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1 Basic

1.1 Shell script

```
g++ -O2 -std=c++17 -Dbbq -Wall -Wextra -Wshadow -o $1
    $1.cpp
chmod +x compile.sh
```

1.2 Default code

```
#include<bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;
#define X first
#define Y second
#define SZ(a) ((int)a.size())
#define ALL(v) v.begin(), v.end()
#define pb push_back
```

1.3 vimrc

```
"This file should be placed at ~/.vimrc"
se nu ai hls et ru ic is sc cul
se re=1 ts=4 sts=4 sw=4 ls=2 mouse=a
syntax on
hi cursorline cterm=none ctermbg=89
set bg=dark
inoremap {<ENTER> {}<LEFT><ENTER><ENTER><UP><TAB>
```

1.4 readchar

```
inline char readchar() {
    static const size_t bufsize = 65536;
    static char buf[bufsize];
    static char *p = buf, *end = buf;
    if (p == end) end = buf + fread_unlocked(buf, 1,
        bufsize, stdin), p = buf;
    return *p++;
}
```

1.5 Black Magic

```
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp> //rb_tree
using namespace __gnu_pbds;
typedef __gnu_pbds::priority_queue<int> heap;
int main() {
    heap h1, h2;
    h1.push(1), h1.push(3);
    h2.push(2), h2.push(4);
    h1.join(h2);
    cout << h1.size() << h2.size() << h1.top() << endl;
    //404
    tree<ll, null_type, less<ll>, rb_tree_tag,
        tree_order_statistics_node_update> st;
    tree<ll, ll, less<ll>, rb_tree_tag,
        tree_order_statistics_node_update> mp;
    for (int x : {0, 2, 3, 4}) st.insert(x);
    cout << *st.find_by_order(2) << st.order_of_key(1) <<
        endl; //31
}
```

1.6 Texas hold'em

```
char suit[4]={'C','D','H','Y'}, ranks[13]={'2','3','4','5','6','7','8','9','T','J','Q','K','A'};
int rk[256];
/*
for(int i=0;i<13;++i)
rk[ranks[i]]=i;
for(int i=0;i<4;++i)
rk[suit[i]]=i;
*/
struct cards{
vector<pii> v;
int suit_count[4], hands;
void reset(){v.clear(), FILL(suit_count, 0), hands=-1;}
void insert(char a, char b){//suit, rank
++suit_count[rk[a]];
int flag=0;
for(auto &i:v)
if(i.Y==rk[b])
{
++i.X, flag=1;
break;
}
if(!flag) v.pb(pii(1, rk[b]));
}
void insert(string s){insert(s[0], s[1]);}
void ready(){
int Straight=0, Flush=(max_element(suit_count, suit_count+4))==5;
sort(ALL(v), [](ii a, ii b){return a>b;});
if(SZ(v)==5&&v[0].Y==v[1].Y+1&&v[1].Y==v[2].Y+1&&v[2].Y==v[3].Y+1&&v[3].Y==v[4].Y+1)
Straight=1;
else if(SZ(v)==5&&v[0].Y==12&&v[1].Y==3&&v[2].Y==2&&v[3].Y==1&&v[4].Y==0)
v[0].Y=3, v[1].Y=2, v[2].Y=1, v[3].Y=0, v[4].Y=-1,
Straight=1;
if(Straight&&Flush) hands=1;
else if(v[0].X==4) hands=2;
else if(v[0].X==3&&v[1].X==2) hands=3;
else if(Flush) hands=4;
else if(Straight) hands=5;
else if(v[0].X==3) hands=6;
else if(v[0].X==2&&v[1].X==2) hands=7;
else if(v[0].X==2) hands=8;
else hands=9;
}
bool operator>(const cards &a) const{
if(hands==a.hands) return v>a.v;
return hands<a.hands;
}
};
```

2 Graph

2.1 BCC Vertex*

```
vector<int> G[N]; //1-base
vector<int> nG[N], bcc[N];
int low[N], dfn[N], Time;
int bcc_id[N], bcc_cnt; //1-base
bool is_cut[N]; //whether is av
bool cir[N];
int st[N], top;

void dfs(int u, int pa = -1) {
int child = 0;
low[u] = dfn[u] = ++Time;
st[top++] = u;
for(int v : G[u])
if(!dfn[v]) {
dfs(v, u), ++child;
low[u] = min(low[u], low[v]);
if(dfn[u] <= low[v]) {
is_cut[u]=1;
bcc[++bcc_cnt].clear();
int t;
```

```
do {
bcc_id[t = st[--top]] = bcc_cnt;
bcc[bcc_cnt].push_back(t);
}while(t != v);
bcc_id[u]=bcc_cnt;
bcc[bcc_cnt].pb(u);
}
}
else if(dfn[v] < dfn[u] && v!=pa)
low[u] = min(low[u], dfn[v]);
if(pa == -1 && child < 2)
is_cut[u] = 0;
}

void bcc_init(int n) {
Time = bcc_cnt = top = 0;
for(int i = 1; i <= n; ++i)
G[i].clear(), dfn[i] = bcc_id[i] = is_cut[i] = 0;
}

void bcc_solve(int n) {
for(int i = 1; i <= n; ++i)
if(!dfn[i])
dfs(i);
// circle-square tree
for(int i = 1; i <= n; ++i)
if(is_cut[i])
bcc_id[i] = ++bcc_cnt, cir[bcc_cnt] = 1;
for(int i = 1; i <= bcc_cnt && !cir[i]; ++i)
for(int j : bcc[i])
if(is_cut[j])
nG[i].pb(bcc_id[j]), nG[bcc_id[j]].pb(i);
}
```

2.2 Bridge*

```
int low[N], dfn[N], Time; // 1-base
vector<pii> G[N], edge;
vector<bool> is_bridge;

void init(int n) {
Time = 0;
for(int i = 1; i <= n; ++i)
G[i].clear(), low[i] = dfn[i] = 0;
}

void add_edge(int a, int b) {
G[a].pb(pii(b, SZ(edge))), G[b].pb(pii(a, SZ(edge)));
edge.pb(pii(a, b));
}

void dfs(int u, int f) {
dfn[u] = low[u] = ++Time;
for(auto i : G[u])
if(!dfn[i.X])
dfs(i.X, i.Y), low[u] = min(low[u], low[i.X]);
else if(i.Y != f)
low[u] = min(low[u], dfn[i.X]);
if(low[u] == dfn[u] && f != -1)
is_bridge[f] = 1;
}

void solve(int n) {
is_bridge.resize(SZ(edge));
for(int i = 1; i <= n; ++i)
if(!dfn[i])
dfs(i, -1);
}
```

2.3 2SAT (SCC)*

```
struct SAT { // 0-base
int low[N], dfn[N], bln[N], n, Time, nScc;
bool instack[N], istrue[N];
stack<int> st;
vector<int> G[N], SCC[N];
void init(int _n) {
n = _n; // assert(n * 2 <= N);
```

```

    for (int i = 0; i < n + n; ++i)
        G[i].clear();
}
void add_edge(int a, int b) {
    G[a].pb(b);
}
void dfs(int u) {
    dfn[u] = low[u] = ++Time;
    instack[u] = 1, st.push(u);
    for (int i : G[u])
        if (!dfn[i]) dfs(i), low[u] = min(low[i], low[u]);
        else if (instack[i] && dfn[i] < dfn[u])
            low[u] = min(low[u], dfn[i]);
    if (low[u] == dfn[u]) {
        int tmp;
        do {
            tmp = st.top(), st.pop();
            instack[tmp] = 0, bln[tmp] = nScc;
        } while (tmp != u);
        ++nScc;
    }
}
bool solve() {
    Time = nScc = 0;
    for (int i = 0; i < n + n; ++i)
        SCC[i].clear(), low[i] = dfn[i] = bln[i] = 0;
    for (int i = 0; i < n + n; ++i)
        if (!dfn[i]) dfs(i);
    for (int i = 0; i < n + n; ++i)
        SCC[bln[i]].pb(i);
    for (int i = 0; i < n; ++i) {
        if (bln[i] == bln[i + n]) return false;
        istrue[i] = bln[i] < bln[i + n];
        istrue[i + n] = !istrue[i];
    }
    return true;
}
};

```

2.4 MinimumMeanCycle*

```

ll road[N][N]; //input here
struct MinimumMeanCycle{
    ll dp[N + 5][N], n;
    pll solve() {
        ll a = -1, b = -1, L = n + 1;
        for(int i = 2; i <= L; ++i)
            for(int k = 0; k < n; ++k)
                for(int j = 0; j < n; ++j)
                    dp[i][j] = min(dp[i - 1][k] + road[k][j], dp[i][j]);
        for(int i = 0; i < n; ++i) {
            if(dp[L][i] >= INF) continue;
            ll ta = 0, tb = 1;
            for(int j = 1; j < n; ++j)
                if(dp[j][i] < INF && ta * (L - j) < (dp[L][i] - dp[j][i]) * tb)
                    ta = dp[L][i] - dp[j][i], tb = L - j;
            if(ta == 0) continue;
            if(a == -1 || a * tb > ta * b)
                a = ta, b = tb;
        }
        if(a != -1) {
            ll g = __gcd(a, b);
            return pll(a / g, b / g);
        }
        return pll(-1LL, -1LL);
    }
}
void init(int _n){
    n = _n;
    for(int i = 0; i < n; ++i)
        for(int j = 0; j < n; ++j)
            dp[i + 2][j] = INF;
}
};

```

2.5 Virtual Tree*

```

vector<int> vG[N];
int top, st[N];

void insert(int u) {
    if(top == -1)
        return st[++top] = u, void();
    int p = LCA(st[top], u);
    if(p == st[top])
        return st[++top] = u, void();
    while(top >= 1 && dep[st[top - 1]] >= dep[p])
        vG[st[top - 1]].pb(st[top]), --top;
    if(st[top] != p)
        vG[p].pb(st[top]), --top, st[++top] = p;
    st[++top] = u;
}

void reset(int u) {
    for(int i : vG[u])
        reset(i);
    vG[u].clear();
}

void solve(vector<int> &v) {
    top = -1;
    sort(ALL(v), [&](int a, int b){return dfn[a] < dfn[b];});
    for (int i : v)
        insert(i);
    while (top > 0)
        vG[st[top - 1]].pb(st[top]), --top;
    //do something
    reset(v[0]);
}

```

2.6 Maximum Clique Dyn*

```

const int N = 150;
struct MaxClique { // Maximum Clique
    bitset<N> a[N], cs[N];
    int ans, sol[N], q, cur[N], d[N], n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; i++) a[i].reset();
    }
    void addEdge(int u, int v) { a[u][v] = a[v][u] = 1; }
    void csort(vector<int> &r, vector<int> &c) {
        int mx = 1, km = max(ans - q + 1, 1), t = 0, m = r.size();
        cs[1].reset(), cs[2].reset();
        for (int i = 0; i < m; i++) {
            int p = r[i], k = 1;
            while ((cs[k] & a[p]).count()) k++;
            if (k > mx) mx++, cs[mx + 1].reset();
            cs[k][p] = 1;
            if (k < km) r[t++] = p;
        }
        c.resize(m);
        if (t) c[t - 1] = 0;
        for (int k = km; k <= mx; k++)
            for (int p = cs[k]._Find_first(); p < N; p = cs[k]._Find_next(p))
                r[t] = p, c[t] = k, t++;
    }
    void dfs(vector<int> &r, vector<int> &c, int l, bitset<N> mask) {
        while (!r.empty()) {
            int p = r.back();
            r.pop_back(), mask[p] = 0;
            if (q + c.back() <= ans) return;
            cur[q++] = p;
            vector<int> nr, nc;
            bitset<N> nmask = mask & a[p];
            for (int i : r)
                if (a[p][i]) nr.push_back(i);
            if (!nr.empty()) {
                if (l < 4) {
                    for (int i : nr) d[i] = (a[i] & nmask).count();
                    sort(nr.begin(), nr.end(), [&](int x, int y) { return d[x] > d[y]; });
                }
            }
        }
    }
};

```

```

    }
    csort(nr, nc), dfs(nr, nc, 1 + 1, nmask);
} else if (q > ans)
    ans = q, copy_n(cur, q, sol);
c.pop_back(), q--;
}
}
int solve(bitset<N> mask = bitset<N>(string(N, '1'))
{ // vertex mask
    vector<int> r, c;
    ans = q = 0;
    for (int i = 0; i < n; i++)
        if (mask[i]) r.push_back(i);
    for (int i = 0; i < n; i++) d[i] = (a[i] & mask).
        count();
    sort(r.begin(), r.end(), [&](int i, int j) { return
        d[i] > d[j]; });
    csort(r, c), dfs(r, c, 1, mask);
    return ans; // sol[0 ~ ans-1]
}
} graph;

```

2.7 Minimum Steiner Tree*

```

// Minimum Steiner Tree
// O(V 3^T + V^2 2^T)
struct SteinerTree{// 0-base
    static const int T = 10, N = 105, INF = 1e9;
    int n, dst[N][N], dp[1 << T][N], tdst[N];
    int vcost[N]; // the cost of vertices
    void init(int _n){
        n = _n;
        for(int i = 0; i < n; ++i) {
            for(int j = 0; j < n; ++j)
                dst[i][j] = INF;
            dst[i][i] = vcost[i] = 0;
        }
    }
    void add_edge(int ui, int vi, int wi) {
        dst[ui][vi] = min(dst[ui][vi], wi);
    }
    void shortest_path() {
        for(int k = 0; k < n; ++k)
            for(int i = 0; i < n; ++i)
                for(int j = 0; j < n; ++j)
                    dst[i][j] = min(dst[i][j], dst[i][k] + dst[k][j]);
    }
    int solve(const vector<int>& ter) {
        shortest_path();
        int t = SZ(ter);
        for(int i = 0; i < (1 << t); ++i)
            for(int j = 0; j < n; ++j)
                dp[i][j] = INF;
        for(int i = 0; i < n; ++i)
            dp[0][i] = vcost[i];
        for(int msk = 1; msk < (1 << t); ++msk){
            if(!(msk & (msk - 1))){
                int who = __lg(msk);
                for(int i = 0; i < n; ++i)
                    dp[msk][i] = vcost[ter[who]] + dst[ter[who]][i];
            }
            for(int i = 0; i < n; ++i)
                for(int submsk = (msk - 1) & msk; submsk; submsk = (submsk - 1) & msk)
                    dp[msk][i] = min(dp[msk][i], dp[submsk][i] +
                        dp[msk ^ submsk][i] - vcost[i]);
            for(int i = 0; i < n; ++i) {
                tdst[i] = INF;
                for(int j = 0; j < n; ++j)
                    tdst[i] = min(tdst[i], dp[msk][j] + dst[j][i]);
            }
            for(int i = 0; i < n; ++i)
                dp[msk][i] = tdst[i];
        }
        int ans = INF;
        for(int i = 0; i < n; ++i)
            ans = min(ans, dp[(1 << t) - 1][i]);
    }
}

```

```

    return ans;
}
};

```

2.8 Dominator Tree*

```

struct dominator_tree{//1-base
    vector<int> G[N], rG[N];
    int n, pa[N], dfn[N], id[N], Time;
    int semi[N], idom[N], best[N];
    vector<int> tree[N]; //dominator_tree
    void init(int _n) {
        n = _n;
        for(int i = 1; i <= n; ++i)
            G[i].clear(), rG[i].clear();
    }
    void add_edge(int u, int v) {
        G[u].pb(v), rG[v].pb(u);
    }
    void dfs(int u) {
        id[dfn[u] = ++Time] = u;
        for(auto v : G[u])
            if(!dfn[v])
                dfs(v), pa[dfn[v]] = dfn[u];
    }
    int find(int y, int x) {
        if(y <= x)
            return y;
        int tmp = find(pa[y], x);
        if(semi[best[y]] > semi[best[pa[y]]])
            best[y] = best[pa[y]];
        return pa[y] = tmp;
    }
    void tarjan(int root) {
        Time = 0;
        for(int i = 1; i <= n; ++i){
            dfn[i] = idom[i] = 0;
            tree[i].clear();
            best[i] = semi[i] = i;
        }
        dfs(root);
        for(int i = Time; i > 1; --i) {
            int u = id[i];
            for(auto v : rG[u])
                if(v < dfn[v]) {
                    find(v, i);
                    semi[i] = min(semi[i], semi[best[v]]);
                }
            tree[semi[i]].pb(i);
            for(auto v : tree[pa[i]]) {
                find(v, pa[i]);
                idom[v] = semi[best[v]] == pa[i] ? pa[i] : best[v];
            }
            tree[pa[i]].clear();
        }
        for(int i = 2; i <= Time; ++i) {
            if(idom[i] != semi[i])
                idom[i] = idom[idom[i]];
            tree[id[idom[i]]].pb(id[i]);
        }
    }
}
};

```

2.9 Minimum Arborescence*

```

struct zhu_liu{//O(VE)
    struct edge{
        int u, v;
        ll w;
    };
    vector<edge> E; //0-base
    int pe[N], id[N], vis[N];
    ll in[N];
    void init() {E.clear();}
    void add_edge(int u, int v, ll w) {
        if (u != v) E.pb(edge{u, v, w});
    }
}

```

```

11 build(int root, int n) {
    ll ans = 0;
    for(;;) {
        fill_n(in, n, INF);
        for (int i = 0; i < SZ(E); ++i)
            if (E[i].u != E[i].v && E[i].w < in[E[i].v])
                pe[E[i].v] = i, in[E[i].v] = E[i].w;
        for (int u = 0; u < n; ++u) //no solution
            if (u != root && in[u] == INF) return -INF;
        int cntnode = 0;
        fill_n(id, n, -1), fill_n(vis, n, -1);
        for (int u = 0; u < n; ++u) {
            if (u != root) ans += in[u];
            int v = u;
            while (vis[v] != u && !~id[v] && v != root)
                vis[v] = u, v = E[pe[v]].u;
            if (v != root && !~id[v]) {
                for (int x = E[pe[v]].u; x != v; x = E[pe[x]].u)
                    id[x] = cntnode;
                id[v] = cntnode++;
            }
        }
        if (!cntnode) break; //no cycle
        for (int u = 0; u < n; ++u)
            if (!~id[u]) id[u] = cntnode++;
        for (int i = 0; i < SZ(E); ++i) {
            int v = E[i].v;
            E[i].u = id[E[i].u], E[i].v = id[E[i].v];
            if (E[i].u != E[i].v) E[i].w -= in[v];
        }
        n = cntnode, root = id[root];
    }
    return ans;
}
};

```

2.10 Vizing's theorem

```

namespace vizing { // returns edge coloring in adjacent
    matrix G, 1 - based
    int C[kN][kN], G[kN][kN];
    void clear(int N) {
        for (int i = 0; i <= N; i++) {
            for (int j = 0; j <= N; j++) C[i][j] = G[i][j] = 0;
        }
    }
    void solve(vector<pair<int, int>> &E, int N, int M) {
        int X[kN] = {}, a;
        auto update = [&](int u) {
            for (X[u] = 1; C[u][X[u]]; X[u]++);
        };
        auto color = [&](int u, int v, int c) {
            int p = G[u][v];
            G[u][v] = G[v][u] = c;
            C[u][c] = v, C[v][c] = u;
            C[u][p] = C[v][p] = 0;
            if (p) X[u] = X[v] = p;
            else update(u), update(v);
            return p;
        };
        auto flip = [&](int u, int c1, int c2) {
            int p = C[u][c1];
            swap(C[u][c1], C[u][c2]);
            if (p) G[u][p] = G[p][u] = c2;
            if (!C[u][c1]) X[u] = c1;
            if (!C[u][c2]) X[u] = c2;
            return p;
        };
        for (int i = 1; i <= N; i++) X[i] = 1;
        for (int t = 0; t < E.size(); t++) {
            int u = E[t].first, v0 = E[t].second, v = v0, c0
                = X[u], c = c0, d;
            vector<pair<int, int>> L;
            int vst[kN] = {};
            while (!G[u][v0]) {
                L.emplace_back(v, d = X[v]);
                if (!C[v][c]) for (a = (int)L.size() - 1; a >= 0; a--) c = color(u, L[a].first, c);
            }
        }
    }
};

```

```

        else if (!C[u][d]) for (a = (int)L.size() - 1;
            a >= 0; a--) color(u, L[a].first, L[a].
                second);
        else if (vst[d]) break;
        else vst[d] = 1, v = C[u][d];
    }
    if (!G[u][v0]) {
        for (; v; v = flip(v, c, d), swap(c, d));
        if (C[u][c0]) {
            for (a = (int)L.size() - 2; a >= 0 && L[a].
                second != c; a--);
            for (; a >= 0; a--) color(u, L[a].first, L[a].
                second);
        } else t--;
    }
}
}
};

```

2.11 Minimum Clique Cover*

```

struct Clique_Cover { // 0-base, O(n^2^n)
    int co[1 << N], n, E[N];
    int dp[1 << N];
    void init(int _n) {
        n = _n, fill_n(dp, 1 << n, 0);
        fill_n(E, n, 0), fill_n(co, 1 << n, 0);
    }
    void add_edge(int u, int v) {
        E[u] |= 1 << v, E[v] |= 1 << u;
    }
    int solve() {
        for (int i = 0; i < n; ++i)
            co[1 << i] = E[i] | (1 << i);
        co[0] = (1 << n) - 1;
        dp[0] = (n & 1) * 2 - 1;
        for (int i = 1; i < (1 << n); ++i) {
            int t = i & -i;
            dp[i] = -dp[i ^ t];
            co[i] = co[i ^ t] & co[t];
        }
        for (int i = 0; i < (1 << n); ++i)
            co[i] = (co[i] & i) == i;
        fwt(co, 1 << n);
        for (int ans = 1; ans < n; ++ans) {
            int sum = 0;
            for (int i = 0; i < (1 << n); ++i)
                sum += (dp[i] * co[i]);
            if (sum) return ans;
        }
        return n;
    }
};

```

2.12 NumberofMaximalClique*

```

struct BronKerbosch { // 1-base
    int n, a[N], g[N][N];
    int S, all[N][N], some[N][N], none[N][N];
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            for (int j = 1; j <= n; ++j)
                g[i][j] = 0;
    }
    void add_edge(int u, int v) {
        g[u][v] = g[v][u] = 1;
    }
    void dfs(int d, int an, int sn, int nn) {
        if (S > 1000) return; // pruning
        if (sn == 0 && nn == 0) ++S;
        int u = some[d][0];
        for (int i = 0; i < sn; ++i) {
            int v = some[d][i];
            if (g[u][v]) continue;
            int tsu = 0, tnn = 0;
            copy_n(all[d], an, all[d + 1]);
            all[d + 1][an] = v;
            for (int j = 0; j < sn; ++j)

```

```

    if(g[v][some[d][j]])
        some[d + 1][tsn++] = some[d][j];
    for(int j = 0; j < nn; ++j)
        if(g[v][none[d][j]])
            none[d + 1][tnn++] = none[d][j];
    dfs(d + 1, an + 1, tsn, tnn);
    some[d][i] = 0, none[d][nn++] = v;
}
}
int solve() {
    iota(some[0], some[0] + n, 1);
    S = 0, dfs(0, 0, n, 0);
    return S;
}
};

```

2.13 Theory

$|\text{Maximum independent edge set}| = |V| - |\text{Minimum edge cover}|$

$|\text{Maximum independent set}| = |V| - |\text{Minimum vertex cover}|$

A sequence of non-negative integers $d_1 \geq \dots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + \dots + d_n$ is even and

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k) \text{ holds for every } k \text{ in } 1 \leq k \leq n.$$

```

    else
        bln[i.Y]=u,dt[u]=edge[i.Y];
}
void cut(int u,int link){
    data[pl[u]=t++]=dt[u],ulink[u]=link;
    if(!mxson[u]) return;
    cut(mxson[u],link);
    for(auto i:G[u])
        if(i.X!=pa[u]&&i.X!=mxson[u])
            cut(i.X,i.X);
}
void build(){
    dfs(1,1,1),cut(1,1),/*build*/;
}
int query(int a,int b){
    int ta=ulink[a],tb=ulink[b],re=0;
    while(ta!=tb)
        if(deep[ta]<deep[tb])
            /*query*/,tb=ulink[b=pa[tb]];
        else
            /*query*/,ta=ulink[a=pa[ta]];
    if(a==b) return re;
    if(pl[a]>pl[b]) swap(a,b);
    /*query*/
    return re;
}
};

```

3 Data Structure

3.1 Leftist Tree

```

struct node{
    ll v,data,sz,sum;
    node *l,*r;
    node(ll k):v(0),data(k),sz(1),l(0),r(0),sum(k){}
};
ll sz(node *p){return p ? p->sz : 0;}
ll V(node *p){return p ? p->v : -1;}
ll sum(node *p){return p ? p->sum : 0;}
node* merge(node *a,node *b){
    if(!a || !b) return a ? a : b;
    if(a->data<b->data) swap(a,b);
    a->r=merge(a->r,b);
    if(V(a->r)>V(a->l)) swap(a->r,a->l);
    a->v=V(a->r)+1,a->sz=sz(a->l)+sz(a->r)+1;
    a->sum=sum(a->l)+sum(a->r)+a->data;
    return a;
}
void pop(node *&o){
    node *tmp=o;
    o=merge(o->l,o->r);
    delete tmp;
}
}

```

3.2 Heavy light Decomposition

```

struct Heavy_light_Decomposition{//1-base
    int n,ulink[10005],deep[10005],mxson[10005],w[10005],
        pa[10005];
    int t,pl[10005],data[10005],dt[10005],bln[10005],edge
        [10005],et;
    vector<pii> G[10005];
    void init(int _n){n=_n,t=0,et=1;
        for(int i=1;i<=n;++i) G[i].clear(),mxson[i]=0;
    }
    void add_edge(int a,int b,int w){
        G[a].pb(pii(b,et)),G[b].pb(pii(a,et)),edge[et++]=w;
    }
    void dfs(int u,int f,int d){
        w[u]=1,pa[u]=f,deep[u]=d++;
        for(auto &i:G[u])
            if(i.X!=f){
                dfs(i.X,u,d),w[u]+=w[i.X];
                if(w[mxson[u]]<w[i.X])
                    mxson[u]=i.X;
            }
    }
}

```

3.3 Centroid Decomposition*

```

struct Cent_Dec { // 1-base
    vector<pll> G[N];
    pll info[N]; // store info. of itself
    pll upinfo[N]; // store info. of climbing up
    int n, pa[N], layer[N], sz[N], done[N];
    ll dis[___lg(N) + 1][N];
    void init(int _n) {
        n = _n, layer[0] = -1;
        fill_n(pa + 1, n, 0), fill_n(done + 1, n, 0);
        for (int i = 1; i <= n; ++i) G[i].clear();
    }
    void add_edge(int a, int b, int w) {
        G[a].pb(pll(b, w)), G[b].pb(pll(a, w));
    }
    void get_cent(int u, int f, int &mx, int &c, int num)
        {
            int mxsz = 0;
            sz[u] = 1;
            for (pll e : G[u])
                if (!done[e.X] && e.X != f) {
                    get_cent(e.X, u, mx, c, num);
                    sz[u] += sz[e.X], mxsz = max(mxsz, sz[e.X]);
                }
            if (mx > max(mxsz, num - sz[u]))
                mx = max(mxsz, num - sz[u]), c = u;
        }
    void dfs(int u, int f, ll d, int org) {
        // if required, add self info or climbing info
        dis[layer[org]][u] = d;
        for (pll e : G[u])
            if (!done[e.X] && e.X != f)
                dfs(e.X, u, d + e.Y, org);
    }
    int cut(int u, int f, int num) {
        int mx = 1e9, c = 0, lc;
        get_cent(u, f, mx, c, num);
        done[c] = 1, pa[c] = f, layer[c] = layer[f] + 1;
        for (pll e : G[c])
            if (!done[e.X]) {
                if (sz[e.X] > sz[c])
                    lc = cut(e.X, c, num - sz[c]);
                else
                    lc = cut(e.X, c, sz[e.X]);
                upinfo[lc] = pll(), dfs(e.X, c, e.Y, c);
            }
        return done[c] = 0, c;
    }
    void build(){cut(1, 0, n);}
    void modify(int u) {

```



```

    for (int a = u, ly = layer[a]; a; a = pa[a], --ly)
    {
        info[a].X += dis[ly][u], ++info[a].Y;
        if (pa[a])
            upinfo[a].X += dis[ly - 1][u], ++upinfo[a].Y;
    }
}
ll query(int u) {
    ll rt = 0;
    for (int a = u, ly = layer[a]; a; a = pa[a], --ly)
    {
        rt += info[a].X + info[a].Y * dis[ly][u];
        if (pa[a])
            rt -= upinfo[a].X + upinfo[a].Y * dis[ly - 1][u];
    }
    return rt;
}
};

```

3.4 Link cut tree*

```

struct Splay { // xor-sum
    static Splay nil;
    Splay *ch[2], *f;
    int val, sum, rev, size;
    Splay (int _val = 0) : val(_val), sum(_val), rev(0),
        size(1) { f = ch[0] = ch[1] = &nil; }
    bool isr() { return f -> ch[0] != this && f -> ch[1] != this; }
    int dir() { return f -> ch[0] == this ? 0 : 1; }
    void setCh(Splay *c, int d) { ch[d] = c; if (c != &nil) c -> f = this; pull(); }
    void push() { if (!rev) return; swap(ch[0], ch[1]); if (ch[0] != &nil) ch[0] -> rev ^= 1; if (ch[1] != &nil) ch[1] -> rev ^= 1; rev = 0; }
    void pull() { // take care of the nil! size = ch[0] -> size + ch[1] -> size + 1; sum = ch[0] -> sum ^ ch[1] -> sum ^ val; if (ch[0] != &nil) ch[0] -> f = this; if (ch[1] != &nil) ch[1] -> f = this; }
} Splay::nil;
Splay *nil = &Splay::nil;
void rotate(Splay *x) {
    Splay *p = x -> f;
    int d = x -> dir();
    if (!p -> isr()) p -> f -> setCh(x, p -> dir());
    else x -> f = p -> f;
    p -> setCh(x -> ch[!d], d);
    x -> setCh(p, !d);
    p -> pull(), x -> pull();
}
void splay(Splay *x) {
    vector<Splay*> splayVec;
    for (Splay *q = x; q; q = q -> f) {
        splayVec.pb(q);
        if (q -> isr()) break;
    }
    reverse(ALL(splayVec));
    for (auto it : splayVec) it -> push();
    while (!x -> isr()) {
        if (x -> f -> isr()) rotate(x);
        else if (x -> dir() == x -> f -> dir()) rotate(x -> f), rotate(x);
        else rotate(x), rotate(x);
    }
}

```

```

}
Splay* access(Splay *x) {
    Splay *q = nil;
    for (; x != nil; x = x -> f)
        splay(x), x -> setCh(q, 1), q = x;
    return q;
}
void root_path(Splay *x) {
    access(x), splay(x);
}
void chroot(Splay *x) {
    root_path(x), x -> rev ^= 1;
    x -> push(), x -> pull();
}
void split(Splay *x, Splay *y) {
    chroot(x), root_path(y);
}
void link(Splay *x, Splay *y) {
    root_path(x), chroot(y);
    x -> setCh(y, 1);
}
void cut(Splay *x, Splay *y) {
    split(x, y);
    if (y -> size != 5) return;
    y -> push();
    y -> ch[0] = y -> ch[0] -> f = nil;
}
Splay* get_root(Splay *x) {
    for (root_path(x); x -> ch[0] != nil; x = x -> ch[0])
        x -> push();
    splay(x);
    return x;
}
bool conn(Splay *x, Splay *y) {
    return get_root(x) == get_root(y);
}
Splay* lca(Splay *x, Splay *y) {
    access(x), root_path(y);
    if (y -> f == nil) return y;
    return y -> f;
}
void change(Splay *x, int val) {
    splay(x), x -> val = val, x -> pull();
}
int query(Splay *x, Splay *y) {
    split(x, y);
    return y -> sum;
}
}

```

3.5 KDTree

```

namespace kdt {
    int root, lc[maxn], rc[maxn], xl[maxn], xr[maxn], yl[maxn], yr[maxn];
    point p[maxn];
    int build(int l, int r, int dep = 0) {
        if (l == r) return -1;
        function<bool(const point &, const point &> f = [
            dep](const point &a, const point &b) {
                if (dep & 1) return a.x < b.x;
                else return a.y < b.y;
            };
        int m = (l + r) >> 1;
        nth_element(p + l, p + m, p + r, f);
        xl[m] = xr[m] = p[m].x;
        yl[m] = yr[m] = p[m].y;
        lc[m] = build(l, m, dep + 1);
        if (~lc[m]) {
            xl[m] = min(xl[m], xl[lc[m]]);
            xr[m] = max(xr[m], xr[lc[m]]);
            yl[m] = min(yl[m], yl[lc[m]]);
            yr[m] = max(yr[m], yr[lc[m]]);
        }
        rc[m] = build(m + 1, r, dep + 1);
        if (~rc[m]) {
            xl[m] = min(xl[m], xl[rc[m]]);
            xr[m] = max(xr[m], xr[rc[m]]);
            yl[m] = min(yl[m], yl[rc[m]]);
            yr[m] = max(yr[m], yr[rc[m]]);
        }
    }
}

```

```

    }
    return m;
}
bool bound(const point &q, int o, long long d) {
    double ds = sqrt(d + 1.0);
    if (q.x < xl[o] - ds || q.x > xr[o] + ds ||
        q.y < yl[o] - ds || q.y > yr[o] + ds) return
        false;
    return true;
}
long long dist(const point &a, const point &b) {
    return (a.x - b.x) * 1ll * (a.x - b.x) +
        (a.y - b.y) * 1ll * (a.y - b.y);
}
void dfs(const point &q, long long &d, int o, int dep =
    0) {
    if (!bound(q, o, d)) return;
    long long cd = dist(p[o], q);
    if (cd != 0) d = min(d, cd);
    if ((dep & 1) && q.x < p[o].x || !(dep & 1) && q.y
        < p[o].y) {
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
    } else {
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
    }
}
void init(const vector<point> &v) {
    for (int i = 0; i < v.size(); ++i) p[i] = v[i];
    root = build(0, v.size());
}
long long nearest(const point &q) {
    long long res = 1e18;
    dfs(q, res, root);
    return res;
}
}

```

4 Flow/Matching

4.1 Kuhn Munkres

```

struct KM{// 0-base
    int w[MAXN][MAXN],hl[MAXN],hr[MAXN],slk[MAXN],n;
    int fl[MAXN],fr[MAXN],pre[MAXN],qu[MAXN],ql,q;
    bool vl[MAXN],vr[MAXN];
    void init(int _n){n=_n;
        for(int i=0;i<n;++i)
            for(int j=0;j<n;++j)
                w[i][j]=-INF;
    }
    void add_edge(int a,int b,int wei){
        w[a][b]=wei;
    }
    bool Check(int x){
        if(vl[x]==1,~fl[x]) return vr[qu[qr++]=fl[x]]=1;
        while(~x) swap(x,fr[fl[x]=pre[x]]);
        return 0;
    }
    void Bfs(int s){
        fill(slk,slk+n,INF);
        fill(vl,vl+n,0),fill(vr,vr+n,0);
        ql=qr=0,qu[qr++]=s,vr[s]=1;
        while(1){
            int d;
            while(ql<qr)
                for(int x=0,y=qu[ql++];x<n;++x)
                    if(!vl[x]&&slk[x]>=(d=hl[x]+hr[y]-w[x][y]))
                        if(pre[x]=y,d) slk[x]=d;
                        else if(!Check(x)) return;
            d=INF;
            for (int x=0;x<n;++x)
                if (!vl[x]&&d>slk[x]) d=slk[x];
            for (int x=0;x<n;++x){
                if(vl[x]) hl[x]+=d;
                else slk[x]-=d;
                if(vr[x]) hr[x]-=d;
            }
        }
    }
}

```

```

        for (int x=0;x<n;++x)
            if(!vl[x]&&slk[x]&&!Check(x)) return;
    }
}
int Solve(){
    fill(fl,fl+n,-1),fill(fr,fr+n,-1),fill(hr,hr+n,0);
    for (int i=0;i<n;++i) hl[i]=*max_element(w[i],w[i]+
        n);
    for (int i=0;i<n;++i) Bfs(i);
    int res=0;
    for (int i=0;i<n;++i) res += w[i][fl[i]];
    return res;
}
}
};

```

4.2 MincostMaxflow

```

struct MCMF{//0-base
    struct edge{
        ll from,to,cap,flow,cost,rev;
    }*past[MAXN];
    vector<edge> G[MAXN];
    bitset<MAXN> inq;
    ll dis[MAXN],up[MAXN],s,t,mx,n;
    bool BellmanFord(ll &flow,ll &cost){
        fill(dis,dis+n,INF);
        queue<ll> q;
        q.push(s),inq.reset(),inq[s]=1;
        up[s]=mx-flow,past[s]=0,dis[s]=0;
        while(!q.empty()){
            ll u=q.front();
            q.pop(),inq[u]=0;
            if(!up[u]) continue;
            for(auto &e:G[u])
                if(e.flow!=e.cap&&dis[e.to]>dis[u]+e.cost){
                    dis[e.to]=dis[u]+e.cost,past[e.to]=&e;
                    up[e.to]=min(up[u],e.cap-e.flow);
                    if(!inq[e.to]) inq[e.to]=1,q.push(e.to);
                }
        }
        if(dis[t]==INF) return 0;
        flow+=up[t],cost+=up[t]*dis[t];
        for(ll i=t;past[i];i=past[i]->from){
            auto &e=*past[i];
            e.flow+=up[t],G[e.to][e.rev].flow-=up[t];
        }
        return 1;
    }
    ll MinCostMaxFlow(ll _s,ll _t,ll &cost){
        s=_s,t=_t,cost=0;ll flow=0;
        while(BellmanFord(flow,cost));
        return flow;
    }
    void init(ll _n,ll _mx){n=_n,mx=_mx;
        for(int i=0;i<n;++i) G[i].clear();
    }
    void add_edge(ll a,ll b,ll cap,ll cost){
        G[a].pb(edge{a,b,cap,0,cost,G[b].size()});
        G[b].pb(edge{b,a,0,0,-cost,G[a].size()-1});
    }
}
};

```

4.3 Maximum Simple Graph Matching*

```

struct GenMatch { // 1-base
    int V, pr[N];
    bool el[N][N], inq[N], inp[N], inb[N];
    int st, ed, nb, bk[N], djs[N], ans;
    void init(int _V) {
        V=_V;
        for(int i = 0; i <= V; ++i) {
            for(int j = 0; j <= V; ++j)
                el[i][j] = 0;
            pr[i] = bk[i] = djs[i] = 0;
            inq[i] = inp[i] = inb[i] = 0;
        }
    }
    void add_edge(int u, int v){

```



```

    el[u][v] = el[v][u] = 1;
}
int lca(int u, int v) {
    fill_n(inp, V + 1, 0);
    while(1)
        if(u = djs[u], inp[u] = true, u == st) break;
        else u = bk[pr[u]];
    while(1)
        if(v = djs[v], inp[v]) return v;
        else v = bk[pr[v]];
    return v;
}
void upd(int u){
    for(int v; djs[u] != nb; ) {
        v = pr[u], inb[djs[u]] = inb[djs[v]] = true;
        u = bk[v];
        if(djs[u] != nb) bk[u] = v;
    }
}
void blo(int u, int v, queue<int> &qe) {
    nb = lca(u, v), fill_n(inb, V + 1, 0);
    upd(u), upd(v);
    if(djs[u] != nb) bk[u] = v;
    if(djs[v] != nb) bk[v] = u;
    for(int tu = 1; tu <= V; ++tu)
        if(inb[djs[tu]])
            if(djs[tu] = nb, !inq[tu])
                qe.push(tu), inq[tu]=1;
}
void flow() {
    fill_n(inq + 1, V, 0), fill_n(bk + 1, V, 0);
    iota(djs + 1, djs + V + 1, 1);
    queue<int> qe;
    qe.push(st), inq[st] = 1, ed = 0;
    while(!qe.empty()) {
        int u = qe.front();
        qe.pop();
        for(int v = 1; v <= V; ++v)
            if(el[u][v] && djs[u] != djs[v] && pr[u] != v)
                {
                    if((v == st) || (pr[v] > 0 && bk[pr[v]] > 0))
                        blo(u, v, qe);
                    else if(!bk[v]) {
                        if(bk[v] = u, pr[v] > 0) {
                            if(!inq[pr[v]])
                                qe.push(pr[v]);
                        }
                        else
                            return ed = v, void();
                    }
                }
    }
}
void aug(){
    for(int u = ed, v, w; u > 0; )
        v = bk[u], w = pr[v], pr[v] = u, pr[u] = v, u = w;
}
int solve() {
    fill_n(pr, V + 1, 0), ans = 0;
    for(int u = 1; u <= V; ++u)
        if(!pr[u])
            if(st = u, flow(), ed > 0)
                aug(), ++ans;
    return ans;
}
};

```

4.4 Minimum Weight Matching (Clique version)*

```

struct Graph { // 0-base (Perfect Match), n is even
    int n, match[N], onstk[N], stk[N], tp;
    ll edge[N][N], dis[N];
    void init(int _n) {
        n = _n, tp = 0;
        for (int i = 0; i < n; ++i)
            fill_n(edge[i], n, 0);
    }
}

```

```

void add_edge(int u, int v, ll w) { edge[u][v] = edge[v][u] = w; }
bool SPFA(int u) {
    stk[tp++] = u, onstk[u] = 1;
    for (int v = 0; v < n; ++v)
        if (!onstk[v] && match[u] != v) {
            int m = match[v];
            if (dis[m] > dis[u] - edge[v][m] + edge[u][v])
                {
                    dis[m] = dis[u] - edge[v][m] + edge[u][v];
                    onstk[v] = 1, stk[tp++] = v;
                    if (onstk[m] || SPFA(m)) return 1;
                    --tp, onstk[v] = 0;
                }
        }
    onstk[u] = 0, --tp;
    return 0;
}
ll solve() { // find a match
    for (int i = 0; i < n; ++i) match[i] = i ^ 1;
    while (1) {
        int found = 0;
        fill_n(dis, n, 0); fill_n(onstk, n, 0);
        for (int i = 0; i < n; ++i)
            if (tp = 0, !onstk[i] && SPFA(i))
                for (found = 1; tp >= 2; ) {
                    int u = stk[--tp];
                    int v = stk[--tp];
                    match[u] = v, match[v] = u;
                }
        if (!found) break;
    }
    ll ret = 0;
    for (int i = 0; i < n; ++i) ret += edge[i][match[i]];
    return ret >> 1;
}
};

```

4.5 SW-mincut

```

// global min cut
struct SW{ // O(V^3)
    static const int MXN = 514;
    int n, vst[MXN], del[MXN];
    int edge[MXN][MXN], wei[MXN];
    void init(int _n){
        n=_n, MEM(edge,0), MEM(del,0);
    }
    void addEdge(int u,int v,int w){
        edge[u][v]+=w, edge[v][u]+=w;
    }
    void search(int &s,int &t){
        MEM(vst,0), MEM(wei,0), s=t=-1;
        while(1){
            int mx=-1, cur=0;
            for(int i=0; i<n; ++i)
                if(!del[i] && !vst[i] && mx<wei[i])
                    cur=i, mx=wei[i];
            if(mx==-1) break;
            vst[cur]=1, s=t, t=cur;
            for(int i=0; i<n; ++i)
                if(!vst[i] && !del[i]) wei[i]+=edge[cur][i];
        }
    }
    int solve(){
        int res=INF;
        for(int i=0, x, y; i<n-1; ++i){
            search(x, y), res=min(res, wei[y]), del[y]=1;
            for(int j=0; j<n; ++j)
                edge[x][j]=(edge[j][x]+=edge[y][j]);
        }
        return res;
    }
}
};

```

4.6 BoundedFlow(Dinic*)

```

struct BoundedFlow { //0-base
    struct edge {
        int to, cap, flow, rev;
    };
    vector<edge> G[N];
    int n, s, t, dis[N], cur[N], cnt[N];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n + 2; ++i)
            G[i].clear(), cnt[i] = 0;
    }
    void add_edge(int u, int v, int lcap, int rcap) {
        cnt[u] -= lcap, cnt[v] += lcap;
        G[u].pb(edge{v, rcap, lcap, SZ(G[v])});
        G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
    }
    void add_edge(int u, int v, int cap) {
        G[u].pb(edge{v, cap, 0, SZ(G[v])});
        G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
    }
    int dfs(int u, int cap) {
        if (u == t || !cap) return cap;
        for (int &i = cur[u]; i < SZ(G[u]); ++i) {
            edge &e = G[u][i];
            if (dis[e.to] == dis[u] + 1 && e.cap != e.flow) {
                int df = dfs(e.to, min(e.cap - e.flow, cap));
                if (df) {
                    e.flow += df, G[e.to][e.rev].flow -= df;
                    return df;
                }
            }
        }
        dis[u] = -1;
        return 0;
    }
    bool bfs() {
        fill_n(dis, n + 3, -1);
        queue<int> q;
        q.push(s), dis[s] = 0;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (edge &e : G[u])
                if (!dis[e.to] && e.flow != e.cap)
                    q.push(e.to), dis[e.to] = dis[u] + 1;
        }
        return dis[t] != -1;
    }
    int maxflow(int _s, int _t) {
        s = _s, t = _t;
        int flow = 0, df;
        while (bfs()) {
            fill_n(cur, n + 3, 0);
            while ((df = dfs(s, INF))) flow += df;
        }
        return flow;
    }
    bool solve() {
        int sum = 0;
        for (int i = 0; i < n; ++i)
            if (cnt[i] > 0) add_edge(n + 1, i, cnt[i]), sum += cnt[i];
            else if (cnt[i] < 0) add_edge(i, n + 2, -cnt[i]);
        if (sum != maxflow(n + 1, n + 2)) sum = -1;
        for (int i = 0; i < n; ++i)
            if (cnt[i] > 0) G[n + 1].pop_back(), G[i].pop_back();
            else if (cnt[i] < 0) G[i].pop_back(), G[n + 2].pop_back();
        return sum != -1;
    }
    int solve(int _s, int _t) {
        add_edge(_t, _s, INF);
        if (!solve()) return -1; //invalid flow
        int x = G[_t].back().flow;
        return G[_t].pop_back(), G[_s].pop_back(), x;
    }
};

```

4.7 Gomory Hu tree

```

struct Gomory_Hu_tree { //0-base
    MaxFlow Dinic;
    int n;
    vector<pii> G[MAXN];
    void init(int _n) { n = _n;
        for (int i = 0; i < n; ++i) G[i].clear();
    }
    void solve(vector<int> &v) {
        if (v.size() <= 1) return;
        int s = rand() % SZ(v);
        swap(v.back(), v[s]), s = v.back();
        int t = v[rand() % (SZ(v) - 1)];
        vector<int> L, R;
        int x = (Dinic.reset(), Dinic.maxflow(s, t));
        G[s].pb(pii(t, x)), G[t].pb(pii(s, x));
        for (int i : v)
            if (~Dinic.dis[i]) L.pb(i);
            else R.pb(i);
        solve(L), solve(R);
    }
    void build() {
        vector<int> v(n);
        for (int i = 0; i < n; ++i) v[i] = i;
        solve(v);
    }
} ght; //test by BZOJ 4519
MaxFlow &Dinic = ght.Dinic;

```

4.8 isap

```

struct Maxflow {
    static const int MAXV = 20010;
    static const int INF = 1000000;
    struct Edge {
        int v, c, r;
        Edge(int _v, int _c, int _r) :
            v(_v), c(_c), r(_r) {}
    };
    int s, t;
    vector<Edge> G[MAXV*2];
    int iter[MAXV*2], d[MAXV*2], gap[MAXV*2], tot;
    void init(int x) {
        tot = x + 2;
        s = x + 1, t = x + 2;
        for (int i = 0; i <= tot; i++) {
            G[i].clear();
            iter[i] = d[i] = gap[i] = 0;
        }
    }
    void addEdge(int u, int v, int c) {
        G[u].push_back(Edge(v, c, SZ(G[v])));
        G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
    }
    int dfs(int p, int flow) {
        if (p == t) return flow;
        for (int &i = iter[p]; i < SZ(G[p]); i++) {
            Edge &e = G[p][i];
            if (e.c > 0 && d[p] == d[e.v] + 1) {
                int f = dfs(e.v, min(flow, e.c));
                if (f) {
                    e.c -= f;
                    G[e.v][e.r].c += f;
                    return f;
                }
            }
        }
        if (--gap[d[p]] == 0) d[s] = tot;
        else {
            d[p]++;
            iter[p] = 0;
            ++gap[d[p]];
        }
        return 0;
    }
    int solve() {
        int res = 0;
        gap[0] = tot;
    }
};

```

```

    for(res = 0; d[s] < tot; res += dfs(s, INF));
    return res;
}
} flow;

```

5 String

5.1 KMP

```

int F[MAXN];
vector<int> match(string A, string B){
    vector<int> ans;
    F[0]=-1, F[1]=0;
    for(int i=1, j=0; i<B.size(); F[++i]=++j){
        if(B[i]==B[j]) F[i]=F[j]; //optimize
        while(j!=-1 && B[i]!=B[j]) j=F[j];
    }
    for(int i=0, j=0; i-j+B.size()<=A.size(); ++i, ++j){
        while(j!=-1 && A[i]!=B[j]) j=F[j];
        if(j==B.size()-1) ans.pb(i-j);
    }
    return ans;
}

```

5.2 Z-value

```

const int MAXN = 1e5 + 5;
int z[MAXN];
void make_z(string s){
    int l = 0, r = 0;
    for(int i = 1; i < s.size(); i++){
        for(z[i] = max(0, min(r - i + 1, z[i - 1]));
            i + z[i] < s.size() && s[i + z[i]] == s[z[i]]; z
                [i]++);
        if(i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
}

```

5.3 Manacher*

```

int z[MAXN];
int Manacher(string tmp){
    string s = "&";
    int l=0, r=0, x, ans;
    for(char c:tmp) s.pb(c), s.pb('%');
    ans=0, x=0;
    for(int i=1; i<SZ(s); ++i){
        z[i]=r > i ? min(z[2*1-i], r-i) : 1;
        while(s[i+z[i]]==s[i-z[i]]) ++z[i];
        if(z[i]+i>r) r=z[i]+i, l=i;
    }
    for(int i=1; i<SZ(s); ++i)
        if(s[i]=='%')
            x=max(x, z[i]);
    ans=x/2*2, x=0;
    for(int i=1; i<SZ(s); ++i)
        if(s[i]!='%')
            x=max(x, z[i]);
    return max(ans, (x-1)/2*2+1);
}

```

5.4 Suffix Array

```

struct suffix_array{
    int box[MAXN], tp[MAXN], m;
    bool not_equ(int a, int b, int k, int n){
        return ra[a]!=ra[b] || b+k>n || b+k>n || ra[a+k]!=ra[b+
            k];
    }
    void radix(int *key, int *it, int *ot, int n){
        fill_n(box, m, 0);
        for(int i=0; i<n; ++i) ++box[key[i]];
    }
}

```

```

partial_sum(box, box+m, box);
for(int i=n-1; i>=0; --i) ot[--box[key[it[i]]]]=it[i];
}
void make_sa(string s, int n){
    int k=1;
    for(int i=0; i<n; ++i) ra[i]=s[i];
    do{
        iota(tp, tp+k, n-k), iota(sa+k, sa+n, 0);
        radix(ra+k, sa+k, tp+k, n-k);
        radix(ra, tp, sa, n);
        tp[sa[0]]=0, m=1;
        for(int i=1; i<n; ++i){
            m+=not_equ(sa[i], sa[i-1], k, n);
            tp[sa[i]]=m-1;
        }
        copy_n(tp, n, ra);
        k*=2;
    }while(k<n && m!=n);
}
void make_he(string s, int n){
    for(int j=0, k=0; j<n; ++j){
        if(ra[j])
            for(; s[j+k]==s[sa[ra[j]-1]+k]; ++k);
        he[ra[j]]=k, k=max(0, k-1);
    }
}
int sa[MAXN], ra[MAXN], he[MAXN];
void build(string s){
    FILL(sa, 0), FILL(ra, 0), FILL(he, 0);
    FILL(box, 0), FILL(tp, 0), m=256;
    make_sa(s, s.size());
    make_he(s, s.size());
}
};

```

5.5 SAIS*

```

class SAIS {
public:
    int *SA, *H;
    // zero based, string content MUST > 0
    // result height H[i] is LCP(SA[i - 1], SA[i])
    // string, length, |sigma|
    void build(int *s, int n, int m = 128){
        copy_n(s, n, _s);
        _h[0] = _s[n++] = 0;
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
        SA = _sa + 1; H = _h + 1;
    }
private:
    bool _t[N * 2];
    int _s[N * 2], _c[N * 2], x[N], _p[N], _q[N * 2], r
        [N], _sa[N * 2], _h[N];
    void mkhei(int n){
        for(int i = 0; i < n; i++) r[_sa[i]] = i;
        for(int i = 0; i < n; i++) if(r[i]) {
            int ans = i > 0 ? max(_h[r[i - 1]] - 1, 0) : 0;
            while(_s[i + ans] == _s[_sa[r[i] - 1] + ans])
                ans++;
            _h[r[i]] = ans;
        }
    }
    void sais(int *s, int *sa, int *p, int *q, bool *t,
        int *c, int n, int z){
        bool uniq = t[n - 1] = 1, neq;
        int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
            lst = -1;
    }
}

```

```

#define MAGIC(XD) \
    fill_n(sa, n, 0); \
    copy_n(c, z, x); \
    XD; \
    copy_n(c, z - 1, x + 1); \
    for(int i = 0; i < n; i++) if(sa[i] && !t[sa[i]
        - 1]) \
        sa[x[sa[i]-1]++] = sa[i] - 1; \
    copy_n(c, z, x); \
}

```

```

for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i]-1]) \
sa[--x[s[sa[i]-1]]] = sa[i] - 1;

fill_n(c, z, 0);
for (int i = 0; i < n; i++) uniq &= ++c[s[i]] < 2;
partial_sum(c, c + z, c);
if (uniq) {
    for (int i = 0; i < n; i++) sa[--c[s[i]]] = i;
    return;
}
for(int i = n - 2; i >= 0; i--)
    t[i] = (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
MAGIC(
    for (int i = 1; i <= n - 1; i++) if (t[i] && !t[i - 1])
        sa[--x[s[i]]] = p[q[i] = nn++] = i
    );
for (int i = 0; i < n; i++) if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
    neq = (lst < 0) || !equal(s + lst, s + lst + p[q[sa[i]] + 1] - sa[i], s + sa[i]);
    ns[q[lst = sa[i]]] = nmzx += neq;
}
sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx + 1);
MAGIC(
    for(int i = nn - 1; i >= 0; i--)
        sa[--x[s[p[nsa[i]]]]] = p[nsa[i]]
    );
}
} sa;

```

5.6 Aho-Corasick Automatan

```

const int len=400000,sigma=26;
struct AC_Automatan{
    int nx[len][sigma],fl[len],cnt[len],pri[len],top;
    int newnode(){
        fill(nx[top],nx[top]+sigma,-1);
        return top++;
    }
    void init(){top=1,newnode();}
    int input(string &s){//return the end_node of string
        int X=1;
        for(char c:s){
            if(!~nx[X][c-'a'])nx[X][c-'a']=newnode();
            X=nx[X][c-'a'];
        }
        return X;
    }
    void make_fl(){
        queue<int> q;
        q.push(1),fl[1]=0;
        for(int t=0;!q.empty();){
            int R=q.front();
            q.pop(),pri[t++]=R;
            for(int i=0;i<sigma;++i)
                if(~nx[R][i]){
                    int X=nx[R][i],Z=fl[R];
                    for(;Z&&!~nx[Z][i];)Z=fl[Z];
                    fl[X]=Z?nx[Z][i]:1,q.push(X);
                }
        }
    }
    void get_v(string &s){
        int X=1;
        fill(cnt,cnt+top,0);
        for(char c:s){
            while(X&&!~nx[X][c-'a'])X=fl[X];
            X=X?nx[X][c-'a']:1,++cnt[X];
        }
        for(int i=top-2;i>0;--i) cnt[fl[pri[i]]]+=cnt[pri[i]];
    }
};

```

5.7 Smallest Rotation

```

string mcp(string s){
    int n=SZ(s),i=0,j=1;
    s+=s;
    while(i<n&&j<n){
        int k=0;
        while(k<n&&s[i+k]==s[j+k]) ++k;
        if(s[i+k]<=s[j+k]) j+=k+1;
        else i+=k+1;
        if(i==j) ++j;
    }
    int ans=i<n?i:j;
    return s.substr(ans,n);
}

```

5.8 De Bruijn sequence*

```

constexpr int MAXC = 10, MAXN = 1e5 + 10;
struct DBSeq {
    int C, N, K, L, buf[MAXC * MAXN]; //K <= C^N
    void dfs(int *out, int t, int p, int &ptr) {
        if (ptr >= L) return;
        if (t > N) {
            if (N % p) return;
            for (int i = 1; i <= p && ptr < L; ++i)
                out[ptr++] = buf[i];
        } else {
            buf[t] = buf[t - p], dfs(out, t + 1, p, ptr);
            for (int j = buf[t - p] + 1; j < C; ++j)
                buf[t] = j, dfs(out, t + 1, t, ptr);
        }
    }
    void solve(int _c, int _n, int _k, int *out) {
        int p = 0;
        C = _c, N = _n, K = _k, L = N + K - 1;
        dfs(out, 1, 1, p);
        if (p < L) fill(out + p, out + L, 0);
    }
} dbs;

```

5.9 SAM

```

const int MAXM = 1000010;
struct SAM{
    int tot, root, lst, mom[MAXM], mx[MAXM];
    int acc[MAXM], nxt[MAXM][33];
    int newNode(){
        int res = ++tot;
        fill(nxt[res], nxt[res]+33, 0);
        mom[res] = mx[res] = acc[res] = 0;
        return res;
    }
    void init(){
        tot = 0;
        root = newNode();
        mom[root] = 0, mx[root] = 0;
        lst = root;
    }
    void push(int c){
        int p = lst;
        int np = newNode();
        mx[np] = mx[p]+1;
        for(; p && nxt[p][c] == 0; p = mom[p])
            nxt[p][c] = np;
        if(p == 0) mom[np] = root;
        else{
            int q = nxt[p][c];
            if(mx[p]+1 == mx[q]) mom[np] = q;
            else{
                int nq = newNode();
                mx[nq] = mx[p]+1;
                for(int i = 0; i < 33; i++)
                    nxt[nq][i] = nxt[q][i];
                mom[nq] = mom[q];
                mom[q] = nq;
                mom[np] = nq;
            }
        }
    }
}

```

```

        for(; p && nxt[p][c] == q; p = mom[p])
            nxt[p][c] = nq;
    }
}
lst = np;
}
void push(char *str){
    for(int i = 0; str[i]; i++)
        push(str[i] - 'a' + 1);
}
} sam;

```

5.10 PalTree

```

struct palindromic_tree{// Check by APIO 2014
    palindrome
    struct node{
        int next[26], fail, len;
        int cnt, num; //cnt: appear times, num: number of pal
                        . suf.
        node(int l=0):fail(0), len(1), cnt(0), num(0){
            for(int i=0; i<26; ++i) next[i]=0;
        }
    };
    vector<node> St;
    vector<char> s;
    int last, n;
    palindromic_tree():St(2), last(1), n(0){
        St[0].fail=1, St[1].len=-1, s.pb(-1);
    }
    inline void clear(){
        St.clear(), s.clear(), last=1, n=0;
        St.pb(0), St.pb(-1);
        St[0].fail=1, s.pb(-1);
    }
    inline int get_fail(int x){
        while(s[n-St[x].len-1] != s[n]) x=St[x].fail;
        return x;
    }
    inline void add(int c){
        s.push_back(c - 'a'), ++n;
        int cur=get_fail(last);
        if(!St[cur].next[c]){
            int now=SZ(St);
            St.pb(St[cur].len+2);
            St[now].fail=St[get_fail(St[cur].fail)].next[c];
            St[cur].next[c]=now;
            St[now].num=St[St[now].fail].num+1;
        }
        last=St[cur].next[c], ++St[last].cnt;
    }
    inline void count(){// counting cnt
        auto i=St.rbegin();
        for(; i!=St.rend(); ++i){
            St[i->fail].cnt+=i->cnt;
        }
    }
    inline int size(){// The number of diff. pal.
        return SZ(St)-2;
    }
};

```

5.11 cyclicLCS

```

#define L 0
#define LU 1
#define U 2
const int mov[3][2]={0, -1, -1, -1, -1, 0};
int al, bl;
char a[MAXL*2], b[MAXL*2]; // 0-indexed
int dp[MAXL*2][MAXL];
char pred[MAXL*2][MAXL];
inline int lcs_length(int r) {
    int i=r+al, j=bl, l=0;
    while(i>r) {
        char dir=pred[i][j];
        if(dir==LU) l++;
        i+=mov[dir][0];

```

```

        j+=mov[dir][1];
    }
    return l;
}
inline void reroot(int r) { // r = new base row
    int i=r, j=1;
    while(j<=bl && pred[i][j]!=LU) j++;
    if(j>bl) return;
    pred[i][j]=L;
    while(i<2*al && j<=bl) {
        if(pred[i+1][j]==U) {
            i++;
            pred[i][j]=L;
        } else if(j<bl && pred[i+1][j+1]==LU) {
            i++;
            j++;
            pred[i][j]=L;
        } else {
            j++;
        }
    }
}
int cyclic_lcs() {
    // a, b, al, bl should be properly filled
    // note: a WILL be altered in process
    // -- concatenated after itself
    char tmp[MAXL];
    if(al>bl) {
        swap(al, bl);
        strcpy(tmp, a);
        strcpy(a, b);
        strcpy(b, tmp);
    }
    strcpy(tmp, a);
    strcat(a, tmp);
    // basic lcs
    for(int i=0; i<=2*al; i++) {
        dp[i][0]=0;
        pred[i][0]=U;
    }
    for(int j=0; j<=bl; j++) {
        dp[0][j]=0;
        pred[0][j]=L;
    }
    for(int i=1; i<=2*al; i++) {
        for(int j=1; j<=bl; j++) {
            if(a[i-1]==b[j-1]) dp[i][j]=dp[i-1][j-1]+1;
            else dp[i][j]=max(dp[i-1][j], dp[i][j-1]);
            if(dp[i][j-1]==dp[i][j]) pred[i][j]=L;
            else if(a[i-1]==b[j-1]) pred[i][j]=LU;
            else pred[i][j]=U;
        }
    }
    // do cyclic lcs
    int clcs=0;
    for(int i=0; i<al; i++) {
        clcs=max(clcs, lcs_length(i));
        reroot(i+1);
    }
    // recover a
    a[al]='\0';
    return clcs;
}

```

6 Math

6.1 ax+by=gcd*

```

pll exgcd(ll a, ll b) {
    if(b == 0) return pll(1, 0);
    else {
        ll p = a / b;
        pll q = exgcd(b, a % b);
        return pll(q.Y, q.X - q.Y * p);
    }
}

```

6.2 floor and ceil

```
int floor(int a, int b){
    return a/b-(a%b&&a<0^b<0);
}
int ceil(int a, int b){
    return a/b+(a%b&&a<0^b>0);
}
```

6.3 floor sum*

```
ll floor_sum(ll n, ll m, ll a, ll b) {
    ll ans = 0;
    if (a >= m)
        ans += (n - 1) * n * (a / m) / 2, a %= m;
    if (b >= m)
        ans += n * (b / m), b %= m;
    ll y_max = (a * n + b) / m, x_max = (y_max * m - b) / m;
    if (y_max == 0) return ans;
    ans += (n - (x_max + a - 1) / a) * y_max;
    ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
    return ans;
} // sum_{i=0}^{n-1} floor((a * i + b) / m) in Log(n + m + a + b)
```

6.4 Miller Rabin*

```
// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pirmes <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool Miller_Rabin(ll a, ll n) {
    if((a = a % n) == 0) return 1;
    if((n & 1) ^ 1) return n == 2;
    ll tmp = (n - 1) / ((n - 1) & (1 - n));
    ll t = __lg(((n - 1) & (1 - n))), x = 1;
    for(; tmp; tmp >>= 1, a = mul(a, a, n))
        if(tmp & 1) x = mul(x, a, n);
    if(x == 1 || x == n - 1) return 1;
    while(--t)
        if((x = mul(x, x, n)) == n - 1) return 1;
    return 0;
}
```

6.5 Big number

```
template<typename T>
inline string to_string(const T& x){
    stringstream ss;
    return ss<<x,ss.str();
}
struct bigN:vector<ll>{
    const static int base=1000000000,width=log10(base);
    bool negative;
    bigN(const_iterator a,const_iterator b):vector<ll>(a,b){}
    bigN(string s){
        if(s.empty())return;
        if(s[0]=='-')negative=1,s=s.substr(1);
        else negative=0;
        for(int i=int(s.size())-1;i>=0;i-=width){
            ll t=0;
            for(int j=max(0,i-width+1);j<=i;j++)
                t=t*10+s[j]-'0';
            push_back(t);
        }
        trim();
    }
    template<typename T>
    bigN(const T &x):bigN(to_string(x)){}
    bigN():negative(0){}
    void trim(){
        while(size()&&back())pop_back();
    }
}
```

```
if(empty())negative=0;
}
void carry(int _base=base){
    for(size_t i=0;i<size();++i){
        if(at(i)>=0&&at(i)<_base)continue;
        if(i+1u==size())push_back(0);
        int r=at(i)%_base;
        if(r<0)r+=_base;
        at(i+1)+=(at(i)-r)/_base,at(i)=r;
    }
}
int abscomp(const bigN &b)const{
    if(size()>b.size())return 1;
    if(size()<b.size())return -1;
    for(int i=int(size())-1;i>=0;--i){
        if(at(i)>b[i])return 1;
        if(at(i)<b[i])return -1;
    }
    return 0;
}
int cmp(const bigN &b)const{
    if(negative!=b.negative)return negative?-1:1;
    return negative?-abscomp(b):abscomp(b);
}
bool operator<(const bigN&b)const{return cmp(b)<0;}
bool operator>(const bigN&b)const{return cmp(b)>0;}
bool operator<=(const bigN&b)const{return cmp(b)<=0;}
bool operator>=(const bigN&b)const{return cmp(b)>=0;}
bool operator==(const bigN&b)const{return !cmp(b);}
bool operator!=(const bigN&b)const{return cmp(b)!=0;}
bigN abs()const{
    bigN res=*this;
    return res.negative=0, res;
}
bigN operator-()const{
    bigN res=*this;
    return res.negative=!negative,res.trim(),res;
}
bigN operator+(const bigN &b)const{
    if(negative)return -(*this)+(-b);
    if(b.negative)return *this-(-b);
    bigN res=*this;
    if(b.size()>size())res.resize(b.size());
    for(size_t i=0;i<b.size();++i)res[i]+=b[i];
    return res.carry(),res.trim(),res;
}
bigN operator-(const bigN &b)const{
    if(negative)return -(*this)-(-b);
    if(b.negative)return *this+(-b);
    if(abscomp(b)<0)return -(b-(*this));
    bigN res=*this;
    if(b.size()>size())res.resize(b.size());
    for(size_t i=0;i<b.size();++i)res[i]-=b[i];
    return res.carry(),res.trim(),res;
}
bigN operator*(const bigN &b)const{
    bigN res;
    res.negative=negative!=b.negative;
    res.resize(size()+b.size());
    for(size_t i=0;i<size();++i)
        for(size_t j=0;j<b.size();++j)
            if((res[i+j]+at(i)*b[j])>=base){
                res[i+j+1]+=res[i+j]/base;
                res[i+j]%=base;
            } // %base carry
    return res.trim(),res;
}
bigN operator/(const bigN &b)const{
    int norm=base/(b.back()+1);
    bigN x=abs()*norm;
    bigN y=b.abs()*norm;
    bigN q,r;
    q.resize(x.size());
    for(int i=int(x.size())-1;i>=0;--i){
        r=r*base+x[i];
        int s1=r.size()<=y.size()?r[y.size()];
        int s2=r.size()<y.size()?r[y.size()-1];
        int d=(ll(base)*s1+s2)/y.back();
        r=r-y*d;
        while(r.negative)r=r+y,--d;
        q[i]=d;
    }
}
```



```

    q.negative=negative!=b.negative;
    return q.trim(),q;
}
bigN operator%(const bigN &b)const{
    return *this-(*this/b)*b;
}
friend istream& operator>>(istream &ss, bigN &b){
    string s;
    return ss>>s, b=s, ss;
}
friend ostream& operator<<(ostream &ss, const bigN &b)
{
    if(b.negative)ss<<"-";
    ss<<(b.empty()?0:b.back());
    for(int i=int(b.size())-2;i>=0;--i)
        ss<<setw(width)<<setfill('0')<<b[i];
    return ss;
}
template<typename T>
operator T(){
    stringstream ss;
    ss<<*this;
    T res;
    return ss>>res,res;
}
};

```

6.6 Fraction

```

struct fraction{
    ll n,d;
    fraction(const ll &n=0, const ll &d=1):n(_n),d(_d){
        ll t=__gcd(n,d);
        n/=t,d/=t;
        if(d<0) n=-n,d=-d;
    }
    fraction operator-()const{
        return fraction(-n,d);
    }
    fraction operator+(const fraction &b)const{
        return fraction(n*b.d+b.n*d,d*b.d);
    }
    fraction operator-(const fraction &b)const{
        return fraction(n*b.d-b.n*d,d*b.d);
    }
    fraction operator*(const fraction &b)const{
        return fraction(n*b.n,d*b.d);
    }
    fraction operator/(const fraction &b)const{
        return fraction(n*b.d,d*b.n);
    }
    void print(){
        cout << n;
        if(d!=1) cout << "/" << d;
    }
};

```

6.7 Simultaneous Equations

```

struct matrix { //m variables, n equations
    int n, m;
    fraction M[MAXN][MAXN + 1], sol[MAXN];
    int solve() { // -1: inconsistent, >= 0: rank
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m) continue;
            for (int j = 0; j < n; ++j) {
                if (i == j) continue;
                fraction tmp = -M[j][piv] / M[i][piv];
                for (int k = 0; k <= m; ++k) M[j][k] = tmp * M[i][k] + M[j][k];
            }
        }
        int rank = 0;
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;

```

```

            if (piv == m && M[i][m].n) return -1;
            else if (piv < m) ++rank, sol[piv] = M[i][m] / M[i][piv];
        }
        return rank;
    }
};

```

6.8 Pollard Rho

```

// does not work when n is prime
ll f(ll x, ll mod){ return add(mul(x,x,mod),1,mod); }
ll pollard_rho(ll n){
    if(!(n&1)) return 2;
    while(1){
        ll y=2,x=rand()%(n-1)+1,res=1;
        for(int sz=2;res==1;y=x,sz*=2)
            for(int i=0;i<sz&&res<=1;++i)
                x=f(x,n),res=__gcd(abs(x-y),n);
        if(res!=0&&res!=n) return res;
    }
}

```

6.9 Simplex Algorithm

```

const int MAXN = 111;
const int MAXM = 111;
const double eps = 1E-10;
double a[MAXN][MAXM], b[MAXN], c[MAXN][MAXM];
double x[MAXN];
int ix[MAXN + MAXM]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b,x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[MAXN][MAXM], double b[MAXN],
    double c[MAXN], int n, int m){
    ++m;
    int r = n, s = m - 1;
    memset(d, 0, sizeof(d));
    for (int i = 0; i < n + m; ++i) ix[i] = i;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m - 1; ++j) d[i][j] = -a[i][j];
        d[i][m - 1] = 1;
        d[i][m] = b[i];
        if (d[r][m] > d[i][m]) r = i;
    }
    for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];
    d[n + 1][m - 1] = -1;
    for (double dd;; ) {
        if (r < n) {
            int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
            d[r][s] = 1.0 / d[r][s];
            for (int j = 0; j <= m; ++j)
                if (j != s) d[r][j] *= -d[r][s];
            for (int i = 0; i <= n + 1; ++i) if (i != r) {
                for (int j = 0; j <= m; ++j) if (j != s)
                    d[i][j] += d[r][j] * d[i][s];
                d[i][s] *= d[r][s];
            }
        }
        r = -1; s = -1;
        for (int j = 0; j < m; ++j)
            if (s < 0 || ix[s] > ix[j]) {
                if (d[n + 1][j] > eps ||
                    (d[n + 1][j] > -eps && d[n][j] > eps))
                    s = j;
            }
        if (s < 0) break;
        for (int i = 0; i < n; ++i) if (d[i][s] < -eps) {
            if (r < 0 ||
                (dd = d[r][m] / d[r][s] - d[i][m] / d[i][s]) < -eps ||
                (dd < eps && ix[r + m] > ix[i + m]))
                r = i;
        }
        if (r < 0) return -1; // not bounded
    }
}

```

```

}
if (d[n + 1][m] < -eps) return -1; // not executable
double ans = 0;
for(int i=0; i<m; i++) x[i] = 0;
for (int i = m; i < n + m; ++i) { // the missing
    enumerated x[i] = 0
    if (ix[i] < m - 1){
        ans += d[i - m][m] * c[ix[i]];
        x[ix[i]] = d[i-m][m];
    }
}
return ans;
}

```

6.10 chineseRemainder

```

LL solve(LL x1, LL m1, LL x2, LL m2) {
    LL g = __gcd(m1, m2);
    if((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    pair<LL,LL> p = gcd(m1, m2);
    LL lcm = m1 * m2 * g;
    LL res = p.first * (x2 - x1) * m1 + x1;
    return (res % lcm + lcm) % lcm;
}

```

6.11 QuadraticResidue

```

int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
    return s;
}

int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    const int jc = Jacobi(a, p);
    if (jc == 0) return 0;
    if (jc == -1) return -1;
    int b, d;
    for (; ; ) {
        b = rand() % p;
        d = (1LL * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (1LL + p) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (1LL * g0 * f0 + 1LL * d * (1LL * g1 * f1 % p)) % p;
            g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (1LL * f0 * f0 + 1LL * d * (1LL * f1 * f1 % p)) % p;
        f1 = (2LL * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}

```

6.12 PiCount

```

int64_t PrimeCount(int64_t n) {
    if (n <= 1) return 0;
    const int v = sqrt(n);
    vector<int> smalls(v + 1);
    for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
}

```

```

int s = (v + 1) / 2;
vector<int> roughs(s);
for (int i = 0; i < s; ++i) roughs[i] = 2 * i + 1;
vector<int64_t> larges(s);
for (int i = 0; i < s; ++i) larges[i] = (n / (2 * i + 1) + 1) / 2;
vector<bool> skip(v + 1);
int pc = 0;
for (int p = 3; p <= v; ++p) {
    if (smalls[p] > smalls[p - 1]) {
        int q = p * p;
        pc++;
        if (1LL * q * q > n) break;
        skip[p] = true;
        for (int i = q; i <= v; i += 2 * p) skip[i] = true;
        int ns = 0;
        for (int k = 0; k < s; ++k) {
            int i = roughs[k];
            if (skip[i]) continue;
            int64_t d = 1LL * i * p;
            larges[ns] = larges[k] - (d <= v ? larges[smalls[d] - pc] : smalls[n / d]) + pc;
            roughs[ns++] = i;
        }
        s = ns;
        for (int j = v / p; j >= p; --j) {
            int c = smalls[j] - pc;
            for (int i = j * p, e = min(i + p, v + 1); i < e; ++i) smalls[i] -= c;
        }
    }
}
for (int k = 1; k < s; ++k) {
    const int64_t m = n / roughs[k];
    int64_t s = larges[k] - (pc + k - 1);
    for (int l = 1; l < k; ++l) {
        int p = roughs[l];
        if (1LL * p * p > m) break;
        s -= smalls[m / p] - (pc + l - 1);
    }
    larges[0] -= s;
}
return larges[0];
}

```

6.13 Primes

```

/*
12721 13331 14341 75577 123457 222557 556679 999983
1097774749 1076767633 100102021 999997771
1001010013 1000512343 987654361 999991231
999888733 98789101 987777733 999991921
1010101333 1010102101 100000000039
100000000000037 2305843009213693951
4611686018427387847 9223372036854775783
18446744073709551557
*/

```

7 Polynomial

7.1 Fast Fourier Transform

```

template<int MAXN>
struct FFT {
    using val_t = complex<double>;
    const double PI = acos(-1);
    val_t w[MAXN];
    FFT() {
        for (int i = 0; i < MAXN; ++i) {
            double arg = 2 * PI * i / MAXN;
            w[i] = val_t(cos(arg), sin(arg));
        }
    }
    void bitrev(val_t *a, int n); // see NTT
    void trans(val_t *a, int n, bool inv = false); // see NTT;
}

```

```
// remember to replace LL with val_t
};
```

7.2 Number Theory Transform

```
//(2^16)+1, 65537, 3
//7*17*(2^23)+1, 998244353, 3
//1255*(2^20)+1, 1315962881, 3
//51*(2^25)+1, 1711276033, 29
template<int MAXN, LL P, LL RT> //MAXN must be 2^k
struct NTT {
    LL w[MAXN];
    LL mpow(LL a, LL n);
    LL minv(LL a) { return mpow(a, P - 2); }
    NTT() {
        LL dw = mpow(RT, (P - 1) / MAXN);
        w[0] = 1;
        for (int i = 1; i < MAXN; ++i) w[i] = w[i - 1] * dw
            % P;
    }
    void bitrev(LL *a, int n) {
        int i = 0;
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (i ^= k) < k; k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
    }
    void operator()(LL *a, int n, bool inv = false) { //0
        if (a[0] < P)
            bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = MAXN / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; ++j, x += dx)
                {
                    LL tmp = a[j + dl] * w[x] % P;
                    if ((a[j + dl] = a[j] - tmp) < 0) a[j + dl]
                        += P;
                    if ((a[j] += tmp) >= P) a[j] -= P;
                }
            }
        }
        if (inv) {
            reverse(a + 1, a + n);
            LL invn = minv(n);
            for (int i = 0; i < n; ++i) a[i] = a[i] * invn %
                P;
        }
    }
};
```

7.3 Fast Walsh Transform*

```
/* x: a[j], y: a[j + (L >> 1)]
or: (y += x * op), and: (x += y * op)
xor: (x, y = (x + y) * op, (x - y) * op)
invop: or, and, xor = -1, -1, 1/2 */
void fwt(int *a, int n, int op) { //or
    for (int L = 2; L <= n; L <= 1)
        for (int i = 0; i < n; i += L)
            for (int j = i; j < i + (L >> 1); ++j)
                a[j + (L >> 1)] += a[j] * op;
}
const int N = 21;
int f[N][1 << N], g[N][1 << N], h[N][1 << N], ct[1 << N]
];
void subset_convolution(int *a, int *b, int *c, int L)
{
    // c_k = \sum_{i+j=k, i&j=0} a_i * b_j
    int n = 1 << L;
    for (int i = 1; i < n; ++i)
        ct[i] = ct[i & (i - 1)] + 1;
    for (int i = 0; i < n; ++i)
        f[ct[i]][i] = a[i], g[ct[i]][i] = b[i];
    for (int i = 0; i <= L; ++i)
        fwt(f[i], n, 1), fwt(g[i], n, 1);
    for (int i = 0; i <= L; ++i)
        for (int j = 0; j <= i; ++j)
```

```
        for (int x = 0; x < n; ++x)
            h[i][x] += f[j][x] * g[i - j][x];
    for (int i = 0; i <= L; ++i)
        fwt(h[i], n, -1);
    for (int i = 0; i < n; ++i)
        c[i] = h[ct[i]][i];
}
```

7.4 Polynomial Operation

```
#define fi(s, n) for (int i = (int)(s); i < (int)(n); ++i)
#define Fi(s, n) for (int i = (int)(n); i > (int)(s); --i)
int n2k(int n) {
    int sz = 1; while (sz < n) sz <= 1;
    return sz;
}
template<int MAXN, LL P, LL RT> // MAXN = 2^k
struct Poly { // coefficients in [0, P)
    static NTT<MAXN, P, RT> ntt;
    vector<LL> coef;
    int n() const { return (int)coef.size(); } // n() >=
        1
    LL *data() { return coef.data(); }
    const LL *data() const { return coef.data(); }
    LL &operator[](size_t i) { return coef[i]; }
    const LL &operator[](size_t i) const { return coef[i]
        ]; }
    Poly(initializer_list<LL> a) : coef(a) { }
    explicit Poly(int _n = 1) : coef(_n) { }
    Poly(const LL *arr, int _n) : coef(arr, arr + _n) { }
    Poly(const Poly &p, int _n) : coef(_n) {
        copy_n(p.data(), min(p.n(), _n), data());
    }
    Poly& irev() { return reverse(data(), data() + n()),
        *this; }
    Poly& isz(int _n) { return coef.resize(_n), *this; }
    Poly& iadd(const Poly &rhs) { // n() == rhs.n()
        fi(0, n()) if ((coef[i] += rhs[i]) >= P) coef[i] -=
            P;
        return *this;
    }
    Poly& imul(LL k) {
        fi(0, n()) coef[i] = coef[i] * k % P;
        return *this;
    }

    Poly Mul(const Poly &rhs) const {
        const int _n = n2k(n() + rhs.n() - 1);
        Poly X(*this, _n), Y(rhs, _n);
        ntt(X.data(), _n), ntt(Y.data(), _n);
        fi(0, _n) X[i] = X[i] * Y[i] % P;
        ntt(X.data(), _n, true);
        return X.isz(n() + rhs.n() - 1);
    }
    Poly Inv() const { // coef[0] != 0
        if (n() == 1) return {ntt.minv(coef[0])};
        const int _n = n2k(n() * 2);
        Poly Xi = Poly(*this, (n() + 1) / 2).Inv().isz(_n);
        Poly Y(*this, _n);
        ntt(Xi.data(), _n), ntt(Y.data(), _n);
        fi(0, _n) {
            Xi[i] *= (2 - Xi[i] * Y[i]) % P;
            if ((Xi[i] %= P) < 0) Xi[i] += P;
        }
        ntt(Xi.data(), _n, true);
        return Xi.isz(n());
    }
    Poly Sqrt() const { // Jacobi(coef[0], P) = 1
        if (n() == 1) return {QuadraticResidue(coef[0], P)
            };
        Poly X = Poly(*this, (n() + 1) / 2).Sqrt().isz(n());
        ;
        return X.iadd(Mul(X.Inv()).isz(n())).imul(P / 2 +
            1);
    }
    pair<Poly, Poly> DivMod(const Poly &rhs) const { // (
        rhs).back() != 0
        if (n() < rhs.n()) return {{0}, *this};
```

```

const int _n = n() - rhs.n() + 1;
Poly X(rhs); X.irev().isz(_n);
Poly Y(*this); Y.irev().isz(_n);
Poly Q = Y.Mul(X.Inv()).isz(_n).irev();
X = rhs.Mul(Q), Y = *this;
fi(0, n()) if ((Y[i] - X[i]) < 0) Y[i] += P;
return {Q, Y.isz(max(1, rhs.n() - 1))};
}
Poly Dx() const {
    Poly ret(n() - 1);
    fi(0, ret.n()) ret[i] = (i + 1) * coef[i + 1] % P;
    return ret.isz(max(1, ret.n()));
}
Poly Sx() const {
    Poly ret(n() + 1);
    fi(0, n()) ret[i + 1] = ntt.minv(i + 1) * coef[i] %
        P;
    return ret;
}
Poly _tmul(int nn, const Poly &rhs) const {
    Poly Y = Mul(rhs).isz(n() + nn - 1);
    return Poly(Y.data() + n() - 1, nn);
}
vector<LL> _eval(const vector<LL> &x, const vector<
    Poly> &up) const {
    const int _n = (int)x.size();
    if (!_n) return {};
    vector<Poly> down(_n * 2);
    down[1] = DivMod(up[1]).second;
    fi(2, _n * 2) down[i] = down[i / 2].DivMod(up[i]).
        second;
    /* down[1] = Poly(up[1]).irev().isz(n()).Inv().irev
        ()._tmul(_n, *this);
    fi(2, _n * 2) down[i] = up[i ^ 1]._tmul(up[i].n() -
        1, down[i / 2]); */
    vector<LL> y(_n);
    fi(0, _n) y[i] = down[_n + i][0];
    return y;
}
static vector<Poly> _tree1(const vector<LL> &x) {
    const int _n = (int)x.size();
    vector<Poly> up(_n * 2);
    fi(0, _n) up[_n + i] = {(x[i] ? P - x[i] : 0), 1};
    Fi(0, _n - 1) up[i] = up[i * 2].Mul(up[i * 2 + 1]);
    return up;
}
vector<LL> Eval(const vector<LL> &x) const {
    auto up = _tree1(x); return _eval(x, up);
}
static Poly Interpolate(const vector<LL> &x, const
    vector<LL> &y) {
    const int _n = (int)x.size();
    vector<Poly> up = _tree1(x), down(_n * 2);
    vector<LL> z = up[1].Dx()._eval(x, up);
    fi(0, _n) z[i] = y[i] * ntt.minv(z[i]) % P;
    fi(0, _n) down[_n + i] = {z[i]};
    Fi(0, _n - 1) down[i] = down[i * 2].Mul(up[i * 2 +
        1]).iadd(down[i * 2 + 1].Mul(up[i * 2]));
    return down[1];
}
Poly Ln() const { // coef[0] == 1
    return Dx().Mul(Inv()).Sx().isz(n());
}
Poly Exp() const { // coef[0] == 0
    if (n() == 1) return {1};
    Poly X = Poly(*this, (n() + 1) / 2).Exp().isz(n());
    Poly Y = X.Ln(); Y[0] = P - 1;
    fi(0, n()) if ((Y[i] = coef[i] - Y[i]) < 0) Y[i] +=
        P;
    return X.Mul(Y).isz(n());
}
Poly Pow(const string &K) const {
    int nz = 0;
    while (nz < n() && !coef[nz]) ++nz;
    LL nk = 0, nk2 = 0;
    for (char c : K) {
        nk = (nk * 10 + c - '0') % P;
        nk2 = nk2 * 10 + c - '0';
        if (nk2 * nz >= n()) return Poly(n());
        nk2 %= P - 1;
    }
    if (!nk && !nk2) return Poly({1}, n());

```

```

Poly X(data() + nz, n() - nz * nk2);
LL x0 = X[0];
return X.imul(ntt.minv(x0)).Ln().imul(nk).Exp()
    .imul(ntt.mpow(x0, nk2)).irev().isz(n()).irev();
}
static LL LinearRecursion(const vector<LL> &a, const
    vector<LL> &coef, LL n) { // a_n = \sum c_j a_{n-
    j}
    const int k = (int)a.size();
    assert((int)coef.size() == k + 1);
    Poly C(k + 1), W({1}, k), M = {0, 1};
    fi(1, k + 1) C[k - i] = coef[i] ? P - coef[i] : 0;
    C[k] = 1;
    while (n) {
        if (n % 2) W = W.Mul(M).DivMod(C).second;
        n /= 2, M = M.Mul(M).DivMod(C).second;
    }
    LL ret = 0;
    fi(0, k) ret = (ret + W[i] * a[i]) % P;
    return ret;
}
};
#undef fi
#undef Fi
using Poly_t = Poly<131072 * 2, 998244353, 3>;
template<> decltype(Poly_t::ntt) Poly_t::ntt = {};

```

8 Geometry

8.1 Default Code

```

typedef pair<double, double> pdd;
typedef pair<pdd, pdd> Line;
struct Cir{pdd O; double R;};
const double eps=1e-8;
pdd operator+(const pdd &a, const pdd &b)
{ return pdd(a.X + b.X, a.Y + b.Y);}
pdd operator-(const pdd &a, const pdd &b)
{ return pdd(a.X - b.X, a.Y - b.Y);}
pdd operator*(const pdd &a, const double &b)
{ return pdd(a.X * b, a.Y * b);}
pdd operator/(const pdd &a, const double &b)
{ return pdd(a.X / b, a.Y / b);}
double dot(const pdd &a, const pdd &b)
{ return a.X * b.X + a.Y * b.Y;}
double cross(const pdd &a, const pdd &b)
{ return a.X * b.Y - a.Y * b.X;}
double abs2(const pdd &a)
{ return dot(a, a);}
double abs(const pdd &a)
{ return sqrt(dot(a, a));}
int sign(const double &a)
{ return fabs(a) < eps ? 0 : a > 0 ? 1 : -1;}
int ori(const pdd &a, const pdd &b, const pdd &c)
{ return sign(cross(b - a, c - a));}
bool collinearity(const pdd &p1, const pdd &p2, const
    pdd &p3)
{ return fabs(cross(p1 - p3, p2 - p3)) < eps;}
bool btw(const pdd &p1, const pdd &p2, const pdd &p3) {
    if(!collinearity(p1, p2, p3)) return 0;
    return dot(p1 - p3, p2 - p3) < eps;
}
bool seg_intersect(const pdd &p1, const pdd &p2, const
    pdd &p3, const pdd &p4) {
    int a123 = ori(p1, p2, p3);
    int a124 = ori(p1, p2, p4);
    int a341 = ori(p3, p4, p1);
    int a342 = ori(p3, p4, p2);
    if(a123 == 0 && a124 == 0)
        return btw(p1, p2, p3) || btw(p1, p2, p4) ||
            btw(p3, p4, p1) || btw(p3, p4, p2);
    return a123 * a124 <= 0 && a341 * a342 <= 0;
}
pdd intersect(const pdd &p1, const pdd &p2, const pdd &
    p3, const pdd &p4) {
    double a123 = cross(p2 - p1, p3 - p1);
    double a124 = cross(p2 - p1, p4 - p1);
    return (p4 * a123 - p3 * a124) / (a123 - a124);
}

```

```

}
pdd perp(const pdd &p1)
{ return pdd(-p1.Y, p1.X);}
pdd foot(const pdd &p1, const pdd &p2, const pdd &p3)
{ return intersect(p1, p2, p3, p3 + perp(p2 - p1));}

```

8.2 Convex hull*

```

void hull(vector<p11> &dots) {
    sort(dots.begin(), dots.end());
    vector<p11> ans(1, dots[0]);
    for (int ct = 0; ct < 2; ++ct, reverse(ALL(dots)))
        for (int i = 1, t = SZ(ans); i < SZ(dots); ans.pb(
            dots[i++]))
            while (SZ(ans) > t && ori(ans[SZ(ans) - 2], ans.
                back(), dots[i]) <= 0)
                ans.pop_back();
    ans.pop_back(), ans.swap(dots);
}

```

8.3 External bisector

```

pdd external_bisector(pdd p1,pdd p2,pdd p3){//213
    pdd l1=p2-p1,l2=p3-p1;
    L2=L2*abs(L1)/abs(L2);
    return L1+L2;
}

```

8.4 Heart

```

pdd excenter(pdd p0,pdd p1,pdd p2,double &radius){
    p1=p1-p0,p2=p2-p0;
    double x1=p1.X,y1=p1.Y,x2=p2.X,y2=p2.Y;
    double m=2.*(x1*y2-y1*x2);
    center.X=(x1*x1*y2-x2*x2*y1+y1*y2*(y1-y2))/m;
    center.Y=(x1*x2*(x2-x1)-y1*y1*x2+x1*y2*y2)/m;
    return radius=abs(center),center+p0;
}

pdd incenter(pdd p1,pdd p2,pdd p3,double &radius){
    double a=abs(p2-p1),b=abs(p3-p1),c=abs(p3-p2);
    double s=(a+b+c)/2,area=sqrt(s*(s-a)*(s-b)*(s-c));
    pdd l1=external_bisector(p1,p2,p3),l2=
        external_bisector(p2,p1,p3);
    return radius=area/s,intersect(p1,p1+l1,p2,p2+l2),
}

pdd escenter(pdd p1,pdd p2,pdd p3){//213
    pdd l1=external_bisector(p1,p2,p3),l2=
        external_bisector(p2,p2+p2-p1,p3);
    return intersect(p1,p1+l1,p2,p2+l2);
}

pdd barycenter(pdd p1,pdd p2,pdd p3){
    return (p1+p2+p3)/3;
}

pdd orthocenter(pdd p1,pdd p2,pdd p3){
    pdd l1=p3-p2,l2=p3-p1;
    swap(L1.X,L1.Y),L1.X*=-1;
    swap(L2.X,L2.Y),L2.X*=-1;
    return intersect(p1,p1+l1,p2,p2+l2);
}

```

8.5 Minimum Circle Cover*

```

pdd Minimum_Circle_Cover(vector<pdd> dots, double &r) {
    pdd cent;
    random_shuffle(ALL(dots));
    cent = dots[0], r = 0;
    for (int i = 1; i < SZ(dots); ++i)
        if (abs(dots[i] - cent) > r) {
            cent = dots[i], r = 0;

```

```

        for (int j = 0; j < i; ++j)
            if (abs(dots[j] - cent) > r) {
                cent = (dots[i] + dots[j]) / 2;
                r = abs(dots[i] - cent);
                for (int k = 0; k < j; ++k)
                    if (abs(dots[k] - cent) > r)
                        cent = excenter(dots[i], dots[j], dots[k]
                            ], r);
            }
        }
    return cent;
}

```

8.6 Polar Angle Sort*

```

pdd center;//sort base
int Quadrant(pdd a) {
    if(a.X > 0 && a.Y >= 0) return 1;
    if(a.X <= 0 && a.Y > 0) return 2;
    if(a.X < 0 && a.Y <= 0) return 3;
    if(a.X >= 0 && a.Y < 0) return 4;
}
bool cmp(p11 a, p11 b) {
    a = a - center, b = b - center;
    if (Quadrant(a) != Quadrant(b))
        return Quadrant(a) < Quadrant(b);
    if (cross(b, a) == 0) return abs2(a) < abs2(b);
    return cross(a, b) > 0;
}
bool cmp(pdd a, pdd b) {
    a = a - center, b = b - center;
    if(fabs(atan2(a.Y, a.X) - atan2(b.Y, b.X)) > eps)
        return atan2(a.Y, a.X) < atan2(b.Y, b.X);
    return abs(a) < abs(b);
}

```

8.7 Intersection of two circles*

```

bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
    pdd o1 = a.O, o2 = b.O;
    double r1 = a.R, r2 = b.R, d2 = abs2(o1 - o2), d =
        sqrt(d2);
    if(d < max(r1, r2) - min(r1, r2) || d > r1 + r2)
        return 0;
    pdd u = (o1 + o2) * 0.5 + (o1 - o2) * ((r2 * r2 - r1
        * r1) / (2 * d2));
    double A = sqrt((r1 + r2 + d) * (r1 - r2 + d) * (r1 +
        r2 - d) * (-r1 + r2 + d));
    pdd v = pdd(o1.Y - o2.Y, -o1.X + o2.X) * A / (2 * d2)
        ;
    p1 = u + v, p2 = u - v;
    return 1;
}

```

8.8 Intersection of polygon and circle

```

// Divides into multiple triangle, and sum up
// test by HDU2892
const double PI=acos(-1);
double _area(pdd pa, pdd pb, double r){
    if(abs(pa)<abs(pb)) swap(pa, pb);
    if(abs(pb)<eps) return 0;
    double S, h, theta;
    double a=abs(pb),b=abs(pa),c=abs(pb-pa);
    double cosB = dot(pb,pb-pa) / a / c, B = acos(cosB);
    double cosC = dot(pa,pb) / a / b, C = acos(cosC);
    if(a > r){
        S = (C/2)*r*r;
        h = a*b*sin(C)/c;
        if (h < r && B < PI/2) S -= (acos(h/r)*r*r - h*sqrt(
            (r*r-h*h)));
    }
    else if(b > r){
        theta = PI - B - asin(sin(B)/r*a);
        S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
    }
}

```

```

else S = .5*sin(C)*a*b;
return S;
}
double area_poly_circle(const vector<pdd> poly, const
    pdd &O, const double r){
    double S=0;
    for(int i=0; i<SZ(poly); ++i)
        S += _area(poly[i]-O, poly[(i+1)%SZ(poly)]-O, r)*ori(0,
            poly[i], poly[(i+1)%SZ(poly)]);
    return fabs(S);
}

```

8.9 Intersection of line and circle

```

vector<pdd> line_interCircle(const pdd &p1, const pdd &
    p2, const pdd &c, const double r){
    pdd ft=foot(p1, p2, c), vec=p2-p1;
    double dis=abs(c-ft);
    if(fabs(dis-r)<eps) return vector<pdd>{ft};
    if(dis>r) return {};
    vec=vec*sqrt(r*r-dis*dis)/abs(vec);
    return vector<pdd>{ft+vec, ft-vec};
}

```

8.10 point in circle

```

// return p4 is strictly in circumcircle of tri(p1, p2,
    p3)
long long sqr(long long x) { return x * x; }
bool in_cc(const pll& p1, const pll& p2, const pll& p3,
    const pll& p4) {
    long long u11 = p1.X - p4.X; long long u12 = p1.Y -
        p4.Y;
    long long u21 = p2.X - p4.X; long long u22 = p2.Y -
        p4.Y;
    long long u31 = p3.X - p4.X; long long u32 = p3.Y -
        p4.Y;
    long long u13 = sqr(p1.X) - sqr(p4.X) + sqr(p1.Y) -
        sqr(p4.Y);
    long long u23 = sqr(p2.X) - sqr(p4.X) + sqr(p2.Y) -
        sqr(p4.Y);
    long long u33 = sqr(p3.X) - sqr(p4.X) + sqr(p3.Y) -
        sqr(p4.Y);
    __int128 det = (__int128)-u13 * u22 * u31 + (
        __int128)u12 * u23 * u31 + (__int128)u13 * u21
        * u32 - (__int128)u11 * u23 * u32 - (__int128)
        u12 * u21 * u33 + (__int128)u11 * u22 * u33;
    return det > eps;
}

```

8.11 Half plane intersection

```

bool isin( Line l0, Line l1, Line l2 ){
    // Check inter(l1, l2) in l0
    pdd p = intersect(l1.X, l1.Y, l2.X, l2.Y);
    return cross(l0.Y - l0.X, p - l0.X) > eps;
}
/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.Y - l.X) ^ (p - l.X) > 0
 */
/* --- Line.X --- Line.Y --- */
vector<Line> halfPlaneInter(vector<Line> lines){
    int sz = lines.size();
    vector<double> ata(sz), ord(sz);
    for(int i=0; i<sz; ++i) {
        ord[i] = i;
        pdd d = lines[i].Y - lines[i].X;
        ata[i] = atan2(d.Y, d.X);
    }
    sort(ord.begin(), ord.end(), [&](int i, int j){
        if( fabs(ata[i] - ata[j]) < eps )
            return (cross(lines[i].Y-lines[i].X,
                lines[j].Y-lines[i].X)<0;
        return ata[i] < ata[j];
    });
}

```

```

vector<Line> fin;
for (int i=0; i<sz; ++i)
    if (!i || fabs(ata[ord[i]] - ata[ord[i-1]]) > eps)
        fin.pb(lines[ord[i]]);
deque<Line> dq;
for (int i=0; i<SZ(fin); i++){
    while(SZ(dq)>=2&&!isin(fin[i], dq[SZ(dq)-2], dq.back
        ()))
        dq.pop_back();
    while(SZ(dq)>=2&&!isin(fin[i], dq[0], dq[1]))
        dq.pop_front();
    dq.push_back(fin[i]);
}
while(SZ(dq)>=3&&!isin(dq[0], dq[SZ(dq)-2], dq.back()))
    dq.pop_back();
while(SZ(dq)>=3&&!isin(dq.back(), dq[0], dq[1]))
    dq.pop_front();
vector<Line> res(ALL(dq));
return res;
}

```

8.12 CircleCover*

```

const int N = 1021;
struct CircleCover {
    int C;
    Cir c[N];
    bool g[N][N], overlap[N][N];
    // Area[i]: area covered by at least i circles
    double Area[ N ];
    void init(int _C){ C = _C; }
    struct Teve {
        pdd p; double ang; int add;
        Teve() {}
        Teve(pdd _a, double _b, int _c):p(_a), ang(_b), add
            (_c){}
        bool operator<(const Teve &a) const
            {return ang < a.ang;}
    }eve[N * 2];
    // strict: x = 0, otherwise x = -1
    bool disjuct(Cir &a, Cir &b, int x)
    {return sign(abs(a.O - b.O) - a.R - b.R) > x;}
    bool contain(Cir &a, Cir &b, int x)
    {return sign(a.R - b.R - abs(a.O - b.O)) > x;}
    bool contain(int i, int j) {
        /* c[j] is non-strictly in c[i]. */
        return (sign(c[i].R - c[j].R) > 0 || (sign(c[i].R -
            c[j].R) == 0 && i < j)) && contain(c[i], c[j],
            -1);
    }
    void solve(){
        fill_n(Area, C + 2, 0);
        for(int i = 0; i < C; ++i)
            for(int j = 0; j < C; ++j)
                overlap[i][j] = contain(i, j);
        for(int i = 0; i < C; ++i)
            for(int j = 0; j < C; ++j)
                g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                    disjuct(c[i], c[j], -1));
        for(int i = 0; i < C; ++i){
            int E = 0, cnt = 1;
            for(int j = 0; j < C; ++j)
                if(j != i && overlap[j][i])
                    ++cnt;
            for(int j = 0; j < C; ++j)
                if(i != j && g[i][j]) {
                    pdd aa, bb;
                    CCinter(c[i], c[j], aa, bb);
                    double A = atan2(aa.Y - c[i].O.Y, aa.X - c[i]
                        ].O.X);
                    double B = atan2(bb.Y - c[i].O.Y, bb.X - c[i]
                        ].O.X);
                    eve[E++] = Teve(bb, B, 1), eve[E++] = Teve(aa
                        , A, -1);
                    if(B > A) ++cnt;
                }
            if(E == 0) Area[cnt] += pi * c[i].R * c[i].R;
            else{
                sort(eve, eve + E);
                eve[E] = eve[0];
            }
        }
    }
}

```



```

    for(int j = 0; j < E; ++j){
        cnt += eve[j].add;
        Area[cnt] += cross(eve[j].p, eve[j + 1].p) *
            .5;
        double theta = eve[j + 1].ang - eve[j].ang;
        if (theta < 0) theta += 2. * pi;
        Area[cnt] += (theta - sin(theta)) * c[i].R *
            c[i].R * .5;
    }
}
};

```

8.13 3Dpoint*

```

struct Point {
    double x, y, z;
    Point(double _x = 0, double _y = 0, double _z = 0): x
        (_x), y(_y), z(_z){}
    Point(Point p) { x = p.X, y = p.Y, z = p.Z; }
};
Point operator-(const Point &p1, const Point &p2)
{ return Point(p1.x - p2.x, p1.y - p2.y, p1.z - p2.z);}
Point cross(const Point &p1, const Point &p2)
{ return Point(p1.y * p2.z - p1.z * p2.y, p1.z * p2.x -
    p1.x * p2.z, p1.x * p2.y - p1.y * p2.x);}
double dot(const Point &p1, const Point &p2)
{ return p1.x * p2.x + p1.y * p2.y + p1.z * p2.z;}
double abs(const Point &a)
{ return sqrt(dot(a, a));}
Point cross3(const Point &a, const Point &b, const
    Point &c)
{ return cross(b - a, c - a);}
double area(Point a, Point b, Point c)
{ return abs(cross3(a, b, c));}
double volume(Point a, Point b, Point c, Point d)
{ return dot(cross3(a, b, c), d - a);}

```

8.14 Convexhull3D*

```

struct CH3D {
    struct face{int a, b, c; bool ok;} F[8 * N];
    double dblcmp(Point &p, face &f)
    {return dot(cross3(P[f.a], P[f.b], P[f.c]), p - P[f.a]
        );}
    int g[N][N], num, n;
    Point P[N];
    void deal(int p, int a, int b) {
        int f = g[a][b];
        face add;
        if (F[f].ok) {
            if (dblcmp(P[p], F[f]) > eps) dfs(p, f);
        }
        else
            add.a = b, add.b = a, add.c = p, add.ok = 1, g[
                p][b] = g[a][p] = g[b][a] = num, F[num++] =
                add;
    }
    void dfs(int p, int now) {
        F[now].ok = 0;
        deal(p, F[now].b, F[now].a), deal(p, F[now].c, F[
            now].b), deal(p, F[now].a, F[now].c);
    }
    bool same(int s, int t){
        Point &a = P[F[s].a];
        Point &b = P[F[s].b];
        Point &c = P[F[s].c];
        return fabs(volume(a, b, c, P[F[t].a])) < eps &&
            fabs(volume(a, b, c, P[F[t].b])) < eps && fabs(
                volume(a, b, c, P[F[t].c])) < eps;
    }
    void init(int _n){n = _n, num = 0;}
    void solve() {
        face add;
        num = 0;
        if(n < 4) return;
        if([&](){

```

```

            for (int i = 1; i < n; ++i)
                if (abs(P[0] - P[i]) > eps)
                    return swap(P[1], P[i]), 0;
            return 1;
        }() || [&](){
            for (int i = 2; i < n; ++i)
                if (abs(cross3(P[i], P[0], P[1])) > eps)
                    return swap(P[2], P[i]), 0;
            return 1;
        }() || [&](){
            for (int i = 3; i < n; ++i)
                if (fabs(dot(cross(P[0] - P[1], P[1] - P[2]), P
                    [0] - P[i])) > eps)
                    return swap(P[3], P[i]), 0;
            return 1;
        }())return;
        for (int i = 0; i < 4; ++i) {
            add.a = (i + 1) % 4, add.b = (i + 2) % 4, add.c =
                (i + 3) % 4, add.ok = true;
            if (dblcmp(P[i], add) > 0) swap(add.b, add.c);
            g[add.a][add.b] = g[add.b][add.c] = g[add.c][add.
                a] = num;
            F[num++] = add;
        }
        for (int i = 4; i < n; ++i)
            for (int j = 0; j < num; ++j)
                if (F[j].ok && dblcmp(P[i], F[j]) > eps) {
                    dfs(i, j);
                    break;
                }
        for (int tmp = num, i = (num = 0); i < tmp; ++i)
            if (F[i].ok) F[num++] = F[i];
    }
    double get_area() {
        double res = 0.0;
        if (n == 3)
            return abs(cross3(P[0], P[1], P[2])) / 2.0;
        for (int i = 0; i < num; ++i)
            res += area(P[F[i].a], P[F[i].b], P[F[i].c]);
        return res / 2.0;
    }
    double get_volume() {
        double res = 0.0;
        for (int i = 0; i < num; ++i)
            res += volume(Point(0, 0, 0), P[F[i].a], P[F[i].b]
                , P[F[i].c]);
        return fabs(res / 6.0);
    }
    int triangle() {return num;}
    int polygon() {
        int res = 0;
        for (int i = 0, flag = 1; i < num; ++i, res += flag
            , flag = 1)
            for (int j = 0; j < i && flag; ++j)
                flag &= !same(i, j);
        return res;
    }
    Point getcent(){
        Point ans(0, 0, 0), temp = P[F[0].a];
        double v = 0.0, t2;
        for (int i = 0; i < num; ++i)
            if (F[i].ok == true) {
                Point p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[
                    i].c];
                t2 = volume(temp, p1, p2, p3) / 6.0;
                if (t2 > 0)
                    ans.x += (p1.x + p2.x + p3.x + temp.x) * t2,
                    ans.y += (p1.y + p2.y + p3.y + temp.y) *
                        t2, ans.z += (p1.z + p2.z + p3.z + temp.z
                            ) * t2, v += t2;
            }
        ans.x /= (4 * v), ans.y /= (4 * v), ans.z /= (4 * v
            );
        return ans;
    }
    double pointmindis(Point p) {
        double rt = 99999999;
        for(int i = 0; i < num; ++i)
            if(F[i].ok == true) {
                Point p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[
                    i].c];

```

```

    double a = (p2.y - p1.y) * (p3.z - p1.z) - (p2.
        z - p1.z) * (p3.y - p1.y);
    double b = (p2.z - p1.z) * (p3.x - p1.x) - (p2.
        x - p1.x) * (p3.z - p1.z);
    double c = (p2.x - p1.x) * (p3.y - p1.y) - (p2.
        y - p1.y) * (p3.x - p1.x);
    double d = 0 - (a * p1.x + b * p1.y + c * p1.z)
        ;
    double temp = fabs(a * p.x + b * p.y + c * p.z
        + d) / sqrt(a * a + b * b + c * c);
    rt = min(rt, temp);
}
return rt;
}
};

```

8.15 DelaunayTriangulation*

```

/* Delaunay Triangulation:
Given a sets of points on 2D plane, find a
triangulation such that no points will strictly
inside circumcircle of any triangle.
find : return a triangle contain given point
add_point : add a point into triangulation
A Triangle is in triangulation iff. its has_chd is 0.
Region of triangle u: iterate each u.edge[i].tri,
each points are u.p[(i+1)%3], u.p[(i+2)%3]
Voronoi diagram: for each triangle in triangulation,
the bisector of all its edges will split the region.
nearest point will belong to the triangle containing it
*/
const ll inf = MAXC * MAXC * 100; // Lower_bound
unknown
struct Tri;
struct Edge {
    Tri* tri; int side;
    Edge(): tri(0), side(0){}
    Edge(Tri* _tri, int _side): tri(_tri), side(_side)
    {}
};
struct Tri {
    pll p[3];
    Edge edge[3];
    Tri* chd[3];
    Tri() {}
    Tri(const pll& p0, const pll& p1, const pll& p2) {
        p[0] = p0; p[1] = p1; p[2] = p2;
        chd[0] = chd[1] = chd[2] = 0;
    }
    bool has_chd() const { return chd[0] != 0; }
    int num_chd() const {
        return !!chd[0] + !!chd[1] + !!chd[2];
    }
    bool contains(pll const& q) const {
        for (int i = 0; i < 3; ++i)
            if (ori(p[i], p[(i + 1) % 3], q) < 0)
                return 0;
        return 1;
    }
} pool[N * 10], *tris;
void edge(Edge a, Edge b) {
    if(a.tri) a.tri -> edge[a.side] = b;
    if(b.tri) b.tri -> edge[b.side] = a;
}
struct Trig { // Triangulation
    Trig() {
        the_root = // Tri should at least contain all
            points
            new(tris++) Tri(pll(-inf, -inf), pll(inf +
                inf, -inf), pll(-inf, inf + inf));
    }
    Tri* find(pll p) { return find(the_root, p); }
    void add_point(const pll &p) { add_point(find(
        the_root, p), p); }
    Tri* the_root;
    static Tri* find(Tri* root, const pll &p) {
        while (1) {
            if (!root -> has_chd())
                return root;

```

```

        for (int i = 0; i < 3 && root -> chd[i]; ++
            i)
            if (root -> chd[i] -> contains(p)) {
                root = root -> chd[i];
                break;
            }
        }
        assert(0); // "point not found"
    }
    void add_point(Tri* root, pll const& p) {
        Tri* t[3];
        /* split it into three triangles */
        for (int i = 0; i < 3; ++i)
            t[i] = new(tris++) Tri(root -> p[i], root
                -> p[(i + 1) % 3], p);
        for (int i = 0; i < 3; ++i)
            edge(Edge(t[i], 0), Edge(t[(i + 1) % 3], 1)
                );
        for (int i = 0; i < 3; ++i)
            edge(Edge(t[i], 2), root -> edge[(i + 2) %
                3]);
        for (int i = 0; i < 3; ++i)
            root -> chd[i] = t[i];
        for (int i = 0; i < 3; ++i)
            flip(t[i], 2);
    }
    void flip(Tri* tri, int pi) {
        Tri* trj = tri -> edge[pi].tri;
        int pj = tri -> edge[pi].side;
        if (!trj) return;
        if (!in_cc(tri -> p[0], tri -> p[1], tri -> p
            [2], trj -> p[pj])) return;
        /* flip edge between tri, trj */
        Tri* trk = new(tris++) Tri(tri -> p[(pi + 1) %
            3], trj -> p[pj], tri -> p[pi]);
        Tri* trl = new(tris++) Tri(trj -> p[(pj + 1) %
            3], tri -> p[pi], trj -> p[pj]);
        edge(Edge(trk, 0), Edge(trl, 0));
        edge(Edge(trk, 1), tri -> edge[(pi + 2) % 3]);
        edge(Edge(trk, 2), trj -> edge[(pj + 1) % 3]);
        edge(Edge(trl, 1), trj -> edge[(pj + 2) % 3]);
        edge(Edge(trl, 2), tri -> edge[(pi + 1) % 3]);
        tri -> chd[0] = trk; tri -> chd[1] = trl; tri
            -> chd[2] = 0;
        trj -> chd[0] = trk; trj -> chd[1] = trl; trj
            -> chd[2] = 0;
        flip(trk, 1); flip(trk, 2);
        flip(trl, 1); flip(trl, 2);
    }
};
vector<Tri*> triang; // vector of all triangle
set<Tri*> vst;
void go(Tri* now) { // store all tri into triang
    if (vst.find(now) != vst.end())
        return;
    vst.insert(now);
    if (!now -> has_chd())
        return triang.push_back(now);
    for (int i = 0; i < now -> num_chd(); ++i)
        go(now -> chd[i]);
}
void build(int n, pll* ps) { // build triangulation
    tris = pool; triang.clear(); vst.clear();
    random_shuffle(ps, ps + n);
    Trig tri; // the triangulation structure
    for (int i = 0; i < n; ++i)
        tri.add_point(ps[i]);
    go(tri.the_root);
}

```

8.16 Triangulation Voronoi*

```

vector<Line> ls[N];
pll arr[N];
Line make_line(pll p, Line l) {
    pdd d = l.Y - l.X; d = perp(d);
    pdd m = (l.X + l.Y) / 2;
    l = Line(m, m + d);
    if (ori(l.X, l.Y, p) < 0)
        l = Line(m + d, m);
}

```

```

    return l;
}
double calc_area(int id) {
    // use to calculate the area of point "strictly in
    // the convex hull"
    vector<Line> hpi = halfPlaneInter(ls[id]);
    vector<pdd> ps;
    for (int i = 0; i < SZ(hpi); ++i)
        ps.pb(intersect(hpi[i].X, hpi[i].Y, hpi[(i + 1)
            % SZ(hpi)].X, hpi[(i + 1) % SZ(hpi)].Y));
    double rt = 0;
    for (int i = 0; i < SZ(ps); ++i)
        rt += cross(ps[i], ps[(i + 1) % SZ(ps)]);
    return fabs(rt) / 2;
}
void solve(int n, pii *oarr) {
    map<pll, int> mp;
    for (int i = 0; i < n; ++i)
        arr[i] = pll(oarr[i].X, oarr[i].Y), mp[arr[i]]
            = i;
    build(n, arr); // Triangulation
    for (auto *t : triang) {
        vector<int> p;
        for (int i = 0; i < 3; ++i)
            if (mp.find(t -> p[i]) != mp.end())
                p.pb(mp[t -> p[i]]);
        for (int i = 0; i < SZ(p); ++i)
            for (int j = i + 1; j < SZ(p); ++j) {
                line l(oarr[p[i]], oarr[p[j]]);
                ls[p[i]].pb(make_line(oarr[p[i]], l));
                ls[p[j]].pb(make_line(oarr[p[j]], l));
            }
    }
}

```

8.17 Tangent line of two circles

```

vector<Line> go( const Cir& c1 , const Cir& c2 , int
    sign1 ){
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    double d_sq = norm2( c1.0 - c2.0 );
    if( d_sq < eps ) return ret;
    double d = sqrt( d_sq );
    Pt v = ( c2.0 - c1.0 ) / d;
    double c = ( c1.R - sign1 * c2.R ) / d;
    if( c * c > 1 ) return ret;
    double h = sqrt( max( 0.0 , 1.0 - c * c ) );
    for( int sign2 = 1 ; sign2 >= -1 ; sign2 -= 2 ){
        Pt n = { v.X * c - sign2 * h * v.Y ,
            v.Y * c + sign2 * h * v.X };
        Pt p1 = c1.0 + n * c1.R;
        Pt p2 = c2.0 + n * ( c2.R * sign1 );
        if( fabs( p1.X - p2.X ) < eps and
            fabs( p1.Y - p2.Y ) < eps )
            p2 = p1 + perp( c2.0 - c1.0 );
        ret.push_back( { p1 , p2 } );
    }
    return ret;
}

```

8.18 minMaxEnclosingRectangle

```

pdd solve(vector<pll> &dots){
    vector<pll> hull;
    const double INF=1e18, qi=acos(-1)/2*3;
    cv.dots=dots;
    hull=cv.hull();
    double Max=0, Min=INF, deg;
    ll n=hull.size();
    hull.pb(hull[0]);
    for(int i=0, u=1, r=1; i<n; ++i){
        pll nw=hull[i+1]-hull[i];
        while(cross(nw, hull[u+1]-hull[i])>cross(nw, hull[u]-
            hull[i]))
            u=(u+1)%n;
        while(dot(nw, hull[r+1]-hull[i])>dot(nw, hull[r]-hull
            [i]))

```

```

            r=(r+1)%n;
        if(!i) l=(r+1)%n;
        while(dot(nw, hull[l+1]-hull[i])<dot(nw, hull[l]-hull
            [i]))
            l=(l+1)%n;
        Min=min(Min, (double)(dot(nw, hull[r]-hull[i])-dot(nw
            , hull[l]-hull[i]))*cross(nw, hull[u]-hull[i])/
            abs2(nw));
        deg=acos((double)dot(hull[r]-hull[l], hull[u]-hull[i]
            ))/abs(hull[r]-hull[l])/abs(hull[u]-hull[i]));
        deg=(qi-deg)/2;
        Max=max(Max, (double)abs(hull[r]-hull[l])*abs(hull[u]
            -hull[i])*sin(deg)*sin(deg));
    }
    return pdd(Min, Max);
}

```

8.19 minDistOfTwoConvex

```

// p, q is convex
double TwoConvexHullMinDist(Point P[], Point Q[], int n
    , int m) {
    int YMinP = 0, YMaxQ = 0;
    double tmp, ans = 999999999;
    for (i = 0; i < n; ++i) if(P[i].y < P[YMinP].y) YMinP
        = i;
    for (i = 0; i < m; ++i) if(Q[i].y > Q[YMaxQ].y) YMaxQ
        = i;
    P[n] = P[0], Q[m] = Q[0];
    for (int i = 0; i < n; ++i) {
        while (tmp = Cross(Q[YMaxQ + 1] - P[YMinP + 1], P[
            YMinP] - P[YMinP + 1]) > Cross(Q[YMaxQ] - P[
            YMinP + 1], P[YMinP] - P[YMinP + 1])) YMaxQ = (
            YMaxQ + 1) % m;
        if (tmp < 0) ans = min(ans, PointToSegDist(P[YMinP
            ], P[YMinP + 1], Q[YMaxQ]));
        else ans = min(ans, TwoSegMinDist(P[YMinP], P[YMinP
            + 1], Q[YMaxQ], Q[YMaxQ + 1]));
        YMinP = (YMinP + 1) % n;
    }
    return ans;
}

```

8.20 Minkowski Sum*

```

vector<pll> Minkowski(vector<pll> A, vector<pll> B) {
    hull(A), hull(B);
    vector<pll> C(1, A[0] + B[0]), s1, s2;
    for(int i = 0; i < SZ(A); ++i)
        s1.pb(A[(i + 1) % SZ(A)] - A[i]);
    for(int i = 0; i < SZ(B); ++i)
        s2.pb(B[(i + 1) % SZ(B)] - B[i]);
    for(int p1 = 0, p2 = 0; p1 < SZ(A) || p2 < SZ(B);)
        if (p2 >= SZ(B) || (p1 < SZ(A) && cross(s1[p1], s2[
            p2]) >= 0))
            C.pb(C.back() + s1[p1++]);
        else
            C.pb(C.back() + s2[p2++]);
    return hull(C), C;
}

```

9 Else

9.1 Mo's Alogrithm(With modification)

```

struct QUERY{//BLOCK=N^{2/3}
    int L, R, id, LBid, RBid, T;
    QUERY(int l, int r, int id, int lb, int rb, int t):
        L(l), R(r), id(id), LBid(lb), RBid(rb), T(t){}
    bool operator<(const QUERY &b) const{
        if(LBid!=b.LBid) return LBid<b.LBid;
        if(RBid!=b.RBid) return RBid<b.RBid;
        return T<b.T;
    }
}

```

```

};
vector<QUERY> query;
int cur_ans, arr[MAXN], ans[MAXN];
void addTime(int L, int R, int T){}
void subTime(int L, int R, int T){}
void add(int x){}
void sub(int x){}
void solve(){
    sort(ALL(query));
    int L=0, R=0, T=-1;
    for(auto q:query){
        while(T<q.T) addTime(L, R, ++T);
        while(T>q.T) subTime(L, R, T--);
        while(R<q.R) add(arr[++R]);
        while(L>q.L) add(arr[--L]);
        while(R>q.R) sub(arr[R--]);
        while(L<q.L) sub(arr[L++]);
        ans[q.id]=cur_ans;
    }
}

```

9.2 Mo's Algorithm On Tree

```

const int MAXN=40005;
vector<int> G[MAXN]; //1-base
int n, B, arr[MAXN], ans[100005], cur_ans;
int in[MAXN], out[MAXN], dfn[MAXN*2], dft;
int deep[MAXN], sp[___lg(MAXN*2)+1][MAXN*2], bln[MAXN], spt;
bitset<MAXN> inset;
struct QUERY{
    int L, R, Lid, id, lca;
    QUERY(int l, int r, int _id):L(l), R(r), lca(0), id(_id){}
    bool operator<(const QUERY &b){
        if(Lid!=b.Lid) return Lid<b.Lid;
        return R<b.R;
    }
};
vector<QUERY> query;
void dfs(int u, int f, int d){
    deep[u]=d, sp[0][spt]=u, bln[u]=spt++;
    dfn[dft]=u, in[u]=dft++;
    for(int v:G[u])
        if(v!=f)
            dfs(v, u, d+1), sp[0][spt]=u, bln[u]=spt++;
    dfn[dft]=u, out[u]=dft++;
}
int lca(int u, int v){
    if(bln[u]>bln[v]) swap(u, v);
    int t=___lg(bln[v]-bln[u]+1);
    int a=sp[t][bln[u]], b=sp[t][bln[v]-(1<<t)+1];
    if(deep[a]<deep[b]) return a;
    return b;
}
void sub(int x){}
void add(int x){}
void flip(int x){
    if(inset[x]) sub(arr[x]);
    else add(arr[x]);
    inset[x]=~inset[x];
}
void solve(){
    B=sqrt(2*n), dft=spt=cur_ans=0, dfs(1, 1, 0);
    for(int i=1, x=2; x<2*n; ++i, x<=1)
        for(int j=0; j+x<2*n; ++j)
            if(deep[sp[i-1][j]]<deep[sp[i-1][j+x/2]])
                sp[i][j]=sp[i-1][j];
            else sp[i][j]=sp[i-1][j+x/2];
    for(auto &q:query){
        int c=lca(q.L, q.R);
        if(c==q.L || c==q.R)
            q.L=out[c==q.L?q.R:q.L], q.R=out[c];
        else if(out[q.L]<in[q.R])
            q.lca=c, q.L=out[q.L], q.R=in[q.R];
        else q.lca=c, c=in[q.L], q.L=out[q.R], q.R=c;
        q.Lid=q.L/B;
    }
    sort(ALL(query));
    int L=0, R=-1;
    for(auto q:query){

```

```

        while(R<q.R) flip(dfn[++R]);
        while(L>q.L) flip(dfn[--L]);
        while(R>q.R) flip(dfn[R--]);
        while(L<q.L) flip(dfn[L++]);
        if(q.lca) add(arr[q.lca]);
        ans[q.id]=cur_ans;
        if(q.lca) sub(arr[q.lca]);
    }
}

```

9.3 DynamicConvexTrick*

```

// only works for integer coordinates!!
struct Line {
    mutable ll a, b, p;
    bool operator<(const Line &rhs) const { return a <
        rhs.a; }
    bool operator<(ll x) const { return p < x; }
};
struct DynamicHull : multiset<Line, less<>> {
    static const ll kInf = 1e18;
    ll Div(ll a, ll b) { return a / b - ((a ^ b) < 0 &&
        a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x -> p = kInf; return 0; }
        if (x -> a == y -> a) x -> p = x -> b > y -> b
            ? kInf : -kInf;
        else x -> p = Div(y -> b - x -> b, x -> a - y
            -> a);
        return x -> p >= y -> p;
    }
    void addline(ll a, ll b) {
        auto z = insert({a, b, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y =
            erase(y));
        while ((y = x) != begin() && (--x) -> p >= y ->
            p) isect(x, erase(y));
    }
    ll query(ll x) {
        auto l = *lower_bound(x);
        return l.a * x + l.b;
    }
};

```

9.4 DLX*

```

template<bool Exact>
struct DLX {
    int lt[NN], rg[NN], up[NN], dn[NN], cl[NN], rw[NN],
        bt[NN], s[NN], head, sz, ans;
    int columns;
    bool vis[NN];
    void remove(int c) {
        if (Exact) lt[rg[c]] = lt[c], rg[lt[c]] = rg[c];
        for (int i = dn[c]; i != c; i = dn[i]) {
            if (Exact) {
                for (int j = rg[i]; j != i; j = rg[j])
                    up[dn[j]] = up[j], dn[up[j]] = dn[j], --s[cl[
                        j]];
            } else {
                lt[rg[i]] = lt[i], rg[lt[i]] = rg[i];
            }
        }
    }
    void restore(int c) {
        for (int i = up[c]; i != c; i = up[i]) {
            if (Exact) {
                for (int j = lt[i]; j != i; j = lt[j])
                    ++s[cl[j]], up[dn[j]] = j, dn[up[j]] = j;
            } else {
                lt[rg[i]] = rg[lt[i]] = i;
            }
        }
        if (Exact) lt[rg[c]] = c, rg[lt[c]] = c;
    }
    void init(int c) {
        columns = c;
    }
};

```

```

    for (int i = 0; i < c; ++i) {
        up[i] = dn[i] = bt[i] = i;
        lt[i] = i == 0 ? c : i - 1;
        rg[i] = i == c - 1 ? c : i + 1;
        s[i] = 0;
    }
    rg[c] = 0, lt[c] = c - 1;
    up[c] = dn[c] = -1;
    head = c, sz = c + 1;
}

void insert(int r, const vector<int> &col) {
    if (col.empty()) return;
    int f = sz;
    for (int i = 0; i < (int)col.size(); ++i) {
        int c = col[i], v = sz++;
        dn[bt[c]] = v;
        up[v] = bt[c], bt[c] = v;
        rg[v] = (i + 1 == (int)col.size() ? f : v + 1);
        rw[v] = r, cl[v] = c;
        ++s[c];
        if (i > 0) lt[v] = v - 1;
    }
    lt[f] = sz - 1;
}

int h() {
    int ret = 0;
    memset(vis, 0, sizeof(bool) * sz);
    for (int x = rg[head]; x != head; x = rg[x]) {
        if (vis[x]) continue;
        vis[x] = true, ++ret;
        for (int i = dn[x]; i != x; i = dn[i]) {
            for (int j = rg[i]; j != i; j = rg[j])
                vis[cl[j]] = true;
        }
    }
    return ret;
}

void dfs(int dep) {
    if (dep + (Exact ? 0 : h()) >= ans) return;
    if (rg[head] == head) return ans = dep, void();
    if (dn[rg[head]] == rg[head]) return;
    int c = rg[head];
    int w = c;
    for (int x = c; x != head; x = rg[x]) if (s[x] < s[w]) w = x;
    if (Exact) {
        remove(w);
        for (int i = dn[w]; i != w; i = dn[i]) {
            for (int j = rg[i]; j != i; j = rg[j]) remove(
                cl[j]);
            dfs(dep + 1);
            for (int j = lt[i]; j != i; j = lt[j]) restore(
                cl[j]);
        }
        restore(w);
    } else {
        for (int i = dn[w]; i != w; i = dn[i]) {
            remove(i);
            for (int j = rg[i]; j != i; j = rg[j]) remove(j);
            dfs(dep + 1);
            for (int j = lt[i]; j != i; j = lt[j]) restore(j);
            restore(i);
        }
    }
}

int solve() {
    for (int i = 0; i < columns; ++i)
        dn[bt[i]] = i, up[i] = bt[i];
    ans = 1e9, dfs(0);
    return ans;
}
};

```