)),edge[et++]=w;
    }
    void dfs(int u,int f,int d){
        w[u]=1,pa[u]=f,deep[u]=d++;
        for(auto &i:G[u])
            if(i.X!=f){
                dfs(i.X,u,d),w[u]+=w[i.X];
                if(w[mxson[u]]<w[i.X])
                    mxson[u]=i.X;
            }
        else
            bln[i.Y]=u,dt[u]=edge[i.Y];
    }
    void cut(int u,int link){
        data[pl[u]=t++]=dt[u],ulink[u]=link;
        if(!mxson[u]) return ;
        cut(mxson[u],link);
        for(auto i:G[u])
            if(i.X!=pa[u]&&i.X!=mxson[u])
                cut(i.X,i.X);
    }
    void build(){
        dfs(1,1,1),cut(1,1),/*build*/;
    }
    int query(int a,int b){
        int ta=ulink[a],tb=ulink[b],re=0;
        while(ta!=tb)
            if(deep[ta]<deep[tb])
                /*query*/,tb=ulink[b=pa[tb]];
            else
                /*query*/,ta=ulink[a=pa[ta]];
        if(a==b) return re;
        if(pl[a]>pl[b]) swap(a,b);
        /*query*/
        return re;
    }
};

```

### 3.3 Smart Pointer

```
#ifndef REFERENCE_POINTER
#define REFERENCE_POINTER
template<typename T>
struct _RefCounter{
    T data;
    int ref;
    _RefCounter(const T&d=0):data(d),ref(0){}
};
template<typename T>
struct reference_pointer{
    _RefCounter<T> *p;
    T *operator->(){return &p->data;}
    T &operator*(){return p->data;}
    operator _RefCounter<T>*(){return p;}
    reference_pointer &operator=(const reference_pointer &t)
    ){
        if(p&&!--p->ref)delete p;
        p=t.p;
        p&&++p->ref;
        return *this;
    }
    reference_pointer(_RefCounter<T> *t=0):p(t){
        p&&++p->ref;
    }
    reference_pointer(const reference_pointer &t):p(t.p){
        p&&++p->ref;
    }
    ~reference_pointer(){
        if(p&&!--p->ref)delete p;
    }
};
template<typename T>
inline reference_pointer<T> new_reference(const T&nd){
    return reference_pointer<T>(new _RefCounter<T>(nd));
}
#endif
//note:
reference_pointer<int> a;
a = new_reference(5);
a = new_reference<int>(5);
a = new_reference((int)5);
reference_pointer<int> b = a;

struct P{
    int a,b;
    P(int _a,int _b):a(_a),b(_b){}
}p(2,3);
reference_pointer<P> a;
c = new_reference(P(1,2));
c = new_reference<P>(P(1,2));
c = new_reference(p);
```

### 3.4 LiChaoST

```
struct LiChao_min{
    struct line{
        LL m, c;
        line(LL _m=0, LL _c=0) { m = _m; c = _c; }
        LL eval(LL x) { return m * x + c; }
    };
    struct node{
        node *l, *r; line f;
        node(line v) { f = v; l = r = NULL; }
    };
    typedef node* pnode;
    pnode root; int sz;
#define mid ((l+r)>>1)
    void insert(line &v, int l, int r, pnode &nd){
        if(!nd) { nd = new node(v); return; }
        LL trl = nd->f.eval(l), trr = nd->f.eval(r);
        LL vl = v.eval(l), vr = v.eval(r);
        if(trl <= vl && trr <= vr) return;
        if(trl > vl && trr > vr) { nd->f = v; return; }
        if(trl > vl) swap(nd->f, v);
        if(nd->f.eval(mid) < v.eval(mid)) insert(v, mid + 1, r, nd->r);
        else swap(nd->f, v), insert(v, l, mid, nd->l);
    }
```

```

    }
    LL query(int x, int l, int r, pnode &nd){
        if(!nd) return LLONG_MAX;
        if(l == r) return nd->f.eval(x);
        if(mid >= x) return min(nd->f.eval(x), query(x, l, mid, nd->l));
        return min(nd->f.eval(x), query(x, mid + 1, r, nd->r));
    }
    /* -sz <= query_x <= sz */
    void init(int _sz){ sz = _sz + 1; root = NULL; }
    void add_line(LL m, LL c){ line v(m, c); insert(v, -sz, sz, root); }
    LL query(LL x) { return query(x, -sz, sz, root); }
};
```

### 3.5 link cut tree

```
const int MXN = 100005;
const int MEM = 100005;
struct Splay {
    static Splay nil, mem[MEM], *pmem;
    Splay *ch[2], *f;
    int val, rev, size;
    Splay(int _val=-1) : val(_val), rev(0), size(1)
    { f = ch[0] = ch[1] = &nil; }
    bool isr()
    { return f->ch[0] != this && f->ch[1] != this; }
    int dir()
    { return f->ch[0] == this ? 0 : 1; }
    void setCh(Splay *c, int d){
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void push(){
        if( !rev ) return;
        swap(ch[0], ch[1]);
        if (ch[0] != &nil) ch[0]->rev ^= 1;
        if (ch[1] != &nil) ch[1]->rev ^= 1;
        rev=0;
    }
    void pull(){
        size = ch[0]->size + ch[1]->size + 1;
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::mem;
Splay *nil = &Splay::nil;
void rotate(Splay *x){
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[!d], d);
    x->setCh(p, !d);
    p->pull(); x->pull();
}
vector<Splay*> splayVec;
void splay(Splay *x){
    splayVec.clear();
    for (Splay *q=x; q=q->f){
        splayVec.push_back(q);
        if (q->isr()) break;
    }
    reverse(begin(splayVec), end(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir()==x->f->dir())
            rotate(x->f), rotate(x);
        else rotate(x), rotate(x);
    }
}
int id(Splay *x) { return x - Splay::mem + 1; }
Splay* access(Splay *x){
    Splay *q = nil;
    for (;x!=nil;x=x->f){
        splay(x);
    }
```



```

    x->setCh(q, 1);
    q = x;
}
return q;
}
void chroot(Splay *x){
    access(x);
    splay(x);
    x->rev ^= 1;
    x->push(); x->pull();
}
void link(Splay *x, Splay *y){
    access(x);
    splay(x);
    chroot(y);
    x->setCh(y, 1);
}
void cut_p(Splay *y) {
    access(y);
    splay(y);
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}
void cut(Splay *x, Splay *y){
    chroot(x);
    cut_p(y);
}
Splay* get_root(Splay *x) {
    access(x);
    splay(x);
    for(; x->ch[0] != nil; x = x->ch[0])
        x->push();
    splay(x);
    return x;
}
bool conn(Splay *x, Splay *y) {
    x = get_root(x);
    y = get_root(y);
    return x == y;
}
Splay* lca(Splay *x, Splay *y) {
    access(x);
    access(y);
    splay(x);
    if (x->f == nil) return x;
    else return x->f;
}
}

```

### 3.6 KDTree

```

template<typename T, size_t kd> //kd???????
class kd_tree{
public:
    struct point{
        T d[kd];
        inline T dist(const point &x) const{
            T ret=0;
            for(size_t i=0; i<kd; ++i) ret+=std::abs(d[i]-x.d[i]);
            return ret;
        }
        inline bool operator==(const point &p){
            for(size_t i=0; i<kd; ++i){
                if(d[i]!=p.d[i]) return 0;
            }
            return 1;
        }
        inline bool operator<(const point &b) const{
            return d[0]<b.d[0];
        }
    };
private:
    struct node{
        node *l,*r;
        point pid;
        int s;
        node(const point &p):l(0),r(0),pid(p),s(1){}
        inline void up(){
            s=(l?l->s:0)+1+(r?r->s:0);
        }
    }
}

```

```

}*root;
const double alpha, loga;
const T INF; //????INF,????
int maxn;
struct __cmp{
    int sort_id;
    inline bool operator()(const node*x, const node*y) const{
        return operator()(x->pid, y->pid);
    }
    inline bool operator()(const point &x, const point &y) const{
        if(x.d[sort_id]!=y.d[sort_id])
            return x.d[sort_id]<y.d[sort_id];
        for(size_t i=0; i<kd; ++i){
            if(x.d[i]!=y.d[i]) return x.d[i]<y.d[i];
        }
        return 0;
    }
}cmp;
void clear(node *o){
    if(!o) return;
    clear(o->l);
    clear(o->r);
    delete o;
}
inline int size(node *o){
    return o?o->s:0;
}
std::vector<node*> A;
node* build(int k, int l, int r){
    if(l>r) return 0;
    if(k==kd) k=0;
    int mid=(l+r)/2;
    cmp.sort_id=k;
    std::nth_element(A.begin()+l, A.begin()+mid, A.begin()+r+1, cmp);
    node *ret=A[mid];
    ret->l=build(k+1, l, mid-1);
    ret->r=build(k+1, mid+1, r);
    ret->up();
    return ret;
}
inline bool isbad(node*o){
    return size(o->l)>alpha*o->s || size(o->r)>alpha*o->s;
}
void flatten(node *u, typename std::vector<node*>::iterator &it){
    if(!u) return;
    flatten(u->l, it);
    *it=u;
    flatten(u->r, ++it);
}
inline void rebuild(node*&u, int k){
    if((int)A.size()<u->s) A.resize(u->s);
    typename std::vector<node*>::iterator it=A.begin();
    flatten(u, it);
    u=build(k, 0, u->s-1);
}
bool insert(node*&u, int k, const point &x, int dep){
    if(!u){
        u=new node(x);
        return dep<=0;
    }
    ++u->s;
    cmp.sort_id=k;
    if(insert(cmp(x, u->pid)?u->l:u->r, (k+1)%kd, x, dep-1)){
        if(!isbad(u)) return 1;
        rebuild(u, k);
    }
    return 0;
}
node *findmin(node*o, int k){
    if(!o) return 0;
    if(cmp.sort_id==k) return o->l?findmin(o->l, (k+1)%kd):o;
    node *l=findmin(o->l, (k+1)%kd);
    node *r=findmin(o->r, (k+1)%kd);
    if(l&&!r) return cmp(l, o)?l:o;
}

```

```

    if(!l&&r)return cmp(r,o)?r:o;
    if(!l&&!r)return o;
    if(cmp(l,r))return cmp(l,o)?l:o;
    return cmp(r,o)?r:o;
}
bool erase(node *&u,int k,const point &x){
    if(!u)return 0;
    if(u->pid==x){
        if(u->r);
        else if(u->l){
            u->r=u->l;
            u->l=0;
        }else{
            delete u;
            u=0;
            return 1;
        }
        --u->s;
        cmp.sort_id=k;
        u->pid=findmin(u->r,(k+1)%kd)->pid;
        return erase(u->r,(k+1)%kd,u->pid);
    }
    cmp.sort_id=k;
    if(erase(cmp(x,u->pid)?u->l:u->r,(k+1)%kd,x)){
        --u->s;return 1;
    }else return 0;
}
inline T heuristic(const T h[])const{
    T ret=0;
    for(size_t i=0;i<kd;++i)ret+=h[i];
    return ret;
}
int qM;
std::priority_queue<std::pair<T,point > >pQ;
void nearest(node *u,int k,const point &x,T *h,T &
    mndist){
    if(u==0||heuristic(h)>=mndist)return;
    T dist=u->pid.dist(x),old=h[k];
    /*mndist=std::min(mndist,dist);*/
    if(dist<mndist){
        pQ.push(std::make_pair(dist,u->pid));
        if((int)pQ.size()==qM+1){
            mndist=pQ.top().first,pQ.pop();
        }
    }
    if(x.d[k]<u->pid.d[k]){
        nearest(u->l,(k+1)%kd,x,h,mndist);
        h[k]=std::abs(x.d[k]-u->pid.d[k]);
        nearest(u->r,(k+1)%kd,x,h,mndist);
    }else{
        nearest(u->r,(k+1)%kd,x,h,mndist);
        h[k]=std::abs(x.d[k]-u->pid.d[k]);
        nearest(u->l,(k+1)%kd,x,h,mndist);
    }
    h[k]=old;
}
std::vector<point>in_range;
void range(node *u,int k,const point&mi,const point
    &ma){
    if(!u)return;
    bool is=1;
    for(int i=0;i<kd;++i)
        if(u->pid.d[i]<mi.d[i]||ma.d[i]<u->pid.d[i]){
            is=0;break;
        }
    if(is)in_range.push_back(u->pid);
    if(mi.d[k]<u->pid.d[k])range(u->l,(k+1)%kd,mi,ma);
    if(ma.d[k]>u->pid.d[k])range(u->r,(k+1)%kd,mi,ma);
}
public:
kd_tree(const T &INF,double a=0.75):root(0),alpha(a),
    loga(log2(1.0/a)),INF(INF),maxn(1){}
inline void clear(){
    clear(root),root=0,maxn=1;
}
inline void build(int n,const point *p){
    clear(root),A.resize(maxn=n);
    for(int i=0;i<n;++i)A[i]=new node(p[i]);
    root=build(0,0,n-1);
}

```

```

inline void insert(const point &x){
    insert(root,0,x,std::__lg(size(root))/loga);
    if(root->s>maxn)maxn=root->s;
}
inline bool erase(const point &p){
    bool d=erase(root,0,p);
    if(root&&root->s<alpha*maxn)rebuild();
    return d;
}
inline void rebuild(){
    if(root)rebuild(root,0);
    maxn=root->s;
}
inline T nearest(const point &x,int k){
    qM=k;
    T mndist=INF,h[kd]={};
    nearest(root,0,x,h,mndist);
    mndist=pQ.top().first;
    pQ=std::priority_queue<std::pair<T,point > >();
    return mndist;/*???x?k????*/
}
inline const std::vector<point> &range(const point&
    mi,const point&ma){
    in_range.clear();
    range(root,0,mi,ma);
    return in_range;/*???mi?ma???vector*/
}
inline int size(){return root?root->s:0;}
};

```

## 4 Flow/Matching

### 4.1 Dinic

```

struct MaxFlow{/*-base
struct edge{
    int to,cap,flow,rev;
};
vector<edge> G[MAXN];
int s,t,dis[MAXN],cur[MAXN],n;
int dfs(int u,int cap){
    if(u==t||!cap) return cap;
    for(int &i=cur[u];i<(int)G[u].size();++i){
        edge &e=G[u][i];
        if(dis[e.to]==dis[u]+1 && e.flow!=e.cap){
            int df=dfs(e.to,min(e.cap-e.flow,cap));
            if(df){
                e.flow+=df;
                G[e.to][e.rev].flow-=df;
                return df;
            }
        }
    }
    dis[u]=-1;
    return 0;
}
bool bfs(){
    FILL(dis,-1);
    queue<int> q;
    q.push(s),dis[s]=0;
    while(!q.empty()){
        int tmp=q.front();
        q.pop();
        for(auto &u:G[tmp])
            if(!dis[u.to] && u.flow!=u.cap){
                q.push(u.to);
                dis[u.to]=dis[tmp]+1;
            }
    }
    return dis[t]!=-1;
}
int maxflow(int _s,int _t){
    s=_s,t=_t;
    int flow=0,df;
    while(bfs()){
        FILL(cur,0);
        while(df=dfs(s,INF)) flow+=df;
    }
}

```



```

    return flow;
}
void init(int _n){n=_n;
for(int i=0;i<n;++i) G[i].clear();
}
void reset(){
for(int i=0;i<n;++i)
for(auto &j:G[i])
j.flow=0;
}
void add_edge(int u,int v,int cap){
G[u].pb(edge{v,cap,0,(int)G[v].size()});
G[v].pb(edge{u,0,0,(int)G[u].size()-1});
}
};

```

## 4.2 Kuhn Munkres

```

struct KM{// 0-base
int w[MAXN][MAXN],hl[MAXN],hr[MAXN],slk[MAXN],n;
int fl[MAXN],fr[MAXN],pre[MAXN],qu[MAXN],ql,q,r;
bool vl[MAXN],vr[MAXN];
void init(int _n){n=_n;
for(int i=0;i<n;++i)
for(int j=0;j<n;++j)
w[i][j]=-INF;
}
void add_edge(int a,int b,int wei){
w[a][b]=wei;
}
bool Check(int x){
if(vl[x]==1,~fl[x]) return vr[qu[qr++]=fl[x]]=1;
while(~x) swap(x,fr[fl[x]=pre[x]]);
return 0;
}
void Bfs(int s){
fill(slk,slk+n,INF);
fill(vl,vl+n,0),fill(vr,vr+n,0);
ql=qr=0,qu[qr++]=s,vr[s]=1;
while(1){
int d;
while(ql<qr)
for(int x=0,y=qu[ql++];x<n;++x)
if(!vl[x]&&slk[x]>=(d=hl[x]+hr[y]-w[x][y]))
if(pre[x]=y,d) slk[x]=d;
else if(!Check(x)) return;
d=INF;
for(int x=0;x<n;++x)
if(!vl[x]&&d>slk[x]) d=slk[x];
for(int x=0;x<n;++x){
if(vl[x]) hl[x]+=d;
else slk[x]-=d;
if(vr[x]) hr[x]-=d;
}
for(int x=0;x<n;++x)
if(!vl[x]&&!slk[x]&&!Check(x)) return;
}
}
int Solve(){
fill(fl,fl+n,-1),fill(fr,fr+n,-1),fill(hr,hr+n,0);
for(int i=0;i<n;++i) hl[i]=*max_element(w[i],w[i]+n);
for(int i=0;i<n;++i) Bfs(i);
int res=0;
for(int i=0;i<n;++i) res += w[i][fl[i]];
return res;
}
};

```

## 4.3 MincostMaxflow

```

struct MCMF{//0-base
struct edge{
ll from,to,cap,flow,cost,rev;
}*past[MAXN];
vector<edge> G[MAXN];

```

```

bitset<MAXN> inq;
ll dis[MAXN],up[MAXN],s,t,mx,n;
bool BellmanFord(ll &flow,ll &cost){
fill(dis,dis+n,INF);
queue<ll> q;
q.push(s),inq.reset(),inq[s]=1;
up[s]=mx-flow,past[s]=0,dis[s]=0;
while(!q.empty()){
ll u=q.front();
q.pop(),inq[u]=0;
if(!up[u]) continue;
for(auto &e:G[u])
if(e.flow!=e.cap&&dis[e.to]>dis[u]+e.cost){
dis[e.to]=dis[u]+e.cost,past[e.to]=&e;
up[e.to]=min(up[u],e.cap-e.flow);
if(!inq[e.to]) inq[e.to]=1,q.push(e.to);
}
}
if(dis[t]==INF) return 0;
flow+=up[t],cost+=up[t]*dis[t];
for(ll i=t;past[i];i=past[i]->from){
auto &e=*past[i];
e.flow+=up[t],G[e.to][e.rev].flow-=up[t];
}
return 1;
}
ll MinCostMaxFlow(ll _s,ll _t,ll &cost){
s=_s,t=_t,cost=0;ll flow=0;
while(BellmanFord(flow,cost));
return flow;
}
void init(ll _n,ll _mx){n=_n,mx=_mx;
for(int i=0;i<n;++i) G[i].clear();
}
void add_edge(ll a,ll b,ll cap,ll cost){
G[a].pb(edge{a,b,cap,0,cost,G[b].size()});
G[b].pb(edge{b,a,0,0,-cost,G[a].size()-1});
}
};

```

## 4.4 Maximum Simple Graph Matching

```

struct GenMatch{//1-base
int V,pr[MAXN];
bool el[MAXN][MAXN];
bool inq[MAXN],inp[MAXN],inb[MAXN];
queue<int> qe;
int st,ed,nb,bk[MAXN],djs[MAXN],ans;
void init(int _V){V=_V;
for(int i=0;i<=V;++i){
for(int j=0;j<=V;++j)
el[i][j]=0;
pr[i]=bk[i]=djs[i]=0;
inq[i]=inp[i]=inb[i]=0;
}
ans=0;
}
void add_edge(int u,int v){
el[u][v]=el[v][u]=1;
}
int lca(int u,int v){
for(int i=0;i<=V;++i) inp[i]=0;
while(1)
if(u=djs[u],inp[u]=true,u==st) break;
else u=bk[pr[u]];
while(1)
if(v=djs[v],inp[v]) return v;
else v=bk[pr[v]];
return v;
}
void upd(int u){
int v;
while(djs[u]!=nb){
v=pr[u],inb[djs[u]]=inb[djs[v]]=true;
u=bk[v];
if(djs[u]!=nb) bk[u]=v;
}
}
void blo(int u,int v){
nb=lca(u,v);

```

```

    for (int i=0;i<V;++i) inb[i]=0;
    upd(u),upd(v);
    if(djs[u]!=nb) bk[u]=v;
    if(djs[v]!=nb) bk[v]=u;
    for(int tu=1;tu<=V;++tu)
        if(inb[djs[tu]])
            if(djs[tu]==nb,!inq[tu])
                qe.push(tu),inq[tu]=1;
}
void flow(){
    for(int i=1;i<=V;++i)
        inq[i]=0,bk[i]=0,djs[i]=i;
    while(qe.size()) qe.pop();
    qe.push(st),inq[st]=1,ed=0;
    while(qe.size()){
        int u = qe.front();
        qe.pop();
        for(int v=1;v<=V;++v)
            if(e[u][v]&&djs[u]!=djs[v]&&pr[u]!=v)
                if((v==st)||pr[v]>0&&bk[pr[v]]>0)
                    blo(u,v);
                else if(!bk[v])
                    if(bk[v]=u,pr[v]>0)
                        if(!inq[pr[v]])
                            qe.push(pr[v]);
                    else;
                else
                    return ed=v,void();
    }
}
void aug(){
    for(int u=ed,v=w;u>0;)
        v=bk[u],w=pr[v],pr[v]=u,pr[u]=v,u=w;
}
int solve(){
    for(int i=0;i<=V;++i) pr[i]=0;
    for(int u=1;u<=V;++u)
        if(!pr[u])
            if(st=u,flow(),ed>0)
                aug(),++ans;
    return ans;
}
};

```

#### 4.5 Minimum Weight Matching (Clique version)

```

struct Graph{//0-base (Perfect Match)
    int n,edge[MAXN][MAXN];
    int match[MAXN],dis[MAXN],onstk[MAXN];
    vector<int> stk;
    void init(int _n){n=_n;
        for(int i=0;i<n;++i)
            for(int j=0;j<n;++j)
                edge[i][j]=0;
    }
    void add_edge(int u,int v,int w){
        edge[u][v]=edge[v][u]=w;
    }
    bool SPFA(int u){
        if(onstk[u]) return 1;
        stk.pb(u),onstk[u]=1;
        for(int v=0;v<n;++v)
            if(u!=v&&match[u]!=v&&!onstk[v]){
                int m=match[v];
                if(dis[m]>dis[u]-edge[v][m]+edge[u][v])
                    {
                        dis[m]=dis[u]-edge[v][m]+edge[u][v];
                        onstk[v]=1,stk.pb(v);
                        if(SPFA(m)) return 1;
                        stk.pop_back(),onstk[v]=0;
                    }
            }
        onstk[u]=0,stk.pop_back();
        return 0;
    }
    int solve(){// find a match
        for(int i=0;i<n;i+=2)

```

```

            match[i]=i+1,match[i+1]=i;
        while(1){
            int found=0;
            for(int i=0;i<n;++i) dis[i]=onstk[i]=0;
            for(int i=0;i<n;++i)
                if(stk.clear(),!onstk[i]&&SPFA(i))
                    for(found=1;stk.size()>=2;){
                        int u=stk.back();
                        stk.pop_back();
                        int v=stk.back();
                        stk.pop_back();
                        match[u]=v,match[v]=u;
                    }
                if(!found) break;
        }
        int ret=0;
        for(int i=0;i<n;++i) ret+=edge[i][match[i]];
        return ret>>1;
    }
};

```

#### 4.6 SW-mincut

```

// global min cut
struct SW{ //  $O(V^3)$ 
    static const int MXN = 514;
    int n,vst[MXN],del[MXN];
    int edge[MXN][MXN],wei[MXN];
    void init(int _n){
        n=_n,MEM(edge,0),MEM(del,0);
    }
    void addEdge(int u,int v,int w){
        edge[u][v]+=w,edge[v][u]+=w;
    }
    void search(int &s,int &t){
        MEM(vst,0),MEM(wei,0),s=t=-1;
        while(1){
            int mx=-1,cur=0;
            for(int i=0;i<n;++i)
                if(!del[i]&&!vst[i]&&mx<wei[i])
                    cur=i,mx=wei[i];
            if(mx==-1) break;
            vst[cur]=1,s=t,t=cur;
            for(int i=0;i<n;++i)
                if(!vst[i]&&!del[i]) wei[i]+=edge[cur][i];
        }
    }
    int solve(){
        int res=INF;
        for(int i=0,x,y;i<n-1;++i){
            search(x,y),res=min(res,wei[y]),del[y]=1;
            for(int j=0;j<n;++j)
                edge[x][j]=(edge[j][x]+edge[y][j]);
        }
        return res;
    }
};

```

#### 4.7 BoundedFlow

```

struct BoundedFlow{//0-base
    struct Edge{
        int to,cap,flow,rev;
    };
    vector<Edge> G[MAXN];
    int n,s,t,dis[MAXN],cur[MAXN],cnt[MAXN];
    void init(int _n){n=_n;
        for(int i=0;i<n+2;++i) G[i].clear(),cnt[i]=0;
    }
    void add_edge(int u,int v,int lcap,int rcap){
        cnt[u]-=lcap,cnt[v]+=lcap;
        G[u].pb(Edge{v,rcap,lcap,(int)G[v].size()});
        G[v].pb(Edge{u,0,0,(int)G[u].size()-1});
    }
    void add_edge(int u,int v,int cap){
        G[u].pb(Edge{v,cap,0,(int)G[v].size()});
        G[v].pb(Edge{u,0,0,(int)G[u].size()-1});
    }
};

```

```

int dfs(int u,int cap){
    if(u==t || !cap) return cap;
    for(int &i=cur[u];i<(int)G[u].size();++i){
        Edge &e=G[u][i];
        if(dis[e.to]==dis[u]+1 && e.flow!=e.cap){
            int df=dfs(e.to,min(e.cap-e.flow,cap));
            if(df){
                e.flow+=df;
                G[e.to][e.rev].flow-=df;
                return df;
            }
        }
    }
    dis[u]=-1;
    return 0;
}

bool bfs(){
    FILL(dis,-1);
    queue<int> q;
    q.push(s),dis[s]=0;
    while(!q.empty()){
        int tmp=q.front();
        q.pop();
        for(auto &u:G[tmp])
            if(!~dis[u.to] && u.flow!=u.cap){
                q.push(u.to);
                dis[u.to]=dis[tmp]+1;
            }
    }
    return dis[t]!=-1;
}

int maxflow(int _s,int _t){
    s=_s,t=_t;
    int flow=0,df;
    while(bfs()){
        FILL(cur,0);
        while(df=dfs(s,INF)) flow+=df;
    }
    return flow;
}

bool solve(){
    int sum=0;
    for(int i=0;i<n;++i)
        if(cnt[i]>0) add_edge(n+1,i,cnt[i]),sum+=cnt[i];
        else if(cnt[i]<0) add_edge(i,n+2,-cnt[i]);
    if(sum!=maxflow(n+1,n+2)) sum=-1;
    for(int i=0;i<n;++i)
        if(cnt[i]>0) G[n+1].pop_back(),G[i].pop_back();
        else if(cnt[i]<0) G[i].pop_back(),G[n+2].pop_back();
    return sum!=-1;
}

int solve(int _s,int _t){
    add_edge(_t,_s,INF);
    if(!solve()) return -1; //invalid flow
    int x=G[_t].back().flow;
    return G[_t].pop_back(),G[_s].pop_back(),x;
}
};

```

## 4.8 Gomory Hu tree

```

struct Gomory_Hu_tree{//0-base
    MaxFlow Dinic;
    int n;
    vector<pii> G[MAXN];
    void init(int _n){n=_n;
        for(int i=0;i<n;++i) G[i].clear();
    }
    void solve(vector<int> &v){
        if(v.size()<=1) return;
        int s=rand()%SZ(v);
        swap(v.back(),v[s]),s=v.back();
        int t=v[rand()%(SZ(v)-1)];
        vector<int> L,R;
        int x=(Dinic.reset(),Dinic.maxflow(s,t));
        G[s].pb(pii(t,x)),G[t].pb(pii(s,x));
        for(int i:v)
            if(~Dinic.dis[i]) L.pb(i);
            else R.pb(i);
    }
};

```

```

    solve(L),solve(R);
}

void build(){
    vector<int> v(n);
    for(int i=0;i<n;++i) v[i]=i;
    solve(v);
}

}gnt;//test by BZOJ 4519
MaxFlow &Dinic=gnt.Dinic;

```

## 4.9 NumberofMaximalClique

```

// bool g[][] : adjacent array indexed from 1 to n
void dfs(int sz){
    int i, j, k, t, cnt, best = 0;
    if(ne[sz]==ce[sz]){ if (ce[sz]==0) ++ans; return; }
    for(t=0, i=1; i<=ne[sz]; ++i){
        for (cnt=0, j=ne[sz]+1; j<=ce[sz]; ++j)
            if (!g[1st[sz][i]][1st[sz][j]]) ++cnt;
        if (t==0 || cnt<best) t=i, best=cnt;
    } if (t && best<=0) return;
    for (k=ne[sz]+1; k<=ce[sz]; ++k) {
        if (t>0){ for (i=k; i<=ce[sz]; ++i)
            if (!g[1st[sz][t]][1st[sz][i]]) break;
        swap(1st[sz][k], 1st[sz][i]);
        i=1st[sz][k]; ne[sz+1]=ce[sz+1]=0;
        for (j=1; j<k; ++j)if (g[i][1st[sz][j]])
            1st[sz+1][++ne[sz+1]]=1st[sz][j];
        for (ce[sz+1]=ne[sz+1], j=k+1; j<=ce[sz]; ++j)
            if (g[i][1st[sz][j]]) 1st[sz+1][++ce[sz+1]]=1st[sz][j];
        dfs(sz+1); ++ne[sz]; --best;
        for (j=k+1, cnt=0; j<=ce[sz]; ++j) if (!g[i][1st[sz][j]]) ++cnt;
        if (t==0 || cnt<best) t=k, best=cnt;
        if (t && best<=0) break;
    }
}

void work(){
    ne[0]=0; ce[0]=0;
    for(int i=1; i<=n; ++i) 1st[0][++ce[0]]=i;
    ans=0; dfs(0);
}

```

## 4.10 isap

```

struct Maxflow {
    static const int MAXV = 20010;
    static const int INF = 1000000;
    struct Edge {
        int v, c, r;
        Edge(int _v, int _c, int _r):
            v(_v), c(_c), r(_r) {}
    };
    int s, t;
    vector<Edge> G[MAXV*2];
    int iter[MAXV*2], d[MAXV*2], gap[MAXV*2], tot;
    void init(int x) {
        tot = x+2;
        s = x+1, t = x+2;
        for(int i = 0; i <= tot; i++) {
            G[i].clear();
            iter[i] = d[i] = gap[i] = 0;
        }
    }
    void addEdge(int u, int v, int c) {
        G[u].push_back(Edge(v, c, SZ(G[v])));
        G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
    }
    int dfs(int p, int flow) {
        if(p == t) return flow;
        for(int &i = iter[p]; i < SZ(G[p]); i++) {
            Edge &e = G[p][i];
            if(e.c > 0 && d[p] == d[e.v]+1) {
                int f = dfs(e.v, min(flow, e.c));
                if(f) {
                    e.c -= f;
                    G[e.v][e.r].c += f;
                    return f;
                }
            }
        }
    }
};

```

```

    }
}
}
if ( (--gap[d[p]]) == 0 ) d[s] = tot;
else {
    d[p]++;
    iter[p] = 0;
    ++gap[d[p]];
}
return 0;
}
int solve() {
    int res = 0;
    gap[0] = tot;
    for (res = 0; d[s] < tot; res += dfs(s, INF));
    return res;
}
} flow;

```

## 5 String

### 5.1 KMP

```

int F[MAXN];
vector<int> match(string A, string B){
    vector<int> ans;
    F[0]=-1, F[1]=0;
    for(int i=1, j=0; i<B.size(); F[++i]=++j){
        if(B[i]==B[j]) F[i]=F[j]; //optimize
        while(j!=-1 && B[i]!=B[j]) j=F[j];
    }
    for(int i=0, j=0; i-j+B.size()<=A.size(); ++i, ++j){
        while(j!=-1 && A[i]!=B[j]) j=F[j];
        if(j==B.size()-1) ans.pb(i-j);
    }
    return ans;
}

```

### 5.2 Z-value

```

const int MAXN = 1e5 + 5;
int z[MAXN];
void make_z(string s){
    int l = 0, r = 0;
    for(int i = 1; i < s.size(); i++){
        for(z[i] = max(0, min(r - i + 1, z[i - 1]));
            i + z[i] < s.size() && s[i + z[i]] == s[z[i]]; z[i]++);
        if(i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
}

```

### 5.3 Manacher

```

int z[MAXN];
int Manacher(string tmp){
    string s = "&";
    int l=0, r=0, x, ans;
    for(char c:tmp) s.pb(c), s.pb('%');
    ans=0, x=0;
    for(int i=1; i<SZ(s); ++i){
        z[i]=r > i ? min(z[2*1-i], r-i) : 1;
        while(s[i+z[i]]==s[i-z[i]]) ++z[i];
        if(z[i]+i>r) r=z[i]+i, l=i;
    }
    for(int i=1; i<SZ(s); ++i)
        if(s[i]=='%')
            x=max(x, z[i]);
    ans=x/2*2, x=0;
    for(int i=1; i<SZ(s); ++i)
        if(s[i]!='%')
            x=max(x, z[i]);
    return max(ans, (x-1)/2*2+1);
}

```

### 5.4 Suffix Array

```

struct suffix_array{
    int box[MAXN], tp[MAXN], m;
    bool not_equ(int a, int b, int k, int n){
        return ra[a]!=ra[b] || a+k>n || b+k>n || ra[a+k]!=ra[b+k];
    }
    void radix(int *key, int *it, int *ot, int n){
        fill_n(box, m, 0);
        for(int i=0; i<n; ++i) ++box[key[i]];
        partial_sum(box, box+m, box);
        for(int i=n-1; i>=0; --i) ot[--box[key[it[i]]]]=it[i];
    }
    void make_sa(string s, int n){
        int k=1;
        for(int i=0; i<n; ++i) ra[i]=s[i];
        do{
            iota(tp, tp+k, n-k, iota(sa+k, sa+n, 0));
            radix(ra+k, sa+k, tp+k, n-k);
            radix(ra, tp, sa, n);
            tp[sa[0]]=0, m=1;
            for(int i=1; i<n; ++i){
                m+=not_equ(sa[i], sa[i-1], k, n);
                tp[sa[i]]=m-1;
            }
            copy_n(tp, n, ra);
            k*=2;
        }while(k<n && m!=n);
    }
    void make_he(string s, int n){
        for(int j=0, k=0; j<n; ++j){
            if(ra[j])
                for(; s[j+k]==s[sa[ra[j]-1]+k]; ++k);
            he[ra[j]]=k, k=max(0, k-1);
        }
    }
    int sa[MAXN], ra[MAXN], he[MAXN];
    void build(string s){
        FILL(sa, 0), FILL(ra, 0), FILL(he, 0);
        FILL(box, 0), FILL(tp, 0), m=256;
        make_sa(s, s.size());
        make_he(s, s.size());
    }
};

```

### 5.5 SAIS

```

struct SA{
#define REP(i,n) for ( int i=0; i<int(n); i++ )
#define REP1(i,a,b) for ( int i=(a); i<=int(b); i++ )
    bool _t[MAXN*2];
    int _s[MAXN*2], _sa[MAXN*2], _c[MAXN*2], x[MAXN], _p[
        MAXN], _q[MAXN*2], hei[MAXN], r[MAXN];
    int operator [] (int i){ return _sa[i]; }
    void build(int *s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
    void mkhei(int n){
        REP(i,n) r[_sa[i]] = i;
        hei[0] = 0;
        REP(i,n) if(r[i]) {
            int ans = i>0 ? max(hei[r[i-1]] - 1, 0) : 0;
            while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans++;
            hei[r[i]] = ans;
        }
    }
    void sais(int *s, int *sa, int *p, int *q, bool *t,
        int *c, int n, int z){
        bool uniq = t[n-1] = true, neq;
        int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
            lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
        memcpy(x, c, sizeof(int) * z); \
        XD; \

```

```

memcpy(x + 1, c, sizeof(int) * (z - 1)); \
REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[s[sa[i]
]-1]]++ = sa[i]-1; \
memcpy(x, c, sizeof(int) * z); \
for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i]
]-1]) sa[-x[s[sa[i]-1]]] = sa[i]-1;
MS0(c, z);
REP(i,n) uniq &= ++c[s[i]] < 2;
REP(i,z-1) c[i+1] += c[i];
if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return; }
for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s[i
+1] ? t[i+1] : s[i]<s[i+1]);
MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[--x[s[i
]]]=p[q[i]=nn++]=i);
REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1]) {
    neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa
[i])*sizeof(int));
    ns[q[lst=sa[i]]]=nmxz+neq;
}
sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmxz
+ 1);
MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[s[p[
nsa[i]]]]] = p[nsa[i]]);
}
}sa;
int H[ MAXN ], SA[ MAXN ];
void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // ip is int array, len is array length
    // ip[0..n-1] != 0, and ip[len] = 0
    ip[len++] = 0;
    sa.build(ip, len, 128);
    for (int i=0; i<len; i++) {
        H[i] = sa.hei[i + 1];
        SA[i] = sa._sa[i + 1];
    }
    // resulting height, sa array \in [0,len)
}

```

## 5.6 Aho-Corasick Automatan

```

const int len=400000,sigma=26;
struct AC_Automatan{
    int nx[len][sigma],fl[len],cnt[len],pri[len],top;
    int newnode(){
        fill(nx[top],nx[top]+sigma,-1);
        return top++;
    }
    void init(){top=1,newnode();}
    int input(string &s){//return the end_node of string
        int X=1;
        for(char c:s){
            if(!nx[X][c-'a'])nx[X][c-'a']=newnode();
            X=nx[X][c-'a'];
        }
        return X;
    }
    void make_fl(){
        queue<int> q;
        q.push(1),fl[1]=0;
        for(int t=0;!q.empty();){
            int R=q.front();
            q.pop(),pri[t++]=R;
            for(int i=0;i<sigma;++i)
                if(~nx[R][i]){
                    int X=nx[R][i],Z=fl[R];
                    for(;Z&&!~nx[Z][i];)Z=fl[Z];
                    fl[X]=Z?nx[Z][i]:1,q.push(X);
                }
        }
    }
    void get_v(string &s){
        int X=1;
        fill(cnt,cnt+top,0);
        for(char c:s){
            while(X&&!~nx[X][c-'a'])X=fl[X];
            X=X?nx[X][c-'a']:1,++cnt[X];
        }
        for(int i=top-2;i>0;--i) cnt[fl[pri[i]]]+=cnt[pri[i]
]];
    }
}

```

```

}
};

```

## 5.7 Smallest Rotation

```

string mcp(string s){
    int n=SZ(s),i=0,j=1;
    s+=s;
    while(i<n&&j<n){
        int k=0;
        while(k<n&&s[i+k]==s[j+k]) ++k;
        if(s[i+k]<=s[j+k]) j+=k+1;
        else i+=k+1;
        if(i==j) ++j;
    }
    int ans=i<n?i:j;
    return s.substr(ans,n);
}

```

## 5.8 De Bruijn sequence

```

constexpr int MAXC = 10, MAXN = 1e5 + 10;
struct DBSeq {
    int C, N, K, L, buf[MAXC * MAXN]; //K <= C^N
    void dfs(int *out, int t, int p, int &ptr) {
        if (ptr>=L) return;
        if (t>N) {
            if (N%p) return;
            for (int i=1;i<=p&&ptr<L;++i) out[ptr++]=
                buf[i];
        } else {
            buf[t]=buf[t-p],dfs(out,t+1,p,ptr);
            for (int j=buf[t-p]+1;j<C;++j) buf[t]=j,dfs
                (out,t+1,t,ptr);
        }
    }
    void solve(int _c, int _n, int _k, int *out) {
        int p=0;
        C=_c,N=_n,K=_k,L=N+K-1;dfs(out,1,1,p);
        if (p<L) fill(out+p,out+L,0);
    }
} dbs;

```

## 5.9 SAM

```

const int MAXM = 1000010;
struct SAM{
    int tot, root, lst, mom[MAXM], mx[MAXM];
    int acc[MAXM], nxt[MAXM][33];
    int newNode(){
        int res = ++tot;
        fill(nxt[res],nxt[res]+33, 0);
        mom[res] = mx[res] = acc[res] = 0;
        return res;
    }
    void init(){
        tot = 0;
        root = newNode();
        mom[root] = 0, mx[root] = 0;
        lst = root;
    }
    void push(int c){
        int p = lst;
        int np = newNode();
        mx[np] = mx[p]+1;
        for(; p && nxt[p][c] == 0; p = mom[p])
            nxt[p][c] = np;
        if(p == 0) mom[np] = root;
        else{
            int q = nxt[p][c];
            if(mx[p]+1 == mx[q]) mom[np] = q;
            else{
                int nq = newNode();
                mx[nq] = mx[p]+1;
                for(int i = 0; i < 33; i++)
                    nxt[nq][i] = nxt[q][i];
            }
        }
    }
}

```

```

        mom[nq] = mom[q];
        mom[q] = nq;
        mom[np] = nq;
        for(; p && nxt[p][c] == q; p = mom[p])
            nxt[p][c] = nq;
    }
    }
    lst = np;
}
void push(char *str){
    for(int i = 0; str[i]; i++)
        push(str[i] - 'a' + 1);
}
} sam;

```

## 5.10 PalTree

```

struct palindromic_tree{// Check by APIO 2014
    palindrome
    struct node{
        int next[26],fail,len;
        int cnt,num;//cnt: appear times, num: number of pal
            . suf.
        node(int l=0):fail(0),len(1),cnt(0),num(0){
            for(int i=0;i<26;++i)next[i]=0;
        }
    };
    vector<node>St;
    vector<char>s;
    int last,n;
    palindromic_tree():St(2),last(1),n(0){
        St[0].fail=1, St[1].len=-1, s.pb(-1);
    }
    inline void clear(){
        St.clear(), s.clear(), last=1, n=0;
        St.pb(0), St.pb(-1);
        St[0].fail=1, s.pb(-1);
    }
    inline int get_fail(int x){
        while(s[n-St[x].len-1]!=s[n])x=St[x].fail;
        return x;
    }
    inline void add(int c){
        s.push_back(c-'a'), ++n;
        int cur=get_fail(last);
        if(!St[cur].next[c]){
            int now=SZ(St);
            St.pb(St[cur].len+2);
            St[now].fail=St[get_fail(St[cur].fail)].next[c];
            St[cur].next[c]=now;
            St[now].num=St[St[now].fail].num+1;
        }
        last=St[cur].next[c], ++St[last].cnt;
    }
    inline void count(){// counting cnt
        auto i=St.rbegin();
        for(;i!=St.rend();++i){
            St[i->fail].cnt+=i->cnt;
        }
    }
    inline int size(){// The number of diff. pal.
        return SZ(St)-2;
    }
};

```

## 5.11 cyclicLCS

```

#define L 0
#define LU 1
#define U 2
const int mov[3][2]={0,-1, -1,-1, -1,0};
int al,bl;
char a[MAXL*2],b[MAXL*2]; // 0-indexed
int dp[MAXL*2][MAXL];
char pred[MAXL*2][MAXL];
inline int lcs_length(int r) {
    int i=r+al,j=bl,l=0;
    while(i>r) {

```

```

        char dir=pred[i][j];
        if(dir==LU) l++;
        i+=mov[dir][0];
        j+=mov[dir][1];
    }
    return l;
}
inline void reroot(int r) { // r = new base row
    int i=r,j=1;
    while(j<=bl&&pred[i][j]!=LU) j++;
    if(j>bl) return;
    pred[i][j]=L;
    while(i<2*al&&j<=bl) {
        if(pred[i+1][j]==U) {
            i++;
            pred[i][j]=L;
        } else if(j<bl&&pred[i+1][j+1]==LU) {
            i++;
            j++;
            pred[i][j]=L;
        } else {
            j++;
        }
    }
}
int cyclic_lcs() {
    // a, b, al, bl should be properly filled
    // note: a WILL be altered in process
    // -- concatenated after itself
    char tmp[MAXL];
    if(al>bl) {
        swap(al,bl);
        strcpy(tmp,a);
        strcpy(a,b);
        strcpy(b,tmp);
    }
    strcpy(tmp,a);
    strcat(a,tmp);
    // basic lcs
    for(int i=0;i<2*al;i++) {
        dp[i][0]=0;
        pred[i][0]=U;
    }
    for(int j=0;j<=bl;j++) {
        dp[0][j]=0;
        pred[0][j]=L;
    }
    for(int i=1;i<2*al;i++) {
        for(int j=1;j<=bl;j++) {
            if(a[i-1]==b[j-1]) dp[i][j]=dp[i-1][j-1]+1;
            else dp[i][j]=max(dp[i-1][j],dp[i][j-1]);
            if(dp[i][j-1]==dp[i][j]) pred[i][j]=L;
            else if(a[i-1]==b[j-1]) pred[i][j]=LU;
            else pred[i][j]=U;
        }
    }
    // do cyclic lcs
    int clcs=0;
    for(int i=0;i<al;i++) {
        clcs=max(clcs,lcs_length(i));
        reroot(i+1);
    }
    // recover a
    a[al]='\0';
    return clcs;
}

```

## 6 Math

### 6.1 ax+by=gcd

```

pii gcd(int a,int b){
    if(b==0) return pii(1,0);
    else{
        int p=a/b;
        pii q=gcd(b,a%b);
        return MP(q.Y,q.X-q.Y*p);
    }
}

```



## 6.2 floor and ceil

```
int floor(int a, int b){
    return a/b-(a%b&&a<0^b<0);
}
int ceil(int a, int b){
    return a/b+(a%b&&a<0^b>0);
}
```

## 6.3 Miller Rabin

```
// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633 4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383 6 : pirmes <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool Miller_Rabin(ll a, ll n){
    if((a=a%n)==0) return 1;
    if(n&1) return n==2;
    ll tmp=(n-1)/((n-1)&(-n+1)), t=__lg((n-1)&(-n+1)), x=1;
    for(; tmp; tmp/=2, a=mul(a, a, n))
        if(tmp&1) x=mul(x, a, n);
    if(x==1 || x==n-1) return 1;
    while(--t)
        if((x=mul(x, x, n))==n-1) return 1;
    return 0;
}
```

## 6.4 Big number

```
template<typename T>
inline string to_string(const T& x){
    stringstream ss;
    return ss<<x, ss.str();
}
struct bigN:vector<ll>{
    const static int base=1000000000, width=log10(base);
    bool negative;
    bigN(const_iterator a, const_iterator b):vector<ll>(a, b){}
    bigN(string s){
        if(s.empty())return;
        if(s[0]=='-')negative=1, s=s.substr(1);
        else negative=0;
        for(int i=int(s.size())-1; i>=0; i-=width){
            ll t=0;
            for(int j=max(0, i-width+1); j<=i; ++j)
                t=t*10+s[j]-'0';
            push_back(t);
        }
        trim();
    }
    template<typename T>
    bigN(const T& x):bigN(to_string(x)){}
    bigN():negative(0){}
    void trim(){
        while(size()&&!back())pop_back();
        if(empty())negative=0;
    }
    void carry(int _base=base){
        for(size_t i=0; i<size(); ++i){
            if(at(i)>=0&&at(i)<_base)continue;
            if(i+1u==size())push_back(0);
            int r=at(i)%_base;
            if(r<0)r+=_base;
            at(i+1)+=(at(i)-r)/_base, at(i)=r;
        }
    }
    int abscmp(const bigN& b) const{
        if(size()>b.size())return 1;
        if(size()<b.size())return -1;
        for(int i=int(size())-1; i>=0; --i){
            if(at(i)>b[i])return 1;
            if(at(i)<b[i])return -1;
        }
        return 0;
    }
}
```

```
int cmp(const bigN& b) const{
    if(negative!=b.negative)return negative?-1:1;
    return negative?-abscmp(b):abscmp(b);
}
bool operator<(const bigN& b) const{return cmp(b)<0;}
bool operator>(const bigN& b) const{return cmp(b)>0;}
bool operator<=(const bigN& b) const{return cmp(b)<=0;}
bool operator>=(const bigN& b) const{return cmp(b)>=0;}
bool operator==(const bigN& b) const{return !cmp(b);}
bool operator!=(const bigN& b) const{return cmp(b)!=0;}
bigN abs() const{
    bigN res=*this;
    return res.negative=0, res;
}
bigN operator-() const{
    bigN res=*this;
    return res.negative=!negative, res.trim(), res;
}
bigN operator+(const bigN& b) const{
    if(negative)return -(*this)+(-b);
    if(b.negative)return *this-(-b);
    bigN res=*this;
    if(b.size()>size())res.resize(b.size());
    for(size_t i=0; i<b.size(); ++i)res[i]+=b[i];
    return res.carry(), res.trim(), res;
}
bigN operator-(const bigN& b) const{
    if(negative)return -(*this)-(-b);
    if(b.negative)return *this+(-b);
    if(abscmp(b)<0)return -(b-(*this));
    bigN res=*this;
    if(b.size()>size())res.resize(b.size());
    for(size_t i=0; i<b.size(); ++i)res[i]-=b[i];
    return res.carry(), res.trim(), res;
}
bigN operator*(const bigN& b) const{
    bigN res;
    res.negative=negative!=b.negative;
    res.resize(size()+b.size());
    for(size_t i=0; i<size(); ++i)
        for(size_t j=0; j<b.size(); ++j)
            if((res[i+j]+at(i)*b[j])>=base){
                res[i+j+1]+=res[i+j]/base;
                res[i+j]%=base;
            }
    return res.trim(), res;
}
bigN operator/(const bigN& b) const{
    int norm=base/(b.back()+1);
    bigN x=abs()*norm;
    bigN y=b.abs()*norm;
    bigN q, r;
    q.resize(x.size());
    for(int i=int(x.size())-1; i>=0; --i){
        r=r*base+x[i];
        int s1=r.size()<=y.size()?0:r[y.size()];
        int s2=r.size()<y.size()?0:r[y.size()-1];
        int d=(ll(base)*s1+s2)/y.back();
        r=r-y*d;
        while(r.negative)r=r+y, --d;
        q[i]=d;
    }
    q.negative=negative!=b.negative;
    return q.trim(), q;
}
bigN operator%(const bigN& b) const{
    return *this-(*this/b)*b;
}
friend istream& operator>>(istream& ss, bigN& b){
    string s;
    return ss>>s, b=s, ss;
}
friend ostream& operator<<(ostream& ss, const bigN& b)
{
    if(b.negative)ss<<"-";
    ss<<(b.empty()?0:b.back());
    for(int i=int(b.size())-2; i>=0; --i)
        ss<<setw(width)<<setfill('0')<<b[i];
    return ss;
}
template<typename T>
operator T() {
```

```

stringstream ss;
ss<<*this;
T res;
return ss>>res,res;
}
};

```

## 6.5 Fraction

```

struct fraction{
    ll n,d;
    fraction(const ll &n=0,const ll &d=1):n(_n),d(_d){
        ll t=__gcd(n,d);
        n/=t,d/=t;
        if(d<0) n=-n,d=-d;
    }
    fraction operator-(const fraction &b)const{
        return fraction(-n,d);
    }
    fraction operator+(const fraction &b)const{
        return fraction(n*b.d+b.n*d,d*b.d);
    }
    fraction operator-(const fraction &b)const{
        return fraction(n*b.d-b.n*d,d*b.d);
    }
    fraction operator*(const fraction &b)const{
        return fraction(n*b.n,d*b.d);
    }
    fraction operator/(const fraction &b)const{
        return fraction(n*b.d,d*b.n);
    }
    void print(){
        cout << n;
        if(d!=1) cout << "/" << d;
    }
};

```

## 6.6 Simultaneous Equations

```

struct matrix { //m variables, n equations
    int n, m;
    fraction M[MAXN][MAXN + 1], sol[MAXN];
    int solve() { //-1: inconsistent, >= 0: rank
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m) continue;
            for (int j = 0; j < n; ++j) {
                if (i == j) continue;
                fraction tmp = -M[j][piv] / M[i][piv];
                for (int k = 0; k <= m; ++k) M[j][k] =
                    tmp * M[i][k] + M[j][k];
            }
        }
        int rank = 0;
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m && M[i][m].n) return -1;
            else if (piv < m) ++rank, sol[piv] = M[i][m]
                / M[i][piv];
        }
        return rank;
    }
};

```

## 6.7 Pollard Rho

```

// does not work when n is prime
ll f(ll x,ll mod){ return add(mul(x,x,mod),1,mod); }
ll pollard_rho(ll n){
    if(!(n&1)) return 2;
    while(1){
        ll y=2,x=rand()%(n-1)+1,res=1;
        for(int sz=2;res==1;y=x,sz*=2)
            for(int i=0;i<sz&&res<=1;++i)

```

```

        x=f(x,n),res=__gcd(abs(x-y),n);
        if(res!=0&&res!=n) return res;
    }
}

```

## 6.8 Simplex Algorithm

```

const int MAXN = 111;
const int MAXM = 111;
const double eps = 1E-10;
double a[MAXN][MAXM], b[MAXN], c[MAXN], d[MAXN][MAXM];
double x[MAXN];
int ix[MAXN + MAXM]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b,x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[MAXN][MAXM], double b[MAXN],
    double c[MAXN], int n, int m){
    ++m;
    int r = n, s = m - 1;
    memset(d, 0, sizeof(d));
    for (int i = 0; i < n + m; ++i) ix[i] = i;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m - 1; ++j) d[i][j] = -a[i][j];
        d[i][m - 1] = 1;
        d[i][m] = b[i];
        if (d[r][m] > d[i][m]) r = i;
    }
    for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];
    d[n + 1][m - 1] = -1;
    for (double dd;; ) {
        if (r < n) {
            int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
            d[r][s] = 1.0 / d[r][s];
            for (int j = 0; j <= m; ++j)
                if (j != s) d[r][j] *= -d[r][s];
            for (int i = 0; i <= n + 1; ++i) if (i != r) {
                for (int j = 0; j <= m; ++j) if (j != s)
                    d[i][j] += d[r][j] * d[i][s];
                d[i][s] *= d[r][s];
            }
        }
        r = -1; s = -1;
        for (int j = 0; j < m; ++j)
            if (s < 0 || ix[s] > ix[j]) {
                if (d[n + 1][j] > eps ||
                    (d[n + 1][j] > -eps && d[n][j] > eps))
                    s = j;
            }
        if (s < 0) break;
        for (int i = 0; i < n; ++i) if (d[i][s] < -eps) {
            if (r < 0 ||
                (dd = d[r][m] / d[r][s] - d[i][m] / d[i][s])
                    < -eps ||
                (dd < eps && ix[r + m] > ix[i + m]))
                r = i;
            if (r < 0) return -1; // not bounded
        }
        if (d[n + 1][m] < -eps) return -1; // not executable
        double ans = 0;
        for(int i=0; i<m; i++) x[i] = 0;
        for (int i = m; i < n + m; ++i) { // the missing
            enumerated x[i] = 0
            if (ix[i] < m - 1){
                ans += d[i - m][m] * c[ix[i]];
                x[ix[i]] = d[i - m][m];
            }
        }
        return ans;
    }
}

```

## 6.9 chineseRemainder

```

LL solve(LL x1, LL m1, LL x2, LL m2) {
    LL g = __gcd(m1, m2);

```

```

if((x2 - x1) % g) return -1; // no sol
m1 /= g; m2 /= g;
pair<LL,LL> p = gcd(m1, m2);
LL lcm = m1 * m2 * g;
LL res = p.first * (x2 - x1) * m1 + x1;
return (res % lcm + lcm) % lcm;
}

```

## 6.10 QuadraticResidue

```

int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
    return s;
}

int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    const int jc = Jacobi(a, p);
    if (jc == 0) return 0;
    if (jc == -1) return -1;
    int b, d;
    for (; ; ) {
        b = rand() % p;
        d = (1LL * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (1LL + p) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (1LL * g0 * f0 + 1LL * d * (1LL * g1
                * f1 % p)) % p;
            g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (1LL * f0 * f0 + 1LL * d * (1LL * f1 * f1
            % p)) % p;
        f1 = (2LL * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}

```

## 6.11 PiCount

```

int64_t PrimeCount(int64_t n) {
    if (n <= 1) return 0;
    const int v = sqrt(n);
    vector<int> smalls(v + 1);
    for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
    int s = (v + 1) / 2;
    vector<int> roughs(s);
    for (int i = 0; i < s; ++i) roughs[i] = 2 * i + 1;
    vector<int64_t> larges(s);
    for (int i = 0; i < s; ++i) larges[i] = (n / (2 * i
        + 1) + 1) / 2;
    vector<bool> skip(v + 1);
    int pc = 0;
    for (int p = 3; p <= v; ++p) {
        if (smalls[p] > smalls[p - 1]) {
            int q = p * p;
            pc++;
            if (1LL * q * q > n) break;
            skip[p] = true;
            for (int i = q; i <= v; i += 2 * p) skip[i]
                = true;
            int ns = 0;
            for (int k = 0; k < s; ++k) {
                int i = roughs[k];

```

```

                if (skip[i]) continue;
                int64_t d = 1LL * i * p;
                larges[ns] = larges[k] - (d <= v ?
                    larges[smalls[d] - pc] : smalls[n /
                        d]) + pc;
                roughs[ns++] = i;
            }
            s = ns;
            for (int j = v / p; j >= p; --j) {
                int c = smalls[j] - pc;
                for (int i = j * p, e = min(i + p, v +
                    1); i < e; ++i) smalls[i] -= c;
            }
        }
        for (int k = 1; k < s; ++k) {
            const int64_t m = n / roughs[k];
            int64_t s = larges[k] - (pc + k - 1);
            for (int l = 1; l < k; ++l) {
                int p = roughs[l];
                if (1LL * p * p > m) break;
                s -= smalls[m / p] - (pc + l - 1);
            }
            larges[0] -= s;
        }
        return larges[0];
    }
}

```

## 6.12 Algorithms about Primes

```

/*
12721 13331 14341 75577 123457 222557 556679 999983
1097774749 1076767633 100102021 999997771
1001010013 1000512343 987654361 999991231 999888733
98789101 987777733 999991921 1010101333 1010102101
1000000000039 100000000000037 2305843009213693951
4611686018427387847 9223372036854775783
18446744073709551557
*/

```

## 7 Polynomial

### 7.1 Fast Fourier Transform

```

template<int MAXN>
struct FFT {
    using val_t = complex<double>;
    const double PI = acos(-1);
    val_t w[MAXN];
    FFT() {
        for (int i = 0; i < MAXN; ++i) {
            double arg = 2 * PI * i / MAXN;
            w[i] = val_t(cos(arg), sin(arg));
        }
    }
    void bitrev(val_t *a, int n) {
        int i = 0;
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (i ^ k) < k; k >>= 1)
                if (j < i) swap(a[i], a[j]);
        }
    }
    void trans(val_t *a, int n, bool inv = false) {
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = MAXN / L;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + (L >> 1)
                    ; ++j, x += dx) {
                    val_t tmp = a[j + (L >> 1)] * w[x];
                    a[j + (L >> 1)] = a[j] - tmp;
                    a[j] += tmp;
                }
            }
        }
    }
}

```

```

    if (inv) {
        reverse(a + 1, a + n);
        for (int i = 0; i < n; ++i) a[i] /= n;
    }
};

```

## 7.2 Number Theory Transform

```

// (2^16)+1, 65537, 3
// 7*17*(2^23)+1, 998244353, 3
// 1255*(2^20)+1, 1315962881, 3
// 51*(2^25)+1, 1711276033, 29
template<int MAXN, LL P, LL RT> // MAXN must be 2^k
struct NTT {
    LL w[MAXN];
    LL mpow(LL a, LL n);
    LL minv(LL a) { return mpow(a, P - 2); }
    NTT() {
        LL dw = mpow(RT, (P - 1) / MAXN);
        w[0] = 1;
        for (int i = 1; i < MAXN; ++i) w[i] = w[i - 1] * dw % P;
    }
    void bitrev(LL *a, int n) {
        int i = 0;
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (i ^ k) < k; k >>= 1)
                if (j < i) swap(a[i], a[j]);
        }
    }
    void operator()(LL *a, int n, bool inv = false) {
        // 0 <= a[i] < P
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = MAXN / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; ++j, x += dx) {
                    LL tmp = a[j + dl] * w[x] % P;
                    if ((a[j + dl] = a[j] - tmp) < 0) a[j + dl] += P;
                    if ((a[j] += tmp) >= P) a[j] -= P;
                }
            }
        }
        if (inv) {
            reverse(a + 1, a + n);
            LL invn = minv(n);
            for (int i = 0; i < n; ++i) a[i] = a[i] * invn % P;
        }
    }
};

```

## 7.3 Fast Walsh Transform

```

/*
x: a[j], y: a[j + (L >> 1)]
or: (x, y + x), (x, y - x)
and: (x + y, y), (x - y, y)
xor: (x + y, x - y), (x + y, x - y)/2
*/
void fwt(val_t *a, int n) { // or
    for (int L = 2; L <= n; L <= 1) {
        for (int i = 0; i < n; i += L) {
            for (int j = i; j < i + (L >> 1); ++j) {
                a[j + (L >> 1)] += a[j]; // inv: a[j + (L >> 1)] -= a[j];
            }
        }
    }
}

```

## 7.4 Polynomial Operation

```

template<int MAXN, LL P, LL RT> // MAXN must be 2^k
struct PolyOp {
    NTT<MAXN, P, RT> ntt;
    const LL INV2 = ntt.minv(2);
    int get_sz(int n) {
        int sz = 1;
        while (sz < n) sz <= 1;
        return sz;
    }
    void mul(LL *a, int n, LL *b, int m, LL *c) {
        static LL buf1[MAXN], buf2[MAXN];
        int sz = get_sz(n + m - 1);
        copy(a, a + n, buf1), fill(buf1 + n, buf1 + sz, 0);
        copy(b, b + m, buf2), fill(buf2 + m, buf2 + sz, 0);
        ntt(buf1, sz), ntt(buf2, sz);
        for (int i = 0; i < sz; ++i) c[i] = buf1[i] * buf2[i] % P;
        ntt(c, sz, true);
    }
    void inv(LL *a, int n, LL *b) { // a[0] != 0
        static LL buf[MAXN];
        if (n == 1) return b[0] = ntt.minv(a[0]), void();
        inv(a, (n + 1) / 2, b);
        int sz = get_sz(n * 2);
        copy(a, a + n, buf), fill(buf + n, buf + sz, 0);
        fill(b + n, b + sz, 0);
        ntt(buf, sz), ntt(b, sz);
        for (int i = 0; i < sz; ++i) {
            b[i] *= (2 - b[i] * buf[i]) % P;
            if ((b[i] % P) < 0) b[i] += P;
        }
        ntt(b, sz, true), fill(b + n, b + sz, 0);
    }
    LL _msqrt(LL x) {
        for (LL i = 0; i <= P / 2; ++i) if (i * i % P == x) return i;
        throw string("BBQube");
    }
    void sqrt(LL *a, int n, LL *b) { // a[0] != 0 && sqrt(a[0]) exists
        static LL invb[MAXN], buf[MAXN];
        if (n == 1) return b[0] = _msqrt(a[0]), void();
        sqrt(a, (n + 1) / 2, b);
        int sz = get_sz(n * 2);
        inv(b, n, invb);
        copy(a, a + n, buf), fill(buf + n, buf + sz, 0);
        ntt(b, sz), ntt(invb, sz), ntt(buf, sz);
        for (int i = 0; i < sz; ++i) {
            if ((b[i] += buf[i] * invb[i] % P) >= P) b[i] -= P;
            b[i] = b[i] * INV2 % P;
        }
        ntt(b, sz, true), fill(b + n, b + sz, 0);
    }
    void div(LL *a, int n, LL *b, int m, LL *q, LL *r) {
        static LL invb[MAXN], buf[MAXN];
        if (n < m) {
            fill(q, q + m, 0), copy(a, a + n, r), fill(r + n, r + m, 0);
            return;
        }
        int mod_sz = n - m + 1;
        copy(b, b + m, buf), reverse(buf, buf + m);
        if (m < mod_sz) fill(buf + m, buf + mod_sz, 0);
        inv(buf, mod_sz, invb);
        copy(a, a + n, buf), reverse(buf, buf + n);
        mul(buf, mod_sz, invb, mod_sz, q);
        fill(q + mod_sz, q + n, 0), reverse(q, q + mod_sz);
        mul(b, m, q, mod_sz, buf);
        for (int i = 0; i < n; ++i) {
            if ((r[i] = a[i] - buf[i]) < 0) r[i] += P;
        }
    }
};

```

```
    }
};
```

## 8 Geometry

### 8.1 Default Code

```
typedef pair<double,double> pdd;
typedef pair<pdd,pdd> Line;
const double eps=1e-8;
pdd operator+(const pdd &a,const pdd &b)
{ return pdd(a.X+b.X,a.Y+b.Y);}
pdd operator-(const pdd &a,const pdd &b)
{ return pdd(a.X-b.X,a.Y-b.Y);}
pdd operator*(const pdd &a,const double &b)
{ return pdd(a.X*b,a.Y*b);}
pdd operator/(const pdd &a,const double &b)
{ return pdd(a.X/b,a.Y/b);}
double dot(const pdd &a,const pdd &b)
{ return a.X*b.X+a.Y*b.Y;}
double cross(const pdd &a,const pdd &b)
{ return a.X*b.Y-a.Y*b.X;}
double abs2(const pdd &a)
{ return dot(a,a);}
double abs(const pdd &a)
{ return sqrt(dot(a,a));}
int ori(const pdd &a,const pdd &b,const pdd &c){
    double res=cross(b-a,c-a);
    if(fabs(res)<eps) return 0;
    return res > 0 ? 1 : -1;
}
bool collinearity(const pdd &p1, const pdd &p2, const
    pdd &p3){
    return fabs(cross(p1-p3,p2-p3))<eps;
}
bool btw(const pdd &p1,const pdd &p2,const pdd &p3){
    if(!collinearity(p1,p2,p3)) return 0;
    return dot(p1-p3,p2-p3)<eps;
}
bool seg_intersect(const pdd &p1,const pdd &p2,const
    pdd &p3,const pdd &p4){
    int a123=ori(p1,p2,p3);
    int a124=ori(p1,p2,p4);
    int a341=ori(p3,p4,p1);
    int a342=ori(p3,p4,p2);
    if(a123==0 && a124==0)
        return btw(p1,p2,p3)||btw(p1,p2,p4)||btw(p3,p4,p1)
            ||btw(p3,p4,p2);
    return a123*a124<=0&&a341*a342<=0;
}
pdd intersect(const pdd &p1,const pdd &p2,const pdd &p3
    ,const pdd &p4){
    double a123=cross(p2-p1,p3-p1);
    double a124=cross(p2-p1,p4-p1);
    return (p4*a123-p3*a124)/(a123-a124);
}
pdd foot(const pdd &p1,const pdd &p2,const pdd &p3){
    pdd tmp=p2-p1;
    swap(tmp.X,tmp.Y),tmp.Y*=-1;
    return intersect(p1,p2,p3,p3+tmp);
}
```

### 8.2 Convex hull

```
struct convex_hull{
    vector<pdd> dots;
    void add_dot(double a,double b){
        dots.pb(pdd(a,b));
    }
    vector<pdd> hull(){
        vector<pdd> ans;
        sort(dots.begin(),dots.end());
        ans.pb(dots[0]),ans.pb(dots[1]);
        for(int i=2;i<SZ(dots);++i){
            while(SZ(ans)>=2)
                if(ori(ans[SZ(ans)-2],ans.back(),dots[i])<=0)
```

```
                ans.pop_back();
            else break;
            ans.pb(dots[i]);
        }
        for(int i=SZ(dots)-2,t=SZ(ans);i>=0;--i){
            while(SZ(ans)>t)
                if(ori(ans[SZ(ans)-2],ans.back(),dots[i])<=0)
                    ans.pop_back();
            else break;
            ans.pb(dots[i]);
        }
        ans.pop_back();
        return ans;
    }
};
```

### 8.3 External bisector

```
pdd external_bisector(pdd p1,pdd p2,pdd p3){//213
    pdd L1=p2-p1,L2=p3-p1;
    L2=L2*abs(L1)/abs(L2);
    return L1+L2;
}
```

### 8.4 Heart

```
pdd excenter(pdd p0,pdd p1,pdd p2,double &radius){
    p1=p1-p0,p2=p2-p0;
    double x1=p1.X,y1=p1.Y,x2=p2.X,y2=p2.Y;
    double m=2.*(x1*y2-y1*x2);
    center.X=(x1*x1*y2-x2*x2*y1+y1*y2*(y1-y2))/m;
    center.Y=(x1*x2*(x2-x1)-y1*y1*x2+x1*y2*y2)/m;
    return radius=abs(center),center+p0;
}
pdd incenter(pdd p1,pdd p2,pdd p3,double &radius){
    double a=abs(p2-p1),b=abs(p3-p1),c=abs(p3-p2);
    double s=(a+b+c)/2,area=sqrt(s*(s-a)*(s-b)*(s-c));
    pdd L1=external_bisector(p1,p2,p3),L2=
        external_bisector(p2,p1,p3);
    return radius=area/s,intersect(p1,p1+L1,p2,p2+L2),
}
pdd escenter(pdd p1,pdd p2,pdd p3){//213
    pdd L1=external_bisector(p1,p2,p3),L2=
        external_bisector(p2,p2+p2-p1,p3);
    return intersect(p1,p1+L1,p2,p2+L2);
}
pdd barycenter(pdd p1,pdd p2,pdd p3){
    return (p1+p2+p3)/3;
}
pdd orthocenter(pdd p1,pdd p2,pdd p3){
    pdd L1=p3-p2,L2=p3-p1;
    swap(L1.X,L1.Y),L1.X*=-1;
    swap(L2.X,L2.Y),L2.X*=-1;
    return intersect(p1,p1+L1,p2,p2+L2);
}
```

### 8.5 Polar Angle Sort

```
pdd c;//sort base
int Quadrant(pdd a){
    if(a.X>0&&a.Y>=0) return 1;
    if(a.X<=0&&a.Y>0) return 2;
    if(a.X<0&&a.Y<=0) return 3;
    if(a.X>=0&&a.Y<0) return 4;
}
bool cmp(pdd a,pdd b){
    a=a-c,b=b-c;
    if(Quadrant(a)!=Quadrant(b))
        return Quadrant(a)<Quadrant(b);
    if(cross(b,a)==0) return abs(a)<abs(b);
    return cross(b,a)>0;
}
```

```
bool cmp(pdd a,pdd b){
    a=a-c,b=b-c;
    if(fabs(atan2(a.Y,a.X)-atan2(b.Y,b.X))>eps)
        return atan2(a.Y,a.X)<atan2(b.Y,b.X);
    return abs(a)<abs(b);
}
```

## 8.6 Intersection of two circles

```
vector<pdd> interCircle(pdd o1 ,double r1 ,pdd o2 ,
    double r2){
    if(abs(o1-o2)<max(r1,r2)-min(r1,r2)) return {};
    double d2=abs2(o1-o2);
    double d=sqrt(d2);
    if(d>r1+r2) return {};
    pdd u = (o1+o2)*0.5 + (o1-o2)*((r2*r2-r1*r1)/(2*d2));
    double A = sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2
        +d));
    pdd v = pdd(o1.Y-o2.Y,-o1.X+o2.X)*A/(2*d2);
    return {u+v, u-v};
}
```

## 8.7 Intersection of polygon and circle

```
// Divides into multiple triangle, and sum up
// test by HDU2892
const double PI=acos(-1);
double _area(pdd pa, pdd pb, double r){
    if(abs(pa)<abs(pb)) swap(pa, pb);
    if(abs(pb)<eps) return 0;
    double S, h, theta;
    double a=abs(pb),b=abs(pa),c=abs(pb-pa);
    double cosB = dot(pb,pb-pa) / a / c, B = acos(cosB);
    double cosC = dot(pa,pb) / a / b, C = acos(cosC);
    if(a > r){
        S = (C/2)*r*r;
        h = a*b*sin(C)/c;
        if (h < r && B < PI/2) S -= (acos(h/r)*r*r - h*sqrt
            (r*r-h*h));
    }
    else if(b > r){
        theta = PI - B - asin(sin(B)/r*a);
        S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
    }
    else S = .5*sin(C)*a*b;
    return S;
}
double area_poly_circle(const vector<pdd> poly,const
    pdd &O,const double r){
    double S=0;
    for(int i=0;i<SZ(poly);++i)
        S+=_area(poly[i]-O,poly[(i+1)%SZ(poly)]-O,r)*ori(0,
            poly[i],poly[(i+1)%SZ(poly)]);
    return fabs(S);
}
```

## 8.8 Intersection of line and circle

```
vector<pdd> line_interCircle(const pdd &p1,const pdd &
    p2,const pdd &c,const double r){
    pdd ft=foot(p1,p2,c),vec=p2-p1;
    double dis=abs(c-ft);
    if(fabs(dis-r)<eps) return vector<pdd>{ft};
    if(dis>r) return {};
    vec=vec*sqrt(r*r-dis*dis)/abs(vec);
    return vector<pdd>{ft+vec,ft-vec};
}
```

## 8.9 Half plane intersection

```
bool isin( Line l0, Line l1, Line l2 ){
    // Check inter(l1, l2) in l0
    pdd p = intersect(l1.X,l1.Y,l2.X,l2.Y);
```

```
    return cross(l0.Y - l0.X,p - l0.X) > eps;
}
/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.Y - l.X) ^ (p - l.X) > 0
 */
/* --- Line.X --- Line.Y --- */
vector<Line> halfPlaneInter(vector<Line> lines){
    int sz = lines.size();
    vector<double> ata(sz),ord(sz);
    for(int i=0; i<sz; ++i) {
        ord[i] = i;
        pdd d = lines[i].Y - lines[i].X;
        ata[i] = atan2(d.Y, d.X);
    }
    sort(ord.begin(), ord.end(), [&](int i,int j){
        if( fabs(ata[i] - ata[j]) < eps )
            return (cross(lines[i].Y-lines[i].X,
                lines[j].Y-lines[j].X)<0;
        return ata[i] < ata[j];
    });
    vector<Line> fin;
    for (int i=0; i<sz; ++i)
        if (!i || fabs(ata[ord[i]] - ata[ord[i-1]]) > eps)
            fin.pb(lines[ord[i]]);
    deque<Line> dq;
    for (int i=0; i<SZ(fin); i++){
        while(SZ(dq)>=2&&!isin(fin[i],dq[SZ(dq)-2],dq.back
            ()))
            dq.pop_back();
        while(SZ(dq)>=2&&!isin(fin[i],dq[0],dq[1]))
            dq.pop_front();
        dq.push_back(fin[i]);
    }
    while(SZ(dq)>=3&&!isin(dq[0],dq[SZ(dq)-2],dq.back()))
        dq.pop_back();
    while(SZ(dq)>=3&&!isin(dq.back(), dq[0], dq[1]))
        dq.pop_front();
    vector<Line> res(ALL(dq));
    return res;
}
```

## 8.10 Convexhull3D

```
struct Point{
    double x,y,z;
    Point(double x=0,double y=0,double z=0):x(x),y(y),z(z)
    {}
    Point operator-(const Point p1){return Point(x-p1.x,y
        -p1.y,z-p1.z);}
    Point operator*(Point p){return Point(y*p.z-z*p.y,z*p
        .x-x*p.z,x*p.y-y*p.x);}
    double operator^(Point p){return (x*p.x+y*p.y+z*p.z)
        ;}
};
struct CH3D{
    struct face{int a,b,c;bool ok;}F[8*MAXN];
    int g[MAXN][MAXN],num,n;
    Point P[MAXN];
    double vlen(Point a){return sqrt(a.x*a.x+a.y*a.y+a.z*a.
        z);}
    Point cross(const Point &a, const Point &b, const Point
        &c){
        return Point((b.y-a.y)*(c.z-a.z)-(b.z-a.z)*(c.y-a.y)
            ,-(b.x-a.x)*(c.z-a.z)-(b.z-a.z)*(c.x-a.x),(b.x-
                a.x)*(c.y-a.y)-(b.y-a.y)*(c.x-a.x));
    }
    double area(Point a,Point b,Point c){return vlen((b-a)
        *(c-a));}
    double volume(Point a,Point b,Point c,Point d){return (
        b-a)*(c-a)^(d-a);}
    double dblcmp(Point &p,face &f){return ((P[f.b]-P[f.a])
        *(P[f.c]-P[f.a]))^(p-P[f.a]);}
    void deal(int p,int a,int b){
        int f=g[a][b];
        face add;
        if(F[f].ok)
            if(dblcmp(P[p],F[f])>EPS) dfs(p,f);
        else
```



```

        add.a=b,add.b=a,add.c=p,add.ok=1,g[p][b]=g[a][p]=
            g[b][a]=num,F[num++]=add;
    }
    void dfs(int p,int now){
        F[now].ok=0;
        deal(p,F[now].b,F[now].a),deal(p,F[now].c,F[now].b),
            deal(p,F[now].a,F[now].c);
    }
    bool same(int s,int t){
        Point &a=P[F[s].a];
        Point &b=P[F[s].b];
        Point &c=P[F[s].c];
        return fabs(volume(a,b,c,P[F[t].a]))<EPS && fabs(
            volume(a,b,c,P[F[t].b]))<EPS && fabs(volume(a,b,c,
                P[F[t].c]))<EPS;
    }
    void init(int _n){n=_n,num=0;}
    void solve(){
        face add;
        bool flag=true;
        num=0;
        if(n<4) return;
        if([&](){
            for(int i=1;i<n;++i)if(vlen(P[0]-P[i])>EPS)
                return swap(P[1],P[i]),0;return 1;}() ||
            [&](){
                for(int i=2;i<n;++i)if(vlen((P[0]-P[i])*(P[1]-P[
                    i]))>EPS)return swap(P[2],P[i]),0;return
                    1;}() || [&](){
                    for(int i=3;i<n;++i)if(fabs((P[0]-P[1])*(P[1]-P[
                        2])^(P[0]-P[i]))>EPS)return swap(P[3],P[i
                            ]),0;return 1;}())return;
                for(int i=0;i<4;++i){
                    add.a=(i+1)%4,add.b=(i+2)%4,add.c=(i+3)%4,add.ok=
                        true;
                    if(dblcmp(P[i],add)>0) swap(add.b,add.c);
                    g[add.a][add.b]=g[add.b][add.c]=g[add.c][add.a]=num
                        ;
                    F[num++]=add;
                }
                for(int i=4;i<n;++i)
                    for(int j=0;j<num;++j)
                        if(F[j].ok && dblcmp(P[i],F[j])>EPS){dfs(i,j);
                            break;}
                for(int tmp=num,i=(num=0);i<tmp;++i)
                    if(F[i].ok) F[num++]=F[i];
            }
        double area(){
            double res=0.0;
            if(n==3)
                return vlen(cross(P[0],P[1],P[2]))/2.0;
            for(int i=0;i<num;++i)
                res+=area(P[F[i].a],P[F[i].b],P[F[i].c]);
            return res/2.0;
        }
        double volume(){
            double res=0.0;
            for(int i=0;i<num;i++)
                res+=volume(Point(0,0,0),P[F[i].a],P[F[i].b],P[F[i]
                    ].c));
            return fabs(res/6.0);
        }
        int triangle(){return num;}
        int polygon(){
            int res=0;
            for(int i=0,flag=1;i<num;++i,res+=flag,flag=1)
                for(int j=0;j<i&&flag;++j)
                    flag&=!same(i,j);
            return res;
        }
        Point getcent(){
            Point ans(0,0,0),temp=P[F[0].a];
            double v = 0.0,t2;
            for(int i=0;i<num;++i)
                if(F[i].ok == true){
                    Point p1=P[F[i].a],p2=P[F[i].b],p3=P[F[i].c];
                    t2 = volume(temp,p1,p2,p3)/6.0;
                    if(t2>0)
                        ans.x += (p1.x+p2.x+p3.x+temp.x)*t2, ans.y += (
                            p1.y+p2.y+p3.y+temp.y)*t2, ans.z += (p1.z+
                                p2.z+p3.z+temp.z)*t2, v += t2;
                }
        }

```

```

        ans.x/=(4*v),ans.y/=(4*v),ans.z/=(4*v);
        return ans;
    }
    double pointmindis(Point fuck){
        double min=99999999;
        for(int i=0;i<num;i++)
            if(F[i].ok==true){
                Point p1=P[F[i].a] , p2=P[F[i].b] , p3=P[F[i].c];
                double a = ( (p2.y-p1.y)*(p3.z-p1.z)-(p2.z-p1.z)
                    *(p3.y-p1.y) );
                double b = ( (p2.z-p1.z)*(p3.x-p1.x)-(p2.x-p1.x)
                    *(p3.z-p1.z) );
                double c = ( (p2.x-p1.x)*(p3.y-p1.y)-(p2.y-p1.y)
                    *(p3.x-p1.x) );
                double d = ( 0-(a*p1.x+b*p1.y+c*p1.z) );
                double temp = fabs(a*fuck.x+b*fuck.y+c*fuck.z+d)/
                    sqrt(a*a+b*b+c*c);
                if(temp<min) min = temp;
            }
        return min;
    }
}hull;

```

## 8.11 CircleCover

```

#define N 1021
struct CircleCover{
    int C; Circ c[ N ];
    bool g[ N ][ N ], overlap[ N ][ N ];
    // Area[i] : area covered by at least i circles
    D Area[ N ];
    void init( int _C ){ C = _C; }
    bool CCinter( Circ& a , Circ& b , Pt& p1 , Pt& p2 ){
        Pt o1 = a.O , o2 = b.O;
        D r1 = a.R , r2 = b.R;
        if( norm( o1 - o2 ) > r1 + r2 ) return {};
        if( norm( o1 - o2 ) < max(r1, r2) - min(r1, r2) )
            return {};
        D d2 = ( o1 - o2 ) * ( o1 - o2 );
        D d = sqrt(d2);
        if( d > r1 + r2 ) return false;
        Pt u=(o1+o2)*0.5 + (o1-o2)*((r2*r2-r1*r1)/(2*d2));
        D A=sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2+d));
        Pt v=Pt( o1.Y-o2.Y , -o1.X + o2.X ) * A / (2*d2);
        p1 = u + v; p2 = u - v;
        return true;
    }
    struct Teve {
        Pt p; D ang; int add;
        Teve() {}
        Teve(Pt _a, D _b, int _c):p(_a), ang(_b), add(_c){}
        bool operator<(const Teve &a)const
            {return ang < a.ang;}
    }eve[ N * 2 ];
    // strict: x = 0, otherwise x = -1
    bool disjunct( Circ& a, Circ &b, int x )
    {return sign( norm( a.O - b.O ) - a.R - b.R ) > x;}
    bool contain( Circ& a, Circ &b, int x )
    {return sign( a.R - b.R - norm( a.O - b.O ) ) > x;}
    bool contain(int i, int j){
        /* c[j] is non-strictly in c[i]. */
        return (sign(c[i].R - c[j].R) > 0 ||
            (sign(c[i].R - c[j].R) == 0 && i < j) ) &&
            contain(c[i], c[j], -1);
    }
    void solve(){
        for( int i = 0 ; i <= C + 1 ; i ++ )
            Area[ i ] = 0;
        for( int i = 0 ; i < C ; i ++ )
            for( int j = 0 ; j < C ; j ++ )
                overlap[i][j] = contain(i, j);
        for( int i = 0 ; i < C ; i ++ )
            for( int j = 0 ; j < C ; j ++ )
                g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                    disjunct(c[i], c[j], -1));
        for( int i = 0 ; i < C ; i ++ ){
            int E = 0, cnt = 1;
            for( int j = 0 ; j < C ; j ++ )
                if( j != i && overlap[j][i] )
                    cnt ++;
        }
    }

```

```

for( int j = 0 ; j < C ; j ++ )
    if( i != j && g[i][j] ){
        Pt aa, bb;
        CCinter(c[i], c[j], aa, bb);
        D A=atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);
        D B=atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
        eve[E ++] = Teve(bb, B, 1);
        eve[E ++] = Teve(aa, A, -1);
        if(B > A) cnt ++;
    }
if( E == 0 ) Area[ cnt ] += pi * c[i].R * c[i].R;
else{
    sort( eve , eve + E );
    eve[E] = eve[0];
    for( int j = 0 ; j < E ; j ++ ){
        cnt += eve[j].add;
        Area[cnt] += (eve[j].p ^ eve[j + 1].p) * .5;
        D theta = eve[j + 1].ang - eve[j].ang;
        if (theta < 0) theta += 2. * pi;
        Area[cnt] +=
            (theta - sin(theta)) * c[i].R*c[i].R * .5;
    }
}
}
};

```

## 8.12 DelaunayTriangulation

```

/* Delaunay Triangulation:
   Given a sets of points on 2D plane, find a
   triangulation such that no points will strictly
   inside circumcircle of any triangle.
find : return a triangle contain given point
add_point : add a point into triangulation
A Triangle is in triangulation iff. its has_chd is 0.
Region of triangle u: iterate each u.edge[i].tri,
each points are u.p[(i+1)%3], u.p[(i+2)%3]
calculation involves  $O(|V|^6)$  */
const double inf = 1e9;
double eps = 1e-6; // 0 when integer
// return p4 is in circumcircle of tri(p1,p2,p3)
bool in_cc(P &p1, P &p2, P &p3, P &p4) {
    int o1 = (abs(p1.x) >= inf * 0.99 || abs(p1.y) >= inf * 0.99);
    int o2 = (abs(p2.x) >= inf * 0.99 || abs(p2.y) >= inf * 0.99);
    int o3 = (abs(p3.x) >= inf * 0.99 || abs(p3.y) >= inf * 0.99);
    int rtrue = o1 + o2 + o3;
    int rfalse = abs(p4.x) >= inf * 0.99 || abs(p4.y) >= inf * 0.99;
    if (rtrue == 3) return true;
    if (rtrue) {
        P in(0, 0), out(0, 0);
        if (o1) out = out + p1; else in = in + p1;
        if (o2) out = out + p2; else in = in + p2;
        if (o3) out = out + p3; else in = in + p3;
        return (p4 - in) * (out - in) > 0;
    }
    if (rfalse) return false;
    // ^ ?
    double u11 = p1.x - p4.x, u12 = p1.y - p4.y;
    double u21 = p2.x - p4.x, u22 = p2.y - p4.y;
    double u31 = p3.x - p4.x, u32 = p3.y - p4.y;
    double u13 = sq(p1.x) - sq(p4.x) + sq(p1.y) - sq(p4.y);
    double u23 = sq(p2.x) - sq(p4.x) + sq(p2.y) - sq(p4.y);
    double u33 = sq(p3.x) - sq(p4.x) + sq(p3.y) - sq(p4.y);
    double det = -u13 * u22 * u31 + u12 * u23 * u31 + u13 * u21 * u32 - u11 * u23 * u32 - u12 * u21 * u33 + u11 * u22 * u33;
    return det > eps;
}
double side(P &a, P &b, P &p) { return (b - a) ^ (p - a); }
struct Tri;
struct Edge {

```

```

    Tri *tri;
    int side;
    Edge() : tri(0), side(0) {}
    Edge(Tri *_tri, int _side) : tri(_tri), side(_side) {}
};
struct Tri {
    P p[3];
    Edge edge[3];
    Tri *ch[3];
    Tri() {}
    Tri(P p0, P p1, P p2) {
        p[0] = p0; p[1] = p1; p[2] = p2;
        ch[0] = ch[1] = ch[2] = 0;
    }
    bool has_ch() { return ch[0] != 0; }
    int num_ch() {
        return ch[0] == 0 ? 0 : ch[1] == 0 ? 1 : ch[2] == 0 ? 2 : 3;
    }
    bool contains(P &q) {
        for (int i = 0; i < 3; ++i)
            if (side(p[i], p[(i + 1) % 3], q) < -eps) return false;
        return true;
    }
} pool[maxn * 10], *tris;
void edge(Edge a, Edge b) {
    if (a.tri) a.tri->edge[a.side] = b;
    if (b.tri) b.tri->edge[b.side] = a;
}
struct Trig {
    Trig() {
        the_root = new (tris++) Tri(P(-inf, -inf), P(inf * 2, -inf), P(-inf, inf * 2));
    } // all p should in
    Tri *find(P p) { return find(the_root, p); }
    void add_point(P &p) { add_point(find(the_root, p), p); }
    Tri *the_root;
    static Tri *find(Tri *root, P &p) {
        while (true) {
            if (!root->has_ch()) return root;
            for (int i = 0; i < 3 && root->ch[i]; ++i)
                if (root->ch[i]->contains(p)) {
                    root = root->ch[i];
                    break;
                }
        }
        assert(false); // "point not found"
    }
    void add_point(Tri *root, P &p) {
        Tri *tab, *tbc, *tca;
        tab = new (tris++) Tri(root->p[0], root->p[1], p);
        tbc = new (tris++) Tri(root->p[1], root->p[2], p);
        tca = new (tris++) Tri(root->p[2], root->p[0], p);
        edge(Edge(tab, 0), Edge(tbc, 1));
        edge(Edge(tbc, 0), Edge(tca, 1));
        edge(Edge(tca, 0), Edge(tab, 1));
        edge(Edge(tab, 2), root->edge[2]);
        edge(Edge(tbc, 2), root->edge[0]);
        edge(Edge(tca, 2), root->edge[1]);
        root->ch[0] = tab; root->ch[1] = tbc; root->ch[2] = tca;
        flip(tab, 2); flip(tbc, 2); flip(tca, 2);
    }
    void flip(Tri *tri, int pi) {
        Tri *trj = tri->edge[pi].tri;
        int pj = tri->edge[pi].side;
        if (!trj) return;
        if (!in_cc(tri->p[0], tri->p[1], tri->p[2], trj->p[pj])) return;
        /* flip edge between tri, trj */
        Tri *trk = new (tris++) Tri(tri->p[(pi + 1) % 3], trj->p[pj], tri->p[pi]);
        Tri *trl = new (tris++) Tri(trj->p[(pj + 1) % 3], tri->p[pi], trj->p[pj]);
        edge(Edge(trk, 0), Edge(trl, 0));
        edge(Edge(trk, 1), tri->edge[(pi + 2) % 3]);
        edge(Edge(trk, 2), trj->edge[(pj + 1) % 3]);
        edge(Edge(trl, 1), trj->edge[(pj + 2) % 3]);
        edge(Edge(trl, 2), tri->edge[(pi + 1) % 3]);
    }
}

```

```

    tri->ch[0] = trk; tri->ch[1] = trl; tri->ch[2] = 0;
    trj->ch[0] = trk; trj->ch[1] = trl; trj->ch[2] = 0;
    flip(trk, 1); flip(trk, 2);
    flip(trl, 1); flip(trl, 2);
}
};
vector<Tri *> triang;
set<Tri *> vst;
void go(Tri *now) {
    if (vst.find(now) != vst.end()) return;
    vst.insert(now);
    if (!now->has_ch()) {
        triang.push_back(now);
        return;
    }
    for (int i = 0; i < now->num_ch(); ++i) go(now->ch[i]);
}
void build(int n, P *ps) {
    tris = pool;
    random_shuffle(ps, ps + n);
    Trig tri;
    for (int i = 0; i < n; ++i) tri.add_point(ps[i]);
    go(tri.the_root);
}

```

### 8.13 Triangulation Voronoi

```

int gid(P &p) {
    auto it = ptoid.find(p);
    if (it == ptoid.end()) return -1;
    return it->second;
}
L make_line(P p, L l) {
    P d = l.pb - l.pa; d = d.spin(pi / 2);
    P m = (l.pa + l.pb) / 2;
    l = L(m, m + d);
    if (((l.pb - l.pa) ^ (p - l.pa)) < 0) l = L(m + d, m);
    return l;
}
double calc_ans(int i) {
    vector<P> ps = HPI[ls[i]];
    double rt = 0;
    for (int i = 0; i < (int)ps.size(); ++i) {
        rt += (ps[i] ^ ps[(i + 1) % ps.size()]);
    }
    return abs(rt) / 2;
}
void solve() {
    for (int i = 0; i < n; ++i) ops[i] = ps[i], ptoid[ops[i]] = i;
    random_shuffle(ps, ps + n);
    build(n, ps);
    for (auto *t : triang) {
        int z[3] = {gid(t->p[0]), gid(t->p[1]), gid(t->p[2])};
        for (int i = 0; i < 3; ++i) for (int j = 0; j < 3; ++j) if (i != j && z[i] != -1 && z[j] != -1) {
            L l(t->p[i], t->p[j]);
            ls[z[i]].push_back(make_line(t->p[i], l));
        }
    }
    vector<P> tb = convex(vector<P>(ps, ps + n));
    for (auto &p : tb) isinf[gid(p)] = true;
    for (int i = 0; i < n; ++i) {
        if (isinf[i]) cout << -1 << '\n';
        else cout << fixed << setprecision(12) << calc_ans(i) << '\n';
    }
}

```

### 8.14 Tangent line of two circles

```

vector<Line> go( const Cir& c1 , const Cir& c2 , int sign1 ){
    // sign1 = 1 for outer tang, -1 for inter tang

```

```

vector<Line> ret;
double d_sq = norm2( c1.0 - c2.0 );
if( d_sq < eps ) return ret;
double d = sqrt( d_sq );
Pt v = ( c2.0 - c1.0 ) / d;
double c = ( c1.R - sign1 * c2.R ) / d;
if( c * c > 1 ) return ret;
double h = sqrt( max( 0.0 , 1.0 - c * c ) );
for( int sign2 = 1 ; sign2 >= -1 ; sign2 -= 2 ){
    Pt n = { v.X * c - sign2 * h * v.Y ,
            v.Y * c + sign2 * h * v.X };
    Pt p1 = c1.0 + n * c1.R;
    Pt p2 = c2.0 + n * ( c2.R * sign1 );
    if( fabs( p1.X - p2.X ) < eps and
        fabs( p1.Y - p2.Y ) < eps )
        p2 = p1 + perp( c2.0 - c1.0 );
    ret.push_back( { p1 , p2 } );
}
return ret;
}

```

### 8.15 minMaxEnclosingRectangle

```

pdd solve(vector<p11> &dots){
    vector<p11> hull;
    const double INF=1e18,qi=acos(-1)/2*3;
    cv.dots=dots;
    hull=cv.hull();
    double Max=0,Min=INF,deg;
    ll n=hull.size();
    hull.pb(hull[0]);
    for(int i=0,u=1,r=1,l;i<n;++i){
        p11 nw=hull[i+1]-hull[i];
        while(cross(nw,hull[u+1]-hull[i])>cross(nw,hull[u]-hull[i]))
            u=(u+1)%n;
        while(dot(nw,hull[r+1]-hull[i])>dot(nw,hull[r]-hull[i]))
            r=(r+1)%n;
        if(!i) l=(r+1)%n;
        while(dot(nw,hull[l+1]-hull[i])<dot(nw,hull[l]-hull[i]))
            l=(l+1)%n;
        Min=min(Min,(double)(dot(nw,hull[r]-hull[i])-dot(nw,hull[l]-hull[i]))*cross(nw,hull[u]-hull[i])/abs2(nw));
        deg=acos((double)dot(hull[r]-hull[l],hull[u]-hull[i])/abs(hull[r]-hull[l])/abs(hull[u]-hull[i]));
        deg=(qi-deg)/2;
        Max=max(Max,(double)abs(hull[r]-hull[l])*abs(hull[u]-hull[i])*sin(deg)*sin(deg));
    }
    return pdd(Min,Max);
}

```

### 8.16 minDistOfTwoConvex

```

// p, q is convex
double TwoConvexHullMinDist(Point P[], Point Q[], int n, int m) {
    int YMinP = 0, YMaxQ = 0;
    double tmp, ans = 999999999;
    for (i = 0; i < n; ++i) if (P[i].y < P[YMinP].y) YMinP = i;
    for (i = 0; i < m; ++i) if (Q[i].y > Q[YMaxQ].y) YMaxQ = i;
    P[n] = P[0], Q[m] = Q[0];
    for (int i = 0; i < n; ++i) {
        while (tmp = Cross(Q[YMaxQ + 1] - P[YMinP + 1], P[YMinP] - P[YMinP + 1]) > Cross(Q[YMaxQ] - P[YMinP + 1], P[YMinP] - P[YMinP + 1]))
            YMaxQ = (YMaxQ + 1) % m;
        if (tmp < 0) ans = min(ans, PointToSegDist(P[YMinP], P[YMinP + 1], Q[YMaxQ]));
        else ans = min(ans, TwoSegMinDist(P[YMinP], P[YMinP + 1], Q[YMaxQ], Q[YMaxQ + 1]));
        YMinP = (YMinP + 1) % n;
    }
}

```

```

    return ans;
}

```

## 8.17 Minkowski Sum

```

/* convex hull Minkowski Sum*/
#define INF 10000000000000LL
int pos( const Pt& tp ){
    if( tp.Y == 0 ) return tp.X > 0 ? 0 : 1;
    return tp.Y > 0 ? 0 : 1;
}
#define N 300030
Pt pt[ N ], qt[ N ], rt[ N ];
LL Lx,Rx;
int dn,un;
inline bool cmp( Pt a, Pt b ){
    int pa=pos( a ),pb=pos( b );
    if(pa==pb) return (a^b)>0;
    return pa<pb;
}
int minkowskiSum(int n,int m){
    int i,j,r,p,q,fi,fj;
    for(i=1,p=0;i<n;i++){
        if( pt[i].Y<pt[p].Y ||
            (pt[i].Y==pt[p].Y && pt[i].X<pt[p].X) ) p=i; }
    for(i=1,q=0;i<m;i++){
        if( qt[i].Y<qt[q].Y ||
            (qt[i].Y==qt[q].Y && qt[i].X<qt[q].X) ) q=i; }
    rt[0]=pt[p]+qt[q];
    r=1; i=p; j=q; fi=fj=0;
    while(1){
        if((fj&&j==q) ||
            ( (!fi||i!=p) &&
              cmp(pt[(p+1)%n]-pt[p],qt[(q+1)%m]-qt[q]) ) ){
            rt[r]=rt[r-1]+pt[(p+1)%n]-pt[p];
            p=(p+1)%n;
            fi=1;
        }else{
            rt[r]=rt[r-1]+qt[(q+1)%m]-qt[q];
            q=(q+1)%m;
            fj=1;
        }
        if(r<=1 || ((rt[r]-rt[r-1])^(rt[r-1]-rt[r-2]))!=0)
            r++;
        else rt[r-1]=rt[r];
        if(i==p && j==q) break;
    }
    return r-1;
}
void initInConvex(int n){
    int i,p,q;
    LL Ly,Ry;
    Lx=INF; Rx=-INF;
    for(i=0;i<n;i++){
        if(pt[i].X<Lx) Lx=pt[i].X;
        if(pt[i].X>Rx) Rx=pt[i].X;
    }
    Ly=Ry=INF;
    for(i=0;i<n;i++){
        if(pt[i].X==Lx && pt[i].Y<Ly){ Ly=pt[i].Y; p=i; }
        if(pt[i].X==Rx && pt[i].Y<Ry){ Ry=pt[i].Y; q=i; }
    }
    for(dn=0,i=p;i!=q;i=(i+1)%n){ qt[dn++]=pt[i]; }
    qt[dn]=pt[q]; Ly=Ry=-INF;
    for(i=0;i<n;i++){
        if(pt[i].X==Lx && pt[i].Y>Ly){ Ly=pt[i].Y; p=i; }
        if(pt[i].X==Rx && pt[i].Y>Ry){ Ry=pt[i].Y; q=i; }
    }
    for(un=0,i=p;i!=q;i=(i+n-1)%n){ rt[un++]=pt[i]; }
    rt[un]=pt[q];
}
inline int inConvex(Pt p){
    int L,R,M;
    if(p.X<Lx || p.X>Rx) return 0;
    L=0;R=dn;
    while(L<R-1){ M=(L+R)/2;
        if(p.X<qt[M].X) R=M; else L=M; }
    if(tri(qt[L],qt[R],p)<0) return 0;
    L=0;R=un;
    while(L<R-1){ M=(L+R)/2;

```

```

        if(p.X<rt[M].X) R=M; else L=M; }
    if(tri(rt[L],rt[R],p)>0) return 0;
    return 1;
}
int main(){
    int n,m,i;
    Pt p;
    scanf("%d",&n);
    for(i=0;i<n;i++) scanf("%Ld%Ld",&pt[i].X,&pt[i].Y);
    scanf("%d",&m);
    for(i=0;i<m;i++) scanf("%Ld%Ld",&qt[i].X,&qt[i].Y);
    n=minkowskiSum(n,m);
    for(i=0;i<n;i++) pt[i]=rt[i];
    scanf("%d",&m);
    for(i=0;i<m;i++) scanf("%Ld%Ld",&qt[i].X,&qt[i].Y);
    n=minkowskiSum(n,m);
    for(i=0;i<n;i++) pt[i]=rt[i];
    initInConvex(n);
    scanf("%d",&m);
    for(i=0;i<m;i++){
        scanf("%Ld %Ld",&p.X,&p.Y);
        p.X*=3; p.Y*=3;
        puts(inConvex(p)? "YES": "NO");
    }
}

```

## 9 Else

### 9.1 Mo's Alogrithm(With modification)

```

struct QUERY{//BLOCK=N^{2/3}
    int L,R,id,LBId,RBId,T;
    QUERY(int l,int r,int id,int lb,int rb,int t):
        L(l),R(r),id(id),LBId(lb),RBId(rb),T(t){}
    bool operator<(const QUERY &b)const{
        if(LBId!=b.LBId) return LBId<b.LBId;
        if(RBId!=b.RBId) return RBId<b.RBId;
        return T<b.T;
    }
};
vector<QUERY> query;
int cur_ans,arr[MAXN],ans[MAXN];
void addTime(int L,int R,int T){}
void subTime(int L,int R,int T){}
void add(int x){}
void sub(int x){}
void solve(){
    sort(ALL(query));
    int L=0,R=0,T=-1;
    for(auto q:query){
        while(T<q.T) addTime(L,R,++T);
        while(T>q.T) subTime(L,R,T--);
        while(R<q.R) add(arr[++R]);
        while(L>q.L) add(arr[--L]);
        while(R>q.R) sub(arr[R--]);
        while(L<q.L) sub(arr[L--]);
        ans[q.id]=cur_ans;
    }
}

```

### 9.2 Mo's Alogrithm On Tree

```

const int MAXN=40005;
vector<int> G[MAXN];//1-base
int n,B,arr[MAXN],ans[100005],cur_ans;
int in[MAXN],out[MAXN],dfn[MAXN*2],dft;
int deep[MAXN],sp[___lg(MAXN*2)+1][MAXN*2],bIn[MAXN],spt;
bitset<MAXN> inset;
struct QUERY{
    int L,R,Lid,id,lca;
    QUERY(int l,int r,int _id):L(l),R(r),lca(0),id(_id){}
    bool operator<(const QUERY &b){
        if(Lid!=b.Lid) return Lid<b.Lid;
        return R<b.R;
    }
}

```

```

};
vector<QUERY> query;
void dfs(int u, int f, int d){
    deep[u]=d, sp[0][spt]=u, bln[u]=spt++;
    dfn[dfn]=u, in[u]=dft++;
    for(int v:G[u])
        if(v!=f)
            dfs(v, u, d+1), sp[0][spt]=u, bln[u]=spt++;
    dfn[dfn]=u, out[u]=dft++;
}
int lca(int u, int v){
    if(bln[u]>bln[v]) swap(u, v);
    int t=__lg(bln[v]-bln[u]+1);
    int a=sp[t][bln[u]], b=sp[t][bln[v]-(1<<t)+1];
    if(deep[a]<deep[b]) return a;
    return b;
}
void sub(int x){}
void add(int x){}
void flip(int x){
    if(inset[x]) sub(arr[x]);
    else add(arr[x]);
    inset[x]=~inset[x];
}
void solve(){
    B=sqrt(2*n), dft=spt=cur_ans=0, dfs(1, 1, 0);
    for(int i=1, x=2; x<2*n; ++i, x<=1)
        for(int j=0; j+x<=2*n; ++j)
            if(deep[sp[i-1][j]]<deep[sp[i-1][j+x/2]])
                sp[i][j]=sp[i-1][j];
            else sp[i][j]=sp[i-1][j+x/2];
    for(auto &q:query){
        int c=lca(q.L, q.R);
        if(c==q.L || c==q.R)
            q.L=out[c==q.L?q.R:q.L], q.R=out[c];
        else if(out[q.L]<in[q.R])
            q.lca=c, q.L=out[q.L], q.R=in[q.R];
        else q.lca=c, c=in[q.L], q.L=out[q.R], q.R=c;
        q.Lid=q.L/B;
    }
    sort(ALL(query));
    int L=0, R=-1;
    for(auto q:query){
        while(R<q.R) flip(dfn[++R]);
        while(L>q.L) flip(dfn[--L]);
        while(R>q.R) flip(dfn[R--]);
        while(L<q.L) flip(dfn[L++]);
        if(q.lca) add(arr[q.lca]);
        ans[q.id]=cur_ans;
        if(q.lca) sub(arr[q.lca]);
    }
}
}

```

```

set<Line, cmp> st;
void ins(val a, val b){
    Flag = 0;
    Line L(a, b);
    auto it = st.lower_bound(L);
    if(it != st.begin() && it != st.end())
        if(!comp((*prev(it))(it->l - 1), L(it->l - 1)) && !comp((*it)(it->l), L(it->l)))
            return;
    while(it != st.end()){
        if(it->a == L.a && !comp(it->b, L.b)) return;
        ;
        if(comp((*it)(it->r), L(it->r))) it = st.erase(it);
    }
    else{
        Line M = *it;
        st.erase(it);
        L.r = max(idiv(L.b - M.b, M.a - L.a), minx);
        M.l = L.r + 1;
        it = st.insert(M).X;
        break;
    }
}
while(it != st.begin()){
    auto pit = prev(it);
    if(comp((*pit)(pit->l), L(pit->l))) st.erase(pit);
    else{
        Line M = *pit;
        st.erase(pit);
        M.r = min(idiv(L.b - M.b, M.a - L.a), maxx - 1);
        L.l = M.r + 1;
        st.insert(M);
        break;
    }
}
st.insert(L);
}
val operator () (val x){
    Flag = 1;
    auto it = st.lower_bound({0, 0, x, x});
    return (*it)(x);
}
}

```

DynamicConvexTrick<> DCT;

### 9.3 DynamicConvexTrick

```

// only works for integer coordinates!!
bool Flag; // 0: insert Line, 1: Lower_bound x
template<class val = ll, class compare = less<val>> //
    sort Lines with comp
struct DynamicConvexTrick{
    static const ll minx = 0, maxx = 1l(1e9) + 5;
    static compare comp;
    struct Line{
        val a, b, l, r; // Line ax + b in [l, r]
        Line(val _a, val _b, val _l = minx, val _r = maxx):a(_a), b(_b), l(_l), r(_r){}
        val operator () (val x) const {
            return a * x + b;
        }
    };
    struct cmp{
        bool operator () (const Line a, const Line b){
            if(Flag == 0) return comp(a.a, b.a);
            return a.r < b.l;
        }
    };
    inline val idiv(val a, val b){
        return a / b - (a % b && a < 0 ^ b < 0);
    }
}

```