法律声明

- 课程详情请咨询
  - 微信公众号：小象学院
  - 新浪微博：小象AI学院

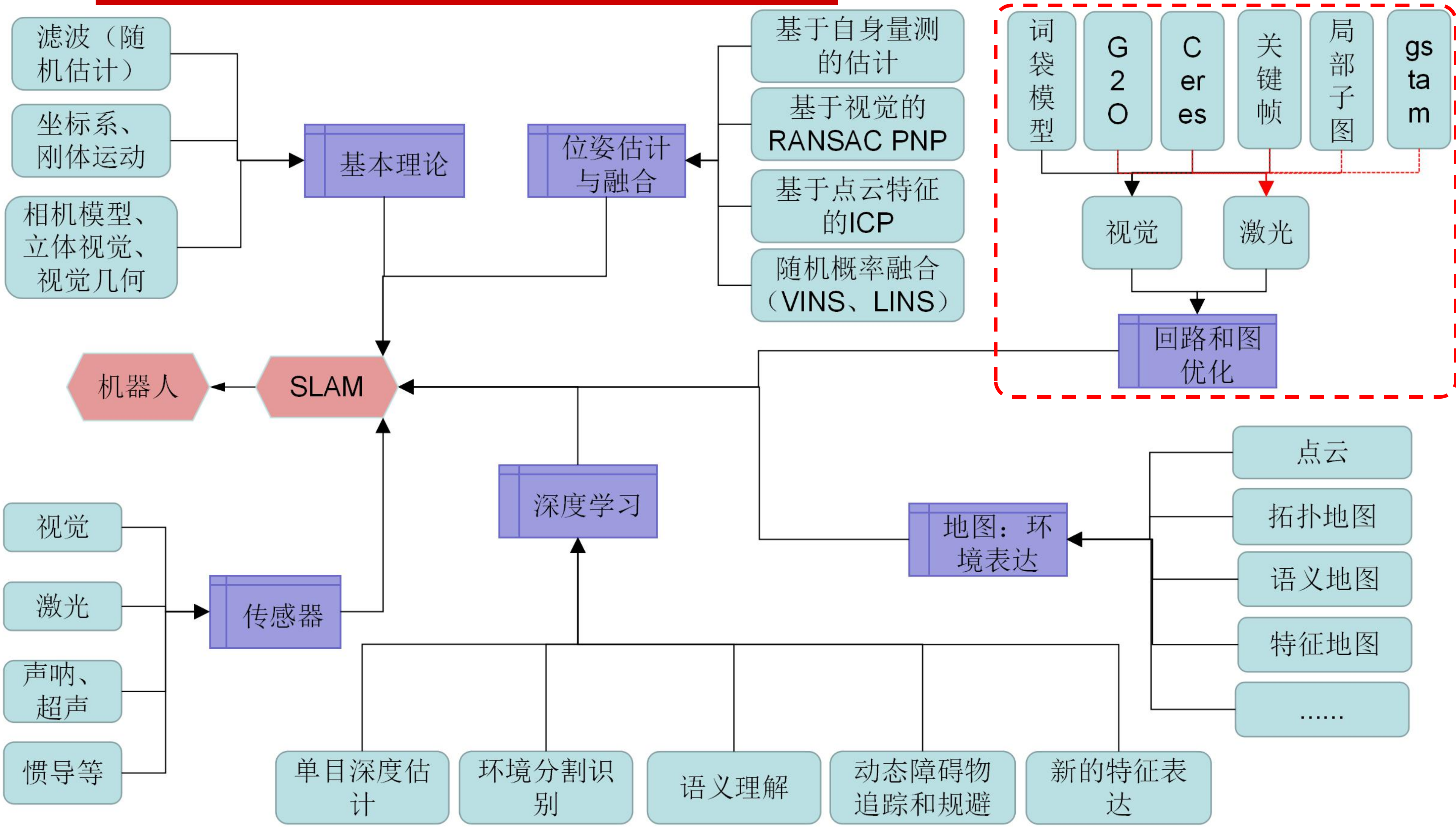
第四讲：

SLAM基本理论三：图优化

主讲：杨亮

GitHub链接：https://github.com/EricLYang/courseRepo

总结



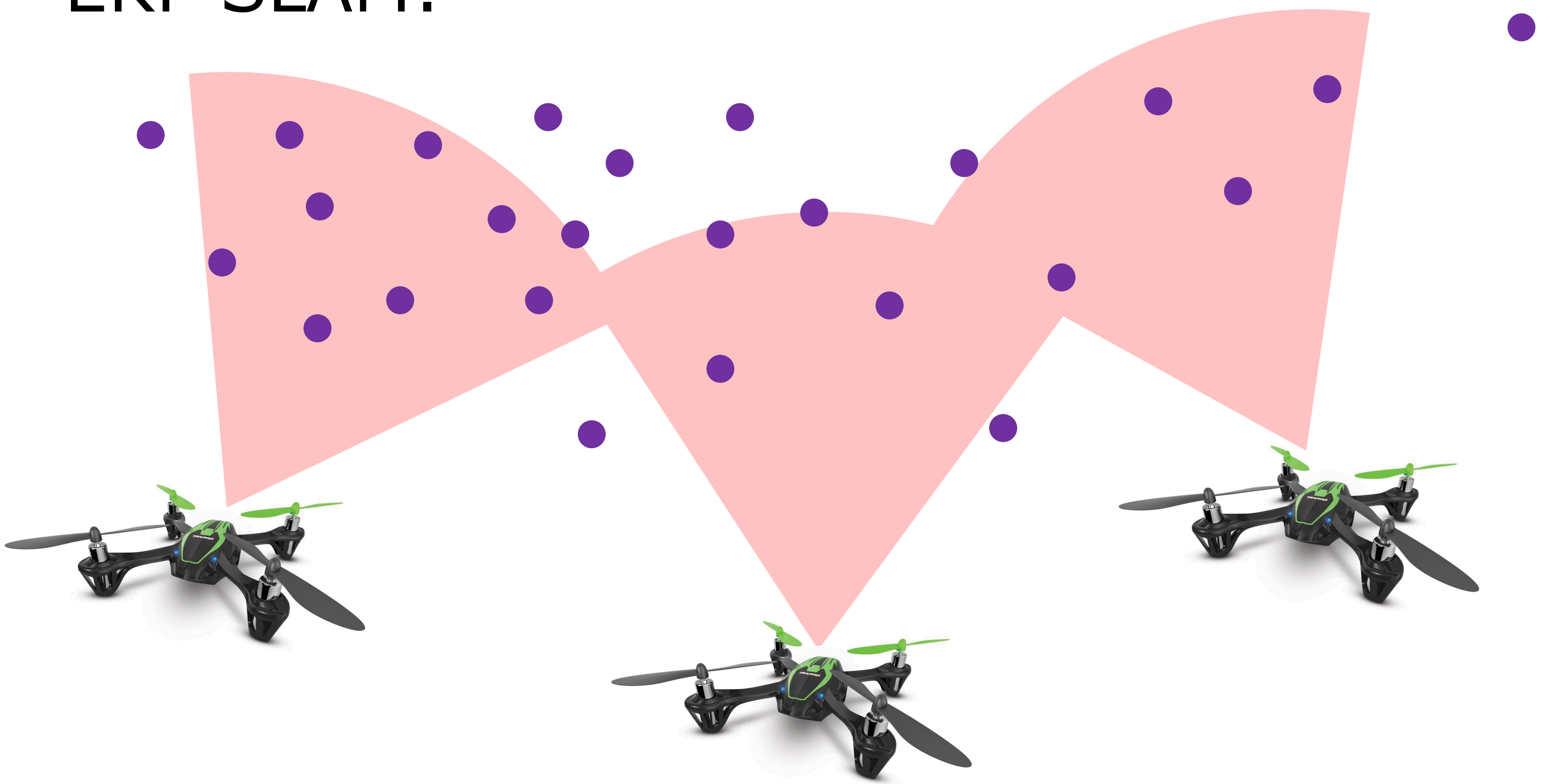
滤波（随机估计）
坐标系、刚体运动
相机模型、立体视觉、视觉几何
基本理论
位姿估计与融合
基于自身量测的估计
基于视觉的RANSAC PNP
基于点云特征的ICP
随机概率融合（VINS、LINS）
词袋模型
G2O
Ceres
关键帧
局部子图
gstam
视觉
激光
回路和图优化
机器人
SLAM
深度学习
地图：环境表达
点云
拓扑地图
语义地图
特征地图
……
视觉
激光
声呐、超声
惯导等
传感器
单目深度估计
环境分割识别
语义理解
动态障碍物追踪和规避
新的特征表达

提纲

- 从滤波器的痛来谈图优化
- Covisibility Graph和最小二乘
- 浅谈Marginlization
- 实例：G2O图优化实战

从滤波器的痛来谈图优化

EKF SLAM:



不仅要维护自身的状态，还需要维护地图（特征）

从滤波器的痛来谈图优化

机器人的状态：

$$x_t = (\underbrace{x, y, \theta}_{\text{robot's pose}}, \underbrace{m_{1,x}, m_{1,y}}_{\text{landmark 1}}, \ldots, \underbrace{m_{n,x}, m_{n,y}}_{\text{landmark n}})^T$$

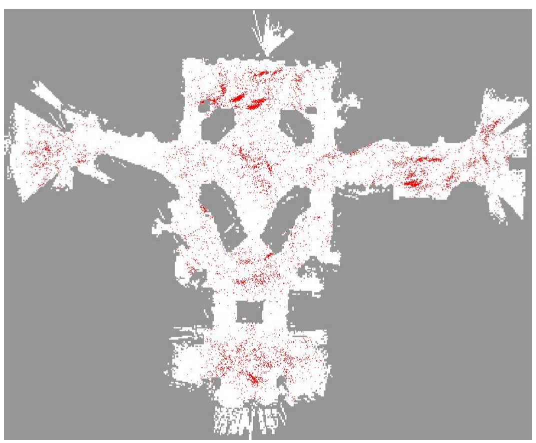
状态的方差矩阵（置信）：

$$\underbrace{\begin{pmatrix} x \\ y \\ \theta \\ m_{1,x} \\ m_{1,y} \\ \vdots \\ m_{n,x} \\ m_{n,y} \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xm_{1,x}} & \sigma_{xm_{1,y}} & \cdots & \sigma_{xm_{n,x}} & \sigma_{xm_{n,y}} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{y\theta} & \sigma_{ym_{1,x}} & \sigma_{ym_{1,y}} & \cdots & \sigma_{m_{n,x}} & \sigma_{m_{n,y}} \\ \sigma_{\theta x} & \sigma_{\theta y} & \sigma_{\theta\theta} & \sigma_{\theta m_{1,x}} & \sigma_{\theta m_{1,y}} & \cdots & \sigma_{\theta m_{n,x}} & \sigma_{\theta m_{n,y}} \\ \sigma_{m_{1,x}x} & \sigma_{m_{1,x}y} & \sigma_{\theta} & \sigma_{m_{1,x}m_{1,x}} & \sigma_{m_{1,x}m_{1,y}} & \cdots & \sigma_{m_{1,x}m_{n,x}} & \sigma_{m_{1,x}m_{n,y}} \\ \sigma_{m_{1,y}x} & \sigma_{m_{1,y}y} & \sigma_{\theta} & \sigma_{m_{1,y}m_{1,x}} & \sigma_{m_{1,y}m_{1,y}} & \cdots & \sigma_{m_{1,y}m_{n,x}} & \sigma_{m_{1,y}m_{n,y}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \sigma_{m_{n,x}x} & \sigma_{m_{n,x}y} & \sigma_{\theta} & \sigma_{m_{n,x}m_{1,x}} & \sigma_{m_{n,x}m_{1,y}} & \cdots & \sigma_{m_{n,x}m_{n,x}} & \sigma_{m_{n,x}m_{n,y}} \\ \sigma_{m_{n,y}x} & \sigma_{m_{n,y}y} & \sigma_{\theta} & \sigma_{m_{n,y}m_{1,x}} & \sigma_{m_{n,y}m_{1,y}} & \cdots & \sigma_{m_{n,y}m_{n,x}} & \sigma_{m_{n,y}m_{n,y}} \end{pmatrix}}_{\Sigma}$$

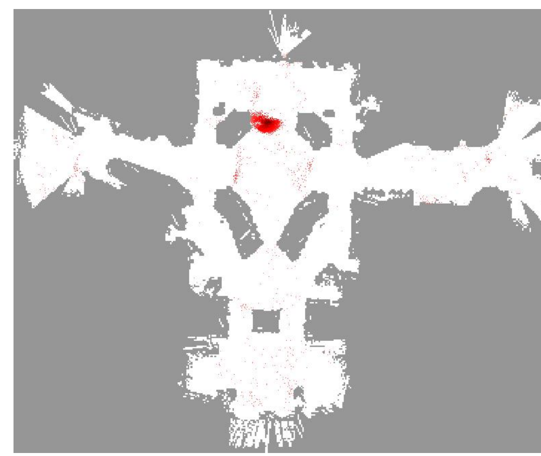
从滤波器的痛来谈图优化

粒子滤波器：

Particle 1	x, y, θ	Landmark 1	Landmark 2	...	Landmark M
Particle 2	x, y, θ	Landmark 1	Landmark 2	...	Landmark M
⋮					
Particle N	x, y, θ	Landmark 1	Landmark 2	...	Landmark M

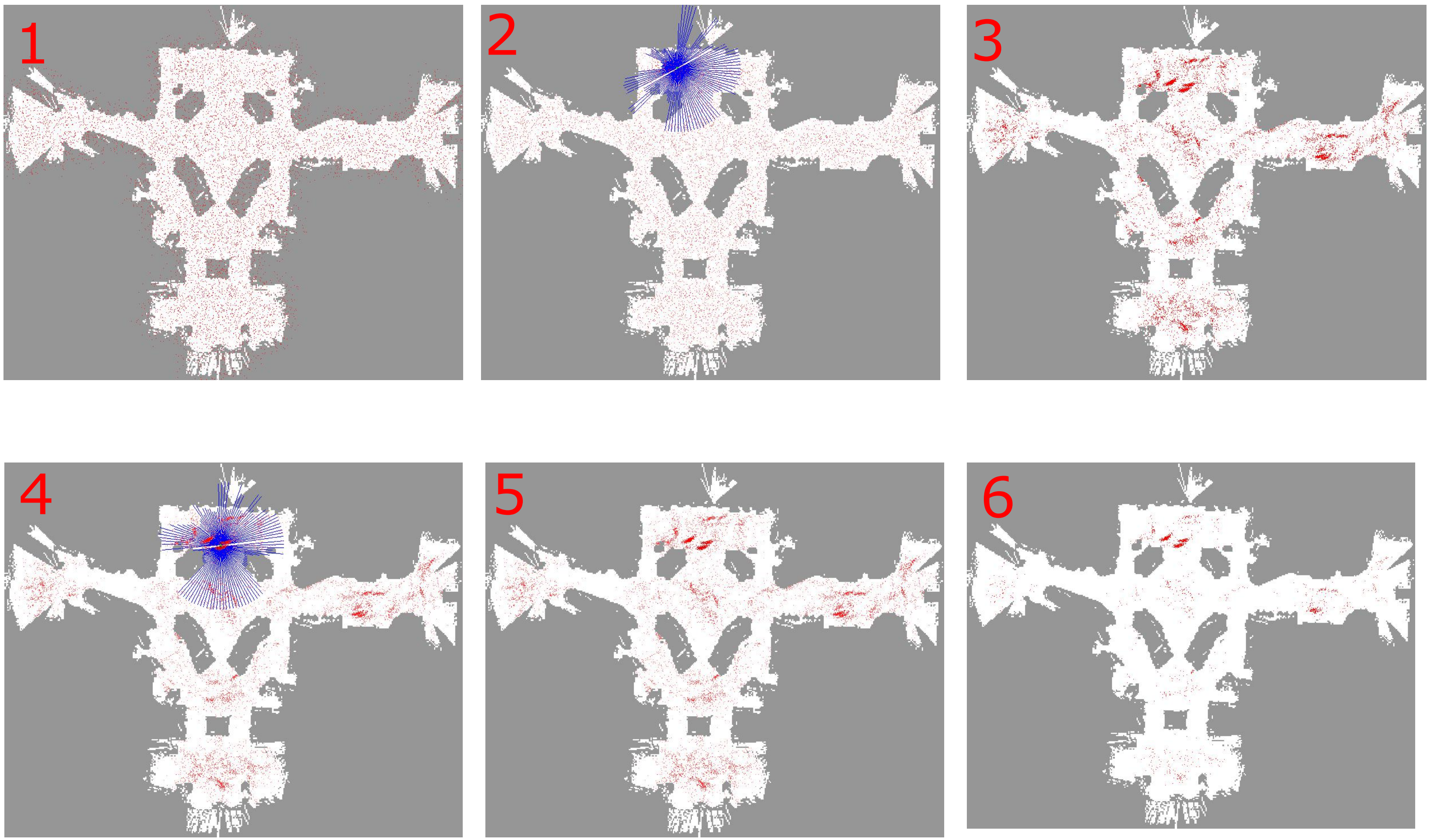


大量的随机采样



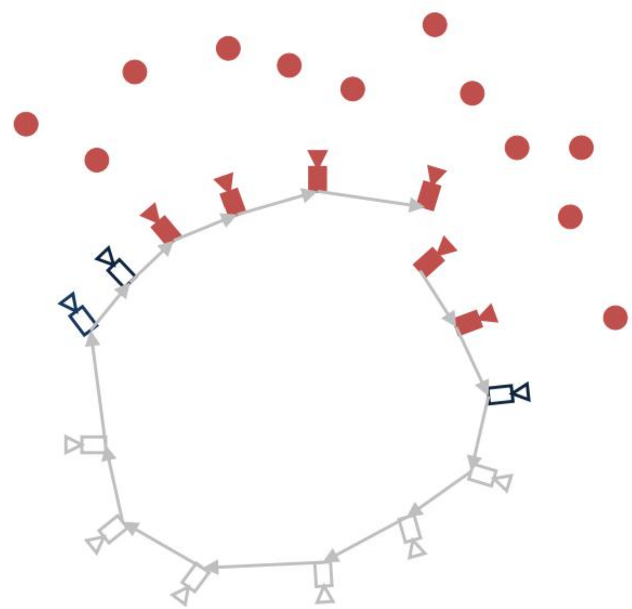
从滤波器的痛来谈图优化

如何解决收敛过慢？？？以及粒子退化？？



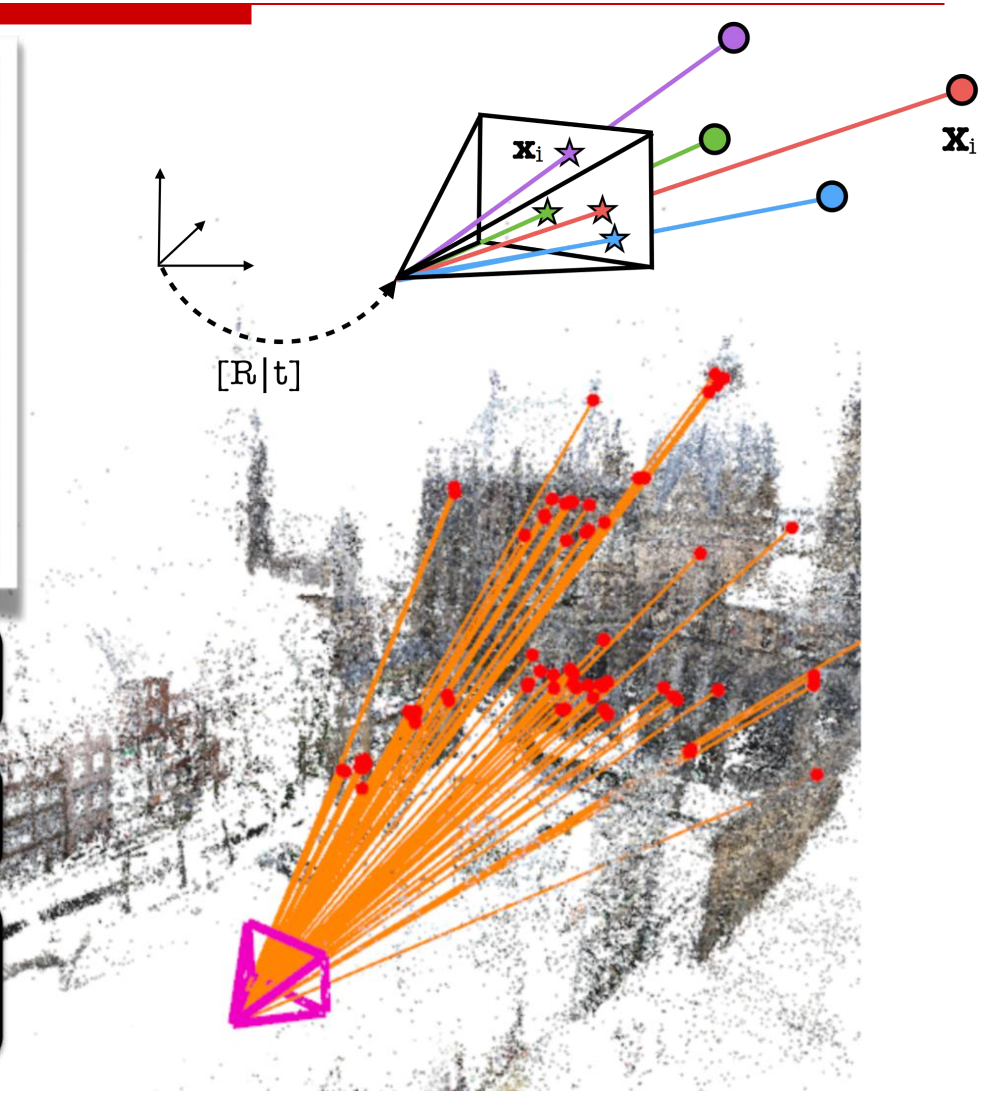
从滤波器的痛来谈图优化

EKF---FAST：　更尴尬的是：如何检测回路？？



想想：他们都是增量式计算！！！

对于基于视觉的单步定位



Extract Local Features

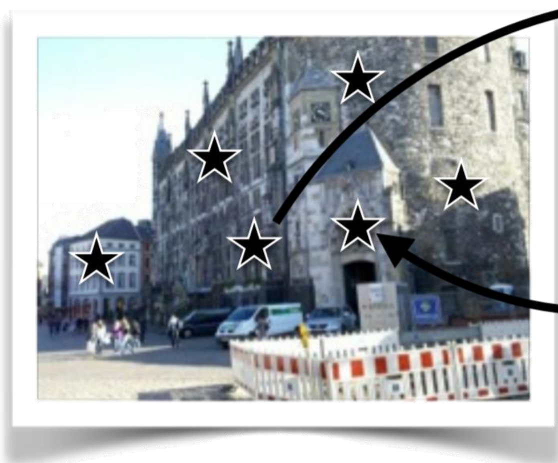
Establish 2D-3D Matches

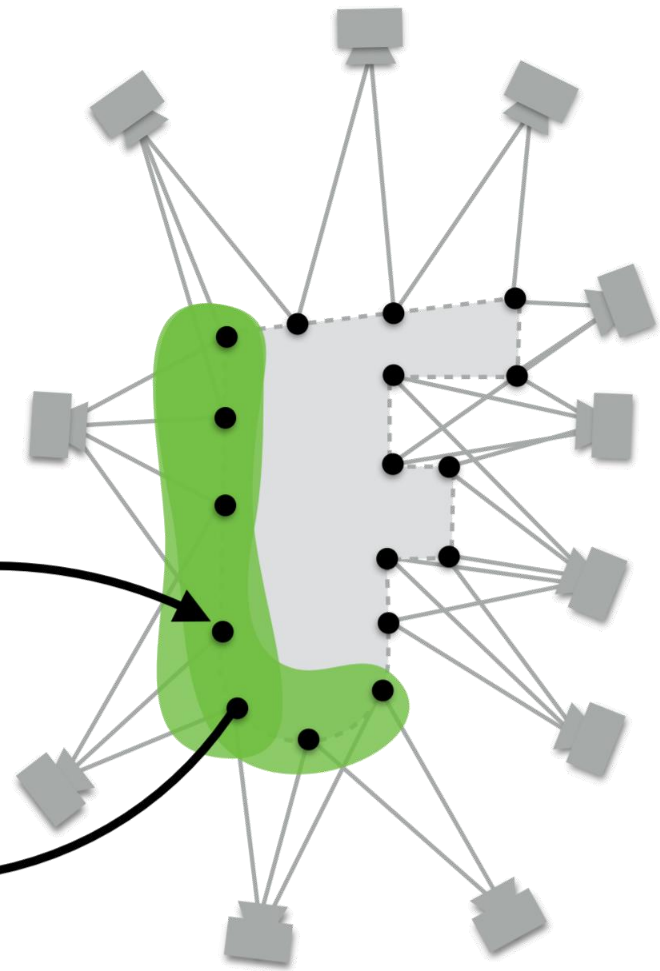
Camera Pose Estimation:
RANSAC + n-Point-Pose Algorithm

为什么叫图

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

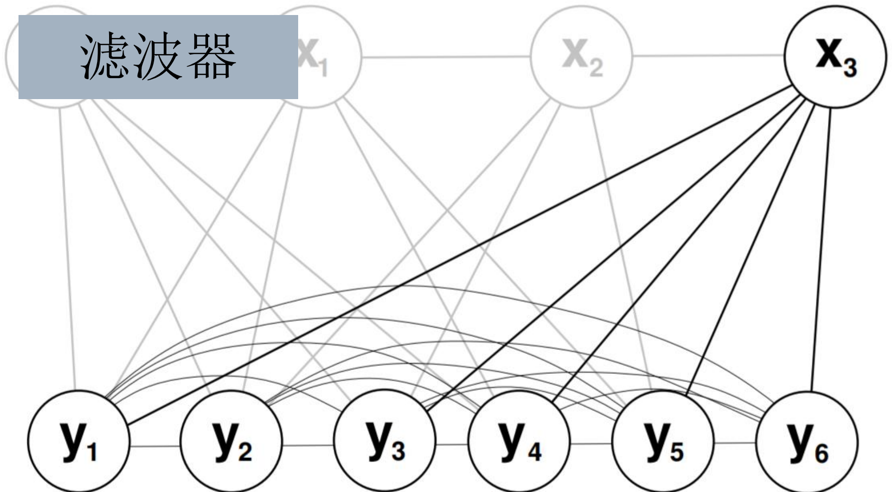
PnP $\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K\boxed{[R \mid t]} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$ 位姿



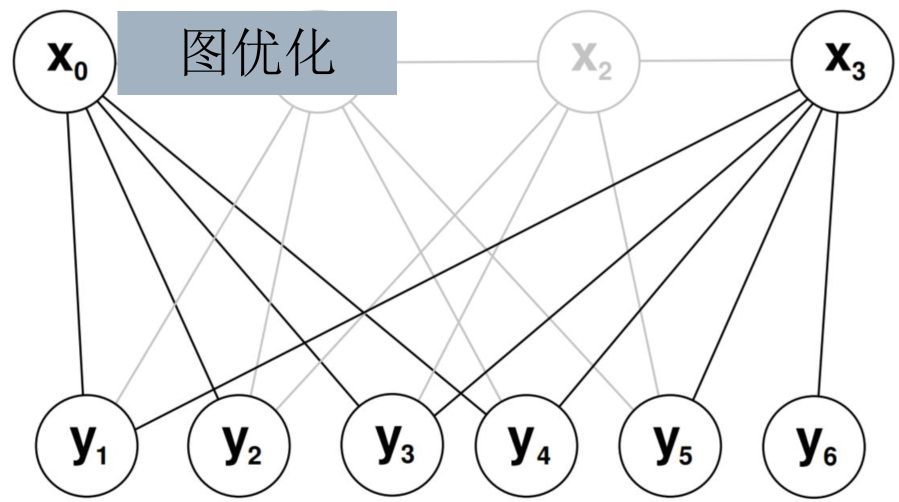


我见过别人
也曾见过

从滤波器的痛来谈图优化

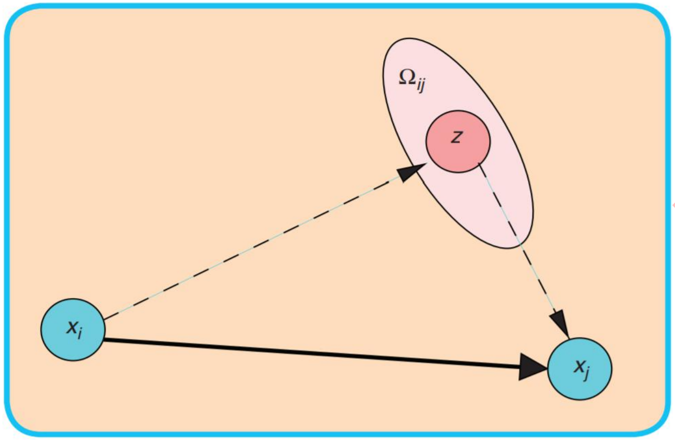


约束太复杂：我们来点简单的



图是怎么回事？

SLAM有运动模型



观测得到地图特征

x

$$h_1(x) = \hat{z}_1 \qquad z_1$$

$$h_2(x) = \hat{z}_2 \qquad z_2$$

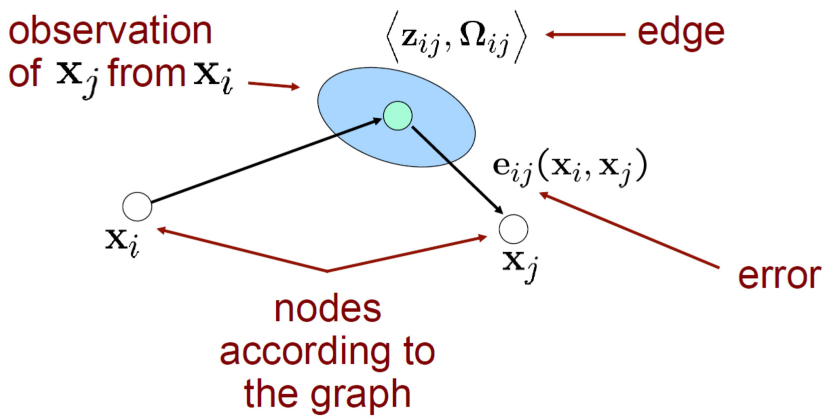
$$h_n(x) = \hat{z}_n \qquad z_n$$

$$e_i(x) = z_i - h(x_i)$$

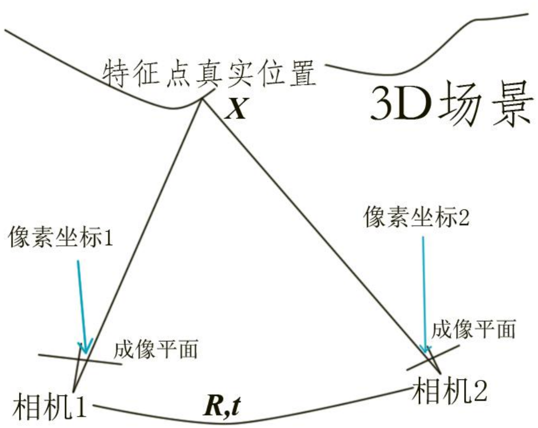
$$e_i(x) = e_i(x)^T \Omega e_i(x)$$

状态 预测 观测

还有那些表示？



$$
\begin{aligned}
\mathbf{x}^* &= \operatorname*{argmin}_{\mathbf{x}} \sum_{ij} \mathbf{e}_{ij}^T(\mathbf{x}_i, \mathbf{x}_j)\mathbf{\Omega}_{ij}\mathbf{e}_{ij}(\mathbf{x}_i, \mathbf{x}_j) \\
&= \operatorname*{argmin}_{\mathbf{x}} \sum_{k} \mathbf{e}_{k}^T(\mathbf{x})\mathbf{\Omega}_{k}\mathbf{e}_{k}(\mathbf{x})
\end{aligned}
$$

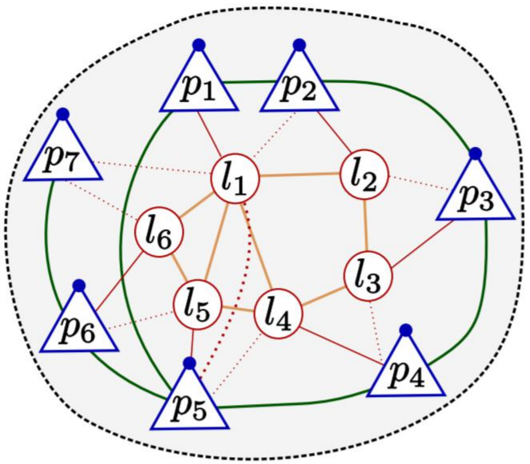


$$
\min_{X^j,R,t} \left\| \frac{1}{\lambda_1} C X^j - \left[z_1^j, 1\right]^T \right\|^2 + \left\| \frac{1}{\lambda_2} C \left(R X^j + t\right) - \left[z_2^j, 1\right]^T \right\|^2
$$

SLAM十四讲

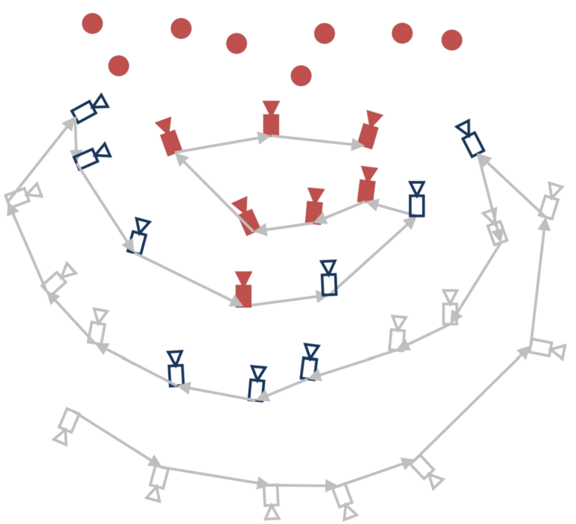
图是怎么回事？

那什么是图呢？



- 状态、特征点作为顶点
- 可见链接作为边
- 边的权重---> 不确定性、共同可见特征点数

如何决定有没有链接？？



- 有相同的特征点：还得足数！

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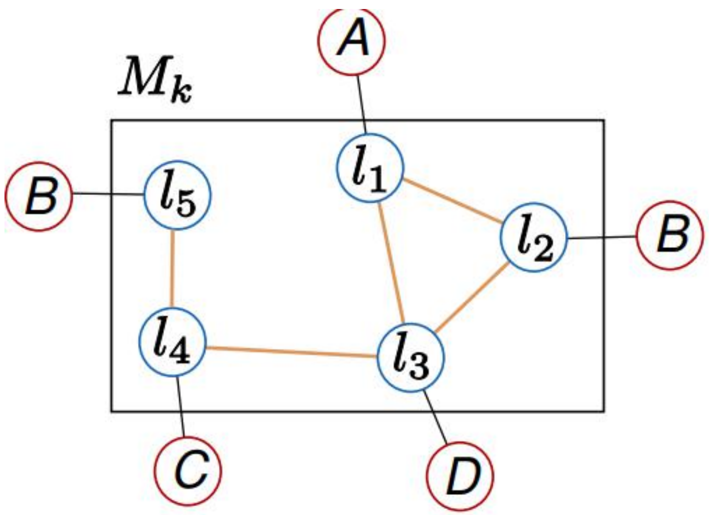
Covisibility Graph和最小二乘

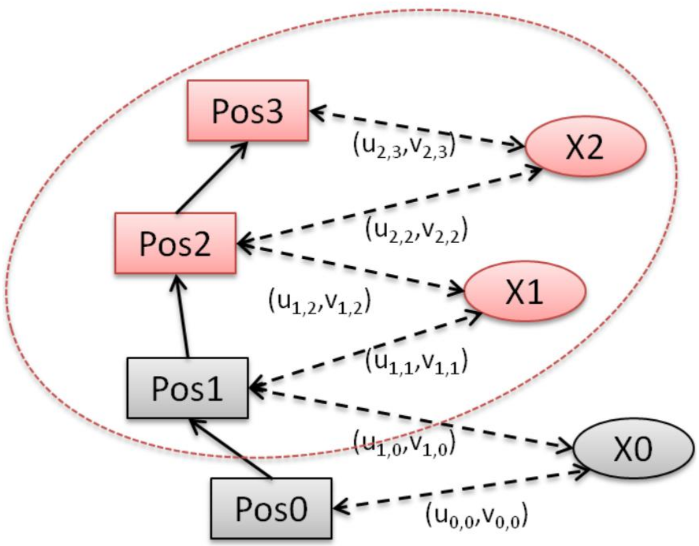
定义：
由一系列的可以相互可见的标志点和状态组成的无向链接。



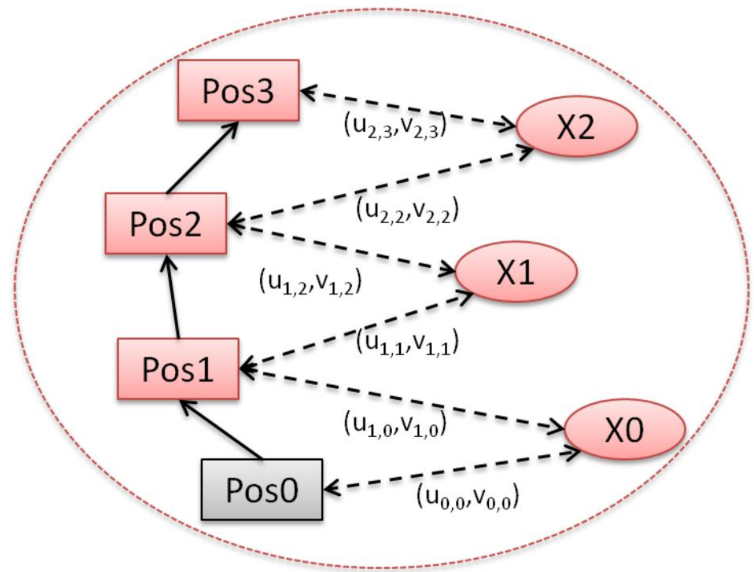
$$
\begin{aligned}
\mathbf{F}(\mathbf{x}) &= \sum_{\langle i,j\rangle\in\mathcal{C}} \underbrace{\mathbf{e}(\mathbf{x}_i,\mathbf{x}_j,\mathbf{z}_{ij})^\top \boldsymbol{\Omega}_{ij}\mathbf{e}(\mathbf{x}_i,\mathbf{x}_j,\mathbf{z}_{ij})}_{\mathbf{F}_{ij}} \\
\mathbf{x}^* &= \operatorname*{argmin}_{\mathbf{x}} \mathbf{F}(\mathbf{x}).
\end{aligned}
$$

Mei, C., Sibley, G., & Newman, P. (2010, October). Closing loops without places. In Intelligent Robots and Systems (IROS), 2010 IEEE/RSJ International Conference on (pp. 3738-3744). IEEE.

Covisibility Graph和最小二乘



Metric Loop Closure：
只和当前帧相关



Large-scale Loop Closure：
全局矫正

Strasdat, H., Davison, A. J., Montiel, J. M., & Konolige, K. (2011, November). Double window optimisation for constant time visual SLAM. In Computer Vision (ICCV), 2011 IEEE International Conference on (pp. 2352-2359). IEEE.

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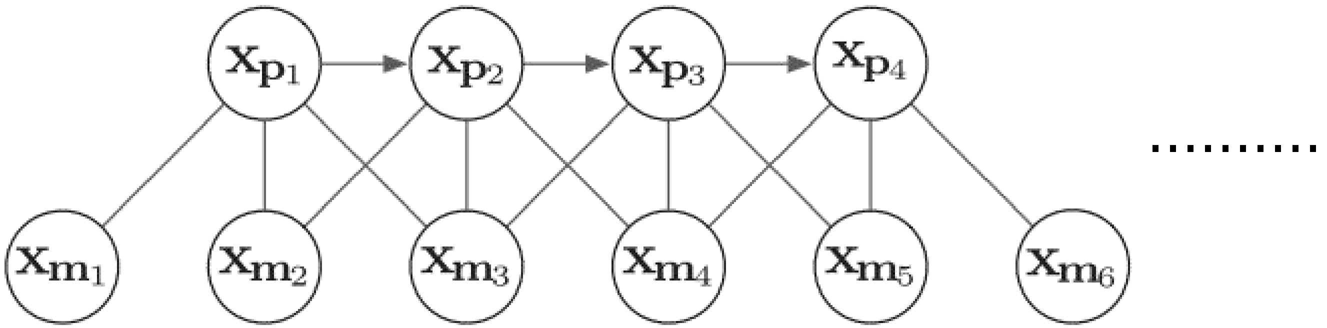
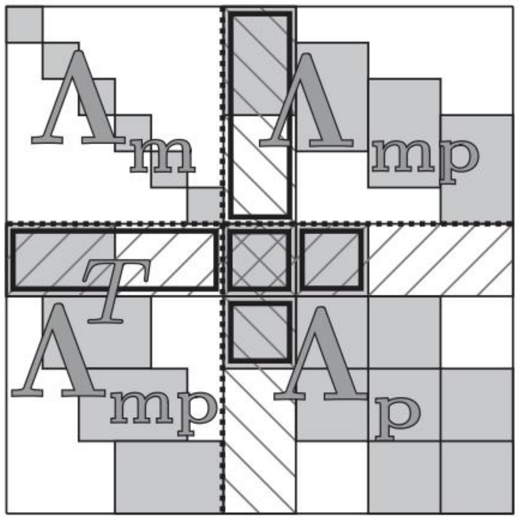
浅谈Marginlization

状态：

特征：





关系矩阵越来越 Sparse

浅谈Marginlization

Schur complement:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \xrightarrow{\text{Block A}} M/A := D - CA^{-1}B$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} I & 0 \\ CA^{-1} & I \end{bmatrix} \begin{bmatrix} A & 0 \\ 0 & \Delta_{\mathbf{A}} \end{bmatrix} \begin{bmatrix} I & A^{-1}B \\ 0 & I \end{bmatrix}$$

$$\Delta_{\mathbf{A}} = D - CA^{-1}B$$

状态a和状态b $\mathbf{x} = \begin{bmatrix} a \\ b \end{bmatrix}$

协方差矩阵 $\mathbf{K} = \begin{bmatrix} A & C^T \\ C & D \end{bmatrix}$

浅谈Marginlization

$$P(a,b) \propto \exp\left(-\frac{1}{2}\begin{bmatrix}a\\b\end{bmatrix}^T\begin{bmatrix}A & C^T\\C & D\end{bmatrix}^{-1}\begin{bmatrix}a\\b\end{bmatrix}\right)$$

$$\propto \exp\left(-\frac{1}{2}\begin{bmatrix}a\\b\end{bmatrix}^T\begin{bmatrix}I & -A^{-1}C^T\\0 & I\end{bmatrix}\begin{bmatrix}A^{-1} & 0\\0 & \Delta_{\mathbf{A}}^{-1}\end{bmatrix}\begin{bmatrix}I & 0\\-CA^{-1} & I\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix}\right)$$

$$\propto \exp\left(-\frac{1}{2}\begin{bmatrix}a & b-A^{-1}C^Tb\end{bmatrix}\begin{bmatrix}A^{-1} & 0\\0 & \Delta_{\mathbf{A}}^{-1}\end{bmatrix}\begin{bmatrix}a\\b-CA^{-1}a\end{bmatrix}\right)$$

$$\propto \exp\left(-\frac{1}{2}\left(a^TA^{-1}a\right)+(b-A^{-1}C^Tb)^T\Delta_{\mathbf{A}}^{-1}(b-A^{-1}C^Tb)\right)$$

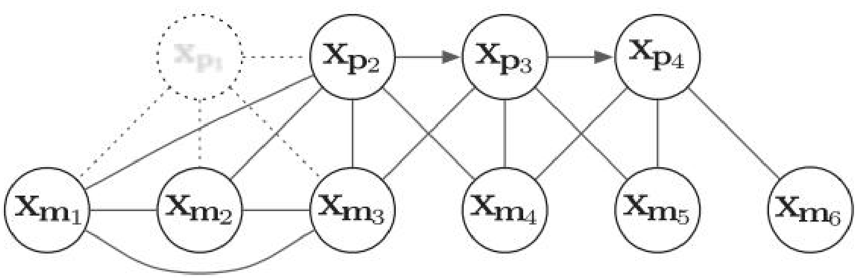
$$\propto \underbrace{\exp\left(-\frac{1}{2}a^TA^{-1}a\right)}_{P(a)}\underbrace{\exp\left(-\frac{1}{2}(b-A^{-1}C^Tb)^T\Delta_{\mathbf{A}}^{-1}(b-A^{-1}C^Tb)\right)}_{P(b)}$$

浅谈Marginlization

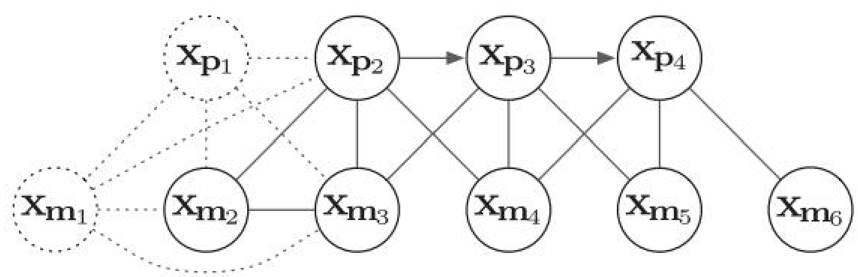
$$\propto \underbrace{\exp\left(-\frac{1}{2}a^T A^{-1} a\right)}_{P(a)} \underbrace{\exp\left(-\frac{1}{2}(b - A^{-1}C^T b)^T \Delta_{\mathbf{A}}^{-1}(b - A^{-1}C^T b)\right)}_{P(b)}$$

我们可以去掉状态a,因为b的信息里面已经包含了a

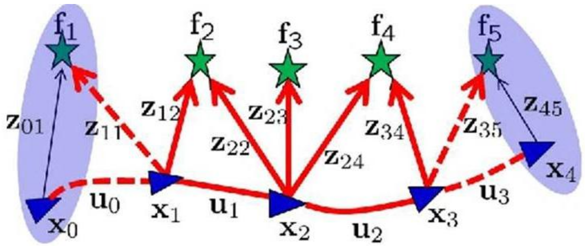
去掉状态



去掉特征

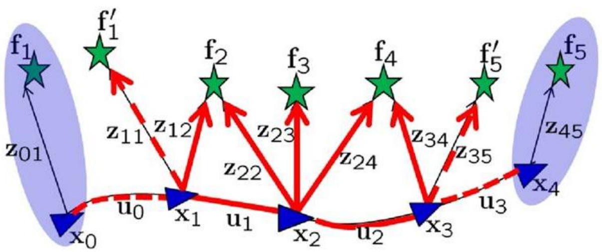


浅谈Marginlization



$$\min \; \mathcal{C}_1(\mathbf{x}_0, \mathbf{x}_4, \mathbf{f}_1, \mathbf{f}_5; z_{01}, z_{45}) + \mathcal{C}_2(\mathbf{x}_{1:3}, \mathbf{f}_{2:4}, \mathbf{x}_0, \mathbf{x}_4, \mathbf{f}_1, \mathbf{f}_5; \mathcal{Z}_{1:3}, u_{0:3})$$

Approximation: Duplicate features
(drop common feature constraints)



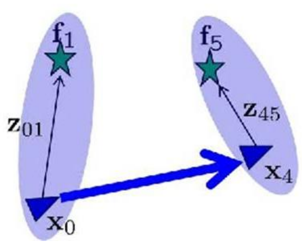
$$\min \; \mathcal{C}_1(\mathbf{x}_0, \mathbf{x}_4, \mathbf{f}_1, \mathbf{f}_5; z_{01}, z_{45}) + \mathcal{C}_2(\mathbf{x}_{1:3}, \mathbf{f}_{2:4}, \mathbf{x}_0, \mathbf{x}_4, \mathbf{f}_1', \mathbf{f}_5'; \mathcal{Z}_{1:3}, u_{0:3})$$

$$\cancel{s.t. \; \mathbf{f}_1 = \mathbf{f}_1' \;\; \mathbf{f}_5 = \mathbf{f}_5'}$$

请看：
状态已经不是原来的状态了

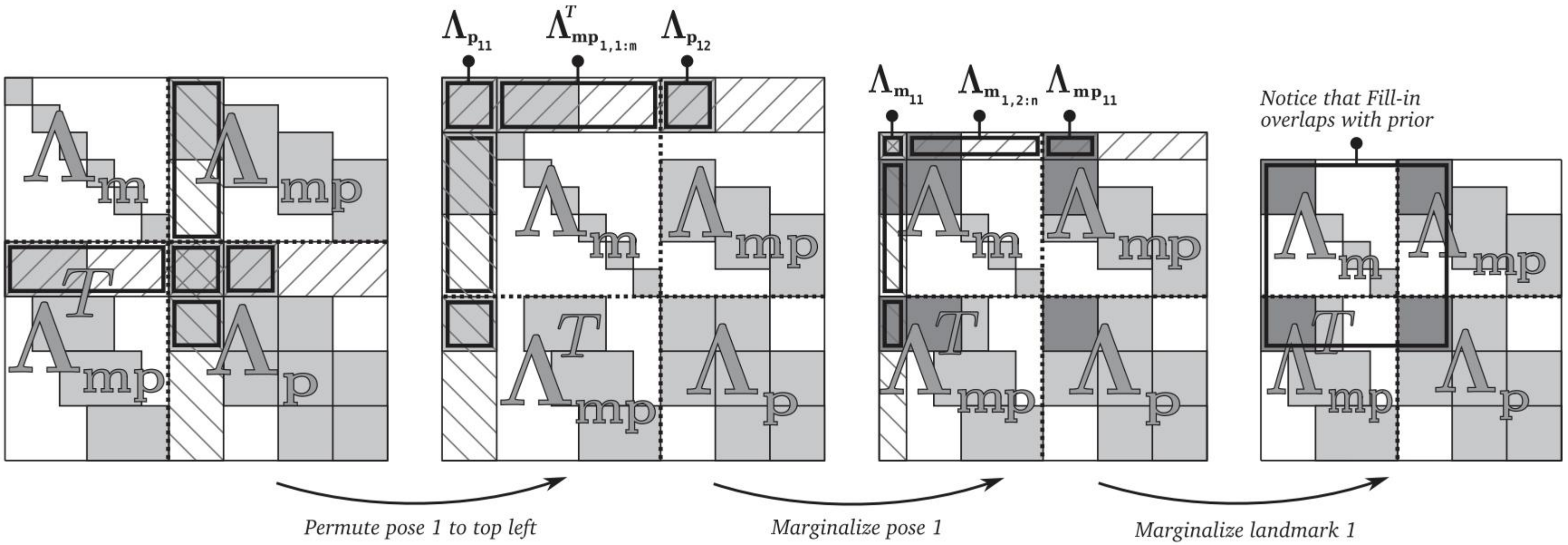
包含了被block的状态

Marginalization



$$\min \; \mathcal{C}_1(\mathbf{x}_0, \mathbf{x}_4, \mathbf{f}_1, \mathbf{f}_5; z_{01}, z_{45}) + \mathcal{C}_2'(\mathbf{x}_0, \mathbf{x}_4; \hat{\mathbf{x}}_0, \hat{\mathbf{x}}_4)$$

浅谈Marginlization



Marginlization让矩阵维度下降，同时也更加Dense

Source: Sibley, Gabe, Larry Matthies, and Gaurav Sukhatme. "Sliding window filter with application to planetary landing." Journal of Field Robotics 27.5 (2010): 587-608.

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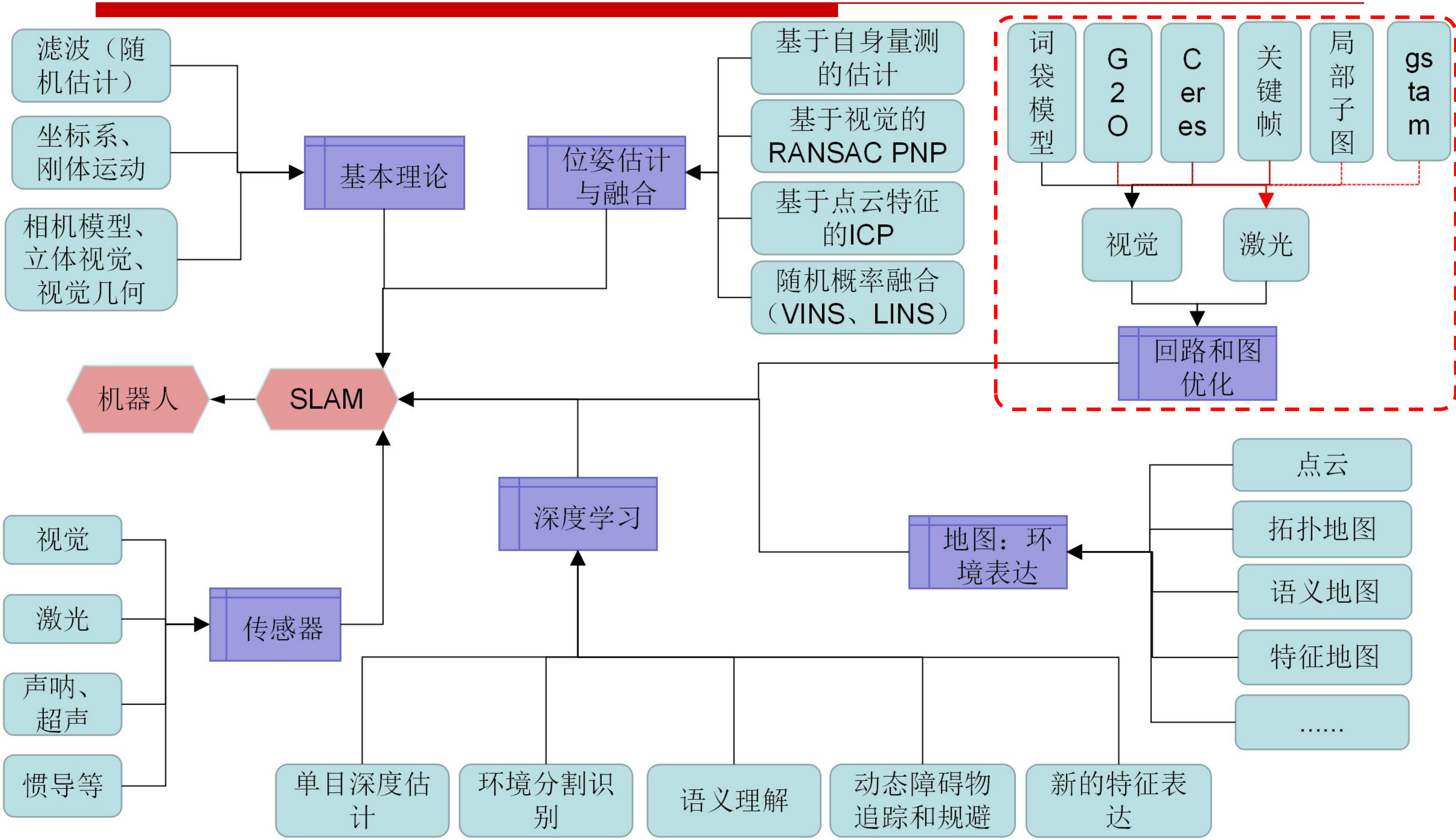

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Q&A