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## 1 Basic

### 1.1 Shell script

```
g++ -O2 -std=c++17 -Dbbq -Wall -Wextra -Wshadow -o $1
    $1.cpp
chmod +x compile.sh
```

### 1.2 Default code

```
#include<bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;
#define X first
#define Y second
#define SZ(a) ((int)a.size())
#define ALL(v) v.begin(), v.end()
#define pb push_back
```

### 1.3 vimrc

```
"This file should be placed at ~/.vimrc"
se nu ai hls et ru ic is sc cul
se re=1 ts=4 sts=4 sw=4 ls=2 mouse=a
syntax on
hi cursorline cterm=none ctermbg=89
set bg=dark
inoremap {<ENTER> {}<LEFT><ENTER><ENTER><UP><TAB>
```

### 1.4 readchar

```
inline char readchar() {
    static const size_t bufsize = 65536;
    static char buf[bufsize];
    static char *p = buf, *end = buf;
    if (p == end) end = buf + fread_unlocked(buf, 1,
        bufsize, stdin), p = buf;
    return *p++;
}
```

### 1.5 Black Magic

```
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp> //rb_tree
using namespace __gnu_pbds;
typedef __gnu_pbds::priority_queue<int> heap;
int main() {
    heap h1, h2;
    h1.push(1), h1.push(3);
    h2.push(2), h2.push(4);
    h1.join(h2);
    cout << h1.size() << h2.size() << h1.top() << endl;
    //404
    tree<ll, null_type, less<ll>, rb_tree_tag,
        tree_order_statistics_node_update> st;
    tree<ll, ll, less<ll>, rb_tree_tag,
        tree_order_statistics_node_update> mp;
    for (int x : {0, 2, 3, 4}) st.insert(x);
    cout << *st.find_by_order(2) << st.order_of_key(1) <<
        endl; //31
}
```

## 1.6 Texas hold'em

```
char suit[4]={'C','D','H','Y'}, ranks[13]={'2','3','4','5','6','7','8','9','T','J','Q','K','A'};
int rk[256];
/*
for(int i=0;i<13;++i)
rk[ranks[i]]=i;
for(int i=0;i<4;++i)
rk[suit[i]]=i;
*/
struct cards{
vector<pii> v;
int suit_count[4], hands;
void reset(){v.clear(), FILL(suit_count, 0), hands=-1;}
void insert(char a, char b){//suit, rank
++suit_count[rk[a]];
int flag=0;
for(auto &i:v)
if(i.Y==rk[b])
{
++i.X, flag=1;
break;
}
if(!flag) v.pb(pii(1, rk[b]));
}
void insert(string s){insert(s[0], s[1]);}
void ready(){
int Straight=0, Flush=(max_element(suit_count, suit_count+4))==5;
sort(ALL(v), [](ii a, ii b){return a>b;});
if(SZ(v)==5&&v[0].Y==v[1].Y+1&&v[1].Y==v[2].Y+1&&v[2].Y==v[3].Y+1&&v[3].Y==v[4].Y+1)
Straight=1;
else if(SZ(v)==5&&v[0].Y==12&&v[1].Y==3&&v[2].Y==2&&v[3].Y==1&&v[4].Y==0)
v[0].Y=3, v[1].Y=2, v[2].Y=1, v[3].Y=0, v[4].Y=-1,
Straight=1;
if(Straight&&Flush) hands=1;
else if(v[0].X==4) hands=2;
else if(v[0].X==3&&v[1].X==2) hands=3;
else if(Flush) hands=4;
else if(Straight) hands=5;
else if(v[0].X==3) hands=6;
else if(v[0].X==2&&v[1].X==2) hands=7;
else if(v[0].X==2) hands=8;
else hands=9;
}
bool operator>(const cards &a) const{
if(hands==a.hands) return v>a.v;
return hands<a.hands;
}
};
```

## 2 Graph

### 2.1 BCC Vertex\*

```
vector<int> G[N]; //1-base
vector<int> nG[N], bcc[N];
int low[N], dfn[N], Time;
int bcc_id[N], bcc_cnt; //1-base
bool is_cut[N]; //whether is av
bool cir[N];
int st[N], top;
```

```
void dfs(int u, int pa = -1) {
int child = 0;
low[u] = dfn[u] = ++Time;
st[top++] = u;
for(int v : G[u])
if(!dfn[v]) {
dfs(v, u), ++child;
low[u] = min(low[u], low[v]);
if(dfn[u] <= low[v]) {
is_cut[u]=1;
bcc[++bcc_cnt].clear();
int t;
```

```
do {
bcc_id[t = st[--top]] = bcc_cnt;
bcc[bcc_cnt].push_back(t);
}while(t != v);
bcc_id[u]=bcc_cnt;
bcc[bcc_cnt].pb(u);
}
}
else if(dfn[v] < dfn[u] && v!=pa)
low[u] = min(low[u], dfn[v]);
if(pa == -1 && child < 2)
is_cut[u] = 0;
}

void bcc_init(int n) {
Time = bcc_cnt = top = 0;
for(int i = 1; i <= n; ++i)
G[i].clear(), dfn[i] = bcc_id[i] = is_cut[i] = 0;
}

void bcc_solve(int n) {
for(int i = 1; i <= n; ++i)
if(!dfn[i])
dfs(i);
// circle-square tree
for(int i = 1; i <= n; ++i)
if(is_cut[i])
bcc_id[i] = ++bcc_cnt, cir[bcc_cnt] = 1;
for(int i = 1; i <= bcc_cnt && !cir[i]; ++i)
for(int j : bcc[i])
if(is_cut[j])
nG[i].pb(bcc_id[j]), nG[bcc_id[j]].pb(i);
}
```

### 2.2 Bridge\*

```
int low[N], dfn[N], Time; // 1-base
vector<pii> G[N], edge;
vector<bool> is_bridge;

void init(int n) {
Time = 0;
for(int i = 1; i <= n; ++i)
G[i].clear(), low[i] = dfn[i] = 0;
}

void add_edge(int a, int b) {
G[a].pb(pii(b, SZ(edge))), G[b].pb(pii(a, SZ(edge)));
edge.pb(pii(a, b));
}

void dfs(int u, int f) {
dfn[u] = low[u] = ++Time;
for(auto i : G[u])
if(!dfn[i.X])
dfs(i.X, i.Y), low[u] = min(low[u], low[i.X]);
else if(i.Y != f)
low[u] = min(low[u], dfn[i.X]);
if(low[u] == dfn[u] && f != -1)
is_bridge[f] = 1;
}

void solve(int n) {
is_bridge.resize(SZ(edge));
for(int i = 1; i <= n; ++i)
if(!dfn[i])
dfs(i, -1);
}
```

### 2.3 Strongly Connected Components\*

```
struct Strongly_CC{//1-base
int low[N], dfn[N], bln[N], sz[N], n, Time, nScc;
bitset<N> instack;
stack<int> st;
vector<int> G[N], SCC[N];
void init(int _n) {
n = _n;
```

```

    for(int i = 1; i <= n; ++i)
        G[i].clear();
}
void add_edge(int a, int b) {
    G[a].pb(b);
}
void dfs(int u) {
    dfn[u] = low[u] = ++Time;
    instack[u] = 1, st.push(u);
    for(int i : G[u])
        if(!dfn[i]) dfs(i), low[u] = min(low[i], low[u]);
        else if(instack[i] && dfn[i] < dfn[u])
            low[u] = min(low[u], dfn[i]);
    if(low[u] == dfn[u]) {
        int tmp;
        do {
            tmp = st.top(), st.pop();
            instack[tmp] = 0, bln[tmp] = nScc;
        } while(tmp != u);
        ++nScc;
    }
}
void solve() {
    Time = nScc = 0;
    for(int i = 1; i <= n; ++i)
        SCC[i].clear(), low[i] = dfn[i] = bln[i] = sz[i] = 0;
    for(int i = 1; i <= n; ++i)
        if(!dfn[i])
            dfs(i);
    for(int i = 1; i <= n; ++i)
        ++sz[bln[i]], SCC[bln[i]].pb(i);
}
};

```

## 2.4 MinimumMeanCycle\*

```

ll road[N][N]; //input here
struct MinimumMeanCycle {
    ll dp[N + 5][N], n;
    pll solve() {
        ll a = -1, b = -1, L = n + 1;
        for(int i = 2; i <= L; ++i)
            for(int k = 0; k < n; ++k)
                for(int j = 0; j < n; ++j)
                    dp[i][j] = min(dp[i - 1][k] + road[k][j], dp[i][j]);
        for(int i = 0; i < n; ++i) {
            if(dp[L][i] >= INF) continue;
            ll ta = 0, tb = 1;
            for(int j = 1; j < n; ++j)
                if(dp[j][i] < INF && ta * (L - j) < (dp[L][i] - dp[j][i]) * tb)
                    ta = dp[L][i] - dp[j][i], tb = L - j;
            if(ta == 0) continue;
            if(a == -1 || a * tb > ta * b)
                a = ta, b = tb;
        }
        if(a != -1) {
            ll g = __gcd(a, b);
            return pll(a / g, b / g);
        }
        return pll(-1LL, -1LL);
    }
}
void init(int _n) {
    n = _n;
    for(int i = 0; i < n; ++i)
        for(int j = 0; j < n; ++j)
            dp[i + 2][j] = INF;
}
};

```

## 2.5 Virtual Tree\*

```

vector<int> vG[N];
int top, st[N];
void insert(int u) {

```

```

    if(top == -1)
        return st[++top] = u, void();
    int p = LCA(st[top], u);
    if(p == st[top])
        return st[++top] = u, void();
    while(top >= 1 && dep[st[top - 1]] >= dep[p])
        vG[st[top - 1]].pb(st[top]), --top;
    if(st[top] != p)
        vG[p].pb(st[top]), --top, st[++top] = p;
    st[++top] = u;
}

void reset(int u) {
    for(int i : vG[u])
        reset(i);
    vG[u].clear();
}

void solve(vector<int> &v) {
    top = -1;
    sort(ALL(v), [&](int a, int b){return dfn[a] < dfn[b];});
    for(int i : v)
        insert(i);
    while(top > 0)
        vG[st[top - 1]].pb(st[top]), --top;
    //do something
    reset(v[0]);
}

```

## 2.6 Maximum Clique Dyn\*

```

const int N = 150;
struct MaxClique { // Maximum Clique
    bitset<N> a[N], cs[N];
    int ans, sol[N], q, cur[N], d[N], n;
    void init(int _n) {
        n = _n;
        for(int i = 0; i < n; i++) a[i].reset();
    }
    void addEdge(int u, int v) { a[u][v] = a[v][u] = 1; }
    void csort(vector<int> &r, vector<int> &c) {
        int mx = 1, km = max(ans - q + 1, 1), t = 0, m = r.size();
        cs[1].reset(), cs[2].reset();
        for(int i = 0; i < m; i++) {
            int p = r[i], k = 1;
            while((cs[k] & a[p]).count()) k++;
            if(k > mx) mx++, cs[mx + 1].reset();
            cs[k][p] = 1;
            if(k < km) r[t++] = p;
        }
        c.resize(m);
        if(t) c[t - 1] = 0;
        for(int k = km; k <= mx; k++)
            for(int p = cs[k]._Find_first(); p < N; p = cs[k]._Find_next(p))
                r[t] = p, c[t] = k, t++;
    }
    void dfs(vector<int> &r, vector<int> &c, int l, bitset<N> mask) {
        while(!r.empty()) {
            int p = r.back();
            r.pop_back(), mask[p] = 0;
            if(q + c.back() <= ans) return;
            cur[q++] = p;
            vector<int> nr, nc;
            bitset<N> nmask = mask & a[p];
            for(int i : r)
                if(a[p][i]) nr.push_back(i);
            if(!nr.empty()) {
                if(l < 4) {
                    for(int i : nr) d[i] = (a[i] & nmask).count();
                    sort(nr.begin(), nr.end(), [&](int x, int y) {
                        return d[x] > d[y];
                    });
                }
                csort(nr, nc), dfs(nr, nc, l + 1, nmask);
            }
            else if(q > ans)
                ans = q, copy_n(cur, q, sol);
        }
    }
};

```

```

        c.pop_back(), q--;
    }
}
int solve(bitset<N> mask = bitset<N>(string(N, '1')))
{
    // vertex mask
    vector<int> r, c;
    ans = q = 0;
    for (int i = 0; i < n; i++)
        if (mask[i]) r.push_back(i);
    for (int i = 0; i < n; i++) d[i] = (a[i] & mask).
        count();
    sort(r.begin(), r.end(), [&](int i, int j) { return
        d[i] > d[j]; });
    csort(r, c), dfs(r, c, 1, mask);
    return ans; // sol[0 ~ ans-1]
}
} graph;

```

## 2.7 Minimum Steiner Tree\*

```

// Minimum Steiner Tree
// O(V 3^T + V^2 2^T)
struct SteinerTree{// 0-base
    static const int T = 10, N = 105, INF = 1e9;
    int n, dst[N][N], dp[1 << T][N], tdst[N];
    int vcost[N]; // the cost of vertices
    void init(int _n){
        n = _n;
        for(int i = 0; i < n; ++i) {
            for(int j = 0; j < n; ++j)
                dst[i][j] = INF;
            dst[i][i] = vcost[i] = 0;
        }
    }
    void add_edge(int ui, int vi, int wi) {
        dst[ui][vi] = min(dst[ui][vi], wi);
    }
    void shortest_path() {
        for(int k = 0; k < n; ++k)
            for(int i = 0; i < n; ++i)
                for(int j = 0; j < n; ++j)
                    dst[i][j] = min(dst[i][j], dst[i][k] + dst[k]
                        [j]);
    }
    int solve(const vector<int>& ter) {
        shortest_path();
        int t = SZ(ter);
        for(int i = 0; i < (1 << t); ++i)
            for(int j = 0; j < n; ++j)
                dp[i][j] = INF;
        for(int i = 0; i < n; ++i)
            dp[0][i] = vcost[i];
        for(int msk = 1; msk < (1 << t); ++msk){
            if(!(msk & (msk - 1))){
                int who = __lg(msk);
                for(int i = 0; i < n; ++i)
                    dp[msk][i] = vcost[ter[who]] + dst[ter[who]]
                        [i];
            }
            for(int i = 0; i < n; ++i)
                for(int submsk = (msk - 1) & msk; submsk;
                    submsk = (submsk - 1) & msk)
                    dp[msk][i] = min(dp[msk][i], dp[submsk][i] +
                        dp[msk ^ submsk][i] - vcost[i]);
            for(int i = 0; i < n; ++i) {
                tdst[i] = INF;
                for(int j = 0; j < n; ++j)
                    tdst[i] = min(tdst[i], dp[msk][j] + dst[j][i]
                        );
            }
            for(int i = 0; i < n; ++i)
                dp[msk][i] = tdst[i];
        }
        int ans = INF;
        for(int i = 0; i < n; ++i)
            ans = min(ans, dp[(1 << t) - 1][i]);
        return ans;
    }
};

```

## 2.8 Dominator Tree\*

```

struct dominator_tree{//1-base
    vector<int> G[N], rG[N];
    int n, pa[N], dfn[N], id[N], Time;
    int semi[N], idom[N], best[N];
    vector<int> tree[N]; //dominator_tree
    void init(int _n) {
        n = _n;
        for(int i = 1; i <= n; ++i)
            G[i].clear(), rG[i].clear();
    }
    void add_edge(int u, int v) {
        G[u].pb(v), rG[v].pb(u);
    }
    void dfs(int u) {
        id[dfn[u] = ++Time] = u;
        for(auto v : G[u])
            if(!dfn[v])
                dfs(v), pa[dfn[v]] = dfn[u];
    }
    int find(int y, int x) {
        if(y <= x)
            return y;
        int tmp = find(pa[y], x);
        if(semi[best[y]] > semi[best[pa[y]]])
            best[y] = best[pa[y]];
        return pa[y] = tmp;
    }
    void tarjan(int root) {
        Time = 0;
        for(int i = 1; i <= n; ++i){
            dfn[i] = idom[i] = 0;
            tree[i].clear();
            best[i] = semi[i] = i;
        }
        dfs(root);
        for(int i = Time; i > 1; --i) {
            int u = id[i];
            for(auto v : rG[u])
                if(v = dfn[v]) {
                    find(v, i);
                    semi[i] = min(semi[i], semi[best[v]]);
                }
            tree[semi[i]].pb(i);
            for(auto v : tree[pa[i]]) {
                find(v, pa[i]);
                idom[v] = semi[best[v]] == pa[i] ? pa[i] : best
                    [v];
            }
            tree[pa[i]].clear();
        }
        for(int i = 2; i <= Time; ++i) {
            if(idom[i] != semi[i])
                idom[i] = idom[idom[i]];
            tree[id[idom[i]]].pb(id[i]);
        }
    }
};

```

## 2.9 Minimum Arborescence\*

```

struct zhu_liu{//O(VE)
    struct edge{
        int u, v;
        ll w;
    };
    vector<edge> E; //0-base
    int pe[N], id[N], vis[N];
    ll in[N];
    void init() {E.clear();}
    void add_edge(int u, int v, ll w) {
        if (u != v) E.pb(edge{u, v, w});
    }
    ll build(int root, int n) {
        ll ans = 0;
        for(;;) {
            fill_n(in, n, INF);
            for (int i = 0; i < SZ(E); ++i)

```

## 2.10 Viziting's theorem

```
}  
if (!G[u][v0]) {  
    for (; v; v = flip(v, c, d), swap(c, d));  
    if (C[u][c0]) {  
        for (a = (int)L.size() - 2; a >= 0 && L[a].second != c; a--);  
        for (; a >= 0; a--) color(u, L[a].first, L[a].second);  
    } else t--;  
}  
}  
}
```

```

struct Clique_Cover { // Θ-base, O(n2^N)
    int co[1 << N], n, E[N];
    int dp[1 << N];
    void init(int _n) {
        n = _n, fill_n(dp, 1 << n, 0);
        fill_n(E, n, 0), fill_n(co, 1 << n, 0);
    }
    void add_edge(int u, int v) {
        E[u] |= 1 << v, E[v] |= 1 << u;
    }
    int solve() {
        for (int i = 0; i < n; ++i)
            co[1 << i] = E[i] | (1 << i);
        co[0] = (1 << n) - 1;
        dp[0] = (n & 1) * 2 - 1;
        for (int i = 1; i < (1 << n); ++i) {
            int t = i & -i;
            dp[i] = -dp[i ^ t];
            co[i] = co[i ^ t] & co[t];
        }
        for (int i = 0; i < (1 << n); ++i)
            co[i] = (co[i] & i) == i;
        for (int i = 0; i < n; ++i)
            dp[i] = (dp[i] & i) == i;
        for (int ans = 1; ans < n; ++ans) {
            int sum = 0;
            for (int i = 0; i < (1 << n); ++i)
                sum += (dp[i] & i) == i;
            if (sum) return ans;
        }
        return n;
    }
};

```

```

struct BronKerbosch { // 1-base
    int n, a[N], g[N][N];
    int S, all[N][N], some[N][N], none[N][N];
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            for (int j = 1; j <= n; ++j)
                g[i][j] = 0;
    }
    void add_edge(int u, int v) {
        g[u][v] = g[v][u] = 1;
    }
    void dfs(int d, int an, int sn, int nn) {
        if (S > 1000) return; // pruning
        if (sn == 0 && nn == 0) ++S;
        int u = some[d][0];
        for(int i = 0; i < sn; ++i) {
            int v = some[d][i];
            if(g[u][v]) continue;
            int tsu = 0, tnn = 0;
            copy_n(all[d], an, all[d + 1]);
            all[d + 1][an] = v;
            for(int j = 0; j < sn; ++j)
                if(g[v][some[d][j]])
                    some[d + 1][tsu++] = some[d][j];
            for(int j = 0; j < nn; ++j)
                if(g[v][none[d][j]])
                    none[d + 1][tnn++] = none[d][j];
        }
    }
};

```

```

    dfs(d + 1, an + 1, tsu, tnn);
    some[d][i] = 0, none[d][nn++] = v;
}
}
int solve() {
    iota(some[0], some[0] + n, 1);
    S = 0, dfs(0, 0, n, 0);
    return S;
}
};

```

## 2.13 Theory

Maximum independent edge set =  $|V| - \text{Minimum edge cover}$

Maximum independent set =  $|V| - \text{Minimum vertex cover}$

A sequence of non-negative integers  $d_1 \geq \dots \geq d_n$  can be represented as the degree sequence of a finite simple graph on  $n$  vertices if and only if  $d_1 + \dots + d_n$  is even and

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k) \text{ holds for every } k \text{ in } 1 \leq k \leq n.$$

```

if(!mxson[u]) return ;
cut(mxson[u],link);
for(auto i:G[u])
    if(i.X!=pa[u]&&i.X!=mxson[u])
        cut(i.X,i.X);
}
void build(){
    dfs(1,1,1),cut(1,1),/*build*/;
}
int query(int a,int b){
    int ta=ulink[a],tb=ulink[b],re=0;
    while(ta!=tb)
        if(deep[ta]<deep[tb])
            /*query*/,tb=ulink[b=pa[tb]];
        else
            /*query*/,ta=ulink[a=pa[ta]];
    if(a==b) return re;
    if(pl[a]>pl[b]) swap(a,b);
    /*query*/
    return re;
}
};

```

## 3 Data Structure

### 3.1 Leftist Tree

```

struct node{
    ll v,data,sz,sum;
    node *l,*r;
    node(ll k):v(0),data(k),sz(1),l(0),r(0),sum(k){}
};
ll sz(node *p){return p ? p->sz : 0;}
ll V(node *p){return p ? p->v : -1;}
ll sum(node *p){return p ? p->sum : 0;}
node* merge(node *a,node *b){
    if(!a || !b) return a ? a : b;
    if(a->data<b->data) swap(a,b);
    a->r=merge(a->r,b);
    if(V(a->r)>V(a->l)) swap(a->r,a->l);
    a->v=V(a->r)+1,a->sz=sz(a->l)+sz(a->r)+1;
    a->sum=sum(a->l)+sum(a->r)+a->data;
    return a;
}
void pop(node *&o){
    node *tmp=o;
    o=merge(o->l,o->r);
    delete tmp;
}

```

### 3.2 Heavy light Decomposition

```

struct Heavy_light_Decomposition{//1-base
    int n,ulink[10005],deep[10005],mxson[10005],w[10005],
        pa[10005];
    int t,pl[10005],data[10005],dt[10005],bln[10005],edge[10005],et;
    vector<pii> G[10005];
    void init(int _n){n=_n,t=0,et=1;
        for(int i=1;i<=n;++i) G[i].clear(),mxson[i]=0;
    }
    void add_edge(int a,int b,int w){
        G[a].pb(pii(b,et)),G[b].pb(pii(a,et)),edge[et++]=w;
    }
    void dfs(int u,int f,int d){
        w[u]=1,pa[u]=f,deep[u]=d++;
        for(auto &i:G[u])
            if(i.X!=f){
                dfs(i.X,u,d),w[u]+=w[i.X];
                if(w[mxson[u]]<w[i.X])
                    mxson[u]=i.X;
            }
            else
                bln[i.Y]=u,dt[u]=edge[i.Y];
    }
    void cut(int u,int link){
        data[pl[u]=t++]=dt[u],ulink[u]=link;
    }
}

```

### 3.3 Centroid Decomposition\*

```

struct Cent_Dec { // 1-base
    vector<pii> G[N];
    pii info[N]; // store info. of itself
    pii upinfo[N]; // store info. of climbing up
    int n, pa[N], layer[N], sz[N], done[N];
    ll dis[lg(N) + 1][N];
    void init(int _n) {
        n = _n, layer[0] = -1;
        fill_n(pa + 1, n, 0), fill_n(done + 1, n, 0);
        for (int i = 1; i <= n; ++i) G[i].clear();
    }
    void add_edge(int a, int b, int w) {
        G[a].pb(pii(b, w)), G[b].pb(pii(a, w));
    }
    void get_cent(int u, int f, int &mx, int &c, int num) {
        int mxsz = 0;
        sz[u] = 1;
        for (pii e : G[u])
            if (!done[e.X] && e.X != f) {
                get_cent(e.X, u, mx, c, num);
                sz[u] += sz[e.X], mxsz = max(mxsz, sz[e.X]);
            }
        if (mx > max(mxsz, num - sz[u]))
            mx = max(mxsz, num - sz[u]), c = u;
    }
    void dfs(int u, int f, ll d, int org) {
        // if required, add self info or climbing info
        dis[layer[org]][u] = d;
        for (pii e : G[u])
            if (!done[e.X] && e.X != f)
                dfs(e.X, u, d + e.Y, org);
    }
    int cut(int u, int f, int num) {
        int mx = 1e9, c = 0, lc;
        get_cent(u, f, mx, c, num);
        done[c] = 1, pa[c] = f, layer[c] = layer[f] + 1;
        for (pii e : G[c])
            if (!done[e.X]) {
                if (sz[e.X] > sz[c])
                    lc = cut(e.X, c, num - sz[c]);
                else
                    lc = cut(e.X, c, sz[e.X]);
                upinfo[lc] = pii(), dfs(e.X, c, e.Y, c);
            }
        return done[c] = 0, c;
    }
    void build(){cut(1, 0, n);}
    void modify(int u) {
        for (int a = u, ly = layer[a]; a; a = pa[a], --ly)
            {
                info[a].X += dis[ly][u], ++info[a].Y;
                if (pa[a])
                    upinfo[a].X += dis[ly - 1][u], ++upinfo[a].Y;
            }
    }
}

```



```

}
ll query(int u) {
    ll rt = 0;
    for (int a = u, ly = layer[a]; a; a = pa[a], --ly)
    {
        rt += info[a].X + info[a].Y * dis[ly][u];
        if (pa[a])
            rt -= upinfo[a].X + upinfo[a].Y * dis[ly - 1][u];
    }
    return rt;
}
};

```

### 3.4 link cut tree

```

const int MXN = 100005;
const int MEM = 100005;
struct Splay {
    static Splay nil, mem[MEM], *pmem;
    Splay *ch[2], *f;
    int val, rev, size;
    Splay (int _val=-1) : val(_val), rev(0), size(1)
    { f = ch[0] = ch[1] = &nil; }
    bool isr()
    { return f->ch[0] != this && f->ch[1] != this; }
    int dir()
    { return f->ch[0] == this ? 0 : 1; }
    void setCh(Splay *c, int d){
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void push(){
        if (!rev) return;
        swap(ch[0], ch[1]);
        if (ch[0] != &nil) ch[0]->rev ^= 1;
        if (ch[1] != &nil) ch[1]->rev ^= 1;
        rev=0;
    }
    void pull(){
        size = ch[0]->size + ch[1]->size + 1;
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::mem;
Splay *nil = &Splay::nil;
void rotate(Splay *x){
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[!d], d);
    x->setCh(p, !d);
    p->pull(); x->pull();
}
vector<Splay*> splayVec;
void splay(Splay *x){
    splayVec.clear();
    for (Splay *q=x;; q=q->f){
        splayVec.push_back(q);
        if (q->isr()) break;
    }
    reverse(begin(splayVec), end(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir()==x->f->dir())
            rotate(x->f), rotate(x);
        else rotate(x), rotate(x);
    }
}
int id(Splay *x) { return x - Splay::mem + 1; }
Splay* access(Splay *x){
    Splay *q = nil;
    for (;x!=nil;x=x->f){
        splay(x);
        x->setCh(q, 1);
        q = x;
    }
}

```

```

}
return q;
}
void chroot(Splay *x){
    access(x);
    splay(x);
    x->rev ^= 1;
    x->push(); x->pull();
}
void link(Splay *x, Splay *y){
    access(x);
    splay(x);
    chroot(y);
    x->setCh(y, 1);
}
void cut_p(Splay *y) {
    access(y);
    splay(y);
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}
void cut(Splay *x, Splay *y){
    chroot(x);
    cut_p(y);
}
Splay* get_root(Splay *x) {
    access(x);
    splay(x);
    for (; x->ch[0] != nil; x = x->ch[0])
        x->push();
    splay(x);
    return x;
}
bool conn(Splay *x, Splay *y) {
    x = get_root(x);
    y = get_root(y);
    return x == y;
}
Splay* lca(Splay *x, Splay *y) {
    access(x);
    access(y);
    splay(x);
    if (x->f == nil) return x;
    else return x->f;
}
}

```

### 3.5 KDTree

```

namespace kdt {
    int root, lc[maxn], rc[maxn], xl[maxn], xr[maxn], yl[
        maxn], yr[maxn];
    point p[maxn];
    int build(int l, int r, int dep = 0) {
        if (l == r) return -1;
        function<bool(const point &, const point &)> f = [
            dep](const point &a, const point &b) {
                if (dep & 1) return a.x < b.x;
                else return a.y < b.y;
            };
        int m = (l + r) >> 1;
        nth_element(p + l, p + m, p + r, f);
        xl[m] = xr[m] = p[m].x;
        yl[m] = yr[m] = p[m].y;
        lc[m] = build(l, m, dep + 1);
        if (~lc[m]) {
            xl[m] = min(xl[m], xl[lc[m]]);
            xr[m] = max(xr[m], xr[lc[m]]);
            yl[m] = min(yl[m], yl[lc[m]]);
            yr[m] = max(yr[m], yr[lc[m]]);
        }
        rc[m] = build(m + 1, r, dep + 1);
        if (~rc[m]) {
            xl[m] = min(xl[m], xl[rc[m]]);
            xr[m] = max(xr[m], xr[rc[m]]);
            yl[m] = min(yl[m], yl[rc[m]]);
            yr[m] = max(yr[m], yr[rc[m]]);
        }
        return m;
    }
}
bool bound(const point &q, int o, long long d) {
}

```

```

double ds = sqrt(d + 1.0);
if (q.x < xl[o] - ds || q.x > xr[o] + ds ||
    q.y < yl[o] - ds || q.y > yr[o] + ds) return
    false;
return true;
}
long long dist(const point &a, const point &b) {
    return (a.x - b.x) * 1ll * (a.x - b.x) +
        (a.y - b.y) * 1ll * (a.y - b.y);
}
void dfs(const point &q, long long &d, int o, int dep =
    0) {
    if (!bound(q, o, d)) return;
    long long cd = dist(p[o], q);
    if (cd != 0) d = min(d, cd);
    if ((dep & 1) && q.x < p[o].x || !(dep & 1) && q.y
        < p[o].y) {
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
    } else {
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
    }
}
void init(const vector<point> &v) {
    for (int i = 0; i < v.size(); ++i) p[i] = v[i];
    root = build(0, v.size());
}
long long nearest(const point &q) {
    long long res = 1e18;
    dfs(q, res, root);
    return res;
}
}

```

## 4 Flow/Matching

### 4.1 Kuhn Munkres

```

struct KM{// 0-base
    int w[MAXN][MAXN], h1[MAXN], hr[MAXN], slk[MAXN], n;
    int fl[MAXN], fr[MAXN], pre[MAXN], qu[MAXN], ql, qr;
    bool vl[MAXN], vr[MAXN];
    void init(int _n){n=_n;
        for(int i=0;i<n;++i)
            for(int j=0;j<n;++j)
                w[i][j]=-INF;
    }
    void add_edge(int a,int b,int wei){
        w[a][b]=wei;
    }
    bool Check(int x){
        if(vl[x]=1,~fl[x]) return vr[qu[qr++]=fl[x]]=1;
        while(~x) swap(x,fr[fl[x]=pre[x]]);
        return 0;
    }
    void Bfs(int s){
        fill(slk,slk+n,INF);
        fill(vl,vl+n,0),fill(vr,vr+n,0);
        ql=qr=0,qu[qr++]=s,vr[s]=1;
        while(1){
            int d;
            while(ql<qr)
                for(int x=0,y=qu[ql++];x<n;++x)
                    if(!vl[x]&&slk[x]>=(d=h1[x]+hr[y]-w[x][y]))
                        if(pre[x]=y,d) slk[x]=d;
                        else if(!Check(x)) return;
            d=INF;
            for (int x=0;x<n;++x)
                if (!vl[x]&&d>slk[x]) d=slk[x];
            for (int x=0;x<n;++x){
                if(vl[x]) h1[x]+=d;
                else slk[x]-=d;
                if(vr[x]) hr[x]-=d;
            }
            for (int x=0;x<n;++x)
                if(!vl[x]&&slk[x]&&!Check(x)) return;
        }
    }
}

```

```

int Solve(){
    fill(fl,fl+n,-1),fill(fr,fr+n,-1),fill(hr,hr+n,0);
    for (int i=0;i<n;++i) h1[i]=*max_element(w[i],w[i]+
        n);
    for (int i=0;i<n;++i) Bfs(i);
    int res=0;
    for (int i=0;i<n;++i) res += w[i][fl[i]];
    return res;
}
};

```

### 4.2 MincostMaxflow

```

struct MCMF{//0-base
    struct edge{
        ll from,to,cap,flow,cost,rev;
    }*past[MAXN];
    vector<edge> G[MAXN];
    bitset<MAXN> inq;
    ll dis[MAXN],up[MAXN],s,t,mx,n;
    bool BellmanFord(ll &flow,ll &cost){
        fill(dis,dis+n,INF);
        queue<ll> q;
        q.push(s),inq.reset(),inq[s]=1;
        up[s]=mx-flow,past[s]=0,dis[s]=0;
        while(!q.empty()){
            ll u=q.front();
            q.pop(),inq[u]=0;
            if(!up[u]) continue;
            for(auto &e:G[u])
                if(e.flow!=e.cap&&dis[e.to]>dis[u]+e.cost){
                    dis[e.to]=dis[u]+e.cost,past[e.to]=&e;
                    up[e.to]=min(up[u],e.cap-e.flow);
                    if(!inq[e.to]) inq[e.to]=1,q.push(e.to);
                }
        }
        if(dis[t]==INF) return 0;
        flow+=up[t],cost+=up[t]*dis[t];
        for(ll i=t;past[i];i=past[i]->from){
            auto &e=*past[i];
            e.flow+=up[t],G[e.to][e.rev].flow-=up[t];
        }
        return 1;
    }
    ll MinCostMaxFlow(ll _s,ll _t,ll &cost){
        s=_s,t=_t,cost=0;ll flow=0;
        while(BellmanFord(flow,cost));
        return flow;
    }
    void init(ll _n,ll _mx){n=_n,mx=_mx;
        for(int i=0;i<n;++i) G[i].clear();
    }
    void add_edge(ll a,ll b,ll cap,ll cost){
        G[a].pb(edge{a,b,cap,0,cost,G[b].size()});
        G[b].pb(edge{b,a,0,0,-cost,G[a].size()-1});
    }
}
};

```

### 4.3 Maximum Simple Graph Matching\*

```

struct GenMatch { // 1-base
    int V, pr[N];
    bool el[N][N], inq[N], inp[N], inb[N];
    int st, ed, nb, bk[N], djs[N], ans;
    void init(int _V) {
        V=_V;
        for(int i = 0; i <= V; ++i) {
            for(int j = 0; j <= V; ++j)
                el[i][j] = 0;
            pr[i] = bk[i] = djs[i] = 0;
            inq[i] = inp[i] = inb[i] = 0;
        }
    }
    void add_edge(int u, int v){
        el[u][v] = el[v][u] = 1;
    }
    int lca(int u, int v) {
        fill_n(inp, V + 1, 0);
    }
}

```



```

while(1)
    if(u = djs[u], inp[u] = true, u == st) break;
    else u = bk[pr[u]];
while(1)
    if(v = djs[v], inp[v]) return v;
    else v = bk[pr[v]];
return v;
}
void upd(int u){
    for(int v; djs[u] != nb;){
        v = pr[u], inb[djs[u]] = inb[djs[v]] = true;
        u = bk[v];
        if(djs[u] != nb) bk[u] = v;
    }
}
void blo(int u, int v, queue<int> &qe) {
    nb = lca(u, v), fill_n(inb, V + 1, 0);
    upd(u), upd(v);
    if(djs[u] != nb) bk[u] = v;
    if(djs[v] != nb) bk[v] = u;
    for(int tu = 1; tu <= V; ++tu)
        if(inb[djs[tu]])
            if(djs[tu] = nb, !inq[tu])
                qe.push(tu), inq[tu]=1;
}
void flow() {
    fill_n(inq + 1, V, 0), fill_n(bk + 1, V, 0);
    iota(djs + 1, djs + V + 1, 1);
    queue<int> qe;
    qe.push(st), inq[st] = 1, ed = 0;
    while(!qe.empty()) {
        int u = qe.front();
        qe.pop();
        for(int v = 1; v <= V; ++v)
            if(el[u][v] && djs[u] != djs[v] && pr[u] != v)
                {
                    if((v == st) || (pr[v] > 0 && bk[pr[v]] > 0))
                        blo(u, v, qe);
                    else if(!bk[v]) {
                        if(bk[v] = u, pr[v] > 0) {
                            if(!inq[pr[v]])
                                qe.push(pr[v]);
                        }
                        else
                            return ed = v, void();
                    }
                }
    }
}
void aug(){
    for(int u = ed, v, w; u > 0;){
        v = bk[u], w = pr[v], pr[v] = u, pr[u] = v, u = w;
    }
}
int solve() {
    fill_n(pr, V + 1, 0), ans = 0;
    for(int u = 1; u <= V; ++u)
        if(!pr[u])
            if(st = u, flow(), ed > 0)
                aug(), ++ans;
    return ans;
}
};

```

#### 4.4 Minimum Weight Matching (Clique version)\*

```

struct Graph { // 0-base (Perfect Match), n is even
    int n, match[N], onstk[N], stk[N], tp;
    ll edge[N][N], dis[N];
    void init(int _n) {
        n = _n, tp = 0;
        for (int i = 0; i < n; ++i)
            fill_n(edge[i], n, 0);
    }
    void add_edge(int u, int v, ll w) { edge[u][v] = edge[v][u] = w; }
    bool SPFA(int u) {
        stk[tp++] = u, onstk[u] = 1;

```

```

        for (int v = 0; v < n; ++v)
            if (!onstk[v] && match[u] != v) {
                int m = match[v];
                if (dis[m] > dis[u] - edge[v][m] + edge[u][v])
                    {
                        dis[m] = dis[u] - edge[v][m] + edge[u][v];
                        onstk[v] = 1, stk[tp++] = v;
                        if (onstk[m] || SPFA(m)) return 1;
                        --tp, onstk[v] = 0;
                    }
            }
        onstk[u] = 0, --tp;
        return 0;
    }
}
ll solve() { // find a match
    for (int i = 0; i < n; ++i) match[i] = i ^ 1;
    while (1) {
        int found = 0;
        fill_n(dis, n, 0); fill_n(onstk, n, 0);
        for (int i = 0; i < n; ++i)
            if (tp = 0, !onstk[i] && SPFA(i))
                for (found = 1; tp >= 2;){
                    int u = stk[--tp];
                    int v = stk[--tp];
                    match[u] = v, match[v] = u;
                }
            if (!found) break;
    }
    ll ret = 0;
    for (int i = 0; i < n; ++i) ret += edge[i][match[i]];
    return ret >> 1;
}
};

```

#### 4.5 SW-mincut

```

// global min cut
struct SW{ // O(V^3)
    static const int MXN = 514;
    int n, vst[MXN], del[MXN];
    int edge[MXN][MXN], wei[MXN];
    void init(int _n){
        n = _n, MEM(edge, 0), MEM(del, 0);
    }
    void addEdge(int u, int v, int w){
        edge[u][v] += w, edge[v][u] += w;
    }
    void search(int &s, int &t){
        MEM(vst, 0), MEM(wei, 0), s = t = -1;
        while(1){
            int mx = -1, cur = 0;
            for(int i = 0; i < n; ++i)
                if(!del[i] && !vst[i] && mx < wei[i])
                    cur = i, mx = wei[i];
            if(mx == -1) break;
            vst[cur] = 1, s = t, t = cur;
            for(int i = 0; i < n; ++i)
                if(!vst[i] && !del[i]) wei[i] += edge[cur][i];
        }
    }
    int solve(){
        int res = INF;
        for(int i = 0, x, y; i < n - 1; ++i){
            search(x, y), res = min(res, wei[y]), del[y] = 1;
            for(int j = 0; j < n; ++j)
                edge[x][j] = (edge[j][x] + edge[y][j]);
        }
        return res;
    }
}
};

```

#### 4.6 BoundedFlow(Dinic\*)

```

struct BoundedFlow { // 0-base
    struct edge {
        int to, cap, flow, rev;
    };

```

```

vector<edge> G[N];
int n, s, t, dis[N], cur[N], cnt[N];
void init(int _n) {
    n = _n;
    for (int i = 0; i < n + 2; ++i)
        G[i].clear(), cnt[i] = 0;
}
void add_edge(int u, int v, int lcap, int rcap) {
    cnt[u] -= lcap, cnt[v] += lcap;
    G[u].pb(edge{v, rcap, lcap, SZ(G[v])});
    G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
}
void add_edge(int u, int v, int cap) {
    G[u].pb(edge{v, cap, 0, SZ(G[v])});
    G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
}
int dfs(int u, int cap) {
    if (u == t || !cap) return cap;
    for (int &i = cur[u]; i < SZ(G[u]); ++i) {
        edge &e = G[u][i];
        if (dis[e.to] == dis[u] + 1 && e.cap != e.flow) {
            int df = dfs(e.to, min(e.cap - e.flow, cap));
            if (df) {
                e.flow += df, G[e.to][e.rev].flow -= df;
                return df;
            }
        }
    }
    dis[u] = -1;
    return 0;
}
bool bfs() {
    fill_n(dis, n + 3, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        for (edge &e : G[u])
            if (!dis[e.to] && e.flow != e.cap)
                q.push(e.to), dis[e.to] = dis[u] + 1;
    }
    return dis[t] != -1;
}
int maxflow(int _s, int _t) {
    s = _s, t = _t;
    int flow = 0, df;
    while (bfs()) {
        fill_n(cur, n + 3, 0);
        while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
}
bool solve() {
    int sum = 0;
    for (int i = 0; i < n; ++i)
        if (cnt[i] > 0) add_edge(n + 1, i, cnt[i]), sum += cnt[i];
        else if (cnt[i] < 0) add_edge(i, n + 2, -cnt[i]);
    if (sum != maxflow(n + 1, n + 2)) sum = -1;
    for (int i = 0; i < n; ++i)
        if (cnt[i] > 0) G[n + 1].pop_back(), G[i].pop_back();
        else if (cnt[i] < 0) G[i].pop_back(), G[n + 2].pop_back();
    return sum != -1;
}
int solve(int _s, int _t) {
    add_edge(_t, _s, INF);
    if (!solve()) return -1; //invalid flow
    int x = G[_t].back().flow;
    return G[_t].pop_back(), G[_s].pop_back(), x;
}
};

```

## 4.7 Gomory Hu tree

```

struct Gomory_Hu_tree{//0-base
    MaxFlow Dinic;
    int n;

```

```

    vector<pii> G[MAXN];
    void init(int _n){n=_n;
        for(int i=0;i<n;++i) G[i].clear();
    }
    void solve(vector<int> &v){
        if(v.size()<=1) return;
        int s=rand()%SZ(v);
        swap(v.back(),v[s]),s=v.back();
        int t=v[rand()%SZ(v)-1];
        vector<int> L,R;
        int x=(Dinic.reset(),Dinic.maxflow(s,t));
        G[s].pb(pii(t,x)),G[t].pb(pii(s,x));
        for(int i:v)
            if(~Dinic.dis[i]) L.pb(i);
            else R.pb(i);
        solve(L),solve(R);
    }
    void build(){
        vector<int> v(n);
        for(int i=0;i<n;++i) v[i]=i;
        solve(v);
    }
}ght;//test by BZOJ 4519
MaxFlow &Dinic=ght.Dinic;

```

## 4.8 isap

```

struct Maxflow {
    static const int MAXV = 20010;
    static const int INF = 1000000;
    struct Edge {
        int v, c, r;
        Edge(int _v, int _c, int _r):
            v(_v), c(_c), r(_r) {}
    };
    int s, t;
    vector<Edge> G[MAXV*2];
    int iter[MAXV*2], d[MAXV*2], gap[MAXV*2], tot;
    void init(int x) {
        tot = x+2;
        s = x+1, t = x+2;
        for (int i = 0; i <= tot; i++) {
            G[i].clear();
            iter[i] = d[i] = gap[i] = 0;
        }
    }
    void addEdge(int u, int v, int c) {
        G[u].push_back(Edge(v, c, SZ(G[v])));
        G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
    }
    int dfs(int p, int flow) {
        if (p == t) return flow;
        for (int &i = iter[p]; i < SZ(G[p]); i++) {
            Edge &e = G[p][i];
            if (e.c > 0 && d[p] == d[e.v] + 1) {
                int f = dfs(e.v, min(flow, e.c));
                if (f) {
                    e.c -= f;
                    G[e.v][e.r].c += f;
                    return f;
                }
            }
        }
        if (--gap[d[p]] == 0) d[s] = tot;
        else {
            d[p]++;
            iter[p] = 0;
            ++gap[d[p]];
        }
        return 0;
    }
    int solve() {
        int res = 0;
        gap[0] = tot;
        for (res = 0; d[s] < tot; res += dfs(s, INF));
        return res;
    }
} flow;

```

## 5 String

### 5.1 KMP

```
int F[MAXN];
vector<int> match(string A, string B){
    vector<int> ans;
    F[0]=-1, F[1]=0;
    for(int i=1, j=0; i<B.size(); F[++i]=++j){
        if(B[i]==B[j]) F[i]=F[j]; //optimize
        while(j!=-1 && B[i]!=B[j]) j=F[j];
    }
    for(int i=0, j=0; i-j+B.size()<=A.size(); ++i, ++j){
        while(j!=-1 && A[i]!=B[j]) j=F[j];
        if(j==B.size()-1) ans.pb(i-j);
    }
    return ans;
}
```

### 5.2 Z-value

```
const int MAXN = 1e5 + 5;
int z[MAXN];
void make_z(string s){
    int l = 0, r = 0;
    for(int i = 1; i < s.size(); i++){
        for(z[i] = max(0, min(r - i + 1, z[i - 1]));
            i + z[i] < s.size() && s[i + z[i]] == s[z[i]]; z
                [i]++);
        if(i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
}
```

### 5.3 Manacher\*

```
int z[MAXN];
int Manacher(string tmp){
    string s = "&";
    int l=0, r=0, x, ans;
    for(char c:tmp) s.pb(c), s.pb('%');
    ans=0, x=0;
    for(int i=1; i<SZ(s); ++i){
        z[i]=r > i ? min(z[2*l-i], r-i) : 1;
        while(s[i+z[i]]==s[i-z[i]]) ++z[i];
        if(z[i]+i>r) r=z[i]+i, l=i;
    }
    for(int i=1; i<SZ(s); ++i)
        if(s[i]=='%')
            x=max(x, z[i]);
    ans=x/2*2, x=0;
    for(int i=1; i<SZ(s); ++i)
        if(s[i]!='%')
            x=max(x, z[i]);
    return max(ans, (x-1)/2*2+1);
}
```

### 5.4 Suffix Array

```
struct suffix_array{
    int box[MAXN], tp[MAXN], m;
    bool not_equ(int a, int b, int k, int n){
        return ra[a] != ra[b] || a+k>n || b+k>n || ra[a+k] != ra[b+k];
    }
    void radix(int *key, int *it, int *ot, int n){
        fill_n(box, m, 0);
        for(int i=0; i<n; ++i) ++box[key[i]];
        partial_sum(box, box+m, box);
        for(int i=n-1; i>=0; --i) ot[--box[key[it[i]]]] = it[i];
    }
    void make_sa(string s, int n){
        int k=1;
        for(int i=0; i<n; ++i) ra[i]=s[i];
    }
}
```

```
do{
    iota(tp, tp+k, n-k), iota(sa+k, sa+n, 0);
    radix(ra+k, sa+k, tp+k, n-k);
    radix(ra, tp, sa, n);
    tp[sa[0]]=0, m=1;
    for(int i=1; i<n; ++i){
        m+=not_equ(sa[i], sa[i-1], k, n);
        tp[sa[i]]=m-1;
    }
    copy_n(tp, n, ra);
    k*=2;
}while(k<n && m!=n);
}
void make_he(string s, int n){
    for(int j=0, k=0; j<n; ++j){
        if(ra[j])
            for(; s[j+k]==s[sa[ra[j]-1]+k]; ++k);
        he[ra[j]]=k, k=max(0, k-1);
    }
}
int sa[MAXN], ra[MAXN], he[MAXN];
void build(string s){
    FILL(sa, 0), FILL(ra, 0), FILL(he, 0);
    FILL(box, 0), FILL(tp, 0), m=256;
    make_sa(s, s.size());
    make_he(s, s.size());
}
```

### 5.5 SAIS\*

```
class SAIS {
public:
    int *SA, *H;
    // zero based, string content MUST > 0
    // result height H[i] is LCP(SA[i - 1], SA[i])
    // string, length, |sigma|
    void build(int *s, int n, int m = 128){
        copy_n(s, n, _s);
        _h[0] = _s[n++] = 0;
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
        SA = _sa + 1; H = _h + 1;
    }
private:
    bool _t[N * 2];
    int _s[N * 2], _c[N * 2], x[N], _p[N], _q[N * 2], r
        [N], _sa[N * 2], _h[N];
    void mkhei(int n){
        for(int i = 0; i < n; i++) r[_sa[i]] = i;
        for(int i = 0; i < n; i++) if(r[i]) {
            int ans = i > 0 ? max(_h[r[i] - 1] - 1, 0) : 0;
            while(_s[i + ans] == _s[_sa[r[i] - 1] + ans])
                ans++;
            _h[r[i]] = ans;
        }
    }
    void sais(int *s, int *sa, int *p, int *q, bool *t,
        int *c, int n, int z){
        bool uniq = t[n - 1] = 1, neq;
        int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
            lst = -1;

```

```
#define MAGIC(XD) \
    fill_n(sa, n, 0); \
    copy_n(c, z, x); \
    XD; \
    copy_n(c, z - 1, x + 1); \
    for(int i = 0; i < n; i++) if(sa[i] && !t[sa[i]
        - 1]) \
        sa[x[s[sa[i]-1]]++] = sa[i] - 1; \
    copy_n(c, z, x); \
    for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[
        i]-1]) \
        sa[--x[s[sa[i]-1]]] = sa[i] - 1;

    fill_n(c, z, 0);
    for(int i = 0; i < n; i++) uniq &= ++c[s[i]] <
        2;
    partial_sum(c, c + z, c);
}
```

```

    if (uniq) {
        for (int i = 0; i < n; i++) sa[--c[s[i]]] = i;
        return;
    }
    for (int i = n - 2; i >= 0; i--)
        t[i] = (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
    MAGIC(
        for (int i = 1; i <= n - 1; i++) if (t[i] && !t[i - 1])
            sa[--x[s[i]]] = p[q[i] = nn++] = i;
        for (int i = 0; i < n; i++) if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
            neq = (lst < 0) || !equal(s + lst, s + lst + p[q[sa[i]] + 1] - sa[i], s + sa[i]);
            ns[q[lst = sa[i]]] = nmzx += neq;
        }
        sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx + 1);
        MAGIC(
            for (int i = nn - 1; i >= 0; i--)
                sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
        );
    }
} sa;

```

## 5.6 Aho-Corasick Automatan

```

const int len=400000,sigma=26;
struct AC_Automatan{
    int nx[len][sigma],fl[len],cnt[len],pri[len],top;
    int newnode(){
        fill(nx[top],nx[top]+sigma,-1);
        return top++;
    }
    void init(){top=1,newnode();}
    int input(string &s){//return the end_node of string
        int X=1;
        for(char c:s){
            if(!nx[X][c-'a'])nx[X][c-'a']=newnode();
            X=nx[X][c-'a'];
        }
        return X;
    }
    void make_fl(){
        queue<int> q;
        q.push(1),fl[1]=0;
        for(int t=0;!q.empty();){
            int R=q.front();
            q.pop(),pri[t++]=R;
            for(int i=0;i<sigma;++i)
                if(~nx[R][i]){
                    int X=nx[R][i],Z=fl[R];
                    for(;Z&&!~nx[Z][i];)Z=fl[Z];
                    fl[X]=Z?nx[Z][i]:1,q.push(X);
                }
        }
    }
    void get_v(string &s){
        int X=1;
        fill(cnt,cnt+top,0);
        for(char c:s){
            while(X&&!~nx[X][c-'a'])X=fl[X];
            X=X?nx[X][c-'a']:1,++cnt[X];
        }
        for(int i=top-2;i>0;--i) cnt[fl[pri[i]]]+=cnt[pri[i]];
    }
};

```

## 5.7 Smallest Rotation

```

string mcp(string s){
    int n=SZ(s),i=0,j=1;
    s+=s;
    while(i<n&&j<n){
        int k=0;

```

```

        while(k<n&&s[i+k]==s[j+k]) ++k;
        if(s[i+k]<=s[j+k]) j+=k+1;
        else i+=k+1;
        if(i==j) ++j;
    }
    int ans=i<n?i:j;
    return s.substr(ans,n);
}

```

## 5.8 De Bruijn sequence\*

```

constexpr int MAXC = 10, MAXN = 1e5 + 10;
struct DBSeq {
    int C, N, K, L, buf[MAXC * MAXN]; //K <= C^N
    void dfs(int *out, int t, int p, int &ptr) {
        if (ptr >= L) return;
        if (t > N) {
            if (N % p) return;
            for (int i = 1; i <= p && ptr < L; ++i)
                out[ptr++] = buf[i];
        } else {
            buf[t] = buf[t - p], dfs(out, t + 1, p, ptr);
            for (int j = buf[t - p] + 1; j < C; ++j)
                buf[t] = j, dfs(out, t + 1, t, ptr);
        }
    }
    void solve(int _c, int _n, int _k, int *out) {
        int p = 0;
        C = _c, N = _n, K = _k, L = N + K - 1;
        dfs(out, 1, 1, p);
        if (p < L) fill(out + p, out + L, 0);
    }
} dbs;

```

## 5.9 SAM

```

const int MAXM = 1000010;
struct SAM{
    int tot, root, lst, mom[MAXM], mx[MAXM];
    int acc[MAXM], nxt[MAXM][33];
    int newNode(){
        int res = ++tot;
        fill(nxt[res], nxt[res]+33, 0);
        mom[res] = mx[res] = acc[res] = 0;
        return res;
    }
    void init(){
        tot = 0;
        root = newNode();
        mom[root] = 0, mx[root] = 0;
        lst = root;
    }
    void push(int c){
        int p = lst;
        int np = newNode();
        mx[np] = mx[p]+1;
        for(; p && nxt[p][c] == 0; p = mom[p])
            nxt[p][c] = np;
        if(p == 0) mom[np] = root;
        else{
            int q = nxt[p][c];
            if(mx[p]+1 == mx[q]) mom[np] = q;
            else{
                int nq = newNode();
                mx[nq] = mx[p]+1;
                for(int i = 0; i < 33; i++)
                    nxt[nq][i] = nxt[q][i];
                mom[nq] = mom[q];
                mom[q] = nq;
                mom[np] = nq;
                for(; p && nxt[p][c] == q; p = mom[p])
                    nxt[p][c] = nq;
            }
        }
        lst = np;
    }
    void push(char *str){
        for(int i = 0; str[i]; i++)

```

```

    push(str[i]-'a'+1);
}
} sam;

```

## 5.10 PalTree

```

struct palindromic_tree{// Check by APIO 2014
    palindrome
    struct node{
        int next[26],fail,len;
        int cnt,num;//cnt: appear times, num: number of pal
            . suf.
        node(int l=0):fail(0),len(1),cnt(0),num(0){
            for(int i=0;i<26;++i)next[i]=0;
        }
    };
    vector<node>St;
    vector<char>s;
    int last,n;
    palindromic_tree():St(2),last(1),n(0){
        St[0].fail=1, St[1].len=-1, s.pb(-1);
    }
    inline void clear(){
        St.clear(), s.clear(), last=1, n=0;
        St.pb(0), St.pb(-1);
        St[0].fail=1, s.pb(-1);
    }
    inline int get_fail(int x){
        while(s[n-St[x].len-1]!=s[n])x=St[x].fail;
        return x;
    }
    inline void add(int c){
        s.push_back(c-'a'), ++n;
        int cur=get_fail(last);
        if(!St[cur].next[c]){
            int now=SZ(St);
            St.pb(St[cur].len+2);
            St[now].fail=St[get_fail(St[cur].fail)].next[c];
            St[cur].next[c]=now;
            St[now].num=St[St[now].fail].num+1;
        }
        last=St[cur].next[c], ++St[last].cnt;
    }
    inline void count(){// counting cnt
        auto i=St.rbegin();
        for(;i!=St.rend();++i){
            St[i->fail].cnt+=i->cnt;
        }
    }
    inline int size(){// The number of diff. pal.
        return SZ(St)-2;
    }
};

```

## 5.11 cyclicLCS

```

#define L 0
#define LU 1
#define U 2
const int mov[3][2]={0,-1, -1,-1, -1,0};
int al,bl;
char a[MAXL*2],b[MAXL*2]; // 0-indexed
int dp[MAXL*2][MAXL];
char pred[MAXL*2][MAXL];
inline int lcs_length(int r) {
    int i=r+al,j=bl,l=0;
    while(i>r) {
        char dir=pred[i][j];
        if(dir==LU) l++;
        i+=mov[dir][0];
        j+=mov[dir][1];
    }
    return l;
}
inline void reroot(int r) { // r = new base row
    int i=r,j=1;
    while(j<=bl&&pred[i][j]!=LU) j++;
    if(j>bl) return;

```

```

    pred[i][j]=L;
    while(i<2*al&&j<=bl) {
        if(pred[i+1][j]==U) {
            i++;
            pred[i][j]=L;
        } else if(j<bl&&pred[i+1][j+1]==LU) {
            i++;
            j++;
            pred[i][j]=L;
        } else {
            j++;
        }
    }
}
int cyclic_lcs() {
    // a, b, al, bl should be properly filled
    // note: a WILL be altered in process
    // -- concatenated after itself
    char tmp[MAXL];
    if(al>bl) {
        swap(al,bl);
        strcpy(tmp,a);
        strcpy(a,b);
        strcpy(b,tmp);
    }
    strcpy(tmp,a);
    strcat(a,tmp);
    // basic lcs
    for(int i=0;i<2*al;i++) {
        dp[i][0]=0;
        pred[i][0]=U;
    }
    for(int j=0;j<=bl;j++) {
        dp[0][j]=0;
        pred[0][j]=L;
    }
    for(int i=1;i<2*al;i++) {
        for(int j=1;j<=bl;j++) {
            if(a[i-1]==b[j-1]) dp[i][j]=dp[i-1][j-1]+1;
            else dp[i][j]=max(dp[i-1][j],dp[i][j-1]);
            if(dp[i][j-1]==dp[i][j]) pred[i][j]=L;
            else if(a[i-1]==b[j-1]) pred[i][j]=LU;
            else pred[i][j]=U;
        }
    }
    // do cyclic lcs
    int clcs=0;
    for(int i=0;i<al;i++) {
        clcs=max(clcs,lcs_length(i));
        reroot(i+1);
    }
    // recover a
    a[al]='\0';
    return clcs;
}

```

## 6 Math

### 6.1 ax+by=gcd\*

```

pll exgcd(ll a, ll b) {
    if(b == 0) return pll(1, 0);
    else {
        ll p = a / b;
        pll q = exgcd(b, a % b);
        return pll(q.Y, q.X - q.Y * p);
    }
}

```

### 6.2 floor and ceil

```

int floor(int a,int b){
    return a/b-(a%b&&a<0^b<0);
}
int ceil(int a,int b){
    return a/b+(a%b&&a<0^b>0);
}

```

### 6.3 floor sum\*

```

11 floor_sum(11 n, 11 m, 11 a, 11 b) {
    11 ans = 0;
    if (a >= m)
        ans += (n - 1) * n * (a / m) / 2, a %= m;
    if (b >= m)
        ans += n * (b / m), b %= m;
    11 y_max = (a * n + b) / m, x_max = (y_max * m - b)
        ;
    if (y_max == 0) return ans;
    ans += (n - (x_max + a - 1) / a) * y_max;
    ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
    return ans;
} // sum_{i=0}^{n-1} floor((a * i + b) / m) in Log(n + m + a + b)

```

### 6.4 Miller Rabin\*

```

// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633 4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383 6 : pirmes <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool Miller_Rabin(11 a, 11 n) {
    if((a = a % n) == 0) return 1;
    if((n & 1) ^ 1) return n == 2;
    11 tmp = (n - 1) / ((n - 1) & (1 - n));
    11 t = __lg((n - 1) & (1 - n)), x = 1;
    for(; tmp; tmp >>= 1, a = mul(a, a, n))
        if(tmp & 1) x = mul(x, a, n);
    if(x == 1 || x == n - 1) return 1;
    while(--t)
        if((x = mul(x, x, n)) == n - 1) return 1;
    return 0;
}

```

### 6.5 Big number

```

template<typename T>
inline string to_string(const T& x){
    stringstream ss;
    return ss<<x,ss.str();
}
struct bigN:vector<11>{
    const static int base=1000000000,width=log10(base);
    bool negative;
    bigN(const_iterator a,const_iterator b):vector<11>(a,
        b){}
    bigN(string s){
        if(s.empty())return;
        if(s[0]=='-')negative=1,s=s.substr(1);
        else negative=0;
        for(int i=int(s.size())-1;i>=0;i-=width){
            11 t=0;
            for(int j=max(0,i-width+1);j<=i;j++)
                t=t*10+s[j]-'0';
            push_back(t);
        }
        trim();
    }
    template<typename T>
    bigN(const T &x):bigN(to_string(x)){}
    bigN():negative(0){}
    void trim(){
        while(size()&&!back())pop_back();
        if(empty())negative=0;
    }
    void carry(int _base=base){
        for(size_t i=0;i<size();++i){
            if(at(i)>=0&&at(i)<_base)continue;
            if(i+1u==size())push_back(0);
            int r=at(i)%_base;
            if(r<0)r+=_base;
            at(i+1)+=(at(i)-r)/_base,at(i)=r;
        }
    }
}

```

```

int abscomp(const bigN &b)const{
    if(size()>b.size())return 1;
    if(size()<b.size())return -1;
    for(int i=int(size())-1;i>=0;--i){
        if(at(i)>b[i])return 1;
        if(at(i)<b[i])return -1;
    }
    return 0;
}
int cmp(const bigN &b)const{
    if(negative!=b.negative)return negative?-1:1;
    return negative?-abscomp(b):abscomp(b);
}
bool operator<(const bigN&b)const{return cmp(b)<0;}
bool operator>(const bigN&b)const{return cmp(b)>0;}
bool operator<=(const bigN&b)const{return cmp(b)<=0;}
bool operator>=(const bigN&b)const{return cmp(b)>=0;}
bool operator==(const bigN&b)const{return !cmp(b);}
bool operator!=(const bigN&b)const{return cmp(b)!=0;}
bigN abs()const{
    bigN res=*this;
    return res.negative=0, res;
}
bigN operator-()const{
    bigN res=*this;
    return res.negative=!negative,res.trim(),res;
}
bigN operator+(const bigN &b)const{
    if(negative)return -(*this)+(-b));
    if(b.negative)return *this-(-b);
    bigN res=*this;
    if(b.size()>size())res.resize(b.size());
    for(size_t i=0;i<b.size();++i)res[i]+=b[i];
    return res.carry(),res.trim(),res;
}
bigN operator-(const bigN &b)const{
    if(negative)return -(*this)-(-b));
    if(b.negative)return *this+(-b);
    if(abscomp(b)<0)return -(b-(*this));
    bigN res=*this;
    if(b.size()>size())res.resize(b.size());
    for(size_t i=0;i<b.size();++i)res[i]-=b[i];
    return res.carry(),res.trim(),res;
}
bigN operator*(const bigN &b)const{
    bigN res;
    res.negative=negative!=b.negative;
    res.resize(size()+b.size());
    for(size_t i=0;i<size();++i)
        for(size_t j=0;j<b.size();++j)
            if((res[i+j]+at(i)*b[j])>=base){
                res[i+j+1]+=res[i+j]/base;
                res[i+j]%=base;
            } // %k%carry·|.·!%
    return res.trim(),res;
}
bigN operator/(const bigN &b)const{
    int norm=base/(b.back()+1);
    bigN x=abs()*norm;
    bigN y=b.abs()*norm;
    bigN q,r;
    q.resize(x.size());
    for(int i=int(x.size())-1;i>=0;--i){
        r=r*base+x[i];
        int s1=r.size()<=y.size()?0:r[y.size()];
        int s2=r.size()<y.size()?0:r[y.size()-1];
        int d=(11(base)*s1+s2)/y.back();
        r=r-y*d;
        while(r.negative)r=r+y,--d;
        q[i]=d;
    }
    q.negative=negative!=b.negative;
    return q.trim(),q;
}
bigN operator%(const bigN &b)const{
    return *this-(*this/b)*b;
}
friend istream& operator>>(istream &ss, bigN &b){
    string s;
    return ss>>s, b=s, ss;
}

```



```

friend ostream& operator<<(ostream &ss, const bigN &b)
{
    if(b.negative) ss<< '-';
    ss<<(b.empty()?0:b.back());
    for(int i=int(b.size())-2; i>=0; --i)
        ss<<setw(width)<<setfill('0')<<b[i];
    return ss;
}
template<typename T>
operator T(){
    stringstream ss;
    ss<<*this;
    T res;
    return ss>>res, res;
}
};

```

## 6.6 Fraction

```

struct fraction{
    ll n,d;
    fraction(const ll &n=0, const ll &d=1):n(_n),d(_d){
        ll t=__gcd(n,d);
        n/=t,d/=t;
        if(d<0) n=-n,d=-d;
    }
    fraction operator-(const fraction &b) const{
        return fraction(-n,d);
    }
    fraction operator+(const fraction &b) const{
        return fraction(n*b.d+b.n*d,d*b.d);
    }
    fraction operator-(const fraction &b) const{
        return fraction(n*b.d-b.n*d,d*b.d);
    }
    fraction operator*(const fraction &b) const{
        return fraction(n*b.n,d*b.d);
    }
    fraction operator/(const fraction &b) const{
        return fraction(n*b.d,d*b.n);
    }
    void print(){
        cout << n;
        if(d!=1) cout << "/" << d;
    }
};

```

## 6.7 Simultaneous Equations

```

struct matrix { //m variables, n equations
    int n, m;
    fraction M[MAXN][MAXN + 1], sol[MAXN];
    int solve() { //-1: inconsistent, >= 0: rank
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m) continue;
            for (int j = 0; j < n; ++j) {
                if (i == j) continue;
                fraction tmp = -M[j][piv] / M[i][piv];
                for (int k = 0; k <= m; ++k) M[j][k] = tmp * M[i][k] + M[j][k];
            }
        }
        int rank = 0;
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m && M[i][m].n) return -1;
            else if (piv < m) ++rank, sol[piv] = M[i][m] / M[i][piv];
        }
        return rank;
    }
};

```

## 6.8 Pollard Rho

```

// does not work when n is prime
ll f(ll x, ll mod){ return add(mul(x,x,mod),1,mod); }
ll pollard_rho(ll n){
    if(!(n&1)) return 2;
    while(1){
        ll y=2,x=rand()%(n-1)+1,res=1;
        for(int sz=2;res==1;y=x,sz*=2)
            for(int i=0;i<sz&&res<=1;++i)
                x=f(x,n),res=__gcd(abs(x-y),n);
        if(res!=0&&res!=n) return res;
    }
}

```

## 6.9 Simplex Algorithm

```

const int MAXN = 111;
const int MAXM = 111;
const double eps = 1E-10;
double a[MAXN][MAXM], b[MAXN], c[MAXN][MAXM];
double x[MAXN];
int ix[MAXN + MAXM]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b, x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[MAXN][MAXM], double b[MAXN],
    double c[MAXN], int n, int m){
    ++m;
    int r = n, s = m - 1;
    memset(d, 0, sizeof(d));
    for (int i = 0; i < n + m; ++i) ix[i] = i;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m - 1; ++j) d[i][j] = -a[i][j];
        d[i][m - 1] = 1;
        d[i][m] = b[i];
        if (d[r][m] > d[i][m]) r = i;
    }
    for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];
    d[n + 1][m - 1] = -1;
    for (double dd;; ) {
        if (r < n) {
            int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
            d[r][s] = 1.0 / d[r][s];
            for (int j = 0; j <= m; ++j)
                if (j != s) d[r][j] *= -d[r][s];
            for (int i = 0; i <= n + 1; ++i) if (i != r) {
                for (int j = 0; j <= m; ++j) if (j != s)
                    d[i][j] += d[r][j] * d[i][s];
                d[i][s] *= d[r][s];
            }
        }
        r = -1; s = -1;
        for (int j = 0; j < m; ++j)
            if (s < 0 || ix[s] > ix[j]) {
                if (d[n + 1][j] > eps ||
                    (d[n + 1][j] > -eps && d[n][j] > eps))
                    s = j;
            }
        if (s < 0) break;
        for (int i = 0; i < n; ++i) if (d[i][s] < -eps) {
            if (r < 0 ||
                (dd = d[r][m] / d[r][s] - d[i][m] / d[i][s]) < -eps ||
                (dd < eps && ix[r + m] > ix[i + m]))
                r = i;
        }
        if (r < 0) return -1; // not bounded
    }
    if (d[n + 1][m] < -eps) return -1; // not executable
    double ans = 0;
    for (int i = 0; i < m; ++i) x[i] = 0;
    for (int i = m; i < n + m; ++i) { // the missing
        enumerated x[i] = 0
        if (ix[i] < m - 1) {
            ans += d[i - m][m] * c[ix[i]];
            x[ix[i]] = d[i - m][m];
        }
    }
}

```

```

    }
}
return ans;
}

```

## 6.10 chineseRemainder

```

LL solve(LL x1, LL m1, LL x2, LL m2) {
    LL g = __gcd(m1, m2);
    if((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    pair<LL, LL> p = gcd(m1, m2);
    LL lcm = m1 * m2 * g;
    LL res = p.first * (x2 - x1) * m1 + x1;
    return (res % lcm + lcm) % lcm;
}

```

## 6.11 QuadraticResidue

```

int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
    return s;
}

int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    const int jc = Jacobi(a, p);
    if (jc == 0) return 0;
    if (jc == -1) return -1;
    int b, d;
    for (; ) {
        b = rand() % p;
        d = (1LL * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (1LL + p) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (1LL * g0 * f0 + 1LL * d * (1LL * g1 * f1 % p)) % p;
            g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (1LL * f0 * f0 + 1LL * d * (1LL * f1 * f1 % p)) % p;
        f1 = (2LL * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}

```

## 6.12 PiCount

```

int64_t PrimeCount(int64_t n) {
    if (n <= 1) return 0;
    const int v = sqrt(n);
    vector<int> smalls(v + 1);
    for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
    int s = (v + 1) / 2;
    vector<int> roughs(s);
    for (int i = 0; i < s; ++i) roughs[i] = 2 * i + 1;
    vector<int64_t> larges(s);
    for (int i = 0; i < s; ++i) larges[i] = (n / (2 * i + 1) + 1) / 2;
    vector<bool> skip(v + 1);
    int pc = 0;
    for (int p = 3; p <= v; ++p) {

```

```

        if (smalls[p] > smalls[p - 1]) {
            int q = p * p;
            pc++;
            if (1LL * q * q > n) break;
            skip[p] = true;
            for (int i = q; i <= v; i += 2 * p) skip[i] = true;
            int ns = 0;
            for (int k = 0; k < s; ++k) {
                int i = roughs[k];
                if (skip[i]) continue;
                int64_t d = 1LL * i * p;
                larges[ns] = larges[k] - (d <= v ? larges[smalls[d] - pc] : smalls[n / d]) + pc;
                roughs[ns++] = i;
            }
            s = ns;
            for (int j = v / p; j >= p; --j) {
                int c = smalls[j] - pc;
                for (int i = j * p, e = min(i + p, v + 1); i < e; ++i) smalls[i] -= c;
            }
        }
    }
    for (int k = 1; k < s; ++k) {
        const int64_t m = n / roughs[k];
        int64_t s = larges[k] - (pc + k - 1);
        for (int l = 1; l < k; ++l) {
            int p = roughs[l];
            if (1LL * p * p > m) break;
            s -= smalls[m / p] - (pc + l - 1);
        }
        larges[0] -= s;
    }
    return larges[0];
}

```

## 6.13 Algorithms about Primes

```

/*
12721 13331 14341 75577 123457 222557 556679 999983
1097774749 1076767633 100102021 999997771
1001010013 1000512343 987654361 999991231
999888733 98789101 987777733 999991921
1010101333 1010102101 1000000000039
100000000000037 2305843009213693951
4611686018427387847 9223372036854775783
18446744073709551557
*/

```

# 7 Polynomial

## 7.1 Fast Fourier Transform

```

template<int MAXN>
struct FFT {
    using val_t = complex<double>;
    const double PI = acos(-1);
    val_t w[MAXN];
    FFT() {
        for (int i = 0; i < MAXN; ++i) {
            double arg = 2 * PI * i / MAXN;
            w[i] = val_t(cos(arg), sin(arg));
        }
    }
    void bitrev(val_t *a, int n); // see NTT
    void trans(val_t *a, int n, bool inv = false); // see NTT;
    // remember to replace LL with val_t
};

```

## 7.2 Number Theory Transform

```

// (2^16)+1, 65537, 3
// 7*17*(2^23)+1, 998244353, 3
// 1255*(2^20)+1, 1315962881, 3
// 51*(2^25)+1, 1711276033, 29
template<int MAXN, LL P, LL RT> //MAXN must be 2^k
struct NTT {
    LL w[MAXN];
    LL mpow(LL a, LL n);
    LL minv(LL a) { return mpow(a, P - 2); }
    NTT() {
        LL dw = mpow(RT, (P - 1) / MAXN);
        w[0] = 1;
        for (int i = 1; i < MAXN; ++i) w[i] = w[i - 1] * dw % P;
    }
    void bitrev(LL *a, int n) {
        int i = 0;
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (i ^ k) < k; k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
    }
    void operator()(LL *a, int n, bool inv = false) { //0
        if (a[0] < P)
            bitrev(a, n);
        for (int L = 2; L <= n; L <<= 1) {
            int dx = MAXN / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; ++j, x += dx) {
                    LL tmp = a[j + dl] * w[x] % P;
                    if ((a[j + dl] = a[j] - tmp) < 0) a[j + dl] += P;
                    if ((a[j] += tmp) >= P) a[j] -= P;
                }
            }
        }
        if (inv) {
            reverse(a + 1, a + n);
            LL invn = minv(n);
            for (int i = 0; i < n; ++i) a[i] = a[i] * invn % P;
        }
    }
};

```

### 7.3 Fast Walsh Transform

```

/* x: a[j], y: a[j + (L >> 1)]
or: (y += x), (y -= x) and: (x += y), (x -= y)
xor: (x+y, x-y), (x+y, x-y)/2 */
void fwt(val_t *a, int n) { //or
    for (int L = 2; L <= n; L <<= 1) {
        for (int i = 0; i < n; i += L) {
            for (int j = i; j < i + (L >> 1); ++j) {
                a[j + (L >> 1)] += a[j];
            }
        }
    }
}

```

### 7.4 Polynomial Operation

```

template<int MAXN, LL P, LL RT> //MAXN must be 2^k
struct PolyOp {
    NTT<MAXN, P, RT> ntt;
    const LL INV2 = ntt.minv(2);
    int get_sz(int n) {
        int sz = 1;
        while (sz < n) sz <<= 1;
        return sz;
    }
    void mul(LL *a, int n, LL *b, int m, LL *c) {
        static LL buf1[MAXN], buf2[MAXN];
        int sz = get_sz(n + m - 1);
        copy(a, a + n, buf1), fill(buf1 + n, buf1 + sz, 0);
        copy(b, b + m, buf2), fill(buf2 + m, buf2 + sz, 0);
        ntt(buf1, sz), ntt(buf2, sz);
    }
};

```

```

for (int i = 0; i < sz; ++i) c[i] = buf1[i] * buf2[i] % P;
ntt(c, sz, true);
}
void inv(LL *a, int n, LL *b) { //a[0] != 0
    static LL buf[MAXN];
    if (n == 1) return b[0] = ntt.minv(a[0]), void();
    inv(a, (n + 1) / 2, b);
    int sz = get_sz(n * 2);
    copy(a, a + n, buf), fill(buf + n, buf + sz, 0);
    fill(b + n, b + sz, 0);
    ntt(buf, sz), ntt(b, sz);
    for (int i = 0; i < sz; ++i) {
        b[i] *= (2 - b[i] * buf[i]) % P;
        if ((b[i] % P) < 0) b[i] += P;
    }
    ntt(b, sz, true), fill(b + n, b + sz, 0);
}
LL _msqrt(LL x) {
    for (LL i = 0; i <= P / 2; ++i) if (i * i % P == x)
        return i;
    throw string("BBQube");
}
void sqrt(LL *a, int n, LL *b) { //a[0] != 0 && sqrt(a[0]) exists
    static LL invb[MAXN], buf[MAXN];
    if (n == 1) return b[0] = _msqrt(a[0]), void();
    sqrt(a, (n + 1) / 2, b);
    int sz = get_sz(n * 2);
    inv(b, n, invb);
    copy(a, a + n, buf), fill(buf + n, buf + sz, 0);
    ntt(b, sz), ntt(invb, sz), ntt(buf, sz);
    for (int i = 0; i < sz; ++i) {
        if ((b[i] += buf[i] * invb[i] % P) >= P) b[i] -= P;
        b[i] = b[i] * INV2 % P;
    }
    ntt(b, sz, true), fill(b + n, b + sz, 0);
}
void div(LL *a, int n, LL *b, int m, LL *q, LL *r) {
    static LL invb[MAXN], buf[MAXN];
    if (n < m) {
        fill(q, q + m, 0), copy(a, a + n, r), fill(r + n, r + m, 0);
        return;
    }
    int mod_sz = n - m + 1;
    copy(b, b + m, buf), reverse(buf, buf + m);
    if (m < mod_sz) fill(buf + m, buf + mod_sz, 0);
    inv(buf, mod_sz, invb);
    copy(a, a + n, buf), reverse(buf, buf + n);
    mul(buf, mod_sz, invb, mod_sz, q);
    fill(q + mod_sz, q + n, 0), reverse(q, q + mod_sz);
    mul(b, m, q, mod_sz, buf);
    for (int i = 0; i < n; ++i) {
        if ((r[i] = a[i] - buf[i]) < 0) r[i] += P;
    }
}
};

```

## 8 Geometry

### 8.1 Default Code

```

typedef pair<double, double> pdd;
typedef pair<pdd, pdd> Line;
struct Cir { pdd O; double R; };
const double eps = 1e-8;
pdd operator+(const pdd &a, const pdd &b) {
    return pdd(a.X + b.X, a.Y + b.Y);
}
pdd operator-(const pdd &a, const pdd &b) {
    return pdd(a.X - b.X, a.Y - b.Y);
}
pdd operator*(const pdd &a, const double &b) {
    return pdd(a.X * b, a.Y * b);
}
pdd operator/(const pdd &a, const double &b) {
    return pdd(a.X / b, a.Y / b);
}
double dot(const pdd &a, const pdd &b) {
    return a.X * b.X + a.Y * b.Y;
}

```

```

double cross(const pdd &a, const pdd &b)
{ return a.X * b.Y - a.Y * b.X; }
double abs2(const pdd &a)
{ return dot(a, a); }
double abs(const pdd &a)
{ return sqrt(dot(a, a)); }
int sign(const double &a)
{ return fabs(a) < eps ? 0 : a > 0 ? 1 : -1; }
int ori(const pdd &a, const pdd &b, const pdd &c)
{ return sign(cross(b - a, c - a)); }
bool collinearity(const pdd &p1, const pdd &p2, const pdd &p3)
{ return fabs(cross(p1 - p3, p2 - p3)) < eps; }
bool btw(const pdd &p1, const pdd &p2, const pdd &p3) {
    if(!collinearity(p1, p2, p3)) return 0;
    return dot(p1 - p3, p2 - p3) < eps;
}
bool seg_intersect(const pdd &p1, const pdd &p2, const pdd &p3, const pdd &p4) {
    int a123 = ori(p1, p2, p3);
    int a124 = ori(p1, p2, p4);
    int a341 = ori(p3, p4, p1);
    int a342 = ori(p3, p4, p2);
    if(a123 == 0 && a124 == 0)
        return btw(p1, p2, p3) || btw(p1, p2, p4) ||
            btw(p3, p4, p1) || btw(p3, p4, p2);
    return a123 * a124 <= 0 && a341 * a342 <= 0;
}
pdd intersect(const pdd &p1, const pdd &p2, const pdd &p3, const pdd &p4) {
    double a123 = cross(p2 - p1, p3 - p1);
    double a124 = cross(p2 - p1, p4 - p1);
    return (p4 * a123 - p3 * a124) / (a123 - a124);
}
pdd perp(const pdd &p1)
{ return pdd(-p1.Y, p1.X); }
pdd foot(const pdd &p1, const pdd &p2, const pdd &p3)
{ return intersect(p1, p2, p3, p3 + perp(p2 - p1)); }

```

## 8.2 Convex hull\*

```

void hull(vector<p11> &dots) {
    sort(dots.begin(), dots.end());
    vector<p11> ans(1, dots[0]);
    for (int ct = 0; ct < 2; ++ct, reverse(ALL(dots)))
        for (int i = 1, t = SZ(ans); i < SZ(dots); ans.pb(dots[i++]))
            while (SZ(ans) > t && ori(ans[SZ(ans) - 2], ans.back(), dots[i]) <= 0)
                ans.pop_back();
    ans.pop_back(), ans.swap(dots);
}

```

## 8.3 External bisector

```

pdd external_bisector(pdd p1, pdd p2, pdd p3) { //213
    pdd L1=p2-p1, L2=p3-p1;
    L2=L2*abs(L1)/abs(L2);
    return L1+L2;
}

```

## 8.4 Heart

```

pdd excenter(pdd p0, pdd p1, pdd p2, double &radius) {
    p1=p1-p0, p2=p2-p0;
    double x1=p1.X, y1=p1.Y, x2=p2.X, y2=p2.Y;
    double m=2.*(x1*y2-y1*x2);
    center.X=(x1*x1*y2-x2*x2*y1+y1*y2*(y1-y2))/m;
    center.Y=(x1*x2*(x2-x1)-y1*y1*x2+x1*y2*y2)/m;
    return radius=abs(center), center+p0;
}

pdd incenter(pdd p1, pdd p2, pdd p3, double &radius) {
    double a=abs(p2-p1), b=abs(p3-p1), c=abs(p3-p2);
    double s=(a+b+c)/2, area=sqrt(s*(s-a)*(s-b)*(s-c));
}

```

```

pdd L1=external_bisector(p1,p2,p3), L2=
    external_bisector(p2,p1,p3);
return radius=area/s, intersect(p1, p1+L1, p2, p2+L2),
}

pdd escenter(pdd p1, pdd p2, pdd p3) { //213
    pdd L1=external_bisector(p1,p2,p3), L2=
        external_bisector(p2,p2+p2-p1,p3);
    return intersect(p1, p1+L1, p2, p2+L2);
}

pdd barycenter(pdd p1, pdd p2, pdd p3) {
    return (p1+p2+p3)/3;
}

pdd orthocenter(pdd p1, pdd p2, pdd p3) {
    pdd L1=p3-p2, L2=p3-p1;
    swap(L1.X, L1.Y), L1.X*=-1;
    swap(L2.X, L2.Y), L2.X*=-1;
    return intersect(p1, p1+L1, p2, p2+L2);
}

```

## 8.5 Minimum Circle Cover\*

```

pdd Minimum_Circle_Cover(vector<pdd> dots, double &r) {
    pdd cent;
    random_shuffle(ALL(dots));
    cent = dots[0], r = 0;
    for (int i = 1; i < SZ(dots); ++i)
        if (abs(dots[i] - cent) > r) {
            cent = dots[i], r = 0;
            for (int j = 0; j < i; ++j)
                if (abs(dots[j] - cent) > r) {
                    cent = (dots[i] + dots[j]) / 2;
                    r = abs(dots[i] - cent);
                    for (int k = 0; k < j; ++k)
                        if (abs(dots[k] - cent) > r)
                            cent = excenter(dots[i], dots[j], dots[k], r);
                }
        }
    return cent;
}

```

## 8.6 Polar Angle Sort\*

```

pdd center; //sort base
int Quadrant(pdd a) {
    if(a.X > 0 && a.Y >= 0) return 1;
    if(a.X <= 0 && a.Y > 0) return 2;
    if(a.X < 0 && a.Y <= 0) return 3;
    if(a.X >= 0 && a.Y < 0) return 4;
}

bool cmp(p11 a, p11 b) {
    a = a - center, b = b - center;
    if (Quadrant(a) != Quadrant(b))
        return Quadrant(a) < Quadrant(b);
    if (cross(b, a) == 0) return abs2(a) < abs2(b);
    return cross(a, b) > 0;
}

bool cmp(pdd a, pdd b) {
    a = a - center, b = b - center;
    if(fabs(atan2(a.Y, a.X) - atan2(b.Y, b.X)) > eps)
        return atan2(a.Y, a.X) < atan2(b.Y, b.X);
    return abs(a) < abs(b);
}

```

## 8.7 Intersection of two circles\*

```

bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
    pdd o1 = a.o, o2 = b.o;
    double r1 = a.R, r2 = b.R, d2 = abs2(o1 - o2), d =
        sqrt(d2);
    if(d < max(r1, r2) - min(r1, r2) || d > r1 + r2)
        return 0;
}

```

```

pdd u = (o1 + o2) * 0.5 + (o1 - o2) * ((r2 * r2 - r1
    * r1) / (2 * d2));
double A = sqrt((r1 + r2 + d) * (r1 - r2 + d) * (r1 +
    r2 - d) * (-r1 + r2 + d));
pdd v = pdd(o1.Y - o2.Y, -o1.X + o2.X) * A / (2 * d2)
    ;
p1 = u + v, p2 = u - v;
return 1;
}

```

## 8.8 Intersection of polygon and circle

```

// Divides into multiple triangle, and sum up
// test by HDU2892
const double PI=acos(-1);
double _area(pdd pa, pdd pb, double r){
    if(abs(pa)<abs(pb)) swap(pa, pb);
    if(abs(pb)<eps) return 0;
    double S, h, theta;
    double a=abs(pb),b=abs(pa),c=abs(pb-pa);
    double cosB = dot(pb,pb-pa) / a / c, B = acos(cosB);
    double cosC = dot(pa,pb) / a / b, C = acos(cosC);
    if(a > r){
        S = (C/2)*r*r;
        h = a*b*sin(C)/c;
        if (h < r && B < PI/2) S -= (acos(h/r)*r*r - h*sqrt
            (r*r-h*h));
    }
    else if(b > r){
        theta = PI - B - asin(sin(B)/r*a);
        S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
    }
    else S = .5*sin(C)*a*b;
    return S;
}
double area_poly_circle(const vector<pdd> poly,const
    pdd &O,const double r){
    double S=0;
    for(int i=0;i<SZ(poly);++i)
        S+=_area(poly[i]-O,poly[(i+1)%SZ(poly)]-O,r)*ori(O,
            poly[i],poly[(i+1)%SZ(poly)]);
    return fabs(S);
}

```

## 8.9 Intersection of line and circle

```

vector<pdd> line_interCircle(const pdd &p1,const pdd &
    p2,const pdd &c,const double r){
    pdd ft=foot(p1,p2,c),vec=p2-p1;
    double dis=abs(c-ft);
    if(fabs(dis-r)<eps) return vector<pdd>{ft};
    if(dis>r) return {};
    vec=vec*sqrt(r*r-dis*dis)/abs(vec);
    return vector<pdd>{ft+vec,ft-vec};
}

```

## 8.10 point in circle

```

// return p4 is strictly in circumcircle of tri(p1,p2,
    p3)
long long sqr(long long x) { return x * x; }
bool in_cc(const p1l& p1, const p1l& p2, const p1l& p3,
    const p1l& p4) {
    long long u11 = p1.X - p4.X; long long u12 = p1.Y -
        p4.Y;
    long long u21 = p2.X - p4.X; long long u22 = p2.Y -
        p4.Y;
    long long u31 = p3.X - p4.X; long long u32 = p3.Y -
        p4.Y;
    long long u13 = sqr(p1.X) - sqr(p4.X) + sqr(p1.Y) -
        sqr(p4.Y);
    long long u23 = sqr(p2.X) - sqr(p4.X) + sqr(p2.Y) -
        sqr(p4.Y);
    long long u33 = sqr(p3.X) - sqr(p4.X) + sqr(p3.Y) -
        sqr(p4.Y);
}

```

```

__int128 det = (__int128)-u13 * u22 * u31 + (
    __int128)u12 * u23 * u31 + (__int128)u13 * u21
    * u32 - (__int128)u11 * u23 * u32 - (__int128)
    u12 * u21 * u33 + (__int128)u11 * u22 * u33;
return det > eps;
}

```

## 8.11 Half plane intersection

```

bool isin( Line l0, Line l1, Line l2 ){
    // Check inter(L1, L2) in l0
    pdd p = intersect(l1.X,l1.Y,l2.X,l2.Y);
    return cross(l0.Y - l0.X,p - l0.X) > eps;
}
/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (L.Y - L.X) ^ (p - L.X) > 0
 */
/* --^-- Line.X --^-- Line.Y --^-- */
vector<Line> halfPlaneInter(vector<Line> lines){
    int sz = lines.size();
    vector<double> ata(sz),ord(sz);
    for(int i=0; i<sz; ++i) {
        ord[i] = i;
        pdd d = lines[i].Y - lines[i].X;
        ata[i] = atan2(d.Y, d.X);
    }
    sort(ord.begin(), ord.end(), [&](int i,int j){
        if( fabs(ata[i] - ata[j]) < eps )
            return (cross(lines[i].Y-lines[i].X,
                lines[j].Y-lines[j].X)<0);
        return ata[i] < ata[j];
    });
    vector<Line> fin;
    for (int i=0; i<sz; ++i)
        if (!i || fabs(ata[ord[i]] - ata[ord[i-1]]) > eps)
            fin.pb(lines[ord[i]]);
    deque<Line> dq;
    for (int i=0; i<SZ(fin); i++){
        while (SZ(dq)>=2&&!isin(fin[i],dq[SZ(dq)-2],dq.back
            ()))
            dq.pop_back();
        while (SZ(dq)>=2&&!isin(fin[i],dq[0],dq[1]))
            dq.pop_front();
        dq.push_back(fin[i]);
    }
    while (SZ(dq)>=3&&!isin(dq[0],dq[SZ(dq)-2],dq.back()))
        dq.pop_back();
    while (SZ(dq)>=3&&!isin(dq.back(), dq[0], dq[1]))
        dq.pop_front();
    vector<Line> res(ALL(dq));
    return res;
}

```

## 8.12 CircleCover\*

```

const int N = 1021;
struct CircleCover {
    int C;
    Cir c[N];
    bool g[N][N], overlap[N][N];
    // Area[i] : area covered by at least i circles
    double Area[ N ];
    void init(int _C){ C = _C; }
    struct Teve {
        pdd p; double ang; int add;
        Teve() {}
        Teve(pdd _a, double _b, int _c):p(_a), ang(_b), add
            (_c){}
        bool operator<(const Teve &a)const {
            return ang < a.ang;
        }
    }eve[N * 2];
    // strict: x = 0, otherwise x = -1
    bool disjuct(Cir &a, Cir &b, int x)
    {return sign(abs(a.O - b.O) - a.R - b.R) > x;}
    bool contain(Cir &a, Cir &b, int x)
    {return sign(a.R - b.R - abs(a.O - b.O)) > x;}
    bool contain(int i, int j) {
}

```

```

    /* c[j] is non-strictly in c[i]. */
    return (sign(c[i].R - c[j].R) > 0 || (sign(c[i].R -
        c[j].R) == 0 && i < j)) && contain(c[i], c[j],
        -1);
}
void solve(){
    fill_n(Area, C + 2, 0);
    for(int i = 0; i < C; ++i)
        for(int j = 0; j < C; ++j)
            overlap[i][j] = contain(i, j);
    for(int i = 0; i < C; ++i)
        for(int j = 0; j < C; ++j)
            g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                disjuct(c[i], c[j], -1));
    for(int i = 0; i < C; ++i){
        int E = 0, cnt = 1;
        for(int j = 0; j < C; ++j)
            if(j != i && overlap[j][i])
                ++cnt;
        for(int j = 0; j < C; ++j)
            if(i != j && g[i][j]) {
                pdd aa, bb;
                CCinter(c[i], c[j], aa, bb);
                double A = atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);
                double B = atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
                eve[E++] = Teve(bb, B, 1), eve[E++] = Teve(aa, A, -1);
                if(B > A) ++cnt;
            }
        if(E == 0) Area[cnt] += pi * c[i].R * c[i].R;
        else{
            sort(eve, eve + E);
            eve[E] = eve[0];
            for(int j = 0; j < E; ++j){
                cnt += eve[j].add;
                Area[cnt] += cross(eve[j].p, eve[j + 1].p) * .5;
                double theta = eve[j + 1].ang - eve[j].ang;
                if (theta < 0) theta += 2. * pi;
                Area[cnt] += (theta - sin(theta)) * c[i].R * c[i].R * .5;
            }
        }
    }
}
};

```

### 8.13 3Dpoint\*

```

struct Point {
    double x, y, z;
    Point(double _x = 0, double _y = 0, double _z = 0): x(_x), y(_y), z(_z){}
    Point(pdd p) { x = p.X, y = p.Y, z = abs2(p); }
};
Point operator-(const Point &p1, const Point &p2)
{ return Point(p1.x - p2.x, p1.y - p2.y, p1.z - p2.z);}
Point cross(const Point &p1, const Point &p2)
{ return Point(p1.y * p2.z - p1.z * p2.y, p1.z * p2.x - p1.x * p2.z, p1.x * p2.y - p1.y * p2.x);}
double dot(const Point &p1, const Point &p2)
{ return p1.x * p2.x + p1.y * p2.y + p1.z * p2.z;}
double abs(const Point &a)
{ return sqrt(dot(a, a));}
Point cross3(const Point &a, const Point &b, const Point &c)
{ return cross(b - a, c - a);}
double area(Point a, Point b, Point c)
{ return abs(cross3(a, b, c));}
double volume(Point a, Point b, Point c, Point d)
{ return dot(cross3(a, b, c), d - a);}

```

### 8.14 Convexhull3D\*

```

struct CH3D {
    struct face{int a, b, c; bool ok;} F[8 * N];

```

```

double dblcmp(Point &p, face &f)
{return dot(cross3(P[f.a], P[f.b], P[f.c]), p - P[f.a]);}
int g[N][N], num, n;
Point P[N];
void deal(int p, int a, int b) {
    int f = g[a][b];
    face add;
    if (F[f].ok) {
        if (dblcmp(P[p], F[f]) > eps) dfs(p, f);
    } else
        add.a = b, add.b = a, add.c = p, add.ok = 1, g[p][b] = g[a][p] = g[b][a] = num, F[num++] = add;
}
void dfs(int p, int now) {
    F[now].ok = 0;
    deal(p, F[now].b, F[now].a), deal(p, F[now].c, F[now].b), deal(p, F[now].a, F[now].c);
}
bool same(int s, int t){
    Point &a = P[F[s].a];
    Point &b = P[F[s].b];
    Point &c = P[F[s].c];
    return fabs(volume(a, b, c, P[F[t].a])) < eps && fabs(volume(a, b, c, P[F[t].b])) < eps && fabs(volume(a, b, c, P[F[t].c])) < eps;
}
void init(int _n){n = _n, num = 0;}
void solve() {
    face add;
    num = 0;
    if(n < 4) return;
    if([&]() {
        for (int i = 1; i < n; ++i)
            if (abs(P[0] - P[i]) > eps)
                return swap(P[1], P[i]), 0;
        return 1;
    }()) || [&]() {
        for (int i = 2; i < n; ++i)
            if (abs(cross3(P[i], P[0], P[1])) > eps)
                return swap(P[2], P[i]), 0;
        return 1;
    }()) || [&]() {
        for (int i = 3; i < n; ++i)
            if (fabs(dot(cross(P[0] - P[1], P[1] - P[2]), P[0] - P[i])) > eps)
                return swap(P[3], P[i]), 0;
        return 1;
    }())return;
    for (int i = 0; i < 4; ++i) {
        add.a = (i + 1) % 4, add.b = (i + 2) % 4, add.c = (i + 3) % 4, add.ok = true;
        if (dblcmp(P[i], add) > 0) swap(add.b, add.c);
        g[add.a][add.b] = g[add.b][add.c] = g[add.c][add.a] = num;
        F[num++] = add;
    }
    for (int i = 4; i < n; ++i)
        for (int j = 0; j < num; ++j)
            if (F[j].ok && dblcmp(P[i], F[j]) > eps) {
                dfs(i, j);
                break;
            }
    for (int tmp = num, i = (num = 0); i < tmp; ++i)
        if (F[i].ok) F[num++] = F[i];
}
double get_area() {
    double res = 0.0;
    if (n == 3)
        return abs(cross3(P[0], P[1], P[2])) / 2.0;
    for (int i = 0; i < num; ++i)
        res += area(P[F[i].a], P[F[i].b], P[F[i].c]);
    return res / 2.0;
}
double get_volume() {
    double res = 0.0;
    for (int i = 0; i < num; ++i)
        res += volume(Point(0, 0, 0), P[F[i].a], P[F[i].b], P[F[i].c]);
    return fabs(res / 6.0);
}

```



```

}
int triangle() {return num;}
int polygon() {
    int res = 0;
    for (int i = 0, flag = 1; i < num; ++i, res += flag, flag = 1)
        for (int j = 0; j < i && flag; ++j)
            flag &= !same(i,j);
    return res;
}
Point getcent(){
    Point ans(0, 0, 0), temp = P[F[0].a];
    double v = 0.0, t2;
    for (int i = 0; i < num; ++i)
        if (F[i].ok == true) {
            Point p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[i].c];
            t2 = volume(temp, p1, p2, p3) / 6.0;
            if (t2 > 0)
                ans.x += (p1.x + p2.x + p3.x + temp.x) * t2,
                ans.y += (p1.y + p2.y + p3.y + temp.y) * t2,
                ans.z += (p1.z + p2.z + p3.z + temp.z) * t2, v += t2;
        }
    ans.x /= (4 * v), ans.y /= (4 * v), ans.z /= (4 * v);
    return ans;
}
double pointmindis(Point p) {
    double rt = 99999999;
    for(int i = 0; i < num; ++i)
        if(F[i].ok == true) {
            Point p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[i].c];
            double a = (p2.y - p1.y) * (p3.z - p1.z) - (p2.z - p1.z) * (p3.y - p1.y);
            double b = (p2.z - p1.z) * (p3.x - p1.x) - (p2.x - p1.x) * (p3.z - p1.z);
            double c = (p2.x - p1.x) * (p3.y - p1.y) - (p2.y - p1.y) * (p3.x - p1.x);
            double d = 0 - (a * p1.x + b * p1.y + c * p1.z);
            double temp = fabs(a * p.x + b * p.y + c * p.z + d) / sqrt(a * a + b * b + c * c);
            rt = min(rt, temp);
        }
    return rt;
}
};

```

## 8.15 DelaunayTriangulation\*

```

/* Delaunay Triangulation:
Given a sets of points on 2D plane, find a
triangulation such that no points will strictly
inside circumcircle of any triangle.
find : return a triangle contain given point
add_point : add a point into triangulation
A Triangle is in triangulation iff. its has_chd is 0.
Region of triangle u: iterate each u.edge[i].tri,
each points are u.p[(i+1)%3], u.p[(i+2)%3]
Voronoi diagram: for each triangle in triangulation,
the bisector of all its edges will split the region.
nearest point will belong to the triangle containing it
*/
const ll inf = MAXC * MAXC * 100; // Lower_bound
unknown
struct Tri;
struct Edge {
    Tri* tri; int side;
    Edge(): tri(0), side(0){}
    Edge(Tri* _tri, int _side): tri(_tri), side(_side) {}
};
struct Tri {
    pll p[3];
    Edge edge[3];
    Tri* chd[3];
    Tri() {}
    Tri(const pll& p0, const pll& p1, const pll& p2) {

```

```

        p[0] = p0; p[1] = p1; p[2] = p2;
        chd[0] = chd[1] = chd[2] = 0;
    }
    bool has_chd() const { return chd[0] != 0; }
    int num_chd() const {
        return !!chd[0] + !!chd[1] + !!chd[2];
    }
    bool contains(pll const& q) const {
        for (int i = 0; i < 3; ++i)
            if (ori(p[i], p[(i + 1) % 3], q) < 0)
                return 0;
        return 1;
    }
} pool[N * 10], *tris;
void edge(Edge a, Edge b) {
    if(a.tri) a.tri -> edge[a.side] = b;
    if(b.tri) b.tri -> edge[b.side] = a;
}
struct Trig { // Triangulation
    Trig() {
        the_root = // Tri should at least contain all
                    points
                    new(tris++) Tri(pll(-inf, -inf), pll(inf +
                    inf, -inf), pll(-inf, inf + inf));
    }
    Tri* find(pll p) { return find(the_root, p); }
    void add_point(const pll &p) { add_point(find(
        the_root, p), p); }
    Tri* the_root;
    static Tri* find(Tri* root, const pll &p) {
        while (1) {
            if (!root -> has_chd())
                return root;
            for (int i = 0; i < 3 && root -> chd[i]; ++i)
                if (root -> chd[i] -> contains(p)) {
                    root = root -> chd[i];
                    break;
                }
        }
        assert(0); // "point not found"
    }
}
void add_point(Tri* root, pll const& p) {
    Tri* t[3];
    /* split it into three triangles */
    for (int i = 0; i < 3; ++i)
        t[i] = new(tris++) Tri(root -> p[i], root
        -> p[(i + 1) % 3], p);
    for (int i = 0; i < 3; ++i)
        edge(Edge(t[i], 0), Edge(t[(i + 1) % 3], 1)
        );
    for (int i = 0; i < 3; ++i)
        edge(Edge(t[i], 2), root -> edge[(i + 2) %
        3]);
    for (int i = 0; i < 3; ++i)
        root -> chd[i] = t[i];
    for (int i = 0; i < 3; ++i)
        flip(t[i], 2);
}
void flip(Tri* tri, int pi) {
    Tri* trj = tri -> edge[pi].tri;
    int pj = tri -> edge[pi].side;
    if (!trj) return;
    if (!in_cc(tri -> p[0], tri -> p[1], tri -> p
    [2], trj -> p[pj])) return;
    /* flip edge between tri, trj */
    Tri* trk = new(tris++) Tri(tri -> p[(pi + 1) %
    3], trj -> p[pj], tri -> p[pi]);
    Tri* trl = new(tris++) Tri(trj -> p[(pj + 1) %
    3], tri -> p[pi], trj -> p[pj]);
    edge(Edge(trk, 0), Edge(trl, 0));
    edge(Edge(trk, 1), tri -> edge[(pi + 2) % 3]);
    edge(Edge(trk, 2), trj -> edge[(pj + 1) % 3]);
    edge(Edge(trl, 1), trj -> edge[(pj + 2) % 3]);
    edge(Edge(trl, 2), tri -> edge[(pi + 1) % 3]);
    tri -> chd[0] = trk; tri -> chd[1] = trl; tri
    -> chd[2] = 0;
    trj -> chd[0] = trk; trj -> chd[1] = trl; trj
    -> chd[2] = 0;
    flip(trk, 1); flip(trk, 2);
    flip(trl, 1); flip(trl, 2);
}
}

```

```

};
vector<Tri*> triang; // vector of all triangle
set<Tri*> vst;
void go(Tri* now) { // store all tri into triang
    if (vst.find(now) != vst.end())
        return;
    vst.insert(now);
    if (!now -> has_chd())
        return triang.push_back(now);
    for (int i = 0; i < now->num_chd(); ++i)
        go(now -> chd[i]);
}
void build(int n, pll* ps) { // build triangulation
    tris = pool; triang.clear(); vst.clear();
    random_shuffle(ps, ps + n);
    Trig tri; // the triangulation structure
    for (int i = 0; i < n; ++i)
        tri.add_point(ps[i]);
    go(tri.the_root);
}

```

## 8.16 Triangulation Voronoi\*

```

vector<Line> ls[N];
pll arr[N];
Line make_line(pdd p, Line l) {
    pdd d = l.Y - l.X; d = perp(d);
    pdd m = (l.X + l.Y) / 2;
    l = Line(m, m + d);
    if (ori(l.X, l.Y, p) < 0)
        l = Line(m + d, m);
    return l;
}
double calc_area(int id) {
    // use to calculate the area of point "strictly in
    // the convex hull"
    vector<Line> hpi = halfPlaneInter(ls[id]);
    vector<pdd> ps;
    for (int i = 0; i < SZ(hpi); ++i)
        ps.pb(intersect(hpi[i].X, hpi[i].Y, hpi[(i + 1)
            % SZ(hpi)].X, hpi[(i + 1) % SZ(hpi)].Y));
    double rt = 0;
    for (int i = 0; i < SZ(ps); ++i)
        rt += cross(ps[i], ps[(i + 1) % SZ(ps)]);
    return fabs(rt) / 2;
}
void solve(int n, pii *oarr) {
    map<pll, int> mp;
    for (int i = 0; i < n; ++i)
        arr[i] = pll(oarr[i].X, oarr[i].Y), mp[arr[i]]
            = i;
    build(n, arr); // Triangulation
    for (auto *t : triang) {
        vector<int> p;
        for (int i = 0; i < 3; ++i)
            if (mp.find(t -> p[i]) != mp.end())
                p.pb(mp[t -> p[i]]);
        for (int i = 0; i < SZ(p); ++i)
            for (int j = i + 1; j < SZ(p); ++j) {
                Line l(oarr[p[i]], oarr[p[j]]);
                ls[p[i]].pb(make_line(oarr[p[i]], l));
                ls[p[j]].pb(make_line(oarr[p[j]], l));
            }
    }
}

```

## 8.17 Tangent line of two circles

```

vector<Line> go(const Cir& c1, const Cir& c2, int
    sign1) {
    // sign1 = 1 for outer tang, -1 for inner tang
    vector<Line> ret;
    double d_sq = norm2(c1.O - c2.O);
    if (d_sq < eps) return ret;
    double d = sqrt(d_sq);
    Pt v = (c2.O - c1.O) / d;
    double c = (c1.R - sign1 * c2.R) / d;
    if (c * c > 1) return ret;

```

```

    double h = sqrt(max(0.0, 1.0 - c * c));
    for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
        Pt n = { v.X * c - sign2 * h * v.Y,
            v.Y * c + sign2 * h * v.X };
        Pt p1 = c1.O + n * c1.R;
        Pt p2 = c2.O + n * (c2.R * sign1);
        if (fabs(p1.X - p2.X) < eps and
            fabs(p1.Y - p2.Y) < eps)
            p2 = p1 + perp(c2.O - c1.O);
        ret.push_back({ p1, p2 });
    }
    return ret;
}

```

## 8.18 minMaxEnclosingRectangle

```

pdd solve(vector<pll> &dots) {
    vector<pll> hull;
    const double INF=1e18, qi=acos(-1)/2*3;
    cv.dots=dots;
    hull=cv.hull();
    double Max=0, Min=INF, deg;
    ll n=hull.size();
    hull.pb(hull[0]);
    for (int i=0, u=1, r=1, l; i<n; ++i) {
        pll nw=hull[i+1]-hull[i];
        while (cross(nw, hull[u+1]-hull[i])>cross(nw, hull[u]-
            hull[i]))
            u=(u+1)%n;
        while (dot(nw, hull[r+1]-hull[i])>dot(nw, hull[r]-hull
            [i]))
            r=(r+1)%n;
        if (!i) l=(r+1)%n;
        while (dot(nw, hull[l+1]-hull[i])<dot(nw, hull[l]-hull
            [i]))
            l=(l+1)%n;
        Min=min(Min, (double)(dot(nw, hull[r]-hull[i])-dot(nw
            , hull[l]-hull[i]))*cross(nw, hull[u]-hull[i])/
            abs2(nw));
        deg=acos((double)dot(hull[r]-hull[l], hull[u]-hull[i]
            )/abs(hull[r]-hull[l])/abs(hull[u]-hull[i]));
        deg=(qi-deg)/2;
        Max=max(Max, (double)abs(hull[r]-hull[l])*abs(hull[u]
            -hull[i])*sin(deg)*sin(deg));
    }
    return pdd(Min, Max);
}

```

## 8.19 minDistOfTwoConvex

```

// p, q is convex
double TwoConvexHullMinDist(Point P[], Point Q[], int n
    , int m) {
    int YMinP = 0, YMaxQ = 0;
    double tmp, ans = 999999999;
    for (i = 0; i < n; ++i) if (P[i].y < P[YMinP].y) YMinP
        = i;
    for (i = 0; i < m; ++i) if (Q[i].y > Q[YMaxQ].y) YMaxQ
        = i;
    P[n] = P[0], Q[m] = Q[0];
    for (int i = 0; i < n; ++i) {
        while (tmp = Cross(Q[YMaxQ + 1] - P[YMinP + 1], P[
            YMinP] - P[YMinP + 1]) > Cross(Q[YMaxQ] - P[
            YMinP + 1], P[YMinP] - P[YMinP + 1])) YMaxQ = (
            YMaxQ + 1) % m;
        if (tmp < 0) ans = min(ans, PointToSegDist(P[YMinP
            ], P[YMinP + 1], Q[YMaxQ]));
        else ans = min(ans, TwoSegMinDist(P[YMinP], P[YMinP
            + 1], Q[YMaxQ], Q[YMaxQ + 1]));
        YMinP = (YMinP + 1) % n;
    }
    return ans;
}

```

## 8.20 Minkowski Sum\*

```
vector<pll> Minkowski(vector<pll> A, vector<pll> B) {
    hull(A), hull(B);
    vector<pll> C(1, A[0] + B[0]), s1, s2;
    for(int i = 0; i < SZ(A); ++i)
        s1.pb(A[(i + 1) % SZ(A)] - A[i]);
    for(int i = 0; i < SZ(B); ++i)
        s2.pb(B[(i + 1) % SZ(B)] - B[i]);
    for(int p1 = 0, p2 = 0; p1 < SZ(A) || p2 < SZ(B);)
        if (p2 >= SZ(B) || (p1 < SZ(A) && cross(s1[p1], s2[p2]) >= 0))
            C.pb(C.back() + s1[p1++]);
        else
            C.pb(C.back() + s2[p2++]);
    return hull(C), C;
}
```

## 9 Else

### 9.1 Mo's Alogrithm(With modification)

```
struct QUERY{//BLOCK=N^{2/3}
    int L,R,id,LBId,RBId,T;
    QUERY(int l,int r,int id,int lb,int rb,int t):
        L(l),R(r),id(id),LBId(lb),RBId(rb),T(t){}
    bool operator<(const QUERY &b)const{
        if(LBId!=b.LBId) return LBId<b.LBId;
        if(RBId!=b.RBId) return RBId<b.RBId;
        return T<b.T;
    }
};
vector<QUERY> query;
int cur_ans,arr[MAXN],ans[MAXN];
void addTime(int L,int R,int T){}
void subTime(int L,int R,int T){}
void add(int x){}
void sub(int x){}
void solve(){
    sort(ALL(query));
    int L=0,R=0,T=-1;
    for(auto q:query){
        while(T<q.T) addTime(L,R,++T);
        while(T>q.T) subTime(L,R,T--);
        while(R<q.R) add(arr[++R]);
        while(L>q.L) add(arr[--L]);
        while(R>q.R) sub(arr[R--]);
        while(L<q.L) sub(arr[L++]);
        ans[q.id]=cur_ans;
    }
}
```

### 9.2 Mo's Alogrithm On Tree

```
const int MAXN=40005;
vector<int> G[MAXN];//1-base
int n,B,arr[MAXN],ans[100005],cur_ans;
int in[MAXN],out[MAXN],dfn[MAXN*2],dft;
int deep[MAXN],sp[___lg(MAXN*2)+1][MAXN*2],b1n[MAXN],spt;
bitset<MAXN> inset;
struct QUERY{
    int L,R,Lid,id,lca;
    QUERY(int l,int r,int _id):L(l),R(r),lca(0),id(_id){}
    bool operator<(const QUERY &b){
        if(Lid!=b.Lid) return Lid<b.Lid;
        return R<b.R;
    }
};
vector<QUERY> query;
void dfs(int u,int f,int d){
    deep[u]=d,sp[0][spt]=u,b1n[u]=spt++;
    dfn[dft]=u,in[u]=dft++;
    for(int v:G[u])
        if(v!=f)
            dfs(v,u,d+1),sp[0][spt]=u,b1n[u]=spt++;
    dfn[dft]=u,out[u]=dft++;
}
```

```
int lca(int u,int v){
    if(b1n[u]>b1n[v]) swap(u,v);
    int t=___lg(b1n[v]-b1n[u]+1);
    int a=sp[t][b1n[u]],b=sp[t][b1n[v]-(1<<t)+1];
    if(deep[a]<deep[b]) return a;
    return b;
}
void sub(int x){}
void add(int x){}
void flip(int x){
    if(inset[x]) sub(arr[x]);
    else add(arr[x]);
    inset[x]=~inset[x];
}
void solve(){
    B=sqrt(2*n),dft=spt=cur_ans=0,dfs(1,1,0);
    for(int i=1,x=2;x<2*n;++i,x<=1)
        for(int j=0;j+x<2*n;++j)
            if(deep[sp[i-1][j]]<deep[sp[i-1][j+x/2]])
                sp[i][j]=sp[i-1][j];
            else sp[i][j]=sp[i-1][j+x/2];
    for(auto &q:query){
        int c=lca(q.L,q.R);
        if(c==q.L||c==q.R)
            q.L=out[c==q.L?q.R:q.L],q.R=out[c];
        else if(out[q.L]<in[q.R])
            q.lca=c,q.L=out[q.L],q.R=in[q.R];
        else q.lca=c,c=in[q.L],q.L=out[q.R],q.R=c;
        q.Lid=q.L/B;
    }
    sort(ALL(query));
    int L=0,R=-1;
    for(auto q:query){
        while(R<q.R) flip(dfn[++R]);
        while(L>q.L) flip(dfn[--L]);
        while(R>q.R) flip(dfn[R--]);
        while(L<q.L) flip(dfn[L++]);
        if(q.lca) add(arr[q.lca]);
        ans[q.id]=cur_ans;
        if(q.lca) sub(arr[q.lca]);
    }
}
```

### 9.3 DynamicConvexTrick

```
// only works for integer coordinates!!

bool Flag; // 0: insert Line, 1: Lower_bound x
template<class val = ll, class compare = less<val>> //
    sort lines with comp
struct DynamicConvexTrick{
    static const ll minx = 0, maxx = 1l(1e9) + 5;
    static compare comp;
    struct Line{
        val a, b, l, r; // Line ax + b in [l, r]
        Line(val _a, val _b, val _l = minx, val _r = maxx):
            a(_a), b(_b), l(_l), r(_r){}
        val operator () (val x) const {
            return a * x + b;
        }
    };
    struct cmp{
        bool operator () (const Line a, const Line b){
            if(Flag == 0)return comp(a.a, b.a);
            return a.r < b.l;
        }
    };
    inline val idiv(val a, val b){
        return a / b - (a % b && a < 0 ^ b < 0);
    }
    set<Line, cmp> st;
    void ins(val a, val b){
        Flag = 0;
        Line L(a, b);
        auto it = st.lower_bound(L);
        if(it != st.begin() && it != st.end())
            if(!comp((*prev(it))(it->l - 1), L(it->l - 1)) &&
                !comp((*it)(it->l), L(it->l)))
                return;
        while(it != st.end()){

```

```

    if(it->a == L.a && !comp(it->b, L.b))return;
    if(comp((*it)(it->r), L(it->r)))it = st.erase(it)
    ;
    else{
        Line M = *it;
        st.erase(it);
        L.r = max(idiv(L.b - M.b, M.a - L.a), minx);
        M.l = L.r + 1;
        it = st.insert(M).X;
        break;
    }
}
while(it != st.begin()){
    auto pit = prev(it);
    if(comp((*pit)(pit->l), L(pit->l)))st.erase(pit);
    else{
        Line M = *pit;
        st.erase(pit);
        M.r = min(idiv(L.b - M.b, M.a - L.a), maxx - 1)
        ;
        L.l = M.r + 1;
        st.insert(M);
        break;
    }
}
st.insert(L);
}
val operator () (val x){
    Flag = 1;
    auto it = st.lower_bound({0, 0, x, x});
    return (*it)(x);
}
};
DynamicConvexTrick<> DCT;

```

## 9.4 DLX\*

```

template<bool Exact>
struct DLX {
    int lt[NN], rg[NN], up[NN], dn[NN], cl[NN], rw[NN],
        bt[NN], s[NN], head, sz, ans;
    int columns;
    bool vis[NN];
    void remove(int c) {
        if (Exact) lt[rg[c]] = lt[c], rg[lt[c]] = rg[c];
        for (int i = dn[c]; i != c; i = dn[i]) {
            if (Exact) {
                for (int j = rg[i]; j != i; j = rg[j])
                    up[dn[j]] = up[j], dn[up[j]] = dn[j], --s[cl[j]];
            } else {
                lt[rg[i]] = lt[i], rg[lt[i]] = rg[i];
            }
        }
    }
    void restore(int c) {
        for (int i = up[c]; i != c; i = up[i]) {
            if (Exact) {
                for (int j = lt[i]; j != i; j = lt[j])
                    ++s[cl[j]], up[dn[j]] = j, dn[up[j]] = j;
            } else {
                lt[rg[i]] = rg[lt[i]] = i;
            }
        }
        if (Exact) lt[rg[c]] = c, rg[lt[c]] = c;
    }
    void init(int c) {
        columns = c;
        for (int i = 0; i < c; ++i) {
            up[i] = dn[i] = bt[i] = i;
            lt[i] = i == 0 ? c : i - 1;
            rg[i] = i == c - 1 ? c : i + 1;
            s[i] = 0;
        }
        rg[c] = 0, lt[c] = c - 1;
        up[c] = dn[c] = -1;
        head = c, sz = c + 1;
    }
    void insert(int r, const vector<int> &col) {

```

```

        if (col.empty()) return;
        int f = sz;
        for (int i = 0; i < (int)col.size(); ++i) {
            int c = col[i], v = sz++;
            dn[bt[c]] = v;
            up[v] = bt[c], bt[c] = v;
            rg[v] = (i + 1 == (int)col.size() ? f : v + 1);
            rw[v] = r, cl[v] = c;
            ++s[c];
            if (i > 0) lt[v] = v - 1;
        }
        lt[f] = sz - 1;
    }
    int h() {
        int ret = 0;
        memset(vis, 0, sizeof(bool) * sz);
        for (int x = rg[head]; x != head; x = rg[x]) {
            if (vis[x]) continue;
            vis[x] = true, ++ret;
            for (int i = dn[x]; i != x; i = dn[i]) {
                for (int j = rg[i]; j != i; j = rg[j])
                    vis[cl[j]] = true;
            }
        }
        return ret;
    }
    void dfs(int dep) {
        if (dep + (Exact ? 0 : h()) >= ans) return;
        if (rg[head] == head) return ans = dep, void();
        if (dn[rg[head]] == rg[head]) return;
        int c = rg[head];
        int w = c;
        for (int x = c; x != head; x = rg[x]) if (s[x] < s[w]) w = x;
        if (Exact) {
            remove(w);
            for (int i = dn[w]; i != w; i = dn[i]) {
                for (int j = rg[i]; j != i; j = rg[j]) remove(
                    cl[j]);
                dfs(dep + 1);
                for (int j = lt[i]; j != i; j = lt[j]) restore(
                    cl[j]);
            }
            restore(w);
        } else {
            for (int i = dn[w]; i != w; i = dn[i]) {
                remove(i);
                for (int j = rg[i]; j != i; j = rg[j]) remove(j);
                dfs(dep + 1);
                for (int j = lt[i]; j != i; j = lt[j]) restore(
                    j);
                restore(i);
            }
        }
    }
    int solve() {
        for (int i = 0; i < columns; ++i)
            dn[bt[i]] = i, up[i] = bt[i];
        ans = 1e9, dfs(0);
        return ans;
    }
};

```