Contents

1 Basic

1.1 Shell script

```
g++ -O2 -std=c++17 -Dbbq -Wall -Wextra -Wshadow -o $1
    $1.cpp
chmod +x compile.sh
```

1.2 Default code

```
#include<bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;
#define X first
#define Y second
#define SZ(a) ((int)a.size())
#define ALL(v) v.begin(), v.end()
#define pb push_back
```

1.3 vimrc

```
"This file should be placed at ~/.vimrc"
se nu ai hls et ru ic is sc cul
se re=1 ts=4 sts=4 sw=4 ls=2 mouse=a
syntax on
hi cursorline cterm=none ctermbg=89
set bg=dark
inoremap {<ENTER> {}<LEFT><ENTER><ENTER><UP><TAB>
```

1.4 readchar

```
inline char readchar() {
  static const size_t bufsize = 65536;
  static char buf[bufsize];
  static char *p = buf, *end = buf;
  if (p == end) end = buf + fread_unlocked(buf, 1,
      bufsize, stdin), p = buf;
  return *p++;
}
```

1.5 Black Magic

```
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp> //rb_tree
using namespace __gnu_pbds;
typedef __gnu_pbds::priority_queue<int> heap;
int main() {
  heap h1, h2;
  h1.push(1), h1.push(3);
  h2.push(2), h2.push(4);
  h1.join(h2);
  cout << h1.size() << h2.size() << h1.top() << endl;
      //404
  tree<ll, null_type, less<ll>, rb_tree_tag,
      tree_order_statistics_node_update> st;
  tree<ll, ll, less<ll>, rb_tree_tag,
      tree_order_statistics_node_update> mp;
  for (int x : {0, 2, 3, 4}) st.insert(x);
  cout << *st.find_by_order(2) << st.order_of_key(1) <<
       endl; //31
}
//__int128_t,__float128_t
```

1.6 Texas hold'em

```
char suit[4]={'C','D','H','Y'},ranks[13]={'2','3','4','
    5','6','7','8','9','T','J','Q','K','A'};
int rk[256];
/*
   for(int i=0;i<13;++i)
   rk[ranks[i]]=i;
   for(int i=0;i<4;++i)
   rk[suit[i]]=i;
   */
struct cards{
  vector<pii> v;
  int suit_count[4],hands;
  void reset(){v.clear(),FILL(suit_count,0),hands=-1;}
  void insert(char a,char b){//suit,rank
    ++suit_count[rk[a]];
    int flag=0;
    for(auto &i:v)
      if(i.Y==rk[b])
      {
        ++i.X,flag=1;
        break;
      }
    if(!flag) v.pb(pii(1,rk[b]));
  }
  void insert(string s){insert(s[0],s[1]);}
  void ready(){
    int Straight=0,Flush=(*max_element(suit_count,
        suit_count+4)==5);
    sort(ALL(v),[](ii a,ii b){return a>b;});
    if(SZ(v)==5&&v[0].Y==v[1].Y+1&&v[1].Y==v[2].Y+1&&v
        [2].Y==v[3].Y+1&&v[3].Y==v[4].Y+1)
      Straight=1;
    else if(SZ(v)==5&&v[0].Y==12&&v[1].Y==3&&v[2].Y
        ==2&&v[3].Y==1&&v[4].Y==0)
      v[0].Y=3,v[1].Y=2,v[2].Y=1,v[1].Y=0,v[0].Y=-1,
          Straight=1;
    if(Straight&&Flush) hands=1;
    else if(v[0].X==4) hands=2;
    else if(v[0].X==3&&v[1].X==2) hands=3;
    else if(Flush) hands=4;
    else if(Straight) hands=5;
    else if(v[0].X==3) hands=6;
    else if(v[0].X==2&&v[1].X==2) hands=7;
    else if(v[0].X==2) hands=8;
    else hands=9;
  }
  bool operator>(const cards &a)const{
    if(hands==a.hands) return v>a.v;
    return hands<a.hands;
  }
};
```

2 Graph

2.1 BCC Vertex*

```
vector<int> G[N]; //1-base
vector<int> nG[N], bcc[N];
int low[N], dfn[N], Time;
int bcc_id[N], bcc_cnt; //1-base
bool is_cut[N]; //whether is av
bool cir[N];
int st[N], top;

void dfs(int u, int pa = -1) {
  int child = 0;
  low[u] = dfn[u] = ++Time;
  st[top++] = u;
  for(int v : G[u])
    if(!dfn[v]) {
      dfs(v,u), ++child;
      low[u] = min(low[u], low[v]);
      if(dfn[u] <= low[v]) {
        is_cut[u]=1;
        bcc[++bcc_cnt].clear();
        int t;
        do {
          bcc_id[t = st[--top]] = bcc_cnt;
          bcc[bcc_cnt].push_back(t);
        }while(t != v);
        bcc_id[u]=bcc_cnt;
        bcc[bcc_cnt].pb(u);
      }
    }
    else if(dfn[v] < dfn[u] && v!=pa)
      low[u] = min(low[u], dfn[v]);
  if(pa == -1 && child < 2)
    is_cut[u] = 0;
}

void bcc_init(int n) {
  Time = bcc_cnt = top = 0;
  for(int i = 1; i <= n ; ++i)
    G[i].clear(), dfn[i] = bcc_id[i] = is_cut[i] = 0;
}

void bcc_solve(int n) {
  for (int i = 1; i <= n; ++i)
    if (!dfn[i])
      dfs(i);
  // circle-square tree
  for(int i = 1; i <= n; ++i)
    if(is_cut[i])
      bcc_id[i] = ++bcc_cnt, cir[bcc_cnt] = 1;
  for(int i = 1; i <= bcc_cnt && !cir[i]; ++i)
    for(int j : bcc[i])
      if(is_cut[j])
        nG[i].pb(bcc_id[j]), nG[bcc_id[j]].pb(i);
}
```

2.2 Bridge*

```
int low[N], dfn[N], Time;// 1-base
vector<pii> G[N], edge;
vector<bool> is_bridge;

void init(int n) {
  Time = 0;
  for (int i = 1; i <= n; ++i)
    G[i].clear(), low[i] = dfn[i] = 0;
}

void add_edge(int a, int b) {
  G[a].pb(pii(b, SZ(edge))), G[b].pb(pii(a, SZ(edge)));
  edge.pb(pii(a, b));
}

void dfs(int u, int f) {
  dfn[u] = low[u] = ++Time;
  for (auto i : G[u])
    if (!dfn[i.X])
      dfs(i.X, i.Y), low[u] = min(low[u], low[i.X]);
    else if (i.Y != f)
      low[u] = min(low[u], dfn[i.X]);
  if (low[u] == dfn[u] && f != -1)
    is_bridge[f] = 1;
}

void solve(int n) {
  is_bridge.resize(SZ(edge));
  for (int i = 1; i <= n; ++i)
    if (!dfn[i])
      dfs(i, -1);
}
```

2.3 Strongly Connected Components*

```
struct Strongly_CC{//1-base
  int low[N], dfn[N], bln[N], sz[N], n, Time, nScc;
  bitset<N> instack;
  stack<int> st;
  vector<int> G[N], SCC[N];
  void init(int _n) {
    n = _n;
```

```
    for(int i = 1; i <= n; ++i)
      G[i].clear();
  }
  void add_edge(int a, int b) {
    G[a].pb(b);
  }
  void dfs(int u) {
    dfn[u] = low[u] = ++Time;
    instack[u] = 1, st.push(u);
    for(int i : G[u])
      if(!dfn[i]) dfs(i), low[u] = min(low[i], low[u]);
      else if(instack[i] && dfn[i] < dfn[u])
        low[u] = min(low[u], dfn[i]);
    if(low[u] == dfn[u]) {
      int tmp;
      do {
        tmp = st.top(), st.pop();
        instack[tmp]=0, bln[tmp] = nScc;
      }while(tmp != u);
      ++nScc;
    }
  }
  void solve() {
    Time = nScc = 0;
    for(int i = 1; i <= n; ++i)
      SCC[i].clear(), low[i] = dfn[i] = bln[i] = sz[i]
          = 0;
    for(int i = 1; i <= n; ++i)
      if(!dfn[i])
        dfs(i);
    for(int i = 1; i <= n; ++i)
      ++sz[bln[i]], SCC[bln[i]].pb(i);
  }
};
```

2.4 MinimumMeanCycle*

```
ll road[N][N];//input here
struct MinimumMeanCycle{
  ll dp[N + 5][N], n;
  pll solve() {
    ll a = -1, b = -1, L = n+1;
    for(int i = 2; i <= L; ++i)
      for(int k = 0; k < n; ++k)
        for(int j = 0;j < n; ++j)
          dp[i][j] = min(dp[i - 1][k] + road[k][j], dp[
              i][j]);
    for(int i = 0; i < n; ++i) {
      if(dp[L][i] >= INF) continue;
      ll ta = 0, tb = 1;
      for(int j = 1; j < n; ++j)
        if(dp[j][i] < INF && ta * (L - j) < (dp[L][i] -
            dp[j][i]) * tb)
          ta = dp[L][i] - dp[j][i], tb= L - j;
      if(ta == 0) continue;
      if(a == -1 || a * tb > ta * b)
        a = ta, b = tb;
    }
    if(a != -1) {
      ll g = __gcd(a, b);
      return pll(a / g, b / g);
    }
    return pll(-1LL, -1LL);
  }
  void init(int _n){
    n = _n;
    for(int i = 0; i < n; ++i)
      for(int j = 0; j < n; ++j)
        dp[i + 2][j] = INF;
  }
};
```

2.5 Virtual Tree*

```
vector<int> vG[N];
int top, st[N];

void insert(int u) {
  if(top == -1)
    return st[++top] = u, void();
  int p = LCA(st[top], u);
  if(p == st[top])
    return st[++top] = u, void();
  while(top >= 1 && dep[st[top - 1]] >= dep[p])
    vG[st[top - 1]].pb(st[top]), --top;
  if(st[top] != p)
      vG[p].pb(st[top]), --top, st[++top] = p;
  st[++top] = u;
}

void reset(int u) {
  for(int i : vG[u])
    reset(i);
  vG[u].clear();
}

void solve(vector<int> &v) {
    top = -1;
    sort(ALL(v), [&](int a, int b){return dfn[a] < dfn[
        b];});
    for (int i : v)
        insert(i);
    while (top > 0)
        vG[st[top - 1]].pb(st[top]), --top;
    //do something
    reset(v[0]);
}
```

2.6 Maximum Clique Dyn*

```
const int N = 150;
struct MaxClique {  // Maximum Clique
  bitset<N> a[N], cs[N];
  int ans, sol[N], q, cur[N], d[N], n;
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; i++) a[i].reset();
  }
  void addEdge(int u, int v) { a[u][v] = a[v][u] = 1; }
  void csort(vector<int> &r, vector<int> &c) {
    int mx = 1, km = max(ans - q + 1, 1), t = 0, m = r.
        size();
    cs[1].reset(), cs[2].reset();
    for (int i = 0; i < m; i++) {
      int p = r[i], k = 1;
      while ((cs[k] & a[p]).count()) k++;
      if (k > mx) mx++, cs[mx + 1].reset();
      cs[k][p] = 1;
      if (k < km) r[t++] = p;
    }
    c.resize(m);
    if (t) c[t - 1] = 0;
    for (int k = km; k <= mx; k++)
      for (int p = cs[k]._Find_first(); p < N; p = cs[k
          ]._Find_next(p))
        r[t] = p, c[t] = k, t++;
  }
  void dfs(vector<int> &r, vector<int> &c, int l,
      bitset<N> mask) {
    while (!r.empty()) {
      int p = r.back();
      r.pop_back(), mask[p] = 0;
      if (q + c.back() <= ans) return;
      cur[q++] = p;
      vector<int> nr, nc;
      bitset<N> nmask = mask & a[p];
      for (int i : r)
        if (a[p][i]) nr.push_back(i);
      if (!nr.empty()) {
        if (l < 4) {
          for (int i : nr) d[i] = (a[i] & nmask).count
              ();
          sort(nr.begin(), nr.end(), [&](int x, int y)
              { return d[x] > d[y]; });
        }
        csort(nr, nc), dfs(nr, nc, l + 1, nmask);
      } else if (q > ans)
        ans = q, copy_n(cur, q, sol);
```

```
      c.pop_back(), q--;
    }
  }
  int solve(bitset<N> mask = bitset<N>(string(N, '1'))
      ) {  // vertex mask
    vector<int> r, c;
    ans = q = 0;
    for (int i = 0; i < n; i++)
      if (mask[i]) r.push_back(i);
    for (int i = 0; i < n; i++) d[i] = (a[i] & mask).
        count();
    sort(r.begin(), r.end(), [&](int i, int j) { return
         d[i] > d[j]; });
    csort(r, c), dfs(r, c, 1, mask);
    return ans;  // sol[0 ~ ans-1]
  }
} graph;
```

2.7 MinimumSteinerTree*

```
// Minimum Steiner Tree
// O(V 3^T + V^2 2^T)
struct SteinerTree{// 0-base
    static const int T = 10, N = 105, INF = 1e9;
    int n, dst[N][N], dp[1 << T][N], tdst[N];
    int vcost[N]; // the cost of vertexs
    void init(int _n){
        n = _n;
        for(int i = 0; i < n; ++i) {
            for(int j = 0; j < n; ++j)
                dst[i][j] = INF;
            dst[i][i] = vcost[i] = 0;
        }
    }
    void add_edge(int ui, int vi, int wi) {
        dst[ui][vi]=min(dst[ui][vi], wi);
    }
    void shortest_path() {
        for(int k = 0; k < n; ++k)
            for(int i = 0; i < n; ++i)
                for(int j = 0; j < n; ++j)
                    dst[i][j] = min(dst[i][j], dst[i][k
                        ] + dst[k][j]);
    }
    int solve(const vector<int>& ter) {
        shortest_path();
        int t = SZ(ter);
        for(int i = 0; i < (1 << t); ++i)
            for(int j = 0; j < n; ++j)
                dp[i][j] = INF;
        for(int i = 0; i < n; ++i)
            dp[0][i] = vcost[i];
        for(int msk = 1; msk < (1 << t); ++msk){
            if(!(msk & (msk - 1))){
                int who = __lg(msk);
                for(int i = 0; i < n; ++i)
                    dp[msk][i] = vcost[ter[who]] + dst[
                        ter[who]][i];
            }
            for(int i = 0; i < n; ++i)
                for(int submsk = (msk - 1) & msk;
                    submsk; submsk = (submsk - 1) & msk
                    )
                    dp[msk][i] = min(dp[msk][i], dp[
                        submsk][i] + dp[msk ^ submsk][i
                        ] - vcost[i]);
            for(int i = 0; i < n; ++i) {
                tdst[i] = INF;
                for(int j = 0; j < n; ++j)
                    tdst[i] = min(tdst[i], dp[msk][j] +
                         dst[j][i]);
            }
            for(int i = 0; i < n; ++i)
                dp[msk][i] = tdst[i];
        }
        int ans = INF;
        for(int i = 0; i < n; ++i)
            ans = min(ans, dp[(1 << t) - 1][i]);
        return ans;
    }
};
```

2.8 Dominator Tree*

```
struct dominator_tree{//1-base
    vector<int> G[N],rG[N];
    int n, pa[N], dfn[N], id[N], Time;
    int semi[N], idom[N], best[N];
    vector<int> tree[N];//dominator_tree
    void init(int _n) {
        n = _n;
        for(int i = 1; i <= n; ++i)
            G[i].clear(), rG[i].clear();
    }
    void add_edge(int u, int v) {
        G[u].pb(v), rG[v].pb(u);
    }
    void dfs(int u) {
        id[dfn[u] = ++Time] = u;
        for(auto v : G[u])
            if(!dfn[v])
                dfs(v), pa[dfn[v]] = dfn[u];
    }
    int find(int y, int x) {
        if(y <= x)
            return y;
        int tmp = find(pa[y], x);
        if(semi[best[y]] > semi[best[pa[y]]])
            best[y] = best[pa[y]];
        return pa[y] = tmp;
    }
    void tarjan(int root) {
        Time = 0;
        for(int i = 1; i <= n; ++i){
            dfn[i] = idom[i] = 0;
            tree[i].clear();
            best[i] = semi[i] = i;
        }
        dfs(root);
        for(int i = Time; i > 1; --i) {
            int u = id[i];
            for(auto v : rG[u])
                if(v = dfn[v]) {
                    find(v, i);
                    semi[i] = min(semi[i], semi[best[v
                        ]]);
                }
            tree[semi[i]].pb(i);
            for(auto v : tree[pa[i]]) {
                find(v, pa[i]);
                idom[v] = semi[best[v]] == pa[i] ? pa[i
                    ] : best[v];
            }
            tree[pa[i]].clear();
        }
        for(int i = 2; i <= Time; ++i) {
            if(idom[i] != semi[i])
                idom[i] = idom[idom[i]];
            tree[id[idom[i]]].pb(id[i]);
        }
    }
};
```

2.9 Minimum Arborescence*

```
struct zhu_liu{//O(VE)
    struct edge{
        int u,v;
        ll w;
    };
    vector<edge> E; //0-base
    int pe[N], id[N], vis[N];
    ll in[N];
    void init() {E.clear();}
    void add_edge(int u, int v, ll w) {
        if (u != v) E.pb(edge{u, v, w});
    }
    ll build(int root, int n) {
```

```
        ll ans = 0;
        for(;;) {
            fill_n(in, n, INF);
            for (int i = 0; i < SZ(E); ++i)
                if (E[i].u != E[i].v && E[i].w < in[E[i
                    ].v])
                    pe[E[i].v] = i, in[E[i].v] = E[i].w
                        ;
            for (int u = 0; u < n; ++u)//no solution
                if (u != root && in[u] == INF) return -
                    INF;
            int cntnode = 0;
            fill_n(id, n, -1), fill_n(vis, n, -1);
            for (int u = 0; u < n; ++u) {
                if (u != root) ans += in[u];
                int v = u;
                while (vis[v] != u && !~id[v] && v !=
                    root)
                    vis[v] = u, v = E[pe[v]].u;
                if (v != root && !~id[v]) {
                    for (int x = E[pe[v]].u; x != v; x
                        = E[pe[x]].u)
                        id[x] = cntnode;
                    id[v] = cntnode++;
                }
            }
            if (!cntnode) break;//no cycle
            for (int u = 0; u < n; ++u)
                if (!~id[u]) id[u] = cntnode++;
            for (int i = 0; i < SZ(E); ++i) {
                int v = E[i].v;
                E[i].u = id[E[i].u], E[i].v = id[E[i].v
                    ];
                if (E[i].u != E[i].v) E[i].w -= in[v];
            }
            n = cntnode, root = id[root];
        }
        return ans;
    }
};
```

2.10 Vizing's theorem

```
namespace vizing { // returns edge coloring in adjacent
     matrix G. 1 - based
  int C[kN][kN], G[kN][kN];
  void clear(int N) {
    for (int i = 0; i <= N; i++) {
      for (int j = 0; j <= N; j++) C[i][j] = G[i][j] =
          0;
    }
  }
  void solve(vector<pair<int, int>> &E, int N, int M) {
    int X[kN] = {}, a;
    auto update = [&](int u) {
      for (X[u] = 1; C[u][X[u]]; X[u]++);
    };
    auto color = [&](int u, int v, int c) {
      int p = G[u][v];
      G[u][v] = G[v][u] = c;
      C[u][c] = v, C[v][c] = u;
      C[u][p] = C[v][p] = 0;
      if (p) X[u] = X[v] = p;
      else update(u), update(v);
      return p;
    };
    auto flip = [&](int u, int c1, int c2) {
      int p = C[u][c1];
      swap(C[u][c1], C[u][c2]);
      if (p) G[u][p] = G[p][u] = c2;
      if (!C[u][c1]) X[u] = c1;
      if (!C[u][c2]) X[u] = c2;
      return p;
    };
    for (int i = 1; i <= N; i++) X[i] = 1;
    for (int t = 0; t < E.size(); t++) {
      int u = E[t].first, v0 = E[t].second, v = v0, c0
          = X[u], c = c0, d;
      vector<pair<int, int>> L;
      int vst[kN] = {};
      while (!G[u][v0]) {
        L.emplace_back(v, d = X[v]);
        if (!C[v][c]) for (a = (int)L.size() - 1; a >=
            0; a--) c = color(u, L[a].first, c);
        else if (!C[u][d]) for (a = (int)L.size() - 1;
            a >= 0; a--) color(u, L[a].first, L[a].
            second);
        else if (vst[d]) break;
        else vst[d] = 1, v = C[u][d];
      }
      if (!G[u][v0]) {
        for (; v; v = flip(v, c, d), swap(c, d));
        if (C[u][c0]) {
          for (a = (int)L.size() - 2; a >= 0 && L[a].
              second != c; a--);
          for (; a >= 0; a--) color(u, L[a].first, L[a
              ].second);
        } else t--;
      }
    }
  }}
```

2.11 Theory

|Maximum independent edge set| $= |V| -$ |Minimum edge cover|
|Maximum independent set| $= |V| -$ |Minimum vertex cover|
A sequence of non-negative integers $d_1 \geq \cdots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + \cdots + d_n$ is even and $\sum_{i-1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k)$ holds for every k in $1 \leq k \leq n$.

3 Data Structure

3.1 Leftist Tree

```
struct node{
  ll v,data,sz,sum;
  node *l,*r;
  node(ll k):v(0),data(k),sz(1),l(0),r(0),sum(k){}
};
ll sz(node *p){return p ? p->sz : 0;}
ll V(node *p){return p ? p->v : -1;}
ll sum(node *p){return p ? p->sum : 0;}
node* merge(node *a,node *b){
  if(!a || !b) return a ? a : b;
  if(a->data<b->data) swap(a,b);
  a->r=merge(a->r,b);
  if(V(a->r)>V(a->l)) swap(a->r,a->l);
  a->v=V(a->r)+1,a->sz=sz(a->l)+sz(a->r)+1;
  a->sum=sum(a->l)+sum(a->r)+a->data;
  return a;
}
void pop(node *&o){
  node *tmp=o;
  o=merge(o->l,o->r);
  delete tmp;
}
```

3.2 Heavy light Decomposition

```
struct Heavy_light_Decomposition{//1-base
  int n,ulink[10005],deep[10005],mxson[10005],w[10005],
      pa[10005];
  int t,pl[10005],data[10005],dt[10005],bln[10005],edge
      [10005],et;
  vector<pii> G[10005];
  void init(int _n){n=_n,t=0,et=1;
    for(int i=1;i<=n;++i) G[i].clear(),mxson[i]=0;
  }
  void add_edge(int a,int b,int w){
    G[a].pb(pii(b,et)),G[b].pb(pii(a,et)),edge[et++]=w;
  }
  void dfs(int u,int f,int d){
    w[u]=1,pa[u]=f,deep[u]=d++;
```

```
    for(auto &i:G[u])
      if(i.X!=f){
        dfs(i.X,u,d),w[u]+=w[i.X];
        if(w[mxson[u]]<w[i.X])
          mxson[u]=i.X;
      }
      else
        bln[i.Y]=u,dt[u]=edge[i.Y];
  }
  void cut(int u,int link){
    data[pl[u]=t++]=dt[u],ulink[u]=link;
    if(!mxson[u]) return ;
    cut(mxson[u],link);
    for(auto i:G[u])
      if(i.X!=pa[u]&&i.X!=mxson[u])
        cut(i.X,i.X);
  }
  void build(){
    dfs(1,1,1),cut(1,1),/*build*/;
  }
  int query(int a,int b){
    int ta=ulink[a],tb=ulink[b],re=0;
    while(ta!=tb)
      if(deep[ta]<deep[tb])
        /*query*/,tb=ulink[b=pa[tb]];
      else
        /*query*/,ta=ulink[a=pa[ta]];
    if(a==b) return re;
    if(pl[a]>pl[b]) swap(a,b);
    /*query*/
    return re;
  }
};
```

3.3 LiChaoST

```
struct LiChao_min{
  struct line{
    LL m, c;
    line(LL _m=0, LL _c=0) { m = _m; c = _c; }
    LL eval(LL x) { return m * x + c; }
  };
  struct node{
    node *l, *r; line f;
    node(line v) { f = v; l = r = NULL; }
  };
  typedef node* pnode;
  pnode root; int sz;
#define mid ((l+r)>>1)
  void insert(line &v, int l, int r, pnode &nd){
    if(!nd) { nd = new node(v); return; }
    LL trl = nd->f.eval(l), trr = nd->f.eval(r);
    LL vl = v.eval(l), vr = v.eval(r);
    if(trl <= vl && trr <= vr) return;
    if(trl > vl && trr > vr) { nd->f = v; return; }
    if(trl > vl) swap(nd->f, v);
    if(nd->f.eval(mid) < v.eval(mid)) insert(v, mid +
        1, r, nd->r);
    else swap(nd->f, v), insert(v, l, mid, nd->l);
  }
  LL query(int x, int l, int r, pnode &nd){
    if(!nd) return LLONG_MAX;
    if(l == r) return nd->f.eval(x);
    if(mid >= x) return min(nd->f.eval(x), query(x, l,
        mid, nd->l));
    return min(nd->f.eval(x), query(x, mid + 1, r, nd->
        r));
  }
  /* -sz <= query_x <= sz */
  void init(int _sz){ sz = _sz + 1; root = NULL; }
  void add_line(LL m, LL c){ line v(m, c); insert(v, -
      sz, sz, root); }
  LL query(LL x) { return query(x, -sz, sz, root); }
};
```

3.4 link cut tree

```
const int MXN = 100005;
const int MEM = 100005;
struct Splay {
  static Splay nil, mem[MEM], *pmem;
  Splay *ch[2], *f;
  int val, rev, size;
  Splay (int _val=-1) : val(_val), rev(0), size(1)
  { f = ch[0] = ch[1] = &nil; }
  bool isr()
  { return f->ch[0] != this && f->ch[1] != this; }
  int dir()
  { return f->ch[0] == this ? 0 : 1; }
  void setCh(Splay *c, int d){
    ch[d] = c;
    if (c != &nil) c->f = this;
    pull();
  }
  void push(){
    if( !rev ) return;
    swap(ch[0], ch[1]);
    if (ch[0] != &nil) ch[0]->rev ^= 1;
    if (ch[1] != &nil) ch[1]->rev ^= 1;
    rev=0;
  }
  void pull(){
    size = ch[0]->size + ch[1]->size + 1;
    if (ch[0] != &nil) ch[0]->f = this;
    if (ch[1] != &nil) ch[1]->f = this;
  }
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::
    mem;
Splay *nil = &Splay::nil;
void rotate(Splay *x){
  Splay *p = x->f;
  int d = x->dir();
  if (!p->isr()) p->f->setCh(x, p->dir());
  else x->f = p->f;
  p->setCh(x->ch[!d], d);
  x->setCh(p, !d);
  p->pull(); x->pull();
}
vector<Splay*> splayVec;
void splay(Splay *x){
  splayVec.clear();
  for (Splay *q=x;; q=q->f){
    splayVec.push_back(q);
    if (q->isr()) break;
  }
  reverse(begin(splayVec), end(splayVec));
  for (auto it : splayVec) it->push();
  while (!x->isr()) {
    if (x->f->isr()) rotate(x);
    else if (x->dir()==x->f->dir())
      rotate(x->f),rotate(x);
    else rotate(x),rotate(x);
  }
}
int id(Splay *x) { return x - Splay::mem + 1; }
Splay* access(Splay *x){
  Splay *q = nil;
  for (;x!=nil;x=x->f){
    splay(x);
    x->setCh(q, 1);
    q = x;
  }
  return q;
}
void chroot(Splay *x){
  access(x);
  splay(x);
  x->rev ^= 1;
  x->push(); x->pull();
}
void link(Splay *x, Splay *y){
  access(x);
  splay(x);
  chroot(y);
  x->setCh(y, 1);
}
void cut_p(Splay *y) {
  access(y);
  splay(y);
```

```
  y->push();
  y->ch[0] = y->ch[0]->f = nil;
}
void cut(Splay *x, Splay *y){
  chroot(x);
  cut_p(y);
}
Splay* get_root(Splay *x) {
  access(x);
  splay(x);
  for(; x->ch[0] != nil; x = x->ch[0])
    x->push();
  splay(x);
  return x;
}
bool conn(Splay *x, Splay *y) {
  x = get_root(x);
  y = get_root(y);
  return x == y;
}
Splay* lca(Splay *x, Splay *y) {
  access(x);
  access(y);
  splay(x);
  if (x->f == nil) return x;
  else return x->f;
}
```

3.5 KDTree

```
template<typename T,size_t kd>//kd???????
class kd_tree{
  public:
    struct point{
      T d[kd];
      inline T dist(const point &x)const{
        T ret=0;
        for(size_t i=0;i<kd;++i)ret+=std::abs(d[i]-x.d[
            i]);
        return ret;
      }
      inline bool operator==(const point &p){
        for(size_t i=0;i<kd;++i){
          if(d[i]!=p.d[i])return 0;
        }
        return 1;
      }
      inline bool operator<(const point &b)const{
        return d[0]<b.d[0];
      }
    };
  private:
    struct node{
      node *l,*r;
      point pid;
      int s;
      node(const point &p):l(0),r(0),pid(p),s(1){}
      inline void up(){
        s=(l?l->s:0)+1+(r?r->s:0);
      }
    }*root;
    const double alpha,loga;
    const T INF;//????INF,?????
    int maxn;
    struct __cmp{
      int sort_id;
      inline bool operator()(const node*x,const node*y)
          const{
        return operator()(x->pid,y->pid);
      }
      inline bool operator()(const point &x,const point
           &y)const{
        if(x.d[sort_id]!=y.d[sort_id])
          return x.d[sort_id]<y.d[sort_id];
        for(size_t i=0;i<kd;++i){
          if(x.d[i]!=y.d[i])return x.d[i]<y.d[i];
        }
        return 0;
      }
    }cmp;
    void clear(node *o){
      if(!o)return;
      clear(o->l);
      clear(o->r);
      delete o;
    }
    inline int size(node *o){
      return o?o->s:0;
    }
    std::vector<node*> A;
    node* build(int k,int l,int r){
      if(l>r)return 0;
      if(k==kd)k=0;
      int mid=(l+r)/2;
      cmp.sort_id=k;
      std::nth_element(A.begin()+l,A.begin()+mid,A.
          begin()+r+1,cmp);
      node *ret=A[mid];
      ret->l=build(k+1,l,mid-1);
      ret->r=build(k+1,mid+1,r);
      ret->up();
      return ret;
    }
    inline bool isbad(node*o){
      return size(o->l)>alpha*o->s||size(o->r)>alpha*o
          ->s;
    }
    void flatten(node *u,typename std::vector<node*>::
        iterator &it){
      if(!u)return;
      flatten(u->l,it);
      *it=u;
      flatten(u->r,++it);
    }
    inline void rebuild(node*&u,int k){
      if((int)A.size()<u->s)A.resize(u->s);
      typename std::vector<node*>::iterator it=A.begin
          ();
      flatten(u,it);
      u=build(k,0,u->s-1);
    }
    bool insert(node*&u,int k,const point &x,int dep){
      if(!u){
        u=new node(x);
        return dep<=0;
      }
      ++u->s;
      cmp.sort_id=k;
      if(insert(cmp(x,u->pid)?u->l:u->r,(k+1)%kd,x,dep
          -1)){
        if(!isbad(u))return 1;
        rebuild(u,k);
      }
      return 0;
    }
    node *findmin(node*o,int k){
      if(!o)return 0;
      if(cmp.sort_id==k)return o->l?findmin(o->l,(k+1)%
          kd):o;
      node *l=findmin(o->l,(k+1)%kd);
      node *r=findmin(o->r,(k+1)%kd);
      if(l&&!r)return cmp(l,o)?l:o;
      if(!l&&r)return cmp(r,o)?r:o;
      if(!l&&!r)return o;
      if(cmp(l,r))return cmp(l,o)?l:o;
      return cmp(r,o)?r:o;
    }
    bool erase(node *&u,int k,const point &x){
      if(!u)return 0;
      if(u->pid==x){
        if(u->r);
        else if(u->l){
          u->r=u->l;
          u->l=0;
        }else{
          delete u;
          u=0;
          return 1;
        }
        --u->s;
        cmp.sort_id=k;
        u->pid=findmin(u->r,(k+1)%kd)->pid;
```

```
      return erase(u->r,(k+1)%kd,u->pid);
    }
    cmp.sort_id=k;
    if(erase(cmp(x,u->pid)?u->l:u->r,(k+1)%kd,x)){
      --u->s;return 1;
    }else return 0;
  }
  inline T heuristic(const T h[])const{
    T ret=0;
    for(size_t i=0;i<kd;++i)ret+=h[i];
    return ret;
  }
  int qM;
  std::priority_queue<std::pair<T,point > >pQ;
  void nearest(node *u,int k,const point &x,T *h,T &
      mndist){
    if(u==0||heuristic(h)>=mndist)return;
    T dist=u->pid.dist(x),old=h[k];
    /*mndist=std::min(mndist,dist);*/
    if(dist<mndist){
      pQ.push(std::make_pair(dist,u->pid));
      if((int)pQ.size()==qM+1){
        mndist=pQ.top().first,pQ.pop();
      }
    }
    if(x.d[k]<u->pid.d[k]){
      nearest(u->l,(k+1)%kd,x,h,mndist);
      h[k]=std::abs(x.d[k]-u->pid.d[k]);
      nearest(u->r,(k+1)%kd,x,h,mndist);
    }else{
      nearest(u->r,(k+1)%kd,x,h,mndist);
      h[k]=std::abs(x.d[k]-u->pid.d[k]);
      nearest(u->l,(k+1)%kd,x,h,mndist);
    }
    h[k]=old;
  }
  std::vector<point>in_range;
  void range(node *u,int k,const point&mi,const point
      &ma){
    if(!u)return;
    bool is=1;
    for(int i=0;i<kd;++i)
      if(u->pid.d[i]<mi.d[i]||ma.d[i]<u->pid.d[i]){
        is=0;break;
      }
    if(is)in_range.push_back(u->pid);
    if(mi.d[k]<=u->pid.d[k])range(u->l,(k+1)%kd,mi,ma
        );
    if(ma.d[k]>=u->pid.d[k])range(u->r,(k+1)%kd,mi,ma
        );
  }
public:
  kd_tree(const T &INF,double a=0.75):root(0),alpha(a
      ),loga(log2(1.0/a)),INF(INF),maxn(1){}
  inline void clear(){
    clear(root),root=0,maxn=1;
  }
  inline void build(int n,const point *p){
    clear(root),A.resize(maxn=n);
    for(int i=0;i<n;++i)A[i]=new node(p[i]);
    root=build(0,0,n-1);
  }
  inline void insert(const point &x){
    insert(root,0,x,std::__lg(size(root))/loga);
    if(root->s>maxn)maxn=root->s;
  }
  inline bool erase(const point &p){
    bool d=erase(root,0,p);
    if(root&&root->s<alpha*maxn)rebuild();
    return d;
  }
  inline void rebuild(){
    if(root)rebuild(root,0);
    maxn=root->s;
  }
  inline T nearest(const point &x,int k){
    qM=k;
    T mndist=INF,h[kd]={};
    nearest(root,0,x,h,mndist);
    mndist=pQ.top().first;
    pQ=std::priority_queue<std::pair<T,point > >();
    return mndist;/*???x?k??????*/
  }
  inline const std::vector<point> &range(const point&
      mi,const point&ma){
    in_range.clear();
    range(root,0,mi,ma);
    return in_range;/*????mi?ma????vector*/
  }
  inline int size(){return root?root->s:0;}
};
```

4 Flow/Matching

4.1 Kuhn Munkres

```
struct KM{// 0-base
  int w[MAXN][MAXN],hl[MAXN],hr[MAXN],slk[MAXN],n;
  int fl[MAXN],fr[MAXN],pre[MAXN],qu[MAXN],ql,qr;
  bool vl[MAXN],vr[MAXN];
  void init(int _n){n=_n;
    for(int i=0;i<n;++i)
      for(int j=0;j<n;++j)
        w[i][j]=-INF;
  }
  void add_edge(int a,int b,int wei){
    w[a][b]=wei;
  }
  bool Check(int x){
    if(vl[x]=1,~fl[x]) return vr[qu[qr++]=fl[x]]=1;
    while(~x) swap(x,fr[fl[x]=pre[x]]);
    return 0;
  }
  void Bfs(int s){
    fill(slk,slk+n,INF);
    fill(vl,vl+n,0),fill(vr,vr+n,0);
    ql=qr=0,qu[qr++]=s,vr[s]=1;
    while(1){
      int d;
      while(ql<qr)
        for(int x=0,y=qu[ql++];x<n;++x)
          if(!vl[x]&&slk[x]>=(d=hl[x]+hr[y]-w[x][y]))
            if(pre[x]=y,d) slk[x]=d;
            else if(!Check(x)) return;
      d=INF;
      for (int x=0;x<n;++x)
        if (!vl[x]&&d>slk[x]) d=slk[x];
      for (int x=0;x<n;++x){
        if(vl[x]) hl[x]+=d;
        else slk[x]-=d;
        if(vr[x]) hr[x]-=d;
      }
      for (int x=0;x<n;++x)
        if(!vl[x]&&!slk[x]&&!Check(x)) return;
    }
  }
  int Solve(){
    fill(fl,fl+n,-1),fill(fr,fr+n,-1),fill(hr,hr+n, 0);
    for (int i=0;i<n;++i) hl[i]=*max_element(w[i],w[i]+
        n);
    for (int i=0;i<n;++i) Bfs(i);
    int res=0;
    for (int i=0;i<n;++i) res += w[i][fl[i]];
    return res;
  }
};
```

4.2 MincostMaxflow

```
struct MCMF{//0-base
  struct edge{
    ll from,to,cap,flow,cost,rev;
  }*past[MAXN];
  vector<edge> G[MAXN];
  bitset<MAXN> inq;
  ll dis[MAXN],up[MAXN],s,t,mx,n;
  bool BellmanFord(ll &flow,ll &cost){
    fill(dis,dis+n,INF);
```

```
    queue<ll> q;
    q.push(s),inq.reset(),inq[s]=1;
    up[s]=mx-flow,past[s]=0,dis[s]=0;
    while(!q.empty()){
      ll u=q.front();
      q.pop(),inq[u]=0;
      if(!up[u]) continue;
      for(auto &e:G[u])
        if(e.flow!=e.cap&&dis[e.to]>dis[u]+e.cost){
          dis[e.to]=dis[u]+e.cost,past[e.to]=&e;
          up[e.to]=min(up[u],e.cap-e.flow);
          if(!inq[e.to]) inq[e.to]=1,q.push(e.to);
        }
    }
    if(dis[t]==INF) return 0;
    flow+=up[t],cost+=up[t]*dis[t];
    for(ll i=t;past[i];i=past[i]->from){
      auto &e=*past[i];
      e.flow+=up[t],G[e.to][e.rev].flow-=up[t];
    }
    return 1;
  }
  ll MinCostMaxFlow(ll _s,ll _t,ll &cost){
    s=_s,t=_t,cost=0;ll flow=0;
    while(BellmanFord(flow,cost));
    return flow;
  }
  void init(ll _n,ll _mx){n=_n,mx=_mx;
    for(int i=0;i<n;++i) G[i].clear();
  }
  void add_edge(ll a,ll b,ll cap,ll cost){
    G[a].pb(edge{a,b,cap,0,cost,G[b].size()});
    G[b].pb(edge{b,a,0,0,-cost,G[a].size()-1});
  }
};
```

4.3 Maximum Simple Graph Matching*

```
struct GenMatch {//1-base
  int V,pr[MAXN];
  bool el[MAXN][MAXN];
  bool inq[MAXN],inp[MAXN],inb[MAXN];
  queue<int> qe;
  int st,ed,nb,bk[MAXN],djs[MAXN],ans;
  void init(int _V){V=_V;
    for(int i=0;i<=V;++i){
      for(int j=0;j<=V;++j)
        el[i][j]=0;
      pr[i]=bk[i]=djs[i]=0;
      inq[i]=inp[i]=inb[i]=0;
    }
    ans=0;
  }
  void add_edge(int u, int v){
    el[u][v]=el[v][u]=1;
  }
  int lca(int u,int v){
    for(int i=0;i<=V;++i) inp[i]=0;
    while(1)
      if(u=djs[u],inp[u]=true,u==st) break;
      else u=bk[pr[u]];
    while(1)
      if(v=djs[v],inp[v]) return v;
      else v=bk[pr[v]];
    return v;
  }
  void upd(int u){
    int v;
    while(djs[u]!=nb){
      v=pr[u],inb[djs[u]]=inb[djs[v]]=true;
      u=bk[v];
      if(djs[u]!=nb) bk[u]=v;
    }
  }
  void blo(int u,int v){
    nb=lca(u,v);
    for (int i=0;i<=V;++i) inb[i]=0;
    upd(u),upd(v);
    if(djs[u]!=nb) bk[u]=v;
    if(djs[v]!=nb) bk[v]=u;
    for(int tu=1;tu<=V;++tu)
      if(inb[djs[tu]])
        if(djs[tu]=nb,!inq[tu])
          qe.push(tu),inq[tu]=1;
  }
  void flow(){
    for(int i=1;i<=V;++i)
      inq[i]=0,bk[i]=0,djs[i]=i;
    while(qe.size()) qe.pop();
    qe.push(st),inq[st]=1,ed=0;
    while(qe.size()){
      int u = qe.front();
      qe.pop();
      for(int v=1;v<=V;++v)
        if(el[u][v]&&djs[u]!=djs[v]&&pr[u]!=v)
          if((v==st)||pr[v]>0&&bk[pr[v]]>0)
            blo(u,v);
          else if(!bk[v])
            if(bk[v]=u,pr[v]>0)
              if(!inq[pr[v]])
                qe.push(pr[v]);
              else;
            else
              return ed=v,void();
    }
  }
  void aug(){
    for(int u=ed,v,w;u>0;)
      v=bk[u],w=pr[v],pr[v]=u,pr[u]=v,u=w;
  }
  int solve(){
    for(int i=0;i<=V;++i) pr[i]=0;
    for(int u=1;u<=V;++u)
      if(!pr[u])
        if(st=u,flow(),ed>0)
          aug(),++ans;
    return ans;
  }
};
```

4.4 Minimum Weight Matching (Clique version)

```
struct Graph{//0-base (Perfect Match)
  int n,edge[MAXN][MAXN];
  int match[MAXN],dis[MAXN],onstk[MAXN];
  vector<int> stk;
  void init(int _n){n=_n;
    for(int i=0;i<n;++i)
      for(int j=0;j<n;++j)
        edge[i][j]=0;
  }
  void add_edge(int u,int v,int w){
    edge[u][v]=edge[v][u]=w;
  }
  bool SPFA(int u){
    if(onstk[u]) return 1;
    stk.pb(u),onstk[u]=1;
    for(int v=0;v<n;++v)
      if(u!=v&&match[u]!=v&&!onstk[v]){
        int m=match[v];
        if(dis[m]>dis[u]-edge[v][m]+edge[u][v]){
          dis[m]=dis[u]-edge[v][m]+edge[u][v];
          onstk[v]=1,stk.pb(v);
          if(SPFA(m)) return 1;
          stk.pop_back(),onstk[v]=0;
        }
      }
    onstk[u]=0,stk.pop_back();
    return 0;
  }
  int solve(){// find a match
    for(int i=0;i<n;i+=2)
      match[i]=i+1,match[i+1]=i;
    while(1){
      int found=0;
      for(int i=0;i<n;++i) dis[i]=onstk[i]=0;
      for(int i=0;i<n;++i)
        if(stk.clear(),!onstk[i]&&SPFA(i))
```

```
          for(found=1;stk.size()>=2;){
            int u=stk.back();
            stk.pop_back();
            int v=stk.back();
            stk.pop_back();
            match[u]=v,match[v]=u;
          }
      if(!found) break;
    }
    int ret=0;
    for(int i=0;i<n;++i) ret+=edge[i][match[i]];
    return ret>>1;
  }
};
```

4.5 SW-mincut

```
// global min cut
struct SW{ // O(V^3)
  static const int MXN = 514;
  int n,vst[MXN],del[MXN];
  int edge[MXN][MXN],wei[MXN];
  void init(int _n){
    n=_n,MEM(edge,0),MEM(del,0);
  }
  void addEdge(int u,int v,int w){
    edge[u][v]+=w,edge[v][u]+=w;
  }
  void search(int &s,int &t){
    MEM(vst,0),MEM(wei,0),s=t=-1;
    while(1){
      int mx=-1,cur=0;
      for(int i=0;i<n;++i)
        if(!del[i]&&!vst[i]&&mx<wei[i])
          cur=i,mx=wei[i];
      if(mx==-1) break;
      vst[cur]=1,s=t,t=cur;
      for(int i=0;i<n;++i)
        if(!vst[i]&&!del[i]) wei[i]+=edge[cur][i];
    }
  }
  int solve(){
    int res=INF;
    for(int i=0,x,y;i<n-1;++i){
      search(x,y),res=min(res,wei[y]),del[y]=1;
      for(int j=0;j<n;++j)
        edge[x][j]=(edge[j][x]+=edge[y][j]);
    }
    return res;
  }
};
```

4.6 BoundedFlow(Dinic*)

```
struct BoundedFlow {//0-base
    struct edge {
        int to, cap, flow, rev;
    };
    vector<edge> G[N];
    int n, s, t, dis[N], cur[N], cnt[N];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n + 2; ++i)
            G[i].clear(), cnt[i] = 0;
    }
    void add_edge(int u, int v, int lcap, int rcap) {
        cnt[u] -= lcap, cnt[v] += lcap;
        G[u].pb(edge{v, rcap, lcap, SZ(G[v])});
        G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
    }
    void add_edge(int u, int v, int cap){
        G[u].pb(edge{v, cap, 0, SZ(G[v])});
        G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
    }
    int dfs(int u, int cap) {
        if (u == t || !cap) return cap;
        for (int &i = cur[u]; i < SZ(G[u]); ++i) {
            edge &e = G[u][i];
            if (dis[e.to] == dis[u]+1 && e.cap != e.
                flow) {
                int df = dfs(e.to, min(e.cap - e.flow,
                    cap));
                if(df) {
                    e.flow += df, G[e.to][e.rev].flow
                        -= df;
                    return df;
                }
            }
        }
        dis[u] = -1;
        return 0;
    }
    bool bfs() {
        fill_n(dis, n + 3, -1);
        queue<int> q;
        q.push(s), dis[s] = 0;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (edge &e : G[u])
                if (!~dis[e.to] && e.flow != e.cap)
                    q.push(e.to), dis[e.to] = dis[u] +
                        1;
        }
        return dis[t] != -1;
    }
    int maxflow(int _s, int _t) {
        s = _s, t = _t;
        int flow = 0, df;
        while(bfs()) {
            fill_n(cur, n + 3, 0);
            while ((df = dfs(s, INF))) flow += df;
        }
        return flow;
    }
    bool solve() {
        int sum = 0;
        for(int i = 0; i < n; ++i)
            if(cnt[i] > 0) add_edge(n + 1, i, cnt[i]),
                sum += cnt[i];
            else if(cnt[i] < 0) add_edge(i, n + 2, -cnt
                [i]);
        if(sum != maxflow(n + 1, n + 2)) sum = -1;
        for(int i = 0; i < n; ++i)
            if(cnt[i] > 0) G[n + 1].pop_back(), G[i].
                pop_back();
            else if(cnt[i] < 0) G[i].pop_back(), G[n +
                2].pop_back();
        return sum != -1;
    }
    int solve(int _s, int _t) {
        add_edge(_t, _s, INF);
        if(!solve()) return -1; //invalid flow
        int x = G[_t].back().flow;
        return G[_t].pop_back(), G[_s].pop_back(), x;
    }
};
```

4.7 Gomory Hu tree

```
struct Gomory_Hu_tree{//0-base
  MaxFlow Dinic;
  int n;
  vector<pii> G[MAXN];
  void init(int _n){n=_n;
    for(int i=0;i<n;++i) G[i].clear();
  }
  void solve(vector<int> &v){
    if(v.size()<=1) return;
    int s=rand()%SZ(v);
    swap(v.back(),v[s]),s=v.back();
    int t=v[rand()%(SZ(v)-1)];
    vector<int> L,R;
    int x=(Dinic.reset(),Dinic.maxflow(s,t));
    G[s].pb(pii(t,x)),G[t].pb(pii(s,x));
    for(int i:v)
      if(~Dinic.dis[i]) L.pb(i);
      else R.pb(i);
```

```
    solve(L),solve(R);
  }
  void build(){
    vector<int> v(n);
    for(int i=0;i<n;++i) v[i]=i;
    solve(v);
  }
}ght;//test by BZOJ 4519
MaxFlow &Dinic=ght.Dinic;
```

4.8 NumberofMaximalClique

```
// bool g[][] : adjacent array indexed from 1 to n
void dfs(int sz){
  int i, j, k, t, cnt, best = 0;
  if(ne[sz]==ce[sz]){ if (ce[sz]==0) ++ans; return; }
  for(t=0, i=1; i<=ne[sz]; ++i){
    for (cnt=0, j=ne[sz]+1; j<=ce[sz]; ++j)
      if (!g[lst[sz][i]][lst[sz][j]]) ++cnt;
    if (t==0 || cnt<best) t=i, best=cnt;
  } if (t && best<=0) return;
  for (k=ne[sz]+1; k<=ce[sz]; ++k) {
    if (t>0){ for (i=k; i<=ce[sz]; ++i)
      if (!g[lst[sz][t]][lst[sz][i]]) break;
      swap(lst[sz][k], lst[sz][i]);
    } i=lst[sz][k]; ne[sz+1]=ce[sz+1]=0;
    for (j=1; j<k; ++j)if (g[i][lst[sz][j]])
      lst[sz+1][++ne[sz+1]]=lst[sz][j];
    for (ce[sz+1]=ne[sz+1], j=k+1; j<=ce[sz]; ++j)
      if (g[i][lst[sz][j]]) lst[sz+1][++ce[sz+1]]=lst[
          sz][j];
    dfs(sz+1); ++ne[sz]; --best;
    for (j=k+1, cnt=0; j<=ce[sz]; ++j) if (!g[i][lst[sz
        ][j]]) ++cnt;
    if (t==0 || cnt<best) t=k, best=cnt;
    if (t && best<=0) break;
  }}
void work(){
  ne[0]=0; ce[0]=0;
  for(int i=1; i<=n; ++i) lst[0][++ce[0]]=i;
  ans=0; dfs(0);
}
```

4.9 isap

```
struct Maxflow {
  static const int MAXV = 20010;
  static const int INF  = 1000000;
  struct Edge {
    int v, c, r;
    Edge(int _v, int _c, int _r):
      v(_v), c(_c), r(_r) {}
  };
  int s, t;
  vector<Edge> G[MAXV*2];
  int iter[MAXV*2], d[MAXV*2], gap[MAXV*2], tot;
  void init(int x) {
    tot = x+2;
    s = x+1, t = x+2;
    for(int i = 0; i <= tot; i++) {
      G[i].clear();
      iter[i] = d[i] = gap[i] = 0;
    }
  }
  void addEdge(int u, int v, int c) {
    G[u].push_back(Edge(v, c, SZ(G[v]) ));
    G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
  }
  int dfs(int p, int flow) {
    if(p == t) return flow;
    for(int &i = iter[p]; i < SZ(G[p]); i++) {
      Edge &e = G[p][i];
      if(e.c > 0 && d[p] == d[e.v]+1) {
        int f = dfs(e.v, min(flow, e.c));
        if(f) {
          e.c -= f;
          G[e.v][e.r].c += f;
          return f;
        }
      }
    }
    if( (--gap[d[p]]) == 0) d[s] = tot;
    else {
      d[p]++;
      iter[p] = 0;
      ++gap[d[p]];
    }
    return 0;
  }
  int solve() {
    int res = 0;
    gap[0] = tot;
    for(res = 0; d[s] < tot; res += dfs(s, INF));
    return res;
  }
} flow;
```

5 String

5.1 KMP

```
int F[MAXN];
vector<int> match(string A,string B){
  vector<int> ans;
  F[0]=-1,F[1]=0;
  for(int i=1,j=0;i<B.size();F[++i]=++j){
    if(B[i]==B[j]) F[i]=F[j];//optimize
    while(j!=-1&&B[i]!=B[j]) j=F[j];
  }
  for(int i=0,j=0;i-j+B.size()<=A.size();++i,++j){
    while(j!=-1&&A[i]!=B[j]) j=F[j];
    if(j==B.size()-1) ans.pb(i-j);
  }
  return ans;
}
```

5.2 Z-value

```
const int MAXn = 1e5 + 5;
int z[MAXn];
void make_z(string s){
  int l = 0, r = 0;
  for(int i = 1;i < s.size();i++){
    for(z[i] = max(0, min(r - i + 1, z[i - l]));
        i + z[i] < s.size() && s[i + z[i]] == s[z[i]];z
            [i]++);
    if(i + z[i] - 1 > r)l = i, r = i + z[i] - 1;
  }
}
```

5.3 Manacher*

```
int z[MAXN];
int Manacher(string tmp){
  string s="&";
  int l=0,r=0,x,ans;
  for(char c:tmp) s.pb(c),s.pb('%');
  ans=0,x=0;
  for(int i=1;i<SZ(s);++i){
    z[i]=r > i ? min(z[2*l-i],r-i) : 1;
    while(s[i+z[i]]==s[i-z[i]])++z[i];
    if(z[i]+i>r)r=z[i]+i,l=i;
  }
  for(int i=1;i<SZ(s);++i)
    if(s[i]=='%')
      x=max(x,z[i]);
  ans=x/2*2,x=0;
  for(int i=1;i<SZ(s);++i)
    if(s[i]!='%')
      x=max(x,z[i]);
  return max(ans,(x-1)/2*2+1);
}
```

5.4 Suffix Array

```
struct suffix_array{
  int box[MAXN],tp[MAXN],m;
  bool not_equ(int a,int b,int k,int n){
    return ra[a]!=ra[b]||a+k>=n||b+k>=n||ra[a+k]!=ra[b+
        k];
  }
  void radix(int *key,int *it,int *ot,int n){
    fill_n(box,m,0);
    for(int i=0;i<n;++i) ++box[key[i]];
    partial_sum(box,box+m,box);
    for(int i=n-1;i>=0;--i) ot[--box[key[it[i]]]]=it[i
        ];
  }
  void make_sa(string s,int n){
    int k=1;
    for(int i=0;i<n;++i) ra[i]=s[i];
    do{
      iota(tp,tp+k,n-k),iota(sa+k,sa+n,0);
      radix(ra+k,sa+k,tp+k,n-k);
      radix(ra,tp,sa,n);
      tp[sa[0]]=0,m=1;
      for(int i=1;i<n;++i){
        m+=not_equ(sa[i],sa[i-1],k,n);
        tp[sa[i]]=m-1;
      }
      copy_n(tp,n,ra);
      k*=2;
    }while(k<n&&m!=n);
  }
  void make_he(string s,int n){
    for(int j=0,k=0;j<n;++j){
      if(ra[j])
        for(;s[j+k]==s[sa[ra[j]-1]+k];++k);
      he[ra[j]]=k,k=max(0,k-1);
    }
  }
  int sa[MAXN],ra[MAXN],he[MAXN];
  void build(string s){
    FILL(sa,0),FILL(ra,0),FILL(he,0);
    FILL(box,0),FILL(tp,0),m=256;
    make_sa(s,s.size());
    make_he(s,s.size());
  }
};
```

5.5 SAIS*

```
class SAIS {
 public:
  int *SA, *H;
  // zero based, string content MUST > 0
  // result height H[i] is LCP(SA[i - 1], SA[i])
  // string, length, |sigma|
  void build(int *s, int n, int m = 128){
    copy_n(s, n, _s);
    _h[0] = _s[n++] = 0;
    sais(_s, _sa, _p, _q, _t, _c, n, m);
    mkhei(n);
    SA = _sa + 1; H = _h + 1;
  }
 private:
  bool _t[N * 2];
  int _s[N * 2], _c[N * 2], x[N], _p[N], _q[N * 2], r[N
      ], _sa[N * 2], _h[N];
  void mkhei(int n){
    for (int i = 0; i < n; i++) r[_sa[i]] = i;
    for (int i = 0; i < n; i++) if(r[i]) {
      int ans = i > 0 ? max(_h[r[i - 1]] - 1, 0) : 0;
      while(_s[i + ans] == _s[_sa[r[i] - 1] + ans]) ans
          ++;
      _h[r[i]] = ans;
    }
  }
  void sais(int *s, int *sa, int *p, int *q, bool *t,
      int *c, int n, int z){
    bool uniq = t[n - 1] = 1, neq;
    int nn = 0, nmxz = -1, *nsa = sa + n, *ns = s + n,
        lst = -1;

#define MAGIC(XD) \
    fill_n(sa, n, 0); \
    copy_n(c, z, x); \
    XD; \
    copy_n(c, z - 1, x + 1); \
    for (int i = 0; i < n; i++) if(sa[i] && !t[sa[i] -
        1]) \
        sa[x[s[sa[i]-1]]++] = sa[i] - 1; \
    copy_n(c, z, x); \
    for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i
        ]-1]) \
        sa[--x[s[sa[i]-1]]] = sa[i] - 1;

    fill_n(c, z, 0);
    for (int i = 0; i < n; i++) uniq &= ++c[s[i]] < 2;
    partial_sum(c, c + z, c);
    if (uniq) {
      for (int i = 0; i < n; i++) sa[--c[s[i]]] = i;
      return;
    }
    for(int i = n - 2; i >= 0; i--)
      t[i] = (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[
          i + 1]);
    MAGIC(
      for (int i = 1; i <= n - 1; i++) if (t[i] && !t
          [i - 1])
        sa[--x[s[i]]] = p[q[i] = nn++] = i
    );
    for (int i = 0; i < n; i++) if (sa[i] && t[sa[i]]
        && !t[sa[i] - 1]) {
      neq = (lst < 0) || !equal(s + lst, s + lst + p[q[
          sa[i]] + 1] - sa[i], s + sa[i]);
      ns[q[lst = sa[i]]] = nmxz += neq;
    }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmxz
        + 1);
    MAGIC(
      for(int i = nn - 1; i >= 0; i--)
        sa[--x[s[p[nsa[i]]]]] = p[nsa[i]]
    );
  }
} sa;
```

5.6 Aho-Corasick Automatan

```
const int len=400000,sigma=26;
struct AC_Automatan{
  int nx[len][sigma],fl[len],cnt[len],pri[len],top;
  int newnode(){
    fill(nx[top],nx[top]+sigma,-1);
    return top++;
  }
  void init(){top=1,newnode();}
  int input(string &s){//return the end_node of string
    int X=1;
    for(char c:s){
      if(!~nx[X][c-'a'])nx[X][c-'a']=newnode();
      X=nx[X][c-'a'];
    }
    return X;
  }
  void make_fl(){
    queue<int> q;
    q.push(1),fl[1]=0;
    for(int t=0;!q.empty();){
      int R=q.front();
      q.pop(),pri[t++]=R;
      for(int i=0;i<sigma;++i)
        if(~nx[R][i]){
          int X=nx[R][i],Z=fl[R];
          for(;Z&&!~nx[Z][i];)Z=fl[Z];
          fl[X]=Z?nx[Z][i]:1,q.push(X);
        }
    }
  }
  void get_v(string &s){
    int X=1;
    fill(cnt,cnt+top,0);
    for(char c:s){
```

```
      while(X&&!~nx[X][c-'a'])X=fl[X];
      X=X?nx[X][c-'a']:1,++cnt[X];
    }
    for(int i=top-2;i>0;--i) cnt[fl[pri[i]]]+=cnt[pri[i
        ]];
  }
};
```

5.7 Smallest Rotation

```
string mcp(string s){
  int n=SZ(s),i=0,j=1;
  s+=s;
  while(i<n&&j<n){
    int k=0;
    while(k<n&&s[i+k]==s[j+k]) ++k;
    if(s[i+k]<=s[j+k]) j+=k+1;
    else i+=k+1;
    if(i==j) ++j;
  }
  int ans=i<n?i:j;
  return s.substr(ans,n);
}
```

5.8 De Bruijn sequence

```
constexpr int MAXC = 10, MAXN = 1e5 + 10;
struct DBSeq {
  int C, N, K, L, buf[MAXC * MAXN]; //K <= C^N
  void dfs(int *out, int t, int p, int &ptr) {
    if (ptr>=L) return;
    if (t>N) {
      if (N%p) return;
      for (int i=1;i<=p&&ptr<L;++i) out[ptr++]=buf[i];
    } else {
      buf[t]=buf[t-p],dfs(out,t+1,p,ptr);
      for (int j=buf[t-p]+1;j<C;++j) buf[t]=j,dfs(out,t
          +1,t,ptr);
    }
  }
  void solve(int _c, int _n, int _k, int *out) {
    int p=0;
    C=_c,N=_n,K=_k,L=N+K-1;dfs(out,1,1,p);
    if (p<L) fill(out+p,out+L,0);
  }
} dbs;
```

5.9 SAM

```
const int MAXM = 1000010;
struct SAM{
  int tot, root, lst, mom[MAXM], mx[MAXM];
  int acc[MAXM], nxt[MAXM][33];
  int newNode(){
    int res = ++tot;
    fill(nxt[res], nxt[res]+33, 0);
    mom[res] = mx[res] = acc[res] = 0;
    return res;
  }
  void init(){
    tot = 0;
    root = newNode();
    mom[root] = 0, mx[root] = 0;
    lst = root;
  }
  void push(int c){
    int p = lst;
    int np = newNode();
    mx[np] = mx[p]+1;
    for(; p && nxt[p][c] == 0; p = mom[p])
      nxt[p][c] = np;
    if(p == 0) mom[np] = root;
    else{
      int q = nxt[p][c];
      if(mx[p]+1 == mx[q]) mom[np] = q;
      else{
        int nq = newNode();
        mx[nq] = mx[p]+1;
        for(int i = 0; i < 33; i++)
          nxt[nq][i] = nxt[q][i];
        mom[nq] = mom[q];
        mom[q] = nq;
        mom[np] = nq;
        for(; p && nxt[p][c] == q; p = mom[p])
          nxt[p][c] = nq;
      }
    }
    lst = np;
  }
  void push(char *str){
    for(int i = 0; str[i]; i++)
      push(str[i]-'a'+1);
  }
} sam;
```

5.10 PalTree

```
struct palindromic_tree{// Check by APIO 2014
    palindrome
  struct node{
    int next[26],fail,len;
    int cnt,num;//cnt: appear times, num: number of pal
        . suf.
    node(int l=0):fail(0),len(l),cnt(0),num(0){
      for(int i=0;i<26;++i)next[i]=0;
    }
  };
  vector<node>St;
  vector<char>s;
  int last,n;
  palindromic_tree():St(2),last(1),n(0){
    St[0].fail=1, St[1].len=-1, s.pb(-1);
  }
  inline void clear(){
    St.clear(), s.clear(), last=1, n=0;
    St.pb(0), St.pb(-1);
    St[0].fail=1, s.pb(-1);
  }
  inline int get_fail(int x){
    while(s[n-St[x].len-1]!=s[n])x=St[x].fail;
    return x;
  }
  inline void add(int c){
    s.push_back(c-='a'), ++n;
    int cur=get_fail(last);
    if(!St[cur].next[c]){
      int now=SZ(St);
      St.pb(St[cur].len+2);
      St[now].fail=St[get_fail(St[cur].fail)].next[c];
      St[cur].next[c]=now;
      St[now].num=St[St[now].fail].num+1;
    }
    last=St[cur].next[c], ++St[last].cnt;
  }
  inline void count(){// counting cnt
    auto i=St.rbegin();
    for(;i!=St.rend();++i){
      St[i->fail].cnt+=i->cnt;
    }
  }
  inline int size(){// The number of diff. pal.
    return SZ(St)-2;
  }
};
```

5.11 cyclicLCS

```
#define L 0
#define LU 1
#define U 2
const int mov[3][2]={0,-1, -1,-1, -1,0};
int al,bl;
char a[MAXL*2],b[MAXL*2]; // 0-indexed
int dp[MAXL*2][MAXL];
```

```
char pred[MAXL*2][MAXL];
inline int lcs_length(int r) {
  int i=r+al,j=bl,l=0;
  while(i>r) {
    char dir=pred[i][j];
    if(dir==LU) l++;
    i+=mov[dir][0];
    j+=mov[dir][1];
  }
  return l;
}
inline void reroot(int r) { // r = new base row
  int i=r,j=1;
  while(j<=bl&&pred[i][j]!=LU) j++;
  if(j>bl) return;
  pred[i][j]=L;
  while(i<2*al&&j<=bl) {
    if(pred[i+1][j]==U) {
      i++;
      pred[i][j]=L;
    } else if(j<bl&&pred[i+1][j+1]==LU) {
      i++;
      j++;
      pred[i][j]=L;
    } else {
      j++;
    }
  }
}
int cyclic_lcs() {
  // a, b, al, bl should be properly filled
  // note: a WILL be altered in process
  //       -- concatenated after itself
  char tmp[MAXL];
  if(al>bl) {
    swap(al,bl);
    strcpy(tmp,a);
    strcpy(a,b);
    strcpy(b,tmp);
  }
  strcpy(tmp,a);
  strcat(a,tmp);
  // basic lcs
  for(int i=0;i<=2*al;i++) {
    dp[i][0]=0;
    pred[i][0]=U;
  }
  for(int j=0;j<=bl;j++) {
    dp[0][j]=0;
    pred[0][j]=L;
  }
  for(int i=1;i<=2*al;i++) {
    for(int j=1;j<=bl;j++) {
      if(a[i-1]==b[j-1]) dp[i][j]=dp[i-1][j-1]+1;
      else dp[i][j]=max(dp[i-1][j],dp[i][j-1]);
      if(dp[i][j-1]==dp[i][j]) pred[i][j]=L;
      else if(a[i-1]==b[j-1]) pred[i][j]=LU;
      else pred[i][j]=U;
    }
  }
  // do cyclic lcs
  int clcs=0;
  for(int i=0;i<al;i++) {
    clcs=max(clcs,lcs_length(i));
    reroot(i+1);
  }
  // recover a
  a[al]='\0';
  return clcs;
}
```

6 Math

6.1 ax+by=gcd

```
pii gcd(int a,int b){
  if(b==0) return pii(1,0);
  else{
    int p=a/b;
    pii q=gcd(b,a%b);
    return MP(q.Y,q.X-q.Y*p);
  }
}
```

6.2 floor and ceil

```
int floor(int a,int b){
  return a/b-(a%b&&a<0^b<0);
}
int ceil(int a,int b){
  return a/b+(a%b&&a<0^b>0);
}
```

6.3 Miller Rabin

```
// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pirmes <= 13
// n < 2^64               7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool Miller_Rabin(ll a,ll n){
  if((a=a%n)==0) return 1;
  if(n&1^1) return n==2;
  ll tmp=(n-1)/((n-1)&(-n+1)),t=__lg((n-1)&(-n+1)),x=1;
  for(;tmp;tmp/=2,a=mul(a,a,n))
    if(tmp&1) x=mul(x,a,n);
  if(x==1 || x==n-1) return 1;
  while(--t)
    if((x=mul(x,x,n))==n-1) return 1;
  return 0;
}
```

6.4 Big number

```
template<typename T>
inline string to_string(const T& x){
  stringstream ss;
  return ss<<x,ss.str();
}
struct bigN:vector<ll>{
  const static int base=1000000000,width=log10(base);
  bool negative;
  bigN(const_iterator a,const_iterator b):vector<ll>(a,
      b){}
  bigN(string s){
    if(s.empty())return;
    if(s[0]=='-')negative=1,s=s.substr(1);
    else negative=0;
    for(int i=int(s.size())-1;i>=0;i-=width){
      ll t=0;
      for(int j=max(0,i-width+1);j<=i;++j)
        t=t*10+s[j]-'0';
      push_back(t);
    }
    trim();
  }
  template<typename T>
    bigN(const T &x):bigN(to_string(x)){}
  bigN():negative(0){}
  void trim(){
    while(size()&&!back())pop_back();
    if(empty())negative=0;
  }
  void carry(int _base=base){
    for(size_t i=0;i<size();++i){
      if(at(i)>=0&&at(i)<_base)continue;
      if(i+1u==size())push_back(0);
      int r=at(i)%_base;
      if(r<0)r+=_base;
      at(i+1)+=(at(i)-r)/_base,at(i)=r;
    }
  }
  int abscmp(const bigN &b)const{
    if(size()>b.size())return 1;
```

```
  if(size()<b.size())return -1;
  for(int i=int(size())-1;i>=0;--i){
    if(at(i)>b[i])return 1;
    if(at(i)<b[i])return -1;
  }
  return 0;
}
int cmp(const bigN &b)const{
  if(negative!=b.negative)return negative?-1:1;
  return negative?-abscmp(b):abscmp(b);
}
bool operator<(const bigN&b)const{return cmp(b)<0;}
bool operator>(const bigN&b)const{return cmp(b)>0;}
bool operator<=(const bigN&b)const{return cmp(b)<=0;}
bool operator>=(const bigN&b)const{return cmp(b)>=0;}
bool operator==(const bigN&b)const{return !cmp(b);}
bool operator!=(const bigN&b)const{return cmp(b)!=0;}
bigN abs()const{
  bigN res=*this;
  return res.negative=0, res;
}
bigN operator-()const{
  bigN res=*this;
  return res.negative=!negative,res.trim(),res;
}
bigN operator+(const bigN &b)const{
  if(negative)return -(-(*this)+(-b));
  if(b.negative)return *this-(-b);
  bigN res=*this;
  if(b.size()>size())res.resize(b.size());
  for(size_t i=0;i<b.size();++i)res[i]+=b[i];
  return res.carry(),res.trim(),res;
}
bigN operator-(const bigN &b)const{
  if(negative)return -(-(*this)-(-b));
  if(b.negative)return *this+(-b);
  if(abscmp(b)<0)return -(b-(*this));
  bigN res=*this;
  if(b.size()>size())res.resize(b.size());
  for(size_t i=0;i<b.size();++i)res[i]-=b[i];
  return res.carry(),res.trim(),res;
}
bigN operator*(const bigN &b)const{
  bigN res;
  res.negative=negative!=b.negative;
  res.resize(size()+b.size());
  for(size_t i=0;i<size();++i)
    for(size_t j=0;j<b.size();++j)
      if((res[i+j]+=at(i)*b[j])>=base){
        res[i+j+1]+=res[i+j]/base;
        res[i+j]%=base;
      }//¾ªk¥carry·|·¸¦
  return res.trim(),res;
}
bigN operator/(const bigN &b)const{
  int norm=base/(b.back()+1);
  bigN x=abs()*norm;
  bigN y=b.abs()*norm;
  bigN q,r;
  q.resize(x.size());
  for(int i=int(x.size())-1;i>=0;--i){
    r=r*base+x[i];
    int s1=r.size()<=y.size()?0:r[y.size()];
    int s2=r.size()<y.size()?0:r[y.size()-1];
    int d=(ll(base)*s1+s2)/y.back();
    r=r-y*d;
    while(r.negative)r=r+y,--d;
    q[i]=d;
  }
  q.negative=negative!=b.negative;
  return q.trim(),q;
}
bigN operator%(const bigN &b)const{
  return *this-(*this/b)*b;
}
friend istream& operator>>(istream &ss,bigN &b){
  string s;
  return ss>>s, b=s, ss;
}
friend ostream& operator<<(ostream &ss,const bigN &b)
    {
  if(b.negative)ss<<'-';
    ss<<(b.empty()?0:b.back());
    for(int i=int(b.size())-2;i>=0;--i)
      ss<<setw(width)<<setfill('0')<<b[i];
    return ss;
  }
  template<typename T>
    operator T(){
      stringstream ss;
      ss<<*this;
      T res;
      return ss>>res,res;
    }
};
```

6.5 Fraction

```
struct fraction{
  ll n,d;
  fraction(const ll &_n=0,const ll &_d=1):n(_n),d(_d){
    ll t=__gcd(n,d);
    n/=t,d/=t;
    if(d<0) n=-n,d=-d;
  }
  fraction operator-()const{
    return fraction(-n,d);
  }
  fraction operator+(const fraction &b)const{
    return fraction(n*b.d+b.n*d,d*b.d);
  }
  fraction operator-(const fraction &b)const{
    return fraction(n*b.d-b.n*d,d*b.d);
  }
  fraction operator*(const fraction &b)const{
    return fraction(n*b.n,d*b.d);
  }
  fraction operator/(const fraction &b)const{
    return fraction(n*b.d,d*b.n);
  }
  void print(){
    cout << n;
    if(d!=1) cout << "/" << d;
  }
};
```

6.6 Simultaneous Equations

```
struct matrix { //m variables, n equations
  int n, m;
  fraction M[MAXN][MAXN + 1], sol[MAXN];
  int solve() { //-1: inconsistent, >= 0: rank
    for (int i = 0; i < n; ++i) {
      int piv = 0;
      while (piv < m && !M[i][piv].n) ++piv;
      if (piv == m) continue;
      for (int j = 0; j < n; ++j) {
        if (i == j) continue;
        fraction tmp = -M[j][piv] / M[i][piv];
        for (int k = 0; k <= m; ++k) M[j][k] = tmp * M[
            i][k] + M[j][k];
      }
    }
    int rank = 0;
    for (int i = 0; i < n; ++i) {
      int piv = 0;
      while (piv < m && !M[i][piv].n) ++piv;
      if (piv == m && M[i][m].n) return -1;
      else if (piv < m) ++rank, sol[piv] = M[i][m] / M[
          i][piv];
    }
    return rank;
  }
};
```

6.7 Pollard Rho

```
// does not work when n is prime
ll f(ll x,ll mod){ return add(mul(x,x,mod),1,mod); }
ll pollard_rho(ll n){
  if(!(n&1)) return 2;
  while(1){
    ll y=2,x=rand()%(n-1)+1,res=1;
    for(int sz=2;res==1;y=x,sz*=2)
      for(int i=0;i<sz&&res<=1;++i)
        x=f(x,n),res=__gcd(abs(x-y),n);
    if(res!=0&&res!=n) return res;
  }
}
```

6.8 Simplex Algorithm

```
const int MAXN = 111;
const int MAXM = 111;
const double eps = 1E-10;
double a[MAXN][MAXM], b[MAXN], c[MAXM], d[MAXN][MAXM];
double x[MAXM];
int ix[MAXN + MAXM]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b,x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[MAXN][MAXM], double b[MAXN],
    double c[MAXM], int n, int m){
  ++m;
  int r = n, s = m - 1;
  memset(d, 0, sizeof(d));
  for (int i = 0; i < n + m; ++i) ix[i] = i;
  for (int i = 0; i < n; ++i) {
    for (int j = 0; j < m - 1; ++j) d[i][j] = -a[i][j];
    d[i][m - 1] = 1;
    d[i][m] = b[i];
    if (d[r][m] > d[i][m]) r = i;
  }
  for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];
  d[n + 1][m - 1] = -1;
  for (double dd;; ) {
    if (r < n) {
      int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
      d[r][s] = 1.0 / d[r][s];
      for (int j = 0; j <= m; ++j)
        if (j != s) d[r][j] *= -d[r][s];
      for (int i = 0; i <= n + 1; ++i) if (i != r) {
        for (int j = 0; j <= m; ++j) if (j != s)
          d[i][j] += d[r][j] * d[i][s];
        d[i][s] *= d[r][s];
      }
    }
    r = -1; s = -1;
    for (int j = 0; j < m; ++j)
      if (s < 0 || ix[s] > ix[j]) {
        if (d[n + 1][j] > eps ||
            (d[n + 1][j] > -eps && d[n][j] > eps))
          s = j;
      }
    if (s < 0) break;
    for (int i = 0; i < n; ++i) if (d[i][s] < -eps) {
      if (r < 0 ||
          (dd = d[r][m] / d[r][s] - d[i][m] / d[i][s])
              < -eps ||
          (dd < eps && ix[r + m] > ix[i + m]))
        r = i;
    }
    if (r < 0) return -1; // not bounded
  }
  if (d[n + 1][m] < -eps) return -1; // not executable
  double ans = 0;
  for(int i=0; i<m; i++) x[i] = 0;
  for (int i = m; i < n + m; ++i) { // the missing
      enumerated x[i] = 0
    if (ix[i] < m - 1){
      ans += d[i - m][m] * c[ix[i]];
      x[ix[i]] = d[i-m][m];
    }
  }
  return ans;
}
```

6.9 chineseRemainder

```
LL solve(LL x1, LL m1, LL x2, LL m2) {
  LL g = __gcd(m1, m2);
  if((x2 - x1) % g) return -1;// no sol
  m1 /= g; m2 /= g;
  pair<LL,LL> p = gcd(m1, m2);
  LL lcm = m1 * m2 * g;
  LL res = p.first * (x2 - x1) * m1 + x1;
  return (res % lcm + lcm) % lcm;
}
```

6.10 QuadraticResidue

```
int Jacobi(int a, int m) {
  int s = 1;
  for (; m > 1; ) {
    a %= m;
    if (a == 0) return 0;
    const int r = __builtin_ctz(a);
    if ((r & 1) && ((m + 2) & 4)) s = -s;
    a >>= r;
    if (a & m & 2) s = -s;
    swap(a, m);
  }
  return s;
}

int QuadraticResidue(int a, int p) {
  if (p == 2) return a & 1;
  const int jc = Jacobi(a, p);
  if (jc == 0) return 0;
  if (jc == -1) return -1;
  int b, d;
  for (; ; ) {
    b = rand() % p;
    d = (1LL * b * b + p - a) % p;
    if (Jacobi(d, p) == -1) break;
  }
  int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
  for (int e = (1LL + p) >> 1; e; e >>= 1) {
    if (e & 1) {
      tmp = (1LL * g0 * f0 + 1LL * d * (1LL * g1 * f1 %
          p)) % p;
      g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
      g0 = tmp;
    }
    tmp = (1LL * f0 * f0 + 1LL * d * (1LL * f1 * f1 % p
        )) % p;
    f1 = (2LL * f0 * f1) % p;
    f0 = tmp;
  }
  return g0;
}
```

6.11 PiCount

```
int64_t PrimeCount(int64_t n) {
  if (n <= 1) return 0;
  const int v = sqrt(n);
  vector<int> smalls(v + 1);
  for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
  int s = (v + 1) / 2;
  vector<int> roughs(s);
  for (int i = 0; i < s; ++i) roughs[i] = 2 * i + 1;
  vector<int64_t> larges(s);
  for (int i = 0; i < s; ++i) larges[i] = (n / (2 * i +
      1) + 1) / 2;
  vector<bool> skip(v + 1);
  int pc = 0;
  for (int p = 3; p <= v; ++p) {
    if (smalls[p] > smalls[p - 1]) {
      int q = p * p;
      pc++;
```

```
      if (1LL * q * q > n) break;
      skip[p] = true;
      for (int i = q; i <= v; i += 2 * p) skip[i] =
          true;
      int ns = 0;
      for (int k = 0; k < s; ++k) {
        int i = roughs[k];
        if (skip[i]) continue;
        int64_t d = 1LL * i * p;
        larges[ns] = larges[k] - (d <= v ? larges[
            smalls[d] - pc] : smalls[n / d]) + pc;
        roughs[ns++] = i;
      }
      s = ns;
      for (int j = v / p; j >= p; --j) {
        int c = smalls[j] - pc;
        for (int i = j * p, e = min(i + p, v + 1); i <
            e; ++i) smalls[i] -= c;
      }
    }
  }
  for (int k = 1; k < s; ++k) {
    const int64_t m = n / roughs[k];
    int64_t s = larges[k] - (pc + k - 1);
    for (int l = 1; l < k; ++l) {
      int p = roughs[l];
      if (1LL * p * p > m) break;
      s -= smalls[m / p] - (pc + l - 1);
    }
    larges[0] -= s;
  }
  return larges[0];
}
```

6.12 Algorithms about Primes

```
/*
   12721 13331 14341 75577 123457 222557 556679 999983
       1097774749 1076767633 100102021 999997771
       1001010013 1000512343 987654361 999991231
       999888733 98789101 987777733 999991921
       1010101333 1010102101 1000000000039
       100000000000037 2305843009213693951
       4611686018427387847 9223372036854775783
       18446744073709551557
   */
```

7 Polynomial

7.1 Fast Fourier Transform

```
template<int MAXN>
struct FFT {
  using val_t = complex<double>;
  const double PI = acos(-1);
  val_t w[MAXN];
  FFT() {
    for (int i = 0; i < MAXN; ++i) {
      double arg = 2 * PI * i / MAXN;
      w[i] = val_t(cos(arg), sin(arg));
    }
  }
  void bitrev(val_t *a, int n); // see NTT
  void trans(val_t *a, int n, bool inv = false); // see
      NTT;
  // remember to replace LL with val_t
};
```

7.2 Number Theory Transform

```
//(2^16)+1, 65537, 3
//7*17*(2^23)+1, 998244353, 3
//1255*(2^20)+1, 1315962881, 3
//51*(2^25)+1, 1711276033, 29
```

```
template<int MAXN, LL P, LL RT> //MAXN must be 2^k
struct NTT {
  LL w[MAXN];
  LL mpow(LL a, LL n);
  LL minv(LL a) { return mpow(a, P - 2); }
  NTT() {
    LL dw = mpow(RT, (P - 1) / MAXN);
    w[0] = 1;
    for (int i = 1; i < MAXN; ++i) w[i] = w[i - 1] * dw
        % P;
  }
  void bitrev(LL *a, int n) {
    int i = 0;
    for (int j = 1; j < n - 1; ++j) {
      for (int k = n >> 1; (i ^= k) < k; k >>= 1);
      if (j < i) swap(a[i], a[j]);
    }
  }
  void operator()(LL *a, int n, bool inv = false) { //0
      <= a[i] < P
    bitrev(a, n);
    for (int L = 2; L <= n; L <<= 1) {
      int dx = MAXN / L, dl = L >> 1;
      for (int i = 0; i < n; i += L) {
        for (int j = i, x = 0; j < i + dl; ++j, x += dx
            ) {
          LL tmp = a[j + dl] * w[x] % P;
          if ((a[j + dl] = a[j] - tmp) < 0) a[j + dl]
              += P;
          if ((a[j] += tmp) >= P) a[j] -= P;
        }
      }
    }
    if (inv) {
      reverse(a + 1, a + n);
      LL invn = minv(n);
      for (int i = 0; i < n; ++i) a[i] = a[i] * invn %
          P;
    }
  }
};
```

7.3 Fast Walsh Transform

```
/* x: a[j], y: a[j + (L >> 1)]
or: (y += x), (y -= x) and: (x += y), (x -= y)
xor: (x+y, x-y), (x+y, x-y)/2                */
void fwt(val_t *a, int n) { //or
  for (int L = 2; L <= n; L <<= 1) {
    for (int i = 0; i < n; i += L) {
      for (int j = i; j < i + (L >> 1); ++j) {
        a[j + (L >> 1)] += a[j];
      }
    }
  }
}
```

7.4 Polynomial Operation

```
template<int MAXN, LL P, LL RT> //MAXN must be 2^k
struct PolyOp {
  NTT<MAXN, P, RT> ntt;
  const LL INV2 = ntt.minv(2);
  int get_sz(int n) {
    int sz = 1;
    while (sz < n) sz <<= 1;
    return sz;
  }
  void mul(LL *a, int n, LL *b, int m, LL *c) {
    static LL buf1[MAXN], buf2[MAXN];
    int sz = get_sz(n + m - 1);
    copy(a, a + n, buf1), fill(buf1 + n, buf1 + sz, 0);
    copy(b, b + m, buf2), fill(buf2 + m, buf2 + sz, 0);
    ntt(buf1, sz), ntt(buf2, sz);
    for (int i = 0; i < sz; ++i) c[i] = buf1[i] * buf2[
        i] % P;
    ntt(c, sz, true);
  }
```

```
  void inv(LL *a, int n, LL *b) { //a[0] != 0
    static LL buf[MAXN];
    if (n == 1) return b[0] = ntt.minv(a[0]), void();
    inv(a, (n + 1) / 2, b);
    int sz = get_sz(n * 2);
    copy(a, a + n, buf), fill(buf + n, buf + sz, 0);
    fill(b + n, b + sz, 0);
    ntt(buf, sz), ntt(b, sz);
    for (int i = 0; i < sz; ++i) {
      b[i] *= (2 - b[i] * buf[i]) % P;
      if ((b[i] %= P) < 0) b[i] += P;
    }
    ntt(b, sz, true), fill(b + n, b + sz, 0);
  }
  LL _msqrt(LL x) {
    for (LL i = 0; i <= P / 2; ++i) if (i * i % P == x)
        return i;
    throw string("BBQube");
  }
  void sqrt(LL *a, int n, LL *b) { //a[0] != 0 && sqrt(
      a[0]) exists
    static LL invb[MAXN], buf[MAXN];
    if (n == 1) return b[0] = _msqrt(a[0]), void();
    sqrt(a, (n + 1) / 2, b);
    int sz = get_sz(n * 2);
    inv(b, n, invb);
    copy(a, a + n, buf), fill(buf + n, buf + sz, 0);
    ntt(b, sz), ntt(invb, sz), ntt(buf, sz);
    for (int i = 0; i < sz; ++i) {
      if ((b[i] += buf[i] * invb[i] % P) >= P) b[i] -=
          P;
      b[i] = b[i] * INV2 % P;
    }
    ntt(b, sz, true), fill(b + n, b + sz, 0);
  }
  void div(LL *a, int n, LL *b, int m, LL *q, LL *r) {
    static LL invb[MAXN], buf[MAXN];
    if (n < m) {
      fill(q, q + m, 0), copy(a, a + n, r), fill(r + n,
          r + m, 0);
      return;
    }
    int mod_sz = n - m + 1;
    copy(b, b + m, buf), reverse(buf, buf + m);
    if (m < mod_sz) fill(buf + m, buf + mod_sz, 0);
    inv(buf, mod_sz, invb);
    copy(a, a + n, buf), reverse(buf, buf + n);
    mul(buf, mod_sz, invb, mod_sz, q);
    fill(q + mod_sz, q + n, 0), reverse(q, q + mod_sz);
    mul(b, m, q, mod_sz, buf);
    for (int i = 0; i < n; ++i) {
      if ((r[i] = a[i] - buf[i]) < 0) r[i] += P;
    }
  }
};
```

8 Geometry

8.1 Default Code

```
typedef pair<double,double> pdd;
typedef pair<pdd,pdd> Line;
const double eps=1e-8;
pdd operator+(const pdd &a,const pdd &b)
{ return pdd(a.X+b.X,a.Y+b.Y);}
pdd operator-(const pdd &a,const pdd &b)
{ return pdd(a.X-b.X,a.Y-b.Y);}
pdd operator*(const pdd &a,const double &b)
{ return pdd(a.X*b,a.Y*b);}
pdd operator/(const pdd &a,const double &b)
{ return pdd(a.X/b,a.Y/b);}
double dot(const pdd &a,const pdd &b)
{ return a.X*b.X+a.Y*b.Y;}
double cross(const pdd &a,const pdd &b)
{ return a.X*b.Y-a.Y*b.X;}
double abs2(const pdd &a)
{ return dot(a,a);}
double abs(const pdd &a)
{ return sqrt(dot(a,a));}
int ori(const pdd &a,const pdd &b,const pdd &c){
  double res=cross(b-a,c-a);
  if(fabs(res)<eps) return 0;
  return res > 0 ? 1 : -1;
}
bool collinearity(const pdd &p1, const pdd &p2, const
    pdd &p3){
  return fabs(cross(p1-p3,p2-p3))<eps;
}
bool btw(const pdd &p1,const pdd &p2,const pdd &p3){
  if(!collinearity(p1,p2,p3)) return 0;
  return dot(p1-p3,p2-p3)<eps;
}
bool seg_intersect(const pdd &p1,const pdd &p2,const
    pdd &p3,const pdd &p4){
  int a123=ori(p1,p2,p3);
  int a124=ori(p1,p2,p4);
  int a341=ori(p3,p4,p1);
  int a342=ori(p3,p4,p2);
  if(a123==0 && a124==0)
    return btw(p1,p2,p3)||btw(p1,p2,p4)||btw(p3,p4,p1)
        ||btw(p3,p4,p2);
  return a123*a124<=0&&a341*a342<=0;
}
pdd intersect(const pdd &p1,const pdd &p2,const pdd &p3
    ,const pdd &p4){
  double a123=cross(p2-p1,p3-p1);
  double a124=cross(p2-p1,p4-p1);
  return (p4*a123-p3*a124)/(a123-a124);
}
pdd foot(const pdd &p1,const pdd &p2,const pdd &p3){
  pdd tmp=p2-p1;
  swap(tmp.X,tmp.Y),tmp.Y*=-1;
  return intersect(p1,p2,p3,p3+tmp);
}
```

8.2 Convex hull

```
struct convex_hull{
  vector<pdd> dots;
  void add_dot(double a,double b){
    dots.pb(pdd(a,b));
  }
  vector<pdd> hull(){
    vector<pdd> ans;
    sort(dots.begin(),dots.end());
    ans.pb(dots[0]),ans.pb(dots[1]);
    for(int i=2;i<SZ(dots);++i){
      while(SZ(ans)>=2)
        if(ori(ans[SZ(ans)-2],ans.back(),dots[i])<=0)
          ans.pop_back();
        else break;
      ans.pb(dots[i]);
    }
    for(int i=SZ(dots)-2,t=SZ(ans);i>=0;--i){
      while(SZ(ans)>t)
        if(ori(ans[SZ(ans)-2],ans.back(),dots[i])<=0)
          ans.pop_back();
        else break;
      ans.pb(dots[i]);
    }
    ans.pop_back();
    return ans;
  }
};
```

8.3 External bisector

```
pdd external_bisector(pdd p1,pdd p2,pdd p3){//213
  pdd L1=p2-p1,L2=p3-p1;
  L2=L2*abs(L1)/abs(L2);
  return L1+L2;
}
```

8.4 Heart

```
pdd excenter(pdd p0,pdd p1,pdd p2,double &radius){
  p1=p1-p0,p2=p2-p0;
  double x1=p1.X,y1=p1.Y,x2=p2.X,y2=p2.Y;
  double m=2.*(x1*y2-y1*x2);
  center.X=(x1*x1*y2-x2*x2*y1+y1*y2*(y1-y2))/m;
  center.Y=(x1*x2*(x2-x1)-y1*y1*x2+x1*y2*y2)/m;
  return radius=abs(center),center+p0;
}

pdd incenter(pdd p1,pdd p2,pdd p3,double &radius){
  double a=abs(p2-p1),b=abs(p3-p1),c=abs(p3-p2);
  double s=(a+b+c)/2,area=sqrt(s*(s-a)*(s-b)*(s-c));
  pdd L1=external_bisector(p1,p2,p3),L2=
      external_bisector(p2,p1,p3);
  return radius=area/s,intersect(p1,p1+L1,p2,p2+L2),
}

pdd escenter(pdd p1,pdd p2,pdd p3){//213
  pdd L1=external_bisector(p1,p2,p3),L2=
      external_bisector(p2,p2+p2-p1,p3);
  return intersect(p1,p1+L1,p2,p2+L2);
}

pdd barycenter(pdd p1,pdd p2,pdd p3){
  return (p1+p2+p3)/3;
}

pdd orthocenter(pdd p1,pdd p2,pdd p3){
  pdd L1=p3-p2,L2=p3-p1;
  swap(L1.X,L1.Y),L1.X*=-1;
  swap(L2,X,L2.Y),L2.X*=-1;
  return intersect(p1,p1+L1,p2,p2+L2);
}
```

8.5 Polar Angle Sort

```
pdd c;//sort base
int Quadrant(pdd a){
  if(a.X>0&&a.Y>=0)  return 1;
  if(a.X<=0&&a.Y>0)  return 2;
  if(a.X<0&&a.Y<=0)  return 3;
  if(a.X>=0&&a.Y<0)  return 4;
}
bool cmp(pdd a,pdd b){
  a=a-c,b=b-c;
  if(Quadrant(a)!=Quadrant(b))
    return Quadrant(a)<Quadrant(b);
  if(cross(b,a)==0) return abs(a)<abs(b);
  return cross(b,a)>0;
}
bool cmp(pdd a,pdd b){
  a=a-c,b=b-c;
  if(fabs(atan2(a.Y,a.X)-atan2(b.Y,b.X))>eps)
    return atan2(a.Y,a.X)<atan2(b.Y,b.X);
  return abs(a)<abs(b);
}
```

8.6 Intersection of two circles

```
vector<pdd> interCircle(pdd o1 ,double r1 ,pdd o2 ,
    double r2){
  if(abs(o1-o2)<max(r1,r2)-min(r1,r2)) return {};
  double d2=abs2(o1-o2);
  double d=sqrt(d2);
  if(d>r1+r2) return {};
  pdd u = (o1+o2)*0.5 + (o1-o2)*((r2*r2-r1*r1)/(2*d2));
  double A = sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2
      +d));
  pdd v = pdd(o1.Y-o2.Y,-o1.X+o2.X)*A/(2*d2);
  return {u+v, u-v};
}
```

8.7 Intersection of polygon and circle

```
// Divides into multiple triangle, and sum up
// test by HDU2892
const double PI=acos(-1);
double _area(pdd pa, pdd pb, double r){
  if(abs(pa)<abs(pb)) swap(pa, pb);
  if(abs(pb)<eps) return 0;
  double S, h, theta;
  double a=abs(pb),b=abs(pa),c=abs(pb-pa);
  double cosB = dot(pb,pb-pa) / a / c, B = acos(cosB);
  double cosC = dot(pa,pb) / a / b, C = acos(cosC);
  if(a > r){
    S = (C/2)*r*r;
    h = a*b*sin(C)/c;
    if (h < r && B < PI/2) S -= (acos(h/r)*r*r - h*sqrt
        (r*r-h*h));
  }
  else if(b > r){
    theta = PI - B - asin(sin(B)/r*a);
    S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
  }
  else S = .5*sin(C)*a*b;
  return S;
}
double area_poly_circle(const vector<pdd> poly,const
    pdd &O,const double r){
  double S=0;
  for(int i=0;i<SZ(poly);++i)
    S+=_area(poly[i]-O,poly[(i+1)%SZ(poly)]-O,r)*ori(O,
        poly[i],poly[(i+1)%SZ(poly)]);
  return fabs(S);
}
```

8.8 Intersection of line and circle

```
vector<pdd> line_interCircle(const pdd &p1,const pdd &
    p2,const pdd &c,const double r){
  pdd ft=foot(p1,p2,c),vec=p2-p1;
  double dis=abs(c-ft);
  if(fabs(dis-r)<eps) return vector<pdd>{ft};
  if(dis>r) return {};
  vec=vec*sqrt(r*r-dis*dis)/abs(vec);
  return vector<pdd>{ft+vec,ft-vec};
}
```

8.9 Half plane intersection

```
bool isin( Line l0, Line l1, Line l2 ){
  // Check inter(l1, l2) in l0
  pdd p = intersect(l1.X,l1.Y,l2.X,l2.Y);
  return cross(l0.Y - l0.X,p - l0.X) > eps;
}
/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.Y - l.X) ^ (p - l.X) > 0
 */
/* --^-- Line.X --^-- Line.Y --^-- */
vector<Line> halfPlaneInter(vector<Line> lines){
  int sz = lines.size();
  vector<double> ata(sz),ord(sz);
  for(int i=0; i<sz; ++i) {
    ord[i] = i;
    pdd d = lines[i].Y - lines[i].X;
    ata[i] = atan2(d.Y, d.X);
  }
  sort(ord.begin(), ord.end(), [&](int i,int j){
      if( fabs(ata[i] - ata[j]) < eps )
      return (cross(lines[i].Y-lines[i].X,
            lines[j].Y-lines[i].X))<0;
      return ata[i] < ata[j];
      });
  vector<Line> fin;
  for (int i=0; i<sz; ++i)
    if (!i || fabs(ata[ord[i]] - ata[ord[i-1]]) > eps)
      fin.pb(lines[ord[i]]);
  deque<Line> dq;
```

```
  for (int i=0; i<SZ(fin); i++){
    while(SZ(dq)>=2&&!isin(fin[i],dq[SZ(dq)-2],dq.back
        ()))
      dq.pop_back();
    while(SZ(dq)>=2&&!isin(fin[i],dq[0],dq[1]))
      dq.pop_front();
    dq.push_back(fin[i]);
  }
  while(SZ(dq)>=3&&!isin(dq[0],dq[SZ(dq)-2],dq.back()))
    dq.pop_back();
  while(SZ(dq)>=3&&!isin(dq.back(), dq[0], dq[1]))
    dq.pop_front();
  vector<Line> res(ALL(dq));
  return res;
}
```

8.10 Convexhull3D

```
struct Point{
  double x,y,z;
  Point(double x=0,double y=0,double z=0):x(x),y(y),z(z
      ){}
  Point operator-(const Point p1){return Point(x-p1.x,y
      -p1.y,z-p1.z);}
  Point operator*(Point p){return Point(y*p.z-z*p.y,z*p
      .x-x*p.z,x*p.y-y*p.x);}
  double operator^(Point p){return (x*p.x+y*p.y+z*p.z)
      ;}
};
struct CH3D{
  struct face{int a,b,c;bool ok;}F[8*MAXN];
  int g[MAXN][MAXN],num,n;
  Point P[MAXN];
  double vlen(Point a){return sqrt(a.x*a.x+a.y*a.y+a.z*
      a.z);}
  Point cross(const Point &a, const Point &b, const
      Point &c){
    return Point((b.y-a.y)*(c.z-a.z)-(b.z-a.z)*(c.y-a.y
        ),-((b.x-a.x)*(c.z-a.z)-(b.z-a.z)*(c.x-a.x)),(b
        .x-a.x)*(c.y-a.y)-(b.y-a.y)*(c.x-a.x));
  }
  double area(Point a,Point b,Point c){return vlen((b-a
      )*(c-a));}
  double volume(Point a,Point b,Point c,Point d){return
       (b-a)*(c-a)^(d-a);}
  double dblcmp(Point &p,face &f){return ((P[f.b]-P[f.a
      ])*(P[f.c]-P[f.a]))^(p-P[f.a]);}
  void deal(int p,int a,int b){
    int f=g[a][b];
    face add;
    if(F[f].ok)
      if(dblcmp(P[p],F[f])>EPS) dfs(p,f);
      else
        add.a=b,add.b=a,add.c=p,add.ok=1,g[p][b]=g[a][p
            ]=g[b][a]=num,F[num++]=add;
  }
  void dfs(int p,int now){
    F[now].ok=0;
    deal(p,F[now].b,F[now].a),deal(p,F[now].c,F[now].b)
        ,deal(p,F[now].a,F[now].c);
  }
  bool same(int s,int t){
    Point &a=P[F[s].a];
    Point &b=P[F[s].b];
    Point &c=P[F[s].c];
    return fabs(volume(a,b,c,P[F[t].a]))<EPS && fabs(
        volume(a,b,c,P[F[t].b]))<EPS && fabs(volume(a,b
        ,c,P[F[t].c]))<EPS;
  }
  void init(int _n){n=_n,num=0;}
  void solve(){
    face add;
    bool flag=true;
    num=0;
    if(n<4) return;
    if([&](){
        for(int i=1;i<n;++i)if(vlen(P[0]-P[i])>EPS)
            return swap(P[1],P[i]),0;return 1;}() ||
            [&](){
        for(int i=2;i<n;++i)if(vlen((P[0]-P[i])*(P[1]-P
            [i]))>EPS)return swap(P[2],P[i]),0;return
            1;}() || [&](){
        for(int i=3;i<n;++i)if(fabs((P[0]-P[1])*(P[1]-P
            [2])^(P[0]-P[i]))>EPS)return swap(P[3],P[i
            ]),0;return 1;}())return;
    for(int i=0;i<4;++i){
      add.a=(i+1)%4,add.b=(i+2)%4,add.c=(i+3)%4,add.ok=
          true;
      if(dblcmp(P[i],add)>0) swap(add.b,add.c);
      g[add.a][add.b]=g[add.b][add.c]=g[add.c][add.a]=
          num;
      F[num++]=add;
    }
    for(int i=4;i<n;++i)
      for(int j=0;j<num;++j)
        if(F[j].ok && dblcmp(P[i],F[j])>EPS){dfs(i,j);
            break;}
    for(int tmp=num,i=(num=0);i<tmp;++i)
      if(F[i].ok) F[num++]=F[i];
  }
  double area(){
    double res=0.0;
    if(n==3)
      return vlen(cross(P[0],P[1],P[2]))/2.0;
    for(int i=0;i<num;++i)
      res+=area(P[F[i].a],P[F[i].b],P[F[i].c]);
    return res/2.0;
  }
  double volume(){
    double res=0.0;
    for(int i=0;i<num;i++)
      res+=volume(Point(0,0,0),P[F[i].a],P[F[i].b],P[F[
          i].c]);
    return fabs(res/6.0);
  }
  int triangle(){return num;}
  int polygon(){
    int res=0;
    for(int i=0,flag=1;i<num;++i,res+=flag,flag=1)
      for(int j=0;j<i&&flag;++j)
        flag&=!same(i,j);
    return res;
  }
  Point getcent(){
    Point ans(0,0,0),temp=P[F[0].a];
    double v = 0.0,t2;
    for(int i=0;i<num;++i)
      if(F[i].ok == true){
        Point p1=P[F[i].a],p2=P[F[i].b],p3=P[F[i].c];
        t2 = volume(temp,p1,p2,p3)/6.0;
        if(t2>0)
          ans.x += (p1.x+p2.x+p3.x+temp.x)*t2, ans.y +=
               (p1.y+p2.y+p3.y+temp.y)*t2, ans.z += (p1
              .z+p2.z+p3.z+temp.z)*t2, v += t2;
      }
    ans.x/=(4*v),ans.y/=(4*v),ans.z/=(4*v);
    return ans;
  }
  double pointmindis(Point fuck){
    double min=99999999;
    for(int i=0;i<num;i++)
      if(F[i].ok==true){
        Point p1=P[F[i].a] , p2=P[F[i].b] , p3=P[F[i].c
            ];
        double a = ( (p2.y-p1.y)*(p3.z-p1.z)-(p2.z-p1.z
            )*(p3.y-p1.y) );
        double b = ( (p2.z-p1.z)*(p3.x-p1.x)-(p2.x-p1.x
            )*(p3.z-p1.z) );
        double c = ( (p2.x-p1.x)*(p3.y-p1.y)-(p2.y-p1.y
            )*(p3.x-p1.x) );
        double d = ( 0-(a*p1.x+b*p1.y+c*p1.z) );
        double temp = fabs(a*fuck.x+b*fuck.y+c*fuck.z+d
            )/sqrt(a*a+b*b+c*c);
        if(temp<min) min = temp;
      }
    return min;
  }
}hull;
```

8.11 CircleCover

```
#define N 1021
struct CircleCover{
  int C; Circ c[ N ];
  bool g[ N ][ N ], overlap[ N ][ N ];
  // Area[i] : area covered by at least i circles
  D Area[ N ];
  void init( int _C ){ C = _C; }
  bool CCinter( Circ& a , Circ& b , Pt& p1 , Pt& p2 ){
    Pt o1 = a.O , o2 = b.O;
    D r1 = a.R , r2 = b.R;
    if( norm( o1 - o2 ) > r1 + r2 ) return {};
    if( norm( o1 - o2 ) < max(r1, r2) - min(r1, r2) )
        return {};
    D d2 = ( o1 - o2 ) * ( o1 - o2 );
    D d = sqrt(d2);
    if( d > r1 + r2 ) return false;
    Pt u=(o1+o2)*0.5 + (o1-o2)*((r2*r2-r1*r1)/(2*d2));
    D A=sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2+d));
    Pt v=Pt( o1.Y-o2.Y , -o1.X + o2.X ) * A / (2*d2);
    p1 = u + v; p2 = u - v;
    return true;
  }
  struct Teve {
    Pt p; D ang; int add;
    Teve() {}
    Teve(Pt _a, D _b, int _c):p(_a), ang(_b), add(_c){}
    bool operator<(const Teve &a)const
    {return ang < a.ang;}
  }eve[ N * 2 ];
  // strict: x = 0, otherwise x = -1
  bool disjuct( Circ& a, Circ &b, int x )
  {return sign( norm( a.O - b.O ) - a.R - b.R ) > x;}
  bool contain( Circ& a, Circ &b, int x )
  {return sign( a.R - b.R - norm( a.O - b.O ) ) > x;}
  bool contain(int i, int j){
    /* c[j] is non-strictly in c[i]. */
    return (sign(c[i].R - c[j].R) > 0 ||
        (sign(c[i].R - c[j].R) == 0 && i < j) ) &&
            contain(c[i], c[j], -1);
  }
  void solve(){
    for( int i = 0 ; i <= C + 1 ; i ++ )
      Area[ i ] = 0;
    for( int i = 0 ; i < C ; i ++ )
      for( int j = 0 ; j < C ; j ++ )
        overlap[i][j] = contain(i, j);
    for( int i = 0 ; i < C ; i ++ )
      for( int j = 0 ; j < C ; j ++ )
        g[i][j] = !(overlap[i][j] || overlap[j][i] ||
            disjuct(c[i], c[j], -1));
    for( int i = 0 ; i < C ; i ++ ){
      int E = 0, cnt = 1;
      for( int j = 0 ; j < C ; j ++ )
        if( j != i && overlap[j][i] )
          cnt ++;
      for( int j = 0 ; j < C ; j ++ )
        if( i != j && g[i][j] ){
          Pt aa, bb;
          CCinter(c[i], c[j], aa, bb);
          D A=atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);
          D B=atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
          eve[E ++] = Teve(bb, B, 1);
          eve[E ++] = Teve(aa, A, -1);
          if(B > A) cnt ++;
        }
      if( E == 0 ) Area[ cnt ] += pi * c[i].R * c[i].R;
      else{
        sort( eve , eve + E );
        eve[E] = eve[0];
        for( int j = 0 ; j < E ; j ++ ){
          cnt += eve[j].add;
          Area[cnt] += (eve[j].p ^ eve[j + 1].p) * .5;
          D theta = eve[j + 1].ang - eve[j].ang;
          if (theta < 0) theta += 2. * pi;
          Area[cnt] +=
            (theta - sin(theta)) * c[i].R*c[i].R * .5;
        }
      }
    }
  }
};
```

8.12 DelaunayTriangulation

```
/* Delaunay Triangulation:
   Given a sets of points on 2D plane, find a
   triangulation such that no points will strictly
   inside circumcircle of any triangle.
find : return a triangle contain given point
add_point : add a point into triangulation
A Triangle is in triangulation iff. its has_chd is 0.
Region of triangle u: iterate each u.edge[i].tri,
each points are u.p[(i+1)%3], u.p[(i+2)%3]
calculation involves O(|V|^6) */
const double inf = 1e9;
double eps = 1e-6;  // 0 when integer
// return p4 is in circumcircle of tri(p1,p2,p3)
bool in_cc(P &p1, P &p2, P &p3, P &p4) {
  int o1 = (abs(p1.x) >= inf * 0.99 || abs(p1.y) >= inf
       * 0.99);
  int o2 = (abs(p2.x) >= inf * 0.99 || abs(p2.y) >= inf
       * 0.99);
  int o3 = (abs(p3.x) >= inf * 0.99 || abs(p3.y) >= inf
       * 0.99);
  int rtrue = o1 + o2 + o3;
  int rfalse = abs(p4.x) >= inf * 0.99 || abs(p4.y) >=
      inf * 0.99;
  if (rtrue == 3) return true;
  if (rtrue) {
    P in(0, 0), out(0, 0);
    if (o1) out = out + p1; else in = in + p1;
    if (o2) out = out + p2; else in = in + p2;
    if (o3) out = out + p3; else in = in + p3;
    return (p4 - in) * (out - in) > 0;
  }
  if (rfalse) return false;
  // ^ ?
  double u11 = p1.x - p4.x, u12 = p1.y - p4.y;
  double u21 = p2.x - p4.x, u22 = p2.y - p4.y;
  double u31 = p3.x - p4.x, u32 = p3.y - p4.y;
  double u13 = sq(p1.x) - sq(p4.x) + sq(p1.y) - sq(p4.y
      );
  double u23 = sq(p2.x) - sq(p4.x) + sq(p2.y) - sq(p4.y
      );
  double u33 = sq(p3.x) - sq(p4.x) + sq(p3.y) - sq(p4.y
      );
  double det = -u13 * u22 * u31 + u12 * u23 * u31 + u13
       * u21 * u32 - u11 * u23 * u32 - u12 * u21 * u33
      + u11 * u22 * u33;
  return det > eps;
}
double side(P &a, P &b, P &p) { return (b - a) ^ (p - a
    ); }
struct Tri;
struct Edge {
  Tri *tri;
  int side;
  Edge() : tri(0), side(0) {}
  Edge(Tri *_tri, int _side) : tri(_tri), side(_side)
      {}
};
struct Tri {
  P p[3];
  Edge edge[3];
  Tri *ch[3];
  Tri() {}
  Tri(P p0, P p1, P p2) {
    p[0] = p0; p[1] = p1; p[2] = p2;
    ch[0] = ch[1] = ch[2] = 0;
  }
  bool has_ch() { return ch[0] != 0; }
  int num_ch() {
    return ch[0] == 0 ? 0 : ch[1] == 0 ? 1 : ch[2] == 0
         ? 2 : 3;
  }
  bool contains(P &q) {
    for (int i = 0; i < 3; ++i)
      if (side(p[i], p[(i + 1) % 3], q) < -eps) return
          false;
```

```
    return true;
  }
} pool[maxn * 10], *tris;
void edge(Edge a, Edge b) {
  if (a.tri) a.tri->edge[a.side] = b;
  if (b.tri) b.tri->edge[b.side] = a;
}
struct Trig {
  Trig() {
    the_root = new (tris++) Tri(P(-inf, -inf), P(inf *
        2, -inf), P(-inf, inf * 2));
  }  // all p should in
  Tri *find(P p) { return find(the_root, p); }
  void add_point(P &p) { add_point(find(the_root, p), p
      ); }
  Tri *the_root;
  static Tri *find(Tri *root, P &p) {
    while (true) {
      if (!root->has_ch()) return root;
      for (int i = 0; i < 3 && root->ch[i]; ++i)
        if (root->ch[i]->contains(p)) {
          root = root->ch[i];
          break;
        }
    }
    assert(false);  // "point not found"
  }
  void add_point(Tri *root, P &p) {
    Tri *tab, *tbc, *tca;
    tab = new (tris++) Tri(root->p[0], root->p[1], p);
    tbc = new (tris++) Tri(root->p[1], root->p[2], p);
    tca = new (tris++) Tri(root->p[2], root->p[0], p);
    edge(Edge(tab, 0), Edge(tbc, 1));
    edge(Edge(tbc, 0), Edge(tca, 1));
    edge(Edge(tca, 0), Edge(tab, 1));
    edge(Edge(tab, 2), root->edge[2]);
    edge(Edge(tbc, 2), root->edge[0]);
    edge(Edge(tca, 2), root->edge[1]);
    root->ch[0] = tab; root->ch[1] = tbc; root->ch[2] =
        tca;
    flip(tab, 2); flip(tbc, 2); flip(tca, 2);
  }
  void flip(Tri *tri, int pi) {
    Tri *trj = tri->edge[pi].tri;
    int pj = tri->edge[pi].side;
    if (!trj) return;
    if (!in_cc(tri->p[0], tri->p[1], tri->p[2], trj->p[
        pj])) return;
    /* flip edge between tri,trj */
    Tri *trk = new (tris++) Tri(tri->p[(pi + 1) % 3],
        trj->p[pj], tri->p[pi]);
    Tri *trl = new (tris++) Tri(trj->p[(pj + 1) % 3],
        tri->p[pi], trj->p[pj]);
    edge(Edge(trk, 0), Edge(trl, 0));
    edge(Edge(trk, 1), tri->edge[(pi + 2) % 3]);
    edge(Edge(trk, 2), trj->edge[(pj + 1) % 3]);
    edge(Edge(trl, 1), trj->edge[(pj + 2) % 3]);
    edge(Edge(trl, 2), tri->edge[(pi + 1) % 3]);
    tri->ch[0] = trk; tri->ch[1] = trl; tri->ch[2] = 0;
    trj->ch[0] = trk; trj->ch[1] = trl; trj->ch[2] = 0;
    flip(trk, 1); flip(trk, 2);
    flip(trl, 1); flip(trl, 2);
  }
};
vector<Tri *> triang;
set<Tri *> vst;
void go(Tri *now) {
  if (vst.find(now) != vst.end()) return;
  vst.insert(now);
  if (!now->has_ch()) {
    triang.push_back(now);
    return;
  }
  for (int i = 0; i < now->num_ch(); ++i) go(now->ch[i
      ]);
}
void build(int n, P *ps) {
  tris = pool;
  random_shuffle(ps, ps + n);
  Trig tri;
  for (int i = 0; i < n; ++i) tri.add_point(ps[i]);
  go(tri.the_root);
}
```

8.13 Triangulation Vonoroi

```
int gid(P &p) {
  auto it = ptoid.find(p);
  if (it == ptoid.end()) return -1;
  return it->second;
}
L make_line(P p, L l) {
  P d = l.pb - l.pa; d = d.spin(pi / 2);
  P m = (l.pa + l.pb) / 2;
  l = L(m, m + d);
  if (((l.pb - l.pa) ^ (p - l.pa)) < 0) l = L(m + d, m)
      ;
  return l;
}
double calc_ans(int i) {
  vector<P> ps = HPI(ls[i]);
  double rt = 0;
  for (int i = 0; i < (int)ps.size(); ++i) {
    rt += (ps[i] ^ ps[(i + 1) % ps.size()]);
  }
  return abs(rt) / 2;
}
void solve() {
  for (int i = 0; i < n; ++i) ops[i] = ps[i], ptoid[ops
      [i]] = i;
  random_shuffle(ps, ps + n);
  build(n, ps);
  for (auto *t : triang) {
    int z[3] = {gid(t->p[0]), gid(t->p[1]), gid(t->p
        [2])};
    for (int i = 0; i < 3; ++i) for (int j = 0; j < 3;
        ++j) if (i != j && z[i] != -1 && z[j] != -1) {
      L l(t->p[i], t->p[j]);
      ls[z[i]].push_back(make_line(t->p[i], l));
    }
  }
  vector<P> tb = convex(vector<P>(ps, ps + n));
  for (auto &p : tb) isinf[gid(p)] = true;
  for (int i = 0; i < n; ++i) {
    if (isinf[i]) cout << -1 << '\n';
    else cout << fixed << setprecision(12) << calc_ans(
        i) << '\n';
  }
}
```

8.14 Tangent line of two circles

```
vector<Line> go( const Cir& c1 , const Cir& c2 , int
    sign1 ){
  // sign1 = 1 for outer tang, -1 for inter tang
  vector<Line> ret;
  double d_sq = norm2( c1.O - c2.O );
  if( d_sq < eps ) return ret;
  double d = sqrt( d_sq );
  Pt v = ( c2.O - c1.O ) / d;
  double c = ( c1.R - sign1 * c2.R ) / d;
  if( c * c > 1 ) return ret;
  double h = sqrt( max( 0.0 , 1.0 - c * c ) );
  for( int sign2 = 1 ; sign2 >= -1 ; sign2 -= 2 ){
    Pt n = { v.X * c - sign2 * h * v.Y ,
      v.Y * c + sign2 * h * v.X };
    Pt p1 = c1.O + n * c1.R;
    Pt p2 = c2.O + n * ( c2.R * sign1 );
    if( fabs( p1.X - p2.X ) < eps and
        fabs( p1.Y - p2.Y ) < eps )
      p2 = p1 + perp( c2.O - c1.O );
    ret.push_back( { p1 , p2 } );
  }
  return ret;
}
```

8.15 minMaxEnclosingRectangle

```
pdd solve(vector<pll> &dots){
  vector<pll> hull;
  const double INF=1e18,qi=acos(-1)/2*3;
  cv.dots=dots;
  hull=cv.hull();
  double Max=0,Min=INF,deg;
  ll n=hull.size();
  hull.pb(hull[0]);
  for(int i=0,u=1,r=1,l;i<n;++i){
    pll nw=hull[i+1]-hull[i];
    while(cross(nw,hull[u+1]-hull[i])>cross(nw,hull[u]-
        hull[i]))
      u=(u+1)%n;
    while(dot(nw,hull[r+1]-hull[i])>dot(nw,hull[r]-hull
        [i]))
      r=(r+1)%n;
    if(!i) l=(r+1)%n;
    while(dot(nw,hull[l+1]-hull[i])<dot(nw,hull[l]-hull
        [i]))
      l=(l+1)%n;
    Min=min(Min,(double)(dot(nw,hull[r]-hull[i])-dot(nw
        ,hull[l]-hull[i]))*cross(nw,hull[u]-hull[i])/
        abs2(nw));
    deg=acos((double)dot(hull[r]-hull[l],hull[u]-hull[i
        ])/abs(hull[r]-hull[l])/abs(hull[u]-hull[i]));
    deg=(qi-deg)/2;
    Max=max(Max,(double)abs(hull[r]-hull[l])*abs(hull[u
        ]-hull[i])*sin(deg)*sin(deg));
  }
  return pdd(Min,Max);
}
```

8.16 minDistOfTwoConvex

```
// p, q is convex
double TwoConvexHullMinDist(Point P[], Point Q[], int n
    , int m) {
  int YMinP = 0, YMaxQ = 0;
  double tmp, ans = 999999999;
  for (i = 0; i < n; ++i) if(P[i].y < P[YMinP].y) YMinP
       = i;
  for (i = 0; i < m; ++i) if(Q[i].y > Q[YMaxQ].y) YMaxQ
       = i;
  P[n] = P[0], Q[m] = Q[0];
  for (int i = 0; i < n; ++i) {
    while (tmp = Cross(Q[YMaxQ + 1] - P[YMinP + 1], P[
        YMinP] - P[YMinP + 1]) > Cross(Q[YMaxQ] - P[
        YMinP + 1], P[YMinP] - P[YMinP + 1])) YMaxQ = (
        YMaxQ + 1) % m;
    if (tmp < 0) ans = min(ans, PointToSegDist(P[YMinP
        ], P[YMinP + 1], Q[YMaxQ]));
    else ans = min(ans, TwoSegMinDist(P[YMinP], P[YMinP
         + 1], Q[YMaxQ], Q[YMaxQ + 1]));
    YMinP = (YMinP + 1) % n;
  }
  return ans;
}
```

8.17 Minkowski Sum

```
/* convex hull Minkowski Sum*/
#define INF 1000000000000000LL
int pos( const Pt& tp ){
  if( tp.Y == 0 ) return tp.X > 0 ? 0 : 1;
  return tp.Y > 0 ? 0 : 1;
}
#define N 300030
Pt pt[ N ], qt[ N ], rt[ N ];
LL Lx,Rx;
int dn,un;
inline bool cmp( Pt a, Pt b ){
  int pa=pos( a ),pb=pos( b );
  if(pa==pb) return (a^b)>0;
  return pa<pb;
}
int minkowskiSum(int n,int m){
  int i,j,r,p,q,fi,fj;
  for(i=1,p=0;i<n;i++){
    if( pt[i].Y<pt[p].Y ||
        (pt[i].Y==pt[p].Y && pt[i].X<pt[p].X) ) p=i; }
  for(i=1,q=0;i<m;i++){
    if( qt[i].Y<qt[q].Y ||
        (qt[i].Y==qt[q].Y && qt[i].X<qt[q].X) ) q=i; }
  rt[0]=pt[p]+qt[q];
  r=1; i=p; j=q; fi=fj=0;
  while(1){
    if((fj&&j==q) ||
        ( (!fi||i!=p) &&
          cmp(pt[(p+1)%n]-pt[p],qt[(q+1)%m]-qt[q]) ) ){
      rt[r]=rt[r-1]+pt[(p+1)%n]-pt[p];
      p=(p+1)%n;
      fi=1;
    }else{
      rt[r]=rt[r-1]+qt[(q+1)%m]-qt[q];
      q=(q+1)%m;
      fj=1;
    }
    if(r<=1 || ((rt[r]-rt[r-1])^(rt[r-1]-rt[r-2]))!=0)
        r++;
    else rt[r-1]=rt[r];
    if(i==p && j==q) break;
  }
  return r-1;
}
void initInConvex(int n){
  int i,p,q;
  LL Ly,Ry;
  Lx=INF; Rx=-INF;
  for(i=0;i<n;i++){
    if(pt[i].X<Lx) Lx=pt[i].X;
    if(pt[i].X>Rx) Rx=pt[i].X;
  }
  Ly=Ry=INF;
  for(i=0;i<n;i++){
    if(pt[i].X==Lx && pt[i].Y<Ly){ Ly=pt[i].Y; p=i; }
    if(pt[i].X==Rx && pt[i].Y<Ry){ Ry=pt[i].Y; q=i; }
  }
  for(dn=0,i=p;i!=q;i=(i+1)%n){ qt[dn++]=pt[i]; }
  qt[dn]=pt[q]; Ly=Ry=-INF;
  for(i=0;i<n;i++){
    if(pt[i].X==Lx && pt[i].Y>Ly){ Ly=pt[i].Y; p=i; }
    if(pt[i].X==Rx && pt[i].Y>Ry){ Ry=pt[i].Y; q=i; }
  }
  for(un=0,i=p;i!=q;i=(i+n-1)%n){ rt[un++]=pt[i]; }
  rt[un]=pt[q];
}
inline int inConvex(Pt p){
  int L,R,M;
  if(p.X<Lx || p.X>Rx) return 0;
  L=0;R=dn;
  while(L<R-1){ M=(L+R)/2;
    if(p.X<qt[M].X) R=M; else L=M; }
  if(tri(qt[L],qt[R],p)<0) return 0;
  L=0;R=un;
  while(L<R-1){ M=(L+R)/2;
    if(p.X<rt[M].X) R=M; else L=M; }
  if(tri(rt[L],rt[R],p)>0) return 0;
  return 1;
}
int main(){
  int n,m,i;
  Pt p;
  scanf("%d",&n);
  for(i=0;i<n;i++) scanf("%lld%lld",&pt[i].X,&pt[i].Y);
  scanf("%d",&m);
  for(i=0;i<m;i++) scanf("%lld%lld",&qt[i].X,&qt[i].Y);
  n=minkowskiSum(n,m);
  for(i=0;i<n;i++) pt[i]=rt[i];
  scanf("%d",&m);
  for(i=0;i<m;i++) scanf("%lld%lld",&qt[i].X,&qt[i].Y);
  n=minkowskiSum(n,m);
  for(i=0;i<n;i++) pt[i]=rt[i];
  initInConvex(n);
  scanf("%d",&m);
  for(i=0;i<m;i++){
    scanf("%lld %lld",&p.X,&p.Y);
    p.X*=3; p.Y*=3;
    puts(inConvex(p)?"YES":"NO");
  }
}
```

9 Else

9.1 Mo's Alogrithm(With modification)

```
struct QUERY{//BLOCK=N^{2/3}
  int L,R,id,LBid,RBid,T;
  QUERY(int l,int r,int id,int lb,int rb,int t):
    L(l),R(r),id(id),LBid(lb),RBid(rb),T(t){}
  bool operator<(const QUERY &b)const{
    if(LBid!=b.LBid) return LBid<b.LBid;
    if(RBid!=b.RBid) return RBid<b.RBid;
    return T<b.T;
  }
};
vector<QUERY> query;
int cur_ans,arr[MAXN],ans[MAXN];
void addTime(int L,int R,int T){}
void subTime(int L,int R,int T){}
void add(int x){}
void sub(int x){}
void solve(){
  sort(ALL(query));
  int L=0,R=0,T=-1;
  for(auto q:query){
    while(T<q.T) addTime(L,R,++T);
    while(T>q.T) subTime(L,R,T--);
    while(R<q.R) add(arr[++R]);
    while(L>q.L) add(arr[--L]);
    while(R>q.R) sub(arr[R--]);
    while(L<q.L) sub(arr[L++]);
    ans[q.id]=cur_ans;
  }
}
```

9.2 Mo's Alogrithm On Tree

```
const int MAXN=40005;
vector<int> G[MAXN];//1-base
int n,B,arr[MAXN],ans[100005],cur_ans;
int in[MAXN],out[MAXN],dfn[MAXN*2],dft;
int deep[MAXN],sp[__lg(MAXN*2)+1][MAXN*2],bln[MAXN],spt
    ;
bitset<MAXN> inset;
struct QUERY{
  int L,R,Lid,id,lca;
  QUERY(int l,int r,int _id):L(l),R(r),lca(0),id(_id){}
  bool operator<(const QUERY &b){
    if(Lid!=b.Lid) return Lid<b.Lid;
    return R<b.R;
  }
};
vector<QUERY> query;
void dfs(int u,int f,int d){
  deep[u]=d,sp[0][spt]=u,bln[u]=spt++;
  dfn[dft]=u,in[u]=dft++;
  for(int v:G[u])
    if(v!=f)
      dfs(v,u,d+1),sp[0][spt]=u,bln[u]=spt++;
  dfn[dft]=u,out[u]=dft++;
}
int lca(int u,int v){
  if(bln[u]>bln[v]) swap(u,v);
  int t=__lg(bln[v]-bln[u]+1);
  int a=sp[t][bln[u]],b=sp[t][bln[v]-(1<<t)+1];
  if(deep[a]<deep[b]) return a;
  return b;
}
void sub(int x){}
void add(int x){}
void flip(int x){
  if(inset[x]) sub(arr[x]);
  else add(arr[x]);
  inset[x]=~inset[x];
}
void solve(){
  B=sqrt(2*n),dft=spt=cur_ans=0,dfs(1,1,0);
  for(int i=1,x=2;x<2*n;++i,x<<=1)
    for(int j=0;j+x<=2*n;++j)
      if(deep[sp[i-1][j]]<deep[sp[i-1][j+x/2]])
        sp[i][j]=sp[i-1][j];
      else sp[i][j]=sp[i-1][j+x/2];
  for(auto &q:query){
    int c=lca(q.L,q.R);
    if(c==q.L||c==q.R)
      q.L=out[c==q.L?q.R:q.L],q.R=out[c];
    else if(out[q.L]<in[q.R])
      q.lca=c,q.L=out[q.L],q.R=in[q.R];
    else q.lca=c,c=in[q.L],q.L=out[q.R],q.R=c;
    q.Lid=q.L/B;
  }
  sort(ALL(query));
  int L=0,R=-1;
  for(auto q:query){
    while(R<q.R) flip(dfn[++R]);
    while(L>q.L) flip(dfn[--L]);
    while(R>q.R) flip(dfn[R--]);
    while(L<q.L) flip(dfn[L++]);
    if(q.lca) add(arr[q.lca]);
    ans[q.id]=cur_ans;
    if(q.lca) sub(arr[q.lca]);
  }
}
```

9.3 DynamicConvexTrick

```
// only works for integer coordinates!!

bool Flag; // 0: insert Line, 1: lower_bound x
template<class val = ll, class compare = less<val>> //
    sort lines with comp
struct DynamicConvexTrick{
  static const ll minx = 0, maxx = ll(1e9) + 5;
  static compare comp;
  struct Line{
    val a, b, l, r; // line ax + b in [l, r]
    Line(val _a, val _b, val _l = minx, val _r = maxx):
        a(_a), b(_b), l(_l), r(_r){}
    val operator () (val x) const {
      return a * x + b;
    }
  };
  struct cmp{
    bool operator () (const Line a, const Line b){
      if(Flag == 0)return comp(a.a, b.a);
      return a.r < b.l;
    }
  };
  inline val idiv(val a, val b){
    return a / b - (a % b && a < 0 ^ b < 0);
  }
  set<Line, cmp> st;
  void ins(val a, val b){
    Flag = 0;
    Line L(a, b);
    auto it = st.lower_bound(L);
    if(it != st.begin() && it != st.end())
      if(!comp((*prev(it))(it->l - 1), L(it->l - 1)) &&
           !comp((*it)(it->l), L(it->l)))
        return;
    while(it != st.end()){
      if(it->a == L.a && !comp(it->b, L.b))return;
      if(comp((*it)(it->r), L(it->r)))it = st.erase(it)
          ;
      else{
        Line M = *it;
        st.erase(it);
        L.r = max(idiv(L.b - M.b, M.a - L.a), minx);
        M.l = L.r + 1;
        it = st.insert(M).X;
        break;
      }
    }
    while(it != st.begin()){
      auto pit = prev(it);
      if(comp((*pit)(pit->l), L(pit->l)))st.erase(pit);
      else{
        Line M = *pit;
        st.erase(pit);
```

```
        M.r = min(idiv(L.b - M.b, M.a - L.a), maxx - 1)
            ;
        L.l = M.r + 1;
        st.insert(M);
        break;
      }
    }
    st.insert(L);
  }
  val operator () (val x){
    Flag = 1;
    auto it = st.lower_bound({0, 0, x, x});
    return (*it)(x);
  }
};

DynamicConvexTrick<> DCT;
```