hw4_51215903008_陈诺

作业链接

https://www.wolai.com/mathskiller/oujwmKGKLrr7j9TZsdwrCa?theme=light

习题部分

$$\text{习题1.判定矩阵}C=\begin{bmatrix}3&2&-1\\-1&0&0\\-1&3&0\end{bmatrix}\text{和}B=\begin{bmatrix}0&2&-1\\-1&4&-1\\1&3&-5\end{bmatrix}\text{能否进行}\boldsymbol{LU}\text{分解}$$
$$\text{,为什么?如果能分解,试分解之。}$$

$$\text{对于}C\text{，所有顺序主子式行列式不为0，可以进行}\mathbf{LU}\text{分解}$$
$$\text{将}C\text{对角化为}\mathbf{U}\text{，有}E_{23}(-\frac{11}{2})E_{13}(\frac{1}{3})E_{12}(\frac{1}{3})C=\mathbf{U}=\begin{pmatrix}3&2&-1\\0&\frac{2}{3}&-\frac{1}{3}\\0&0&\frac{3}{2}\end{pmatrix}$$
$$\text{对}E_{23}(-\frac{11}{2})E_{13}(\frac{1}{3})E_{12}\text{求逆，有}E_{12}(-\frac{1}{3})E_{13}(-\frac{1}{3})E_{23}(\frac{11}{2})I=\mathbf{L}$$
$$\text{故}\mathbf{L}=E_{12}(-\frac{1}{3})E_{13}(-\frac{1}{3})E_{23}(\frac{11}{2})I=\begin{pmatrix}1&0&0\\-\frac{1}{3}&1&0\\-\frac{1}{3}&\frac{11}{2}&1\end{pmatrix}$$
$$\mathbf{U}=\begin{pmatrix}3&2&-1\\0&\frac{2}{3}&-\frac{1}{3}\\0&0&\frac{3}{2}\end{pmatrix}$$
$$C=\mathbf{LU}$$
$$\text{对于}B\text{，一阶主子式为0，不能进行}\mathbf{LU}\text{分解}$$

$$\text{习题2.求矩阵}\boldsymbol{A}=\begin{bmatrix}2&1&1\\1&2&1\\1&1&0\end{bmatrix}\text{的}\boldsymbol{LU}\text{分解。}$$

$$\begin{pmatrix}2&1&1\\1&2&1\\1&1&0\end{pmatrix}\xrightarrow[E_{13}\left(-\frac{1}{2}\right)]{E_{12}\left(-\frac{1}{2}\right)}\begin{pmatrix}2&1&1\\0&\frac{3}{2}&\frac{1}{2}\\0&\frac{1}{2}&-\frac{1}{2}\end{pmatrix}\xrightarrow{E_{23}\left(-\frac{1}{3}\right)}\begin{pmatrix}2&1&1\\0&\frac{3}{2}&\frac{1}{2}\\0&0&-\frac{2}{3}\end{pmatrix}$$

$$\mathbf{U}=E_{23}\left(-\frac{1}{3}\right)E_{13}\left(-\frac{1}{2}\right)E_{12}\left(-\frac{1}{2}\right)A=\begin{pmatrix}2&1&1\\0&\frac{3}{2}&\frac{1}{2}\\0&0&-\frac{2}{3}\end{pmatrix}$$
$$\mathbf{L}=E_{12}\left(\frac{1}{2}\right)E_{13}\left(\frac{1}{2}\right)E_{23}\left(\frac{1}{3}\right)I=\begin{pmatrix}1&0&0\\\frac{1}{2}&1&0\\\frac{1}{2}&\frac{1}{3}&1\end{pmatrix}$$
$$A=\mathbf{LU}=\begin{pmatrix}1&0&0\\\frac{1}{2}&1&0\\\frac{1}{2}&\frac{1}{3}&1\end{pmatrix}\begin{pmatrix}2&1&1\\0&\frac{3}{2}&\frac{1}{2}\\0&0&-\frac{2}{3}\end{pmatrix}$$

注意！在对角化的过程中一定不要使用互换初等阵 E_{ij} ，且倍加初等阵 $E_{ij}(k)$ 的 i 一定要小于 j ，不然会出现如下所示的情况！

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{pmatrix} \xrightarrow[E_{13}\left(-\frac{1}{2}\right)]{E_{12}\left(-\frac{1}{2}\right)} \begin{pmatrix} 2 & 1 & 1 \\ 0 & \frac{3}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \xrightarrow{E_{23}\left(-\frac{1}{3}\right)} \begin{pmatrix} 2 & 1 & 1 \\ 0 & \frac{3}{2} & \frac{1}{2} \\ 0 & 0 & -\frac{2}{3} \end{pmatrix}$$

$$\xrightarrow[E_{32}\left(-\frac{1}{2}\right)]{E_{3}\left(-\frac{3}{2}\right)} \begin{pmatrix} 2 & 1 & 1 \\ 0 & \frac{3}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow[E_{21}(-1)]{E_{2}\left(\frac{2}{3}\right)} \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow[E_{1}\left(-\frac{1}{2}\right)]{E_{31}(-1)} I$$

$$\mathbf{U} = E_1\left(-\frac{1}{2}\right) E_{31}(-1) E_{21}(-1) E_2\left(\frac{2}{3}\right) E_{32}\left(-\frac{1}{2}\right)$$
$$E_3\left(-\frac{3}{2}\right) E_{23}\left(-\frac{1}{3}\right) E_{13}\left(-\frac{1}{2}\right) E_{12}\left(-\frac{1}{2}\right) A = I$$
$$\mathbf{L} = E_{12}\left(\frac{1}{2}\right) E_{13}\left(\frac{1}{2}\right) E_{23}\left(\frac{1}{3}\right) E_3\left(-\frac{2}{3}\right)$$
$$E_{32}\left(\frac{1}{2}\right) E_2\left(\frac{3}{2}\right) E_{21}(1) E_{31}(1) E_1(-2) I = A$$
$$A = \mathbf{LU} = AI$$

习题3.求对称正定矩阵

$$A = \begin{bmatrix} 5 & 2 & -4 \\ 2 & 1 & -2 \\ -4 & -2 & 5 \end{bmatrix}$$

的不带平方根的$Cholesky$分解。

$$\mathbf{L} = \begin{pmatrix} \sqrt{A_{11}} & 0 & 0 \\ A_{21}/L_{11} & \sqrt{A_{22} - L_{21}^2} & 0 \\ A_{31}/L_{11} & (A_{32} - L_{31}L_{21})/L_{22} & \sqrt{A_{33} - L_{31}^2 - L_{32}^2} \end{pmatrix}$$

L的每个元素服从：

$$L_{j,j} = (\pm)\sqrt{A_{j,j} - \sum_{k=1}^{j-1} L_{j,k}^2}$$

$$L_{i,j} = \frac{1}{L_{j,j}}\left(A_{i,j} - \sum_{k=1}^{j-1} L_{i,k} L_{j,k}\right) \quad \text{for } i > j.$$

$$\text{先求}G\text{第一列元素：}\begin{pmatrix} \sqrt{5} & & \\ \frac{2}{\sqrt{5}} & & \\ -\frac{4}{\sqrt{5}} & & \end{pmatrix}$$

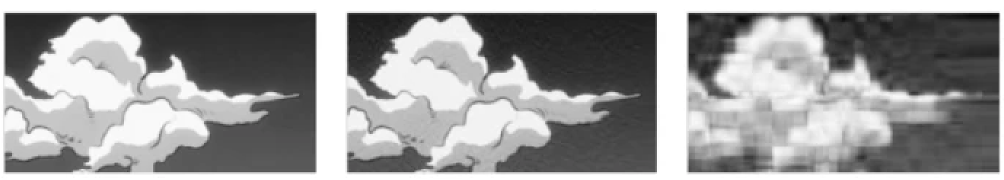
$$\text{第二列：}\begin{pmatrix} \sqrt{5} & & \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \\ -\frac{4}{\sqrt{5}} & -\frac{2}{\sqrt{5}} & \end{pmatrix}$$

$$\text{第三列：}\begin{pmatrix} \sqrt{5} & & \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \\ -\frac{4}{\sqrt{5}} & -\frac{2}{\sqrt{5}} & 1 \end{pmatrix}$$

$$A = \mathbf{L}\mathbf{L}^T = \begin{pmatrix} \sqrt{5} & & \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \\ -\frac{4}{\sqrt{5}} & -\frac{2}{\sqrt{5}} & 1 \end{pmatrix} \begin{pmatrix} \sqrt{5} & \frac{2}{\sqrt{5}} & -\frac{4}{\sqrt{5}} \\ & \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ & & 1 \end{pmatrix}$$

实操部分

习题4.将一张图片（图片自选）利用奇异值分解完成图像的压缩。示例图如下：



(a) 原图　(b) 提取 50 个奇异值的图像　(c) 提取 10 个奇异值的图像

图 1: 使用 SVD 进行图像压缩

```python
import cv2
```

```python
import numpy as np
def restore(u, sigma, v, k):
    # 重构图像
    print(u,sigma,v,k)
    a = np.dot(u[:, :k], np.diag(sigma[:k])).dot(v[:k, :])
    # 上述语句等价于：
    # for i in range(k):
    #     ui = u[:, i].reshape(m, 1)
    #     vi = v[i].reshape(1, n)
    #     a += sigma[i] * np.dot(ui, vi)
    a[a < 0] = 0
    a[a > 255] = 255
    return np.rint(a).astype('uint8')

def compression(origin,k):
    image=np.array(origin)
    for i in range(3):
        u,sigma,v=np.linalg.svd(image[:,:,i])
        image[:,:,i]=restore(u,sigma,v,k)
    return image

origin=cv2.imread('dase.png')
result=compression(origin,10)
cv2.imwrite('dase_swd_10.jpg',result)
result=compression(origin,50)
cv2.imwrite('dase_swd_50.jpg',result)
```

Python

原图

提取 50 个奇异值的图像

提取 10 个奇异值的图像