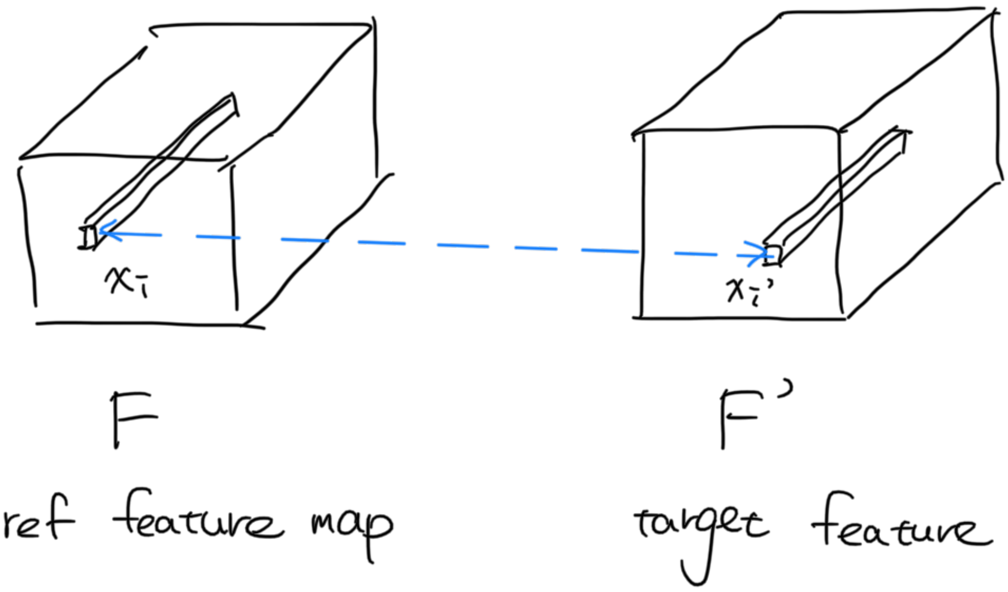
GN-Net Note:



$$\vec{r_i} = F'(x_i') - F(x_i) \quad \text{feature residual}$$

Background you should know before you go:

- Jacobian matrix

$$J = \left[\frac{\partial \vec{f}}{\partial x_1}, \frac{\partial \vec{f}}{\partial x_2}, \cdots, \frac{\partial \vec{f}}{\partial x_n} \right]$$

$$= \underbrace{\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \cdots & \frac{\partial f_2}{\partial x_n} \\ & \vdots & \\ & & \end{bmatrix}}_{n} \Bigg\} m$$

$$= [J_1, J_2, \cdots, J_n]$$

or

$$= \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_m \end{bmatrix}$$

$$J_1 \rightarrow \left(\frac{\partial f_1}{\partial x_1}, \frac{\partial f_1}{\partial x_2}, \cdots, \frac{\partial f_1}{\partial x_n} \right) = \frac{df_1}{d\vec{x}}$$

- Hessian matrix $\quad H_{ij} = \dfrac{\partial^2 \vec{f}}{\partial x_i \partial x_j}$

- Relation between them

$$J^T J = \begin{bmatrix} \frac{\partial \vec{f}}{\partial x_1} \\ \vdots \\ \frac{\partial \vec{f}}{\partial x_n} \end{bmatrix} \begin{bmatrix} \frac{\partial \vec{f}}{\partial x_1} & \cdots & \frac{\partial \vec{f}}{\partial x_n} \end{bmatrix}$$

$$= \left[\frac{\partial^2 \vec{f}}{\partial x_i \partial x_j} \right]_{ij} = H$$

or

$$J^T J = [J_1^T, J_2^T, \cdots, J_m^T] \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_m \end{bmatrix}$$

$$= \underbrace{J_1^T J_1}_{H_1} + \underbrace{J_2^T J_2}_{H_2} + \cdots + \underbrace{J_m^T J_m}_{H_m} = H$$

(This paper follows this "blue" convention)

$$\underset{\text{jacobian element}}{J_i} = \frac{dr_i}{d\xi} = \frac{dF'(x_i')}{d\xi} - \frac{dF(x_i)}{d\xi}$$

relative pose

$$= \frac{dF'(x_i')}{dx_i'} \cdot \frac{dx_i'}{d\xi} - \frac{dF(x_i)}{dx_i} \cdot \cancel{\frac{dx_i}{d\xi}}^{0}$$

ref point does not change

$$= \underbrace{\frac{dF'(x_i')}{dx_i'}} \cdot \frac{dx_i'}{d\xi}$$

Hey! How about do it on location (x_i') directly?

$$J_i' = \frac{dF'(x_i')}{dx_i'}$$

Since we know

$$J_i^T J_i = H_i$$

We can derive

$$J_i^T J_i = \left(\frac{dx_i'}{d\xi}\right)^T \underbrace{J_i'^T J_i'}_{H_i'} \cdot \left(\frac{dx_i'}{d\xi}\right) = H_i'$$

So

$$H = \sum_i \left(\frac{dx_i'}{d\xi}\right)^T H_i' \left(\frac{dx_i'}{d\xi}\right)$$

consider only spatial location, no pose info.

both pose and location are considered.

The authors conclude this is why

good matches $\rightarrow$ good pose

since the pose can be correct and the matches could be wrong

How can we replace covariance matrix into Hessian matrix? Any limitation?

$$\mathrm{prob}(x) = (2\pi)^{-\frac{1}{2}} \, |\Sigma_x|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(x-\mu)\Sigma_x^{-1}(x-\mu)\right]$$

covariance matrix

$$E(x) = -\log \mathrm{prob}(x) = \frac{1}{2}\log(2\pi \cdot |\Sigma_x|) + \frac{1}{2}(x-\mu)\Sigma_x^{-1}(x-\mu)$$

unknown function

$$H_E^{(i,j)}(\mu) = \left.\frac{\partial^2 E(x)}{\partial x_i \partial x_j}\right|_{x=\mu} = \Sigma_x^{-1\,(i,j)}$$

But if

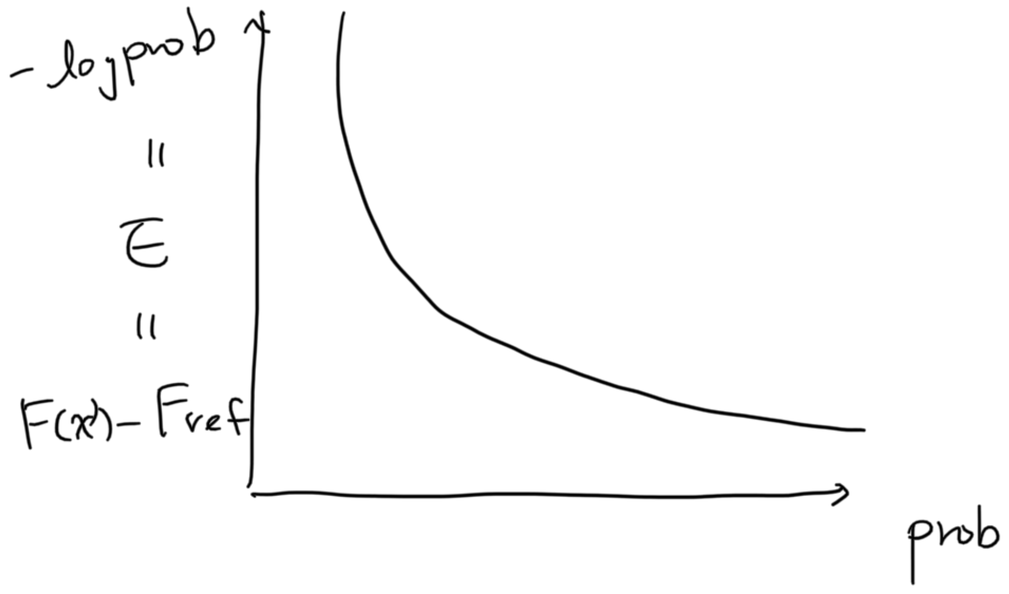
$$\boxed{E = F(x') - F_{ref}(x) = -\log \mathrm{prob}(x')}$$

$$H_E(x') = \frac{\partial^2 (F(x') - F_{ref}(x))}{\partial x_i' \partial x_j'} = \frac{\partial^2 F(x')}{\partial x_i' \partial x_j'} = \Sigma_{x'}^{-1}$$

which means

minizing error E $\Rightarrow$ prob ↺

(align the correct matches)



- Gauss-Newton

unknown function : $r(\beta)$

solution: $\beta^{(s+1)} = \beta^{(s)} - (J_r^T J_r)^{-1} J_r^T r$

where $J_{r,ij} = \dfrac{\partial r_i(\beta^{(s)})}{\partial \beta_j}$

Now we define such unknown function as

$$r(\mu) = -\log prob(\mu) = E = F(p') - F(p)$$

$$\mu = \mu - H^{-1} b$$

$H^{-1} \rightarrow (J^T J)^{-1}$, $b \rightarrow J^T r$

$$J_\mu = \frac{\partial}{\partial \mu} (-\log prob(\mu))$$

the link

$$= \frac{\partial}{\partial \mu} \left(F'(x') - F(x) \right)$$

$$= \frac{\partial F'(x')}{\partial \mu} \approx \frac{\partial F'(x')}{\partial x'} = J_F$$

This is why you can use the

Jacobian of feature : J_F

to replace

Jacobian of μ : J_μ

and use Gauss-Newton to update μ.

If you don't have "the link", you can still do Gauss-Newton, but you cannot have a relation with the feature we want to optimize.