

# Contents

<b>1 Basic</b>	<b>1</b>	<b>9 Else</b>	<b>24</b>
1.1 Shell script	1	9.1 Mo's Alogrithm(With modification)	24
1.2 Default code	1	9.2 Mo's Alogrithm On Tree	24
1.3 vimrc	1	9.3 DynamicConvexTrick	25
1.4 readchar	1		
1.5 Black Magic	1		
1.6 Texas hold'em	2		
<b>2 Graph</b>	<b>2</b>		
2.1 BCC Vertex	2		
2.2 Bridge	2		
2.3 Strongly Connected Components	2		
2.4 MinimumMeanCycle	3		
2.5 Virtual Tree	3		
2.6 Maximum Clique	3		
2.7 MinimumSteinerTree	3		
2.8 Dominator Tree	4		
2.9 Minimum Arborescence	4		
2.10Vizing's theorem	5		
2.11Theory	5		
<b>3 Data Structure</b>	<b>5</b>		
3.1 Leftist Tree	5		
3.2 Heavy light Decomposition	5		
3.3 Smart Pointer	6		
3.4 LiChaoST	6		
3.5 link cut tree	6		
3.6 KDTree	7		
<b>4 Flow/Matching</b>	<b>8</b>		
4.1 Dinic	8		
4.2 Kuhn Munkres	9		
4.3 MincostMaxflow	9		
4.4 Maximum Simple Graph Matching	9		
4.5 Minimum Weight Matching (Clique version)	10		
4.6 SW-mincut	10		
4.7 BoundedFlow	10		
4.8 Gomory Hu tree	11		
4.9 NumberofMaximalClique	11		
4.10isap	11		
<b>5 String</b>	<b>12</b>		
5.1 KMP	12		
5.2 Z-value	12		
5.3 Manacher	12		
5.4 Suffix Array	12		
5.5 SAIS	12		
5.6 Aho-Corasick Automatan	13		
5.7 Smallest Rotation	13		
5.8 De Bruijn sequence	13		
5.9 SAM	13		
5.10PalTree	14		
5.11cyclicLCS	14		
<b>6 Math</b>	<b>14</b>		
6.1 ax+by=gcd	14		
6.2 floor and ceil	15		
6.3 Miller Rabin	15		
6.4 Big number	15		
6.5 Fraction	16		
6.6 Simultaneous Equations	16		
6.7 Pollard Rho	16		
6.8 Simplex Algorithm	16		
6.9 chineseRemainder	17		
6.10QuadraticResidue	17		
6.11PiCount	17		
6.12Algorithms about Primes	17		
<b>7 Polynomial</b>	<b>17</b>		
7.1 Fast Fourier Transform	17		
7.2 Number Theory Transform	18		
7.3 Fast Walsh Transform	18		
7.4 Polynomial Operation	18		
<b>8 Geometry</b>	<b>19</b>		
8.1 Default Code	19		
8.2 Convex hull	19		
8.3 External bisector	19		
8.4 Heart	19		
8.5 Polar Angle Sort	19		
8.6 Intersection of two circles	20		
8.7 Intersection of polygon and circle	20		
8.8 Intersection of line and circle	20		
8.9 Half plane intersection	20		
8.10Convexhull3D	20		
8.11CircleCover	21		
8.12DelaunayTriangulation	22		
8.13Triangulation Voronoi	23		
8.14Tangent line of two circles	23		
8.15minMaxEnclosingRectangle	23		
8.16minDistOfTwoConvex	23		
8.17Minkowski Sum	24		

## 1 Basic

### 1.1 Shell script

```
g++ -O2 -std=c++17 -Dbbq -Wall -Wextra -Wshadow -o $1
    $1.cpp
chmod +x compile.sh
```

### 1.2 Default code

```
#include<bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;
#define X first
#define Y second
#define SZ(a) ((int)a.size())
#define ALL(v) v.begin(), v.end()
#define pb push_back
```

### 1.3 vimrc

```
"This file should be placed at ~/.vimrc"
se nu ai hls et ru ic is sc cul
se re=1 ts=4 sts=4 sw=4 ls=2 mouse=a
syntax on
hi cursorline cterm=none ctermbg=89
set bg=dark
inoremap {<ENTER> {}<LEFT><ENTER><ENTER><UP><TAB>
```

### 1.4 readchar

```
inline char readchar() {
    static const size_t bufsize = 65536;
    static char buf[bufsize];
    static char *p = buf, *end = buf;
    if (p == end) end = buf + fread_unlocked(buf, 1,
        bufsize, stdin), p = buf;
    return *p++;
}
```

### 1.5 Black Magic

```
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp> //rb_tree
using namespace __gnu_pbds;
typedef __gnu_pbds::priority_queue<int> heap;
int main() {
    heap h1, h2;
    h1.push(1), h1.push(3);
    h2.push(2), h2.push(4);
    h1.join(h2);
    cout << h1.size() << h2.size() << h1.top() << endl;
    //404
    tree<ll, null_type, less<ll>, rb_tree_tag,
        tree_order_statistics_node_update> st;
    tree<ll, ll, less<ll>, rb_tree_tag,
        tree_order_statistics_node_update> mp;
    for (int x : {0, 2, 3, 4}) st.insert(x);
    cout << *st.find_by_order(2) << st.order_of_key(1) <<
        endl; //31
}
```

## 1.6 Texas hold'em

```
char suit[4]={'C','D','H','Y'}, ranks[13]={'2','3','4','5','6','7','8','9','T','J','Q','K','A'};
int rk[256];
/*
for(int i=0;i<13;++i)
rk[ranks[i]]=i;
for(int i=0;i<4;++i)
rk[suit[i]]=i;
*/
struct cards{
vector<pii> v;
int suit_count[4], hands;
void reset(){v.clear(), FILL(suit_count, 0), hands=-1;}
void insert(char a, char b){//suit, rank
++suit_count[rk[a]];
int flag=0;
for(auto &i:v)
if(i.Y==rk[b])
{
++i.X, flag=1;
break;
}
if(!flag) v.pb(pii(1, rk[b]));
}
void insert(string s){insert(s[0], s[1]);}
void ready(){
int Straight=0, Flush=(max_element(suit_count, suit_count+4))==5;
sort(ALL(v), [](ii a, ii b){return a>b;});
if(SZ(v)==5&&v[0].Y==v[1].Y+1&&v[1].Y==v[2].Y+1&&v[2].Y==v[3].Y+1&&v[3].Y==v[4].Y+1)
Straight=1;
else if(SZ(v)==5&&v[0].Y==12&&v[1].Y==3&&v[2].Y==2&&v[3].Y==1&&v[4].Y==0)
v[0].Y=3, v[1].Y=2, v[2].Y=1, v[3].Y=0, v[4].Y=-1,
Straight=1;
if(Straight&&Flush) hands=1;
else if(v[0].X==4) hands=2;
else if(v[0].X==3&&v[1].X==2) hands=3;
else if(Flush) hands=4;
else if(Straight) hands=5;
else if(v[0].X==3) hands=6;
else if(v[0].X==2&&v[1].X==2) hands=7;
else if(v[0].X==2) hands=8;
else hands=9;
}
bool operator>(const cards &a) const{
if(hands==a.hands) return v>a.v;
return hands<a.hands;
}
};
```

## 2 Graph

### 2.1 BCC Vertex

```
vector<int> G[N]; //1-base
vector<int> nG[N], bcc[N];
int low[N], dfn[N], Time;
int bcc_id[N], bcc_cnt; //1-base
bool is_cut[N]; //whether is av
bool cir[N];
int st[N], top;
```

```
void dfs(int u, int pa = -1) {
int child = 0;
low[u] = dfn[u] = ++Time;
st[top++] = u;
for(int v : G[u])
if(!dfn[v]) {
dfs(v, u), ++child;
low[u] = min(low[u], low[v]);
if(dfn[u] <= low[v]) {
is_cut[u]=1;
bcc[++bcc_cnt].clear();
int t;
```

```
do {
bcc_id[t = st[--top]] = bcc_cnt;
bcc[bcc_cnt].push_back(t);
}while(t != v);
bcc_id[u]=bcc_cnt;
bcc[bcc_cnt].pb(u);
}
}
else if(dfn[v] < dfn[u] && v!=pa)
low[u] = min(low[u], dfn[v]);
if(pa == -1 && child < 2)
is_cut[u] = 0;
}

void bcc_init(int n) {
Time = bcc_cnt = top = 0;
for(int i = 1; i <= n; ++i)
G[i].clear(), dfn[i] = bcc_id[i] = is_cut[i] = 0;
}

void bcc_solve(int n) {
for (int i = 1; i <= n; ++i)
if (!dfn[i])
dfs(i);
// circle-square tree
for(int i = 1; i <= n; ++i)
if(is_cut[i])
bcc_id[i] = ++bcc_cnt, cir[bcc_cnt] = 1;
for(int i = 1; i <= bcc_cnt && !cir[i]; ++i)
for(int j : bcc[i])
if(is_cut[j])
nG[i].pb(bcc_id[j]), nG[bcc_id[j]].pb(i);
}
```

### 2.2 Bridge

```
int low[N], dfn[N], Time; // 1-base
vector<pii> G[N], edge;
vector<bool> is_bridge;

void init(int n) {
Time = 0;
for (int i = 1; i <= n; ++i)
G[i].clear(), low[i] = dfn[i] = 0;
}

void add_edge(int a, int b) {
G[a].pb(pii(b, SZ(edge))), G[b].pb(pii(a, SZ(edge)));
edge.pb(pii(a, b));
}

void dfs(int u, int f) {
dfn[u] = low[u] = ++Time;
for (auto i : G[u])
if (!dfn[i.X])
dfs(i.X, i.Y), low[u] = min(low[u], low[i.X]);
else if (i.Y != f)
low[u] = min(low[u], dfn[i.X]);
if (low[u] == dfn[u] && f != -1)
is_bridge[f] = 1;
}

void solve(int n) {
is_bridge.resize(SZ(edge));
for (int i = 1; i <= n; ++i)
if (!dfn[i])
dfs(i, -1);
}
```

### 2.3 Strongly Connected Components

```
struct Strongly_CC{//1-base
int low[N], dfn[N], bln[N], sz[N], n, Time, nScc;
bitset<N> instack;
stack<int> st;
vector<int> G[N], SCC[N];
void init(int _n) {
n = _n;
```

```

    for(int i = 1; i <= n; ++i)
        G[i].clear();
}
void add_edge(int a, int b) {
    G[a].pb(b);
}
void dfs(int u) {
    dfn[u] = low[u] = ++Time;
    instack[u] = 1, st.push(u);
    for(int i : G[u])
        if(!dfn[i]) dfs(i), low[u] = min(low[i], low[u]);
        else if(instack[i] && dfn[i] < dfn[u])
            low[u] = min(low[u], dfn[i]);
    if(low[u] == dfn[u]) {
        int tmp;
        do {
            tmp = st.top(), st.pop();
            instack[tmp] = 0, bln[tmp] = nScc;
        } while(tmp != u);
        ++nScc;
    }
}
void solve() {
    Time = nScc = 0;
    for(int i = 1; i <= n; ++i)
        SCC[i].clear(), low[i] = dfn[i] = bln[i] = sz[i] = 0;
    for(int i = 1; i <= n; ++i)
        if(!dfn[i])
            dfs(i);
    for(int i = 1; i <= n; ++i)
        ++sz[bln[i]], SCC[bln[i]].pb(i);
}
};

```

## 2.4 MinimumMeanCycle

```

ll road[N][N]; //input here
struct MinimumMeanCycle{
    ll dp[N + 5][N], n;
    pll solve() {
        ll a = -1, b = -1, L = n+1;
        for(int i = 2; i <= L; ++i)
            for(int k = 0; k < n; ++k)
                for(int j = 0; j < n; ++j)
                    dp[i][j] = min(dp[i - 1][k] + road[k][j], dp[i][j]);
        for(int i = 0; i < n; ++i) {
            if(dp[L][i] >= INF) continue;
            ll ta = 0, tb = 1;
            for(int j = 1; j < n; ++j)
                if(dp[j][i] < INF && ta * (L - j) < (dp[L][i] - dp[j][i]) * tb)
                    ta = dp[L][i] - dp[j][i], tb = L - j;
            if(ta == 0) continue;
            if(a == -1 || a * tb > ta * b)
                a = ta, b = tb;
        }
        if(a != -1) {
            ll g = __gcd(a, b);
            return pll(a / g, b / g);
        }
        return pll(-1LL, -1LL);
    }
    void init(int _n){
        n = _n;
        for(int i = 0; i < n; ++i)
            for(int j = 0; j < n; ++j)
                dp[i + 2][j] = INF;
    }
};

```

## 2.5 Virtual Tree

```

vector<int> vG[N];
int top, st[N];
void insert(int u) {

```

```

    if(top == -1)
        return st[++top] = u, void();
    int p = LCA(st[top], u);
    if(p == st[top])
        return st[++top] = u, void();
    while(top >= 1 && dep[st[top - 1]] >= dep[p])
        vG[st[top - 1]].pb(st[top]), --top;
    if(st[top] != p)
        vG[p].pb(st[top]), --top, st[++top] = p;
    st[++top] = u;
}

void reset(int u) {
    for(int i : vG[u])
        reset(i);
    vG[u].clear();
}

void solve(vector<int> &v) {
    top = -1;
    sort(ALL(v), [&](int a, int b){return dfn[a] < dfn[b];});
    for(int i : v)
        insert(i);
    while(top > 0)
        vG[st[top - 1]].pb(st[top]), --top;
    //do something
    reset(v[0]);
}

```

## 2.6 Maximum Clique

```

struct Maximum_Clique{
    typedef bitset<MAXN> bst;
    bst N[MAXN], empty;
    int p[MAXN], n, ans;
    void BronKerbosch2(bst R, bst P, bst X){
        if(P==empty && X==empty)
            return ans=max(ans, (int)R.count()), void();
        bst tmp=P|X;
        int u;
        if((R|P|X).count()<=ans) return;
        for(int uu=0; uu<n; ++uu){
            u=p[uu];
            if(tmp[u]==1) break;
        }
        //if (double(cLock())/CLOCKS_PER_SEC > .999) return;
        bst now2=P&~N[u];
        for(int vv=0; vv<n; ++vv){
            int v=p[vv];
            if(now2[v]==1){
                R[v]=1;
                BronKerbosch2(R, P&N[v], X&N[v]);
                R[v]=0, P[v]=0, X[v]=1;
            }
        }
    }
    void init(int _n){n=_n;
        for(int i=0; i<n; ++i) N[i].reset();
    }
    void add_edge(int u, int v){N[u][v]=N[v][u]=1;}
    int solve(){//remember srand
        bst R, P, X;
        ans=0, P.flip();
        for(int i=0; i<n; ++i) p[i]=i;
        random_shuffle(p, p+n), BronKerbosch2(R, P, X);
        return ans;
    }
};

```

## 2.7 MinimumSteinerTree

```

// Minimum Steiner Tree
// O(V 3^T + V^2 2^T)
struct SteinerTree{// 0-base
    static const int T = 10, N = 105, INF = 1e9;
    int n, dst[N][N], dp[1 << T][N], tdst[N];

```

```

int vcost[N]; // the cost of vertexs
void init(int _n){
    n = _n;
    for(int i = 0; i < n; ++i) {
        for(int j = 0; j < n; ++j)
            dst[i][j] = INF;
        dst[i][i] = vcost[i] = 0;
    }
}
void add_edge(int ui, int vi, int wi) {
    dst[ui][vi] = min(dst[ui][vi], wi);
}
void shortest_path() {
    for(int k = 0; k < n; ++k)
        for(int i = 0; i < n; ++i)
            for(int j = 0; j < n; ++j)
                dst[i][j] = min(dst[i][j], dst[i][k]
                                + dst[k][j]);
}
int solve(const vector<int>& ter) {
    shortest_path();
    int t = SZ(ter);
    for(int i = 0; i < (1 << t); ++i)
        for(int j = 0; j < n; ++j)
            dp[i][j] = INF;
    for(int i = 0; i < n; ++i)
        dp[0][i] = vcost[i];
    for(int msk = 1; msk < (1 << t); ++msk){
        if(!(msk & (msk - 1))){
            int who = __lg(msk);
            for(int i = 0; i < n; ++i)
                dp[msk][i] = vcost[ter[who]] + dst[
                    ter[who]][i];
        }
        for(int i = 0; i < n; ++i)
            for(int submsk = (msk - 1) & msk;
                submsk; submsk = (submsk - 1) & msk)
                dp[msk][i] = min(dp[msk][i], dp[
                    submsk][i] + dp[msk ^ submsk][i]
                                - vcost[i]);
        for(int i = 0; i < n; ++i) {
            tdst[i] = INF;
            for(int j = 0; j < n; ++j)
                tdst[i] = min(tdst[i], dp[msk][j] +
                                dst[j][i]);
        }
        for(int i = 0; i < n; ++i)
            dp[msk][i] = tdst[i];
    }
    int ans = INF;
    for(int i = 0; i < n; ++i)
        ans = min(ans, dp[(1 << t) - 1][i]);
    return ans;
}
};

```

## 2.8 Dominator Tree

```

struct dominator_tree{//1-base
    int n;
    vector<int> G[MAXN], rG[MAXN];
    int pa[MAXN], dfn[MAXN], id[MAXN], dfnCnt;
    int semi[MAXN], idom[MAXN], best[MAXN];
    vector<int> tree[MAXN]; //dominator_tree
    void init(int _n){
        n = _n;
        for(int i=1; i<=n; ++i) G[i].clear(), rG[i].clear();
    }
    void add_edge(int u, int v){
        G[u].push_back(v);
        rG[v].push_back(u);
    }
    void dfs(int u){
        id[dfn[u]=++dfnCnt]=u;
        for(auto v:G[u]) if(!dfn[v]){
            dfs(v), pa[dfn[v]]=dfn[u];
        }
    }
    int find(int y, int x){

```

```

        if(y<=x) return y;
        int tmp=find(pa[y],x);
        if(semi[best[y]]>semi[best[pa[y]]])
            best[y]=best[pa[y]];
        return pa[y]=tmp;
    }
    void tarjan(int root){
        dfnCnt=0;
        for(int i=1; i<=n; ++i){
            dfn[i]=idom[i]=0;
            tree[i].clear();
            best[i]=semi[i]=i;
        }
        dfs(root);
        for(int i=dfnCnt; i>1; --i){
            int u=id[i];
            for(auto v:rG[u]) if(v=dfn[v]){
                find(v,i);
                semi[i]=min(semi[i], semi[best[v]]);
            }
            tree[semi[i]].push_back(i);
            for(auto v:tree[pa[i]]){
                find(v, pa[i]);
                idom[v] = semi[best[v]]==pa[i] ? pa[i] : best[v];
            }
            tree[pa[i]].clear();
        }
        for(int i=2; i<=dfnCnt; ++i){
            if(idom[i]!=semi[i]) idom[i]=idom[idom[i]];
            tree[id[idom[i]]].push_back(id[i]);
        }
    }
};

```

## 2.9 Minimum Arborescence

```

#define INF 0x3f3f3f3f
template<typename T>
struct zhu_liu{//O(VE)
    static const int MAXN=110;
    struct edge{
        int u,v;
        T w;
        edge(int u=0, int v=0, T w=0):u(u),v(v),w(w){}
    };
    vector<edge> E; // 0-base
    int pe[MAXN], id[MAXN], vis[MAXN];
    T in[MAXN];
    void init(){E.clear();}
    void add_edge(int u, int v, T w){
        if(u!=v) E.push_back(edge(u,v,w));
    }
    T build(int root, int n){
        T ans=0;
        for(;;){
            for(int u=0; u<n; ++u) in[u]=INF;
            for(size_t i=0; i<E.size(); ++i)
                if(E[i].u!=E[i].v && E[i].w<in[E[i].v]){
                    pe[E[i].v]=i, in[E[i].v]=E[i].w;
                }
            for(int u=0; u<n; ++u) //no solution
                if(u!=root && in[u]==INF) return -INF;
            int cntnode=0;
            memset(id, -1, sizeof(int)*n);
            memset(vis, -1, sizeof(int)*n);
            for(int u=0; u<n; ++u){
                if(u!=root) ans+=in[u];
                int v=u;
                for(; vis[v]!=u && id[v]==-1 && v!=root; v=E[pe[v]].u)
                    vis[v]=u;
                if(v!=root && id[v]==-1){
                    for(int x=E[pe[v]].u; x!=v; x=E[pe[x]].u)
                        id[x]=cntnode;
                    id[v]=cntnode++;
                }
            }
            if(!cntnode) break; //no cycle
            for(int u=0; u<n; ++u) if(id[u]==-1) id[u]=cntnode++;
            for(size_t i=0; i<E.size(); ++i){

```

```

    int v=E[i].v;
    E[i].u=id[E[i].u];
    E[i].v=id[E[i].v];
    if(E[i].u!=E[i].v)E[i].w-=in[v];
}
n=cntnode;
root=id[root];
}
return ans;
}
};

```

## 2.10 Vizing's theorem

```

namespace vizing { // returns edge coloring in adjacent
matrix G. 1 - based
int C[kN][kN], G[kN][kN];
void clear(int N) {
    for (int i = 0; i <= N; i++) {
        for (int j = 0; j <= N; j++) C[i][j] = G[i][j] =
            0;
    }
}
void solve(vector<pair<int, int>> &E, int N, int M) {
    int X[kN] = {}, a;
    auto update = [&](int u) {
        for (X[u] = 1; C[u][X[u]]; X[u]++);
    };
    auto color = [&](int u, int v, int c) {
        int p = G[u][v];
        G[u][v] = G[v][u] = c;
        C[u][c] = v, C[v][c] = u;
        C[u][p] = C[v][p] = 0;
        if (p) X[u] = X[v] = p;
        else update(u), update(v);
        return p;
    };
    auto flip = [&](int u, int c1, int c2) {
        int p = C[u][c1];
        swap(C[u][c1], C[u][c2]);
        if (p) G[u][p] = G[p][u] = c2;
        if (!C[u][c1]) X[u] = c1;
        if (!C[u][c2]) X[u] = c2;
        return p;
    };
    for (int i = 1; i <= N; i++) X[i] = 1;
    for (int t = 0; t < E.size(); t++) {
        int u = E[t].first, v0 = E[t].second, v = v0, c0
            = X[u], c = c0, d;
        vector<pair<int, int>> L;
        int vst[kN] = {};
        while (!G[u][v0]) {
            L.emplace_back(v, d = X[v]);
            if (!C[v][c]) for (a = (int)L.size() - 1; a >=
                0; a--) c = color(u, L[a].first, c);
            else if (!C[u][d]) for (a = (int)L.size() - 1;
                a >= 0; a--) color(u, L[a].first, L[a].
                second);
            else if (vst[d]) break;
            else vst[d] = 1, v = C[u][d];
        }
        if (!G[u][v0]) {
            for (; v; v = flip(v, c, d), swap(c, d));
            if (C[u][c0]) {
                for (a = (int)L.size() - 2; a >= 0 && L[a].
                    second != c; a--);
                for (; a >= 0; a--) color(u, L[a].first, L[a]
                    .second);
            } else t--;
        }
    }
}
}
}

```

## 2.11 Theory

$|\text{Maximum independent edge set}| = |V| - |\text{Minimum edge cover}|$

$|\text{Maximum independent set}| = |V| - |\text{Minimum vertex cover}|$

A sequence of non-negative integers  $d_1 \geq \dots \geq d_n$  can be represented as the degree sequence of a finite simple graph

on  $n$  vertices if and only if  $d_1 + \dots + d_n$  is even and  $\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$  holds for every  $k$  in  $1 \leq k \leq n$ .

## 3 Data Structure

### 3.1 Leftist Tree

```

struct node{
    ll v,data,sz,sum;
    node *l,*r;
    node(ll k):v(0),data(k),sz(1),l(0),r(0),sum(k){}
};
ll sz(node *p){return p ? p->sz : 0;}
ll V(node *p){return p ? p->v : -1;}
ll sum(node *p){return p ? p->sum : 0;}
node* merge(node *a,node *b){
    if(!a || !b) return a ? a : b;
    if(a->data<b->data) swap(a,b);
    a->r=merge(a->r,b);
    if(V(a->r)>V(a->l)) swap(a->r,a->l);
    a->v=V(a->r)+1,a->sz=sz(a->l)+sz(a->r)+1;
    a->sum=sum(a->l)+sum(a->r)+a->data;
    return a;
}
void pop(node *&o){
    node *tmp=o;
    o=merge(o->l,o->r);
    delete tmp;
}

```

### 3.2 Heavy light Decomposition

```

struct Heavy_light_Decomposition{//1-base
    int n,ulink[10005],deep[10005],mxson[10005],w[10005],
        pa[10005];
    int t,pl[10005],data[10005],dt[10005],bln[10005],edge
        [10005],et;
    vector<pii> G[10005];
    void init(int _n){n=_n,t=0,et=1;
        for(int i=1;i<=n;++i) G[i].clear(),mxson[i]=0;
    }
    void add_edge(int a,int b,int w){
        G[a].pb(pii(b,et)),G[b].pb(pii(a,et)),edge[et++]=w;
    }
    void dfs(int u,int f,int d){
        w[u]=1,pa[u]=f,deep[u]=d++;
        for(auto &i:G[u])
            if(i.X!=f){
                dfs(i.X,u,d),w[u]+=w[i.X];
                if(w[mxson[u]]<w[i.X])
                    mxson[u]=i.X;
            }
        else
            bln[i.Y]=u,dt[u]=edge[i.Y];
    }
    void cut(int u,int link){
        data[pl[u]=t++]=dt[u],ulink[u]=link;
        if(!mxson[u]) return;
        cut(mxson[u],link);
        for(auto i:G[u])
            if(i.X!=pa[u]&&i.X!=mxson[u])
                cut(i.X,i.X);
    }
    void build(){
        dfs(1,1,1),cut(1,1),/*build*/;
    }
    int query(int a,int b){
        int ta=ulink[a],tb=ulink[b],re=0;
        while(ta!=tb)
            if(deep[ta]<deep[tb])
                /*query*/,tb=ulink[b=pa[tb]];
            else
                /*query*/,ta=ulink[a=pa[ta]];
        if(a==b) return re;
        if(pl[a]>pl[b]) swap(a,b);
    }
}

```

```

    /*query*/
    return re;
}
};

```

### 3.3 Smart Pointer

```

#ifndef REFERENCE_POINTER
#define REFERENCE_POINTER
template<typename T>
struct _RefCounter{
    T data;
    int ref;
    _RefCounter(const T&d=0):data(d),ref(0){}
};
template<typename T>
struct reference_pointer{
    _RefCounter<T> *p;
    T *operator->(){return &p->data;}
    T &operator*(){return p->data;}
    operator _RefCounter<T>*(){return p;}
    reference_pointer &operator=(const reference_pointer
        &t){
        if(p&&!--p->ref)delete p;
        p=t.p;
        p&&+p->ref;
        return *this;
    }
    reference_pointer(_RefCounter<T> *t=0):p(t){
        p&&+p->ref;
    }
    reference_pointer(const reference_pointer &t):p(t.p){
        p&&+p->ref;
    }
    ~reference_pointer(){
        if(p&&!--p->ref)delete p;
    }
};
template<typename T>
inline reference_pointer<T> new_reference(const T&nd){
    return reference_pointer<T>(new _RefCounter<T>(nd));
}
#endif
//note:
reference_pointer<int> a;
a = new_reference(5);
a = new_reference<int>(5);
a = new_reference((int)5);
reference_pointer<int> b = a;

struct P{
    int a,b;
    P(int _a,int _b):a(_a),b(_b){}
}p(2,3);
reference_pointer<P> a;
c = new_reference(P(1,2));
c = new_reference<P>(P(1,2));
c = new_reference(p);

```

### 3.4 LiChaoST

```

struct LiChao_min{
    struct line{
        LL m, c;
        line(LL _m=0, LL _c=0) { m = _m; c = _c; }
        LL eval(LL x) { return m * x + c; }
    };
    struct node{
        node *l, *r; line f;
        node(line v) { f = v; l = r = NULL; }
    };
    typedef node* pnode;
    pnode root; int sz;
#define mid ((l+r)>>1)
    void insert(line &v, int l, int r, pnode &nd){
        if(!nd) { nd = new node(v); return; }
        LL trl = nd->f.eval(l), trr = nd->f.eval(r);
        LL vl = v.eval(l), vr = v.eval(r);

```

```

        if(trl <= vl && trr <= vr) return;
        if(trl > vl && trr > vr) { nd->f = v; return; }
        if(trl > vl) swap(nd->f, v);
        if(nd->f.eval(mid) < v.eval(mid)) insert(v, mid +
            1, r, nd->r);
        else swap(nd->f, v), insert(v, l, mid, nd->l);
    }
    LL query(int x, int l, int r, pnode &nd){
        if(!nd) return LLONG_MAX;
        if(l == r) return nd->f.eval(x);
        if(mid >= x) return min(nd->f.eval(x), query(x, l,
            mid, nd->l));
        return min(nd->f.eval(x), query(x, mid + 1, r, nd->
            r));
    }
}
/* -sz <= query_x <= sz */
void init(int _sz){ sz = _sz + 1; root = NULL; }
void add_line(LL m, LL c){ line v(m, c); insert(v, -
    sz, sz, root); }
LL query(LL x) { return query(x, -sz, sz, root); }
};

```

### 3.5 link cut tree

```

const int MXN = 100005;
const int MEM = 100005;
struct Splay {
    static Splay nil, mem[MEM], *pmem;
    Splay *ch[2], *f;
    int val, rev, size;
    Splay (int _val=-1) : val(_val), rev(0), size(1)
    { f = ch[0] = ch[1] = &nil; }
    bool isr()
    { return f->ch[0] != this && f->ch[1] != this; }
    int dir()
    { return f->ch[0] == this ? 0 : 1; }
    void setCh(Splay *c, int d){
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void push(){
        if( !rev ) return;
        swap(ch[0], ch[1]);
        if (ch[0] != &nil) ch[0]->rev ^= 1;
        if (ch[1] != &nil) ch[1]->rev ^= 1;
        rev=0;
    }
    void pull(){
        size = ch[0]->size + ch[1]->size + 1;
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::
    mem;
Splay *nil = &Splay::nil;
void rotate(Splay *x){
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[!d], d);
    x->setCh(p, !d);
    p->pull(); x->pull();
}
vector<Splay*> splayVec;
void splay(Splay *x){
    splayVec.clear();
    for (Splay *q=x; q=q->f){
        splayVec.push_back(q);
        if (q->isr()) break;
    }
    reverse(begin(splayVec), end(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir()==x->f->dir())
            rotate(x->f), rotate(x);
        else rotate(x), rotate(x);
    }
}

```



```

}
int id(Splay *x) { return x - Splay::mem + 1; }
Splay* access(Splay *x){
    Splay *q = nil;
    for (;x!=nil;x=x->f){
        splay(x);
        x->setCh(q, 1);
        q = x;
    }
    return q;
}
void chroot(Splay *x){
    access(x);
    splay(x);
    x->rev ^= 1;
    x->push(); x->pull();
}
void link(Splay *x, Splay *y){
    access(x);
    splay(x);
    chroot(y);
    x->setCh(y, 1);
}
void cut_p(Splay *y) {
    access(y);
    splay(y);
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}
void cut(Splay *x, Splay *y){
    chroot(x);
    cut_p(y);
}
Splay* get_root(Splay *x) {
    access(x);
    splay(x);
    for(; x->ch[0] != nil; x = x->ch[0])
        x->push();
    splay(x);
    return x;
}
bool conn(Splay *x, Splay *y) {
    x = get_root(x);
    y = get_root(y);
    return x == y;
}
Splay* lca(Splay *x, Splay *y) {
    access(x);
    access(y);
    splay(x);
    if (x->f == nil) return x;
    else return x->f;
}
}

```

### 3.6 KDTree

```

template<typename T,size_t kd>//kd???????
class kd_tree{
public:
    struct point{
        T d[kd];
        inline T dist(const point &x)const{
            T ret=0;
            for(size_t i=0;i<kd;++i)ret+=std::abs(d[i]-x.d[i]);
            return ret;
        }
        inline bool operator==(const point &p){
            for(size_t i=0;i<kd;++i){
                if(d[i]!=p.d[i])return 0;
            }
            return 1;
        }
        inline bool operator<(const point &b)const{
            return d[0]<b.d[0];
        }
    };
private:
    struct node{
        node *l,*r;

```

```

        point pid;
        int s;
        node(const point &p):l(0),r(0),pid(p),s(1){}
        inline void up(){
            s=(l?l->s:0)+1+(r?r->s:0);
        }
    }*root;
    const double alpha,loga;
    const T INF;//????INF,????
    int maxn;
    struct __cmp{
        int sort_id;
        inline bool operator()(const node*x,const node*y)
            const{
            return operator()(x->pid,y->pid);
        }
        inline bool operator()(const point &x,const point
            &y)const{
            if(x.d[sort_id]!=y.d[sort_id])
                return x.d[sort_id]<y.d[sort_id];
            for(size_t i=0;i<kd;++i){
                if(x.d[i]!=y.d[i])return x.d[i]<y.d[i];
            }
            return 0;
        }
    }cmp;
    void clear(node *o){
        if(!o)return;
        clear(o->l);
        clear(o->r);
        delete o;
    }
    inline int size(node *o){
        return o?o->s:0;
    }
    std::vector<node*> A;
    node* build(int k,int l,int r){
        if(l>r)return 0;
        if(k==kd)k=0;
        int mid=(l+r)/2;
        cmp.sort_id=k;
        std::nth_element(A.begin()+l,A.begin()+mid,A.
            begin()+r+1,cmp);
        node *ret=A[mid];
        ret->l=build(k+1,l,mid-1);
        ret->r=build(k+1,mid+1,r);
        ret->up();
        return ret;
    }
    inline bool isbad(node*o){
        return size(o->l)>alpha*o->s||size(o->r)>alpha*o->s;
    }
    void flatten(node *u,typename std::vector<node*>::
        iterator &it){
        if(!u)return;
        flatten(u->l,it);
        *it=u;
        flatten(u->r,++it);
    }
    inline void rebuild(node*&u,int k){
        if((int)A.size()<u->s)A.resize(u->s);
        typename std::vector<node*>::iterator it=A.begin
            ();
        flatten(u,it);
        u=build(k,0,u->s-1);
    }
    bool insert(node*&u,int k,const point &x,int dep){
        if(!u){
            u=new node(x);
            return dep<=0;
        }
        ++u->s;
        cmp.sort_id=k;
        if(insert(cmp(x,u->pid)?u->l:u->r,(k+1)%kd,x,dep
            -1)){
            if(!isbad(u))return 1;
            rebuild(u,k);
        }
        return 0;
    }
    node *findmin(node*o,int k){

```

```

    if(!o)return 0;
    if(cmp.sort_id==k)return o->l?findmin(o->l,(k+1)%kd):o;
    node *l=findmin(o->l,(k+1)%kd);
    node *r=findmin(o->r,(k+1)%kd);
    if(l&&!r)return cmp(l,o)?l:o;
    if(!l&&r)return cmp(r,o)?r:o;
    if(!l&&!r)return o;
    if(cmp(l,r))return cmp(l,o)?l:o;
    return cmp(r,o)?r:o;
}
bool erase(node *&u,int k,const point &x){
    if(!u)return 0;
    if(u->pid==x){
        if(u->r);
        else if(u->l){
            u->r=u->l;
            u->l=0;
        }else{
            delete u;
            u=0;
            return 1;
        }
        --u->s;
        cmp.sort_id=k;
        u->pid=findmin(u->r,(k+1)%kd)->pid;
        return erase(u->r,(k+1)%kd,u->pid);
    }
    cmp.sort_id=k;
    if(erase(cmp(x,u->pid)?u->l:u->r,(k+1)%kd,x)){
        --u->s;return 1;
    }else return 0;
}
inline T heuristic(const T h[])const{
    T ret=0;
    for(size_t i=0;i<kd;++i)ret+=h[i];
    return ret;
}
int qM;
std::priority_queue<std::pair<T,point > >pQ;
void nearest(node *u,int k,const point &x,T *h,T &mndist){
    if(u==0||heuristic(h)>=mndist)return;
    T dist=u->pid.dist(x),old=h[k];
    /*mndist=std::min(mndist,dist);*/
    if(dist<mndist){
        pQ.push(std::make_pair(dist,u->pid));
        if((int)pQ.size()==qM+1){
            mndist=pQ.top().first,pQ.pop();
        }
    }
    if(x.d[k]<u->pid.d[k]){
        nearest(u->l,(k+1)%kd,x,h,mndist);
        h[k]=std::abs(x.d[k]-u->pid.d[k]);
        nearest(u->r,(k+1)%kd,x,h,mndist);
    }else{
        nearest(u->r,(k+1)%kd,x,h,mndist);
        h[k]=std::abs(x.d[k]-u->pid.d[k]);
        nearest(u->l,(k+1)%kd,x,h,mndist);
    }
    h[k]=old;
}
std::vector<point>in_range;
void range(node *u,int k,const point&mi,const point &ma){
    if(!u)return;
    bool is=1;
    for(int i=0;i<kd;++i)
        if(u->pid.d[i]<mi.d[i]||ma.d[i]<u->pid.d[i]){
            is=0;break;
        }
    if(is)in_range.push_back(u->pid);
    if(mi.d[k]<u->pid.d[k])range(u->l,(k+1)%kd,mi,ma);
    if(ma.d[k]>u->pid.d[k])range(u->r,(k+1)%kd,mi,ma);
}
public:
kd_tree(const T &INF,double a=0.75):root(0),alpha(a),loga(log2(1.0/a)),INF(INF),maxn(1){}
inline void clear(){
    clear(root),root=0,maxn=1;
}

```

```

}
inline void build(int n,const point *p){
    clear(root),A.resize(maxn=n);
    for(int i=0;i<n;++i)A[i]=new node(p[i]);
    root=build(0,0,n-1);
}
inline void insert(const point &x){
    insert(root,0,x,std::lg(size(root))/loga);
    if(root->s>maxn)maxn=root->s;
}
inline bool erase(const point &p){
    bool d=erase(root,0,p);
    if(root&&root->s<alpha*maxn)rebuild();
    return d;
}
inline void rebuild(){
    if(root)rebuild(root,0);
    maxn=root->s;
}
inline T nearest(const point &x,int k){
    qM=k;
    T mndist=INF,h[kd]={};
    nearest(root,0,x,h,mndist);
    mndist=pQ.top().first;
    pQ=std::priority_queue<std::pair<T,point > >();
    return mndist;/*???x?k????*/
}
inline const std::vector<point> &range(const point&mi,const point&ma){
    in_range.clear();
    range(root,0,mi,ma);
    return in_range;/*???mi?ma???vector*/
}
inline int size(){return root?root->s:0;}
};

```

## 4 Flow/Matching

### 4.1 Dinic

```

struct MaxFlow{//0-base
    struct edge{
        int to,cap,flow,rev;
    };
    vector<edge> G[MAXN];
    int s,t,dis[MAXN],cur[MAXN],n;
    int dfs(int u,int cap){
        if(u==t||!cap)return cap;
        for(int &i=cur[u];i<(int)G[u].size();++i){
            edge &e=G[u][i];
            if(dis[e.to]==dis[u]+1&&e.flow!=e.cap){
                int df=dfs(e.to,min(e.cap-e.flow,cap));
                if(df){
                    e.flow+=df;
                    G[e.to][e.rev].flow-=df;
                    return df;
                }
            }
        }
        dis[u]=-1;
        return 0;
    }
    bool bfs(){
        FILL(dis,-1);
        queue<int> q;
        q.push(s),dis[s]=0;
        while(!q.empty()){
            int tmp=q.front();
            q.pop();
            for(auto &u:G[tmp])
                if(!dis[u.to]&&u.flow!=u.cap){
                    q.push(u.to);
                    dis[u.to]=dis[tmp]+1;
                }
        }
        return dis[t]!=-1;
    }
    int maxflow(int _s,int _t){

```



```

s=_s,t=_t;
int flow=0,df;
while(bfs()){
    FILL(cur,0);
    while(df=dfs(s,INF)) flow+=df;
}
return flow;
}
void init(int _n){n=_n;
for(int i=0;i<n;++i) G[i].clear();
}
void reset(){
for(int i=0;i<n;++i)
for(auto &j:G[i])
j.flow=0;
}
void add_edge(int u,int v,int cap){
G[u].pb(edge{v,cap,0,(int)G[v].size()});
G[v].pb(edge{u,0,0,(int)G[u].size()-1});
}
};

```

## 4.2 Kuhn Munkres

```

struct KM{// 0-base
int w[MAXN][MAXN],h1[MAXN],hr[MAXN],slk[MAXN],n;
int fl[MAXN],fr[MAXN],pre[MAXN],qu[MAXN],ql,qr;
bool vl[MAXN],vr[MAXN];
void init(int _n){n=_n;
for(int i=0;i<n;++i)
for(int j=0;j<n;++j)
w[i][j]=-INF;
}
void add_edge(int a,int b,int wei){
w[a][b]=wei;
}
bool Check(int x){
if(vl[x]=1,~fl[x]) return vr[qu[qr++]=fl[x]]=1;
while(~x) swap(x,fr[fl[x]=pre[x]]);
return 0;
}
void Bfs(int s){
fill(slk,slk+n,INF);
fill(vl,vl+n,0),fill(vr,vr+n,0);
ql=qr=0,qu[qr++]=s,vr[s]=1;
while(1){
int d;
while(ql<qr)
for(int x=0,y=qu[ql++];x<n;++x)
if(!vl[x]&&slk[x]>=(d=h1[x]+hr[y]-w[x][y]))
if(pre[x]=y,d) slk[x]=d;
else if(!Check(x)) return;
d=INF;
for(int x=0;x<n;++x)
if(!vl[x]&&d>slk[x]) d=slk[x];
for(int x=0;x<n;++x){
if(vl[x]) h1[x]+=d;
else slk[x]-=d;
if(vr[x]) hr[x]-=d;
}
for(int x=0;x<n;++x)
if(!vl[x]&&slk[x]&&!Check(x)) return;
}
}
int Solve(){
fill(fl,fl+n,-1),fill(fr,fr+n,-1),fill(hr,hr+n,0);
for(int i=0;i<n;++i) h1[i]=*max_element(w[i],w[i]+n);
for(int i=0;i<n;++i) Bfs(i);
int res=0;
for(int i=0;i<n;++i) res += w[i][fl[i]];
return res;
}
};

```

## 4.3 MincostMaxflow

```

struct MCMF{//0-base
struct edge{
ll from,to,cap,flow,cost,rev;
}*past[MAXN];
vector<edge> G[MAXN];
bitset<MAXN> inq;
ll dis[MAXN],up[MAXN],s,t,mx,n;
bool BellmanFord(ll &flow,ll &cost){
fill(dis,dis+n,INF);
queue<ll> q;
q.push(s),inq.reset(),inq[s]=1;
up[s]=mx-flow,past[s]=0,dis[s]=0;
while(!q.empty()){
ll u=q.front();
q.pop(),inq[u]=0;
if(!up[u]) continue;
for(auto &e:G[u])
if(e.flow!=e.cap&&dis[e.to]>dis[u]+e.cost){
dis[e.to]=dis[u]+e.cost,past[e.to]=&e;
up[e.to]=min(up[u],e.cap-e.flow);
if(!inq[e.to]) inq[e.to]=1,q.push(e.to);
}
}
if(dis[t]==INF) return 0;
flow+=up[t],cost+=up[t]*dis[t];
for(ll i=t;past[i];i=past[i]->from){
auto &e=*past[i];
e.flow+=up[t],G[e.to][e.rev].flow-=up[t];
}
return 1;
}
ll MinCostMaxFlow(ll _s,ll _t,ll &cost){
s=_s,t=_t,cost=0;ll flow=0;
while(BellmanFord(flow,cost));
return flow;
}
void init(ll _n,ll _mx){n=_n,mx=_mx;
for(int i=0;i<n;++i) G[i].clear();
}
void add_edge(ll a,ll b,ll cap,ll cost){
G[a].pb(edge{a,b,cap,0,cost,G[b].size()});
G[b].pb(edge{b,a,0,0,-cost,G[a].size()-1});
}
};

```

## 4.4 Maximum Simple Graph Matching

```

struct GenMatch{//1-base
int V,pr[MAXN];
bool el[MAXN][MAXN];
bool inq[MAXN],inp[MAXN],inb[MAXN];
queue<int> qe;
int st,ed,nb,bk[MAXN],djs[MAXN],ans;
void init(int _V){V=_V;
for(int i=0;i<V;++i){
for(int j=0;j<V;++j)
el[i][j]=0;
pr[i]=bk[i]=djs[i]=0;
inq[i]=inp[i]=inb[i]=0;
}
ans=0;
}
void add_edge(int u,int v){
el[u][v]=el[v][u]=1;
}
int lca(int u,int v){
for(int i=0;i<V;++i) inp[i]=0;
while(1)
if(u=djs[u],inp[u]=true,u==st) break;
else u=bk[pr[u]];
while(1)
if(v=djs[v],inp[v]) return v;
else v=bk[pr[v]];
return v;
}
void upd(int u){
int v;
while(djs[u]!=nb){
v=pr[u],inb[djs[u]]=inb[djs[v]]=true;
u=bk[v];
}
}
}

```

```

    if(djs[u]!=nb) bk[u]=v;
}
}
void blo(int u,int v){
    nb=lca(u,v);
    for (int i=0;i<V;++i) inb[i]=0;
    upd(u),upd(v);
    if(djs[u]!=nb) bk[u]=v;
    if(djs[v]!=nb) bk[v]=u;
    for(int tu=1;tu<=V;++tu)
        if(inb[djs[tu]])
            if(djs[tu]==nb,!inq[tu])
                qe.push(tu),inq[tu]=1;
}
void flow(){
    for(int i=1;i<=V;++i)
        inq[i]=0,bk[i]=0,djs[i]=i;
    while(qe.size()) qe.pop();
    qe.push(st),inq[st]=1,ed=0;
    while(qe.size()){
        int u = qe.front();
        qe.pop();
        for(int v=1;v<=V;++v)
            if(e1[u][v]&&djs[u]!=djs[v]&&pr[u]!=v)
                if((v==st)||pr[v]>0&&bk[pr[v]]>0)
                    blo(u,v);
            else if(!bk[v])
                if(bk[v]=u,pr[v]>0)
                    if(!inq[pr[v]])
                        qe.push(pr[v]);
            else;
            else
                return ed=v,void();
    }
}
void aug(){
    for(int u=ed,v,w;u>0;){
        v=bk[u],w=pr[v],pr[v]=u,pr[u]=v,u=w;
    }
}
int solve(){
    for(int i=0;i<=V;++i) pr[i]=0;
    for(int u=1;u<=V;++u)
        if(!pr[u])
            if(st=u,flow(),ed>0)
                aug(),++ans;
    return ans;
}
};

```

#### 4.5 Minimum Weight Matching (Clique version)

```

struct Graph{//0-base (Perfect Match)
    int n,edge[MAXN][MAXN];
    int match[MAXN],dis[MAXN],onstk[MAXN];
    vector<int> stk;
    void init(int _n){n=_n;
        for(int i=0;i<n;++i)
            for(int j=0;j<n;++j)
                edge[i][j]=0;
    }
    void add_edge(int u,int v,int w){
        edge[u][v]=edge[v][u]=w;
    }
    bool SPFA(int u){
        if(onstk[u]) return 1;
        stk.pb(u),onstk[u]=1;
        for(int v=0;v<n;++v)
            if(u!=v&&match[u]!=v&&!onstk[v]){
                int m=match[v];
                if(dis[m]>dis[u]-edge[v][m]+edge[u][v]){
                    dis[m]=dis[u]-edge[v][m]+edge[u][v];
                    onstk[v]=1,stk.pb(v);
                    if(SPFA(m)) return 1;
                    stk.pop_back(),onstk[v]=0;
                }
            }
        onstk[u]=0,stk.pop_back();
        return 0;
    }
};

```

```

}
int solve(){// find a match
    for(int i=0;i<n;i+=2)
        match[i]=i+1,match[i+1]=i;
    while(1){
        int found=0;
        for(int i=0;i<n;++i) dis[i]=onstk[i]=0;
        for(int i=0;i<n;++i)
            if(stk.clear(),!onstk[i]&&SPFA(i))
                for(found=1;stk.size()>=2;){
                    int u=stk.back();
                    stk.pop_back();
                    int v=stk.back();
                    stk.pop_back();
                    match[u]=v,match[v]=u;
                }
            if(!found) break;
    }
    int ret=0;
    for(int i=0;i<n;++i) ret+=edge[i][match[i]];
    return ret>>1;
}
};

```

#### 4.6 SW-mincut

```

// global min cut
struct SW{ // O(V^3)
    static const int MXN = 514;
    int n,vst[MXN],del[MXN];
    int edge[MXN][MXN],wei[MXN];
    void init(int _n){
        n=_n,MEM(edge,0),MEM(del,0);
    }
    void addEdge(int u,int v,int w){
        edge[u][v]+=w,edge[v][u]+=w;
    }
    void search(int &s,int &t){
        MEM(vst,0),MEM(wei,0),s=t=-1;
        while(1){
            int mx=-1,cur=0;
            for(int i=0;i<n;++i)
                if(!del[i]&&!vst[i]&&mx<wei[i])
                    cur=i,mx=wei[i];
            if(mx==-1) break;
            vst[cur]=1,s=t,cur;
            for(int i=0;i<n;++i)
                if(!vst[i]&&!del[i]) wei[i]+=edge[cur][i];
        }
    }
    int solve(){
        int res=INF;
        for(int i=0,x,y;i<n-1;++i){
            search(x,y),res=min(res,wei[y]),del[y]=1;
            for(int j=0;j<n;++j)
                edge[x][j]=(edge[j][x]+edge[y][j]);
        }
        return res;
    }
};

```

#### 4.7 BoundedFlow

```

struct BoundedFlow{//0-base
    struct Edge{
        int to, cap, flow, rev;
    };
    vector<Edge> G[MAXN];
    int n,s,t,dis[MAXN],cur[MAXN],cnt[MAXN];
    void init(int _n){n=_n;
        for(int i=0;i<n+2;++i) G[i].clear(),cnt[i]=0;
    }
    void add_edge(int u,int v,int lcap,int rcap){
        cnt[u]-=lcap,cnt[v]+=lcap;
        G[u].pb(Edge{v,rcap,lcap,(int)G[v].size()});
        G[v].pb(Edge{u,0,0,(int)G[u].size()-1});
    }
    void add_edge(int u,int v,int cap){

```

```

G[u].pb(Edge{v,cap,0,(int)G[v].size()});
G[v].pb(Edge{u,0,0,(int)G[u].size()-1});
}
int dfs(int u,int cap){
    if(u==t || !cap) return cap;
    for(int &i=cur[u];i<(int)G[u].size();++i){
        Edge &e=G[u][i];
        if(dis[e.to]==dis[u]+1 && e.flow!=e.cap){
            int df=dfs(e.to,min(e.cap-e.flow,cap));
            if(df){
                e.flow+=df;
                G[e.to][e.rev].flow-=df;
                return df;
            }
        }
    }
    dis[u]=-1;
    return 0;
}
bool bfs(){
    FILL(dis,-1);
    queue<int> q;
    q.push(s),dis[s]=0;
    while(!q.empty()){
        int tmp=q.front();
        q.pop();
        for(auto &u:G[tmp])
            if(!~dis[u.to] && u.flow!=u.cap){
                q.push(u.to);
                dis[u.to]=dis[tmp]+1;
            }
    }
    return dis[t]!=-1;
}
int maxflow(int _s,int _t){
    s=_s,t=_t;
    int flow=0,df;
    while(bfs()){
        FILL(cur,0);
        while(df=dfs(s,INF)) flow+=df;
    }
    return flow;
}
bool solve(){
    int sum=0;
    for(int i=0;i<n;++i)
        if(cnt[i]>0) add_edge(n+1,i,cnt[i],sum+=cnt[i];
        else if(cnt[i]<0) add_edge(i,n+2,-cnt[i]);
    if(sum!=maxflow(n+1,n+2)) sum=-1;
    for(int i=0;i<n;++i)
        if(cnt[i]>0) G[n+1].pop_back(),G[i].pop_back();
        else if(cnt[i]<0) G[i].pop_back(),G[n+2].pop_back();
    return sum!=-1;
}
int solve(int _s,int _t){
    add_edge(_t,_s,INF);
    if(!solve()) return -1; //invalid flow
    int x=G[_t].back().flow;
    return G[_t].pop_back(),G[_s].pop_back(),x;
}
};

```

## 4.8 Gomory Hu tree

```

struct Gomory_Hu_tree{//0-base
    MaxFlow Dinic;
    int n;
    vector<pii> G[MAXN];
    void init(int _n){n=_n;
        for(int i=0;i<n;++i) G[i].clear();
    }
    void solve(vector<int> &v){
        if(v.size()<=1) return;
        int s=rand()%SZ(v);
        swap(v.back(),v[s]),s=v.back();
        int t=v[rand()%SZ(v)-1];
        vector<int> L,R;
        int x=(Dinic.reset(),Dinic.maxflow(s,t));
        G[s].pb(pii(t,x)),G[t].pb(pii(s,x));
    }
};

```

```

for(int i:v)
    if(~Dinic.dis[i]) L.pb(i);
    else R.pb(i);
solve(L),solve(R);
}
void build(){
    vector<int> v(n);
    for(int i=0;i<n;++i) v[i]=i;
    solve(v);
}
}glt; //test by BZOJ 4519
MaxFlow &Dinic=glt.Dinic;

```

## 4.9 NumberofMaximalClique

```

// bool g[][] : adjacent array indexed from 1 to n
void dfs(int sz){
    int i, j, k, t, cnt, best = 0;
    if(ne[sz]==ce[sz]){ if (ce[sz]==0) ++ans; return; }
    for(t=0, i=1; i<=ne[sz]; ++i){
        for (cnt=0, j=ne[sz]+1; j<=ce[sz]; ++j)
            if (!g[lst[sz][i]][lst[sz][j]]) ++cnt;
        if (t==0 || cnt<best) t=i, best=cnt;
    } if (t && best<=0) return;
    for (k=ne[sz]+1; k<=ce[sz]; ++k) {
        if (t>0){ for (i=k; i<=ce[sz]; ++i)
            if (!g[lst[sz][t]][lst[sz][i]]) break;
            swap(lst[sz][k], lst[sz][i]);
        } i=lst[sz][k]; ne[sz+1]=ce[sz+1]=0;
        for (j=1; j<k; ++j)if (g[i][lst[sz][j]])
            lst[sz+1][++ne[sz+1]]=lst[sz][j];
        for (ce[sz+1]=ne[sz+1], j=k+1; j<=ce[sz]; ++j)
            if (g[i][lst[sz][j]]) lst[sz+1][++ce[sz+1]]=lst[sz][j];
        dfs(sz+1); ++ne[sz]; --best;
        for (j=k+1, cnt=0; j<=ce[sz]; ++j) if (!g[i][lst[sz][j]]) ++cnt;
        if (t==0 || cnt<best) t=k, best=cnt;
        if (t && best<=0) break;
    }
}
void work(){
    ne[0]=0; ce[0]=0;
    for(int i=1; i<=n; ++i) lst[0][++ce[0]]=i;
    ans=0; dfs(0);
}

```

## 4.10 isap

```

struct Maxflow {
    static const int MAXV = 20010;
    static const int INF = 1000000;
    struct Edge {
        int v, c, r;
        Edge(int _v, int _c, int _r):
            v(_v), c(_c), r(_r) {}
    };
    int s, t;
    vector<Edge> G[MAXV*2];
    int iter[MAXV*2], d[MAXV*2], gap[MAXV*2], tot;
    void init(int x) {
        tot = x+2;
        s = x+1, t = x+2;
        for(int i = 0; i <= tot; i++) {
            G[i].clear();
            iter[i] = d[i] = gap[i] = 0;
        }
    }
    void addEdge(int u, int v, int c) {
        G[u].push_back(Edge(v, c, SZ(G[v])));
        G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
    }
    int dfs(int p, int flow) {
        if(p == t) return flow;
        for(int &i = iter[p]; i < SZ(G[p]); i++) {
            Edge &e = G[p][i];
            if(e.c > 0 && d[p] == d[e.v]+1) {
                int f = dfs(e.v, min(flow, e.c));
                if(f) {

```

```

        e.c -= f;
        G[e.v][e.r].c += f;
        return f;
    }
}
if( (--gap[d[p]]) == 0) d[s] = tot;
else {
    d[p]++;
    iter[p] = 0;
    ++gap[d[p]];
}
return 0;
}
int solve() {
    int res = 0;
    gap[0] = tot;
    for(res = 0; d[s] < tot; res += dfs(s, INF));
    return res;
}
} flow;

```

## 5 String

### 5.1 KMP

```

int F[MAXN];
vector<int> match(string A, string B){
    vector<int> ans;
    F[0]=-1, F[1]=0;
    for(int i=1, j=0; i<B.size(); F[++i]=++j){
        if(B[i]==B[j]) F[i]=F[j]; //optimize
        while(j!=-1 && B[i]!=B[j]) j=F[j];
    }
    for(int i=0, j=0; i-j+B.size()<=A.size(); ++i, ++j){
        while(j!=-1 && A[i]!=B[j]) j=F[j];
        if(j==B.size()-1) ans.pb(i-j);
    }
    return ans;
}

```

### 5.2 Z-value

```

const int MAXn = 1e5 + 5;
int z[MAXN];
void make_z(string s){
    int l = 0, r = 0;
    for(int i = 1; i < s.size(); i++){
        for(z[i] = max(0, min(r - i + 1, z[i - 1]));
            i + z[i] < s.size() && s[i + z[i]] == s[z[i]]; z
                [i]++);
        if(i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
}

```

### 5.3 Manacher

```

int z[MAXN];
int Manacher(string tmp){
    string s = "&";
    int l=0, r=0, x, ans;
    for(char c:tmp) s.pb(c), s.pb('%');
    ans=0, x=0;
    for(int i=1; i<SZ(s); ++i){
        z[i]=r > i ? min(z[2*i-i], r-i) : 1;
        while(s[i+z[i]]==s[i-z[i]]) ++z[i];
        if(z[i]+i>r) r=z[i]+i, l=i;
    }
    for(int i=1; i<SZ(s); ++i)
        if(s[i]=='%')
            x=max(x, z[i]);
    ans=x/2*2, x=0;
    for(int i=1; i<SZ(s); ++i)
        if(s[i]!='%')

```

```

        x=max(x, z[i]);
    return max(ans, (x-1)/2*2+1);
}

```

### 5.4 Suffix Array

```

struct suffix_array{
    int box[MAXN], tp[MAXN], m;
    bool not_equ(int a, int b, int k, int n){
        return ra[a]!=ra[b] || a+k>n || b+k>n || ra[a+k]!=ra[b+k];
    }
    void radix(int *key, int *it, int *ot, int n){
        fill_n(box, m, 0);
        for(int i=0; i<n; ++i) ++box[key[i]];
        partial_sum(box, box+m, box);
        for(int i=n-1; i>=0; --i) ot[--box[key[it[i]]]]=it[i];
    }
    void make_sa(string s, int n){
        int k=1;
        for(int i=0; i<n; ++i) ra[i]=s[i];
        do{
            iota(tp, tp+k, n-k), iota(sa+k, sa+n, 0);
            radix(ra+k, sa+k, tp+k, n-k);
            radix(ra, tp, sa, n);
            tp[sa[0]]=0, m=1;
            for(int i=1; i<n; ++i){
                m+=not_equ(sa[i], sa[i-1], k, n);
                tp[sa[i]]=m-1;
            }
            copy_n(tp, n, ra);
            k*=2;
        }while(k<n && m!=n);
    }
    void make_he(string s, int n){
        for(int j=0, k=0; j<n; ++j){
            if(ra[j])
                for(; s[j+k]==s[sa[ra[j]-1]+k]; ++k);
            he[ra[j]]=k, k=max(0, k-1);
        }
    }
    int sa[MAXN], ra[MAXN], he[MAXN];
    void build(string s){
        FILL(sa, 0), FILL(ra, 0), FILL(he, 0);
        FILL(box, 0), FILL(tp, 0), m=256;
        make_sa(s, s.size());
        make_he(s, s.size());
    }
};

```

### 5.5 SAIS

```

struct SA{
#define REP(i,n) for ( int i=0; i<int(n); i++ )
#define REP1(i,a,b) for ( int i=(a); i<=int(b); i++ )
    bool _t[MAXN*2];
    int _s[MAXN*2], _sa[MAXN*2], _c[MAXN*2], x[MAXN], _p[
        MAXN], _q[MAXN*2], hei[MAXN], r[MAXN];
    int operator [] (int i){ return _sa[i]; }
    void build(int *s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
    void mkhei(int n){
        REP(i,n) r[_sa[i]] = i;
        hei[0] = 0;
        REP(i,n) if(r[i]) {
            int ans = i>0 ? max(hei[r[i-1]] - 1, 0) : 0;
            while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans++;
            hei[r[i]] = ans;
        }
    }
    void sais(int *s, int *sa, int *p, int *q, bool *t,
        int *c, int n, int z){
        bool uniq = t[n-1] = true, neq;

```

```

    int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
        lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
    memcpy(x, c, sizeof(int) * z); \
    XD; \
    memcpy(x + 1, c, sizeof(int) * (z - 1)); \
    REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[s[sa[i]
        ]-1]]++ = sa[i]-1; \
    memcpy(x, c, sizeof(int) * z); \
    for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i]
        ]-1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
    MS0(c, z);
    REP(i,n) uniq &= ++c[s[i]] < 2;
    REP(i,z-1) c[i+1] += c[i];
    if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return; }
    for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s[i
        +1] ? t[i+1] : s[i]<s[i+1]);
    MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[--x[s[i
        ]]] = p[q[i]=nn++] = i);
    REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1]) {
        neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa
            [i])*sizeof(int));
        ns[q[lst=sa[i]]]=nmzx+=neq;
    }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx
        + 1);
    MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[s[p[
        nsa[i]]]]] = p[nsa[i]]);
}
}sa;
int H[ MAXN ], SA[ MAXN ];
void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // ip is int array, len is array length
    // ip[0..n-1] != 0, and ip[len] = 0
    ip[len++] = 0;
    sa.build(ip, len, 128);
    for (int i=0; i<len; i++) {
        H[i] = sa.hei[i + 1];
        SA[i] = sa._sa[i + 1];
    }
    // resulting height, sa array \in [0,len)
}

```

## 5.6 Aho-Corasick Automatan

```

const int len=400000,sigma=26;
struct AC_Automatan{
    int nx[len][sigma],fl[len],cnt[len],pri[len],top;
    int newnode(){
        fill(nx[top],nx[top]+sigma,-1);
        return top++;
    }
    void init(){top=1,newnode();}
    int input(string &s){//return the end_node of string
        int X=1;
        for(char c:s){
            if(!nx[X][c-'a'])nx[X][c-'a']=newnode();
            X=nx[X][c-'a'];
        }
        return X;
    }
    void make_fl(){
        queue<int> q;
        q.push(1),fl[1]=0;
        for(int t=0;!q.empty();){
            int R=q.front();
            q.pop(),pri[t++]=R;
            for(int i=0;i<sigma;++i)
                if(~nx[R][i]){
                    int X=nx[R][i],Z=f1[R];
                    for(;Z&&!~nx[Z][i];)Z=f1[Z];
                    fl[X]=Z?nx[Z][i]:1,q.push(X);
                }
        }
    }
    void get_v(string &s){
        int X=1;
        fill(cnt,cnt+top,0);
    }
}

```

```

for(char c:s){
    while(X&&!~nx[X][c-'a'])X=f1[X];
    X=X?nx[X][c-'a']:1,++cnt[X];
}
for(int i=top-2;i>0;--i) cnt[f1[pri[i]]]+=cnt[pri[i]
    ]];
}
};

```

## 5.7 Smallest Rotation

```

string mcp(string s){
    int n=SZ(s),i=0,j=1;
    s+=s;
    while(i<n&&j<n){
        int k=0;
        while(k<n&&s[i+k]==s[j+k]) ++k;
        if(s[i+k]<s[j+k]) j+=k+1;
        else i+=k+1;
        if(i==j) ++j;
    }
    int ans=i<n?i:j;
    return s.substr(ans,n);
}

```

## 5.8 De Bruijn sequence

```

constexpr int MAXC = 10, MAXN = 1e5 + 10;
struct DBSeq {
    int C, N, K, L, buf[MAXC * MAXN]; //K <= C^N
    void dfs(int *out, int t, int p, int &ptr) {
        if (ptr>=L) return;
        if (t>N) {
            if (N%p) return;
            for (int i=1;i<=p&&ptr<L;++i) out[ptr++]=buf[i];
        } else {
            buf[t]=buf[t-p],dfs(out,t+1,p,ptr);
            for (int j=buf[t-p]+1;j<C;++j) buf[t]=j,dfs(out,t
                +1,t,ptr);
        }
    }
    void solve(int _c, int _n, int _k, int *out) {
        int p=0;
        C=_c,N=_n,K=_k,L=N+K-1;dfs(out,1,1,p);
        if (p<L) fill(out+p,out+L,0);
    }
} dbs;

```

## 5.9 SAM

```

const int MAXM = 1000010;
struct SAM{
    int tot, root, lst, mom[MAXM], mx[MAXM];
    int acc[MAXM], nxt[MAXM][33];
    int newNode(){
        int res = ++tot;
        fill(nxt[res],nxt[res]+33, 0);
        mom[res] = mx[res] = acc[res] = 0;
        return res;
    }
    void init(){
        tot = 0;
        root = newNode();
        mom[root] = 0, mx[root] = 0;
        lst = root;
    }
    void push(int c){
        int p = lst;
        int np = newNode();
        mx[np] = mx[p]+1;
        for(; p && nxt[p][c] == 0; p = mom[p])
            nxt[p][c] = np;
        if(p == 0) mom[np] = root;
        else{
            int q = nxt[p][c];
            if(mx[p]+1 == mx[q]) mom[np] = q;
        }
    }
}

```

```

    else{
        int nq = newNode();
        mx[nq] = mx[p]+1;
        for(int i = 0; i < 33; i++){
            nxt[nq][i] = nxt[q][i];
            mom[nq] = mom[q];
            mom[q] = nq;
            mom[np] = nq;
            for(; p && nxt[p][c] == q; p = mom[p])
                nxt[p][c] = nq;
        }
        lst = np;
    }
    void push(char *str){
        for(int i = 0; str[i]; i++)
            push(str[i]- 'a'+1);
    }
} sam;

```

## 5.10 PalTree

```

struct palindromic_tree{// Check by APIO 2014
    palindrome
    struct node{
        int next[26],fail,len;
        int cnt,num;//cnt: appear times, num: number of pal
            . suf.
        node(int l=0):fail(0),len(l),cnt(0),num(0){
            for(int i=0;i<26;++i)next[i]=0;
        }
    };
    vector<node>St;
    vector<char>s;
    int last,n;
    palindromic_tree():St(2),last(1),n(0){
        St[0].fail=1, St[1].len=-1, s.pb(-1);
    }
    inline void clear(){
        St.clear(), s.clear(), last=1, n=0;
        St.pb(0), St.pb(-1);
        St[0].fail=1, s.pb(-1);
    }
    inline int get_fail(int x){
        while(s[n-St[x].len-1]!=s[n])x=St[x].fail;
        return x;
    }
    inline void add(int c){
        s.push_back(c-'a'), ++n;
        int cur=get_fail(last);
        if(!St[cur].next[c]){
            int now=SZ(St);
            St.pb(St[cur].len+2);
            St[now].fail=St[get_fail(St[cur].fail)].next[c];
            St[cur].next[c]=now;
            St[now].num=St[St[now].fail].num+1;
        }
        last=St[cur].next[c], ++St[last].cnt;
    }
    inline void count(){// counting cnt
        auto i=St.rbegin();
        for(;i!=St.rend();++i){
            St[i->fail].cnt+=i->cnt;
        }
    }
    inline int size(){// The number of diff. pal.
        return SZ(St)-2;
    }
};

```

## 5.11 cyclicLCS

```

#define L 0
#define LU 1
#define U 2
const int mov[3][2]={0,-1, -1,-1, -1,0};
int al,bl;
char a[MAXL*2],b[MAXL*2]; // 0-indexed

```

```

int dp[MAXL*2][MAXL];
char pred[MAXL*2][MAXL];
inline int lcs_length(int r) {
    int i=r+al,j=bl,l=0;
    while(i>r) {
        char dir=pred[i][j];
        if(dir==LU) l++;
        i+=mov[dir][0];
        j+=mov[dir][1];
    }
    return l;
}
inline void reroot(int r) { // r = new base row
    int i=r,j=1;
    while(j<=bl&&pred[i][j]!=LU) j++;
    if(j>bl) return;
    pred[i][j]=L;
    while(i<2*al&&j<=bl) {
        if(pred[i+1][j]==U) {
            i++;
            pred[i][j]=L;
        } else if(j<bl&&pred[i+1][j+1]==LU) {
            i++;
            j++;
            pred[i][j]=L;
        } else {
            j++;
        }
    }
}
int cyclic_lcs() {
    // a, b, al, bl should be properly filled
    // note: a WILL be altered in process
    // -- concatenated after itself
    char tmp[MAXL];
    if(al>bl) {
        swap(al,bl);
        strcpy(tmp,a);
        strcpy(a,b);
        strcpy(b,tmp);
    }
    strcpy(tmp,a);
    strcat(a,tmp);
    // basic lcs
    for(int i=0;i<2*al;i++) {
        dp[i][0]=0;
        pred[i][0]=U;
    }
    for(int j=0;j<=bl;j++) {
        dp[0][j]=0;
        pred[0][j]=L;
    }
    for(int i=1;i<2*al;i++) {
        for(int j=1;j<=bl;j++) {
            if(a[i-1]==b[j-1]) dp[i][j]=dp[i-1][j-1]+1;
            else dp[i][j]=max(dp[i-1][j],dp[i][j-1]);
            if(dp[i][j-1]==dp[i][j]) pred[i][j]=L;
            else if(a[i-1]==b[j-1]) pred[i][j]=LU;
            else pred[i][j]=U;
        }
    }
    // do cyclic lcs
    int clcs=0;
    for(int i=0;i<al;i++) {
        clcs=max(clcs,lcs_length(i));
        reroot(i+1);
    }
    // recover a
    a[al]='\0';
    return clcs;
}

```

## 6 Math

### 6.1 ax+by=gcd

```

pii gcd(int a,int b){
    if(b==0) return pii(1,0);
}

```



```

else{
    int p=a/b;
    pii q=gcd(b,a%b);
    return MP(q.Y,q.X-q.Y*p);
}
}

```

## 6.2 floor and ceil

```

int floor(int a,int b){
    return a/b-(a%b&&a<0^b<0);
}
int ceil(int a,int b){
    return a/b+(a%b&&a<0^b>0);
}

```

## 6.3 Miller Rabin

```

// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pirmes <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool Miller_Rabin(ll a,ll n){
    if((a=a%n)==0) return 1;
    if(n&1^1) return n==2;
    ll tmp=(n-1)/((n-1)&(-n+1)),t=__lg((n-1)&(-n+1)),x=1;
    for(;tmp;tmp/=2,a=mul(a,a,n))
        if(tmp&1) x=mul(x,a,n);
    if(x==1 || x==n-1) return 1;
    while(--t)
        if((x=mul(x,x,n))==n-1) return 1;
    return 0;
}

```

## 6.4 Big number

```

template<typename T>
inline string to_string(const T& x){
    stringstream ss;
    return ss<<x,ss.str();
}
struct bigN:vector<ll>{
    const static int base=1000000000,width=log10(base);
    bool negative;
    bigN(const_iterator a,const_iterator b):vector<ll>(a,b){}
    bigN(string s){
        if(s.empty())return;
        if(s[0]=='-')negative=1,s=s.substr(1);
        else negative=0;
        for(int i=int(s.size())-1;i>=0;i-=width){
            ll t=0;
            for(int j=max(0,i-width+1);j<=i;j++)
                t=t*10+s[j]-'0';
            push_back(t);
        }
        trim();
    }
    template<typename T>
    bigN(const T &x):bigN(to_string(x)){}
    bigN():negative(0){}
    void trim(){
        while(size()&&!back())pop_back();
        if(empty())negative=0;
    }
    void carry(int _base=base){
        for(size_t i=0;i<size();++i){
            if(at(i)>=0&&at(i)<_base)continue;
            if(i+1u==size())push_back(0);
            int r=at(i)%_base;
            if(r<0)r+=_base;
            at(i+1)+=(at(i)-r)/_base,at(i)=r;
        }
    }
    int abscmp(const bigN &b)const{

```

```

        if(size()>b.size())return 1;
        if(size()<b.size())return -1;
        for(int i=int(size())-1;i>=0;--i){
            if(at(i)>b[i])return 1;
            if(at(i)<b[i])return -1;
        }
        return 0;
    }
    int cmp(const bigN &b)const{
        if(negative!=b.negative)return negative?-1:1;
        return negative?-abscmp(b):abscmp(b);
    }
    bool operator<(const bigN&b)const{return cmp(b)<0;}
    bool operator>(const bigN&b)const{return cmp(b)>0;}
    bool operator<=(const bigN&b)const{return cmp(b)<=0;}
    bool operator>=(const bigN&b)const{return cmp(b)>=0;}
    bool operator==(const bigN&b)const{return !cmp(b);}
    bool operator!=(const bigN&b)const{return cmp(b)!=0;}
    bigN abs()const{
        bigN res=*this;
        return res.negative=0, res;
    }
    bigN operator-()const{
        bigN res=*this;
        return res.negative=!negative,res.trim(),res;
    }
    bigN operator+(const bigN &b)const{
        if(negative)return -(*this)+(-b);
        if(b.negative)return *this-(-b);
        bigN res=*this;
        if(b.size()>size())res.resize(b.size());
        for(size_t i=0;i<b.size();++i)res[i]+=b[i];
        return res.carry(),res.trim(),res;
    }
    bigN operator-(const bigN &b)const{
        if(negative)return -(*this)-(-b);
        if(b.negative)return *this+(-b);
        if(abscmp(b)<0)return -(b-(*this));
        bigN res=*this;
        if(b.size()>size())res.resize(b.size());
        for(size_t i=0;i<b.size();++i)res[i]-=b[i];
        return res.carry(),res.trim(),res;
    }
    bigN operator*(const bigN &b)const{
        bigN res;
        res.negative=negative!=b.negative;
        res.resize(size()+b.size());
        for(size_t i=0;i<size();++i)
            for(size_t j=0;j<b.size();++j)
                if((res[i+j]+=at(i)*b[j])>=base){
                    res[i+j+1]+=res[i+j]/base;
                    res[i+j]%=base;
                }
        return res.trim(),res;
    }
    bigN operator/(const bigN &b)const{
        int norm=base/(b.back()+1);
        bigN x=abs()*norm;
        bigN y=b.abs()*norm;
        bigN q,r;
        q.resize(x.size());
        for(int i=int(x.size())-1;i>=0;--i){
            r=r*base+x[i];
            int s1=r.size()<=y.size()?r[y.size()]:
            int s2=r.size()<y.size()?0:r[y.size()-1];
            int d=(ll(base)*s1+s2)/y.back();
            r=r-y*d;
            while(r.negative)r=r+y,--d;
            q[i]=d;
        }
        q.negative=negative!=b.negative;
        return q.trim(),q;
    }
    bigN operator%(const bigN &b)const{
        return *this-(*this/b)*b;
    }
    friend istream& operator>>(istream &ss,bigN &b){
        string s;
        return ss>>s, b=s, ss;
    }
    friend ostream& operator<<(ostream &ss,const bigN &b)
    {

```

```

    if(b.negative)ss<<'-';
    ss<<(b.empty()?0:b.back());
    for(int i=int(b.size())-2;i>=0;--i)
        ss<<setw(width)<<setfill('0')<<b[i];
    return ss;
}
template<typename T>
operator T(){
    stringstream ss;
    ss<<*this;
    T res;
    return ss>>res,res;
}
};

```

## 6.5 Fraction

```

struct fraction{
    ll n,d;
    fraction(const ll &n=0,const ll &d=1):n(_n),d(_d){
        ll t=__gcd(n,d);
        n/=t,d/=t;
        if(d<0) n=-n,d=-d;
    }
    fraction operator-(const fraction &b)const{
        return fraction(-n,d);
    }
    fraction operator+(const fraction &b)const{
        return fraction(n*b.d+b.n*d,d*b.d);
    }
    fraction operator-(const fraction &b)const{
        return fraction(n*b.d-b.n*d,d*b.d);
    }
    fraction operator*(const fraction &b)const{
        return fraction(n*b.n,d*b.d);
    }
    fraction operator/(const fraction &b)const{
        return fraction(n*b.d,d*b.n);
    }
    void print(){
        cout << n;
        if(d!=1) cout << "/" << d;
    }
};

```

## 6.6 Simultaneous Equations

```

struct matrix { //m variables, n equations
    int n, m;
    fraction M[MAXN][MAXN + 1], sol[MAXN];
    int solve() { //-1: inconsistent, >= 0: rank
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m) continue;
            for (int j = 0; j < n; ++j) {
                if (i == j) continue;
                fraction tmp = -M[j][piv] / M[i][piv];
                for (int k = 0; k <= m; ++k) M[j][k] = tmp * M[i][k] + M[j][k];
            }
        }
        int rank = 0;
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m && M[i][m].n) return -1;
            else if (piv < m) ++rank, sol[piv] = M[i][m] / M[i][piv];
        }
        return rank;
    }
};

```

## 6.7 Pollard Rho

```

// does not work when n is prime
ll f(ll x,ll mod){ return add(mul(x,x,mod),1,mod); }
ll pollard_rho(ll n){
    if(!(n&1)) return 2;
    while(1){
        ll y=2,x=rand()%(n-1)+1,res=1;
        for(int sz=2;res==1;y=x,sz*=2)
            for(int i=0;i<sz&&res<=1;++i)
                x=f(x,n),res=__gcd(abs(x-y),n);
        if(res!=0&&res!=n) return res;
    }
}

```

## 6.8 Simplex Algorithm

```

const int MAXN = 111;
const int MAXM = 111;
const double eps = 1E-10;
double a[MAXN][MAXM], b[MAXN], c[MAXM], d[MAXN][MAXM];
double x[MAXM];
int ix[MAXN + MAXM]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b,x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[MAXN][MAXM], double b[MAXN],
    double c[MAXM], int n, int m){
    ++m;
    int r = n, s = m - 1;
    memset(d, 0, sizeof(d));
    for (int i = 0; i < n + m; ++i) ix[i] = i;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m - 1; ++j) d[i][j] = -a[i][j];
        d[i][m - 1] = 1;
        d[i][m] = b[i];
        if (d[r][m] > d[i][m]) r = i;
    }
    for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];
    d[n + 1][m - 1] = -1;
    for (double dd;; ) {
        if (r < n) {
            int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
            d[r][s] = 1.0 / d[r][s];
            for (int j = 0; j <= m; ++j)
                if (j != s) d[r][j] *= -d[r][s];
            for (int i = 0; i <= n + 1; ++i) if (i != r) {
                for (int j = 0; j <= m; ++j) if (j != s)
                    d[i][j] += d[r][j] * d[i][s];
                d[i][s] *= d[r][s];
            }
        }
        r = -1; s = -1;
        for (int j = 0; j < m; ++j)
            if (s < 0 || ix[s] > ix[j]) {
                if (d[n + 1][j] > eps ||
                    (d[n + 1][j] > -eps && d[n][j] > eps))
                    s = j;
            }
        if (s < 0) break;
        for (int i = 0; i < n; ++i) if (d[i][s] < -eps) {
            if (r < 0 ||
                (dd = d[r][m] / d[r][s] - d[i][m] / d[i][s]) < -eps ||
                (dd < eps && ix[r + m] > ix[i + m]))
                r = i;
        }
        if (r < 0) return -1; // not bounded
    }
    if (d[n + 1][m] < -eps) return -1; // not executable
    double ans = 0;
    for(int i=0; i<m; i++) x[i] = 0;
    for (int i = m; i < n + m; ++i) { // the missing
        enumerated x[i] = 0
        if (ix[i] < m - 1){
            ans += d[i - m][m] * c[ix[i]];
            x[ix[i]] = d[i - m][m];
        }
    }
    return ans;
}

```

```
}
}
```

## 6.9 chineseRemainder

```
LL solve(LL x1, LL m1, LL x2, LL m2) {
    LL g = __gcd(m1, m2);
    if((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    pair<LL, LL> p = gcd(m1, m2);
    LL lcm = m1 * m2 * g;
    LL res = p.first * (x2 - x1) * m1 + x1;
    return (res % lcm + lcm) % lcm;
}
```

## 6.10 QuadraticResidue

```
int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
    return s;
}

int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    const int jc = Jacobi(a, p);
    if (jc == 0) return 0;
    if (jc == -1) return -1;
    int b, d;
    for (; ; ) {
        b = rand() % p;
        d = (1LL * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (1LL + p) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (1LL * g0 * f0 + 1LL * d * (1LL * g1 * f1 % p)) % p;
            g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (1LL * f0 * f0 + 1LL * d * (1LL * f1 * f1 % p)) % p;
        f1 = (2LL * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}
```

## 6.11 PiCount

```
int64_t PrimeCount(int64_t n) {
    if (n <= 1) return 0;
    const int v = sqrt(n);
    vector<int> smalls(v + 1);
    for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
    int s = (v + 1) / 2;
    vector<int> roughs(s);
    for (int i = 0; i < s; ++i) roughs[i] = 2 * i + 1;
    vector<int64_t> larges(s);
    for (int i = 0; i < s; ++i) larges[i] = (n / (2 * i + 1) + 1) / 2;
    vector<bool> skip(v + 1);
    int pc = 0;
    for (int p = 3; p <= v; ++p) {
        if (smalls[p] > smalls[p - 1]) {
            int q = p * p;
            pc++;
        }
    }
}
```

```
if (1LL * q * q > n) break;
skip[p] = true;
for (int i = q; i <= v; i += 2 * p) skip[i] = true;
int ns = 0;
for (int k = 0; k < s; ++k) {
    int i = roughs[k];
    if (skip[i]) continue;
    int64_t d = 1LL * i * p;
    larges[ns] = larges[k] - (d <= v ? larges[smalls[d] - pc] : smalls[n / d]) + pc;
    roughs[ns++] = i;
}
s = ns;
for (int j = v / p; j >= p; --j) {
    int c = smalls[j] - pc;
    for (int i = j * p, e = min(i + p, v + 1); i < e; ++i) smalls[i] -= c;
}
}
}

for (int k = 1; k < s; ++k) {
    const int64_t m = n / roughs[k];
    int64_t s = larges[k] - (pc + k - 1);
    for (int l = 1; l < k; ++l) {
        int p = roughs[l];
        if (1LL * p * p > m) break;
        s -= smalls[m / p] - (pc + l - 1);
    }
    larges[0] -= s;
}
return larges[0];
}
```

## 6.12 Algorithms about Primes

```
/*
12721 13331 14341 75577 123457 222557 556679 999983
1097774749 1076767633 100102021 999997771
1001010013 1000512343 987654361 999991231
999888733 98789101 987777733 999991921
1010101333 1010102101 100000000039
100000000000037 2305843009213693951
4611686018427387847 9223372036854775783
18446744073709551557
*/
```

## 7 Polynomial

### 7.1 Fast Fourier Transform

```
template<int MAXN>
struct FFT {
    using val_t = complex<double>;
    const double PI = acos(-1);
    val_t w[MAXN];
    FFT() {
        for (int i = 0; i < MAXN; ++i) {
            double arg = 2 * PI * i / MAXN;
            w[i] = val_t(cos(arg), sin(arg));
        }
    }
    void bitrev(val_t *a, int n) {
        int i = 0;
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (i ^ k) < k; k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
    }
    void trans(val_t *a, int n, bool inv = false) {
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = MAXN / L;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + (L >> 1); ++j, x += dx) {

```

```

        val_t tmp = a[j + (L >> 1)] * w[x];
        a[j + (L >> 1)] = a[j] - tmp;
        a[j] += tmp;
    }
}
if (inv) {
    reverse(a + 1, a + n);
    for (int i = 0; i < n; ++i) a[i] /= n;
}
};

```

## 7.2 Number Theory Transform

```

// (2^16)+1, 65537, 3
// 7*17*(2^23)+1, 998244353, 3
// 1255*(2^20)+1, 1315962881, 3
// 51*(2^25)+1, 1711276033, 29
template<int MAXN, LL P, LL RT> // MAXN must be 2^k
struct NTT {
    LL w[MAXN];
    LL mpow(LL a, LL n);
    LL minv(LL a) { return mpow(a, P - 2); }
    NTT() {
        LL dw = mpow(RT, (P - 1) / MAXN);
        w[0] = 1;
        for (int i = 1; i < MAXN; ++i) w[i] = w[i - 1] * dw % P;
    }
    void bitrev(LL *a, int n) {
        int i = 0;
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (i ^= k) < k; k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
    }
    void operator()(LL *a, int n, bool inv = false) { // 0
        <= a[i] < P
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = MAXN / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; ++j, x += dx) {
                    LL tmp = a[j + dl] * w[x] % P;
                    if ((a[j + dl] = a[j] - tmp) < 0) a[j + dl] += P;
                    if ((a[j] += tmp) >= P) a[j] -= P;
                }
            }
        }
        if (inv) {
            reverse(a + 1, a + n);
            LL invn = minv(n);
            for (int i = 0; i < n; ++i) a[i] = a[i] * invn % P;
        }
    }
};

```

## 7.3 Fast Walsh Transform

```

/*
x: a[j], y: a[j + (L >> 1)]
or: (x, y + x), (x, y - x)
and: (x + y, y), (x - y, y)
xor: (x + y, x - y), (x + y, x - y)/2
*/
void fwt(val_t *a, int n) { // or
    for (int L = 2; L <= n; L <= 1) {
        for (int i = 0; i < n; i += L) {
            for (int j = i; j < i + (L >> 1); ++j) {
                a[j + (L >> 1)] += a[j]; // inv: a[j + (L >> 1)] -= a[j];
            }
        }
    }
}

```

## 7.4 Polynomial Operation

```

template<int MAXN, LL P, LL RT> // MAXN must be 2^k
struct PolyOp {
    NTT<MAXN, P, RT> ntt;
    const LL INV2 = ntt.minv(2);
    int get_sz(int n) {
        int sz = 1;
        while (sz < n) sz <= 1;
        return sz;
    }
    void mul(LL *a, int n, LL *b, int m, LL *c) {
        static LL buf1[MAXN], buf2[MAXN];
        int sz = get_sz(n + m - 1);
        copy(a, a + n, buf1), fill(buf1 + n, buf1 + sz, 0);
        copy(b, b + m, buf2), fill(buf2 + m, buf2 + sz, 0);
        ntt(buf1, sz), ntt(buf2, sz);
        for (int i = 0; i < sz; ++i) c[i] = buf1[i] * buf2[i] % P;
        ntt(c, sz, true);
    }
    void inv(LL *a, int n, LL *b) { // a[0] != 0
        static LL buf[MAXN];
        if (n == 1) return b[0] = ntt.minv(a[0]), void();
        inv(a, (n + 1) / 2, b);
        int sz = get_sz(n * 2);
        copy(a, a + n, buf), fill(buf + n, buf + sz, 0);
        fill(b + n, b + sz, 0);
        ntt(buf, sz), ntt(b, sz);
        for (int i = 0; i < sz; ++i) {
            b[i] *= (2 - b[i] * buf[i]) % P;
            if ((b[i] % P) < 0) b[i] += P;
        }
        ntt(b, sz, true), fill(b + n, b + sz, 0);
    }
    LL _msqrt(LL x) {
        for (LL i = 0; i <= P / 2; ++i) if (i * i % P == x)
            return i;
        throw string("BBQube");
    }
    void sqrt(LL *a, int n, LL *b) { // a[0] != 0 && sqrt(a[0]) exists
        static LL invb[MAXN], buf[MAXN];
        if (n == 1) return b[0] = _msqrt(a[0]), void();
        sqrt(a, (n + 1) / 2, b);
        int sz = get_sz(n * 2);
        inv(b, n, invb);
        copy(a, a + n, buf), fill(buf + n, buf + sz, 0);
        ntt(b, sz), ntt(invb, sz), ntt(buf, sz);
        for (int i = 0; i < sz; ++i) {
            if ((b[i] += buf[i] * invb[i] % P) >= P) b[i] -= P;
            b[i] = b[i] * INV2 % P;
        }
        ntt(b, sz, true), fill(b + n, b + sz, 0);
    }
    void div(LL *a, int n, LL *b, int m, LL *q, LL *r) {
        static LL invb[MAXN], buf[MAXN];
        if (n < m) {
            fill(q, q + m, 0), copy(a, a + n, r), fill(r + n, r + m, 0);
            return;
        }
        int mod_sz = n - m + 1;
        copy(b, b + m, buf), reverse(buf, buf + m);
        if (m < mod_sz) fill(buf + m, buf + mod_sz, 0);
        inv(buf, mod_sz, invb);
        copy(a, a + n, buf), reverse(buf, buf + n);
        mul(buf, mod_sz, invb, mod_sz, q);
        fill(q + mod_sz, q + n, 0), reverse(q, q + mod_sz);
        mul(b, m, q, mod_sz, buf);
        for (int i = 0; i < n; ++i) {
            if ((r[i] = a[i] - buf[i]) < 0) r[i] += P;
        }
    }
};

```

## 8 Geometry

### 8.1 Default Code

```
typedef pair<double,double> pdd;
typedef pair<pdd,pdd> Line;
const double eps=1e-8;
pdd operator+(const pdd &a,const pdd &b)
{ return pdd(a.X+b.X,a.Y+b.Y);}
pdd operator-(const pdd &a,const pdd &b)
{ return pdd(a.X-b.X,a.Y-b.Y);}
pdd operator*(const pdd &a,const double &b)
{ return pdd(a.X*b,a.Y*b);}
pdd operator/(const pdd &a,const double &b)
{ return pdd(a.X/b,a.Y/b);}
double dot(const pdd &a,const pdd &b)
{ return a.X*b.X+a.Y*b.Y;}
double cross(const pdd &a,const pdd &b)
{ return a.X*b.Y-a.Y*b.X;}
double abs2(const pdd &a)
{ return dot(a,a);}
double abs(const pdd &a)
{ return sqrt(dot(a,a));}
int ori(const pdd &a,const pdd &b,const pdd &c){
    double res=cross(b-a,c-a);
    if(fabs(res)<eps) return 0;
    return res > 0 ? 1 : -1;
}
bool collinearity(const pdd &p1, const pdd &p2, const
    pdd &p3){
    return fabs(cross(p1-p3,p2-p3))<eps;
}
bool btw(const pdd &p1,const pdd &p2,const pdd &p3){
    if(!collinearity(p1,p2,p3)) return 0;
    return dot(p1-p3,p2-p3)<eps;
}
bool seg_intersect(const pdd &p1,const pdd &p2,const
    pdd &p3,const pdd &p4){
    int a123=ori(p1,p2,p3);
    int a124=ori(p1,p2,p4);
    int a341=ori(p3,p4,p1);
    int a342=ori(p3,p4,p2);
    if(a123==0 && a124==0)
        return btw(p1,p2,p3) || btw(p1,p2,p4) || btw(p3,p4,p1)
            || btw(p3,p4,p2);
    return a123*a124<=0&&a341*a342<=0;
}
pdd intersect(const pdd &p1,const pdd &p2,const pdd &p3
    ,const pdd &p4){
    double a123=cross(p2-p1,p3-p1);
    double a124=cross(p2-p1,p4-p1);
    return (p4*a123-p3*a124)/(a123-a124);
}
pdd foot(const pdd &p1,const pdd &p2,const pdd &p3){
    pdd tmp=p2-p1;
    swap(tmp.X,tmp.Y),tmp.Y*=-1;
    return intersect(p1,p2,p3,p3+tmp);
}
```

### 8.2 Convex hull

```
struct convex_hull{
    vector<pdd> dots;
    void add_dot(double a,double b){
        dots.pb(pdd(a,b));
    }
    vector<pdd> hull(){
        vector<pdd> ans;
        sort(dots.begin(),dots.end());
        ans.pb(dots[0]),ans.pb(dots[1]);
        for(int i=2;i<SZ(dots);++i){
            while(SZ(ans)>=2)
                if(ori(ans[SZ(ans)-2],ans.back(),dots[i])<=0)
                    ans.pop_back();
                else break;
            ans.pb(dots[i]);
        }
        for(int i=SZ(dots)-2,t=SZ(ans);i>=0;--i){
```

```
            while(SZ(ans)>t)
                if(ori(ans[SZ(ans)-2],ans.back(),dots[i])<=0)
                    ans.pop_back();
                else break;
            ans.pb(dots[i]);
        }
        ans.pop_back();
        return ans;
    }
};
```

### 8.3 External bisector

```
pdd external_bisector(pdd p1,pdd p2,pdd p3){//213
    pdd L1=p2-p1,L2=p3-p1;
    L2=L2*abs(L1)/abs(L2);
    return L1+L2;
}
```

### 8.4 Heart

```
pdd excenter(pdd p0,pdd p1,pdd p2,double &radius){
    p1=p1-p0,p2=p2-p0;
    double x1=p1.X,y1=p1.Y,x2=p2.X,y2=p2.Y;
    double m=2.*(x1*y2-y1*x2);
    center.X=(x1*x1*y2-x2*x2*y1+y1*y2*(y1-y2))/m;
    center.Y=(x1*x2*(x2-x1)-y1*y1*x2+x1*y2*y2)/m;
    return radius=abs(center),center+p0;
}
pdd incenter(pdd p1,pdd p2,pdd p3,double &radius){
    double a=abs(p2-p1),b=abs(p3-p1),c=abs(p3-p2);
    double s=(a+b+c)/2,area=sqrt(s*(s-a)*(s-b)*(s-c));
    pdd L1=external_bisector(p1,p2,p3),L2=
        external_bisector(p2,p1,p3);
    return radius=area/s,intersect(p1,p1+L1,p2,p2+L2),
}
pdd escenter(pdd p1,pdd p2,pdd p3){//213
    pdd L1=external_bisector(p1,p2,p3),L2=
        external_bisector(p2,p2+p2-p1,p3);
    return intersect(p1,p1+L1,p2,p2+L2);
}
pdd barycenter(pdd p1,pdd p2,pdd p3){
    return (p1+p2+p3)/3;
}
pdd orthocenter(pdd p1,pdd p2,pdd p3){
    pdd L1=p3-p2,L2=p3-p1;
    swap(L1.X,L1.Y),L1.X*=-1;
    swap(L2.X,L2.Y),L2.X*=-1;
    return intersect(p1,p1+L1,p2,p2+L2);
}
```

### 8.5 Polar Angle Sort

```
pdd c;//sort base
int Quadrant(pdd a){
    if(a.X>0&&a.Y>=0) return 1;
    if(a.X<=0&&a.Y>0) return 2;
    if(a.X<0&&a.Y<=0) return 3;
    if(a.X>=0&&a.Y<0) return 4;
}
bool cmp(pdd a,pdd b){
    a=a-c,b=b-c;
    if(Quadrant(a)!=Quadrant(b))
        return Quadrant(a)<Quadrant(b);
    if(cross(b,a)==0) return abs(a)<abs(b);
    return cross(b,a)>0;
}
bool cmp(pdd a,pdd b){
    a=a-c,b=b-c;
    if(fabs(atan2(a.Y,a.X)-atan2(b.Y,b.X))>eps)
        return atan2(a.Y,a.X)<atan2(b.Y,b.X);
    return abs(a)<abs(b);
}
```

```
}
}
```

## 8.6 Intersection of two circles

```
vector<pdd> interCircle(pdd o1, double r1, pdd o2,
    double r2){
    if(abs(o1-o2)<max(r1,r2)-min(r1,r2)) return {};
    double d2=abs2(o1-o2);
    double d=sqrt(d2);
    if(d>r1+r2) return {};
    pdd u = (o1+o2)*0.5 + (o1-o2)*((r2*r2-r1*r1)/(2*d2));
    double A = sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2
        +d));
    pdd v = pdd(o1.Y-o2.Y,-o1.X+o2.X)*A/(2*d2);
    return {u+v, u-v};
}
```

## 8.7 Intersection of polygon and circle

```
// Divides into multiple triangle, and sum up
// test by HDU2892
const double PI=acos(-1);
double _area(pdd pa, pdd pb, double r){
    if(abs(pa)<abs(pb)) swap(pa, pb);
    if(abs(pb)<eps) return 0;
    double S, h, theta;
    double a=abs(pb), b=abs(pa), c=abs(pb-pa);
    double cosB = dot(pb,pb-pa) / a / c, B = acos(cosB);
    double cosC = dot(pa,pb) / a / b, C = acos(cosC);
    if(a > r){
        S = (C/2)*r*r;
        h = a*b*sin(C)/c;
        if (h < r && B < PI/2) S -= (acos(h/r)*r*r - h*sqrt
            (r*r-h*h));
    }
    else if(b > r){
        theta = PI - B - asin(sin(B)/r*a);
        S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
    }
    else S = .5*sin(C)*a*b;
    return S;
}
double area_poly_circle(const vector<pdd> poly,const
    pdd &O,const double r){
    double S=0;
    for(int i=0;i<SZ(poly);++i)
        S+=_area(poly[i]-O,poly[(i+1)%SZ(poly)]-O,r)*ori(O,
            poly[i],poly[(i+1)%SZ(poly)]);
    return fabs(S);
}
```

## 8.8 Intersection of line and circle

```
vector<pdd> line_interCircle(const pdd &p1,const pdd &
    p2,const pdd &c,const double r){
    pdd ft=foot(p1,p2,c),vec=p2-p1;
    double dis=abs(c-ft);
    if(fabs(dis-r)<eps) return vector<pdd>{ft};
    if(dis>r) return {};
    vec=vec*sqrt(r*r-dis*dis)/abs(vec);
    return vector<pdd>{ft+vec,ft-vec};
}
```

## 8.9 Half plane intersection

```
bool isin( Line l0, Line l1, Line l2 ){
    // Check inter(l1, l2) in l0
    pdd p = intersect(l1.X,l1.Y,l2.X,l2.Y);
    return cross(l0.Y - l0.X,p - l0.X) > eps;
}
/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.Y - l.X) ^ (p - l.X) > 0
```

```
*/
/* --^-- Line.X --^-- Line.Y --^-- */
vector<Line> halfPlaneInter(vector<Line> lines){
    int sz = lines.size();
    vector<double> ata(sz),ord(sz);
    for(int i=0; i<sz; ++i) {
        ord[i] = i;
        pdd d = lines[i].Y - lines[i].X;
        ata[i] = atan2(d.Y, d.X);
    }
    sort(ord.begin(), ord.end(), [&](int i,int j){
        if( fabs(ata[i] - ata[j]) < eps )
            return (cross(lines[i].Y-lines[i].X,
                lines[j].Y-lines[i].X)<0;
            return ata[i] < ata[j];
        });
    vector<Line> fin;
    for (int i=0; i<sz; ++i)
        if (!i || fabs(ata[ord[i]] - ata[ord[i-1]]) > eps)
            fin.pb(lines[ord[i]]);
    deque<Line> dq;
    for (int i=0; i<SZ(fin); i++){
        while(SZ(dq)>=2&&!isin(fin[i],dq[SZ(dq)-2],dq.back
            ()))
            dq.pop_back();
        while(SZ(dq)>=2&&!isin(fin[i],dq[0],dq[1]))
            dq.pop_front();
        dq.push_back(fin[i]);
    }
    while(SZ(dq)>=3&&!isin(dq[0],dq[SZ(dq)-2],dq.back()))
        dq.pop_back();
    while(SZ(dq)>=3&&!isin(dq.back(), dq[0], dq[1]))
        dq.pop_front();
    vector<Line> res(ALL(dq));
    return res;
}
```

## 8.10 Convexhull3D

```
struct Point{
    double x,y,z;
    Point(double x=0,double y=0,double z=0):x(x),y(y),z(z)
    {}
    Point operator-(const Point p1){return Point(x-p1.x,y
        -p1.y,z-p1.z);}
    Point operator*(Point p){return Point(y*p.z-z*p.y,z*p
        .x-x*p.z,x*p.y-y*p.x);}
    double operator^(Point p){return (x*p.x+y*p.y+z*p.z)
        ;}
};
struct CH3D{
    struct face{int a,b,c;bool ok;}F[8*MAXN];
    int g[MAXN][MAXN],num,n;
    Point P[MAXN];
    double vlen(Point a){return sqrt(a.x*a.x+a.y*a.y+a.z*
        a.z);}
    Point cross(const Point &a, const Point &b, const
        Point &c){
        return Point((b.y-a.y)*(c.z-a.z)-(b.z-a.z)*(c.y-a.y
            ),-((b.x-a.x)*(c.z-a.z)-(b.z-a.z)*(c.x-a.x)),(b
                .x-a.x)*(c.y-a.y)-(b.y-a.y)*(c.x-a.x));
    }
    double area(Point a,Point b,Point c){return vlen((b-a
        )*(c-a));}
    double volume(Point a,Point b,Point c,Point d){return
        (b-a)*(c-a)^(d-a);}
    double dblcmp(Point &p,face &f){return ((P[f.b]-P[f.a
        ])*(P[f.c]-P[f.a]))^(p-P[f.a]);}
    void deal(int p,int a,int b){
        int f=g[a][b];
        face add;
        if(F[f].ok)
            if(dblcmp(P[p],F[f])>EPS) dfs(p,f);
        else
            add.a=b,add.b=a,add.c=p,add.ok=1,g[p][b]=g[a][p
                ]=g[b][a]=num,F[num++]=add;
    }
    void dfs(int p,int now){
        F[now].ok=0;
```



```

deal(p,F[now].b,F[now].a),deal(p,F[now].c,F[now].b)
,deal(p,F[now].a,F[now].c);
}
bool same(int s,int t){
    Point &a=P[F[s].a];
    Point &b=P[F[s].b];
    Point &c=P[F[s].c];
    return fabs(volume(a,b,c,P[F[t].a]))<EPS && fabs(
        volume(a,b,c,P[F[t].b]))<EPS && fabs(volume(a,b
        ,c,P[F[t].c]))<EPS;
}
void init(int _n){n=_n,num=0;}
void solve(){
    face add;
    bool flag=true;
    num=0;
    if(n<4) return;
    if([&](){
        for(int i=1;i<n;++i)if(vlen(P[0]-P[i])>EPS)
            return swap(P[1],P[i]),0;return 1;}() ||
        [&](){
            for(int i=2;i<n;++i)if(vlen((P[0]-P[i])*(P[1]-P
            [i]))>EPS)return swap(P[2],P[i]),0;return
            1;}() || [&](){
                for(int i=3;i<n;++i)if(fabs((P[0]-P[1])*(P[1]-P
                [2])^(P[0]-P[i]))>EPS)return swap(P[3],P[i
                ]),0;return 1;}()return;
            for(int i=0;i<4;++i){
                add.a=(i+1)%4,add.b=(i+2)%4,add.c=(i+3)%4,add.ok=
                true;
                if(dblcmp(P[i],add)>0) swap(add.b,add.c);
                g[add.a][add.b]=g[add.b][add.c]=g[add.c][add.a]=
                num;
                F[num++]=add;
            }
            for(int i=4;i<n;++i)
                for(int j=0;j<num;++j)
                    if(F[j].ok && dblcmp(P[i],F[j])>EPS){dfs(i,j);
                    break;}
            for(int tmp=num,i=(num=0);i<tmp;++i)
                if(F[i].ok) F[num++]=F[i];
        }
        double area(){
            double res=0.0;
            if(n==3)
                return vlen(cross(P[0],P[1],P[2]))/2.0;
            for(int i=0;i<num;++i)
                res+=area(P[F[i].a],P[F[i].b],P[F[i].c]);
            return res/2.0;
        }
        double volume(){
            double res=0.0;
            for(int i=0;i<num;i++)
                res+=volume(Point(0,0,0),P[F[i].a],P[F[i].b],P[F[
                i].c]);
            return fabs(res/6.0);
        }
    }
    int triangle(){return num;}
    int polygon(){
        int res=0;
        for(int i=0,flag=1;i<num;++i,res+=flag,flag=1)
            for(int j=0;j<i&&flag;++j)
                flag&=!same(i,j);
        return res;
    }
    Point getcent(){
        Point ans(0,0,0),temp=P[F[0].a];
        double v = 0.0,t2;
        for(int i=0;i<num;++i)
            if(F[i].ok == true){
                Point p1=P[F[i].a],p2=P[F[i].b],p3=P[F[i].c];
                t2 = volume(temp,p1,p2,p3)/6.0;
                if(t2>0)
                    ans.x += (p1.x+p2.x+p3.x+temp.x)*t2, ans.y +=
                    (p1.y+p2.y+p3.y+temp.y)*t2, ans.z += (p1
                    .z+p2.z+p3.z+temp.z)*t2, v += t2;
            }
        ans.x/=(4*v),ans.y/=(4*v),ans.z/=(4*v);
        return ans;
    }
    double pointmindis(Point fuck){
        double min=99999999;

```

```

for(int i=0;i<num;i++)
    if(F[i].ok==true){
        Point p1=P[F[i].a] , p2=P[F[i].b] , p3=P[F[i].c
        ];
        double a = ( (p2.y-p1.y)*(p3.z-p1.z)-(p2.z-p1.z
        )*(p3.y-p1.y) );
        double b = ( (p2.z-p1.z)*(p3.x-p1.x)-(p2.x-p1.x
        )*(p3.z-p1.z) );
        double c = ( (p2.x-p1.x)*(p3.y-p1.y)-(p2.y-p1.y
        )*(p3.x-p1.x) );
        double d = ( 0-(a*p1.x+b*p1.y+c*p1.z) );
        double temp = fabs(a*fuck.x+b*fuck.y+c*fuck.z+d
        )/sqrt(a*a+b*b+c*c);
        if(temp<min) min = temp;
    }
    return min;
}
}hull;

```

## 8.11 CircleCover

```

#define N 1021
struct CircleCover{
    int C; Circ c[ N ];
    bool g[ N ][ N ], overlap[ N ][ N ];
    // Area[i] : area covered by at least i circles
    D Area[ N ];
    void init( int _C ){ C = _C; }
    bool CCinter( Circ& a , Circ& b , Pt& p1 , Pt& p2 ){
        Pt o1 = a.O , o2 = b.O;
        D r1 = a.R , r2 = b.R;
        if( norm( o1 - o2 ) > r1 + r2 ) return {};
        if( norm( o1 - o2 ) < max(r1, r2) - min(r1, r2) )
            return {};
        D d2 = ( o1 - o2 ) * ( o1 - o2 );
        D d = sqrt(d2);
        if( d > r1 + r2 ) return false;
        Pt u=(o1+o2)*0.5 + (o1-o2)*((r2*r2-r1*r1)/(2*d2));
        D A=sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2+d));
        Pt v=Pt( o1.Y-o2.Y , -o1.X + o2.X ) * A / (2*d2);
        p1 = u + v; p2 = u - v;
        return true;
    }
    struct Teve {
        Pt p; D ang; int add;
        Teve() {}
        Teve(Pt _a, D _b, int _c):p(_a), ang(_b), add(_c){}
        bool operator<(const Teve &a)const
            {return ang < a.ang;}
    }eve[ N * 2 ];
    // strict: x = 0, otherwise x = -1
    bool disjuct( Circ& a, Circ &b, int x )
    {return sign( norm( a.O - b.O ) - a.R - b.R ) > x;}
    bool contain( Circ& a, Circ &b, int x )
    {return sign( a.R - b.R - norm( a.O - b.O ) ) > x;}
    bool contain(int i, int j){
        /* c[j] is non-strictly in c[i]. */
        return (sign(c[i].R - c[j].R) > 0 ||
            (sign(c[i].R - c[j].R) == 0 && i < j) ) &&
            contain(c[i], c[j], -1);
    }
    void solve(){
        for( int i = 0 ; i <= C + 1 ; i ++ )
            Area[ i ] = 0;
        for( int i = 0 ; i < C ; i ++ )
            for( int j = 0 ; j < C ; j ++ )
                overlap[i][j] = contain(i, j);
        for( int i = 0 ; i < C ; i ++ )
            for( int j = 0 ; j < C ; j ++ )
                g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                    disjuct(c[i], c[j], -1));
        for( int i = 0 ; i < C ; i ++ ){
            int E = 0, cnt = 1;
            for( int j = 0 ; j < C ; j ++ )
                if( j != i && overlap[j][i] )
                    cnt ++;
            for( int j = 0 ; j < C ; j ++ )
                if( i != j && g[i][j] ){
                    Pt aa, bb;
                    CCinter(c[i], c[j], aa, bb);

```

```

    D A=atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);
    D B=atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
    eve[E++] = Teve(bb, B, 1);
    eve[E++] = Teve(aa, A, -1);
    if(B > A) cnt++;
}
if( E == 0 ) Area[ cnt ] += pi * c[i].R * c[i].R;
else{
    sort( eve , eve + E );
    eve[E] = eve[0];
    for( int j = 0 ; j < E ; j ++ ){
        cnt += eve[j].add;
        Area[cnt] += (eve[j].p ^ eve[j + 1].p) * .5;
        D theta = eve[j + 1].ang - eve[j].ang;
        if (theta < 0) theta += 2. * pi;
        Area[cnt] +=
            (theta - sin(theta)) * c[i].R*c[i].R * .5;
    }
}
}
};

```

## 8.12 DelaunayTriangulation

```

/* Delaunay Triangulation:
   Given a sets of points on 2D plane, find a
   triangulation such that no points will strictly
   inside circumcircle of any triangle.
find : return a triangle contain given point
add_point : add a point into triangulation
A Triangle is in triangulation iff. its has_chd is 0.
Region of triangle u: iterate each u.edge[i].tri,
each points are u.p[(i+1)%3], u.p[(i+2)%3]
calculation involves  $O(\sqrt{V})$  */
const double inf = 1e9;
double eps = 1e-6; // 0 when integer
// return p4 is in circumcircle of tri(p1,p2,p3)
bool in_cc(P &p1, P &p2, P &p3, P &p4) {
    int o1 = (abs(p1.x) >= inf * 0.99 || abs(p1.y) >= inf * 0.99);
    int o2 = (abs(p2.x) >= inf * 0.99 || abs(p2.y) >= inf * 0.99);
    int o3 = (abs(p3.x) >= inf * 0.99 || abs(p3.y) >= inf * 0.99);
    int rtrue = o1 + o2 + o3;
    int rfalse = abs(p4.x) >= inf * 0.99 || abs(p4.y) >= inf * 0.99;
    if (rtrue == 3) return true;
    if (rtrue) {
        P in(0, 0), out(0, 0);
        if (o1) out = out + p1; else in = in + p1;
        if (o2) out = out + p2; else in = in + p2;
        if (o3) out = out + p3; else in = in + p3;
        return (p4 - in) * (out - in) > 0;
    }
    if (rfalse) return false;
    // ^ ?
    double u11 = p1.x - p4.x, u12 = p1.y - p4.y;
    double u21 = p2.x - p4.x, u22 = p2.y - p4.y;
    double u31 = p3.x - p4.x, u32 = p3.y - p4.y;
    double u13 = sq(p1.x) - sq(p4.x) + sq(p1.y) - sq(p4.y);
    double u23 = sq(p2.x) - sq(p4.x) + sq(p2.y) - sq(p4.y);
    double u33 = sq(p3.x) - sq(p4.x) + sq(p3.y) - sq(p4.y);
    double det = -u13 * u22 * u31 + u12 * u23 * u31 + u13 * u21 * u32 - u11 * u23 * u32 - u12 * u21 * u33 + u11 * u22 * u33;
    return det > eps;
}
double side(P &a, P &b, P &p) { return (b - a) ^ (p - a); }
struct Tri;
struct Edge {
    Tri *tri;
    int side;
    Edge() : tri(0), side(0) {}

```

```

    Edge(Tri *_tri, int _side) : tri(_tri), side(_side) {}
};
struct Tri {
    P p[3];
    Edge edge[3];
    Tri *ch[3];
    Tri() {}
    Tri(P p0, P p1, P p2) {
        p[0] = p0; p[1] = p1; p[2] = p2;
        ch[0] = ch[1] = ch[2] = 0;
    }
    bool has_ch() { return ch[0] != 0; }
    int num_ch() {
        return ch[0] == 0 ? 0 : ch[1] == 0 ? 1 : ch[2] == 0 ? 2 : 3;
    }
    bool contains(P &q) {
        for (int i = 0; i < 3; ++i)
            if (side(p[i], p[(i + 1) % 3], q) < -eps) return false;
        return true;
    }
} pool[maxn * 10], *tris;
void edge(Edge a, Edge b) {
    if (a.tri) a.tri->edge[a.side] = b;
    if (b.tri) b.tri->edge[b.side] = a;
}
struct Trig {
    Trig() {
        the_root = new (tris++) Tri(P(-inf, -inf), P(inf * 2, -inf), P(-inf, inf * 2));
    } // all p should in
    Tri *find(P p) { return find(the_root, p); }
    void add_point(P &p) { add_point(find(the_root, p), p); }
    Tri *the_root;
    static Tri *find(Tri *root, P &p) {
        while (true) {
            if (!root->has_ch()) return root;
            for (int i = 0; i < 3 && root->ch[i]; ++i)
                if (root->ch[i]->contains(p)) {
                    root = root->ch[i];
                    break;
                }
        }
        assert(false); // "point not found"
    }
    void add_point(Tri *root, P &p) {
        Tri *tab, *tbc, *tca;
        tab = new (tris++) Tri(root->p[0], root->p[1], p);
        tbc = new (tris++) Tri(root->p[1], root->p[2], p);
        tca = new (tris++) Tri(root->p[2], root->p[0], p);
        edge(Edge(tab, 0), Edge(tbc, 1));
        edge(Edge(tbc, 0), Edge(tca, 1));
        edge(Edge(tca, 0), Edge(tab, 1));
        edge(Edge(tab, 2), root->edge[2]);
        edge(Edge(tbc, 2), root->edge[0]);
        edge(Edge(tca, 2), root->edge[1]);
        root->ch[0] = tab; root->ch[1] = tbc; root->ch[2] = tca;
        flip(tab, 2); flip(tbc, 2); flip(tca, 2);
    }
    void flip(Tri *tri, int pi) {
        Tri *trj = tri->edge[pi].tri;
        int pj = tri->edge[pi].side;
        if (!trj) return;
        if (!in_cc(tri->p[0], tri->p[1], tri->p[2], trj->p[pj])) return;
        /* flip edge between tri, trj */
        Tri *trk = new (tris++) Tri(tri->p[(pi + 1) % 3], trj->p[pj], tri->p[pi]);
        Tri *trl = new (tris++) Tri(trj->p[(pj + 1) % 3], tri->p[pi], trj->p[pj]);
        edge(Edge(trk, 0), Edge(trl, 0));
        edge(Edge(trk, 1), tri->edge[(pi + 2) % 3]);
        edge(Edge(trk, 2), trj->edge[(pj + 1) % 3]);
        edge(Edge(trl, 1), trj->edge[(pj + 2) % 3]);
        edge(Edge(trl, 2), tri->edge[(pi + 1) % 3]);
        tri->ch[0] = trk; tri->ch[1] = trl; tri->ch[2] = 0;
        trj->ch[0] = trk; trj->ch[1] = trl; trj->ch[2] = 0;
        flip(trk, 1); flip(trk, 2);
    }

```

```

        flip(tr1, 1); flip(tr1, 2);
    }
};
vector<Tri *> triang;
set<Tri *> vst;
void go(Tri *now) {
    if (vst.find(now) != vst.end()) return;
    vst.insert(now);
    if (!now->has_ch()) {
        triang.push_back(now);
        return;
    }
    for (int i = 0; i < now->num_ch(); ++i) go(now->ch[i]);
}
void build(int n, P *ps) {
    tris = pool;
    random_shuffle(ps, ps + n);
    Trig tri;
    for (int i = 0; i < n; ++i) tri.add_point(ps[i]);
    go(tri.the_root);
}

```

### 8.13 Triangulation Voronoi

```

int gid(P &p) {
    auto it = ptoid.find(p);
    if (it == ptoid.end()) return -1;
    return it->second;
}
L make_line(P p, L l) {
    P d = l.pb - l.pa; d = d.spin(pi / 2);
    P m = (l.pa + l.pb) / 2;
    l = L(m, m + d);
    if (((l.pb - l.pa) ^ (p - l.pa)) < 0) l = L(m + d, m);
    return l;
}
double calc_ans(int i) {
    vector<P> ps = HPI[ls[i]];
    double rt = 0;
    for (int i = 0; i < (int)ps.size(); ++i) {
        rt += (ps[i] ^ ps[(i + 1) % ps.size()]);
    }
    return abs(rt) / 2;
}
void solve() {
    for (int i = 0; i < n; ++i) ops[i] = ps[i], ptoid[ops[i]] = i;
    random_shuffle(ps, ps + n);
    build(n, ps);
    for (auto *t : triang) {
        int z[3] = {gid(t->p[0]), gid(t->p[1]), gid(t->p[2])};
        for (int i = 0; i < 3; ++i) for (int j = 0; j < 3; ++j) if (i != j && z[i] != -1 && z[j] != -1) {
            L l(t->p[i], t->p[j]);
            ls[z[i]].push_back(make_line(t->p[i], l));
        }
    }
    vector<P> tb = convex(vector<P>(ps, ps + n));
    for (auto &p : tb) isinf[gid(p)] = true;
    for (int i = 0; i < n; ++i) {
        if (isinf[i]) cout << -1 << '\n';
        else cout << fixed << setprecision(12) << calc_ans(i) << '\n';
    }
}

```

### 8.14 Tangent line of two circles

```

vector<Line> go( const Cir& c1 , const Cir& c2 , int sign1 ){
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    double d_sq = norm2( c1.O - c2.O );
    if( d_sq < eps ) return ret;
    double d = sqrt( d_sq );

```

```

    Pt v = ( c2.O - c1.O ) / d;
    double c = ( c1.R - sign1 * c2.R ) / d;
    if( c * c > 1 ) return ret;
    double h = sqrt( max( 0.0 , 1.0 - c * c ) );
    for( int sign2 = 1 ; sign2 >= -1 ; sign2 -= 2 ){
        Pt n = { v.X * c - sign2 * h * v.Y ,
                v.Y * c + sign2 * h * v.X };
        Pt p1 = c1.O + n * c1.R;
        Pt p2 = c2.O + n * ( c2.R * sign1 );
        if( fabs( p1.X - p2.X ) < eps and
            fabs( p1.Y - p2.Y ) < eps )
            p2 = p1 + perp( c2.O - c1.O );
        ret.push_back( { p1 , p2 } );
    }
    return ret;
}

```

### 8.15 minMaxEnclosingRectangle

```

pdd solve(vector<pll> &dots){
    vector<pll> hull;
    const double INF=1e18, qi=acos(-1)/2*3;
    cv.dots=dots;
    hull=cv.hull();
    double Max=0, Min=INF, deg;
    ll n=hull.size();
    hull.pb(hull[0]);
    for(int i=0, u=1, r=1, l=1; i<n; ++i){
        pll nw=hull[i+1]-hull[i];
        while(cross(nw, hull[u+1]-hull[i])>cross(nw, hull[u]-hull[i]))
            u=(u+1)%n;
        while(dot(nw, hull[r+1]-hull[i])>dot(nw, hull[r]-hull[i]))
            r=(r+1)%n;
        if(!i) l=(r+1)%n;
        while(dot(nw, hull[l+1]-hull[i])<dot(nw, hull[l]-hull[i]))
            l=(l+1)%n;
        Min=min(Min, (double)(dot(nw, hull[r]-hull[i])-dot(nw, hull[l]-hull[i]))*cross(nw, hull[u]-hull[i])/abs2(nw));
        deg=acos((double)dot(hull[r]-hull[l], hull[u]-hull[i])/abs(hull[r]-hull[l])/abs(hull[u]-hull[i]));
        deg=(qi-deg)/2;
        Max=max(Max, (double)abs(hull[r]-hull[l])*abs(hull[u]-hull[i])*sin(deg)*sin(deg));
    }
    return pdd(Min, Max);
}

```

### 8.16 minDistOfTwoConvex

```

// p, q is convex
double TwoConvexHullMinDist(Point P[], Point Q[], int n, int m) {
    int YMinP = 0, YMaxQ = 0;
    double tmp, ans = 999999999;
    for (i = 0; i < n; ++i) if (P[i].y < P[YMinP].y) YMinP = i;
    for (i = 0; i < m; ++i) if (Q[i].y > Q[YMaxQ].y) YMaxQ = i;
    P[n] = P[0], Q[m] = Q[0];
    for (int i = 0; i < n; ++i) {
        while (tmp = Cross(Q[YMaxQ + 1] - P[YMinP + 1], P[YMinP] - P[YMinP + 1]) > Cross(Q[YMaxQ] - P[YMinP + 1], P[YMinP] - P[YMinP + 1])) YMaxQ = (YMaxQ + 1) % m;
        if (tmp < 0) ans = min(ans, PointToSegDist(P[YMinP], P[YMinP + 1], Q[YMaxQ]));
        else ans = min(ans, TwoSegMinDist(P[YMinP], P[YMinP + 1], Q[YMaxQ], Q[YMaxQ + 1]));
        YMinP = (YMinP + 1) % n;
    }
    return ans;
}

```

## 8.17 Minkowski Sum

```

/* convex hull Minkowski Sum*/
#define INF 1000000000000000LL
int pos( const Pt& tp ){
    if( tp.Y == 0 ) return tp.X > 0 ? 0 : 1;
    return tp.Y > 0 ? 0 : 1;
}
#define N 300030
Pt pt[ N ], qt[ N ], rt[ N ];
LL Lx,Rx;
int dn,un;
inline bool cmp( Pt a, Pt b ){
    int pa=pos( a ),pb=pos( b );
    if(pa==pb) return (a^b)>0;
    return pa<pb;
}
int minkowskiSum(int n,int m){
    int i,j,r,p,q,fi,fj;
    for(i=1,p=0;i<n;i++){
        if( pt[i].Y<pt[p].Y ||
            (pt[i].Y==pt[p].Y && pt[i].X<pt[p].X) ) p=i; }
    for(i=1,q=0;i<m;i++){
        if( qt[i].Y<qt[q].Y ||
            (qt[i].Y==qt[q].Y && qt[i].X<qt[q].X) ) q=i; }
    rt[0]=pt[p]+qt[q];
    r=1; i=p; j=q; fi=fj=0;
    while(1){
        if((fj&&j==q) ||
            ( !fi || i==p) &&
            cmp(pt[(p+1)%n]-pt[p],qt[(q+1)%m]-qt[q]) ) ){
            rt[r]=rt[r-1]+pt[(p+1)%n]-pt[p];
            p=(p+1)%n;
            fi=1;
        }else{
            rt[r]=rt[r-1]+qt[(q+1)%m]-qt[q];
            q=(q+1)%m;
            fj=1;
        }
        if(r<=1 || ((rt[r]-rt[r-1])^(rt[r-1]-rt[r-2]))!=0)
            r++;
        else rt[r-1]=rt[r];
        if(i==p && j==q) break;
    }
    return r-1;
}
void initInConvex(int n){
    int i,p,q;
    LL Ly,Ry;
    Lx=INF; Rx=-INF;
    for(i=0;i<n;i++){
        if(pt[i].X<Lx) Lx=pt[i].X;
        if(pt[i].X>Rx) Rx=pt[i].X;
    }
    Ly=Ry=INF;
    for(i=0;i<n;i++){
        if(pt[i].X==Lx && pt[i].Y<Ly){ Ly=pt[i].Y; p=i; }
        if(pt[i].X==Rx && pt[i].Y<Ry){ Ry=pt[i].Y; q=i; }
    }
    for(dn=0,i=p;i!=q;i=(i+1)%n){ qt[dn++]=pt[i]; }
    qt[dn]=pt[q]; Ly=Ry=-INF;
    for(i=0;i<n;i++){
        if(pt[i].X==Lx && pt[i].Y>Ly){ Ly=pt[i].Y; p=i; }
        if(pt[i].X==Rx && pt[i].Y>Ry){ Ry=pt[i].Y; q=i; }
    }
    for(un=0,i=p;i!=q;i=(i+n-1)%n){ rt[un++]=pt[i]; }
    rt[un]=pt[q];
}
inline int inConvex(Pt p){
    int L,R,M;
    if(p.X<Lx || p.X>Rx) return 0;
    L=0;R=dn;
    while(L<R-1){ M=(L+R)/2;
        if(p.X<qt[M].X) R=M; else L=M; }
    if(tri(qt[L],qt[R],p)<0) return 0;
    L=0;R=un;
    while(L<R-1){ M=(L+R)/2;
        if(p.X<rt[M].X) R=M; else L=M; }
    if(tri(rt[L],rt[R],p)>0) return 0;
    return 1;
}

```

```

int main(){
    int n,m,i;
    Pt p;
    scanf("%d",&n);
    for(i=0;i<n;i++) scanf("%Ld%Ld",&pt[i].X,&pt[i].Y);
    scanf("%d",&m);
    for(i=0;i<m;i++) scanf("%Ld%Ld",&qt[i].X,&qt[i].Y);
    n=minkowskiSum(n,m);
    for(i=0;i<n;i++) pt[i]=rt[i];
    scanf("%d",&m);
    for(i=0;i<m;i++) scanf("%Ld%Ld",&qt[i].X,&qt[i].Y);
    n=minkowskiSum(n,m);
    for(i=0;i<n;i++) pt[i]=rt[i];
    initInConvex(n);
    scanf("%d",&m);
    for(i=0;i<m;i++){
        scanf("%Ld %Ld",&p.X,&p.Y);
        p.X*=3; p.Y*=3;
        puts(inConvex(p)? "YES": "NO");
    }
}

```

## 9 Else

### 9.1 Mo's Alogrithm(With modification)

```

struct QUERY{//BLOCK=N^{2/3}
    int L,R,id,LBId,RBId,T;
    QUERY(int l,int r,int id,int lb,int rb,int t):
        L(l),R(r),id(id),LBId(lb),RBId(rb),T(t){}
    bool operator<(const QUERY &b)const{
        if(LBId!=b.LBId) return LBId<b.LBId;
        if(RBId!=b.RBId) return RBId<b.RBId;
        return T<b.T;
    }
};
vector<QUERY> query;
int cur_ans,arr[MAXN],ans[MAXN];
void addTime(int L,int R,int T){}
void subTime(int L,int R,int T){}
void add(int x){}
void sub(int x){}
void solve(){
    sort(ALL(query));
    int L=0,R=0,T=-1;
    for(auto q:query){
        while(T<q.T) addTime(L,R,++T);
        while(T>q.T) subTime(L,R,T--);
        while(R<q.R) add(arr[++R]);
        while(L>q.L) add(arr[--L]);
        while(R>q.R) sub(arr[R--]);
        while(L<q.L) sub(arr[L++]);
        ans[q.id]=cur_ans;
    }
}

```

### 9.2 Mo's Alogrithm On Tree

```

const int MAXN=40005;
vector<int> G[MAXN];//1-base
int n,B,arr[MAXN],ans[100005],cur_ans;
int in[MAXN],out[MAXN],dfn[MAXN*2],dft;
int deep[MAXN],sp[___lg(MAXN*2)+1][MAXN*2],bIn[MAXN],spt;
bitset<MAXN> inset;
struct QUERY{
    int L,R,Lid,id,lca;
    QUERY(int l,int r,int _id):L(l),R(r),lca(0),id(_id){}
    bool operator<(const QUERY &b){
        if(Lid!=b.Lid) return Lid<b.Lid;
        return R<b.R;
    }
};
vector<QUERY> query;
void dfs(int u,int f,int d){
    deep[u]=d,sp[0][spt]=u,bIn[u]=spt++;
}

```

```

dfn[dfn]=u,in[u]=dfn++;
for(int v:G[u])
    if(v!=f)
        dfs(v,u,d+1),sp[0][spt]=u,bln[u]=spt++;
dfn[dfn]=u,out[u]=dfn++;
}
int lca(int u,int v){
    if(bln[u]>bln[v]) swap(u,v);
    int t=__lg(bln[v]-bln[u]+1);
    int a=sp[t][bln[u]],b=sp[t][bln[v]-(1<<t)+1];
    if(deep[a]<deep[b]) return a;
    return b;
}
void sub(int x){}
void add(int x){}
void flip(int x){
    if(inset[x]) sub(arr[x]);
    else add(arr[x]);
    inset[x]=~inset[x];
}
void solve(){
    B=sqrt(2*n),dft=spt=cur_ans=0,dfs(1,1,0);
    for(int i=1,x=2;x<2*n;++i,x<=1)
        for(int j=0;j+x<=2*n;++j)
            if(deep[sp[i-1][j]]<deep[sp[i-1][j+x/2]])
                sp[i][j]=sp[i-1][j];
            else sp[i][j]=sp[i-1][j+x/2];
    for(auto &q:query){
        int c=lca(q.L,q.R);
        if(c==q.L||c==q.R)
            q.L=out[c==q.L?q.R:q.L],q.R=out[c];
        else if(out[q.L]<in[q.R])
            q.lca=c,q.L=out[q.L],q.R=in[q.R];
        else q.lca=c,c=in[q.L],q.L=out[q.R],q.R=c;
        q.Lid=q.L/B;
    }
    sort(ALL(query));
    int L=0,R=-1;
    for(auto q:query){
        while(R<q.R) flip(dfn[++R]);
        while(L>q.L) flip(dfn[--L]);
        while(R>q.R) flip(dfn[R--]);
        while(L<q.L) flip(dfn[L++]);
        if(q.lca) add(arr[q.lca]);
        ans[q.id]=cur_ans;
        if(q.lca) sub(arr[q.lca]);
    }
}
}

```

```

auto it = st.lower_bound(L);
if(it != st.begin() && it != st.end())
    if(!comp((*prev(it))(it->l - 1), L(it->l - 1)) &&
        !comp((*it)(it->l), L(it->l)))
        return;
while(it != st.end()){
    if(it->a == L.a && !comp(it->b, L.b))return;
    if(comp((*it)(it->r), L(it->r)))it = st.erase(it);
    ;
    else{
        Line M = *it;
        st.erase(it);
        L.r = max(idiv(L.b - M.b, M.a - L.a), minx);
        M.l = L.r + 1;
        it = st.insert(M).X;
        break;
    }
}
while(it != st.begin()){
    auto pit = prev(it);
    if(comp((*pit)(pit->l), L(pit->l)))st.erase(pit);
    else{
        Line M = *pit;
        st.erase(pit);
        M.r = min(idiv(L.b - M.b, M.a - L.a), maxx - 1);
        ;
        L.l = M.r + 1;
        st.insert(M);
        break;
    }
}
st.insert(L);
}
val operator () (val x){
    Flag = 1;
    auto it = st.lower_bound({0, 0, x, x});
    return (*it)(x);
}
};
DynamicConvexTrick<> DCT;

```

### 9.3 DynamicConvexTrick

```

// only works for integer coordinates!!
bool Flag; // 0: insert Line, 1: Lower_bound x
template<class val = ll, class compare = less<val>> //
    sort Lines with comp
struct DynamicConvexTrick{
    static const ll minx = 0, maxx = 1l(1e9) + 5;
    static compare comp;
    struct Line{
        val a, b, l, r; // Line ax + b in [l, r]
        Line(val _a, val _b, val _l = minx, val _r = maxx){
            a(_a), b(_b), l(_l), r(_r){}
        }
        val operator () (val x) const {
            return a * x + b;
        }
    };
    struct cmp{
        bool operator () (const Line a, const Line b){
            if(Flag == 0)return comp(a.a, b.a);
            return a.r < b.l;
        }
    };
    inline val idiv(val a, val b){
        return a / b - (a % b && a < 0 ^ b < 0);
    }
    set<Line, cmp> st;
    void ins(val a, val b){
        Flag = 0;
        Line L(a, b);
    }
}

```